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Communicating mathematics: a historical and personal journey

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For the past forty years I have attempted to communicate mathematics to a wide range of audiences – through teaching at the Open University and elsewhere, public lectures, books, articles, television, and other means. This illustrated lecture explores these in the context of the wide range of ways in which mathematics has been communicated over the past 4000 years.

Introduction

The topic of this plenary is 'Communicating mathematics' - something we all try to do through our various teaching activities and publications.

I was first directed to think about what this involves (as opposed to actually doing it) by a 1990 examination paper for our history of maths course, which I was studying as an Open University student. While sweating it out in the exam room I saw Question 15 that began: "Describe some of the ways in which mathematicians have communicated their results to each other." - a question I greatly enjoyed answering.

I won't have time today to cover every form of communication, but here are some ways that maths has been propagated over the centuries. I've divided my lecture into two main parts - *the spoken word*, on everything from lectures and TV broadcasts to casual conversations in the corridor, and *the written word*, on everything from research papers and books to newspaper articles and websites. In each part I'll try to give both a historical and a personal account, with a wide range of examples covering 4000 years.

Part I: The spoken word

So let's start with the spoken word, and it seems appropriate to begin with *inaugural lectures*. Historically, these seem to be mainly of two types: those that presented and explained original research in a wider context than is usually possible, and others that were more of an expository nature.

An example of the latter was that of Christopher Wren, appointed at age 27 to the Astronomy Chair at Gresham College, who noted that London was particularly favoured

> 'with so general a relish of mathematicks and the liberal philosophia in such measure as is hardly to be found in the academies themselves'.

and concluded by enthusing that:

> Mathematical demonstrations being built upon the impregnable Foundations of Geometry and Arithmetick are the only truths that can sink into the Mind of Man, void of all Uncertainty; and all other Discourses participate more or less of Truth according as their Subjects are more or less capable of Mathematical Demonstration.

An example of the former type of inaugural was G. H. Hardy's 1920 lecture as Savilian Professor of Geometry in Oxford. Hardy was interested in an old number theory result of Edward Waring, that *every positive whole number can be written as the sum of at most four perfect squares, nine perfect cubes, and so on*, and much of his inaugural lecture was devoted to it, leading to several celebrated papers on the problem with his colleague J. E. Littlewood of Cambridge.

Some 60 years earlier, in Germany, two inaugural lectures had transformed the world of geometry. At the beginning of the 19th century there had been essentially only one type of geometry around - *Euclid's geometry*, the one we're all familiar with. But as the century progressed, other types of geometry appeared: *projective geometry*, with its origins in perspective painting, and *non-Euclidean geometry*, that defied one of Euclid's postulates. In his celebrated inaugural lecture at Göttingen in 1854, Bernhard Riemann explained a new distance-based geometry, now called *Riemannian geometry*, which was to be of great importance in Einstein's theory of relativity long after Riemann's death. Eighteen years later, in Erlangen, Felix Klein presented what is now called the *Erlanger Programm* in which he tried to sort out the mess that geometry had become - *so many geometries: how can we make sense of them?* His answer involved the algebraic idea of a *group*, a subject that is now studied by our undergraduate students.

Connoisseurs of inaugural lectures have a soft spot for the two given by James Joseph Sylvester. At Oxford he'd been appointed to the Savilian Geometry Chair at the age of 69 (no fixed retirement age then), and gave an inaugural lecture in 1885 in which he outlined the current state of algebra, with particular reference to his work on invariant theory, and recited a sonnet he'd specially composed on noticing that an expected term in b^4d did not appear in a table of reciprocantive protomorphs (whatever they are):

Lone and discarded one! Divorced by fate,

From thy wished-for fellows — whither art thou flown?

Where lingerest thou in thy bereaved estate,

Like some lost star, or buried meteor stone? …

Nine years earlier, he'd been appointed the first professor of mathematics at the newly-founded Johns Hopkins University in Baltimore. He based his inaugural lecture on a supposed link between chemistry and algebra, and later described how he came up with the idea in a wonderful paragraph of the type that you no longer find in mathematical journals. Paragraph 2 of his lengthy research paper on chemistry and algebra consists of just one sentence:

> Casting about, as I lay in bed last night, to discover some means of conveying an intelligible conception of the objects of modern algebra to a mixed society, mainly

> composed of physicists, chemists and biologists, interspersed only with a few mathematicians … (the sentence rambles on) I was agreeably surprised to find, all of a sudden, distinctly pictured on my mental retina a chemico-graphical image, etc., etc.,

This discovery led to his using the word *graph* for the first time (in the graph theory sense), and to his use of graphs to represent algebraic invariants.

