Maths At Work –
Mathematics in a Changing World

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These proceedings include both refereed and non-refereed papers.

People writing papers for the conference were able to choose whether or not they wanted to go through the formal, blind, peer refereeing process. Those papers which went through this refereeing process are marked in the table of contents with an asterisk (*) next to the title.

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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established in July 1994 as an international research forum with the following aim:

> To promote the learning of mathematics by adults through an international forum, which brings together those engaged and interested in research and development in the field of adult mathematics learning and teaching.

Charitable status

ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number 3901346). The company address is 26, Tennyson Road, London NW6 7SA.

Objectives of ALM

The Charity's objectives are the advancement of education by the establishment of an international research forum in the lifelong learning of mathematics and numeracy by adults by:

- Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;
- Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.

ALM Activities

ALM members work in a variety of educational settings both as practitioners and research, improving the learning of mathematics at all levels. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world.

ALM does not foster one particular theoretical framework but encourages discussion on research methods and findings.

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Anyone who is interested in joining ALM should contact the membership secretary. Contact details are on the ALM website: www.alm-online.net

Membership fees for 2011

	Sterling	Euro	U.S.Dollar
Individual	£20	€24	$32
Institution	£40	€48	$80
Student/unwaged	£4	€6	$8
Low waged – contribute between full & unwaged			

Preface

The 17th ALM conference was held in Oslo, Norway in 2010. It was attended by 50 researchers, practitioners and policymakers from 16 nations (Australia, Canada, Denmark, Greece, Ireland, the Netherlands, New Zealand, Norway, Poland, Qatar, Spain, Sweden, Switzerland, Thailand, the United Kingdom and the United States of America).

This was the first time that the conference met in Norway, and the meeting offered a good opportunity for national and international participants to come together to share and discuss issues concerning adults learning mathematics. The conference title *Maths at work – mathematics in a changing world* indicated the focus of the discussion; the skills needed for work and life in a changing world, including mathematics for industry, and with a special focus on the "whats, whys and hows" of adult numeracy.

This year the ALM conference programme included invited plenary lectures as well as a wide variety of parallel sessions, all contributing to an enriching dialogue during the days in Oslo. We are grateful to all participants and presenters who wanted to share ideas and join in discussions at ALM 17.

The conference was organised and hosted by Vox, Norwegian Agency for Lifelong Learning. We want to thank Kees Hoogland especially for his helpful and patient assistance in the process.

All conference presentations that are submitted for publication and meet the editors' requirements for style and presentation are published in the conference proceedings.

Presentations for which no paper was submitted are represented by their programme abstract.

Two kinds of contributions to the proceedings of ALM 17 were possible:

1. **Refereed Papers**
   Papers that have been peer reviewed are identified by an asterisk (*) alongside their title.

2 **Non-refereed Contributions**
   Papers or workshop reports whose authors did not request their contribution to be refereed have no identification marking.

Acknowledgement

Javier Palomar-Diez and Christopher M. Klinger have been responsible for the peer review, and Joanne Kantner has been of invaluable assistance in the technical editing process.

Their work is greatly appreciated and hereby acknowledged.

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Plenary Presentations

Contextualised Mathematics

Eva Jablonka

Luleå University of Technology, Sweden

Eva Jablonka is professor of mathematics education at Luleå University of Technology, Sweden. She earned a PhD in mathematics education at the University of Technology in Berlin (1996) and a Habilitation in educational science at the Freie Universität Berlin (2004). She has worked in Germany and as visiting researcher in Australia and New Zealand.

As adult mathematics education does not primarily function as an introduction into the esoteric domain of academic mathematics, but rather aims at developing adults' basic numeracy skills and problem solving related to everyday practices and work places, there is commonly a focus on "mathematics in context" and "contextualised mathematics tasks". As in teaching and learning school mathematics, transfer of learning between contexts poses a challenge for researchers and for teachers who attempt to relate the curriculum to the learners' contexts. On the other hand, there is evidence that „[P]rojects employing constructivist theories of learning and with a 'connectionist' orientation to teaching and learning, making connections with the world beyond the classroom and with other elements of mathematics, demonstrate improvements in attitude and attainment." (Coben et al., 2003, p. 7). In school mathematics textbooks, contextualised mathematics in the form of traditional word problems often constitutes an intermediary domain that serves as a route to access academic mathematics. This function is less salient in texts used in workplace related mathematics education. In such materials, depending on the degree of specialisation of the work, mathematical notions, technical terms from other fields, specific ways of displaying information and references to very specific tasks included in the work, establish complex significations. This mix creates a type of contextualised mathematics tasks different from school mathematics. In the talk, I attempt to problematise some difficulties with contextualised mathematics tasks, which are related to the learners' orientations towards meanings. Through a discussion of different types of such tasks, I will address the confrontation and transformation of meanings involved in the mathematical recontextualisation of domestic and workplace practices. I will also touch upon the ideological underpinnings of conceptions of numeracy reflected in different versions of contextualised mathematics.

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Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). Adult numeracy: Review of the research and related literature. London, England: National Research and Development Centre. Retrieved June 10, 2010, from http://www.nrdc.org.uk/uploads/documents/doc_2802.pdf.

Why teach mathematics to Adults - in a changing world?

Lene Johansen

Aalborg University, Denmark

Lene Johansen is an associate professor at Aalborg University. Her research interests relate to basic mathematical skills of adults and their relationships to literacy, school learning, and societal demands, and she aims to link numeracy education, democracy and citizenship using the concept of "Bildung". She has looked at political and policy aspects of setting a new national mathematics curriculum for low-skilled adults in Denmark following the results of the Second International Adult Literacy survey results, and at curriculum planning issues and dilemmas in this regard. She also has a great interest in learning difficulties and she is a member of "The Nordic Research Network on Special Needs Education in Mathematics". She is also a member of the International Numeracy Expert Team as well as of the Danish Expert team for the coming OECD Survey PIAAC.

10 years ago in May 2000 the Danish Government adopted a reform of the vocational education and continuing training system for adults and introduced "Preparatory Adult Education" (PAE). The question "Why teach numeracy to adults with lack of basic mathematical skills?" has been the core question in my research since. I used the political discussion of Preparatory Adult Education and the development of the New National Adult Numeracy Curriculum to find answers to the question.

Comparing the arguments provided by the politicians and by the researchers' showed that there were shared discourses. The picture the politicians and the researchers' constructed of the society and the future society was shared. Both talked about a society which demands skills and competences from the adults. The ideal of human development was also shared both saw active citizens as a result of PAE. Furthermore, they shared the understanding that formal education was a way and a means to reach the ideals as well as fulfill the demands from society and the labour market. But there were also great differences between the politicians and the researchers. Firstly, the understanding of the content in the new education and the understanding of "mathematics" was quite different. The politicians want to offer a course where the content was basic skills. They saw basic skills as context free and that the result of PAE was to provide the adults with a toolbox filled with mathematical tools to be used everywhere. On the other hand, the researchers saw numeracy as a context dependent competence which should be taught in context. The researchers also saw numeracy as consisting of functional mathematical skills and understandings that in principle all adults needed to have. The last area of conflicting discourses was about the target group of PAE. The politicians saw adults with lack of basic skills as adults on the edge of society, I call them the "excluded", adults who were unemployed or soon would lose their jobs, unable to participate in the life of the society. The politicians also described the target group as adults unaware of their own inabilities. On the other hand the researchers described the target group

as a group of competent adults able to cope with their lives but unaware of their own abilities according to mathematics. Ten years has past and the society has changed and so has the lives of many adults. Having that in mind, I will in my presentation discuss “Why teach mathematics to Adults - in a changing world?”

A door to lifelong learning for all

Lena Lindenskov

Danish Scool of Education, Danmark

Lena Lindenskov, is Professor in mathematics and science education, Danish School of Education, the Copenhagen campus of Aarhus University. Member of local organiser team for ALM 8, 2001. Developer on national curriculum and teacher guidance materials for PAE, preparatory adult education in mathematics 2000-02. Participant in EU-projects ALMAB, EMMA, MIA, and in Asia-Europe meeting ASEM Education and Research Hub for Lifelong Learning.

In many reports being numerate (/mathematical literate) is seen as relevant for all and is illustrated by examples from everyday settings and educational and work-place settings. At the same time stories of children, adolescents and adults experiencing failure in mathematics and even math anxiety are told. The problem area of relevance and the problem area of attitudes and feelings towards numeracy and mathematics are not new. They have been studied and discussed for decades, but still failure is experienced in many contexts and still mathematics acts in some contexts as a door keeper instead of a door handle to education and learning.

In the plenary I will try to look at the issues from the perspective of lifelong learning. In a changing world nobody can at the end of school assume to stop learning and some of this later learning will include some mathematics. Supporting people to continuously being capable and motivated to learn – with and without mathematics – will be a growing need, I think.

I'll look at the issues by asking 'how can adults learning mathematics be a door to lifelong learning for all?' I'll draw on observations and interviews from a number of projects, on discussions on competence concepts in ALM and ASEM, and on James Paul Gee's notion of *doing being an identity* (Gee 2001: 27). I'll try to conclude with some recommendations and questions for research and practice.

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Gee, J. P. (2001). Critical Literacy/Socially Perspective Literacy: A Study of Language in Action. In: H. Fehring & P. Green (Eds.), *Critical Literacy*. *A Collection of Articles from the Australian Literacy Educators' Association*. Delaware & Norwood: International Reading Association, 15-39.
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http://www.dyskalkuli.dk
http://www.cormea.org

Adults Learning Mathematics – Plotting a New Course

Terry Maguire

Institute of Technology Tallaght, Ireland

Terry Maguire is currently the Head of Lifelong Learning at the Institute of Technology Tallaght, Dublin Ireland. She has been involved in adult education for over 25 years. She has been a member of ALM for 10 years. Her research interest includes identifying good models of professional development for tutors of adult mathematics; uncovering the hidden mathematics of the workplace and contributing to setting up a coherent framework for adult mathematics in Ireland.

Research on Adult Learning Mathematics has progressed exponentially since ALM – A Research Forum was formed in the early 90's. At that time one of the central challenges that needed to be addressed was the need to clearly define numeracy as a concept and to ensure numeracy had an identity separate from literacy.

Research on defining numeracy as a concept has culminated in its inclusion as a clearly defined domain in international surveys. The Adult Literacy and Lifeskills International survey (ALLS) (2006) and the forthcoming Programme for the Programme for the International Assessment of Adult Competencies (PIAAC) (Data collection 20011/12) clearly define numeracy as a construct and place a clearly defined concept of numeracy on the agenda of over many countries internationally. The results of these international surveys are powerful drivers in terms of policy development in relation to provision for adult learning mathematics. Since the results of ALLS have been published, a number of countries have developed or are in the process of developing adult mathematics qualification frameworks.

As the environment in which Adult Learning Mathematics – A research forum operates has changed dramatically since its inception, this paper explores the need to consider if we need to plot a new course and to reappraise the role and function of the forum in a changing environment to ensure it has a central and influential role to play in terms of shaping the policy and provision for adult learning mathematics into the future.

Tearing through the tartan maths ceiling

Daniel Sellers

Learning and Teaching Scotland, UK

Daniel Sellers has been involved in numeracy as a tutor, manager and national development co-ordinator. He taught numeracy and managed an adult learning centre in Liverpool at the time of the introduction of the Core Curricula for Adult Literacy and Numeracy in England. He then helped establish literacy and numeracy as a core learning support service in a further education college in Scotland. For the past five years, Daniel has led national developments in adult numeracy in Scotland. His priorities include embedding professional development for a cross-sectoral workforce, and supporting an emerging community of practice around non-formal financial learning/financial capability for adults and young people. He enjoys travelling, reading and playing with his iPhone (especially the numeracy apps!).

New Light on Adult Literacy and Numeracy in Scotland (2008) suggested that as many as 71% of Scots in their thirties lacked the numeracy skills they needed in order to fulfil their potential as individuals, family members and workers. But, with only 7% of the sample study having any true perception of their own maths ability, this "maths ceiling" remains invisible until individuals hit it – sometimes quite hard! The Scottish Government funds national developments in adult numeracy, focusing on research and development, resource development, partnership support, awareness raising and – its priority – professional development for tutors. These national developments must take account of a diverse, sometimes insecure, workforce employed in a range of sectors, supporting learners with widely different learning needs and abilities. This session describes some of the characteristics of adult numeracy learning in Scotland, outlines some of its successes and continuing challenges, and proposes a vision for tearing holes in the tartan maths ceiling.

Maths matters at work

Sue Southwood

NIACE, UK

Sue Southwood is the Programme Director with particular responsibility for developing literacy, language and numeracy with employers and employees in the workplace. Much of Sue's work involves developing innovative approaches to support learners in the workplace or to enter the workplace in conjunction with union colleagues. Before joining NIACE in August 2004, Sue set up and managed workplace basic skills programmes for Northern Foods, Ford Motor Company and Transport for London.

Advancing the case for adult learning in public policy and debate lies at the heart of NIACE's charitable purpose. With our advocacy work we influence and persuade others that improving, increasing and extending opportunities for adults to learn throughout their lives, has real public as well as private benefits.

The UK's refreshed Skills for Life strategy "Skills for Life: Changing Lives" (DIUS 2009) focuses on employability, aiming to ensure that mathematical skills will support adults to find, stay and progress in work but it is not a straightforward task. There are differing perceptions between education and training organisations, employers and individuals of the type of maths that matters in the workplace. It is not as simple as a list of skills that can be easily learnt and assessed but a more complex matter of how mathematical skills and knowledge can be understood and applied in the context of the workplace. The focus should perhaps be less on skills-based tests or qualifications but more about developing mathematical thinking in a workplace context.

This presentation looks at how we are addressing the issues of poor numeracy in the UK and tackling our culture that says "it's ok to be bad at maths".

Paper Presentations

Examples of Good Practice

Våril Bendiksen

Since the introduction of the Norwegian competence goals for basic skills for adults the need to link the basic skills training to authentic situations has been emphasized. The providers of the numeracy courses which are funded by the program for Basic Competence in Working Life are encouraged to create their own teaching materials, based on situations which are relevant for the particular course. For many of the numeracy teachers this means a new way of thinking, for example identifying the numeracy situations and including them in the teaching and training. The extent to which providers have managed this challenge varies. In this talk some of the best examples will be presented.

The Relationship between Students' Characteristics, Use of Mathematics Media and Achievement in Distance Education

Sakorn Boondao

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In providing distance education, "A" Open University has used textbooks as the main medium and a range of other support methods to motivate students to learn. This paper discusses the relationship of the students' characteristics and their use of mathematics teaching media to achievement. Students' characteristics were their demography, mathematics background and computer skills.

The population was 5,584 students who studied one of four Mathematics courses in the first semester of 2008. The sample was 2,746 randomly selected students. The instrument was a questionnaire asking students' satisfaction about the media used for mathematics courses. Every student had received textbooks and information about the support media for each course. The media were classified into four types, textbooks, assignment, face-to-face tutorial and electronic media. At the end of the semester, students were asked to fill in the questionnaire. A total of 44.8% of students returned completed questionnaires.

Textbooks and the assignment were the most popular forms of support. Students' achievements were significantly influenced by their use of study materials and their characteristics: gender, career, income, education, mathematical background and computer skills. It is concluded that students' characteristic are related to their use of the study media and influence their achievement.

Key words: Mathematics media, students' characteristics, Distance Education

Introduction

"A" Open University ("A"OU) has provided a distance education system since 1978 by using textbooks as the main medium and other supporting media to motivate students to learn. Different media types were used to deliver information. Face-to-face tutorials, radio and television programs, and audio and videocassettes have been frequently used for more than two decades. As new technologies became readily available, computer assisted instruction (CAI) VCD and DVD were introduced to the system. In 2001, an assignment accounting for 20% of the assessment was introduced. As Internet access is now available to students, online learning, using "ATutor", a web board, and eTutorials were also used to support the teaching and learning. Each medium and each technology has its own strengths and weaknesses. Nanda, V.K., (1998) suggested that teaching at a distance had to use more than one medium to balance between the weak points and the good points of this type of teaching and to provide reinforcement. He said it might be boring if only print and assignments with feedback were used.

There were four core mathematics courses: Mathematics and Statistics; Mathematics and Statistics for Business; Mathematics and Statistics for Science and Technology; and Mathematics for Social Science; for four different groups of students. Each course had some common topics. Every "A"OU student has textbooks delivered by the university. Supporting media were provided for some courses depending on the management of the course team and "A"OU regulations. Since "A"OU delivered via a variety of media, but no one had investigated what types of media were most popular and whether the students' preferences were influenced by student characteristics or not, this research was undertaken. This paper discusses the relationship of the students' characteristics and their use of mathematics teaching media to achievement. Students' characteristics were their demography, mathematics backgrounds and computer skills.

Methodology

The population was 5,584 students who each studied one of four Mathematics courses in semester 1/2008. The sample was 2,746 randomly selected students. The instrument was a questionnaire asking students' satisfaction about the media used for mathematics courses. All students had textbooks and schedules for face-to-face tutorials and other support such as television and eTutorial sessions for each course. The sample received extra information about the support available for each of the mathematics courses and an invitation to use media available for other courses but not provided for their specific course. The support was classified into four types, textbooks, assignments, face-to-face tutorials and electronic media consisting of a VCD for Mathematics and Statistics for Science and Technology, a Web board, past eTutorial sessions, past VCD eTutorial sessions, live eTutorial sessions and Television programs. After the final examination, students were asked to fill in the questionnaire. A total of 44.8% of students returned completed questionnaires.

Findings

The findings were divided into two topics, students' characteristics and the use of mathematics media.

Students' characteristics:

There were more females than males. More than a half were in the range of 30-39 years old and worked in the private sector. Nearly half held a high school certificate or a vocational certificate. About 20% of the students lived in each of the Central, Northeast and Bangkok regions. More than a half had incomes of less than 10,000 baht per month. The details are in table 1.

Table 1. **Students' demography.**

Items	n	%
Gender	**1,240**	**100.0**
Male	838	67.6
Female	402	32.4
Age Min 17 yrs, Max 65 yrs, $\bar{X}$= 29.43 yrs, S.D. = 7.565 yrs	**1,211**	**100.0**
17-19 yrs	47	3.9
20-29 yrs	672	55.5
30-39 yrs	349	28.8
40-49 yrs	125	10.3
> 49 yrs	18	1.5
Career	**1,043**	**100.0**
Private sector employee	571	54.7

Unemployed/Others (included students from traditional institution, housewives, monks and prisoners)	173	16.6
Own business/Merchant/Agriculture	150	14.4
Government/ Government Enterprise	149	14.3
Education	**1,051**	**100.0**
Lower than high school certificate	40	3.8
High school certificate /vocational certificate	505	48.0
Vocational diploma	364	34.6
Bachelor degree or higher	142	13.5

Table 1 **(Cont.)**

Items	**n**	**%**
Region	**1,239**	**100.0**
Central	283	22.8
Northeast	264	21.3
Bangkok	249	20.1
North	203	16.4
South	132	10.7
East	108	8.7
Income (per month) (1 USD ≈ 33 baht)	**1,044**	**100.0**
< 10,000 baht	552	52.9
10,001-15,000 baht	272	26.1
15,001-20,000 baht	77	7.4
20,001-25,000 baht	55	5.3
25,001-30,000 baht	25	2.4
> 30,000 baht	63	66

Students were asked about their mathematics knowledge background and computer skills. More than a half of the students evaluated their mathematical knowledge as at a fair level followed by poor. Nearly a half of the students felt that they had computer/internet skills at a fair level followed by a good level. Details are in table 2.

Table 2. **Students' self-assessed mathematical knowledge and computer/internet skills.**

Items	**n**	**%**
Mathematical knowledge	**1,053**	**100.0**
Very good	7	0.7
Good	85	8.1
Fair	581	55.2
Poor	314	29.8
Very poor	66	6.3
Computer/Internet Skill	**1,054**	**100.0**
Very good	107	10.2
Good	346	32.8
Fair	467	44.3
Not good	79	7.5
No skill	55	5.2

The use of mathematics media:

The most popular means of study used by students were textbooks followed by assignments. VCD for Mathematics and Statistics for Science and Technology, Face-to-face tutorial and Web board were also used by a few. The rest were seldom used by students. Details are in table 3.

Table 3. **Percentage of students who used mathematical media.**

Type of media	n	Using media (n, %)	
		Used	Did not use
Textbooks	641	582 (90.8)	59 (9.2)
Assignment	641	530 (82.7)	111 (17.3)
VCD for Mathematics and Statistics for Science and Technology	641	157 (24.5)	484 (75.5)
Face to face tutorial	641	134 (20.9)	507 (79.1)
Web board	641	106 (16.5)	535 (83.5)
Past eTutorial sessions	641	89 (13.9)	552 (86.1)
Past VCD eTutorial sessions	641	59 .(9.2)	582 (90.8)
Live eTutorial sessions	641	54 (8.4)	587 (91.6)
Television programs	641	47 (7.3)	594 (92.7)

Students indicated their satisfaction with the mathematical media used. Electronic media included all media that used electronics. It was found that on the average, from a five point scale: where 5 was highly satisfied and 1 was highly dissatisfied, students were satisfied with the media at a fair level. The assignment gave the most satisfaction followed by the face-to-face tutorial and the textbook. The electronic media provided the least satisfaction. Details are in Table 4.

Table 4. **The average score on students satisfaction and the mathematical media used.**

Mathematical media	n	$\bar{X}$	S.D.
Assignment	498	3.61	.809
Face-to-face tutorial	120	3.53	.896
Textbooks	551	3.49	.969
Electronic media	37	3.19	.772

Students' characteristics were compared with the mathematical media used and with their achievement. It was revealed that students' scores varied significantly with their characteristics and the mathematical media used at the .05 level. (Details are described below). Gender was affected by textbooks, face-to-face tutorial, assignment and eLearning. Career was affected by only assignments. Income was affected by textbooks, assignment and Electronic media. Education was affected by all four mathematical media. Mathematical background was affected by textbooks, assignments and eLearning. Computer skill was affected by only electronic media. The summary is in table 5.

Table 5. **Summary of the relation between students' characteristics and their use of mathematical media.**

Mathematical media	Students' Characteristics					
	Gender	Career	Income	Education	Mathematical background	Computer skill
Textbooks	✓	-	✓	✓	✓	-
Face to face tutorial	-	-	-	✓	-	-
Assignment	✓	✓	✓	✓	✓	-
Electronic media	-	-	✓	✓	✓	✓

The relationship between the students' achievement within each characteristic of the mathematical media used is shown by some examples.

When comparing students' achievement and the use of textbooks with gender, it was revealed that males who used textbooks did significantly better than females who used textbooks at a level of .05. Details are in table 6.

Table 6. **Comparison of students' achievement and the use of textbooks with gender.**

Gender	n	$\bar{X}$	S.D		S.S	df	M.S	F	p
Male	184	34.28	13.970	Between groups	1585.835	1	1585.835	13.440	.000
Female	341	30.64	8.744	Within group	61711.702	523	117.996		
Total	**525**	**31.92**	**10.991**	**Total**	**63297.538**	**524**			

When comparing students' achievement and the use of textbooks with income, it was found that there was significant difference at a level of .05. Details are in table 7.

Table 7. **Comparison of students' achievement and the use of textbooks with income.**

Income	n	$\bar{X}$	S.D		S.S	df	M.S	F	P
< 10,000 baht	201	31.17	9.155	Between groups	1519.018	2	759.509	7.164	.001
10,000-15,000 baht	100	29.95	10.877	Within group	38485.074	363	106.019		
> 15,000 baht	65	35.92	12.507	**Total**	40004.092	365			
Total	**366**	**31.68**	**10.469**						

When comparing students' achievement and the use of textbooks with income by pairing, it was found that students who had incomes more than 15,000 baht had better performances than those who had incomes between 10,000 and 15,000 baht at a level of .05. Details are in table 8.

Table 8. **Comparison of students' achievement and the use of textbooks with income by pairing.**

I	J	M.D. (I-J)	S.E.	P
< 10,000 baht	10,000-15,000 baht	1.218	1.260	.334
	>15,000 baht	-4.749(*)	1.469	.001
10,000-15,000 baht	< 10,000 baht	-1.218	1.260	.334
	>15,000 baht	-5.967(*)	1.641	.000
>15,000 baht	< 10,000 baht	4.749(*)	1.469	.001
	10,000-15,000 baht	5.967(*)	1.641	.000

When students' characteristics in terms of education with the use of textbooks were compared with students' achievement, it was found that the average scores were significantly different at a level of .05. Details are in table 9.

Table 9. **Comparison of students' achievement and the use of textbooks with education**

Education	n	$\bar{X}$	S.D		S.S	df	M.S	F	p
Lower than senior high school cert.	32	30.94	6.908	Between groups	12305.493	3	4101.831	45.997	.000
Senior high school cert./Voc cert	410	30.97	8.549	Within group	73659.011	826	89.176		
Voc dip	292	27.59	7.943	**Total**	85964.504	829			
Bachelor degree and higher	96	40.64	15.870						
Total	**830**	**30.90**	**10.183**						

When comparing students' achievement and the use of textbooks with education by pairing, it was found that students who had bachelor degrees and higher had significantly better performances than those who had lower education at a .05 level. Similarly, students with a senior high school certificate or a vocational certificate had significantly higher results than those with a vocational diploma at a level of .05. Details are in table 10.

Table 10. **Comparison of students' achievement and the use of textbooks with education by pairing**

I	J	M.D. (I-J)	S.E.	P
Lower than senior high school cert	Senior high school cert./Voc cert	-.033	1.733	1.000
	Voc dip	3.349	1.758	.305
	Bachelor degree and higher	-9.699(*)	1.928	.000
Senior high school cert./Voc cert	Lower than senior high school cert.	.033	1.733	1.000
	Voc dip	3.382(*)	.723	.000
	Bachelor degree and higher	-9.666(*)	1.071	.000
Voc dip	Lower than senior high school cert	-3.349	1.758	.305
	Senior high school cert./Voc cert	-3.382(*)	.723	.000
	Bachelor degree and higher	-13.048(*)	1.111	.000
Bachelor degree and higher	Lower than senior high school cert.	9.699(*)	1.928	.000
	Senior high school cert./Voc cert	9.666(*)	1.071	.000
	Voc dip	13.048(*)	1.111	.000

The achievements of students who used textbooks were compared with their self-assessed mathematical backgrounds. It appeared that there were significantly different scores at a level of .05. Details are shown in table 11.

Table 11. **Comparison of students' achievement and the use of textbooks with self-assessed mathematical background**

Mathematical background	n	$\bar{X}$	S.D		S.S	df	M.S	F	p
Very good/good	28	40.87	14.955	Between groups	3298.921	3	1099.640	10.969	.000
Fair	207	32.01	10.532	Within group	36790.734	367	100.247		
Poor	106	29.60	7.834	**Total**	40089.655	370			
Very poor	30	27.81	7.088						
Total	**371**	**31.65**	**10.409**						

A comparison of students' self-assessed mathematical backgrounds with their final results showed that students who were very good/good in mathematical background received significantly higher scores than the rest at a level of .05. Details are shown in table 12.

Table 12. **Comparison of students' achievement and the use of textbooks with self-assessed Mathematical background by pairing**

I	J	M.D. (I-J)	S.E.	p
Very good/good	Fair	8.863(*)	2.016	.000
	Poor	11.277(*)	2.127	.000
	Very poor	13.065(*)	2.631	.000
Fair	Very good/good	-8.863(*)	2.016	.000
	Poor	2.414	1.196	.255
	Very poor	4.202	1.956	.204
Poor	Very good/good	-11.277(*)	2.127	.000
	Fair	-2.414	1.196	.255
	Very poor	1.788	2.071	.862
Very poor	Very good/good	-13.065(*)	2.631	.000
	Fair	-4.202	1.956	.204
	Poor	-1.788	2.071	.862

Careers of students who submitted assignment were compared with their achievement. It was found that there was a significant difference at a level of .05. Details are in table 13.

Table 13. **Comparison of students' achievements and the use of assignment with career.**

Career	n	X	S.D		S.S	df	M.S	F	p
Government/Government enterprise	46	35.41	13.468	Between groups	1114.909	3	371.636	3.608	.014
Private sector	146	31.85	10.015	Within group	35532.297	345	102.992		
Business owner/merchant/Agriculture	94	30.03	8.912	**Total**	36647.206	348			
Unemployed/Others	63	33.90	9.367						
Total	**349**	**32.20**	**10.262**						

Careers of students who submitted assignments were compared with their achievement by pairing. It was found that those who worked in government/government enterprise had better scores than those who were business owners/merchants/agricultural workers at a level of .05. Details are in table 14.

Table 14. **Comparison of students' achievements and the use of assignment with career by pairing**

I	J	M.D. (I-J)	S.D	p
Government/Government enterprise	Private sector	3.560	1.716	.232
	Business owner/merchant/Agriculture	5.378(*)	1.826	.035
	Unemployed/Others	1.513	1.968	.898
Private sector	Government/Government enterprise	-3.560	1.716	.232
	Business owner/merchant/Agriculture	1.818	1.342	.608
	Unemployed/Others	-2.047	1.530	.618
Business owner/merchant/Agriculture	Government/Government enterprise	-5.378(*)	1.826	.035
	Private sector	-1.818	1.342	.608
	Unemployed/Others	-3.865	1.652	.143
Unemployed/Others	Government/Government enterprise	-1.513	1.968	.898
	Private sector	2.047	1.530	.618
	Business owner/merchant/Agriculture	3.865	1.652	.143

Students who had different self-assessed computer skills and used electronic media had significantly different achievement at a level of .05. Details are in table 15.

Table 15. **Comparison of students' achievements and the use of electronic media with self-assessed computer skills**

Computer skills	n	X	S.D		S.S	df	M.S	F	p
Very good	34	32.46	9.280	Between groups	1175.451	4	293.863	2.588	.039
Good	65	33.60	12.781	Within group	19988.075	176	113.569		
Fair	63	27.92	9.698	**Total**	21163.526	180			
Poor	11	30.51	7.480						
No skill	8	28.19	6.558						
Total	**181**	**30.98**	**10.843**						

Comparison using pairing showed that students who regarded themselves as very good in computer skills had significantly higher scores than those who thought they were fair at a .05 level. Similary, students who considered themselves good in computer skills had scores significantly higher than those who thought they were fair at a level of .05. Details are in table 16.

Table 16. **Comparison of students' achievement and the use of electronic media with self-assessed computer skills by pairing**

I	J	M.D. (I-J)	S.E.	p
Very good	Good	-1.139	2.256	.614
	Fair	4.542(*)	2.268	.047
	Not good	1.952	3.697	.598
	No skill	4.263	4.188	.310
Good	Very good	1.139	2.256	.614
	Fair	5.681(*)	1.884	.003
	Not good	3.091	3.474	.375
	No skill	5.402	3.993	.178
Fair	Very good	-4.542(*)	2.268	.047
	Good	-5.681(*)	1.884	.003
	Not good	-2.590	3.482	.458
	No skill	-.279	4.000	.944
Not good	Very good	-1.952	3.697	.598
	Good	-3.091	3.474	.375
	Fair	2.590	3.482	.458
	No skill	2.310	4.952	.641
No skill	Very good	-4.263	4.188	.310
	Good	-5.402	3.993	.178
	Fair	.279	4.000	.944
	Not good	-2.310	4.952	.641

In summary, students' characteristics: gender, career, income, education, perceived mathematical background and computer skills affect their achievement.

Discussion

The relationship between students' characteristics, use of mathematics media and their achievement:

Students' achievements were significantly influenced by their use of study materials and their six characteristics: gender, career, income, education, perceived mathematical background and computer skills.

Gender: Males had higher scores than females. This corresponds to the finding that over the past 33 years, the average male score on SAT: M has consistently been around 35 points higher than the average female score. (Allen, Caitilyn, 2010)

Career: Students who worked in government or government enterprises had significantly better scores than those who were business owners, merchants or those employed in agriculture. As government officials are required to undertake regular studies in order to gain promotion, they may have developed better study skills.

Income: Students who had incomes more than 15,000 baht had significantly better performances than those who had incomes less than 15,000 baht. Incomes in Thailand tend to be based on levels of education and experience. Those on higher salaries are likely to have spent more time studying in the past.

Education: Students who had Bachelor degrees or higher had significantly better performances than those who had lower education. Again, those who had higher education have had more experience in managing study and can be expected to outperform those lacking such experience. Students with a senior high school certificate or a vocational certificate had significantly higher results than those with a vocational diploma. Students at senior high

school generally study mathematics at a higher level than those completing a vocational diploma.

Mathematical background: Students who believed they were good or very good in mathematical background received significantly higher scores than the rest. Confidence in a strong mathematical background is a considerable advantage.

Computer skills: Students who had self-perceived computer skills at a good or very good level had significantly higher scores than those who had only a fair level. Widespread use of computers is a relatively recent phenomenon in Thailand. Those who have developed strong skills in the use of computers are possibly active learners.

The popularity of media:

Textbooks are the most popular because they are produced with great care and expertise by contents experts and help students to study on their own without any help. The popularity of print depends on its ease of use and its quality. Textbooks are also the most commonly used medium because they are traditionally the main study materials of "A"OU and all students are required to have them. They are easy to carry, reasonably priced and convenient to use. This corresponds to the writing of Ramaiah, Y, R, (2001) who said that print was an essential component for all education processes especially distance education. Distance education could not operate without textbooks which are the main medium. Similarly Ely, Donald P (2003), claimed that the most common medium for learning at a distance is still paper-books, study guides, and bibliographies.

Assignments were the second most preferred which might be due to the fact that marks for assignments could be worth up to 20% of the final assessment. Assignments also encourage students to study. Earlier studies on assignments at "A"OU revealed that students who submitted assignments received significantly higher results on the study than those who did not (Author, 2005, 1992, and Angsuchoti and Wipassilpa, 2008).

The face-to-face tutorial was the third preference that students used to support their study. Students are able to learn from the tutor. They may understand more when the tutor explained in more details. They might be field-dependent type who likes to be cared for and guided in their study as discussed by Author (1992). It was not as popular as the first two media because it was arranged at only a few centers throughout the country with enrolments high enough to justify the expense. Students who had work engagements or lived far from the centers may not find attendance convenient.

The electronic media may not be as popular as other media because it was primarily offered to the students of only two courses with fewer students though it did contain material that was relevant to all of the courses. Although they were informed that they were welcome to use media from other courses, those did not enroll in the mathematics courses with electronic media might not have felt that it would be relevant.

Conclusion and Suggestions

It is concluded that students' characteristics and the use of mathematical media influenced their study result. Therefore, it is vital to pay attention to producing mathematical media to suit students' characteristics and learning styles. It is especially important to try out the media before delivery. Also team work is essential to produce all media to improve the quality. At present, only textbooks were formally produced by teams.

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Facilitating Mathematics Teaching and Learning within the Irish Prison Education Service: A model for future development – Work in Progress

Catherine Byrne

The strategic aim of prison education in Ireland is to provide access to high quality, broad and flexible programmes at all levels. Prison educators must provide an environment that ensures that individuals engage in learning, within unique constraints. In mathematics teaching, it is important to ensure that the service offered is appropriate to the individual's needs.

This research will develop a diagnostic approach that in a holistic way will identify an individual's mathematical skills, knowledge and competence, and monitor the learning that takes place including the individual's development as a reflective lifelong learner. It will also develop and validate a number of pedagogical approaches and resources to facilitate mathematics teaching and learning in prison and enable individuals become self-directed learners. This will form the basis of a roadmap/model for facilitating effective mathematics teaching and learning.

The questions guiding this research are:

- How can real life mathematics ability and learning be effectively monitored in Irish prison education?
- What pedagogical resources enable mathematics learning?
- Can mathematics learning promote prisoner self efficacy?

High stakes assessment: Assessing numeracy for nursing in two recent projects

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In this paper I report on two recently completed interdisciplinary projects on numeracy for nursing undertaken in the UK: 'Benchmark assessment of numeracy for nursing: medication dosage calculation at point of registration', and 'Evaluating the assessment of student nurses' numeracy for nursing'. These projects raise important issues about the development of competence in numeracy for nursing and potentially also other safety-critical areas of work which I hope to explore in further research.

Introduction

There is growing international concern about the competence in numeracy of some nurses, both qualified staff and nursing students (Sabin, 2001). The importance of numeracy for professional practice in nursing is increasingly recognized in many countries, but there is still no consensus on what is meant by numeracy for nursing, nor on how it should be taught, learned and assessed.

In response to growing concern, from September 2008 nursing students in the UK must achieve 100% in a test of numerical competence in the practice setting before being allowed to register as nurses (NMC, 2007). But in the absence of a national standard in numeracy for nursing this begs the question: 100% of what? Without a benchmark, any measure of numerical competence is "*in the eye of the recipient of evidence of that competence, be it higher education institutions, regulators, employers or service users" (Hutton, 2004).*

Against this background this paper outlines what has been achieved in two interdisciplinary projects in numeracy for nursing: a study creating and testing a proposed benchmark assessment in numeracy for nursing in Scotland, specifically in medication dosage calculation, and an investigation of the assessment of numeracy in a large university school of nursing in England. I reflect on the implications of these studies for the assessment of numeracy for nursing.

The 'Benchmark' project: 'Benchmark assessment of numeracy for nursing: medication dosage calculation at point of registration'

The 'Benchmark' project was funded by NHS Education for Scotland (NES) and undertaken by a team brought together and led by Mike Sabin of NES, comprising Diana Coben and Meriel Hutton of Kings College London, Carol Hall, University of Nottingham, David Rowe

of the University of Strathclyde and Keith Weeks and Norman Woolley of the University of Glamorgan.[1]

In preliminary work we made the case for such a benchmark, adopted a definition of numeracy (Coben, 2000, p. 35), and developed evidence-based principles, criteria and content for the authentic assessment of numeracy for nursing in line with our definition (Coben, Hall, et al., 2008). On the basis of this rubric we created a comprehensive authentic assessment of medicine dosage calculations for nursing covering the calculation of sub-, multiple- and unit dosage of tablets, capsules and liquid medicines, drip rates and intravenous (IV) infusions (Weeks, 2001). The assessment was produced in two parallel forms requiring the same calculations: a computer-based assessment and a simulated practice assessment. Simulated practice assessment in a controlled environment, often called an objective structured clinical examination (OSCE), is commonly used in healthcare education because for practical and ethical reasons it may not be acceptable to assess students' skills with actual patients. Our simulated practice assessment was a modified OSCE and our computer-based assessment was adapted from selected items from the Authentic World® item bank. We compared the performance of a sample of final year nursing students in universities in Scotland on the practical activity and the computer-based simulation.

We found that the criterion-related validity of the computer simulation was supported both in terms of putting participants in a similar order of competence and in terms of participants getting the same number of questions correct on the computer as they did in the practical situation. We were also able to identify aspects of medicine dosage calculation that were not assessed in the computer-based test as currently constituted, but which were assessed in the practical simulation. This is important as it opens the possibility of much faster, cheaper and more efficient authentic and comprehensive summative and formative assessment, with feedback to the student built in, using the computer programme, while identifying those areas which still require practical assessment and freeing up time for them to be taught and assessed in practice.

In further ongoing work we are attempting to determine the acceptability to learners of the assessment tools in terms of authenticity, relevance, fidelity and value, and developing a sophisticated model of competence, as steps towards creating a proposeed benchmark for numeracy for nursing.

Evaluating the assessment of numeracy for nursing in the absence of an agreed benchmark

The second study, funded by King's College London, investigated the assessment of numeracy in a large school of nursing in a university in England. The team consisted of Diana Coben, Jeremy Hodgen, Meriel Hutton and Sherri Ogston-Tuck, all of King's College London. Two members of the team, Diana Coben and Meriel Hutton, were also members of the Benchmark project.

Universities, including the one in our study, are incorporating numeracy tests into their pre-registration nurse education programmes in order to meet the requirement for a 100% pass in a test of numerical competence in the practice setting, as outlined above (NMC, 2007). We had anecdotal evidence that these tests may be of variable quality and at worst, some may be

[1] The report of the project (Coben, et al., 2010) together with sample test items and other material, is available on the project website: http://www.nursingnumeracy.info/index.html.

mathematically and/or professionally inappropriate. In a safety-critical context such as nursing this may have serious negative consequences, not least for the patient but also for the student, who may not be adequately prepared for the numeracy demands of their chosen profession or who may be unable to qualify as a nurse because they have failed the numeracy test. We decided to evaluate the assessment of numeracy in one school of nursing in order to provide a case study complementing the work in Scotland outlined above and to inform further research in numeracy for nursing.

Students in the second study were taking an integrated programme in undergraduate nursing studies leading to professional registration after three years full-time study. We decided to focus on the mandatory 'Calculations for Nursing' module which covers the development of mathematical skills, reviews some basic mathematics principles and applies these to calculations encountered in clinical practice. Summative assessment of the module comprises ten multiple-choice questions chosen at random from an item bank; these are attempted online under examination conditions over 30 minutes. Students have three opportunities to achieve 100% and those who do not achieve this may be required to leave the course.

We analysed the performance of a sample comprising 378 Year One students on a total of 41 test items. We undertook a Rasch analysis[2] of the difficulty of these items in relation to the candidates' performance and we also analysed item difficulty in mathematical terms by comparing the items to standards established in GAIM (Graded Assessment in Mathematics) (Brown, 1992).

Of the 378 candidates, 199 were unsuccessful at their first attempt. Item facilities (i.e., the percentage of correct answers) were generally high but the tests taken by individual candidates varied considerably in difficulty depending on which 10 items happened to be drawn from the item bank. The Rasch analysis revealed a large overlap between candidates' ability estimates for the pass and fail groups, with very large confidence intervals for estimates of candidates' abilities. As a result, relatively minor changes to the pass ability level would have major effects on the number of candidates passing the test. The test items were found to be relatively easy for the candidates, despite the large number who failed to achieve 100%. We also found the fit between the levels of difficulty of the test items and GAIM levels to be problematic, largely because of the poor construction of many of the items. In particular, multiple choice 'answers' mainly involved choices involving orders of magnitude, something which is only appropriate where such mistakes could be dangerous in nursing practice; it also makes many items 'easier' than they might be.

