# Adults Learning Mathematics

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## Objectives

**Adults Learning Mathematics (ALM) – An International Research Forum** has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/ numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

**Adults Learning Mathematics – An International Journal** is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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**Adults Learning Mathematics** – An International Journal

## In this Volume 13(1)

# Editorial.

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I am glad to introduce this new issue of ALMIJ. Mathematics is crucial knowledge for adult people to manage situations in the everyday life. In a recent speech, Mr. Stephaan Hermans, Director of Policy Strategy and Evaluation in the Directorate-General for Education, Youth, Sport and Culture at the European Commission, claimed that one out of five Europeans fails in literacy skills, including mathematics. Being numerate is nowadays a challenge for many people. The digital era introduces many changes, as reported by Evans in his article, in the ways that adults need to manage, interpret and use data in our everyday lives. Mathematics and numeracy are two important aspects to be socially included. The fact is that mathematics and numeracy are embedded everywhere, thus being mathematically or numerically literate is a requirement to fully participate in modern societies. According to the scientific literature, adults who have some degree of numerical capability are more likely to make more informed decisions in relevant aspects of their lives than adults with poor numeracy skills (Peters, 2012).

The OECD is very worried about this fact, and recently they launched the second cycle of the Programme for the International Assessment of Adult Competencies (PIAAC) survey (Hoogland, Díez-Palomar, Maguire, 2018; Hoogland, Díez-Palomar, & Vliegenthart, 2018; Díez-Palomar, Hoogland, & Geiger, 2018).

This special issue of ALMIJ includes some of the papers presented at ALM24, in Rotterdam (NL). The conference theme builds on the idea of all hands-on math, providing examples of inspiring and practical mathematical activities for adults.

The first article of this special issue is devoted precisely to the concept of numeracy, in the case of refugees. Joana Caniglia introduces a cultural approach to this topic. She discusses the idea of "functional mathematics skills" drawing on the evidence provided by 12 women participating in the study reported in her article. Caniglia presents mathematics in context, in practical situations, mediated by cultural and linguistic constraints, as the case of using different units of measures when performing everyday activities such as shopping. Refugees must deal not only with prejudice, abuse and many times resistance from governments and some individuals as well but also with different languages, social structures, institutions, etc. Mathematics is universal. However, the way in which we use mathematics is culturally situated, as Alan Bishop wrote in his Mathematical Enculturation book (1991).

In the second article, Katherine Safford-Ramus and Diana Coben introduce the new publication after the two topic working groups devoted to adults learning mathematics at ICME-13. They narrate the process of receiving, reviewing and editing the different chapters for the book. It is a good overview of how to produce a relevant publication for the field. It also contains important information about previous ICMEs, giving the reader an overview of the development in the conferences of our field.

The third article introduces the concept of "innate mathematical ability." Colleran draws on classic philosophers as the basis for this concept. The contribution of this article is very relevant since it provides good evidence suggesting that intelligence, in terms of mathematical thinking, is innate to human beings. No one is lacking the intelligence to think mathematically, thus mathematics has to be inclusive: mathematics must be for all. This statement has an incredible revolutionary potential in the field of adult education since many adult learners feel they are not able to learn mathematics. Drawing on Kant's works, Colleran claims that "mathematical ability" is innate. Thinking mathematically is a form of perception typical to human beings, due to their own human nature. According to him, mathematical thinking is like space and time in Kant’s terms. As those two categories are not emerging from experience, but are innate to all human knowers, mathematical thinking as well. Readers would find in this article arguments to transform the idea that mathematics is not for everyone; on the contrary, Colleran has been able to present a really egalitarian approach to mathematics education.

The next article goes back to the concept of numeracy, but focusing on the field of statistics. In this article, Evans discusses how people feel disconnected from "expert" statistical methods, with terrible consequences as result. What Evans calls "disenchantment" about statistics, becomes a huge barrier excluding people from participating actively and critically in society. The availability of big data would seem to make life easier for us, allowing us to make more informed and rational decisions, with better results for all. However, the fact is that many people rely on expert systems (based on unclear statistical algorithms and procedures) without having the competences to critically interpret the data. "Disenchantment" reminds me of Max Weber’s metaphor, the "Iron cage." Weber analyzed the consequences of bureaucratization (as an unexpected consequence of Modernity), highlighting that the rationalization of expert systems of the lifeworld (in Schütz's terms) produce a loss of enchantment in the social structures and processes. For instance, when schooling is mandatory for everyone, children do not appreciate attending school; but two or three generations ago, when going to school was a privilege for a few people, attending the school was a dream for many children who would never have that opportunity in their lives. In a similar vein, statistics was a democratic instrument to make decisions in modern societies. Social policies (as affirmative action) relied on statistical information for their evaluation and legitimation. However, when people forget statistics for different reasons (like statistics are deeply embedded in routines managed by "artificial intelligence" items), then they become uncritical. Evans's article also elaborates on the crisis that controlling and manipulating information would mean for citizens, exposed towards malpractices. Probably this is one of the main reasons for passing the new European law about the use of personal data in electronic sources, the GDPR.

The fifth article is also devoted to statistics. In this case, Kontogianni and Tatsis discuss the notion of statistical literacy, following in a way Evans's postulates. They focus on the ability that adult learners have to interpret critical information in a graphical representation. The Greek authors claim that although some of the adults that they observed in a Second Chance School in Greece were able to interpret data displayed as graphs, they were not able to develop a critical sense of the data. This is a serious result, since data (and visual representation of this data) is permanently available every day, everywhere. People must deal with situations in which numeracy is embedded, and statistical information (and procedures) is one of the most common types of situations. In this sense, the gap identified by Kontogianni and Tatsis between those students' personal opinions and the actual data displayed in the graphs is a serious problem that has to make us reflect on our work as educators of adults.

The last article included in this special issue is devoted to discussing what people like about mathematics. In this article, Ionica Smeets introduces some curious situations and anecdotes about mathematics that can awaken people's interest in mathematics, as the starting point. Then, she elaborates on a survey that she conducted through her column published weekly in De Volkskrant in the Netherlands. The questions were: "which number they thought deserved its own column and why." She obtained 203 responses to that question, raising an interesting range of topics. Further research would be needed to develop more in-depth her findings, which opens a promising field of research for the future.

All in all, the six articles included in this special issue provide a good opportunity for researchers in the adults learning mathematics field to discuss and elaborate further lines of research. I encourage the audience to enjoy the reading.

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# Teaching Function Mathematics Skills to Refugees

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## Abstract

In the United States, after approximately 90 days of English language programs, refugee adults are placed in the mainstream, where they face enormous challenges including navigating bus schedules, buying groceries with a different monetary system, shopping for necessities, and applying for needed social services. The complexity of the mathematics that these activities require poses a significant barrier for adult refugees with limited English and interrupted education. This action research study reports on a year-long project that sought to uncover a mathematics educator's assumptions and misconceptions brought to the teaching of functional mathematics skills for small groups of refugee women. Strategies to address teaching functional mathematics to refugees are provided within each section with an emphasis on knowing the stories of students and honoring them by creating an environment of welcome and high expectations for their success.

Keywords: functional mathematics skills, misconceptions, refugees

## Introduction

In a recent TED Talk, author and poet, Chimamanda Adichie tells the story of how she found her "authentic cultural voice"—and warns that if we hear only a single story about another person or culture we risk a critical misunderstanding (Adichie, 2009). Adichie's story describes the major concerns of refugees and their teachers—that of making assumptions and critiques of other cultures before teachers get to know their students' stories. The purpose of this study was to expand the stories of mathematics educators of refugees. From a year-long study of adult refugee men and women who were learning functional mathematics, this study investigated strategies that are effective and at the same time could expand the single stories that teachers may carry with them into a classroom.

### The Plight of Refugees

Who are refugees and where are they from? According to the 1951 United Nations Convention Relating to the Status of Refugees a refugee is "a person who owing to a well-founded fear of being persecuted for reasons of race, religion, nationality, membership of a particular social group, or political opinion, is outside the country of their nationality, and is unable to or, owing to such fear, is unwilling to avail him/herself of the protection of that country" (United Nations, 1951, p. 137). Statistics on refugees are provided by the United Nations High Commissioner on Refugees (UNHCR, 2016). Syria remains a leader in numbers of displaced persons: 12 million people, or two thirds of the population is leaving. Colombia has 7.7 million refugees, then Afghanistan (4.7 million) and Iraq (4.2 million). South Sudan follows with 3.3 million with the fastest growing number of refugees. While refugees may flee to other countries, many may spend years in camps waiting to return to their own country or to another one selected by the United Nations Immigration Division (UNHCR, 2016).

> The human tragedy of massive forced displacement continued to unfold around the world during the first half of 2016 with conflict, persecution, generalized violence, and violations of human rights causing forced displacement to increase further. The first half of the year saw persistent conflict in many regions, notably Nigeria, Yemen, South Sudan and the Syrian Arab Republic (Syria), leading millions to flee their homes, most remaining displaced within their own country but many also leaving for other countries (UNHCR, 2016, Introduction).

The refugee statistics are both daunting and in a constant state of flux. The United Nations Refugee Agency's annual Global Trends study found that 65.6 million people were forcibly displaced worldwide at the end of 2016 – approximately 300,000 more than in the previous year (United Nations High Commissioner for Refugees (UNHCR, 2016)). It noted that the pace at which individuals are becoming displaced remains very high. On average, 20 people were driven from their homes every minute last year, or one person every three seconds. The total number of refugees includes 40.3 million people uprooted within the borders of their own countries, about 500,000 fewer than in 2015. Meanwhile, the total number seeking asylum globally was 2.8 million, about 400,000 fewer than in the previous year. However, the total seeking safety across international borders as refugees topped 22.5 million, the highest number seen since UNHCR was founded in 1950 in the aftermath of the Second World War (Ratha, Eigen-Zucchi, & Piaz, 2016).

An examination of where refugees to the U.S. have come from and their numbers provide a glimpse into global events and the U.S.'s role in providing a safe haven (Pew Research, 2017). Of the 84,995 refugees admitted to the United States in fiscal year 2016, the largest numbers came from the Democratic Republic of Congo, Syria, Burma (Myanmar) and Iraq. Although the countries of origin have not significantly changed, in spite of shifts in policy, the end of 2017 found only 24, 559 refugees resettled in the United States.

Table 1.
*Functional Mathematics Skills Delineated by NRS Levels*

| Functional Math Skills | NRS Learning Standard |
|---|---|
| Support Services, Cardinal Numbers, Money, Ordinal Numbers, Personal Information, Shopping, Telephone Use, Telling Time (hours), Bus Time Table | Beginning literacy |
| Telephone Use, Leaving a Message, 911 Calls, Before and Now, Telling Time (1/2 hour and beyond) Liquid Measure | Low Beginning ELL |
| Using the Newspaper to Buy Groceries | High Beginning ELL |
| Using the Newspaper to Study Restaurant Ads Banking | Low Intermediate ELL |
| Job Applications Warning Labels | High Intermediate ELL |
| Getting the Facts Using the Newspaper to Buy a Car, Using the Newspaper to Find an Apartment Insurance: Medical and Dental | Advanced ELL |

## Functional Mathematics

For the purposes of this study, functional mathematics skills are those defined by the National Reporting System for Adult Education (NRS, 2016). This is an outcome-based reporting system for the state administered, federally funded adult education program. Developed by the U.S. Department of Education's Division of Adult Education and Literacy (DAEL), the NRS continues a cooperative process through which state adult education directors and DAEL manage a reporting system that demonstrates learner outcomes for adult education. The project is conducted by the American Institute for Research (AIR).

The NRS divides educational functioning into six levels for English Language Learners (ELL). The six levels are beginning literacy, low beginning ELL, high beginning ELL, low and high intermediate ELL, and advanced ELL. The ELL levels describe speaking and listening skills and basic reading, writing, and functional workplace skills that can be expected from a person functioning at a particular level. The skill descriptors illustrate the types of skill students at a given level are likely to have. The descriptors of functional mathematics skills do not provide a complete or comprehensive delineation of all of the skills at a given level but provide examples to guide assessment and instruction. Upon DAEL approval, states may also use additional educational levels and skill.

The contents of functional math skills are listed in Table 1. Each entry includes both the content and the NRS category. As the above list of functional skills attempts to do, it is imperative that there be ways to help refugee adults and students both to survive the trauma of the refugee experience, and to gain competence as new members of the United States. This research seeks to answer the question: What are some of the myths and misconceptions that a functional mathematics teacher brings to her work and what strategies appear to be successful given understanding gleaned from in-depth tutoring and conversations?

## Methodology

### Participants

The demographic variables for the 12 women in this study including age, status within the community, country of origin, and length of stay in the U.S., are found in Table 2.

Table 2.
*Demographic variables and frequency of refugee women*

| *Demographic Variables* | *Refugee Information* |
|---|---|
| Age | 24-59 |
| Purpose of Attending Education | Citizenship (n=2)<br>Livelihood (n=10 |
| Country of Origin | Iraq (n=3)<br>Bhutan/Nepal (n=2)<br>Uganda/Congo (n=3)<br>Somalia (n=1)<br>Afghanistan (n=1)<br>Syria (n=2) |
| Length of time in United States | 10 days to 4 years |
| Number of Children | Each woman had between 1to 5 children.<br>(all school age)<br>All children were with their mothers except 2 who had their children left behind |
| Educational Experiences before they came to US | Little to no education (n=4)<br>Elementary Education (n=5)<br>Secondary Education (n=3) in Refugee Camps. |
| Employment | Part-time jobs afternoon shifts (n=5)<br>Unemployed (n=7) |

### Data Sources

Data sources consisted primarily of participants' work of basic functional mathematics lessons, author's notes within a research journal and conversations with refugees, support staff, and ESL teachers. This study was conducted between January 2017 and January 2018. This period was significant because of the multiple judicial challenges of presidential executive orders made by the United States executive branch. In addition, multiple agencies within a large, Midwestern urban area with extensive networks of supporting refugees during their resettlement period provided background information on lands of the refugees' origins.

### Data Analysis

As is consistent with qualitative action research, data analysis began with reading through the data multiple times and identifying initial themes. Coding procedures from grounded theory were utilized (Strauss & Corbin, 1998). The author began with open coding, a process through which data “are broken down into discrete parts, closely examined, and compared for similarities and differences” (Strauss & Corbin, 1998, p. 102). During the open coding stage, data were reviewed to understand what individuals were expressing in their responses. The second step in the coding process was to use axial coding, for the purpose of reassembling the data to develop connections and categories within the data across participants, and between student and teacher responses. This process supports research triangulation of data, a method of increasing trustworthiness in the data (Lincoln & Guba, 1985).

## Findings

The findings will be presented in three sections. The first is an examination of myths identified by refugee agencies and those found by the author as she taught functional mathematics. The second and third sections describe the content knowledge and cultural themes the author found while teaching the 12 refugee women. Within a discussion of each theme, strategies for teaching functional mathematics will emerge.

### Myths of Refugees

Daily images of refugees and migrants seeking safety in countries often far from their own have shocked the world. Countries and continents are confronted with tragic images of refugee-filled boats sinking. Host countries’ responses have been unpredictable; policies change nearly daily; people travel between borders and thousands die in the Mediterranean while others are saved. Given this chaos, many myths about migrants and refugees persist. By understanding these myths, teachers can better understand how students and citizens of all countries can lessen their misconceptions of policies. The following list of misconceptions and myths (Table 3) was prepared by the United Nations Regional Information Centre (UNRIC); International Organization of Migration (IOM); United Nations Development Programme (UNDP), UN Refugee Center (UNRC), United Nations High Commissioner for Refugees.

Table 3.
*Myths, Facts, and Answers to Questions*

| |
|---|
| **Migration is bad for the economy and economies in origin countries** |
| The proven reality is that migration brings benefits, fuelling growth, innovation and entrepreneurship in both the countries people come from, and in those they move to, if managed smartly. Migrants and refugees contribute to the economy both as employees and as entrepreneurs, creating new firms and businesses. |
| **Stricter border controls and measures like fences reduce irregular migration** |
| Building fences does not stop the refugee influx; it merely shifts it to other countries and increases human misery. Migrants and asylum seekers are more likely to resort to entering a country irregularly when there are no legal alternatives. |
| **Migrants and refugees take jobs away from local people** |
| Migrants accounted for 47% of the increase in the workforce in the United States and 70% in Europe over the past ten years according to the OECD. Migrants often take jobs that others are less willing to do or take, and can help fill gaps in the job market. |
| **Migrants and refugees want to come to Europe and the US? Is Europe is facing the world’s heaviest refugee burden?** |
| Turkey, Pakistan, and Lebanon are now home to 30% of refugees worldwide, followed by Iran, Ethiopia, Jordan and Kenya. |

### *Myth #1. The Resettlement Process is Straightforward*

> I thought everyone wanted to come to the U.S. We have many of the best advantages here. Then why are they so sad? I never believed that refugees may not want to come. It seems like a straightforward process to me. (Author 's Research Journal, Day 2)

In actuality, the migration process for refugees is initiated by the United Nations, and refugees have little input as to where their destination may be. The process of verification is not brief. It actually takes years before a person or family member may be allowed to migrate to other lands. Persons' identification documents are scrutinized. According to the U.S. State Department, 20 steps are necessary for those who wish to enter another country. These steps are included in Table 4. Because of the length of time between the initial screening and departure, officials conduct a final check before the refugee leaves for a final destination.

