Behaviourism, cognitivism, constructivism, or connectivism? Tackling mathematics anxiety with 'isms' for a digital age

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One of the major challenges facing practitioners in any field of adult mathematics learning is to achieve effective learning outcomes in the face of prevailing negative attitudes in their students, often present as a consequence of unsatisfactory early mathematics learning experiences. At all levels of numeracy and mathematics education the issues are essentially the same: traditional approaches to mathematics teaching do not work for math-averse students and there is a need to find new ways to tackle old problems. Here, some stereotypes of traditional methodologies, which have their roots in behaviourism and cognitivism, are considered in the context of behavioural and cognitive characteristics common among math-averse and math-anxious students. This motivates a re-framing of the practitioner's approach by outlining strategies for effective practice that align with a connectivist paradigm.

Introduction

There is a very clear link between endemic adult innumeracy in modern Western societies and the spectrum of mathematics anxiety, negative mathematics attitudes and aversion to the learning of mathematics that practitioners so often encounter among adult learners. It has been shown (Klinger, 2009) that these aversive affective behaviours are typically founded on negative mathematics learning experiences in the later years of primary education, during the transition from instruction in concrete procedures to increasingly sophisticated abstract concepts. This is a time when pupils' difficulties can be aggravated or even caused by the 'guidance' they receive from teachers who themselves may be math-averse and even covertly innumerate. Indeed, pre-service primary teachers with such negative attributes have been found to be disproportionately over-represented among undergraduate cohorts (*ibid*).

The challenge for teachers and practitioners in adult mathematics education at any level is to find effective ways to break through the barriers of anxiety and disaffection and to allow students to experience success, often for the first time, at a level that is at least sufficient for their immediate needs. Ultimately, the greatest achievement will arise when students can overcome their anxiety and aversion to become independent learners with the capacity to extend willingly their engagement with mathematics. It will be argued here that at all levels of adult mathematics teaching the issues are essentially the same: 'traditional' approaches simply do not work for math-averse students. This will not surprise most practitioners but what, specifically, does not work? Or, rather, what works better? To respond to this, the basis of traditional pedagogies and their underlying epistemologies will be considered in the context of behavioural and cognitive learning characteristics that typify math-averse and math-anxious students and which tend to undermine their learning goals.

The practitioner's challenge

Many adult learners have only a vague concept of what mathematics is really all about. They lack confidence in their abilities, often failing to realize that they actually and routinely engage in essentially mathematical thinking in their daily lives, and display characteristics that cause them to experience difficulty in their mathematics learning endeavours. The math-averse adult learner is typically overwhelmed by the task at hand of acquiring specific skills, dealing with math content, and developing effective learning strategies. The needed insights have to be facilitated from outside but they are blocked by the barriers of anxiety, negative beliefs and stereotypes (Klinger, 2004): students who are unclear about what is required of them tend to over- or under-estimate the difficulty of subject material and display little insight about the extent and relevance of their prior knowledge and skills. This is the practitioner's particular challenge: effective teaching strategies for these students exceed the 'usual' scope of mathematics teaching.

Commonly, confusion dominates the learners' behaviour because they do not know what it is that they 'don't know'. Repeated lack of success undermines their confidence and manifests in low self-efficacy beliefs, often accompanied by strong negative emotions of embarrassment, self-deprecation, and helplessness (reported also by Karabenick and Knapp, 1988). So, low expectations are common, as are lack of persistence and little interest in acquiring deeper understanding, with negative perceptions contributing to low self-esteem. The influence of such affective dimensions is strong, resulting in students' obvious lack of strategies to cope with their difficulties. With little capacity to generalise and make knowledge connections to aid understanding, motivation is likely to be assessment-driven rather than intrinsic. In particular, while students may recognize and accept the use of specialized language or jargon to describe mathematical concepts, they have little or no appreciation of the converse that, whatever else it may be, first and foremost, mathematics *is* language**.**

These characteristics are often associated in the literature with shallow, or surface, learning styles as well as avoidance behaviour. But, while students may exhibit such characteristics in association with their mathematics learning, it should not be assumed that the characteristics are generally typical of their learning style in other situations – I contend that in many instances an individual's mathematics learning style is anything but intrinsic and is instead directed by anxiety and prior mathematics learning experiences. In many respects, then, these characteristics are *reactive* and lead to self-defeating behaviours that undermine the learning situation unless the teacher/practitioner intervenes successfully. Since the cause of the behaviours is likely to be strongly related to past learning situations, any form of instruction that derives from a deficit model of remediation will just tend to repeat or re-visit negative early encounters in the mathematics classroom and tend to validate the student's poor perceptions. Rather, an entirely different framework is needed so that students have real opportunities to experience the epiphany they need to shed their history and to construct new understandings. This is a matter of good practice for which, as Swain (n.d.) pointed out, 'it is vital to understand the epistemological basis that underlies the teaching of numeracy in the adult classroom.

