Real life aspects in number stories of an adult math-avoider – A case study*

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One central aspect of my practical work with math avoiders is to encourage them to make sense of abstract mathematical terms, for example by translating them into a context. One method to make it easier to get the sense is to weave number stories around an arithmetical problem. This paper presents two number stories of a 41 year old woman which deal with problems of division. It is shown that what she does goes far beyond just transferring the arithmetical problem into a story. Three central aspects seem to be crucial for her process of producing and using the story: her identification with the stories' characters, objects and actions and their legitimation, as well as the dealing with discrepancies. Furthermore, these aspects seem to be comparable to aspects that play a central role when analysing childrens' work on and with number stories.

Mathematical everyday life experiences are suitable starting points for reducing learning obstacles and for developing mathematical comprehension (Schlöglmann 2002). Even low-numerate adults do not enter settings of mathematical education without mathematical experiences. Their knowledge, especially experiences of everyday life, can be used as starting point for mathematical instruction. They deal with money, they hang up pairs of socks on the clothesline, they are involved in explicit or implicit mathematical actions and questions. It is necessary to give flexibility to their mathematical knowledge to make it transferable for using it in everyday life situations. This argumentation leads to the question of when and how everyday life experience connects to specific situations of mathematical thinking and acting.

My research focuses on adults who have acquired only rudimentary mathematical knowledge which is not always suitable for managing simple everyday life problems. Everyday life behaviour of those people is often characterized by the avoidance of mathematical thinking and acting so that the term "maths-avoiders" is used (see also the expression "*Nichtrechner*" in Nolte 2002). The practical outcome of my work shall be a teaching concept for this specific group of learners. The aim is to enable individuals access to mathematics and to develop mathematical literacy by incorporating the learners' mathematical experiences into everyday life situations (see an overview of my concept in Langpaap, 2005). Although the use of everyday life experiences in mathematical instruction of adult maths-avoiders is often suggested, little research has been done on the question how these experiences connect in specific situations to their mathematical thinking, acting and learning.

A crucial aim of my teaching concept is to encourage learners to make sense to abstract mathematical terms. For this, one method is the use of number stories that are constructed and discussed by the students. Thus, in this paper I would like to present a case study of a 41 years old lady who constructs number stories for dealing with problems of division. The kind of everyday life experiences that occur in her number stories and their functions can be observed. Furthermore, I will show that there are similarities to childrens' work on number stories.

Number Stories

The formal multiplication 30*8 (30×8) can be solved by a procedure like "multiply 3 and 8 and afterwards add zero". This procedure leads to a result but reduces the aspects of numbers to operations with digits. Following this procedure, an inner understanding of the numbers and the operation is not necessary (Radatz & Schipper, 1983). One method to make sense of a formal mathematical problem is to weave a story around it. For example the problem 30*8 can be translated into a story that deals with quantities instead of formal numbers:

> At first 3 children take 8 lollipops, together 24 lollipops. Afterwards the group expands by the factor 10 to be ten times as large, so the quantity of 24 lollipops is needed 10 times.

This story gives sense to the effect of factor 10. Radatz and Schipper see the advantage that such number stories make aspects and relationships between numbers clearer and meaningful (Radatz & Schipper, 1983). Selter (1993) regards number stories as a method that fits with the conception of productive learning. This conception implies that students often should be given the opportunities to use the functional aspects of their individual cognitive structures and to participate in organizing their own learning process in productive ways (Selter, 1993). Self productions may be suitable to enable learners to develop their own solution strategies and their own ways of documenting them. The students decide, on their own, how to go forward and how to present results.