Table 4.
*Steps of Refugee Resettlement*

| | |
|---|---|
| 1. **Registration** with the United Nations.<br>2. **Interview** with the United Nations.<br>3. **Refugee status** granted by the United Nations.<br>4. **Referral** for resettlement in the United States. The United Nations decides if the person fits the definition of a refugee and whether to refer the person to the United States or to another country for resettlement. Only the most vulnerable are referred, accounting for less than 1 percent of refugees worldwide. Some people spend years waiting in refugee camps.<br>5. **Interview** with State Department contractors.<br>6. First **background check**.<br>7. **Higher-level** background check for some.<br>8. Another **background check**.<br>9. First **fingerprint screening**; photo taken.<br>10. Second **fingerprint screening**. | 11. Third **fingerprint screening**. The refugee's fingerprints are screened against F.B.I. and Homeland Security databases, which contain watch list information and past immigration encounters, including if the refugee previously applied for a visa at a United States embassy. Fingerprints are also checked against those collected by the Defense Department during operations in Iraq.<br>12. **Case reviewed** at United States immigration headquarters.<br>13. Some cases referred for additional review.<br>14. Extensive, in-person interview with Homeland Security officer.<br>15. Homeland Security **approval is required**<br>16. **Screening** for contagious diseases.<br>17. **Cultural orientation** class.<br>18. Matched with a **resettlement agency**.<br>19. **Multi-agency security check** before leaving for the United States<br>20. Final **security check** at an American airport. |

Although the refugee women in this study never shared which countries they would have liked to have been resettled in, they unanimously wanted to go "home." Statistics are essential to see a larger picture and to support advocacy efforts for refugees, yet the women's stories built a "bridge of empathy" (Fleming, 2017) that helped people to understand why refugees take the risks to come to this country.

The following strategies were developed by Canadian teachers as they met students who had experienced trauma during their lengthy resettlement (Calgary Board of Education).

- Build safety through routines. Create predictable environments and responses. Use routines to assist students to know what will happen next and why they are asked to do something.
- Establish regular activities, with consistent greetings and good-byes, daily reviews, transition point markers, calming activities, etc.
- Choose important routine events to celebrate (e.g., birthdays, holidays)
- Recognize and avoid triggers that may remind refugees of traumatic past events.

### *Myth #2: If One Knows Social Language Then One Knows Academic Language*

"What do you mean, 'quarter past two' when you said a quarter was 25 cents." (Refugee adult after being asked to show 2:15 on an analog clock)

The author assumed that if an individual could communicate socially, academic language would be an automatic by-product and easily assimilated. But it became evident that not only do English Language Learners have difficulty in acquiring academic language skills, but teachers from various disciplines (mathematics education included) have difficulty preparing students for academic and professional achieved with academic language. Functional mathematics skills require academic language abilities. The vocabulary of measurement alone can be daunting given that all refugees are from countries that use the metric system.

Cummins (1984, 1991,1994, 2000) explained that Basic Interpersonal Communicative Skills (BICS) and Cognitive/Academic Language Proficiency (CALP) are qualitatively different skills. BICS include skills such as pronunciation, basic vocabulary, and grammar required in everyday communication situations. Most immigrant students can develop these skills rapidly, with the result that "teachers prematurely assume that minority children have attained sufficient English proficiency to exit to an English-only program" (p. 27). In addition, Cummins criticized policymakers' demands for a quick transition to English-only instruction by stating that the policies are "veneers for the xenophobic belief that minority languages threaten social cohesion" (p. 27). In contrast to BICS, which involve contextual processing of language, CALP is a cognitively demanding process that is not embedded in a meaningful interpersonal context. Cummins (1981, 1992) reviewed numerous research studies that point to the interdependence of native and second language learning in advancing CALP skills and indicated that second language CALP takes five years or more to develop.

Strategies for teaching BICS and CALP are to enable the learner to communicate in simple language and to understand the meaning of what is heard. Asking questions and answering them takes much practice in functional mathematics skills. Collaborative learning cultures, role-playing, interviews and games make the language-building activities of BICS and CALP helpful in writing, reading, speaking, and listening, and creating a sense of community as well. Discussing current events has the whole classroom involved in conversations informally.

### *Myth #3: Math is a Universal Language*

"This is the way we do maths in my country." (Students were adding \$.34 + \$.45. One woman wrote tally marks to show tenths, and hundredths. while completing a page of addition and subtraction problems.)
If I did not know mathematics as well as I do, I would never consider their solutions and processes as correct. It took two and three reviews of their work to determine that they were correct. It was not incorrect, it was just different. (Authors' Research Journal).

Although some mathematical calculations and processes may be similar, once students begin to solve word problems or more complex problems, they encounter difficulties in academic language. Mathematical language presents them with words and symbols that have double meanings, like "table," and English expressions, such as questions asking for the "difference" between two numbers. Instead of an answer to a subtraction problem, some may respond by stating similarities and differences in numbers. The level of complexity and high degree of emphasis on academic language makes it more difficult to grasp thus needing more support.

Steinhardt NYU researchers have identified additional difficulties and confusions often faced by English Language Learners (ELLs and Mathematics, 2009). These include:

- Students must learn to associate mathematical symbols with concepts and the language used to express those concepts. Example: the symbol / expresses the idea of something 'divided by'.
- Mathematical texts frequently use the passive voice, a complex and difficult structure for many non-English speakers. For example: ten (is) divided by two and when 15 is added to a number, the result is 21; find the number.

- Mathematics also uses strings of words to create complex phrases with specific meanings, such as a measure of central tendency and square root.

Even if mathematical language can be considered universal, the language of 'doing mathematics within the classroom' is far from universal. The language of exploratory discussions, the discourse-specific mathematical talk, and the mathematical talk and writing taking place in the language of instruction, make it unique to each culture (Planas, 2001). Whether an English language learner or a native speaker, each one faces a challenge in learning to converse within the mathematics language. Moschkovich writes, "The communicative competence necessary and sufficient for competent participation in mathematical discourse practices… [involves] specialized vocabulary, syntax, organization, register and discourse practices" (Moschkovich, 2012, p. 22). Moschkovich suggests that the presence of ELL learners in the classroom can help build an awareness of the linguistic challenges we face as classroom teachers. Thus, instead of considering ELL as problematic, she considers English learners as a gift, because when one hears imperfect language with an accent, or has incorrect tense, students and teachers are reminded that even if you are in a monolingual English class, with students who are native English speakers, there are language issues going on there as well (Moschkovich, 2012).

### Content and Cultural Themes

In addition to encountering myths and misconceptions while teaching mathematics to refugees, the author found content and cultural themes that when examined, could provide greater insight for those who teach functional mathematics. Discussions within each theme will also describe strategies and research that may support adult learners.

#### *Content Theme 1: A Picture Is Worth a Thousand Words, But An Object and Gestures Are Worth More.*

"I like it when I can see what you are talking about."

"I can't understand the money without real money. I can't see the numbers"

"I liked it when you brought in a pizza and we found ½ and ¼ of the pizza, then you cut out papers and you showed us ½, ¼." (Student after three sessions on money and common fractions)

To develop vocabulary, the refugee women appreciated multiple representations in functional math. The Universal Design for Learning (UDL) framework emphasizes multiple means of representation, multiple means of expression, and multiple means of engagement (Rappolt-Schlichtmann, Daley, & Rose, 2012). UDL provides educators with a framework for all kinds of learners in mind. ELLs, while limited in their English proficiency, come to school with variability in their home language skills, from full oral and literate proficiency, to very limited skill sets (Meyer, Rose, & Gordon, 2014). In using UDL, the author was able to guide the development of measurement terminology and basic cooking by using pictures, utensils, recipes, bus schedules, and newspaper advertisements.

#### *Content Theme 2: Awareness of Sources of Confusion: Multiple Meanings of Words*

You said a quarter past five, but a quarter was 25 cents. Why isn't it 25 minutes after five? (Adult Refugee)

Adult learners in this study consistently were confused over the meaning of mathematics vocabulary. For example, the meaning of 'quarter' was difficult because of its dual meaning. Polysemous words, which are words with the same spelling and pronunciation but different meanings, can be confusing for adults to understand. Many words are used in math textbooks and teaching which differ from their everyday life meanings. Instruction in specific vocabulary is crucial because vocabulary knowledge correlates with math reading comprehension (Smith, 1997; Sidek, Rahim, 2015). Students also found that words functioning as a verb, a noun, or an adjective also have different definitions.

As refugees advance in their language skills, strategies to become competent in examining the context to decide whether the meaning of the conversion is closely related to the meaning they already know are essential. The author found that her role was not only to teach children about this language phenomenon, but also to help the women develop confidence in their ability to infer the meaning of a conversion. This requires us to help readers attend closely to context. It also means helping students identify the grammatical function of a word, which can be difficult for beginning English speakers and writers.
In one relevant study, Carlo et al. (2004) taught fifth graders about how English words work. Topics included learning about polysemy, learning the structure of morphologically complex words and understanding the nature of academic language. On the polysemy post-test, the ELL group made significant improvement compared to their pre-test scores, yet despite this gain they did not match the progress of students who spoke English. natively However, both groups -- ELLs and native English speakers -- made significant gains over the control group, who did not receive the intervention. Within the current study, students appeared to enjoy the experience!

### *Content Theme 3: Repetition, Repetition, Repetition…*

> "1 O'Clock, 2 O'Clock, 3 O'Clock…" Rote repetition of time. I think they got the sound and meaning of hours." (Authors' Research Journal—Week 6)

> "Oh no, they did not remember the times on the clock nor did they get what the numbers around the clock represented." (Authors Research Journal, Week 7)

Among the language learners in the study, repetition was essential. But the author soon discovered the importance of "deliberate practice." Deliberate practice is not the same as rote repetition. Rote repetition—simply repeating a task—will not by itself improve performance. This is what I found after one week with the adult learners. When using deliberate practice, which involved attention, rehearsal and repetition leading to new knowledge or skills, I was able to make the concepts understood. (Hambrick, et. al., 2014). Although other factors are necessary, deliberate practice with meaning appeared to be helpful in learning.

### *Cultural Theme 1: Understanding Background*

> "Why are some of the adults so sad? I asked for information from social workers and staff. They mentioned that many of the refugees are reminded of traumatic experiences. It seems that functional mathematics does not seem to be important to them. I need to pursue their background knowledge."
> (Author Journal Entry 26)

Upon returning from sessions of functional mathematics skills, I searched for available information on the refugees' backgrounds and their educational backgrounds. Linking to students' personal life experiences is beneficial for a number of reasons. Personal life experiences can help students find meaning in content learning, and linking to an experience can provide clarity and promote retention of learning. Relating content to students' personal lives and experiences also serves the purpose of validating students' lives, culture and experiences.

For example, Haynes & Zacarian (2010) note that in general, members of the dominant U.S. culture believe that children should be raised to think independently. The goal of education is to have children learn to think like adults when they are still children. Children's efforts to think and use their independent thinking skills are praised and rewarded. Their wants, needs, and desires are often viewed as of primary concern in the family.

However, many refugees come from collectivistic cultures in which the good of the individual is sacrificed to the good of the group. A person's moral worth is judged by how much he or she sacrifices for the group. Students from this type of culture work best when they can form a relationship with the group. They are "we" rather than "I" oriented. Because of their subtle influence, these factors are important for all teachers and administrators to know (Zwiers, et.al, 2017).

### *Cultural Theme 3: Parents Desire to Succeed*

> When adult students introduced themselves, and spoke of their children, they often said that their children were helping them learn at home. Their children often help the parents. (Author Research Journal).

It became clear that the adults in the study were eager to learn since they wanted to help their children with homework. Many children of these refugee women supported their parents in language development. This role reversal can be a source of culture shock. Because of different cultural beliefs, many parents may understand the concept of parental involvement differently from the way that U.S. parents do. These conditions may increase cultural dissonance.

Children may be encouraged to practice language skills by teaching their parents. The exchange of ideas and information encourages communication skills in listening, speaking, writing, and reading. When students become the “teacher” to family members, they are participating in a process termed “language brokering” (Tse, 1996). Language brokering refers to translation between linguistically and culturally different parties. Language brokers are often the children of immigrant and refugee families who serve as interpreters and translators for their parents and family members (Morales & Hanson, 2005; Tse, 1996). These children may accompany their parents to a doctor’s appointment to interpret conversations, help translate the content of a letter sent home in English, or speak on the phone (on behalf of a relative) to school personnel. Language brokers rarely receive formal training as translators or interpreters, yet their day-to-day experiences often draw on their bilingualism. In this study, the author encouraged parents to bring their children to class to demonstrate to children how their parents are learning and the importance of respecting each other’s ability to communicate.

## Conclusion

As Chimamanda Adichie summarizes in her TED Talk,

> Stories matter. Many stories matter. Stories have been used to dispossess and to malign, but stories can also be used to empower and to humanize. Stories can break the dignity of a people, but stories can also repair that broken dignity.

This action research study points to the importance of teachers' and administrators' understanding of the refugee experience and the dangers of assuming a single story. Misunderstanding the dire situations of parents, the role of trauma in refugees' behaviors, cultural differences, and best practices in language acquisition can hinder the teaching and learning processes. (Birman et al., 2001; Timm, 1994; Trueba et al., 1990). Although the list of strategies presented in this paper may be incomplete, this research strongly recommends the importance of knowing the stories of students and honoring them by creating an environment of welcome and high expectations for their success.

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# The Fruits of Research
# Editors' Perspectives on Publishing Work from ICME-13

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## Abstract

The Thirteenth Meeting of the International Congress on Mathematical Education (ICME-13) convened in Hamburg, Germany, in July, 2016. There were two Topic Study Groups (TSGs) dedicated exclusively to adult learners and the authors of this paper served on the organizing committees of these groups. Arrangements were made by the congress committee for the publication of peer-reviewed papers from each TSG by Springer International Publishing AG in a series of edited books. In this paper we focus on our experiences as editors of the monographs resulting from our two TSGs.

## Learning the Landscape

The International Congress on Mathematical Education meets every four years. The thirteenth congress (ICME-13) was held in Hamburg, Germany, from July 24 to 31, 2016. Among the sessions offered on the program are TSGs whose purpose includes the promotion of high-standard discussions of a variety of perspectives on the theme of the TSG as well as giving a broad overview on the state-of-the-art for that specific topic. TSGs serve as mini-conferences and are intended to display the progress of the discussion in the intervening years since the previous ICME, enabling the newcomer to get a broad overview on the state-of-the-art and allowing the experts to lead discussions at a high level; they represent 'the fruits of research' on each topic. The 56 ICME-13 TSGs covered a broad range of topics from pre-school to university mathematics education and included TSGs with historical, theoretical and philosophical foci, for example, TSG11 covered 'Teaching and Learning of Algebra' and TSG25's focus was on 'The Role of History of Mathematics in Mathematics Education'. Each TSG organizing team provides the members of their TSG with an overview on the international discussion as broadly as possible and allows for insight into less well-known strands of the discussion from under-represented countries. For ICME-13, the TSG was the major arena for participation. Participants were expected to associate themselves with one TSG and to stay in that group for all sessions (http://www.icme13.org/topic_study_groups).

ICME-13 was the fifth congress to recognize adult learners as a viable category of mathematics learners. There were 'adult' Working Groups for Action (WGAs) or TSGs at ICME-8 (Seville, Spain), ICME-9 (Tokyo/Makuhuri, Japan), ICME-10 (Copenhagen, Denmark), and ICME-11 (Monterrey, Mexico) and a TSG on 'Mathematics Education In and For Work' at ICME-12 (Seoul, S. Korea) as well as at ICME-13. The practical and financial organisation of an ICME is the independent responsibility of a Local Organizing Committee, operating under the auspices and principles of the International Commission on Mathematical Instruction (ICMI). Consequently each ICME has both broad international scope and a distinctly local flavour.

The International Programme Committee (IPC) for ICME-13 decided to offer two TSG topics specifically related to adult learners and invited people to join an organizing committee for each TSG. TSG3, entitled "Mathematics Education In and For Work", addressed vocational mathematics education. TSG6 embraced any aspect of adult mathematics education, as shown in its title, "Adult

Learning of Mathematics – Lifelong Learning”. Each TSG was encouraged to publish an optional pre-congress survey of their topic area. TSG6 did so and the TSG6 survey can be found at http://www. springer. com/us/book/9783319328072. There are 25 other topical surveys available on a variety of aspects of mathematics education and all are open access so readers may wish to view the series catalogue in addition to accessing the TSG6 volume.

It is common practice for TSG organizing committee members to publish congress papers after the conclusion of the congress. Edited books emanated from the ‘adult’ TSGs of ICME-8, ICME-9 and ICME-11 through a range publishers (Coben & O’Donoghue, Eds., 2011; FitzSimons, 1997; FitzSimons, O’Donoghue & Coben, 2001). ICME-13 differed in that each TSG’s monograph[1] was assured in advance of publication by Springer. Also, all initial versions of papers to be presented had to be submitted in advance via the ICME-13 portal. These were strictly limited to eight pages in length with a uniform editing protocol in place. This should have made the eventual monograph editing easier since these papers were, in effect,early versions of the final chapters. However, a variety of factors meant that this was not necessarily so. For example, not everyone took readily to the online system for submission of papers. Also, many authors were not native speakers of English and ICME-13 had not funded a language editor for the series. But that is getting ahead of the story.

