Bet and lose: learning mathematics or losing money

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A lot of people risk money with bets on sports or other events. The bookmakers offering such bets earn a lot of money. Together with a colleague from Salzburg University, Dr. Hans-Stefan Siller, I put together a proposal - or better: a concept for part of a basic mathematics course - for learning the mathematics 'behind the screen' (internet bets in particular are very popular). It starts with a simple simulation of a game or sports event in a mathematics lesson. Learners organize a sports event (or a simulation of an event), create several different types of bet offices and offer odds. Other participants get game money and bet. When the event is over and the outcomes are fixed the learners calculate wins and losses. This way, they learn how to calculate odds and why bookmakers earn so much money – if they know enough about mathematics and a few other things they need to know.

Starting point: Some facts about gambling, money and risks

The German word “Spiel” (= game and play) is a word that covers very many different activities including children playing, sports, chess, poker, and games where “luck” or destiny and professional knowledge are responsible for winning or losing money. (For an overview see http://www.spielforschung.at/ menu “Publikationen”). In this paper I will concentrate on games where one wins or loses money betting on the outcome of sports events, for example a tennis match or a soccer championship. When I did some internet research for this paper I was astonished how much money is spent on - and earned with - such bets (especially in the World Wide Web). An official German website (http://www.dhs.de/web/datenfakten/glueckksspiel.php – DHS is The German Centre for Addiction Issues) offers some interesting information: In the year 2007 people in Germany spend 28.000.000.000 Euro on gambling. The German government earns 4.250.000.000 Euro through taxes. This might point to why there is a centre for addiction issues but not a general law against gambling.

Is there any reason to think about a general law against gambling? Yes, there is. In Germany there are about 220.000 *pathological gamblers*! Pathological? Are they ill? Yes, they are, according to the WHO (World Health Organisation), in "ICD Version 2007" Chapter V:

> Mental and behavioural disorders (F00-F99): Habit and impulse disorders. This category includes certain disorders of behaviour that are not classifiable under

> other categories. They are characterized by repeated acts that have no clear rational motivation, cannot be controlled, and generally harm the patient's own interests and those of other people. The patient reports that the behaviour is associated with impulses to action. The cause of these disorders is not understood and they are grouped together because of broad descriptive similarities, not because they are known to share any other important features.

One of these is F 63.0:

> Pathological gambling: The disorder consists of frequent, repeated episodes of gambling that dominate the patient's life to the detriment of social, occupational, material, and family values and commitments.
> (WHO, http://apps.who.int/classifications/apps/icd/icd10online/)

If you are looking for examples you can find them in literature – e.g. The Gambler by Fyodor Mikhailovich Dostojevsky - and in the newspapers: “An online gambler has pleaded guilty to stealing more than £1m from his employer to feed an "out-of-control" gambling habit...” (http://www.vnunet.com/vnunet/news/2160083/online-gambler-guilty-theft).

Why is gambling a topic for teaching mathematics?

So there is a problematic situation for many adults called “pathological gambling”, which includes high risks for those who gamble, their families and society. But why is this a topic for teaching mathematics? One reason is quite simple, from my point of view: bets/odds, quotas and wins or losses are calculated. It is relatively easy to understand these calculations with basic mathematical knowledge - the four fundamental operations of mathematics and easy stochastics are used. If you want to understand the mathematical background of the daily work of a bookmaker better, you gain insight into a lot of statistical models.

A more complicated reason – from my point of view – is one related to the general aims of education. Maybe the most important aim is going back to I. Kant and “Enlightenment” (see: "Answering the Question: What is Enlightenment?" German: "Beantwortung der Frage: Was ist Aufklärung?". This is the title of an essay by the philosopher Immanuel Kant published in the year 1784). In modern words and including a little shift of meaning the general aim of teaching mathematics is this: Teaching should enable students to become critical citizens. They should be able to use their mathematical knowledge for analysing situations, finding rational solutions for problems and to see structures and the influence of different factors.

I propose to introduce this topics in mathematics courses for adults in the hope that the students enjoy these lessons. They are a good example for the usefulness of mathematics to understand what happens in daily life. I hope that students learn something for their life - especially not to risk their money and happiness by gambling. This more general aim leads me to propose a good method for these lessons or part of a course.