A current principle of teaching mathematics in primary schools is to deal with mathematical ideas in different forms of representation. The students should have experiences with mathematical ideas, operations and relationships in connection with different forms of representation (Radatz, 1993). In a first phase the mathematical subject will be made accessible by counting stories and real life experiences (experience based learning). These experiences become consolidated through the dealing with didactical and illustrative materials (enactive representation) then the interpretation of illustrating pictures (visual representation) and finally dealing with numbers and signs (symbolic representations) (Radatz, 1993). Illustrative mathematical materials are used to build up mental comprehension networks. Students deal with such materials, find out their inner mathematical structure and then come to mathematical concepts by a process of internalization and abstraction. But dealing with illustrative materials is not self explanatory and nor does comprehension develop from these materials directly. Dealing with illustrative materials has to be learned and comprehension has to be built up through action in a constructive process (Lorenz, 1992).

Therefore, it is not surprising that number stories as one technique of making sense to arithmetical statements do not necessarily lead to improved comprehension. Radatz (1993) examined number stories of students in primary schools. These students of different third grade classes wrote number stories to the equation 38 + 7 = 45. Radatz found that higher performing students often write texts similar to schoolbooks or that their stories fit with the equation but are not very realistic (38 elephants walk through the city, they meet 7 elephants, together 45 elephants.). Low performing students often write unsolvable stories (Two boys play together. One is 5 years the other one 7 years old. One asks: Can you count up to 100?). Some of them write imaginative and longer stories but without relationship to the equation (Radatz, 1993).

One reason for low performance of low numerate students is that they view an equation with numbers and mathematical symbols more like a secret code with context free signs and operations that have to be manipulated by certain rules. For most themes in mathematic lessons these students rarely develop mental representations or connections to real life (Lorenz & Radatz, 1992).

In my sessions with adult math-avoiders I found these problems too. Often students have no idea how to begin. What is the meaning of an arithmetical problem? How can it be translated into a story? What must students put into a story? Fictional aspects, realistic aspects, aspects of own real life experience? How can I use the setting of a story to solve the arithmetical problem? In the following I'd like to present three cases where numbers stories support the solving of an arithmetical problem in fruitful ways. Two case studies are with an adult maths-avoider, one with primary school children. The focus is on the way the stories are built up and how they are discussed.

Scene "distribution of money"

The data of the following scenes have been interpreted by sequential text analysis following concepts of interpretative instruction research (Krummheuer & Naujok, 1999; Voigt 1984, 1991). In the first scene Mary (maths-avoider, 45 years old) tries to solve the arithmetical problem, 40315÷3, by creation of a number story:

> Mary: My father gives this amount of money [40315 €] as a present to me and my sisters and we shall share it fairly among the three sisters. … He had found this [money] and doesn't need it [because] as he never had it before. [12]

Mary introduces characters who are part of her own real life. She even made herself part of the story. This allows her a special identification with the characters and the scenery. Mary translates the problem of division into a context of distribution. She stresses that the distribution should be fair. The characteristic of equally sized distribution legitimates the story's setting for it fits with the special demands of division. Mary also implicitly legitimates the money as part of the scenery. She introduces the money as found by the father. Although it is not necessary to mention the money's origin, as it has nothing to do with the arithmetical aspect of the problem, she mentions this in order to

[12] Original text: Mary: [...] mein vater schenkt meinen schwestern und mir diesen betrag (zeigt auf "40315") und wir sollen den jetzt gerecht aufteilen unter den 3 schwestern. #ja/ prima\# ok? (.) hat er gefunden braucht er nicht. hat er vorher auch nicht gehabt. sagt sich könnt ihr euch gut\ duellieren. jetzt duellieren wir uns also. (..) also ich würde sagen also erstmal kriegt jeder 105 euro und dann haben wir noch 40000 übrig.

legitimate the moneys' existence. Moreover she describes why her father doesn't need the money. By this she legitimates that the money is used and given away.

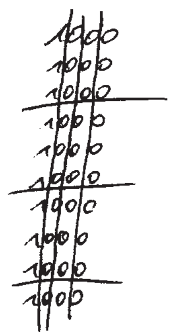
After initializing the story Mary first writes down the solution of 315:3. Then she tries to solve 40000÷3 by formal calculation.

315 : 3 = 105
40000 : 3 =

Because of her problems, the teacher asked her to imagine the 40000€ lying on the table.

