Descriptive and depictive representations in mathematical problems – the effect on vocational students' results

ALM24: All hands on math

Kees Hoogland, ECRW

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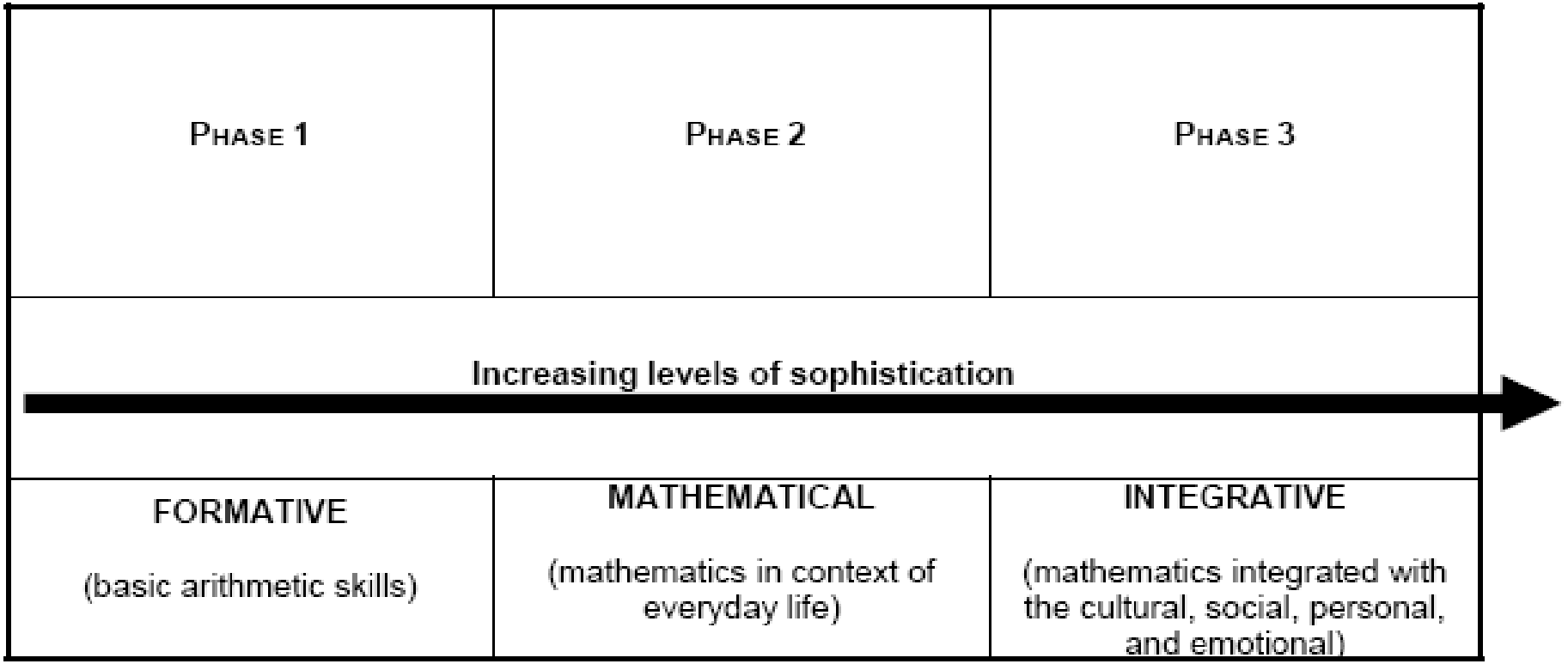
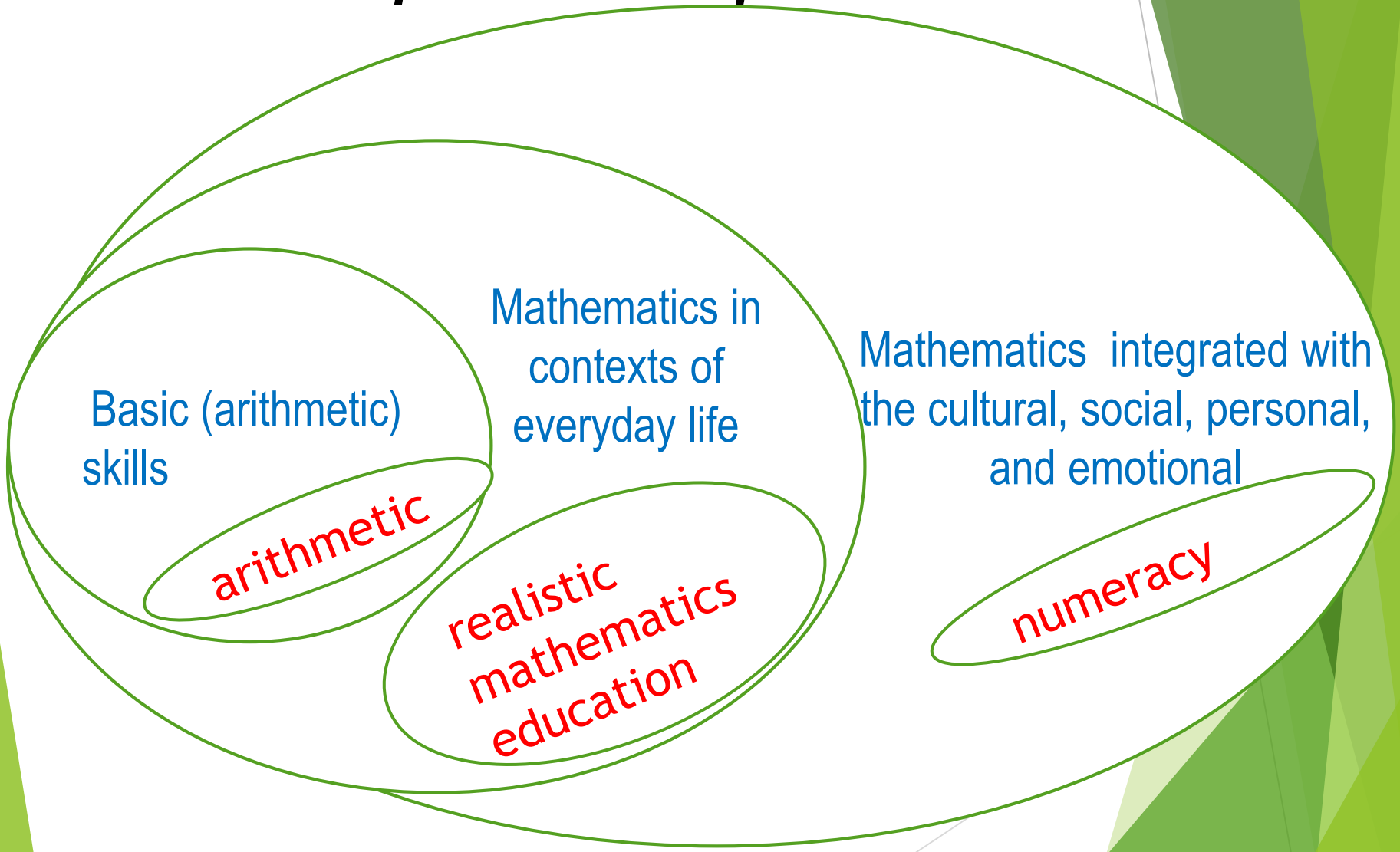
keeshoogland1@outlook.com