Proposed method of teaching: self exploring

Looking at educational activities whose aim it is that students or others should learn to avoid risks (caused by drugs, smoking, consuming alcohol, etc.) we know that changing behaviour in many cases is not (only) the result of enlightenment or

information. Smokers do not stop smoking because they are told that they risk cancer. Many adults tell children that they should be careful if they cross a road – but every year children die crossing a road.

If people discover on their own how to get information, to analyse situations and structures, and to draw conclusions for behaviour, the chances that they really change their behaviour are much better. Therefore it seems to be a very good idea to use a self-directed method for this theme. Learners should explore the situation and find their own way to handle it.

Proposed course concept

The first step of a learning project should be an agreement between teacher(s) and learners about the topic and a draft of the structure. In my opinion it is a good idea that the teacher (or a student) starts with a proposal and some source of motivation. This could be a headline from the newsletter like the one I quoted at the beginning or a little story like this one: "Last week I lost some money because I thought that my favourite soccer team would win the cup. I bet on them but they have lost the match and I have lost all my money!"

In any case the draft structure for this project should include

- some research (or exploration or collecting information) about betting, bookmakers and rules,
- a simulation of a sports event and betting with game money as main content,
- a documentation of the data of the simulation which will be analysed,
- collaborative project planning, organizing and responsibility for success,
- collaborative reflection on the results/outcomes.

The second step is the collaborative planning and organising of the learning process. This is the main aim of a group, but I do not want to plan the complete project for a group here. Therefore I will only give some tips on important aspects. Starting research about betting means to list questions: What do we want to know? Where do we find information? Who is going to look where? What is a good time-frame for completing the first overview and presenting it to the others?

If the group does not really know what they are looking for at the beginning they will formulate very open questions and may be overwhelmed by the flood of information which can be found on the internet. This is not the worst case scenario however.

Reflecting on their experiences, the group should learn something: It is a very good idea to start with concrete questions and clear aims. This will make it easier to sort out a lot of information which is maybe very interesting in other situations but not now in this project. Then the second attempt to find out about "something" will be more directed by the aims of their work and therefore much more successful.

My second tip is to simulate a sports event. This should be a very simple match involving two people and can be realized quickly and easily. What works well in my experience is kicking something into a "goal" - i.e. the space between two books on a table. Hitting the goal is easy if you can concentrated and train a little to gain some

skill with your fingers. If there are too many hits just move the goal for one or two (deci-)meters until just a few of ten trials are hits.

Maybe some students will ask “Why do we need a sports simulation?” “Why don’t we take a coin or dice to get something like match results?” This is a good question that leads to a very important point. Bets are based on estimations or expectations about the outcome. People have different reasons to estimate or expect that team A or team B will win – and bet following their estimations and emotions. If the “match” is a random event decided by a coin it is easy and boring to estimate what will happen. The chance is 50% that one of the sides of the coin is “winning”. Therefore it is rational that the bets are distributed equally. There is no favourite and no outsider. I think it is a good idea to hand this question back to the students. After some research and some practice with being a bookmaker in the classroom experiment they will be able to answer this question themselves.

A third tip relates to the documentation of the data. Many students like to play a game or do a simulation but very few are happy with having to document it. At this point there are two ways forward: one is to leave it open – students will experience for themselves that they have difficulties reflecting on the results of the simulation if they do not remember them. They will need a second attempt at the simulation to correct the situation. The other way is that the teacher is argues the case of documenting what happens before the experiment starts and convinces the students. The use of a computer and a spread sheet could support the documentation and the analysis of the results.

Many teachers believe that students are not able to plan their own learning process well. The paradox in this situation is that teachers will be right as long as they believe this and do the planning without students. If the students never have the chance to do it – be it planning or something else – they will not learn it. But if the teacher is convinced that students will be able to learn the planning learning process on their own, they will be able to do it. Some teachers might say “this is not mathematics” – but this is wrong: project planning includes a lot of mathematics. It is applied mathematics.