> Mary: OK. So we three sisters are sitting their together and first everyone gets … 10,000. … Then we have only … 10,000. OK, now we distribute it to the four, no, three sisters.[13]

In the next step, Mary distributes the remaining 10,000 € by dividing it up it into ten amounts of 1,000 €. Now each person gets 3,000 €, and 1,000 € is left. Analogous to this procedure, Mary distributes the whole money step by step.



Because she herself receives an amount of money, Mary is part of the scene. The distribution of money is introduced by a first step (distribution of 30,000 €) which is followed by further acts of distribution. These acts form a natural part of the story and support sequential thinking and action. This shows how the step-by-step-procedure of division is legitimated by the setting.

Mary makes a comment on this large-scale procedure:

> Mary: … (laughs) oh how terrible. I hope my father never would be so generous. But anyway he isn't. So no matter.[14]

The father is described as not generous. Although this fact is not necessary for the solving process or the ongoing of the story, Mary uses this statement to express implicitly her wish that the mathematical problem she has to cope with within the scope of the story should never become part of her real life. To express this, she uses a discrepancy of the stories' setting and her real life.

The procedure of step-by-step distribution leads Mary finally to the remainder 1€.

> Mary: There is one left, yes (laughs). This we donate to amnesty international.[15]

Mary tries to find a finishing step for the remainder. Her solution legitimates the remainder as a consistent part of the story.

Summarizing the results, three central aspects are found.

There are aspects of identification:

- Mary introduces characters who are part of her real life.

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[13] Original text: Mary: ja/ also wir drei schwestern sitzen da jetzt zusammen/ also dann kriegt jeder schon mal 1000. dann haben wir schon mal #sie meinen 10000# 10000 #einverstanden# jede kriegt schon mal ähm 10000 das heißt (schreibt unter A2.6 einen Strich) wir haben jetzt nur noch 1000 äh 10000 einmal 10000 übrig (schreibt in A1.8 "10000") (9) gut jetzt teilen wir das durch die 4 schwestern quatsch 3 schwestern. (.)

[14] Original text: Mary: ja (..) oder ich mach das so (setzt neben A1.8 zum schreiben oder zeichnen an) das ist ja n bisschen viel das jetzt das alles da hin zu schreiben. ich kann 10 (5) 5000 ne das nützt mir nichts. (schreibt in A1.9-18 zehnmal "1000" untereinander) gut jetzt kriegt jede drei davon (malt 3 waagerechte Linien in A1.9-18) plus 3000 (schreibt in A1.3 "+3000") dann haben wir noch diesen übrig (zeigt auf A1.18) 1000 (schreibt in A1.19 "1000") (lacht) oh wie schrecklich. hoffentlich (lacht) ist mein Vater nicht mal irgendwann so großzügig. ist er sowieso nicht. also keine bange. #(schmunzelt)# (lacht)

[15] Original text: Mary: ich habe noch einen übrig ja (lacht) (.) den spenden wir doch amnesty.

- She makes herself part of story

There are aspects of legitimation:

- Mary legitimates the moneys' existence.
- She legitimates the moneys' use (its giving away).
- Mary legitimates the stories' setting being conformal with the special demands of division by calling the distribution fair.
- She legetimates the remainder of division the as conformal with the story by integrating it.
- Her calculating strategy (step-by-step-procedure) is natural part of the story and so legitimated by the setting.

There is an aspect of dealing with discrepancies:

- Mary uses the discrepancy of the stories' setting and her real life to express implicitly her emotional problems with the mathematical task.