Epistemology and pedagogy

Behaviourism to social constructivism

'Skill and drill' teaching epitomises *behaviourism* in mathematics education – the 'ideal' learning environment focuses on hierarchical procedures and outcomes so that mastery of basic skills provides a scaffold to progressively more advanced activities. In this framework, mathematical knowledge is external, absolute, and teaching is didactic. Learning is seen as the correct application of appropriate algorithms to obtain correct answers, practiced by studying worked examples, with behaviour conditioned and reinforced positively by 'rewards' of success and approval or negatively by failure and disapproval (even to extremes of physical and psychological punishment, as commonly reported by sufferers of more severe mathematics anxiety). According to Orton (2004, p29), '...Exposition by the teacher followed by practise of skills and techniques is a feature which most people remember when they think of how they learned mathematics'. Orton goes on to explain that while the objective is to establish strong stimulus-response bonds, as teachers well know, these are usually short-lived – having a 'use it or lose it' impermanence – and uses the example of the addition of fractions, the algorithm of which is taught, re-taught, and practiced throughout early schooling only to be forgotten repeatedly. There is a long tradition of teachers adopting an essentially behaviourist approach in their mathematics teaching when they would reject such methods in other curriculum areas. Sometimes these are inexpert teachers. Sometimes they are teachers with highly-developed mathematical ability, whose unconscious competence and lack of awareness of the actual complexity of their expertise can lead them to pursue what they see as efficiency in the transmission of what is, to them, 'obvious' knowledge (Golding, 1990). This is not to suggest an absolute rejection of behaviourism but, instead, to question the means rather than the ends.

Cognitivism arose largely in response to behaviourism, with learning being seen as an adaptive process where knowledge may be transmitted between individuals but is stored as internal mental constructs or representations. S*ocial cognitivism,* which recognizes that learning is at least as much a *social* activity as it is behavioural and cognitive, fuses elements of behaviourism and cognitivism with social aspects of learning (Bandura, 1986). The theory emphasizes the importance of observational learning, by which behavioural and/or cognitive changes are brought about by the learner's comparative observations of others and of self, a process incorporated in the concept of self-efficacy beliefs (Bandura, 1997), which have a prominent role in the learning activities of math-averse students (Klinger, 2004-2009). For mathematics education, the influence of cognitivism places an emphasis on learning by problem solving as a recursive process, whereby a problem is interpreted by assigning it to existing internal representations or *schema* (Bartlett, 1932). It is an approach that has been shown to yield superior learning outcomes for more experienced learners, for whom worked examples become increasingly redundant (Kalyuga, Chandler, Tuovinen, and Sweller, 2001). The cognitivist approach does not supplant behaviourist practices, it augments them and behaviourist practices persist as the dominant mode of instruction when new (and, particularly, fundamental or foundational) procedures are introduced.

For at least three decades, *constructivism* has dominated as a learning theory and mathematics and science instruction is 'increasingly grounded in constructivist theories of learning' (Alenezi, 2008 p17). The central principle is that knowledge cannot be transmitted because it is a construct of the mind and learners have an active role in building understanding so as to make sense of the world, with learning resulting from an ongoing process of hypothesizing, rule-creation and reflection. The teacher surrenders the role of didactic authority to become instead an information conduit and facilitator of the learning process by providing students with opportunities to discover, explore and apply ideas that will satisfy their learning objectives. *Social constructivism* adds the further proposition that there can be no sensible definition of knowledge that ignores its social context. That is, knowledge must necessarily be grounded in the social values, standards, mores, language and culture by which the learner acquires an understanding of the world. In accepting that learning is a social activity, it follows that social interaction extends the location of knowledge from the individual to the community via shared understandings.

While constructivism dominates current pedagogy, I suggest that there are profound flaws in the context of mathematics and numeracy education. First, it demands curriculum outcomes that are all but identical to those of behaviourists and cognitivists – that is, a demonstrated ability to *perform* by applying appropriate procedures to a given situation to arrive at a correct result according to agreed conventions. Second, there are implicit assumptions that self-directed learners have 'sufficient prior knowledge and skills … to engage effectively and productively' (Rowe, 2006 p101) with their learning activities. However, elementary or foundational concepts, in particular, are not reasonably accessible to exploration and discovery. They need to be learned in essentially the same way as vocabulary and rules of grammar – no matter how they arrive at their ideas, students must know *what* to write and *how* to write it in order to reliably record and communicate them.

