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Understanding Mathematics

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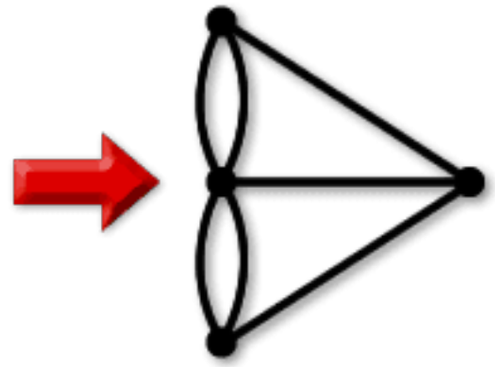
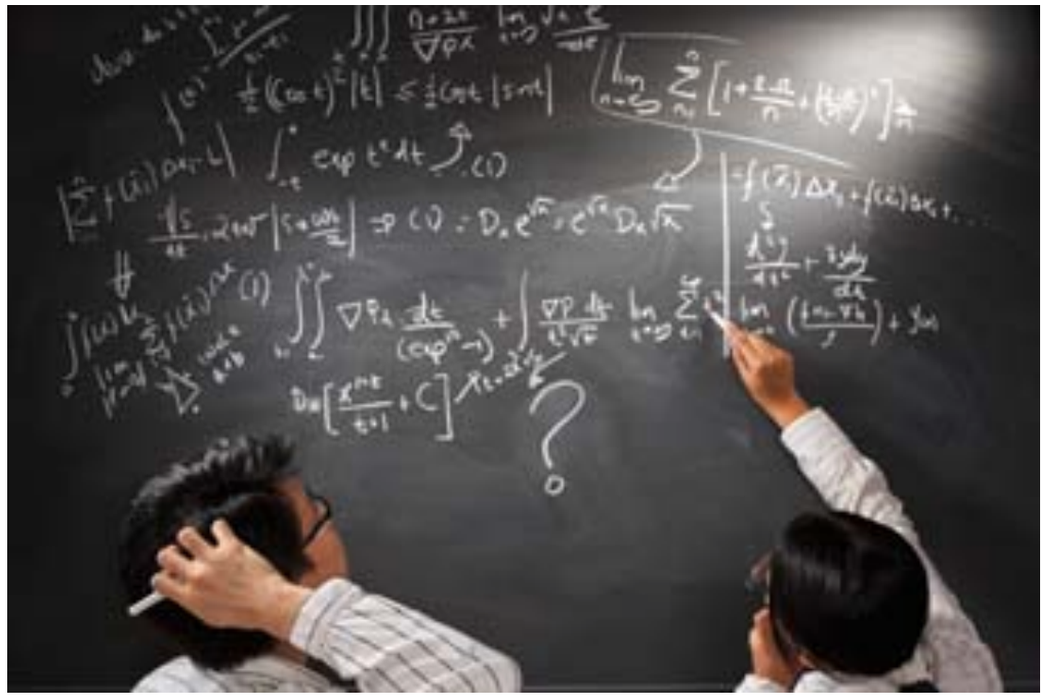
Philosophy of Mathematics





Philosophy and Education

- Teasing apart different concepts
- Offering new perspectives on familiar phenomena
- Point to areas of discussion that might otherwise go unnoticed

Different teaching goals

- What do we want students to walk away with at the end of the school year?
- Do we want them to know certain things?
- Or do we want something more?

Understanding

- Understanding in literature on mathematics education
- Understanding a theory
- Understanding why something is true

In mathematics education

- “Divisibility and Multiplicative Structure of Natural Numbers: Preservice Teachers’ Understanding”, Journal for Research in Mathematics Education 1996

- “Secondary School Students’ Understanding of Mathematical Induction: Structural Characteristics and the Process of Proof Construction”, International Journal of Science and Mathematics Education 2012

- “High school students’ understanding of the function concept”, Journal of Mathematical Behaviour 2013

What is this ‘understanding’?

