Using the history of mathematics in teaching adult numeracy

David Kaye
LLU+, London South Bank University
d.kaye@lsbu.ac.uk

There is a popular image that mathematics (particularly school mathematics) is unchanging, based on knowing the correct method, and that all questions have one correct answer. In current teacher education for adult numeracy these images are challenged. For example, trainee teachers are encouraged to be aware of the many ways of doing 'long multiplication'. One of the most powerful techniques available for challenging these beliefs is introducing and using topics from the history of mathematics. By exposing trainee teachers and numeracy learners to the mathematics that has evolved over the last 5000 years all over the world, from many contrasting cultures, makes many aspects of mathematics more accessible. This workshop will introduce some key themes from the history of mathematics and demonstrate how they deepen an understanding of the nature of mathematical knowledge and provide access to mathematical concepts at all levels.

Introduction

This workshop was a practical event that provided the participants with the opportunity to work collaboratively on activities which introduced some aspects of the history of mathematics. There was also an underpinning assumption that using (rather than teaching) the history of mathematics is a powerful tool in teaching mathematics and adult numeracy.

The literature about using the history of mathematics in teaching is complex and far-reaching; sometimes focusing on this as a specific pedagogy, but sometimes having an epistemological emphasis. The workshop itself did not enter into this debate, but by way of introduction some of these issues will be considered here. During the last 20 years interest in the broader use of the history of mathematics in teacher education has been nurtured by a comparatively small group of practitioners and researchers. Key publications that identify this are the special edition of *flm* in June 1991 (Fauvel, ed.) the *flm* edition of February 1997 and the report of the ICME study conference held in April 1998 (Fauvel & van Maanen, eds. 2000). A summary of suggested 'whys' and 'hows' of using the history of mathematics has recently been published (Jankvist, 2009).

Theoretical background

In all these papers arguments are proposed and countered about the purpose of using history in teaching mathematics. This unfortunately seems to continue to be necessary. I say unfortunately, as in very few other subjects is it ever considered necessary to explain or justify why the history of the subject is a necessary part of learning it.

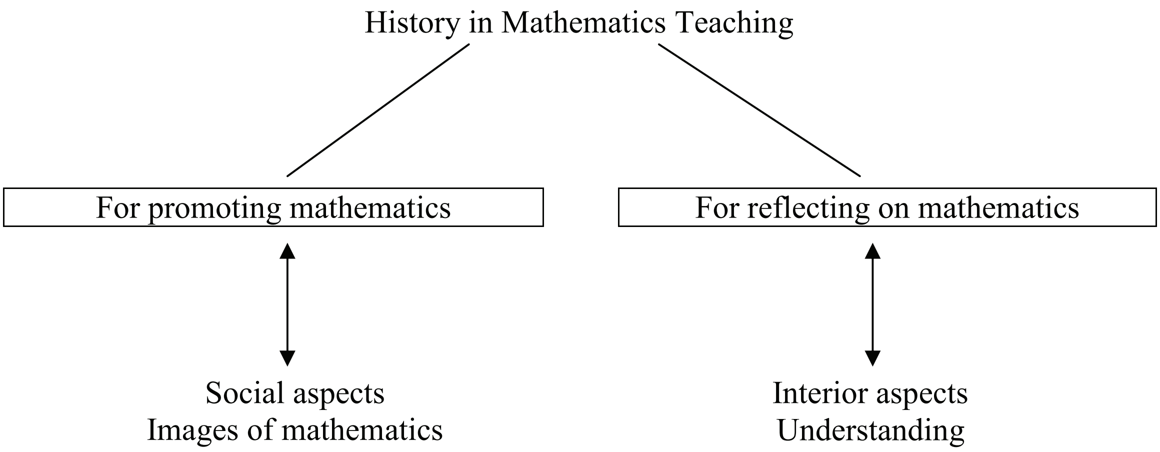
In a recent paper Fulvia Furinghetti (2007) has re-visited these reasons in the context of teacher training, which suits my purpose exactly. She summarises the views previously published in the ICME study (referred to above) in which three reasons are succinctly labelled as 'cultural understanding', replacement' and 'reorientation'. These all serve to create new ways of looking at the mathematics under consideration.

Cultural understanding for example makes connections with the history of ideas, 'replacement' indicates an approach that contrasts familiar and recognisable mathematical techniques with equivalent techniques from the past. 'Re-orientation' urges teacher educators to help their trainees to re-construct the most familiar and elementary mathematics.

She explains her particular purpose is:

> … to make the participants reflect on the meaning of mathematical objects through experiencing historical moments of their construction. It was intended that this reflection would promote an appropriation of meaning for teaching mathematical objects that counteracts the passive reproduction of the style of mathematics teaching the prospective teachers have experienced as students.

