Hands, feet and other body parts

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This workshop will discuss the development of collaborative mathematical activities, suitable for small class tutorials, and based on current affairs. It will briefly describe The Big Foot Problem, originating from a case study overseen by Professor Richard Lesh of Purdue University, as an example of such an activity. Participants will then experience the New Zealand version of a similar task.

Introduction

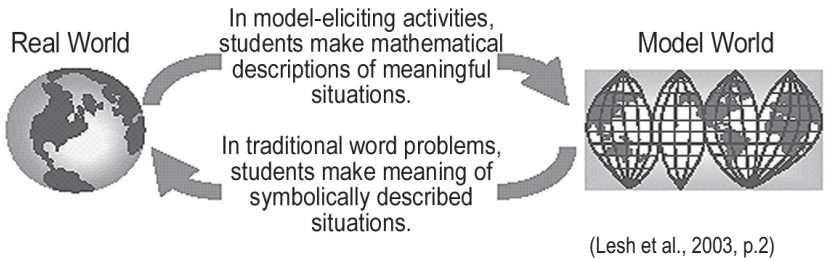
The University of Auckland offers the Tertiary Foundation Certificate Programme at a pre-degree level for students who need preparation for university study. The students fall into two groups: mature students who are lacking in confidence in their ability to succeed in first year degree courses, and those under twenty who have no recognised entry qualification. To be awarded the certificate, all students must enrol in, and pass, a Mathematics course. As part of the assessment for the Mathematics course, students undertake fortnightly tasks in a collaborative tutorial. One of the challenges facing tutors is the development of a bank of suitable tasks.

Task development

Theoretical background

During 2003, the Mathematics Education Unit at the University of Auckland was visited by Professor Richard Lesh from Purdue University, West Lafayette, Indiana, USA. He gave a series of lectures that provided the Tertiary Foundation Certificate (TFC) mathematics tutors with a new perspective on the writing of collaborative tasks. He described these tasks as Model-Eliciting Activities (MEA), which have an underlying purpose of revealing the 'Big Ideas' in mathematic. The tasks involve mathematising, not decoding (Figure 1).

When creating a 'Model-Eliciting Activity' the development of a 'Model Development Sequence' may be a more appropriate learning experience for both the teacher and the students.



(Lesh et al., 2003, p.2)

Figure 1. Mathematising versus decoding.

There are three steps to a sequence:

- The *Model-Eliciting Activity* is the first activity of the sequence, and is designed to encourage students to think about how to find a possible solution to a real life problem. In this process they will need to discuss, describe, predict, design, construct and model the situation. Often students who previously have not achieved highly, standout as having the skills required for this type of activity.

- The *Model-Exploration Activity* is the second activity in the sequence, the goal of which is to develop a representational system for making sense of the targeted conceptual system (Lesh et al, 2003). This is often of great assistance to students when they continue on to the Model-Adaptation Activity.
- This *Model-Adaptation Activity* is similar to the Model-Eliciting Activity in that it again requires the student to relate to a real life situation. The students need to use the conceptual tool they refined in the Model-Exploration Activity, to deal with a problem that would be difficult to handle otherwise.

An Example of a Model-Eliciting Activity: The Big Foot Problem

The example in Figure 2 was used in a case study overseen by Lesh:

Early this morning, the police discovered that, sometime late last night, some nice people rebuilt the old brick drinking fountain in the park where lots of neighborhood children like to play. The parents in the neighborhood would like to thank the people who did it. But, nobody saw who it was. All the police could find were lots of footprints. One of the footprints is shown here. The person who made this footprint seems to be very big. But, to find this person and his friends, it would help if we could figure out how big he is?…. Your job is to make a "HOW TO" TOOL KIT that police can use to make good guesses about how big people are just by looking at their footprints. Your tool kit should work for footprints like the one shown here. But, it should also work for other footprints.

(Lesh et al., 2003, p.6)

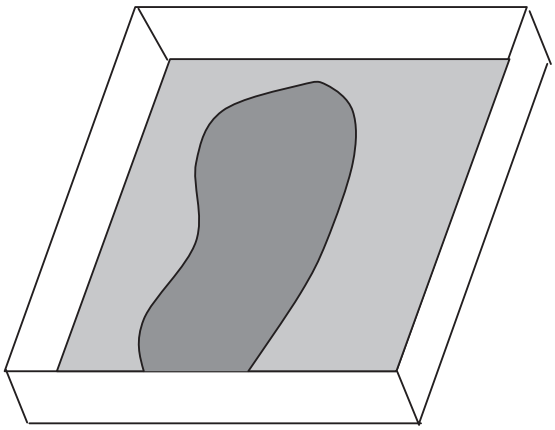
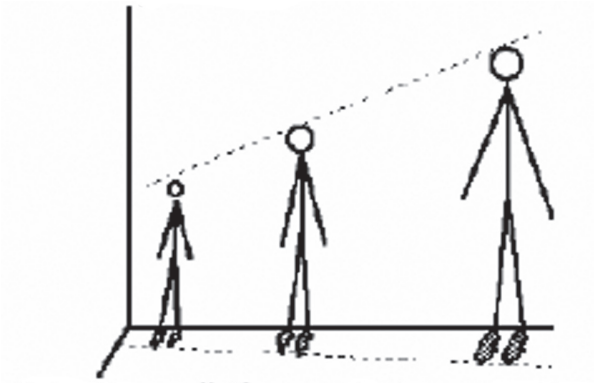