Scene "ingots of gold"

In another lesson Mary also gives an example for division as distribution. At the beginning of the session she remembers an extensive questionnaire she got some days before. Based on this, she generates a mathematical problem: "Distribute 604 questions to 6 days evenly". Her first solution was "100 questions per day and a remainder of 4 days". The teacher (me) asks her to calculate 604:6 again but this time to resolve the remainder. To do so, Mary constructs a story:

> Mary: ... One could say there are ingots of gold. ... and these are six people. They have to distribute it evenly. So if there are some unpleasant questions one would say: Oh no, I don't care, you could take the other four, the remained ones. But if there are 604 ingots of gold, then every one of the six people want to settle it very fairly.[16]

Mary changes the object of distribution. For distribution ingots of gold seem to be more attractive to her than unpleasant questions. Mary argues from the characters' point of view, so she identifies herself with their needs. The ingots of gold guarantee the aspect of even distribution because they fit the characters' needs and motivations. Mary regards the action of even distribution legitimated only by attractive objects. The introduction of "unpleasant questions" as object of distribution would produce a discrepancy between the object and the intention of fair distribution.

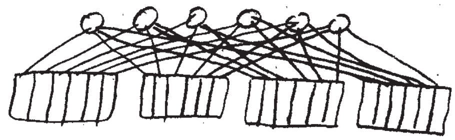
Mary distributes 100 ingots to each person. Four ingots are left. She draws these four ingots and tries to distribute them into 6 sections each.

> Mary: Ok, now I have 4 ingots of gold and 6 persons. […] Everyone wants to get a fair sized number of pieces. What does everyone get? I don't know. […] So first everyone can get one. That would be the easiest way now. And so on. But then I don't know how much everyone has got (laughts). Everyone has 4. Isn't it terrible if you are absolutely so…[17]

She develops the picture:

[16] Original text: Mary: also ich habe jetzt gedacht man könnte jetzt zum beispiel auch sagen das sind goldbarren. #ja/# und das sind 6 leute. die sollen das gerecht aufteilen. also wenn das irgendwelche unangenehmen fragen sind/ dann sagt einer ach nö ist mir egal nimm du ruhig die anderen 4 die überflüssigen. wenn es aber 604 goldbarren sind dann wollen die 6 leute das ganz gerecht(!) haben.

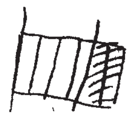
[17] Original text: Mary: ok jetzt habe ich also 4 goldbarren/ (zeigt auf A1.6) und 6 personen. 1 2 (malt über die Kästchen in A1.6 sechs kleine Kreise) 3 4 5 6. jeder will gerecht viel haben. #hmhm/# was kriegen die denn jetzt (..) ja. (7) das weiß ich auch nicht. (7) hm. (3) 1 2 3 4 5 6 (zählt in A1.6 im ersten Kästchen die Abteilungen ab). Habe ich ja schon (..) jetzt bin ich echt überfragt (lacht) #hm/# oh gott. (..) also jeder kann ja erstmal ein kriegen. das wäre jetzt das am einfachsten (verbindet in A1.6 jede Abteilung des ersten Kästchens mit einem der 6 Kreise). so. ein bisschen umständlich/ aber geht. #L schmunzelt# gut. und das gleiche so weiter. #ja/# aber dann wüsste ich beim besten willen nicht wie viel jeder hat (lacht). 4 hat dann ja hm. (..) ist das nicht schrecklich wenn man so völlig



Again Mary mentions the characters' motivation to distribute fairly. She prefers a step by step distribution. Also she feels unsure about the result of this procedure she doesn't criticize the procedure itself. For her the step-by-step procedure seems to be logical. In other words the procedure is legitimated by the setting.

After finding the result 4/6 Mary remains unsure about its meaning. The teacher (me) asks her to visualize 4/6.

> Mary: so if this is my ingot of gold now [… Mary finishes the picture …] everyone has this piece [on the left side].[18]



Mary speaks of "my [!] ingot of gold". So she does not work on an anonymous object but on an object she could identify herself with. The importance of identification is also indicated when Mary finally reflected her doings again:

> Mary: Yes. That's why I thought if you take something more attractive than silly questions you can put yourself in the six peoples' position. Why what why do I want to get it. If I don't want it I'm happy if I don't have to distribute it fairly.[19]

Here the attractiveness of the ingots is ambiguous. On the one hand, the object is attractive to the stories characters'. On the other hand, it is attractive for the one who deals with the mathematical problem. The user of the number story can put himself in the characters' position and can identify himself with their needs.

