albeda

All Hands-on Math

July 2nd – July 5th 2017

Proceedings of the 24th International Conference of Adults Learning Mathematics – A Research Forum (ALM)

Hosted by Albeda College Rotterdam

Edited by **Kees Hoogland, David Kaye and Beth Kelly**

ALM

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Edited by:
Kees Hoogland, David Kaye and Beth Kelly

Conference hosted by Albeda College
Rotterdam, The Netherlands

July 2nd – July 5th, 2017

Local Organisers:
Kooske Franken, Kees Hoogland and Rinske Stelwagen

Conference Convenor

Local Conference Host

albeda college

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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established in July, 1994 as an international research forum with the following aim:

> *To promote the learning of mathematics by adults through an international forum that brings together those engaged and interested in research and development in the field of mathematics learning and teaching.*

Charitable Status

ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number: 3901346). The company address is: 26 Tennyson Road, London NW6 7SA, UK.

Aims of ALM

ALM's aims are to promote the advancement of education by supporting the establishment and development of an international research forum for adult mathematics and numeracy by:

> *Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit.*
>
> *Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.*

ALM's vision is to be a catalyst for the development and dissemination of theory, research and best practices in the learning of mathematics by adults, and to provide an international identity for the profession through an international conference that helps to promote and share knowledge of adults' mathematics teaching and learning for the public benefit.

ALM Activities

ALM members work in a variety of educational settings, as practitioners and researchers, to improve the teaching and learning of mathematics at all levels. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers, and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world. ALM does not foster one particular theoretical framework, but encourages discussion on research methods and findings from multiple frameworks.

ALM holds an international conference each year at which members and delegates share their work, meet each other, and network. ALM produces and disseminates Conference Proceedings and a multi-series online Adults Learning Mathematics – International Journal (ALM - IJ).

ALM website

On the ALM website http://www.alm-online.net, you will also find pages of interest for teachers, experienced researchers, new researchers and graduate students, and policy makers.

Teachers: The work of members includes many ideas for the development and advancement of practice, which is documented in the Proceedings of ALM conferences and in other ALM publications.

Experienced Researchers: The organization brings together international academics, who promote the sharing of ideas, publications, and dissemination of knowledge via the conference and academic refereed journal.

New Researchers and Ph.D. Students: ALM annual conferences and other events allow a friendly and interactive environment of exchange between practitioners and researchers to examine ideas, develop work, and advance the field of mathematics teaching and learning.

Policymakers: The work of the individuals in the organization helps to shape policies in various countries around the world.

ALM Members

ALM Members live and work all over the world. See the ALM members' page at www.almonline.net for more information on regional activities and representatives, and for information on contacting your regional representative. How to become a member: Anyone who is interested in joining ALM should contact the membership secretary. Contact details are on the ALM website: www.alm-online.net.

Membership fees for 2018

	Sterling	Euro	US Dollar
Individual	20	24	32
Institution	40	48	64
Student/unwaged	4	6	7
Low waged	Contribute between full and unwaged		

Board of Trustees

ALM is managed by a Board of Trustees elected by the members at the Annual General Meeting (AGM), which is held at the annual international conference.

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Open access publication

ALM24 Proceedings Editors: Kees Hoogland, David Kaye and Beth Kelly

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These proceedings include both refereed and non-refereed papers. Peer reviewed papers are indicated with an asterisk (*) next to the title in the table of contents. They have been published earlier in the ALM International Journal 13(1), under chief editor Javier Díez-Palomar.

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Preface: About ALM24

The 24th International Conference of Adults Learning Mathematics – A Research Forum (ALM 24) was held in Rotterdam, The Netherlands.

The conference was organized by Albeda College, Rotterdam and was funded by Albeda College, SLO, CINOP, and the City of Rotterdam.

The conference was attended by almost hundred researchers, practitioners and policymakers from thirteen countries: Aruba, Germany, Greece, Hungary, Ireland, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, the United Kingdom, and the United States of America.

The main theme of the conference was 'All hands-on Math" with the following additional themes.

All Hands-on Math: Examples of inspiring and practical mathematical activities for adults

Mathematics in and for the Workplace

Future Trends in Adults Learning Mathematics

Mathematical Applications for Adult Education

At the conference there were six plenary sessions with thought-provoking themes and ideas. Next to that, there were 40 presentations, workshops and discussion groups in parallel sessions.

As a special experience, participants could engage for 1 hour in

"The Albeda Mathematical Escape Room" which was a real challenge for individual thinking and collaborative working.

The outcomes of the conference are included in these proceedings.

Papers were invited based on conference presentations, and these could be submitted for peer review or non-peer-review. All submissions which met editors' requirements for style and presentation are published in this ALM 24 Conference Proceedings.

Papers marked with * are papers which have been peer reviewed and are also included in ALM-IJ special Issue 13(1) under the responsibility of Chief Editor Javier Díez-Palomar.

For further information on presentations for which no paper was submitted, the abstract is inserted. For presentation slides and photos of the conference please refer to the ALM-website (http://www.alm-online.net).

These proceedings are published as a pdf-version on the ALM's website (http://www.alm-online.net/alm-conferenceproceedings/) and as hardcopy for members and conference delegates on request.

Conference host

Albeda College, the largest vocational college in Rotterdam and one of the major colleges in the Netherlands, was proud to host this event.

The conference took place in Rotterdam, Holland's second biggest city and Europe's largest port.

This conference is supported by the City of Rotterdam: Rotterdam Education City 2016-2017

Rotterdam is known for its innovative ambitions, multicultural population and a hands-on and no-nonsense approach to work, education and life in general. This Rotterdam mentality will reflect in the conference organisation and in the overall theme: All hands-on math.

Theme of the conference

The overall theme of ALM 24 was:

All Hands-on Math

The Conference themes are:

All Hands-on Math: Examples of inspiring and practical mathematical activities for adults

Mathematics in and for the Workplace

Future Trends in Adults Learning Mathematics

Mathematical Applications for Adult Education

Plenary speakers- Papers and abstracts

In order of appearance.

Ionica Smeets, The Netherlands - What do people like about mathematics?

Ionica Smeets

Science Communication & Society, Leiden University

<i.smeets@biology.leidenuniv.nl>

Published before

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Smeets, I. (2018). What do people like about mathematics? Adults Learning Mathematics: An International Journal, 13(1), p. 58-64

Abstract

In this discussion paper we look at questions that adults have about numbers. Many of their questions are not about pure mathematics, but about personal, cultural or societal issues. We discuss how to connect mathematical topics with things people are interested in, based on theoretic knowledge from the field of science communication. We focus on using narratives to make mathematics more personal, how to use games as demonstrations and different ways to present the same mathematical problem in different societal settings.

Introduction

When I tell people I studied mathematics, the most common responses are 'I was very bad at maths in school' or a plain 'Really? I hate mathematics.' However, when you actually talk to people, it turns out that quite a few of them are in fact quite keen on learning more about mathematics. And adults consistently name mathematics as the school subject with the most value to their lives (Gallup, 2013).

In science communication it is very important to talk with your target group instead of talking about them (Arnstein, 1969). This paper discusses what kind of mathematics the general public is interested in by asking them questions about numbers via mass media.

Let's just ask people what they like

In 2011 the British journalist and mathematician Alex Bellos asked people what their favourite number is (Bellos, 2014). He set up a website where people could send in their favourite and tell him why they liked this number.

Over 44,000 people responded and Bellos reported that the world's favourite number is 7, with 3 and 8 as runners up. Almost half of the submissions were for a number between 1 and 10 and the lowest whole number that did not receive any votes was 110. The most common reason for picking a favourite number was it being a birthday. Even more interesting were the properties that people connected with their favourite numbers: 7 was described as sacred, magical, good, unalterable, overconfident and awkward. While 8 was called neat, feminine, reliable, kind, unassuming and huggable. People seemed to anthropomorphize numbers and ascribe entire personalities to them. This begged for a follow-up question where people could explain more about which numbers matter to them personally.

A broader question

Since 2014 I have been writing a weekly column for the national paper De Volkskrant under the title *Ionica saw a number*. Every week I discuss a number that was somehow in the news,

the column can be about anything from politics to literature and from classic math puzzles to recent research. I often get suggestions from readers, or nice additional information about the subjects I wrote about. About a quarter of my columns are inspired by reader's suggestions.

In my 100thcolumn I decided to something similar to what Alex Bellos did: I asked people which number they thought deserved its own column and why (Smeets, 2017). Readers could send in their suggestions by e-mail. I promised to write my 101th column about one of their suggestions.

The response

Within one weekend the question yielded 203 responses from readers. Their suggestions ranged from very funny to very serious. A fun question came from someone who wanted to know if there was anything interesting about the number six, since Bert proclaims in Sesame Street that his favourite number is six and Ernie keeps protesting that six is a very boring number. A more serious question came from a reader who was wondering why the Dutch House of Representatives has 150 seats. Why was this number chosen? Should the number of seats be expanded as the population grows? And do other countries use a similar ratio of representatives compared to the population?

One of the most serious questions, and the one I wrote my next column about, was very personal. A reader wrote about a friend who was in the hospital with leukaemia. To survive he needed stem cell therapy with suitable donor material. For siblings the probability that they are a match is 25%. The friend in the hospital had three siblings and his family and friends were desperately trying to calculate the probability that at least one of them would be a suitable donor. This is mathematics that really matters to people's lives and I explained in my column how to calculate that the probability of at least one match amongst the three siblings is 58%. I also wrote about more general odds of finding a match and encouraged readers to register as donors, since the odds are much lower than you would like.

Categories of responses

As the previous examples show the subjects of the 203 responses varied wildly. We categorised the responses with an inductive method, following the basic principles of grounded theory (Martin et al, 1986). First we coded all of the responses, then we grouped comparable codes in concepts and in the final coding round we combined similar concept in six overarching categories. These categories are:

1. Personal: this category included questions and anecdotes about lucky numbers, special dates and times plus personal connections people felt with a number.
2. Cultural: this category included questions and anecdotes about music, books, language, history and games.
3. Societal: the questions and anecdotes in this category were mainly about why certain numbers in the society are chosen as they are and how the occurrences of societal phenomena are distributed.
4. Mathematical: this category contained nice facts about numbers and theoretical questions about infinity, pi and prime numbers.
5. Scientific: this category was for all questions or stories about biology, physics, astronomy and the other sciences.
6. Other: Everything that could not fit in one of the above categories. For instance because readers only sent in a number without further comment.

Some responses mentioned different numbers or would fit into multiple categories, in those cases we chose the category best fitting the first topic mentioned.

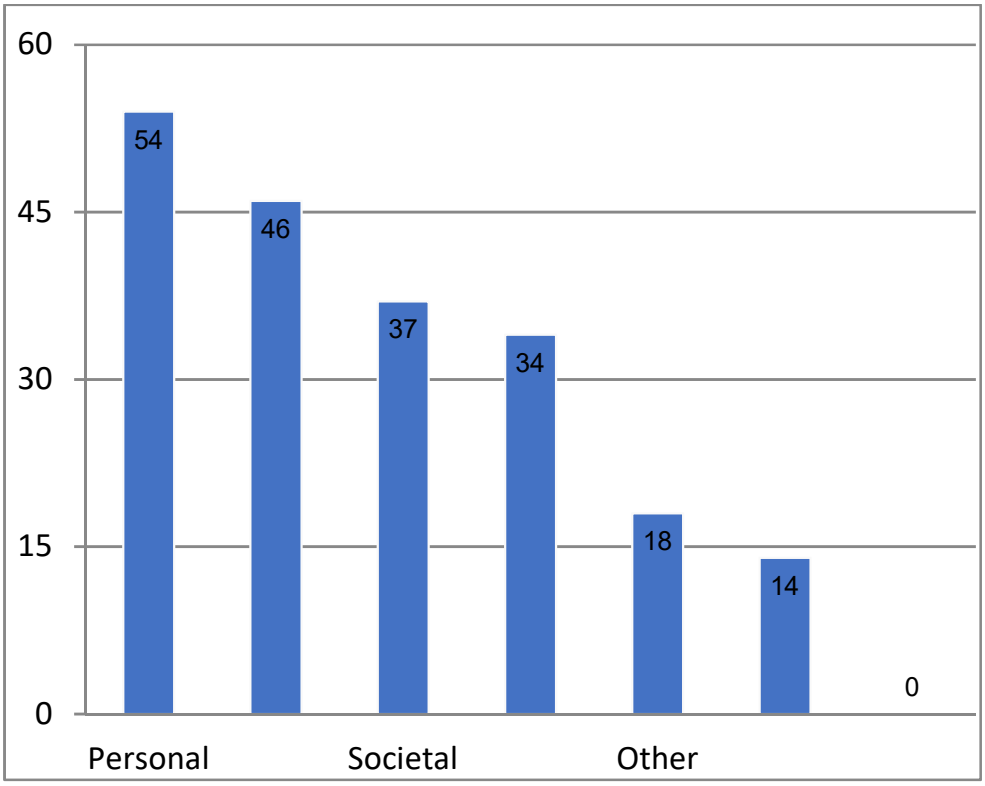

Figure 1. Reader's responses grouped by category

Figure 1 shows the distribution of the readers responses over these six categories. The three biggest categories are Personal, Cultural and Societal questions. In the rest of this paper we will discuss ways to connect people to mathematics using one of these themes.

Personal mathematics

People give numbers personalities and connect personally with a clock time like 22:22. There are many stories behind the numbers. Yet when we present mathematics we too often start from the facts and use a structure where we first introduce a topic, give all the necessary background and definitions, derive the results and only then (if there is still time) talk about why this matters. While science communication literature generally shows that is easier to convince people with stories than with logical arguments (Dahlstrom, 2014).

Telling it like Kurt Vonnegut, one way to make mathematics more personal, is using techniques from storytelling. The American writer Kurt Vonnegut describes how you can build a story based on a simple graph:

> Now let me give you a marketing tip. The people who can afford to buy books and magazines and go to the movies don't like to hear about people who are poor or sick, so start your story up here [indicates top of the Good fortune -Ill fortune axis]. You will see this story over and over again. People love it, and it is not copyrighted. The story is 'Man in Hole,' but the story needn't be about a man or a hole. It's: somebody gets into trouble, gets out of it again [draws a graph]. It is not accidental that the line ends up higher than where it began. This is encouraging to readers.
>
> (VONNEGUT, 2011)

Even the most difficult mathematics can be presented as a story that connects with people. One of the nicest examples I have ever seen was a graduate student who was working on elliptical curves who presented his work as a will-they-make-it-in-time-adventure with a co-author who had to catch a plane to his institute where he would be very hard to reach by e-mail. The entire audience was so eager to hear if they made it, that they happily listened to all kind of details about elliptical curves.

Small tricks from storytelling can be useful to communicate, even within science itself. A recent study showed that scientific papers on climate change that use narrative techniques are cited more often (Hillier, 2016). These basic narrative techniques are relatively easy to incorporate in texts or lectures about mathematics and include using sensory language and using conjunctions to logically order the reasoning. Furthermore making a direct appeal to the audience is a successful narrative tool for helping people understand why what you are telling them is important. You can do this for instance by asking your audience to imagine something or giving them a clear recommendation for action.

Cultural mathematics

There are many ways to connect mathematics to cultural subjects. When you want to introduce fractals you can use the fact that they are used to test the authenticity of Jackson Pollock's drip paintings (Taylor et al, 1999). Or if you have a less high-brow audience you could use the computer-generated backgrounds of animation movies like Up. There's mathematics to be found in popular books and movies and you can use them to engage math-haters.

In this section I am going to focus on another cultural phenomenon: games. Readers sent in many questions and suggestions about numbers in games, so this seems to be something that is on people's minds. In the last few years gamification has proven to be useful in many educational contexts (Hamari et al. 2014). There are many examples of beautiful games in mathematics, here I will focus on a simple version of Nim and the different ways you can present the same game.

Nim

Nim is a game where two players take turns and remove objects from distinct heaps (Berlekamp et al, 1982). In the simplified 21-version there is one heap with 21 objects, in each turn a player can take away 1,2 or 3 objects and the person who has to take the last object loses the game. In this case there is a winning strategy for the second player, which is not too hard to figure out.

In 2006 I played this game with a bunch of mathematicians at a science festival. We played the game with matches on a table and invited the audience to try and beat us. Since we were polite and let the other person start the game, none of them stood a chance. We usually played a few games, until they figured out the strategy for themselves. This worked well and afterwards I used the game in many talks and workshops.

A few years later I was asked to give a maths show for a group of two hundred primary school kids. I wanted to play Nim with them, but realised that the matches on a stage would be very unpractical for such a big audience. We decided to use 21 brightly coloured balloons. One volunteer out of the audience would play against me and instead of removing the balloons we would pop them with a pin while the audience shouted their advice. I used this set-up for many more talks and demonstrations.

After the balloon-game I explained why the volunteer would always lose, even if she was the smartest person in the world. If someone from the audience figured out the strategy, I let them explain it. Otherwise I would urge the audience to consider what happens if it is your turn and

there are only five balloons left. Whatever you do, the other player can make sure that in your next turn there is only one balloon left, so you will always lose. After this step we can inductively work back to the starting point with 21 balloons.

Are there more ways to use this same game in an educational setting? Of course there are. Marcus du Sautoy plays a similar game with 13 chocolates and one chili pepper (Du Sautoy, 2011). He keeps playing against opponents until they figure out how to make him eat the chili (he is fair enough to let them start in the position with a winning strategy). This set-up works really well in smaller groups and is great for making participants figuring out the strategy on their own. The take home message is that you should think about the best way to present a game for your audience.

Societal mathematics

In the previous section we saw that there are many ways to present the same game. One of the nice things about mathematics is that you quite often can present the same idea in many different societal settings. It has been shown that personalising mathematics has a positive effect on both learning gain and interest in mathematics (Bernacki et al, 2018). The underlying mathematics stays the same, but it will be much easier for people to relate to an example from a field they are interested in. My favourite paradox can for instance be introduced in many different ways.

Simpsons paradox

If I am presenting Simpsons paradox to an audience of teachers, or other people interested in education, I start with the example of a gender bias case at Berkeley University (Bickel et al, 1975).

Table 1.
Admittance rates of the six biggest departments

Department	Men		Women	
	Applications	Admitted	Applications	Admitted
A	825	62%	108	82%
B	560	63%	25	68%
C	325	37%	593	34%
D	417	33%	375	35%
E	191	28%	393	24%
F	272	6%	341	7%

In 1973 thousands of students applied for admission at Berkeley. Of the 8.442 men who applied, 44% was admitted. Of the 4.321 women who applied, only 35% was admitted. It seems that there is some gender bias, since women have a significantly lower probability of being admitted. However, a closer look at the admission process reveals that each department did their own admissions. Table 1 shows the admittance rates of the six biggest departments.

We see that in most departments women have a higher probability of being admitted than men. This pattern held for the entire university, yet when we look at the total admittance rates, women seem to have a lower probability of being accepted.

In lectures this is the point where I ask the audience to think about explanations. Sometimes people guess that it might be because there are more men applying than women, but this does not explain the odds as they are. Young students usually guess what is happening here: women and men pick different studies. The most popular department for men is A with an overall admittance rate of 64% . However, amongst women the most popular department is C with a much lower overall admittance rate of 35%. So even though women have a slightly higher probability of being admitted to a department, in total over the university they have a lower probability of being admitted.

This is Simpson's paradox: A trend in different groups is reversed when the groups are combined. If I am talking to an audience of medical professionals, I introduce the very same paradox with another real-life example. A study compared two treatments on kidney stones: the expensive method A and the cheaper method B. The study found that method A had the best results for patients with small kidney stones. It also found that method A had the best results for patients with large kidney stones. Finally it concluded that averaged over all patients method B was the better one. How is this possible? Once again, the clue is that patients are not randomly distributed over the methods. The patients with small kidney stones usually got method B (and had little complications) while the patients with large kidney stones were given method A (and had more complications, since they came in with a larger problem).

Simpsons paradox pops up in batting averages in baseball, payment gender gaps, survival rates for the Titanic, delayed flights from different carriers and death-penalty sentences. You can probably construct a realistic example of Simpson's paradox for any field of interest to explain the basic idea behind it (Wagner, 1982). This is true for many mathematical concepts: you can translate them to apply to any subject your audience is intrinsically interested in.

Discussion

We described ways to connect people to mathematics, based on the three most occurring categories of suggestions in a small non-random survey of newspaper readers. Their preference for these categories might not generalize to the general population and one could also describe cases on how to connect with a public that is interested in other categories. For instance, when communicating with people who are already interested in mathematical topics it might be good to focus on a more abstract case. A great example of this is a video about infinite sums by YouTube channel Numberphile that currently has over 6 million views and more than 12 thousand comments (Haran et al, 2014). For people interested in the category of broader scientific concepts, it will be useful to consider cases where mathematics leads to applications in other fields. One nice example is Braess' paradox applied to traffic networks, where adding a new road actually impedes the traffic flow (Braess et al, 2005).

However, whichever category of mathematical questions people are interested in, the general science communication theories we describe will apply in these contexts. A broader review of science communication literature can give more advice on the use of jargon and target audiences (Hut et al, 2016).

Conclusions

When people are invited to ask questions about numbers, many of their responses are not about pure mathematics, but about personal, cultural and societal issues. We discussed ways to relate

mathematics to these kind of questions to make it easier to connect with people. Remember that you can steal narrative tricks from writers, how games can be a playful way to introduce concepts and that there are many ways to present the same mathematical idea.

Acknowledgements

Thanks to all the readers who sent in their favourite numbers and explained what they mean to them personally. Many thanks to Stefanie Brackenhoff for assisting in coding the responses - and for replying to their most urgent questions. Thanks to the reviewer and editor for suggestions that helped improving this manuscript.

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Katherine Safford-Ramus, USA & Diana Coben, UK - The Fruits of Research: Editors' Perspectives on Publishing Work from ICME-13,

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Abstract

The Thirteenth Meeting of the International Congress on Mathematical Education (ICME-13) convened in Hamburg, Germany, in July, 2016. There were two Topic Study Groups (TSGs) dedicated exclusively to adult learners and the authors of this paper served on the organizing committees of these groups. Arrangements were made by the congress committee for the publication of peer-reviewed papers from each TSG by Springer International Publishing AG in a series of edited books. In this paper we focus on our experiences as editors of the monographs resulting from our two TSGs.

Learning the Landscape

The International Congress on Mathematical Education meets every four years. The thirteenth congress (ICME-13) was held in Hamburg, Germany, from July 24 to 31, 2016. Among the sessions offered on the program are TSGs whose purpose includes the promotion of high-standard discussions of a variety of perspectives on the theme of the TSG as well as giving a broad overview on the state-of-the-art for that specific topic. TSGs serve as mini-conferences and are intended to display the progress of the discussion in the intervening years since the previous ICME, enabling the newcomer to get a broad overview on the state-of-the-art and allowing the experts to lead discussions at a high level; they represent 'the fruits of research' on each topic. The 56 ICME-13 TSGs covered a broad range of topics from pre-school to university mathematics education and included TSGs with historical, theoretical and philosophical foci, for example, TSG11 covered 'Teaching and Learning of Algebra' and TSG25's focus was on 'The Role of History of Mathematics in Mathematics Education'. Each TSG organizing team provides the members of their TSG with an overview on the international discussion as broadly as possible and allows for insight into less well-known strands of the discussion from under-represented countries. For ICME-13, the TSG was the major arena for participation. Participants were expected to associate themselves with one TSG and to stay in that group for all sessions (http://www.icme13.org /topic_study_groups).

ICME-13 was the fifth congress to recognize adult learners as a viable category of mathematics learners. There were 'adult' Working Groups for Action (WGAs) or TSGs at ICME-8 (Seville, Spain), ICME-9 (Tokyo/Makuhuri, Japan), ICME-10 (Copenhagen, Denmark), and ICME-11 (Monterrey, Mexico) and a TSG on 'Mathematics Education In and For Work' at ICME-12 (Seoul, S. Korea) as well as at ICME-13. The practical and financial organisation of an ICME is the independent responsibility of a Local Organizing Committee, operating under the auspices and principles of the International Commission on Mathematical Instruction (ICMI). Consequently each ICME has both broad international scope and a distinctly local flavour.

The International Programme Committee (IPC) for ICME-13 decided to offer two TSG topics specifically related to adult learners and invited people to join an organizing committee for each TSG. TSG3, entitled "Mathematics Education In and For Work", addressed vocational mathematics education. TSG6 embraced any aspect of adult mathematics education, as shown in its title, "Adult

Learning of Mathematics – Lifelong Learning". Each TSG was encouraged to publish an optional pre-congress survey of their topic area. TSG6 did so and the TSG6 survey can be found at http://www.springer.com/us/book/9783319328072. There are 25 other topical surveys available on a variety of aspects of mathematics education and all are open access so readers may wish to view the series catalogue in addition to accessing the TSG6 volume.

It is common practice for TSG organizing committee members to publish congress papers after the conclusion of the congress. Edited books emanated from the 'adult' TSGs of ICME-8, ICME-9 and ICME-11 through a range publishers (Coben & O'Donoghue, Eds., 2011; FitzSimons, 1997; FitzSimons, O'Donoghue & Coben, 2001). ICME-13 differed in that each TSG's monograph[1] was assured in advance of publication by Springer. Also, all initial versions of papers to be presented had to be submitted in advance via the ICME-13 portal. These were strictly limited to eight pages in length with a uniform editing protocol in place. This should have made the eventual monograph editing easier since these papers were, in effect, early versions of the final chapters. However, a variety of factors meant that this was not necessarily so. For example, not everyone took readily to the online system for submission of papers. Also, many authors were not native speakers of English and ICME-13 had not funded a language editor for the series. But that is getting ahead of the story.

Filling the dance card[2]

The invitation to submit a paper to a Topic Study Group was issued in the announcement for the congress. Potential participants completed a paper or poster for consideration by the TSG organizing committee. Each paper was independently peer-reviewed by two people and the organizing committee then developed a program of presentations that spanned the assigned time slots on the congress program. All TSGs met at the same time; those with a large number of papers were allotted additional slots. TSGs with the opposite problem, a small number of submissions, were permitted to invite authors from other TSGs, or even people not attending the congress, to submit papers for their monograph but not to present in the TSG program. The organizing committee selected papers for peer review with a view to subsequent publication by Springer.

[1] A monograph is a publication that focuses on a limited and specific topic.

[2] In earlier times, women attending a dance held a card that contained the names of their partners for the evening's dance programme; similarly, at ICME-13 the TSG organisers had the task of organising their programmes.

TSG3 was well-subscribed but the TSG6 organizers encountered two problems while 'filling their dance cards'. The first was the result of the overlap of the three TSGs that would attract adult mathematics educators since research in one area does not preclude its utility for the other two. The field is small compared to other mathematics education research and arguably congress participants were spread too thin. This was tied to the second problem, namely that attendees had to commit exclusively to one TSG. Participants were not allowed to submit papers to more than one TSG and invited plenary speakers (a special category within the congress strata) could not be invited to publish a paper in a TSG monograph, even on a topic radically different from that of their plenary talk.

Herding the cats[3]

While it was determined before the conference who would edit the post-conference publication, the real work on the volumes commenced in August, 2016, with authors submitting revised versions of their eight-page papers. The procedure to be followed was helpfully set out by Prof. Dr Gabriele Kaiser, the Convenor of ICME-13, in guidance issued to TSG organising committees. Accordingly, each paper was double peer-reviewed and this work was shared between members of the respective TSG organizing committees and external reviewers as required.

Record keeping was critical in order to monitor progress though the editorial process, keep track of submitted papers and reviews and chase up any that were missing. The TSG editors created spreadsheets modeled on the one used to monitor contributions to the ALM International Journal. These were used to track the peer reviewers assigned, the dates papers were sent out for review and returned, and the dates authors received their reviewers' anonymous comments and then returned their amended papers. For TSG6, only one author declined the opportunity to amend a paper and this was omitted from the final manuscript submitted for publication.

There were, of course, problems with intended deadlines and at times we felt we were herding cats. The authors, editors and reviewers were all gainfully employed and these work responsibilities necessarily took precedence over their original and amended submissions. The TSG6 editor took a light-hearted approach to prompting the authors, sending a reminder of publication deadlines at the beginning of each month. For example, the November, 2016, entry read:

> Dear Authors,
> Can you believe that another month has flown by? I hope that your article is taking shape and that you have a tentative date for submitting it to me. Please let me know how you are doing.

This approach kept both the editors and authors aware of the progress of the volumes.

Working with idioms[4]

Good communication between editor and authors, and understanding, or not, of the language used was crucial. English was the language of the congress and also of the TSG monographs. Each TSG team was allowed to choose a variety of English; the TSG6 team chose American English and the TSG3 team chose British English.

The international nature of the conference meant that many of the authors were not native speakers of English. The TSG6 editor-in-chief was the only native English speaker of the three

[3] Cats, or academics in this case, are notoriously difficult to organize and control.

[4] An idiom is an expression whose meaning cannot be gleaned by a direct translation of the individual words that comprise it.

editors so had the task of editing all submissions by authors whose manuscripts might need to be edited for clarity. This is always a delicate task. The editor walks the narrow line between correcting and clarifying - or altering - the author's intended message. In the event, all the authors graciously accepted suggestions and the editor emerged confident that she had not sullied the gist of the author's concept.

A further complication stemming from the mix of languages and underlining the delicacy of the editorial task was the potential for misunderstanding through the use of idioms. A humorous example of the gulf that can exist between the editor's or author's intention and the recipient's understanding of a message is shown below in a table widely circulated on the internet (Richards, 2015). Other languages could no doubt produce their own versions of the table, which serves to indicate the ease with which lines of communication can get crossed.

Table 1.
What the British say, what they really mean and what others understand

What The British Say	What The British Mean	What Others Understand
I hear what you say	I disagree and do not want to discuss it further	She accepts my point of view
With the greatest respect	You are an idiot	She is listening to me
That's not bad	That's good	That's poor
That is a very brave proposal	You are insane	She thinks I have courage
Quite good	A bit disappointing	Quite good
I would suggest	Do it or be prepared to justify yourself	Think about the idea, but do what you like
Oh, incidentally / by the way	The primary purpose of our discussion is	That is not very important
I was a bit disappointed that	I am annoyed that	It doesn't really matter
Very interesting	That is clearly nonsense	They are impressed
I only have a few minor comments	Please rewrite completely	She has found a few typos
Could we consider some other options?	I don't like your idea	They have not yet decided

Dealing with radio silence

Despite valiant efforts to keep things light, there is a fine line between 'reminding' and 'hounding' the reviewers, fellow editors and authors. Email was the main medium for communication and it is like an arrow - you shoot it off but never know if it hits the mark.

Authors - and sometimes also editors - who have fallen behind may be embarrassed or feel harried or be unwell - for a variety of reasons they may not respond to messages. They can, literally, be halfway around the world so the editor does not have the option to appear in their office door and ask "So, how's it going?" or "Can I help?".

Assembling the furniture

While the papers are being finalized, the editorial teams have decisions to make about the ordering, organization and formatting of the volume chapters. How should the volume be organized? Are there categories or themes that emerged from the submissions? This was perhaps a greater dilemma for the TSG6 editorial team as the topics of the papers submitted for the TSG6 book varied widely. After discussion, they decided that the chapters fell into four broad categories: Numeracy; Student Focus; Teacher Focus; and The Crossroads. By contrast, the TSG3 book broadly follows the structure used in the TSG3 meetings in Hamburg and is arranged around four key questions:

> What makes for authenticity in mathematics education in and for work?
> How do we make sense of mathematics in and for work using different research methodologies and theoretical approaches?
> What is the role and place of mathematics in education in and for work?
> What are the advantages and challenges of interdisciplinary approaches to mathematics education in and for work?

As the deadline for manuscript submission loomed, the next challenge was the existence of 'missing authors'. Diplomacy worked and the final TSG6 manuscript was compiled on time. Then it was time to check and double-check the formatting of each chapter. Formatting specifications had been sent from both the congress and Springer and these did not always align. Correspondence with the Springer staff clarified the discrepancies and a compromise template was defined which will be used for both books.