In an earlier paper Furinghetti (1997) produced a schematic summary identifying a dual approach.



Many of those writing about history in teaching mathematics focus on school education (see for example the introduction to TSG 23 for ICME 11, 2008). Within school mathematics, the examples are focused on those who have already demonstrated an understanding and facility with a wide range of mathematical concepts.

Another concern is given by Luis Redford (1997), who focuses on a critique of the history of mathematics within the broader context of the history of science. He considers that the history is often presented as a series of events in which "the books unfold episodic narratives implicitly underlain by an apriorist epistemology of platonistic style". This leads, as many traditional histories do, to reinforcing what we now do is 'right' and history merely shows the various twists and turns taken to get to what is right (now).

This theoretical introduction firmly places the inspiration of this workshop into an evolving epistemology of numeracy. This approach borrows large themes from histories of mathematics and sifts from them snippets of words and methods relevant to those just discovering the germ of mathematical experience inside them.

As Luis Redford (1997) says

> Data is never interesting in itself. Historical data will always be interesting with regards to the conceptual framework upon which the research program relies. Furthermore, the framework makes it possible to offer a theoretical explanation of the data. In the case of epistemological inquiries about the history of mathematics, the search for interesting historical data will be shaped particularly by our own perception of how mathematical knowledge grows.

However, unlike most who use the history of mathematics in education, I have focused on the elementary and basic mathematics that falls within the current remit of most adult numeracy teaching practice. Working with students who have very little grasp of mathematics as a discipline, and who doubt even their ability to use the most basic concepts of number, these theoretical debates are non-existent. This does not mean these learners should be excluded from the history of mathematics. Quite the opposite: the history of mathematics should be used to beat down some of the barriers these students have built up through a lifetime of poor mathematical experiences which have led to fear and avoidance of mathematics education (see for example Coben et al, 2003).

To encourage the use of the history of mathematics with adult numeracy learners, activities need to be introduced to adult numeracy teachers on training or professional development courses to encourage their interest in the history of mathematics . (This short statement begs a lot of questions. The development of the history of mathematics in maths education programmes continues to be the main focus of much research and occasional experimentation; see for example the topic group reports from ICME Conferences.) Some other strategies for using the history of mathematics in teacher training courses for the post-16 sector have been previously published; see for example the recent edition of *Mathematics in Schools* (Kaye, 2010).

One key activity developed to address this adult numeracy context is a ‘human time line’; a kinaesthetic ordering activity. This is not only a beginners' introduction to the history of mathematics, but also models using training techniques which are focused on working to a range of learning preferences or styles. See for example the report on learning styles and adult numeracy (Kirby and Sellars, 2006).

Time line of mathematical events

The key activity in the workshop was a human mathematics time line. The selection of ‘knowledge’ begins with the choice of events for the time line cards. The events chosen take into account the target audience who are not expected to have much familiarity with higher level mathematics or current mathematical research. At a minimum level the events chosen provide a demonstration of the history of mathematics across 5000 years and many cultures. Currently there are 25 items in the time line and the full list of event cards is given in the appendix. This list shows both the ‘time line card’ text in the middle column and the brief explanation that is given afterwards (besides the date and order of events) in the right hand column. For example “Defining a point, a line and a surface” is the text handed out on the card. The brief explanation given on the handout and on the Power Point presentation is

"Euclid's elements – classical Greek geometry", and the date in the left-hand column is -300 BCE.

The activity itself is to give each person a mathematical event card and instruct the group as whole to try to sort themselves into the chronological order in which these events (as given on each card) occurred. This means that a reasonable attempt can be made at a rough order without having to have specific knowledge about even the century in which something happened. The time line activity can serve as an introduction to the history of mathematics in many situations. It is in the discussion that follows and any additional activities, that the teaching event or session can be focused on the needs of a particular group of participants. Observing the participants sort themselves into some sort of order provides an excellent opportunity to assess the level of historical knowledge of the group and individuals.

As referenced earlier, this can be seen as an example of what Furinghetti has called 're-orientation'- providing a new or original view of something considered familiar or ordinary. Certain topics are picked out for this. One of the most exquisite is making the connection between the Babylonian sexagesimal counting system and our current system of time measurements, which still uses a base 60 system. To indicate to numeracy trainees and learners that when they are converting hours to minutes they are performing 5000-year-old mathematics, is a powerful demonstration of 're-orientation'.

One other re-orientation concerns fractions. Any adult numeracy specialist knows working with common fractions is famously difficult; see for example the NRDC Fraction booklet (McLeod & Newmarch, 2006). If anything brings out 'maths phobia', it is manipulating common fractions. The introduction of the term 'broken numbers', rather than 'fractions', is inspired by 17^{th} century English arithmetic text books (such as Robert Recorde's *The Ground of Artes*). This simple change of label produces a re-orientation. The fear associated with 'fractions' is minimised, to be replaced by a more holistic view which can encompass whole and broken numbers with equanimity.

