Aggregate, investigate, and organise: the process of mathematical investigation and teachers of adult numeracy

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In 2007/2008 we began a new programme of teacher training at LLU+ / LSBU using new specifications devised at a national level. As part of this programme, we requested that trainees deliver a presentation outlining a problem and the stages of its solution. In another assignment, we asked trainees to undertake an investigation in one or two areas of mathematics and reflect on the process. With both assignments we are attempting to get trainees to focus on the process, rather than content, of mathematics. This follows similar attempts in many curricula internationally – from GCSE to functional skills in England and the 'reform' agenda of the USA. In (2009) the authors noted some of the issues that can be involved in presentations during the first year of delivery. In this paper, we will consider some further examples of assignments drawn from our second year of delivery, and consider what issues for research and teacher training this data suggests.

Introduction and background

In this paper we consider the assignment submissions of trainee teachers who were asked to write a report on their experiences of undertaking a mathematical investigation of their choice. The task was to discuss the experience and process of doing the investigation, as well as discussing the implications for numeracy teaching. (see appendix A). We asked trainees to concentrate and reflect on the process of doing the task and to use these reflections as a vehicle to discuss the nature of mathematics itself. This focus on process for the trainees was intended to reflect changes in a variety of mathematics and numeracy curricula in the UK and internationally (for example see NCTM 2000).

We reflect on the responses to the task, how this may parallel more generally the experience of the mathematics teaching profession towards seeing mathematics as process and consider how we may further investigate this issue in the future.

Mathematical Investigations

The trainee teachers were asked to write a report about the process of a mathematical investigation. What do we understand by this? Why is it relevant for adult numeracy teachers?

In this context an investigation is an open ended problem that can be solved using mathematical skills, where the learner is given little guidance as to how to find a solution or in which direction to take lines of enquiry. Rather than involving the research into some aspect of mathematics, an investigation involves some form of 'systematic inquiry' (Oxford English Dictionary, Simpson 2008). Investigations are used in different areas of numeracy and mathematics curricula as a means of developing learners' skills in problem solving and the processes of doing maths.

Since investigations are about the processes involved in doing maths we felt they could be usefully used as an assignment on the training course both to develop teachers' own awareness and understanding of process and also to enable us to get insights into the teachers' own beliefs about mathematics.

Investigative problems can take different forms, some are more open than others. In the GCSE curriculum they took the form of a simple question (see Figure 1), learners were expected to answer the question and then continue to explore other aspects of the situation or related questions. There were no single right answers and the situations were rich enough for learners at all skills levels to be able to develop and learn. The process of exploration and reaching solutions is as important as the solutions themselves, both for learning and for assessment.

Trios

Three whole numbers, greater than zero, can be used to form a trio.

For example: (1,2,2) is a trio whose sum is 1+2+2=5

And (2,1,2) is a different trio whose sum is also 5

How many trios can you find with a sum of 5?

Investigate further.

Fig. 1 GCSE Course Work Question (AQA 2006)

Teachers' own views and beliefs about mathematics and their understanding of process are likely to influence their ability to support these activities. As the authors noted in an earlier paper (Griffiths, Kaye and Moulton 2009), many teachers find it difficult to understand and to discuss the process of doing mathematics, that is understand mathematics at a meta-level, as opposed to simply doing it.

The place of process in maths education

In 1982 a committee chaired by W.H. Cockroft published a seminal report on mathematics education in the UK (Cockroft 1982) recommending that investigations play a part in maths learning at every level:

> The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems.
>
> ... much of the value of an investigation can be lost unless the outcome of the investigation is discussed. Such discussion should include consideration not only of the method which has been used and the results which have been obtained but also of false trails which have been followed and mistakes which may have been made in the course of the investigation.
>
> ... general strategies directed towards problem solving and investigations can start during the primary years.

However we believe there is evidence that some teachers, both in school and in adult environments, have continued to be reluctant to use investigations in their teaching and as instruments of assessment for a variety of reasons. We note below some evidence taken from the website of the National Centre for Excellence in the Teaching of Mathematics (NCETM).