The study highlighted the dangers of high stakes testing without a benchmark. We concluded:

> Our analysis points up the dangers of high stakes testing with a 100% pass mark in the absence of a reliable and valid assessment instrument set to an agreed standard and reflecting the scope of numeracy for nursing. While this is intended to indicate mastery of a safety-critical aspect of nursing practice, our analysis shows that the test in our study does not warrant this. The test is neither reliable nor valid: it does not consistently test what it is intended to test. (Coben, Hodgen, Hutton, & Ogston-Tuck, 2008, p40)

The school in our study scores highly on measures of the quality of its nurse education. Yet its numeracy assessment has been found to be flawed and, in the absence of an agreed evidence-

[2] http://www.rasch-analysis.com/

based benchmark for numeracy for nursing, such as that being developed in the 'Benchmark' project in Scotland, tests used elsewhere may also be flawed. If so, an avoidable and unjust wastage of potential nurses is occurring just at a time when qualified nurses are in short supply in the UK (Finlayson, Dixon, Meadows, & Blair, 2002), the USA (HRSA, 2004) and elsewhere (Oulton, 2006).

The findings of our evaluation study bear out the need for a benchmark in numeracy for nursing such as that developed and tested in the 'Benchmark' project. More generally, the findings of these projects raise important issues about the development of competence in numeracy for nursing, and potentially also other safety-critical areas of work, which we are exploring in our ongoing research.

In a forthcoming paper Meriel Hutton and I reflect on these issues and conclude that:

> Where mathematics is situated in professional/vocational practice it should be taught, learned and assessed in relation to that practice, both directly *in practice* and through authentic and comprehensive *simulation of practice*; the latter enables individuals to be exposed to the full range of problems associated with the use of mathematics in their professional practice, something which may be impossible to do safely, comprehensively and effectively in real world, real time contexts. (Coben & Hutton, forthcoming)

Such an approach requires a recognition that nursing can be stressful and calculations may have to be done when "knowledge is limited, time is pressing, and deep thought is often an unattainable luxury" (Gigerenzer, Todd, & ABC Research Group, 1999). Assessment of numeracy for nursing must be authentic, comprehensive and pitched at an appropriate level, both mathematically and with respect to nursing content and the sometimes stressful nursing context.

Ultimately, the decision as to whether to establish a benchmark in numeracy for nursing, the nature of that benchmark and its relationship to assessment regimes must be made by the regulator (in the UK: the Nursing and Midwifery Council, NMC); it must also be owned by the profession and be acceptable to patients and the wider public. The stakes are high in numeracy for nursing for all concerned. Its assessment must do justice to the students, their future careers and their prospective patients.

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Changing classroom culture: If everybody does something different we all work better together

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In this paper I describe how, using a tool initially developed to combat plagiarism in a classroom setting where I was teaching early intervention second chance learners, I transformed a class of reluctant adult mathematics students into active participants in an active and vibrant learning environment.

In Ireland, mathematics education at second level (High school) is dominated by the "Leaving cert" syllabus, which is assessed by an annual examination, which takes place in June. However an alternative is provided by the FETAC L3 and L4 mathematics syllabus and assessment mechanism. In this mechanism, assessment is based on a portfolio of work. This course is more commonly found within the second chance and adult education sectors, where a mathematics module may form a component of a broader vocational qualification. To give an idea of the standard involved, L3 students would be expected to handle operations with simple fractions and decimals, while L4 students would be expected to compute a pay slip with a range of deductions, and handle some simple statistics such as the mean and the standard deviation of a set of numbers. More details of these standards can be found on the Irish awards council website (Further Education and Training Awards Council, 2010)

The tool and teaching method described here was initially developed by me in response to my perception that a risk of plagiarism existed when I initially taught these courses in Youthreach (Irish Government, 2010), an Irish second chance education program for early school leavers. At level three, assessment consists of perhaps 38 or 40 SLO's each of which is assessed by completing examples of each mathematical task involved. This assessment is delivered via a series of briefs, or problem sets, which the students complete and the instructor certifies. When this assessment system is delivered in a less than fully controlled classroom situation, a student may ask to see another student's work to see how they have worked a problem and in that process find the correct answer without understanding where it came from. To reduce the risk of this occurring, I decided to developed a set of individualized briefs where all students gets similar problems, each with different sets of numbers.

Developing the tool

Individualized briefs were developed within the Microsoft office suite, using excel primarily. Individual worksheets were set up in the spreadsheet program for each brief, and the built in random number generator was used to produce appropriate numerical values in the problems. This was not always totally simple to do.

As an example I discuss setting up a two digit addition sum with carry, showing the formulas present in each field of the spreadsheet. I'm running through this example in detail to give a flavor of the issues involved in setting up problem sheets.
We require two digits in the top row. The leading digit should be non zero. In the second row digits should be large enough so that a carry will be required if possible.

This is set up as nine cells of a spread sheet with using "border formating" on two of the cells to draw the line under the summation. The following table shows the excel formulae involved.

Table 1. **Spreadsheet formula and layout used to generate a 2 digit addition problem with carry.**

A	B	C
	=Int(1+8.99*rand())	=Int(9.99*Rand())
+	=Int((10-B1)+(B1-1)*Rand())	=Int((9.99-C1)+(C1)*Rand())

Here the rows are numbered starting with 1 as the first row, and the columns are labeled sequentially A,B, C....
The formula in the top right hand corner generates pseudo random integers between 0 and 9 inclusive. The formula at the top of the next column to the left, produces pseudo random integers between 1 and 9 inclusive.
On the next row, in the formula at row 2, column 2 the first term inside the brackets, (10-B1) ensures that the column total here will always be greater than or equal to 10, and the second term, (B1-1)*Rand() ensures cell value will not go into double digits by generating a random number who's maximum value is (B1-1).
The final formula, Int((9.99-C1)+(C1)*Rand()), in cell C3, attempts to ensure a carry in the units column of the addition, but as we allow a zero value at the top of this column this is not always possible.

The example developed here setting up a two digit addition is one of the most involved, because there are pedagogic restrictions on each digit of the sum. It is my experience, as the maths being taught grows more complex, the spreadsheet set up becomes simpler.
For word problems at the higher level, FETAC L4, where there were a couple of sentences involved in stating the problems in most cases, excel was used to generate the random numbers and mail merge was used to read this data into selected fields in a word document.
For example in setting up a tax problem where the student calculates tax deductions for a payslip, I generate either the total hours worked or the rate of pay in an excel spreadsheet and read this value into the word document using mail merge with the spreadsheet as a data source.

Deploying the tool

The tool was first deployed with classes of adult education students taking part in a "back to education initiative program" BTEI (Dept. of Education and Skills, 2010), designed to up skill adults with a low level of educational attainment.
This group were initially disengaged from my class, as when they enrolled in the programs vocational courses in computer skills or advanced woodwork, or similar, mathematics was not described as being an essential part of coursework. Thus, when I started teaching them maths, they did not want to do it, couldn't see the point, and felt that it had been foisted upon them.
The average age of the group was in the early fifties, and these were classical adult learners as described in :(Knowles, Holton, & Swanson, 2005). More specifically unless they were interested in the material they were not prepared to learn.

Reaction

When I presented the individualised assignments, the class immediately saw it as an attack on their honesty. “You think we cheat” was one immediate comment. I explained that this was not a slur on their honesty, but rather an acknowledgment of the fact that they were adults who would talk to each other and help each other out, and this would enable that interaction without putting them in a position where they were just copying each other’s work.

Classes were an hour long, I would start with a short demonstration of a mathematical procedure on the board, taking no more than five minutes, and would then pass out the individualized briefs. Students were expected to work on these individually, and when they got stuck they would call on me or a fellow student for help. Once a student was finished they would swap their work with that of a neighbour and each would correct the other persons work. All being well we would then move on to the next topic.

To my surprise Initial scepticism turned to enthusiasm as the classes adjusted to the new system.

What changed? The class became energized, students tended to interact a lot more with each other and with me as a teacher, and they became engaged with the subject. They were now learning the maths as an interesting skill, mathematics for the fun of it rather than just as a course requirement. As evidence of this I noted that at the end of the year they wanted to know by what further course or avenue they could precede further with the subject.

In practice the class became a series of self organized groups, teaching themselves how to do the problem sets and discussing the mathematics in general. This was group learning and problem solving at its best.

Some Conclusions

I addressed this class as adult learners telling them that adults talked to each other, adults helped each other out and that that was what I expected them to do in this class. The class responded to this.

Initially some students felt insecure about their level mathematical skill, and while they would have used some mathematics in their everyday life, many of them felt that their mathematics knowledge fell below that of the class. The group work allowed them to re-evaluate this, particularly when helping other students or discussing a problem communally. Once the barrier of insecurity about their knowledge level was removed, they enjoyed the experience and the sense of contributing and being involved.

A further positive factor was that if a student self identified as being no good at maths, but then found themselves teaching it to other class members there was a element of cognitive dissonance (Festinger, 1957) present, which was generally resolved by the student concluding that they had been wrong and that they were good at maths.

Finally why did these students work? As Dan Pink puts it, (*RSA Animate - Drive*, 2010),(Pink, 2010) once we have removed the other barriers to education, people work for challenge and mastery and making a contribution. The class work challenged them, they mastered it and they could make a contribution by helping each other. This, I think, explains the success of this intervention.

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When parents become learners: a critical approach to family involvement in mathematics

Javier Diez-Palomar (presented by Sikunder Ali)

In this paper we draw on parents' voices to look at their believes and social representations (Moscovici, 1984; Gorgorió, 2007) regarding on how they become involved in their children's mathematics education. Prior researches suggest that parents face many difficulties when doing mathematics with their children, because their lack of knowledge in terms of reform mathematics, the change in teachers' methods to teach mathematics, and so forth. Authors such as Civil (2001), Civil and Bernier (2006) point out how parents learn (in many cases again) mathematics. Drawing on Freire's (1998) and Flecha's (2000) approaches to adult learning, we discuss with parents how they see their learning in mathematics. We use the grounded theory approach (Glasser and Strauss, 1967) as a methodology to build with parents new critical approaches to adult mathematics learning in the context of family involvement. We also report on socio-cultural practices that parents use to do to create meaning on their children' mathematics.

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New theoretical trends for adults learning mathematics in the context of family involvement

Javier Diez-Palomar Clive Kanes

(presented by Sikunder Ali)

Mathematics teaching schemes that have the family as the key site of educational intervention have attracted the ongoing attention of policy makers in Europe and other parts of the world. Such programmes offer a way forward that promise to simultaneously meet the educative needs of both young people in the compulsory education sector, and their legal guardians in the post-compulsory and adult sectors. In this paper, structured as a dialogue between the authors, we focus on exploring some key theoretical approaches to understanding and analyzing family mathematics schemes. First, we take account of studies that point to the large and complex varieties of family relationships of current and emerging important in educational contexts, we aim to problematise and suitably enlarge the notions of "family" under consideration. Next, we build on Wedege (2009) and Zevenbergen's (2005) ideas concerning the applicability of Bourdieu's sociological concepts applied to ALM. In particular, we discuss the appropriateness of using *habitus* as a mean to analyze the effect of social background in parents' attitudes, social representations and experiences in doing mathematics with their children. In this light, we critically review the contributions from scholars such as Civil (2001), Abreu & Cline (2005), Allesxhat-Snider (2006), whose large experience working with parents and mathematics is of key importance in our growing understanding of the potential of family oriented programmes in achieving educational ends. We argue that other sociological concepts are also of help in these contexts. For example, Bernstein's (1973a, 1973b, 1977, 1990) ideas of class, codes and control, offer useful ways to conceptualise the rules of recognition and realisation and distribution of knowledge that characterise family pedagogical practices. In this context, some remarks of Dowling (1994) will also prove helpful. We conclude the paper by briefly considering what these sociological accounts leave out that might also need to be considered in the context of family mathematics programmes. Here the work of Knijnik (2007), and the work Kanes, Morgan & Tsatsaroni (2010), drawing on theoretical resources inspired by Michel Foucault, will prove helpful.

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Evaluating the Impact of a Refresher Course in Mathematics on Adult Learners

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In response to the growing numbers of adult learners in attendance at the University of Limerick (UL) and the increasing demand on services provided by the Mathematics Learning Centre (MLC), the author initiated a one-week mathematics revision course in August 2007 to bridge some of the gaps in knowledge long forgotten by many. In 2008 the author carried out an investigation on the impact of the course on participants' mathematics self-concept. While the study showed no significant findings, due in part to a small sample, the anecdotal feedback from students was positive. In this paper, the author presents findings from a more recent evaluation in which a larger sample was used. Furthermore, results from a focus group held with participants having completed a year or two of formal mathematics education at UL are presented.

In the academic year 2009/10 adult learners of mathematics constituted 14% of the entire student population, an increase of 49% on the previous year (Coveney – O'Beirne, 2010). Mathematics learning support in the form of a drop-in centre (fully supervised for 20 hours each week), support tutorials and pre-examination revision programmes are provided for all students who study service mathematics in the University of Limerick (UL) by the Mathematics Learning Centre (MLC). Prior to 2007, front-end tutorials provided adult learners with the opportunity to catch up on mathematics fundamentals they would require for their mathematics modules. These short tutorials were replaced in 2007 with a week long intensive revision programme which takes place in August before students' orientation week, and more recently in January for adult learners who do not study mathematics until the second semester of first year. The programme is entitled 'Head Start Maths'. In 2008 the author was seconded by sigma in the UK to compile a set of workbooks specifically for this programme[3]. A previous endeavour to measure the effectiveness of the programme was carried out in 2008. The author distributed pre-and post questionnaire to 18 participants to investigate if participation had a positive impact on their mathematics self concept (Gourgey, 1982). In addition qualitative questions were given to acquire anecdotal feedback from the students on their perceptions of the course and the set of notes and to determine their main anxieties/concerns about mathematics at third level. While the mathematics self concept scores showed an increase, the differences were not significant. The feedback however from the students was unanimously positive ([Author], 2009). In this paper, the author presents the findings from a larger sample. In addition results from a focus group held with participants a year or two years after completing the 'Head Start Maths' programme are disclosed.

[3] This teaching and learning package is available at http://www.sigma-cetl.ac.uk/index.php?section=108.

Background to 'Head Start Maths'

Head Start Maths is a one week revision programme covering 9 different mathematical topics: Number Systems, Natural Numbers and Integers, Rational Numbers, Algebra, Equations, Factorising, Graphing lines, Problem Solving, Quadratics and Other Special Functions, Logs and Indices. The programme is offered to all adult learners who are enrolled on a degree programme in UL, the week before their university induction. In January 2010, however, the programme was offered in between the first and second semesters for adult learners enrolled on the Business Studies degree programme who were about to take their first mathematics module in UL. Each topic session in 'Head Start Maths' lasts for 2 hours; a one hour lecture given by a qualified mathematics teacher followed by a one hour workshop/problem session where 3 or 4 tutors are available to help with any difficulties the participants may have ([Author], 2009). To date 131 students have completed the course at UL. The author started measuring the mathematics self concept of students in 2008. A database of 69 students' results formed the basis of this study.

Mathematics Self Concept (MSC)

Bridging courses may be evaluated simply on students' mathematical performance on the course itself (Boland, 2002) or in their degree modules having completed a bridging course (Youl, Read, George and Schmid, 2006). The students in this study present with very diverse backgrounds as displayed in Figures 1 and 2.

For this reason it seems pointless giving a summative written test at the end of the programme. Furthermore the author wants to provide an environment that conquers rather than nourishes maths anxiety, a sentiment that has been well documented in adult mathematics education literature (Macrae, 2003; Cockroft, 1982).

Mathematical self concept (MSC) is the perception one has of their mathematical skills and abilities and feelings towards mathematics (Gourgey, 1982). Correlations have been demonstrated between MSC and student motivation (Githua and Mwangi, 2003) and between MSC and student achievement (Liston, 2008).

Methodology

Research Design

The research was carried out in two main stages.

Stage One

- The Author designed and taught the programme in August 2007.
- In 2008 the first empirical investigation carried out by the author on the effectiveness of course. Mathematics Self Concept (Gourgey, 1982) scale chosen. No statistical improvement shown due to small sample (N=18).

Stage Two

- Course retaught in 2009 and 2010.
- 2010 second evaluation carried out on larger sample (N=69) using Mathematics Self Concept scale.
- Ethical approval attained to carry out focus group with 7 participants from 'Head Start Maths' .

Research Sample

69 students (47 male, 22 male) took part in this study. They were all registered on one of 24 degree programmes at UL on completion of the 'Head Start Maths' programme. The

backgrounds of these students were very diverse (10 did not speak English as a first language). Table 1 outlines the gender and age of the participants in the sample.

***Table 1*. Age and Gender Breakdown of Sample (N= 69)**

			age				
			21-25	26-35	36-45	over 45	Total
gender	Female	Count	6	6	5	5	22
		% within gender	27.3%	27.3%	22.7%	22.7%	100.0%
	Male	Count	9	19	18	1	47
		% within gender	19.1%	40.4%	38.3%	2.1%	100.0%
	Total	Count	15	25	23	6	69
		% within gender	21.7%	36.2%	33.3%	8.7%	100.0%

The length of time since the group had studied mathematics varied considerably as Figure 1 shows. 31.8% of the female participants had not studied mathematics in any formal sense in 11-20 years while the 38.3% of the male cohort had not studied mathematics for 6-10 years.

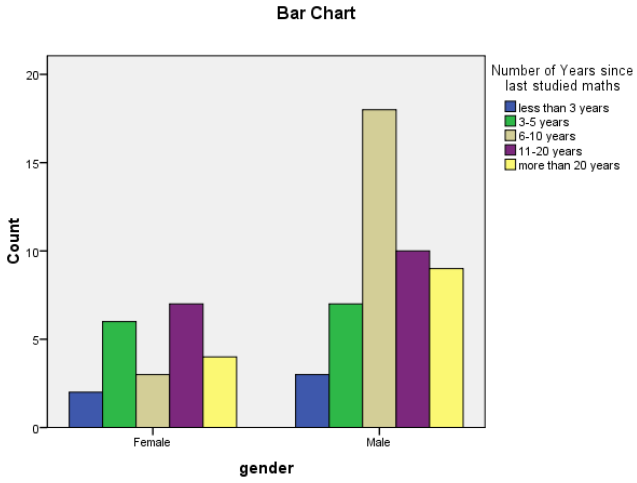

Figure 1. Gender and Mathematical Background of Sample

Figure 2 illustrates the mathematical qualifications of the participants. The majority of the sample (69.6%) had sat their Leaving Certificate examination (final state examination at completion of second level education in Ireland), the requirement for traditional age students entering third level education in Ireland. Many adult learners, learners who are 23 years of age or over, get accepted into UL programmes via interview so do not need to have a Leaving Certificate qualification. Approximately 10 of the sample fall into this category with the remainder having completed the UL access programme or present with international qualifications.

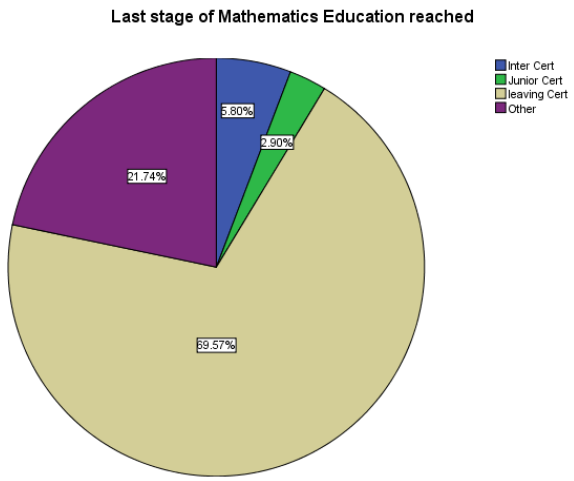

Figure 2. Mathematical Qualifications of Sample

Data Collection

The author uses a 12 item scale (adapted from Gourgey's self concept scale by Miller-Reilly, 2005) to investigate if there is an increase in students' MSC through participation in 'Head Start Maths'. A Cronbach's Alpha of 0.81 indicates high reliability on the 12 (6 positively worded and 6 negatively worded) statement scale With scoring reversed for negatively worded statements, the maximum score achievable for MSC is 60. The higher the score, the higher the MSC of a student. A database of scores was constructed for the purpose of this study. SPSS was used to analyse this data.

Data Analysis

Mathematics Self Concept (MSC)

Table 2 illustrates the pre and post scores per statement. It can be seen from the table that the scores for all of the statements either stay the same or increase after participation in the week of 'Head Start Maths'.

Table 2. **Pre and Post Mean Scores of Mathematical Self Concept (N= 69)**

MSC Statements	Pre Mean(SD)	Post Mean(SD)
1. It takes me much longer to understand mathematical concepts than the average person.	3.3(.87)	3.5 (.92)

2. If I can understand a maths problem, then it must be an easy one.	3.4(.89)	3.5 (.87)
3. I don't ask questions in maths classes because mine sound so stupid.	3.7 (.95)	3.7 (1.0)
4. I have never been able to think mathematically.	3.8 (.82)	3.8 (.91)
5. I don't have a good enough memory to learn maths.	3.8 (.79)	3.9 (.75)
6. Whenever I do a maths problem, I am sure that I have made a mistake.	3.5 (.85)	3.5 (.85)
7. I have never felt myself incapable of learning maths.	3.4 (.93)	3.5 (.95)
8. I have a good mind for maths.	3.3 (.87)	3.5 (.83)
9. I can understand maths better than most people.	2.6 (.85)	2.7 (.89)
10. I have no more trouble understanding maths than any other subject.	3.1 (.96)	3.1 (1.0)
11. When I have difficulties with maths, I know I can handle them if I try.	3.7 (.70)	4.0 (.69)
12. When I do maths, I feel confident that I have done it correctly.	3.0 (.84)	3.1 (.84)

* Significant difference $p < 0.05$

Paired sample t-tests were conducted between pre and post total MSC scores. The pre MSC score was calculated at 40.45 (SD = 6.7) and the post MSC was 41.74 (SD = 7.2), an overall increase of 1.29, significant at $p< 0.05$. This displays evidence of a statistically significant increase in students' MSC having participated in 'Head Start Maths'.

Focus Group

At the end of the academic year 2009/2010, having attained ethical approval from the UL Research Ethics Committee, a focus group was held with 7 participants (3 Science, 1 Technology, 2 Engineering, 1 Applied Physics, 2 female, 5 male) who participated in Head Start Maths in 2009 (5 students) and 2008 (2 students) to investigate the impact, if any, the course had on their mathematics studies at UL. The session was video recorded and transcribed. The data was then analysed using NVIVO. Some of the emerging themes and findings from the focus group are now reported.

Benefits of participation

The students who participated in the focus group indicated that there were a number of benefits, other than mathematical benefits, to participating in the programme. Meeting other students similar to themselves was a huge advantage.

> '…see the range of mature students which amazed me because I would have been the oldest in the mature access course, coming up on 50 and there were lads of 23 or 24. Other mature students didn't realise that it can be like that. You're talking from 23 up to 50/55 years of age like and you get to see faces like. It's lovely to see someone that you recognise, that you can latch on a little bit to'

(Female Engineering Student.

The gentle introduction to university life in small groups was welcomed by some.

> '..the smaller classes that we were in, before you joined the big melee, so you got to meet people and I think I probably met male student 5 that week and we've been hanging round together since. It's a good environment to break people in for sure.'
>
> (Male Science Student).

Participation in the course also informed some students as to what was in store in a few weeks time.

> 'But at least I had an idea what was coming at me by doing the course and at least you knew it wasn't going to be easy. You had that information coming in, it's up to you then to do something about it you know?'
>
> (Male Science Student)

Delivery of programme

The majority of the students (6) were pleased with the delivery of the programme and the number of tutors available to help in the problem sessions and their discretion.

> 'I loved the tutors' part, the [tutors] coming in and helping with the questions going around. First of all they showed straight away how discrete they could be. Like they were just standing around but they were watching and the minute they saw someone struggling they would just lean in nice and gently and the guy next to you didn't have to know what you were on about. If you were stuck on the numberline it didn't matter. The fact that the lads gave us that much respect that does wonders for your confidence down along the line. That is important. The lectures were all presented very clearly really weren't they? I know it's very hard to get people to ask questions but there were very few questions that we could actually ask at the end of a session. It's as simple as that. They were actually very well presented and the notes were A1, A1'.
>
> (Female Engineering Student)

One of the male engineers, however, was disappointed with what he referred to as 'spoonfeeding'.

> 'It was good revision wise, but study skills wise, no. It was spoon feeding'.
>
> (Male Engineering Student)

Diversity in mathematical backgrounds

This was evident to all present.

> 'Yeah like I'd say people that had a decent or a half decent knowledge of maths might have found the first piece of it a bit basic you know and are probably looking around thinking 'god I came in all the way for this!!' … whereas other people like me were sitting there delighted thinking 'I wish I could have had another week of it''.
>
> (Male Science Student)

The students who graduated from second level not so long ago did not find the course challenging enough.

> 'I noticed, some of the lads who weren't long out of school, they were kind of a little bit bored'.
>
> (Female Engineering Student)
>
> I think a lot of it as you were saying about people being bored I'd say, if you came in with a decent knowledge of maths, maybe a leaving cert from 6/7 years ago, you possibly would have been bored, personally once I got to page 2, I was in for the long haul.
>
> (Male Science Student)

One student suggested giving different levels of homework to counter this problem.

> '…but if you're going to do that, the thing is you're going to have to tailor that homework to…well what I would suggest is either give people a couple of sheets whatever, and say look do the level that you're happiest with, the one you think you're kind of maybe you know challenging yourself. If you don't want to do all the other boring stuff, stuff you're really happy with, or comfortable with, then leave it aside but do the section that you're going to get something out of, something that you want to practice, to feel you're going to need the practice.
>
> (Male Science Student)

Preparation for third level mathematics

3 out of the 4 students who did the Science and Technology based programmes were satisfied with the level of 'Head Start Maths' and how it prepared them for mathematics at UL.

> 'I'd say people who came in from the access or people came in with a good knowledge of maths, they might have found the first couple of days maybe a bit basic. Personally I didn't, I needed it… and then some'.
>
> (Male Science Student)

The engineering students however felt that it was not enough. The step up to first year engineering mathematics was too steep according to the engineers and 'Head Start Maths' gave them a false sense of security.

> 'I didn't find it that much benefit to be honest with you because it was such a huge leap into the engineering maths so it was grand to come in and meet the people and stuff like that but from a mathematics point of view. I don't think it really did help me to be honest with you because I think by the time I was using the stuff we covered I had forgotten it again anyway'.
>
> (Male Engineering Student)
>
> 'I agree, what was covered was pretty much what myself and (Female Student) covered on the access course last year in the second semester. While it was good to revise that, it in no way prepared us for what came a couple of weeks later once we started into the course. I do the engineering maths and the applied physics course and we really could have done with a step up again from that'.

(Male Applied Physics Student)

A second more advanced week was suggested by these students and this recommendation has been taken on board. In August 2010, the course will run over a two week period at two levels. Students are advised what material will be covered in each week and they can decide if they want to attend one week or two.

Mathematics anxiety

Mathematics anxiety was something that cropped up frequently.

> 'Truthfully when I came in the door, I was speaking to a lot of the lads in the class, they felt the same like. The one subject that we were really afraid of was maths. We just didn't know what was coming at us'
>
> (Male Science Student)
>
> '…when you're coming in cold, it evokes all sorts of thoughts like 'Jesus they'll all be geniuses up there, I'll only be a fool inside there like'.
>
> (Male Science Student)

For all the adult learners, it appears that mathematics was the main worry entering UL.

> "…you know the day you come in before you start and the 2nd year or 3rd year student brings you round on the induction day , it was the first question on every mature student's lips. 'What's the maths like?', it's the 1st question….every single mature student….every single traditional student asks 'where is the pub?' From a mature student point of view it was the very first question. Everybody!! There are 7 mature students in my class and we actually spoke about it one day. The 7 of us asked 'what's the maths like?' You know.. so it is,.. it's the one subject that plays on your mind coming in, you're nervous about it coming in and the more help you can get before you're coming in ideally (the better)…."
>
> (Male Science Student)

Participation on mathematics courses has been reported as one means of dealing with mathematics anxiety (Peskoff and Khazanov, 2006). This appears to be the case with the participants in this focus group. All agreed that attending the course before the 10,000 other students returned was a great advantage. According to them the adult learners who chose not to take part were sorry.

> It took the intimidating, the whole intimidation thing, that whole fear, those fears it kind of eliminated those. At least you kind of felt you had a little soft introduction and you might have had a step in the door before them, it kind of gave you a bit of confidence as well. The other mature students that didn't do it, I think there was a few in our class, they regretted it after it.
>
> (Male Science Student)

Mathematics learning support

In the academic year 2009/10, adult learners of mathematics were responsible for 54% of the attendance at the MLC drop-in centre, despite comprising 14% of the overall population. The author, who is also manager of the MLC, suspected that participation in 'Head Start Maths'

was partly responsible for this. The students did state that having been through the programme, they were unafraid to ask for help having met all the MLC tutors at 'Head Start Maths'.

> Interviewer: Did [participating in 'Head Start Maths'] encourage you to seek support?
>
> It did yeah, like say because Female Tutor was there and Male Tutor 2 was there and Male Tutor 1 I think was there and a couple more of them you know and even if you didn't know the name, you knew the face. You felt a bit more comfortable then asking questions because you have been asking them all the week beforehand you know. Whereas it might slow down dome people if they didn't do ['Head Start Maths'] and they might be a little bit insular about asking or [going to the centre is it].
>
> (Male Science Student)

Conclusion

The author presents both quantitative and qualitative evidence that participation in a mathematics revision course prior to starting university can impact positively on students' mathematics self concept and their whole mathematics education experience. A statistical investigation into students' mathematical self concepts displayed a significant difference after participation in a one week intensive course at the University of Limerick. A follow up focus group with some participants revealed a positive experience for these students at the transition to third level education. Findings showed anecdotal evidence that while student who proceed to science and technology based degree programmes were satisfied with the level of the programme offered, students who undertake engineering degree programmes require a more advanced programme than the one currently offered at UL.

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Ethical Issues of Learning Manual Mathematics Computations: When Pocket Calculators Can Do Better

Lynroy Grant

Teaching mathematics may seem socially benign, free from political, socio-cultural and ethical controversy. However, as competition for students intensifies, there is an emerging pedagogical schism among faculty, students, and administrators of professional colleges (engineering, finance, technology etc.) about teaching and learning mathematics skills that pocket computers (calculators) are capable of carrying out. This pedagogical schism is also fueling an energetic ethical debate about mathematics curriculum content. This paper reviews some specific issues in these debates and provides some explanation of the issues using a socio-cultural model of tradition.

Making a Drama Out of a Crisis: Using Theatrical Scenes in an Adult Numeracy Classroom

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The authors have felt for some time that there is a place in adult numeracy education (or indeed mathematics education) for some theatrical element. Developments in the learning and teaching of numeracy and mathematics have increasingly involved some innovative approaches such as those involved in embedded practice, ethnomathematics and the cognitive focus of collaborative learning. We felt that it is possible to develop a script for a drama about adult numeracy that could be used in teaching for a range of learners. There are dangers that such a script would be too worthy or clichéd or indeed unreal (not unlike positions that we teachers give to learners about the subject).. This workshop will investigate the development of a script through the discussion of some short dialogues that we have prepared and we invite all to contribute to the first level of development of the script.

This approach draws on many philosophies: the humanistic tradition of education, the tradition of agit-prop combining politics and the creative arts, and the current pedagogy of strategies for active learning. The dialectic of 'drama' and 'mathematics' provides the opportunity to re-focus views – about mathematics – about numeracy – about teaching strategies. This is an experiment to explore, explode and explicate the models, misconceptions and mystification of mathematics – it is the Melodramatic Manifestation of Mathematical Manipulation.

Introduction

While it has been recognised for some time that drama may provide a useful learning experience in the learning of mathematics[4], this is a seldom seen aspect either in schools or for adults. Following the work of Vygotsky (eg see Vygotsky 1986) the link between language and learning has been noted and explored. A number of authors have argued that discussion in mathematics is a significant part of learning. For example, Hoyles (1985) made the case a number of years ago and Swan (2006) with others have produced a range of resources (see DfES 2005) including some aimed at adult numeracy (NRDC 2007). There have also been a number of studies that look at how to understand and encourage discussion in school (see Edwards and Westgate 1994, Mercer and Sams 2006) and some for adult numeracy (Oughton 2009). Nevertheless, although some aspects of 'effective practice' are beginning to emerge (see Coben et al (2007) it is far from clear what type of activities are 'winners' in the numeracy classroom.

[4] See for example, the TES website http://www.tes.co.uk/article.aspx?storycode=6047242 and the theatre company Complicite for examples www.complicite.org/swf/adn/ADN_online_workpack.pdf

Why drama?

The authors feel that dramatical scenes played out and discussed will form a part of effective teaching practice (see Hardy 2008). The benefits of such activities have been variously identified and we outline the following issues.

1. **The use of drama to explore the meanings underlying mathematics**. At advanced levels Kallman and Aulls (2003) identify the rehearsal of historical argument as a place for the development of conceptual understanding. To some extent one could argue that that Lakatos (1976) is undertaking a similar process in Proofs and Refutations, using characters taking historical positions to provide the vehicle for his position on how mathematics is constructed.

2. **The use of a medium in which some learners feel more comfortable**. Many learners are not naturally drawn to mathematics and their strengths may lie in alternative activity which can involve the performing arts. This can be argued to play to the strengths of those with kinaesthetic learning styles (Dunn, Dunn and Price (1984)).

3. **Drama can allow for a range of cultural perspectives to be taken on board**. The history of mathematics involves a range of cultures scattered across time and space. The ability to use events from these sources allows the recognition of many cultures to ways of thinking (see Ponza (1998) and Kaye (2011) in this volume).

Reflections on the use of Lakatos with trainees

In the update of our teacher training course in 2007, we decided to include a session which used a segment of the dialogue scenes in Proofs and Refutations by Imre Lakatos (1976). The session was intended to introduce a number of different types of problem in mathematics and to consider ways of solving these problems. After having attempted to find connections between the vertices, edges and faces of a variety of shapes, the solutions that Lakatos discusses in his book are investigated. Trainees are given the individual roles to play and everyone follows the script. For example, see the following extract which follows one proposed proof of the result.

> Pupil Alpha; I wonder. I see that this experiment can be performed for a cube or for a tetrahedron, but how am I to know that it can be performed for *any* polyhedron? For instance, are you sure, Sir, that *any polyhedron, after having a face removed, can be stretched flat on the blackboard?* I am dubious of your first step.
>
> Pupil Beta: Are you sure that in *triangulating the map will always get a new face for any new edge?* I am dubious of your second step.
>
> Pupil Gamma: are you sure that *there are only two alteratives – the disappearance of one edge or else of two edges and a vertex – when one drops the triangles one by one?* Are you even sure that *one is left with a single triangle at the end of this process?* I am dubious of your third step.
>
> Teacher: of course I am not sure. (Lakatos, 1976, p. 8)

The session is paused at times to discuss the meanings. The session has been run five times with different groups. Overall, the impression given from the trainees is that (a) the session

was enjoyable and (b) the main points were not understood. In feedback from trainees one can see that trainees thought that "as a learning experience, I found it very engaging. On reflection, I guess it must have been because we were taking part (albeit more as observers than participants) in a realistic discussion (academic)" and "perhaps this underlines the reason why polemical plays can be such powerful tools for change ~ the audience 'participates' in the discourse, and is convinced (?) by the case." The positive opinion of the activity (related to the 'distancing' effect of Brecht (1964)) appears to have been the majority view with only one experience of a trainee with an apparently very visual style of learning who struggled with the verbalisation of large amounts of text. But despite these predominantly positive views, it appeared that few of the trainees understood the point of the text – at least at the time of the experience.

Some scenarios

What exists out there to be used in such dialogue scenes? We are aware of two scenes from existing broadcast media that could be used to provoke interesting discussions and one scene from existing literature.

1. The opening scene of Rosencrantz And Guildenstern Are Dead is one such scene.

R: Heads, getting a bit of a bore, isn't it?
G: A bore? Well... What about the suspense?
R: What suspense?
G: It must be the law of diminishing returns. I feel the spell about to be broken.
G ; Well, it was an even chance.
R: Seventy eight in a row. A new record, I imagine.
G: Is that what you imagine? A new record? No questions? Not a flicker of doubt?
R: I could be wrong.
G: No fear?
R: Fear?
G: Fear!
R: Seventy nine.
G: I think I have it. Time has stopped dead.
The single experience of one coin being spun once has been repeated.
A hundred and fifty six times.
On the whole, doubtful.
Or, a spectacular vindication of the principle.
That each individual coin spun individually is... as likely to come down heads as tails and therefore should cause no surprise each individual time it does.

R: Heads... I've never known anything like it.

G: He has never known anything like it. But he has never known anything to write home about. Therefore it's just nothing to write home about. (Stoppard, 1991)

The scene outlines a discussion about the likelihood of throwing a large number of heads in a row. This is the beginning of a drama that is 'intensely conscious of its own theatricality' (according to Hunter (2000)) and questions what 'reality' is. The text notes the artificiality of the play by elaborating on such an unlikely event. In any case, this enables a discussion of the alternative perspectives proposed by the characters and relate to probability.

2. A scene in Season 1 Episode 8 of The Wire

The scene shows how a child having difficulties with calculations of her school homework finds it easier to understand in the context of drug sales.

> **W:** This one here? A bus travelling on Central Avenue begins its route by picking up 8 passengers, at the next stop it picks up 4 more and an additional 2 at the 3rd stop while discharging 1. The next to last stop, 3 passengers get off the bus while another 2 get on. How many passengers are still on the bus when the last stop is reached? …. Just do it in your head. [tosses book away]
>
> After a discussion with a third character about a deal.
>
> **C:** Eight?
>
> **W:** Damn, Sara. Look. You work in the ground stash, twenty picks, two picks come out for you and ask for two each, another one cops 3, then Bodie hands you up 10 more, but some white guy rolls up in a car, wiggles you down a piece for 8. How many vials you got left?
>
> **C:** [thinks for a bit] fi'teen
>
> **W:** how … you able to keep The Count but you not able to do the book problem then.
>
> **C:** The Count be wrong, they [cut] you up. (Simon & Burns, 2002)

This interchange shows the type of thinking that has followed such studies as Nunes et al (1993) and Lave et al (1984). These studies have been argued to show that context appears to affect thinking and learning and that textbook problems are not 'real' problems.

3. Swan dialogue scene

The following scene was produced by Swan (2006) as material for CPD from a task intended to promote discussion of percentages. The task involved the increase and decrease of train fares with a 20% increase followed by a 20% decrease. A character called Sue states that 'the fares are back to where they were before … the increase' and learners are asked whether they agree or not. The following scene is intended to allow discussion of issues around percentages. The scene was generated by Swan as an example of the type of discussion that could occur.

> Harriet ; that's wrong, because … they went up by 205, say you had 3100 that's 5 , no 10.
>
> Andy ; yes, £10 so its 90 quid, no 20% so that's 380. 20% of 100 is 80, … no, 20.
>
> Harriet : five twenties are in a hundred.
>
> Dan; say thr fare was 100 and it went up by 205, that's 120.
>
> Sara: then it went back down, so that's the same.
>
> Harriet : no, because 20% of £120 is more than 20% of £100. It will go down by more so it will be less. Are you with me?
>
> Andy: Would it go down by more?

Harriet; Yes because 20% of 120 is more than 20% of 100.

Andy: What is 20% of 120?

Dan: 96…

Harriet: It will go down more so it will be less than 100.

Dan: it will go down to 96. (Swan, 2006)

In addition to these scenes, we decided to construct our own scene reflecting the views of some learners about why they should study mathematics.

4. A scene discussing the point of mathematical study

To give a flavour, we include an extract here with the full text in the Appendix.

Teacher : mathematics helps us to understand how to build bridges, send submarines to the bottom of the ocean and rockets to the moon.

Jo(e) : didn't the millennium bridge have to be closed down because they hadn't worked out that it would wobble?

Toni/y : and didn't NASA mess up with metres and yards and lost a satellite.

Sam: and I'm not going to build bridges or send people to the moon anyway.

Teacher : aren't there other subjects that you do where you might not use it straight away.

Alex : yeah, I think this is interesting

These four scenes allow for discussion of a range of aspects related to adult numeracy. We have discussions of various topics (probability, percentages, addition / subtraction), how mathematics works in context and the purpose of mathematical study. As such these scenes are chosen to enable rich sets of data from their use.

Deploying iteratively

One of the difficulties with using such dramatic scenes is that they are not really representations of reality but are included to drive plot or make points. As such they are unlikely to be convincing to be used in making direct points. What they could be useful for doing is allowing discussion of issues.

What about writing our own scenes, or indeed asking learners to write scenes (Ponza 1998), would this not help? If we ignore the fact that we (and the learners) are not professional screen writers and are unlikely to find it easy to produce such scenes, it is still likely that such scenes would contain (the wrong sort of) artificiality which may be counter-productive.

At this point the authors considered making advantage of the weakness by incorporating discussion by participants about scenes into a new scene. This appealed to the post modern

sensibilities of the authors and in some ways reflected the iterative nature of some elements of mathematics.

Using the scenes

These scenes were used and discussed with two sets of students. One, a group of 12 teachers of adult numeracy undertaking a mathematics 'up-skilling' professional development course, the other three volunteers from a set of adults having completed an adult numeracy course.

The scenes created some interesting discussion among the two groups. To encourage the discussions we displayed the following questions although the responses tended to be more free-flowing rather than addressing these systematically.

Questions for the drama and mathematics

How might this help learning mathematics / numeracy?

What do you think of what is said?

What would you say?

What sort of scene would you write?

The following contains some extracts from discussion with the volunteer adult numeracy learners. These learners found the scenes interesting and had a number of interesting points to make.

About the drug scene 2

Learner 1 - The child is used to the second calculation … its in its everyday life. The first bit probably doesn't happen very often. But the second part is probably like us going to the shops and buying bread everyday.

Researcher - a teacher the other day … said that you can't talk about drugs with a class

Learner 2 – no not really

Learner 3 – your dealing with adults, you can talk about anything with adults

About the classroom scene 4

It will not come easily in our minds that constructing a bridge needs mathematics … to build an effective and sound quality bridge that will last for a number of years.

And it will be building bridges between your mental ability as well, … yes some people believe maths is difficult … and if they think maths is difficult I want to build a bridge where they can have fun and at the same time learn real maths.

In contrast, the discussion from some of the teachers was a little less enthusiastic.