- How students understand mathematical induction is researched by seeing how they approach problems involving mathematical induction and how they think about induction

- Students’ strategies and definitions of mathematical induction are grouped together to give an idea of how students understand mathematical induction

- This question, ‘*how* do students understand X?’ is (in philosophical terminology) asking for a conception

Conceptions

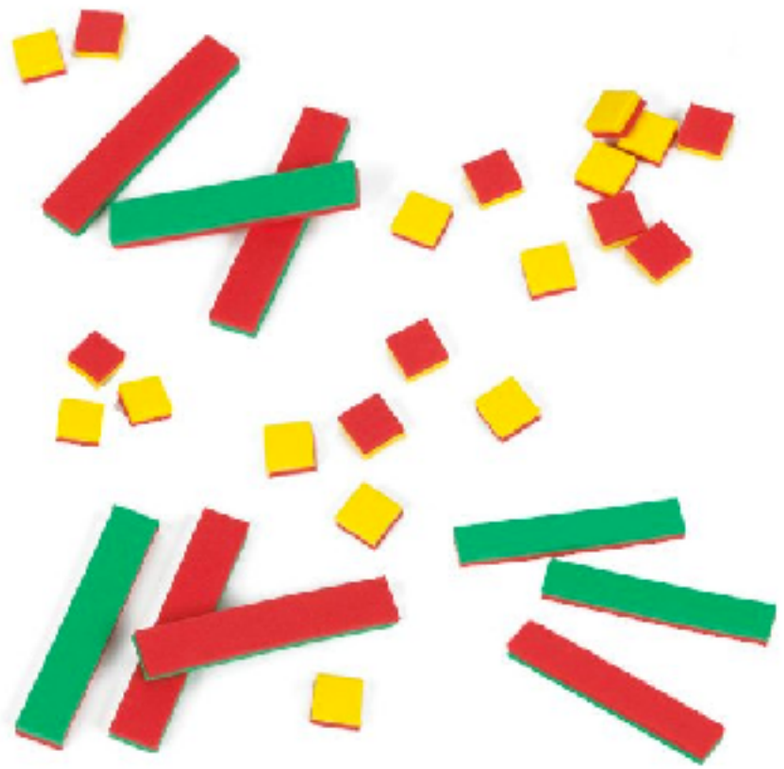
- Concepts: what is shared
- Conception: what is in the head of an individual; the associations and views he/she has related to the concept
- It seems that often research into *how we understand* something is research into what conceptions people have

Theories

- Not just single concepts are relevant for education; sometimes we want students to see how different concepts work together to form a part of mathematics

- 1/4 = 0.25

- 16% of 25



Understanding and explanation

- Often we can understand *why* something is true, over and above *that* it is true

Explanations in mathematics

- We see the same kind of thing in mathematics

123321

753357

963369

All divisible by 37!

Not always …

- 9 is the thirteenth digit of both π and *e:*

  3.14 159 265 358 979 3

  2.71 828 182 845 904 5

- *Why* is 123321 divisible by 37?

… but when there's symmetry…

- A number *p* between 0 and 1 is randomly generated to give the bias for a coin. So, *p* is the chance the the coin lands on heads. What is the chance that the coin lands on heads 1000 times, out of 2000 coin tosses?

- 1/2001, for any number of heads

- The number *p* can be interpreted as the number of heads over 2000 tosses. Since *p* has an equal chance to deliver 0 head as 1000 heads, the chance for any number of heads (between 0 and 2000) is equal.

… or some unifying factor

- Claim: the product of any three consecutive nonzero numbers (e.g. 3, 4, 5 or 14, 15, 16) is divisible by 6

- First, at least one of the three numbers will be even (so divisible by 2)

- Second, since there are three numbers exactly one of them is divisible by 3

- So, their product is divisible by 2 x 3 = 6

And maybe in more cases

- 0.999999… = 1, but *why?*

- 0.333333… = 1/3, so just multiply by 3

- 0.999999… = 1 because you can’t add anything to 0.999999… to get to 1

Back to understanding

- In cases where it makes sense to ask for an explanation of a mathematical fact we can understand *why* that fact is true

- So understanding *why* is more than just knowing that something is true. It involves being able to give explanations and therefore requires some idea of what underlies the mathematical fact

- We can get some sense of how well one understands why something is true based on how well one can give explanations

Benefits of understanding

- We seem to remember better (and our beliefs are more stable) when we don't just know that something is true, but understand *why* something is true

- Explanations, because they use aspects such as symmetry, often show the mathematical structure more clearly

- Explanations also often generalise more easily than other types of reasoning, meaning that once one understands why something is true it is often easy to understand why similar results are also true

Generalising an explanation

- As an example, take the fact that the sum $1 + 2 + 3 + \ldots + n = n(n + 1)/2$

- $1 + 2 + 3 + \ldots + n$
  $n + \ldots + 3 + 2 + 1$

- The same symmetry can be used to figure out
  $1 + 2 + 3 + \ldots + 2n$
  or any other such sum

Understanding in adult education

- We don’t just learn more when we understand something. We also learn the same things better, remembering them more accurately and with more confidence.

- Since techniques that can be used in explanations also might be remembered better, including them in adult education could benefit

- In general, being aware of different ways in which students might now something is helpful in thinking about what exactly we expect from students