Adult Numeracy Concept Continuum of Development



A continuum of development of the concept of numeracy showing increased level of sophistication from left to right (from Maguire & O'Donoghue, 2002)

Numeracy & Functional Mathematics Conceptual Development



9.65

a. $\frac{5}{33} + \frac{9}{22} =$

b. $\frac{7}{24} - \frac{3}{16} =$

c. $\frac{13}{12} + \frac{4}{15} =$

d. $\frac{4}{9} \times \frac{4}{11} =$

e. $\frac{7}{5} : \frac{5}{7} =$

9.68

a. $\dfrac{\frac{5}{6} + \frac{2}{5}}{\frac{2}{3} + \frac{1}{6}} =$

b. $\dfrac{\frac{3}{4} + \frac{4}{3}}{\frac{3}{4} - \frac{1}{3}} =$

31 Jan knapt zijn slaapkamer op. Hij gaat de drie wanden lichtblauw verven. Hij heeft 5 liter verf gekocht, genoeg voor 30 m^2.

a De eerste muur heeft een oppervlakte van 4 m^2. Hoeveel verf heeft Jan voor deze muur nodig? Gebruik een verhoudingstabel.

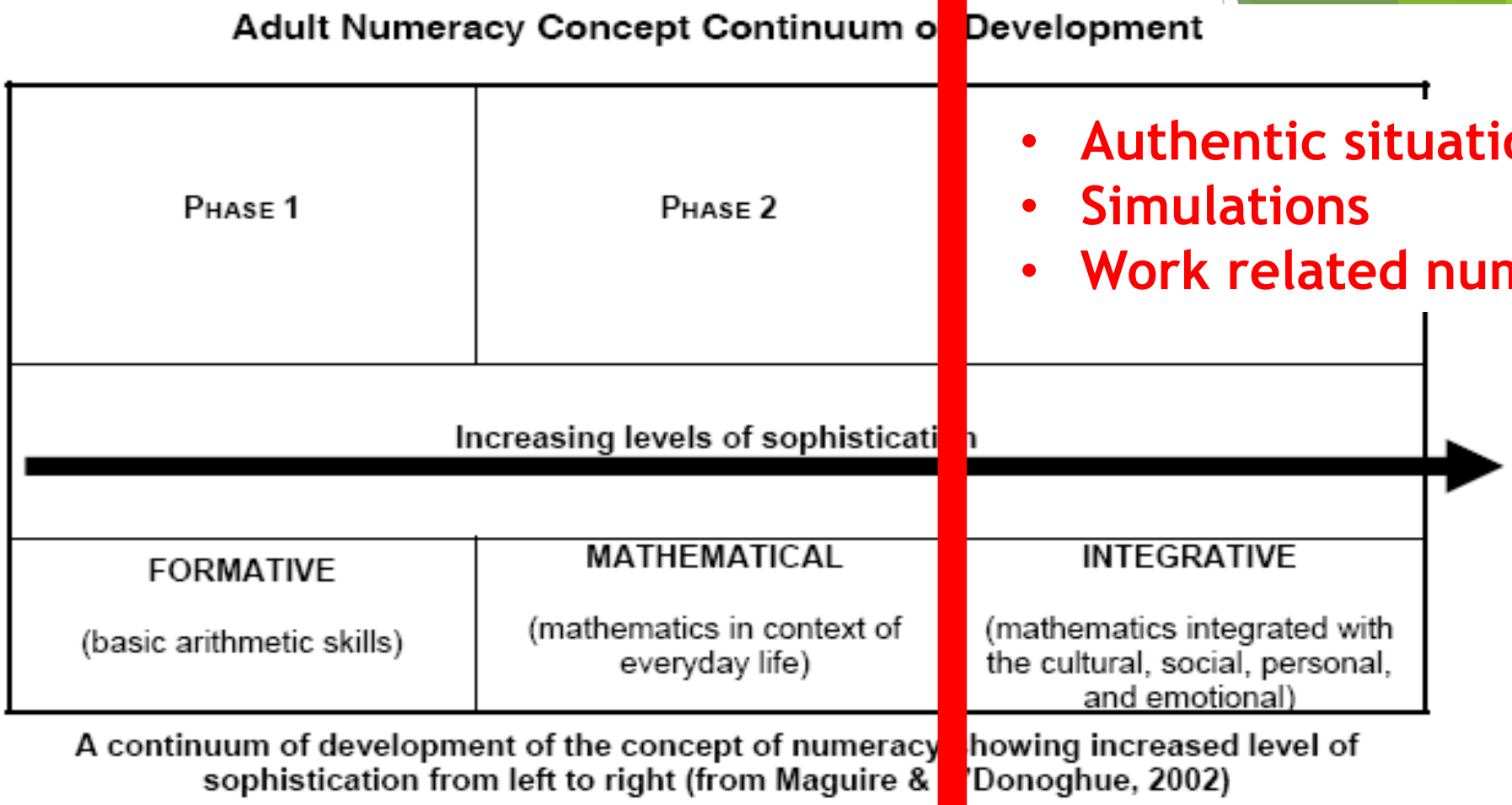
b De volgende muur heeft een oppervlakte van 10 m^2.

realistic mathematics education

arithmetic

numeracy

Gecijferdheid

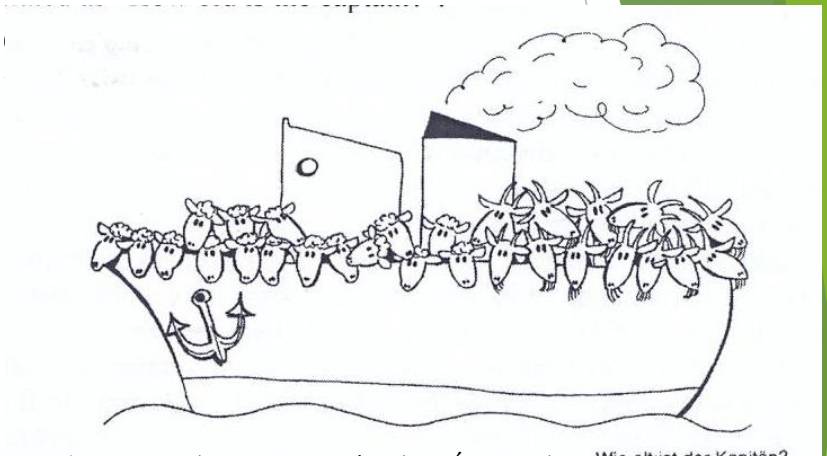


- Authentic situation
- Simulations
- Work related nume

Age of the captain

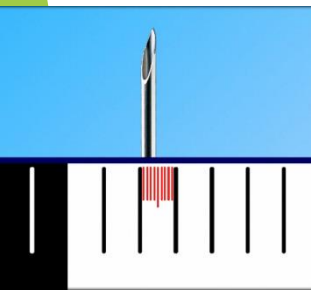
A captain owns 26 sheep and 10 goats.
How old is the captain?