Common fractions is in fact a good example of how a theme can be extended from a time line 'event'. Common fractions are too extended and diverse a topic to be easily identified with a single event. The 'broken numbers' link can be introduced in connection with Robert Recorde, who appears in the time line as the 'inventor' of the equals sign.

One other important aspect of items chosen for the time line are various events that can be connected to the evolution of what is introduced in school mathematics as algebra. Here various events such as word problems in the Ahmes Papyrus (1550 BCE), the origin of the word 'algebra' (825) and the introduction of letter symbols (1580) can be used to disassociate algebraic techniques for solving problems from a system of writing 'calculations' with letters. It also shows that such complex structures as symbolic algebra took a long time, across many cultures, to develop.

Additional activities

The time line can be used to prompt a wide range of topics. The appendix includes an activity sheet designed to introduce ten topics that were particularly chosen for those aiming to use the history of mathematics in a context where the culture aspects are significant. These are categorised under three headings: *writing numbers*, *calculating aids and methods* and *origins of current mathematics.*

The introduction of different numeral systems (the written symbols that are used to represent numbers) has considerable potential for many different types of learners. Those familiar with written sources from other cultures, including different alphabets either from the past or other parts of the world, can feel comfortable with alternative numerals. Employed this way cultural artefacts add richness and depth to the learners' development of fundamental numeracy concepts. The use of place value underpins most of the calculating procedures evolved using our current system. Looking at an alternative system, such as Roman numerals, quickly exposes the fact that some numeral systems (such as the Roman) do not use place value. (A quick note of explanation: in our system a numeral can take different powers of ten; in 51 the five is five tens i.e. fifty, however in 105 it is five; the V in the Roman system is worth five (or quinque) wherever it is written.) Having established this significant difference teaching and learning can develop more deeply in many directions. The contrast between a place value and non-place value system may be a method that helps expose the learner's deep misconceptions about calculating techniques. The place value system remains hidden from most people as it is learnt alongside counting. The few lessons that focus on this in primary education are probably long forgotten. Trying to do calculations (such as addition and subtraction) with Roman numerals can begin to expose calculating habits that have become automatic, and therefore to begin the re-construction of more efficient habits.

The examples of calculating aids can also be developed in many potential directions. Perhaps at the heart of most of them is the question that anyone teaching any level of numeracy or mathematics faces at some time in their career: the calculator question. Is the calculator an essential labour saving tool which enables mathematics to be used effectively or is a calculator at best a prop or at worst a cheat for those unable to do or understand their mathematics properly? Placing the calculator in the context of historical aids for calculating helps to put this question into perspective. Mathematical tables have an extended and worthy history, though most are now considered largely redundant. However, multiplication tables and the multiplication square are still exceptionally valid tools. Placing these in the context of a mathematical tool with a long history can raise their status, and therefore make their use more acceptable. The use of the electronic calculator can then be re-examined alongside the use of other calculating aids used throughout history.

Conclusion

This account of a workshop on using the history of mathematics in teaching adult numeracy has introduced and described a key group activity – a human time line. The reasons for using the history of mathematics in adult numeracy are considered , with some reference given to some key debates and research positions which explore connections between maths education and the history of mathematics. The debate identifies two focii: the role the history of mathematics should have in teacher education programmes and the influence the history of mathematics has on the nature of mathematical knowledge. These can be seen as issues of pedagogy and of epistemology respectively. Finally a number of activities are identified to demonstrate the usage of the history of mathematics in the adult numeracy context.

References

Coben, D. (with Colwell, D., Macrae, S., Boaler J., Brown M., & Rhodes, V.) (2003) *Adult numeracy: review of research and related literature*. London: NRDC pp 92-109

Fauve, J. and van Maanen, J. Eds. (2000) *History in Mathematics Education: the ICMI Study.* Dordecht: Kluwer

Fauvel, J. (Ed.) (1991) *for the learning of mathematics 11.2*

Furinghetti, F. (1997) History of Mathematics, Mathematics Education, School Practice: Case Studies in Linking Different Domains in Wheeler, D (Ed.) *for the learning of mathematics 17.1*

Furinghetti, F. (2007) Teacher Education through the history of mathematics Educational Studies in Mathematics Springer Netherlands _Volume 66, Number 2 / October, 2007

Jankvist, U. T. (2009) A categorization of the "whys" and "hows" of using history in mathematics education *Educational Studies in Mathematics* Springer Netherlands Volume 71, Number 3 / July, 2009

Kaye, D. (2010) Penetrating the Inevitable Certainty of Error - Using the History of Mathematics in Teacher Training for the Post-16 Sector *Mathematics in Schools May 2010 Vol. 39 No. 3* (pp 9 – 11)

Kirby, J. & Sellers, T. (2006) *Using Learning Styles in adult numeracy: an exploration of their value and potential in teacher training.* London: NRDC.