Picking the fruit

Once chapters were selected they were checked for the chosen variant of English spelling and final grammar checks were run. Font size and section numbering were made uniform across the chapters. Each chapter was then pasted into the manuscript document. A table of contents was constructed for each book after perusing other volumes published by Springer.

Giving birth

The TSG3 book has had a difficult gestation, beset by illness and other delaying factors, so it lags well behind that of TSG6. Either way, once the manuscript has been sent off there is nothing to do but wait for the Springer contract as well as the result of external reviews. Based on these reviews, authors may be asked to make further amendments to their work. That having been done the revised manuscript will be submitted, proofs reviewed, and the volumes published and marketed by Springer.

When published the monographs will add to the growing literature on adults learning mathematics in a range of contexts - a literature to which ALM and ALM members have made and continue to make an important contribution at successive ICMEs and elsewhere. As editors we will be proud 'parents', waving our 'offspring' off to make their way in the world.

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Stefan Buijsman, The Netherlands - Understanding Mathematics

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Abstract

I look at different uses of the term 'understanding', going through three different uses. One is from the literature on mathematics education, namely that of how we understand different mathematical notions. The other two are from the philosophical literature. The first is that of understanding a mathematical theory, the second is understanding why some mathematical result is true. Each of these three looks at a different aspect of what students might get out of their mathematics lessons. Being more aware of those different aspects may help both researchers and practitioners in mathematics education.

Introduction

A philosopher may not be the most obvious person to ask to speak at a conference on mathematics education. Philosophers have very little experience teaching, and usually no experience whatsoever teaching mathematics to adults. We don't even do experiments related to mathematics education. Yet, what philosophers can do is look at the work in an area and suggest ways of thinking about that work. That is exactly what I will try to do here. There is, in a sense, nothing new in the material that I will present. Most of it is probably familiar to practitioners. However, the way of thinking about that material is hopefully new. What philosophers can do is offer new perspectives on familiar phenomena. The particular phenomena that I focus on are those related to types of understanding in mathematics.

Why look at understanding? One of the reasons is that understanding, in its various forms, is something we strive for. Intuitively there doesn't seem to be any higher goal than to understand the material that you are supposed to learn. Understanding is 'better' than merely being able to produce the right answers as a result of rote memorization. Perhaps we don't require that students understand everything that they learn. As long as there are some parts that we want them to understand it is relevant to have some idea what this understanding is. It should be easier to reach a goal if we know more about what that goal is. What it means to understand something.

Another reason is that the term 'understanding' is already frequently used in the literature on mathematics education. Just to name a few places where it pops up, there is "Divisibility and Multiplicative Structure of Natural Numbers: Preservice Teachers' Understanding" (Zazkis and Campbell, 1996), "Secondary School Students' Understanding of Mathematical Induction: Structural Characteristics and the Process of Proof Construction" (Palla et al., 2012) and "High School Students' Understanding of the Function Concept" (Dubinsky and Wilson, 2013). The situation was no different at ALM24 itself, as one of the presentations had the title "Using Algebra Tiles to Help Integer Understanding". Not only is understanding something that we might want to achieve in our students, it is also something that has figured in the work of researchers in mathematics education.

Understanding in mathematics education

When one looks at the research that falls under the heading of 'understanding' that I just cited, there is a general trend in the kind of phenomenon that they are studying. As the title of Dubinsky and Wilson (2013) suggests, they look at concepts. That can be the function concept, the concept of divisibility of natural numbers, or some other mathematical concept. What they research in relation to these concepts is *how* certain people understand these notions. In other words, how they think about them. They look at students' understanding of mathematical induction by studying their approach to solving problems. Regardless of whether the students do so correctly, the results are formulated as a description of their thinking. Students who clearly miss the mark when it comes to producing mathematical induction proofs still count as having *an* understanding of mathematical induction.

Clearly this is not the kind of understanding that might be set as a teaching goal. Just the fact that students have some way of thinking about mathematical induction is hardly sufficient. If they can't even produce the right answers in a reasonable number of cases, then there is a need for more teaching. While this research talks about understanding, it focuses rather on concepts and how people think with those concepts. Within the philosophical literature there is a convenient distinction for this: that between concepts and conceptions (cf. Margolis and Laurence, 2014 for literature on concepts). Concepts, for a philosopher, are used to account for the idea that several people can think about the same thing. If everyone in a classroom is thinking about addition, then there must be something in common between all these students that explains that it is the *same* thing they are thinking about. There are different ways of thinking about what this common thing is, but whatever one prefers, it is that which philosophers call a concept.

Conceptions, on the other hand, are the peculiar way in which an individual thinks about that which is shared. Being able to think with a concept happens in virtue of something in the mind of an individual. That particular state of mind is the conception. So, while all the students in a classroom may think about the same concept of addition, each will do so in a slightly different way. This way of thinking about addition is captured by their individual conceptions of addition. It is those conceptions that researchers in mathematics education seem to be studying when they look at *how* subjects understand a certain part of mathematics. They try to categorize the different types of conceptions that students have, so that teachers know how students might think about the concepts that they are teaching about. They do not look at whether or not students understand the material, in the more substantive sense of understanding that I will look at in the rest of this paper.

Understanding theories

The first of the two notions of understanding that I will discuss is that linked to theories. A mathematical theory is generally given by an axiomatic system, which is how they tend to be identified. We have, for example, different versions of Peano arithmetic, that deal with arithmetic involving the natural numbers. The notion of theory comes directly from mathematics itself: take set theory as an example. There is, of course, not just one set theory. Just as with arithmetic there are different set theories, depending on one's choice of axioms. Whether or not you accept the continuum hypothesis is a choice that determines the theory you work in. So, a mathematical theory specifies fairly precisely what part of mathematics you are talking about.

This is where a notion of understanding can come in. We can talk about students understanding a part of mathematics, in the sense that they are able to work with it with facility. The idea of this kind of understanding was raised in the philosophical literature in connection with scientific theories (De Regt and Diecks, 2005; De Regt, 2009). Their idea was that in the case of a scientific theory one can understand how the different parts of the theory work together

to generate predictions. For example, one understands classical mechanics if one is in a position to recognize what kinds of predictions classical mechanics will make in different situations. What type of trajectory classical mechanics will predict for a canon ball shot at a certain angle, with a certain force. One understands how this works according to classical mechanics if one can reason in a qualitative way (without performing the actual calculations) about the different influences on the distance that the canon ball will travel. In that case what one understands is not any one particular fact. Rather, this understanding of classical mechanics is an understanding of how the different parts of the theory fit together to make predictions about the natural world.

How to apply this idea in the context of mathematics? Mathematical theories do not make predictions about the world in the way that scientific theories do. However, they do consist of theorems, i.e. of true statements. The way in which those theorems come about is something that one might be more or less aware of. Knowing some of the basic rules for calculating with percentages is of course sufficient for working with them, yet it is not always the most convenient way. An example that came up during one of the talks at ALM24 is that of calculating 16% of 25. On the face of it this is a calculation that will take a bit of work. Yet percentages have a nice structural feature. 16% of 25 is the same as 25% of 16. The latter is much easier to compute, giving a quick response that the answer is 4.

This trick with percentages works, since 16% of 25 can also be written as 16/100 x 25. In other words, as (16 x 25) / 100. Just take out the 16 now, and we get 25/100 x 16. There is nothing more to it than using the associativity of multiplication. Being aware of features of a mathematical theory of this kind is similar to the kind of qualitative awareness of the aspects of a scientific theory that De Regt and Diecks talk about. Understanding a mathematical theory can be seen as a qualitative understanding of the aspects of a theory that are responsible for the validity of results. One can interchange 16% of 25 for 25% of 16 because of the associativity of multiplication. Other results are dependent on other parts of the theory. The end result of this kind of understanding is that it is easier to work within a mathematical theory.

This is not just a theoretical difference with few visible consequences. Students who start out learning a new area of mathematics, say arithmetic, do not yet know how to work with the different rules of arithmetic. They do not know which rule to apply when, nor which mental shortcuts are admissible and which lead to the wrong results. This kind of general ability to work with a theory, an ability to recognize the solutions to problems, is what I have described as understanding the theory.

Furthermore, it is hardly the case that practitioners are not already aware of this notion of understanding. A presentation at ALM24 that I have already mentioned, "Using Algebra Tiles to Help Integer Understanding", used exactly this notion of understanding. It also helps to make the idea of understanding a theory slightly less mysterious. In this case, the tool to help understanding was a set of physical objects that could be moved around, each object representing a certain number or variable. By rearranging them into various configurations, students could figure out which transformations work (do not change the number of tiles) and which transformations are not allowed. Through manipulating representations of different sums, including the procedure of adding zero (by adding both a tile representing a positive value and a tile representing the corresponding negative value), students are put in a position where they get a better idea of how the theory of integer arithmetic works. In contrast to the notion of *how* students understand something that amounts to their conceptions of mathematical concepts, this is the kind of understanding that teachers might strive to achieve in their students.

Understanding *why*

A third notion of understanding, which has been actively discussed in philosophical circles, is that of understanding *why* something is the case. One way in which this is different from understanding a theory is that understanding why is always understanding why this one thing is true. Instead of focusing on a mathematical theory as a whole, the focus is now in particular truths, such as that 0.9999... = 1.

A detour via science is once again the easiest way to get to the type of understanding I have in mind here. In both science and mathematics, the notion of understanding why is linked to explanation. We understand why something is true when we are able to explain why it is true. Only 'explanation' here is meant in a very particular way. Not just anything that someone might say in response to 'why is that?' is an explanation. For example, think of the situation where a baseball has hit a window, shattering the glass. We can now ask two questions: 'why did the glass break?' and 'why did the baseball hit the window?'

A natural answer to the first question is that the glass broke because the baseball hit it at a certain velocity. If you want, you can even add more details from physics describing the transfer of energy from the baseball to the glass, and so on, but the basic answer 'it was hit by a baseball' is already quite satisfying. It not only leads us to the conclusion that the glass broke, but also makes it clearer why that happened. Compare that answer now to a possible answer to 'why did the baseball hit the window?'. Someone could say that the baseball hit the window because the glass broke. After all, you can reason from the fact that the window broke that it was hit by a baseball. If you have enough information, you might even deduce from the way in which the glass broke how fast the baseball was travelling. Yet the answer sounds somehow wrong. It doesn't really enlighten us on *why* the baseball was there in the first place. That sense, that the first answer is helpful but the second is not, is what underlies the philosophers' use of the word 'explanation' (cf. Woodward, 2017 for a general overview of theories of scientific explanation).

The example of the glass relies on something that we don't find in mathematics: cause and effect. It's strange to answer a question of the sort 'why did this happen?' by pointing to an effect, rather than to a cause. Yet even without cause and effect there are a number of clear examples of what look like explanations in mathematics (borrowed from Lange (2014, 2016)). One such example is that of the so-called calculator numbers. The way to construct these numbers is to take the keypad of a calculator, ignore the zero, and to start with one of the numbers at the edge (so anything but 5). Then go in a horizontal, vertical or diagonal direction so that you get three numbers in a row. Finally go back in the same direction, so that you end up with a number with six digits. Examples of such numbers are 123321, 753357 and 963369. As it turns out, every such number is divisible by 37.

This may be a bit of a surprise, but then again, there are numbers a plenty that are divisible by 37. Why is this case special? It is special, because here there is a reason why all of them can be divided by 37. Each one of these numbers is constructed in the same way. You start at a certain number, a, and either add or subtract the same number d twice. So, you go from 1 to 1 + 1, to 1 + 1 + 1. Or from 7 to 7 - 2, to 7 - 2 - 2. In addition to that, you make use of the base-10 system, so that the first number ends up with a value ten times that of the second, and so on. Putting this into a formula gives that every calculator number is an instance of the following formula:

$$10^5a + 10^4(a + d) + 10^3(a + 2d) + 10^2(a + 2d) + 10(a + d) + a$$

Rearranging the terms eventually gives you the formula

(3 x 11 x 37) (91a + 10d)

So, every calculator number is divisible by 37 (and 3 and 11), because the way in which they are constructed is equivalent to a construction involving multiplication by 37. It is not just that it happens to be the case that each one of them is divisible by 37, for no further reason. There is an actual story that one can tell as to *why* they are divisible by 37.`

There is not always such a story to tell. If we look at the decimal expansion of π and e then there is a similarity between the two:

π = 3.14 159 265 358 979 3

e = 2.71 828 182 845 904 5

Both have a 9 as their thirteenth digit. But there's nothing more that one can say than that. There is nothing like the explanation for the divisibility of calculator numbers by 37.

Philosophers are as of yet unsure what it is that makes some mathematical reasons explanations. There are a few suggestions, though, such as that we get explanations when there is a kind of symmetry in a mathematical problem. An example of that kind of explanation is Zeitz's biased coin. This is a coin for which we generate a random bias. So, a random number is generated between 0 and 1 that gives the bias of the coin: at 0, the coin will only land on tails and at 1 the coin will only land on heads. The one time when the coin is fair, is when the random number happens to be 0.5. Now the coin is tossed 2000 times. What is the chance that it lands on heads exactly 1000 times? As it turns out, the chance for 1000 heads is 1/2001. The chance for 999 heads is 1/2001. In fact, for any number of heads from 0 to 2000, the chance is 1/2001. You can of course prove this using a number of integrals involving binomial coefficients, but the symmetry (the fact that the answer is the same for any number of heads) suggests that a simpler way to the answer is available. Instead of thinking of the random number as giving the bias of the coin, you can think of it as telling us how many times the coin will land on heads. If the random number is 0, then the coin will never land on heads. If the number is 1, then the coin will land 2000 times on heads. Then, since the number is generated randomly, it will randomly select one of the 2001 options: form 0 heads all the way up to 2000 heads. Since the selection is random, the chance of any one outcome happening is the same. So, the odds that you see 1000 heads are 1 divided by the number of options; 1 divided by 2001.

Another feature of mathematics that seems to help is when there is something that brings together a number of seemingly unrelated results. For example, it is the case that the product of any three consecutive numbers (e.g. 3 x 4 x 5 and 16 x 17 x 18) is divisible by six. For an explanation, we can look at what it is that makes all of these cases the same. For one, every three consecutive numbers is guaranteed to contain an even number. That means that at least one of the three numbers is divisible by 2. Another point to note is that, since there are *three* consecutive numbers, one of them must be divisible by 3. This means that, if you look at the product of all three numbers, it is divisible by 2 x 3 = 6. By looking at what all the different cases have in common, we arrive at a piece of reasoning that seems to not just convince us that the claim is correct, but that also explains the truth of this claim.

When it comes to explanations, they are not always there in mathematics. Not every single mathematical fact has an explanation. A number of them do have one though, and it is for

those where the idea of understanding can come in. In those cases we can ask more than just 'is this true?'. We can ask 'why is this true?'. If someone is able to give an explanation in answer to that question, then that person has (some) understanding *why* it is true. Not every explanation is equal; some explanations may be better than others. So, not everyone needs to understand why something is true to the same degree. The important point is, though, that we have this third way of talking about understanding, which relates to single truths in mathematics.

In contrast to the other two notions of understanding it is difficult to find traces of understanding *why* in the literature on mathematics education. The one place that I have managed to find which hints at this notion of understanding is in Weller et al. (2009, 2011). This was a series of experiments related to the thinking of preservice teachers about fractions and decimal expansions. In particular, one of the things they tested was whether teachers believe that 0.9999... = 1 (which it is). They talk of 'conceptual justifications', which seem to be more explanatory than 'procedural justifications'. For example, as a conceptual justification they list the reasoning that 0.9999... = 1 because there is no non-zero number that can be added to 0.9999... to get to 1. If the numbers are to be different, there must be some non-zero difference between the two. By being unable to find one, it follows that they have to be the same. This contrasts with a procedural justification such as believing that 0.9999... = 1 because that is what a calculator tells you.

Interestingly, when Weller et al. (2011) tested how confident students were in their belief that 0.9999... = 1 four months after they had received lessons on the topic, the results suggested that those giving the more explanatory reasons had stronger and more stable beliefs. They felt surer that they remembered the right answer, and more of them still believed that 0.9999... = 1. So, those who ended up giving explanations had real benefits from being able to do so. At least, that is the suggestion. Without more experiments, that are more clearly designed to verify this, it is hard to be sure. I expect that such experiments will reach this same conclusion. Understanding why, i.e. being able to give explanations, signals that students have a better grip on the material. If they are no longer sure that they remember the right conclusion, then they can always fall back on the explanation that they have to lead them back to the correct answer. This is thus one reason to prefer students to have explanations available whenever possible.

A second reason is that explanations have a tendency to generalize. Explanations, while aimed at one result in mathematics, may with some small changes apply to other results. The reasoning for why 0.9999... = 1, namely that there is no non-zero number that separates the two, can be applied for any identity between two numbers. Another case is the reasoning to prove that $1 + 2 + 3 + ... + (n-1) + n = n (n + 1)/2$, supposedly due to a young Gauss. The proof works by imagining two copies of the sum to be computed, only one starts from 1 and the other starts from n:

$1 + 2 + 3 + ... + (n-2) + (n-1) + n$

$n + (n-1) + (n-2) + ... + 3 + 2 + 1$

This lines up 1 and n, 2 and (n-1), 3 and (n-2), and so on. Each of those, when added together, sum to n + 1. And, since there are exactly n numbers, that means that there are n of those pairs that add to n + 1. The sum of these two series is therefore n times n + 1. Now, since we added the sum twice, all we need to do is divide by two for the answer: $n (n + 1)/2$. A nice piece of explanatory reasoning, but also a general technique for finding the value of sums of this kind. The same type of reasoning works for $1 + 2 + 3 + ... + 2n$, $1 + 3 + 5 + ... + 2n + 1$, and so on. The explanation generalizes, in the sense that with some small changes it can be turned into

an explanation for another true mathematical statement. You get lots of results for the price of one.

Conclusions

I have talked about three different ways in which you can use the word 'understanding' in the context of mathematics. You can use it when thinking about *how* people understand something. What their conceptions are for certain mathematical concepts. Alternatively, you can think about understanding a mathematical theory, which means that you look at how well someone is able to see how all the different parts of a theory fit together. Finally, you can look at whether someone understands why something is true. While that's not always possible, when it is it means that they can give you an explanation of a mathematical result.

Each of these notions of understanding is important. Yet they are also very different, even though the same word is used to refer to them. Therefore it is relevant to keep them apart, and to be aware of their differences. Hopefully that will make it possible to be more specific in formulating research questions and results. Furthermore, it may help when formulating teaching goals. Being aware in particular of the option of understanding why, and the possible benefits of having explanations, may help in everyday teaching. Finding such explanations is difficult work, especially since there may not always be an explanation to be found. If the reward turns out to be stronger and more stable beliefs on the part of the student, it may be worth the effort though.

As always, there is a lot more research to be done. What I have presented here are the basic notions. Philosophers pretty much agree that there are these different kinds of understanding, and that there are such things as mathematical explanations. What they disagree about is what makes something an explanation, and what the different kinds of understanding really are. Besides, it is still unclear if the benefits of understanding that I have highlighted are indeed there. Being aware of the different notions as a researcher or practitioner in mathematics education, despite the need for this further research, is something that I hope will still be helpful. If so, then it has been one way in which philosophy turns out to be of use in mathematics education.

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Stefan Buijsman received a PhD in philosophy at Stockholm University in 2016, at the age of 20. He now holds a post-doc at Stockholm University. His research is in the philosophy of mathematics and looks primarily at how children and adults learn mathematics in/before school.

John Poppelaars, The Netherlands - Mathematics at Work

John Poppelaars is senior manager and practice leader Advanced Analytics of BearingPoint in Amsterdam.

Abstract

In a data driven society computers and advanced algorithms will take over many of our tasks. This will reduce the need for the skills that are now at the core of our math education. However, the demand for creative and collaborative problem solving skills will rise as working with large data sets and complex algorithms will become more commonplace. To prepare well for this 2nd machine age, students of all ages need to be able to detect patterns and relations in data, also to solve practical problems by selecting or adapting the appropriate algorithms. As data availability and algorithmic progress is accelerating, focus of math education should change to adaptive learning to prepare students better for the future. During the lecture John will use examples, ranging from simple to complex, from his work as a mathematical consultant to illustrate.

Mark Mieras, The Netherlands - Nine Misunderstandings About Learning

Mark Mieras is a Dutch science journalist and best-selling author on brain-development.

Abstract

Downloading information to the brain... that is one of the nine painful misunderstandings about learning that science journalist Mark Mieras unravels. In his lecture he takes us deep inside the neurobiology of the learning process. How does it really work? How does one prepare the brain for learning? And what do these new insights mean for math education?

Parallel sessions – Papers and abstracts

We first start with the presenters who submitted an article. Articles labelled with an asterisk are peer-reviewed and were published before in the special conference issue of the ALM-international Journal: ALMIJ 13(1).

After the articles you will find the abstracts of all other presentations at ALM24..

Charlotte Arkenback, Sweden - Adult retail apprenticeships – Logbook and Group -supervision as learning tools in workplace-based education

Abstract

In this paper, I investigate the significance of structured logbook and group-supervision for the development of mathematics containing vocational competencies in adult retail apprenticeships.

In the autumn of 2014, I conducted a preliminary study of mathematics containing activities in adult retail apprenticeships through action research. The study was conducted during twelve weeks in collaboration with a group of retail apprentices at South School with the aim to find out what activities in the learning practices in retail that could be mathematics containing. Among the most important research tools were apprenticeships logbook, group-supervision, and reflective dialogue conversations. Group-supervision can be described as a combination of group coaching and study circle (Arkenback-Sundström, 2013). The apprentices aim to be involved in the action research study were to 1) identify mathematical activities at work and practice on the practical applications of the educational mathematical content (budget, shop mathematical, schedule, economic calculations) and 2) use their knowledge, skills and experiences from the workplace-based learning to achieve the mathematics containing course objectives of the retail education.

Vocational Education and Training and Apprenticeships in Sweden

The Swedish model of adult apprenticeships, lärlingsvux, was implemented in municipal adult education in 2012 (Skolverket, 2012) and can be described as a form of study where the school and the vocational teachers are responsible for planning, and evaluation of students learning at workplaces. The municipalities can apply for government funding for adult apprenticeships, the requirement is that at least 70 % of the education must be conducted at a workplace. The range of adult VET programs and apprenticeships is governed by the labor market needs and allocation of government funding. The content of the vocational programs is expected to match the skilling demands in the specific vocation, and with the outcome that students who have completed education shall be employable.

Vocational education and training, VET, can sometimes be said to have a dual purpose; contribute to competence supply and to increase the proportion who take upper secondary diploma and thereby better opportunities in the labor market (Skolverket, 2015). VET in the Swedish municipal adult education is compressed vocationally-oriented educations that consist of a selection of vocational subject from the corresponding upper secondary vocational program. The length of programs ranges from a few months up to a maximum of two years. Adult vocational programs follow the upper secondary school syllabuses, core content and knowledge requirements, however, the overall aim of the corresponding upper secondary vocational program cannot be applied. Mathematics, or numeracy, is generally not included as a subject in the adult vocational educations, however, mathematical content can be embedded in the course objectives. It´s not uncommon for vocational teachers to rehearse basic mathematics to prepare students for the mathematics used in the vocational courses and in work activities that characterize the profession (Noss, Hoyles, &Pozzi, 2000; Williams & Wakefield, 2007; Triantafillou &Potari, 2010). For example, rehearsing calculating with fractions, percentages, and VAT is a preparation for budget calculations in retail apprenticeships.

One of the intentions with the new vocational programs in upper secondary school (Skolverket, 2011) and VET in adult education is an increased cooperation between schools,

industry councils, employers, workplaces, supervisors, and teachers. This emerges for example in the knowledge requirements of vocational subjects “In consultation with the supervisor, students assess with some certainty their own ability and the requirements of the situation” (Skolverket, 2011). The importance of partnerships to develop work-based education and training is also discussed internationally (e.g. Choy, Brennan Kemmis, & Green, 2016, p. 350): “Without partnerships, the process for meeting the skilling demands of individuals, enterprises and communities, and the outcomes of education and training, remain distributed and disparate”.

If VET in adult education is supposed to meet the skilling demands of individuals, enterprises, and communities then we must get deeper knowledge about what happens in workplace-based learning practices; how mathematics is used in workplace activities and what enables and restrains the development of mathematics containing working competencies. A knowledge that also can contribute to the development of mathematics, and other subjects in school and for work.

Research aims

The aim of this study is to develop knowledge of mathematics containing activities in practices of adult retail apprenticeships. The aim is also to understand how the development of mathematics containing professional skills is enabled and constrained by the practice architectures found or brought to the site of practice, in other words, the cultural-discursive, economic-material and social-political arrangements that inter subjectively shapes and prefigures learning practices in retail.

The study's research questions:

- How is the math containing vocational competencies for sales assistants described in education policy documents?
- Which learning practices can be identified in the workplace-based part of apprenticeships?
- What are apprentices and supervisors doing in learning practices of retail?
- Which mathematics containing activities can be identified?
- How is the development of mathematics containing vocational competencies enabled and constrained through the apprenticeships learning practices?

To increase my knowledge of what workplace-based learning in adult retail apprenticeships may involve and what work activities that might be mathematics containing, a pilot study was conducted in autumn 2014. An action research project that was carried out in collaboration with a group of apprentices at South School.

Theoretical and methodological framework

The theory of practice architectures, TPA, (Kemmis et al., 2014) is used both as a theoretical lens to explore the nature and conduct of learning practices in retail apprenticeships, and as an analysis tool, together with parts of Nicolini´s “toolkit” (2012, p.221) and a mathematical framework developed for the study (Arkenback-Sundström, 2017). A practice is understood as a socially established and cooperative human activity, which includes opinions and forms of understanding (sayings), ways of acting (doings) and how people relate to one another and the world (relatings), that “hang together” in a characteristic way in a distinctive "project"

(Kemmis et al., 2014). As the sayings, doings, and relatings may occur independently of practices it´s, according to Kemmis et al., essential that they "hang together" when a specific practice is studied. The arrangements or recourses that forms and are formed by practice is cultural-discursive, material-economic and social-political arrangements. These practice architectures constrain and enable the learning that occurs in practice (see figure 1).

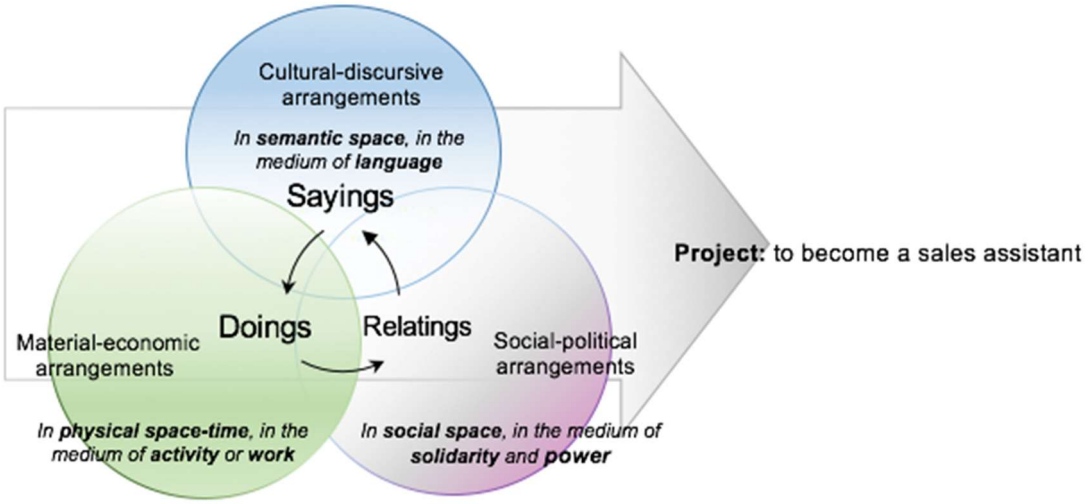

Figure 1. The media and spaces in which sayings, doings, and relatings exist (freely translated from Kemmis et al., 2014, p.34).

Adult Retail Apprenticeships at South School

South School is an adult education provider in a larger Swedish municipality. In spring 2014, the school started a new vocational education at upper secondary level, retail apprenticeships (lärlingsvux till butikssäljare), that lasted until October 2015 when South school lost the municipal procurement of vocational training for adults. As a specialist teacher and educational leader at South School, I was involved in planning and developing the 40-week apprenticeships education, which consisted of four days' work-based learning and one day per week was school-based learning. The educational content comprised commerce subjects from the corresponding upper secondary program, Business and Administration Program.

The starting point for the planning of the retail apprenticeships was the experiences of an earlier conducted action research study, Group-supervision in mathematics. The dialogue a way to develop adults' mathematical competencies (Arkenback-Sundström, 2013). To establish and develop the connection between school-based learning and apprenticeships practices at various workplaces, scheduled group-supervision was introduced on the theme: "Learning to learn at work" (90 min/week). Based on the aims of education and knowledge requirements an apprenticeship logbook was developed, and the dialogues in the supervision group emanated from the students' logbook notes. The remaining part of the school-based part of the education comprised lectures, group assignments and examinations. During the period that South School offered the apprenticeship education, I was the moderator of the supervision group.

Something that emerged over time was that the apprentices never described the activities that involved mathematics, for example, the core content of Commerce: "Retail maths e.g. order-point and conversion rates, margins, markups, and risk assessments" or "Different techniques

and methods of making financial calculations". They did not recognize concepts like "forms of payment", "financial calculations", "financial ratio", "calculations" or "retail math" from their learning practices in retail (Colwell, 1997; Coben, 2000), and were therefore not able to develop: "The ability to carry out financial calculations, make risk assessments and solve problems" (Skolverket, 2011). All apprentices but one expressed concern they would not pass the "math test" in Commerce, the background to this anxiety was past failures in school mathematics. They could not see that the mathematics they worked with on the subject of Commerce was used in the workplace activities, at least not by the sales assistants. There was also great anxiety about how to cope with the checkout regarding calculating exchange and discount.

An action research study – mathematic containing activities in retail apprenticeships

To get to know mathematics containing activities in the workplace-based learning in adult retail apprenticeships, I conducted a shorter action research study (12 weeks) in collaboration with the retail apprentices at South School. Because of the continuous admissions system, the number of group participants varied between 4 and 8 participants. The action research study was a pilot study for another study (Arkenback-Sundström, 2017) and the research aim was to get 1) increased knowledge of vocational practices and activities at workplaces that retail apprentices get the opportunity to take part of and 2) knowledge of work activities that may be mathematics containing.

The apprentices aim to be involved in the action research study were to 1) identify mathematical activities at work and practice on the practical applications of the educational mathematical content (budget, shop mathematical, schedule, economic calculations) and 2) use their knowledge, skills and experience from the workplace-based learning to achieve the mathematical containing course objectives of the retail education, which turned out to be a great motivator for participating in the study. The theme of the research project was initially Mathematics containing retail activities, but it turned out to be difficult to discover the mathematics in work activities. Therefore, the theme, in dialogue with participants, was changed to Activities involving and communicating digits, numbers and mathematical relationships, which led to apprentices discovering a variety of mathematical containing activities in the stockroom, on the sales floor and at checkout.

In action research, it is common with ethnographic research methods (Colwell, 1997; Rönnerman, 2011) and the study used participant observation, structured logbook, dialogue conversation, group interview, field notes, photography and sound recording. The apprentices were already accustomed to logbook writing, and they also had the benefit of logbook notes from previous discussions in the group; however, several of them frequently forgot to write them. This was partly because the vocational teacher did not show interest in their reflections from work practice, but focused on the examinations and the supervisors' evaluations. Also they did not see any difference between planning and doing, they experienced it as something that was done simultaneously. To improve the quality of dialogues and logbook notes and so the log book could serve as a real learning tool it was decided that the tripartite talks (apprentice, supervisor, and teacher) at the workplace should be based on the logbook notes. The apprentices would thus have greater opportunities to take ownership of and shape the workplace-based learning. The altered role of the apprenticeship logbook as a learning tool continued throughout.

