Gresham College: Over four centuries of adult education

Raymond Flood

Slide: Title

Gresham College, in the City of London, was established in 1597 with the founding principle of accessible free education for all. It was funded from the will of the Elizabethan financier Sir Thomas Gresham. In my presentation I will first briefly talk about him and what he wanted to achieve. One of his original seven professorships was in Geometry and this is the oldest professorship of mathematics in England. I will talk about some of the holders of the chair: from Henry Briggs, the first Geometry professor and creator of common logarithms to some of the more recent professors.

The final part of the lecture will be about my lectures and some of the excitement, achievements and challenges in giving lectures in mathematics to a general adult audience and I will show some of the approaches I've taken

Slide: Sir Thomas Gresham

Gresham served all but one of the Tudor monarchs. First Henry VIII, then Mary I followed by Edward VI and finally Elizabeth I. He was a very successful merchant banker involved with trade between London and Antwerp and did well for himself and his clients while serving these four monarchs.

Slide: Career

Gresham began his public career by purchasing gunpowder for Henry VIII. It wasn't until the end of the century that gunpowder was manufactured in quantity in England. When Henry VIII died he left the country's finances in a mess due to his extravagance and mismanagement. Gresham was called in to help and was very successful and was appointed Royal Agent under Edward VI. On the catholic Queen Mary's accession in 1553 Gresham, because he was a protestant, went out of fashion for a while but he was so valuable in his ability to raise loans at a good rate of interest that he was soon reinstated. Gresham was a shrewd economist with a deep understanding of currencies and coinage. His Gresham's Law is usually stated that 'bad money drives out good' and his efforts to restore respect for English currency strengthened England nationally and internationally.

He served Elizabeth when she became Queen in 1558 and was knighted for his service. He continued to amass his fortune becoming one of the richest if not the richest man in England and had the grandest mansion in London. He built in London in 1565 the Royal Exchange, where merchants could meet to transact business, such as buy and sell cargoes, share business deals and join together to fund ships which were in effect joint stock companies. It was the centre for commerce in London. The Royal Exchange burnt down twice and the third one is now no longer a financial centre but an upmarket retail centre and the rents from it fund the Gresham Professorships via the Corporation of London and the Mercer's Company.

Slide: The Will: the Corporation

In his will Gresham divided most of his assets between the Corporation of London and the Mercers' Livery Company. He wrote:

I Will and Dispose that one Moiety.. shall be unto the Mayor and Commonalty and Citizens of London … and the other to the Mercers … and from thence, so long as they and their Successors shall by any means or title have hold or enjoy the same , they and their successors, shall give and distribute, to and for the sustenation, maintenance and Finding Four persons, from Tyme to Tyme to be chosen, nominated and appointed …. And their successors to read the Lectures of Divinity, Astronomy, Musick and Geometry…

Slide: The Mercers responsible for the other three chairs

The Mercers were responsible for the appointment of the other three original professorships in Law, Physic (Medicine) and Rhetoric.

As Valerie Shrimplin, Academic Registrar at Gresham has written

Slide: Valerie Shrimplin quote

… an important fact for the history of science in England is that the Chairs for Astronomy and Geometry at Gresham were the first Chairs in those subjects at any English university. In choosing these subjects, Thomas Gresham

appeared to have clearly understood and recognised their importance as separate disciplines in scholarship, many years earlier than either Oxford or Cambridge, where they continued to be studied only as part of a broader classical curriculum. Gresham recognised the importance of applying in practice the knowledge gained from theoretical study. In astronomy for example, the emphasis was on its use for mariners in navigation and geography generally.

Slide: Original Gresham College and list of Geometry Professors

Here we see the first Gresham College which was created in Sir Thomas Gresham's mansion. The list is of the first 31 Professors of Geometry. They had to give their lectures in English as well as Latin which considerably widened their audience to many members of the community for example merchants and navigators not familiar with Latin. The requirement to repeat the lectures in Latin was abandoned in 1811.