Summarizing the results, we also find the three aspects identification, legitimation and dealing with discrepancies.

There are aspects of identification:

- Mary identifies herself with the characters' needs and motivations.
- She identifies herself with the object as her ingot of gold.
- Mary describes the function of the setting: The user of the number story can put himself into the characters' position and can identify himself with their needs

There are aspects of legitimation:

- Mary views the action of even distribution legitimated only by attractive objects
- The step-by-step procedure of division is legitimated by the setting.

There is an aspect of dealing with discrepancies:

- Mary tries to avoid discrepancy between the object of distribution and the intention of fair distribution

Scene "Mr. Dröge buys a present"

Dröge (1991) suggests that real life situations which deal with context referred mathematical problems should be put into the centre of primary school lessons. She supposes lessons that are orientated to the focus on real life situations, motivate students to use their individual real life experiences. An analysis of one of her lessons shows that her students also deal with aspects of identification, legitimation discrepancies when they deal with text based

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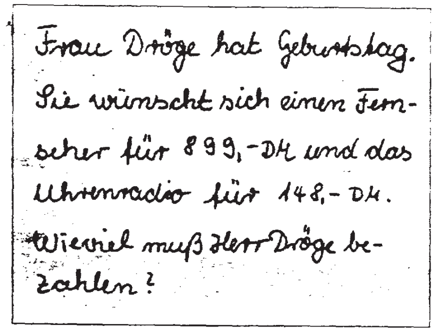
[18] Original text: Mary: also wenn das jetzt mein goldbarren ist (zeichnet in A1.14 ein Kästchen) ähm (halbiert das Kästchen, dann drittelt sie durch zwei Striche die linke Hälfte, in der rechten Hälfte zeichnet sie nur einen Strich) dann ist das hier weg (streicht die 5. Abteilung aus). also dieses stück (verstärkt zwei Striche, indem sie sie über den Kästchenrand hinweg verlängert, zeigt auf den dadurch markierten Bereich) hat jeder.

[19] Original text: Mary: ja. deshalb habe ich mir ja gedacht wenn man jetzt was attraktiveres nimmt als so blöde psychofragen/ hat man ja überhaupt ein also da kann man sich in diese 6 people da überhaupt reinversetzen warum was warum will ich das überhaupt haben. Wenn ich das gar nicht haben will bin ich ja froh, wenn ichs nicht gerecht aufteilen muss.

mathematical problems. This leads to the hypothesis that there are similarities between adults and children in dealing with number stories.

Dröge (1991) describes how third grade students of her primary school dealt with number stories they had produced on a brochure:

> "It's Mrs. Dröges birthday. She'd like to have a TV set that costs 899,- DM and a clock radio that costs 148,- DM. How much has Mr. Dröge to pay?"


Frau Dröge hat Geburtstag.
Sie wünscht sich einen Fern-
seher für 899,-DM und das
Uhrenradio für 148,-DM.
Wieviel muß Herr Dröge be-
zahlen?

The text looks like a simple schoolbook text. Many researchers found that students often deal with such schoolbook problems by focusing on numbers without including the context. For instance, the appearance of two similar sized numbers leads those students to addition or subtraction whether the appearance of different sized numbers lead to multiplication or division (Lorenz/Radatz 1993). What makes the text shown above different? The difference to schoolbook problems is that the text is related to aspects of the students' real life. Mrs. Dröge is their teacher and becomes the main character of the story. The students discussed the text

> Yvonne: If we would add this up… perhaps what his wife like to have would be too expensive for Mr. Dröge and he wouldn't buy it.
>
> Melanie: What television set this must be for that price.
>
> Kathrin: If I'm Mr. Dröge I would wait and see whether I could get the set cheaper.
>
> Ulrike: But Mr. Dröge has to order the set soon to get it early enough for the birthday.