Figure 2. The Big Foot Problem

The students who were observed in the case study progressed through the three steps of the model-development sequence. Their final results were summarized in Figure 3.



Note: Here, all three students are working together to measure heights, and the measurements are getting to be much more precise and accurate.

(Lesh et al., 2003, p.21)

Figure 3. Children's Representation of Results

The children's final conclusion was that a person's height can be estimated at about six times the length of their foot print. This gave rise to the comment 'everybody's a six-footer!'

The New Zealand experience: Big Foot to Giant Hand (see Appendix)

On Saturday 20th September 2003 the New Zealand Herald published an article promoting that night's Rugby League semifinal clash between the New Zealand Warriors and an Australian team, the Canberra Raiders. One quarter of the page was taken up with an actual-size photograph of one of the Warrior's hands:

> 'The biggest mitt in the team belongs to Awen Guttenbeil, and stretches 23cm from wrist to tip of the middle finger.' (NZ Herald, 2003).

The Tertiary Foundation Certificate tutors saw an opportunity to adapt Lesh's 'Big Foot' problem to a New Zealand context, with proportional reasoning the 'Big Idea' to be promoted. In the Tertiary Foundation Certificate mathematics course, the second topic covered is Rate, Ratio and Proportion, so the adapted problem fitted well into this section. The level of mathematics required was not high, but the content was appropriate. Furthermore the context was topical as the students were in the fourth week of their course, in late March/early April, just as pre-season publicity for the 2004 Rugby League season was getting underway. In addition, the students gathered data about themselves, so the task became personalised.

The activity that was developed required students to investigate Awen Guttenbeil's size, in particular his height and the size of his feet. It was successfully used as a collaborative tutorial task in 2004. Unfortunately, however, it should be noted that time constraints meant the activity was presented to students in a more structured form than is suggested by Lesh. The ideal three step sequence of development was compressed into a thirty minute session in such a way that the students really only experienced the application of the model. It was hoped that the process of developing the model was a transparent outcome against the background of previous teaching in the topic area.

The experience the Australians could have had : Flipper Foot

The Tertiary Foundation Certificate tutors have another adaptation of the same problem. A newspaper report from the NZ Herald during the previous Olympics provided a topical context for the exploration of Ian Thorpe's (The Thorpedo) size, given his foot length. His shoe size was given as 39.8cm, and from this it was possible to produce a drawing of his foot. Students then investigated his size in an activity similar to the one above.

Summary

There are rich sources of material available in newspapers and other print material once awareness is raised of the possibilities. The activities should maintain a focus on appropriate content, topical context and the potential to personalize data. Ideally, time should not be a constraint in allowing students the freedom to experience the three full steps of the process advocated by Lesh.

Reference

Lesh, R., Cramer, K., Doerr, H. M., Post, T., & Zawojewski, J. S. (2003). Model Development Sequences. In R. Lesh, & H. M. Doerr (Eds.*) Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching.* (pp. 35-58). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Further Information from http: //tcct.soe.purdue.edu/lesh

Appendix: The Workshop Task

Giant Hand

Awen Guttenbeil who plays in the second row of the Warriors has a hand length of 23 centimetres (see over for an actual sized picture of his right hand).

Your task today is to estimate Awen's height and the length of his feet.

One possible method we could use is:

Part A

1. Measure the hand length of each person in your group from the wrist to the tip of the middle finger. Record your measurements in the table:

Name	Hand length (L)	Height (H)	First ratio (L:H)	Second ratio (1: …)

2. Measure the height of each person in your group. Record.
3. Write down the ratio of hand length: height for each person.
4. Change the ratio into the form 1: …. for each person.
5. Work out the average ratio for your group.
6. Based on your group average, estimate how tall you think Awen is.

Part B

Repeat Part A, but this time, measure the foot length of each person in your group.

By considering the hand to foot length ratio, estimate the length of Awen's feet.

Name	Hand length (L)	Height (H)	First ratio (L:H)	Second ratio (1: …)