Unfortunately, inaugural lectures don't always take place, for a variety of reasons. In 1827 the newly appointed Savilian Professor at Oxford expected to give his inaugural lecture: this was the Revd. Baden Powell, father of the founder of the Boy Scout movement. But mathematics in Oxford was then in low esteem and Powell was advised not to give his lecture as he'd most surely not attract an audience.

Of course, these aren't the only one-off lectures. Gresham College in London has been presenting lectures to the general public since 1597, while mathematicians immediately think of the famous lecture by David Hilbert at the Paris International Congress in 1900, where he set the mathematical agenda for the next century by discussing a number of important problems that he wished to see solved.

Another celebrated lecture took place in Cambridge in 1993, when Andrew Wiles announced that he'd proved Fermat's so-called 'last theorem', a result that had remained unproven for 350 years. Although his announcement turned out to be premature - it took two more years to finish the proof - his lecture caused excitement in the national press, who are not usually very interested in mathematics.

Indeed, many invited plenary lectures at conferences take one of the two forms of lectures I've described: the presentation of recent research by a mathematician who has made a notable discovery, or an overview of a whole area of interest to the audience. Both types of lecture are important and valuable.

Sometimes, it's a whole series of lectures that's remembered many years later. One example was the 1893 Gresham College lectures on probability by Karl Pearson, entitled *Laws of Chance*, in which the familiar terms 'standard deviation' and 'histogram' were used for the first time.

Another example, many years earlier, were the lectures given by Hypatia, probably the most celebrated woman mathematician of the ancient world. Although mainly remembered for the grisly death she suffered at the hands of the Roman soldiers, she was a geometer of some renown, being Head of the Neoplatonic School in Alexandria, and is credited with impressive commentaries on many classic texts, such as Apollonius' *Conics* and Ptolemy's *Almagest*. She was apparently such a fine expositor that people came from miles around to hear her geometry lectures.

However, not all lectures have been so popular. Those of Isaac Newton, the second Lucasian Professor of Mathematics in Cambridge (the post now held by Stephen Hawking) were apparently so abstruse or poorly delivered that, according to his assistant, "so few went to hear Him, & fewer that understood him, that oft-times he did in a manner, for want of hearers, read to the Walls". And his bitter rival Robert Hooke, probably my most distinguished predecessor as Gresham Professor of Geometry, was very conscientious about his lectures, but frequently failed to attract an audience. As his diary records:

> No auditory came morning or afternoon so I read not … No lecture but a rusty old fellow walked in the hall from 2 until almost 3 … Only one came, peeped into the hall but stayed not …

These days, the lecture is the usual vehicle in universities for transferring material from the notebook of the lecturer to the notebook of the student, sometimes (it is said) without going through the mind of either. It's considered an efficient vehicle for communicating the subject, especially when there are large numbers of students, but has the obvious disadvantage that it forces everyone to move at the same speed.

Indeed, this is always our challenge: *how do we design a lecture that's not too fast for the weaker students, or too slow for the stronger ones*? Or, for a general lecture such as this one, *how do I choose material that's accessible to most of the audience, yet also contains something of interest to the specialists*?

In spite of the inherent disadvantages of lectures, I don't propose that we get rid of them. We've all been to lectures that excited and inspired us, and there's no substitute for them.

Meeting informally, such as happens at summer schools or conferences, is an important means for communicating. Informal chats over lunch, or in the corridor or the bar, can be as productive as the more structured teaching sessions, such as classes or tutorials, just as networking at meetings and conferences can be more important than the organised programme. Indeed, we should never underestimate the casual conversation as a way of communicating maths, whether it's with our colleagues in a corridor or in a brief discussion with a distinguished mathematician at a conference.

A nice example of this occurred in 1871. A well-known problem, solved in the 18th century by Leonhard Euler, concerned *Königsberg*, a city of four land areas linked by seven bridges: the problem was to find a walk that crossed each of the seven bridges just once. Euler proved that no such route exists, and produced a recipe for deciding, for any given arrangement of land areas and bridges, whether such a route is possible. But he failed to show *how* to find it, and it wasn't until 130 years later that a young Viennese mathematician named Karl Hierholzer filled in the gap, described his proof to a colleague in a corridor, and then promptly died. Fortunately this colleague, Christian Wiener, had understood the proof and wrote it up for publication.

