Using spreadsheets for algebra

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Generic programs such as Excel can be used in many ways to support student learning and to provide a classroom tool for mathematical exploration. The formulae in Excel provide a model for algebra where a cell (or a column) acts as a variable and the students can see that the cell may contain a range of numbers. Other cells and numbers can then operate with this cell of columns of cells. The use of a spreadsheet in this way does not require a high level of computer skills but provides a context for students where the unknown or variable has meaning.

Algebra is one area of Mathematics in which many students experience difficulty and which adults often claim as the cause or beginning of their lack of success in mathematics. Students learning algebra have difficulties with seeing its relevance, using the language and understanding the basic concepts. Many different approaches have been and are still being tried in an attempt to enhance understanding. The concepts and context need to be presented in a manner enabling the students to attach meaning to the symbolic language of algebra.

Problems learners encounter in their early algebraic learning fall into three main categories: Those associated with attaching meaning to letters used in algebra, those related to the structure and conventions of algebra and corresponding arithmetical structure and those associated with translation between different forms of representation (Horne, 1996). Henry (2001) categorised the obstacles also as three but differently. Her first category, algebra sign systems, encompasses the meaning attached to the letters but also to the other symbols which Horne associated partly with arithmetical structure. The second category, "transitioning from means-end strategies to continuous translation strategies" is "the move students must make from working just in steps to setting up the entire problem first" (Henry, 2001, p.301). The third category is the transition from focussing just on the process to having an object based conception of algebra.

Difficulties in the algebraic sign system (Chalouh & Herscovics, 1988; Küchemann 1981, Kieran 1989; 1997; Wood, 1998) include:

- difficulty attaching meaning to letters which in different circumstances can have different meanings.
- perceived differences between arithmetic and algebra such as the meaning attached to $3a$ by reading $3a$ when $a = 2$ as 32 showing a place value understanding of the $3a$. Horne (1994) found that inability to complete a substitution correctly was due to the lack of understanding of arithmetic rather than the algebra involved.
- the understanding of the '=' sign in arithmetic as an operation meaning action is now required (MacGregor 1991) which leads to conjoining or over-generalisation such as students simplifying the expression $6y - 4$ to $2y$. This need to reduce to a single entity has been discussed widely and also called the process/product dilemma; the idea of suspended operations; and the acceptance of lack of closure (Collis 1975).
- The use of brackets and the order of operations. For example, when the context was not clearly defined students worked from left to right regardless of the operations (Kieran, 1989; Booth, 1984).

Many people have suggested the computer as being of assistance in teaching algebra and currently the use of dedicated Computer Algebra Systems (CAS) are being explored to support the teaching of algebra both on calculators and on the computer. Thomas and Tall (1988) approached using the computer in developing algebraic understanding initially through the use of BASIC programming. Sutherland (1987) looked at Logo and children's understanding of variables. Many have thought of or have used spreadsheets particularly at years 9 and 10 (Asp et al, 1992; Friedlander & Tabach, 2001; Kaput, 1992; Peasey, 1985). Sutherland (1991) developed a program to work with year 10 students who had dropped out of traditional mathematics and found that the environment of the spreadsheet enabled them to develop algebraic concepts. Following this, Sutherland and Rojano (1992) used similar material with year 6 pre-algebra students. Care needs to be taken however, for, as Thomas and Tall warned back in 1988, if the ideas of variable that are presented are simplistic, problems will occur later.

Rationale for the Use of Spreadsheets

The spreadsheet is available on nearly all computers without the purchase of extra software. It is an environment which encourages, and in fact requires, the use of a symbolic language. Inherent in this language is the concept of the unknown (letter) as a variable. Now with refinements in the programming, spreadsheets such as Excel allow dynamic investigations where tables of figures, formulae and graphs interact. It is also possible to set up the spreadsheet using symbols such as *x* and *y* instead of the normal cell references. The development of different representations in algebra, including visual, alongside one another is seen as important in the translation between forms (Janvier, 1989). This is a strength of the spreadsheet approach which allows the spreadsheet to become an algebraic micro-world in which students can construct meaning. Within this world the concept of the unknown is as a variable. This should overcome some of the cognitive conflict which arises between ways of viewing the unknown in generalised arithmetic.