About classroom scene 4

> Teacher 1 - Is it an all male group? … the topics seem all bloke based … submarines and bridges and sending people to the moon.
>
> Teacher 2 - Its more than bloke based – it's too far away from our students experience

About percentage calculation 3

> Teacher 3 - If the purpose was to show to learners … then hiding behind a character is a less threatening way. [But] perhaps we teachers of mathematics or numeracy should be able to create an atmosphere where learners can raise their concerns. Then again, if its for teacher training then perhaps that would be a good.

Reflections

The discussions following the use of the scenes is encouraging for our purpose. In the playing of scenes, the researchers observed a good engagement in the activity even from those teachers who later expressed some concerns. The use of these scenes appeared to follow the previous use of the Lakatos dialogue and demonstrated a positive experience. Nevertheless the authors accept that this requires a more systematic study.

Some of the teachers expressed serious concerns about the use of these scenes with learners. They were concerned that the scenes would either not seem real (eg the classroom scene), would be unusable (eg drug scenes) or that it would be better to discuss their own calculations rather than taken from other sources.

The learners we saw did not appear to have these problems. They were far more positive about the scenes and had some interesting discussions about them. One learner did consider the Stoppard scene a little 'boring' but still had useful things to say about the meaning.

Of course, we are aware that we have only used are approach with two small groups of participants and making generalisations about these scenes and their use is clearly problematic. Nevertheless, we feel that we have observed that such scenes can be used both with teachers in professional development and with learners. We have also seen that these scenes could be extended using the comments from our participants as extra, interesting dialogue. Indeed, one of the learners was rather lyrical in using the metaphor of the bridge when discussing the classroom scene.

We are aware that there may be difficulties in using one type of 'social practice' when engaged in another (see Wenger 1999, Papen 2005). Nevertheless, we feel that it is important to make connections and that the employment of one type of practice (in this case the use of drama scenes) may assist the development of thinking about mathematics (and hence rethinking mathematics itself).

Conclusion

The authors have undertaken a small scale exploratory study of the use of drama in mathematics classes. We feel that this has been a successful exercise that has outlined a number of ways forward.

i. The scenes could be used with other groups to add to data.

ii. The scenes could be extended by the addition of extra dialogue taken from the participants so far.

iii. A study of the types of discussions that take place when such scenes are used could be undertaken.

We propose that each of these is important and should be investigated.

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Appendix : A scene in a classroom

Teacher : ok then. are there any questions?

Jo(e) : I don't want to be difficult but how will any of this help me?

Teacher : An interesting question Jo(e), mathematics is all around us. There are mathematical patterns in the way that plants grow, in the stripes on a tiger, in the way that bees create their honeycombs.

Toni/y : Are you saying that bees do geometry?

Teacher : Toni/y, it isn't the bee that is doing geometry but that the structures that are around us forcing the world to work in certain ways. If you try building a honeycomb then you would realise that the structure that works easiest is in hexagons.

Toni/y : ok, so why do we have to learn it then if it happens naturally.

Alex : but, isn't it interesting? I hadn't thought about maths like that before.

Toni/y : hmmm, maybe. There are other interesting things

Teacher : mathematics helps us to understand how to build bridges, send submarines to the bottom of the ocean and rockets to the moon.

Jo(e) : didn't the millennium bridge have to be closed down because they hadn't worked out that it would wobble?

Toni/y : and didn't NASA mess up with metres and yards and lost a satellite.

Sam: and I'm not going to build bridges or send people to the moon anyway.

Teacher : aren't there other subjects that you do where you might not use it straight away.

Alex : yeah, I think this is interesting

Sam : but other subjects aren't compulsory.

Teacher: Did you realise that there is a strong link between the level of your mathematics qualification and how much you will earn in the future.

Jo(e) : My mate's dad didn't have any qualifications when he left school and he now runs his own business.

Alex : the world has changed.

Sam : there are plenty of jobs that don't need maths.

Teacher: maybe, but the better paid ones usually expect maths qualifications.

Sam : I want to work in the film industry, there's not much call for actors to know maths is there?

The New Adult Education Movement in the United States: Transitioning English Language Learners. "What Role does Mathematical Literacy play?"

Anestine Hector Mason

The education of English language learners (ELL) who transition from advance English as a second language (ESL) classes into adult Second Education (ASE), General Education Diploma (GED), or adult basic education (ABE) classes is of major concern in the United States today as the need for ESL intensify amidst global economic and workforce challenges. To address this concern, in 2008, the US department of Education funded the Transitioning English Language Learner (TELL) Project to lay the foundation for future works designed to enhance the quality of services that support ELL transition to secondary education credential and ultimately to college.

The TELL project investigates and describes policies and instructional and programmatic strategies that support advanced adult ELLs continued development of English proficiency, including cognitive academic language proficiency in order to successfully transfer into ABE or ASE, complete a high school equivalency program, and become prepared for postsecondary education and the 21st century workplace. A preliminary collection of extant data showed that there is a critical need for programmatic and instructional systems that will enhance educational practices in numeracy for transitioning English language learners; final findings from interviews and extant data showed that there is a significant need for numeracy instruction and programming to help ELLs make a successful transition to advanced educational arenas.

The purpose of this presentation is to report the final findings of TELL research project related to adult numeracy, describe the state of literature in the US relative to educational practices in numeracy with TELLs, and to prompt discussions about promising practices in numeracy with similar students in other countries.

Realistic Numeracy Problems

Kees Hoogland

Concepts of adult numeracy education can be arranged along a continuum of increasing levels of sophistication. According to a review of AIR (2006) all of the most recent influential approaches to defining adult numeracy fall into the so called integrative phase of this continuum. In this phase numeracy is viewed as a complex multifaceted and sophisticated construct, incorporating the mathematics, communications, cultural, social, emotional and personal aspects of each individual in context.

A closer look however at learning or test materials used in many different countries reveals that most materials consist of word problems or of exercises with formal arithmetic skills. One could say that the sophistication of the concepts runs way ahead of the sophistication of the learning and testing materials. In this era of technology and multimedia a next step can and should be made to bring real quantitative problems – problems as individuals face them – into learning or test materials using real life images.

The presentation will focus on the first results to compare students' performance on word problem with their performance on more realistic numeracy problems.

Defining Numeracy – the story continues

David Kaye

My exploration of defining numeracy began in 2002 at ALM9. I looked at how "numeracy" and "mathematics" were used or defined in papers published in ALM conference proceedings.

This is still a live debate and the terms used (often to avoid using "numeracy") now include 'quantitative literacy', 'mathematical literacy', and 'functional mathematics'.

In evaluating the usage of the term "numeracy" I have posed a simple dichotomy – 'small' numeracy (bound by low levels, part of mathematics, revisiting school mathematics, limited) and 'big' numeracy (any level, mathematics "plus", experiential, includes context).

I have continued to explore how numeracy and mathematics are used and defined; there is very little agreement about what numeracy is, and its relationship to mathematics is far from simple. The use of the word "numeracy" has to be continually defended, and the argument is often a political one, rather than academic or pedagogical.

Finally I pose the hypothesis that the ambiguity of "numeracy" enables questioning and inclusion, rather than acceptance and exclusion.

Looking at the Workplace through Mathematical Eyes - Challenges and Solutions

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'Looking at the Workplace through Mathematical Eyes – Challenges and Solutions', builds on work introduced at ALM 16. It is concerned with the idea that people use a range of mathematical knowledge, skills and competence (MKSC) in the normal course of their work, yet may not recognise them as such. Furthermore, the MKSC that underpin the performance of work tend to be obscured from view by a combination of many factors, enmeshed in a job and deeply embedded in an organisation. This paper will describe the challenges encountered by this research and the innovative solutions developed in response. It reports on the research methodology adopted and the enquiry instruments designed. It suggests how aspects of mathematics invisibility may be distilled into fewer, pivotal factors, and explains the role of Rasch Measurement Theory employed to communicate the research findings to a general audience.

People encounter and manage mathematics containing situations in the normal course of their work. The mathematics information may be presented in a variety of ways and elicit a range of responses (Gal et al., 2009) that are underpinned by mathematics knowledge, skills and competence (MKSC). That these may be denied, or dismissed as commonsense, (Coben & Thumpston, 1995) indeed anything but mathematics, is a key concern of this research. The everyday experience of work features the routine application of procedures which comprise a collection of tasks, mediated by artefacts and reinforced by training (Keogh, Maguire, & O'Donoghue, 2010) . That jobs are surrounded by a stream of preceding and following jobs, constructs a lattice of interdependence that tends to deter change and render the embedded MKSC invisible to 'everyday eyes'.

The performance of work, viewed through a lens that is sensitised to mathematics, may reveal the MKSC that temper the responses to mathematics containing situations. This opening of one's 'Mathematical Eyes' (Keogh et al., 2010; Benedicty & Donahoe, 1997) to make MKSC more visible, may challenge the self concept of 'not being a maths person', for the benefit of the individual. Work practices, thus enriched by understanding, may create a platform that facilitates change and encourages development by promoting creativity and innovation, all of which are highly valued by prospective employers (Expert group on Future Skills Needs, 2009; O'Donoghue, 2000).

Researching mathematics in the workplace is difficult, especially in challenging economic circumstances in which employers pursue cost efficiencies and workers fear for their jobs. Mathematics concepts in the workplace are not neatly packaged, conveniently labelled and easily recognisable in the language of the schoolroom. That they may be denied or dismissed by the worker intensifies the extent to which the MKSC may be concealed.

This paper describes the challenges prompted by the questions initiating this research. Continuing the work presented to conference at ALM 16 (Keogh, Maguire, & O'Donoghue, 2010), it briefly recapitulates the enfolding literature, explains a range of innovative solutions developed and adapted to meet these challenges and outlines the guiding methodology. Finally, the role of Rasch Measurement Theory is introduced as a key component of this present work to establish validity, reliability and communicate findings.

Research Questions

The questions driving his research are concerned with the identification of mathematical knowledge, skills and competence (MKSC) that underpin numerate behaviours, however they may be contextualised in the workplace. It is anticipated that this may be problematic, as the mathematics concepts may be rendered invisible to the worker, due to the interaction of many factors, varying in degree and intensity from one work context to the next. This research seeks to develop a methodology that will uncover the 'hidden' MKSC, calibrate them, and make them more visible in terms of the National Framework of Qualifications, by portable rather than generalisable means.

Mathematics contextualisations

The struggle to encapsulate the nature of the mathematics deployed in the workplace is reflected by the variety of ways it has been discussed in the literature. It is being described variously as techno-mathematical literacy (Hoyles, Noss, Kent, & Bakker, 2007), functional mathematics (Wake, 2005), quantitative literacy (van der Kooij & Strasser, 2004; Coben, 2009; Steen, 1997), numeracy (OECD, 2008; Maguire & O'Donoghue, 2003), and realistic mathematics (van den Heuvel-Panhizen, 1998; Treffers, 1987). The proliferation of such similar terms seeking to nuance mathematical content, tends to exacerbate attempts to achieve shared meaning and some measure of generalisation. More than ever, problem solving, spatial awareness, estimation, interpretation and communication skills, are highly valued in the modern worker, as being essential to support change, reaction and response (Expert group on Future Skills Needs, 2009; O'Donoghue, 2000), especially given the pervasiveness of ICT and 'black boxes'. However the MKSC that underpin work, may be dismissed as 'just part of the job' (Coben & Thumpston, 1995). Skills deployed from a 'common sense' perspective may tend to conceal mathematical ability rather than expose it for development (Coben, 2009). To dismiss mathematical skills, rendering them invisible, poses significant challenges for education and training programmes for want of a starting point, i.e. the so-called 'bootstrap problem' (Klinger, 2009). This is problematic for the worker, since, in the view of the National Development Plan (NDP) in Ireland, a lack of appropriate MKSC can have a critical impact on a person's continuing employability (NDP/CSF Information Office, 2007).

Mathematics Invisibility

There are many factors that may exacerbate MKSC invisibility e.g. Language / Jargon (Wake & Williams, 2007); Habitus (Wedege, 1999; Bourdieu, 1977); Training (Brown, Collins, & Duguid, 1989); Common sense (Coben, 2000); Group Status (Gal et al., 2009) (Lave & Wenger, 1991); Artefact (Marr & Hagston, 2007; Strasser, 2003); Black-box (Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002; Wake & Williams, 2007);

Culture (Colleran, O'Donoghue, & Murphy, 2003) and Anxiety (Evans, 2000; McLeod, 1992), operating to obscure or deny such skills, however unwittingly. Just as human vision perceives only a narrow spectrum of light, so too will many overlook the mathematical concepts that pervade everyday work and life.

Within organisational settings, the performance of work is orchestrated toward the achievement of shared goals and is necessarily repetitive on a daily basis. Seeing things through what might be called 'Everyday Eyes', gives prominence to what workers 'do', rather then what they 'know', and adds to the self-perception of not being a 'maths person'.

Identifying the Mathematics

For the purposes of this research, we have equated a workplace setting with what is commonly referred to as a 'job'. In the modern workplace, skills deployed in the performance of work lie at the kernel of a number of encapsulating layers, each exerting its own influence to hide, wholly or partially, the underpinning MKSC. Businesses comprise chains of activity that are triggered in different combinations. Activities encapsulate work practices, which themselves are a combination of tasks, normally considered to be a 'job'. Underpinning a 'job' is a collection of knowledge, skills and competence, the mathematical content of which may lie under the cover of a range of factors, whether internal, external, personal, contextual or other, Fig. 1.

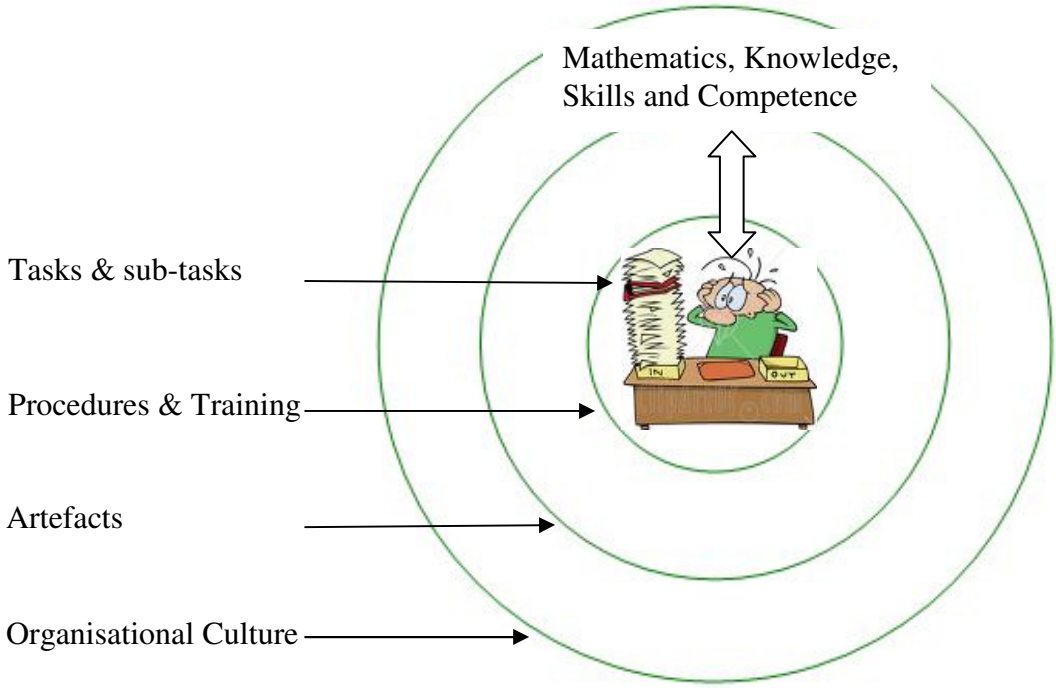

Figure 1. Job: Tasks formed into Procedures, reinforced by Training, supported by Artefacts, surrounded by Organisational Culture and Expectations

The challenge posed by the first question is threefold viz., to identify and set boundaries to a 'job' in a given context, to peel away the succession of occluding layers and finally to identify the underpinning MKSC and calibrate them as to domain, complexity and

complicatedness. Such tools have been developed and tested and are fully described in Methodology hereunder.

Impact of Mathematics Invisibility

The second research question deals with how Mathematics Invisibility might be expressed in the workplace. While many factors are said to contribute to mathematics invisibility, to some extent at least, this research argues that these affectors vary in intensity from one individual to the next, even between those sharing the same workplace. While the encounter between the worker and the work may be an entirely different experience due to personal background factors, the tasks, procedures, artefacts, and work practices are necessarily shared by people doing the same job.

It is beyond the scope of this present work to differentiate in any depth between competing theories of the nature of logical thinking, whether it is supported by whole integrated structures (Piaget & Inhelder, 1959), an inventory of schemata from which inferences may be drawn (O'Brien, 1998), a series of mental models of the real world (Johnson-Laird, 1983) or an innate, common sense ability (Colleran et al., 2003). However, it seems that mathematics concepts are not applied as discrete units of cognition in the initial stages of solving a problem or managing a situation. In this way, degrees of MKSC invisibility may indicate that mathematical ideas and ability have been successfully subsumed into an integrated cognitive resource with which people manage everyday situations. The MKSC may lurk in the background awaiting a suitable stimulus, being brought to the fore as and when needed.

From this perspective, it appears that the continuous use of a set of knowledge and skills, in pursuit of a specific goal, may relegate the awareness of mathematics concepts in a person's work, but not necessarily the associated numerate behaviour. We have developed an instrument that will profile the contrast between the self-reported and the observed use of MKSC in the workplace, in order to make links between dimensions of mathematics invisibility and the possible explanations.

Methodology

This research is guided by the 'Building Theory from Case Study Research' (BTCSR) methodology, described by Eisenhardt (2002). In developing BTCSR, Eisenhardt draws together work previously done on 'Grounded Theory' (Glaser & Strauss, 1967) and 'Case Study Design' (Yin, 1984). The resulting platform includes construct definition, triangulation, within and across case analysis, and the role of pertinent literature (Eisenhardt, 2002).

The richness of detail, anticipated from relatively few cases studied, rather than being a weakness of BTCSR (Huberman & Miles, 2002), supports within and across case analysis, comparison with enfolding literature and triangulation of multiple sources.

Research Plan

The overall research plan is divided into three phases, appropriate to each research question, Fig. 2. At the core of the research are 4 case studies, each of which will be profiled to produce a detailed task analysis (phase 1), which the researcher will use to

complete the validated survey instrument (phase 2). The person whose work is so profiled will also complete the same survey

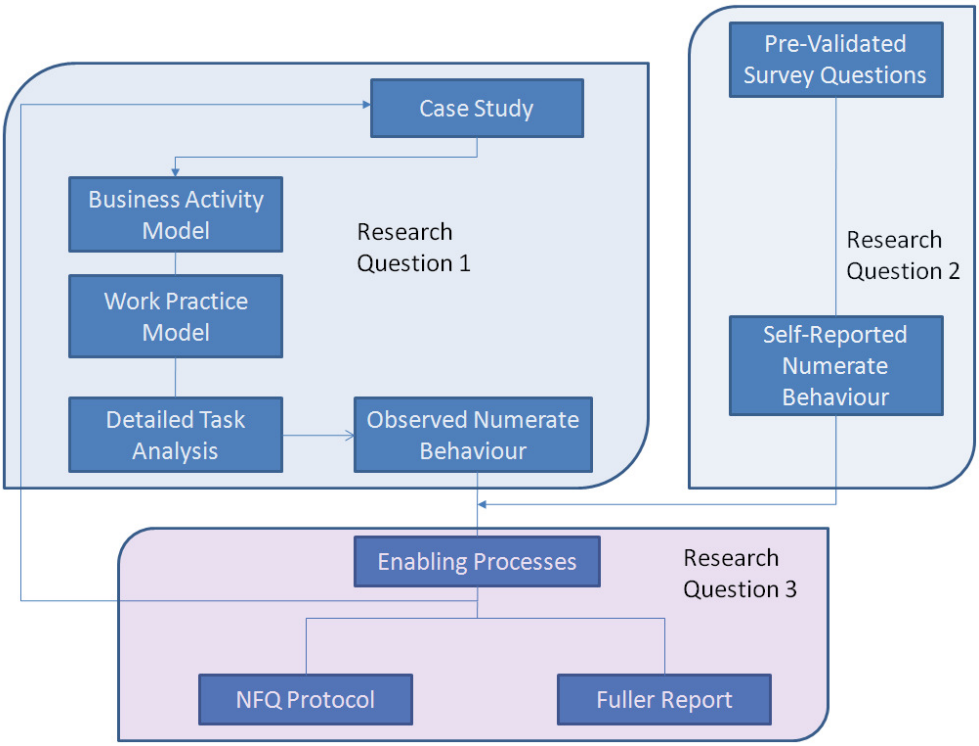

Figure 2. Research Plan Overview

The third phase will make links between the reported and observed incidence of Mathematics Invisibility and suggested possible causes. The process by which mathematics in the workplace will be made more visible is dependant on the findings that emerge from preceding phases and is still under development. While early indications are encouraging at this stage, they serve only to attest to the potential of these instruments to capture and compare the relevant data.

Research Instruments

On first contact, a workplace may seem to be chaotic, replete with a particular use of language and people absorbed in their work. The uninitiated may struggle to make sense of the purpose and processes on view. However, it is crucial for the researcher to become promptly orientated and be equipped, not only to ask clarifying questions, but also to understand the meaning of the answers.

Business Activity Model (BAM)

We have adapted Business Activity Modelling tool (Ericsson, 2004), as an initial step in bring order to this apparent chaos.

A Business Activity Model, Fig.2, is a high-level representation of the architecture of an organisation. Its technical origins may be traced to Leonard Euler's solution to the 'Seven Bridges of Konigsberg' problem in 1765, wherein all non-essential factors were stripped away so as to expose the problem to uncluttered scrutiny. Its principles have been applied in scientific approaches to management such as Taylorism, Fordism, Total Quality Management, Lean Manufacturing and latterly, Lean Thinking. It is adapted to the needs of this research for its ability to communicate succinctly regarding;

- The location and interdependence of organisational units e.g. Sales, Delivery, Accounts
- The sequence of activities triggered by business events
- The context in which activities occur

It further serves as an *aide memoire,* supporting recall and orientation of current and following researchers, and guides the selection of 'jobs' for closer examination. Although the fictional company, 'Paper 2 Picture Services' depicted in Fig. 3 below as an exemplar, typifies the services sector, the same principles may be applied to any sector, including manufacturing and ecommerce.

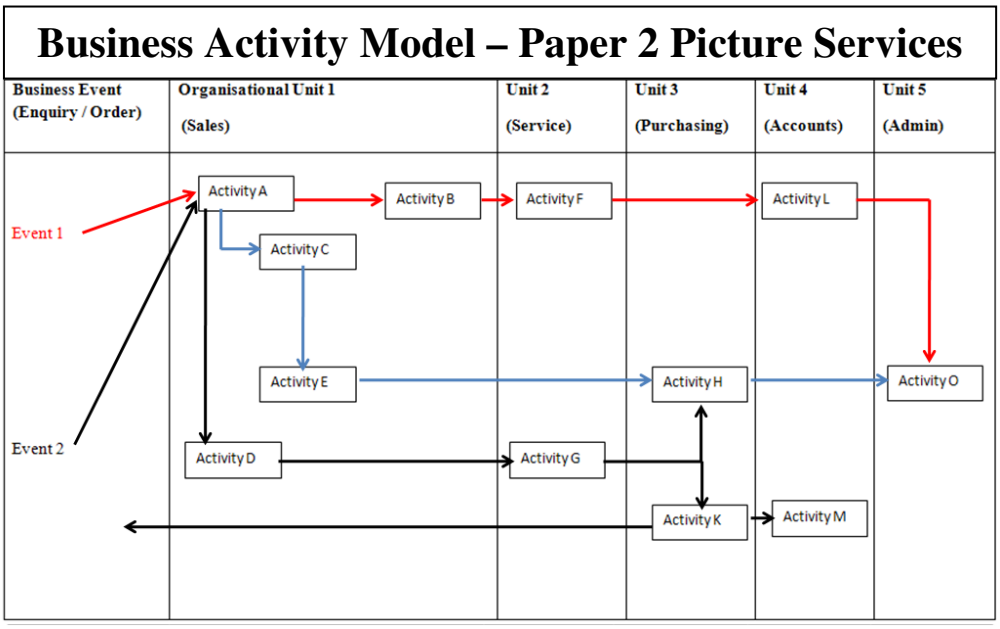

Figure 3. Business Activity Model

Business events, such as a service order or enquiry, trigger different combinations of activities in response to prevailing terms, conditions and process outcomes. For example, the Red arrows depict a transaction being initiated by a telephone call. Activity A makes a decision about the activity most appropriate to respond on behalf of the company. Activity B checks if the caller has an account and whether it is a billable transaction. Activity F, gathers details of the service required and arranges for it to be provided. Activity L, checks the caller's credit position, issues an invoice and sets terms and conditions. Activity O records the incident for statistics and market research.

The Black arrows trace the progress of an order to supply, routed by Activity A to Activity D, Documents Inwards Control, the result of which is a detailed list of the Services to be supplied and routed accordingly. Activity G, deals with scanning a wide range of document types received in bulk by post from a multi-national retail chain. Activities H and K are internal customers providing Warehousing and Shredding services, which may be invoked, depending on the terms of the Service Level Agreement. Activity M, is responsible for internal billing and arranges settlement with creditors. Finally the scanned images are uploaded to a shared storage location and released to the customer. Internally stimulated activities such as R&D and planning are not included above in the interests of clarity.

The BAM diagrams the flow of work through an organisation, from initial stimulus e.g. order or enquiry, through service delivery, administration and accounting. As a model of an organisation's activities, it has the capacity to orientate the observer as to overall context in broad terms, crucial when undertaking business process re-engineering or automation projects.

The BAM serves to suggest activities that may be of interest to the research. However, selecting a specific 'job' for observation requires the finer degree of granularity provided by the Work Practice Model.

Work Practice Model (WPM)

Having identified the business activities and their interdependencies, a more detailed profile of a specific activity is required in order to access the constituent tasks. This is accomplished by means of a Work Practice Model (Sierhuis, Clancey, & de Hoog, 2009), similar in intent to the BAM, yet bearing a tighter focus by defining a 'job' and picking it apart.

The WPM identifies all of the actions required to complete the selected function, in finer detail. Such actions may be performed by an individual, or team, or in collaboration with entities that are physically, if not logically, external to the workplace. The tasks are characterised by active verbs, and are augmented by arrows to indicate dependencies, set boundaries to the 'job' and aid selection for closer observation. This method of 'job' targeting can be replicated in any site, by any researcher and thus adds significantly to reliability.

Alerted to the possibility of tension between what people say and what they do (Strauss & Corbin, 1998):32, the research plan provides for the validation of the WPM by the worker.

The minimal amount of explicative narrative is so because the schematic provides an ideal scaffold to support meaning, context and recall. An example of a WPM describing Activity G of a fictional company is shown in Fig.4.

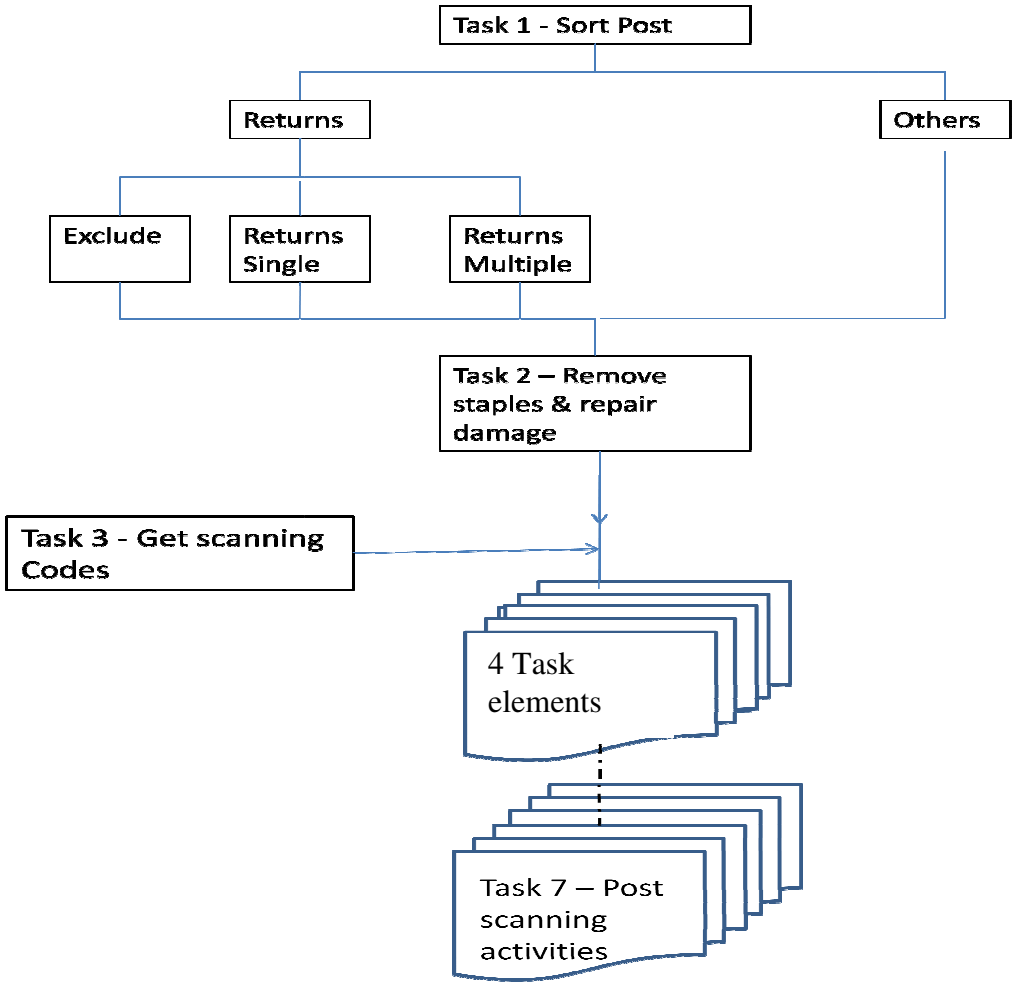

Figure 4. Sample Work Practice Model

The Work Practice Model (WPM) provides the researcher with a platform from which to observe work in process. Furthermore, as some of the worker's responses may be habitual and non explicit, it may be necessary to ask questions in order to clarify the purpose of these actions and to make sense of them in the context of the overall aims of the work.

In the course of the observation, the researcher constructs a Detailed Task Analysis of the actual processes engaged in by the worker, which may contrast with those reported at the outset.

Detailed Task Analysis (DTA)

Informed by the WPM, and observation through MKSC sensitised i.e. 'mathematical' eyes, the researcher lists all of the major tasks and their minor elements detected in the performance of the 'job'. These are then categorised as to their Mathematics Domain, i.e. Pattern & Relationship (PR), Quantity & Number (QN), Dimension and Shape (DS) Data Handling & Chance (DC), an extract of which is illustrated in part by Observation Summary Instrument Fig. 5.

Observation Instrument: Summary Site Code: PS0012009 Job: Bulk Post Participant: PSLE		Mathematics Domain					Non-Math
		P/R	Q/N		D/S	D/C	
4.1	Verify that barcode is appropriate to batch and week / day number. The scanner operator checks that the barcode header sheet is the correct one for the batch prepared for scanning		√				
4.2	Call Software select job type, initialise Scanner. Turn on the scanning computer and select the scanning software. Select 'new batch' and select whether single or multipage documents are input.		√		√		comptr skills
4.3	Place documents in the in tray hopper and view scanned images in process. When the job is setup on the computer, place the barcode header on the top of the bundle being scanned, all documents facing upwards. Larger bundles are processed in manageable quantities, estimated by the operator. As each bundle is finished, they are removed from the out tray and placed face down on the operators worktop to indicate they have been scanned. On completion of the bundle is reassembled including the barcode header, held together by an elastic band and repacked into a filestores box, near at hand. Should a document emerge in another bundle that belongs to a bundled already completed that day, it is replaced in the sorting cages for processing the following						Manual Dexterity
4.4	Detect and solve scanning problems, paper jams, multi feeds, processing speed. Problems can arise in 3 distinct areas, maechanical handling, unacceptable image recorded and computer perormance. Mechanical handling is typified by paper jams, multiple feeds or the presence of debris in the handling mechanism. The machine will stop and require attention. PSLE investigates each event, and uses tried and tested procedures to clear the fault, e.g. open scanner hood, remove staples and other debris, and clears sensors with the help of canned compressed air. Forms with uneven edges or overly creased, cause paper jams and are removed and repaired, before being re-fed. PSLE retraces the procedure to the last good feed and placing the problem document on the top, holds the following pages so that only a single document is fed. PSLE must observe the images being captured by the scanning operation. Unacceptable images and all following images are deleted, the problem causing them rectified the computer is reset and restarted at the last good image capture. At the end of each batch PSLE notes the number of pages scanned on a manually maintained table.	√	√		√	√	Manual Dexterity

Figure 5. Observation Summary Instrument

Each mathematics domain is then taken in turn and detailed as to complexity, complicatedness, Knowledge, Skills and Competence as illustrated in Fig. 6

Observation Instrument: Quantity and Number									
Site Code: PS0012009 Job: Bulk Post Participant: PSLE		Knowledge		Skills		Competence			
Task	Description	Breadth	Kind	Range	Selectivity	Context	Role	Learning to Learn	Insight
4.1	Check that the week number / day number combination (3 digits decimal) are correct. Check that the name on the barcode header sheet is correct	2	3/F3	3	1	1	1	1	1
4.2	Choose 'number of documents in the set' setting, single digit number, from a range offered by the computer software	2	2/F1	3	2	2	1	1	1
4.4	PSLE manually creates a table on which to note the quantity of documents processed for each bundle. A 3 digit decimal number is read from the computor monitor and written into the table as work	1	1/F2	1	1	1	1	1	2
6.1	Create and maintain a table to record the quantiy of pages scanned in each bundle. The quantity typically ranges from few up to several hundred. Only natutal numbers are used.There is no time component.	1	2/F2	1	2	1	1	1	2
6.2	Total a column of 3 digit numbers, comprising up to 10 rows and enter the data on the Dailywork Log. There is no time component as PSLE is usually asigned to this work for whole days.	1	2/F3	2	2	1	1	1	2
7.2	Finished boxes of scanned documents are marked with the week number / day number to indicate to Filestores personnel that the	1	1/F3	1	1	1	1	1	1

Figure 6. Detailed Task Analysis by reference to mathematics Domain and NFQ level.

Survey Instrument

The impact of Mathematics Invisibility measured by this research is exemplified by the possible tension between the awareness of role of mathematics reported by the worker, and that observed and profiled in the Detailed Task Analysis.

We have developed, and are currently validating, a Likert style, survey instrument that enquires about one's awareness of mathematical behaviour and surrounding beliefs and attitudes in the course of one's work. Some questions refer to the 'school' nomenclature of Geometry, Algebra and so on, to gauge the extent to which re-contextualisation to the workplace may have occurred, and if this correlates with mathematics education level.

The test sample is drawn from across a spectrum of workers whose mathematics education ranges from the rudimentary, through the standards expected from compulsory second level schooling, to third level professional and vocation specific. The jobs targeted by the survey range from those containing little or no apparent mathematics to the overt and specific use of mathematics as may be expected from engineering professionals The survey items are derived from a framework of numeracy and numerate behaviour and are modelled along the lines of the Mathematics Anxiety Rating Scale (Fennema & Sherman, 1976).

The survey instrument is designed to be clear and unambiguous, enquiring about one dimension of numerate behaviour at a time. The range of workers included in the sample reflects the workforce in Ireland active in 2009, and although they may respond in different ways, will be monitored for consistency. An inherent difficulty in providing response options graded from Strongly Disagree at one extreme to Strongly Agree at the other is that it implies a scale comprising equal intervals. Assigning quantities to different response categories, values one response as a multiple of another and suggests that perhaps six Strongly Disagrees might somehow outweigh one Strongly Agree, which is incongruent with Social Science. It may also be seen that not all 'strongly' expressed opinions index the same degree of intensity concerning the object of enquiry (Bond & Fox, 2007).

This research has adopted the Rasch Measurement Theory (RMT) as a means of;

- interpreting these response intervals,
- identifying poorly defined elements of the construct of numerate behaviour, and
- exposing faulty enquiry items for further refinement.

We acknowledge that RMT does not enjoy universal acceptance, and that it is a relatively new entrant in the field of Social Science. However, we will establish the credentials of the survey data by traditional methods only then deploying Rasch Analysis techniques to interpret our findings and to enhance communication with specialist and non-specialist audiences alike.

Rasch Measurement Theory

The Rasch Measurement Theory (RMT), is an item response theory developed by George Rasch in the 1960s, on the premise that there exists an ideal measurement instrument in the Social Sciences. However, this ideal is not always attainable, being frustrated by incomplete construct definition and flawed instruments of enquiry. Of several existing Rasch Models, the most commonly encountered is the Dichotomous, which identifies uncharacteristic patterns of response in pass/fail tests. It is based on the idea that the outcome of the encounter between a test item and a person, is a function of the person's ability and the item difficulty – nothing more. The probability of a correct response increases in line with the extent to which a person's ability exceeds the item difficulty. The corollary is that the probable outcome is 50/50 when the person's ability and the item difficulty are exactly matched. When these responses are converted to logits (Log Odds Units), it can be seen that the increase in ability necessary to achieve an improvement from 85%-95% is more than twice that required to improve from 25% to 35%. To this extent, Rasch has the potential to identify not only that there is more evidence of a latent trait, but to indicate how much more.

Rating Scale Model

Extending the principles of the Dichotomous Model to the Polytomous or Rating Scale Model, allows for the transformation of ordinal data into interval data. In contrast with the dichotomous context, there is no right or wrong response to Likert style items, only opinions expressed within the terms of the construct and the response structure. Typically, participants are invited to choose the response that most closely matches their opinion from a 5 point scale, ranging across Strongly Disagree, Disagree, Neutral, Agree and Strongly Agree. In keeping with the Rasch probabilistic outlook, the response options are interspaced with thresholds (Bond & Fox, 2007). These are the points at which the chosen response has a 50/50 chance of falling in an adjacent lower category. This forms the basis of the pattern of probable responses, and the means by which Rasch Analysis deals with missing data and the associated Standard Error.

A guiding principle of RMT is that the data match the model (Linacre, 2010). This is commonly misconstrued as an exhortation to eliminate data that is considered to be inconvenient. On the contrary, Rasch approaches data 'Fit' in three ways. Over-fitting data is thought to be too predictable, offering little in the way of new information. Under-fitting data may be too unpredictable, exerting an undue, distorting influence. However, the reported 'Under-fit' statistic draws attention to the structure of the survey item, rather than the nature of the responses. Items that are ambiguously or negatively worded are open to interpretation and tend to undermine consistency. Data collected by flawed means may be tainted beyond repair, threaten validity, and, being of no assistance in the analysis of survey data by any method, may be properly excised from the data set. Finally, 'Fitting' data, holds out the promise that the items comprising the survey, are sharply constructed, consistently interpreted and uni-dimensional (Bond & Fox, 2007).

Application of Rasch Analysis

A Rasch analysis of the validated survey data, will develop a cumulative, probability scale calibrated in logits, of a numerate behaviour response to a mathematics containing situation e.g. Acting Upon - Quantity and Number, Fig. 7. It is possible to locate the self- reported responses of a specific individual to survey items regarding this numerate behaviour along that scale. Supported by the Detailed Task Analysis, the researcher will complete the survey on behalf of the observed individual, and locate those responses on the same scale. It may be possible to link the apparent contrast between the self-reported/observed and the relevant beliefs and attitudes data elicited from the same individual.

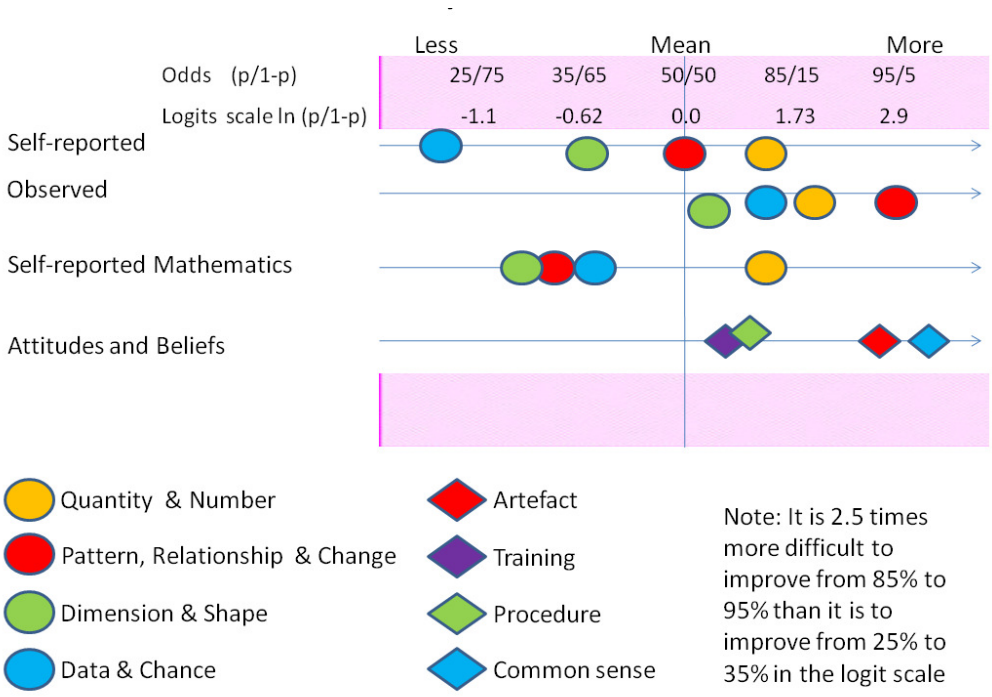

Figure 7. Bond & Fox Pathway, adapted to contrast the observed with self-reported numerate behaviours and surrounding beliefs and attitudes.

It is not the intention of this figure to represent the candidate's mathematical abilities on an absolute scale. Rather it seeks to profile the tension between the self-reported and the observed numerate behaviour in response to mathematical concepts encountered in the course of work. For this reason, the survey does not include a series of tests, across a range of difficulty designed to capture the relative abilities of the participants. Some of the survey items are expressed in the language of 'school' mathematics in anticipation that respondents may display varying degrees of success in linking their formal learning experience with workplace contextualisations of mathematics.