Group-supervision

Group-supervision is a form of study, that can be described as a mix between the Swedish study circle and group coaching, where democratic working methods are central. The group makes agreements concerning meeting structure, conversation rules, content, time and the ways in which issues raised will be taken care of (Handal & Lauvås, 1982, 2000). With the support of logbook notes (text, photo, film, audio) one participant at a time narrates on a subject or work activity from the past week, followed by a discussion on the activity and the ways in which it may relate to course content, objectives and knowledge requirements.

Many of the apprentices in the action research study felt that it was difficult to find time to write their logbook entries during retail practice, therefore the group-supervision always started with five minutes of reflection and logbook-writing. The apprentices' narratives developed knowledge of situations, working methods and learning activities at various workplaces within retail. Issues and development areas were caught up, discussed, and could then form the basis for the school-based teaching. For example, a five-week project was designed to cover all mathematics containing elements and aspects of the retail education, Digits, numbers, (mathematical) relationships and problem-solving in retail.

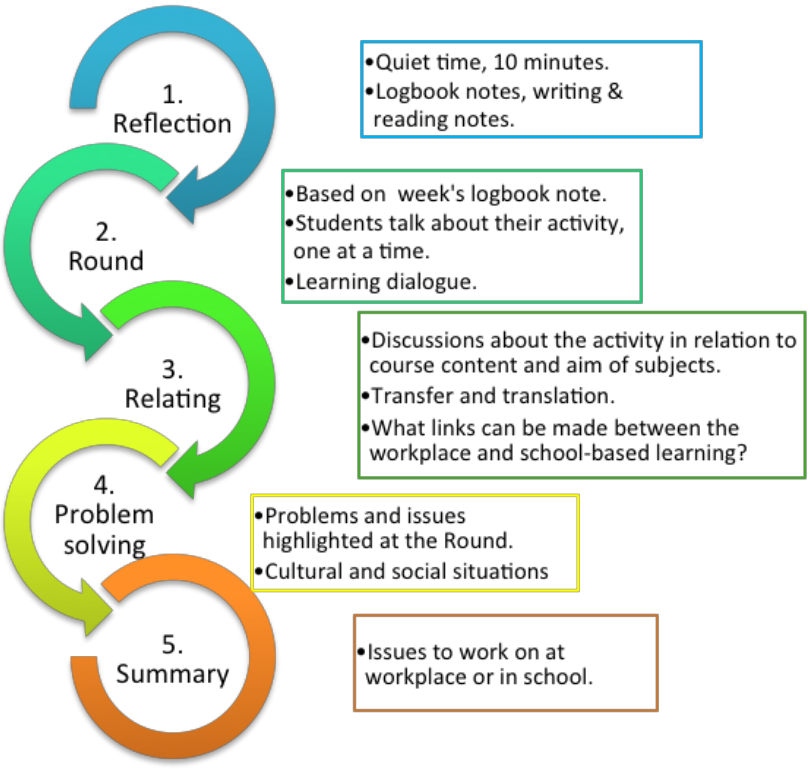

Figure 2. Structure of the Group-supervision (90 min/week)

Apprenticeships Logbook

The structured apprenticeships logbook was in its design, based on the knowledge requirements of retail apprenticeships education (Skolverket, 2011). Students would choose one single work activity they participated in or observed the past week, and then describe it with help of some questions:

- *What?*

  - *What was the activity about?*
- *Why?*
  - *What was the purpose of the activity?*
  - *What would it lead to?*
- *Planning*
  - *How was the activity planned?*
  - *What information did you need?*
  - *How did you get it?*
  - *Was there anything that you had to consider (e.g. laws, rules, ergonomics ...)?*
- *Carrying through*
  - *Describe shortly how the activity was carried out*
  - *What knowledge, skills, working methods, tools were needed to carry out the activity?*
- *Evaluating*
  - *What was the outcome?*
  - *What are you most / least satisfied with?*
  - *What could be done differently to improve the outcome?*
  - *How can you proceed to develop vocational knowledge and skills?*

Initially, the logbook notes were very short, just a few sentences, but over time the content and quality of notes developed. Regarding fullness of the notes, they were enough to provide rich descriptions of work activities during the "round of the dialogue".

Results

The action research study identified five physical sites in shops and stores: office, staff room, stockroom, sales floor and checkout, where sales assistants along with visual merchandisers, cashiers and store managers have their work. The different vocational practices overlap (Kemmis et al, 2014) and it is the store´s size and organization that determines the work and responsibilities of sales assistants (eg some shops lack a stockroom). Working as a sales assistant may include tasks from different professions that is carried out at one or more sites in the shop. Many stores have limited space, which means that the same space can be both staff room, office, and stockroom. The consequences for the apprentice's education is that it is difficult for the supervisor and apprentice to sit down in peace and quiet for reflection, planning or supervision. The study's participants said that they sometimes resolved the issue by going to a neighboring cafe to sit down with a cup of coffee. During their education, the apprentices were given the opportunity to take part in some or all the mathematics containing activities in learning practices in the stockroom, on the sales floor or at checkout. Large parts

of the educational course content were found to belong to the work of the store manager and the visual merchandiser (or marketing department).

Following dialogues between apprentices from the South School and their supervisors are excerpts from the narratives of apprenticeships in the fictional department store Delta (Arkenback-Sundström, 2017). The narratives take place a couple of months after the action research study was completed, and we meet the fictional apprentices Diana and David who are at the end of their educational programme to become sales assistants. Diana discusses the content of the retail maths course (Skolverket, 2011) with her supervisor at checkout:

> *(Diana) - Retail mathematics is probably when one counting on percent, and budget. Cash register and the budget board in the staff room is math, and when you count the checkout. We have metric percent when we figure out how it goes.*
>
> *(Supervisor) - Mm, but that´s not math, it is part of the job. I was really bad at math when I was in school, but I don´t recognize retail mathematics.*
>
> *(Diana) - When we got the logbook in the project, both I and Dilba said there's no math in the job, it's just the checkout. But when we sat down and thought, we discovered that we calculate all the time".*
>
> *(Supervisor) - Have you thought of Diana, now it's you who compiles and reports the results of the Skimra Shops every week?*
>
> *(Diana) – Oh, that's right, I forgot. I had never imagined I could do that six months ago. But we have practiced with "smartbudget" and other software programs in school, like scheduling and interior design programs.*

The dialogue between them indicates that the logbook-writing enabled the "discovery" of mathematics. It also shows that working with various software programs in school had helped her to develop the knowledge and skills required to make the weekly result report for the management team. The logbook also proved to be a learning tool to develop the ability to plan, carry out and evaluate activities in learning practices in the stockroom, on the sales floor, and at checkout. Here, Diana and her supervisor are discussing a new shop display Diana has completed at Skimra:

> *(Supervisor) - You're very good at planning at home and have a clear picture of how you want it. When you come here, you know what to do, and then you "run on hard" and make it great.*
>
> *(Diana) - I think it has helped me a lot that I got to make the logbook in school, and know that you must think before you do something. We discussed trying to see what you need and how to do to practice it ... To plan, organize, carry out and evaluate. I noticed that yesterday when I and Disa worked together since we started to look at the display drawing, how it would look like in the shop. We planned what we would take down and then where to put it away.*

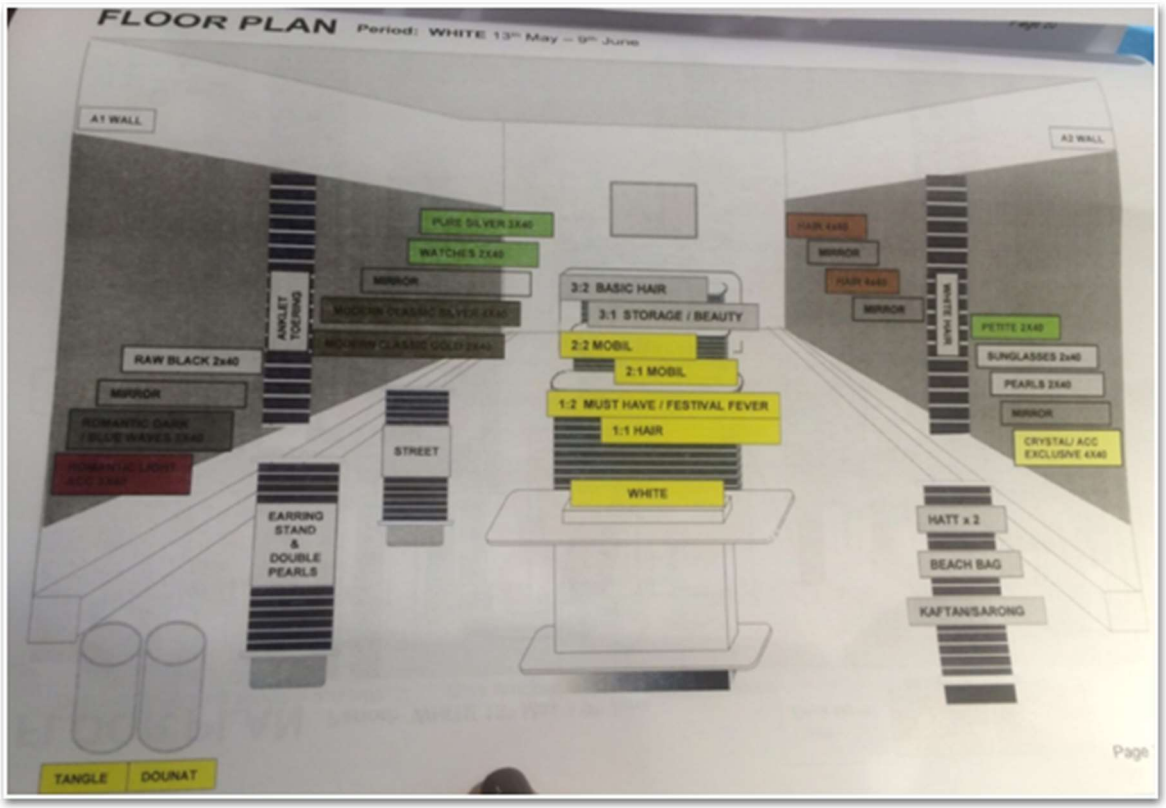

Figure 3. 3D-drawing of the new shop display at Skimra.

David has completed a display of TV Sound Bars at Audi & Video and is also evaluating the result with his supervisor:

> *(Supervisor) - Actually, the sales assistants do not usually plan the store´s displays, we get ready campaigns from the marketing department / ... / We have some sales managers who work with it, then, of course, everyone helps. There is much to be carried back and forth from the stockroom.*
>
> *(David) - Yes, but they've got another and much longer education, so then they should be able to make budgets, planning, and stuff. Before this project, I thought the planning and the carrying out was the same thing, you do it all at once. But then we had to plan this graduation project at school, in the logbook. It changed the way of thinking.*
>
> *(Supervisor) - I think we've noticed a difference, from the different tasks you made in the beginning when you got here, and it got better and better with each task. On the one hand, you have planned better and thought about it more, all the way in advance ...*
>
> *(David) - Before, I could forget to print out price signs, turn on the alarm or that any TV just stood there on the floor. For it was like I felt that it was 90 per cent complete, and you did not make the final 10 percent of it because there was a customer and you forgot about it. Now I have done it a few times and then it will be that you think about it in advance. What was the outcome last time and what can I do better? I have been involved in re-making Sound, an X number of times now.*

It also emerged that the logbook notes and the help questions in the logbook were used in discussions between the apprentice and the supervisor, but also between the apprentice and

the employees. Discussions concerning, for example, different tasks, budget, retail maths, economic models, marketing, and service. One of the apprentices, Disa, was at the end of her education supervisor for a trainee from another school, she had chosen the assignment as her project in the course Business development and leadership (Skolverket, 2011). She made a copy of the logbook for the trainee and used her experiences from the supervision group when supervising the trainee.

However, the apprentices are not talking about the group-supervision, just merely about school. This may be due to the traditional lessons and examinations were replaced by "project time" after the action research study completed. This meant that issues and development areas that emerged during group-supervision steered the content of the project time. For the apprentices, school came to be synonymous with group-supervision, that is a form of study. The logbook, however, was something they associated with the development of mathematical containing vocational competencies. For example, the ability to organize, orientate, localize, handle data, estimate, measure, count, modeling and spatial thinking (Arkenback-Sundström, 2017).

Conclusions

The didactic implications that emerged from the study of mathematics containing activities in retail learning practices draw attention to the dialogue (group supervision, supervision, trialogue) and systematic reflection (logbook) of activities in learning practices as important for the development of spatial thinking and mathematics containing vocational skills. For example, the ability to plan, organize, implement and evaluate campaigns, exposures, and reorganizations in the stockroom. Likewise, the knowledge and skills to understand and use digital tools and software are a prerequisite for being able to communicate numerical and non-numerical information in the activities at the stockroom, the sales floor and at checkout.

The results indicate that apprentices that kept a logbook and continuously met in the school for group-supervision, to a greater extent discovered mathematics containing activities at the workplace, than those who didn´t reflect on the workplace learning systematically. The structured logbook proved to be an effective learning tool, questions that normally would not be in the workplace-based learning, were discussed as they were in the logbook. The apprentices did not have to be afraid to experience themselves as ignorant, as they could "blame" the logbook. At the same time, it surfaced that some hourly employees felt that the apprentices were given too much attention. The questions in the logbook and its links to course content, course objectives, and knowledge requirements were perceived as a threat to their employment.

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Joanne Caniglia, USA - Teaching Function Mathematics Skills to Refugees

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Abstract

In the United States, after approximately 90 days of English language programs, refugee adults are placed in the mainstream, where they face enormous challenges including navigating bus schedules, buying groceries with a different monetary system, shopping for necessities, and applying for needed social services. The complexity of the mathematics that these activities require poses a significant barrier for adult refugees with limited English and interrupted education. This action research study reports on a year-long project that sought to uncover a mathematics educator's assumptions and misconceptions brought to the teaching of functional mathematics skills for small groups of refugee women. Strategies to address teaching functional mathematics to refugees are provided within each section with an emphasis on knowing the stories of students and honoring them by creating an environment of welcome and high expectations for their success.

Keywords: functional mathematics skills, misconceptions, refugees

Introduction

In a recent TED Talk, author and poet, ChimamandaAdichie tells the story of how she found her "authentic cultural voice"—and warns that if we hear only a single story about another person or culture we risk a critical misunderstanding (Adichie, 2009). Adichie's story describes the major concerns of refugees and their teachers—that of making assumptions and critiques of other cultures before teachers get to know their students' stories. The purpose of this study was to expand the stories of mathematics educators of refugees. From a year-long study of adult refugee men and women who were learning functional mathematics, this study investigated strategies that are effective and at the same time could expand the single stories that teachers may carry with them into a classroom.

The Plight of Refugees

Who are refugees and where are they from? According to the 1951 United Nations Convention Relating to the Status of Refugees a refugee is "a person who owing to a well-founded fear of being persecuted for reasons of race, religion, nationality, membership of a particular social group, or political opinion, is outside the country of their nationality, and is unable to or, owing to such fear, is unwilling to avail him/herself of the protection of that country" (United Nations,

1951, p. 137). Statistics on refugees are provided by the United Nations High Commissioner on Refugees (UNHCR, 2016). Syria remains a leader in numbers of displaced persons: 12 million people, or two thirds of the population is leaving. Colombia has 7.7 million refugees, then Afghanistan (4.7 million) and Iraq (4.2 million). South Sudan follows with 3.3 million with the fastest growing number of refugees. While refugees may flee to other countries, many may spend years in camps waiting to return to their own country or to another one selected by the United Nations Immigration Division (UNHCR, 2016).

> The human tragedy of massive forced displacement continued to unfold around the world during the first half of 2016 with conflict, persecution, generalized violence, and violations of human rights causing forced displacement to increase further. The first half of the year saw persistent conflict in many regions, notably Nigeria, Yemen, South Sudan and the Syrian Arab Republic (Syria), leading millions to flee their homes, most remaining displaced within their own country but many also leaving for other countries (UNHCR, 2016, Introduction)

he refugee statistics are both daunting and in a constant state of flux. The United Nations Refugee Agency's annual Global Trends study found that 65.6 million people were forcibly displaced worldwide at the end of 2016 – approximately 300,000 more than in the previous year (United Nations High Commissioner for Refugees (UNHCR, 2016)). It noted that the pace at which individuals are becoming displaced remains very high. On average, 20 people were driven from their homes every minute last year, or one person every three seconds. The total number of refugees includes 40.3 million people uprooted within the borders of their own countries, about 500,000 fewer than in 2015. Meanwhile, the total number seeking asylum globally was 2.8 million, about 400,000 fewer than in the previous year. However, the total seeking safety across international borders as refugees topped 22.5 million, the highest number seen since UNHCR was founded in 1950 in the aftermath of the Second World War (Ratha, Eigen-Zucchi, &Piaz, 2016).

An examination of where refugees to the U.S. have come from and their numbers provide a glimpse into global events and the U.S.'s role in providing a safe haven (Pew Research, 2017). Of the 84,995 refugees admitted to the United States in fiscal year 2016, the largest numbers came from the Democratic Republic of Congo, Syria, Burma (Myanmar) and Iraq. Although the countries of origin have not significantly changed, in spite of shifts in policy, the end of 2017 found only 24, 559 refugees resettled in the United States.

Table 1.
Functional Mathematics Skills Delineated by NRS Levels

Functional Math Skills	NRS Learning Standard
Support Services, Cardinal Numbers, Money, Ordinal Numbers, Personal Information, Shopping, Telephone Use, Telling Time (hours), Bus Time Table	Beginning literacy
Telephone Use, Leaving a Message, 911 Calls, Before and Now, Telling Time (1/2 hour and beyond) Liquid Measure	Low Beginning ELL
Using the Newspaper to Buy Groceries	High Beginning ELL

Using the Newspaper to Study Restaurant Ads Banking	Low Intermediate ELL
Job Applications Warning Labels Getting the Facts	High Intermediate ELL
Using the Newspaper to Buy a Car, Using the Newspaper to Find an Apartment Insurance: Medical and Dental	Advanced ELL

Functional Mathematics

For the purposes of this study, functional mathematics skills are those defined by the National Reporting System for Adult Education (NRS, 2016). This is an outcome-based reporting system for the state administered, federally funded adult education program. Developed by the U.S. Department of Education's Division of Adult Education and Literacy (DAEL), the NRS continues a cooperative process through which state adult education directors and DAEL manage a reporting system that demonstrates learner outcomes for adult education. The project is conducted by the American Institute for Research (AIR).

The NRS divides educational functioning into six levels for English Language Learners (ELL). The six levels are beginning literacy, low beginning ELL, high beginning ELL, low and high intermediate ELL, and advanced ELL. The ELL levels describe speaking and listening skills and basic reading, writing, and functional workplace skills that can be expected from a person functioning at a particular level. The skill descriptors illustrate the types of skill students at a given level are likely to have. The descriptors of functional mathematics skills do not provide a complete or comprehensive delineation of all of the skills at a given level but provide examples to guide assessment and instruction. Upon DAEL approval, states may also use additional educational levels and skill.

The contents of functional math skills are listed in Table 1. Each entry includes both the content and the NRS category. As the above list of functional skills attempts to do, it is imperative that there be ways to help refugee adults and students both to survive the trauma of the refugee experience, and to gain competence as new members of the United States. This research seeks to answer the question: What are some of the myths and misconceptions that a functional mathematics teacher brings to her work and what strategies appear to be successful given understanding gleaned from in-depth tutoring and conversations?

Methodology

Participants

The demographic variables for the 12 women in this study including age, status within the community, country of origin, and length of stay in the U.S., are found in Table 2.

Table 2.
Demographic variables and frequency of refugee women

Demographic Variables	*Refugee Information*
Age	24-59
Purpose of Attending Education	Citizenship (n=2) Livelihood (n=10
Country of Origin	Iraq (n=3) Bhutan/Nepal (n=2) Uganda/Congo (n=3) Somalia (n=1) Afghanistan (n=1) Syria (n=2)
Length of time in United States	10 days to 4 years
Number of Children	Each woman had between 1to 5 children. (all school age) All children were with their mothers except 2 who had their children left behind
Educational Experiences before they came to US	Little to no education (n=4) Elementary Education (n=5) Secondary Education (n=3) in Refugee Camps.
Employment	Part-time jobs afternoon shifts (n=5) Unemployed (n=7)

Data Sources

Data sources consisted primarily of participants' work of basic functional mathematics lessons, author's notes within a research journal and conversations with refugees, support staff, and ESL teachers. This study was conducted between January 2017 and January 2018. This period was significant because of the multiple judicial challenges of presidential executive orders made by the United States executive branch. In addition, multiple agencies within a large, Midwestern urban area with extensive networks of supporting refugees during their resettlement period provided background information on lands of the refugees' origins.

Data Analysis

As is consistent with qualitative action research, data analysis began with reading through the data multiple times and identifying initial themes. Coding procedures from grounded theory were utilized (Strauss & Corbin, 1998). The author began with open coding, a process through which data "are broken down into discrete parts, closely examined, and compared for similarities and differences" (Strauss & Corbin, 1998, p. 102). During the open coding stage, data were reviewed to understand what individuals were expressing in their responses. The second step in the coding process was to use axial coding, for the purpose of reassembling the data to develop connections and categories within the data across participants, and between student and teacher responses. This process supports research triangulation of data, a method of increasing trustworthiness in the data (Lincoln & Guba, 1985).

Findings

The findings will be presented in three sections. The first is an examination of myths identified by refugee agencies and those found by the author as she taught functional mathematics. The second and third sections describe the content knowledge and cultural themes the author found while teaching the 12 refugee women. Within a discussion of each theme, strategies for teaching functional mathematics will emerge.

Myths of Refugees

Daily images of refugees and migrants seeking safety in countries often far from their own have shocked the world. Countries and continents are confronted with tragic images of refugee-filled boats sinking. Host countries' responses have been unpredictable; policies change nearly daily; people travel between borders and thousands die in the Mediterranean while others are saved. Given this chaos, many myths about migrants and refugees persist. By understanding these myths, teachers can better understand how students and citizens of all countries can lessen their misconceptions of policies. The following list of misconceptions and myths (Table 3) was prepared by the United Nations Regional Information Centre (UNRIC); International Organization of Migration (IOM); United Nations Development Programme (UNDP), UN Refugee Center (UNRC), United Nations High Commissioner for Refugees.

Table 3.
Myths, Facts, and Answers to Questions

Migration is bad for the economy and economies in origin countries The proven reality is that migration brings benefits, fuelling growth, innovation and entrepreneurship in both the countries people come from, and in those they move to, if managed smartly. Migrants and refugees contribute to the economy both as employees and as entrepreneurs, creating new firms and businesses.
Stricter border controls and measures like fences reduce irregular migration Building fences does not stop the refugee influx; it merely shifts it to other countries and increases human misery. Migrants and asylum seekers are more likely to resort to entering a country irregularly when there are no legal alternatives.
Migrants and refugees take jobs away from local people Migrants accounted for 47% of the increase in the workforce in the United States and 70% in Europe over the past ten years according to the OECD. Migrants often take jobs that others are less willing to do or take, and can help fill gaps in the job market.
Migrants and refugees want to come to Europe and the US? Is Europe is facing the world's heaviest refugee burden?

Turkey, Pakistan, and Lebanon are now home to 30% of refugees worldwide, followed by Iran, Ethiopia, Jordan and Kenya.

Myth #1. The Resettlement Process is Straightforward

> I thought everyone wanted to come to the U.S. We have many of the best advantages here. Then why are they so sad? I never believed that refugees may not want to come. It seems like a straightforward process to me. (Author's Research Journal, Day 2)

In actuality, the migration process for refugees is initiated by the United Nations, and refugees have little input as to where their destination may be. The process of verification is not brief. It actually takes years before a person or family member may be allowed to migrate to other lands. Persons' identification documents are scrutinized. According to the U.S. State Department, 20 steps are necessary for those who wish to enter another country. These steps are included in Table 4. Because of the length of time between the initial screening and departure, officials conduct a final check before the refugee leaves for a final destination.

Table 4.
Steps of Refugee Resettlement

1. **Registration** with the United Nations.
2. **Interview** with the United Nations.
3. **Refugee status** granted by the United Nations.
4. **Referral** for resettlement in the United States.

   The United Nations decides if the person fits the definition of a refugee and whether to refer the person to the United States or to another country for resettlement. Only the most vulnerable are referred, accounting for less than 1 percent of refugees worldwide. Some people spend years waiting in refugee camps.
5. **Interview** with State Department contractors.
6. First **background check**.
7. **Higher-level** background check for some.
8. Another **background check**.
9. First **fingerprint screening**; photo taken.
10. Second **fingerprint screening**.
11. Third **fingerprint screening**. The refugee's fingerprints are screened against F.B.I. and Homeland Security databases, which contain watch list information and past immigration encounters, including if the refugee previously applied for a visa at a United States embassy. Fingerprints are also checked against those collected by the Defense Department during operations in Iraq.
12. **Case reviewed** at United States immigration headquarters.
13. Some cases referred for additional review.
14. Extensive, in-person interview with Homeland Security officer.
15. Homeland Security **approval is required**
16. **Screening** for contagious diseases.
17. **Cultural orientation** class.
18. Matched with a **resettlement agency**.
19. **Multi-agency security check** before leaving for the United States
20. Final **security check** at an American airport.

Although the refugee women in this study never shared which countries they would have liked to have been resettled in, they unanimously wanted to go "home." Statistics are essential to see a larger picture and to support advocacy efforts for refugees, yet the women's stories built a "bridge of empathy" (Fleming, 2017) that helped people to understand why refugees take the risks to come to this country.

The following strategies were developed by Canadian teachers as they met students who had experienced trauma during their lengthy resettlement (Calgary Board of Education).

- Build safety through routines. Create predictable environments and responses. Use routines to assist students to know what will happen next and why they are asked to do something.
- Establish regular activities, with consistent greetings and good-byes, daily reviews, transition point markers, calming activities, etc.
- Choose important routine events to celebrate (e.g., birthdays, holidays)
- Recognize and avoid triggers that may remind refugees of traumatic past events.

Myth #2: If One Knows Social Language Then One Knows Academic Language

> "What do you mean, 'quarter past two' when you said a quarter was 25 cents." (Refugee adult after being asked to show 2:15 on an analog clock)

The author assumed that if an individual could communicate socially, academic language would be an automatic by-product and easily assimilated. But it became evident that not only do English Language Learners have difficulty in acquiring academic language skills, but teachers from various disciplines (mathematics education included) have difficulty preparing students for academic and professional achieved with academic language. Functional mathematics skills require academic language abilities. The vocabulary of measurement alone can be daunting given that all refugees are from countries that use the metric system.

Cummins (1984, 1991, 1994, 2000) explained that Basic Interpersonal Communicative Skills (BICS) and Cognitive/Academic Language Proficiency (CALP) are qualitatively different skills. BICS include skills such as pronunciation, basic vocabulary, and grammar required in everyday communication situations. Most immigrant students can develop these skills rapidly, with the result that "teachers prematurely assume that minority children have attained sufficient English proficiency to exit to an English-only program" (p. 27). In addition, Cummins criticized policymakers' demands for a quick transition to English-only instruction by stating that the policies are "veneers for the xenophobic belief that minority languages threaten social cohesion" (p. 27). In contrast to BICS, which involve contextual processing of language, CALP is a cognitively demanding process that is not embedded in a meaningful interpersonal context. Cummins (1981, 1992) reviewed numerous research studies that point to the interdependence of native and second language learning in advancing CALP skills and indicated that second language CALP takes five years or more to develop.

Strategies for teaching BICS and CALP are to enable the learner to communicate in simple language and to understand the meaning of what is heard. Asking questions and answering them takes much practice in functional mathematics skills. Collaborative learning cultures, role-playing, interviews and games make the language-building activities of BICS and CALP helpful in writing, reading, speaking, and listening, and creating a sense of community as well. Discussing current events has the whole classroom involved in conversations informally.

Myth #3: Math is a Universal Language

> "This is the way we do maths in my country." (Students were adding $.34 + $.45. One woman wrote tally marks to show tenths, and hundredths. while completing a page of addition and subtraction problems.)

> If I did not know mathematics as well as I do, I would never consider their solutions and processes as correct. It took two and three reviews of their work to determine that they were correct. It was not incorrect, it was just different. (Authors' Research Journal).

Although some mathematical calculations and processes may be similar, once students begin to solve word problems or more complex problems, they encounter difficulties in academic language. Mathematical language presents them with words and symbols that have double meanings, like "table," and English expressions, such as questions asking for the "difference" between two numbers. Instead of an answer to a subtraction problem, some may respond by stating similarities and differences in numbers. The level of complexity and high degree of emphasis on academic language makes it more difficult to grasp thus needing more support.

Steinhardt NYU researchers have identified additional difficulties and confusions often faced by English Language Learners (ELLs and Mathematics, 2009). These include:

- Students must learn to associate mathematical symbols with concepts and the language used to express those concepts. Example: the symbol / expresses the idea of something 'divided by'.
- Mathematical texts frequently use the passive voice, a complex and difficult structure for many non-English speakers. For example: ten (is) divided by two and when 15 is added to a number, the result is 21; find the number.
- Mathematics also uses strings of words to create complex phrases with specific meanings, such as a measure of central tendency and square root.

Even if mathematical language can be considered universal, the language of 'doing mathematics within the classroom' is far from universal. The language of exploratory discussions, the discourse-specific mathematical talk, and the mathematical talk and writing taking place in the language of instruction, make it unique to each culture (Planas, 2001). Whether an English language learner or a native speaker, each one faces a challenge in learning to converse within the mathematics language. Moschkovich writes, "The communicative competence necessary and sufficient for competent participation in mathematical discourse practices… [involves] specialized vocabulary, syntax, organization, register and discourse practices" (Moschkovich, 2012, p. 22). Moschkovich suggests that the presence of ELL learners in the classroom can help build an awareness of the linguistic challenges we face as classroom teachers. Thus, instead of considering ELL as problematic, she considers English learners as a gift, because when one hears imperfect language with an accent, or has incorrect tense, students and teachers are reminded that even if you are in a monolingual English class, with students who are native English speakers, there are language issues going on there as well (Moschkovich, 2012).

Content and Cultural Themes

In addition to encountering myths and misconceptions while teaching mathematics to refugees, the author found content and cultural themes that when examined, could provide greater insight for those who teach functional mathematics. Discussions within each theme will also describe strategies and research that may support adult learners.

Content Theme 1: A Picture Is Worth a Thousand Words, But An Object and Gestures Are Worth More.

> "I like it when I can see what you are talking about."

> "I can't understand the money without real money. I can't see the numbers"

> "I liked it when you brought in a pizza and we found ½ and ¼ of the pizza, then you cut out papers and you showed us ½, ¼." (Student after three sessions on money and common fractions)

To develop vocabulary, the refugee women appreciated multiple representations in functional math. The Universal Design for Learning (UDL) framework emphasizes multiple means of representation, multiple means of expression, and multiple means of engagement (Rappolt-Schlichtmann, Daley, & Rose, 2012). UDL provides educators with a framework for all kinds of learners in mind. ELLs, while limited in their English proficiency, come to school with variability in their home language skills, from full oral and literate proficiency, to very limited skill sets (Meyer, Rose, & Gordon, 2014). In using UDL, the author was able to guide the development of measurement terminology and basic cooking by using pictures, utensils, recipes, bus schedules, and newspaper advertisements.

Content Theme 2: Awareness of Sources of Confusion: Multiple Meanings of Words

> You said a quarter past five, but a quarter was 25 cents. Why isn't it 25 minutes after five? (Adult Refugee)

Adult learners in this study consistently were confused over the meaning of mathematics vocabulary. For example, the meaning of 'quarter' was difficult because of its dual meaning. Polysemous words, which are words with the same spelling and pronunciation but different meanings, can be confusing for adults to understand. Many words are used in math textbooks and teaching which differ from their everyday life meanings. Instruction in specific vocabulary is crucial because vocabulary knowledge correlates with math reading comprehension (Smith, 1997; Sidek, Rahim, 2015). Students also found that words functioning as a verb, a noun, or an adjective also have different definitions.