Following the Cockcroft report, mathematics processes i.e. 'using and applying mathematics' were included in the national curriculum for mathematics (first published in 1988) for all levels of study. It is evident in formal assessment as well as encouraged in classroom practice (Ofsted 2008) in academic and vocational curricula.

In 1986 investigations were introduced into GCSE[1] assessment in the form of course work: usually a statistical analysis of 'real life' data and an abstract mathematical investigation (See Fig 1 above). Marks were allocated to evidence of the decisions taken in exploring the problem, i.e. to the mathematical processes. In our experience adult learners on GCSE courses often enjoy the opportunity to carry out an extended investigation and have been 'turned onto' maths by the experience. Many teachers have understood investigations in the narrow sense of the types of problems set for GCSE course work:

> Most pupils do not regard the idea of mathematical investigation as an integral part of their ongoing mathematical education but a bolt-on extra for the exam. (NCETM 2008b)

Although Cockcroft envisaged such work as open ended in many cases learners understand that in fact their teachers do have in mind particular ways of carrying out the investigation which therefore leads to a set of 'correct' answers. Learners 'do' investigations as a separate part of mathematics rather than as a way of developing problem solving techniques (Hewitt 1994, Robinson 2004). Course work has recently been dropped from maths GCSEs by the QCA (Johnson 2006), who nonetheless retain a commitment to assessing problem solving skills through extended examination questions. In the school environment many teachers had found difficulty in managing the course work process.

The Association of Teachers of Mathematics (ATM) ran a pilot GCSE which was assessed entirely through coursework between 1986 and 1994 (Ollerton and Watson 2007). Assessment criteria focused on process:

> The criteria provide a detailed description of mathematical behaviour and a mathematical mindset unlike anything else we have seen produced for subsequent assessment systems.

Teachers taking part felt the assessment method to be very successful "assessment for learning and assessment of learning in action", but the project was dropped by the then secretary of education who

> … failed to recognise this as an innovative, robust and profound development in mathematics education. Neither was it recognised as a way to develop students' problem-solving skills, flexible approaches to learning and the knowledgeable, adaptable, confident use of mathematics for which employers and universities now call. (Ollerton and Watson 2007*)*

Process skills were the focus of attention and assessment in vocational courses and work-based training at the same period. The development in this sector can be seen to have its

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[1] General Certificate of Secondary Education is the major assessment for year 11 (approximately 16 year old) school pupils in the UK.

origin in the 1989 report *Towards a Skills Revolution* by the CBI (quoted in Jessop, 1991) which introduces 'Common Learning Outcomes as core elements'. The full list of these outcomes / elements is worth noting:

- values and integrity
- effective communication
- application of number
- application of technology
- understanding of work and the world
- personal and interpersonal skills
- problem solving
- positive attitudes to change.

Those familiar with the recent literature surrounding the introduction of functional skills will find these familiar, perhaps surprisingly so. These ideas evolved into what were at first called cores skills, and were seen as an add-on to the newly introduced NVQ qualifications. Gilbert Jessup says:

> The idea that there are basic skills in areas such as communication, numeracy, problem solving, which can be applied to a wide range of activities, is of course not new. … The new methods of formulating outcomes in statements of competence and attainments has raised the question whether core skills can be assessed in this way. (Jessup 1991)

There was a revision of Core Skills at the end of the 1990s when the new Key Skills were piloted. The introduction to the units (QCA 1997) re-states the aims of the new units, which include Application of Number for levels 1 to 4:

- contain skills which are essential to effective performance in a wide variety of settings
- help individuals to identify the skill and knowledge requirements which underpin each Key Skill
- emphasise the 'application' of skills
- encourage an holistic approach to learning and assessment

The Key Skills qualifications, introduced in 2000, also saw the introduction of the national tests (alongside the portfolio). These tests were adopted as the assessment for the newly developed adult numeracy qualification. Key Skills are now (2010) being replaced by Functional Skills (of which Functional Maths is one). These three versions – Core, Key and Functional - all incorporate similar numeracy process skills: recognising that number is a feature of a task and deciding how to deal with it (mathematising); carrying out the mathematical operations and procedures; arriving at a result or making judgements e.g. by estimating; deciding what the numbers mean i.e. interpreting and presenting (Levy 1987 annex 14).