Concluding Remarks

The difficulty in conducting this research is highlighted by the multiple affectors that may contribute to invisibility, each in their own way and extent. Moreover, that the

MKSC sought are to be found at the individual task level, 'buried' under a number of layers, present particular challenges of identification. The approach to observing the workplace described above, consists of a set of tools for the exploration of mathematics in the workplace, to overcome these obstacles and meet the needs of the research.

The **BAM** rapidly orientates the researcher in the workplace, providing sufficient familiarity to make an informed and independent selection of candidate jobs. In this way, the possible contaminating affects of self-selectors and employer influence can be avoided.

The **WPM**, developed initially from the Management point of view and corroborated by that of Operations, builds internal validity and sets boundaries to the job under observation. The researcher can absorb the details of the work practices described and be better positioned to appreciate the purpose and meaning of each task that might otherwise be masked by the operator, however unintentionally.

The Bond & Fox Pathway graphic, Fig. 7, adapted to the needs of this research, will illustrate the contrast between the self-reported and observed, **DTA** supported, instances of numerate behaviour and the surrounding beliefs and attitudes. By reflecting the job-MKSC content back to the participants, it may be possible to triangulate the extent and possible causes of mathematics invisibility in the workplace. These may provide an index of invisibility, pointing to the factors to be addressed in order to open a person's 'Mathematical Eyes'.

The benefits accruing to the individual of being made aware of their facility with MKSC may include a review of their self perception of not being a maths person, and their being encouraged to pursue further development. Acknowledgement of their skills, may support the Recognition of Prior Learning, and empower them in their work and elsewhere. They need no longer equate what they know with what they do, and become aware of their inter-sector mobility. The employer, accurately informed by the MKSC requirement of existing, changing and planned jobs will be more likely to recruit suitable personnel. Providing learning and training opportunities, specific to the identified need could be more efficient use of time and money. Personnel, matched to the requirements of their job, whose potential to develop is leveraged by the company, provides fulfilment and satisfaction, with a consequent positive affect on retention and absence. Workplace learning and training providers will benefit from a clearer indication of the skills required by the company, and, being equipped with an audience profile based on their work, can adjust their content and delivery methods accordingly.

The linking of possible causes to the incidence of Mathematics Invisibility, may underpin the development of teaching and learning strategies that support the re-contextualisation, rather than the transfer, of mathematics from formal acquisition to non-formal application in workplace settings.

Finally, the survey instrument, taken together with the BAM, WPM and DTA tools developed at the root of this work, represents an integrated methodology which may offer support to future research in the workplace.

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Automated online homework vs traditional paper and pencil

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Automated online homework is an increasing trend in college mathematics courses over traditional paper and pencil. One university's college algebra(pre-calculus) courses have moved the majority of homework assignments into the online environment, allowing for immediate feedback. We examined the degree to which online homework, specifically for algebra courses, affects perceptions of learning and motivation to learn. Online homework is an increasing trend in college mathematics courses over traditional paper and pencil. One such instance of this is at a large University of Qatar, where the college algebra courses have moved to comprising almost all homework as online assignments. Studies have been conducted on the use of online homework in physics courses (Bonham, Beichner, & Deardorff, 2001) and chemistry courses (Cole & Todd, 2003), but parallel research was not found in the field of mathematics. The aforementioned studies investigate exam scores and difference in online versus traditional methods of completing homework. Due to the considerable use of online homework in courses such as the college algebra course, it is necessary to investigate its effectiveness in a mathematics class. Furthermore, the investigation needs to go beyond test scores and examine its effects on student learning and motivation to learn mathematics. We examined the degree to which online homework, specifically for algebra courses, affects perceptions of learning and motivation to learn for this group of college students. The investigation, surveys involved all of the college algebra students enrolled in the course for one semester (n ~ 180). The Motivated Strategies for Learning Questionnaire (MSLQ) using both Likert-scale and open-ended response items.

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The adult numeracy conundrum

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The history of adult numeracy reveals decades of investment in efforts to improve functional numeracy skills in the workplace and beyond yet, for all the training programs and initiatives, the problem of poor adult numeracy skills and overt adult innumeracy remains largely intractable, with little real change in adult numeracy rates. This, then, is the adult numeracy conundrum. There are numerous agendas, ideologies, and viewpoints held by a raft of interested parties all of which might be regarded as factions whose influences drive public policy. It is argued here that such factionalism serves mostly to cloud the real issue, which is that the usual conceptions of the adult numeracy problem – and the various efforts to find a solution – are fundamentally flawed and it is this that lies at the heart of the conundrum. Progress in the natural sciences has often demonstrated that intractable problems may signal a need for a radical change of perspective, or paradigm shift. Therein, it is suggested, lies a lesson for adult numeracy.

Introduction

The adult numeracy conundrum is that despite decades of investment on efforts to improve functional numeracy skills in the workplace and beyond, for all the training programs and initiatives, the intractable problems of poor adult numeracy skills and overt adult innumeracy persist, with little real change having occurred in adult numeracy rates and essentially the same issues arise over and over again, as evidenced by recurring media reports, for example, bemoaning the poor state of adult numeracy and the extent to which vast numbers of adults lack mathematical skills beyond those expected of children at the end of primary school education. Collectively, these efforts present a phenomenology that is reminiscent of the history of intractable problems in the natural and social sciences, which have yielded only to a radical change of perspective, the so-called ‘paradigm shift’, a term coined by Thomas Kuhn (1970).

In this sense, then, it is proposed here that there are fundamental flaws in the conception of the ‘adult numeracy problem’, and consequently in the many and various tried and proposed solutions, that signal the need for a new paradigm. To that end, this paper first reflects on some examples of paradigm shifts in the natural sciences, which is the domain on which Kuhn focused, and also examples from the social sciences, an entirely different domain but one for which the concept of the paradigm shift has been adapted and applied extensively. Next, the broad nature of the current conception of the adult numeracy problem is outlined, together with a brief consideration of the driving factors that influence that conception. Finally, I present some ‘random thoughts’ – speculations that might help a reframing of the problem and thus seed the much-needed paradigm shift that could see an end to the adult numeracy conundrum. My purpose throughout is not to argue rigorously but to prod and, hopefully, to stimulate.

Paradigm shift

Originally, as defined by Kuhn (1970), a 'paradigm shift' describes a fundamental change in basic assumptions within a ruling, or dominant, scientific theory. There are many examples, such as:

- the departure from Ptolemaic geocentric cosmology in favour of the Copernican heliocentric system
- Newton's unification of classical physics into a coherent mechanical worldview
- the transition from Maxwellian electromagnetics to Einsteinian Relativity
- Lamarckian theories of evolution (inheritance of acquired characteristics) being overthrown by Darwinian theory of evolution (natural selection)
- acceptance of Plate tectonics to explain large-scale geological changes
- Newtonian physics yielding to the Einsteinian Relativistic worldview
- classical mechanics giving way to quantum mechanics

Even within a single disciplinary area, there can be multiple shifts in paradigm. Cosmology underwent a paradigm shift from the 'steady state' theory of the universe – a model developed by Hoyle in 1948 – to the now dominant 'big bang' theory, which is currently regarded as the 'standard model' but which itself was subject to a radical re-formulation via the inflation theory proposed by Guth in 1980. The theories of quantum mechanics and general relativity, each regarded as highly successful in their respective domains, were soon found to be irreconcilable under conditions in which both theories had to be invoked; the ensuing conflicts gave rise to new paradigms which sought to reconcile the differences – relativistic cosmology, string theory, *m*-theory, and the 'many worlds interpretation', to name but a few. Each has its adherents and detractors but, nonetheless, they are radically different conceptions to those that previously held sway.

Drawing on Kuhn's work, Handa introduced the idea of a 'social paradigm' for social, rather than scientific, contexts, looking particularly at the affect on social institutions (including education) of fundamental perceptual and conceptual changes and the social dynamics that bring about such radical shifts. Examples of social paradigm shifts include:

- the so-called 'cognitive revolution' from behaviourist approaches to psychological study and the acceptance of cognition as central to studying human behaviour
- the 'Keynesian revolution' in economics, the driving force for which was the economic crisis of the Great Depression
- industrialisation – from field to factory; from artisan to mass production

The term 'paradigm shift' has taken on broader and broader application than intended by Kuhn's scientific application. Educators are wont to see paradigm shifts in 'teachable moments' – the time at which learning becomes possible or easiest. In principle, this seems to be quite apt since those 'light bulb moments' experienced by learners often signal not merely a point where the last piece of the jig-saw clicks into place but, rather, denote a fundamental

shift in their perception whereby they see material in an entirely new and different way so that previously intractable ideas suddenly assume an unanticipated clarity. Such experiences mark vastly more than just a breakthrough in understanding but stem from a dramatic and radical change of perspective that makes understanding *possible* and it is this that is the hallmark of a true paradigm shift.

It is important to note another defining characteristic of paradigm shift: in every instance described by Kuhn, profound changes in prevailing thinking were preceded by almost insurmountable opposition.

The problem of adult (in)numeracy

As with cosmology, there is a 'standard model' of adult numeracy. It is a defacto standard, perhaps, but no less powerful in its influence. It is a model of *deficit* and *remediation* that in practice operates mostly from the basic premise that innumerate adults are 'broken' and in need of 'repair'. Various causes are attributed to their deficiency, ranging from a lack of diligence, or even laziness, as school pupils, to insufficient or inadequate school-based mathematics education, often related to educational disadvantage due to social, economic, cultural, and ethnic factors (for example) but also including criticism of school curricula and lack of school teachers with appropriate qualifications. This deficit model of adult learners is something for which the *Skills for Life* ideology has been criticized, along with assumptions that literacy and numeracy can be viewed as sets of 'autonomous skills which can be transferred unproblematically to different contexts of use in adult lives' (Oughton 2008 p. 40).

The customary solution is to seek to remedy the deficit by programs of 're-education' that present school-like mathematics curricula (albeit sometimes disguised) in schools, colleges, community education, work-place training, and on-line learning situations. An example is the introduction in England of national curricula for adult literacy, language and numeracy, which derive from the National Curriculum for schools, as part of the Skills for Life strategy (BSA 2001, DfES 2001). While said to be non-prescriptive, the documents are very much associated with national qualifications, for which funding is tied to learners' attainment – thus in terms of funded provision, the curricula are essentially obligatory (Oughton, 2008).

The motivation for such programs is entirely reasonable, being intended (for instance) to permit adults to work better, to get work, or get better work; to manage their finances; or to help their children with homework. Or, driven by social justice agendas, to effect greater numeracy so as to promote greater equality in terms of opportunity, welfare, health, and life expectancy, and to further ideals of 'critical citizenship'. In industry and commerce, programs for workers are aimed at achieving productivity gains and reduced liability or losses. At a national policy level, programs and initiatives may involve all of the above but, fundamentally, are aimed at delivering gains in social and economic capital thus developing a more competitive workforce to meet the needs of a market-driven global economy.

Regardless of the motivation, the general persistence of low adult numeracy rates in Western first-world economies is a strong indicator that this remedial approach does not work. The question, then, becomes 'why not?' This is far from trivial – clearly, if an obvious solution existed, it would have been found and there would be no adult numeracy 'problem'. But perhaps it may be that there *is* an obvious solution, only that it is obscured by the current paradigm. There are several factors to which this paradigm is intimately tied: there are numerous agendas, ideologies, and viewpoints in play by numerous stakeholders – employers, advertising and marketing agencies, educators, politicians, unions, social groups, charities,

conservationists, environmentalists, the media, parental groups, and so on. All of these interested parties might be regarded as 'factions', whose influences drive public policy. These often competing factional or vested interests operate in a common environment, dominated by the 'standard model' of the current paradigm, and, while they present many 'solutions' (in the form of programs, initiatives, and incentives), no answers have been forthcoming.

Considerable activity might be generated by virtue of the involvement of so many interested parties but one might argue that factional dynamics are such as to focus attention on the means while obscuring the goal. That is, the object of purported concern becomes secondary, with its value to the factional parties being relegated merely to that of a vehicle that serves other interests, whether these be educational, social, ideological, commercial, or political. Paradoxically, the investment in all that activity can imbue it with substantial inertia, notwithstanding that the activity fails to deliver its primary objective.

Random thoughts ...

As the heading suggest, there is no particular structure to what follows. Rather, in contemplating the adult numeracy conundrum, I have been led to various reflections, which are shared here:

- Maybe it doesn't really matter that adults can't 'do fractions' or read graphs and charts (etc) – society still gets along pretty well as it is and there are plenty of ways for adults to learn stuff *if they want to.*

This goes to the heart of the 'how', 'why', and 'what' of adult numeracy programs. In an information-rich society, where computers and the internet are readily accessible to most adults, there is no shortage of resources and expertise if adults *want* to learn more mathematics. Many of the resources available to them also mean that a lot of adults do not experience the need to develop their own 'know how' – they are content to rely on the expertise of others. If it is given that, nonetheless, adults *should* become more quantitatively competent, the issue then is to determine what factors would motivate them to do so. For the vast majority, it appears that exposure to 'school mathematics' (again), in no matter what guise, will not deliver in this regard. Something else must come first, at least.

- Maybe the problem doesn't actually reside with adult learners & the numeracy programs intended to help them. Maybe it's more to do with *who* sets the adult numeracy agenda, *what*'s in it, the *purpose* it's supposed to serve, and how *outcomes* are determined...

As Oughton (2008) has pointed out:

> Lerman (2000) applies Bernstein's theories of recontextualisation and ideology to mathematics education, and suggests that curriculum development may be driven by: an authoritarian view, which involves the 'selection of culturally valued knowledge'; a neo-liberal view, 'producing citizens prepared for useful, wealth producing lives'; an 'old-liberal' agenda of enabling people to fulfil their lives; or a radical agenda of preparing people to critique and change their world.

What is missing here is the idea that *adult* learners should be capable of setting their own curriculum agenda or, at least, voicing their views to set the direction of that curriculum.

- Maybe the 'adult numeracy problem' is a fiction – a consequence of how numeracy is conceived and assessed rather than a reflection of adults' competencies in general. Numeracy issues are complex and multi-dimensional: the appearance of the adult numeracy problem is an illusion of aggregation.

The underlying assumption of the adult numeracy curriculum, that learners need to acquire 'functional' numeracy to help them in their daily lives, is challenged in a study by Swain, which concluded that one of the main reason adults wish to learn numeracy is:

> …to prove to themselves that they have the ability to study and succeed in a high-status subject, which they perceive to be a signifier of intelligence. The other main reasons are for learners to help their children, and for understanding, engagement and enjoyment. (Swain 2005, p305)

While adult literacy and numeracy in England have achieved considerably greater prominence as a result of the *Skills for Life* strategy, Oughton (2008) points out that there have been 'many concerns about its neo-liberal emphasis on economic effectiveness and the functional model it assumes for literacy and numeracy' (p. 40). There are few, if any, consistent models to assess adult numeracy in any meaningful sense and so as draw meaningful comparisons latitudinally (across population groups defined by social, economic, cultural, or national characteristics) or longitudinally over time, which would permit a clearer assessment of the nature and scope of the issues.

- Maybe we can't actually resolve the adult numeracy problem by focussing on ('fixing') adults now. Perhaps the only real answer is to do a better job with maths education in schools – teach it better or differently so that future generations of school leavers become more numerate adults

Ten years ago, Lerman pointed out that much of the mathematics curriculum in England was very similar to that of 50 years previously (Lerman, 2000; Bernstein, 1996) and that the adult numeracy curriculum was largely based on that 50 year-old school mathematics curriculum. It is not difficult to argue that nothing is substantially different today.

Conclusion

If we, as practitioners, theorists, educators and policy-makers, keep trying to do essentially the same things and continue to observe that the outcomes are unsatisfactory, when do we get to the point of tiring of knocking our heads against the wall? At what point do we stop and ask, "What exactly *are* we doing here, and why? Is this the *only* way to proceed?" When do we say, "Is it just me or is there something wrong, here?!"

By coming to such realisations and struggling to ask better questions, according to the scenarios described by Kuhn, one might reach the threshold whereby the prevailing paradigm is sufficiently challenged that it gives way to a new world view. The overwhelming lesson from Kuhn's essay is that repeatedly asking the wrong questions, no matter how carefully framed, cannot elicit the right answer. Repeated efforts to answer the wrong question, no matter how well-crafted the attempt, will not yield the desired understanding. Truly intractable problems demand that one be prepared to subvert prevailing views and adopt a new stance.

The challenge is not to find the answer to the 'adult numeracy problem'. The challenge is to find the right question to ask, and thereby dispose of the adult numeracy conundrum once and for all.

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The language of mathematics: lessons from English language proficiency

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There are as many reasons why adults choose to learn mathematics as there are levels of mathematics education in which they engage, from developing functional numeracy skills for workplace or vocational imperatives, through to the study of mathematics to satisfy higher education prerequisites, and beyond. Epistemologically, what does mathematical knowledge mean? One might know that '2+2=4' and even how to carry out the procedure to add two numbers together but is that enough? Propositional knowledge and procedural knowledge may be functionally necessary but are they sufficient? In particular, do they equip the learner with the capacity to understand, generalise, and communicate effectively their mathematical knowledge and interpretation of quantitative and mathematically relational states? So often, mathematics and numeracy pedagogy focuses on propositional and procedural knowledge, leaving learners to develop for themselves a capacity for interpretative fluency. In this, there are parallels with issues faced by linguists concerned over the English language competence levels of university entrants, both NESB (non-English speaking background) and ESB (English speaking background), most notably in terms of the distinction between competence and performance and differences in formulaic versus structural approaches. This paper explores some of those parallels and considers possible implications for adult mathematics education.

Introduction

Mathematics remains a staple in general education yet is largely unappreciated by many, until they need it. A Google search on 'reasons why adults learn mathematics' returns more than thirty million 'hits' containing lists full of examples such as figuring out how to feed the family, balancing the family budget, planning travel arrangements, calculating and comparing sports statistics, and so on. Most are trivial – hardly worthy of being considered 'mathematics'. How many people actually calculate the total surface area of the walls so as to determine the amount of paint they need to buy? Typically, people do not behave in this manner; rather, they mostly attend to choosing colour schemes and considering logistical options to keep children and pets separate from wet paint – the actual quantity of paint purchased most likely results from a vendor's advice. The 'mathematics' in such scenarios is largely fictional or regarded simply as 'common sense'. In similar vein, while it is desirable that adults have an appreciation of fractional representations of quantity, in reality the need to add, subtract, multiply or divide fractions rarely arises outside of school classrooms and most adults can live full and productive lives never having to re-live the horrors of 'doing fractions'.

For all that, there are as many reasons why some adults choose to learn mathematics 'proper' as there are levels of mathematics education in which they engage, from developing functional numeracy skills for the workplace through to the study of mathematics to satisfy higher education prerequisites and beyond. Yet vast numbers of adults encounter impediments to their learning and still more struggle with the mathematics of everyday lives, as evidenced by recurrent media accounts (for instance) of poor adult numeracy rates, which have remained essentially static for decades despite considerable investment in countless government-funded programmes. Consideration of such aspects, though, is beyond the present scope; rather, the focus here is on epistemological impediments to adult mathematics learning. In this, various conceptions linking mathematics and language are examined to develop the proposition that, whatever else it may be, first and foremost mathematics is language. Next, the nature of mathematical knowledge is addressed from a language perspective, which leads to a consideration of language learning and acquisition, and observations of some significant parallels between the learning of mathematics and natural language development. In this, some essential aspects of linguistics and language teaching theory are drawn upon with the aim of revealing their implications for adult mathematics education.

The confluence of mathematics and language

The words 'mathematics' and 'language' have a long association in the literature, whether rhetorically, as in the simile: mathematics is *like* a language; metaphorically – mathematics *as* language; or literally – mathematics *is* a language. Less directly in terms of the sense that I intend, but perhaps more prolific in the literature, is discussion concerning the language *of* mathematics – that is, the lexicon and usage associated with the mathematics register as a superset of natural language. It is worth looking at each of these aspects so as to illustrate their similarities and differences.

The rhetorical case

Nesher and Katriel (1986) allude to the intersection of language and mathematics, suggesting there are language-like aspects of mathematics. Of mathematician Haskell Brooks Curry, Seldin (2009, p44) wrote, 'For Curry, mathematics is like language, in the sense that it is the result of human activity.' Klemeš (2002, pp110,111) explains that mathematics is like language because: '… it can help us formulate what we know, and ask questions about, or hide, what we do not – but the use of correct grammar offers no clue to the value of our statements, not does it even guarantee that they make any sense at all.' That is, mathematics has a communicative or descriptive function which is not explicitly interpretive since mathematical models are only as good as the understanding of those who generate them.

The metaphorical case

Culik (1993, p132), referring to Beckett's 'aesthetic agenda', writes of 'the metaphoric power of non-literary fields such as … mathematics' and the allusions highlight that the metaphorical association of mathematics *as* language can be far from trivial. More directly, Pimm (1987, p201) maintains that valuable insights are to be gained by considering mathematics *as* language – an expressly metaphorical association which is to be recognized explicitly, else 'there is no work the metaphor can do'. Bullock (1994, p735) writes of the 'intricate linguistic structure' of mathematics and the 'richness of the literature composed with it'. Sarukkai (2003) draws on the work of Steiner by identifying the function of numbers as proper nouns in mathematics whereas their role is adjectival in natural language – that is,

the association is raised to that of metaphor via 'a more general understanding of mathematics as language, particularly in terms of its use.' (p3664).

The literal case

Adams (2003, p786) asserts that 'mathematics *is* a language… of words, numerals, and symbols', while Sarukkai (2003) adds that 'mathematics is a language and should be dealt with as such' (p3670). Wakefield directs our attention to attributes common to both mathematics and natural language but examples of common ground do not, however, constitute proof that mathematics *is* language and Wakefield finally adopts a fall-back position of metaphorical association, expressed in terms of conceiving of mathematics 'as a living, breathing language with a culture of ideas expressed in numbers…' (Wakefield, 2000, p278).

Without further appeal to the substantial body of literature, it seems self evident that mathematics provides a medium of communication that is utterly precise and efficient in its capacity to express very specific concepts, particularly those for which natural language is inadequate or imprecise. Ignoring for the moment considerations of the natural language terminology encompassed by the mathematical register and considering only the 'pure' symbolic formalism of written mathematics, it also seems self evident that this satisfies the usual definitions of a formal language: it comprises a set of words, being finite strings taken from a defined alphabet, the manipulation of which is governed by a defined grammar, and comprising a system for the encoding and decoding of information. Compared to natural languages, mathematics is grammatically simple – there are no tenses, pronouns, propositions, infinitives, participles, gerunds and so on. Simplistically, the 'parts of speech' are mostly nouns and verbs – objects and actions between or upon objects: numbers are nouns, as are pronumerals and well-formed grammatical structures that are reducible, at least in principle, to number; in essence, verbs include the symbolic operators of addition, subtraction, multiplication, and division and positional operators of exponents. Of course, in developing mathematics beyond arithmetic, further notations are defined to extend or embellish these fundamental elements.

Accepting, then, the premise that mathematics is *language*, the task now is to examine the inherent implications proceed to epistemological and pedagogical considerations.

Epistemology – the meaning of mathematical knowledge

Learners of mathematics, at any level, encounter two forms of knowledge: *propositional* or declarative knowledge (knowledge that…); and *procedural* knowledge (knowledge how…). Propositional knowledge includes mathematical facts, such as knowing that:

- the symbols '1', '2', '3'… denote notions of abstract quantity, serving as labels (nouns) for the results of counting exercises, independent from any detail of that which is counted
- the square root of 81 is 9
- '½' is more than zero but less than one; that it is equivalent to the words 'one half' and the symbols '0.5' and '50%'; and that it corresponds to understanding of the physical world (half a pie, etc)
- $x^2 + 3x + 2$ is equivalent to $(x + 1)(x + 2)$ and that both forms are ultimately reducible to a specific quantity once x is decided upon.

The last example also provides an illustration of procedural knowledge – the 'know how' to manipulate symbols correctly. Procedural knowledge includes such things as knowing how to:

- add, subtract, multiply and divide fractions
- extract relevant information from a word problem, translate it into a representation using mathematical symbols, and 'do the maths' to solve the problem
- compute the arithmetic mean, media and mode
- solve a quadratic equation

Both propositional and procedural knowledge extend to knowing the meaning of technical terms comprising the vocabulary of the mathematics register in natural language and how to apply them. For instance, knowing that 'mean', 'median', and 'mode' are measures of central tendency within a statistical distribution; knowing what 'central tendency' means; having the 'know how' and ability to compute these values; understanding that 'mean' and average' may be synonyms, and that in this context 'mean' is to be understood as the 'arithmetic mean', rather than the 'geometric mean' or 'harmonic mean', for example.

Becoming deft in manipulating the symbols of mathematical notation is challenging to learners. In contrast with natural languages, the 'simple' mathematics grammar demands rigorous syntactical rules that allow little scope for the imprecision that can be sustained by natural language without loss of meaning; application of procedural knowledge frequently means performing set algorithms precisely. The mathematics register introduces a whole new level of difficulty, not least because of its extensive and often complex vocabulary, with the added challenge that common natural language words can take on different meanings in the mathematical context. To the learner, many of these can be counter-intuitive: for the mathematician, what the lay person calls a straight line is a 'line segment', which is properly a curve with zero curvature; a 'function' is neither *soirée* nor *purpose* but rather a mapping between sets; and 'integration' has little to do with *fitting in*, in a sociological sense, rather it is the summation of infinitesimal values, equivalent to the signed area of a specified region under the graph of a curve.

While it is clear that propositional and procedural knowledge are functionally necessary, they are far from sufficient for these alone amount to little more than a set of habits. True facility and confidence in the application of mathematics requires the ability to understand, interpret, generalise and communicate mathematical knowledge and the goal of every learner should be the attainment of *interpretative fluency*, a pragmatic competence whereby the reading and writing of mathematical expressions and statements makes sense so that the reader understands both the meaning being conveyed and the processes and procedures whereby that meaning is to be actualised – for instance, by carrying out arithmetic, expanding and simplifying an expression to solve an equation, or correctly interpreting graphs. The mathematics that a learner reads may be correct in its 'virtual meaning' but if the learner lacks sufficient skill and interpretative fluency, its 'actual meaning' will remain obscure – the mathematics will not 'speak' to the reader. Conversely, while inexpert learners lacking such fluency might know what they wish to 'utter', the mathematics they write may be incomplete, ambiguous, or nonsensical. The goal of every mathematics educator, then, must be not only to attend to and facilitate their students' acquisition of propositional and procedural knowledge but to foster their development of interpretative fluency, without which any degree of know-what and know-how is impotent. This distinction between virtual and actual meaning is more commonly articulated in language learning theory as that between sentence meaning and

utterance meaning respectively. It is informative to explore in more depth these parallels between language and mathematics learning.

Linguistics – language learning and mathematics learning parallels

In speaking of natural language and referring to Bialystok & Sharwood-Smith (1985), Widdowson (1990, p18) observes, '…there is a difference between knowledge of language and the ability to access that knowledge effectively in different contexts of use. … accuracy and acquisition do not match'. This highlights the distinction between sentence meaning (inherent in the words themselves) and utterance meaning (the intended meaning, or message). This distinction applies equally to learners of mathematics. When faced with the pressure to *perform* – for instance, when practice of algorithms gives way to problem solving activities – students can become so concerned with their efforts to 'make meaning' by communicating their answers that they tend not to monitor whether or not what they write 'makes sense': focussed on output, they fail to attend to the semantic accuracy or appropriateness of their written mathematics.

In language acquisition and learning, there are two essential processes at work: '…*acquisition*, which is natural, unconscious, primary, and causative; and *learning*, which is unnatural, conscious, auxiliary, and corrective' (Widdowson, 1990, p20; emphasis added). That is, the unconscious and natural process of language acquisition occasions the need or desire to *perform* using the new language, thereby invoking a second process – that of conscious learning, which is concerned with *competence*, whereby performance is self-monitored with the aim of ensuring that learned elements of formal rules are appropriately brought into play. This duality is consistent with the interactionist view of language acquisition that it is the interaction between cognition and linguistic environment that gives rise to learning progress so that the key factor lies in *discourse*, rather than utterance alone (Ellis, 1986). This contrasts with the behaviourist view that 'reduces language learning to habit formation' (Widdowson, 1990, p25). As with the learning of mathematics, according to Ellis (1986) such views about language learning point to the importance of recognizing the distinction between two types of knowledge possessed by second language learners: the propositional or declarative knowledge ('knowing that') by which the learner possesses internalized second-language rules and memorized chunks of language, and procedural knowledge ('knowing how') whereby the learner processes second-language information, both for acquisition and for use. Such procedural knowledge provides the learner with strategies and devices to process received information, to form utterances, and to 'compensate for inadequate resources' (*ibid*, p165).

This distinction between *competence* and *performance* arises often in the study of language (see Chomsky (1965), for example). Referring to performance accuracy, Widdowson (1990, p18) advises that Bialystok and Sharwood-Smith explicitly separate a learner's knowledge of language (in a propositionally competent sense) from the ability to *apply* that knowledge appropriately in context. This divide is an enormously challenging aspect of second language acquisition (as it is for mathematics learning) because '…to the extent that language learners, by definition, are deficient in competence they cannot authenticate the language they deal with in the manner of the native speaker' (*ibid*, p45). Mathematics teaching and learning practitioners will see that this observation is equally true of their students when it is paraphrased by replacing 'language' and 'native speaker' with 'mathematics' and 'mathematician', respectively. Widdowson further notes that various approaches to language teaching are inclined to stress either competence or performance, frequently each to the detriment of the other. For instance, 'the so-called "structural approach", focuses attention on

knowing… semantic capsules… The assumption is that once learners have achieved this semantic knowledge, then they will be able to use it pragmatically to do things' (*ibid*, p157). That is, *form* (grammar) takes precedence, with the implicit notion that investment to promote sound understanding of the language code will lead eventually to both maximum productivity in language use and maximum generalisability. This is reflected in the pedagogy, where tasks do not mirror authentic, real life communication but instead consist of activities designed to further the learning of grammatical rules through memorization, drills, and repetition – often involving contrived and 'unnatural' usage (Murray, 2010, pers. comm. 4 June).

This *structural* approach is in contrast with the *functional* or 'communicative' approaches that followed, which place emphasis on the way in which language is used so that it is purposeful and communicative for learners, resembling the ways in which they use their first language. This represents a reversal of the previous assumption, so that now it is pragmatic performance that leads learning through contextual practice to draw necessary inferences and thus learners acquire new knowledge of the language itself (Widdowson, 1990). Functional approaches are about promoting language as a means to getting things done in real life and so syllabi reflect activities centred on language use in the 'real world', including such functional learning goals as: 'offering an apology; showing appreciation; introducing yourself; apologising; etc.' (Murray, 2010, pers. comm. 4 June). In this paradigm, *form* is invoked to serve. Again, there are parallels here for mathematics learning, where the equivalent of the structural approach is the rôte learning of algorithms and procedures – 'capsules' of mathematical knowledge such as the quadratic formula – and the communicative (functional) approach finds its mathematics education counterpart in practice problems and investigative approaches to learners' 'discovery' of mathematical knowledge.

Widdowson (1990) notes that the communicative approach to language learning is appealing because it shifts the teaching focus to the learner and better aligns the ends with the means, but it is not without problems. For one, the primary focus on *function* diverts attention from the need to attend to *form* to the extent that, in reality, teaching of grammar is too easily neglected so that, while students gain a 'repertoire of authentic expressions and a reasonable sense of when to use them appropriately', this is constraining and ungeneralisable and so they do not develop the competence to be creative with language and exceed that which has been explicitly taught (Murray, 2010, pers. comm. 4 June). Thus, students may indeed gain fluency but they attain rather less in terms of grammatical accuracy, which requires supplementary formal instruction – casting doubt over notions that they automatically infer and internalize detailed knowledge necessary to make language available for general use. In other words, 'learners do not very readily infer knowledge of the language system from their communicative activities' (Widdowson, 1990, p161) – the grammar they need as a resource is elusive and they end up with a limited and fragmented performance repertoire with no underlying competence to support it.

In essence, the same is true of mathematics learners – the practice of undertaking real or pseudo-real investigations, while similarly shifting the focus to the learner, does not in itself yield a comprehensive and cohesive body of knowledge and is an inefficient method of acquiring mathematical knowledge (engaging though the practice may be); nor can it generate the underlying competence in the *use* of formal mathematical symbolism and associated language, without which learners cannot sensibly communicate the results of their investigations.

In summary, then, we have the following observations:

Language learners	**Mathematics learners**
teaching vocabulary and grammar does not, in itself, enable learners to *use* language in real situations	teaching the 'rules' of syntax and algorithms does not, in itself, equip learners to *use* mathematics in real situations
teaching a repertoire of phrases etc to use in varying contexts/scenarios does not give learners the fundamental competence to generalize	teaching by practice on set problems and developing a repertoire of techniques does not give learners fundamental competence to generalize
communicative exposure in 'real' settings (learning by doing) can result in fragmented knowledge and weak grammar, which needs to be taught explicitly	'real' investigative approaches may help to develop an appreciation of mathematical relations but do little, if anything, to develop notational/syntactical fluency, which needs to be taught explicitly

In the teaching of both language and mathematics, there are obvious parallels and similar pedagogical challenges. What appears to be missing in each is the need for some combination of approaches. To an extent, this is precisely what has occurred in language teaching. In what might be called the post-communicative era, such a combination has emerged as a compromise between structural and functional approaches. As Murray (2010, pers. comm. 4 June) puts it:

> Functional syllabi have remained but grammar (form) has re-emerged as a focus of pedagogy; however it tends now to be taught implicitly through teachers designing tasks in such a way that they get learners to notice the particular structure(s) that are to be learned.

For language learners, this means that learning is authentic and thus engaging, while language form is highlighted 'incidentally' through artfully designed tasks. For mathematics learners, there is no less an imperative to devise and pursue pedagogical initiatives in a similar vein, developing students' abilities to read and write mathematics so that it, too, 'makes sense'; that is, to attain at least some level of interpretative fluency.

I contend that at any level, whether school-based or for adults, mathematics education generally fails to give learners an adequate appreciation of the linguistic dimension of the subject. Often, even advanced students struggle to 'read' mathematics; they cannot readily translate or explain what is 'going on' in a piece of written mathematics – while they might learn to interpret mathematics symbolism as a set of computation instructions, when their computations go wrong they frequently have little idea about what has occurred. When learners write mathematics, their reluctance to embellish their symbolic expressions with plain English explanations or connecting phrases suggests that they do not think of the written mathematics as *communication* to 'tell the story' and explain their reasoning but as a means of satisfying their teacher's admonitions to 'show your working'.

As an illustration, a very common flaw in learners' written mathematics is the abuse of the equality symbol. Novices are often seen to pepper the 'equals' sign between every expression, or step, in their work, oblivious of its literal interpretation. Hersch (1997, p50) gives the

example of a calculus student's response to the question, "What is the minimum of the function, $y = x^2 + 2x + 5$?" The student wrote:

$$"x^2 + 2x + 5 = 2x + 2 = -1 = 4"$$

Clearly, the student understood the need to find the function's first derivative, set this to zero, solve for x, and evaluate the original function at that x-value to obtain the corresponding y-value. Quite apart from the lack of effort to express these ideas so as to better communicate or explain his/her answer, the misuse of '=' in the little that *was* written demonstrates that, while clearly procedurally able, the students has no obvious sense of what '=' means and has used it merely as an indicator of the conceptual flow. Perhaps some explanation towards this lies in unfortunate choices of language in primary school instruction: Austin and Howson (1979, p177) note the conflict between natural language and mathematical symbolism by comparing '6 + 2 = 8' with '8 – 2 = 6', where the '=' in the first may be translated as 'makes' whereas in the second it may be translated as 'leaves', as in teachers' utterances that "Six and two makes eight" and "Eight take two leaves six", neither of which provides any direction to indicate, let alone emphasize, that these are *equivalence relations* made explicit by the use of the '=' symbol. One might wonder how many learners ever come to the realization that '6 + 2' is a *synonym* for the quantity most simply named '8'.

A key missing ingredient for mathematics learners to attain interpretative fluency is 'metalinguistic awareness' – the ability to reflect on and analyze language, which is invoked when *form* and *function* become the focus, rather than literal meaning; when there are decisions to be made about manipulating language and how best to convey meaning. MacGregor and Price (1999) observe that the need to analyze structure, manipulate expressions, and make choices about representation is inherent in formal mathematics, especially algebra, and conclude that 'it seems likely that metalinguistic awareness in ordinary language has an equivalent in algebraic language' (p451). I go considerably further in asserting that the need to develop learners' metalinguistic awareness must be a *fundamental* objective of mathematics educators. It is as vital for learners of mathematics as it is for learners of language since it is the mechanism by which self-correction occurs and through which learners are able to intuit meaning to make sense of input, make inferences, and internalize their knowledge as a generative system so that it is available for contingent use.

Implications and Conclusion

What, then, does language teaching suggest for the teaching of mathematics? For all the commonalties noted thus far, these are distinctly separate curriculum areas. It may be that the *real* commonality for effective teaching in each (as in any discipline) is to ensure that practitioners are fully versed in pedagogy that is firmly rooted in sound epistemology and that they provide a supportive learning environment to enthuse and inspire their students. But if this rather obvious prescription is sound then, surely we should expect students to struggle no more with mathematics than with any other subject. And yet they do. Thus this simple prescription lacks some critical detail. Inspired teaching certainly promotes learning but, again, this is true in any classroom. Modern pedagogical practices for language teaching and mathematics teaching adhere to similar, if not identical, general tenets – arguably, with behaviourism and cognitivism having somewhat yielded to constructivist principles that address collaborative and social dimensions of learning in authentic, learner-directed contexts. This leaves epistemology. While linguistic conceptions of mathematics have their place in modern philosophy of mathematics, they are peripheral, being vastly overshadowed by traditional schools of thought concerning the fundamental nature of mathematical knowledge.

Thus it may be argued that there is little imperative to drive language perspectives to the forefront of pedagogical practices in mathematics. Yet, from empirical observations as a practitioner, I am convinced of the effectiveness of doing so.

To many students, I have likened it thus: suppose one is planning a brief visit to a foreign land; it may be useful to acquire a phrase book to cope with basic communicative needs, such as finding a restaurant. A few key phrases might even be committed to memory but their utility will be at best limited and uncertain. Now suppose that a longer sojourn is contemplated; one might prepare by attending conversational language classes or using language-learning software. With sufficient effort it might be possible to cope with most routine encounters, having acquired a basic working vocabulary and practised a range of 'authentic' scenarios. But it is likely that much would depend on recall ability and not straying too far from 'safe' contexts. Certainly, one would not expect to be able to converse freely with a native speaker or engage in political debate, for example. Such activities require fluency approaching that of a native speaker. So it is with mathematics. One may learn facts, fragments and fundamental rules of grammar – such as knowing how to add fractions and perform basic arithmetic operations – and this may be enough for a few specific contexts but it will prove inadequate for more challenging or unfamiliar situations. One may go further and gain a level of functional fluency sufficient for most day-to-day purposes. But to truly appreciate the beauty and power of mathematics one must be able to throw away the 'phrase book' and embrace the language. I am happy to say that most of my students 'get it' – eventually. They come to realize that they need not be dependent on uncertain recall of memorized rules and routine procedures once they acquire the capacity to *read* the mathematics and understand what is 'going on', that the language itself dictates the 'rules' they once tried to rely upon while lacking understanding.

I have tried here to outline a view of mathematics education that might suggest some useful teaching and learning strategies. There is much scope to explore these and related notions in considerable detail, which is an agenda for the future.

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Dynamics of face-to-face mathematics teaching and learning in workplace training

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This paper examines the dynamics of an adult student learning to read fractions-of-an-inch on a measuring tape during an impromptu one-on-one tutoring session within a trades training program. Analysis is framed using Radford's cultural-semiotic theory of knowledge objectification which reveals the complexity of this multi-semiotic interaction. Specifically, this perspective calls attention to the intense and responsive one-on-one interaction between the student and tutor in this mathematics learning activity as well as the important role of a variety of semiotic resources in mediating this. It follows that workplace training efforts would be served by attending to these dimensions of mathematics learning.

Adults' mathematics learning for and within the workplace is an important yet relatively new and developing area of research. As Coben (2006) explains, "research in adult numeracy and mathematics teaching and learning is still in the exploratory phase of development" (p. 29) and the field "is beset by conceptual difficulties" (p. 18). Furthermore, as FitzSimons, Mlcek, Hull and Wright (2005) state, "there has been little attention ... focused on how numeracy is learned in the workplace, taking into account the complex issues which surround apparently simple calculations, and the importance of social, cultural, and historical contexts" (p. 26). In an effort to provide some insights on mathematics learning for the workplace this paper reports part of a larger study (LaCroix, 2010b) of a single pipe trades pre-apprentice making sense of fractions-of-an-inch on a measuring tape, an essential skill in his chosen vocation. This impromptu one-on-one tutoring session, with the researcher serving as tutor, takes place within a work place training program conducted at a trade union run school. It is part of a larger project, the University of British Columbia Workplace Numeracy Project[5], undertaken in partnership with the BC Construction Industry Skills Improvement Council (SkillPlan). The project examines mathematics learning and practice in a number of workplace training programs related to the construction trades.

The analysis presented here is framed using Radford's (2002, 2006, 2007, 2008b) theory of knowledge objectification (TO), a recent elaboration of activity theory (Leont'ev, 1978). Radford developed this cultural-semiotic theory to focus on the social, mediated, embodied and cultural and historical situated dimensions of mathematics thinking and learning and thereby overcome limitations of existing rationalist and individualistic views of cognition and social interaction. This paper focuses specifically on the social and semiotic dimensions generally, and the dynamics of iconicity and semiotic nodes in particular—that Radford (2005, 2008a, 2008b, 2009) has identified as central elements of the mathematics learning process—within this one-on-one encounter. The enactment of iconicity and semiotic nodes both reflect and contribute to the student's developing awareness of cultural logic of the fraction-of-an-inch pattern used on the measuring tape.