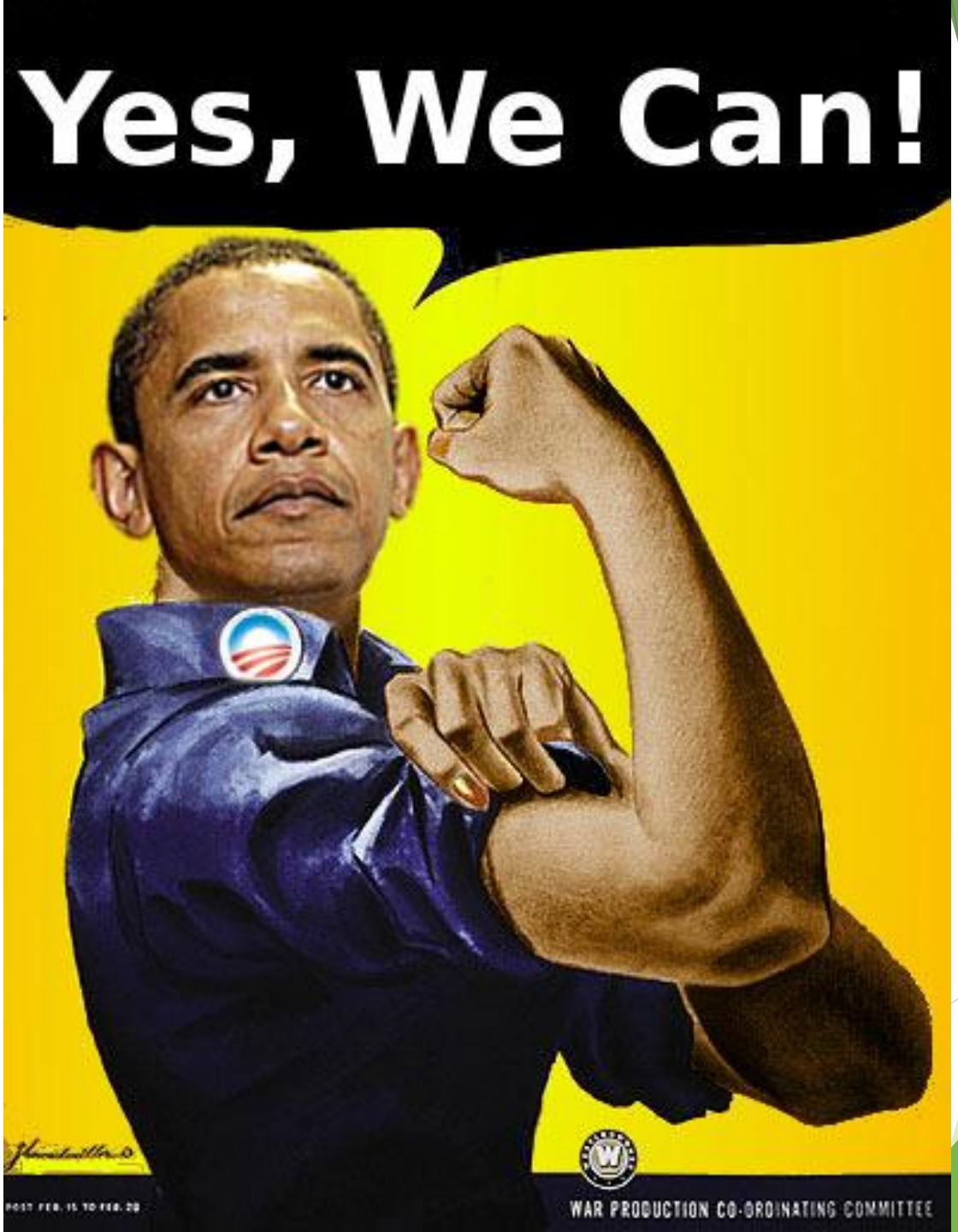
- Suspension of sense makin
- Calculational approach



Puisque tu fais de la géométrie et de la trigonométrie, je vais te donner un problème : Un navire est en mer, il est parti de Boston chargé de coton, il jauge 200 tonneaux. Il fait voile vers le Havre, le grand mât est cassé, il y a un mousse sur le gaillard d'avant, les passagers sont au nombre de douze, le vent souffle N.-E.-E., l'horloge marque 3 heures un quart d'après-midi, on est au mois de mai… On demande l'**âge du capitaine** ?
Flaubert, 1841

Can we improve “regular“ lesson materials and “regular” tests to fit more sophisticated numeracy concepts?




Yes, We Can!
WAR PRODUCTION CO-ORDINATING COMMITTEE

A

Apples are sold in 2.5 kilogram bags.
You weigh one apple and find 157 grams.

About how many apples are there in one bag?
[ ] apples

A

You buy groceries for a total of € 21.30
You pay with a 50 euro bill and two coins of 1 euro.

How much is the change?
€ [ ]

A

The bath room has two windows.
They both are 0,90 m in width and 1,35 m in height.
You want to double glaze these windows.
Double glazing costs € 148,- per m^2

What is the cost of double glazing these windows?
€ [ ]

B

About how many apples are there in one bag?

[ ] apples

B

Je moet betalen	Je betaalt met

You have to pay:

You pay with:

SUPERMARKT

Daliastraat 4
5707 SJ Helmond
0492-527384

15	blik cola 330ml	0.90	13.50
13	chips flav. pnt. light	0.60	7.80
aantal art. 28		subtotaal	21,30
TOTAAL			21.30

How much is the change?

€ [ ]

B

dubbelglas

€148,- per m²

Double glazing
€ 148.- per m²

What is the cost of double glazing these windows?
€ [ ]

Images of Numeracy

Investigating the effects of visual representations of problem situations in contextual mathematical problem solving

From a descriptive representation of reality to a depictive representation of reality.

Je koopt boodschappen voor € 21,30.
Je betaalt met een biljet van 50 euro en twee munten van een euro.