The first Gresham Professor of Geometry was Henry Briggs who was the creator of common or base 10 logarithms. They were of great use in calculations and proved a great boon to navigators and astronomers.

Briggs was not the first to introduce logarithms as an aid to calculation. That was John Napier but it was Briggs who devised a new form of logarithms which satisfied:

Slide: Logarithms and Chilias Prima

$\log_{10} 1 = 0$ and $\log_{10} 10 = 1$.

Then to multiply two numbers one simply added their logarithms.

$\log_{10} ab = \log_{10} a + \log_{10} b$

In 1624, after he had left London to become the first Savilian Professor of Geometry in Oxford, Briggs published an extensive collection of logarithms to base 10 of the integers from 1 to 20,000 and 90,000 to 100,000, all calculated by hand to fourteen decimal places.

The invention of logarithms quickly led to the development of mathematical instruments based on a logarithmic scale. Most notable among these was the *slide rule,* versions of which first appeared around

1630 and were widely used for over 300 years until the advent of the pocket calculator in the 1970s.

So the first professor, Henry Briggs, satisfied the wishes of Gresham in developing and disseminating mathematics of immediate and long-lasting importance.

The next professor I want to mention is Robert Hooke, one of the world's greatest scientists and inventors, who was appointed in 1665. Hooke was professor of geometry at Gresham College for over thirty-five years, giving lectures on mathematics to the general public. As well as this the Royal Society held its meetings at the College - it had been founded at Gresham a few years earlier. Hooke was the Society's Curator of Experiments and was required to design and present experiments on a regular basis. In this way, Gresham became an important centre for scientific research and debate. It was probably the highest point in the College's history.

Hooke was interested in the mathematical principles underlying many of his experiments, and designed a number of mathematical instruments. He was also interested in the design of clocks and watches and formulated 'Hooke's law for springs': if a weight is attached to a spring, the resulting extension of the spring is proportional to the weight added.

Slide: **A plaque on the top left commemorates the High Street site in Oxford of Robert Boyle and Hooke's laboratory (top right), where they worked on the gas and atmospheric-pressure laws.**

Below on the left is Hooke's universal joint. Bottom right is Hooke's diary for 21st August 1678 which records a visit to a coffee house with Sir Christopher Wren: here they exchanged information on their recent inventions, including Hooke's 'philosophicall spring scales'.

Sir Christopher Wren was a lifelong friend of Hooke's and had been Professor of Astronomy at Gresham before moving to Oxford. Indeed it was after a lecture of Wren's at Gresham that the Royal Society was founded. After the great fire of London, Hooke helped Wren in the rebuilding of the city.

But the work of Hooke that I like the best is his *Micrographia* means *little pictures.*

Slide: Micrographia and microscope and flea and compound eye

It was described by the diarist Samuel Pepys as *the most ingenious book that I ever read,* and was the first English book to be devoted to microscopy. The historian Lisa Jardine has commented that the subject of microscopy:

> *required exactly Hooke's combination of instrument-making ability, experimental dexterity and sheer showmanship of which his flair as a draughtsman made a further important contribution*

The slide shows some of Hooke's beautiful illustrations. The one bottom left is an engraving of his microscope. The main one is that of a flea and below that the compound eye of a fly. Hooke also suggested that light was a wave.

The eighteenth century and first half of the nineteenth was a poor time in the history of the college mainly due to the behaviour of the professors who often misused their positions and produced very little. By 1811 attendance at the public lectures had fallen to very low levels. But the second half of the nineteenth century saw improvement partly due to the improved calibre of the professors but also to the change in attitude to state funding of education and increased emphasis on public access and education.

In 1890 a truly outstanding appointment was made.

Slide: Karl Pearson

Karl Pearson is the founder of modern mathematical statistics and if you had attended his lectures at Gresham you would have heard many modern statistical concepts being introduced, often for the first time and in a very visual and hands on way.