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Theoretical framework

According to the TO, "thinking is a *reflection*, that is, a dialectical movement between a historically and culturally constituted reality and an individual who refracts (as well as modifies it) according to his/her own subjective interpretations, actions and feelings" (Radford, 2008b, p. 219). Learning from this perspective is conceptualized as the active and creative acquisition of historically and culturally constituted forms of thinking. Such an acquisition is considered to be a matter of *objectification*, that is, a process of becoming conscious of, and critically conversant with, the cultural logic with which mathematical and other objects have been endowed. (Radford 2002, 2003, 2005). To put it another way, objectification, is the process through which a mathematical object or way of thinking becomes an object of consciousness for the learner. The idea of objectification bears a close relationship with the Vygotskian concept of consciousness and the mediated nature of it (Vygotsky, 1979, also Leont'ev, 1978). From this perspective, consciousness itself is formed through encounters with other voices and the historical intelligence embodied in the artifacts and signs used to mediate our own actions and reflections. The efforts that the pre-apprentice undertakes with the help of the researcher to become aware of the mathematics of a measuring tape in the present study constitute a process of objectification. Central to this is the question of how the cultural meaning of the mathematics behind the measuring tape becomes "recognized." Here the TO departs from other approaches to mathematics learning in that the question is not only the manner in which personal and cultural meanings become tuned, for personal meanings can only arise and evolve against the backdrop of forms of activity. Understanding mathematics learning from this perspective involves examining very social formation and evolution of personal meanings of mathematical objects as they evolve within goal directed activity framed by the cultural meanings conveyed by signs and artifacts within historical-cultural contexts.

As mentioned earlier, Radford has identified two main processes within the objectification (learning) process, namely *iconicity* and *semiotic nodes*. Iconicity is a link between past and present action. It refers to the process of noticing and re-enacting significant parts of previous semiotic activity for the purpose of orienting one's actions and deepening one's own objectification (Radford, personal communication, September 29, 2008). Semiotic nodes refer to the use of multiple semiotic resources together in a coordinated way to achieve and/or sustain knowledge objectification. "Since knowledge objectification is a process of becoming aware of certain conceptual states of affairs, [changes in] semiotic nodes are associated with the progressive course of becoming conscious of something. They are associated with layers of objectification" (Radford, 2005). Furthermore, as the learner becomes increasingly aware of a mathematical object the use of language becomes more precise and central within his or her enactment of semiotic nodes (Radford, 2009).

Method

The pre-apprenticeship course that is the focus of this study was targeted initially based upon convenience from the larger body of data collected as part of the UBC Workplace Numeracy Project. It involved both a higher proportion of class time spent on mathematics related work and considerably more video data of individual students and small groups of students working to make sense of the mathematics-related work assigned to them in comparison to any of the other courses that were observed. The students in this particular class also engaged more readily with the researcher on camera than did those in the other classes. The course was designed to give pre-apprentices a head start with important practical skills that would be

addressed in the first two years of their formal apprenticeship training in one of the pipe trades—plumbing, steam fitting, sprinkler fitting or gas fitting. It addressed basic practical skills in the workshop including tool use and basic construction tasks specific to the pipe trades, as well as workplace safety. The classroom component addressed basic trade-related knowledge including mathematical applications. Throughout the course the researcher observed all of the mathematics related parts of the program—both in the classroom and the workshop, engaged in discussions with pre-apprentices about their mathematics related work while they were completing it, and provided mathematics help for any pre-apprentice who requested it. Pre-apprentices were video-recorded while they worked with the researcher or on their own and copies of the course print materials related to mathematics and copies of pre-apprentices' written work were also retained for analysis. Student-specific data collection was restricted to those members of the class who had given prior written consent, which included the great majority of these adult students.

After an initial review of the data collected from this class, the different kinds of mathematics related tasks assigned to students were identified and all instances of these from the 35 hours of video data were categorized accordingly. At this point it was decided that the 33 minute episode involving one student (who will henceforth be referred to as C) working with the researcher-as-tutor (who will henceforth be referred to as L) in an impromptu tutoring provided a uniquely rich, intensely focused, and sustained episode of mathematics learning activity for analysis. Furthermore, the focus of this episode—on understanding the cultural logic behind an imperial measuring tape—reveals the complexity of learning the mathematics behind reading a measuring tape, a ubiquitous part of measurement activity in the construction trades.

C was a secondary school graduate. He had been in the workforce and completed some courses in an electronics-technician training program at a community college after secondary school and prior to beginning the pre-apprenticeship program in the pipe-trades. Throughout the pre-apprenticeship course C actively sought out L for help with his mathematics related work. Furthermore, not unlike many Canadians of his age group, C had learned to measure exclusively in metric units during his elementary and secondary schooling. However, at the present time the use of imperial measure (e.g. feet, inches, and fractions-of-an-inch) is not uncommon within Canadian industry, especially construction.

Drawing upon the activity theoretical work of Engeström (1987, 1993, 1999, 2001) the larger study identifies and analyses mediating elements within C and L's activity including: artifacts and signs used, rules or cultural-historical norms enacted, divisions of labour relating to the conduct of the tutoring session, conflict and contradictions within this activity, as well as some of the relationships between these. Drawing upon Radford's TO, analysis, the larger study also addressed C's progressive objectification of the fraction-of-an-inch pattern on the measuring tape as reflected by social and semiotic processes identified by the TO, his changing subjectivity as a participant within this activity, and finally a general comparison of epistemological and ontological underpinnings and goals of workplace mathematics activity with the activity of school mathematics.

Methodologically, activity theory provides a set of powerful conceptual tools and general principles for the analysis of human practices as situated, mediated, and developmental processes with the task of applying these in any particular research context dependent upon the particular activity being investigated and, therefore, left to the researcher (Engeström 1993; Kuutti, 1996; Nardi, 1996). In Engeström's (1993) words "methods should be developed or derived from the substance as one enters and penetrates deeper into the object of

study" (p. 99). Therefore, the methodology used in this study can be characterized as theoretically grounded, interpretive, and emergent. Furthermore, within activity theoretical research it is not uncommon that researchers are recognized as participants within the activities being investigating, either as part of the collective subject of the activity or as a member of the community within which the activity occurs (e.g., Engeström, 1993). This stems from the view that all of the participants connected to an activity, including a researcher, are unavoidably related in a dialectical way with all of the other parts of the activity system being investigated. This recognition allows for activity theory to be used for analyzing and understanding events as they occur without the need to engineer situations for data collection in ways that enable researchers to argue that their presence has been controlled or rendered insignificant as the case in other research traditions. (The larger study from which this paper is drawn not only accommodates the researcher as a participant, but includes an analysis of his involvement and learning while working with C.)

This pre-apprenticeship course was one of a number of trades training programs that this researcher visited over a 21-month period as part of the larger project including an earlier session of this pre-apprenticeship course with the same instructor, the final year school component (level four) of a plumbing apprenticeship course, and all three levels of an ironworker apprenticeship program. The researcher's analysis of the target activity was informed by his experience conducting the field work in these programs, extensive and ongoing consultation with experienced workplace educators from SkillPlan as well as a number of the course instructors during this time, along with his previous work experience in industry and as a mathematics educator.

Close multi-semiotic analysis of the video recording of C and L's shared activity was conducted. This involved the construction of an annotated transcript of the dialogue from the video recording along with a detailed account of significant actions, semiotic resources, and artifacts used. The different aspects of fractions and measuring in fractions-of-an-inch with the measuring tape being attended to at each stage of the session were also noted. At times this process entailed slow motion and frame-by-frame analysis of videotape to assess the use and coordination of various semiotic resources. The transcript was reviewed thoroughly from start to finish for and revised repeatedly during this process, with adjustments made for accuracy as needed during the analysis process.

Results and discussion

C and L's encounter that is the focus of this analysis began in the workshop with L discovering that C was having difficulty reading fractions of-an-inch on his measuring tape while completing a pipe-fitting fabrication project. C and L move to a table in a quiet classroom immediately thereafter to focus exclusively on making sense of the measuring tape. The series of selected excerpts to be discussed here show clearly C's use of iconicity, his use of multiple semiotic resources (semiotic nodes), as well as the dynamics of these processes as his objectification of the measuring tape develops during the tutoring session.

During the tutoring session L and C focus together exclusively on C's learning of the fractions-of-an-inch pattern on the measuring tape. L draw's C's awareness to, and prompts him to explain various aspects of the pattern on the measuring tape, systematically explains details of conventional fraction notation and how to read various fractions on the measuring tape, and C asks questions himself relating various aspects of the fraction-of-an-inch pattern. L makes use of a number of semiotic resources in the process including: spoken and written language; mathematics notation; the pattern of fractions-of-an-inch divisions printed on the

measuring tape and on a set of rulers prepared on transparencies for use as a teaching aid by L; counting; indexical or pointing gestures; sweeping or hopping gestures through or over intervals on the measuring tape respectively; chopping gestures in reference to the fraction-of-an-inch division lines on the measuring tape; indexical inscriptions such as circling or underlining existing inscriptions; a line drawn to represent a length of five-eighths-of-an-inch; the physical positioning, orientation and alignment of the physical objects being discussed; the use of rhythm in voice and gestures; and voice inflection and changes in volume when speaking. He also adapts his instruction on an ongoing basis throughout the discourse in response to his perceptions of C's needs.

The first episode begins three minutes into the encounter as L and C sit down together at a table in the classroom. L starts by asking C what difference he notices between the region of the measuring tape below 12 inches, where is its marked to thirty-seconds-of- an-inch, and the region above 12 inches, where it is marked only to sixteenths-of-an-inch. While posing this question, L uses a series of gestures to distinguish these two regions of the measuring tape. The first of these is a sweeping gesture with the index finger of his left hand up the measuring tape to the 12 inch point where he pauses briefly to point at the 12 inch point—the boundary between these two regions. He then changes fingers to use the fourth finger of his right hand to point at the 12 inch point before making a second sweeping gesture with this finger up the measuring tape from 12 inches.

C responds using a series of nine gestures within a span of only nine seconds accompanied by a few words to express his thinking to L. Consider first what C says and note that, taken by itself, his spoken expression does not come close to providing a complete response to L's question. He says, "There's, there's more. It's like, it's more spread out when you pass one, one foot. And when you're before one foot its more, um, ...(silence)." In stark contrast to this incomplete verbal description, C's use of different forms of iconicity—gestures reflecting the forms of gesture that L had just used as well as gestures reflecting the physical pattern of markings on the measuring tape—juxtaposed with his use of spoken words do indeed provide a detailed and thorough explanation of the differences in the marking patterns below and above 12 inches.

To start, C sweeps the fourth finger of his left hand up through the first few inches of the measuring tape, in much the same way as L had done, while saying "There's, there's more." C then makes two chopping motions with his left hand aligned with the markings or divisions on the measuring tape in reference to the markings on the measuring tape below 12 inches (see figure 1) and then points to the 12 inch mark with his left fourth finger. Next he says, "It's like, it's more spread out," while replacing his previous indexical gesture with the index finger of his right hand. To this point in the discourse, C's use of contrasting pointing gestures while sweeping through each of the regions of the measuring tape (below and above 12 inches respectively) serves to draw a distinction between them. Furthermore, his use of a chopping gesture positioned over the region of the measuring tape between zero and 12 inches accompanied by the words "there's more" highlights the greater linear density of markings on this part of the measuring tape in comparison with that above 12 inches. C then makes an approximately 2.5" wide interval using his thumb and first finger (see figure 2) and sweeps this up the measuring tape starting with his right thumb at 12" thus indicating the wider intervals between adjacent markings on this region of the measuring tape. C continues by saying "when you pass one, one foot," and grasping the measuring tape on each side in a pinching action at 12 inches again using his right thumb and first finger, then sweeps his hand in this configuration upwards a short distance from 12", and then repeats this action a second time. These gestures serve again to distinguish the corresponding region of the measuring

tape. The episode ends with C making a very brief narrow-interval gesture with the thumb and first finger of his right hand with this hand now positioned above the region on the tape measure between 0" and 12" (see figure 3) while saying "And when you're before one foot its more." This narrow-interval gesture serves to contrast the size of the intervals between adjacent markings on the measuring tape below 12 inches with those just described using a wide interval gesture above 12 inches. This intense use of iconicity serves both to sustain C's attention on and to deepen his experience of the pattern of markings on his measuring tape. (For a more comprehensive analysis of this single episode, including a discussion of the different types of iconicity enacted see LaCroix, 2010a.)

Figure 1. C makes chopping motions with his left hand.

Figure 2. C begins sweeping a wide interval up from twelve inches.

Figure 3. C makes a very brief narrow-interval gesture.

The discourse in this episode continues with L and C discussing the relative sizes of different unit fractions (halves, quarters, eighths, and sixteenths), L drawing C's attention to fractions-of-an-inch as intervals between markings on the measuring tape rather than the marking lines themselves, C pointing to and counting various fraction intervals on the measuring tape in response to direction from L to do so, L drawing C's attention to the relationship between the denominator of a fraction-of-an-inch and the number needed to span one inch, and L drawing C's attention to the relationship between the sizes of the different fractions-of-an-inch sub-units found on the measuring tape using a set of five ruler transparencies that he had prepared earlier for use as a teaching tool. (Each of these ruler transparencies has a different fraction-of-an-inch interval indicated, from whole inches to sixteenths, and they are designed so that they can be superimposed to produce the fraction-of-an-inch pattern of markings found on a measuring tape or ruler).

In the second episode, that occurs 18 minutes into the tutoring session, C seeks clarification from L about reading the markings on his measuring tape by posing the question, "So then, little ones, the little ones, all the little ones (referring here to the smallest intervals on his measuring tape), if I have three sixteenths I would just count. If I have one inch and three sixteenths, I'd just count the little ones, one, two, three." C accompanies the words "all the little ones" at the start of this with two muted up and down motions with his left hand (flexing from his wrist while his arm is resting on the tabletop). He follows this with three chopping motions in the air with his right hand away from the tape measure, in sync with each syllable of the words "three sixteenths" as he says "one and three sixteenths." (See figure 4.) His final set of gestures here are three beats in the air now with the fourth finger of his right hand in unison with each syllable of the numbers "one, two, three" at the end of his utterance as if he was counting intervals on a measuring tape. (See figure 5.)

Figure 4. C begins a series of chopping gestures with his right hand as he says "one and three sixteenths."

Figure 5. C makes three beat motions in the air with the fourth finger of his right hand as he counts "the little ones–one, two, three."

Two things should be noted here. First, C's gestures shown in figures 4 and 5 bear an unmistakeable resemblance to the gestures that L and he had used earlier (in the first episode), thus illustrating the role of these actions (which Radford [2002, 2005, 2008b, 2009] refers to as *semiotic means of objectification*) in the development and stabilization of his thinking about the measuring tape. Second, C's use of spoken words at this point reflects more completely the thinking that he is trying to express while his use of gestures is coordinated with the words spoken. This reflects a deepening of C's objectification of the fractions-of-an-inch pattern on the measuring tape.

The third episode comes one minute later as C poses the following question to L, "Okay. So if it was 32nds then you'd count the little small ones." As he says this, C makes six rhythmic hopping motions with his left hand in the air (flexing from his wrist) as if counting intervals and then a pointing gesture in the air as if to an imaginary measuring tape in front of his left hand. Here again the use of words in C's explanations is becoming more prominent while his use of gesture is becoming more subdued, serving now only to emphasize the words he speaks. Following this, C sets for himself a number of fraction-of-an-inch lengths to locate on his measuring tape and seeks L's assistance to verify that he has done each one correctly. L uses this as an opportunity to review the relationship between the denominator of a fraction-of-an-inch, the number required to span one inch again using the set of transparency rulers, and the reliable strategy of verifying the various kinds of fraction intervals on the measuring tape by counting how many span one inch.

The fourth episode comes six minutes after that previous one. In response to a request from L to put his thinking into words, C re-enacts words and gestures from L's explanation of counting intervals to identify interval sizes on the measuring tape as he explains how to identify thirty-seconds- and sixteenths-of-an-inch on the measuring tape. C says, "Each little interval (while bringing the tip of his pen to a point on the measuring tape to illustrate what his he saying, much as L had just done in his explanation of this to C), it's, you said it's thirty-seconds. From, like if you count, if I were to count, every one, every one would be thirty-second." While he refers to counting, C sweeps the tip of his pen over a one-inch interval on tape measure and then sweeps his pen again over this interval touching down on a few successive markings in rapid succession during the sweep as he says "every one." C ends his explanation by saying, "And then you said the next one would be every sixteen," while pointing to a number of successive sixteenth-of-an-inch divisions on his measuring tape as he says "every sixteen." Here again, C expresses himself primarily in words. Now, however, his use of gestures serves to complement his efforts to convey his thinking to L in that they are directed precisely towards the particular features of the measuring tape that C is speaking about and are well coordinated with his speech.

In the final episode, which comes two minutes later in the tutorial session (24 minutes after the first example discussed above), C provides a systematic explanation of how to identify each of the different kinds of fractions-of-an-inch that can be read on his measuring tape. While he does this he points precisely with the tip of his pen to each type of interval as he explains them in turn. He says, "Like, like now I understand it because every two [intervals] would be sixteenths, every four would be eighths, every space would be quarters, every two spaces would be halves."

We can see in the series of episodes described here the different aspects of the fractions-of-an-inch pattern that need to be understood and coordinated in order to read the measuring tape reliably and efficiently, L's initial use of multiple semiotic systems in order to draw and sustain C's attention to the customary way of seeing fractions-of-an-inch on the measuring tape, and then C's use of these same forms of semiotic resources in his own thinking and explanations back to L (iconicity). We can also see in C's explanations to L the clear shift in his combined use of gestures and words (semiotic nodes), from the predominant use of iconic gestures—reflecting both the gestures that L had used earlier in his explanations as well as physical markings on the measuring tape with incomplete verbalization of the ideas being conveyed—to the predominance of spoken language with gestures becoming much less pronounced and playing only a supporting role.

Another feature of C and L's interaction throughout the 33 minute tutorial session illustrates the intensity of this encounter most poignantly. Specifically, C repeats or re-enacts what L has just said or done relating to the task at hand on at least 60 occasions.

Concluding remarks

One-on-one interaction using a variety of semiotic resources can play an important, if not essential, role in mathematics learning given the opportunities that it provides for complex and intense interaction between student and teacher/tutor. In the case presented here L plays an active role attending and responding with a variety of semiotic resources to C's thinking as it develops and C responds back using a variety of semiotic resources both to sustain his attention on, and express his thinking about, the mathematics that he is learning. L and C made use of multiple semiotic resources in combination (semiotic nodes) and, in C's case, this changed over the duration of the tutoring session reflecting the development of his thinking about the fraction-of-an-inch pattern on the measuring tape. Furthermore, the recurrence of the particular forms of gestures, that L had introduced to the discourse, in C's subsequent explanations throughout the tutoring session, while diminishing as a primary means for conveying meaning nevertheless indicates their central place in the development of C's thinking about the measuring tape. The findings here also illustrate the mathematical complexity of this common workplace tool and the work involved to unpack the mathematical meaning embedded in it for novice users.

While it is not suggested that these dynamics are limited to mathematics learning within workplace training, this analysis does point to the importance of provisions being in place to respond using a variety of semiotic resources to the particular mathematics learning needs/difficulties of individuals in any setting. This is of particular importance in workplace training programs where it is not uncommon to encounter students who are not strong in mathematics, have limited practical and/or work experience, face challenges with the language of instruction, etc. Furthermore, these insights should be an important consideration in the planning for the "delivery" of workplace training using alternative means such as non-interactive video packages and the internet as is increasingly the case.

Further research is certainly needed to further our understanding of essential requirements and processes for mathematics learning in workplace training. As evident in the preceding pages, the theory of knowledge objectification provides a useful theoretical lens through which to better understand unique features of workplace mathematics practice and learning.

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The activity of a pre-apprentice learning to measure

Lionel LaCroix

This paper examines the activity of a pre-apprentice, with the researcher serving his tutor, as he learns to read fractions-of-an-inch on his measuring tape as needed for his chosen vocation as a plumber. This involves the semiotically and socially mediated process of becoming progressively aware of the pattern of divisions on the measuring tape and the cultural logic behind this, a process that Radford calls objectification.

Drawing upon elements from both cultural-historical activity theory and Radford's cultural-semiotic theory of knowledge objectification, the analysis identifies the pre-apprentice and the tutor's sequence of the learning sub-goals throughout the tutoring session, a wide range of semiotic systems and semiotic means of objectification employed to meet these goals, the mediating role of workplace norms of measurement practice, as well as conflicts and contradictions that serve to motivate this activity.

Recommendations from this analysis are made for teaching mathematics in vocational training generally and for teaching measurement in particular. An example of a particular tool for teaching measurement that was developed directly from the analysis of the actions used within this tutoring session will be provided to illustrate the utility of this type of analysis.

What is the role of mathematics in VET programmes in Sweden and who decides?

Lisbeth Lindberg

This paper focuses on the status of mathematics in Swedish VET programmes within upper secondary education. Policy and ideas of what kind of content students in vocational programmes are to learn have differed over the years, and there have been different "owners" of mathematics in the vocational education, "owners" in the sense of policymakers, trade unions, writers of the syllabus, text book authors and educators.

In the upcoming years there will be a new Curriculum in Sweden with a new organization of the upper secondary school with a stronger separation between the theoretical and the vocational programmes. This also means a new change in the content and goals for mathematics.

In this paper the major ideas of VET mathematics over the years are presented and related to research on these issues. In what respect will the mathematics content differ in the new syllabus of 2011 and what are the reasons for this change?

Developing creative mathematical activities among mathematics teachers

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Independent and creative thinking of students is an integral part of mathematics learning. However, the results of previous research have shown that mathematics teachers do not have enough experience and skills in undertaking creative mathematical activities and consequently they cannot provoke these activities and they cannot organize them while working with students. This paper presents a research project conducted among mathematics teachers. The aim of the project was to improve the teachers' ability to develop creative mathematical activities. In the paper we focus on one creative mathematical activity: discipline of thinking and critical thinking, describing a workshop which was conducted among mathematics teachers. The results show a considerable improvement in the teachers' ability and their attitude towards creative mathematical activities.

Introduction

Considering the huge amount of information which is available from so many sources like internet, television, everyday interactions and transactions, the ability to evaluate them is of great importance. According to Skovmose (1994), mathematics education can contribute to the creation of a critical citizenship and support democratic ideals, although this should not be taken for granted. Thus, the role of training mathematics teachers comes to the fore. One of the mathematics teachers' duties is to educate their students on discipline of thinking (to evaluate your own reasoning) and critical thinking (to evaluate the others' reasoning).

This paper presents a research project conducted among mathematics teachers. The aim of the project was to improve the teachers' ability to develop particular creative mathematical activities, such as: hypotheses formulation and verification; transfer of a method (of reasoning or solutions of the problem onto similar, analogous or general problem); discipline of thinking and critical thinking; problems' prolonging (Klakla, 2002). The results of previous research (e.g., Maj, 2006; Klakla, 2008) have shown that mathematics teachers do not have enough experience and skills in undertaking such activities. For that purpose, diagnostic activities, workshops and lessons' observations were organized.

In the paper we focus on one mathematical activity: discipline of thinking and critical thinking, by describing a workshop which was conducted among mathematics teachers.

Theoretical framework

Discipline of thinking and critical thinking constitute a very important mental mathematical activity. It appears as a component of many different kinds of creative mathematical activities. In the process of problem solving it usually occurs together with other kinds of activities; this

gives the impression that its isolation is artificial. However, it is necessary to characterise its specifications.

Both discipline of thinking and creative thinking are related to the same kind of creative mathematical activity, but they are considered in two different situations: the first is related to evaluating your own reasoning at the moment of its conducting and the second is connected to evaluating a ready reasoning conducted by you or the others (Klakla, 2003). In the first case it is combined with the habit of self-control and the skill of its practicing. The second aspect is connected to the teacher's work but also to the interactions in a learners' community.

We can say that discipline of thinking and creative thinking rely on overcoming the conflict between formal thinking and the other streams of mathematical thinking. These streams may include e.g., an intuition, a strengthened habit or suggestion of a name whose meaning in everyday language is similar – but not the same – with that which is included in a mathematical definition (Klakla, 2003). We can say that overcoming the conflict means the differentiation between the concept definition and the concept image (Vinner, 1991) and the differentiation between the intuition that a hypothesis is true and formal reasoning. We can observe such a conflict e.g., in learning geometry and all situations where the visualisation of mathematical content concretised on a drawing or a diagram plays a significant role.

According to Klakla (2003), discipline of thinking and critical thinking is a skill of:

- proper using a definition in reasoning if it is in a conflict with another factor, e.g., an intuition, a previous experience, a habit, an observed analogy, a model, a representation, a particular case, a term with everyday meaning, etc.;

- evaluating the correctness of your own or the others' reasoning, by paying special attention to the content and the logic;

- active and spontaneous reaction to contradiction or absurdity which appear during reasoning or conclusions.

In our opinion critical thinking in the field of mathematics, which should and can be developed at the mathematics lessons, can be transferred into critical thinking in general. Critical analysing of the logical relations can support critical thinking in real life situations.

Methodology

In this paper we present a piece of a wider research (Maj, 2009) carried among a group of mathematics teachers. We mainly analyse fragments of the workshops, which show the work of the teachers in the direction of discipline of thinking and critical thinking.

A group of seven teachers of mathematics (of gymnasium and high schools[6]) took part in a series of workshops from March to September 2006. The workshops were organised as part of the Professional Development of Teachers Researchers (PDTR) project[7], during the mathematics course. The main content of that course was solving different kind of mathematical problems which were supposed to be challenging for the teachers. The workshops were organised around three multistage tasks (Klakla, 2002). They consisted of

[6] The students are at the age of 13-16 and 16-19.
[7] 226685-CD-1-1-2005–PL-Comenius-C2.1

solving some chosen tasks – open-ended problems and discussing the possibilities of introducing the students to the particular problems. The didactical comments produced were related to the organisation of the work with the students and some reflections on different kinds of mathematical activities which the students had the chance to undertake in the process of solving those tasks.

The main aims of the workshops were:

- developing the skills of undertaking creative mathematical activities among mathematics teachers,
- raising the mathematics teachers' awareness of the need to develop creative mathematical activities among students,
- developing the mathematics teachers' skills of provoking these activities among students,
- presenting a model of working with students.

Our purpose was to influence the development of the teachers' skills in organising situations that – under certain circumstances – can lead to creative mathematical activities which are favourable to be undertaken by the students. The workshops were preceded and followed by some diagnostic activities.

We will present an analysis of two fragments of a single workshop; these fragments are related to the second multistage task called 'Butterflies' (the teachers already had some experience with such work). That analysis was conducted in the direction of answering the following questions:

Was discipline of thinking and critical thinking evident in teachers':

- reasoning?
- way of solving problems?
- verifying hypotheses?

In particular, we wanted to answer the following:

- Did the teachers invoke known theorems and definitions during the verifying of the hypotheses (in case there was such a need)?
- Did the teachers evaluate their own reasoning, were they able to find and correct their errors?
- Did the teachers evaluate the others' reasoning, were they able to react, find and correct the errors?
- Did the teachers have the ability to communicate in precise mathematical language?

The data collected comprised of an audio recording of the workshops (24 lesson hours) and notes. After this, a full transcription of the audio recording and analysis of this transcription was made.

Example of the task

The initial situation of the second multistage task called ‘Butterflies’ was the following:

An acute triangle ABC is given. Through any point P which belongs to the inside of the triangle, three lines parallel to the every side of the triangle are drawn. The lines divide the triangle ABC of the area S into six parts. Three of them are triangles of areas S_1, S_2, S_3. The figure which is a sum of these three triangles with the common vertex P is called ‘butterfly’. The three triangles are called the ‘wings’ of the ‘butterfly’ (Figure 1).

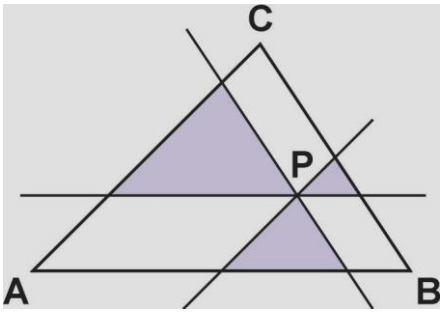

Figure 1. The initial situation of the task ‘Butterfly’.

Without any suggestions on the direction of enquiry the teachers were asked: “What questions would you like to ask to that situation?”.

Results

One of the questions which the teachers asked was if the ‘wings’ of the ‘butterfly’ can be congruent triangles. Then the question was specified: “where should be then the point P?”. Then they found that the point P should be the point of intersection of the medians of the triangle.

We will present an analysis of two fragments of the workshop. The dialogs were realised either between the instructor (I) conducting the workshops and the teachers (T) (fragment 1) or between the teachers (fragment 2).

Fragment 1. The instructor is talking with the teachers T6, T7 and T3:

I: but what would you like to prove? Tell me what you want to prove.

T7: that P is the point of intersection of the medians in the triangle ABC.

I: wait, what is your assumption and what do you want to prove?

T3: if...

I: exactly, what theorem do you want to prove?

T7: that... I mean, I understood that I have to say in what position is..., what is the position of the point P in order for the wings of the butterfly to be congruent to each other.

I: ok.

T6: if the point P is the point of intersection of the medians then the newly-created triangles are congruent.

This part of the dialogue shows the teachers' difficulties in using mathematical language. This is surprising because as teachers they should be able to educate their students to communicate in mathematical language. For the question "what would you like to prove?" the teacher replied only by giving the conclusion without mentioning the assumptions. Only when the second question appeared, the teacher T3 started her sentence with the word 'if'. The instructor wanted to enforce putting the hypothesis in the form 'if…, then…' , but T7 was so involved in her reasoning, that she only explained what was the aim of the search, although she had found the solution before (in the previous talk). Then T6 formulated the correct theorem so the instructor finally got the answer to her question. We can notice the special role of the instructor, whose aim was to develop creative mathematical activities, in which discipline of thinking and critical thinking has significant meaning. Without this interference, the teachers would prove a hypothesis in which the assumption was not clear and they would start solving the next problems. By focusing on the activity itself, the 'determination' of the instructor made them aware of the importance of using precise mathematical language.

Fragment 2. The talk is between three teachers T4, T6 and T1:

T4: but exactly what has to be proved? That point P belongs to the median? No, if these are medians then the ratio is 1 to 2. Is it enough?

T6: No, it's not! For me not, because I would prove after all that this is the half of this side, that this side is divided into halves. It isn't enough, that ratio, for me personally. I know that if it is a median then..., but I don't know that if it is divided (in the ratio 2:1) then it is a median.

T1: Yes, that theorem says that if these are medians then they are divided in the ratio 2 to 1. But it is not vice versa – if they are divided in that ratio then they are medians.

T6: A median for me is a ray which is drawn in a special way, which fulfils a condition, that when it is drawn from a vertex, it divides the opposite side into halves. So if I want to prove that this is a median, I think I should prove that.

T1: But it is a segment! (not a ray).

The talk is between three teachers. T4 was not sure that in order to prove that the segments in a triangle are medians, if it is enough to prove that they intersect in the ratio 2:1. The teachers did not remember exactly the theorem about medians. They concluded that it is safer to prove that the segment divides the opposite side into halves. So, they referred to the definition of the median in a triangle. They tried to specify their talk and to use mathematical language – which was missing in the previous discussions. What is more – they enforced that formal language on themselves. T6 formulated: "I know that if it is a median then ... , but I don't know that if it is divided (in the ratio 2:1) then it is a median". T1 immediately specified:

"that theorem says that if these are medians then they are divided in the ratio 2 to 1. But it is not vice versa – if they are divided in that ratio then they are medians". T6 then quoted the definition of the median of a triangle, but she made a mistake by saying that the median is a ray, which was noticed and corrected by T1.

If the teachers would not use mathematical language this time, the essence of the problem could be missed. Whereas, using that language resulted in all of them talking and thinking about the same thing. The everyday language could be a source of misunderstanding. Precision of the talk helped them understand the problem. This time the situation in which they were – and not the instructor – imposed on them the discipline of thinking and critical thinking.

In Fragment 2 we have observed how mathematically rich the problems which were considered during workshops are and how important is the interaction between the participants. If that problem was solved by one person, such consideration could not appear. When you solve the problem by yourself, you don't use formal language. In that case the teachers worked by stimulating each other and by considering some very important mathematical issues: how to understand a theorem, when to use it, if and when to refer to a definition and a theorem.

Conclusions

By analysing the process of work on the multistage tasks we could identify some evidence of discipline of thinking and critical thinking in the teachers' reasoning, solving problems and communicating. The considered situations favoured the undertaking of that activity; moreover, the teachers acquired successive experience and, in spite of the fact that the process was slow, we could notice the progress in the quality of mathematical thinking.

It can be discerned that there was a hierarchy among different kinds of creative mathematical activities. That hierarchy was related to the order of becoming aware of different areas of creativity in mathematics. The awareness of discipline of thinking and critical thinking proved to be the most difficult compared to the other creative mathematical activities. The skills of undertaking this activity were developed gradually; this was noticeable in the fragments of the dialogs which were presented. The teachers had the occasion to appreciate the value of undertaking that activity, thus they started to do it consciously.

Creative mathematical activity does not develop spontaneously. The conscious attitude of a teacher can be formed through the personal experience of different forms of such activities. Creativity requires conscious didactical methods and tools on every educational level. The multistage tasks used during the workshops support the development of discipline of thinking and critical thinking, but they are not sufficient. The discipline of thinking and critical thinking has to be considered comprehensively, it should be formed in a conscious way in every opportunity. Thus, the skills in that area depend on the teachers' previous knowledge and mathematical skills.

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Numeracy for all College Graduates? Challenges in Designing a College Competency Requirement in Quantitative Reasoning

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In this paper, we will discuss the challenges of designing a numeracy course for college students that would serve their needs and help them meet the university's competency requirement in quantitative reasoning. The challenges surfaced in conversations and debates within the designing committee composed of faculty with diverse mathematical backgrounds, and include the negotiation of (1) the meaning of numeracy, (2) the meaning of a "real-life" problem, (3) teaching methodology and (4) the format of course materials. We will conclude with connecting parts of our discussions to the literature.

In 2010 the Academic Senate of Central Michigan University (CMU) approved a new General Education Program. One of the new requirements is that all students must complete a course in quantitative reasoning for graduation. In this paper, we will discuss the challenges of designing such a course, as they surfaced in conversations and debates within the designing committee, which was composed of faculty with diverse mathematical backgrounds (mathematics, statistics, mathematics education). We will conclude this paper with connecting our discussions to the literature.

Opinions of committee members are not presented in a verbatim form; paraphrases resulting from discussions *a posteriori* are provided. Throughout the text, we use the terms *numeracy* and *quantitative literacy* (QL) interchangeably with quantitative literacy referring mostly to terms and designations that originated in the United States. We use the term quantitative reasoning whenever we refer to the new competency requirement at CMU.

Historical background

For more than two decades, several professional mathematical organizations in the United States have grappled with the question, "What quantitative literacy requirements should be established for all students who receive a bachelor's degree?" (MAA, 1998, Summary, para. 1). One of these organizations, the Mathematical Association of America (MAA), began to address this question in 1989 by establishing a Subcommittee of the Committee on the Undergraduate Program in Mathematics (CUPM) on Quantitative Literacy. This Subcommittee produced four recommendations, which they refer to as "conclusions," in which they recommend quantitative literacy as playing an essential role in every student's undergraduate experience.

In addition to creating and supporting the above-mentioned Subcommittee on Quantitative Literacy, in January of 2004 the MAA also approved the formation of a special interest group on quantitative literacy, which is referred to as SIGMAA QL. The purpose of SIGMAA QL is to assist the mathematics community in determining what prerequisite knowledge is needed

for developing quantitative literacy, finding ways to implement QL into the college curricula, and facilitating colleagues in other disciplines in incorporating QL into their courses as well (SIGMAA QL, 2010).

These national trends inspired many colleges and universities to begin designing their own numeracy programs and graduation requirements. In this spirit, Central Michigan University also began to consider the need for a numeracy requirement for graduation. In the spring of 2005, CMU established its Quantitative Literacy committee within the Mathematics Department.

QL committee in the CMU Mathematics Department

At the time of its establishment, the Quantitative Literacy committee in the Mathematics Department at CMU was composed of faculty with diverse areas of research and teaching – mathematicians with various research interests, statisticians, and mathematics educators. The main goal of the QL committee, at that time, was to learn more about the national trends in numeracy and to explore what numeracy programs were being developed or implemented by other colleges and universities. The ultimate goal for this committee was to make recommendations to the university as to how CMU should proceed with implementing a QL graduation requirement.

Parallel to the creation of the QL committee within the Mathematics Department was the creation of a Steering Committee to Study General Education at CMU. Because General Education at CMU had not been drastically revised since the late 1970's, the university believed that it was time to take a critical look at our General Education Program and make any updates necessary to the program. This Steering Committee consisted of faculty and administrators from all over the university. In November of 2006, the Steering Committee presented their Final Report to CMU's Academic Senate. The Final Report of the Steering Committee recommended including a graduation requirement in which students would need to complete a course in Quantitative Reasoning.

Although it took many years for the new General Education Program to be adopted by CMU, by 2008 it became clear that a new competency requirement in Quantitative Reasoning would be required of all CMU graduates (in addition to the already existing competency requirements in English, speech, and mathematics). As such, the work of the QL committee within the Mathematics Department began to focus on designing a specific course that would serve the students' needs and meet the new competency requirement at CMU.

By this time, there was a general agreement by the QL Committee on a theoretical perspective of what constitutes quantitative literacy and the committee had adopted the SIGMAA QL's definition of quantitative literacy as its working definition: "Quantitative literacy can be described as the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual's private, civic, and work life" (Gillman, 2004).

The committee also agreed on many general principles that should permeate a course in numeracy for college students. These principles were of a diverse nature – some referred to a desirable teaching methodology (active learning, group work, projects, journals, etc.), some specified desirable aspects of the content (context is dominant, mathematics topics should be clustered around specific situations and not the other way around, students must be provided

with realistic problems relevant to their lives, etc.), yet others spoke to the overall course organization (student/instructor ratio, role and format of assessment, etc.).

Challenges in course design

As the focus of the committee shifted from general considerations about numeracy to more specific tasks of course design, more thorough negotiations of differences in the individual members' ideas of numeracy and a numeracy course were necessary. We identified four major areas of conflict: the meaning of numeracy, the meaning of a "real-life" problem, teaching methodology and the format of course materials.

Negotiation of the meaning of numeracy

Although originally there was a general consensus on what numeracy is and what the students should be doing and not doing in the course (as opposed to traditional mathematics classes), the consensus appeared to be more problematic when it came to discussing and designing specific activities. Preferences for content areas ranged from a rather narrow understanding of numeracy as statistical literacy to holistic approaches that extend the use of quantitative reasoning to discussions of environmental issues and issues of social justice, for example.

It also transpired that using a single definition of numeracy did not ensure sharing the same construct of numeracy amongst the committee members. The *interpretation* of the definition turned out to be critical and the differences in interpretations did not surface until specific issues had been discussed.

The numeracy definition that the committee adopted had been interpreted two ways: (1) The student should (first) gain the "ability to adequately use elementary mathematical tools to interpret quantitative data and ideas" that may and will (later) be employed in his/her "private, civic and work life" or (2) We cannot separate this ability to use elementary mathematical tools in any way from the "private, civic and work life" part, in other words, we cannot simply teach or discuss *some* elementary mathematical tools regardless of a context, for it is the context and *only* the context that provides opportunities to develop such ability. It is not an ability to use elementary mathematical tools that the definition talks about; it is the ability to use them in our "private, civic and work life". These different interpretations played out in discussions of teaching methodology, as we will discuss later.

Negotiation of the meaning of a "real-life" problem

The meaning of "everyday mathematics" and a "real-life problem" appeared to cause another unexpected stumble in our process of course design. There was an original consensus that we need to provide the students with "real-life problems", but the interpretations of what a "real-life problem" is started to diverge as the discussion became more and more specific. It was very beneficial to realize that by discussing *everyday math* and *real-life problems*, we, in fact, were referring to two separate issues:

Authenticity: Is this problem authentic? Does it appear exactly in this form in real life or was it modified or simplified to make it mathematically solvable with a "nice" solution?

Relevance: Is this problem relevant to students' lives? Do they really deal with this kind of problem in their lives? Or at least, will the problem be likely relevant later, when the student enters his/her "civic and work life"?

We have realized that our disagreements and inability to find or design a problem that everybody would agree is both *authentic* and *relevant*, had been, at least partially, stemming from the fact that different members viewed these categories (authenticity and relevance) differently. The following example illustrates the issue.

A member of the committee with a statistical background proposed the following question to introduce a classroom activity: *Is hand size a good predictor of person's height?* Undoubtedly, the activity entails valuable quantitative and statistical reasoning by having the students:

- Discuss and negotiate ways in which "hand size" can be represented (What is hand *size*?)
- Deal with and interpret statistical indicators (How can I tell if there is a *relationship* between the two measures?)
- Deal with variance, bias, error (How do I take measurement error into account?)

However, the activity was not unanimously accepted as a good activity for a numeracy course. The committee members who did believe that this was a good problem argued that this problem is relevant because it models the type of thinking that one needs when considering the relationship between two variables. Furthermore, they argued that it was authentic because it utilizes real-life data, such as the measurement of peoples' height and hand size. To the contrary, other committee members claimed that this activity was neither relevant nor authentic. They argued that while the mathematical thought processes used may be important for further mathematics or statistics classes, the problem itself was not relevant to the lives of students. In other words, it is not a problem that they may actually face in everyday life. Moreover, they argued that the problem is not authentic because why would anyone ever need to know if hand size can predict height? For example, if someone were standing in front of you for you to measure his/her hand size, you could just as easily measure his/her height. Such conflicting interpretations of the terms authentic and relevant will be further discussed in the last section.

Negotiation of teaching methodology

While our committee shared the general idea of having a project-based, problem-based course, with units built around contextual, authentic, and relevant real-life problems or situations, there were conflicting views of what would be the appropriate pedagogy and methodology used to teach those units. One of the main pedagogical disagreements among the team members was with respect to the role that expository texts in mathematics should have on our materials.

The objective of teaching for quantitative literacy is, for some committee members, to increase students' ability to use mathematics to interpret and manipulate real-life situations, and therefore our focus should be on students' ability to ask pertinent questions on their own and to locate or retrieve relevant mathematical resources that may lead to solving the problem. For these individuals, a focus on applying previously covered or reviewed mathematical knowledge would not help students to develop numeracy. Moreover, they believed students already know some elementary mathematical concepts that can be used to interpret real-life situations quantitatively, and what they need to develop is the habits of mind of using them. With respect to developing class materials, this would mean that a unit should be opened by

the description of the context or problem to be analyzed. The teacher would act as a mediator and guide the students as they actively and collaboratively work on framing the problem and developing the mathematics necessary to solve it. This view parallels the way in which Niss, Blum, and Galbraith (2007) view mathematical *modeling*, which they describe as taking the direction "reality → mathematics".