## Filling the dance card[2]

The invitation to submit a paper to a Topic Study Group was issued in the announcement for the congress. Potential participants completed a paper or poster for consideration by the TSG organizing committee. Each paper was independently peer-reviewed by two people and the organizing committee then developed a program of presentations that spanned the assigned time slots on the congress program. All TSGs met at the same time; those with a large number of papers were allotted additional slots. TSGs with the opposite problem, a small number of submissions, were permitted to invite authors from other TSGs, or even people not attending the congress, to submit papers for their monograph but not to present in the TSG program. The organizing committee selected papers for peer review with a view to subsequent publication by Springer.

TSG3 was well-subscribed but the TSG6 organizers encountered two problems while ‘filling their dance cards’. The first was the result of the overlap of the three TSGs that would attract adult mathematics educators since research in one area does not preclude its utility for the other two. The field is small compared to other mathematics education research and arguably congress participants were spread too thin. This was tied to the second problem, namely that attendees had to commit exclusively to one TSG. Participants were not allowed to submit papers to more than one TSG and invited plenary speakers (a special category within the congress strata) could not be invited to publish a paper in a TSG monograph, even on a topic radically different from that of their plenary talk.

## Herding the cats[3]

While it was determined before the conference who would edit the post-conference publication, the real work on the volumes commenced in August, 2016, with authors submitting revised versions of their eight-page papers. The procedure to be followed was helpfully set out by Prof. Dr Gabriele Kaiser, the Convenor of ICME-13, in guidance issued to TSG organising committees. Accordingly, each paper was double peer-reviewed and this work was shared between members of the respective TSG organizing committees and external reviewers as required.

Record keeping was critical in order to monitor progress though the editorial process, keep track of submitted papers and reviews and chase up any that were missing. The TSG editors created spreadsheets modeled on the one used to monitor contributions to the ALM International Journal. These were used

[1] A monograph is a publication that focuses on a limited and specific topic.
[2] In earlier times, women attending a dance held a card that contained the names of their partners for the evening’s dance programme; similarly, at ICME-13 the TSG organisers had the task of organising their programmes.
[3] Cats, or academics in this case, are notoriously difficult to organize and control.

to track the peer reviewers assigned, the dates papers were sent out for review and returned, and the dates authors received their reviewers' anonymous comments and then returned their amended papers. For TSG6, only one author declined the opportunity to amend a paper and this was omitted from the final manuscript submitted for publication.

There were, of course, problems with intended deadlines and at times we felt we were herding cats. The authors, editors and reviewers were all gainfully employed and these work responsibilities necessarily took precedence over their original and amended submissions. The TSG6 editor took a light-hearted approach to prompting the authors, sending a reminder of publication deadlines at the beginning of each month. For example, the November, 2016, entry read:

> "Dear Authors,
> Can you believe that another month has flown by? I hope that your article is taking shape and that you have a tentative date for submitting it to me. Please let me know how you are doing."

This approach kept both the editors and authors aware of the progress of the volumes.

## Working with idioms[4]

Good communication between editor and authors, and understanding, or not, of the language used was crucial. English was the language of the congress and also of the TSG monographs. Each TSG team was allowed to choose a variety of English; the TSG6 team chose American English and the TSG3 team chose British English.

The international nature of the conference meant that many of the authors were not native speakers of English. The TSG6 editor-in-chief was the only native English speaker of the three editors so had the task of editing all submissions by authors whose manuscripts might need to be edited for clarity. This is always a delicate task. The editor walks the narrow line between correcting and clarifying - or altering - the author's intended message. In the event, all the authors graciously accepted suggestions and the editor emerged confident that she had not sullied the gist of the author's concept.

A further complication stemming from the mix of languages and underlining the delicacy of the editorial task was the potential for misunderstanding through the use of idioms. A humorous example of the gulf that can exist between the editor's or author's intention and the recipient's understanding of a message is shown below in a table widely circulated on the internet (Richards, 2015). Other languages could no doubt produce their own versions of the table, which serves to indicate the ease with which lines of communication can get crossed.

Table 1.
*What the British say, what they really mean and what others understand*

| What The British Say | What The British Mean | What Others Understand |
|---|---|---|
| I hear what you say | I disagree and do not want to discuss it further | She accepts my point of view |
| With the greatest respect | You are an idiot | She is listening to me |
| That's not bad | That's good | That's poor |
| That is a very brave proposal | You are insane | She thinks I have courage |

[4] An idiom is an expression whose meaning cannot be gleaned by a direct translation of the individual words that comprise it.

| | | |
|---|---|---|
| Quite good | A bit disappointing | Quite good |
| I would suggest | Do it or be prepared to justify yourself | Think about the idea, but do what you like |
| Oh, incidentally / by the way | The primary purpose of our discussion is | That is not very important |
| I was a bit disappointed that | I am annoyed that | It doesn't really matter |
| Very interesting | That is clearly nonsense | They are impressed |
| I only have a few minor comments | Please rewrite completely | She has found a few typos |
| Could we consider some other options? | I don't like your idea | They have not yet decided |

## Dealing with radio silence

Despite valiant efforts to keep things light, there is a fine line between 'reminding' and 'hounding' the reviewers, fellow editors and authors. Email was the main medium for communication and it is like an arrow - you shoot it off but never know if it hits the mark. Authors - and sometimes also editors - who have fallen behind may be embarrassed or feel harried or be unwell - for a variety of reasons they may not respond to messages. They can, literally, be halfway around the world so the editor does not have the option to appear in their office door and ask "So, how's it going?" or "Can I help?".

## Assembling the furniture

While the papers are being finalized, the editorial teams have decisions to make about the ordering, organization and formatting of the volume chapters. How should the volume be organized? Are there categories or themes that emerged from the submissions? This was perhaps a greater dilemma for the TSG6 editorial team as the topics of the papers submitted for the TSG6 book varied widely. After discussion, they decided that the chapters fell into four broad categories: Numeracy; Student Focus; Teacher Focus; and The Crossroads. By contrast, the TSG3 book broadly follows the structure used in the TSG3 meetings in Hamburg and is arranged around four key questions:

What makes for authenticity in mathematics education in and for work?
How do we make sense of mathematics in and for work using different research methodologies and theoretical approaches?
What is the role and place of mathematics in education in and for work?
What are the advantages and challenges of interdisciplinary approaches to mathematics education in and for work?

As the deadline for manuscript submission loomed, the next challenge was the existence of 'missing authors'. Diplomacy worked and the final TSG6 manuscript was compiled on time. Then it was time to check and double-check the formatting of each chapter. Formatting specifications had been sent from both the congress and Springer and these did not always align. Correspondence with the Springer staff clarified the discrepancies and a compromise template was defined which will be used for both books.

## Picking the fruit

Once chapters were selected they were checked for the chosen variant of English spelling and final grammar checks were run. Font size and section numbering were made uniform across the chapters.

Each chapter was then pasted into the manuscript document. A table of contents was constructed for each book after perusing other volumes published by Springer.

## Giving birth

The TSG3 book has had a difficult gestation, beset by illness and other delaying factors, so it lags well behind that of TSG6. Either way, once the manuscript has been sent off there is nothing to do but wait for the Springer contract as well as the result of external reviews. Based on these reviews, authors may be asked to make further amendments to their work. That having been done the revised manuscript will be submitted, proofs reviewed, and the volumes published and marketed by Springer.

When published the monographs will add to the growing literature on adults learning mathematics in a range of contexts - a literature to which ALM and ALM members have made and continue to make an important contribution at successive ICMEs and elsewhere. As editors we will be proud 'parents', waving our 'offspring' off to make their way in the world.

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## Appendix A. Draft Table of Contents for TSG3 Volume

## Appendix B. Draft Table of Contents for TSG6 Volume

**Contemporary Research in Adult and Lifelong Learning of Mathematics: International perspectives**

Katherine Safford-Ramus

**Conclusions and looking ahead**

Juergen Maasz

Colleran, N. (2018). Exploring the genealogy of the concept of “innate mathematical ability” and its potential for an egalitarian approach to mathematics education. *Adults Learning Mathematics: An International Journal*, 13(1), 25-37.

# Exploring the genealogy of the concept of ‘innate mathematical ability’ and its potential for an egalitarian approach to mathematics education

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## Abstract

Recent work by a number of researchers has argued that the capacity for mathematical thinking is innate to human intelligence. Much of the evidence for this conclusion is based on findings in fields as diverse as linguistics, genetics, evolution, archaeology, psychology, and philosophy. This paper argues that the genealogy for this development is sourced in the philosophy of the Enlightenment, particularly the work of Immanuel Kant. Kant’s seminal idea suggests that human intelligence had a natural and necessary capacity for mathematical thinking in the forms of space and time. This paper will explore the ideas of Immanuel Kant regarding space and time, particularly his views that the intuition of space provides the source for geometry while the intuition of time provides the source for number. A limited, yet sufficient, evaluation of recent relevant literature will be employed to illustrate that ‘new insights’ regarding innate mathematics ability can be ‘genealogically’ traced to the work of Immanuel Kant. Ultimately, this paper argues for the debunking of generally accepted agreement among some that many mathematics students have an innate capacity to do mathematics while others are innately incapable in this regard. With an acknowledgement of this ‘initial state’ regarding universal mathematics ability among young, as well as adult students, an egalitarian perspective regarding students’ expectations and achievements in mathematics is in view.

Key words: Enlightenment, Kant, space and time, innate maths, maths gene, egalitarian education.

## 1. Introduction

It is my contention that there are many who believe that people in general, and students in particular, are either good or bad at mathematics. On the basis of this belief students are regularly encouraged or discouraged from doing higher level and more challenging mathematics in schools and colleges. There is no doubt that students come to school, and adults return to education, with proficiency in mathematics which is wide-ranging, however, I will argue that this range, spanning from what might be called ‘prodigies’, on one side, to those who have ‘math phobia’, on the other, results neither from the learners’ giftedness nor from intellectual deficiencies, but from the amount of time they spend thinking mathematically and doing mathematics. I will argue that the element that encourages mathematical practice, on the one hand, and discourages practice, on the other, is accounted for through socialisation processes experienced by each individual. This paper will provide evidence that all of us, young and old, have an innate, natural ability to do mathematics to the highest level: we are all endowed with a ‘maths gene’. And, if Peuquet’s (2002) is correct when she asserts that adults learn in a similar fashion to younger students when confronted with novel situations, my argument is relevance to both adult and younger learners of mathematics. The basis for this argument is derived from the work of the Enlightenment philosopher Immanuel Kant. His philosophy provided revolutionary insights regarding space and time and many of his foundational ideas are continuing to reverberate nearly 250 years later. In the first instance, I will argue that Kant provides the 'genealogical' source for the concept of innate mathematical ability among humans. This will be followed by more current research, which will support Kant's most important ideas, and provide sufficient evidence to reasonably postulate that the Enlightenment thinker provides the most influential source for the concept of innate mathematical ability. I will then provide evidence that the amount of time spent doing mathematics is directly proportional to the proficiency levels achieved. I believe that the acceptance and employment of these two principles - innate mathematics ability, and practice makes perfect – among teachers of mathematics will, no doubt, lead to a re-evaluation of practice with young and, in particular, adult students of mathematics. Ultimately, I believe that practice based on these principles can provide for a more egalitarian approach to mathematics education for all.

## 2. Immanuel Kant's contribution to the concept of innate mathematical ability

Most serious philosophers will agree, to a greater or lesser extent, that Immanuel Kant brought about a transformation in western philosophy the likes of which had not been seen since the ancient Greeks: 'and [Kant's] work did indeed change philosophy permanently' (Hatfield, (2004 p. ix); 'within a few years of the publication of his Critique of Pure Reason in I78I, Immanuel Kant was recognised…as one of the great philosophers of all time' Guyer and Wood, (1998 p. vii); 'the Critique of Pure Reason is a philosophical classic that marks a turning-point in the history of philosophy' Kemp Smith (1918 p. viii); 'the most important phenomenon which has appeared in philosophy for two thousand years… the principal works of Kant' Schopenhauer (1818 p. xv). Much of this reputation is based on his most famous publication in 1781 entitled A Critique of Pure Reason. The metaphysical transformation that Kant brought about with his Critique was centred on the question; what is the range of human understanding? Or, from a negative perspective, what are the limits of human understanding? Accordingly, he turned his attention not to the product of human understanding but the producer; the instrument by which human understanding is generated i.e. human rationality. At the outset Kant was satisfied that human understanding and knowledge were constituted by both sensed experiences and reason, and both had a range within which they operated effectively – outside this range human understanding and knowledge was vulnerable to attack and could not be defended. The philosophical clearing in which Kant's position regarding the range and limits of human cognition is a good place to start, in particular the manner in which he distinguishes the noumenal and the phenomenal world.

### 2.1 The Noumenal and Phenomenal World

Central to Kant's argument in the Critique is his contention that there are two distinct versions of the world: the noumenal world and the phenomenal world. The noumenal world is the world as-it-is-in-itself; the world of beliefs, spirituality, feelings, etc., which are not accessible to human sense organs. And while we may speculate about the noumenal world, humans cannot know it. Humans can understand and know the phenomenal world because we have mediated access to this world through our senses. Knowledge of the phenomenal world is limited by human senses and our capacity to cognise perceptions mediated through those senses. Therefore, the range and limits of human cognition and knowledge lie within the phenomenal world. However, while the human faculty for knowledge is 'limited' to the world of phenomena, it seems to have 'limitless' capacities to generate knowledge within this context, particularly in the sciences, mathematics, information technology, etc., and we must remain ever vigilant of our own limitations and refrain from stepping outside the bounds of the phenomenal world.

## 2.2 Empiricism and Rationalism

The philosophical milieu from which Kant's Critique emerged was dominated by a spectrum of two competing doctrines regarding what constituted genuine human knowledge. This comprised, at one end, a form of radical empiricism, positing that there is an objective out-there-now-real world that we engage with and know, not immediately but mediately, through our five senses i.e. seeing, hearing tasting, touching, and smelling. According to Locke (1690), the human mind enters the world as a 'tabula rasa' (a blank slate) and human experiences cover this blank slate with our knowledge of the world. There are obvious difficulties with this approach because all humans see, hear, etc., differently and the development of any knowledge based on subjective experiences could never approach general or universal understanding. However, the empiricists were convinced that the only true source of human knowledge is through human experiences and were satisfied to push the breaks at this point and conclude that humans know through individual perceptions that aggregate and combine into ever more complex ideas and knowledge. While Kant accepted that our senses provide access to the phenomenal world he rejected the idea that knowledge is an aggregation of increasingly complex sensed experiences. Without some non-empirical faculty that forms, unifies, establishes coherence, and makes sense of these impressions, there is no possibility of knowledge. The empiricists were adamant that these forming and binding capacities are not given in experience and anything not so derived should be 'committed to the flames' (Hume 1748).

On the opposite side of the knowledge spectrum was a form of radical rationalism and chiefs among the rationalists were Descartes and Leibnitz. Like the empiricists, they too questioned the validity of human sensed experience in providing objective knowledge. However, instead of accepting that human senses experiences provide access to knowledge of the world, the rationalists rejected human experiences as much too vulnerable. The rationalists relied on the capacity of human intelligence alone to provide such knowledge. Rationalists:

> '… held that it is possible to determine from pure a priori principles [thinking and speculating without reference to vulnerable human experiences] of the ultimate nature of God, of the soul, and of the material universe'
>
> (KEMP SMITH, 1912, p. 13).

Descartes and Leibnitz contended that human thought, unfettered by subjective sensed experience, can determine objective reality. Again, Kant was satisfied that there was some validity in this view as human intellectual capacities play a fundamental role in forming and synthesising human perceptions to constitute human understanding. He contended, while the empiricists had stopped short, the rationalists had gone too far. Kant agreed that the rationalists provided the cognitive capacities to 'interpret' human experiences, and significantly, these capacities are available without reference to sensed experience. Fundamentally, the human knowing process is available innately, or, in Kant's term, a priori; unadulterated and without reference to human experiences and so 'pure'. While accepting the innate presence of pure reason, a nod of sorts to the rationalists, Kant equally accepts the empiricists' view that the context and source of knowledge is the world of human experiences. Kant provides an accommodation of the limiting aspects of both perspectives by accepting that human knowledge is derived only subsequent to human experiences - a posteriori - and these sensations are necessarily categorised and synthesised by an a priori, innate intellectual capacity, awaiting stimulation.

> 'But, though all our knowledge begins with experience, it by no means follows that all arises out of experience'
>
> (CRITIQUE, B 15).

[5] All references to the *Critique* henceforth will have a prefix 'B' to suggest the second edition in 1787. The Politis translation of 1993 is the version used throughout unless otherwise stated.

His conclusion is that humans have a capacity for receptivity through the senses (content), and a capacity for conceptualising through the intellect (concepts). Both are universal and necessary for the possibility of human knowledge.

> Thoughts without content are empty; intuitions without concepts are blind
>
> (B 74).

And so, as a result of Kant's distinction of the noumenal and phenomenal worlds, and his exploration of the competing aspects of the contemporary field of philosophy, he began his critique of the relevant organ: human intelligence. In the section entitled the Transcendental Aesthetic ('aesthetic' here refers to senses), which consists of not more than forty pages, he describes the mediating role played by human senses and, more importantly for this paper, his contention that all human experiences are grasped by the natural, pure, and innate forming capacities of space and time. According to Guyer and Wood, (1998, p. 7):

> … the "Transcendental Aesthetic" [has] been the subject of a very large proportion of the scholarly work devoted to the Critique in the last two centuries.