As refugees advance in their language skills, strategies to become competent in examining the context to decide whether the meaning of the conversion is closely related to the meaning they already know are essential. The author found that her role was not only to teach children about this language phenomenon, but also to help the women develop confidence in their ability to infer the meaning of a conversion. This requires us to help readers attend closely to context. It also means helping students identify the grammatical function of a word, which can be difficult for beginning English speakers and writers.
In one relevant study, Carlo et al. (2004) taught fifth graders about how English words work. Topics included learning about polysemy, learning the structure of morphologically complex words and understanding the nature of academic language. On the polysemy post-test, the ELL group made significant improvement compared to their pre-test scores, yet despite this gain they did not match the progress of students who spoke English. natively However, both groups -- ELLs and native English speakers -- made significant gains over the control group, who did not receive the intervention. Within the current study, students appeared to enjoy the experience!

Content Theme 3: Repetition, Repetition, Repetition…

> "1 O'Clock, 2 O'Clock, 3 O'Clock…" Rote repetition of time. I think they got the sound and meaning of hours." (Authors' Research Journal—Week 6)

> "Oh no, they did not remember the times on the clock nor did they get what the numbers around the clock represented." (Authors Research Journal, Week 7)

Among the language learners in the study, repetition was essential. But the author soon discovered the importance of "deliberate practice." Deliberate practice is not the same as rote repetition. Rote repetition—simply repeating a task—will not by itself improve performance. This is what I found after one week with the adult learners. When using deliberate practice, which involved attention, rehearsal and repetition leading to new knowledge or skills, I was able to make the concepts understood. (Hambrick, et. al., 2014). Although other factors are necessary, deliberate practice with meaning appeared to be helpful in learning.

Cultural Theme 1: Understanding Background

> "Why are some of the adults so sad? I asked for information from social workers and staff. They mentioned that many of the refugees are reminded of traumatic experiences. It seems that functional mathematics does not seem to be important to them. I need to pursue their background knowledge."
> (Author Journal Entry 26)

Upon returning from sessions of functional mathematics skills, I searched for available information on the refugees' backgrounds and their educational backgrounds. Linking to students' personal life experiences is beneficial for a number of reasons. Personal life experiences can help students find meaning in content learning, and linking to an experience can provide clarity and promote retention of learning. Relating content to students' personal lives and experiences also serves the purpose of validating students' lives, culture and experiences.

For example, Haynes & Zacarian (2010) note that in general, members of the dominant U.S. culture believe that children should be raised to think independently. The goal of education is to have children learn to think like adults when they are still children. Children's efforts to think and use their independent thinking skills are praised and rewarded. Their wants, needs, and desires are often viewed as of primary concern in the family.

However, many refugees come from collectivistic cultures in which the good of the individual is sacrificed to the good of the group. A person's moral worth is judged by how much he or she sacrifices for the group. Students from this type of culture work best when they can form a relationship with the group. They are "we" rather than "I" oriented. Because of their subtle influence, these factors are important for all teachers and administrators to know (Zwiers, et.al, 2017).

Cultural Theme 3: Parents Desire to Succeed

> When adult students introduced themselves, and spoke of their children, they often said that their children were helping them learn at home. Their children often help the parents. (Author Research Journal).

It became clear that the adults in the study were eager to learn since they wanted to help their children with homework. Many children of these refugee women supported their parents in language development. This role reversal can be a source of culture shock. Because of different cultural beliefs, many parents may understand the concept of parental involvement differently from the way that U.S. parents do. These conditions may increase cultural dissonance.

Children may be encouraged to practice language skills by teaching their parents. The exchange of ideas and information encourages communication skills in listening, speaking, writing, and reading. When students become the "teacher" to family members, they are participating in a process termed "language brokering" (Tse, 1996). Language brokering refers to translation between linguistically and culturally different parties. Language brokers are often the children of immigrant and refugee families who serve as interpreters and translators

for their parents and family members (Morales & Hanson, 2005; Tse, 1996). These children may accompany their parents to a doctor's appointment to interpret conversations, help translate the content of a letter sent home in English, or speak on the phone (on behalf of a relative) to school personnel. Language brokers rarely receive formal training as translators or interpreters, yet their day-to-day experiences often draw on their bilingualism. In this study, the author encouraged parents to bring their children to class to demonstrate to children how their parents are learning and the importance of respecting each other's ability to communicate.

Conclusion

As Chimamanda Adichie summarizes in her TED Talk,

> Stories matter. Many stories matter. Stories have been used to dispossess and to malign, but stories can also be used to empower and to humanize. Stories can break the dignity of a people, but stories can also repair that broken dignity.

This action research study points to the importance of teachers' and administrators' understanding of the refugee experience and the dangers of assuming a single story. Misunderstanding the dire situations of parents, the role of trauma in refugees' behaviors, cultural differences, and best practices in language acquisition can hinder the teaching and learning processes. (Birman et al., 2001; Timm, 1994; Trueba et al., 1990). Although the list of strategies presented in this paper may be incomplete, this research strongly recommends the importance of knowing the stories of students and honoring them by creating an environment of welcome and high expectations for their success.

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Noel Colleran, Ireland - Exploring the genealogy of the concept of 'innate mathematical ability' and its potential for an egalitarian approach to mathematics education

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Abstract

Recent work by a number of researchers has argued that the capacity for mathematical thinking is innate to human intelligence. Much of the evidence for this conclusion is based on findings in fields as diverse as linguistics, genetics, evolution, archaeology, psychology, and philosophy. This paper argues that the genealogy for this development is sourced in the philosophy of the Enlightenment, particularly the work of Immanuel Kant. Kant's seminal idea suggests that human intelligence had a natural and necessary capacity for mathematical thinking in the forms of space and time. This paper will explore the ideas of Immanuel Kant regarding space and time, particularly his views that the intuition of space provides the source for geometry while the intuition of time provides the source for number. A limited, yet sufficient, evaluation of recent relevant literature will be employed to illustrate that 'new insights' regarding innate mathematics ability can be 'genealogically' traced to the work of Immanuel Kant. Ultimately, this paper argues for the debunking of generally accepted agreement among some that many mathematics students have an innate capacity to do mathematics while others are innately incapable in this regard. With an acknowledgement of this 'initial state' regarding universal mathematics ability among young, as well as adult students, an egalitarian perspective regarding students' expectations and achievements in mathematics is in view.

Key words: Enlightenment, Kant, space and time, innate maths, maths gene, egalitarian education.

Introduction

It is my contention that there are many who believe that people in general, and students in particular, are either good or bad at mathematics. On the basis of this belief students are regularly encouraged or discouraged from doing higher level and more challenging mathematics in schools and colleges. There is no doubt that students come to school, and adults return to education, with proficiency in mathematics which is wide-ranging, however, I will argue that this range, spanning from what might be called 'prodigies', on one side, to those who have 'math phobia', on the other, results neither from the learners' giftedness nor from intellectual deficiencies, but from the amount of time they spend thinking mathematically and doing mathematics. I will argue that the element that encourages mathematical practice, on the one hand, and discourages practice, on the other, is accounted for through socialisation processes experienced by each individual. This paper will provide evidence that all of us, young and old, have an innate, natural ability to do mathematics to

the highest level: we are all endowed with a 'maths gene'. And, if Peuquet's (2002) is correct when she asserts that adults learn in a similar fashion to younger students when confronted with novel situations, my argument is relevance to both adult and younger learners of mathematics. The basis for this argument is derived from the work of the Enlightenment philosopher Immanuel Kant. His philosophy provided revolutionary insights regarding space and time and many of his foundational ideas are continuing to reverberate nearly 250 years later. In the first instance,

will argue that Kant provides the 'genealogical' source for the concept of innate mathematical ability among humans. This will be followed by more current research, which will support Kant's most important ideas, and provide sufficient evidence to reasonably postulate that the Enlightenment thinker provides the most influential source for the concept of innate mathematical ability. I will then provide evidence that the amount of time spent doing mathematics is directly proportional to the proficiency levels achieved. I believe that the acceptance and employment of these two principles - innate mathematics ability, and practice makes perfect – among teachers of mathematics will, no doubt, lead to a re-evaluation of practice with young and, in particular, adult students of mathematics. Ultimately, I believe that practice based on these principles can provide for a more egalitarian approach to mathematics education for all.

Immanuel Kant's contribution to the concept of innate mathematical ability

Most serious philosophers will agree, to a greater or lesser extent, that Immanuel Kant brought about a transformation in western philosophy the likes of which had not been seen since the ancient Greeks: 'and [Kant's] work did indeed change philosophy permanently' (Hatfield, (2004 p. ix); 'within a few years of the publication of his Critique of Pure Reason in I78I, Immanuel Kant was recognised…as one of the great philosophers of all time' Guyer and Wood, (1998 p. vii); 'the Critique of Pure Reason is a philosophical classic that marks a turning-point in the history of philosophy' Kemp Smith (1918 p. viii); 'the most important phenomenon which has appeared in philosophy for two thousand years… the principal works of Kant' Schopenhauer (1818 p. xv). Much of this reputation is based on his most famous publication in 1781 entitled A Critique of Pure Reason. The metaphysical transformation that Kant brought about with his Critique was centred on the question; what is the range of human understanding? Or, from a negative perspective, what are the limits of human understanding? Accordingly, he turned his attention not to the product of human understanding but the producer; the instrument by which human understanding is generated i.e. human rationality. At the outset Kant was satisfied that human understanding and knowledge were constituted by both sensed experiences and reason, and both had a range within which they operated effectively – outside this range human understanding and knowledge was vulnerable to attack and could not be defended. The philosophical clearing in which Kant's position regarding the range and limits of human cognition is a good place to start, in particular the manner in which he distinguishes the noumenal and the phenomenal world.

The Noumenal and Phenomenal World

Central to Kant's argument in the Critique is his contention that there are two distinct versions of the world: the noumenal world and the phenomenal world. The noumenal world is the world as-it-is-in-itself; the world of beliefs, spirituality, feelings, etc., which are not accessible to human sense organs. And while we may speculate about the noumenal world, humans cannot know it. Humans can understand and know the phenomenal world because we have mediated access to this world through our senses. Knowledge of the phenomenal world is limited by human senses and our capacity to cognise perceptions mediated through those senses. Therefore, the range and limits of human cognition and knowledge lie within the phenomenal

world. However, while the human faculty for knowledge is 'limited' to the world of phenomena, it seems to have 'limitless' capacities to generate knowledge within this context, particularly in the sciences, mathematics, information technology, etc., and we must remain ever vigilant of our own limitations and refrain from stepping outside the bounds of the phenomenal world.

Empiricism and Rationalism

The philosophical milieu from which Kant's Critique emerged was dominated by a spectrum of two competing doctrines regarding what constituted genuine human knowledge. This comprised, at one end, a form of radical empiricism, positing that there is an objective out-there-now-real world that we engage with and know, not immediately but mediately, through our five senses i.e. seeing, hearing tasting, touching, and smelling. According to Locke (1690), the human mind enters the world as a 'tabula rasa' (a blank slate) and human experiences cover this blank slate with our knowledge of the world. There are obvious difficulties with this approach because all humans see, hear, etc., differently and the development of any knowledge based on subjective experiences could never approach general or universal understanding. However, the empiricists were convinced that the only true source of human knowledge is through human experiences and were satisfied to push the breaks at this point and conclude that humans know through individual perceptions that aggregate and combine into ever more complex ideas and knowledge. While Kant accepted that our senses provide access to the phenomenal world he rejected the idea that knowledge is an aggregation of increasingly complex sensed experiences. Without some non-empirical faculty that forms, unifies, establishes coherence, and makes sense of these impressions, there is no possibility of knowledge. The empiricists were adamant that these forming and binding capacities are not given in experience and anything not so derived should be 'committed to the flames' (Hume 1748).

On the opposite side of the knowledge spectrum was a form of radical rationalism and chiefs among the rationalists were Descartes and Leibnitz. Like the empiricists, they too questioned the validity of human sensed experience in providing objective knowledge. However, instead of accepting that human senses experiences provide access to knowledge of the world, the rationalists rejected human experiences as much too vulnerable. The rationalists relied on the capacity of human intelligence alone to provide such knowledge. Rationalists:

> '… held that it is possible to determine from pure a priori principles [thinking and speculating without reference to vulnerable human experiences] of the ultimate nature of God, of the soul, and of the material universe'
>
> (KEMP SMITH, 1912, p. 13).

Descartes and Leibnitz contended that human thought, unfettered by subjective sensed experience, can determine objective reality. Again, Kant was satisfied that there was some validity in this view as human intellectual capacities play a fundamental role in forming and synthesising human perceptions to constitute human understanding. He contended, while the empiricists had stopped short, the rationalists had gone too far. Kant agreed that the rationalists provided the cognitive capacities to 'interpret' human experiences, and significantly, these capacities are available without reference to sensed experience. Fundamentally, the human knowing process is available innately, or, in Kant's term, a priori; unadulterated and without reference to human experiences and so 'pure'. While accepting the innate presence of pure reason, a nod of sorts to the rationalists, Kant equally accepts the empiricists' view that the context and source of knowledge is the world of human experiences. Kant provides an accommodation of the limiting aspects of both perspectives by accepting that human knowledge is derived only subsequent to human experiences - a posteriori - and these

sensations are necessarily categorised and synthesised by an a priori, innate intellectual capacity, awaiting stimulation.

> 'But, though all our knowledge begins with experience, it by no means follows that all arises out of experience'
>
> (CRITIQUE, B 15).

His conclusion is that humans have a capacity for receptivity through the senses (content), and a capacity for conceptualising through the intellect (concepts). Both are universal and necessary for the possibility of human knowledge.

> Thoughts without content are empty; intuitions without concepts are blind
>
> (B 74).

And so, as a result of Kant's distinction of the noumenal and phenomenal worlds, and his exploration of the competing aspects of the contemporary field of philosophy, he began his critique of the relevant organ: human intelligence. In the section entitled the Transcendental Aesthetic ('aesthetic' here refers to senses), which consists of not more than forty pages, he describes the mediating role played by human senses and, more importantly for this paper, his contention that all human experiences are grasped by the natural, pure, and innate forming capacities of space and time. According to Guyer and Wood, (1998, p. 7):

> … the "Transcendental Aesthetic" [has] been the subject of a very large proportion of the scholarly work devoted to the Critique in the last two centuries.

Innate Space and Time

The first key contribution of Kant's Critique, mentioned above, was his distinction between the noumenal and phenomenal world and all that flowed from that position. His second contribution, directly relevant to this paper, is the one element of the 'bridge' he constructs to accommodate his version of empiricism and rationalism. This element is provided by the 'forming' intuitions of space and time. According to Kant, the intuitions of space and time, necessary for human understanding, are not derived from experience. This means that space and time are cognitively available prior to any experiences of the world and so must be innately and necessarily available to all human knowers. Space and time provide the necessary and only faculties by which human experiences are formed, shaped, grasped, or, according to Robinson (2011), how we come to 'behold' the external world. This provides the basis for my hypothesis that the intuitions of space and time are there in the 'initial state' (Chomsky, 2000 p. 7), unlearned, innately accessible to all irrespective of one's experiences in the world. Later I will refer to Kant's contention that space and time provide the foundations for the sciences of geometry and number respectively, and this extra layer provides the source of my argument that the genealogy of the concept of innate mathematical ability originates with Kant's assertion that all human understanding requires the forming intuitions of space and time and, because of their relationship to geometry and number, all humans have an innate capacity to understand and do mathematics.

Exploring Space

The empiricists argue our understanding is derived from human senses and if it is not so derived then it is baseless. Kant, however, suggests that while all our knowledge arises out of human senses it is not the source of all our understanding. The concept of space, which he

[5] All references to the *Critique* henceforth will have a prefix 'B' to suggest the second edition in 1787. The Politis translation of 1993 is the version used throughout unless otherwise stated.

argues is not sourced through the senses, is non-empirical and therefore a priori. Without this a priori forming intuition, Kant argues, human understanding is impossible.

> By means of the external sense, we represent to ourselves objects as outside us, and these all in space. Therein alone are their shape, dimensions, and relations to each other determined or determinable
>
> (B 35).

Kant postulated that space is not derived from the relations of external objects but that external experiences are possible only with the a priori intuition of space. He builds his argument for the innate faculty of space by suggesting a number of thought experiments such as: we can never contemplate the non-existence of space, while we can imagine empty space. We can only think of one space, and when we talk of different spaces they remain parts of the one same space: space cannot be built out of parts of space (B 37). Space is intuitively infinite in quantity; it can be bigger or smaller by an extra measure no matter how large or small (B 39). Space provides the intuition by which we put a shape on the world external to us and 'through space alone is it possible for things to be outer objects for us' (Guyer and Wood, 1998, B 44).

One of Kant's most innovative arguments regarding the innate intuition of space with regard to the appearance of objects was his employment of chiral objects: an object is said to be chiral if it cannot be superimposed onto or does not coincide with its mirror image. Kant employs left-handed and right-handed objects like human hands and gloves, spiral shells, etc., which 'obviously' appear to be chiral or incongruent counterparts when we look at them.

> …for the left hand cannot, after all, be enclosed within the same boundaries as the right (they cannot be made congruent), despite all reciprocal equality and similarity; one hand's glove cannot be used on the other
>
> (KANT, 1783 pp. 37-38).

If one describes a left hand in detail each detail is similar for a left hand as it is for a right hand, yet they appear different. And while they appear incongruent, the incongruence is not amenable to rational explanation. The apparent, yet 'obvious', difference can only be accounted for by the innate spatial intuition. (For an in-depth discussion on incongruent counterparts and chiral objects see Severo, R. (2005) and Bennett, J. (1970).

Moving on to Time

As for time Kant argues that ideas such as co-existence, succession or change could not be perceived were it not for the foundational and a priori intuition of time (B 45). All appearances are connected to time and cannot be contemplated outside the substratum of time i.e. past, present, and future. Principles of time that cannot be derived from sense organs, i.e. experience, such as: ''Time has only one dimension', 'Different times are not co-existent but successive'' demonstrate the a priori nature of time (B 46). And like space, different times are part of the one and the same time; time progresses infinitely into the future and regresses infinitely into the past; and so, is unlimited (ibid). Kant emphasised:

> …the concept of change, and with it the concept of motion, as change of place, is possible only through … and in time
>
> (IBID).

Space and Time as the basis for geometry and number

Kant makes the plausible connection between human intuition of space with the more formal science of Euclidian geometry. He also connects human intuition of time with number and motion.

> Geometry bases itself on the pure intuition of space. Even arithmetic forms its concepts of numbers through successive addition of units in time, but above all pure mechanics can form its concepts of motion only by means of the representation of time
>
> (KANT, 1783 p. 35).

Space and Time as Causality

The concept of causality, or cause-and-effect, is, by its nature, structured according to the sequence of time, i.e. succession of events, and provides the answer to 'why' questions with such answers beginning with 'be-cause'.

> The… causality of a thing is the real which, when posited, is always followed by something else. It consists in the succession of the manifold insofar as that succession is subject to a rule
>
> (B 183).

Schopenhauer's *The world as Will and Representation* (1819) acknowledged the importance of a priori space and time, however, he collapsed the remaining categories, identified by Kant in the Critique, into the single notion of 'causality'. Schopenhauer went further by arguing that causality is constituted by space and time.

> What is determined by the law of causality is therefore not the succession of states in mere time, but that succession in respect of a particular space, and not merely the existence of states at a particular place, but in this place at a particular point in time. Thus change, i.e., variation occurring according to the causal law always concerns a particular part of space and a particular part of time, simultaneously and in union. Consequently, causality unites space and time
>
> (SCHOPENHAUER, 1818 p. 10).

It entails the logical succession of events of time in space and provides the ground for the, sometimes unconscious yet necessary, 'if-then' structuring in mathematics.

Summary

Implications regarding the growth of the science of geometry from the roots of a priori space, and the science of number, sequences, series, motion, etc., deriving from a priori time; and furthermore supported by the view that causality, a concept indispensable to mathematics, is constitutive of space and time, cumulatively provide the basis for my hypothesis that the genealogy of the concept of innate mathematical ability begins with Kant. Taken together, these innate knowledge-constituting intuitions provide an empowering worldview regarding human mathematical ability.

After concluding my argument that Kant provides the genealogical source for the concept of innate mathematical ability, I will now turn to more recent research, which argues, in a more focused manner, that human intelligence is innately mathematical. I will begin with the work of Donna Peuquet's (2002) Representations of Space and Time: her research articulates with Kant's foundational insights i.e. that space and time provide necessary intellectual concepts for the creation of human understanding and knowledge.

The necessity of space and time for human understanding

Donna Peuquet's (2002) Representations of Space and Time is a book about geographic space and the dynamics that occur in that space. While the context is computer-based geographic data processing, there is substantial content on theories of how humans acquire, store, and use spatial knowledge. She believes that things change in space over time; both space and time provide an integrated representation of our experiential world. This sounds a lot like the spatial and temporal intuitions proffered by Kant (1781), and indeed Peuquet does make much

reference to the work of Kant. Peuquet suggests that space and time, being the most fundamental of notions … 'provide that basis for ordering all modes of thought and belief … Kant's space and time are concepts that we possess at birth (pp. 11 and 21). And while both concepts are innate it does not follow that all will make optimal use of these intellectual resources.

> The various ways that space, time, and their properties may appear to individuals are due to differences in attention to detail … access to technology, education … [etc.] (p. 25)

And while it is evident that people experience the world subjectively there is growing evidence:

> ... that the processes used to organise information are innate and either largely independent of the environmental input or dependant on kinds of environmental input that no human can avoid encountering (p. 28).

In her section on Schema: The Link between Percepts and Concepts, she again refers to Kant's view that schemata allow what we gain through our senses (perceptions) to be interpreted (concepts) and thus to be given meaning: schemata provide the bridge between experiences and meaning (p. 85). Research in cognitive linguistics has identified over twenty-four different image schemata and many of these intellectual bridging concepts are fundamentally spatial and temporal in nature. The schemata include:

> … container, balance, … path, cycle, centre-periphery, and link. Although these are called schemata, they are fundamentally spatial in nature. Our schemata for spatial and temporal orientation are so fundamental and pervasive in our experience that they are usually taken for granted (p. 87).

She refers to studies in visual cognition by Johansson (1973) to illustrate the importance of time in understanding. Given this understanding of schemata she concludes that even the youngest children employ space-time schemata to enable learning about the complex world they experience. Furthermore, learning is a similar process for adults and children in contexts where pre-existing knowledge is unavailable; however, what is significant of an increased capability among adults in a novel context is a larger store of knowledge:

> …Our basic notions of space [and time] are fundamental to learning and understanding [for young and old] in all domains (p. 88).

This insight has clear implications for teachers of mathematics to adults: while the learning process is similar for young and old, adults regularly employ a larger store of knowledge, including mathematical knowledge, not available to younger students. (This sophisticated, and often undervalued 'common sense mathematics' resource available to adults is explored in some detail in Colleran and O'Donoghue (2007).

In her analysis of our perceptual field she reminds us that all our sense organs operate in a temporarily sequential manner. While most attention in the psychological literature has focused on visual perception, which provides information about size, distance, shape, and texture, she points out that hearing provides information about size and distance, and all senses provide information regarding pattern. All our senses are temporally extended because no single event affords the sequence of perceptions that provide the basis for the emergence of a pattern.

> All our senses are temporally extended…. [With regard to listening, which] is perhaps a more temporally extended activity than other senses... there is typically no single

> moment in which one hears anything, because sound waves themselves are a space-time phenomenon (105).

The fundamental requirement of pattern in the creation of understanding demands, at a conscious or unconscious level, attending to sequences of sounds, tastes, touches, etc., so that we can relate a particular perception to familiar categories. It is on this basis only that one can identify familiar sounds, images, tastes etc. Obviously, if there is no pattern recognition a new pattern category is developed. All our senses operate within a temporally sequential series of perceptions.

Over time attention has been paid to the contribution of individual senses however Peuquet suggests that an holistic analysis of the contribution of all senses to human knowledge creation can provide a more fruitful approach.

> The current body of evidence supports the view that our senses provide a unified and interrelated suite of sensations and that we understand how these sensations are related very early in life. (p. 108)

Research in psychology provides evidence supporting this unified and interrelated process in gathering spatial information. Furthermore, because of this process, people with deficiencies in one sense area, for example vision, compensate with other senses, such as touch and hearing. Current thinking suggests that… 'encoding our spatial knowledge is innate and not keyed to any particular sensory modality' (p. 110).

In her analysis of language as a symbolic system Peuquet finds that while there are many cultural variations when it comes to languages there is an structural invariance regarding spatial expressions. She suggests that this invariant structure indicates a 'common cognitive structure of spatial knowledge at some deep fundamental level' (p. 166). All languages are constituted predominantly by nouns, verbs, and adjectives, and these can be augmented as the evolving situation demands, for example new technologies. The grammatical elements of a language include prepositions, conjunctions, etc., and these are limited in number. With regard to spatial relationships there are between 80 and 100 relevant prepositions. Spatial and temporal relationships are invariably included within the grammatical structure of a given language (p. 168). The English language provides the verb-ending –ed to indicate past tense; and prepositions 'above' and 'below'; 'near' and 'far'; to refer to space. Temporal relations are referred to with the words 'before'; 'during'; and 'after'. There are also space-time prepositions referring to motion including 'across'; 'through'; 'into'. Peuquet concludes that 'it does seem to be the case that spatial language encodes the world' (p. 175). Furthermore, the fact that the number of spatial and temporal terms is very limited and difficult to increment, plausibly implies an invariant, and fundamental structure essential to the manner in which we perceive and understand the world. Further supporting evidence that mathematical concepts, and therefore, mathematical thinking, is integral to language is provided by Devlin (2000) below.

Having created connections between Kant's contention of the innateness of the mathematical concepts of space and time with more contemporary work, I will now explore recent research providing substantial evidence that our capacity for mathematics is innate and universally available to all humans. This will include Devlin's *The Maths Gene* (2001) and Butterworth's The Mathematical Brain (2000). I will first turn to Devlin's work.

Mathematics ability available to all humans

Devlin (2001) provides a human-evolution approach to his argument that mathematical thinking is innately available to all human knowers. His argument is based on the view that the human language faculty is there in the 'initial state' not unlike our capacity to walk, or

become men and women through puberty: the language faculty just happens. Devlin's source for this hypothesis is derived from the work of Bickerton (1995), however, many would argue that the seminal work on linguistics was done by Chomsky, (a summary is provided in Berwick and Chomsky, 2016). Devlin argues that the language faculty and the human ability to think mathematically are derived from a single mental human ability: the ability to think off-line.

> The two faculties [mathematics and language] are not separate: both are made possible by the same feature of the human brain … [our] genetic predisposition for language is precisely what you require to do mathematics … thinking mathematically is just a specialised form of using our language faculty
>
> (DEVLIN, 2001, pp. 3-4).

Devlin, like Chomsky, has a difficulty with the proposition that language is an evolutionary development derived from the need to communicate more effectively. While there is no doubt that language is the most effective means we have to communicate it certainly is not the only medium. We can communicate by the way we dress, the way we do our hair and make-up, our body language, our facial expressions, and so on. Devlin plausibly argues that language is the externalisation of human thoughts, speculations, plans, understandings, etc. In this view language is the most useful means to communicate complex, and not so complex ideas, among humans, however that is not the original evolutionary purpose for language; language was developed because we needed it to think off-line and so, language is primarily the process by which we think. So, in one of those quirky, yet fortunate evolutionary accidents, human thinking, externalised in the form of speech, provided an extraordinary advantage regarding human development in the last 70,000 years. Integral to the development of language, Devlin argues, was the development of our ability to think mathematically: '…mathematical ability is nothing other than linguistic ability used in a slightly different way' (ibid, p. 22).

Devlin refers to substantial research suggesting an innate mathematical capacity among young children. He concludes that it is not just a correlation but, in fact, a symbiotic relationship between language development and our ability to think mathematically.

> I do not believe that a basic mathematical ability is any more unusual than an ability to talk (p. 126)

Devlin supports his position by referring to evidence that all human languages (known presently) have the same universal grammar. Chomsky's observation that children cannot learn complex syntactic structures because they are not given or taught particular examples by parents, or anyone else that has those structures, leads to the inescapable conclusion that we must be born with the capacity for language.

> '… [G]rammatical structure is innate in much the same way that spinning webs is hard-wired into the spider's brain' (p. 157).

The synthetic structures inherent in language provided the essential resource for off-line thinking i.e. the capacity to reason in an abstract fashion. This in turn provides the capacity for mathematical thought (p. 162). Furthermore, he argues that while humans have been using language for nearly 200,000 years, with no apparent mathematical uses or developments, it is the mathematical structures inherent in human language that provided the natural source for the development of 'formal' mathematics over the past 3000 years.

Devlin's research points to a two-stage development in the evolution of the human brain: the size of the brain increased over 3,000,000 years to allow for the development of more patterns and capacity to respond in a survival manner to new and various patterns. The second stage - 200,000 to 70,000 years ago – the brain didn't increase in size but it changed structure.

> Those structural changes… gave us symbolic (i.e. off-line) thought … language, a sense of time, the ability to formulate and follow complex plans of action, and…. to design a … growing array of artefacts (p. 178).

However, even as far back as homo habilis (the size-change phase) there was evidence of capacities around number sense, spatial reasoning, cause and effect, and relational reasoning. It was the brain's structural change that provided for abstract thinking, and this was the game changer: not a change in degree but a change in kind. An so our basic number sense, developed over 3,000.000 years, and now with the capacity for language from 70,000 years ago, the conditions were ripe for mathematical thinking in the form of numerical ability, algorithmic ability, and logical reasoning ability.

In an exploration of the necessary features of language to represent real-world situations Devlin asks: 'which features of the world are absolutely necessary … and hence will be incorporated into the syntax and which can be optional? He concludes that 'subjects', 'verbs', 'objects', 'tense', gender', 'singular-plural' are elementary to a thinking process capable of representing the world, i.e. off-line thinking.

> Off-line thinking provided the ability to think about past, present and future events, create tools [future orientated]… formulate and follow … plans of future action … logical reasoning (p. 236).

From this description of the necessary elements of syntax coupled with the ability provided by off-line thinking it is clear that many are related to space, time, and causality: verbs and tense are always related to time; singular-plural is related to differentiating space, while logical reasoning is essential in thinking mathematically. Devlin concludes... 'the maths gene and the language gene are one and the same [and] mathematics is an automatic consequence of off-line thinking' (237).

And so how is it that language has been used widely for more than 70,000 years while the development of mathematics stretches back less than 4,000 years? While keeping in mind that mathematical thinking is integral to language and language evolved primarily as a thinking process and not as a communication process, Devlin suggests language was hijacked by gossipers, and gossip was used to understand and care more about each other as humans, members of families, groups, tribes etc. Caring more for each other was the result of finding out more about each other. And caring for each other was a definite evolutionary advantage. In this understanding, the use of language developed a caring attitude among humans leading to group cohesion and the obvious advantages arising – language provided a major evolutionary advantage.

And so, for thousands of years this mathematical ability employed in gossip remained active yet invisible and undetected until a few thousand years ago when social and cultural developments, as well as the emergence of unique and exceptional thinkers, developed formal and abstract models to achieve the relevant mathematical outcomes. Gossipers, then and now, remained unaware and unburdened by the mathematical thinking integral to the language used to gossip. Gossip addresses similar questions to those of the mathematician – what is the relationship between? How many are there? What type? Are they the same? Are they equal? What is the property of... what characteristics does he/she have? and so on. Building an understanding of the relationships between people and the characteristics of each person/group is the material of both gossip and mathematics.