The development of these forms of curriculum and assessment has shown the tension between aiming to achieve measurable process skills and efficient independent assessment. The Core Skills depended on competence-based portfolio evidence, which turned to a mixture of written paper tests and portfolio evidence with Key Skills. Now, with Functional Skills, assessment is purely by written paper tests. This has had its attendant effect on, or perhaps has been influenced by, attitudes of teachers. Early on a decision was taken not to include Key Skills in A Level[2] qualifications, and more recently not to tie Functional Skills to GCSEs. The former decision was taken partly because of practical difficulties, but also not to 'dilute' A levels (Hodgson, Spours, and Savory 2001), the latter not to hinder pupils' access to the more prestigious GCSE qualification. For adult qualifications only the Key Skills written assessment was used, a multiple-choice paper which has practically no assessment of mathematics processes. The shift to paper-based assessment together with pressures to support learners to pass the assessment can lead to ‘teaching to the test’ rather than taking an investigative approach in the classroom, especially when other local constraints such as short programmes and short lessons are taken into consideration (Stone 2009).

A teacher writing on the Adult-Numeracy (Adult-Numeracy 2010) email list (adult-numeracy@jiscmail.ac.uk) comments

> *What can we achieve in one hour a week? Most of the time I get the question from my students: "Miss, can we have more time for maths? It is really important for us." By the time we get into discovering, learning and actually starting to do some very useful work on a new topic, the lesson had ended.*

One of the aims of Functional Mathematics (FM) is to develop learners' real life mathematical skills using written assessments in which methods and processes gain marks. FM is based on a problem solving approach to learning, the standard defines process skills as mentioned above, and appears to be another opportunity to introduce open ended problem solving into teaching and assessment. A support document explains:

> Clearly, teachers cannot know what ...mathematics... their learners will use as they move through their lives. This means that we cannot identify a curriculum core that every learner will use. Instead, and much more powerfully, learners should be taught to use and apply the ...mathematics... that they know, and to ask for help with the areas with which they are less confident. (FSSP 2008 p8)

Examples of assessment materials show a range of styles, from those which appear to be using standard mathematics problems in a context, to those which pose more open ended practical problems, although even the latter's mark scheme is expecting standard maths answers with some marks for explanations or description of strategies. Comments from teachers in briefing meetings reflect those noted on the NCETM website, some of which are noted below, in that that many prefer a more conventional style of assessment.

Free Standing Maths Qualifications (FSMQs), introduced in the late 90s, are also designed to give students an opportunity to develop real-world mathematical understanding and are assessed by portfolio work and a written test, but are only used in a small number of contexts. They cover aspects of mathematics likely to be of relevance to learners using contexts from other aspects of learners' experience such as other subjects, the workplace and vocational areas (see the QCDA website www.qcda.gov.uk).

Recently several resources have been produced to encourage and support teachers in developing the use of collaborative learning in the class room[3]. These resources demonstrate

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[2] A Level is the major assessment for University Entrance in schools in the UK.

[3] As examples *Improving Learning in Mathematics* (DfES 2005) and *Thinking Through Mathematics* (NRDC 2007), resources that model collaborative, investigative approaches to learning mathematics, and two web-sites, Bowland maths (www.bowlandmaths.org.uk/index.htm) and NRICH (nrich.maths.org), which both provide interesting activities to enrich maths learning.

that there is much interest in investigative styles of teaching and that there has been ongoing support from government departments.