## 2.3 Innate Space and Time

The first key contribution of Kant's Critique, mentioned above, was his distinction between the noumenal and phenomenal world and all that flowed from that position. His second contribution, directly relevant to this paper, is the one element of the 'bridge' he constructs to accommodate his version of empiricism and rationalism. This element is provided by the 'forming' intuitions of space and time. According to Kant, the intuitions of space and time, necessary for human understanding, are not derived from experience. This means that space and time are cognitively available prior to any experiences of the world and so must be innately and necessarily available to all human knowers. Space and time provide the necessary and only faculties by which human experiences are formed, shaped, grasped, or, according to Robinson (2011), how we come to 'behold' the external world. This provides the basis for my hypothesis that the intuitions of space and time are there in the 'initial state' (Chomsky, 2000 p. 7), unlearned, innately accessible to all irrespective of one's experiences in the world. Later I will refer to Kant's contention that space and time provide the foundations for the sciences of geometry and number respectively, and this extra layer provides the source of my argument that the genealogy of the concept of innate mathematical ability originates with Kant's assertion that all human understanding requires the forming intuitions of space and time and, because of their relationship to geometry and number, all humans have an innate capacity to understand and do mathematics.

## 2.4 Exploring Space

The empiricists argue our understanding is derived from human senses and if it is not so derived then it is baseless. Kant, however, suggests that while all our knowledge arises out of human senses it is not the source of all our understanding. The concept of space, which he argues is not sourced through the senses, is non-empirical and therefore a priori. Without this a priori forming intuition, Kant argues, human understanding is impossible.

> By means of the external sense, we represent to ourselves objects as outside us, and these all in space. Therein alone are their shape, dimensions, and relations to each other determined or determinable
>
> (B 35).

Kant postulated that space is not derived from the relations of external objects but that external experiences are possible only with the a priori intuition of space. He builds his argument for the innate faculty of space by suggesting a number of thought experiments such as: we can never contemplate the non-existence of space, while we can imagine empty space. We can only think of one space, and when we talk of different spaces they remain parts of the one same space: space cannot be built out of parts of space (B 37). Space is intuitively infinite in quantity;

it can be bigger or smaller by an extra measure no matter how large or small (B 39). Space provides the intuition by which we put a shape on the world external to us and 'through space alone is it possible for things to be outer objects for us' (Guyer and Wood, 1998, B 44).

One of Kant's most innovative arguments regarding the innate intuition of space with regard to the appearance of objects was his employment of chiral objects: an object is said to be chiral if it cannot be superimposed onto or does not coincide with its mirror image. Kant employs left-handed and right-handed objects like human hands and gloves, spiral shells, etc., which 'obviously' appear to be chiral or incongruent counterparts when we look at them.

> …for the left hand cannot, after all, be enclosed within the same boundaries as the right (they cannot be made congruent), despite all reciprocal equality and similarity; one hand's glove cannot be used on the other
>
> (KANT, 1783 pp. 37-38).

If one describes a left hand in detail each detail is similar for a left hand as it is for a right hand, yet they appear different. And while they appear incongruent, the incongruence is not amenable to rational explanation. The apparent, yet 'obvious', difference can only be accounted for by the innate spatial intuition. (For an in-depth discussion on incongruent counterparts and chiral objects see Severo, R. (2005) and Bennett, J. (1970).

### 2.5 Moving on to Time

As for time Kant argues that ideas such as co-existence, succession or change could not be perceived were it not for the foundational and a priori intuition of time (B 45). All appearances are connected to time and cannot be contemplated outside the substratum of time i.e. past, present, and future. Principles of time that cannot be derived from sense organs, i.e. experience, such as: ''Time has only one dimension', 'Different times are not co-existent but successive'' demonstrate the a priori nature of time (B 46). And like space, different times are part of the one and the same time; time progresses infinitely into the future and regresses infinitely into the past; and so, is unlimited (ibid). Kant emphasised:

> …the concept of change, and with it the concept of motion, as change of place, is possible only through … and in time
>
> (IBID).

### 2.6 Space and Time as the basis for geometry and number

Kant makes the plausible connection between human intuition of space with the more formal science of Euclidian geometry. He also connects human intuition of time with number and motion.

> Geometry bases itself on the pure intuition of space. Even arithmetic forms its concepts of numbers through successive addition of units in time, but above all pure mechanics can form its concepts of motion only by means of the representation of time
>
> (KANT, 1783 p. 35).

### 2.7 Space and Time as Causality

The concept of causality, or cause-and-effect, is, by its nature, structured according to the sequence of time, i.e. succession of events, and provides the answer to 'why' questions with such answers beginning with 'be-cause'.

> The… causality of a thing is the real which, when posited, is always followed by something else. It consists in the succession of the manifold insofar as that succession is subject to a rule
>
> (B 183).

Schopenhauer's *The world as Will and Representation* (1819) acknowledged the importance of a priori space and time, however, he collapsed the remaining categories, identified by Kant in the Critique, into the single notion of 'causality'. Schopenhauer went further by arguing that causality is constituted by space and time.

> What is determined by the law of causality is therefore not the succession of states in mere time, but that succession in respect of a particular space, and not merely the existence of states at a particular place, but in this place at a particular point in time. Thus change, i.e., variation occurring according to the causal law always concerns a particular part of space and a particular part of time, simultaneously and in union. Consequently, causality unites space and time
> (SCHOPENHAUER, 1818 p. 10).

It entails the logical succession of events of time in space and provides the ground for the, sometimes unconscious yet necessary, 'if-then' structuring in mathematics.

**2.8 Summary**

Implications regarding the growth of the science of geometry from the roots of a priori space, and the science of number, sequences, series, motion, etc., deriving from a priori time; and furthermore supported by the view that causality, a concept indispensable to mathematics, is constitutive of space and time, cumulatively provide the basis for my hypothesis that the genealogy of the concept of innate mathematical ability begins with Kant. Taken together, these innate knowledge-constituting intuitions provide an empowering worldview regarding human mathematical ability.

After concluding my argument that Kant provides the genealogical source for the concept of innate mathematical ability, I will now turn to more recent research, which argues, in a more focused manner, that human intelligence is innately mathematical. I will begin with the work of Donna Peuquet's (2002) Representations of Space and Time: her research articulates with Kant's foundational insights i.e. that space and time provide necessary intellectual concepts for the creation of human understanding and knowledge.

## 3. The necessity of space and time for human understanding

Donna Peuquet's (2002) Representations of Space and Time is a book about geographic space and the dynamics that occur in that space. While the context is computer-based geographic data processing, there is substantial content on theories of how humans acquire, store, and use spatial knowledge. She believes that things change in space over time; both space and time provide an integrated representation of our experiential world. This sounds a lot like the spatial and temporal intuitions proffered by Kant (1781), and indeed Peuquet does make much reference to the work of Kant. Peuquet suggests that space and time, being the most fundamental of notions … 'provide that basis for ordering all modes of thought and belief … Kant's space and time are concepts that we possess at birth (pp. 11 and 21). And while both concepts are innate it does not follow that all will make optimal use of these intellectual resources.

> The various ways that space, time, and their properties may appear to individuals are due to differences in attention to detail … access to technology, education … [etc.] (p. 25)

And while it is evident that people experience the world subjectively there is growing evidence:

> ... that the processes used to organise information are innate and either largely independent of the environmental input or dependant on kinds of environmental input that no human can avoid encountering (p. 28).

In her section on Schema: The Link between Percepts and Concepts, she again refers to Kant's view that schemata allow what we gain through our senses (perceptions) to be interpreted (concepts) and thus to be given meaning: schemata provide the bridge between experiences and meaning (p. 85). Research in cognitive linguistics has identified over twenty-four different image schemata and many of these intellectual bridging concepts are fundamentally spatial and temporal in nature. The schemata include:

> … container, balance, … path, cycle, centre-periphery, and link. Although these are called schemata, they are fundamentally spatial in nature. Our schemata for spatial and temporal orientation are so fundamental and pervasive in our experience that they are usually taken for granted (p. 87).

She refers to studies in visual cognition by Johansson (1973) to illustrate the importance of time in understanding. Given this understanding of schemata she concludes that even the youngest children employ space-time schemata to enable learning about the complex world they experience. Furthermore, learning is a similar process for adults and children in contexts where pre-existing knowledge is unavailable; however, what is significant of an increased capability among adults in a novel context is a larger store of knowledge:

> …Our basic notions of space [and time] are fundamental to learning and understanding [for young and old] in all domains (p. 88).

This insight has clear implications for teachers of mathematics to adults: while the learning process is similar for young and old, adults regularly employ a larger store of knowledge, including mathematical knowledge, not available to younger students. (This sophisticated, and often undervalued ‘commonsense mathematics’ resource available to adults is explored in some detail in Colleran and O’Donoghue (2007).

In her analysis of our perceptual field she reminds us that all our sense organs operate in a temporarily sequential manner. While most attention in the psychological literature has focused on visual perception, which provides information about size, distance, shape, and texture, she points out that hearing provides information about size and distance, and all senses provide information regarding pattern. All our senses are temporally extended because no single event affords the sequence of perceptions that provide the basis for the emergence of a pattern.

> All our senses are temporally extended…. [With regard to listening, which] is perhaps a more temporally extended activity than other senses... there is typically no single moment in which one hears anything, because sound waves themselves are a space-time phenomenon (105).

The fundamental requirement of pattern in the creation of understanding demands, at a conscious or unconscious level, attending to sequences of sounds, tastes, touches, etc., so that we can relate a particular perception to familiar categories. It is on this basis only that one can identify familiar sounds, images, tastes etc. Obviously, if there is no pattern recognition a new pattern category is developed. All our senses operate within a temporally sequential series of perceptions.

Over time attention has been paid to the contribution of individual senses however Peuquet suggests that an holistic analysis of the contribution of all senses to human knowledge creation can provide a more fruitful approach.

> The current body of evidence supports the view that our senses provide a unified and interrelated suite of sensations and that we understand how these sensations are related very early in life. (p. 108)

Research in psychology provides evidence supporting this unified and interrelated process in gathering spatial information. Furthermore, because of this process, people with deficiencies in one sense area, for example vision, compensate with other senses, such as touch and hearing. Current thinking suggests that… ‘encoding our spatial knowledge is innate and not keyed to any particular sensory modality’ (p. 110).

In her analysis of language as a symbolic system Peuquet finds that while there are many cultural variations when it comes to languages there is an structural invariance regarding spatial expressions. She suggests that this invariant structure indicates a ‘common cognitive structure of spatial knowledge at some deep fundamental level’ (p. 166). All languages are constituted predominantly by nouns, verbs, and adjectives, and these can be augmented as the evolving situation demands, for example new technologies. The grammatical elements of a language include prepositions, conjunctions, etc., and these are limited in number. With regard to spatial relationships there are between 80 and 100 relevant prepositions. Spatial and temporal relationships are invariably included within the grammatical structure of a given language (p. 168). The English language provides the verb-ending –ed to indicate past tense; and prepositions ‘above’ and ‘below’; ‘near’ and ‘far’; to refer to space. Temporal relations are referred to with the words ‘before’; ‘during’; and ‘after’. There are also space-time prepositions referring to motion including ‘across’; ‘through’; ‘into’. Peuquet concludes that ‘it does seem to be the case that spatial language encodes the world’ (p. 175). Furthermore, the fact that the number of spatial and temporal terms is very limited and difficult to increment, plausibly implies an invariant, and fundamental structure essential to the manner in which we perceive and understand the world. Further supporting evidence that mathematical concepts, and therefore, mathematical thinking, is integral to language is provided by Devlin (2000) below.

Having created connections between Kant’s contention of the innateness of the mathematical concepts of space and time with more contemporary work, I will now explore recent research providing substantial evidence that our capacity for mathematics is innate and universally available to all humans. This will include Devlin’s *The Maths Gene* (2001) and Butterworth’s The Mathematical Brain (2000). I will first turn to Devlin’s work.

## 4. Mathematics ability available to all humans

Devlin (2001) provides a human-evolution approach to his argument that mathematical thinking is innately available to all human knowers. His argument is based on the view that the human language faculty is there in the ‘initial state’ not unlike our capacity to walk, or become men and women through puberty: the language faculty just happens. Devlin’s source for this hypothesis is derived from the work of Bickerton (1995), however, many would argue that the seminal work on linguistics was done by Chomsky, (a summary is provided in Berwick and Chomsky, 2016). Devlin argues that the language faculty and the human ability to think mathematically are derived from a single mental human ability: the ability to think off-line.

> The two faculties [mathematics and language] are not separate: both are made possible by the same feature of the human brain … [our] genetic predisposition for language is precisely what you require to do mathematics … thinking mathematically is just a specialised form of using our language faculty
>
> (DEVLIN, 2001, pp. 3-4).

Devlin, like Chomsky, has a difficulty with the proposition that language is an evolutionary development derived from the need to communicate more effectively. While there is no doubt that language is the most effective means we have to communicate it certainly is not the only medium. We can communicate by the way we dress, the way we do our hair and make-up, our body language, our facial expressions, and so on. Devlin plausibly argues that language is the externalisation of human thoughts, speculations, plans, understandings, etc. In this view language is the most useful means to communicate complex, and not so complex ideas, among humans, however that is not the original evolutionary purpose for language; language was developed because we needed it to think off-line and so, language is primarily the process by which we think. So, in one of those quirky, yet fortunate evolutionary accidents, human thinking, externalised in the form of speech, provided an extraordinary advantage regarding human development in the last 70,000 years. Integral to the development of language, Devlin argues, was the development of our ability to think mathematically: ‘…mathematical ability is nothing other than linguistic ability used in a slightly different way’ (ibid, p. 22).

Devlin refers to substantial research suggesting an innate mathematical capacity among young children. He concludes that it is not just a correlation but, in fact, a symbiotic relationship between language development and our ability to think mathematically.

> I do not believe that a basic mathematical ability is any more unusual than an ability to talk (p. 126)

Devlin supports his position by referring to evidence that all human languages (known presently) have the same universal grammar. Chomsky’s observation that children cannot learn complex syntactic structures because they are not given or taught particular examples by parents, or anyone else that has those structures, leads to the inescapable conclusion that we must be born with the capacity for language.

> ‘… [G]rammatical structure is innate in much the same way that spinning webs is hard-wired into the spider’s brain’ (p. 157).

The synthetic structures inherent in language provided the essential resource for off-line thinking i.e. the capacity to reason in an abstract fashion. This in turn provides the capacity for mathematical thought (p. 162). Furthermore, he argues that while humans have been using language for nearly 200,000 years, with no apparent mathematical uses or developments, it is the mathematical structures inherent in human language that provided the natural source for the development of ‘formal’ mathematics over the past 3000 years.

Devlin’s research points to a two-stage development in the evolution of the human brain: the size of the brain increased over 3,000,000 years to allow for the development of more patterns and capacity to respond in a survival manner to new and various patterns. The second stage - 200,000 to 70,000 years ago – the brain didn’t increase in size but it changed structure.

> Those structural changes… gave us symbolic (i.e. off-line) thought … language, a sense of time, the ability to formulate and follow complex plans of action, and…. to design a … growing array of artefacts (p. 178).

However, even as far back as homo habilis (the size-change phase) there was evidence of capacities around number sense, spatial reasoning, cause and effect, and relational reasoning. It was the brain’s structural change that provided for abstract thinking, and this was the game changer: not a change in degree but a change in kind. An so our basic number sense, developed over 3,000.000 years, and now with the capacity for language from 70,000 years ago, the conditions were ripe for mathematical thinking in the form of numerical ability, algorithmic ability, and logical reasoning ability.

In an exploration of the necessary features of language to represent real-world situations Devlin asks: ‘which features of the world are absolutely necessary … and hence will be incorporated into the syntax and which can be optional? He concludes that ‘subjects’, ‘verbs’, ‘objects’, ‘tense’, gender’, ‘singular-plural’ are elementary to a thinking process capable of representing the world, i.e. off-line thinking.

> Off-line thinking provided the ability to think about past, present and future events, create tools [future orientated]… formulate and follow … plans of future action … logical reasoning (p. 236).

From this description of the necessary elements of syntax coupled with the ability provided by off-line thinking it is clear that many are related to space, time, and causality: verbs and tense are always related to time; singular-plural is related to differentiating space, while logical reasoning is essential in thinking mathematically. Devlin concludes... ‘the maths gene and the language gene are one and the same [and] mathematics is an automatic consequence of off-line thinking’ (237).

And so how is it that language has been used widely for more than 70,000 years while the development of mathematics stretches back less than 4,000 years? While keeping in mind that mathematical thinking is integral to language and language evolved primarily as a thinking process and not as a communication process, Devlin suggests language was hijacked by gossipers, and gossip was used to understand and care more about each other as humans, members of families, groups, tribes etc. Caring more for each other was the result of finding out more about each other. And caring for each other was a definite evolutionary advantage. In this understanding, the use of language developed a caring attitude among humans leading to group cohesion and the obvious advantages arising – language provided a major evolutionary advantage.

And so, for thousands of years this mathematical ability employed in gossip remained active yet invisible and undetected until a few thousand years ago when social and cultural developments, as well as the emergence of unique and exceptional thinkers, developed formal and abstract models to achieve the relevant mathematical outcomes. Gossipers, then and now, remained unaware and unburdened by the mathematical thinking integral to the language used to gossip. Gossip addresses similar questions to those of the mathematician – what is the relationship between? How many are there? What type? Are they the same? Are they equal? What is the property of... what characteristics does he/she have? and so on. Building an understanding of the relationships between people and the characteristics of each person/group is the material of both gossip and mathematics.