Last (and far from least), the word 'basic' is often applied to the everyday arithmetic of addition, subtraction, multiplication, division, fractions, decimals, and percentages. Those who 'can do' tend to use the word in a dismissive or belittling fashion with those who 'can't do' – "You should be able to do *that* [at least]… it's just basic arithmetic". Math-averse learners have heard statements like this over and over, often accompanied by disparagement, derision, frustration, anger and sometimes physical chastisement. While concepts of number, number representation, and arithmetic operations are certainly *fundamental,* I suggest that there is nothing *basic* (in the sense of 'simple' or 'obvious') about them – our species has taken millennia to invent, discover and formalize these ideas. It is neither reasonable nor sensible to expect or require students at any level to *actually* discover 'basic' mathematical concepts and corresponding procedures by pursuing a literal constructivist agenda.

The constructivist approach in mathematics teaching is illustrated by modern elementary mathematics text books, which look like they have changed considerably in recent years, tending to be colourful and inviting, with many diagrams and there are lots of 'real world' scenarios to supplement more formal aspects of the text. Also, practical exercises presented as 'experiments' identify opportunities for learners to verify various aspects by quasi-independent enquiry. While the idea seems to be to motivate students' engagement by providing social grounding for the material, closer inspection shows that changes are largely superficial: core materials (definitions, rules, procedures, algorithms and 'drill' exercises) persist, largely indistinguishable from texts compiled during pre-constructivist eras; that is, constructivist principles have been applied as a *veneer*. Teachers must still be able to demonstrate, via skills tests and league tables, that their practices satisfy benchmarked standards, while learners must demonstrate through standardized assessments that they have acquired sufficient mastery of the curriculum.

There are problems that can be masked by constructivist thinking. Davis and Maher (1990) considered a 'gifted' Grade 5 student's work (Figure 1). The pupil, Ling, was given a problem to solve along with a range of materials, including Pattern Blocks, which she used to correctly solve the problem. Her teacher then asked if she could write her solution and Ling first recorded the diagram. We can infer that she started by drawing a hexagon, indicating that she *knows* the correct answer at this stage – she's 'done the maths'. Ling's troubles began when she tried to write the solution, beginning with the first line in the figure, 1/3 ÷1/2 = 1/3 ×2/1 = 2/3. Davis and Maher wrote that: 'Ling is clearly a good student; she has the "invert and multiply" rule correctly. But she has produced an incorrect answer! How come? Because she has not called upon the correct algorithmic solution procedure. There is no reliable way to go from a problem statement to a solution procedure unless you get a correct representation of the problem.' (Davis and Maher, 1990 p75)

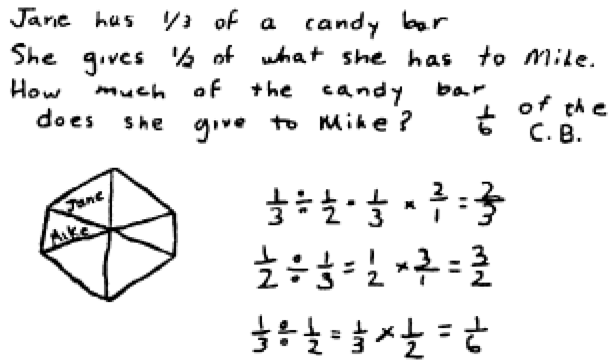

Fig. 1 The Candy Bar Problem (Davis and Maher, 1990 p75)

Ling's first attempt at writing the problem and solution did not match her experience – she failed to get the answer she knew she needed so she made two further attempts, finally appearing satisfied that she had at last written something that produced the answer that she 'knows' is correct. But of course, no doubt out of desperation, she has claimed that division and multiplication are the same thing in this instance. Now, this example is interesting not because of Ling's approach or (directly) because of the constructivist pedagogy – although it is indicative of constructivist thinking. Rather, it is interesting here because of Davis and Maher's observations, which focus on the student not having called upon 'the correct algorithmic procedure' (*ibid.*) instead of identifying that this example is not about problems with mathematics but, rather, problems for Ling with the language of mathematics, which is a different thing entirely. So Davis and Maher totally miss the real point! Ling's mistake was not in calling upon the wrong *procedure* but in her failure to correctly *translate* the English expression, 'half of what she has' (i.e. one half of one third), into the corresponding mathematics expression. This was neither a procedural error nor a retrieval error (as in faulty memory recall of a rule), it was a *language* error. No-one with a fluent grasp of mathematics as language could make such a mistake without seeing it immediately or on reflection, at least – which is something that Ling never did, apparently.