Other members of the committee were very uncomfortable with this idea, and argued that it would be impossible or impractical to present a context or situation and expect students to handle it without prior classroom instruction. For them, some form of introduction or review of the mathematics involved would have to take place *before* the students were able to tackle the problem. For these individuals, the mathematics had to be taught, otherwise the students would not know it. The focus was put back on the mathematical concepts which they viewed should be taught directly. This is analogous to the way in which Niss, Blum, and Galbraith (2007) view the process of *applying* mathematics, which they describe as taking the direction "mathematics → reality".

The distinction Niss, Blum, and Galbraith (2007) make between mathematical applications and modeling can be helpful in understanding these two views. For them, in applications of mathematics the focus is on mathematics concepts, and reality or real life is a source of problems in which those concepts can be applied. The authors contrast this to the process of modeling. For them the starting point should be the context, "since modelling requires a context in which to 'frame' the problem and 'develop' the mathematics" (p. 5). They picture the distinction by saying that in modeling we are placed outside the field of mathematics, looking into it, and asking: "Where can I find some mathematics to help me with this problem?" (p. 10). The illustration continues by saying that in applications we are looking outward from within the field of mathematics, and asking: "Where can I use this particular piece of mathematical knowledge?" (p. 11)

Negotiation of the format of the course materials

As part of planning for the course, the committee needed to decide on what sort of student materials we would adopt for the course. The textbook options available and differences of opinions on what makes a good textbook for numeracy courses, however, disenabled the committee to arrive at a consensus.

As a first approach it seemed natural for us to review any textbook written specifically for quantitative literacy courses, or any other texts used in similar courses elsewhere. The committee reviewed the few textbooks we could find that explicitly mentioned quantitative reasoning or quantitative literacy in the title. We also reviewed other books suggested by representatives of publishing companies that they thought might be useful for the course.

The majority of textbooks employed the traditional format of a mathematics textbook, in which chapters were arranged around different mathematical topics; the lessons started with an exposition of a mathematical concept and then "application problems" followed.

Some committee members felt that these textbooks would undermine the philosophy of our course and would cause it to end up being a traditional mathematics course, and not one with the main goal of improving students' numeracy. Specifically, the committee had agreed that our units would not be set around mathematical topics (unit on probability, unit on geometry, etc.), but around selected real-life situations (reading a newspaper, buying a car, remodeling a room, etc.), which could involve mathematics from diverse conceptual fields.

Another problem the committee encountered with respect to these textbooks was that while we might find an interesting unit or chapter in one book, we would dislike the rest of the chapters. At this point the committee considered creating a custom book, something that is done by some publishers and consists of a compilation of materials of different texts, copyright issues regarded.

This approach was also unfruitful. Some committee members felt that the lack of consistency with regards to the formatting of the chapters was a problem. Others still had problems finding chapters that presented the material in a manner that they desired.

At this point, some committee members proposed that the committee would write our own materials, based on articles collected from the media or other real-life situations. This way our materials would be authentic, relevant, project-based, and up-to-date. Furthermore, we could then present the material in a way that we felt would promote numeracy.

This view was strongly opposed by other members of the committee, who thought it would be impractical or even unfeasible to compile our own course text and keep it always up-to-date, relevant, and authentic. They also thought that using a published textbook would provide necessary structure for the instructor and students. Yet others argued that the use made of the textbook by the instructor was what was going to characterize our course as a numeracy course and not a mathematics course.

We should point out that this difficulty in translating the philosophy of numeracy into a textbook is mirrored by the very fact that textbooks that promote themselves to be QL texts still generally provide the traditional mathematics textbook format. We will use as an example the custom edition of "Case Studies for Quantitative Reasoning: A Casebook of Media Articles" by B. L. Madison and S. Dingman, two influential names in numeracy debates in the U.S.

The book is introduced by a "Course Philosophy" section that provides a description of course characteristics. In this section, the authors clearly state how they view the pedagogy of a quantitative literacy course as being different from that of a mathematics course: "Pedagogy is changed from presenting abstract (finished) mathematics and then applying the mathematics to developing or calling up the mathematics after looking at contextual problems first" (Madison & Dingman, 2008, p.3)

However, when the committee viewed the development of particular units within this textbook, we found that the textbook did not appear to uphold this view. For example, each unit typically began with a mathematical exposition, then included traditional mathematics exercises, and concluded with a newspaper clipping related to the mathematical topic. In this format, it appears as if the finished mathematics is taught first and the contextual problems are provided at the end of the unit so the students can apply the mathematics that they just learned. Such a format models the pedagogy of a traditional mathematics class, which we believe to be contradictory to the statement made by the authors in the "Course Philosophy" section of their book.

Theoretical considerations

The above discussion describes challenges of designing a numeracy course by a group of faculty with various research backgrounds and interests. The arduous negotiation processes within the committee echoed many of the issues of numeracy education that appear in the

literature. We will briefly discuss some of these connections without aspirations to provide a complete theoretical framework.

The problem with inconsistent interpretations of a single definition of numeracy exemplifies the evasiveness of the numeracy concept. As Coben noted, "[t]here is no shortage of definitions but there is, crucially, a shortage of consensus" (as cited in Kaye, 2002, Diana Coben section, para. 1). Although seemingly unproblematic when discussed in general terms, numeracy and the goal of educating a numerate citizen get much more complicated and divisive when a specific path of achieving such a goal is debated.

Our next concern was the negotiation of which problems were authentic and relevant to students' lives. Contextualization of mathematics problems in real life has been problematic over the years. Palm (2006) points out that there has been a movement in mathematics education in many countries towards the inclusion of real-life problems in schools, but that nonetheless many researchers and students complain that the tasks developed as a result of these recommendations lack realism. "The concern is that many of them are not 'real' simulations of out-of-school situations but merely ordinary school mathematics tasks 'dressed up' with an out-of-school figurative context" (p. 42).

This difficulty may stem from the fact that the terms "real", "realistic", and "authentic" may carry different meanings to different people. For example, for the Dutch mathematics education program known as Realistic Mathematics Education, a "realistic" problem is one that the student is able to imagine, to "realize": "the fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves" (Van den Heuvel-Panhuizen, 2005, p. 2). For others, such as Palm (2007), an authentic task is one that describes "a situation from real life outside mathematics and that has occurred or that might very well happen" (p. 203).

If the conceptualization of "realistic" and "authentic" can vary, it is important that we examine the impact that these different conceptions have on students' learning. Palm (2007) points out that it is not enough to say that inauthentic problems have a low impact on students' learning, but that we need empirical evidence to be able to affirm that authentic problems play a significant role in students' learning. He comments that although developing authentic tasks takes a lot of time, effort, and money, many teachers and curriculum developers engage in this work, often motivated by a belief in the positive impact this will have in students' education. He reminds us that others do not share this belief. His point is that basically the decisions in this matter have been made based on assumptions only. He urges us to do more research to clarify the issue: "A more extensive body of research about the consequences of authenticity of mathematical applications is needed so practitioners will have better possibilities to base their decisions about task development on empirical evidence grounded in scientific research" (Palm, 2007, p. 207).

As for relevance, when we choose a problem for the class, we are necessarily making a statement about what we think is relevant for the students' lives. Jablonka (2007) points out that when we choose particular contexts we are advocating a particular life-style that may not be that of the majority of students. But she argues that if students and teachers engage in genuine modeling activities, this will involve not only mathematical understanding "but also contextual knowledge, political awareness and judgments based on values" (p. 197).

In our discussion of teaching methodologies and teaching materials, the issue of the order in which we should organize "delivery" was most controversial: Should we teach the mathematics first, then bring in application problems? Or should we start with a situation and then work with students on developing the mathematics to solve it?

It is interesting to note that this dichotomy is one of those that have been identified by Steen, Turner and Burkhardt (2007) as "contentious issues" when teaching QL. They ask the question "chicken or egg – which comes first?" and argue that:

> Many people believe that skills must precede applications and that once learned, mathematical skills can be applied whenever needed (…). Considerable evidence about the associative nature of learning suggests that the skills-first approach works imperfectly, at best. For many students, skills learned free of context are skills devoid of meaning and utility. (p. 291)

We also saw in the section about teaching methodology that these two views correspond to what has been characterized as the distinction between a "modeling" and an "applications" approach.

Mathematical modeling in its broadest sense is undoubtedly present whenever we deal with numeracy. When we use quantitative reasoning and arguments to solve a problem in real life, we work with our internal representation of the situation (model) that allows us to approach the problem. As such, the ability to model the world around us using mathematical means is an inevitable component of our numeracy.

Modeling has also been adopted by PISA as one of the "cognitive mathematical competencies" needed for what they call mathematical literacy (OECD, 2009, p. 106). In its assessment framework, it explains that the program aims at assessing students' abilities to solve authentic problems from various contexts and for which there is not a "pre-described" solving strategy, because "in real-world problem solving there is no well-known strategy available" (p. 89). In PISA's framework, mathematical literacy involves *identifying* the role of mathematics in real-life situations, making "well-founded judgments", and using the mathematics in "reflective and insight-based ways"(p. 84), processes that are not included in mere applications of given strategies.

One may ask if we should just refer to the modeling literature for the development of numeracy courses and adopt mathematical modeling textbooks for numeracy classes. We should note that courses and textbooks in modeling do not necessarily follow the "modelling" approach pointed out by Niss, Blum, and Galbraith (2007). Often mathematics modeling textbooks use an "applications" approach, teaching in separate chapters some well-known models and then proposing, in a list of exercises at the end of each chapter, situations that can be modeled by the particular model described in that section. Instead of teaching "modelling", one would be teaching "models" (Niss, Blum, and Galbraith, 2007). The student is deprived of the crucial faculty in real-life problem solving, which is choosing mathematical tools to model a problem.

Another question worth asking is which teaching approach would be best to prepare students for the complexities of solving realistic problems. Legé (2007) set out to answer this question with an experimental study. He created two groups, from two different high schools with similar economic and ethnic characteristics. One group was introduced to a mathematical analysis of the problem by a sequence of five activities, each having a description of models

that described distinct aspects of the situation presented. The activities included questions that guided students into comparing and analyzing the models that had been previously given to them. The second group worked on the same problem by actively exploring it. Students were provided with organizers that helped structure their task, but they had to do all the calculations, observations, and decision making. In an open-ended assessment, the second group performed significantly better in four performance goals, including critiquing the models given, varying their key assumptions and comparing the resulting models, and suggesting better sources of information upon which to base predictions. Note that these are abilities and habits that are desirable when confronting real-life situations.

It seems that the problem of negotiation of teaching methodology reflects the degree in which members of the QL committee at Central Michigan University believed in students' active learning. We should point out that one of the big factors present in innumeracy is that adults often see themselves as not capable of solving problems for themselves. Solomon (2009), for example, explores the intensity with which issues of identity and agency are crucial factors in fostering student empowerment and mathematical literacy. She analyzes several studies that suggest that it is possible to develop pedagogies that foster students' agency, and that give them the chance to be the primary "knowers" and "producers of knowledge" in the classroom (Solomon, 2009).

In many aspects the challenges we faced within our committee mirror discussions and debates that appear in the literature. The process of negotiating various aspects of numeracy, a numeracy course, and numeracy education within a small, yet diverse committee, seems to model the exchange of ideas and opinions that have been present in a broad community of researchers and practitioners in the area of adult numeracy and exposes the elusiveness of our goal of educating a numerate citizen and the concept of numeracy itself.

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Enhancing research relationships in adult numeracy between New Zealand and Europe: a beginning

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There is a lack of research in New Zealand into adults learning mathematics. With the support of a UK researcher (Professor Diana Coben[8]), I received funding from the Royal Society of New Zealand (NZ) which allowed me to spend some time based in London investigating UK research expertise and experience in this field. The primary purpose of the funding, to support the development and enhancement of research relationships with other countries, has been fulfilled. Future plans and possibilities for international collaboration in research have arisen.

Introduction

The funding which I received was from the Royal Society of New Zealand (RSNZ) in the Bilateral Research Activities Programme (BRAP)[9]. In the Applicant's Guide (RSNZ, 2009) some of the goals this Programme is designed to support are discussed:

> The primary purpose of this programme is to support the development and enhancement of research relationships with other countries with an emphasis on supporting new activities and relationships. … The programme contributes to enhanced global connectedness by providing the means for NZ researchers or overseas visitors to participate in the processes that precede the establishment of a joint research programme. … Such linkages are expected to bring benefits in terms of greater access to foreign research expertise, technologies and research infrastructure, and the learning that flows from engagement.

Through discussions with key researchers and representatives of relevant organisations I hoped to draw on experience and knowledge of the field, both internationally and specifically in the UK, to enhance research relationships between NZ and the UK. In this way my project addresses the dearth of research in this area in NZ.

This paper is informed by my visit to the UK in the period April-June 2010. The main focus of this paper is adult numeracy in England.

My main stated initial objective was to develop a collaborative research project between New Zealand (myself – the NZ Principal Investigator (PI)) and the UK (Professor Diana Coben – the UK PI) in order to inform the next stage of research and development in this area in NZ. Such collaboration should provide valuable impetus for current and emerging NZ researchers in the neglected area of *adults* learning mathematics in NZ (neglected despite its importance for the individual, the economy and society in general).

[8] I wish to express my thanks to the UK PI of this proposal, Professor Diana Coben, for facilitating this project in the UK. In particular, I wish to thank her for using her good offices to obtain invitations for me to talk with leading practitioners and researchers in the field.

[9] The BRAP Programme was initiated in 1994 but, from 2011, is unfortunately no longer funded.

As a result of this visit to the UK, and with the help of Professor Diana Coben, I am much more familiar with the complex arrangements for the delivery of, and key research in, adult numeracy/mathematics[10] education in the UK. My activities in London were an important part of the preliminary processes which have facilitated our bilateral research project and enabled us to fulfill our main stated objective.

The following two sections focus on initiatives addressing the low skills of some adults. Section 2 gives the policy context which led to the establishment of a range of initiatives, for example the Skills for Life strategy, to improve adults literacy, language and numeracy skills. Section 3 discusses the establishment of the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) to meet the need for further focused research and development to support the Skills for Life strategy.

Section 4 and 5 discuss two organizations that focus on mathematics education, mainly in the compulsory schooling age group, the Advisory Committee on Mathematics Education (ACME) who established the National Centre for Excellence in Teaching Mathematics (NCETM). However, in these organizations, some attention is given to the mathematics education of young adults in the final years of secondary school and in Further Education (FE) i.e., in post-compulsory education. Mathematics education, including adult numeracy, has been assisted by the Science, Technology, Engineering and Mathematics (STEM) Programme, discussed in Section 6, which aims to improve the provision of support for learners and staff in STEM subjects.

Sections 7 and 8 discuss two new initiatives focusing particularly on the mathematical needs of adults. Firstly, the *Maths4 Us* campaign which aims to train Union Learning Representatives to create a cadre of 'maths messengers' to act as advocates for parents, colleagues and workmates. Secondly, *Numeracy Counts*, the National Institute of Adult Continuing Education's (NIACE) inquiry into adult numeracy in England, initiated in June 2010 in London.

Two new surveys of literacy and numeracy skills in adult populations will be discussed in Section 9: the OECD-sponsored Programme for the International Assessment of Adult Competencies (PIAAC) survey and the second Skills for Life survey.

Section 10 notes two new publications launched mid-2010 focusing particularly on the mathematical needs of adults. I attended the launch in May of a new book entitled *Improving Mathematics in the Workplace: The Need for Techno-Mathematical Literacies* (Hoyles, Noss, Kent, & Bakker, 2010). In April, New Philanthropy Capital (NPC) published a report entitled *Count me in: Improving numeracy in England, a guide for charities and funders* (Paterson, Stringer, & Vernon, 2010).

[10] The definition that the NZ and UK PI are working with is the following: 'to be numerate means to be competent, confident, and comfortable with one's judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate and what the answer *means* in relation to the context'

(Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (p. 35). Cresskill, NJ: Hampton Press).

Courses for qualifications for teachers of adult numeracy are taught in many institutions in the UK. Section 11 is informed by my discussions with numeracy lecturers at one such department, LLU+ at London South Bank University.

Adult Literacies in Scotland is the topic of Section 12.

Background - the policy context

The report of a committee chaired by Sir Claus Moser (now Lord Moser) and known as the 'Moser Report', A Fresh Start: Improving literacy and numeracy (DfEE, 1999), set the scene for the field outlined in this report, following the UK's poor showing in the International Adult Literacy Survey (IALS) of 1996 (OECD & Statistics Canada, 2000). The Moser Report signaled the urgent need for improved adult numeracy and literacy education in the UK, defining basic skills as 'the ability to read, write and speak in English and to use mathematics at a level necessary to function at work and in society in general' (DfEE, 1999 p. 1).

As a result of the Moser Report, a range of initiatives was established in each of the four UK home nations (see Appendix B). The largest of these, the Skills for Life strategy to improve adult literacy and numeracy in England, was established in 2001 (DfEE, 2001). Skills for Life introduced: ambitious national targets; an entitlement to learn; guidance, assessment and publicity; better opportunities for learning; quality; a new curriculum based on new national standards; a new system of qualifications; teacher training and improved inspection; the benefits of new technology; planning of delivery (DfES, 2003b). A national survey of the basic skills needs of the adult population undertaken in connection with the Skills for Life strategy found that nearly one in two adults of working age in England (15 million adults) were classified at or below the level expected of an average 11 year old in numeracy (DfES, 2003a).

A report by the Inspectorate in 2003 found that: numeracy is taught less frequently than literacy; there is less demand for numeracy despite equivalent levels of need ('in fact, greater need, according to the Skills for Life Survey', as noted by (Coben, 2006 p. 1); there is a need for greater expertise in teaching numeracy; and 'numeracy is too often taught by rote learning rather than by developing understanding of numerical concepts' (Ofsted & ALI, 2003 p. 14).

Meanwhile, a major report on mathematics education, The Smith Report, was published in 2004. It acknowledges that the adult numeracy strategy is challenging and demanding for teachers and learners alike and states that progress could easily be undermined by uncertainties surrounding the teaching and assessment of mathematics; the limited pool of competent and confident teachers of mathematics and numeracy; and the lack of employer engagement in raising the skill base of new employees (Smith, 2004 para. 4.41).

In 2006, the Leitch Review of the UK's long-term adult skills needs set challenging objectives for 2020, including "95 per cent of adults to achieve the basic skills of functional literacy and numeracy, an increase from levels of 85 per cent literacy and 79 per cent numeracy in 2005" (Leitch, 2006 p. 3).

By June 2008, two years ahead of the Skills for Life target for 2.25m achievements, 5.7 million adults had taken up 12 million Skills for Life learning opportunities with 2,276,000 learners achieving their first Skills for Life qualification in literacy, language or numeracy. However, it appears that the Leitch targets for 2020 will be very challenging to meet (Coben,

O'Hagan, Newmarch, & McDonnell, 2009 (unpublished); House of Commons Public Accounts Committee, 2009).

Against this background[11], the following sections outline some of my visits to and discussions with individuals and organizations involved in research and practice in adult numeracy education in the UK, especially England (see Appendix A for a list of these organizations/groups).

Research and development - National Research and Development Centre for Adult Literacy and Numeracy (NRDC)

The Moser Committee also reported that further focused research was needed so the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) was set up as part of the overall Skills for Life strategy in order to conduct this research and development.

The NRDC was established in 2002 as part of the Skills for Life strategy, the national strategy for improving adult literacy and numeracy skills. The Centre was dedicated to conducting research and development projects to improve literacy, numeracy, language and related skills and knowledge. Details of the NRDC's principles and aims are on the portal (NRDC, 2010). Much of this section has been informed by a discussion with Dr John Vorhaus, NRDC Research Director, and Professor Diana Coben, Professor of Adult Numeracy at King's College London, who was involved in the establishment and management of the NRDC.

NRDC aims and processes

NRDC has always aimed to improve teaching practice and inform government policy through the generation of knowledge, by creating a strong research culture and by developing professional practice. The values, principles and aims of NRDC's activity are set out in full in the strategy document "Strategy 2003-2007: generating knowledge and transforming it into practice" (NRDC, 2003).

The NRDC is committed to ensuring that their research is "both methodologically rigorous and grounded in the needs of learners, practitioners and employers" (NRDC, 2010). Linking research and development enabled the creation of a "strong evidence base, promoting the use of, and engagement in, research by practitioners and policy makers". The staged, cyclical approach used in NRDC's programmes of work ensure that the programmes have a "strong and positive impact, embedding findings and messages in practice and informing the future development of policy" (NRDC, 2010). NRDC prioritises the engagement of practitioners in its work and is focused on the needs and successes of learners.

NRDC research

By 2003 six research projects funded by the NRDC had been completed; by 2010, one hundred and forty eight (148) publications are listed on the NRDC website (NRDC,

[11] My visit coincided with the UK General Election on 6th May which returned a new Conservative-Liberal Democrat coalition government. The *Skills for Life* strategy was established under the previous, Labour, government (1997-2010) and while the strategy and the Leitch targets remain in place at the time of writing, it should be borne in mind that the policy context outlined in this report may change.

2010). The research that NRDC conducts is broken down into five areas of study (or programmes):

- Economic Development, Impact of Basic Skills and Social Inclusion
- Motivating Learners to Succeed - Increasing Participation, Retention and Achievement
- Raising Quality - Effective Teaching and Learning
- Professional Development and the Quality of the Skills for Life Workforce
- The Context, Infrastructure and Impact of Skills for Life on Provision and Learners

(NRDC, 2010)

NRDC funding

The core funding for NRDC was initially given for three years, with the possibility to bid for an extension of the funding for two years. In the event the core funding was extended for two more years, which was subsequently extended for a further year. Hence, core funding extended from 2002 to 2008. During the six years of core funding, the NRDC was a consortium of university and other partners who were specialists in the fields of literacy, numeracy and ESOL. Core funding was augmented by funding from other sources, including the European Social Fund (ESF).

Since 2008 NRDC has been reliant on competitive tender to research councils and charitable bodies, with far fewer studies able to be funded. There is capacity for development activity with very experienced people in the Further Education (FE) sector[12] producing guidance, support, teaching and learning materials. NRDC is still funding some sizeable projects, for example, a 3-year study on the impact of literacy and numeracy training in the armed forces. This project is in its second year.

The lasting legacy of the NRDC is the body of evidence generated. For the future, the question is - how can the body of evidence generated be shared and continually organized? How can practitioners be reached and resources shared?

The Advisory Committee on Mathematics Education (ACME)

Two organizations that focus on mathematics education are discussed in this section and the following section. Their focus extends to the mathematics education of young adults in the final years of secondary school and in Further Education (FE).

This Section is informed by a discussion with Dr Nick Bowes, the Head of Secretariat of ACME. The Advisory Committee on Mathematics Education (ACME) was established in 2002 by the Joint Mathematical Council and the Royal Society, with the explicit support of all (about 30) major mathematics organizations, and is funded by the Gatsby Charitable Foundation. There had been no single Learned Society in England which focused on mathematics education policy issues. One of the key concerns of the UK Labour Government (1997-2010) was mathematics and it was deemed important that this government have one body to have dialogue with on a regular basis.

ACME is an independent committee, based at the Royal Society and operating under its auspices, that aims to influence Government strategy and policies with a view to improving

[12] The Further Education sector consists of post-compulsory education in the tertiary sector outside universities.

the outcomes of mathematics teaching and learning in England and so secure a mathematically enabled population.[13]

ACME has unrivalled access to policy makers and politicians and has a strong track record of influencing mathematics education policy and giving policy advice to government. "Representatives of the (ACME) committee sit on all of the key governmental STEM committees, and perform active roles on other groups specifically established to over see particular areas of policy (such as curriculum changes, new qualifications or teacher training)" (ACME, 2010a p. 1-2). Recent examples of the interaction between ACME and politicians and policy makers, all available to peruse on the website (ACME, 2010d), include:

- Letters to Michael Gove MP and David Laws MP on the Science and Learning Expert Group Report (10 May 2010) and
- Issues with New Funding Arrangements in Further Education (10 May 2010).

ACME is required to focus on ages 5-19, so some of its work impacts on young adults in the post-16 (i.e., post-compulsory) sector. A report published in 2006, 'Mathematics in Further Education Colleges' (ACME, 2006), is a case in point. In the Foreword, Sir Peter Williams states:

> Mathematics teaching in the Further Education sector plays a critical role in the delivery of post-16 education, but evidence suggests that this sector remains relatively unknown to policymakers. In view of this, the Advisory Committee on Mathematics Education (ACME) commissioned a study into mathematics teaching in Further Education colleges, with the aim of identifying best practice in the management and provision of mathematics within institutions.
>
> This report asserts that the government, its agencies and colleges should clearly acknowledge the dual importance of mathematics as an academic subject in its own right and as a subject that underpins vocational disciplines, and ensure that future policies implemented within the Further Education sector do not result in a reduction in the provision of mathematics teaching. With the National Centre for Excellence in the Teaching of Mathematics (NCETM) now launched, the report also recommends the establishment of mathematics-based regional centres to ensure both the development of high quality mathematics teaching and professional training for mathematics teachers in the Further Education sector.
>
> (ACME, 2006 p. 2)

The 2010 annual ACME conference also considered funding issues in FE Colleges (ACME, 2010b). New funding arrangements in FE Colleges seem to be having a detrimental effect on the choice of programme for some students seeking to start on 5 AS-Level subjects. It was agreed that a review of the disparity between the funding of schools and the funding of FE colleges is needed.

Much of ACME's work is focussed on implementing the recommendations of the Smith report (Smith, 2004). Ongoing ACME projects, which may impact on the post-16 sector, are focused on the following areas (ACME, 2010a):

- The Mathematical Needs of the Learners

  This is a 12-month project looking at assessing the mathematical needs of a variety of post-compulsory education routes (e.g. employment, higher education) and reflecting

[13] Details of ACME's activities are available at www.acme-uk.org.

this back on to the current curriculum in order to judge whether the existing system is equipping the nation with the required mathematics. There is possibly a mismatch between what mathematics is being taught and the mathematics that people need.

- Promotion of post-16 mathematical pathways
  The intention is to create a suite of qualifications such that there becomes an expectation on all students to continue with mathematics up to the age of 18. ACME emphasizes, whenever it can, that not doing mathematics post-16 restricts your choices. As recently as early August 2010 ACME have released a position paper *Post-16 in 2016* which contains proposals for 16-19 Mathematics in anticipation of the review of qualifications scheduled for 2013 (ACME, 2010c).

Existing ACME activities which impact on the post-16 sector are:

- Pressing for the introduction of a pair of mathematics GCSEs for all pupils in England, which subsequently has received ministerial support, and is now being piloted from 2010 for 5 years, the longest and biggest pilot that has been undertaken for some time in England.
- The creation of the National Centre for Excellence in the Teaching of Mathematics (NCETM) in 2006, which has reinvigorated continuing professional development (CPD) for mathematics teachers, including those in the post-16 sector. Since its inception, ACME wanted to ensure a high quality, localized infrastructure for the CPD of teachers of mathematics and proposed the establishment of NCETM in 2002, a proposal which was then endorsed by Professor Adrian Smith in his report (Smith, 2004).

The National Centre for Excellence in the Teaching of Mathematics (NCETM)

The National Centre for Excellence in the Teaching of Mathematics (NCETM) aims to support and encourage mathematics-specific Continuing Professional Development (CPD), for all teachers of mathematics. On the NCETM portal[14] there is a wealth of resources and tools to help teachers to realize their full potential. In order for CPD recipients to have more confidence in the providers they choose, NCETM has also recently developed a Standard. Providers who commit to provide CPD to this Standard can also expect more participants.

The NCETM portal has a microsite for Teachers of Mathematics and Numeracy in FE[15], providing material for those teaching mathematics and numeracy in further education. On the microsite, there are not only helpful resources but also ways to discuss successful practice and share experiences with other numeracy teachers. Regular updates are provided by the monthly FE Magazine for post-16 mathematics and numeracy educators[16] as well as information about further professional development resources.

The NCETM has undertaken a major consultation, *Mathematics Matters*, to review and describe the values and practices considered to be most important and effective by the mathematics education community (NCETM, 2008). One outcome of this consultation was

[14] https://www.ncetm.org.uk/
[15] https://www.ncetm.org.uk/numeracy
[16] https://www.ncetm.org.uk/resources/14609

that there was general agreement that teaching should not only value 'fluency in recalling facts and performing skills' but also value other important learning outcomes. These include 'conceptual understanding and interpretations for representations'; 'strategies for investigation and problem solving'; 'awareness of the nature and values of the educational system', and 'appreciation of the power of mathematics in society' (NCETM, 2008 p. 2).

The popular "Thinking Through Mathematics" resource has now been made available online. This resource was originally designed for teachers of Adult Numeracy working with learners from Entry Level to Level 2, but would also be useful to teachers of Mathematics and numeracy working in schools with pupils at Key Stages 1, 2 and 3.

The Science, Technology, Engineering and Mathematics (STEM) Programme

The STEM Programme aims to improve the provision of support for learners and staff in STEM subjects by improving the quality of teaching and learning, providing direct funding across the learning and skills sector for self-improvement activity, and supporting providers to address priority areas covered by the STEM agenda. The National STEM Centre site[17] contains information about what is offered by the STEM Programme, its background, current news and more. The 2004 *Science, Technology, Engineering and Mathematics (STEM) Programme Report* considered ways to "enhance the effectiveness of Government funding in two areas: the flow of qualified people into the STEM workforce and STEM literacy in the population" (DfES and DTI, 2004 p. 4) by delivering STEM support in the most effective way to "every school, college, learning provider and learner" (DfES and DTI, 2004 p. 3).

Settings where this support is delivered include Further Education colleges, work-based learning, prison units adult education and more (LSIS STEM, 2010), hence offering support for mathematics/numeracy teachers and learners in the post-compulsory sector.

In the booklet *Progression through STEM*, it is emphasized that "the FE sector has a key role to play in taking forward the STEM agenda" (LSIS STEM, 2009 p.1)

> The LSIS STEM Programme aims to provide teachers and managers within the further education system with the resources and the opportunities to meet the challenge of improving their performance, unlocking their talent and the talent of those who teach them. These resources and opportunities include:
> - Subject specific teaching and learning resources
> - Tools for senior managers and STEM leaders
> - Summer schools and network meetings for teachers of STEM subjects
> - Funding for action research projects
>
> (LSIS STEM, 2009 p. 11)

One particular set of resources, which has been developed, is called *Mathematical Moments*[18], which offers professional development to teachers of mathematics. Each Mathematical Moment focuses on a particular mathematical topic and offers suggestions for activities to use with learners.

[17] http://www.nationalstemcentre.org.uk/
[18] For access go to http://tlp.excellencegateway.org.uk/tlp/stem/stem-mm.html

Joint campaign of the National Institute of Adult Continuing Education (NIACE) with the National Centre for Teaching Excellence in Mathematics (NCETM) and Unionlearn to support potential numeracy learners and intermediaries to take up and support numeracy learning

A joint campaign[19] of the National Institute of Adult Continuing Education (NIACE) with the National Centre for Teaching Excellence in Mathematics (NCETM) and Unionlearn[20] has been organised to support potential numeracy learners and intermediaries to take up and support numeracy learning. I was invited to attend a meeting with representatives of NIACE (Sue Southward), NCETM (Joan O'Hagan), STEM in NCETM (Norma Honey) and Unionlearn (Judith Swift) who discussed this campaign. The aim is to encourage people to take a second chance to get to grips with maths. The joint campaign is making links with Union Learning Representatives and front line workers in a range of community settings to challenge the culture that 'it's OK to be bad at maths'.

NIACE is focussing on changing attitudes to mathematics in its work this year (NIACE, 2010). NIACE explains the reasons on the portal. Since it is ten years since the Moser Report 'A Fresh Start' alerted the government to the problem, and despite the Government's *Skills for Life* strategy, improving the numeracy skills of adults is a particular challenge. The *Skills for Life* Strategy has been very successful but has failed to engage those with the most significant numeracy requirements. Almost half of the working age population in England has low numeracy skills and a high proportion of adults think they are bad at maths or that it's not relevant to their lives. Joan O'Hagan, an Associate of the NCETM, is quoted as saying that

> People who work out complicated bets in their head often call it commonsense, not maths; they keep the "maths" word for things they can't do. Many of us have a very narrow range of what we call maths but it can be a very creative subject.
>
> (NIACE, 2010)

A national seminar for numeracy-learning specialists - NIACE – June 3, 2010

NIACE has begun a review of numeracy education in England with a national one-day seminar. I was extremely fortunate to be invited to observe at this national seminar entitled "Numeracy Counts for Adults: a review of adult numeracy learning in the UK - *Expert Seminar on Adult Numeracy Learning"*. NIACE expects this seminar, and the subsequent report, will give fresh impetus to efforts to create a numerate society. The necessity for such a review is indicated on the NIACE portal:

> In 2009, the UK Commission for Employment and Skills report, Ambition 2020: World Class Skills and Jobs, indicated that while the literacy target would be achieved by 2020, the numeracy goal would not be reached without 'a step change in numbers over the next decade'.
>
> (NIACE, 2010)

NIACE's national call for evidence asked for views on the following:

[19] A new website has been developed for this joint campaign at http://maths4us.org/.
[20] http://www.unionlearn.org.uk/

- *What the evidence tells us about the current situation regarding numeracy learning*
- *Where the gaps are in current numeracy provision*
- *What the key messages are for numeracy learning.*

(NIACE, 2010)

The seminar provided a mix of listening to experts in the field and round table discussions on the following questions:

- *What would a really numerate society look like?*
- *What kinds of numeracy does our society and UK economy really need?*
- *What actions can we take to support a numeracy-active culture?*
- *What levers can we use to create a culture change around numeracy?*
- *Is weak numeracy mainly an issue among adults whose general employability skills and life chances are low, or is it an issue for everybody to address?*

(NIACE, unpublished p. 2-3)

I include the press release of this event, which appeared on the National Institute of Adult Continuing Education (NIACE) portal in Appendix B. Professor Diana Coben was asked to address this seminar. Her handout is included in Appendix C. She emphasized that we need to shift the focus away from simplistic notions of competence in numeracy expressed in 'can do' lists of tasks, and away from assessing numeracy through de-contextualised mathematical operations (e.g., lists of 'sums'), towards a more holistic notion of competence such as that being developed in her interdisciplinary research on numeracy for nursing (Hutton, et al., 2010) (www.nursingnumeracy.info).

An unpublished interim report discusses some issues and ideas which emerged from the Seminar. Firstly, "we need to develop and articulate a vision for a numerate England", with a "positive attitude and active approach to numeracy/mathematical thinking" (NIACE, unpublished p. 3). The society would then have a deeper understanding of many challenges, for example, global financial systems, climate change, poverty and inequality. Secondly, adopt a more challenging and liberating definition of numeracy[21]. Finally, diversify "delivery models" to address the "concerns about low recruitment to adult numeracy learning opportunities" by funding, for example, "issue-focused learning" and people who support this type of learning. Establish a new organization with a "remit to push numeracy up the agendas of social policy players and to support the development of a numeracy-positive culture" (NIACE, unpublished p. 4).

Two new surveys of adults

Surveys of adults' skills have had huge impacts in a number of countries, often alerting governments to areas of great need. For example, the International Adult Literacy Survey (IALS) results resulted in the Moser Report and then *Skills for Life*. Section 9.1 and 9.2 discuss two new surveys underway in 2010.

[21] 'To be numerate means to be competent, confident, and comfortable with one's judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate and what the answer *means* in relation to the context' (Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult Numeracy Development: Theory, research, practice* (p. 35). Cresskill, NJ: Hampton Press).

The new international adult numeracy survey: the design of PIAAC

Much of this section has been informed by a discussion with Dr Jeff Evans, a member of the PIACC Numeracy Expert Group, and Professor Diana Coben.

Under the auspices of the OECD, a new international comparative study of adults' competencies in basic skills, and in mathematics / numeracy in particular is underway. The new survey is PIAAC (the Programme for the International Assessment of Adult Competencies) and contains three scales - literacy, numeracy, and problem solving in technologically-rich environments (PS/TREs).

> PIAAC aims to follow earlier versions of such surveys, e.g. IALS and ALL, but with some crucial developments, discussed below. The questions have been designed to allow comparisons within countries over time with results from the Adult Literacy and Lifeskills Survey (ALL). It is also hoped that the results can be related to those of PISA, the OECD's assessments for 15 year olds.
> (Evans, Close, & Macquire, forthcoming)

The design of items for the numeracy scale in PIAAC has involved the PIACC Numeracy Expert Group: Iddo Gal (Israel, Chair), Silvia Alatorre (Mexico), Sean Close (Ireland), Jeff Evans (UK), Lene Johansen (Denmark), Terry Maguire (Ireland), Myrna Manly (USA), and Dave Tout (Australia).

The Numeracy Expert Group needed to consider several key issues:

- the way of conceptualising numeracy for this study
- the methods of assessment, including computer administration
- issues of comparability with earlier studies, especially ALL and PISA
- the ways in which PIAAC results are likely to be used to address policy issues
  (Evans, et al., forthcoming).

An overview of the conceptual framework for the assessment of numeracy developed for the OECD's Programme for the International Assessment of Adult Competencies (PIAAC) has recently been published (PIAAC Numeracy Expert Group, 2009). This development phase was under the auspices of the OECD. The numeracy scale was completed in Spring 2009. The field trials of the PIAAC survey are underway now.

Since representative random sampling is subcontracted to the participating countries, there is some concern that appropriate expertise is available in all these countries. Sampling problems can also be exacerbated in surveys of adults since there are no 'captive' populations of adults. The main survey is scheduled for 2011, with results to be available in 2013 / 2014.

The second *Skills for Life* Survey

The House of Commons Public Accounts Committee (Session 2008-09) report *Skills for Life: Progress in Improving Adult Literacy and Numeracy* (House of Commons Public Accounts Committee, 2009) indicates that the Department for Innovation, Universities and Skills (Further Education and Skills) had decided to go ahead with a second *Skills for Life* survey on literacy and numeracy needs. They were considering when and what sort of survey they would undertake. It seems that the second *Skills for Life* Survey is now underway (Coben, 2010), with data collection due to happen in the Autumn, but its present status is unknown.

Teacher Education - Initial Teacher Training at LLU+, London South Bank University

I visited LLU+, London South Bank University, and met with David Kaye and Beth Kelly from the LLU+ Numeracy section, together with Diana Coben. LLU+ is a national consultancy and professional development centre for staff working in the areas of literacy, numeracy, dyslexia, family learning and English for Speakers of Other Languages (ESOL). There are specialists in learning support, language and mathematics, and the application of learning styles approaches to teaching and learning. Within these fields the unit offers consultancy and advice, project development, trainer education, research and development and professional development networks. The unit also has an extensive list of publications and training videos developed through their work. For more than 25 years, they have pioneered work in training and advice in these areas. It is the largest professional organisation of its kind in the UK and a centre of international reputation.

Programmes in adult numeracy offered by LLU+ include:

- **PTLLS (Preparing to teach in the Lifelong Learning Sector): Numeracy focus**
  PTLLS is the preparatory qualification required by teachers, trainers and tutors delivering publicly-funded programmes in the Lifelong Learning Sector (i.e. post-16). It confers the threshold 'Licence to Teach' and can lead on to the Level 5 'Diploma in Teaching in the Lifelong Learning Sector' (DTLLS).

- **CertLL/Additional Diploma (Certificate in Lifelong Learning for Adult Numeracy Specialists (Level 5)**
  This course is for qualified, experienced teachers, who wish to qualify as numeracy specialists. It leads to a qualification meeting the new Subject Specialist requirements and is accredited by London South Bank University.

- **DTLLS (Diploma in Teaching in the Lifelong Learning Sector): Numeracy Focus**
  This 2-year course teaches the knowledge and skills required of an adult Numeracy teacher. Course topics cover: teaching and learning theory and factors affecting learning; the national context and the place of numeracy in life, approaches to teaching, learning and assessment, session planning skills and delivering inclusive sessions; practical teaching skills, effective communication skills relating to numeracy and professional development including job search skills

Some examples of "Training for the Workplace" courses offered by LLU+:

- MANAGERS – "Integrating Language, Literacy & Numeracy (LLN) Development in the Workplace"
- TEACHERS – "Integrating Language, Literacy & Numeracy (LLN) Development in the Workplace" (A new training programme in response to the Leitch Report and the *Skills for Life* Agenda.)
  Each of these courses is a two-day accredited training programme to enable managers/teachers to develop a strategic direction for training in their organisation to raise the skills level of their workforce.

Examples of the one-day training courses offered at LLU+:

> "Getting Your Message Across: Training for Health Professionals", "Strategies to Promote Numeracy in the Workplace" (an interactive session with an opportunity to explore numeracy resources developed by LLU+), or "Analysis of LLN in Job Tasks to Plan Learning in the Workplace".

Techno-mathematical literacies

In May I attended the launch of a new book: 'Improving Mathematics in the Workplace: The Need for Techno-Mathematical Literacies' (Hoyles, et al., 2010). The overarching theme in this book is to highlight a problem concerning "perceptions of mathematics", a "subject most often described in terms of numeracy (arithmetic) and algebra (symbols and equations)" (Hoyles, et al., 2010 p. 183). They argue that

> the major skills problems for workplaces is the understanding of systems, not an ability to calculate or manipulate. Employees need to be able to appreciate models of how IT systems work, or the methodological systems such as those that are used to implement process improvement in workplaces.
>
> (Hoyles, et al., 2010 p. 183)

They argue that there has been a radical shift in the nature of mathematical skills required for work – a shift which has still not been fully recognized by either the formal education system or by employers and managers. The book presents case studies in the manufacturing and financial services sectors.

In April, New Philanthropy Capital (NPC) published a report entitled *Count me in: Improving numeracy in England, a guide for charities and funders* (Paterson, et al., 2010). *Count me in* looks at approaches to dealing with poor numeracy among children and adults in England. The report, aimed at charities and funders, calls for a new National Numeracy Trust to be set up to raise numeracy standards across the whole population.

Adult Literacies in Scotland

This section is informed by discussions with Daniel Sellers, the Adult Literacies Development Officer, Learning and Teaching Scotland, and with Diana Coben.