> 'The mental abilities required for gossip – even the most socially denigrated variety – are highly sophisticated, and already structurally adequate to support mathematical thinking… Mathematicians are not born with an ability that no one else possesses. Practically everyone has 'the maths gene' (pp. 249-250).

If mathematical thinking is as natural as learning a language or walking upright, why then do so many people find mathematics so difficult? The first part of the answer, according to Devlin, is that mathematical thinking is highly conceptual and abstract and what distinguishes a great mathematician from a high school student struggling in a geometry class 'is the degree to which the mathematician can cope with abstraction' (p. 253).

The second part of the answer is that we can become proficient at anything in life only by repeated practice. We become good musicians and writers by playing and writing... we become good mathematicians by repeating and practicing, seeing new angles and approaches for doing. Repeated practice is driven by, sometimes obsessive, interest and passion and it is this passion that differentiates those who can do mathematics well and those who claim to find it impossible.

> But for all its difficulty, doing mathematics does not require any special ability not possessed by every one of us (p. 258).

## 5. Humans have a 'Number Module' located in our brain

*The Mathematical Brain* (2000) by Brian Butterworth approaches the thesis that all humans have, what he terms, a Number Module, from an evolutionary, historical, neurological, and psychological perspective. Butterworth is a neuropsychologist and his interest in mathematics resulted from tests he carried out with people who had severe inabilities when it came to using numbers. Some of his patients had suffered stoke and other suffered brain injuries, while others, without injuries, appeared to be succeeding quite well but suffered a severe dysfunction with numbers. His hypothesis is that the Number Module is genetically provided and provides the basis for our ability to use numbers to interpret and operate in the world. And while some cultures are more advanced mathematically, Butterworth argues that the sophistication of number use is consistent with the technological levels achieved by that culture.

> Our mathematical brain … contains two elements: a Number Module and our ability to use the mathematical tools supplied by our culture' (p. 7)

However, people without access to the Number Module through injury or dysfunction cannot develop number skills to any level of sophistication and are grossly incapable when dealing with numbers. He proceeds to compare number deficiency i.e. dyscalculia, with dyslexia and colour blindness, as these too, result from a similar type of dysfunction in the brain. Consequently, dyscalculia is derived from the lack of a Number Module.

Butterworth sets the standard for the scientific veracity of his hypothesis - that all who function effectively with numbers have an innate Number Module - by presenting plausible evidence regarding a number of premises including the following:

1. Everybody should show evidence of ability to use nemerositie (categorising the world in terms of numbers of things)

2. Evidence must be shown among infants
3. Brain imaging should be able to locate the Number 'hot spots'
4. The Module must be encoded in our genes and must ten have been passed on by our ancestors
5. This may lead to an understanding why some people a very good while others are hopeless (pp. 9-10).

In his exploration of the history of the use of numbers he concludes that the variety of techniques and sophistication levels used across many cultures provide two conclusions:

1. Number techniques were not invented in one location and then disseminated to other cultures across the globe
2. This localised, cultural variety of number techniques provides plausible evidence that, like language, humans have an innate capacity to employ numbers and to appreciate how numbers can improve the way we live in the world (pp. 23 – 103)

Evidence related to experiments with babies, often as young as three months old, illustrate a capacity to differentiate groups of numbers, to recognise changes by adding and subtracting, and ordering numbers by size. It is these three elements that constitute the basic numerical capacities embedded in our Number Module.

In his study of the anatomy of the brain he discovers that the left side of the brain provides the capacity for mathematics, specifically in the left parietal lobe. He goes on to report on a number of case studies of individuals who had very serious difficulties with numbers while simultaneously being capable of operating very effectively where numbers were not concerned. He describes Charles who had A Levels and a university degree in Psychology. He concluded that Charles was deficient when it came to the innate Number Module and this led to his, and other case study subjects', inability with numbers. Consequently, if the Number Module is working effectively, it would seem that all humans can reach a proficiency equal in sophistication and expertise. However, we all know that this is untrue.

Some of us with a perfect genetically endowed Number Module find mathematics very difficult while others see no limits to what they can accomplish with numbers (while Butterworth did not include geometry skills, he did not directly exclude it either). As mentioned above the Number Module has to have something to work with i.e. culturally provided conceptual tools. And while this creates the limits to which all can reach, the overwhelming evidence is that most of us, in a culture that has developed, and continues to develop very sophisticated mathematics, do not reach those standards. There is something other than the Number Module and the cultural affordances required to ensure that all can become proficient mathematicians.

One of the stops put on the 'natural' development of number skills to the highest levels is maths phobia: a learned fear, specific to a situation and accompanied by physiological signs such as increased heart rate, sweating, etc." (p. 333) Students with this infliction do much worse at mathematics and avoid taking mathematics courses. Whether doing badly causes anxiety, or anxiety causes students to do badly is difficult to establish, however, the result is that students are drawn into a vicious cycle of poor performance, external discouragement, internal discouragement, anxiety, avoidance, no improvement, and so on. While the Number Module is available, phobia makes it inaccessible with the result that we avoid spending time with numbers.

On the other hand, Butterworth, like Devlin (2001), argues that differentiation is the result of training. He refers to Ericsson et al (1993) who suggest that the variation in ability to do well at any endeavour, be it music, sport, or mathematics, is 'drive' within the person. And this is manifest in 'deliberate practice, which is usually solitary' (p. 290). He goes so far as to say that 'obsession, however, does seem to be a necessary ingredient' (p. 294). As a result of developments in neuroscience and brain mapping technology, the concept of the 'plastic brain' suggests:

> '…long term, repeated practice at a skill will increase the number of neurons that the brain assigns to that skill on a more or less permanent basis … Long-lasting structural changes in the brain are dependent on practice. Use it or lose it!' … Most of us are born to count, but beyond that the only established limits to mathematical achievement are … zeal and very laborious work (pp. 313 - 314).

In summary, Butterworth is convinced that all of us, excepting those with brain injuries and brain dysfunctions, have a genetic predisposition to use numbers to a level of proficiency limited only by the sophistication of the cultural development one is born into and – agreeing specifically with Devlin (2001) - the amount of time and practice an individual invests.

## 6. Summarising the argument for innate mathematical ability

The basis for my argument that humans have an innate ability to be proficient with mathematics is sourced in the ideas of the Enlightenment philosopher Immanuel Kant. Kant argued that we are endowed with innate temporal and spatial intuitions. These intuitions provide the basis for the development of all scientific understanding and knowledge while simultaneously providing that basis for the science of numbers and geometry. Racing forward by nearly 250 years Peuquet, in the context of geographical research, provided evidence of the innate nature of space and time. She builds on this conclusion derived from Kant and argues that the structure of human language is also spatial and temporal in nature. Devlin's mathematical research, derived primarily from developments in linguistics, provides persuasive evidence that all of us have a 'maths gene'. He argues that language, currently employed quite effectively as an interpersonal communication medium, was initially used to carry out off-line (conceptual) thinking. Integral to off-line thinking is mathematical thinking employing spatial and temporal concepts among others, and the evidence for this is presented in normal language structures. Both Peuquet and Devlin link innate mathematical abilities to human language which all of us use relatively proficiently.

Butterworth presents evidence regarding a Number Module located in the brain and this Module is innately available to all of us. However, if the Module is damaged, or deficient in any way, the individual will have serious difficulties with numbers. Butterworth points out that while we all have the Number Module, all of us do not reach similar levels of proficiency. He suggests 'maths phobia' will create serious difficulties in developing number potential. Agreeing with Devlin, Butterworth concludes that the level of proficiency is directly proportional to the amount of time given to the practice of mathematics. And so, while we do have the innate capability, it takes drive, even obsession, to become extremely proficient.

## 7. Innate mathematical ability as the basis for a more egalitarian approach to mathematical education

I have argued that all of us, excepting those with particular intellectual deficiencies, are naturally capable of becoming proficient mathematicians. If this is the case, as adult mathematics educators we need to re-consider the manner in which we approach our profession. There is no doubt that our students come to us with a range of levels of mathematical proficiency. However, there seems to be a prevailing worldview among some teachers of adult mathematics that some people have mathematics abilities while other do not. The preceding argument calls this assumption into question. The evidence provided suggests that the range of proficiency is the result of the amount of time spent doing and practicing mathematics. We can speculate as to why some adult students do very little practice: math phobia, home environments, educational experiences, etc. We can also speculate why other students spend a lot of time practicing mathematics: a love of mathematics, home environment, educational experiences, etc. The challenge for adult mathematics educators is, firstly, to understand that all our learners are capable mathematicians irrespective of the level at which we meet them; and secondly, to provide a learning environment where students can unlearn the negative emotions, derived from various socialisation processes, that produces reduced expectations and motivation when it comes to mathematical thinking and doing mathematics. This is particularly important when it comes to adults as they have had more time than children and younger students to galvanise the negative and confidence-sapping beliefs of their inability to do mathematics. In this way, we can provide a more egalitarian mathematics education for adult as well as younger students.

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Evans, J. (2018). Statistics in public policy debates: Present crises and adult mathematics education .
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# Statistics in public policy debates: Present crises and adult mathematics education

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## Abstract

Statistics is one of the important branches of mathematics taught in schools, colleges and universities. It is also an important tool in public policy discussions. This paper focuses on the use of statistics in the latter context, rather than its use in adult mathematics education research. I review the key characteristics of the statistical approach to constructing public knowledge, and give a very brief history of key points in its development. I discuss how what I call the "overt crisis of statistics", the apparent disenchantment of large sections of the public with the "expert" statistical methods, outputs and pronouncements, leads to dilemmas both for citizens and for democratic governments. Recently "Big Data" and data analytics seem to many to offer new solutions to problems resulting from the essential lack of certainty surrounding efforts to understand society, and from the need to make quick decisions in a rapidly changing world. These approaches have potential, but also limitations. This leads me to consider a second, "covert" crisis of statistics, resulting from a struggle between proponents of freely available public information and public argument, and those aiming to profit from the appropriation and sequestering of information for private ends. I finish by considering what can be done by ourselves, as citizens, at adult mathematics teachers, and as researchers.

Key words: statistics; public policy; big data; data analytics; technology corporations

## Introduction

During most of our lifetimes, it has been accepted that, in most countries with a developed civil society, citizens and policy makers could rely on statistics – those produced by government agencies, or those from well-designed surveys from other agencies – to set a baseline for discussion and decision making. Thus we have quoted official statistics, results from the European Labour Force Survey, or figures compiled by the World Bank, in discussions of unemployment. Or we have used national household surveys, to estimate the numbers of victims of crime, or of those who rate their own health as poor.

Now, given events of the last two years, and subsequent public reactions, these previously accepted resources are facing new challenges. For example, a recent article by Will Davies in The Guardian, "Have statistics lost their power in public policy discussions?", has raised challenging questions regarding the role of statistics in public discussions:

> Rather than diffusing controversy and polarisation, it seems as if statistics are actually stoking them. Antipathy to statistics has become one of the hallmarks of the populist right, with statisticians and economists chief among the various "experts" ostensibly rejected by voters in 2016.
> (DAVIES, 2017)

Davies, writing just after the Brexit vote and the Trump victory, focuses on the UK and the US, but the issues apply more widely. Here, I first briefly consider key points of the statistical approach, and its historical development. I then explore the idea of a "crisis" in statistics, and argue that it is actually two different crises, based in different social groups. This leads me to consider the recently voguish notion of "Big Data", and the sorts of data analytics used with it. I finish with some suggestions about how citizens can withstand the most challenging features of the society that the large technology / media companies have established, and consider some ways that these ideas can be highlighted in the adult mathematics classroom.

## The statistical approach, and a brief historical development

In order to have a clear discussion, we need to understand that "statistics" can refer to three different aspects, though they are related:

(i) statistical data, and / or

(ii) statistical techniques of data analysis (e.g. averages, measures of spread and correlation, statistical models), and / or

(iii) the particular discipline, which of course includes "experts" in its ideas and procedures.

Key examples of the ideas and procedures of statistics include:

- The importance of investigating a representative sample from a specified population about which one wishes to draw conclusions, and familiarity with the methods of representative sampling and with the drawing of inferences from samples
- The importance of comparable and stable measurements of all the members of the sample, and knowledge of ways to assure the quality of such measures
- The *difference* between correlation and causation, and ways to design studies so as to be able to construct more dependable explanations for what is observed

From the late 17th century, the idea gained ground that statistics should be used to understand an entire population (not only potential soldiers, or tax-payers). Originally, this was not necessarily to be done using numbers, as in geographical descriptions of various German states, pre-unification. In England William Petty & John Graunt introduced the estimation of population size via counting of deaths, rather than via a census (costly).

In 18th and 19th century, in post- Revolutionary France, statistics began to be produced by trained cadres in a centralised statistical office. Across Europe and beyond, in data analysis, the normal distribution was found to be surprisingly powerful for supporting the growth of scientific knowledge, in quantifying and understanding apparently unrelated phenomena:

(i) errors of measurement (Gauss),

(ii) approximations to probabilities of gambling outcomes (de Moivre), and

(iii) the distribution of physical (and mental) characteristics (Quetelet, Galton).

This distribution was argued to underlie variation in a large number of natural phenomena, and so became an assumption of much data analysis well into the 20th century.

In the 19th and 20th centuries, around the world, specific indicators, clearly defined and systematically produced, were constructed for simplifying description of diverse and complex populations. Examples include: population size and vital statistics (births, marriages, deaths);

classifications of disease, national income statistics (e.g. GDP). Surveys and opinion polls of representative samples of the population, and of subgroups, using variations of simple random sampling (itself an advance on haphazard sampling) were introduced. Experimental designs (nowadays called Randomised Controlled Trials - RCTs) were introduced for agricultural trials and extended into the study of medicine and psychology; quasi-experimental designs were introduced from the 1960s, to increase their applicability to contexts where experimental designs were ethically or practically impossible. In addition, in line with a widespread general concern with comparative methods in the social sciences and history, there were efforts in statistical data production to enhance comparability across time, and across nations and subgroups. Overall, statistical data have allowed democratic countries, in particular, to sharpen their political agendas, and to design progressive policies, when the will and the resources to do so were available.

## The "overt crisis" of statistics and resulting dilemmas for citizens and democratic governments

Some dimensions of the current crisis include an increasing lack of trust in statistical data, and a consequent decline in their authority. For various reasons this has become particularly evident in the UK and the USA over recent years. For example, Davies (2017) cites survey results in the US which indicated that 68% of Trump supporters distrusted government economic statistics; and in the UK, that 55% distrusted data on "the number of immigrants living here"; see also Pew Research Center (2018, 14 May). This leads people to brand any evidence that seems contradictory to their preferred worldview as "fake news", or as something fabricated by "experts". Thus there is evidence of a lack of generally accepted baselines for discussing competing claims about society; and consequently a resort to "speaking one's own truth", and drawing on "intuition" and emotion as alternative bases of knowledge.

We can consider further some important aspects of these contemporary reactions to statistics. A key dilemma arises from the need to govern the population as a whole vs. (increasing) pressures to respond to feelings of particular citizens in a particular place and time. This can lead for example to a mismatch between what politicians say about the general state of the labour market, and local experience of the labour market, by individuals or by neighbourhood groups. Recently, such problems have been aggravated by a difficulty of satisfactorily portraying the state of the nation, with the use of summary statistics – because of the fragmentation of available identities and the foregrounding of differences within society. Even if one tries to be sensitive to social differences, by avoiding an overly crude use of averages, the available measures of spread, such as the standard deviation or the range, cannot capture the full quality of the differences currently emerging, say in sexual identity or political allegiance.

Thus there have been strains on existing classifications and definitions, due to changes in cultural politics – more fluid identities, attitudes and beliefs (emotions), and the reshaping of global economy and society. This has made various definitions more complex e.g. of unemployment, or GDP, or even gender. There has been an evident need not only to classify, but also to measure, say *intensity* of employment, or *commitment* to actually exercising one's "voting preference" on election day.

There have also been challenges in ensuring comparability across time, as the governance of states has changed (or fragmented), and especially comparability across nations, for example as the number and variety of countries participating in PISA has changed. For example, it is one thing to rank 10th in a set of 23 countries in the PIAAC survey; it means something different to rank 10th in a group of 62 countries in PISA.

## And now … Here come Big Data and Data Analytics

### What is Big Data? What are Data Analytics?

Big Data and data analytics are seen as possible solutions to pressing problems, such as limited research capability or the difficulty in producing the results of complex analyses in a timely fashion. Big Data can be characterised as the availability of exceedingly large amounts of data. However, these are accumulated by default, as a by-product of other processes, usually without attention to research design (e.g. sampling), but requiring the extensive use of electronic technology for capture, analysis, and presentation.

Examples of *Big Data* include the use of speed cameras or other video cameras, for behaviour monitoring, and for storage of alleged proof of mis-behaviour (allowing efficient legal prosecution). The use of loyalty cards allows monitoring of purchasing behaviour, plus correlation of such data with a number of demographic variables - "freely" produced by the card-holders themselves – so as to facilitate the targeting of marketing communications – with an option of experimenting with differential "special offers" (or experimental treatments). A further example is the harvesting of electronic texts – from individual acts of communication, which in an earlier time might have been assumed to be private, e.g. information searches, social media posts (and possibly emails and internet phone calls?). These texts can now be subjected to *data analytics*; this collection of techniques includes *data mining*, where many of the data analysis decisions are made by "artificial intelligence" – algorithms run by machines, rather than by human analysts. These are supplemented by *data linkage* (linking of data on a person from several databases), and *sentiment analysis*, used to striking effect by certain companies in the US election and the UK referendum (Cadwalladr, 2017).