Mathematicians have gathered together since earliest times. In Plato's Academy men and women met to discuss geometry, number theory, philosophy and other subjects of interest: over the entrance were the words "Let no-one ignorant of geometry enter these doors" (an inscription I'm still trying to get installed at the OU). Another early example was the Pythagorean brotherhood, while modern gatherings include the British Society for the History of Mathematics, the European Mathematical Society and the London Mathematical Society.

Some interesting gatherings took place in the 1620s when Marin Mersenne initiated regular meetings in Paris at which mathematicians could meet to talk about their latest findings. Mersenne himself described it as:

> the most noble academy in the world which has recently been formed in this town. It will no doubt be the most noble for it is wholly mathematical.

Mersenne's meetings led eventually, after his death, to the founding by Louis XIV of the French Royal Academy of Sciences.

In London, the Royal Society, founded at Gresham College in 1660, provided a forum for developing the experimental sciences and presenting practical experiments to the general public, while two hundred years later saw the formation of the London Mathematical Society, with Augustus De Morgan as its first president. Today the LMS presents a range of communal activities, from specialist mathematical talks by world experts to popular lectures for the general public. In 2002 I was particularly pleased to be able to organise a special LMS meeting to celebrate the 150th anniversary of the *four-colour problem*, which asks you to prove that the countries of every map, however complicated, can be coloured with just four colours so that neighbouring countries are differently coloured. This problem took 124 years to solve, and Appel and Haken (the first to solve it) both spoke at the meeting.

Following the success of his 2005 TV programme on *The Music of the Primes*, which achieved TV audiences of well over a million, Marcus du Sautoy has recently travelled the world filming an exciting series of four one-hour BBC-OU TV documentaries on *The Story of Maths*, due to be broadcast this autumn. The OU is planning a short course and some schools material to accompany these programmes, and I'm hoping to show two of them at the Joint Winter meeting of the American mathematics societies in Washington next January.

Other TV programmes have communicated mathematics to the general public — Jakob Bronowski's TV programmes in the 1950s, for example, and the Royal Institution Christmas lectures, which three times have featured maths - with Christopher Zeeman in 1978, Ian Stewart in 1979, and Marcus du Sautoy in 2006.

There's even the occasional film. There is a graph theory question I've sometimes set to students that appeared in *Good Will Hunting*. Some plays have featured mathematics too, such as *Proof* and Tom Stoppard's *Arcadia*.

And on radio, Melvyn Bragg's programmes *In our Time* regularly deal with mathematical topics. Some recent examples have been on Archimedes, probability, maths and music, symmetry, Newton's *Principia*, and Poincaré: such programmes bring mathematical ideas to some two million listeners.

There are many other examples of verbal communication of mathematics which I won't have time for today - from the teaching of scribes in ancient Egypt to the modern school classrooms of today. But I'll end this part of the talk with a bit of a moan.

Over the past 40 years I've given some 1400 one-off lectures (recently over 50 per year) to a wide range of audiences: university seminars, conferences, groups of sixth-formers or undergraduates, and audiences (both general and specialist) of interested adults. I've always regarded a lecture as a performance, and my topics have usually been from graph theory or the history of mathematics, both of which conveniently lend themselves to popular treatment.

Several of my lectures have been to audiences who don't normally encounter mathematics, and this provides a good opportunity to spread the word - to 'raise the public awareness of mathematics' in a small way. But this is something that the mathematical community is generally rather bad at doing. There are the occasional exceptions - for example, there's been a great deal of mathematical activity in Germany as they celebrated 2008 as their 'Year of Mathematics', and there's also a travelling exhibition called *Experiencing Mathematics* which has been touring the world for the past three years attracting crowds numbering hundreds of thousands.

But so often we miss the boat. As a trivial example of a missed opportunity, our trains, buses and planes are full of people enjoying the delights of combinatorial mathematics as they work on their latest sudoku puzzles, but the mathematical world has hardly taken up the opportunity to capitalise on this. There's still a widespread belief, which we've failed to counteract, that sudoku has no connection with mathematics, apart from the appearance of the numbers 1–9. Indeed, a couple of years ago I was invited by the Danish Mathematical Society to give a lecture on sudoku: the Society was in dispute with a national newspaper whose sudoku column assured readers that 'No knowledge of maths is involved - it's all logic', as if there were a difference. The bargain they eventually struck was that the paper would cease printing that sentence if I would give a sudoku lecture for their readers, and it was duly reported on nationally the next morning.