Hoeveel krijg je terug?
€

Research design

- 24 items in two equivalent versions A and B:
  - A = descriptive representation
  - B = depictive representation
- Large scale testing as a numeracy test:
  - random 12 A + 12 B, random order
  - controlled randomized trial
- Research question: What is the effect of item characteristics on students' results?

Theoretical notions

- Contextual (word) problems & their difficulties

  (Boaler, 1993), (Gravemeijer, 1994), (Verschaffel et al. 2000, 2009), Greer (1997) and many more

- Cognitive psychology
  - descriptive versus depictive representations

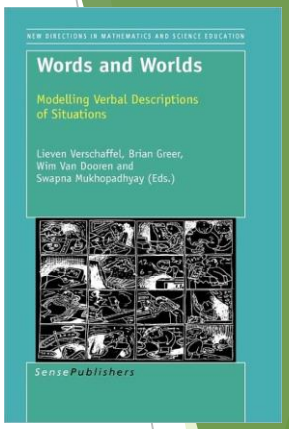
    (Schnotz et al., 2002, 2010)

- Cognitive load theory

  (Sweller, 1996, 1999; Mayer, 2005)

- "Answer getting mindset" versus "Problem solving mindset"

  (Boaler, 2016), (Daro, 2013)



Different “models” of modelling and problem solving

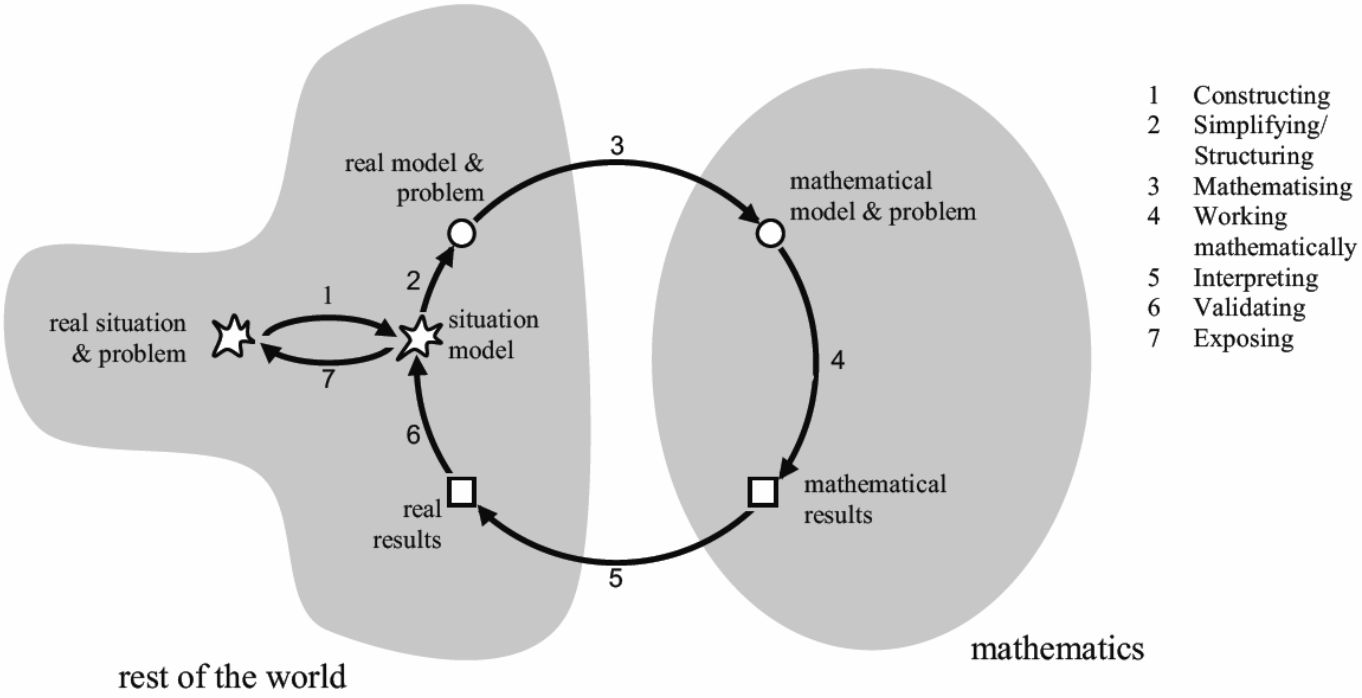


Figure 1 – Modelling cycle

Blum, Werner, Galbraith, Peter L., Henn, Hans-Wolfgang, & Niss, Mogens (Eds.). (2007). *Modelling and applications in mathematics education - The 14th ICMI study. New York, USA: Springer Science & Business Media B.V.*