Other examples are perhaps more positive: "Citizen science" (e.g. astronomical observation by many citizens) and "Citizen maths" (performing time-consuming calculations / simulations by many citizens). In contrast, Mass Observation, begun in 1937 and continuing in various forms to the present, was not electronically supported, and relied on named volunteers to do the interviews and the observations (Hubble, 2010).

### Issues with Big Data and Data Analytics

In methodological terms, the data involved is "big" indeed, i.e. not limited in the ways relevant to the pre-electronic period, but there are several serious limitations. First, the approach involves "haphazard" harvesting of large amounts of data – indeed impressive amounts. However, a huge sample can still be biased (e.g. Marsh, 1979) and, if there is no known sampling design, generalisation to any recognisable population will be difficult in principle.

In many cases too, the data comes without settled categories, since people can take on self-selected identities. This means that data from one database may be hard to "link" with data from another, and it thus may be difficult to analyse even degrees of correlation. Further, even if you have access to a huge data set, and that data shows a very *high correlation* between A and B, that still does not prove that A causes B!

Other more political issues arise for the responsible citizen – to do with freedom of the consumer (data provider), privacy and ownership of data. The "freely chosen" declarations of "informed consent" (EULAs) that individuals are asked to sign in order to use a range of applications provided by the technology companies – and that many sign in an inappropriately off-hand way – may be agreed to long before some particular data is extracted from the "user", and the permissions thereby granted are considered currently to be for forever. Data linkage raises not only technical issues (about how to do it accurately), but also issues of privacy: Would you want data from your medical records to be linked to your income tax return information, or to your Facebook page? If this sounds far-fetched, see the striking novel, *The Circle* (Eggers, 2014), which describes a fictional company, with a resemblance to a combination of Facebook and

Google, which proclaims a commitment to “total transparency” … with instructive consequences for the idea of privacy!

Much data nowadays is *appropriated* by private companies, for their own uses, in much the same way that common lands in English villages were appropriated by private landowners since the 17th century, during periods of “Enclosures”; see e.g. Polyani, (2001). These private companies have few or no obligations towards openness or transparency – though much rhetoric is often heard. Thus the user of the services, who is also of course the provider of the data, may never know what the data says about them – much less how it might be interpreted later by an unknown, and perhaps suspicious, user.

## The covert crisis: from a “logic of statistics” to a “logic of data analytics”

Thus, we have aspects of a second, “covert”, crisis of statistics, based on opposing ideas of knowledge. On the one side, we have the “experts” of the Office of National Statistics – bound by research ethics, and monitored by UK Statistics Authority – and on the other, the experts of Google, Facebook, and other less known policy actors, such as Cambridge Analytica (Cadwalladr, 2017-18, e.g. 2017). These latter appropriate data from unsuspecting individuals, link it with information available from public or privatised databases, analyse it (sometimes) and sell it on to a range of customers, to be used for purposes, including “tailored messaging” – by marketers, politicians, “opinion formers”. Some of these interests are oriented to maximising the appropriation of other people’s data, so as to maximise advertising revenues – the ‘media corporations’. Others may be oriented to undermining rational, open, public discussion of values and policy – the “ideologues”.

There are currently (July 2018) official investigations ongoing into the way these methods were used by the Brexit campaigns in the UK, by the Trump campaign in the USA, and by the media corporations themselves. This is clearly a continuing process, with many landmarks. An important one is the establishment of the General Data Protection Regulations by the EU in May 2018; see https://www.eugdpr.org/ .

## Summary

The “overt crisis” of statistics appears to result from the public’s disenchantment with the provision of statistics to be used as a basis for public discussions of policy. I have also aimed to describe a “covert crisis” lying behind the overt one, where certain interest groups are stoking the overt crisis for their own ends. For without statistics, and social research more generally, made available publicly and discussed freely (without interference or manipulation from unknown human beings, and non-human “bots”), we cannot construct unambiguous, objective, potentially consensus-forming claims about society – nor can we provide a corrective to faulty claims. In such a situation, there will be few mechanisms to prevent people from instinctive reactions and emotional prejudices.

Many have pinned their hopes on certain Open Data initiatives offered by state statistics and certain agencies. However, these public initiatives seem unlikely to be mirrored by the sharing of the results of data analytics by private corporations. In Davies’s (2017) judgment, data analytics is “suited to detecting trends, sensing the mood, spotting things bubbling up” – but not so much for the type of social explanation that many feel is necessary in an advanced democracy. Further, the numbers produced by data analytics are “generated behind our backs and beyond our knowledge”. And the results are appropriated, owned and sold on by private concerns – without the original providers’ knowledge!

Thus, the battle is not between “an elite-led politics of facts versus a populist politics of feeling” (Davies, 2017). Rather, it is between those committed to public knowledge and argument versus those who profit from the privatisation of information and “the ongoing disintegration” of public knowledge and argument.

## Conclusion: What might be done?

Here, we can focus on what might be done (a) by ourselves as citizens; (b) by teachers of adults' mathematics / numeracy; and (c) by researchers.

As citizens, it is important to rethink our relationship with IT and media companies, especially the "FAANGs" (Facebook, Amazon, Apple, Netflix, Google – and many users of Windows may not want to exclude Microsoft!).

a1. "There is no such thing as a free lunch.". So we need to read the EULA (End User Licensing Agreement) before we click to "Accept" the "free access" to software offered by many companies on the web. You are signing a contract, and you are giving something away in return: it is worth thinking about what that something is!

a2. Maybe there are still "free searches"? How many details of your life are on the file-server, of Google? Use gmail? Always "google" when you are searching? (There are alternatives: the search engine DuckDuckGo calls itself "The search engine that doesn't track you.")

a3. Maybe there is still "free" news? Of course, every news source must be selective. But the more they know about you and your "likes", the more selective they can be, so as not to disturb your bubble, and so as to keep you "clicking" (and providing them with income). The alternative is to get news from professional journalists, who take a somewhat broader view and will often have a long-term commitment to, and knowledge of, an issue - and they may occasionally come up with something surprising, like the Panama papers or the Paradise papers. Many good newspapers support the International Consortium of Investigative Journalists. But good journalism requires funds. In most countries, you can support a newspaper, by subscribing online, taking out a paid membership - or even by buying a copy, once in a while.

a4. "Think globally; act locally." Many things can still be bought at a local store, which employs local people, perhaps even some that you know. You can keep your Amazon account for the truly hard-to-find commodities.

a5. Many countries have "fact-checkers", e.g. agencies that check the more important claims made in the political and social sphere: e.g. Full Fact in UK (https://fullfact.org/). They are often charities that depend on financial support from members of the public.

As teachers of adults' mathematics / numeracy, we can encourage our students to consider their positions with respect to the trends described above, with the help of available statistics, and using surveys that can be done in the classroom.

b1. Many countries have available on the web a wealth of statistics produced by government or other agencies. For example, one could consider the data available on unemployment, and ask what it tells us about the current state of work, and "precarity" of employment (e.g. Evans, Ruane and Southall, 2019; Frankenstein, 2014). Or we could ask what is the level of migration into and from our country, and whether we could estimate the numbers of refugees, "economic migrants", and so on (e.g. Tyler, 2017). These are challenging questions, and we can expect one result to be that the students find that an apparently "objective" number comes with a lot of assumptions in these areas of discussion and indeed controversy!

b2. Examples can be given of cases in the era before "big data" where a very large sample could be very biased indeed (e.g. Marsh, 1979).

b3. Many examples of the difference between correlation and causation can be found in a good newspaper; a notorious example is the correlation over time between the number of storks in Germany and the number of human births – seeming to provide corroboration for the view that storks bring babies; see for example,

https://www.researchgate.net/publication/227763292_Storks_Deliver_Babies_p_0008 b4. There is scope for a group of students researching themselves, as to the level of their use of Facebook, Twitter, and Amazon – and their reasons for their use, as well as their beliefs about how their date is used.

As researchers, we might be interested in several types of research.

c1. Researchers might do the type of survey described above, but with a more wide-ranging questionnaire, and a larger and more representative sample.

c2. Further research and analysis is needed to investigate which feelings are most crucial in the "new politics of feeling" mentioned above. This need is most pressing for the groups characterised as "those left behind" in various traditionally democratic societies. on these issues. The most important would seem to be:

- Anxiety / Fear vs. Hope / Love
- Trust vs. Distrust
- Anger / Discrimination vs. Solidarity / Inclusion

Some forms of these feelings will be recognisable from the classroom, by mathematics educators and researchers. They are of course inter-related. For example, anger is often born of fear and anxiety and can be directed against recognisable "Others" (Mishra, 2017; Fraser, 2017).

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# Investigating adults' statistical literacy in a Second Chance School through the teaching of graphs

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## Abstract

Based on the significance of graph comprehension within statistical literacy, we present the findings of a study that took place in a Second Chance School in Greece. Our aim was to assess the progress of adults' comprehension of graphs. In order to offer to our adult students tasks with realistic context we used graphs, published in the media, some of them being potentially misleading. Our results showed that some of the adults have managed to move from the mere reading of the graphs to their interpretation. However, their critical sense was not so well developed since they often based their interpretations on their dispositions and not on their statistical knowledge.

Key words: statistical literacy; adult students; graph comprehension; Second Chance School

## Introduction

The skills that adults need in order to solve everyday problems which contain mathematical and statistical elements has been a topic for research over the last decades (Tout & Gal, 2015). This research area, which might be called Adult's Mathematics Education or Adults Learning Mathematics, can be placed in the borderland between mathematics education and adult education (Wedege, 2010). According to Wedege, its key concept is numeracy and the research field is related to adults, mathematics and lifelong education in a societal context. This means that there are different kinds of adult education settings (Evans, Wedege & Yasukawa, 2013), like adults' basic education (ABE) either in formal or informal contexts.

Acknowledging the importance of a modern citizen's ability to interpret visual data coming from the media, especially data representing statistical information, we organised a study with adults studying in a Greek "Second Chance School" (SCS), described more fully below. In particular, our aim was to monitor the progress of adults' statistical knowledge during the teaching of basic statistical concepts.

Although there are studies about students' graph comprehension in secondary education (e.g. Aoyama, 2007) or tertiary education (e.g. Monteiro & Ainley, 2007), there are not studies about students in adults' basic education programs. In this paper we focus on the adult students' graph comprehension and how this develops when they have to interpret media graphs and misleading graphs. Our research questions were formulated as follows:

- What is the students' level of graph comprehension?
- Do students demonstrate a critical sense towards the statistical information presented by the graphs?

## Numeracy and Statistical literacy

Although it is not easy to discriminate between the different notions of numeracy, it is a fact that numeracy can serve as a connection between mathematics and adult life (Evans, Wedege & Yasukawa, 2013). Similarly, Dalby (2017) concludes that numeracy does not refer to a simplified type of mathematics but concerns the way a person uses it. Developing the analysis, four dimensions of "numerate behavior" were identified for the PIAAC survey:

> (a) context (everyday life, work, societal, further learning), (b) response (identify/locate/access (information); act on/use; interpret/evaluate, (c) mathematical content (quantity and number, dimension and shape, pattern and relationships, data and chance) and (d) representations of mathematical/statistical information for example text, tables and graphs
>
> (EVANS, 2014, pp. 39-40, citing OECD, 2012).

Among these dimensions there are direct references to statistical knowledge and as a consequence to statistical literacy. There is no consensus at this time about the notion of statistical literacy in the relevant literature (Budgett, 2017). Gal (2002), referring to adults, describes statistical literacy as the "ability to interpret and critically evaluate" (p. 2) statistical information, as well as their ability to "discuss or communicate their reactions" (p. 3) to statistical information. He takes into account the impact that statistical literacy has for the effective citizenship since citizens are overwhelmed with statistics in modern societies. Gal divides statistical literacy into two components, a knowledge component and a dispositional one. For the knowledge component general literacy, mathematical skills and the ability to interpret graphs and tables are required. The dispositional component refers to the critical stance that adults should have towards statistical information presented to them, as coupled with certain attitudes and beliefs that would help adults to support their actions.

According to Watson (2006), graph construction and interpretation constitute a crucial part of statistical literacy. For the purpose of our research we focused on the definition of statistical literacy that concerns the "consumers" (Gal, 2002, p. 3), in other words, on statistics and the way that statistical literacy is expressed through the interpretation of graphs by users or readers. We focused on graphs that originate from media sources and may be misleading.

## Graph comprehension literature review

The research about adults' graph comprehension is focused on in-service/pre-service teachers (Gonzalez, Espinel & Ainley, 2011) or students of vocational education (e.g. Bakker & Akkerman, 2014). Moreover, in national surveys aimed to adults, like PIAAC (Programme for the International Assessment of Adult Competencies, OECD, 2012) or ALL (Adult Literacy and Lifeskills Survey) there are tasks related with graph comprehension. However, we did not locate many studies about adults' graph comprehension in settings similar to a Second Chance School. Specifically, for ABE programs there is the study of Conti and Carvalho (2014), who investigated the teaching and learning of statistics in adult education mathematics classes. The researchers designed and implemented a project that was carried out with grade 7 students, aged from 16 to 43, in a public state elementary school in Brazil. Throughout this project some of the students constructed statistical graphs while most of them managed to establish an initial level of statistical literacy.

Monteiro and Ainley (2006; 2007) used media graphs as an assessment tool for the graph comprehension of pre-service teachers. These researchers concluded that for the interpretation of graphs not only a certain level of statistical knowledge was needed but also a kind of 'critical sense', especially given that their participants relied on their personal opinions and dispositions for their interpretations. Respectively, Queiroz et al. (2015) investigated the way that students with different academic backgrounds interpreted media graphs. These researchers found that the students mostly expressed their feelings and their opinions rather than making an objective analysis based on statistical knowledge.

## Methodology

One of the main public institutions related to Adults' Basic Education programs in Greece is the Second Chance School (SCS). Its aim is to combat the social exclusion of adults who have not completed the compulsory secondary education and do not have the appropriate qualifications and skills to adapt to modern vocational requirements. SCSs are considered to be innovative, since they operate without pre-specified curricula, implement new teaching and evaluation methods and offer counselling to students (Efstathiou, 2009). For the Greek educational system, being conservative and rather rigid, the instruction and training in SCSs constitute an exceptional novelty (Koutrouba et. al., 2011). The duration of the studies in a Second Chance School is two academic years. The weekly programme consists of 25 teaching hours and the courses take place during the evening of all weekdays. Mathematics is taught for three hours per week in both cycles (grades) A and B. The main goal of mathematics teaching is the development of the students' numeracy and consequently their statistical literacy.

Our study took place in May 2015 and we used the methodology of a teaching research experiment (Steffe & Thompson, 2000). In particular, we designed an educational approach in which the basic statistical notions served as the basis for the development of adult students' statistical literacy. Initially, we designed a 12-hour sequence of lessons in accordance with the general guidelines for the teaching of statistics in SCSs (Lemonidis & Maravelakis, 2013). Our lessons contained: (1) data collection, data interpretation and organization, (2) reading and interpretation of basic data representations, (3) data description with statistical terminology, and (4) evaluation of arguments based on misleading graphs or incorrect statistical information. In this paper, we focus only to the lesson that concerned the graph comprehension.

For the purpose of this lesson, with a duration of two teaching hours (40 minutes each), we designed a series of eight tasks that consisted of graphs and their interpretation. These tasks were either adapted from relevant studies (PIAAC, PISA) or constructed from scratch for the purposes of the study. We based the task design on the following principles: (a) to include graphs within context, that show data from real-world situations and (b) to include misleading graphs since they are considered to be excellent cases for the students' motivation and assessment (Watson, 1997). Apart from their content, the tasks were formulated according to the learning goals and to a prediction of how the students' graph comprehension will evolve in the given contexts.

Although, according to the teaching experiment methodology an observer might be of great help to the teacher-researcher (Steffe & Thompson, 2000), in our case this was impossible due to the students' opposition. As a consequence, the mathematics teacher (first author) played the role of the researcher and tried as much as possible to share all the data with her fellow researcher. She was acquainted with the students' ways and means of operating since she had taught them for the previous six months. The teacher introduced new material with brief lectures at the beginning of the lesson, during which she posed questions for investigation and gave tasks to the students. For most of the course's duration, the students worked together with the teacher in order to answer specific questions included in the tasks. The students discussed statistical concepts, made conjectures, discussed the validity of specific arguments and applied the newly acquired knowledge to the next task. The teacher provided guidance, but she tried to interfere as little as possible and to promote with constant questions the students' active participation. The teaching episodes were audio taped and transcribed. Since the teaching was implemented to four different classes we had the opportunity to continuously discuss on the students' learning processes and on the role of the teaching materials we had constructed. This gave us the opportunity to make changes when it was needed and to revise some of the tasks or the related questions.