Many teachers are still reluctant to embrace investigation as a teaching method, seeing mathematics not as process but as a set of skills to be 'mastered' before broader investigative work can be undertaken. A recent conversation with a teacher who was worried at how long it was taking to teach her learners how to divide, and felt she could not move on to investigations until they had learned this, illustrates the point. Comments about FM from NCETM blogs (Allan 2008) are positive but also illustrate what some teachers do not like about the new assessment:

> *...students not taken through steps of mathematical modelling ... steps too open-ended ... difficult to teach ... difficult to mark*

Commenting on the more traditional style FM papers teachers like the fact that:

> *...not too much reading ... easier to teach to test ... limited processing required ... will help success rate (from league table point-of-view)*

but in more open style papers do not like:

> *...planning sheet before the question ... very open ended ... amount of reading ... need for statements of assumption (vague) .... too much talk and not enough Maths!*

and from the Adult Numeracy e-mail list (Adult-Numeracy 2009):

> *I really welcome the idea, but I'm afraid I can see it fall apart in a few years time as well as key skills had, as no one is actually addressing the real issues here, which are the lack of basic skills, such as confident knowledge of the times tables and lack of time for teaching these essential problem solving skills, ... In my experience we have the same problem key skills had with a couple of improvements that are the removal of the portfolio and the students now allowed for some demonstration of their knowledge.*

Discussion of the beliefs of teachers evidenced from the assignment

In the academic year 2007/2008, trainee teachers at LLU+ / LSBU were set an assignment with a focus on mathematics at the 'edge of their comfort zone'. This led to a range of interpretations of the task and variability in the discussions of aspects of the process. In response, the assignment and the accompanying guidance were re-written, to focus more on the reflection of the processes involved in solving a mathematical problem (see Appendix A).

The resulting assignments presented during 2008/2009 clearly identified issues related to process. Had we succeeded in our task of emphasising process aspects of mathematics? Were we pleased with the outcome? In part yes; but we were concerned that the trainees may have been producing assignments that 'pleased us' rather than really engaging with their investigations.

We had hoped that the assignments would provide an opportunity for the trainees to reveal their ideas and beliefs about mathematics. The authors considered the assignment responses and came to the conclusion that the submissions were ambiguous in relation to the beliefs of teachers. The following extracts from the 08/09 draft assignments demonstrate the difficulties of interpreting such assignment submissions. The reader should note that the references to both mathematical and statistical investigations are in response to the criteria for assessment (Appendix A).

> **Teacher A**
> A general strategy for this investigation [how many lines are used when joining a given number of dots] is to look for a pattern by investigating.
> To investigate I needed to draw diagrams. When I drew it, I found that the question seemed much more concrete. I could see more clearly what was involved. Drawing a diagram is making the question more concrete, more specific, and more manageable.
>
> Beginning with the fewest number of dots I made diagrams with dots of 2, 3, 4 and 5. I noticed that the total number of lines increased by one more than the previous difference each time. I have obtained 1, 3, 6 and 10 which suggested that I might be able to find a pattern. Before diving in further, I paused to recollect what I want. I want to know whether any pattern can appear. I recognised from my subject knowledge and experience that this was a sequence of triangular numbers. I have not finished yet, because I want a more concrete proof. Therefore I decided to do one more with 6 dots. This gave 15 lines to confirm my findings. Now I can reveal a number pattern which may be used to extend the sequence and enable the determination of the number of lines for any given number of dots. It also possible to use the pattern to obtain a formula for the number of lines in terms of number of dots.

From this extract we could interpret the trainee as having an understanding of how to be systematic, choosing to carefully build the cases one by one until a pattern is spotted. Alternatively, one could see the response as a near replica of examples provided. It may be that the trainee is treating investigations as a new topic for mathematics with its own algorithm (as warned by Hewitt 1994).

> **Teacher B**
> Using investigations with learners provides the opportunity for discussion, for linking areas of maths and finding different ways of approaching a problem. These need not be complex, lengthy investigations but opportunities to discover relationships, experiment, and construct formulas for themselves, rather than being given algorithms to use. Research seems to suggest that, in statistical investigations, too much emphasis is given to the mathematics and not enough to consideration of the problem itself and analysis of conclusions from data. The relationship between teachers' views of maths and teaching methods may be speculative as Ernest points out, but clearly teaching through investigations requires a 'letting go' of the 'traditional' method and 'allowing' many different approaches and interpretations of the problem. The 'traditional' method becomes one possible method of solving a problem.

One interpretation is that this is a call to action, bringing together the literature with their own experience. Alternatively, this may be seen as a contribution written specifically for the assessor. Are they saying what they want us to hear?