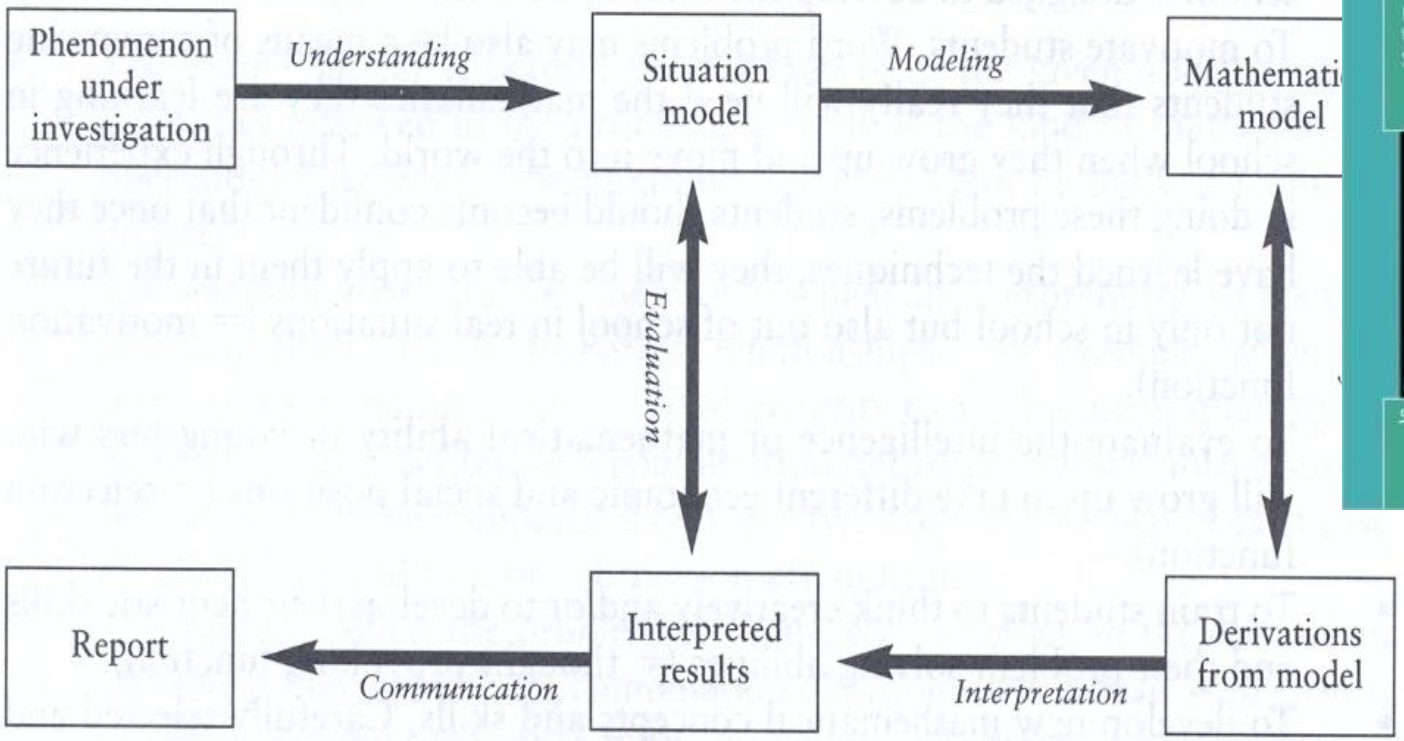
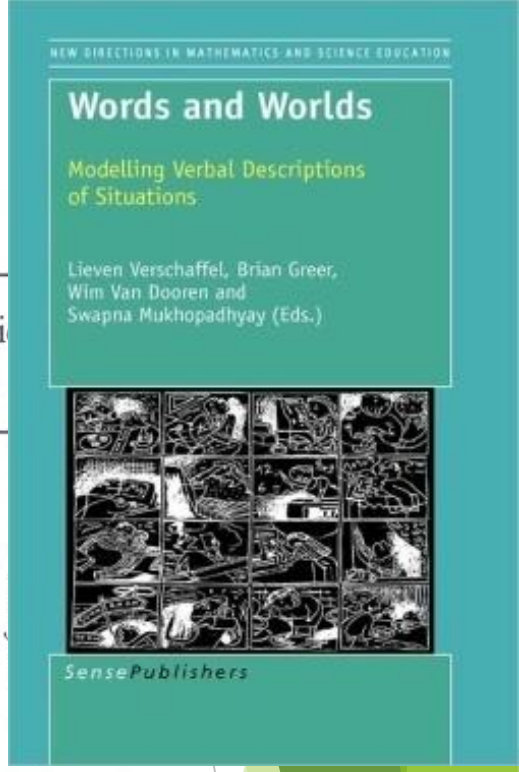


Fig. 0.1 Schematic diagram of the process of modeling.