His lectures on 'The Geometry of Statistics and The Laws of Chance' were very popular, drawing audiences of as many as 300 students.

Reports tell us that in some lectures he used dice, roulette results and 10,000 pennies scattered on the floor!

Pearson first of all wanted to standardize the way frequency distributions were represented, introducing the term *histogram*. He then wanted to quantify variation introducing the term *standard deviation* and indeed higher moments of a distribution. One of his major motivations was to provide a mathematical underpinning to Charles Darwin's ideas on natural selection and evolution based around individual biological variation – *an exact measure to the problem of evolution* as he put it.

In his Gresham lecture of 13th November 1893 Pearson encouraged his students to develop an awareness that they were living:

> *in an essentially critical period of science, when more exact methods and more sound logic were upsetting or modifying many of the whole statements, which had been taken for years as scientific gospel*

Pearson resigned his chair in 1894 due to ill health and was replaced by Henry Wagstaff who was in post for 45 years until the start of the World War II and gave over 500 lectures and also taught in other London schools.

Slide: List of professors

After the War the first appointment was the applied mathematician Milne-Thompson and then Alan Broadbent who had been President of the Mathematical Association and longstanding editor the *Mathematical Gazette.* Next was Sir Bryan Thwaites who was the founding director of the School Mathematics Project (SMP) and was an enthusiastic promoter of computers. The next holder, Clive Kilmister, was also President of the British Society for the History of Mathematics.

Slide: Zeeman, Stewart, Penrose and Thimbleby

This slide shows the next four professors. Sir Christopher Zeeman, Ian Stewart and Roger Penrose are well known popularisers of mathematics. Zeeman and Stewart gave the Royal Institution Christmas lectures – indeed Zeeman was the first mathematician to do so. Ian Stewart has written many popular books such as *Does God Play Dice?* So has Sir Roger Penrose with his *Emperor's New Mind, Shadows of the mind* and *The Road to Reality – a complete Guide to the Laws of the Universe.* Harold Thimbleby is a computer scientist with a passion for designing dependable systems to accommodate human error, especially in healthcare and medicine.

Slide: Robin Wilson

Many recent lectures are on the Gresham website – in fact nearly 2000 of them - and this is a subset showing some of Robin Wilson's. Robin's specialities are history of mathematics, graph theory and combinatorics. He has this year produced the Introduction to Combinatorics in Oxford University Press's Very Short Introduction series. He had a long career at the Open University and Robin and I have collaborated on many projects together.

He was succeeded by John Barrow

Slide: John Barrow

John Barrow is a cosmologist and mathematician based in Cambridge and is only the second person to have been both Gresham Professor of Astronomy and Gresham Professor of Geometry. The other was Laurence Rooke in the middle of the seventeenth Century!

John took advantage of the London Olympics of 2012 to focus his lectures on sporting topics and mathematics as you can see on the slide. They are very interesting and introduce many important mathematical properties in an accessible way.

Slide: Current Gresham Professors

That rather quick tour of 400 years brings us up to the present and here are the present holders of the professorships. In addition to the original seven, Astronomy, Divinity, Geometry, Law, Music, Physic and Rhetoric there are three new ones: Commerce, Environment and Information Technology.

As well as these there are also Visiting Professors who help to cover topics not easily covered by the current ten chairs. Examples of such fields are demographics, the built environment, literature, ageing, climate change, international affairs, local and regional government, neurology and the EU referendum.

The major purpose of the College is to provide high quality free public lectures aimed at a general audience and intended to appeal to both generalists and specialists. Each year over 20,000 people attend these lectures and now thanks to the internet the lectures (over 1900 of them) are available through our website: www.gresham.ac.uk.

There are approaching seven million views and downloads each year and increasing activity on Facebook and Twitter as well as a smartphone app.