For the content analysis of the transcribed episodes, we used the levels of graph comprehension (Friel et. al., 2001; Shaughnessy, 2007) and the notion of critical sense (Monteiro & Ainley, 2007) as subcategories of graph sense. According to Friel et al. (2001) and to the additions made by Shaughnessy (2007), graph comprehension evolves through three ascending levels:

1. Reading the data: extraction of elementary information, recognizing components of graphs and detecting arithmetical information on graphs.
2. Reading between the data: understanding relationships among tables, graphs, and data, making sense of a graph, but avoiding personalization and maintaining an objective stance while talking about the graphs. Also, evaluating the graph for its constructive characteristics and detecting if it is misleading.
3. Reading beyond the data: making inferences from the graph in order to interpret the data, e.g., to compare and contrast data sets, to make a prediction about an unknown case, to generalize to a population, or to identify a trend. Recognizing appropriate graphs for a given data set and its context.

For the purpose of our analysis, critical sense refers to the act of mobilization (Monteiro & Ainley, 2007, p. 16) of diverse kinds of knowledge and experience during the act of graph interpretation. These researchers based their research on the levels of graph comprehension as proposed by Friel et al. (2001). In our study, we considered the critical sense and the graph comprehension levels as interrelated subcategories of graph sense. Graph sense refers to the general ability to read and deeply understand already constructed graphs found in the media (Friel et al., 2001).

## Participants

In our study, 47 adult students aged from 20 to 62 took part, in four different classes with 10 to 14 students each. They all had a primary school leaving certificate and they attended the first year of a SCS (lower secondary) in a small town, with mainly rural population, in North-West Greece. The mathematical skill levels of these students varied from basic elementary level through secondary level, since some of them had completed the primary school while others had attended the first class of the gymnasium (lower secondary education). Most of these people were unemployed or unskilled workers. The following table includes the demographic data of the participants.

Table 1.

*Demographic data of the participants.*

| | N | % of total | | N | % of total | | N | % of total | | N | % of total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gender | | | Age | | | Nationality | | | Occupational status | | |
| Male | 28 | 60% | [20,28) | 3 | 6% | Greek | 44 | 94% | Unemployed | 25 | 53% |
| Female | 19 | 40% | [28,36) | 10 | 21% | Albanian | 3 | 6% | Employees | 22 | 47% |
| | | | [36,44) | 20 | 43% | | | | | | |
| | | | [44,52) | 11 | 24% | | | | | | |
| | | | [52,60) | 2 | 4% | | | | | | |
| | | | [60,68) | 1 | 2% | | | | | | |

## Results

We have chosen to refer to specific tasks due to space limitations. One of the tasks was about a bar-graph (translated to Greek) with a truncated scale which is given below. This task was based on a graph taken from the internet and we constructed the question.

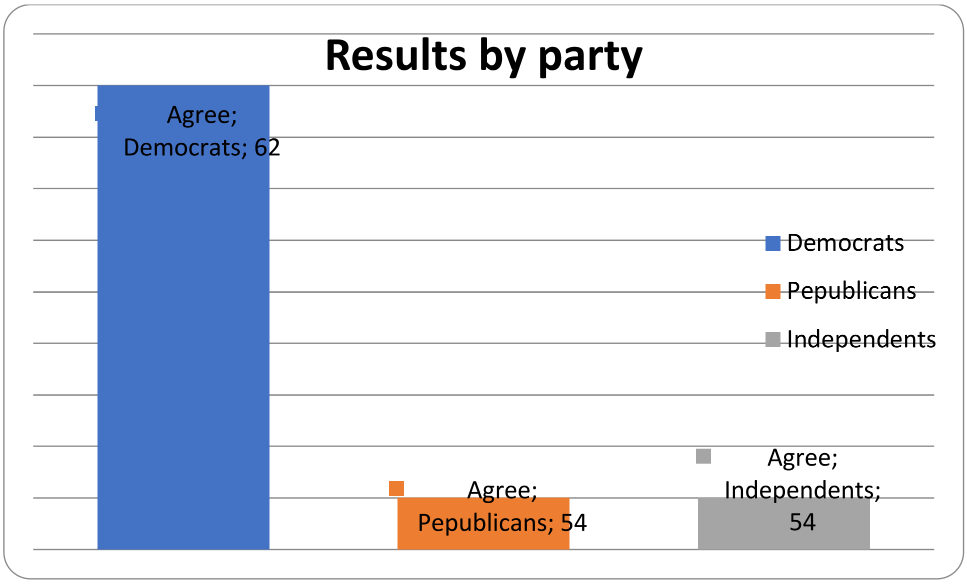


*Figure 1.* Third task. (adapted from Media Matters, 2005)

**Question – Third task:** In the United States, in 2005, CNN conducted a poll to test whether the voters for each party were in agreement with the court's decision about the Terry Schiavo's case and the next bar-graph was published. The source of the data was interviews conducted by telephone in March 18-20, 2005, with 909 adults in the United States. What is your conclusion based on this graph?

The first exchange comes from a class where the students discussed about the bar-graph and its accuracy. All adults' names appearing in the transcripts are pseudonyms. John responded negatively to the question posed by another student about the graph's correctness. He based his answer to the sum of the numbers that exceeds 100. He confused the bar-graph with the pie-graph where the sum of the sections must be equal to 100%. Helen asked about the 170 which is a number that the students mentioned before. Sophie answered to her by stating that 170 people say that they agree. She came to this conclusion by adding the numbers of the graph since 62+54+54=170.

John: No, it's more than 100.

Teacher: We have asked 909.

Helen: No, from 909 people, where does refer the 170?

Gregory: Maybe the percentage of the people asked is small?

Sophie: 170 say that they agree.

Gregory: Maybe the percentage of the people asked is small and we can't draw good conclusions?

Based on the above transcript we could say that the students' graph comprehension corresponds to the first level of reading the data since they read literally the graph focusing only to the given numbers. They extracted data directly from the graph and they tried to answer by relying only on the data shown in it.

In the second class during the same lesson the students interpreted the graph by observing its characteristics (bars) and the numbers represented by the bars. When Harry stated that we have 10 he referred to the 10 lines that he saw on the graph. In this way George understood the range of the numbers that are represented with the vertical axis. He realized that the wrong impression of this graph depends on the fact that the vertical axis' numbering does not start from 0 but from 53. George concluded that if the numbering was correct the bars' length would be different and the bar of 62 wouldn't look so much bigger than the bars of 54.

> George and Harry: (they discuss) In fact, it isn't true.
>
> Teacher: What do you mean?
>
> Harry: Because in fact we have 10…yes 10…
>
> George: Yes 10 lines. In fact if we notice it…I believe if there weren't any numbers …if we had began from 0 in order to go up it shows much bigger than 62. 54 has nothing to do with 62.

Another student (Eva), during the same discussion, trying to understand what her classmates were saying she confused the numbers on the bars with percentages. This was a problem for most of the students since they assumed that the numbers on the graphs referred to percentages. This was due to the fact that the students were not experienced graph readers; actually, this was their first attempt to read graphs after two lessons on the construction and the properties of basic graphs (bar-graphs, pie-graphs and line-graphs). It is obvious that the students of this particular class evaluated the graph's accuracy by noticing that the vertical axis is truncated. Thus, their graph comprehension corresponds to the second level of reading between the data since they detected the misleading effect:

> Eva: This starts from the middle, the 53, if it started from 0 wouldn't the percentage be bigger? Or not?
>
> Harry: No it wouldn't. The graph would be just bigger.
>
> George: By looking at the graph, what we can imagine…. That the first bar looks much bigger than 54…The difference is not so…They should be bigger (They talk simultaneously)

In the third class during the same lesson a student immediately found the inconsistency between the numbers and the bars. Joanna connected the numbers with the bars by comparing the bars' length and the difference among the numbers. She was able to respond at the second level of graph comprehension, reading between the data as it is evident in the next transcript.

> Teacher: (She reads the task). What can we conclude based on this graph?
>
> Joanna: Hold on a second… First of all, why the 54 is so down and the 62 is so up? It isn't correct; if it was reasonable then close to the 62 should have been the 54 and the other 54 of the independent.

Then the teacher asked the students to propose ways of reconstructing the bar-graph in order to correct its misleading effect. Joanna proposed a different scale in order to have a more accurate graph. In her answer it becomes obvious that she can understand the effect that the correct choice of scale has on the construction of the graph. Additionally, Joanna proposed that the numbering in the vertical axis should start from 0.

> Teacher: How would you make it in order to be correct?
>
> Ken: Without numbers.

Joanna: If you ascend 5 to 5 or 10 to 10 (she refers to the scale) it is impossible to have 54. This bar is too high (the one of 62), actually 62 is only 8 (units) more than 54. If we used 10 to 10 then 62 would be close to 54.

Teacher: So, you propose by ten…

Joanna: Yes, 10, 20, 30 and so on. This is what I mean.

Teacher: Yes, but where would you start from?

Joanna: From 0.

In this class during the same lesson the students had to solve the next task.

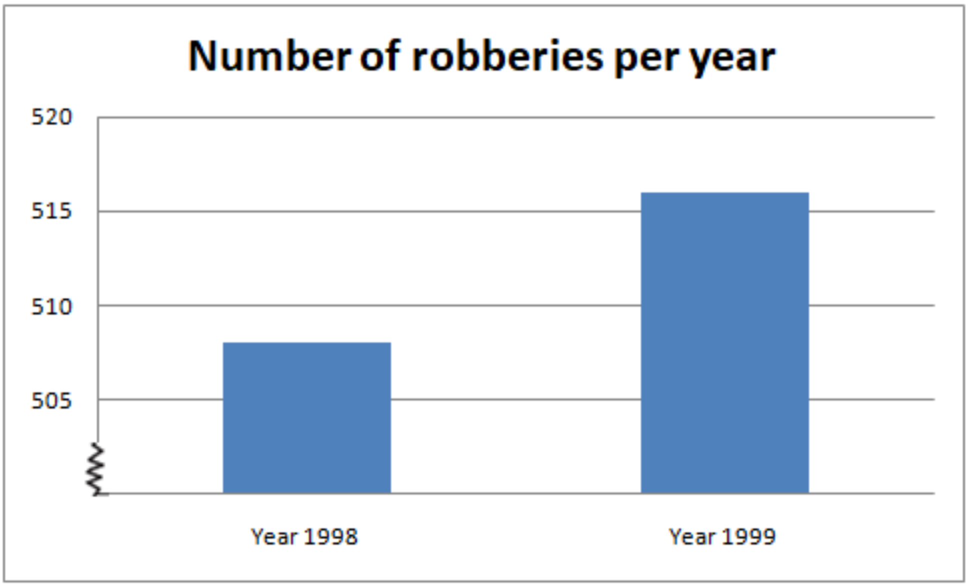


*Figure 2*. Fifth Task. (OECD, 2009)

**Question-Fifth task:** A TV reporter showed the above graph and said:
"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."
Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

## Subsection 1 in section 1, using heading 2

In this part you type the text for subsection 1 within section 1 with ALMIJ body text.
Although Joanna responded correctly to the third task, which was familiar to her, she could not respond to this task which was familiar to all students. The students referred only to the numbers and they did not justify their answers. Joanna referred only to the relation between the bars and she did not connect their length with the numbers. Instead she focused on the numbers and confused them with percentages. When she could not justify her answer, she was confined to express her personal view about the graph.

Joanna: No

Steven, Ken: No.

Teacher: You have to justify your answers.

Joanna: It was 505 and goes to 520, how can we count its percentage?

Teacher: You can't since you don't know the whole.

Joanna: So, what? How would I answer?

Teacher: He said that the increase was great.

Steven: Is the difference big?

Joanna: It is two, it is two times.

Leo: No, it isn't correct.

Joanna: We don't like it.

Another student, Leo, who was silent from the beginning of the lesson, noticed the inconsistency between the numbers and the bar's length. He concluded that the graph is not correct since the second bar is twice the first one while the relevant numbers do not follow the same pattern. Leo managed to read between the data and his comments assisted the other students to express their opinions. Joanna concluded that this graph was used by someone to deceive the readers and Robert expressed a reasonable conclusion about it.

Leo: Because here it is 505, after it is 510 and then the double of it (he refers to the bars) goes to 520.

Teacher: So?

Leo: So, it can't be correct.

Teacher: Do you say that if I look at the numbers the robberies are double?

Leo: No, the numbers aren't double.

Joanna: So, they are trying to deceive us.

Teacher: Robert do you agree?

Robert: I believe the graph is wrong since the number of robberies isn't double as it looks on the graph.

The next exchanges are about the sixth task which contained a line-graph and was provided during the second hour of the same lesson.

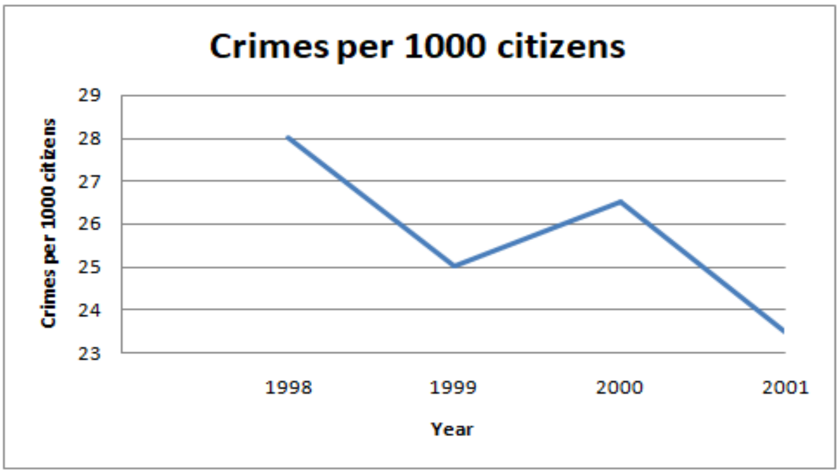


*Figure 3.* Sixth Task. (adapted from Harper, 2004)

> **Question-Sixth task:** In a local newspaper in 2001 we found the above graph about crime rate in the region from 1998 to 2001. The article concludes that crime rate had been reduced to a great extent. Do you agree with that? Justify your answer.

In the fourth class during the second hour of this lesson Maria based her answer on her personal views. She expressed her opposition to the conclusion that accompanied the graph. It seemed that she did not even read the graph, but she merely expressed her disapproval. At the same, time Paul stated that he agreed with the graph since he read literally the graph focusing only to the given numbers.

Teacher: Let's go to the sixth task now (she reads it). The article concluded that criminality had declined to a great extent. Do you agree with this?

Maria: Nonsense. No.

Teacher: Do you agree with this claim? Justify your answer.

Maria: No. As the years go by, criminality becomes greater, not less.

Paul: I agree with the graph, because the graph is correct. I see that in 1998 we had 28 crimes, in 1999 we had 25, in 2000 we had 27 and in 2001 we had 24 hence... I agree.

Afterwards the teacher tried to urge the students to notice the graphs' numerical information in order to understand the inconsistencies. Yanni with his question referred to the effect of the previous tasks. Maria concluded that it should have started from 0 and the scale should have been 1. At the end Yanni concluded that the reduction is very small.

Teacher: Was the decline so great?

Paul: As it seems here, it is not a big reduction but…

Yianni: Isn't it the same again?

Paul: So isn't the graphic representation right?

Teacher: Why is it not right?

Maria: Because it should have started from 0 and maybe go one by one here.

Teacher: Maybe one. If I understood what you said, in 98 was 28 and in 2001 was 24, what is the difference of 28 in 24, how much do we have?

Yanni: 4

Teacher: 4 out of 1000 inhabitants, is this a big reduction?

Yanni: It's nothing.

In the other classes most students interpreted this graph correctly and evaluated its accuracy by using their knowledge from the previous tasks. Some of them – coming from the first class – managed to propose its reconstruction, by using their knowledge about graphs. Eva proposed to change the line-graph to a bar-graph and Helen imputed the wrong effect of the graph to the choice of the scale.

Eva: If we used bars wouldn't it look better?

Theo: It looks that the decrease was huge.

Helen: But the difference isn't as huge at it looks like because the scale is 1. If the scale was 10 it would be…

Eva: If we used bars from 28 to 24…Wouldn't it be the same?

Teacher: If I construct it again and start from 0 what scale should I use?

Helen: By 5.

> Teacher: (Constructs it in the blackboard according to the students' suggestions). So, now, how it does it look?
>
> Eva: Now the decline doesn't look so big. While in the first graph (task) it looks like the decline was very big. Now it looks okay….

We may say that these students partially demonstrated the highest level of graph comprehension, reading beyond the data, since they were able to connect the same data with another graph in an attempt to produce a more accurate presentation of them in relation with the graph's context.

## Conclusions – Discussion

Numerical activities take place every day since people have to confront numerate situations all the time (Diez-Palomar, 2011). Additionally, numeracy is highly connected to statistical literacy; at least to the extent that statistical literacy refers to the abilities that adults as 'consumers' of statistical data should have in order to be active citizens. One of the situations that statistical literacy is needed is graph interpretation. In light of these considerations we designed our research. Our underlying goal was not only to assess the way that adult students interpret graphs in context or media graphs but also to enrich their experiences by providing them with statistical notions which were new for them. These notions were new for our students since they were inexperienced readers of graphs. Our intention was to encourage our students to approach every task as a part of their everyday life. In this way they would appreciate statistics and the impact it has for their effective participation in society.

Concerning the way that our students interpreted the graphs, most of them were able to read the graphs and to extract numerical information from these. They managed to use their previous knowledge to solve the tasks and they realised that a graph is a data representation that must meet certain requirements. Therefore, they could read these data. The students demonstrated to some extent the ability to critically evaluate graphs and to identify the deliberate use of misleading graphs that results to wrong interpretations. Not all of them were able to read between the data. With respect to the third level of graph comprehension – reading beyond the data – very few students generalized from the sample to the population based on specific graphs or were able to reach a conclusion.