(Verschaffel , Greer, & De Corte, 2000, p. ix)

A model of mathematical literacy in practice

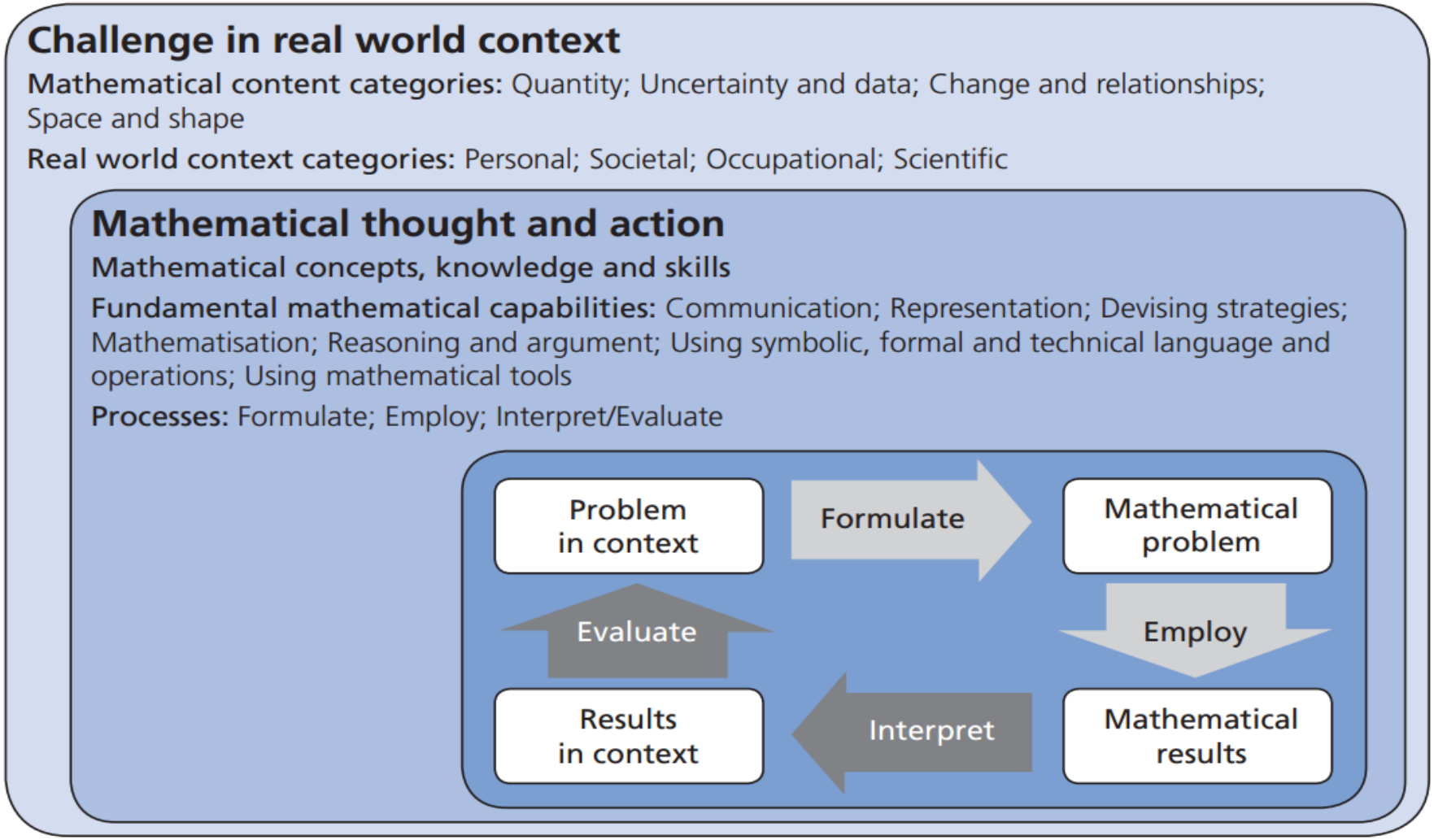
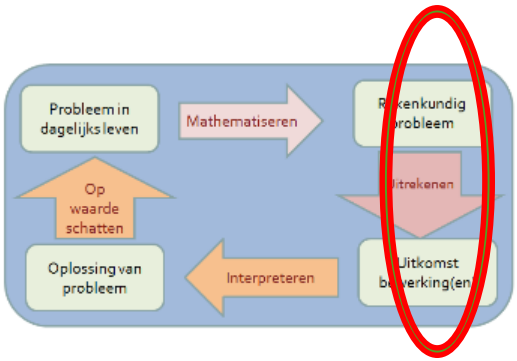


Fig. 1. A model of mathematical literacy in practice. From OECD (2013) (p. 26).

OECD (2013). PISA 2015 draft mathematics framework. Paris, France, OECD Publishing.

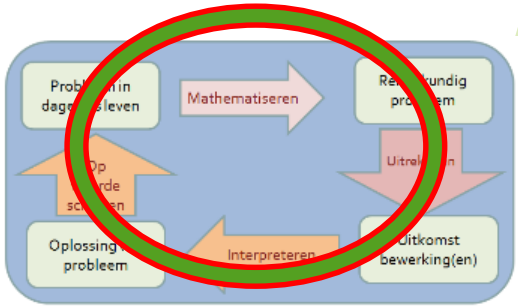
Answer getting

- Only right hand side
- The other parts are just "noise".
- It is all about the operations.



Problem solving

- Always the whole cycle.
- It is an organic system.
- Horizontal steps are the essence of mathematical thinking.