The College is open to all, regardless of age, background or previous educational qualifications. There are no entry or admission requirements. The College has no registered students, does not undertake any assessment and does not confer any academic awards or qualifications.

I want to finish by describing how in mathematics I have addressed the challenge of giving high quality public lectures available to the general public. Lectures, moreover, which are recorded and for which the slides and transcripts are also made available.

Slide: Background, Expectations and accessibility

- Background of the audience

I have said that the audience is a general adult audience but it is by no means homogeneous. There are people from a wide range of backgrounds some very knowledgeable in their own areas, while others might have studied mathematics but a long time ago. Others just have an interest in mathematics with little background and there are some who are knowledgeable mathematicians.

So what expectations are there? I believe the audience want to hear a stimulating and accessible lecture on an important and interesting topic. They do not expect to understand everything or every step but want very much to know where the lecture is going and what path is being taken and if possible why take that path or approach.

As for my expectations, I want to do something that interests me – this is one of the advantages of not teaching to a syllabus and not having to assess or examine anyone. I also always want to look at an important topic in mathematics. It takes me about 50 hours to prepare one of these lectures so it vital it is something I want to learn about and share with others. I'm not unusual in taking this amount of time as other colleagues say they spend about that length of time in preparation.

Well, I used the word accessible. How do I try to achieve accessibility?

There are two things I try to avoid and that is assuming the audience is familiar either with the techniques of mathematics or mathematical notation.

I always use the history of mathematics as a background for the lecture, introducing key figures giving some of their background, achievements and motivation. Doing this also provides a break in the mathematical narrative and also gives another strand to get something out of the lecture if getting lost in the mathematical arguments. The other thing I do is to take a visual approach where possible and I will give some examples of how I do this.

Here are some guidelines I use in trying to achieve accessibility.

Selection of Lecture Content

First, I am free to choose the lecture topics – subject to discussion with the Gresham College Academic Board.

In my first year I took as a theme *Shaping Modern Mathematics.*

The 19th century saw the development of a mathematics profession with people earning their living from teaching, examining and researching and with the mathematical centre of gravity moving from France to Germany. A lot of the mathematics taught at university today was initiated at that time. Whereas in the 18th century one would use the term *mathematician*, by the end of the nineteenth one had specialists in *analysis, algebra, geometry, number theory, probability and statistics*, and *applied mathematics*.

The series of lectures I offered looked at the shaping of each of these mathematical areas and at the people who were involved. The lecture titles were:

Ghost of Departed Quantities: Calculus and its limits
Polynomials and their Roots
From One to Many Geometries
The Queen of Mathematics
Are Averages typical?
Modelling the World

For my next series the theme was *Applying Modern Mathematics*, illustrating how mathematics has developed more recently and what mathematics can, cannot and hopes to achieve. The titles were:

Butterflies, Chaos and Fractals
Public Key Cryptography: Secrecy in Public
Symmetries and Groups
Surfaces and Topology
Probability and its Limits
Modelling the Spread of Infectious Diseases

For the last two years my theme was *Great Mathematicians, Great Mathematics.* Each of these lectures started with a famous mathematician, from Fermat in the seventeenth century to Turing in the twentieth century, and described them and some of their most important work and then discussed more recent developments and applications. The titles were:

Fermat's Theorems
Newton's Laws
Euler's Exponentials
Fourier's Series
Mobius and his Band
Cantor's Infinities

While in my last year they were:

Einstein's *Annus Mirabilis,* 1905
Hamilton, Boole and their Algebras
Babbage and Lovelace
Gauss and Germain

Hardy, Littlewood and Ramanujan
Turing and von Neumann

Visual Aids

I think it is important and beneficial to the audience to make use of visual aids. For example, Memorials can be helpful in introducing a person's life work and impact as seen by contemporaries.

Slide: Isaac Newton's memorial in Westminster Abbey

Here using Isaac Newton's memorial in Westminster Abbey I can illustrate some of his achievements by looking at his tomb in Westminster Abbey. The base bears a Latin inscription and supports a sarcophagus with large scroll feet and a relief panel.