> **Teacher C**
> Although my thinking went through the steps that I have written down above, this does not capture the way in which I worked on the problem.
> Having considered the starter during the session, I did not think about the problem again until a couple of nights later – it was actually the middle of the night. I just thought about the algebraic form of what I wanted to look at and made the connection with the binomial expansion. I started on the expansions mentally, but found it had to hold the algebra in my head with the complication of inequalities, which always feel a bit slippery to me. It was several days later that I started to write the solution down and it fell out very quickly.

This has a ring of authenticity in the description of timings and the feelings and may be helpful in considering the non-mathematical elements of the process. Nevertheless, the description does not help in describing how the solution was obtained.

> **Teacher D**
> With the mathematical investigation I started off with a goal that had been given to me, to find out how many triangles were in different sized polygons.
> With the statistical investigation I decided what the investigation was going to be.
> For the mathematical investigation I used primary data because I was carrying out the investigation myself. The data was from the mathematical world. For the statistical investigation I had to use secondary data, which came from the real world.
> In order to make the mathematical investigation systematic I started with the smallest sized polygon and continued by adding one side at a time. With the statistical investigation I used a name of a country without knowing the size or population.
> I was given criteria to follow for the mathematical investigation, which eliminated other triangles that could have been drawn. This is an essential part of a mathematical investigation. For the statistical investigation I set my own criteria.
> I put the mathematical results in a table, I predicted an outcome and tested that it worked. I was able to use controlled steps in a logical order. I completed the investigation, bringing it to a conclusion.

This extract shows aspects of choice in undertaking an investigation. Nevertheless, it is interesting that the trainee appears not to see a 'mathematical investigation' as a task within their control but something 'given'.

> **Teacher E**
> A search of the Tower of Hanoi literature soon reveals that the solution is well known to be recursive. Let us now explore what this means and why a recursive algorithm should prove that the simple solution described above should hold good for all possible towers.
> We need to understand that if we can solve a 99 discs (which we can, following our simple solution) we can solve a tower of 98, 97, 96 and so forth until we reach a "tower" (if we can so call it) of one disc. We certainly know how to move one disc, so the solution is recursive. To put this in another way, we are breaking the problem down into smaller and smaller parts - something our learners could well be encouraged to do. We will need a system of notation to describe our moves. We can call the three pegs A, B and C, remembering that these labels may move at different steps.

This extract, while outlining a systematic approach, appears to be about inductive proof rather than really describing the process of understanding and problem solving.

This selection of extracts shows that trainees are able to discuss some sort of process in solving the mathematical problem. We have chosen examples that show a range of competence with mathematical thinking, which the authors feel is representative of the 2008/09 cohort. Some have a strong focus on an algebraic solution (like example C) and demonstrate a comfort with formal mathematical content, in this case binomial series. Others describe their investigation as a data collecting activity (like example D).

We were satisfied the trainees had responded appropriately to the task by considering, undertaking (mostly) and reflecting upon a mathematical investigation. Though all achieved this, there was considerable difference in the level of mathematical insight the learners were able to demonstrate. This is hypothesised by the authors as being dependent on their experience with formal, academic mathematics. In addition, we identified the following two items as key issues to follow up.

- Reflecting on "doing mathematics" did not lead trainees to discuss what they thought mathematics is; or even to imagine that such a question was worth considering.
- Trainees made few connections between how they did mathematics and how they taught adult numeracy.

As part of the preparation for the assignment we presented 7 views of mathematics (Brown et al 1981) (Appendix C) such as 'mathematics as logic' and 'mathematics as intellectual passion'. Though this was well received during the training sessions it is rarely identified as having any connection to the investigative assignment task.

We feel we have successfully assessed our trainees within the context of the qualification standards. However, we wanted to do more. We wanted to raise awareness of the inter-relation between a teacher's conceptual map of mathematics (or possibly numeracy) and their approach to teaching. At this stage we believe we have established that very few trainees have an awareness that their image of mathematics is (may be) different from others. It is as though they simply 'know' what mathematics is (although they may question what numeracy is!). However, we are aware that trainees' images of mathematics differ greatly from each other. Currently the evidence for this arises from observation in an educational setting and not from structured research data. Some work is being done on this for school teachers, such as the work of Klinger in Australia (Klinger, 2009), though this has an emphasis on teachers' competence in mathematics, rather than an image of what the subject is.