> 'The mental abilities required for gossip – even the most socially denigrated variety – are highly sophisticated, and already structurally adequate to support mathematical

> thinking… Mathematicians are not born with an ability that no one else possesses. Practically everyone has 'the maths gene' (pp. 249-250).

If mathematical thinking is as natural as learning a language or walking upright, why then do so many people find mathematics so difficult? The first part of the answer, according to Devlin, is that mathematical thinking is highly conceptual and abstract and what distinguishes a great mathematician from a high school student struggling in a geometry class 'is the degree to which the mathematician can cope with abstraction' (p. 253).

The second part of the answer is that we can become proficient at anything in life only by repeated practice. We become good musicians and writers by playing and writing... we become good mathematicians by repeating and practicing, seeing new angles and approaches for doing. Repeated practice is driven by, sometimes obsessive, interest and passion and it is this passion that differentiates those who can do mathematics well and those who claim to find it impossible.

> But for all its difficulty, doing mathematics does not require any special ability not possessed by every one of us (p. 258).

Humans have a 'Number Module' located in our brain

The Mathematical Brain (2000) by Brian Butterworth approaches the thesis that all humans have, what he terms, a Number Module, from an evolutionary, historical, neurological, and psychological perspective. Butterworth is a neuropsychologist and his interest in mathematics resulted from tests he carried out with people who had severe inabilities when it came to using numbers. Some of his patients had suffered stoke and other suffered brain injuries, while others, without injuries, appeared to be succeeding quite well but suffered a severe dysfunction with numbers. His hypothesis is that the Number Module is genetically provided and provides the basis for our ability to use numbers to interpret and operate in the world. And while some cultures are more advanced mathematically, Butterworth argues that the sophistication of number use is consistent with the technological levels achieved by that culture.

> Our mathematical brain … contains two elements: a Number Module and our ability to use the mathematical tools supplied by our culture' (p. 7)

However, people without access to the Number Module through injury or dysfunction cannot develop number skills to any level of sophistication and are grossly incapable when dealing with numbers. He proceeds to compare number deficiency i.e. dyscalculia, with dyslexia and colour blindness, as these too, result from a similar type of dysfunction in the brain. Consequently, dyscalculia is derived from the lack of a Number Module.

Butterworth sets the standard for the scientific veracity of his hypothesis - that all who function effectively with numbers have an innate Number Module - by presenting plausible evidence regarding a number of premises including the following:

1. Everybody should show evidence of ability to use nemerositie (categorising the world in terms of numbers of things)
2. Evidence must be shown among infants
3. Brain imaging should be able to locate the Number 'hot spots'
4. The Module must be encoded in our genes and must ten have been passed on by our ancestors
5. This may lead to an understanding why some people a very good while others are hopeless (pp. 9-10).

In his exploration of the history of the use of numbers he concludes that the variety of techniques and sophistication levels used across many cultures provide two conclusions:

1. Number techniques were not invented in one location and then disseminated to other cultures across the globe
2. This localised, cultural variety of number techniques provides plausible evidence that, like language, humans have an innate capacity to employ numbers and to appreciate how numbers can improve the way we live in the world (pp. 23 – 103)

Evidence related to experiments with babies, often as young as three months old, illustrate a capacity to differentiate groups of numbers, to recognise changes by adding and subtracting, and ordering numbers by size. It is these three elements that constitute the basic numerical capacities embedded in our Number Module.

In his study of the anatomy of the brain he discovers that the left side of the brain provides the capacity for mathematics, specifically in the left parietal lobe. He goes on to report on a number of case studies of individuals who had very serious difficulties with numbers while simultaneously being capable of operating very effectively where numbers were not concerned. He describes Charles who had A Levels and a university degree in Psychology. He concluded that Charles was deficient when it came to the innate Number Module and this led to his, and other case study subjects', inability with numbers. Consequently, if the Number Module is working effectively, it would seem that all humans can reach a proficiency equal in sophistication and expertise. However, we all know that this is untrue.

Some of us with a perfect genetically endowed Number Module find mathematics very difficult while others see no limits to what they can accomplish with numbers (while Butterworth did not include geometry skills, he did not directly exclude it either). As mentioned above the Number Module has to have something to work with i.e. culturally provided conceptual tools. And while this creates the limits to which all can reach, the overwhelming evidence is that most of us, in a culture that has developed, and continues to develop very sophisticated mathematics, do not reach those standards. There is something other than the Number Module and the cultural affordances required to ensure that all can become proficient mathematicians.

One of the stops put on the 'natural' development of number skills to the highest levels is maths phobia: a learned fear, specific to a situation and accompanied by physiological signs such as increased heart rate, sweating, etc." (p. 333) Students with this infliction do much worse at mathematics and avoid taking mathematics courses. Whether doing badly causes anxiety, or anxiety causes students to do badly is difficult to establish, however, the result is that students are drawn into a vicious cycle of poor performance, external discouragement, internal discouragement, anxiety, avoidance, no improvement, and so on. While the Number Module is available, phobia makes it inaccessible with the result that we avoid spending time with numbers.

On the other hand, Butterworth, like Devlin (2001), argues that differentiation is the result of training. He refers to Ericsson et al (1993) who suggest that the variation in ability to do well at any endeavour, be it music, sport, or mathematics, is 'drive' within the person. And this is manifest in 'deliberate practice, which is usually solitary' (p. 290). He goes so far as to say that 'obsession, however, does seem to be a necessary ingredient' (p. 294). As a result of developments in neuroscience and brain mapping technology, the concept of the 'plastic brain' suggests:

> '…long term, repeated practice at a skill will increase the number of neurons that the brain assigns to that skill on a more or less permanent basis … Long-lasting structural

> changes in the brain are dependent on practice. Use it or lose it!' … Most of us are born to count, but beyond that the only established limits to mathematical achievement are … zeal and very laborious work (pp. 313 - 314).

In summary, Butterworth is convinced that all of us, excepting those with brain injuries and brain dysfunctions, have a genetic predisposition to use numbers to a level of proficiency limited only by the sophistication of the cultural development one is born into and – agreeing specifically with Devlin (2001) - the amount of time and practice an individual invests.

Summarising the argument for innate mathematical ability

The basis for my argument that humans have an innate ability to be proficient with mathematics is sourced in the ideas of the Enlightenment philosopher Immanuel Kant. Kant argued that we are endowed with innate temporal and spatial intuitions. These intuitions provide the basis for the development of all scientific understanding and knowledge while simultaneously providing that basis for the science of numbers and geometry. Racing forward by nearly 250 years Peuquet, in the context of geographical research, provided evidence of the innate nature of space and time. She builds on this conclusion derived from Kant and argues that the structure of human language is also spatial and temporal in nature. Devlin's mathematical research, derived primarily from developments in linguistics, provides persuasive evidence that all of us have a 'maths gene'. He argues that language, currently employed quite effectively as an interpersonal communication medium, was initially used to carry out off-line (conceptual) thinking. Integral to off-line thinking is mathematical thinking employing spatial and temporal concepts among others, and the evidence for this is presented in normal language structures. Both Peuquet and Devlin link innate mathematical abilities to human language which all of us use relatively proficiently.

Butterworth presents evidence regarding a Number Module located in the brain and this Module is innately available to all of us. However, if the Module is damaged, or deficient in any way, the individual will have serious difficulties with numbers. Butterworth points out that while we all have the Number Module, all of us do not reach similar levels of proficiency. He suggests 'maths phobia' will create serious difficulties in developing number potential. Agreeing with Devlin, Butterworth concludes that the level of proficiency is directly proportional to the amount of time given to the practice of mathematics. And so, while we do have the innate capability, it takes drive, even obsession, to become extremely proficient.

Innate mathematical ability as the basis for a more egalitarian approach to mathematical education

I have argued that all of us, excepting those with particular intellectual deficiencies, are naturally capable of becoming proficient mathematicians. If this is the case, as adult mathematics educators we need to re-consider the manner in which we approach our profession. There is no doubt that our students come to us with a range of levels of mathematical proficiency. However, there seems to be a prevailing worldview among some teachers of adult mathematics that some people have mathematics abilities while other do not. The preceding argument calls this assumption into question. The evidence provided suggests that the range of proficiency is the result of the amount of time spent doing and practicing mathematics. We can speculate as to why some adult students do very little practice: math phobia, home environments, educational experiences, etc. We can also speculate why other students spend a lot of time practicing mathematics: a love of mathematics, home environment, educational experiences, etc. The challenge for adult mathematics educators is, firstly, to understand that all our learners are capable mathematicians irrespective of the level at which we meet them; and secondly, to provide a learning environment where students can unlearn the negative emotions, derived from various socialisation processes, that produces reduced

expectations and motivation when it comes to mathematical thinking and doing mathematics. This is particularly important when it comes to adults as they have had more time than children and younger students to galvanise the negative and confidence-sapping beliefs of their inability to do mathematics. In this way, we can provide a more egalitarian mathematics education for adult as well as younger students.

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Rebecca Wooley, UK, - Connected Assessment

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Abstract

Rebecca Woolley is a Teacher Educator at the University of Bolton, specialising in mathematics and numeracy for the post compulsory sector. She is a Professional Development Lead for the National Centre for Excellence in Teaching Mathematics and a reviewer of Professional Formation leading to QTLS for the Education and Training Foundation.

Current summative assessment models for mathematics and numeracy are dominated by high stakes, timed examinations that do not reflect how maths is used outside the classroom. This article summarises early findings from PhD research investigating alternatives to timed mathematics examinations for adults in England. In this research, a new 'Connected' model for summative assessment is proposed based on a view of mathematics and numeracy as social practices. The views of adult learners and teachers have been sought to inform the model and the model was subsequently trialled with a small group of adults over several weeks.

Introduction

The traditional way to assess an individual's mathematical 'ability' is through a timed examination or test. This practice is widely accepted and largely uncontested by successive governments, employers, students, teachers and parents. In this paper, I challenge this situation and question the validity of a timed written test because it is unrepresentative of the way maths is used in everyday life and work in the 21st century. Very often numerate problem solving involves collaboration and discussion with other people and use of all and any available information (increasingly making use of digital sources). The mathematics needed for life and work is 'simple maths in complex settings' (Hogden and Marks 2013:5) and this is not reflected in current assessment regimes which instead reward memorisation and speed of calculation (Boaler 2014). In addition, a timed test does not allow all students to demonstrate their best performance, perhaps due to mathematics anxiety or test anxiety (Ashcraft and Krause 2007). This paper summarises some early findings from a piece of PhD research that draws on the theoretical framework conceptualising mathematics/numeracy as a social practice (often referred to as the New Literacy Studies for example Street 1984, 1993). The research aims to develop, implement and evaluate more holistic ways to assess mathematics/numeracy for adult learners that acknowledge and include some aspects of collaboration and use of all available sources. The proposed 'connected' assessment model attempts to relate summative assessment[6] more clearly to adults' preferences, lives and purposes.

Positionality

When I think back to my own experiences of timed examinations, it is clear to me that my results did not reflect my best performances; I always did very well in homework and classwork but usually performed a grade or two lower than expected in timed examinations. I do not seem to have mathematics anxiety or test anxiety but I do remember experiencing

[6] The purpose of summative assessment is to recognise or accredit learning, often at the end of a period of study (Learning Connections nd:21)

wandering thoughts during examinations and finding it hard to concentrate on the task in hand. Put simply, a timed test is probably not the best way for me, personally, to show what I know and can do. If this is the case for me, a 'successful' product of the English education system, then I am sure it is also the case for a significant number of other people.

I also have a history with summative assessment from a different perspective; I have been employed by an awarding body to write exam papers for Functional Mathematics[7] and have been an external moderator for summative mathematics assignments that are marked by teachers. I have experienced first-hand how hard it is to write 'good' examination questions that are clearly written in accessible language yet are also 'functional'. This tension between realistic scenarios and accessibility is exacerbated by the medium of a written examination. For a question to be realistic the scene must be set using written explanations and information but this makes the question more difficult in terms of literacy.

I am a teacher of mathematics and educator of current and future teachers of mathematics for both young people aged 14 – 19 and adults. I see a great deal of classroom teaching in further and adult education being completely assessment driven; teaching to the test is common and the high stakes and high accountability attached to examination passes is largely responsible for this (Mansell et al 2009). I envision learning mathematics as potentially empowering for individuals, supporting them to see themselves as people who understand numbers and their power and who can then critique social and political situations if that is what they choose to do or to serve any other life purposes they may have. Learning maths should be inclusive and accessible to most people. I view mathematics and numeracy as social practices and this is a value laden stance that is denied in my practice as a teacher educator as this position is not generally reflected in the classroom cultures I observe. Teachers of mathematics in further and adult education are under great pressure to get students through examinations and I observe their difficulties with covering so may 'topics' in a short period of time; giving students a potentially poor experience of learning mathematics to add to their previous poor experience in many cases.

Thus, as a teacher educator, I feel conflicted about the need to prepare trainee teachers for the realities of the further and adult education system and how to be 'successful' within it. This means sometimes teaching to the test and getting people through examinations to obtain funding and achieve targets and benchmarks for attainment. However, I also have a responsibility to develop critical thinking in trainee teachers – I need to 'show them the box' that we all work within and then help them to work creatively and critically either within the box or to expand the box. We all need to acknowledge that we are not stuck in a flawed education system, we *are* that flawed system[8]. Thus, in terms of timed tests, the pressure to teach to the test affects my professional role via the experiences of trainee teachers. Teaching placements sometimes expect trainees to be able to cover lots of material for mathematics classes using traditional teaching methods that are not always consistent with the well-researched, more active and collaborative, approaches being modelled in their university classes. As part of their Initial Teacher Training, trainees are encouraged to employ teaching and learning approaches that promote collaborative problem solving and conceptualise mathematics as a network of ideas (Swan 2006, Swain and Swan 2007). However, these

[7] Functional Mathematics is a qualification in England for learners aged 14 and above involving 'practical' rather than 'academic' maths.

[8] From the Tom Tom Sat Nav advertisement 'you are not stuck in traffic; you <u>are</u> traffic'. (Good reads nd)

approaches are not reflected in current summative assessment regimes such as Functional Mathematics or GCSE[9]mathematics which are both timed written examinations.

My conflict relates clearly to two types of professionalism; 'responsive professionalism' based on issues of social justice and a desire to provide what is relevant for learners' lives and purposes and the 'new professionalism' of achieving targets and a more systematic approach to teaching and learning (Ivanic et al 2006: 36). Each year I see trainee teachers go through this same conflict themselves; trying to become a responsive professional within an educational system that requires them to demonstrate the attributes of new professionalism.

Whitehead (1989) suggests that experiencing negation of our values in our practice is a good place to start an action research project and proposes a simple action research cycle;

> I experience problems when my educational values are negated in my practice.
>
> I imagine ways of overcoming my problems.
>
> I act on a chosen solution.
>
> I evaluate the outcomes of my actions.
>
> I modify my problems, ideas and actions in the light of my evaluations... (and the cycle continues). (Whitehead 1989 p43)

As the preceding section makes clear, I am not a separate, objective observer in this research; my frame of reference is an important factor and will inevitably shape every aspect of it including the general area of research; the research questions; methodological approaches and, of course, how data is interpreted and results evaluated.

Background and definitions

There is no consensus on the definitions and differences between the terms 'numeracy' and 'mathematics' and, for convenience, I tend to use them interchangeably here. Although this is an important debate in itself[10], this research is more concerned with assessment that supports individuals to develop their numerical skills and practices rather than deciding whether to call these skills and practices 'mathematics' or 'numeracy'. I acknowledge that there is a perceived difference between the two terms and that 'mathematics' has more status than 'numeracy' and therefore attempt to avoid using a 'value-based distinction' (Johansson 2014:71).

The following definition of numeracy resonates with the general approach taken in the research;

> Numeracy is considered as an individual and dynamic attribute…numeracy is regarded as an integrated web of interacting elements (communication (including literacy), personal and social development, attitudes, beliefs, values, life experience and motivation) with mathematics at its core.(Safford-Ramus et al 2016: 21)

[9]The General Certificate in Secondary Education (GCSE) is a qualification available in various subjects, usually taken at age 16, that was originally a school leaving qualification in England.

[10] For a detailed analysis of the two terms see O'Donoghue 2002

In 2013 the Gordon Commission published its report setting out a new vision for Assessment for the United States of America (USA) where students experience one of the most intensive testing regimes in education in the world (The Gordon Commission 2013). This was a comprehensive and detailed report commissioned by the Education Testing Service in the USA on the assessment needed for 21st century education. One of the themes emerging relates to the need to move away from 'one-off' timed tests towards assessment that takes place on more than one occasion and in more than one way;

The causes and manifestations of intellectual behavior are pluralistic, requiring that the assessment of intellectual behavior also be pluralistic (i.e., conducted from multiple perspectives, by multiple means, at distributed times, and focused on several different indicators of the characteristics of the subject(s) of the assessment). (The Gordon Commission 2013: 19)

Another theme related to the use of technology to assist in capturing assessment information and there is also an acknowledgement in the report that intelligence is a product of social interaction and that collaboration and acknowledgement of social context need to be included as part of assessment regimes (ibid). This report resonated with my own views and was the catalyst that started me thinking about alternatives to a timed test for mathematics assessment, particularly for adult learners.

I have focussed on adult learners in the fieldwork for this study for two reasons; firstly, adults have more life experience and are more likely to have areas in their lives outside the classroom that they can draw on for opportunities to demonstrate their knowledge and skills in numeracy; what Moll refers to as 'Funds of Knowledge'(Moll et al 1992); secondly, adults are outside the compulsory education system and are more likely to have the time and inclination to take part in this research. Young people aged 16 – 19 and their teachers are under pressure to complete qualifications in maths and I thought it would be much harder to recruit teachers and students for this age group. However, my view is that alternatives to timed tests are likely to be just as valid for young people in Further Education and even perhaps those still in school. Although it was realistic in this research to focus on the local context of adult education in the England, the topics of assessment, accountability and high stakes timed examinations resonate with debates around the world relating to all stages of education and in subjects other than mathematics (Boud 2002, Mansell 2007, Maughan and Cooper 2010, Mazur 2013).

Recent government policy in England has focussed mainly on the education of young people and this has resulted in a narrowing of curricula for adults and a convergence with curricula and assessment with those on offer for young people aged 14 – 19. However, it is not always appropriate for adults to take an examination designed for 16-year-old school leavers. Many adults don't want or need to gain a qualification (Torrance and Coultas 2004, Ward and Edwards 2002) but the current funding system strongly encourages them to take accredited courses. I see this as a negative direction that has reduced choice and increased anxiety for some adults who want to improve their use of mathematics. Largely speaking the requirements for summative assessment have *become* the curriculum for many adult classes (similarly for young people) to the detriment of a wider, broader, inclusive approach to teaching mathematics that could be based more closely on the needs and interests of adults themselves (Coffield 2014).

Early findings

This research takes a qualitative approach to data collection within an action research methodology. In the early stages of fieldwork, thirteen adult learners of mathematics were

interviewed and 51 teachers of mathematics/numeracy completed a survey about their views and experiences with summative assessment.

Although analysis is not yet complete, the following themes have emerged for adult learners;

- *Time pressure during examinations leading to anxiety and reduced performance for some adults*
- *Difficulty thinking of any alternatives to a timed test for many adults*
- *Strong interest from some adults in alternatives to a timed test*
- *A wide variety of motivations for attending classes*
- *A wide variety of barriers to progress in adult numeracy*

The survey of teachers effectively highlighted potential strengths and weaknesses of methods such as coursework, portfolio work and project work. The survey also suggested that;

- *About 2/3 of the teachers who responded would prefer to be able to offer an alternative to a timed test to their students.*
- *Over 70% of respondents do not think a timed exam gives everyone the same chance to show what maths they can do.*
- *Only 31% consider a timed test to be the best way to summatively assess mathematics*
- *However; 53% think timed tests are necessary for mathematics*

The last bullet point suggests to me the idea that a timed test is seen by some teachers as a 'necessary evil'. Taken together, these early findings encouraged and informed the development of the proposed assessment model outlined below.

Connected assessment

There is merit in conceptualising mathematics as a web of connected ideas where learning in one area supports and enhances understanding of other, related, areas (Askew et al 2003). Summative assessment should ideally include tasks that both support and assess the ability of learners to make such connections. What I suggest is a summative assessment model that involves learners using any or all the numeracy they have learned through a choice of untimed assessment tasks that could take place over a series of sessions at the end of a period of study.

I propose a model of *connected* assessment that could be offered to adult numeracy learners as an alternative to a timed test. This model reflects the need to support learners to make connections between different facets of numeracy when solving problems, to make connections between numeracy and their own lives and purposes, to make connections between numeracy, literacy and the use of technology and to make connections with other learners and teachers through collaboration and discussion. I also anticipate that such a model will promote connection between teachers' values and their practice.

The key features of connected assessment include;

- *a choice of assessment tasks*
- *learners developing their own assessment tasks to reflect their preferences and interests*

- *an extended period for assessment to take place over several sessions/weeks*
- *no arbitrary time limit for individual assessment tasks*
- *'open book' assessment reducing the need for memorisation*
- *collaboration between learners on some tasks*
- *gathering a wide range of 'evidence' not just written work*
- *use of technology to support assessment.*

A cohort of adult learners was recruited to try out this model of summative assessment. This took the form of a five week pilot 'course' called 'Making Connections' in which the model above was developed, trialled and evaluated by and with the adults involved. Some of the assessment tasks used were projects (topics suggested by learners themselves), creation of puzzles such as Tarsia[11], investigations using prompt cards provided by the teacher, creation of posters and handouts and completion of online quizzes and challenges. The data collected over the duration of the course consisted of;

- *Written evidence generated by learners while completing tasks*
- *Posters created by learners*
- *Digital evidence of tasks completed online*
- *Written comments in learners' journals*
- *Completed course evaluation forms*
- *Fieldnotes written by researcher*

Some typical examples of completed tasks are shown in Figure 1;

[11]Tarsia is free software for making jigsaw puzzles. Available from http://www.mmlsoft.com/index.php/products/tarsia

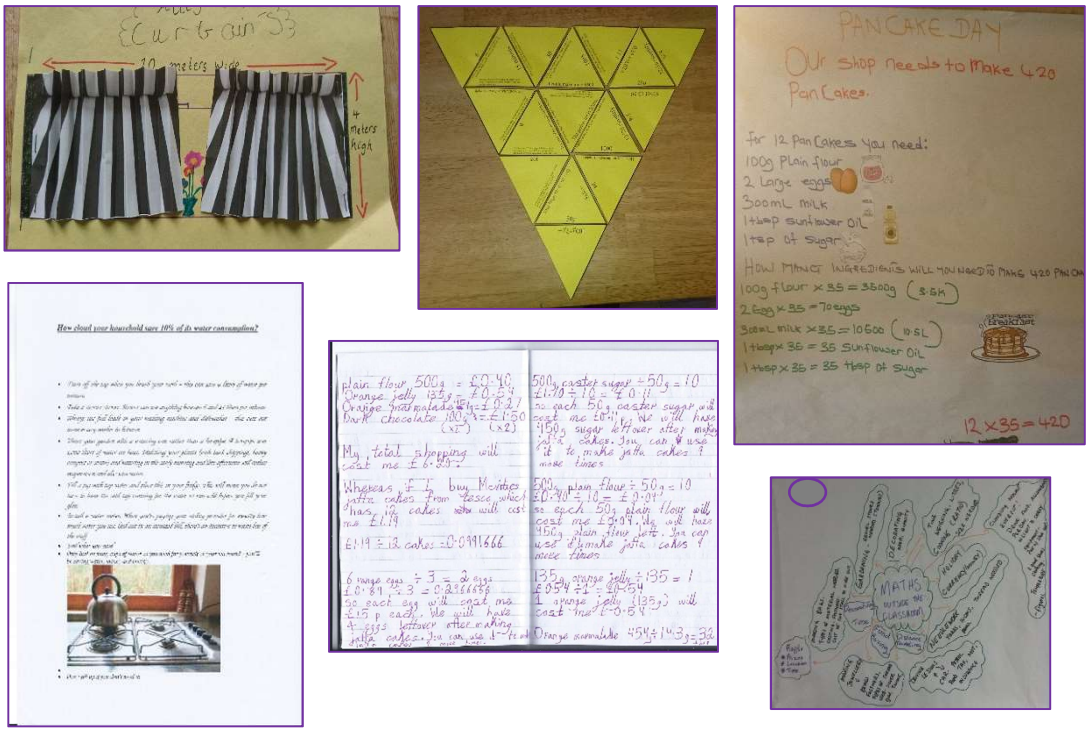

Figure 1. Examples of completed tasks from Connected Assessment model

The adults involved in the pilot were asked to evaluate their experiences on the course;

Adults enjoyed the sessions and they were able to show their maths 'skills'

Adults learned new maths during the sessions (not just an assessment opportunity)

Some adults enjoyed being able to work together on maths tasks

Some adults commented that they liked not having time pressure to complete the tasks

One adult commented;

"*Really enjoyed this hands on learning activity – allowing me to understand where and why certain maths are being used."*

A second 'Making Connections' course is currently running with a different group of adults and the results will feed into the ongoing analysis and final write up of the research.

Conclusion

Debates around assessment usually involve policy makers, academics and sometimes teachers but rarely learners themselves. This research has begun to contribute to knowledge by filling this gap and revealing learners' own ideas, attitudes and opinions concerning summative assessment. This research has started to show what assessment tools learners can develop and use themselves that they feel demonstrate and reflect their personal learning progress. Further analysis of the data collected will help to show whether it is possible to include collaborative aspects in summative assessment and whether learners are able to draw on their lives outside the classroom for meaningful contexts. It may also be possible to find out whether being involved in decisions about assessment will motivate learners and help them to engage more fully with mathematics and numeracy in a positive way and to succeed in solving the everyday numerical problems they come across in their lives.

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Jeff Evans, UK - Statistics in public policy debates: Present crises and adult mathematics education

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Abstract

Statistics is one of the important branches of mathematics taught in schools, colleges and universities. It is also an important tool in public policy discussions. This paper focuses on the use of statistics in the latter context, rather than its use in adult mathematics education research. I review the key characteristics of the statistical approach to constructing public knowledge, and give a very brief history of key points in its development. I discuss how what I call the "overt crisis of statistics", the apparent disenchantment of large sections of the public with the "expert" statistical methods, outputs and pronouncements, leads to dilemmas both for citizens and for democratic governments. Recently "Big Data" and data analytics seem to many to offer new solutions to problems resulting from the essential lack of certainty surrounding efforts to understand society, and from the need to make quick decisions in a rapidly changing world. These approaches have potential, but also limitations. This leads me to consider a second, "covert" crisis of statistics, resulting from a struggle between proponents of freely available public information and public argument, and those aiming to profit from the appropriation and sequestering of information for private ends. I finish by considering what can be done by ourselves, as citizens, at adult mathematics teachers, and as researchers.

Key words: statistics; public policy; big data; data analytics; technology corporations

Introduction

During most of our lifetimes, it has been accepted that, in most countries with a developed civil society, citizens and policy makers could rely on statistics – those produced by government agencies, or those from well-designed surveys from other agencies – to set a baseline for discussion and decision making. Thus we have quoted official statistics, results from the European Labour Force Survey, or figures compiled by the World Bank, in discussions of unemployment. Or we have used national household surveys, to estimate the numbers of victims of crime, or of those who rate their own health as poor.

Now, given events of the last two years, and subsequent public reactions, these previously accepted resources are facing new challenges. For example, a recent article by Will Davies in The Guardian, "Have statistics lost their power in public policy discussions?" has raised challenging questions regarding the role of statistics in public discussions:

> Rather than diffusing controversy and polarisation, it seems as if statistics are actually stoking them. Antipathy to statistics has become one of the hallmarks of the populist

> right, with statisticians and economists chief among the various "experts" ostensibly rejected by voters in 2016.
>
> (DAVIES, 2017)

Davies, writing just after the Brexit vote and the Trump victory, focuses on the UK and the US, but the issues apply more widely. Here, I first briefly consider key points of the statistical approach, and its historical development. I then explore the idea of a "crisis" in statistics, and argue that it is actually two different crises, based in different social groups. This leads me to consider the recently voguish notion of "Big Data", and the sorts of data analytics used with it. I finish with some suggestions about how citizens can withstand the most challenging features of the society that the large technology / media companies have established, and consider some ways that these ideas can be highlighted in the adult mathematics classroom.

The statistical approach, and a brief historical development

In order to have a clear discussion, we need to understand that "statistics" can refer to three different aspects, though they are related:

(i) statistical data, and / or

(ii) statistical techniques of data analysis (e.g. averages, measures of spread and correlation, statistical models), and / or

(iii) the particular discipline, which of course includes "experts" in its ideas and procedures.

Key examples of the ideas and procedures of statistics include:

- The importance of investigating a representative sample from a specified population about which one wishes to draw conclusions, and familiarity with the methods of representative sampling and with the drawing of inferences from samples
- The importance of comparable and stable measurements of all the members of the sample, and knowledge of ways to assure the quality of such measures
- The *difference* between correlation and causation, and ways to design studies so as to be able to construct more dependable explanations for what is observed

From the late 17th century, the idea gained ground that statistics should be used to understand an entire population (not only potential soldiers, or tax-payers). Originally, this was not necessarily to be done using numbers, as in geographical descriptions of various German states, pre-unification. In England William Petty & John Graunt introduced the estimation of population size via counting of deaths, rather than via a census (costly).

In 18th and 19th century, in post- Revolutionary France, statistics began to be produced by trained cadres in a centralised statistical office. Across Europe and beyond, in data analysis, the normal distribution was found to be surprisingly powerful for supporting the growth of scientific knowledge, in quantifying and understanding apparently unrelated phenomena:

(i) errors of measurement (Gauss),

(ii) approximations to probabilities of gambling outcomes (de Moivre), and

(iii) the distribution of physical (and mental) characteristics (Quetelet, Galton).

This distribution was argued to underlie variation in a large number of natural phenomena, and so became an assumption of much data analysis well into the 20th century.

In the 19th and 20th centuries, around the world, specific indicators, clearly defined and systematically produced, were constructed for simplifying description of diverse and complex populations. Examples include: population size and vital statistics (births, marriages, deaths); classifications of disease, national income statistics (e.g. GDP). Surveys and opinion polls of representative samples of the population, and of subgroups, using variations of simple random sampling (itself an advance on haphazard sampling) were introduced. Experimental designs (nowadays called Randomised Controlled Trials - RCTs) were introduced for agricultural trials and extended into the study of medicine and psychology; quasi-experimental designs were introduced from the 1960s, to increase their applicability to contexts where experimental designs were ethically or practically impossible. In addition, in line with a widespread general concern with comparative methods in the social sciences and history, there were efforts in statistical data production to enhance comparability across time, and across nations and subgroups. Overall, statistical data have allowed democratic countries, in particular, to sharpen their political agendas, and to design progressive policies, when the will and the resources to do so were available.

The "overt crisis" of statistics and resulting dilemmas for citizens and democratic governments

Some dimensions of the current crisis include an increasing lack of trust in statistical data, and a consequent decline in their authority. For various reasons this has become particularly evident in the UK and the USA over recent years. For example, Davies (2017) cites survey results in the US which indicated that 68% of Trump supporters distrusted government economic statistics; and in the UK, that 55% distrusted data on "the number of immigrants living here"; see also Pew Research Center (2018, 14 May). This leads people to brand any evidence that seems contradictory to their preferred worldview as "fake news", or as something fabricated by "experts". Thus there is evidence of a lack of generally accepted baselines for discussing competing claims about society; and consequently a resort to "speaking one's own truth", and drawing on "intuition" and emotion as alternative bases of knowledge.