Slide: Sarcophagus

The sarcophagus depicts boys using instruments related to Newton's mathematical and optical work, including the telescope and prism, and his activity as Master of the Mint. The boys are playing on the left with the reflecting telescope which Newton invented, on the right a furnace and newly minted coins relating to his time at the Royal Mint, while in the middle one boy weighs the Sun and planets and to the right of it another boy is playing with a prism.

Here is an illustration from a popular book published in 1728 where Newton explains the motion of the moon as follows.

Slide: From Newton's *A Treatise of the System of the World*

Let a body be projected horizontally from the top of a high mountain and do it repeatedly with greater and greater velocity.

The first time it falls and ends up at D. Now project it with a greater speed and it will go further falling and ending at E. Now project it with even greater speed and it will fall at F and then project it faster and it still falls to earth, this time landing at G.

But now increase the velocity again and this time the projectile continues to fall but the curvature of the earth causes it to curve away from the projectile and the projectile arrives back at the mountain top where it performs the same motion again. I'm supposing that there is no air resistance or other retarding force. So we can think of a satellite, such as the moon, that orbits the earth as continually falling towards it but never landing because the earth is curving away from it. Newton calculated this rate of falling for the moon and showed it was as described by his law of gravitation which also gives the rate of fall of bodies on the earth's surface. It was this result which showed that his law was a *universal* law of gravitation in that it applied equally to the earth and the heavens.

Slide: First appearance of the integral sign

In 1675 Leibniz introduced two symbols that would forever be used in calculus. One was his *d* (or *dy*/*dx*) notation for differentiation, referring to a decrease in dimension − for example, from areas (x^2) to lengths (x). The other was the integral sign: attempting to find areas under curves by summing lines, which he then represented by an elongated *S* for sum: this is the symbol $\int$ for the integral sign. *It will be useful to write* $\int$ *for omn*$\cdots$

And on this slide we see where he first wrote it down!

Slide: Woolsthorpe Manor

This, Woolsthorpe Manor, is the birthplace of Newton, and is well worth a visit. I think it is ironic that the shape chosen for the tie bars is one that you might recognise as one similar to Leibniz's integral sign!

Slide: Boole and Hamilton

Here we see Boole at the bottom sharing the window with Aristotle and Euclid – distinguished company indeed. And on the right is the plaque on the bridge where Hamilton scratched his formula for quaternions. These memorials mark the work of two mathematicians who can fairly be said to have liberated algebra from the constraints of arithmetic.

Computer Simulation

I want to present an example of visualization using the graphical capabilities of spreadsheets.

The first is based on the simple experiment of continually tossing a coin.

If we were to toss a coin a large number of times we observe some interesting and, I think, counterintuitive features and I want to introduce them using the idea of a random walk.

Slide: Symmetric random walk.

At each step you move one unit up with probability ½ or move one unit down with probability ½.

An example is given by tossing a coin where if you get heads move up and if you get tails move down and where heads and tails have equal probability.

It can also be used as a model for two queues where going up means the queue you are in is moving ahead and moving down means that the queue you are in falling behind.

Slide: Coin Tossing

I've done some simulations using a spreadsheet to illustrate the behaviour obtained.

The first column generates a +1 or -1 with equal probability. The second column keeps a running total of the numbers generated to that point.

On the graph the vertical axis is the difference between the number of heads and the number of tails. The horizontal axis is the time in the form of the number of tosses. I have done it for 500 tosses.

Take a moment to think about the type of path you would expect.

Most people think that in a long coin tossing game each player will be on the winning side for about half the time or that half the path will be above the horizontal axis and half below. They also believe that the lead should change frequently.

The path you see does have heads in the lead about half the time and half the time tails is in the lead but it took me quite a few tries to get this result because it is not the usual path you get.