In 2001 the current strategy for Adult Literacy and Numeracy (ALN) in Scotland was set out in the *Adult Literacy and Numeracy in Scotland* report (Scottish Executive, 2001). Consultation with experts in the field indicated strong agreement that an 'adequate' standard of literacy and numeracy were "skills whose sufficiency may only be judged within a specific social, cultural, economic or political context". In this context, an adequate standard is: "the ability to read, write and use numeracy, to handle information, to express ideas and opinions, to make decisions and solve problems, as family members, workers, citizens and lifelong learners"(Scottish Executive, 2001 p. 10).

A curriculum framework, *An Adult Literacy and Numeracy Curriculum Framework for Scotland* (ALN CF), was published in 2005, building on previous guidance in the field (Adult Literacies in Scotland, 2000). The term 'adult literacies', used in Scotland, encompasses literacy and numeracy reflecting "dynamic and diverse ways in which adults encounter and use words and numbers in their written form" (Adult Literacies in Scotland, 2000).

Daniel Sellars, in his keynote talk at the 2010 Adults Learning Mathematics Conference, indicated that a special feature of the Scottish system is its 'social practice approach' where "effective literacies learning should take account of its social, cultural, economic and political contexts. The emphasis is on how individuals and groups use literacy and numeracy in their everyday lives". This approach is "at odds with the 'dipstick' approach to assessing 'levels' – why would you need to have numeracy before you need to use it?" (Sellers, 2010 p. 3).

Diana Coben was commissioned to review the present position in adult numeracy in Scotland and to give her recommendations on the development of an effective strategy. Coben found that the Scottish "adult literacies framework supports teaching and learning in adult numeracy". However,

> attention to date has focussed more strongly on literacy than numeracy, with the result that adult numeracy is less well-developed, in policy and in practice. Numeracy teaching, learning and tutor training issues have been largely subsumed within literacy.'
>
> (Coben, 2005 p. 6)

Coben adds that her "recommendations are offered in the hope of stimulating debate and shifting the focus so that adult numeracy in Scotland becomes more clearly visible in adult literacies policy and practice" (Coben, 2005 p. 6). Coben recommended:

> there is a need to raise the profile of numeracy within a learner-centred, research-informed approach to literacies that suit adults' needs, rights and purposes for learning. This entails building awareness and developing the capacity both to do and review research amongst practitioners and learners, and to reflect on practice, something already encouraged in adult literacies tutor training in Scotland.
>
> (Coben, 2005 p. 6)

Coben's recommendations have spearheaded a number of national developments in Scotland. Daniel Sellars discussed several in his keynote talk at the 2010 ALM Conference: the development of a practitioner network (real and virtual); action research in the use of ICT in numeracy teaching (Coben, et al., 2007); professional development using many resources developed by NIACE and NCETM; national conferences; links to financial capability; and numeracy for healthcare workers (Sellers, 2010).

Findings of a recent survey have been published in a report, *Scottish Survey of Adult Literacies 2009*. Adapting the IALS survey, which was completed in 1996, almost 2000 Scottish adults were surveyed. Some of the main findings are:

> Literacy skills in Scotland are comparable with many of the world's leading economies. … Three-quarters (73.3%) of the Scottish population have a level of skills that has been recognised internationally as appropriate for a contemporary society. …Stronger skills are associated with many other forms of advantage, such as better paying jobs and living in a less deprived area. … There are strong links between measured literacies scores and educational qualifications, being employed, and the skill level of that employment.
>
> (St. Clair, Tett, & Maclachlan, 2010 p. 3)

Conclusions

As is the case in NZ, issues in adult numeracy are not as effectively addressed in the UK as those in adult literacy. It is therefore particularly important to NZ that this funding has enhanced research relationships between NZ and the UK in the field of adult numeracy, with the possibility of other collaborative international projects coming to fruition in the future.

This funding initiated the research project collaboration of the NZ PI with Professor Diana Coben from King's College London. Out of the discussions the NZ PI had with European researchers during her BRAP-sponsored visit to the UK, there are also other possible collaborative projects being explored. In particular, two research proposals are beginning to take shape.

The first project is a proposed collaboration between three European countries (Norway, Ireland and the UK) together with New Zealand and Australia. The topic is the professional development of teachers and trainers in adult numeracy. The second project is a proposed comparative research project in adult numeracy between at least four countries of similar population size, including NZ. The project is at a preliminary discussion stage of development. These projects would inform the development of any future research-led adult literacy, language and numeracy strategy for NZ, with a particular focus on adult numeracy.

So, the unanticipated achievements of this funded project include the possibility of more international collaborative projects involving NZ in the future, as well as bringing the annual ALM international conference to NZ for the first time in 2012.

This BRAP project has clearly contributed to 'enhanced global connectedness', is of 'definable benefit to NZ' and 'has met the objectives of the Programme' (RSNZ, 2009). It has also enabled me to learn a great deal about the adult numeracy field in England, to share my knowledge and understanding with ALM colleagues and around the world and, I hope, to contribute to raising the profile of adult numeracy in New Zealand.

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Appendix A

Author's Appointments, May-June 2010

I accessed UK research expertise and experience in the field of adults learning mathematics by holding discussions with key researchers and representatives of relevant organisations: National Research and Development Centre for Adult Literacy and Numeracy (NRDC) http://www.nrdc.org.uk/; Advisory Committee on Mathematics Education (ACME), Royal Society, London http://www.acme-uk.org/; Learning and Teaching Scotland, Glasgow, Scotland; Lifelong Learning, Institute of Technology (ITT), Tallaght, Dublin, Ireland; BBC Skillswise; VOX Norwegian Agency for Lifelong Learning, Norway http://www.vox.no/templates/CommonPage.aspx?id=2598&epslanguage=NO; LLU+, a professional development centre, London South Bank University, http://www1.lsbu.ac.uk/lluplus/ourexpertise/numeracy.shtml; PIAAC (Programme for the International Assessment of Adult Competencies) Numeracy Expert Group,

Invited as an observer to a meeting held to organise the launch of the **Maths4 Us** campaign http://maths4us.org/ , I met representatives of the National Institute of Adult Continuing Education (NIACE), the National Centre for Excellence in the Teaching of Mathematics (NCETM) and the Trades Union Congress (TUC) Unionlearn. This campaign aims to train Union Learning Representatives to combat the poor numeracy skills blighting more than a fifth of the adult population and create a cadre of 'maths messengers' to act as advocates for parents, colleagues and workmates www.Maths4us.org.

Invited as an observer at the NIACE Expert Seminar, held on Thursday 3 June, I met experts in the adult numeracy field. This one-day, by invitation, seminar was held to initiate *Numeracy Counts*, the NIACE inquiry into adult numeracy in England http://www.niace.org.uk/news/contribute-to-a-national-review-of-adult-numeracy

Attending a book launch allowed me to meet several authors of a new book launched in May 2010 entitled *Improving Mathematics in the Workplace: The Need for Techno-Mathematical Literacies* (Hoyles, Noss, Kent, & Bakker, 2010).

At two conferences I was also able to talk with a number of researchers in the field: Founding Conference of National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL) at the University of Limerick, Ireland, http://www.nce-mstl.ie/; ALM 17, 17th Annual Conference of Adults Learning Mathematics – A Research Forum, 'Maths at Work - mathematics in a changing world', 28-30 June 2010, VOX, Oslo, Norway, http://www.alm-online.net/.

I wish to express my thanks to these colleagues for being so willing to meet and talk with me.

Appendix B:

National seminar for numeracy-learning experts Thursday, June 3, 2010 – NIACE News release

Over ten years on from his report to the government on basic skills, A Fresh Start, Lord Moser participated in a NIACE-led, cross-organisational seminar on Thursday 3 June to explore why the progress in literacy and language learning for adults has not been matched in numeracy learning.
Chaired by Dame Mary Marsh, the seminar unanimously agreed that the current situation was unacceptable and, if the new Government's plan for a Big Society are to become a reality, then urgent steps need to be taken to engage more adults in numeracy learning.

The seminar was part of NIACE's national review into numeracy which is assessing the current situation - where about half of the working age population in England have very low numeracy skills - and how to create a more numerate society.

Today's seminar held in London, gave experts in the numeracy-learning field the chance to contribute to the review - *Numeracy Counts for Adults* - and a final report will be published later on this year.
Lord Moser, whose landmark 1999 review of basic skills - *A Fresh Start* - led to the government's Skills for Life strategy, is taking part in the seminar; and Dame Mary Marsh, Director of the Clore Social Leadership Programme, is the Chair.

Lord Moser said:
"Adult numeracy remains one of the most crucial priorities for the government and, because of how it impacts on individuals, families, communities and the whole of society. This review is essential at a crucial time for the economy and the many challenges that lie ahead."

Dame Mary Marsh, said:
"In so many different ways we all have to make sense of numerical information every day. Those people who have never been given the right help to develop sufficient confidence and competence to do this, face many challenges and frustrations. I am pleased to Chair this important review of the action needed to ensure more people can gain these vital skills."

Sue Southwood, NIACE Programme Director for Numeracy, said:
"More than ten years on from the Moser report and despite the impressive successes of the Skills for Life strategy, improving the numeracy skills of adults still creates a particular challenge. While things are certainly moving in the right direction, this review will support key players to create the step change called for in Skills for Growth."

Strong numeracy skills not only benefit individual adults, but they have a positive effect on families, the society and the economy. Recent government research - *Economic Impact of Training and Education in Basic Skills* - illustrates the impact that adult numeracy can have on employment and earnings, stating that:
• adults who have good numeracy skills have higher earnings and better employment chances than adults with lower skills;
• people who improve their numeracy skills between the ages of 21 and 34 are more likely to own their own homes and to have savings, and are less likely to be on benefits; and

• there is evidence that acquiring numeracy skills in adulthood brings earnings and employment benefits, even though these benefits may be slow to come to adults who often have additional barriers to overcome.

Organisations and individuals were also invited to contribute to NIACE's review, by submitting their policy papers, research, and current practice.

Appendix C

Presentation by Professor Diana Coben at the NIACE Numeracy Expert Seminar. London 3rd June, 2010

What do we need to know (and do) to create a numerate society?

If we are to achieve an ethical, equitable, numerate society, and achieve the Leitch targets of 95 per cent of adults to achieve functional numeracy by 2020 (Leitch, 2006, p. 3) we need:

- Dialogue based on mutual respect between policy-makers and policy-implementers, researchers, teacher, intermediaries, learners and potential learners
- A clearer idea of the nature and scope of the nation's numeracy (better national and international survey techniques based on a deeper, more contextualised notion of competence in numeracy)
- A better appreciation of what both strong and weak numeracy mean for the individual and for society
- Good quality accessible numeracy provision geared to citizenship and a full life, including employment, informed by an understanding of the demands and affordances of education in Domains 1 and 2 (Coben, 2002)
- Teachers and intermediaries who are well-prepared, work in well-managed contexts and undertake regular CPD, with decent pay and conditions and career progression, whose work is grounded in research-based principles for effective teaching (Coben, O'Hagan, Newmarch, & McDonnell, 2009 (unpublished))
- An adult numeracy strategy and implementation plan across England
- Mathematics to be 'naturalised' – regarded as a normal part of the culture

A way forward:
We need to shift the focus away from simplistic notions of competence in numeracy expressed in 'can do' lists of tasks, and away from assessing numeracy through de-contextualised mathematical operations (e.g., lists of 'sums'), towards a more holistic notion of competence such as that being developed in my interdisciplinary research on numeracy for nursing (Coben, et al., 2010, www.nursingnumeracy.info).

Gigerenzer *et al* (1999) help us to focus on the requirements, affordances and exigencies of the context in which numeracy is practised. What is required is the capability:

- to see through to the context-specific mathematics required in a situation
- to appreciate the scale and scope of problems
- to produce and evaluate possible solutions

i.e., to make sensible judgements on:

> "***whether*** to use mathematics in a particular situation and if so, ***what*** mathematics to use, ***how*** to do it, what ***degree of accuracy*** is appropriate and what the answer ***means*** in relation to the context" (Coben, 2000, p. 35).

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E-learning to improve numeracy

Zaeed Mohammed

I have carried out a project this year to see whether peer learning via ILT improves numeracy attainment. The project involved 15 learners aged between 16-19. The learners took part in an initial assessment to give me a starting point as to where the learners were on the numeracy scale. It was followed with 6 weeks of the learners recording different techniques and strategies they researched and producing video recordings to help peers understand various numeracy topics. At the end of the academic year, the students will take another initial assessment to verify whether progress has been made via this route of learning.

Numeracy Requirements for Adult Learners Pursuing a Business Degree

Barbara Poole

Numeracy, the ability to think and express oneself effectively in quantitative terms, is essential for success in an undergraduate business program. Finance, typically a core course requirement, demands that students possess the skills to use mathematics not only for solving equations, but also for analyzing business problems and communicating and justifying recommendations. Graduates who become practitioners in a wide range of financial areas including banking, real estate, investment, tax, and insurance, must also apply these skills.

Adult learners engaged in a long-term part-time effort to complete a business degree can be particularly challenged by numeracy requirements. Significant time may have elapsed between completing the prerequisite quantitative courses and applying those skills in a finance course. Educators who emphasize a process of thinking mathematically, in addition to mechanically solving instructor-created equations, may better equip these learners to apply their learning in their real life as well as in their later course work.

The purpose of this paper is to help educators working with adult learners to understand the numeracy expectations in the required undergraduate introductory finance course. The paper describes the quantitative applications in the topic list used by the Educational Testing Service major field test in business, a frequently used assessment tool. As a result of this work, adult educators may be better equipped to link development of their students' mathematical thinking to their academic and career aspirations.

BBC Skillswise: new numeracy resources for the workplace at http://www.bbc.co.uk/skillswise/numbers/

Michael Rumbelow

BBC Skillswise http://www.bbc.co.uk/skillswise/ is a set of free-to-access online resources used by most UK tutors of adult numeracy and literacy classes and many of their students.

The BBC is now starting to work closely with tutors to develop new additional resources, including videoclips, tools, games and printable worksheets which connect numeracy with a range of workplaces, such as catering, car mechanics, the army, retail and nursing.

This presentation is a demo of some of the new resources in development for online and mobile phones and an invitation for feedback from people involved in adult numeracy tuition to help shape the new resources.

Professional Development for Middle-School Teachers: A Summary of Recent Research in the United States

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At ALM-16 the author presented a paper that described the growing interest in and need for professional development for mathematics teachers of students aged 11 to 14 in the United States. During the academic year 2009-2010 a research project was conducted to identify completed projects funded by the United States government that addressed this audience. This paper summarizes the findings of that research as well as earlier work that led to the conception of the project. The mathematics studied in middle school mirrors the content of adult basic and secondary education so the professional development efforts described translate well to adult tutor/teacher training.

Introduction

In the United States education system the upper primary grades, six through eight, are often referred to as "middle school" whether they are taught in a building that is physically separate from the lower grades or housed within the same structure. Students in those grades are usually eleven to fourteen years old. The university mathematics requirements to teach middle school students vary by state but there has been a steady movement across the nation to increase the university mathematics content course requirement for middle school certification.

Funding for research to formulate professional development projects at the middle school level originate from two major sources, the United States Department of Education and the National Science Foundation. Dissemination of project results is uneven and many projects die a quiet death at their completion. A meta-analysis is needed to look at all the results and combine the artifacts into a coherent catalog that can be accessed by researchers implementing pre-service or in-service teacher education. During the academic year 2009-2010, a search was conducted to identify relevant projects from the two agencies and determine which projects had published findings in journals. This paper recounts the history of the project, details of which are provided in earlier Adults Learning Mathematics proceedings.

Teachers as Adult Mathematics Students

The professional development reported in this paper oscillates between the two elementary education delivery systems prevalent in the United States: the kindergarten through secondary system that services children and adolescents and the adult education system that provides K-12 content to adult audiences. In both systems the teacher certification and mathematics

content backgrounds are similar, leading to analogous professional development needs at the middle school level. Findings from one arena inform research and practice in the other.

Viewed from a different perspective, the teachers attending the professional development courses are themselves adults learning mathematics. Therefore, the instructors of such courses have a dual charge. They must not only teach mathematics but do so in a way that conveys "good practice" for the teaching of mathematics. Student content needs must be assessed and respected, activities that promote deep conceptual understanding are imperative, and appropriate assessment of progress are vital. The author knew that many projects had been undertaken to address these criteria. Finding them was the challenge.

Alm-14 – The *Adult Numeracy Initiative*

During the period October, 2005 through April, 2007, the American Institutes for Research (AIR) oversaw a United States Department of Education grant that investigated numeracy education in the United States via three tasks: a literature review, an environmental scan, and an expert panel. Among the research questions posed to the grant team were these three:

> 1. How does adult numeracy develop and how does it differ from the development of quantitative literacy in children?
>
> 2. What instructional practices exist in mathematics education for adult learners that are worthy of replication?
>
> 3. What practices exist in professional development and certification requirements for teachers of adult mathematics education that are worthy of replication?

While the three tasks considered all the research questions, it was the environmental scan that focused on professional development. The recommendations drew heavily on the K-12 literature base and, in particular, the Eisenhower study report conducted in the 1990's by AIR personnel. That original report and the *Adult Numeracy Initiative* environmental scan suggested the following earmarks for effective professional development:

- Substantial Contact Hours Offered in Multiple-Sessions over an Extended Period of Time
- Incorporate a Distance Learning/Internet Component
- Use Instructional Modeling and Good-Practice Demonstrations
- Develop Local Learning Communities
- Implement Standards-based PD
- Integrate with other State Activities
- Couple Expert Model with Train-the-Trainer
- Implement Active Learning (Sherman, et al., 2007)

These criteria would therefore seem to serve as good practice for professional development across systems, traditional or adult education.

ALM-15 – What Mathematics Should Adults Learn?

At ALM-15 in Philadelphia, the author summarized recommendations from various groups advocating mathematical topics that adults should know. A representative list generated from a conference on quantitative literacy suggests:

> Adult mathematical behaviors can be categorized using six major aspects:
>
> - Data representation and interpretation
> - Number and operation sense
> - Measurement
> - Variables and relations
> - Geometric shapes and spatial visualization
> - Chance (Steen, 1997, p. 173)

A rearrangement of these yields the topics: number and operations, algebraic reasoning, geometry, and statistics and probability as areas that should be the foci of adult mathematics instruction and, by extension, the professional development experiences offered to teachers.

ALM-16 - Professional Development for Middle School Teachers: A Growing Adult Student Audience

In the United States there has been a growing strengthening of the credential requirements for elementary school teachers in *middle school*, grades 6 through 8. Historically, the teachers may have studied little mathematics as a requirement for certification and may not have been taught statistics and probability at all. Adult education instructors typically hold the same certification, therefore advancing the mathematics content knowledge of both groups can be viewed as parallel missions.

Certification in the United States is a state responsibility so requirements vary widely. Saint Peter's College is located in New Jersey; therefore its mathematics and education faculties concentrated on New Jersey standards in order to design a middle school certification program to meet the need of local teachers who sought the new endorsement. At the same time, national recommendations were taken into account.

The resulting program required applicants to possess six credits of college level mathematics. The Saint Peter's middle school certificate sequence of courses was Elementary Mathematical Functions and Models for Middle School; Statistics, Probability and Discrete Mathematics for Middle School; and Geometry for Middle School. Courses were offered over sequential trimesters during the 2009-2010 academic year. They were cross-listed as graduate credit for practicing teachers and undergraduate credit for pre-service teachers. The fall and winter offerings were repeated during the summer sessions I and II. This arrangement facilitated the acquisition of the certificate before the beginning of the next academic year.

ALM17 – Professional Development for Middle-School Teachers: A Summary of Recent Research in the United States

The interest in middle school professional development activities is not new and the author was aware that many federally funded projects had been completed in that area. Documentation for these, however, is elusive as much of the dissemination effort occurs at conferences that produce no proceedings. Unlike dissertations, whose abstracts are centralized by Dissertation Abstracts International (DAI), or journal articles that are indexed by databases, there is no central repository for project abstracts.

For this reason the author directed a search of the United States Department of Education and National Science Foundation (NSF) websites during the academic year, 2009-2010. The two agencies award grants in different ways so the searches differed. The Department of Education distributes funds through the states on an annual basis with the potential for renewal. The award history records covered the years 2006 to 2009. The site www.ed.gov was searched state by state using the Boolean search "Mathematics AND Middle School" and "Mathematics AND Middle Grades." Ninety-seven awards were identified, 40 of these from California.

The National Science Foundation projects are awarded through divisions of the NSF, are often multi-year, and funded through institutions, sometimes involving collaboration across institutions and states. The website www.nsf.gov was searched for expired (completed) awards using the search term "Middle School Mathematics." Expired awards were sought because the principal investigators would have filed final reports that might contain copies of artifacts useful to the mathematics teacher educators involved in professional development activities. Seven hundred sixty-five awards were identified. Abstracts that indicated publications had been produced were noted and contact information for the Principal Investigators was saved.

An internet search was conducted to locate the 92 publications noted in the NSF abstracts. Sixty-eight articles were found in electronic format. Six additional articles were obtained through the university Inter-Library Loan (ILL) system. Eighteen articles could not be located. Many of the abstracts produced multiple articles that varied only slightly. Others proved to be tangentially related to the goal of this project. For instance, articles about the use of artificial intelligence to tutor middle school students provided no insight into the professional development focus on this inquiry.

Finally, the Dissertation Abstracts International database was queried to locate university sanctioned research about middle school mathematics teaching that reflected the reforms in mathematics education. Because there is a typical incubation period of several years between the time that a dissertation concept is conceived to the defense and publication of the dissertation, only the DAI abstracts from 2000 to 2010 were searched using the index term/code "Professional Development AND Mathematics AND Education AND Middle School." Four abstracts were returned as a result of this effort.

During the summer of 2010 the findings of this multi-pronged search are being reviewed. Each abstract is being reviewed to determine whether it truly describes a project in professional development for middle school teachers (pre-service or in-service). Published articles are undergoing the same review.

Future Plans - Academic Year 2010-2011

The next phase of this project will take place in the upcoming academic year. All abstracts will be re-evaluated and a spreadsheet of projects deemed useful will be constructed. The Department of Education and National Science Foundation databases, updated each fall, will be re-visited to locate abstracts from projects completed in the past year and these will be reviewed for candidacy for the spreadsheet.

Principal investigators for projects sieved through this distillation process will be contacted and asked to share any artifacts that might be useful to the greater mathematics teacher educator community. These may be interview protocols, assessment tool, or classroom materials for teachers and their students. The results of this phase will be a catalog of materials that can be used in future research and professional development efforts.

Conclusion

There is a long-standing concern about the mathematical content knowledge of middle school teachers in the United States. Over the past twenty years professional organizations dedicated to mathematics education and state credentialing authorities have addressed this predicament by tightening certificate requirements and offering professional development seminars and courses for middle school teachers. The Saint Peter's College response reflects a review of teacher educator efforts across the country as well as the jurisdictional requirements of the State of New Jersey. The current project aspires to inform the national mathematics teacher educator audience about the availability of materials that promote advances in teacher knowledge for the benefit of students in the middle grades.

Acknowledgement

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Attitude, Perception, Reality – Use of Numeracy/Mathematics At Work

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This workshop presented some of the findings over the last few years into mathematics at work and introduced perceptions of the relevance of school mathematics to current workplace tasks from individuals in a variety of workplaces.

The presenter of this workshop referred to research from Hoyles et al. which had studied use of mathematics in workplaces of bank employees, pilots and nurses. All had comparable mathematics entry requirements and the examples used were involved in "error critical activities". The study examined the relationship between practical, professional and mathematical knowledge and how each was used together or separately at work. Visible mathematics was associated with routine activities but idiosyncratic mental strategies were often seen to solve particular problems.

Wedege's study of people's mathematics in working life discussed 2 paradoxes-1)the objective relevance of mathematics in society versus subjectively experienced irrelevance and 2) adult reasons for studying mathematics. The presenter introduced the research from Straesser who looked at mathematics at/for the workplace comparing mathematics in the workplace, articulation of mathematics taught and mathematics used. He highlighted 2 pedagogies 1) Mathematics for the workplace and 2) Modelling Mathematics for the classroom.

Following discussion on these studies a presentation of the author's mini research project was given. Seven individuals from different occupational areas were interviewed about their attitudes and perceptions to mathematics and use of mathematics in their workplaces. (See appendix 1)

Occupational areas were accountancy (Jenny), redundant car factory worker, now after - school club assistant (Andy), IT professional (Robert), Childminder, self employed(Val), Immigration advisory worker(Rachel),Bookkeeper (Alison) Project worker on mental health programme in university (Marion). The author was interested in the mathematical history of the individuals, their current use of and attitudes to mathematics and their perception of relevance of school based mathematics to working and home life.

The responses to the questions at interview are presented verbatim in appendix 2. Out of 7 individuals, 4 had some anxiety level thinking about mathematics, the remaining 3 had positive attitudes with reasonable confidence levels. Memories of school mathematics evoked 4 negative responses. School mathematics levels varied from CSE grade 3 to good GCSE level passes (including those with negative reactions to study).

During the workshop participants were asked to look at similar occupations and discuss what mathematics they perceived as necessary in that occupation. Long lists of mathematical skills

were identified in all those occupations discussed (fitting in with headings used in school and post compulsory level mathematics syllabus/curriculum).

However in the mini project there was a very different response. Jenny, in accountancy responded that only minimum mathematics was applied although everyday is using algorithms, basic number and statistical data. Andy uses mathematics with young people up to age 9 in an after school club using games to introduce mathematical concepts, making mathematics fun. He had considered himself poor at mathematics but attempted to produce positive images of the need for mathematics skills. Robert acknowledged use of mathematical skills and knowledge in different workplaces and in everyday life.

Val had a very positive attitude both to school mathematics, use of mathematics in the workplace and everyday life as well as promulgating a positive approach to learning number with young children. Rachel used mathematics frequently at work but lacked confidence requiring checking mechanisms to confirm her own calculations. Although very competent in her supervisory role, checking other people's calculations she lacked confidence in herself. Alison was very positive and accepted her school mathematics had been useful in both her current occupation and everyday life. Marion presented herself as competent and confident articulating her understanding and use of mathematical language.

There was a wide variance in the responses to the questionnaire but some agreement on limited relevance of school mathematics.

The author then introduced the new "Functional Skills" curriculum currently being piloted in England before introduction in September 2010. The change from some of the previous initiatives in mathematics/numeracy in England includes more emphasis on process rather than product in assessment. The intention of the new programmes being offered in mathematics, IT and English is to develop an ability to use mathematics, or IT or English in everyday life. In Mathematics, it is to understand and make sense of mathematical information, to use and process information, to interpret and analyse the results of the activity and to present the results. These are defined as process skills and apply to all levels. The three headings in assessment are Representing, Analysing and Interpreting. Differentiation of levels refers to complexity of situations assessed. The details are available from http://www.qca.org.uk/qca.

The author has discussed the implementation with individuals from examining boards introducing the programmes. The delivery of programmes means a different approach to teaching emphasising process not just product. Final assessments must be functional situations which can be difficult as the programmes are available for learners aged 14+ where limitations on life experience can limit situations or mean contrived "functional settings".

The final pilot study is not yet complete although implementation is imminent.

During the mini project the author was concerned that there was still an issue in school mathematics teaching and development of the transferability of skills, knowledge and understanding to the workplace.

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APPENDIX 1

Mathematics in the workplace – attitude, perception and reality

1.Your initial reaction to thinking about mathematics/numeracy?

2.Your memories of school mathematics?

3.Highest mathematics/numeracy qualification at school leaving age?

4.Additional mathematics/numeracy qualification post school?

5.Vocational qualification /level/mathematics/numeracy included?

6.Current employment?

7.Do you consider you use mathematics in your workplace?

8.If so examples?

9.Relevance of school based learning in mathematics to your current uses of mathematics?

10 Any perceived gaps in mathematics/numeracy competency?

11.Rating of your individual mathematics/numeracy competency?

12.Your use of mathematics in other sphere of life?

APPENDIX 2

Jenny – accountant

1.Never liked Maths, can't remember anything but cope on a daily basis. I'm no mathematician!

2.Not good, didn't like GCE O Level Maths, just didn't enjoy because not something easy or interesting.

3.O level pass

4.No

5.Started certified accountancy course. Did not see maths relevant to any extent.

6.In accountancy/bookkeeping high level Maths not in it. "Accountancy regime rules." I use calculations, computer programmes (EXCEL).Most I do in my head so don't consider it maths.

7/8No. I do total up in my head but basic maths with 3 or 4 figures I do in my head. 4 figures I do in my head not 10000's I use a calculator.

9.Only school maths relevant add, take away, times and divide, some decimals. I do use formulas but not maths. On my old computer I used "ability" programme-not maths and it was easier than EXCEL.

10.Would like additional accountancy qualification but don't need any more maths.

11.I think my maths is quite poor especially algebra and equations (that's what I see as maths), my basics are OK.

12. I use some maths in supermarket each week. I play a game when I go round ,I mentally add up as I put in trolley and am usually accurate to a couple of pounds. For example if I get something for 45p I round up so 2 at 45p I say about £1 then a £1.20 item I round to £1 and say a chicken for £4.50 I say £5 and lettuce for 50p I say £1.I usually account it to near a £2-£3 over estimate.

Andy – redundant car factory worker now after school club worker

1.Worrying, I struggled at school, not my strongest subject (loved history, geography and science. Maths no!)

2.School memories-I was in a low group but I see schools now concentrating on weaknesses and that would have helped me.

3. CSE grade 3

4.No extra qualifications with maths.

5.Diploma in Youth Community and Playwork had assignments with some basic number and my NVQ 3 in childcare in 2009 I did an initial test in literacy and numeracy.

6.I work in an after school club now after being made redundant and retrained.

7/8.in car industry I worked in the paint shop, spraying and tagging cars to be sprayed. I had some stock control in the "scruff" bay-gloves, bags and sandpaper. No problems with numbers. I did things in my own way and no real time limit.

In my job now I work with kids and play dominoes, cards and other games to encourage using really basic numbers and some adding, taking away, times-ing and dividing. The kids are all school age some of them are aged 9 and some have difficulties already at school with Maths so I try to help. They try to do percentages but struggle.

9.I didn't need it at Rover, some of my mates could not read and write but they didn't need to. In my youth work I need it -mainly basics. I do use CD's to sing tables with the 5-8 year olds.

10.I don't really think there are gaps, I see people who struggle ,some kids are streetwise but they have a better chance than in my day and even then they still struggle. The word Maths is seen as a "naughty" word not like history, it's a frightening word, see a red light!

11.I am not the same as I was at school –I am a parent, I am seen as valuable with school things to my kids. It is important to do modern technology, my 16 year old daughter is way ahead..I need to use pen and paper when working out family finances but I usually get it right.

12.Budgets, time sheets, I like baking so do a lot of measuring. I drive and I think there is some Maths there.

Robert – retired IT professional

1.I see maths as a working tool.

2.I remember school Maths as awful at first where they insisted things must be exact and right. Up to 14 it was a drag but after that I enjoyed it especially logs.

3.O level and additional maths (calculus was in that in the old days)

4.Since school I did a Philosophy degree learnt about Russell and sets; and truth tables and binary systems. I later did a level 4 (35 years later)

5.Degree then later certificate in distribution management and some IT modules

6.Was an IT systems analyst programmer

7/8Did not use maths much (add times take away and divide for checking) .I did programming lots of logic, codifying .Is that Maths? I did use hex in programmes.

9.Relevance of school based maths –bored learnt little that I use now only 4 basic rules.

10.I don't see any gaps

11.I rate myself against peers as quite good.

12.I use percentages in election results for the local political party and I do in family finances.

Other jobs I have done included arithmetic not maths, used calculus once in a real problem in sorting out a temporary outdoor storage area to be fenced while store was being rebuilt. I used calculus and enjoyed it (but needed a book!).

Val – Childminder

1.Reaction to talking about maths –it doesn't phase me at all.

2.Memories of school maths I was always good at it “engaged” is the word now!

3.GCE O level grade 3

4/5/6.I did numeracy 2 years ago via my level 2 vocational qualification. It was easy and I think it helps me with the children as childminder.

7/8 I do use in my work. I am self-employed and do taxes and invoices to parents but I think of it as doing my job not doing maths.

9.The basics were and are useful. I use them a lot. Logs, trigonometry I have never used, if I needed to I would need a refresher.

10.No gaps. Apart from kids in school learning long multiplication and division they do it differently so I need to learn new ways.

11.Rate myself as good.

12.I use Maths in everyday living, family finance not really maths I use maths based activities with children. Low level memory and problem solving activities eg. ask to do something and show different ways of doing it. Questions like “how do I get this out of the jar -turn the lid this way etc. I help develop lateral thinking, rhymes for learning 1-5 and then backwards.

Rachel – Immigration advisory worker

1.Fear-not very good at Maths

2.Maths- when i went to school-or was there! I didn’t mind it. I understood except when I didn’t concentrate. Not good at equations, I couldn’t get my head round them.

3.GCSE grade B but only ‘cos my mum helped me revise before exams.

4.None

5.Qualifications-A levels worked in legal service completed conversion and immigration courses. No maths

6.Case work team leader in immigration advice

7/8Yes i assess eligibility for legal aid-do income calculations, thresholds. I don’t struggle I use a calculator. Other team workers do struggle I have to help.

9.Relevance only the four rules which I use a lot.

10.Gaps –oh! Er! percentages not very good I get confused.

11. Rate myself –not very competent when I do calculations for clients and they are with me I need to use the calculator as I don’t trust my mental calculations.

12. I use in everyday things now- baby bottles (how much milk to sort for each feed each day. I need to budget more carefully for baby and for all my bills

Alison – Bookkeeper

1. Apprehensive
2. School maths I enjoyed but the teacher changed and I went downhill but coped with it.
3. I did GCSE in its first year and got a B or C
4. No
5. I did NNEB and this had core maths
6. Current employment mum and bookkeeper for husband.
7. Yes
8. VAT for invoices, bank reconciliations and records
9. Very relevant for basics especially long division and multiplication but not algebra
10. gaps? Percentages I struggle
11. OK I passed my maths
12. I do SODUKO a lot

Marion – project worker on mental health programmes in university

1.Initial reaction? how much do I remember-probably a lot.

2.Loved it - algebra and geometry and mental arithmetic

3.O level

4.No

5.no

6.Mental health project in university

7/8.No

9.Relevance-see as fairly indirect, I am confident with numbers, calculations ,money, know order of magnitude. Know everyday life how to calculate I think I have a mathematical awareness in general" Never see algebra, it must have some transferable skills but I don't know where. I think maths informs the rest of your life.

10.No

11.Quite good I remember long division!

12.Budget in everyday life and household things.

Exploring the influence of life and school on mathematical problem solving

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The mathematical knowledge embedded in the activities that constitute adults' lives has been researched in a variety of contexts, revealing both the prevalence of situated mathematical knowledge and the difference between mathematical activity outside of school and within school mathematics. In this paper, preliminary findings from a study of adult students' solutions to an open-ended mathematics problem are presented. The data were collected from students enrolled in one of three pre-college level algebra courses offered at an urban community college in the United States. Using a framework developed to analyze students' work on this problem, solutions written by traditional and non-traditional students are compared. Although non-traditional students submitted correct and efficient solutions to the problem at a rate comparable to the non-traditional students surveyed, these adult students more frequently made explicit reference to their out-of-school experience in their solutions. This study is part of a project designed to understand mathematical knowledge that adult students bring to school.

"*Problems in the real world are generally complicated, not in terms of their mathematical demand, but in the relative influence of subjectivity, experience, communication, process and content*" (Boaler, 1993, p. 371).

Introduction

The mathematical knowledge embedded in the activities that constitute adults' lives has been researched in a variety of contexts (e.g., Carraher, Carraher, & Schliemann, 2004; Gainsburg, 2007; Hoyles, Noss, & Pozzi, 2001; Lave, 1977; Martin, LaCroix, & Fownes, 2006; Scribner, 1984) revealing both the prevalence of situated mathematical knowledge and the difference between mathematical activity outside of school and within school mathematics. This suggests that many adult students return to school with mathematical knowledge that is difficult both to qualify and to quantify. This paper presents preliminary findings from a study that set out to explore the influence of an adult's life and experience in school on mathematical problem solving.

Three questions framed the design of the study:

- What ways of thinking do adult students bring to bear on mathematical problem solving?
- Do adult students use qualitatively different solution strategies than those used by more traditional college age students[22]?
- How does school mathematical knowledge influence students' solutions to open-ended mathematics problems situated in a context?

The preliminary findings reported in this paper address the first of these questions.

A Brief Literature Review

Evidence that the same person performs differently in different settings indicates that knowledge is situated (Boaler, 1993; Lave, 1988). This evidence also suggests that adult students who return to school with experience that developed their mathematical abilities may find these abilities irrelevant in the context of school mathematics. Distinctions between mathematical knowledge outside of school and school mathematical knowledge has been described by Masingila, Davidenko & Prus-Wisniowska (1996), and by Resnick (1987). Masingila and her colleagues outline three distinctions between mathematics in and out of school: (i) the goals of the activity; (ii) the conceptual understanding of the person engaged in the activity; and (iii) flexibility in dealing with constraints. Resnick highlights the following distinctions (or, as she describes them, discontinuities): (i) individual cognition in school versus shared cognition outside school; (ii) pure mentation in school versus tool manipulation outside school; (iii) symbol manipulation in school versus contextualized reasoning outside school; and, (iv) generalized learning in school versus situation-specific competencies outside school. Of these distinctions, the current study is interested in exploring the ways in which adult students deal with constraints implicit in problem solving (Masingila et al.'s third distinction) and whether or not adult students undertake contextualized reasoning when presented with problems in school (Resnick's fourth discontinuity).

There is little research on the mathematical problem solving abilities of community college students. Greer, Verschaffel and deCorte (2002) report that community college students, like the school children and pre-service teachers that they also studied, tended to suspend sense-making when presented with the typical word problems of school mathematics. That is, students might provide an answer that involves a fractional part when a whole number response would be more reasonable in the context of the problem. Greer et al. explain this phenomenon in terms of the students' beliefs about mathematics and its relationship to the real world. More recently, as part of a larger study of developmental mathematics at community colleges, Stigler, Givven and Thompson (2010) analyzed students' responses to the questions on one placement test used at many California community colleges. The researchers found that students tended to throw poorly recalled procedures at problems rather than using number sense to reason through a problem. Although these researchers report that older returning students were more successful on one particular problem, their conclusions point to deficiencies in K-12 education rather than looking at the mathematical abilities adult students may bring to school that may not be measured by placement tests. The study described here seeks to explore whether adult students have mathematical problem-solving

[22] Traditional college age students are typically 18-22 years old, although not all students in this age group would be considered traditional college students. For example, a 20-year-old student who has finished military service brings different experiences to the classroom that a 20-year-old student who enrolled in higher education directly after finishing secondary school.

abilities that may be attributable to their experience outside of school mathematics, experiences in which adults neither suspend sense making nor throw procedures at problems.

The research setting

The study was conducted at a large, multi-campus, urban community college. Community colleges are an educational institution found predominantly in the United States, serving purposes similar to institutions of further education in the U.K. Community colleges provide vocational training, as well as the academic foundation necessary for higher education. In addition, community colleges serve the members of the local community by providing classes in a wide variety of subjects, including cooking, software, local geography and physical fitness. Many adult students attend community colleges and these colleges are understood by many as an institution offering adult students a second chance at education. (Cohen & Brawer, 2003).

Methodology

Nearly 500 students from 16 class sections of one of three pre-college level algebra courses completed a survey for this study. These 16 sections were taught by 10 instructors on three campuses of a four-campus community college[23]. The instructors volunteered to administer the survey in their sections, so no claims can be made that the students surveyed are a representative sample of the students enrolled in these three courses, but there is also no reason to expect that this particular sample is biased any way[24]. The surveys were administered during the first week of the term, so that solutions written by students taking their first community college math course could be compared to solutions written by students who had been back in school for one or more terms. However, students taking their first community college mathematics course turned out to be a small fraction of the students enrolled in these courses.

The survey used in the study solicited some brief biographical information and asked students to solve two open-ended problems. Biographical data was collected to identify non-traditional students, although the instrument used posed some difficulties (see Preliminary Results section). The two open-ended problems were adapted by Ching-Chia Ko (Oregon State University) from problems used by Jo Boaler in her study of two high schools in London (Boaler, 1993). These problems were selected so that the community college students' responses might, in a future study, be compared to the responses to these same questions made by more traditional students at a large public university. Although these types of problems are not atypical within school mathematics, they are different from many of the problems students are asked to solve in algebra classes. Also, Boaler (ibid) notes that open-ended problems offer students the opportunity to determine their own goals and that, "[t]his formation of goals means that students bring their own 'context' to a task" (p. 370). Given the goals of the study, open-ended problems seemed particularly well suited.

Students' solutions to each of these two problems were sorted by the author into categories that emerged from the data. Due to the large number of students surveyed, this first sorting

[23] The college also offers these courses via distance learning and at several satellite locations.

[24] The researcher is also a mathematics teacher at the community college. It is certain that some of the instructors volunteered to help a colleague, but two of the instructors were unknown to the researcher. Also, surveys were administered at several locations across the school district and at classes that met both during the day and during evening hours.

classified solutions in terms of the reasonableness of the presented solutions. The categories were further refined during a second sorting, during which reasonable solutions, for example, were categorized in terms of their efficiency. (A description of these categories is presented in the next section.) Although some categorization at the level of the strategies used was developed at this point, the following focuses more on the reasonableness of solutions. Finally, the numbers of traditional college age and non-traditional students whose solutions fall into each of these categories were tallied. (The category of traditional college age and non-traditional students will be described in the next section.)

Preliminary Results

Students' responses to one of the two problems, The Board Cutting Problem, are discussed in this section.

The Board Cutting Problem

A small wood shop has lots of planks of wood in stock, each of which is 20m long. A customer building a cupboard needs the following size planks of wood:

9m, 8m, 7m, 7m, 5m, 5m, 4m, 4m, 4m, 4m, 4m, 4m, 3m, 3m, 3m, 3m, 3m.

Show how the shop could cut the 20m planks into the sizes the customer wants. You should try not to waste any of the 20m planks. Write down all the decisions you make when you are working out your results.

Surveys were collected from 476 students. Of these 476 surveys, 50 were submitted with no response to The Board Cutting Problem. The Board Cutting Problem was on the last page of the survey, so it is difficult to know whether students left this page blank due to time constraints[25] or whether students found the problem too challenging. These non-responses are not included in the following discussion; all percents are computed using a total of 426 surveys.