McLeod, R. & Newmarch, B. (2006) *Fractions*. London: NRDC also available from www.ncetm.org.uk/files/257666/fractions_booklet.pdf

Radford, L. (1997) On Psychology, Historical Epistemology, and the Teaching of Mathematics: Towards a Socio-Cultural History of Mathematics in Wheeler, D (Ed.) *for the learning of mathematics 17.1*

Topic Study Group 23 (2008) *Aims and Focus* ICME 11 http://tsg.icme11.org/tsg/show/24 (accessed June 2010)

Wheeler, D. (Ed.) (1997) *for the learning of mathematics 17.1*

Appendix

Time Line Activity

Key themes from the history of mathematics in teaching Language and Numeracy

Writing numbers
- different symbols (numerals)
- with or without place value (eg Roman numerals do not have place value or zero)
- different base (eg time, like Babylonian number system uses base 60)

Calculating aids and methods
- abacus or counting board
- tables (like multiplication tables)
- calculating methods (ways of setting out long multiplication)

Origins of current mathematics from many sources
- early counting from Africa
- current numeral system and zero from India
- using algebraic methods from Middle East and North Africa

- development of calculus from Europe

Consider in your groups how one of these key themes from the history of mathematics can help to make connections between language and numeracy teaching.

It may help to consider a particular group or student you have worked with.

Mathematics in History and Society

Summary of Time Line Events

DATE (approx.)	TOPIC	DETAILS
-3000 BCE	Counting in 60s (like time)	Babylonian counting system in base 60 found on clay tablets
-1550 BCE	Word problems – finding missing amount	Problems that may now be solved by algebra in Ahmes papyrus, written by an Egyptian scribe
-1500 BCE	12 hour day	Dividing the day into 12 hours – ancient Egypt
-1000 BCE	Chinese magic squares - "Lo Shu"	The origin of these three by three arrangements of numbers are mythical, but they have been recorded continuously since 200 BCE
-300 BCE	Defining a point, a line and a surface	Euclid's elements – the basis of classical Greek geometry
-200 BCE	Chinese rod numbers for calculating on a checkerboard	A system for recording numbers and calculating in a base 10 system. Colour coding used to identify negative numbers at a later date
-100 BCE	Development of 'Roman Numerals'	The Roman numerals as we know them developed over a very long period of time, and continued to develop into the Middle Ages. Most signs were in use by this time
600 CE	Discovery of Zero	India Zero as place holder 458 CE Zero as a number 628 CE
700 CE	Finger counting in Britain	Bede's publication explaining finger 'numerals' – North East England
825 CE	Origins of the word algebra	The origins of the word algebra in Al Khowarizme's work - Baghdad
1200 CE	Base 20 counting system in Africa	Complex counting system using subtraction with base 20 from Yoruba
1150 CE	Algebra (of Al Khowarizme) into Western Europe	Arabic knowledge reaches western Europe mainly through contact made in the crusades
1200 CE	Current numerals (with zero) introduced to Western Europe	The currently used system of numerals, with zero introduced in Fibonacci's '*Liber abaci*' – Pisa, Italy
1478 CE	Lattice method of multiplying	Lattice method (gelosia) first printed description in Treviso - Italy
1500 (about)	Knotted strings (called 'quipu') to	The system of recording numbers by the Inca civilisation in South America (mainly Peru). The

	record numerical information	system may have been in use for some time before this date.
1557	The equals sign	Introduced by Robert Recorde in his *Whetstone of Witte* a book on algebra published in London in English
1580	Introduction of symbolic algebra	Use of symbols (vowels for unknowns, consonants for knowns) in work of Viète (Brittany and Paris)
1642	First mechanical calculator	Invented by Blaise Pascal (1623-62) who is also associated with the earliest work on probability
1687	Invention of the calculus	Published by Newton (England) and Leibnitz (Germany) 'independently' - major priority dispute
1800	Metric system of measurements	The metric system designed during the French revolution
1830	Non-Euclidean geometry	Part of the beginning of modern mathematics in 19th century – Lobachevsky (Russia) and Bolyai (Hungary)
1848	Development of Boolean algebra	Created by George Boole in a book describing "a calculus of deductive reasoning". It formed the origins of 'abstract algebra' and is now important in using internet search engines.
1855	Polar Area Diagrams for mortality figures	Florence Nightingale created the Polar Area diagrams, or "coxcombs" as she called them. These were used to give a graphical representation of the mortality figures during the Crimean War (1854 - 56).
1937	Definition of the decibel	The measures of the intensity of sound agreed at a meeting
1960	Establishment of the SI units	The international system of units was agreed for use in science and technology