Another opportunity we missed for communicating our subject was the 300th anniversary last year of Leonhard Euler, the most prolific mathematician of all time, but one who's largely unknown to the general public. The three places he was mainly associated with (Basel, Berlin and St Petersburg) celebrated his anniversary in style, as did the United States, but we did very little.

As I said earlier, in order to spread the word further, I sometimes lecture to audiences that don't normally encounter mathematics. I've lectured to philatelists about mathematical stamps, to the University of the Third Age about Isaac Newton, and to the Lewis Carroll Society about the mathematics that pervades his works; indeed, I have published a book on the subject. I've also written a one-hour play about Carroll's mathematical life, which has been performed to various audiences around the world.

A particularly fruitful area for disseminating mathematical ideas is through music, an idea that goes back to Pythagoras' time. Marcus du Sautoy has given several radio talks on the links between maths and music, and I've given live presentations on the subject as a Royal Institution Friday Evening discourse (a scary experience) and at the 2006 Cambridge Music Festival, where due to its popularity I offered to lecture at 3 pm as well as the advertised time of 5 pm, and the poster announced, 'Due to popular demand, Robin Wilson's lecture will be repeated two hours earlier'.

I was also fortunate to be invited to take part in the Radio 3 programme *Private Passions* (a sort of up-market *Desert Island Discs*), where I described some connections between the two subjects. One of the pieces I chose has a nice mathematical structure: first, the two recorder parts are in exact canon right through the piece, with the second part repeating the first one exactly two bars later, and then, underlying all this, is a ground bass, a six-bar phrase that repeats throughout.

Part II: The written word

We now move on briefly to the written word. From earliest times people have communicated their mathematical ideas in writing - indeed, some of the earliest examples of writing are Mesopotamian financial accounts imprinted on clay tablets. Other clay tablets were used to teach mathematical ideas[1] while in Egypt the famous Rhind papyrus was probably used for the instruction of scribes[2].

An important means of written communication was by letter. A well-known example, still in print, was Euler's famous *Letters to a German Princess*, in which he described a range of scientific and mathematical topics to the Princess of Anhalt-Dessau.

Earlier, around 1600, Mersenne (whom you met earlier) carried out an extensive correspondence with all the principal scientists of his day, such as Descartes and Fermat, acting as a sort of European clearing house for scientific discoveries. Other examples of letters are one from Archimedes describing the squaring of the parabola, and a well-known one from 1832, written by Evariste Galois to his friend Chevalier on the night before he was killed in a duel, containing many of his discoveries in what later became known as group theory.

Two letters of particular interest to me relate to combinatorics. The first, sent by Euler in 1750 to his St Petersburg friend and colleague Christian Goldbach, contained the first appearance of the polyhedron formula: *in any polyhedron, the number of vertices plus the number of faces is equal to the number of edges plus 2* - for example, a solid shape with 24 vertices, 14 faces and 36 edges has 24 + 14 = 36 + 2. The other was the letter that initiated the four colour problem, when De Morgan wrote in 1852 to his friend William Rowan Hamilton in Dublin, about 'a fact, which I did not know was a fact, and do not yet'.

But letters don't always communicate. Hand writing was notoriously difficult to read, as this letter from J. J. Sylvester to Felix Klein shows:

> Dear Professor Klein: I find some difficulty in making out some of the words in your highly esteemed letter and would take it as a great favour if you could write a little more clearly for my behalf as I am not very familiar with German handwriting.

These days, letters have largely given way to e-mail and the web as convenient vehicles for communicating mathematics. These have the advantage of being much quicker, but will cause problems for the historians of the future who won't have access to them.

In fact, mathematical writings can take many forms - articles such as this one, scholarly texts, research papers in journals, popular books, and Open University correspondence texts.

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[1] Delegates were shown an image of clay tablets with a 9-times and a 5-times table (eds.)

[2] Delegates were shown an image of part of the Rhind Papyrus, showing some problems from geometry (eds.)

Like lectures, mathematical books range widely from research works, designed for the specialist, to popular accounts. An example of the former is Newton's *Principia Mathematica*, one of the most important scientific books of all time.

Since the invention of printing in the 15th century, books have been accessible to all at low cost. The earliest maths book from Oxford University Press is the *Compotus* of 1520, from which (among other things) you can learn how to calculate the date of Easter on your fingers.

Early printed texts were increasingly written in the vernacular rather than in Latin, such as those of Robert Recorde, the 450th anniversary of whose death was commemorated in 2008. Recorde communicated his mathematical ideas in the form of a dialogue between teacher and pupil, much like those in Plato's writings. The writing of popular texts necessitates the introduction of suitable terminology and notation, and Recorde introduced the term *straight line*, as well as his most famous legacy - the *equals sign*, which first appeared in his 1557 algebra book, *The Whetstone of Witte*.