Here is a short list of some aspects of the project planning that should be mentioned:

- Organisation of the simulation of the sports event: Participants, rules, plan for the matches (like tennis or like soccer?), preparing useful computer support for the documentation.
- Rules for the classroom bookmakers and also the betting with game money.
- Ethical aspects: What is allowed?

The third step is preparing for the next stages. Here I will concentrate on the **mathematical aspects of bets**. How can they be calculated? Let me start with a simple example: a tennis match and a calculation **after** all bets are done. Let us assume that 60% of the money was bet on player A and 40 % on B. What is the next calculation? The question is: How much money does a winner get? To calculate this a **quota (or dividend)** is defined by a division: 100/60 = 1.66 is the quota for A and 100/40 = 2.5 is the quota for B. This means that a winner gets 166 units of money for a 100 unit bet on A and 250 units of money for a 100 unit bet on B. If you bet on A and B wins you lose your money.

Is this calculation realistic? If students ask this please ask them back: Do you think so? What does the bookmaker earn? The first answer might be bookmakers have a lot of information about sports events and therefore they win most of their own bets. This is a good idea but a bookmaker likes to live more comfortably than that. There is a better source of income for him! Let us have a look on the internet and recalculate some quotas. I start with an example from Austria that I found in March: A bookmaker offered for the soccer match Rapid Wien s. Sturm Graz: 1.47 : 3.95 : 5.95. What he offered in words is to pay 1.47 units for one paid unit if Rapid Wien wins, 3.95 for one paid unit if the result is draw and 5.95 for one if Sturm Graz wins. We divide and add:

$1/1.47 = 0.68$
$1/3.95 = 0.25$
$1/5.95 = 0.16$
$0.68 + 0.25 + 0.16 = 1.09$

Wow! The result should be 1, not 1.09. What happened? We calculate again and divide 1 : 1.09 = 0.9174. In other words: 91.74 % of the money is paid back to those who placed bets and 8.36 % is the commission the bookmaker takes. He earns a percentage of the money as commission! If you recalculate more bets and offered quotas you will find 5 to 10 percent commission in most cases. You can find this commission if you search for websites that provide information for gamblers (e.g. http://www.mr-bet.net) – many websites compare quotas. In Austria there is a law that fixes the commission to “no more then 10%”.

Several new questions arise from this and others are still open. My next step is to reflect on my first simple example. What happens if I recalculate it including commission? 100% of the money is paid for bets, 60% for A and 40% for B as winner. The bookmaker takes his commission – let us say 10%. 90% of the money is left and the bookmaker’s offer is calculated as shown now: I will give 90% of the total sum to those who bet on the win of A if A wins, and I will give 90% of the total sum to those who bet on the win of B if B wins. My quotas are: $1.66 \cdot 0{,}9 = 1.50$ (A) and $2.5 \cdot 0.9 = 2.25$ (B). Is this calculated correctly? Let us try! If A wins $60\% \cdot 1{,}50 = 90\%$ are paid. If B wins $40\% \cdot 2{,}25 = 90\%$ are paid. Correct.

The next question addresses the fact that the calculation seems to be correct but the method is not realistic. A bookmaker offers bets with quotas days or weeks before the event happens. He doesn't wait until it is over. Yes! This is what makes the life of a bookmaker thrilling. In reality we have two different types of bookmakers. One is actually offering fair odds which are calculated after all bets have been placed. This type is called “totaliser”. You won't find this type of bookmaker too often. He hasn't got that many (or very few) customers and his life is not all that thrilling because he gets his commission independently from the result of the match. Again: Whoever wins the match and however the bets are situated he will get his commission. Gamblers do not like totalisers because they do not make mistakes with quotas.

The type of bookmaker we see on the internet has to start with an offer. If he accepts a bet he has to pay the winner the offered quota – even if he changes the quota after a while. This opens the door for a lot of statistical calculations: What would happen if... I will come back to this later. Students will learn a lot about calculating quotas if they start to offer them when they play the role of a bookmaker in the classroom.

Step number four is the first trial run! I think it is a good idea to start with a simple match: Two students train for the kicking simulation, other students play bookmaker and offer quotas. Let us imagine that Mary and Jeff train to hit the goal and Mary is more successful: She has 6 hits out of 10 trials and Jeff 3 of 10. Now the bookkeeping students (organized in groups of two or three) should offer their first quotas. What should they offer?