We can consider further some important aspects of these contemporary reactions to statistics. A key dilemma arises from the need to govern the population as a whole vs. (increasing) pressures to respond to feelings of particular citizens in a particular place and time. This can lead for example to a mismatch between what politicians say about the general state of the labour market, and local experience of the labour market, by individuals or by neighbourhood groups. Recently, such problems have been aggravated by a difficulty of satisfactorily portraying the state of the nation, with the use of summary statistics – because of the fragmentation of available identities and the foregrounding of differences within society. Even if one tries to be sensitive to social differences, by avoiding an overly crude use of averages, the available measures of spread, such as the standard deviation or the range, cannot capture the full quality of the differences currently emerging, say in sexual identity or political allegiance.

Thus there have been strains on existing classifications and definitions, due to changes in cultural politics – more fluid identities, attitudes and beliefs (emotions), and the reshaping of global economy and society. This has made various definitions more complex e.g. of unemployment, or GDP, or even gender. There has been an evident need not only to classify,

but also to measure, say *intensity* of employment, or *commitment* to actually exercising one's "voting preference" on election day.

There have also been challenges in ensuring comparability across time, as the governance of states has changed (or fragmented), and especially comparability across nations, for example as the number and variety of countries participating in PISA has changed. For example, it is one thing to rank 10th in a set of 23 countries in the PIAAC survey; it means something different to rank 10th in a group of 62 countries in PISA.

And now … Here come Big Data and Data Analytics

What is Big Data? What are Data Analytics?

Big Data and data analytics are seen as possible solutions to pressing problems, such as limited research capability or the difficulty in producing the results of complex analyses in a timely fashion. Big Data can be characterised as the availability of exceedingly large amounts of data. However, these are accumulated by default, as a by-product of other processes, usually without attention to research design (e.g. sampling), but requiring the extensive use of electronic technology for capture, analysis, and presentation.

Examples of *Big Data* include the use of speed cameras or other video cameras, for behaviour monitoring, and for storage of alleged proof of mis-behaviour (allowing efficient legal prosecution). The use of loyalty cards allows monitoring of purchasing behaviour, plus correlation of such data with a number of demographic variables - "freely" produced by the card-holders themselves – so as to facilitate the targeting of marketing communications – with an option of experimenting with differential "special offers" (or experimental treatments). A further example is the harvesting of electronic texts – from individual acts of communication, which in an earlier time might have been assumed to be private, e.g. information searches, social media posts (and possibly emails and internet phone calls?). These texts can now be subjected to *data analytics*; this collection of techniques includes *data mining*, where many of the data analysis decisions are made by "artificial intelligence" – algorithms run by machines, rather than by human analysts. These are supplemented by *data linkage* (linking of data on a person from several databases), and *sentiment analysis*, used to striking effect by certain companies in the US election and the UK referendum (Cadwalladr, 2017).

Other examples are perhaps more positive: "Citizen science" (e.g. astronomical observation by many citizens) and "Citizen maths" (performing time-consuming calculations / simulations by many citizens). In contrast, Mass Observation, begun in 1937 and continuing in various forms to the present, was not electronically supported, and relied on named volunteers to do the interviews and the observations (Hubble, 2010).

Issues with Big Data and Data Analytics

In methodological terms, the data involved is "big" indeed, i.e. not limited in the ways relevant to the pre-electronic period, but there are several serious limitations. First, the approach involves "haphazard" harvesting of large amounts of data – indeed impressive amounts. However, a huge sample can still be biased (e.g. Marsh, 1979) and, if there is no known sampling design, generalisation to any recognisable population will be difficult in principle.

In many cases too, the data comes without settled categories, since people can take on self-selected identities. This means that data from one database may be hard to "link" with data from another, and it thus may be difficult to analyse even degrees of correlation. Further, even if you have access to a huge data set, and that data shows a very *high correlation* between A and B, that still does not prove that A causes B!

Other more political issues arise for the responsible citizen – to do with freedom of the consumer (data provider), privacy and ownership of data. The "freely chosen" declarations of "informed consent" (EULAs) that individuals are asked to sign in order to use a range of applications provided by the technology companies – and that many sign in an inappropriately off-hand way – may be agreed to long before some particular data is extracted from the "user", and the permissions thereby granted are considered currently to be for forever. Data linkage raises not only technical issues (about how to do it accurately), but also issues of privacy: Would you want data from your medical records to be linked to your income tax return information, or to your Facebook page? If this sounds far-fetched, see the striking novel, *The Circle* (Eggers, 2014), which describes a fictional company, with a resemblance to a combination of Facebook and Google, which proclaims a commitment to "total transparency" … with instructive consequences for the idea of privacy!

Much data nowadays is *appropriated* by private companies, for their own uses, in much the same way that common lands in English villages were appropriated by private landowners since the 17th century, during periods of "Enclosures"; see e.g. Polyani, (2001). These private companies have few or no obligations towards openness or transparency – though much rhetoric is often heard. Thus the user of the services, who is also of course the provider of the data, may never know what the data says about them – much less how it might be interpreted later by an unknown, and perhaps suspicious, user.

The covert crisis: from a "logic of statistics" to a "logic of data analytics"

Thus, we have aspects of a second, "covert", crisis of statistics, based on opposing ideas of knowledge. On the one side, we have the "experts" of the Office of National Statistics – bound by research ethics, and monitored by UK Statistics Authority – and on the other, the experts of Google, Facebook, and other less known policy actors, such as Cambridge Analytica (Cadwalladr, 2017-18, e.g. 2017). These latter appropriate data from unsuspecting individuals, link it with information available from public or privatised databases, analyse it (sometimes) and sell it on to a range of customers, to be used for purposes, including "tailored messaging" – by marketers, politicians, "opinion formers". Some of these interests are oriented to maximising the appropriation of other people's data, so as to maximise advertising revenues – the 'media corporations'. Others may be oriented to undermining rational, open, public discussion of values and policy – the "ideologues".

There are currently (July 2018) official investigations ongoing into the way these methods were used by the Brexit campaigns in the UK, by the Trump campaign in the USA, and by the media corporations themselves. This is clearly a continuing process, with many landmarks. An important one is the establishment of the General Data Protection Regulations by the EU in May 2018; see https://www.eugdpr.org/ .

Summary

The "overt crisis" of statistics appears to result from the public's disenchantment with the provision of statistics to be used as a basis for public discussions of policy. I have also aimed to describe a "covert crisis" lying behind the overt one, where certain interest groups are stoking the overt crisis for their own ends. For without statistics, and social research more generally, made available publicly and discussed freely (without interference or manipulation from unknown human beings, and non-human "bots"), we cannot construct unambiguous, objective, potentially consensus-forming claims about society – nor can we provide a corrective to faulty claims. In such a situation, there will be few mechanisms to prevent people from instinctive reactions and emotional prejudices.

Many have pinned their hopes on certain Open Data initiatives offered by state statistics and certain agencies. However, these public initiatives seem unlikely to be mirrored by the sharing of the results of data analytics by private corporations. In Davies's (2017) judgment, data analytics is "suited to detecting trends, sensing the mood, spotting things bubbling up" – but not so much for the type of social explanation that many feel is necessary in an advanced democracy. Further, the numbers produced by data analytics are "generated behind our backs and beyond our knowledge". And the results are appropriated, owned and sold on by private concerns – without the original providers' knowledge!

Thus, the battle is not between "an elite-led politics of facts versus a populist politics of feeling" (Davies, 2017). Rather, it is between those committed to public knowledge and argument versus those who profit from the privatisation of information and "the ongoing disintegration" of public knowledge and argument.

Conclusion: What might be done?

Here, we can focus on what might be done (a) by ourselves as citizens; (b) by teachers of adults' mathematics / numeracy; and (c) by researchers.

As citizens, it is important to rethink our relationship with IT and media companies, especially the "FAANGs" (Facebook, Amazon, Apple, Netflix, Google – and many users of Windows may not want to exclude Microsoft!).

a1. "There is no such thing as a free lunch.". So we need to read the EULA (End User Licensing Agreement) before we click to "Accept" the "free access" to software offered by many companies on the web. You are signing a contract, and you are giving something away in return: it is worth thinking about what that something is!

a2. Maybe there are still "free searches"? How many details of your life are on the file-server, of Google? Use gmail? Always "google" when you are searching? (There are alternatives: the search engine DuckDuckGo calls itself "The search engine that doesn't track you.")

a3. Maybe there is still "free" news? Of course, every news source must be selective. But the more they know about you and your "likes", the more selective they can be, so as not to disturb your bubble, and so as to keep you "clicking" (and providing them with income). The alternative is to get news from professional journalists, who take a somewhat broader view and will often have a long-term commitment to, and knowledge of, an issue - and they may occasionally come up with something surprising, like the Panama papers or the Paradise papers. Many good newspapers support the International Consortium of Investigative Journalists. But good journalism requires funds. In most countries, you can support a newspaper, by subscribing online, taking out a paid membership - or even by buying a copy, once in a while.

a4. "Think globally; act locally." Many things can still be bought at a local store, which employs local people, perhaps even some that you know. You can keep your Amazon account for the truly hard-to-find commodities.

a5. Many countries have "fact-checkers", e.g. agencies that check the more important claims made in the political and social sphere: e.g. Full Fact in UK (https://fullfact.org/). They are often charities that depend on financial support from members of the public.

As teachers of adults' mathematics / numeracy, we can encourage our students to consider their positions with respect to the trends described above, with the help of available statistics, and using surveys that can be done in the classroom.

b1. Many countries have available on the web a wealth of statistics produced by government or other agencies. For example, one could consider the data available on unemployment, and ask what it tells us about the current state of work, and "precarity" of employment (e.g. Evans, Ruane and Southall, 2019; Frankenstein, 2014). Or we could ask what is the level of migration into and from our country, and whether we could estimate the numbers of refugees, "economic migrants", and so on (e.g. Tyler, 2017). These are challenging questions, and we can expect one result to be that the students find that an apparently "objective" number comes with a lot of assumptions in these areas of discussion and indeed controversy!

b2. Examples can be given of cases in the era before "big data" where a very large sample could be very biased indeed (e.g. Marsh, 1979).

b3. Many examples of the difference between correlation and causation can be found in a good newspaper; a notorious example is the correlation over time between the number of storks in Germany and the number of human births – seeming to provide corroboration for the view that storks bring babies; see for example,

https://www.researchgate.net/publication/227763292_Storks_Deliver_Babies_p_0008

b4. There is scope for a group of students researching themselves, as to the level of their use of Facebook, Twitter, and Amazon – and their reasons for their use, as well as their beliefs about how their date is used.

As researchers, we might be interested in several types of research.

c1. Researchers might do the type of survey described above, but with a more wide-ranging questionnaire, and a larger and more representative sample.

c2. Further research and analysis is needed to investigate which feelings are most crucial in the "new politics of feeling" mentioned above. This need is most pressing for the groups characterised as "those left behind" in various traditionally democratic societies. on these issues. The most important would seem to be:

- Anxiety / Fear vs. Hope / Love
- Trust vs. Distrust
- Anger / Discrimination vs. Solidarity / Inclusion

Some forms of these feelings will be recognisable from the classroom, by mathematics educators and researchers. They are of course inter-related. For example, anger is often born of fear and anxiety and can be directed against recognisable "Others" (Mishra, 2017; Fraser, 2017).

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Aristoula Kontogianni, Konstantinos Tatsis, Greece - Investigating adults' statistical literacy in a Second Chance School through the teaching of graphs

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Abstract

Based on the significance of graph comprehension within statistical literacy, we present the findings of a study that took place in a Second Chance School in Greece. Our aim was to assess the progress of adults' comprehension of graphs. In order to offer to our adult students tasks with realistic context we used graphs, published in the media, some of them being potentially misleading. Our results showed that some of the adults have managed to move from the mere reading of the graphs to their interpretation. However, their critical sense was not so well developed since they often based their interpretations on their dispositions and not on their statistical knowledge.

Key words: statistical literacy; adult students; graph comprehension; Second Chance School

Introduction

The skills that adults need in order to solve everyday problems which contain mathematical and statistical elements has been a topic for research over the last decades (Tout & Gal, 2015). This research area, which might be called Adult's Mathematics Education or Adults Learning Mathematics, can be placed in the borderland between mathematics education and adult education (Wedege, 2010). According to Wedege, its key concept is numeracy and the research field is related to adults, mathematics and lifelong education in a societal context. This means that there are different kinds of adult education settings (Evans, Wedege & Yasukawa, 2013), like adults' basic education (ABE) either in formal or informal contexts.

Acknowledging the importance of a modern citizen's ability to interpret visual data coming from the media, especially data representing statistical information, we organised a study with adults studying in a Greek "Second Chance School" (SCS), described more fully below. In particular, our aim was to monitor the progress of adults' statistical knowledge during the teaching of basic statistical concepts.

Although there are studies about students' graph comprehension in secondary education (e.g. Aoyama, 2007) or tertiary education (e.g. Monteiro & Ainley, 2007), there are not studies about students in adults' basic education programs. In this paper we focus on the adult students' graph comprehension and how this develops when they have to interpret media graphs and misleading graphs. Our research questions were formulated as follows

What is the students' level of graph comprehension?

Do students demonstrate a critical sense towards the statistical information presented by the graphs?

Numeracy and Statistical literacy

Although it is not easy to discriminate between the different notions of numeracy, it is a fact that numeracy can serve as a connection between mathematics and adult life (Evans, Wedege & Yasukawa, 2013). Similarly, Dalby (2017) concludes that numeracy does not refer to a simplified type of mathematics but concerns the way a person uses it. Developing the analysis, four dimensions of "numerate behavior" were identified for the PIAAC survey:

(a) context (everyday life, work, societal, further learning), (b) response (identify/locate/access(information); act on/use; interpret/evaluate, (c) mathematical content (quantity and number, dimension and shape, pattern and relationships, data and chance) and (d) representations of mathematical/statistical information for example text, tables and graphs (Evans, 2014, pp. 39-40, citing OECD, 2012).

Among these dimensions there are direct references to statistical knowledge and as a consequence to statistical literacy. There is no consensus at this time about the notion of statistical literacy in the relevant literature (Budgett, 2017). Gal (2002), referring to adults, describes statistical literacy as the "ability to interpret and critically evaluate" (p. 2) statistical information, as well as their ability to "discuss or communicate their reactions" (p. 3) to statistical information. He takes into account the impact that statistical literacy has for the effective citizenship since citizens are overwhelmed with statistics in modern societies. Gal divides statistical literacy into two components, a knowledge component and a dispositional one. For the knowledge component general literacy, mathematical skills and the ability to interpret graphs and tables are required. The dispositional component refers to the critical stance that adults should have towards statistical information presented to them, as coupled with certain attitudes and beliefs that would help adults to support their actions.

According to Watson (2006), graph construction and interpretation constitute a crucial part of statistical literacy. For the purpose of our research we focused on the definition of statistical literacy that concerns the "consumers" (Gal, 2002, p. 3), in other words, on statistics and the way that statistical literacy is expressed through the interpretation of graphs by users or readers. We focused on graphs that originate from media sources and may be misleading.

Graph comprehension literature review

The research about adults' graph comprehension is focused on in-service/pre-service teachers (Gonzalez, Espinel & Ainley, 2011) or students of vocational education (e.g. Bakker & Akkerman, 2014). Moreover, in national surveys aimed to adults, like PIAAC (Programme for

the International Assessment of Adult Competencies, OECD, 2012) or ALL (Adult Literacy and Lifeskills Survey) there are tasks related with graph comprehension. However, we did not locate many studies about adults' graph comprehension in settings similar to a Second Chance School. Specifically, for ABE programs there is the study of Conti and Carvalho (2014), who investigated the teaching and learning of statistics in adult education mathematics classes. The researchers designed and implemented a project that was carried out with grade 7 students, aged from 16 to 43, in a public state elementary school in Brazil. Throughout this project some of the students constructed statistical graphs while most of them managed to establish an initial level of statistical literacy.

Monteiro and Ainley (2006; 2007) used media graphs as an assessment tool for the graph comprehension of pre-service teachers. These researchers concluded that for the interpretation of graphs not only a certain level of statistical knowledge was needed but also a kind of 'critical sense', especially given that their participants relied on their personal opinions and dispositions for their interpretations. Respectively, Queiroz et al. (2015) investigated the way that students with different academic backgrounds interpreted media graphs. These researchers found that the students mostly expressed their feelings and their opinions rather than making an objective analysis based on statistical knowledge.

Methodology

One of the main public institutions related to Adults' Basic Education programs in Greece is the Second Chance School (SCS). Its aim is to combat the social exclusion of adults who have not completed the compulsory secondary education and do not have the appropriate qualifications and skills to adapt to modern vocational requirements. SCSs are considered to be innovative, since they operate without pre-specified curricula, implement new teaching and evaluation methods and offer counselling to students (Efstathiou, 2009). For the Greek educational system, being conservative and rather rigid, the instruction and training in SCSs constitute an exceptional novelty (Koutrouba et. al., 2011). The duration of the studies in a Second Chance School is two academic years. The weekly programme consists of 25 teaching hours and the courses take place during the evening of all weekdays. Mathematics is taught for three hours per week in both cycles (grades) A and B. The main goal of mathematics teaching is the development of the students' numeracy and consequently their statistical literacy.

Our study took place in May 2015 and we used the methodology of a teaching research experiment (Steffe & Thompson, 2000). In particular, we designed an educational approach in which the basic statistical notions served as the basis for the development of adult students' statistical literacy. Initially, we designed a 12-hour sequence of lessons in accordance with the general guidelines for the teaching of statistics in SCSs (Lemonidis & Maravelakis, 2013). Our lessons contained: (1) data collection, data interpretation and organization, (2) reading and interpretation of basic data representations, (3) data description with statistical terminology, and (4) evaluation of arguments based on misleading graphs or incorrect statistical information. In this paper, we focus only to the lesson that concerned the graph comprehension.

For the purpose of this lesson, with a duration of two teaching hours (40 minutes each), we designed a series of eight tasks that consisted of graphs and their interpretation. These tasks were either adapted from relevant studies (PIAAC, PISA) or constructed from scratch for the purposes of the study. We based the task design on the following principles: (a) to include graphs within context, that show data from real-world situations and (b) to include misleading graphs since they are considered to be excellent cases for the students' motivation and assessment (Watson, 1997). Apart from their content, the tasks were formulated according to the learning goals and to a prediction of how the students' graph comprehension will evolve in the given contexts.

Although, according to the teaching experiment methodology an observer might be of great help to the teacher-researcher (Steffe & Thompson, 2000), in our case this was impossible due to the students' opposition. As a consequence, the mathematics teacher (first author) played the role of the researcher and tried as much as possible to share all the data with her fellow researcher. She was acquainted with the students' ways and means of operating since she had taught them for the previous six months. The teacher introduced new material with brief lectures at the beginning of the lesson, during which she posed questions for investigation and gave tasks to the students. For most of the course's duration, the students worked together with the teacher in order to answer specific questions included in the tasks. The students discussed statistical concepts, made conjectures, discussed the validity of specific arguments and applied the newly acquired knowledge to the next task. The teacher provided guidance, but she tried to interfere as little as possible and to promote with constant questions the students' active participation. The teaching episodes were audio taped and transcribed. Since the teaching was implemented to four different classes we had the opportunity to continuously discuss on the students' learning processes and on the role of the teaching materials we had constructed. This gave us the opportunity to make changes when it was needed and to revise some of the tasks or the related questions.

For the content analysis of the transcribed episodes, we used the levels of graph comprehension (Friel et. al., 2001; Shaughnessy, 2007) and the notion of critical sense (Monteiro & Ainley, 2007) as subcategories of graph sense. According to Friel et al. (2001) and to the additions made by Shaughnessy (2007), graph comprehension evolves through three ascending levels:

Reading the data: extraction of elementary information, recognizing components of graphs and detecting arithmetical information on graphs.

Reading between the data: understanding relationships among tables, graphs, and data, making sense of a graph, but avoiding personalization and maintaining an objective stance while talking about the graphs. Also, evaluating the graph for its constructive characteristics and detecting if it is misleading.

Reading beyond the data: making inferences from the graph in order to interpret the data, e.g., to compare and contrast data sets, to make a prediction about an unknown case, to generalize to a population, or to identify a trend. Recognizing appropriate graphs for a given data set and its context.

For the purpose of our analysis, critical sense refers to the act of mobilization (Monteiro & Ainley, 2007, p. 16) of diverse kinds of knowledge and experience during the act of graph interpretation. These researchers based their research on the levels of graph comprehension as proposed by Friel et al. (2001). In our study, we considered the critical sense and the graph comprehension levels as interrelated subcategories of graph sense. Graph sense refers to the general ability to read and deeply understand already constructed graphs found in the media (Friel et al., 2001).

Participants

In our study, 47 adult students aged from 20 to 62 took part, in four different classes with 10 to 14 students each. They all had a primary school leaving certificate and they attended the first year of a SCS (lower secondary) in a small town, with mainly rural population, in North-West Greece. The mathematical skill levels of these students varied from basic elementary level through secondary level, since some of them had completed the primary school while others had attended the first class of the gymnasium (lower secondary education). Most of these people were unemployed or unskilled workers. The following table includes the demographic data of the participants.

Table 1.
Demographic data of the participants.

	N	% of total		N	% of total		N	% of total		N	% of total
Gender			Age			Nationality			Occupational status		
Male	28	60%	[20,28)	3	6%	Greek	44	94%	Unemployed	25	53%
Female	19	40%	[28,36)	10	21%	Albanian	3	6%	Employees	22	47%
			[36,44)	20	43%						
			[44,52)	11	24%						
			[52,60)	2	4%						
			[60,68)	1	2%						

Results

We have chosen to refer to specific tasks due to space limitations. One of the tasks was about a bar-graph (translated to Greek) with a truncated scale which is given below. This task was based on a graph taken from the internet and we constructed the question.

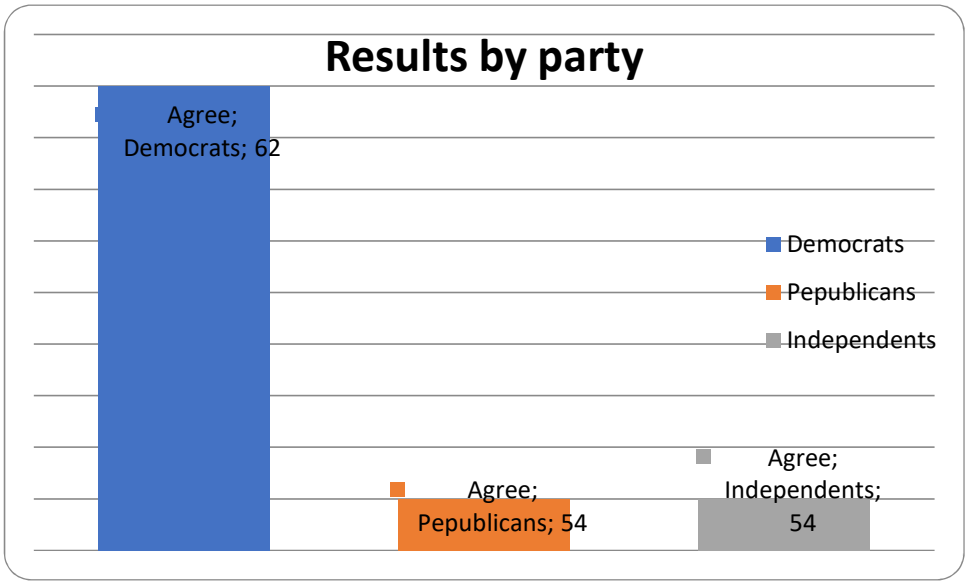

Figure 1. Third task. (adapted from Media Matters, 2005)

Question – Third task: In the United States, in 2005, CNN conducted a poll to test whether the voters for each party were in agreement with the court's decision about the Terry Schiavo's case and the next bar-graph was published. The source of the data was interviews conducted by telephone in March 18-20, 2005, with 909 adults in the United States. What is your conclusion based on this graph?

The first exchange comes from a class where the students discussed about the bar-graph and its accuracy. All adults' names appearing in the transcripts are pseudonyms. John responded negatively to the question posed by another student about the graph's correctness. He based his answer to the sum of the numbers that exceeds 100. He confused the bar-graph with the pie-graph where the sum of the sections must be equal to 100%. Helen asked about the 170 which is a number that the students mentioned before. Sophie answered to her by stating that 170 people say that they agree. She came to this conclusion by adding the numbers of the graph since 62+54+54=170.

John: No, it's more than 100.

Teacher: We have asked 909.

Helen: No, from 909 people, where does refer the 170?

Gregory: Maybe the percentage of the people asked is small?

Sophie: 170 say that they agree.

Gregory: Maybe the percentage of the people asked is small and we can't draw good conclusions?

Based on the above transcript we could say that the students' graph comprehension corresponds to the first level of reading the data since they read literally the graph focusing only to the given numbers. They extracted data directly from the graph and they tried to answer by relying only on the data shown in it.

In the second class during the same lesson the students interpreted the graph by observing its characteristics (bars) and the numbers represented by the bars. When Harry stated that we have 10 he referred to the 10 lines that he saw on the graph. In this way George understood the range of the numbers that are represented with the vertical axis. He realized that the wrong impression of this graph depends on the fact that the vertical axis' numbering does not start from 0 but from 53. George concluded that if the numbering was correct the bars' length would be different and the bar of 62 wouldn't look so much bigger than the bars of 54.

> George and Harry: (they discuss) In fact, it isn't true.
>
> Teacher: What do you mean?
>
> Harry: Because in fact we have 10…yes 10…
>
> George: Yes 10 lines. In fact if we notice it…I believe if there weren't any numbers …if we had began from 0 in order to go up it shows much bigger than 62. 54 has nothing to do with 62.

Another student (Eva), during the same discussion, trying to understand what her classmates were saying she confused the numbers on the bars with percentages. This was a problem for most of the students since they assumed that the numbers on the graphs referred to percentages. This was due to the fact that the students were not experienced graph readers; actually, this was their first attempt to read graphs after two lessons on the construction and the properties of basic graphs (bar-graphs, pie-graphs and line-graphs). It is obvious that the students of this particular class evaluated the graph's accuracy by noticing that the vertical axis is truncated. Thus, their graph comprehension corresponds to the second level of reading between the data since they detected the misleading effect:

> Eva: This starts from the middle, the 53, if it started from 0 wouldn't the percentage be bigger? Or not?
>
> Harry: No it wouldn't. The graph would be just bigger.
>
> George: By looking at the graph, what we can imagine…. That the first bar looks much bigger than 54…The difference is not so…They should be bigger (They talk simultaneously)

In the third class during the same lesson a student immediately found the inconsistency between the numbers and the bars. Joanna connected the numbers with the bars by comparing the bars' length and the difference among the numbers. She was able to respond at the second level of graph comprehension, reading between the data as it is evident in the next transcript.

Teacher: (She reads the task). What can we conclude based on this graph?

Joanna: Hold on a second… First of all, why the 54 is so down and the 62 is so up? It isn't correct; if it was reasonable then close to the 62 should have been the 54 and the other 54 of the independent.

Then the teacher asked the students to propose ways of reconstructing the bar-graph in order to correct its misleading effect. Joanna proposed a different scale in order to have a more accurate graph. In her answer it becomes obvious that she can understand the effect that the correct choice of scale has on the construction of the graph. Additionally, Joanna proposed that the numbering in the vertical axis should start from 0.

> Teacher: How would you make it in order to be correct?

Ken: Without numbers.

Joanna: If you ascend 5 to 5 or 10 to 10 (she refers to the scale) it is impossible to have 54. This bar is too high (the one of 62), actually 62 is only 8 (units) more than 54. If we used 10 to 10 then 62 would be close to 54.

Teacher: So, you propose by ten…

Joanna: Yes, 10, 20, 30 and so on. This is what I mean.

Teacher: Yes, but where would you start from?

Joanna: From 0.

In this class during the same lesson the students had to solve the next task.

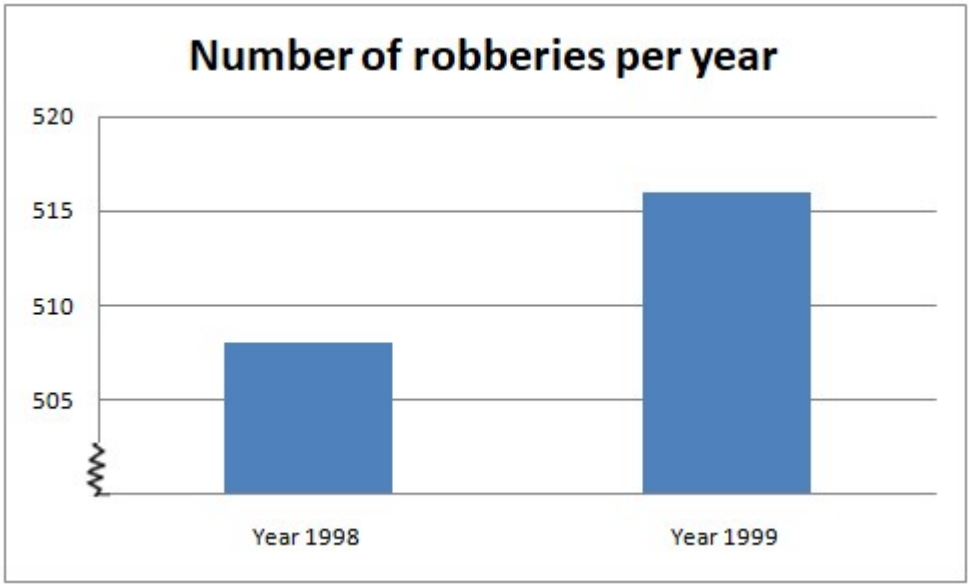

Figure 2. Fifth Task. (OECD, 2009)

Question-Fifth task: A TV reporter showed the above graph and said:
"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."
Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

Although Joanna responded correctly to the third task, which was familiar to her, she could not respond to this task which was familiar to all students. The students referred only to the numbers and they did not justify their answers. Joanna referred only to the relation between the bars and she did not connect their length with the numbers. Instead she focused on the numbers and confused them with percentages. When she could not justify her answer, she was confined to express her personal view about the graph.

Joanna: No

Steven, Ken: No.

Teacher: You have to justify your answers.

Joanna: It was 505 and goes to 520, how can we count its percentage?

Teacher: You can't since you don't know the whole.

Joanna: So, what? How would I answer?

Teacher: He said that the increase was great.

Steven: Is the difference big?

Joanna: It is two, it is two times.

Leo: No, it isn't correct.

Joanna: We don't like it.

Another student, Leo, who was silent from the beginning of the lesson, noticed the inconsistency between the numbers and the bar's length. He concluded that the graph is not correct since the second bar is twice the first one while the relevant numbers do not follow the same pattern. Leo managed to read between the data and his comments assisted the other students to express their opinions. Joanna concluded that this graph was used by someone to deceive the readers and Robert expressed a reasonable conclusion about it.

Leo: Because here it is 505, after it is 510 and then the double of it (he refers to the bars) goes to 520.

Teacher: So?

Leo: So, it can't be correct.

Teacher: Do you say that if I look at the numbers the robberies are double?

Leo: No, the numbers aren't double.

Joanna: So, they are trying to deceive us.

Teacher: Robert do you agree?

Robert: I believe the graph is wrong since the number of robberies isn't double as it looks on the graph.

The next exchanges are about the sixth task which contained a line-graph and was provided during the second hour of the same lesson.

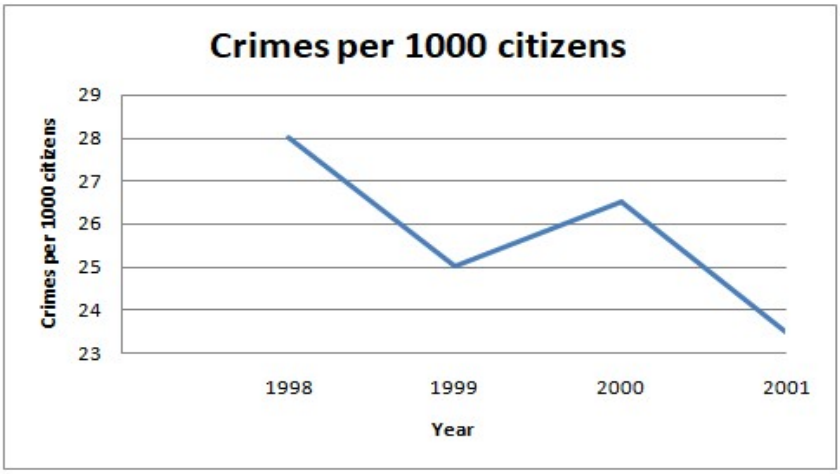

Figure 3. Sixth Task. (adapted from Harper, 2004)

Question-Sixth task: In a local newspaper in 2001 we found the above graph about crime rate in the region from 1998 to 2001. The article concludes that crime rate had been reduced to a great extent. Do you agree with that? Justify your answer.