But undoubtedly the most widely used of all mathematics books has been Euclid's *Elements*, written in Alexandria around 300 BC, and in constant use for the next 2000 years. It's been claimed as the most printed book of all time, after the Bible, with the first printed version appearing in Venice in 1482. In the 19th century alone hundreds of English editions were published, some selling half a million copies or more. There is Oliver Byrne's attractive full-colour version from 1847: a copy of it is in the OU library, in the John Fauvel collection.

In the last few years there's been an explosion in so-called 'popular' mathematics books: here's a list of some of them. These include books on the general nature of mathematics, books of mathematical puzzles, books on history or biography, and novels with a mathematical theme or context, such as the recently filmed *Oxford Murders*. There are also books on specific topics such as Simon Singh's *Fermat's Last Theorem*, Marcus du Sautoy's *The Music of the Primes* on the Riemann hypothesis, and Ian Stewart's *Nature's Numbers*. Just recently there have been four books on symmetry and the classification of simple groups.

I've found that I enjoy writing for such an audience - people who get a kick out of maths - and eight years ago I was invited to write a book on the *four-colour problem*. This has a complicated but fascinating history, and I enjoyed the challenge of trying to explain the proof in accessible terms without overloading the reader.

I was once asked why I wrote and edited so many books, and my answer was quite clear: all the books I produce are ones I want to have on my shelf, and when no-one else seems to be doing them for me, I've had to do them myself.

Several of my books have included contributions by experts from around the world, and it's been a great privilege to work with some of the 'stars' in their respective fields, even if they were somewhat shaken at first when their carefully written contributions received the Open University treatment, with as much red ink on the page as what they originally wrote. Fortunately, most agreed that the final product was better than either of us could have produced separately.

Writing a student textbook can also be an enjoyable but exacting experience. I learned my trade with *Introduction to Graph Theory* — the first edition is from 1972, and there have been three further editions since. As any teacher knows, the first thing when writing or lecturing is to *know your audience*, as far as possible, and this applies just as much when you're writing a research paper for your peers as when you're producing a book for the general public. We all write in different ways for different audiences, but this doesn't absolve us from making every effort to be as lucid as possible, with a careful choice of terminology and notation, appropriate use of diagrams, and the use of suitable examples to motivate the theory and then illustrate it afterwards: all these are crucial when we write for our audiences 'out there'.

This is particularly important for us at the Open University who have to communicate our maths at a distance. The OU has a long history of producing attractive correspondence material that's a pleasure to learn from. For our first *Graphs, Networks and Design* course TM361, a course team of a dozen mathematicians and technologists worked for three years to produce some 1600 pages of printed text, as well as 16 TV programmes and audio-tapes, which were then studied by many thousands of students over the years.

But times change. From the very beginning the Open University has embraced the computer. Our teaching and assessment increasingly embrace the electronic, with internet teaching, podcasts and whatever. I submit my assignments electronically for the course I'm studying this year, and the course I'm currently writing will be delivered as pdfs over the web. At Gresham College all lectures are webcast and watched round the world. It's an exciting and challenging time, and if our future is to be as bright as our past, we need encouragement to experiment and innovate, even if some of our bolder experiments may fail.

I'd like to conclude with a further conversation I had in a corridor at the Open University. In the 1970s my colleague Norman Gower, later Principal of Royal Holloway College, said to me something I never forgot: "It doesn't matter what you do, as long as: (1) it's worthwhile, (2) you do it well, and (3) all the jobs that **have** to be done are done." We all have different talents and in general the Open University is better than many at recognising these.

At a time when so much of our funding is tied to narrow RAE-type research (even though most pure maths research papers are read by only a handful of people), it is regrettable that so little official recognition is given by the powers-that-be for communicating our work to the wider world. Research is important, but it's not the only thing we do. In some recent words of Ian Stewart:

> *It is becoming increasingly necessary, and important, for mathematicians to engage with the general public.*

Our subject is widely misunderstood, and its vital role in today's society goes mostly unobserved. Many mathematicians are now convinced that writing about mathematics is at least as valuable as writing new mathematics. In fact, many of us feel that it is pointless to invent new theorems unless the public gets to hear of them.

The various styles of communication I've described today require from us different, but no less valuable, talents - and when it comes down to it, we're all in the same game.

Thank you very much.