Where next

Despite the relative infancy of teacher training for adult numeracy in the UK (at least compared to school-teacher education) we are beginning to get a picture of teachers of adult numeracy (at least in London). Indeed, as we noted in an earlier paper (Griffiths, Kaye and Moulton 2009) our sample was in broad agreement with Swan's sample of teachers in his studies. We see a range of practitioners dedicated to the development of adult numeracy in their learners. It is perhaps not surprising to note that these teachers appear to have different views of what mathematics is and what it is for. In our previous paper we had argued that some individuals struggled to see the process aspect of mathematics and really responded to their tasks in relation to the content of mathematics. We hope that we have shown in our investigation assignment a similar picture is emerging. It is interesting that the trainees are learning to produce assignments that address the criteria required of them although the extracts that we have chosen above suggest that some of teachers are not seeing 'investigation' in the ways intended by the training team. But where does that leave us?

We feel that it is important to capture some more data on teachers' views of mathematics. To this end we have devised a questionnaire which we will pilot with a future group. This questionnaire has been split into three sections.

Section 1. To position the results of the questionnaire, we will use a part of Mathematics Matters questionnaire (NCETM 2008c). We hope that will enable us to say something about how typical or otherwise our groups are. We note that the sample used by Swan and the research team were self-selected and not representative of teachers as a whole. Nevertheless, we feel that the results from this section will act as a comparator.

Section 2. We would like to ask teachers to describe what they think mathematics is. We expect that this might be difficult without some further prompts. We have therefore used some of the extracts from the investigation assignments and will ask teachers how the statements fit with their own views. The statements chosen represent some different aspects of mathematics as investigation.

Section 3. Finally, after offering a number of views of mathematics and teaching, we will ask teachers to describe what they think mathematics and mathematics teaching is.

In the first instance, we expect to categorise the responses in a number of ways. We are interested in seeing how much the descriptions include mathematics-as-process. We will also see how these fit the framework of Swan et al.

Conclusion

There has been an increase in the importance of seeing the process aspect of mathematics in a range of curricula. This has impacted upon the National Curriculum for schools, the Core, Key and Functional Skills curricula in the UK. Internationally, the USA reform agenda has identified process as a key element of mathematics.

We had hoped to see evidence of the beliefs about mathematics of trainee teachers within their assignments. This evidence turned out to be more ambiguous than we had expected. We suspect that assignment submissions are not the best place for discovering views as they are bound up with passing a course and may be written to send a message to the assessors.

In the future, we will investigate the issues raised more directly by trialling a questionnaire developed during our reflections on the trainee responses to their assignments.

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Appendix A

NKU2 Assessment
Write an account of mathematical investigation, using one or two examples, including a reflection on the process of mathematical activity.
(1500-2000 words)

Assessment criteria

- Describe the nature of argument and proof in the selected investigation
- Describe the use, interpretation and representation of data in the investigative process

  These two criteria should be addressed in the problem and its solution or in the comparison to other types of problem.
- Reflect on how the investigative process relates to views on the nature of mathematics

  This will involve an outline of different perspectives on the nature of mathematics (with reference to relevant literature). You should then consider which perspective relates to the use of investigative processes, and explain why.
- Discuss the implications for the use of investigations with numeracy learners

  Discuss how the use of investigative approaches with numeracy learners can help or hinder the learning process, with examples.

Guidance
In the guidance below you are asked to choose investigations. These may be chosen from those used or referenced in the sessions (or may be chosen from elsewhere).
Either

- Choose a mathematical or statistical investigation
- Identify a solution to the investigation.
- Compare the investigation and solution to the other approach to investigations (i.e. mathematical compared to statistical investigations or vice versa).