Spreadsheets can be quickly accessed by students without a great deal of time being spent teaching the use of the program. Material for using the spreadsheet to develop algebra, based on the work of Sutherland and Rojano, has been used with year 7 students (Horne, 1994) and with teachers and teacher trainees. Two teachers, who were involved with early use of the material, felt that the use of the computer environment enabled a change of teacher role within the classroom and that the process also encouraged greater student control of learning (Horne & Jeppesen, 1993). There were motivational aspects of the spreadsheet as an environment for learning algebra as well. The students saw the use of the computer as being very relevant. In writing about the methods they used some students who had been classified as weak in mathematics demonstrated sound comprehension of the algebraic ideas of substitution and equivalence but were unable to correctly perform the arithmetic calculation required to answer a question correctly and needed more practice to be confident with their algebraic skills.

The teaching approach

Students are introduced to the spreadsheet environment initially with the writing of a formula in cell C1 which uses A1 as the variable. They are then required to predict the effect on C1 of altering the number in cell A1. For example, the number 5 in A1 and the formula '= A1 + 5' will show the number 10 in C1. If the number in A1 is changed to 31, what number will show in C1? Students become good at predicting that it would show 36 as they see the relationship between the two numbers. From this they build a table of values and describe in common language the effect of the formula in C1. This activity requires very little instruction on the use of the spreadsheet. The aim of the series of lessons is to explore some aspects of algebra and so the instruction on spreadsheet use is minimal, although students do learn a great deal about the spreadsheet as the need arises and as their interest dictates. In using this, I always include a range of numbers in the cell A1 so that students become used to the idea that the variable (A1) in the formula might be a decimal, a large number, a very small number, or even a negative number. In using the fraction form of a number such as ¾ it is necessary to type it in as '=3/4'. This approach also provides the opportunity for much mental arithmetic, as students are encouraged to predict before they allow the spreadsheet to do the calculation.

The second lesson follows with the students being required to translate from a verbal description of a formula, such as 'double one more than the number in A1' or 'three less than half of A1', into an algebraic expression, then create a table of values in order to check their work.

The approach used in these lessons focuses on the language used for a formula. For example rather than read the formula C1 = 2 × A1 + 5 as 'C one equals two multiplied by A one plus five' it is better to use relational language which reinforces the meaning of the symbols. One way to do this would be 'the number in C one will be five more than twice the number in A one'.

One important aspect of the teaching approach is that the students are asked to reflect on what they have learned and to explain it verbally (either orally or in written form) to someone else. Class discussion led by the teacher, where students share their understandings, contributes to the overall group learning within the classroom. Language and communication are an important part of all learning so as well as this the students always work in pairs (although they might have their own computer) to share their ideas as they develop. The aim of the lesson is for the students to construct their own understandings of the concepts involved, but this is enhanced by the sharing of ideas within the class.

Equivalent expressions are introduced in a way which allows students to develop their own rules for simplification of algebraic expression. A series of expressions, such as those listed in figure 1 below, are written in a number of cells by the students so they are aware of the formulae. Initially the number 5 is written in the cell A1. All of the formula listed here will give the same number with A1 = 5.

	B	*C*
1	*= A1 + A1 + 5*	*= 3 * A1*
2	*= 3 + 2 * (A1 + 1)*	*= 4 * A1 – 2 + 7 – A1 * 2*
3	*= A1 + 2 * A1 – 5 + A1*	*= (4 * A1 + 10) / 2*
4	*= 8 + 2 * A1 – 3*	*= A1 + 4 + 2 * A1 + 1 – A1*

Figure 1. Examples of exploration of equivalent expressions task

Students are then asked to change the number in the cell A1 and observe what happens. Some of the formula give the same answers but some are different. Figure 2 shows the effect of making A1 = 2. The 'odd ones out' are the formula in B3 and C1.

The students are asked to test the formulae with a range of numbers including very large numbers, decimal numbers, very small numbers and negative numbers and to try to explain what they observe. They are then asked to write two more equivalent expressions belonging in the group in B1, B2, B4 etc. Finally after doing this with some different groups of expressions, and looking for what is the same about the formulae that belong in the group of equivalent expressions each time they are asked to explain how it is possible to tell from the written formulae whether they will be equivalent.