In the fourth class during the second hour of this lesson Maria based her answer on her personal views. She expressed her opposition to the conclusion that accompanied the graph. It seemed that she did not even read the graph, but she merely expressed her disapproval. At the same, time Paul stated that he agreed with the graph since he read literally the graph focusing only to the given numbers.

> Teacher: Let's go to the sixth task now (she reads it). The article concluded that criminality had declined to a great extent. Do you agree with this?
>
> Maria: Nonsense. No.
>
> Teacher: Do you agree with this claim? Justify your answer.
>
> Maria: No. As the years go by, criminality becomes greater, not less.
>
> Paul: I agree with the graph, because the graph is correct. I see that in 1998 we had 28 crimes, in 1999 we had 25, in 2000 we had 27 and in 2001 we had 24 hence... I agree.

Afterwards the teacher tried to urge the students to notice the graphs' numerical information in order to understand the inconsistencies. Yanni with his question referred to the effect of the previous tasks. Maria concluded that it should have started from 0 and the scale should have been 1. At the end Yanni concluded that the reduction is very small.

> Teacher: Was the decline so great?
>
> Paul: As it seems here, it is not a big reduction but…
>
> Yianni: Isn't it the same again?
>
> Paul: So isn't the graphic representation right?

Teacher: Why is it not right?

Maria: Because it should have started from 0 and maybe go one by one here.

Teacher: Maybe one. If I understood what you said, in 98 was 28 and in 2001 was 24, what is the difference of 28 in 24, how much do we have?

Yanni: 4

Teacher: 4 out of 1000 inhabitants, is this a big reduction?

Yanni: It's nothing.

In the other classes most students interpreted this graph correctly and evaluated its accuracy by using their knowledge from the previous tasks. Some of them – coming from the first class – managed to propose its reconstruction, by using their knowledge about graphs. Eva proposed to change the line-graph to a bar-graph and Helen imputed the wrong effect of the graph to the choice of the scale.

Eva: If we used bars wouldn't it look better?

Theo: It looks that the decrease was huge.

Helen: But the difference isn't as huge at it looks like because the scale is 1. If the scale was 10 it would be…

Eva: If we used bars from 28 to 24…Wouldn't it be the same?

Teacher: If I construct it again and start from 0 what scale should I use?

Helen: By 5.

Teacher: (Constructs it in the blackboard according to the students' suggestions). So, now, how it does it look?

Eva: Now the decline doesn't look so big. While in the first graph (task) it looks like the decline was very big. Now it looks okay….

We may say that these students partially demonstrated the highest level of graph comprehension, reading beyond the data, since they were able to connect the same data with another graph in an attempt to produce a more accurate presentation of them in relation with the graph's context.

Conclusions – Discussion

Numerical activities take place every day since people have to confront numerate situations all the time (Diez-Palomar, 2011). Additionally, numeracy is highly connected to statistical literacy; at least to the extent that statistical literacy refers to the abilities that adults as 'consumers' of statistical data should have in order to be active citizens. One of the situations that statistical literacy is needed is graph interpretation. In light of these considerations we designed our research. Our underlying goal was not only to assess the way that adult students interpret graphs in context or media graphs but also to enrich their experiences by providing them with statistical notions which were new for them. These notions were new for our

students since they were inexperienced readers of graphs. Our intention was to encourage our students to approach every task as a part of their everyday life. In this way they would appreciate statistics and the impact it has for their effective participation in society.

Concerning the way that our students interpreted the graphs, most of them were able to read the graphs and to extract numerical information from these. They managed to use their previous knowledge to solve the tasks and they realised that a graph is a data representation that must meet certain requirements. Therefore, they could read these data. The students demonstrated to some extent the ability to critically evaluate graphs and to identify the deliberate use of misleading graphs that results to wrong interpretations. Not all of them were able to read between the data. With respect to the third level of graph comprehension – reading beyond the data – very few students generalized from the sample to the population based on specific graphs or were able to reach a conclusion.

It is noteworthy that the students' personal opinions occasionally overwhelmed their knowledge of the graphs; we may say that in these cases they did not operate well because they acted based on a drive provided by their emotions (Buxton, 1991) and they did not manage to overcome their prejudices against their statistical knowledge of graph interpretation. The context of the graph played a significant role in the students' interpretations; they reached some conclusions without considering the meaningfulness of their answers. Thus, the graphs' realistic context in our study had lead to "unexpected responses and outcomes" (Dalby, 2015, p. 88). The students' critical sense was not so developed since a balance between their knowledge, beliefs and experience in interpreting the graphs was not achieved.

In his seminal paper on the role of visual representations, Arcavi (2003) mentions a phrase attributed to Goethe: "We don't know what we see, we see what we know" (p. 230) to refer to the cases in which students do not see what teachers or researchers see. Especially, the second part of this phrase – "We see what we know" – may refer to all those adults who have to interpret different forms of visual data, including statistical graphs or media graphs without having a statistical background. Some of our students have managed to overcome their prejudices, while others not. Thus, we believe that our study has provided some evidence on how the endeavour of providing a statistical background might look like.

Further research is needed with larger groups of adults, adults with different levels of knowledge and possibly a bigger variety of contexts. This could provide support for the work of Monteiro & Ainley (2007), since the different features of reading contexts has helped us to highlight that the interpretation of graphs is related with school and out-of school knowledge. The teaching of correctly reading statistical graphs seems to be of crucial importance in a world where the average adult has to confront with graphs in almost every strand of his daily life. This is especially true for the adult learners who return to their studies carrying their own experiences from in and out of school, and at the same time they need to confront with all their prejudices on their attempt to interpret an ever-changing world.

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Catherine Paulson-Ellis, UK - Can't see it: won't address it. How perceptions of adult numeracy lead to gaps in skills policy in England

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Introduction

One of the most intriguing and important policy puzzles of our time is why numeracy rates in the working population in England remain stubbornly static while literacy rates have improved over the same period. Numeracy seems to resist advancement, despite policies targeted at school age children and further and adult education. The last major national skills survey of the working population aged 16-64, the Skills for Life Survey, showed a general improvement in literacy rates between 2003 and 2011, with the proportion with skills below level 2 going from 55% to 43%, but a slight decline in numeracy rates, 78% below level 2 rather than 73% (The Department for Business, Innovation and Skills, 2012). Nearly half the population has numeracy levels at or below that expected of primary school children and the performance of the youngest people surveyed was particularly poor (BIS, 2012) . This situation was brought into sharp relief when the OECD published the results of its PIAAC survey in 2013 and England found itself close to the bottom of the rankings for 16-17 year olds and below average overall (The Department for Business, Innovation and Skills , 2013)..

What most disturbed policymakers and many others involved in adult education and training was that this situation did not arise from neglect or deliberate intent. Quite the opposite. Right back to the concerns that lead to the Moser Report in 1999 and through the entire period covered by the Skills for Life Survey, there was substantial investment in school, college and adult education. Between 2001 and 2010, around £3bn was spent on the adult basic skills. New standards, qualifications and development programmes were created and millions of adults took English, maths and ICT courses, many achieving qualifications for the first time since leaving school. Schools and further education colleges were rebuilt and equipped with the latest technology and the national school curriculum was overhauled.

It was precisely because so much time, energy and money had been invested that the disappointing findings of both the Skills for Life Survey and PIAAC lead to a creeping sense that improving adult numeracy was too difficult a problem to solve. The political wheel had turned, the post-2008 crash brought recession and the Coalition government of 2010 to 2015 responded with austerity, significant spending cuts and a narrowing of the policy focus onto schools and apprenticeships, that traditional academic/vocational divide.

Impact of vocational qualifications

Yet, a closer look at analysis of the work that has been done on the impact of vocational qualifications reveals some interesting results that should cause us to think again about the impact of the Skills for Life programme. This work, commissioned by the former UK Department of Business, Innovation and Skills and led by Peter Urwin, showed a positive impact on employment and earnings and a reduced likelihood of being on social security

benefits for those who took virtually any maths or English qualification at every level up to and including level 2 (The Department for Business, Innovation and Skills, 2016). In essence, adult learners who take and complete a maths qualification are, subsequent to the course, more likely to be in work or more likely to have higher earnings than those who started but did not complete or who took a course without doing English and maths. For these students, the educational experience clearly made a substantial difference, even if they had been out of education for a long time and were working well below the standard expected of 16 year olds in England (that tested through the GCSE). Furthermore, the qualifications the adults were taking over the period from which the data was taken for the analysis were those same qualifications which were subsequently described as too simple and educationally flawed and are now withdrawn or on their way to being reformed which raises the interesting question of what the results would be in the study was run again on more recent data.

If you line up the results from a variety of studies now available, a reasonable conclusion might be that, despite its flaws, Skills for Life was far from a wasted investment but actually worked quite well for those people that took part and that to have a population-wide impact it needed to be bigger and keep going.

If poor numeracy is such a large and intransigent problem that it needs continuing attention and there is a track record and good evidence base of effective practice, why does England not have a national numeracy strategy? Indeed, as the skills profile of the working population will not have changed significantly since 2012 and there is a well-established, positive link between skills and productivity and interventions that can be shown to work, why are the numeracy levels of the workforce not considered a major problem or even a national scandal? There is much talk about the need to invest in training, but, outside of apprenticeships and technical education for teenagers, publicly funded training for adults is in decline; participation in adult maths courses last year was down again last academic year, by another 6% (Department of Education, 2017). It's a hard question to answer but this situation provides a good case study in how perceptions and assumptions can drive political behaviours and policy decisions which will fall far short of delivering any results.

Generally speaking politicians, like most people, tend to act on those things that matter to them, so it must be that poor numeracy is not translating into a problem either directly or through their constituents, employers or others. There are some reasonable explanations for why this might be the case. Automation removes the need for any sort of calculating – not only do people not have to do mental arithmetic, they can often get away with not understanding the data that they are handling, if indeed, they are handling it at all. When they do have to use numbers or other mathematical operations, people often find workarounds and get help with or pass over these tasks to "experts" (Department for Business, Innovation and Skills, 2016). People often think that their skills are good enough for their jobs (and often they are correct about this) and some, even with quite low numeracy levels, overestimate their abilities (BIS, 2012). Employers do increasingly use GCSE attainment as a filter in job recruitment, but these qualifications are not a reliable indicator of numeracy and their use in this way as a rough proxy for being educated is a long way from understanding what kind of numeracy, and indeed higher mathematics, is needed within their industry.

The finding at Figure 1, from OECD's recently published *Skills Outlook 2017* (OECD, 2017), is interesting. Is numeracy a completely different kind of skill from reading, writing, ICT and problem-solving? Or is its use being systematically under-reported?

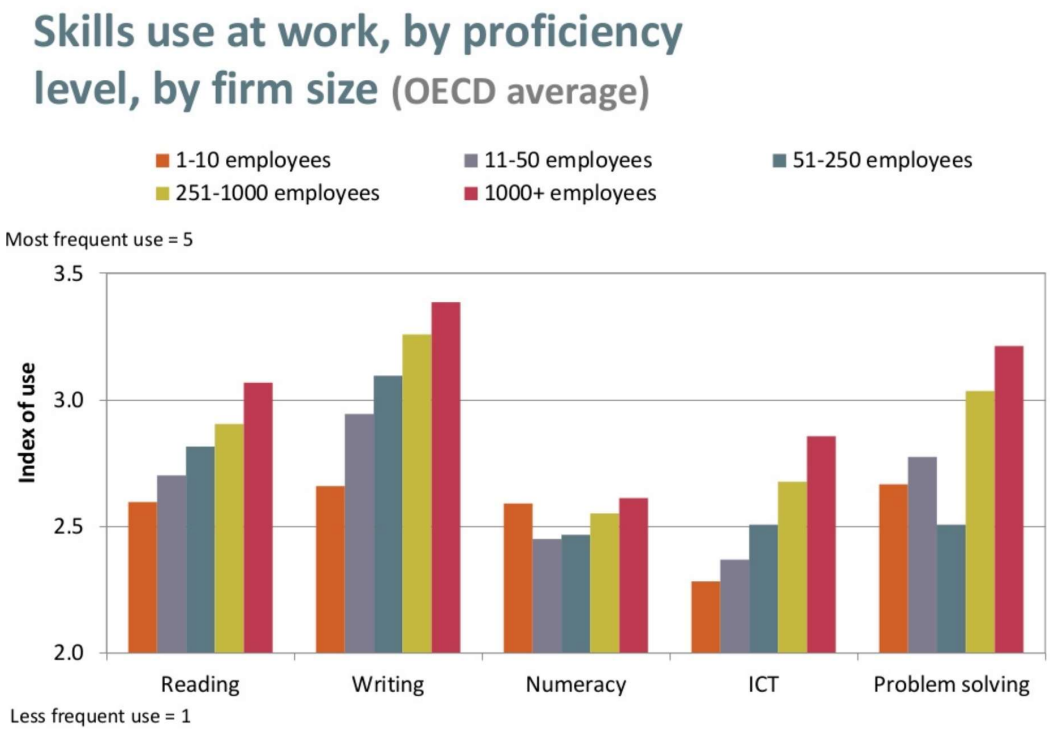

Figure 1. Skills use at work (OECD Skills Outlook 2017. Global Launch. 4 May 2017 Slide 52)

Perhaps we should conclude that we don't need the majority of the workforce to be highly numerate after all? This seems counter-intuitive and short-termist. Successive employer surveys show significant skill shortages for higher level technical and leadership and management positions (See for example: UK Commission for Employment and Skills (2016)). If people are not competent at, and confident in, number-based problem-solving and applying the maths relevant in their industry, how will they be able to achieve the level of education and training and occupational competence needed to be successful at these levels? Furthermore, research with employers does indicate that they underestimate the impact that poor numeracy has in their businesses and that when this it addressed there are positive outcomes for both staff and the bottom line (Department for Business, Innovation and Skills, 2016). It's much more likely that in there is in the English labour market a widespread culture of low expectations by both employers and employees – lots of "making do". It's hard not to surmise that this might be helping to hold down the UK's indifferent rates of productivity.

The hidden nature of numeracy

The hidden nature of numeracy poses a difficult policy challenge. How do you address an issue which can be described theoretically but which is not experienced as a problem by the myriad actors in the real world, especially when it is politically expedient to turn away from previous solutions because of political differences and financial challenges?

People tend to start with what they know and can easily conceptualise as needing changing but when many think of maths, they tend to think about school. Over the last few years there has been a continued drive to improve numeracy through early years, primary and secondary education, another round of qualifications reform and a big push to get all 16 and 17 year olds to pass the same GCSE exams that the majority achieve by age 16. This is all important and welcome and will help move England towards the practices of higher performing OECD countries, but too often numeracy is seen as something that is fixed at age 16 rather than something which develops as people apply their education in work and everyday life. What's

more there will be little impact on adult numeracy levels in the short and medium term. The policy approach is less an attempt to understand the nature of numeracy and how it manifests in normal working life than one that sees the education system as the sole location of any solution and the role of government as bearing down on low standards, as tested through formal examinations.

Even if it were possible to recreate a much larger Skills for Life programme, it would soon reach its limits. You can put on classes and create online courses; some people will come but many won't, especially if they are expected to take exams. You can persuade employers to take part in workplace training schemes, especially if all the leg-work is done for them, but these rarely generate spontaneous and ongoing demand for numeracy training. You can try to provide better information for individuals and employers on the benefits of raising skills, but while this might be acknowledged, action too easily gets squeezed out in the face of more pressing and immediate demands.

Conclusion

I would suggest that what policymakers should be doing is starting to change the terms of debate. There needs to be some serious analysis of what good and poor numeracy actually looks like today's labour market. Surveys on their own can never really capture the complexity of using skills in the workplace and this may be a bigger issue for numeracy than for other basic skills. While commissioning research can be an easy option for government, in the case of understanding how maths is perceived and used day to day it would be justified.

Government can also do what no single body can do alone – really bear down on the "I can't do maths" culture. This needs the weight of the state to generate momentum and support. As employer groups start to develop the technical education routes, there is a great opportunity for some thorough thinking on what being numerate looks like sector by sector and to put maths (and English too) at the heart of the new T levels rather than treating it as an extra subject for students who haven't passed their GCSE.

Finally, not least because of Brexit, government, LEPs and combined authorities all have to start asking the question, "where will we find our future workers?" and come up with some practical and palatable ways of ensuring that everyone in is numerate enough to get a job, train and retrain, potentially over and over. We can't just rely on our youngest people to carry us through the next twenty years.

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About the author

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Jenny Stacey, UK - Mathematics and Examination Anxiety in Adult Learners: findings of surveys of GCSE Maths students in a UK Further Education college

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Abstract

I teach adult learners in an FE college in the Midlands of the UK. These learners are usually between 16 and 60 years old, and they attend classes for many reasons, but 75% are in their 20s and 30s looking for a career change, usually via Access courses and Higher Education (HE). Learners are working towards a GCSE qualification, which is the exam taken in school at 16 years of age; it is used as a marker for HE, so the grade is critical. Learners have a 3 hour lesson for 30 weeks, and are supported with a wide range of resources, such as computer and paper based learning materials, before sitting the final exam in June.

As part of my work at the college I run a survey at the start of each academic year on the attitudes of adult learners around mathematics and exams. This helps inform my teaching: once I know the learners confidence or anxiety levels, it helps me to target their needs. Usually around a third of the learners each year describe themselves as anxious or very anxious about maths, and up to a half have the same responses for exams, out of a cohort of approximately 70 learners, 20% of whom are not first language English speakers.

In this paper I share the results of a further survey to investigate whether there are any patterns in confidence levels around age, gender, or nationality, and whether learners identify any key moments or events that shaped their opinions. It seems, in this action research study, that GB educated females have a proportionally higher rate of anxiety about maths and exams, than their non-GB educated or male peers. Age does not seem to be a significant factor for anxiety levels.

Most comments centred round feeling stupid or humiliated in class, being unable to ask questions, and needing more explanation or time, thus there is some agreement between the literature and learners on factors that have caused their maths anxiety, namely lack of time, lack of trust, and perceived teacher attitudes. Maths and exam anxiety were linked for only 3 learners, and there was little difference between the comments on exam anxiety of the maths anxious and non-maths anxious groups, with most comments identifying pressure, tension and fear.

Learners' views are changing as a result of their experience in an adult classroom, becoming more positive, and less fearful.

Literature review

Maths anxiety has been written about in some depth since the 1980s, and publications range from those centred around education, to mathematics journals, to publications on psychology. I have only focussed on material from the English speaking world in this paper.

There are a number of themes that emerge from the literature review, which I have reflected here: those which identify causes for maths anxiety, those which identify the effects of that anxiety, and lastly those which discuss interventions to reduce anxiety. Of course, a number of publications fall into more than one category.

It is interesting to note the lack of work specifically on exam anxiety, but it does feature as a sub-section of maths anxiety.

Causes of maths anxiety

Timed testing, or time framed exams, is identified by a number of publications as a cause of maths anxiety (Tobias, 1993) (Ashcraft, 2002) (Boaler, 2009) (Boaler, 2016). The pressure to perform within a specified time frame can cause panic, and reduce the ability to think. In the classroom teachers may often take answers from the fastest students, but "Mathematical thinking is about depth, not speed", and "we need students to think deeply, connect methods, reason and justify." (Boaler, 2016, p. 275).

Another cause of maths anxiety is believed to be early streaming, which is particularly present in the UK and USA (Boaler, 2016), as it can induce a fixed mind-set, where those in lower sets believe they cannot do maths, and stop trying.

The need for many learners to vocalise to encourage, extend and consolidate knowledge is identified as another issue, as in many classrooms a quiet environment is preferred (Tobias, 1993) (Sfard, 2008). This may particularly affect females if we believe that they "are more talkative by nature" (Tobias, 1993, p. 78).

Mathematics learning can be seen as a mix of performance of mathematical procedures and contextualised problem solving, which is used to test conceptual understanding. Unrealistic or irrelevant problems may not appeal, or even make sense, to maths students, and may lead to confusion and disengagement from mathematics (Boaler, 2009) (Dalby, Nov 2012).

The mathematics curriculum in the UK seems structured towards end of year or course exams, so there is a constant drive to move on, whether students have gained sufficient skills or not. Topics are revisited in a cycle, in the hope that more students may understand it at subsequent visits. This lack of time in the curriculum is identified by a number of authors as a factor in causing maths anxiety (Swain, Newmarch, & Gormley, 2007) (Boaler, 2009). Success and confidence are closely linked, as are low attainment and disaffection (Dalby, Nov 2012). The Shanghai Maths project can be seen as an attempt to break this time-pressured cycle (Boylan, 2016).

A lack of trust in the classroom can be another factor in poor maths confidence. The relationship between students and teachers can be a critical factor in how well students are able to engage with mathematics in the classroom, and how they feel about the subject (Warner Weil, PhD thesis 1989; 2015) (Tobias, 1993) (Dalby, Nov 2012). Teacher attitudes are also critical, and 16+ learners in FE maths classes valued enthusiastic and committed teachers, who developed positive relationships with their students (Dalby, Nov 2012), using resources that learners can see are useful and relevant to their lives (Dalby, Nov 2012) (Barton & Stone, 2013). The negative effect of maths anxiety amongst Primary School teachers on the children they teach has also been documented (Macrae, 2003) (Beilock & Willingham, Summer 2014)

The attitude of parents is another factor which can affect students, namely, if parents are openly dismissive of mathematics, or their own maths skills, or are themselves anxious about mathematics, this can affect the attitude and perception of the young person, and these beliefs are hard to change (Macrae, 2003) (Beilock & Willingham, Summer 2014).

The language of mathematics has also been identified as a contributory factor for anxiety, as many words have conflicting meanings in maths and 'real life' (Tobias, 1993), and mathematical meanings may be unfamiliar, and resulted in negative experiences in school classrooms (Woolley, 2013).

Effects of maths anxiety

There are a number of potential effects of maths anxiety, which can be present in many adults, including highly qualified ones, and which in extreme forms can be "sometimes verging on math phobia" (Macrae, 2003, p. 104). It can particularly effect the self-efficacy and self-esteem of adults (Boylan & Povey, 2009) (Dalby, Nov 2012) (Lewis, 2013), including those already on university courses (Kinead, 2015).

The psychological and neurological effects of maths anxiety are seen by some as amounting to a disability, as changes in the brain can determine career choices, employment, and professional success (Young, Wu, & Menon, 2012).

Interventions

The focus of interventions designed to reduce maths anxiety and increase the enjoyment and engagement of learners may come in two forms: either delivery content (Swain, Newmarch, & Gormley, 2007) (Boaler, 2009) (Boaler, 2016), or delivery methods (Tobias, 1993), including encouraging discussion in the classroom (Sfard, 2008), building better relationships with learners (Dalby, Nov 2012), or developing matching activities to overcome a fear of statistics in a university course (Kinead, 2015).

The importance of failure as necessary for growth in mathematics is also encouraging learners to attempt questions, rather than reject engaging with them (Beilock & Willingham, Summer 2014) (Boaler, 2016)

Many of the above publications use some combination of interventions, but although these works have identified maths anxiety as an issue, and attempted to counteract it, in fact no maths book, teacher training guidance, or CPD material has ever been written with the intention of causing anxiety in learners. They will all have been produced with the intention of informing, enlightening, and inspiring; that this does not always happen is what keeps teachers of adults in work!

Method and Ethics

Each September since 2014 I have issued a short survey form to learners, and asked them to grade their maths and exam anxiety separately on a Likert Scale, from 1 to 5, in response to what were clearly some very anxious learners in the classes. Learners have also had room to put comments about why they felt the way they did. This was with the idea that they could share their concerns with me, and so leave them in my care, and move on. Some learners covered both sides of an A5 sheet in writing, but none of those comments are included here, as I did not have written permission to share them. I have included the statistical information, however.

Written approval for a second survey, with the intention of sharing learners' comments, was obtained from the college Principal, who requested a guarantee of student anonymity.

Subsequently, in April 2017, I handed out a second questionnaire, asking learners if they could identify any general or specific events that led to their fear of maths or exams. I also asked if their views had changed, and if so, how they now feel. This was a longitudinal study, as the same learners were surveyed in September 2016.

Sampling for the study was completely non-random; anyone who was present on the days chosen was invited to take part. All learners were given a letter requesting consent, which included a tear off form to sign and return. All learners were give a written guarantee of their anonymity, and a 14 day cooling off period, in which they were free to withdraw from the study.

There are a number of surveys on maths anxiety in the public domain, including the Trends in International Mathematics and Science Studies (TIMSS) survey (Lewis, 2013), the Fennema-Sherman survey (Tobias, 1993), and the Peskoff-Khasanov survey (Peskoff & Khazanov, 2015), but I chose to use one of my own, which was quite open in terms of what could be written, but hopefully avoided any suggestion of prompting for responses. It could be seen as lacking intellectual rigor, as there is no cross checking of responses, but for this small scale project, designed to support my reflective practice, I felt that was appropriate.

There was one major change in the language used on the survey forms between 2014 and the other years, in response to discussions with Peskoff and Khasanov, as they highlighted the need not to prompt for maths anxiety: subsequent forms were changed to survey for maths and exam confidence, using the same Likert scale of 1 to 5, where 1 is very confident, 2 is confident, 3 is neutral, 4 is anxious, and 5 is very anxious.

Findings

In September 2014 I surveyed 29 learners for maths and exam anxiety, on the 5 part Likert scale, as discussed above. The results were that 21% described themselves as very anxious, 34% as anxious, 34% as neutral, and 10% as confident. No one described themselves as very confident. The results were the same for exam anxiety, except that two learners moved their responses from anxious to very anxious, changing the percentage points to 28 for each category.

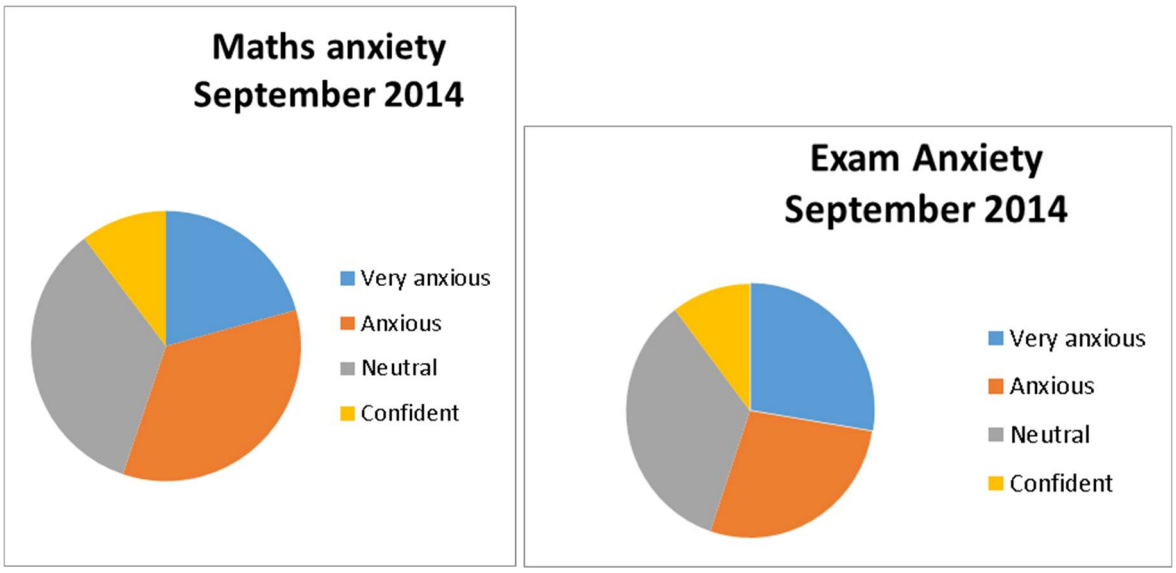

Figure 1. Maths and exam anxiety percentages September 2014

In this cohort over half of the learners identified as anxious or very anxious about exams and/or mathematics, and no one said they were very confident about either.

In 2015 there were a number of differences from 2014. Firstly the wording was changed so that confidence always came first, and the survey form itself was entitled 'Maths and Exam Confidence' , secondly the survey was conducted in the second week of the term, and lastly, there were 73 participants.

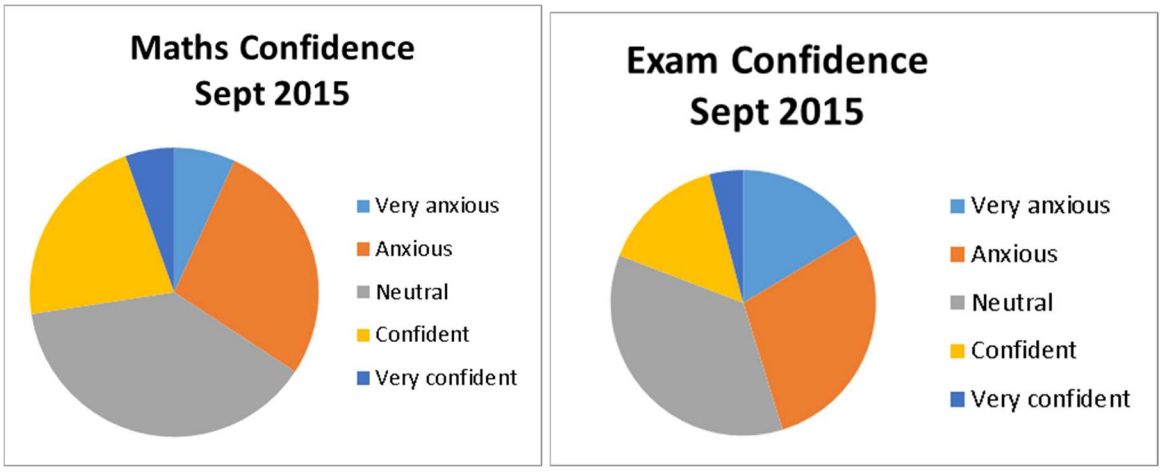

Figure 2. Maths and exam anxiety percentages September 2015

In 2015 around a third of learners described themselves as anxious or very anxious about maths, with 45% having concerns about exams. There are now a few learners who feel very confident.

The difference between 2014 and 2015 led to speculation about the reasons for the difference, that might not be based on the wording, but on the timing of the survey: has the work in Week 1 already had an effect on learners, or are Week 1 surveys negatively impacted by the anxiety of learners in a class, where the teacher, other students, format and timings are all new and unknown?

In 2016 there were 77 learners who completed the survey, all in Week 1.