Or

- undertake one mathematical investigation and one statistical investigation
- identify the two solutions

Then

- Reflect on the process of investigation and identify issues that relate to learning mathematics / numeracy including:
  - strategies for learning and teaching;
  - curriculum development.
- When thinking about implications for practice it is unlikely that you will have a direct link to learner issues. You should consider the parallels between the mathematics that you have studied in the course and the mathematics that your learners undertake. For example, the difficulties you face and how you resolved these difficulties should tell you something about similar issues for learners when solving their own mathematical problems.
- Comment on the nature of mathematical knowledge and investigation and aspects of the wider curriculum (i.e. content, process, assessment, in a variety of subjects / contexts), with reference to relevant literature.

Appendix B

Nature of Mathematics. Adapted from Brown, S. Fauvel, J. and Finnegan, R (eds) (1981) *Conceptions of Inquiry* London. Methuen & Open University Press

Mathematics as Eternal Truth (Plato)
Mathematics as aesthetic creation (Henri Poincaré)
Mathematics as logic (Douglas R. Hofstadter)
Mathematics as language (Lancelot Hogben)
Mathematics as art (G.H.Hardy)
Mathematics as intellectual passion (Michael Polanyi)
Mathematics as social practice (Luke Hodgkin)

Appendix B

Questionnaire on teachers' views of mathematics

Please answer the following three sections as best you can. We expect that the data we collect will inform our understanding of the views of existing and trainee teachers and influence our training.

Section 1 The following questionnaire came from the Mathematics Matters project of the NCETM. Answering these questions will enable us to reference our training groups to this survey.
https://www.ncetm.org.uk/files/309231/Mathematics+Matters+Final+Report.pdf

Principles Ideal and implemented values

Write "**A**" in the appropriate box on each row to show your vision for an ideal mathematics curriculum.

Write "**B**" in the appropriate box on each row to show the values implied by the curriculum that is currently implemented in most schools and other institutions.

4=almost all mathematics lessons should contain this aspect
3=most mathematics lessons should contain this aspect
2=less than half of mathematics lessons should contain this aspect
1=few mathematics lessons should contain this aspect

	More often ⟷ Less often			
Types of outcome and types of activity	**4**	**3**	**2**	**1**
Fluency in recalling facts and performing skills *For example:* Memorising names and notations Practising routine procedures				
Interpretations for concepts and representations *For example:* Discriminating between examples/non-examples Generating representations Constructing relationships Translating between representations				
Strategies for investigation and problem solving *For example:* Formulating questions/problems Developing/comparing strategies for solution Monitoring progress Interpreting/evaluating solutions Communicating results				
Awareness of the nature and values of the educational system *For example:* Recognising the purposes of learning maths Developing learning/reviewing strategies Knowing what others value				
Appreciation of the power of mathematics in society *For example:* Appreciate history/cultural foundations Creating/critiquing models of real situations Recognising uses/abuses of maths in society Gaining power over problems in ones own life				

Section 2
The following extracts are from teachers assignments about the investigation aspect of mathematics. Identify how well these extracts fit with your own views of doing mathematics. Please add any comments you wish.

	Close to ←→ Far from own view			
Extract	4	3	2	1
Teacher A I needed to draw diagrams. When I drew it, I found that the question seemed much more concrete. I could see more clearly what was involved. Drawing diagram is making the question more concrete, more specific, and more manageable.				
Teacher B These need not be complex, lengthy investigations but opportunities to discover relationships, experiment, and construct formula for themselves, rather than being given algorithms to use.				
Teacher C Having considered the starter during the session, I did not think about the problem again until a couple of nights later – it was actually the middle of the night. It was several days later that I started to write the solution down and it fell out very quickly.				
Teacher D I started off with a goal that had been given to me, to find out how many triangles were in different sized polygons. I used primary data because I was carrying out the investigation myself. The data was from the mathematical world.				
Teacher E To put this in another way, we are breaking the problem down into smaller and smaller parts- something our learners could well be encouraged to do. We will need a system of notation to describe our moves.				
Comments				

Section 3 (actually on a separate page with space for extended text)
Finally, it would be helpful to know what are you own views of mathematics and mathematics teaching.

For me, mathematics is …

For me, mathematics learning and teaching is…

Thank you for your time.