Data

- Main run
  - 179 schools
  - 31,842 students
  - Primary (11-12 yr)
    - 1,150
  - Secondary (12-16/18 yr)
    - 29,500
  - Sec. vocational (16-20 ytr)
    - 1,000

- Collected data
  - Scores on items
  - Answers to items
  - Grade level
  - School track level
  - Age
  - SES
  - Gender
  - Ethnicity
  - Time spent on items in ms
  - Last math grade

Statistical analysis

Probit – analysis is a sophisticated multivariate analysis

How do the variables v (version) and x (others) contribute to the ability y.
The chance P(z = 1) that the question is solved correctly equals
the chance P(y ≥ δ) that ability y surpasses a treshold δ.

$$y = \alpha_0 + \alpha_1 v + \alpha_2 x + \varepsilon$$

$$z = 1 \text{ if } y \geq \delta$$
$$z = 0 \text{ otherwise}$$

$$P(z = 1) = P(y \geq \delta) = P(\varepsilon \geq -\alpha_0^* - \alpha_1 v - \alpha_2 x)$$
$$P(z = 0) = 1 - P(y \geq \delta)$$
$$\text{with } \alpha_0^* = -\delta + \alpha_0$$

v = {0 = descriptive repr., 1 = depictive repr. }
x_1 = {0 = male,1 = female}
x_2 = {0 = not, 1 = migrant family}
x_3 = {relative age within level grade}
x_4 = {last school math grade}
x_5 … x_9 = {school track level}

E = not observed variables (m=0, s=1)

From the data the parameters α_0^*, α_1, α_2 are estimated by maximum likelihood.

Results

		Model 1	Model 2	Model 3
Independent variables		marginal effect (std.err.)	marginal effect (std.err.)	marginal effect (std.err.)
v	variant (verbal/image-rich)	0.0207(0.0013)(*)	0.0207(0.0013)(*)	0.0197(0.0025)(*)
x_1	gender	0.0524(0.0013)(*)	0.0509(0.0013)(*)	[illegible]
x_2	etnicity	-0.0288(0.0015)(*)	-0.0267(0.0016)(*)	-0.0266(0.0022)(*)
x_3	age (relative)		-0.0124(0.0010)(*)	-0.0124(0.0010)(*)
x_4	math grade		0.0208(0.0005)(*)	0.0208(0.0005)(*)
x_5	primary education (rel.)	-0.1900(0.0034)(*)	-0.2183(0.0047)(*)	-0.2185(0.0052)(*)
x_6	pre-vocational (rel.)	-0.1810(0.0015)(*)	-0.1856(0.0016)(*)	-0.1863(0.0022)(*)
	general secondary	reference		
x_7	secondary vocational (rel.)	0.0859(0.0035)(*)	0.1020(0.0033)(*)	0.1006(0.0053)(*)
x_8	pre-university (rel.)	0.1136(0.0017)(*)	0.1029(0.0018)(*)	0.1021(0.0025)(*)
x_9	school level	0.0845(0.0007)(*)	0.0899(0.0008)(*)	0.0899(0.0008)(*)
v * x_2	variant *etnicity			-0.0001(0.0032)
v * x_5	variant * primary education			0.0006(0.0093)
v * x_6	variant * pre-vocational			0.0015(0.0032)
	variant * general secondary			reference
v * x_7	variant * secondary vocational			0.0029(0.0075)
v * x_8	variant * pre-university			0.0017(0.0035)
	N	646,275	605,430	605,430
	Pseudo R2	0.0647	0.0676	0.0676

Results

- B > A statistically significant, with a (very) small effect size.
- B > A on a significant number of problems (11/21)
- Bigger effect in domain of measurement & geometry

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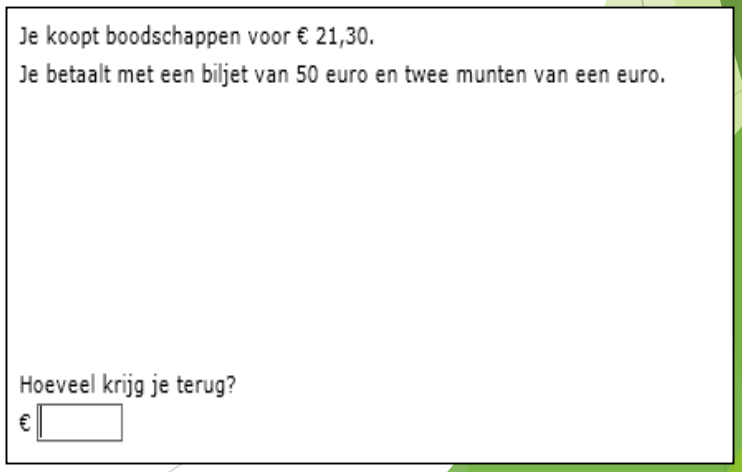

- Further research
  - Interdependency on other variables
  - Actual student behavior
  - Eye-tracking
  - Teaching focus

Broader perspective

- Simulating "reality" in classroom context
  - Verbal / descriptive
  - Visual / photographs / depictive
  - Visual / video clips / animated
  - Visual / augmented reality /

Contact

Thank you for your attention !!

More information or suggestions?

keeshoogland1@outlook.com

https://nl.linkedin.com/in/khoogland