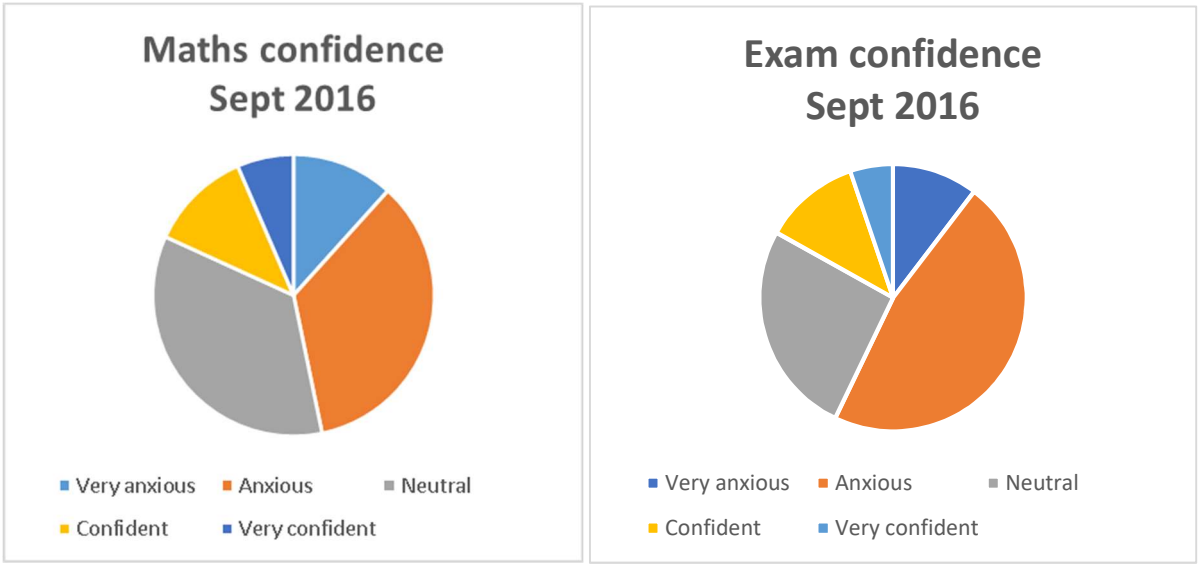

Figure 3. Maths and exam anxiety percentages September 2016

Almost half (47%) of the 77 learners felt anxious or very anxious about maths, and almost 60% were anxious about exams (57%). Comparing 2015 with 2016, there is a difference of approximately 15% between the two years, which may be the effect of surveying in Week 1 rather than Week 2.

An analysis of the original survey results for the respondents of the second survey revealed that 12 of the 46 had no concerns about either maths or exams (26%), but 20 were anxious about both (44%). Of the remaining 14 who responded, eight were only anxious about maths, and six only worried about exams, as can be seen in Table 1.

Table 1.

Maths and exam anxiety levels

Maths(M) and exam(E) anxiety	Number	Percentage
Neither	12	26
Both MA and EA	20	44
MA only	8	17
EA only	6	13
Total respondents	46	100

The results of an analysis by gender and nationality are shown in Table 2.

Table 2.

Maths and exam anxiety levels by gender and nationality

Group of 46 respondents	Percentage of group	Percentage anxious or very anxious: Maths	Percentage anxious or very anxious: Exams
Whole group	100	59	57
Male	20	10	10
Female	80	90	90
GB	90	96	96
Non-GB	10	>4%	>4%

Thus we can see that GB educated females seem to have a proportionally higher level of anxiety than their male, or non-GB educated peers, in this cohort.

A further analysis of learners by age seems to indicate that age is not a factor in anxiety levels for this cohort.

Table 3.

Maths and exam anxiety levels shown by age

Age bands in years in September 2016	Number of learners out of 46	Percentage anxious or very anxious: Maths	Percentage anxious or very anxious: Exams
Up to 21	5	60	60
21 to 30	14	50	57
31 to 40	20	65	50
41 to 50	7	57	71

The April questionnaire was very open ended in terms of the responses that could be given, for instance the opening sentence starts 'Describe your feelings….', but there were some patterns that emerged from the comments. These are the comments for maths anxiety:

- Felt stupid or humiliated in class, by teachers or other pupils: 13 comments
- Issues around asking questions, needing more explanation or more time: 11 comments
- Poor performance/loss of confidence: 7 comments
- Clashed with staff: 6 comments
- Disengaged from school/maths classes/coursework/maths exams: 6 comments
- Streaming: disruptive lower classes (3), or too low a level (2): 5 comments
- Other: unenthusiastic teaching (2), over faced (2), time gap since school (2), language (non-GB 2)

I split the comments about exam anxiety between those that were maths anxious, and those that were not, in case they were dissimilar, but there seems to be little difference between the two groups. Group 1 consists of the 20 learners with both, and group 2 consists of the six learners with just exam anxiety.

- Group 1: 22 comments; exam pressure (6), only/more anxious in maths exams (3), fear of failure (3), lack of experience (3), tension and fear (3), intimidated by others working away (2), too many people in the exam room (1), gap since last exam (1)
- Group 2: 13 comments; exam pressure (7); tension and fear (3), fear of failure (1), too many people in the exam room (1), lack of confidence in answers (1)

Learners were then asked to comment on how they felt now, to see if views had changed over the eight months of the course, and they were specifically asked if they had done a functional skills (FS) year prior to GCSE, if that had helped. All but one of the learners made very positive comments on the FS year, and said that it really helped; the one learner ignored this question. This seems to give support for a two year programme for GCSE Maths for adult learners.

Several learners mentioned that they had been diagnosed with specific learning issues, such as colour preferences for text or background, and dyslexia. This sometimes resulted in extra time for the exams, which was very helpful for those learners.

Other specific comments included:

"I am no longer afraid of Maths, I rather enjoy it now" "relaxed and comfortable environment" "motivated fellow learners" "I'm a lot more positive about maths now" "It has helped a lot at work" "The teacher is understanding" "There is no pressure" "Willing to push myself" "I know it needs time" "I look forward to Maths" "Good when it clicks" "I know I need to work at it", and my favourite: "I love maths", which was written by two learners. There were no negative comments

Conclusions

Whilst there is some agreement between the literature review and the learners' comments on the causes of math anxiety, namely lack of time, lack of trust, and perceived teacher attitudes, there are also some differences, such as early streaming, where it was not the streaming itself that was a problem, but issues around it (identified by five learners: three for disruptive classes, two for being put in too low a group), and realistic or relevant problems, which was not mentioned at all. This may be because adults have more life experiences, or have accepted problem solving as a pertinent vehicle for testing.

From the learners' point of view, with the benefit of hindsight, the relationship between staff and students seems to have been critical factor for my learners, but timed testing, or exams, was linked for only three of the respondents, so in this cohort timed testing was an issue, but rarely only a maths issue.

GB educated females had a proportionally higher rate of maths and exam anxiety than their non-GB educated or male peers. Age does not seem to be a factor for anxiety, and there was little variation in the responses on exam anxiety from the maths anxious and not maths anxious groups.

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Tanja Aas, Norway - Numeracy Counts

Numeracy is a critical skill, which touches several areas in everyday life. Whether you are at work, at home, at class, or shopping you will need Numeracy skills. Lacking these skills as adults, may negatively affect quality of life, labor market possibilities and participation in lifelong learning. The question is how to get this message out to those who need it the most? This poster presentation will present examples of good practice by using financial literacy as a mean to motivate adults to learn Numeracy skills. There are several resources already available to teaching financial literacy and using practical mathematics to enhance Numeracy skills. In addition, the poster will give a heads up about the Managing Money project, which finalizes in 2018. Managing Money is an Erasmus+ project, which is focusing on financial literacy in adults. The project involve 8 partners in 7 different countries and the main outcome is an app and a website full of different resources. The app itself will be available in several languages, including English, French, Dutch and German.

Charlotte Arkenback, Sweden - Problem: The Numbers of Squares on a Chessboard – But, what if you don´t see where the squares on the chessboard start and end?

In this workshop, we are going to show how we work with spatial thinking and geometry in an orientation course in mathematics at Swedish tuition for immigrants (sfi). It has been found that many of our students with a short education background studying math at basic level have difficulty reading, interpreting and constructing 2D and 3D representations. In addition, when they begin to study mathematics, Swedish proficiency is limited and many lack words and expressions to describe, compare, analyze and evaluate solutions to mathematical problems. Knowledge and skills they need to develop to meet the mathematical requirements in the curriculum.

The purpose of the orientation course, which is an action research study, is to develop knowledge of the students' visual-spatial thinking and learning while providing students with the opportunity to develop spatial thinking, visual, mathematical and digital literacy. At the workshop, we will work with one of the practical parts of the course.

Lydia Balomenou, Greece - Adults solving realistic problems

The need for understanding the large amount of information by the use of mathematics is widely accepted. Furthermore, it is important to educate children in order to think critically and use their knowledge to improve their lives. These views are based on Critical Mathematics Education (CME) approach. Beyond this approach, Realistic Mathematics Education (RME) suggests the embodiment of real life situations in the teaching process. From our perspective, realistic problem solving is a way to join these approaches. Particularly, when the solver reads a problem s/he has to think critically in order to process the necessary data and find the relationships between them. After that, the solver has to make some realistic considerations in order to provide the answer. The aim of this study is to observe the strategies that 20 adults used to understand and solve seven realistic problems. More specifically, the adults completed a questionnaire, in which they stated their beliefs about mathematics and their uses. Subsequently, they tried to solve some realistic problems. Most adults had difficulties in solving problems by making their own assumptions. Additionally, they were unable to easily assume the solver’s position. In conclusion, the survey has revealed the difficulties that adults have in using mathematics and think “out of the box” in order to face real life situations.

Mirjam Bos, The Netherlands - Start up your lesson: How to start up your lesson.

In this interactive workshop we are going to experience some lesson starters. Why should we use lesson starters?

- *Some are just for fun.*
- *Other starters are meant to activate prior knowledge, for instant when you start a new subject.*
- *Starters are also useful to differentiate. The way in which the student solves the assignment, says something about his mathematics level.*
- *Starters are so helpful, you can use them every lesson.*

We will not just talk about starters, but most of all we're going to experience them.

Arjan van den Broek, The Netherlands- Say hi to Eddy the educational robot

Eddy helps students in an interactive way by practising numeracy. We show you a mathematics lesson in which Eddy will be helpful. You also can try Eddy by doing a quiz with mathematical questions.

Eddy can be used in educational practice with students during calculation lessons. It's an innovative didactical approach. The big advantage is that students can work independently and using the robot adheres to the modern technological experience world of the student. The expectation of using Eddy is twofold:

- The calculation skill level of students will rise

- Students will be introduced to new forms of technology, which they enjoy

During the demonstration we will show the listeners how the robot works, how it is used in classroom, and we will engage in a quiz with mathematical problems.

Joanne Caniglia, USA - Teaching Mathematics to Adult Learners Using Stations

This session will allow participants to experience stations in teaching adult learners. Teaching with stations is not a new pedagogical innovation. Rather it is a common classroom practice used by k-12 teachers. There are many different modes for designing and implementing "math stations" during a class for adults (GED or Adult Education Classes). In this session we will demonstrate by using 4 stations: Video Station (short videos with practice), Independent Investigation Station (with technology assistance), , "Ask the Expert" (Teacher Station) and a "What to Do When You Get Done Station." (for students who need or want more challenge). Not only will the "nuts and bolts" of stations be explained, but research describing the advantages of stations will also be discussed. The advantages of stations include: ease of implementation, differentation, movement that helps keep interest, and student autonomy is developed. After participants "travel" to each station, a question and answer period will further clarify this strategy.

Diane Dalby, UK - How can I become a better mathematics teacher? The affordance and constraints of professional learning communities in changing classroom practice.

Many teachers are keen to develop their classroom practice but face setbacks due to external constraints or conflicts with the dominant culture of their department, school or college. Changing teacher practice is also a fundamental part of most national strategies for improvement but implementing such changes remains a major difficulty. There is evidence however, that when teachers work collaboratively in professional learning communities, there is the potential for effective and sustainable change in classrooms. In this session we will commence by discussing a case study of a group of mathematics teachers who work across vocational and adult learning programmes in the same college. By tracing the journey of this small professional community, we will see how they adapt and develop their teaching approaches through times of institutional and national change. Our discussion will include practical examples and highlight ways in which participation in a supportive professional learning community can facilitate both shared and individual professional learning, despite external challenges. Finally, by exploring some of the key characteristic of such communities, such as collaboration, inquiry and reflection, we will explore how individual teachers can become better teachers through active participation in these types of groups.

Susanna East, UK - Developing independent learning skills among adult learners

Adults returning to learning often struggle with the required levels of self-study that are essential to achieve success. In 2015 Adult Learning Lewisham elected to offer GCSE qualifications to our 19+ learners most of whom have had a significant break since school with a significant number never having studied maths at this level or sat formal examinations. The content of GCSE syllabus is extensive, it is the standard accreditation for school leavers who generally spend at least two years preparing for GCSE assessment at the end of their compulsory schooling at age 16. Adult learners (and teachers) are faced with a significant challenge to cover the same course content in a period of nine months (September-May). The demands on both parties are high.

Tutors are under pressure to deliver a full scheme of work with little time for revision and review and are often reliant on learners completing study tasks and developing their skills outside of the classroom.

The maths team at ALL have utilised several strategies to support learners developing independent study skills with varying levels of success. My poster will reflect on the strategies employed and their impact.

Jeff Evans, UK; John O'Donoghue, Ireland; Kathy Safford-Ramus, USA; Javier Diez-Palomar, Spain - New Directions for ALM-International Journal

Following John O'Donoghue's plenary at ALM-23, which advanced a number of suggestions for revitalising ALM's work, and prior to a change of Editor(s) for ALM-IJ, this workshop will discuss a number of ideas for developing the journal. These will include:

(i) the appointment of a new Editorial Team, of say three colleagues with complementary expertise;
(ii) refreshing the membership of the Editorial Board;
(iii) reviewing the priorities for topics for articles;
(iv) related to forming links with other branches of mathematics education, organising a Special Issue of ALM-IJ, themed on a topic from maths education, and co-edited with an expert from mathematics education, e.g. the earlier issue on Gender (with the late Christine Keitel in 2008 ca.); etc.

Graham Griffiths, UK - Thinking about the use of alternative dialogue scenes when developing adult mathematics

This paper is a continuation of work investigating the reading aloud use of a scene of dialogue with adult mathematics learners. Through the use of the 'real world', hands-on context of rail ticket prices, the scene and its associated task have been produced to encourage learners to engage with mathematical ideas. Nevertheless, in exploring the use of the dialogue scene with adult learners (Griffiths 2014) through reading aloud, it was noticed that the scene contains features that suggest the 'correct' response to the task without the need to fully engage with the mathematical concepts involved. In order to analyse the responses of adult learners to such mathematical scenes and narrow down my research questions, I will explore the features of some alternative, but linked, scenes of dialogue. The alternative scenes will be examined using discursive approaches to mathematics education (Kieron, Forman and Sfard 2002) and informed by empirical data (Oughton 2009)

Graham Griffiths, UK - Citizen Maths – a free online mathematics course designed for adults. Is it working?

Citizen Maths (www.citizenmaths.com) is a free and open online maths course for self-motivated adults covering five "powerful ideas" in mathematics, and involving between 25 and 50 hours of study. It does not lead to a formal qualification. The development of Citizen Maths was funded by the Ufi Charitable Trust. The work was done by Calderdale College, with the UCL Institute of Education and OCR, with advice from the Google Course Builder team. Thousands of people have signed up for Citizen Maths, at a current rate of nearly 200/week. This workshop will investigate some of the key features of Citizen Maths and discuss the data collected by the team during the development of the resource.

Giel Hanraets, The Netherlands - Digital tools in Dutch vocational mathematics classes

PISA 2012 showed that the Netherlands has one of the smallest shares of 15-year-old students who find learning mathematics interesting or enjoyable among the participating OESO-countries. Dutch students are also less willing to work through problems that are difficult, they do not remain interested in the tasks that they start, and, more than in other countries, they are likely to shy away from complex problems (OECD, 2013-page 84). This means a big challenge for Dutch math-teachers, especially in the secondary vocational education, where Math isn't always considered a high-valued subject amongst students.

In this short workshop, I'll be giving an introduction in the use of (mobile) devices in the math class. Not only to enhance student-motivation, but also to make learning visible and create the opportunity to differentiate. Make sure you're tablet of mobile device is fully charged and has

access to the Albeda-wifi and we'll have a blast. Interactive learning trough Nearpod or go out for a math-walk with Seppo. The main goal is to inspire co-teachers and give an example in the activities teacher have to make to enhance student-motivation and learning.

Kees Hoogland, The Netherlands, Diana Coben, New Zealand; Lynda Ginsburg, USA -Reviewing the PIAAC Assessment Framework

This presentation will report on the results of a recent review of the Programme for the International Assessment of Adult Competencies (PIAAC) Numeracy Assessment Framework. This framework was used in the first cycle of PIAAC. The review was carried out on request of the OECD by an international team of numeracy experts with a long experience in ALM. The presentation will show the results of this review and discuss the recommendations on the assessment of adults' capacity to undertake successfully the range of numeracy tasks they will face in their everyday and working lives in the third decade of the 21st century.

Kees Hoogland, The Netherlands - Descriptive and depictive representations in mathematical problems – the effect on vocational students' results.

In my dissertation the following research question was addressed: "In presenting contextual mathematical problems, what is the effect on student performance of changing a descriptive representation of the problem situation to a mainly depictive one?"

Investigating this research question shed some light on an issue in mathematics education: why do students on so many occasions show a suspension of sense-making in answering contextual mathematical problems, and as result perform poorly, and develop inadequate problem-solving skills?

We argued that a mainly depictive representation of the problem situation, especially with photographs from real problem situations, might increase the likelihood that students address the problem with a problem-solving attitude, that is, taking into account all aspects and constraints that the problem situation demands. I will present some results with a focus on vocational students.

Brooke Istas, USA - Mathematics for the Criminal Justice Field

In the 2016-2017 academic year, I worked with a group of Criminal Justice Experts to contextualize the competencies associated with College Algebra level mathematics. This curriculum was then taught to a group of students who were interested in the field of criminal justice and were considered to be low-numeracy learners. This course/curriculum was taught twice once in the Fall and again in the Spring. I will discuss the results of this type of instruction, the curriculum and focus, as well as, discuss changes that were made to the instruction in the classroom.

Beth Kelly, UK - Adults learning mathematics in the workplace through their trade unions: what motivates them?

My research explores adults' motivation to learn mathematics and focuses on learners who are overcoming many barriers to study in the workplace; in this study through classes organised and funded by trade unions.
The research identifies:
Different types of motivation: Initial motivation to re-engage with learning and motivation to continue learning.

Motivation as a dynamic interplay between the personal needs or goals of individual learners, the influence of other members of face-to-face learning groups within the context of wider UK society.
Learning approaches used in the trade union classes as a key aspect of adult learner motivation. The use of 'collective' learning approaches, develops positive social and emotional encounters in the classroom that are different from their previous experiences.
Successfully developing mathematical skills develops the adults confidence, which helps to shape their identities and has considerable influence on their motivations both inside and outside the classroom.
The idea of an 'Affective Mathematical Journey' developed through the adult's use of emotional language when reporting changes in their feelings and motivation towards mathematics.

Maryam Kiani, USA - Critical Mathematics in a Post-Truth Era: Mathematics is a prerequisite to philosophy and therefore access to wisdom and truth.

"Let No One Ignorant Of Geometry Enter Here"
This phrase, reportedly engraved at the door of Plato's Academy, conveys the ancient idea that is still fueling our lives today: mathematics is a prerequisite to philosophy and therefore access to wisdom and truth.
The idea that mathematics is not only practically useful and theoretically beautiful but also broadly intellectually and cognitively beneficial is still predominant today. Indeed, numbers do thinking for us and no important aspect of life is beyond their reach (Porter, 1997). Numbers have long been important in the management of our life from what to eat to what policy to support. 21stcentury contains statistics which more often are just perplexing array of numbers producing confusion rather than clarity. Everyday we are being bombarded with massive range of data, which some call them fake news, and some the pure truth. In this turbulent century, we must learn to interpret the numbers for ourselves. This talk is about how learning mathematics empowers us to evaluate data rather than just relying on arguments (mostly sound very rational) made by the press, the government, and our fellow citizens. Mathematics is a tool to think suitably and earn aptitude to understand national and global issues.

Maryam Kiani, USA - Numeracy and Mathematics in Brain: The Untapped Potential of Learning Disabled Employees in the Workplace

A new interdisciplinary field of Mind, Brain and Education has been evolving in recent years. Researchers try to answer educationally-relevant questions such as "How the brain of math learning disabilities work?"(Ansari, 2016)
Although generally mathematics learning disability such as dyscalculia considered a childhood disorder, they can persist into adulthood and impede achievement in the workplace. Dyscalculia symptoms can be associated with poor organization, time management, and interpersonal relationships. Furthermore, the costs of employing dyscalculic individuals are higher because of work absences and lost productivity.
Dyscalculia **is** severe difficulty in comprehending mathematics, presumed to be due to a specific impairment in brain function. This disorder is sometimes referred to as a "mathematical learning disability" and can be as complex and damaging as a reading

disability (dyslexia), which tends to be more usually diagnosed. Mathematics learning disabilities do not often occur with clarity and simplicity. Rather they can be combinations of difficulties, which may include language processing problems, visual spatial confusion, memory and sequence difficulties, and, or unusually, high anxiety. Dyscalculia is often manifested in struggles with conceptual understanding, counting sequences, written number symbol systems, the language of math, basic number facts, procedural steps of computation, application of arithmetic skills, and problem-solving.
This review describes the brain of a dyscalculic individual, how dyscalculia symptoms in adulthood affect workplace behaviors, the effect of Dyscalculia on employment and workplace performance, and the management of dyscalculia in working adults.

Suehye Kim, Germany - Adult numeracy within a lifelong learning metric: Action Research to Measure Literacy Programme Participants' Learning Outcomes (RAMAA)

Adult literacy is a foundational means of further learning as nearly all structured learning opportunities to live well in increasingly changing societies (UNESCO, 2017). In this context, adult numeracy is essential in addressing mathematical problems associated with measures to implement the Sustainable Development Goals (SDGs) agenda (UNESCO, 2016). A new vision of SDGs for education proposed adult numeracy be assessed within a learning metric to ensure quality of education.
In the lifelong literacy framework, UNESCO Institute for Lifelong Learning (UIL) has initiated a multi-country action research to measure learning outcomes for literacy programme participants (Recherche-action sur la mesure des apprentissages des bénéficiaires des programmes d'alphabétisation; RAMAA). RAMAA action research started to meet a special need for increasing adult literacy and basic skills in French-speaking African countries. Particularly, adult numeracy is measured as a key component of the skills from level 1 (counting objects or currency and writing numbers) to level 3 (performing operation with quantities or measurements of space tor time). In the current second phase of RAMAA, 12 African countries set to having harmonized tools to measure adult skills. This presentation will highlight two key features of RAMAA; 1) in developing contextualized measurement tools of adult numeracy as a learning outcome and 2) in strengthening national capacities within a country-specific context.

Nárcisz Kulcsár, Hungary - Studying mathematics through texts or images? The importance of visuality in university coursebooks

We are part of a specific iconic revolution in which the dominancy of the written language expiry and the importance of the visuality increases. Globalization catalyzes the visual language with its universality and internationality to become a world language. The dramatic rise of the role of visual culture has impact on education culture as well. Physiological processes in the brain (myelination process) of adolescence and young adults cause that they learn typically differently subsequently often utilize visual aids in order to help them better comprehend difficult subjects. These reasons lead us to pay more attention in higher education on visuality. The preferable ration between text and images in the textbooks should be between 30 and 50 % but the question is that what shows the reality. Miguel de Guzmán Spanish mathematician dealt with visuality in mathematics. He distinguished three different types of

visualization in mathematics due to the strength of relation between the object and its visual mapping. I investigated some coursebooks that students use in our university and I made some comparisons from a visual point of view based on Guzmán's theory and on other visual aspects.

Wim Matthijsse, Monica Wijers, The Netherlands - A Dutch Numeracy approach: Succes! Rekenen

Over 2 million people in the Netherlands have numeracy problems. They have to deal with daily challenges such as paying (debts), measuring (jobs, healthcare), interpreting schedules, schemes, numbers and graphics etc. etc.

A team from Utrecht University has developed commissioned by and in cooperation with the Reading and Writing Foundation, a series of six thematic booklets the lplow-educated adults to become more numerate in daily life and work. The series is called: Succes! Rekenen. Themes are: Money, Healthy living, Do-it-yourself, Cooking, Appointments.

Some of the characteristics of the approach are: Functional authentic situations, focus on students own solutions, interactive guidance, integrative approach of numeracy, literacy and digital competencies, suitable for use by trained volunteers. Volunteers and coordinators were trained for a pilot using the materials in non-formal settings with low-educated adults. In this pilot research was done using a survey, observations, assessment and interviews to answer question like: Are the materials helpful for volunteers and the target group? What works well? What problems do they encounter?

In this workshop you will learn more about the materials and get a chance to work with one of the booklets. We will also present and discuss the results of the pilot.

Sorcha Moran, Ireland - Add1ng Num8er5 t0 L1fe, and Multiply the Possibilities

I teach in an Adult Education setting and for many of my learners, the education system failed regarding their maths learning the first time around. They are very often left with an opinion that they can't do maths. They lack self-belief in their ability to learn maths. Maths is also linked with a perception of intelligence, so has an impact on their overall opinion of their own intellect.

My teaching approach is to NEVER teach using 'rules'. Learners always learn from the very basics of the concept. They learn using visuals, manipulatives, relating the material to real-life, and concepts they have already grasped. They learn through understanding, linking knowledge, and presenting the same thing in many ways. They learn that maths is not just a jumble of numbers, that the order of the numbers combined with symbols gives them meaning. They learn to read maths and understand the language by matching their visuals/manipulatives, English, and mathematical symbols.

In this workshop, I will give examples of hands-on learning activities which have helped learners become proficient at fractions, algebra, and geometry. They developed their problem solving skills and express a confidence in their maths ability, and their ability to help their children.

Leah Rineck, USA - Accelerating Students to Credit Bearing Mathematics Classes

Some students come to the university very underprepared for credit bearing mathematics. How do you help adult students understand numeracy and beginning algebra if they have never understood it before? This presentation will discuss a different way to prepare students that have not succeeded in mathematics before. The class sets students up for success. An accelerated, comprehensive class that ranges from basic math all the way to beginning algebra. The class incorporates growth 'mindset', study skills, and conceptual understanding. The class is flipped and has a vertical redesign. The vast majority of students are successful in this class and are also successful in credit bearing math classes. Come see how to set your students up for success!

Leah Rineck, USA - Using Algebra Tiles to Help Integer Understanding

One of the major stumbling blocks for adult students in basic mathematic classes is understanding integer computations. Having students use many representations of integers to understand operations with integers helps develop conceptual understanding. I have been using algebra tiles for the last two years to help students understand what and how integers work. In this workshop I will discuss some different ways to use algebra tiles, and let participants use the tools to understand how to bring them back to their schools.

Kathy Safford-Ramus, USA - Power in Numbers: Advancing Math for Adult Learners: A Project of the United States Department of Education

In August, 2016, the United States Department of Education Office of Career, Technical, and Adult Education launched a new initiative to promote the use of high-quality Open Education Resources in the teaching and learning of mathematics by adult students. The scope of the three-year project includes the formation of a technical working group of subject matter experts, an environmental scan of available OERs, and the recruitment of Teacher User Groups who will review and evaluate OER for their utility in the adult classroom. The ultimate goal of the project is the appropriate incorporation of OER resources into adult mathematics education in order to aid adults studying mathematics.

This presentation will describe the work of the project thus far as it nears the year-one mark and share the completed products with attendees. Future project goals will be outlined and participants will be invited to contribute information about OER research in their own country that may meld with this project to advance the use of OERs as an essential instructional tool.

Uwe Schallmaier, Germany - Mathematical competences in workplace activities of trained salespeople from different perspectives

Our study aims to detect mathematical competences trained retail salespersons use for coping with mathematical requirements in their professional everyday life. Hence, it is necessary to consider the different approaches to the concept of competence in vocational education and mathematics education.

From the perspective of mathematics education, the general model of vocational mathematical competence (Siebert & Heinze, 2015) describes mathematical competences in a vocational environment. It is based on the German education standards in mathematics and, hence, refers to a cognitive under-standing of competence. This model identifies domain-related

mathematical competences as the intersection of general mathematical competences and vocational action competence.

In the area of vocational education, the term vocational action competence describes skills, knowledge, and abilities to act successfully in work related situations. Being linked to domain-specific situations, vocational action competence is unique to every profession. Focusing on salespeople, the integrative model of commercial competence (Winter & Achtenhagen, 2008) distinguishes between general and vocational competences which differentiate domain-specific from domain-linked economic competence. Domain-linked economic competence, in turn, embraces economic numeracy, which is similar to the conception of domain-related mathematical competences.

Our poster tries to depict this linking between the conceptualisations of mathematical competence invocational and in mathematics education.

William Speer, USA - Mining the Richness of "I Don't Know" Responses

We can't ask our students to be seekers if we aren't seekers ourselves. More time should be spent asking questions which the students don't already know the answers. Students need to spend a greater proportion of instructional time seeking the answers than remembering ones they already know or that we expect them to know. This research-based, practice-oriented, hands-on session explores the benefits of productive struggle with questions that often initially yield a response of "I don't know" to help students shake up naïve or loose thinking and to nurture the ability to construct "new" knowledge by encouraging connections to, and transfer of, related content knowledge. Not everyone wants to be a mathematician, but everyone can learn to experience and appreciate mathematics in the ways that mathematicians do. Mathematicians consistently voice "I don't know" when faced with a real problem. In fact, being good at mathematics is evidenced by what *else* you choose to do when you don't know what to do. We must help people construct their own "new" knowledge, and, most importantly, apply that knowledge in ways different from the situation in which it was first encountered or learned.

Marissa van der Valk, The Netherlands - Electronic Platform of Adult Learning in Europe: Find out about 5 lifehacks of EPALE (and how you can share one too…)

EPALE is a multilingual open membership community for teachers, trainers, researchers, academics, policy makers and anyone else with a professional role in adult learning across Europe. Also for numeracy professionals!

Members of the community can engage with adult learning colleagues across Europe through the site's features, including forums, blogs, events and a news-section. You can also interact with your peers across Europe through the thematic areas which provide structured content according to topic. You can find projects and make professional connections using the partner search repository.

After this active workshop you'll wonder why you haven't joined before! You'll find out about this community, what you can find there, how you can use it in your daily (professional) life. Oh, and you'll add one of your own eye-openers, tips or tricks. No diploma's or certificates necessary.

Shin Watanabe, Japan - Connecting In school and Out School

First we define the 'In' school and 'Out' school. In Japan we have a system of schools, but we do not learn mathematics out of school. I want to explore the idea of lifelong learning.. In Japan, so many people stop learning mathematics when they graduate from a school, we want to change this system. So we want to establish a system of lifelong learning of mathematics. It is important for everyone to use and learn it. Now we think "what is mathematics for lifelong learning?", and "where do we learn it out of school?" Our aim is to create a new way of using mathematics and find a method necessary to solve our problem. It is important to Japan's creativity and to help solve our problems in life. We want to develop a lifelong learning of mathematics.

Hans de Zeeuw, The Netherlands - The Math Games

Games are a way to motivate students to engage in numeracy activities. In this workshop we present two games that are designed for (young) adults. We will report on our classroom experiences with these games.

We will also give the participants the opportunity to play these games and discuss ways in which they might be used in class.

1. The Math game

The Math Game is developed to improve numerical skills. It is in English and comes with translations of the rules in Dutch, German and Spanish. The Math Game is a digital game, but is also available as a card game. The game can be used any time of day, at school or at home, and it only needs 3-5 minutes to finish. It's an addictive educational game that players don't want to stop playing.

The game has four levels, from easy to expert. It has a limited time to finish. There's a setting with additional time for players who require more time to understand the questions and/or lack confidence to play. To challenge the players there's a top score list as well as a top 10.

2. The game, Calculation Match

Calculation Match, is a game that trains players to link concepts, to calculate or count with them and apply them in certain situations. The typical aspect of the game is that all the players are counting even when it's not their turn. The player has to make pairs by matching two cards: one card with an assignment and one card with the solution. The game has four levels. Four types of counting are addressed: basic counting, counting in contexts, geometry and relations in figures. It's a game for two to four players. The game is aimed at vocational school students but can be played at secondary schools too. By placing the cards face down, each player needs to remember not just which two cards belong together, but they also need to remember their place on the table! This helps to create a better memory and makes this an exciting game for everyone to enjoy.

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