Students' solutions to The Board Cutting Problem were sorted into one of the five categories shown in Table 1. There are several complete and efficient (in that no wood is wasted) solutions to The Board Cutting Problem. Students were not asked whether or not there is more than one way to solve the problem, so students who submitted a complete and efficient solution tended to just arrive at one solution and stop there. Inefficient strategies recommended using five, and in one case six, 20m boards to fill the customer's order. Most of the students who submitted a solution of this type acknowledged the waste, but either did not take the time to look for an efficient solution or thought an efficient solution not possible. The "off by two meters" solution was interesting. In this case, a student would suggest cutting one board into requested lengths totaling 19m and cutting another board into requested lengths totaling 21m. These students had not made arithmetic errors, but they did not observe how this proposed solution is not possible. Finally, some students totaled the requested lengths, found this total to be 80m and concluded that four 20m boards were needed without

[25] Instructors administered the survey during the last 10-15 minutes of class.

determining how the four boards should be cut. Table 2 shows the percentage of students who submitted solutions of each type.

Table 1. Types of problem solutions submitted.

A complete and efficient solution[26]
Guess and test strategy
Subtraction strategy
Visual model strategy
A complete but inefficient solution
A solution is off by two meters
A solution is obtained from reasoning using only the total number of meters needed
A solution was attempted but not arrived at[27]
Stated "No idea"
Stated "Not enough time"
General strategy described but not executed
Incomplete solution, but student demonstrates understanding of the problem
Minimal attempt showing little understanding

Table 2. Tally of students' submitted solution types to The Board Cutting Problem.

Solution type	**Number (*N* = 426)**	**Percent**
Complete/Efficient	261	61%
Complete/Inefficient	31	7%
"Off by two" difficulty	34	8%
Total Only	32	8%
Attempted but no solution	68	16%

The biographical data from the survey[28] was used to sort the students into one of three categories: traditional college age students, borderline, and non-traditional students. Students classified as traditional college age students checked "Under 18" or "18-25" as their age range and indicated that they have primarily been in school or have not been doing other things like working, raising a family or serving in the military. Some young students are generally considered non-traditional in that they are taking courses at the community college, rather than taking similar courses at a high school, or they dropped out of school and completed a GED. However, these students are considered traditional for this study since they have not been removed from school mathematics for a significant amount of time. Traditional college age students comprised 24% of the sample.

Students were classified as borderline if they fell into the 18-25 year old age range, but indicated that they had been away from school mathematics for 1 to 5 years before taking a

[26] Three different strategies used to arrive at a complete and efficient solution to the problem are listed, although analysis at the level of strategy has not yet been completed.

[27] This category is a catchall category and some sample responses are given to illustrate the category.

[28] The part of the survey with the questions used to classify a student as traditional or non-traditional is included in the appendix.

math class at the community college and/or they indicated that they have non-school experience, such as working, raising a family or serving in the military. A significant drawback to using the survey to distinguish traditional college age students from non-traditional students was an 18-25 year old age range as one choice for participants to select. Traditional college-age students tend to fall into the 18-22 year old range. It is perhaps more accurate to report that the students in this category could not be classified as traditional or non-traditional. These borderline cases comprised 22% of the sample.

The remaining 54% of the students were classified as non-traditional. These students either indicated that they are over 25, have been away from school mathematics for more than 5 years (prior to taking their first math class at the community college, although students may have misinterpreted the question that addressed this) or have significant out-of-school experience (military service, or raising a family). Whether or not a student has been working is, one its own, difficult to interpret; many teenagers in the U.S. hold part-time jobs, but are still supported financially by a parent or guardian.

Table 3 shows the percentage of solution types submitted by each category of student. The reader will note that percentages within each type of solution mirror the breakdown of the population as a whole (traditional, 24%; borderline, 22% and non-traditional, 54%).

Table 3.
Breakdown of selected problem solution types by student category

Complete/Efficient (*N* = 261)		
Traditional	Borderline	Non-Traditional
64 (24%)	49 (19%)	148 (57%)
Complete/Inefficient (*N* = 31)		
Traditional	Borderline	Non-Traditional
6 (19%)	8 (26%)	17 (55%)
Off by two difficulty (*N* = 34)		
Traditional	Borderline	Non-Traditional
10 (29%)	7 (21%)	17 (50%)
Total Only (*N* = 32)		
Traditional	Borderline	Non-Traditional
8 (25%)	8 (25%)	16 (50%)

Table 4 shows the same data from a slightly different perspective. This table lists the percentage of selected solution types submitted by the students in each of the three groups. Here, we see similarity between the types of solution types submitted by traditional college age and non-traditional students.

Table 4.
Breakdown of each student category by selected problem solution types

	Complete/ Efficient	**Complete/ Inefficient**	**Off by two difficulty**	**Total Only**	**Attempted/ No solution**
Traditional (*N* = 102)	64 (63%)	6 (6%)	10 (10%)	8 (8%)	14 (14%)

Non-traditional (*N* = 230)	148 (64%)	17 (7%)	17 (7%)	16 (7%)	32 (14%)
Borderline (*N* = 94)	49 (52%)	8 (9%)	7 (7%)	8 (9%)	22 (23%)

Discussion

The percentage of solution types submitted by non-traditional students is consistent with the percentage of non-traditional students in the sample, indicating that adult students neither dominate nor are they underrepresented in any category of solution type. For example, the data do not suggest that non-traditional students persevered more than traditional-age college students to find a correct and efficient solution, nor do the data suggest that adult students become frustrated and were unable to solve problems any more than their younger counterpoints. This finding does not help answer the first research question – What ways of thinking do adult students bring to bear on mathematical problem solving? – but the finding is, in itself, interesting in that it demonstrates that for a problem like The Board Cutting Problem neither traditional nor non-traditional students are privileged when it comes to solving the problem; adults' experiences outside of school and traditional students more recent experiences with school mathematics did not appear to play a role.

However, when discussing these preliminary results with the researcher who used these two problems in a related study at a public university, she commented that none of the students surveyed at the public university mentioned their out-of-school experience in their responses to The Board Cutting Problem (C. Ko, personal communication, May 18, 2010). Yet several community college students acknowledged that their solution did or did not account for the loss due to sawing the wood or they commented that their work experience made a solution easy. For example, one student wrote, "The customer should buy 4 20m planks of wood. Unles [sic] he uses a ¼" saw in witch [sic] case he could loose [sic] up to an inch and a half of wood. So he might want an extra plank, just in case." These comments are denoted *real-world tags* and these appeared to be a feature of the work submitted by the community college students, but not the traditional college age university students. Table 4 shows that the majority of these real-world tags were made by non-traditional students, but it is also noted that the number of *real-world tags* is small compared to the size of the sample.

Table 4. Real-world tags

Real-world tags for all solution types (*N* = 13)		
Traditional	Borderline	Non-Traditional
2 (15%)	2 (15%)	9 (70%)

Summary

The preliminary findings presented in this paper start to address the first of the three research questions (What ways of thinking do adult students bring to bear on mathematical problem solving?) in that some non-traditional students explicitly referenced their out-of-school experience when solving a problem like The Board Cutting Problem although the number of

these references were small. Since the preliminary analysis was not done at the level of strategies (several different strategies for finding a complete and efficient solution were noted in Table 1), the current findings do not address the second question (Do adult students use qualitatively different solution strategies than those used by more traditional students?). Yet further analysis of the data may start to answer this question as well. However, it was demonstrated that the adult students are not more or less successful solving an open-ended problem than more traditional students. Finally, it was expected that a larger fraction of the adult students surveyed would be taking their first mathematics class at the community college. The number of students in this category was insignificant. Therefore, it was not reasonable to compare these students' solutions to the solutions submitted by adult students with more recent experience with school mathematics to address the third research question (How does school mathematical knowledge influence students' solutions to open-ended mathematics problems situated in a context?).

Adult students' tendency to include real-world tags in their solution to The Board Cutting Problem suggests that there are experiences that adult students are drawing upon when presented with a problem in the context of school mathematics. This suggests a further question: In what ways and in what situations do adult students use experience from outside the mathematics classroom to help them solve problems in school? Clearly, the data collected in this study are insufficient to answer this question.

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APPENDIX

Part 1 of the survey was used for data collection. Part 2 contained the two open-ended problems, including The Board Cutting Problem.

IF YOU ARE UNCOMFORTABLE ANSWERING ANY QUESTION ON THIS SURVEY, IT IS OKAY TO LEAVE THE QUESTION BLANK.

1. Have you taken a mathematics course at this college or another college before? If so, what course(s) have you taken and when?

2. Before enrolling in your first mathematics course in college, the last time you took a math class in school was

   ☐ more than 20 years ago. ☐ 10-20 years ago. ☐ 5-10 years ago.

   ☐ 1-5 years ago. ☐ less than 1 year ago.

3. Since you were last in school, you have been (select as many choices as apply)

   ☐ working ☐ serving in the military

   ☐ raising a family

   ☐ other _____

4. I am ☐ male/ ☐ female, and I am

   ☐ under 18, ☐ 18-25, ☐ 26-35, ☐ 36-45, ☐ over 46.

The Development Project Dyscalculia

Jens Storm
CSV SydØstfyn

Lena Lindenskov
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In the workshop we will try to illustrate - through a number of activities - how everyday life and opportunities seem to be affected by severe difficulties with numbers and mathematics and how the term dyscalculia is being interpreted.

In the text below, we sum up the Danish development project Experienced Dyscalculia –an explorative study with around 80 adult and adolescent participants.

PROJECT DESCRIPTION

Does dyscalculia exist?
Is there a difference between "having severe difficulties in mathematics" and "suffering from dyscalculia"?
Are dyslexia and dyscalculia connected?
How do persons experience dyscalculia?
Can dyscalculia be tested?
Can persons with dyscalculia learn mathematics?

Our answer to that is maybe...
That was the common starting point for us as members of the working group on dyscalculia. We also all recognized that a group of persons with specific difficulties in mathematics are not adequately benefitting from the established options within preparatory adult education in mathematics (PAE-mathematics) and special education for adults.

Though we are all theoretically and practically working on a daily basis in the education of children, adolescents, and adults with difficulties in mathematics, we had to recognize that a great deal of uncertainty creeps in on the subject of dyscalculia.

That being the case, it may not seem so strange that little scientific research has been done on dyscalculia in Denmark. We were thus moving into an area of as yet unexplored territory when we started on our project one year ago.

The long term goals of this project group are:

- that research should be established to find answers to the above-mentioned questions
- that educational testing material should be developed, which on the one hand can reveal the specific problems of the persons with dyscalculia, and on the other hand, can reveal the learning potential of these individual persons.
- that didactic and educational methods should be developed for persons with dyscalculia.
- that the teaching of mathematics should be generally qualified.

Our immediate contributions to the answers for these questions are:

- establishing a project group
- collecting knowledge and exchange of experience on theory and practice, including "best practice"
- testing existing test material related to dyscalculia/difficulties in mathematics
- screening of a large number of people with possible dyscalculia
- giving information through a professional homepage, a conference, and this paper

A grant from the Danish Ministry of Education of 200.000 DKR made it possible for us to go some of the way towards this pilot project. We hope here to have taken the first steps towards leading to proper research on the subject matter.

Thanks to the numerous test persons who contributed to the project. Without you it wouldn´t have been possible.

Recommendations, views and provisional conclusions

One of the few persons who has worked with dyscalculia in Nordic countries is the neurologist Bjørn Adler. Part of his work has been to develop screening material which can give a hint about adolescents and adults who possibly have difficulties with mathematics (dyscalculia). Very early in the process we decided to use Bjørn Adler´s screening material as a starting point in our investigation into possible dyscalculia.

85 persons have thus participated in our use of Bjørn Adler´s materials for screening. Some of the persons being tested were already "in-house". The rest took contact by themselves when we advertized for persons with possible dyscalculia . We could have tested many more but 85 was the highest number we could manage in this project. 10 of the test persons had other diagnoses. Most of the final 75 test persons have been interviewed as part of the screening and some of them have been exposed to elements from other tests. 10 test persons were examined on a more elaborate group of tests with the use of standardized psychological tests and validated educational tools of assessment. The project therefore provides a large and important body of data material.

During the project we have tried to carry out a state of the art research investigation – in education programs for young people. We sent a questionnaire to all commercial schools, trade schools and Sosu-schools in Funen about students who have been diagnosed as having dyscalculia, dyslexia and facing problems with learning mathematics etc. Several institutions answered that they did not have the information we were asking for.

CSV SydØstfyn, as the body responsible for the project, has been in charge of the practical matters and the coordination of the work of the project. The project group has been running four day-meetings in Nyborg and between the meetings, has communicated by e-mail and a blog on the internet. Moreover we have established the homepage www.dyskalkuli.dk. Another result of the research was the conference about dyscalculia on June 3rd in Nyborg, where approximately 200 interested participants will be informed about the work of the project group and will be encouraged to focus further on the problems of dyscalculia.

As a provisional result of the project we have the following recommendations, views and provisional conclusions:

- Persons who experience dyscalculia have a need for societal recognition and acknowledgement of their handicap.

- Within a society such as in Denmark, where mathematical skills and understanding are necessary, being labeled as "stupid" in mathematics has great human cost.

- There are economic benefits in research-based improvement and soundly-principled reorganization of the education of people facing problems in mathematics. This applies not at least to the basic schools.

- Teachers of mathematics need support if the educational system is to produce fewer students with problems in mathematics. Teacher training and teacher-in-service training should include theory and practice of learning problems in mathematics.

- There is a need for a higher degree of differentiation in the teaching of persons whose difficulties in mathematics are general or specific, respectively.

- Dyscalculia is not an easy matter. Research and further exposure of the subject matter are needed in a Danish context.

- Testing of a large number of persons with possible dyscalculia by the use of available testing material does not give an unambiguous picture of the group. Further investigation is needed.

- Research, as well as evaluation and registration of "best practice", would be an effective way of boosting the topic.

- It seems actually possible to develop progressive educational screening material for immediate use in the education of people who experience dyscalculia.

- This Dyscalculia Development Project is phase one of a large- scale analysis of this field of investigation.

About dyscalculia

The word dyscalculia and the word number-blindness which also sometimes is used in English are contested words. You see uncertainty in the spheres of education and research as well as among people experiencing severe problems with numbers. For some children, adolescents, and adults it is difficult to make sense of numbers and to understand quantities in the real-world and written with symbols. Also, they can have problems with imagining what to do with numbers and quantities, and with actually doing it. They may experience numbers and mathematics as insurmountable obstacles, yet important in education, in work, and in daily living (domestic finance, time and routines etc.) They may experience uncertainty

- whether they should try to come to terms with the difficulties and the consequences: that´s how it is in our family!

- as to how individuals and their family can deal with and relate to the difficulties. For instance, should the family assist with homework or not? And how can you support self-confidence and a feeling of ownership?

- whether more time, or another kind of teaching of mathematics, is needed. Is more time, other aids, other methods of teaching needed? And what is the most important to be learnt?

- whether there are other ways of communication about numbers and mathematics. Who should, for instance, take the initiative, ask questions, narrate, examine, discuss?

- what kind of support is needed and how does it work?

Research has dealt far less with mathematical difficulties and dyscalculia than with reading problems and dyslexia. Uncertainty prevails. In research several schools of thought exist, which means, amongst other things, that estimates of percentage of people who have the problem vary. However, everybody using the term dyscalculia focuses on numbers and arithmetic and all reckon that the problems and potentials that appear can vary from one person to the other. There may be differences in terms of what you are especially good or bad at (profiles of performance) and other distinctive traits may vary. But taken together we must reckon that there are "far more unanswered questions than answers" (D. C. Geary, 2006).

The main schools of thought could be described as follows:

1. Some people think that dyscalculia is caused by a missing or partial lack of a neurobiological number sense. Throughout evolution humans and other species have developed a kind of core capacity to recognize and deal with small numerosities from birth onwards. That is why some people talk about "the mathematical brain", "an innate number sense", and "the brain's number module" (B. Butterworth, 1999; S. Dehaene, 1997) making even the infant able to distinguish between one, two and three objects. Dyscalculia would thus imply that you perceive objects one by one, and have to count 1,2, etc. while other people perceive, for instance, 3 objects as an entity and instantly recognize the number of objects is 3 (substituting, eye-tracking studies, Moeller et al. 2009).

Some people with dyscalculia have no problems learning other school subjects than numbers and mathematics. Then others may have problems in non-mathematical subjects, for example, reading, but it does not mean that these problems cause dyscalculia nor that the basic causes are the same, leading to problems with reading and dyscalculia at the same time. If the neurobiological number sense is lacking or impaired, the child may not be able to pay attention to and gain interest in numbers and quantities – and will not interact and communicate very much with other people about numbers and quantities. Even if you are dyscalculic you can still learn numbers and mathematics but teaching should be individually adapted.

Butterworth has expressed a definition of dyscalculia, which the UK Government Department for Education and Science adopted in The National Numeracy Strategy 2001: "A condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures: Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence."

2. Other people think that dyscalculia should be understood as related to more general cognitive functions, and not to a core biological number sense. This could be semantic memory, working memory, spatial sense. It could be related to attention control or irrelevant associations, or it could be difficulties representing information, and to process that information linguistically.

3. Other people have a functional definition, saying that a person has dyscalculia when he or she – in spite of education and good intelligence – does not learn numbers and arithmetic well and has specific, serious and persistent problems (without reference to the cause). They think the scientific discussions about the existence of the dyscalculia phenomenon and the definition and the specificity are of great theoretical importance, but they see these discussions as not so important for the practical aspects of initiatives dealing with learners with difficulties. In the sphere of education the everyday use of the word is seen as more useful. Here the word dyscalculia means a lack of, or poor, understanding of numbers and calculation skills. In educational institutions one can start by presuming that everybody struggling with numbers has dyscalculia to some extent, whereas the causes are of less interest. (Gross, 2007).

4. Some people think that the term dyscalculia, being something biological, in addition to the uncertainty as far as the definition, testing, frequency, explanations and treatments are concerned, is of no use in the educational world. (Engstrøm, 2009).

5. Others think that the term dyscalculia is too restricted to grasp the present social and individual problems in learning numbers and mathematics, whereas the terms "mathematical difficulties' and 'number holes/mathematical holes' are broader and more adequate.

In the sphere of education you see great uncertainty about the term dyscalculia for many reasons:

- In terms of educational economy it is worrying whether the term itself can be as costly as dyslexia without securing more people to better learn mathematics with increased motivation. On the other hand, there may be societal and individual economic gains if more people get more successful learning experiences.

- One may be concerned from an educational philosophical point of view whether the term increases class division and excludes pupils from school community. On the other hand, this can be an advantage as long as more students will receive an educational environment that makes them feel included – instead of feeling excluded.

- One may be concerned from an educational psychological point of view that the term enhances the idea that "people like us" are incapable of learning numbers and mathematics, and there they give up even trying. On the other hand, the term may be motivating if negative feelings of being lazy, irresponsible and stupid are eliminated successfully.

- One may be concerned seen from a subject didactical perspective whether the term highlights the individual learner`s impediments and deficiencies, and if the term deprioritizes the individual learner`s potential, the social learning of the subject together with others, and the academic creativity and relevance. One may also be

concerned whether the term focuses too much on neurocognition and steals attention from the social aspect, the psychological aspect and the didactical aspect of the learning, and whether the term encourages the use of numbers and arithmetic skills in special teaching as a stop-gap for better mathematics teaching. On the other hand it may be educationally progressive to acknowledge the problems of learners and parents according to their own understanding.

How we carried out the testing

Within the development project on dyscalculia we have screened 85 persons with Bjørn Adler´s mathematic screening. 10 of the tested persons had other diagnoses. A large number of the rest of the people have been interviewed as part of the screening, and some of them have been submitted to elements from other tests. 10 of these persons underwent a more elaborate round of testing, using standardized psychological tests and validated educational assessment tools. There is no doubt the majority of the tested persons in our research that is about 60 persons, have great difficulties in mathematics.

We have experienced that many of our tested persons need even the most basic functions and automatised skills in mathematics. They use a lot of effort in maintaining a single strategy and see no other options. Generally the tested persons spent a lot of time solving the problems – they used fingers, pencils and self-invented strategies to solve the problems. These strategies seem to be very fixed and it appears that the test subjects have difficulties changing them, and finding other ways of solving even the simplest tasks. It seems as if only a very few mathematical skills are learnt.

Through all the testing we have done with Bjørn Adler´s material, almost every tested person appears to have severe difficulties in mathematics. To find out if these difficulties are general, we have tested with extra material for a more precise analysis of the person's cognitive functions. It is found that many of the tested persons have severe difficulties naming numbers such as, for example, 476,893 – they know the individual digits and they can name them but they can not name the number as an entity. The tested persons also have distinct difficulties telling the time from an analogue clock. They are able to name the numbers on the clock but they cannot put the arms on a given time, neither can they tell what time it is. Most of the persons can tell the time from a digital clock. The lack of understanding of a clock causes severe problems in dealing with time. The tested persons experience difficulties in planning and organizing for tasks and jobs.

The Bjørn Adler´s test is not standardized. The test is constructed so that we look at different cognitive functions, such as the value of the numbers, practical everyday tasks, time, the clock, reading numbers, the position of numbers, memory, geometrical figures etc. When we test, the result depends very much on the one who test and on his/her judgement and interpretation. There are only two options, either a 0 (error) or X (acceptable). When, during the project, we became uncertain about the way Bjørn Adler interprets the test result, some of the participants were being tested with parts of other testing material, and fewer of them were called in for extra and more elaborate testing.

The project background

A series of coincidences made CSV SydØstfyn want to look for answers to a number of questions about difficulties people face with mathematics.

Over a period we discovered that some of the people we tested for dyslexia also referred to massive difficulties with mathematics. When we tried to find information so as to be able to prepare goal-directed teaching for these students, we realized that hardly any proper research had been made in this area in Denmark. Therefore, the possibilities of getting the necessary knowledge for the teaching are limited.

We were invited to launch a research process, and the Ministry of Education granted us 200,000 DKR for a preliminary study on dyscalculia. Leading experts in the area agreed to participate in the project, and in August 2009 the development project was launched with a project seminar. We applied various funds for further grants, but the result was negative, so CSV SydØstfyn and the other project institutions have therefore provided the project with the necessary extra means.

The project group

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The Framework for Basic Skills

Margrethe Marstrøm Svensrud

Several studies have shown that there is a large group of adults in Norway whose skills in reading, numeracy and digital competence are insufficient to handle the challenges of contemporary labour markets and social life. Such basic skills constitute a precondition for further learning and competence development. Many adults will therefore require training to strengthen one or more of these skills.

To facilitate adapted training programmes, Vox was assigned the task of elaborating a framework for basic skills for adults, including descriptions of skill levels and competence goals for reading and writing, numeracy, digital competence and oral communication for adults. The framework describes goals for acquired competence. They have been adapted to the life and work situation of adults. The competence goals define the proficiency in the basic skills that adults at various training levels are expected to have. The framework has been developed with a view to adults who need to improve one or more basic skills. By directing attention to the experiences and specific needs of adults, the framework will strengthen the quality and the flexibility of training programmes in basic skills for adults. In this manner the framework could act as an aid to provision of training that is adapted to the life and working situation of adults, in the workplace, in a training centre or in other contexts.

The Basic Competence in Working Life programme

Jan Sørlie

This programme is mainly directed towards improving basic skills among the working population. The lack of functional skills in reading, writing, numeracy and ICT may lead not only to reduced employability but also to absenteeism/illness and early retirement.

The basic idea behind the programme is to train people's basic skills while they are still employed so that they can avoid unemployment. The training should preferably take place where people work, and the content of the training should involve texts and challenges that are linked to their everyday job situations. Avoiding similarities with school situations is also a priority.

Private and public enterprises can apply for funding from the programme. Training providers can also apply on behalf of one or more enterprises. The length of courses and their organisation can vary considerably. There is no formal certification at the end of the courses.

Assessing in-service teachers' modeling activities: Issues of content and complexity

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This paper presents the discussion and the realisation of the assessment of modeling activities designed by 55 in-service teachers participating in a training course. These teachers had no prior experience in modeling and they were initially frustrated once they were faced with the modeling tasks; later on, they engaged in the tasks deeply but in some cases they did not manage to 'move beyond' their instructor's guidance, i.e. design original modeling tasks when they were asked to do so. During that course important issues arose both for the teachers and their instructor: one issue was how the teachers' designed tasks should be assessed. After discussion all participants agreed that two factors are the most crucial: content and complexity.

Models and modeling activities are considered a substantial part of mathematics education, since they create a bridge between mathematics and real life (e.g. Lesh & Doerr, 2003). The use of models is extended in fields that vary from meteorology to finance, thus they are somehow present – or even affect – our everyday lives. The mathematics teacher who wishes to use models in the classroom needs to be aware not only of how to use models as a teaching and learning tool, but also of their importance in her own everyday life – including her own teaching activity. Bearing all these in mind, we organised a training course for in-service primary school teachers. The processes involved in teachers' modelling activities provided a rich source for analysis, reflection and revision of the training. An important part of that process was the assessment of the teachers' work; the in-service teachers were aware of it, thus their participation and their work was influenced by it. Before we present the evolution of the assessment process we will briefly describe the theoretical assumptions underlying it.

Modeling and its assessment in mathematics education

The use of models in mathematics is a relatively recent trend, although modeling is an old mathematical process. In fact, one could characterize even our arithmetical system as a model. Ancient philosophers have introduced mathematical models to explain the movement of many astronomical objects and to predict events like sun and moon eclipses. Nowadays, however, the use of models is much more extended in various and considerably distinct areas. One of the first questions that arise is: What is a model? According to Doerr and English (2003) models are defined as "systems of elements, relations and operations that can be used to describe, explain or predict the behavior of some other familiar systems" (p. 112). Given the importance of the above activities in our modern world, the next question might be: How can the (mathematics) teacher use the modeling approach in her classroom? The so-called model-eliciting activities,

> … present situations where children (a) are confronted with the need to develop a model, (b) clearly recognise the need to revise or refine their current ways of

> thinking about the given problem situation, (c) are challenged to express their understandings in ways that they can test themselves and revise as often as necessary, and (d) develop models that can be shared with others and that can be applied in other problem situations. (English, 2004, p. 208; Lesh & Yoon, 2004)

Thus, solving meaningful problems requires a number of activities, including selecting information and revising initial ideas through discussion, which go beyond the mere 'construction' of knowledge (Lesh et al., 2003). That is why 'traditional' problem solving is seen as a special case of model-eliciting activities (Lesh & Doerr, 2003). Regardless of how one views modeling activities, in any classroom the need for assessment will eventually appear. Lesh et al. (2003) state clearly that usefulness and generalizability are the main factors for judging the appropriateness of a model:

> … useful models (or theories, or conceptual systems) are considered to be those that: (a) begin with assumptions (or "axioms") that are simple and clearly understood, and (b) generate conclusions (or "theorems") that are powerful and not obvious. In particular, the model should be useful to the learner, a user of the model, and a user of the model in a related situation. Usefulness is also determined by the purposes for developing the model. Generalizability is assessed by determining how useful the developed model is for the client in other circumstances that differ from the original given situation. (pp. 225-226)

However, given the complexity of the teaching process, no assessment can be free from revisions, negotiations, problems or even drawbacks. Niss (1993) points out that the assessment of modelling could be problematic; Blum (1993) summarizes the obstacles in assessment from three points of view: (a) the point of view of instruction and assessment, because of the non-standard form of modelling, (b) the student's point of view, because of the more demanding and less predictable nature of modelling and (c) the teacher's point of view, because of the more demanding role of the teacher which is required. Additionally, we cannot ignore the importance of self-assessment and reflection. In this context, when the teacher training course was organized, we had to bear in mind all the above considerations, together with contextual demands related to the teachers' characteristics, as they were gradually expressed in interaction.

Context of the study

Fifty five in-service teachers (35 female and 20 male) had enrolled in the obligatory course named *Didactics of Mathematics*, which is placed in the second – and last – year of their training. In order to participate in the training course they had to meet two requirements: over five years (and less than 25 years) of teaching in schools and passing a national exam. The duration of *Didactics of Mathematics* was one semester (three hours weekly). As with any in-service teacher training course, we had to consider various factors in order to organize and realize the course more effectively. The modeling approach seemed promising – and at the same time challenging – in this aspect, since the particular teachers had no experience of such activities. A recent reform in Greek education, followed by new textbooks in all subjects, had introduced interdisciplinarity and realistic mathematics approach, but these teachers were far from well-informed about these 'novel' approaches. At the same time, they sometimes used their teaching experience to justify claims about their insistence on 'traditional' teaching approaches or to criticize the new textbooks.

The course was organized according to the following scheme: initiation to modeling – modeling – design – assessment. In the initiation to modeling the teachers were introduced to examples of modeling activities from relevant sources. Particularly, they worked on the swimmers' problem (English & Watters, 2005), the car problem (English, 2004) and the Antarctica task from PISA 2000 (OECD, 2002). During these sessions there was also discussion on the solution strategies that emerged, together with suggestions on behalf of the teachers for possible implementation. In all sessions the teachers worked in groups of two to four, according to their preference. Then, the teachers worked on the following tasks:

1. A friend of yours claims that the area of Greece is 40000 square km. Do you think that it is a satisfactory estimation? Justify your opinion.

2. When I entered the post office I got the ticket shown in the image (see Figure 1). The machine also indicated that there are 22 customers waiting and I saw that the customer with the number 398 was the last being served. There were four cashiers operating at that time. The ticket showed that the estimated time of my waiting would be 13 minutes. Write the formula which is used by the machine to estimate the waiting time, given the number of waiting customers and operating cashiers.

Figure 1. Post office ticket.

Finally, the teachers were asked to design a modeling activity that could fit in their own class and comply with all contextual demands including time limitations, students' possibilities to participate and cooperate and their own possibilities to observe and assess. This final task provided the main ground for the discussion on assessment. Surely, as mentioned before, there were ongoing discussions during all sessions; but the fact that the teachers' overall assessment would be mostly based on that design task was enough to trigger their interest and involvement. The outcomes of these discussions together with the results of the assessment are presented in the next section.

Outcomes of discussion and assessment

The discussion on the most important factors that ought to be considered while assessing one's modeling task moved around two topics, content and complexity. This is not to say that other factors were not brought up during talk; for example, classroom management factors, which could be assessed by the following questions:

Does the task 'fit' into an hourly lesson? If not, can it be used as an interdisciplinary task?

How are the students expected to work? Does the task encourage group work?

Is the teacher's role clear, consistent and feasible?

Coming back to the main factors, the discussion on content and complexity moved around some remarks and related questions. Particularly, the content of the task has to be:

- *realistic*: a basic question arises: is the task supposed to represent reality or does it constitutes *a* reality by itself? In the first case, one has to bear in mind that it is impossible to include all parameters of a real situation or problem, thus the representation is incomplete from its very beginning. This is not a problem, as long as the designer is aware of it, and thus does not try to include as many contextual elements as possible. In the second case, one has to consider that the situation described does not try to represent reality, thus the designer has the freedom to create meaningful and rich contexts, according to the task's aims.

- *interesting*: here the question is for whom is the task interesting? Is it interesting just for the designer? Is it interesting just for a hypothetical and ideal well-informed student? The designer is expected to consider the socio-cultural background of the students, together with other contextual factors, such as time and space.

- *within the students' zone of proximal development* (Vygotski, 1978): surely, this is a hard to achieve, since it is not easy to be aware of each student's abilities and potential, especially in our times of multi-cultural classrooms. However, the assessment of the task could be based on a pre-agreed (between the teachers and their instructor) level of difficulty.

- *original*: here the issue of mere copying and pasting of ready-made tasks was raised.

- *included in the official curriculum*: we decided that we could not ignore the official documents and recommendations, simply because their actual assessment would be based in such sources.

The complexity of the task is inherent in the task's content, but we decided to focus on it, following the contemporary discussion about complexity in education (e.g. Davis, Sumara & Luce-Kapler, 2008). From the discussion, it emerged that the complexity of a task can be assessed by:

- *the number of parameters*: here the basic question that came up is if there is a 'rule' for the appropriate number of parameters. Moreover, some teachers raised the question if there is a limit to that number. It was concluded that such concerns cannot be free from the previous issues on content; and, finally, it is always the teacher-designer's choice to revise and restructure the task if needed.

- *the designer's linguistic choices*: the way a task is verbally formulated can play a vital role in the modeling process. The teacher-designer has to be very careful in the language s/he uses, and at the same time be prepared to ask questions and regulate the discussion.

- *the possible interpretations*: this factor is closely related to the linguistic choices made by the task designer, but it is also affected by the nature of the task itself. In fact, modeling activities are expected to lead to more than a single model; by talking about possible interpretations we rather referred to the possibility of misinterpreting the task.

Moving to the actual assessment of the teachers' designed tasks, we will firstly sketch the topics covered by them. These included health (e.g. diets), ecology (e.g. environmental pollution), finance (e.g. investing money or maximizing profit), constructions and managing space (e.g. of a car parking), geography (e.g. mapping), demography (population growth),

fuel consumption, decision making (e.g. on buying a car, on the optimal location of a school or on the best travel option), design (e.g. number of tiles needed for a pattern), everyday activities (e.g. sharing goods) and finally pure mathematical (e.g. finding the number of a polygon's diagonals). An example of how our assessment scheme was implemented in a particular task follows.

The task was entitled "The area of Egypt" and included six parts: introduction, mathematical aims, concept map, main part, assessment and references. The main part included a map of Egypt plain and with lines drawn by the teacher (Figure 2).

Figure 2. The map of Egypt.

The prompts and questions made by the teacher were the following:

a) Find the area of Egypt

b) What does the shape of Egypt remind you of?

c) Do you think that we could separate it into more than one figures? If yes, what could be those figures?

d) Draw on the map the figures that Egypt can be separated into.

e) Find the areas of these figures.

Looking back to our assessment scheme, we can try to reply or comment to the issues raised on content and complexity. Firstly, the task looks realistic, since the context is real, i.e. a map of a real country. Looking more closely, we might note that the students are asked to estimate the area of Egypt, but without any justification. This in turn affects how interesting the task might be for the students. However, the particular drawback could be avoided if the task was included in a larger transdisciplinary task, which could bring together geography with mathematics. The originality of the task can be questioned, since it resembles the task that was given to the students during a previous session about the area of Greece. The mathematical concepts and processes included can fit into many different levels and comply with the curriculum recommendations. Concerning complexity, we might say that the task is flexible enough, so that the teacher may decide anytime to increase its difficulty, e.g. by asking students to use more accurate figures to cover the area of Egypt. Misinterpretations are rather unlikely to occur, and verbal formulation is somehow of less importance because of the representational clarity of the image given.

Concluding remarks

When we looked into all the tasks designed by the teachers, we noticed that some of them seemed not to have grasped the main assumptions of the modeling approach. In some tasks modeling was present as merely a systematic way to perform calculations. Other tasks only implied that a model should be used, but without giving any hints or directions to the students. Surely, our teachers' lack of experience in modeling affected their participation throughout the course. Initially, they were frustrated once they were faced with the modeling tasks; they even deployed some strategies in order to either avoid working on the tasks or getting as much assistance as possible. Later on, they gained some confidence and engaged in the tasks deeply, sometimes providing insightful models and solutions. However, there were some who did not manage to 'move beyond' their instructor's guidance, i.e. design original modeling tasks when they were asked to do so. Concerning assessment, although for the content it was agreed that originality and resemblance to real-life situations are the most important factors, this was not the case with complexity. Particularly, the participants were caught between the need to impose more complexity in the models (by including more parameters in order to make the model more 'accurate') and the need to create an easy-to-handle model (which is closer to their own practice as teachers and citizens). This resulted in some highly complex tasks and few very simple ones. Regardless of all these, assessing these tasks proved to be a highly challenging process for all participants. And we all concluded that assessment should at all times involve both the assessor and the learner in a continuous discussion on the important factors to be considered. In our case, content and complexity proved sufficient for establishing a common ground for a fruitful discussion, negotiation and, hopefully, some meaningful training on modeling and on assessing modeling.

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Topic Group Paper

Topic Group B: Planning for the Future of ALM

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What direction should ALM be in to position itself for the future? What strengths, opportunities and resources (time, talent, money) can the organization allocate to promote the mission and vision of ALM? This topic group used our collective intelligence and wisdom to conduct a SWOT analysis and develop a strategic plan for the future of the organization. The SWOT framework will review the position and direction of ALM by identifying the organization's strengths, weaknesses, opportunities, and threats.

What direction should ALM be in to position itself for the future? Where is the organization going over the next few years? How is it going to get there? In Oslo, Topic Group B met to establish a vision for the association and a strategic plan to clearly establish realistic goals and objectives consistent with the ALM mission. The Trustees facilitated a session using the collective intelligence and wisdom of the ALM membership to conduct a Strengths-Weaknesses-Opportunities-Threats (SWOT) analysis. This planning technique is credited to Albert Humphrey, who led a research project at Stanford University in the 1960s and 1970s using data from the Fortune 500 companies (Haberberg, 2000).

SWOT Procedure

The SWOT analysis compiles an organization's internal and external environment with the aim of leveraging strengths and opportunities to address challenges coming from known weaknesses and threats. Figure 1 below illustrates the SWOT model.

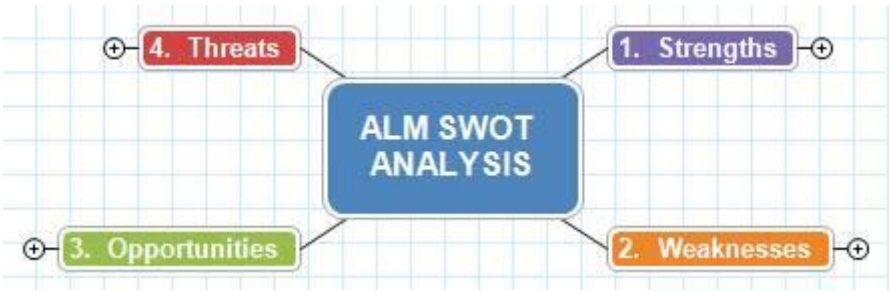

Figure 1. SWOT analysis model.

Data Collection

First, looking internally at ALM, the strengths and weaknesses, under our control, were identified by Topic Group B participants. The Strengths were attributes which were helpful to achieving the mission of the advancement of public education by the establishment and development of an international research forum in the life-long learning of mathematics and numeracy by adults. The value-chain of the strengths was evaluated to determine the value each adds to ALM's services. The Weaknesses identified were competencies and resources which were thought harmful to achieving our mission. After the controllable internal factors were exhausted, the external changes in the Opportunities, which aide the accomplishment of the ALM mission, and the Threats, which damage the mission, were scanned.

Data Analysis: SWOT Matrix

The relationships between the internal strengths and weaknesses and between the external opportunities and threats from the environment are analyzed using a matrix format. A SWOT matrix (Chapman, 2005) assists in generating a series of alternatives based on combinations of the four sets of strategic factors. After the data was collected, the identified attributes were placed into the analysis matrix for ALM below in Figure 2.

Strengths	**Weaknesses**
Unique perspective on Adult Math Education Global/International-many countries Many Countries Strong Intercultural exchange Researchers & Practitioners Strong numeracy conceptual basis Strong numeracy advocacy function Live from year to year Limited expectations Family-like Small Friendly Non-threatening/open Practice & research together Fluid & less rigidity Publications Journal Conference Proceedings Large body of work Concrete Ideas	Small No graduate students Few practitioners-need more No policymakers Need broader audience Visibility low Hard to find Low visibility between conferences 'House of ALM' ALM as 'address' Website *Hyperlinks* *Portal leaflet/flyer* Volunteer organization Weak mechanisms to produce outputs English language issues Poor communications No newsletter Need bibliography of work Lack of esteem parity in some institutions No critical inquiry Lack of arguement Lack of confrontation Over emphasis on ALM
Opportunities	**Threats**
Ready access to publications & bibliography Revitalization New members Beginning researchers New thinkers Redefine/rethink ALM organization *Reinvent ALM* *Reinterpret the brief* Workshops on methods Affiliate with other associations ALM visible at relevant conferences Funding Emphasis on numeracy in many countries Attract more practitioners ALM as a concept	Economic climate *Limitations in funding* Unclear membership benefits Weak sustainability *Overtaken by other organizations* *Senior members retiring* Website Vision © Alan Chapman 2005-08

Figure 2. Adults Learning Mathematics: SWOT Matrix.

The matrix above illustrates how the external opportunities and threats can be matched with the internal strengths and weaknesses to result in four possible strategic actions. First, the Strengths-Opportunity (SO) strategies focus on how to use strengths of ALM to take advantage of opportunities, both current and future. Second, the Strengths-Threats (ST) strategies illustrate how the strengths of ALM can be utilized to avoid identified threats. Third, Weaknesses-Opportunities (WO) strategies propose areas to eliminate weaknesses to open new opportunities. Finally, Weaknesses-Threats (WT) strategies are defensive acts available to minimize the organization's weaknesses and avoid threats.

Next Steps

The work done in Topic Group B provides the Trustees with a working vision for the Adults Learning Mathematics. The next step in the process is to come to conclusions about the major issue and opportunities faced by ALM. These conclusions, outlined below, include the strategic goals to accomplish (What do we have to do to get to our mission?), the critical success factors to achieve these goals (How do we get there?), and the critical actions to position ALM in the 21st century.

Mission: *The advancement of public education by the establishment and development of an international research forum in the life-long learning of mathematics and numeracy by adults (Articles of Association)*

Strategic Goal #1: Encourage research into adults learning mathematics at all levels and disseminate the results of this research for the public benefit.

Critical Success Factors

1.1 .....

1.2 .....

Critical Actions

1.1 .....

1.2 .....

Strategic Goal #2: Promote and share knowledge, awareness and understanding of adults learning mathematics at all levels, for the public benefit.

Critical Success Factors

2.1 .....

2.2 .....

Critical Actions

2.1 .....

2.2 .....

Strategic Goal #3: Encourage the development of the teaching of mathematics to adults at all levels, for the public benefit.

Critical Success Factors

3.1 .....

3.2 .....

Critical Actions

3.1 .....

3.2 .....

In the time between ALM17 (Norway) and ALM18 (Ireland), Trustees will be using the following working vision for ALM:

> *The Adults Learning Mathematics: International Research Forum will be a catalyst for the development and dissemination of theory, research, and best practices in the learning of mathematics by adults; providing identity for the profession; and internationally promoting and sharing knowledge of adults mathematics learning for the public benefit.*

Within this framework, Trustees will continue conversations to answer the question, 'What are our critical success factors which we can leverage to meet our vision?' and working to determine and develop the corresponding critical actions.

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