This simple example illustrates a very common feature of students' mathematics work (even those operating at advanced levels). They write to create or mimic the appearance of mathematics rather than writing to express meaning using maths language. Even many advanced-level students will write mathematics by manipulating symbols according to rules rather than, again, writing to express meaning so that the written mathematics tells a 'story'. The teaching of mathematics and numeracy generally focuses on the doing of it, without explicitly attending to mathematics as language and without developing students' fluency in that language.

Connectivism

The latest contender in educational theory, *connectivism*, was proposed by George Siemens (2005) as a 'learning theory for a digital age'. In proposing connectivism, he borrows, rather speculatively (even extravagantly), from the science of complexity, including chaos theory, networking, and self-organization. While the metaphor is technically flawed (for reasons that will not be pursued here), there is nonetheless considerable appeal in many aspects of the proposal: self-organization and emergent higher-order phenomena from self-referential complex systems are universal principles (Klinger, 2005). The complex and self-referential nature of the human mind is, perhaps, obvious, and in simplistic terms the human brain is certainly a network of interconnected neurons with memory and other cognitive activities being associated with changes to neural connectivity. But, rather than further considering such ideas now, I want to concentrate on just one aspect, which is encapsulated in Siemens' statement that connectivism 'posits that knowledge is distributed across networks and the act of learning is largely one of forming a diverse network of connections and recognizing attendant patterns' (Siemens, 2008 p10).

A connectivist approach to reframe practice

What I am suggesting here is that the particular value of the connectivism paradigm in mathematics and numeracy teaching lies in exploiting the properties of network connectivity in complex systems. By actively pursuing opportunities for students to forge links that promote an understanding of mathematics as *language*, we might help them to make connections that provide mappings between mathematical concepts and their various skills and understandings of the world. That is, mathematics language is to be understood in terms of things and language that the learner already knows.

The idea is that the connectivity attained by linking mathematical know-how, language and other skills from the student's existing knowledge base serves to build understanding and understanding forges fluency. When that happens, dependence on mathematical rules becomes redundant. The rules become obvious because they are just consequences of the mathematics language rather than algorithmic procedures to be applied mechanically.

As new connections incorporate more and more nodes of both congruent and disparate knowledge and experience, the internal and self-referential (reflective) knowledge network grows, undergoing periods of self-organizing criticality that might be thought of as *cognitive phase transitions* (to borrow physics terminology). That is, there are spontaneous flashes of emergent deeper understanding (epiphanies or 'ah-ha' moments) and increasingly, the learner is empowered to undertake self-directed learning according to need or inclination. Of course, this scenario is entirely speculative but, nevertheless, in terms of pedagogical practice, I am suggesting that there is substantial merit in considering mathematics first and foremost as language and focussing on ways and means to develop students' fluency while utilising their existing skills and knowledge-base as leverage.

For the teacher/practitioner working with math-averse and mathematically anxious adult learners, the learning situation, then, needs to be reframed so as to move away from traditional practices towards techniques that explain and demonstrate how the context and methods of mathematics are revealed through its application as *language*. That is, any mathematics that the student reads or writes must make sense – it must 'say something'. It must always be possible to translate freely in either direction: mathematics language to natural language and vice versa.

As with the learning of natural languages, students need to cultivate an 'ear' (or eye) for dissonance between what is understood and what is written or read and to develop the ability to self-correct. Whenever meaning is obscured, both the instructor and the learner should be alert for signs of inappropriate language construction or interpretation which may prompt the need to unpack whatever is being attempted to examine, test, and fix underlying language weaknesses or misunderstanding. I believe that such attention to language is essential as a first step in reducing confusion and anxiety and to broaden students' focus. With each advance, students gain confidence, overcoming their negative perceptions to discover intrinsic motivation for pursuing further learning as an end in itself, rather than simply to satisfy assessment requirements. Math-aversion and anxiety may never disappear entirely but future encounters with mathematics can at least be directed by informed, adult decisions rather than dominated by attitudes and emotions imposed by an unfortunate history.

Conclusion

On one hand, in the context of mathematics and numeracy teaching and learning for math-averse and mathematically anxious adult learners, the traditional 'isms' underpinning conventional pedagogy are in deficit and directly associated with aversive affective behaviours. On the other hand, many of the ideas associated with connectivism resonate with techniques and approaches that are known to be successful with math-averse and maths-anxious students. By invoking some of the properties of network connectivity in complex systems, connectivism provides a theoretical framework to explain mathematics learning and to reframe adult mathematics and numeracy teaching and learning practices. The paradigm of connectivism, at least along the lines indicated here, needs considerable research to become established as significantly more than a rhetorical device. Nonetheless, there is obvious intuitive appeal in the insights suggested by this new 'ism' that surely warrants further investigation.

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