Let me show you ten other paths in the order I obtained them.

Slide: Ten Paths

Path 1. In this case Heads is always in the lead.

Path 2. Here a bit of oscillation but then tails is always in the lead

Path 3. Here heads most of the time then tails for a short time and then back to heads

Path 4. Here heads all the time but there is a short time when tails took the lead

Path 5. Here mainly tails but on occasions heads goes ahead for a short time.

Path 6. At the start it is heads and then tails ahead

Path 7. Heads always ahead here.

Path 8. Tails is ahead most of the time here

Path 9. Heads all the time here

Path 10. Again heads all the time.

These results are typical.

Slide: Law of long leads or arcsine law

The law of long leads, more properly known as the arcsine law, says that in a coin-tossing games, a surprisingly large fraction of sample paths leave one player in the lead almost all the time, and in very few cases will the lead change sides and fluctuate in the manner that is intuitively expected of a well-behaved coin.

The numbers are quite remarkable.

- In *one case out of five* the path stays for about *97.6%* of the time on the same side of the axis.

Slide: one case out of ten and once per second

- In *one case out of ten* the path stays for about *99.4%* on the same side of the axis.
- A coin is tossed once per second for a year.

In *one in twenty cases* the more fortunate player is in the lead for *364 days 10 hours.*

In *one in a hundred cases* the more fortunate player is in the lead for *all but 30 minutes.*

The symmetry of the fair coin is still here. But the symmetry does not show itself in half the path being above the axis and half below.

Rather the symmetry shows itself in that half the sample paths are mostly above the axis and half the sample paths are mostly below the axis.

Modelling the World

The one I wish to use for illustration of modelling the world is the machine for predicting tides which computed the depth of water over a period of years, for any port for which the 'tidal constituents have been found from harmonic analysis of tide-gauge observations' — that is, from the coefficients of the trigonometric series representing the rise and fall of the tide.

Slide: William Thomson, Record at River Clyde and Tidal Predicting Machine.

Modern tidal analysis and prediction in all its mathematical and mechanical detail is due to the Belfast born William Thomson from around 1867.

If we know the tidal record for say a year at a particular spot we will be able to determine the amount of each tidal component at that spot.

The machine could then be set up to predict the tide a year ahead in a matter of days and was in use until after the Second World War and used for the D day landings in Normandy.

Proof/Framework

I always like to include a proof of something in every lecture I do. For me proof is the essence of mathematics and is what gives it so much of its beauty.

The example I want to show you is from topology or rubber sheet geometry and is due to the eighteenth century Swiss mathematician Leonhard Euler (1707–1783)

Slide: Euler

Euler was the most prolific mathematician of all time. He produced over eight hundred books and papers in a wide range of areas, from such ‘pure’ topics as number theory and the geometry of a circle, via mechanics, logarithms, infinite series and calculus, to such practical concerns as optics, astronomy and the stability of ships. He also introduced the symbols *e* for the exponential number, *f* for a function and *i* for $\sqrt{-1}$.

Euler published 228 papers *after he died* making the deceased Euler still one of the world’s most prolific mathematicians!

In the words of the great French mathematician Pierre-Simon Laplace:

Read Euler, read Euler, he is the master of us all!

Slide: Polyhedra

Polyhedra are solid shapes bounded by plane faces such as a cube which is bounded by six squares or a square pyramid bounded by a square and three triangles. They have been studied since ancient times, for example Eulid concludes his *Elements* by showing how to construct the *regular polyhedra* and proving that there are only five of them.

The word polyhedra comes from the Greek roots, *poly* meaning *many* and *hedra* meaning *seat* and on the right of the slide we see many examples.

Initially we will be dealing with *Convex* polyhedra which have the property that the line joining any 2 points in the object is contained in the polyhedron, or an alternative way of thinking about it is that a convex polyhedron can rest on any of its faces.