The students talk about the general conditions and implications of the scenery. This seems to be more important for the students than solving the arithmetical problem quickly. Yvonne, Melanie and Ulrike put themselves in Mr. Dröges place (Is the TV set to expensive? Could it be cheaper? When is the right moment to order it?) Kathrin identifies herself with Mr. Dröge ("If I'm Mr. Dröge"). So, dealing with the solution of the problem is done by identification with the stories' characters and situation.

The next discussion contains aspects of dealing with discrepancies:

> Bettina: This is strange. First the text says Mrs. Dröge and then comes the question: How much has Mr. Dröge to pay?
>
> Frauke: Sure! Because she should not know about it and Mr. Dröge has to do it secretly.

Also here the students do not discuss the arithmetical problem. Bettina and Frauke discuss the discrepancy of two competing characters. Also resolving the discrepancy of the scenery seems to be more important for the students than to deal with the arithmetical problem.

After discussing number stories during the lesson, the students painted price tags, wrote invoices and played buying. Although in schoolbooks dealing with big notes like 500 € or 1000 € is usual, none of the students has ever paid with such a big note. Sabrina gave an explanation:

> Sabrina: With such a big note I wouldn't go out. I would have too much fear to loose it.

Sabrina pays attention to realistic behaviour in real life settings. The setting of an usual buying situation doesn't legitimate the use of 500€ or 1000€ notes. The other way round, this statement shows that students legitimate objects and actions for the scenery.

The students discussions about general conditions and implications of the context are also characterized by the three aspects identification, legitimation and dealing with discrepancies:

- The students identify themselves with the setting's characters, objects and conditions.
- The students discuss discrepancies of the scenery and try to resolve them.
- The students legitimate characters, action and object of the setting and prefer objects that correspond to their everyday life experiences.

Summary and Hypothesis

Comparing the three cases we find three central aspects of dealing with number stories:

- The students identify themselves with characters, object and actions of the story.
- The students discuss discrepancies of the setting and try to resolve them.
- The students reflect the implications of characters, objects and actions for the stories' credibility and try to legitimate elements of the story.

Table 1. Summary of aspects of number stories

	distribution of money	**ingots of gold**	**birthday present**
identification with characters, objects and actions	"me and my sisters" (characters, be part of the story)	"wanting ingots" (identification with the characters' needs) "my ingot of gold" (object) "put yourself in the six peoples' position" (characters)	"if I'm Mr. Dröge" (character) "what a TV set" (object) "get it early enough" (condition)
discussion/resolving of discrepancies	"father never would be so generous" (discrepancy between story and real life)	"unpleasant questions" don't fit the intention of fair distribution (object)	"Mrs. Dröge demands, Mr. Dröge buys" (discrepancy) "buy secretly" (resolving)
legitimation of characters, objects and actions	"he found the money" (object) "doesn't need it" (legitimates giving away, action) step-by-step procedure (action) "donate to amnesty international" (action)	"the six people want to settle it very fairly" (action) step-by-step procedure (action)	"using common notes" (object, action)

The students' work and discussion on the texts contain aspects that seem to be unnecessary to solve the arithmetical problem. The question is why the students behave that way. Is it a warming up to become closer to the arithmetical problem? Are "additional" aspects of the story essential to get along with the story itself? Are the additional aspects necessary for the transfer between the mathematical area of experience and the real life experiences? I suppose that a successful identification with the different aspects of the story like characters, actions and objects supports the process of sense making. Identification with the story depends on its consistency. Resolving discrepancies by the students is a necessary reaction to inconsistency. Also the legitimation of the story's elements is an expression of their needs for consistent settings.

The students succeed in dealing with number stories and text based problems because the contents touch their personal experiences. Mary' approach to dealing with number stories is similar to that of the children. This leads to the hypothesis, that the approach of adult maths-avoiders in general is similar to the approach of children. Further I suppose that the development of comprehension should be linked to aspects of problem solving competencies in similar ways. Dröges' approach to put situations that deal with aspects close to the students' experiences and imagination into the lessons' centre seems to be transferrable to adults.

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