It is noteworthy that the students' personal opinions occasionally overwhelmed their knowledge of the graphs; we may say that in these cases they did not operate well because they acted based on a drive provided by their emotions (Buxton, 1991) and they did not manage to overcome their prejudices against their statistical knowledge of graph interpretation. The context of the graph played a significant role in the students' interpretations; they reached some conclusions without considering the meaningfulness of their answers. Thus, the graphs' realistic context in our study had lead to "unexpected responses and outcomes" (Dalby, 2015, p. 88). The students' critical sense was not so developed since a balance between their knowledge, beliefs and experience in interpreting the graphs was not achieved.

In his seminal paper on the role of visual representations, Arcavi (2003) mentions a phrase attributed to Goethe: "We don't know what we see, we see what we know" (p. 230) to refer to the cases in which students do not see what teachers or researchers see. Especially, the second part of this phrase – "We see what we know" – may refer to all those adults who have to interpret different forms of visual data, including statistical graphs or media graphs without having a statistical background. Some of our students have managed to overcome their prejudices, while others not. Thus, we believe that our study has provided some evidence on how the endeavour of providing a statistical background might look like.

Further research is needed with larger groups of adults, adults with different levels of knowledge and possibly a bigger variety of contexts. This could provide support for the work of Monteiro & Ainley (2007), since the different features of reading contexts has helped us to highlight that the interpretation of graphs is related with school and out-of school knowledge. The teaching of correctly reading statistical graphs seems to be of crucial importance in a world where the average adult has to confront with graphs in almost every strand of his daily life. This is especially true for the adult learners who return to their studies carrying their own experiences from in and out of school, and at the same time they need to confront with all their prejudices on their attempt to interpret an ever-changing world.

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# What do people like about mathematics?

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## Abstract

In this discussion paper we look at questions that adults have about numbers. Many of their questions are not about pure mathematics, but about personal, cultural or societal issues. We discuss how to connect mathematical topics with things people are interested in, based on theoretic knowledge from the field of science communication. We focus on using narratives to make mathematics more personal, how to use games as demonstrations and different ways to present the same mathematical problem in different societal settings.

## Introduction

When I tell people I studied mathematics, the most common responses are ‘I was very bad at maths in school’ or a plain ‘Really? I hate mathematics.’ However, when you actually talk to people, it turns out that quite a few of them are in fact quite keen on learning more about mathematics. And adults consistently name mathematics as the school subject with the most value to their lives (Gallup, 2013).

In science communication it is very important to talk with your target group instead of talking about them (Arnstein, 1969). This paper discusses what kind of mathematics the general public is interested in by asking them questions about numbers via mass media.

## Let’s just ask people what they like

In 2011 the British journalist and mathematician Alex Bellos asked people what their favourite number is (Bellos, 2014). He set up a website where people could send in their favourite and tell him why they liked this number.

Over 44,000 people responded and Bellos reported that the world’s favourite number is 7, with 3 and 8 as runners up. Almost half of the submissions were for a number between 1 and 10 and the lowest whole number that did not receive any votes was 110. The most common reason for picking a favourite number was it being a birthday. Even more interesting were the properties that people connected with their favourite numbers: 7 was described as sacred, magical, good, unalterable, overconfident and awkward. While 8 was called neat, feminine, reliable, kind, unassuming and huggable. People seemed to anthropomorphize numbers and ascribe entire personalities to them. This begged for a follow-up question where people could explain more about which numbers matter to them personally.

### A broader question

Since 2014 I have been writing a weekly column for the national paper De Volkskrant under the title *Ionica saw a number*. Every week I discuss a number that was somehow in the news, the column can be about anything from politics to literature and from classic math puzzles to recent research. I often get suggestions from readers, or nice additional information about the subjects I wrote about. About a quarter of my columns are inspired by reader’s suggestions.

In my 100th column I decided to something similar to what Alex Bellos did: I asked people which number they thought deserved its own column and why (Smeets, 2017). Readers could send in their suggestions by e-mail. I promised to write my 101th column about one of their suggestions.

## The response

Within one weekend the question yielded 203 responses from readers. Their suggestions ranged from very funny to very serious. A fun question came from someone who wanted to know if there was anything interesting about the number six, since Bert proclaims in Sesame Street that his favourite number is six and Ernie keeps protesting that six is a very boring number. A more serious question came from a reader who was wondering why the Dutch House of Representatives has 150 seats. Why was this number chosen? Should the number of seats be expanded as the population grows? And do other countries use a similar ratio of representatives compared to the population?

One of the most serious questions, and the one I wrote my next column about, was very personal. A reader wrote about a friend who was in the hospital with leukaemia. To survive he needed stem cell therapy with suitable donor material. For siblings the probability that they are a match is 25%. The friend in the hospital had three siblings and his family and friends were desperately trying to calculate the probability that at least one of them would be a suitable donor. This is mathematics that really matters to people's lives and I explained in my column how to calculate that the probability of at least one match amongst the three siblings is 58%. I also wrote about more general odds of finding a match and encouraged readers to register as donors, since the odds are much lower than you would like.

## Categories of responses

As the previous examples show the subjects of the 203 responses varied wildly. We categorised the responses with an inductive method, following the basic principles of grounded theory (Martin et al, 1986). First we coded all of the responses, then we grouped comparable codes in concepts and in the final coding round we combined similar concept in six overarching categories. These categories are:

1. Personal: this category included questions and anecdotes about lucky numbers, special dates and times plus personal connections people felt with a number.
2. Cultural: this category included questions and anecdotes about music, books, language, history and games.
3. Societal: the questions and anecdotes in this category were mainly about why certain numbers in the society are chosen as they are and how the occurrences of societal phenomena are distributed.
4. Mathematical: this category contained nice facts about numbers and theoretical questions about infinity, pi and prime numbers.
5. Scientific: this category was for all questions or stories about biology, physics, astronomy and the other sciences.
6. Other: Everything that could not fit in one of the above categories. For instance because readers only sent in a number without further comment.

Some responses mentioned different numbers or would fit into multiple categories, in those cases we chose the category best fitting the first topic mentioned.

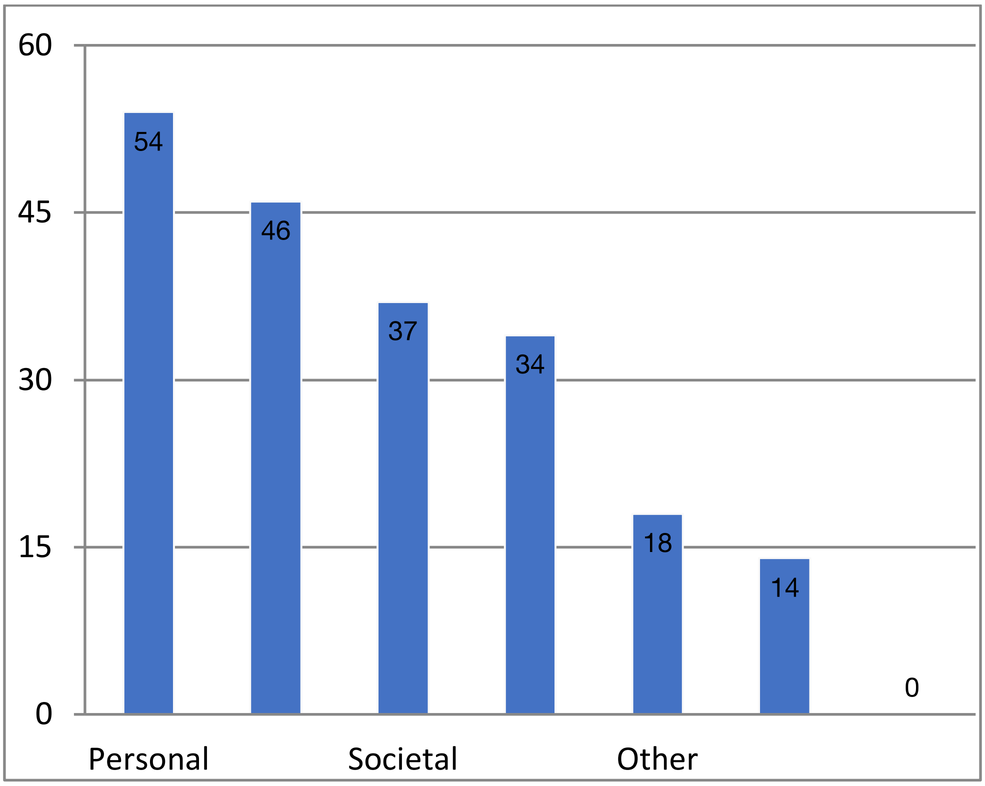


*Figure 1*. Reader's responses grouped by category

Figure 1 shows the distribution of the readers responses over these six categories. The three biggest categories are Personal, Cultural and Societal questions. In the rest of this paper we will discuss ways to connect people to mathematics using one of these themes.

## Personal mathematics

People give numbers personalities and connect personally with a clock time like 22:22. There are many stories behind the numbers. Yet when we present mathematics we too often start from the facts and use a structure where we first introduce a topic, give all the necessary background and definitions, derive the results and only then (if there is still time) talk about why this matters. While science communication literature generally shows that is easier to convince people with stories than with logical arguments (Dahlstrom, 2014).

Telling it like Kurt Vonnegut, one way to make mathematics more personal, is using techniques from storytelling. The American writer Kurt Vonnegut describes how you can build a story based on a simple graph:

> Now let me give you a marketing tip. The people who can afford to buy books and magazines and go to the movies don't like to hear about people who are poor or sick, so start your story up here [indicates top of the Good fortune -Ill fortune axis]. You will see this story over and over again. People love it, and it is not copyrighted. The story is 'Man in Hole,' but the story needn't be about a man or a hole. It's: somebody gets into trouble, gets out of it again [draws a graph]. It is not accidental that the line ends up higher than where it began. This is encouraging to readers.
>
> (VONNEGUT, 2011)

Even the most difficult mathematics can be presented as a story that connects with people. One of the nicest examples I have ever seen was a graduate student who was working on elliptical curves who presented his work as a will-they-make-it-in-time-adventure with a co-author who had to catch a plane to his institute where he would be very hard to reach by e-mail. The entire audience was so eager to hear if they made it, that they happily listened to all kind of details about elliptical curves.

Small tricks from storytelling can be useful to communicate, even within science itself. A recent study showed that scientific papers on climate change that use narrative techniques are cited more often (Hillier, 2016). These basic narrative techniques are relatively easy to incorporate in texts or lectures about mathematics and include using sensory language and using conjunctions to logically order the reasoning. Furthermore making a direct appeal to the audience is a successful narrative tool for helping people understand why what you are telling them is important. You can do this for instance by asking your audience to imagine something or giving them a clear recommendation for action.

## Cultural mathematics

There are many ways to connect mathematics to cultural subjects. When you want to introduce fractals you can use the fact that they are used to test the authenticity of Jackson Pollock's drip paintings (Taylor et al, 1999). Or if you have a less high-brow audience you could use the computer-generated backgrounds of animation movies like Up. There's mathematics to be found in popular books and movies and you can use them to engage math-haters.

In this section I am going to focus on another cultural phenomenon: games. Readers sent in many questions and suggestions about numbers in games, so this seems to be something that is on people's minds. In the last few years gamification has proven to be useful in many educational contexts (Hamari et al. 2014). There are many examples of beautiful games in mathematics, here I will focus on a simple version of Nim and the different ways you can present the same game.

### Nim

Nim is a game where two players take turns and remove objects from distinct heaps (Berlekamp et al, 1982). In the simplified 21-version there is one heap with 21 objects, in each turn a player can take away 1,2 or 3 objects and the person who has to take the last object loses the game. In this case there is a winning strategy for the second player,  which is not too hard to figure out.

In 2006 I played this game with a bunch of mathematicians at a science festival. We played the game with matches on a table and invited the audience to try and beat us. Since we were polite and let the other person start the game, none of them stood a chance. We usually played a few games, until they figured out the strategy for themselves. This worked well and afterwards I used the game in many talks and workshops.

A few years later I was asked to give a maths show for a group of two hundred primary school kids. I wanted to play Nim with them, but realised that the matches on a stage would be very unpractical for such a big audience. We decided to use 21 brightly coloured balloons. One volunteer out of the audience would play against me and instead of removing the balloons we would pop them with a pin while the audience shouted their advice. I used this set-up for many more talks and demonstrations.

After the balloon-game I explained why the volunteer would always lose, even if she was the smartest person in the world. If someone from the audience figured out the strategy, I let them explain it. Otherwise I would urge the audience to consider what happens if it is your turn and there are only five balloons left. Whatever you do, the other player can make sure that in your next turn there is only one balloon left, so you will always lose. After this step we can inductively work back to the starting point with 21 balloons.

Are there more ways to use this same game in an educational setting? Of course there are. Marcus du Sautoy plays a similar game with 13 chocolates and one chili pepper (Du Sautoy, 2011). He keeps playing against opponents until they figure out how to make him eat the chili (he is fair enough to let them start in the position with a winning strategy). This set-up works really well in smaller groups and is great for making participants figuring out the strategy on their own. The take home message is that you should think about the best way to present a game for your audience.

## Societal mathematics

In the previous section we saw that there are many ways to present the same game. One of the nice things about mathematics is that you quite often can present the same idea in many different societal settings. It has been shown that personalising mathematics has a positive effect on both learning gain and interest in mathematics (Bernacki et al, 2018). The underlying mathematics stays the same, but it will be much easier for people to relate to an example from a field they are interested in. My favourite paradox can for instance be introduced in many different ways.

### Simpsons paradox

If I am presenting Simpsons paradox to an audience of teachers, or other people interested in education, I start with the example of a gender bias case at Berkeley University (Bickel et al, 1975).

Table 1.

*Admittance rates of the six biggest departments*

| Department | Men | | Women | |
|---|---|---|---|---|
| | Applications | Admitted | Applications | Admitted |
| A | 825 | 62% | 108 | 82% |
| B | 560 | 63% | 25 | 68% |
| C | 325 | 37% | 593 | 34% |
| D | 417 | 33% | 375 | 35% |
| E | 191 | 28% | 393 | 24% |
| F | 272 | 6% | 341 | 7% |

In 1973 thousands of students applied for admission at Berkeley. Of the 8.442 men who applied, 44% was admitted. Of the 4.321 women who applied, only 35% was admitted. It seems that there is some gender bias, since women have a significantly lower probability of being admitted.However, a closer look at the admission process reveals that each department did their own admissions. Table 1 shows the admittance rates of the six biggest departments.

We see that in most departments women have a higher probability of being admitted than men. This pattern held for the entire university, yet when we look at the total admittance rates, women seem to have a lower probability of being accepted.

In lectures this is the point where I ask the audience to think about explanations. Sometimes people guess that it might be because there are more men applying than women, but this does not explain the odds as they are. Young students usually guess what is happening here: women and men pick different studies. The most popular department for men is A with an overall admittance rate of 64% However, amongst women the most popular department is C with a much lower overall admittance rate of 35%. So even though women have a slightly higher probability of being admitted to a department, in total over the university they have a lower probability of being admitted.

This is Simpson's paradox: A trend in different groups is reversed when the groups are combined. If I am talking to an audience of medical professionals, I introduce the very same paradox with another real-life example. A study compared two treatments on kidney stones: the expensive method A and the cheaper method B. The study found that method A had the best results for patients with small kidney stones. It also found that method A had the best results for patients with large kidney stones. Finally it concluded that averaged over all patients method B was the better one. How is this possible? Once again, the clue is that patients are not randomly distributed over the methods. The patients with small kidney stones usually got method B (and had little complications) while the patients with large kidney stones were given method A (and had more complications, since they came in with a larger problem).

Simpsons paradox pops up in batting averages in baseball, payment gender gaps, survival rates for the Titanic, delayed flights from different carriers and death-penalty sentences. You can probably construct a realistic example of Simpson's paradox for any field of interest to explain the basic idea behind it (Wagner, 1982). This is true for many mathematical concepts: you can translate them to apply to any subject your audience is intrinsically interested in.

## Discussion

We described ways to connect people to mathematics, based on the three most occurring categories of suggestions in a small non-random survey of newspaper readers. Their preference for these categories might not generalize to the general population and one could also describe cases on how to connect with a public that is interested in other categories. For instance, when communicating with people who are already interested in mathematical topics it might be good to focus on a more abstract case. A great example of this is a video about infinite sums by YouTube channel Numberphile that currently has over 6 million views and more than 12 thousand comments (Haran et al, 2014). For people interested in the category of broader scientific concepts, it will be useful to consider cases where mathematics leads to applications in other fields. One nice example is Braess' paradox applied to traffic networks, where adding a new road actually impedes the traffic flow (Braess et al, 2005).

However, whichever category of mathematical questions people are interested in, the general science communication theories we describe will apply in these contexts. A broader review of science communication literature can give more advice on the use of jargon and target audiences (Hut et al, 2016).

## Conclusions

When people are invited to ask questions about numbers, many of their responses are not about pure mathematics, but about personal, cultural and societal issues. We discussed ways to relate mathematics to these kind of questions to make it easier to connect with people. Remember that you can steal narrative tricks from writers, how games can be a playful way to introduce concepts and that there are many ways to present the same mathematical idea.

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