Slide: Convex and non-convex polyhedra

On the left we have an icosidodecahedron. It is convex because it is possible to set it down on any of its faces or alternatively if you take any two points in it the line joining those two points also lies in the icosidodecahedron.

On the right we have a hexagonal torus or doughnut shape, made up of six blocks each consisting of four quadrilaterals. It is not convex because it is not possible to set it down on one of the inner quadrilaterals or if you take a point towards the front face and a point towards the back then then joining them crosses the “hole” of the torus and so does not lie in the torus.

We will concentrate on convex polyhedrons.

Now to discuss Euler’s formula for convex polyhedral:

Slide: Euler’s formula for convex polyhedra, V – E + F = 2

In this formula V is the number of vertices, E is the number of edges and F is the number of faces.

It is an alternating sum: number of vertices minus number of edges plus number of faces.

Also note that a vertex is a point so zero dimensional, an edge is a line so one dimensional and a face is part of a plane so two dimensional.

The formula assigns plus to the even dimensions 0 and 2 and minus to the odd dimension 1.

Let me run through some examples to illustrate the formula.

Slides: Tetrahedron, Cube, Octahedron, Dodecahedron and Icosahedron.

Let me now give an outline if the proof of Euler’s polyhedron formula.

It is going to make use of the nice structure of the expression V – E + F and that it is an alternating sum.

Slide: V – E + F

- If we remove an edge and a face at the same time then number of vertices – number of edges + number of faces stays the same. Because you are talking away one less edge but adding on one less face

- Similarly we remove an edge and a vertex at the same time then number of vertices – number of edges + number of faces
  stays the same. Because you are talking away one less edge but adding on one less vertex.

The structure of the proof is that we are going to keep doing these operations so that V – E + F remains unchanged – we say it is an invariant – until we get to something simple enough to calculate and that must also be the original value because V – E + F did not change!

Let us see how we can do it.

Slide: Deform the convex polyhedron into a sphere

The first step is to deform the convex polyhedron into a sphere. This why we assumed the polyhedron was convex to allow us to do this. Imagine the polyhedron is made out of wire then surround it with a sphere, place a light bulb inside it and look at the shadow of the polyhedron on the sphere. Then being convex, allows us to say that the image on the sphere leaves V – E + F the same or invariant.

Slide: Remove an edge so as to merge two faces.

Step 2 is to remove an edge so as to merge two faces. Leaves V – E + F unchanged. Keep doing this step until only one face is left.

Slide: End up with only 1 face

When we end up with only 1 face the remaining edges and vertices forming a graph with no loops because if there was a loop there would be an inside and outside face and we would remove an edge of the loop to merge those faces together.

Slide: remove a terminating vertex and edge from the tree

In step 4 we remove a terminating vertex and edge from the tree. Leaves V – E + F unchanged. Keep doing this until you are left with only one vertex.

Slide: As F = 1, E = 0 and V = 1, V – E + F = 2

We can now calculate V – E + F for this simple situation as we have one vertex, no edges and one face so the answer is 2. As V – E + F has been invariant the whole way through this must also have been its starting value so we have proved the result.

As an application of this result it is fairly easy to prove the theorem that there are **only** five regular polyhedra which was how I continued the lecture.

In a ***regular*** polyhedron all the faces are the same and the arrangement of faces at each vertex is the same. They are:

Tetrahedron – four faces each an equilateral triangle

Cube – six faces, each a square

Octahedron – eight faces each an equilateral triangle

Dodecahedron – twelve faces each a regular pentagon

Icosahedron – twenty faces each an equilateral triangle

Let me finish.

I have tried to give an insight into how a College such as Gresham was initiated and developed, the renowned mathematicians who have held the Chair of Geometry there and the contributions they have made to the world of mathematics and to the general public who have benefitted from their lectures for over more than 400 years.

I hope I have also shared with you my own enjoyment in finding different ways of making mathematics accessible to a general audience.

Thank you!