Different solutions are possible. Is it a good idea to take the training results as if they were probabilities? Is a chance of 6 : 3 that A will win? This is not correct from a mathematical point of view because probability-values have to be 1 or less. Next trial, better formulation: 6 : 3 can be translated as A will win 2 of 3 games. This is a probability of 2/3 or 66.67%. The quota is calculated as explained above as reciprocal, 1 : 2/3 = 3/2 = 1.5. This might be realistic. The quota for B is 1 : 1/3 = 3 in this case. Controlling the quotas we find that 2/3 + 1/3 = 1 – that is correct. But the commission is not included. If the student bookmaker wants to get 10% commission they should offer these quotas: 1.5 · 0.9 = 1.35 for A and 3 · 0.9 = 2.7 for B. After that the student bookmakers must **hope** that the other students make their bets as estimated – 2 of 3 bet on A and 1 of 3 bet on B.

Other student bookmakers might go in a different direction: A is much better then B. Everyone will bet on A – of course. What is a good quota for this situation? If A wins with a probability of 1 the quota for A is 1 : 1 = 1. In this case the quota for B as winner must be 1 : 0 = ??? (a very, very or infinitely high quota?). Both quotas (for A and B) will cause problems: If the quota for A is 1 and the bookmaker likes to get "his" 10% commission he will only pay the winner 90 percent of his money back. What will be the reaction of the winner? If against all odds B wins someone who has made a bet on him should get a infinite amount of money – which is impossible. The bookmaker cannot promise to pay the winner more than all the money in world. So the bookmaker has to offer a very high quota, for example 100. If you look on the internet, you cannot find such high quotas. If a bookmaker is thinking in such a direction he has to look out for practical limits - and he will have to offer at least 1,1 or something like that for A and no more then maybe 20 or 30 for B. The student bookmakers must hope that the other students make their bets as estimated – according to the offered quotas.

Step five is reflecting on the results of the first trial and planning the next steps. After the first experiments it is good to have a look back and reflect on the situation together. Students playing bookmakers have to be doubly aware of expectations: What do they expect that betting students are expecting? A second question is: does the offered quota have any influence on the gamblers? A third – and in reality the most important point – is: How much does the bookmaker want to earn? A first or later summary might be that a bookmaker offers good quotas if he gets his commission without risks (that is whoever wins). The offered quotas should be the same as they would be if they are calculated afterwards (as a totaliser does it). The risk of a bookmaker grows the more his quotas differ from those of the totaliser!

Maybe the students decide on doing more trials now that they have better background knowledge, to gain more experience and gather more documented data. Maybe they want to do more mathematics first. What are possible mathematical questions?

What happens if the first quota of a bookmaker is wrong? Wrong means that people don't bet as estimated. Let us calculate an example. A bookmaker offers 1.35 for A and 2.7 for B with the idea that the probability for A winning is 2/3 and that he wants to earn 10% commission as explained above. After a while this bookmaker realizes that 50% of the incoming bets go to A and 50% to B. What would happen if he does not react and change quotas? If A is the winner he has to pay 1.35 units for each unit of the 50% that have bet on A. In other words: He has to pay 67.5% of the money to those who have bet on A. This would be a very good situation for him – he would earn 32.5% of the money. But if B is the winner, he has to pay 2.7 units to each of the winners, which would be 135 % of the incoming money in total. In other words: If B is the winner, the bookmaker has to pay 35% more then he has earned! He has to pay those who bet on B some (or a lot) of his own money. This is the worst case scenario for him!

The calculations need more mathematical knowledge if we try to understand what would happen if the bookmaker changed his quota every day or each time 1000 units have come in. In fact this is much easier if we take a spread sheet and type in the data and the corresponding formula. The results of such an experiment are easy to understand. The bookmaker has the best chances to earn money by commission if he changes the quota very flexibly, following the incoming bets.

Expected results

Students know more about the calculation of bets and odds. They should have learned why and how a bookmaker earns money and who gives them money – the gamblers. I hope that they decide to avoid gambling. Last but not least they have learnt to organize themselves as learners.

References

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All internet links were checked in July 2009

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