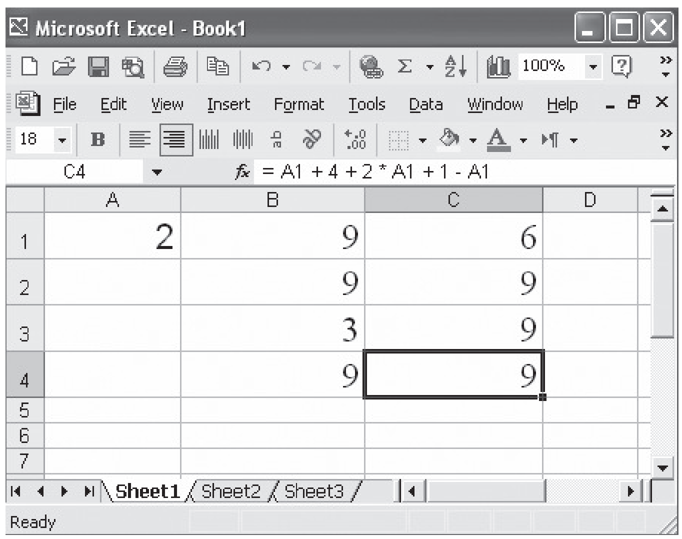

Figure 2. Equivalent expressions in Excel

A discussion on the meaning of the simplification follows. This is also followed up away from the computers by asking the students just to write down three expressions equivalent to one such as $5x + 2$ or $3x – 6$.

Once the idea of equivalent expressions is understood, translating from a table of values to common language and an algebraic rule through a game for the discovery of a formula are possible. One group sets a formula up in the cell C1 using A1 as the independent variable. I have always instructed the students at first that they are not to use A1 * A1, to raise A1 to a power or to divide by A1. The other group then tries to discover the formula by changing the values in A1 until they think they know the rule. It is a good idea to have the students record their findings in a table. When they think they know the rule they write it into cell B1. If they are correct, changing the value in A1 will always return the same numbers in B1 and C1. Students again should be encouraged to test their rule with a range of numbers including decimals and negative numbers. The rule they find may look very different to the original but it will be equivalent (Contact the author for further information).

One feature of Excel, the fill down, can be taught here and used to keep the tabular record on the computer. Once the rule is set the students who set the problem can copy the cells A1 and C1 down 10 rows or so. All 10 rows should read the same. The pair trying to find the rule can then change the values in column A, thus keeping a record. When the rule is written in cell B1 it can also be copied down the column and the decimals and negative numbers added to the bottom.

The final stages of the introductory program include the idea of inverse operations undoing the process and the solution of worded problems by designing a spreadsheet with appropriate labelling and structure. This latter word problem type of activity introduces an equation as a special case or specific value of a function.

The spreadsheet skills students develop alongside the algebra include the copying of cells through the fill down command; the idea of cells containing a label, a number or a formula; and layout and design.

The representations of formula or function used in this approach are the symbolic formula, a table of values and language. The other representation, the graph of a function, can also be easily introduced through the use of the graph capabilities of a spreadsheet. Spreadsheet programs differ but the more recent ones all allow a graph to be shown on the same screen as the cells, allowing the graph to develop as the table of values is extended. This is a dynamic way to see a straight line or a curved graph develop as students can enter any value of the variable they wish. The early lessons encouraged them to include decimal values and negative numbers as well. This is important as too often students working from very traditional approaches are restricted in the early stages to whole number operations and thus their views of the variable are limited.

Comments from research using this program with early algebra learners

The use of this program to introduce algebra was studied in three schools. Classes were observed, teachers were interviewed, and an interview assessment consisting of a series of questions which were administered verbally and on cards in a one-on-one situation, was done with the students at least six weeks after the unit had been taught. Some of the findings are described here.

It is not only the nature of the program but also the commitment teachers have to it that influences the outcomes. This was evident at all schools but showed very clearly at one. One teacher entered into the program but was not really committed to the approach and this showed in the outcomes from his class in comparison to the other class from that school. It is also clear from the interviews that one teacher taught the students algorithms rather than allowing the students to develop their own approaches.

No gender differences showed overall but there were significant differences between the schools with the boys' school showing up more poorly, due in part to the large number of students who did not attempt some questions asked in the interview. The only section that the boys' school students showed greater achievement (though not significantly) was in some of the tasks finding the rule in the table. This was due largely to one class who had been taught an algorithmic approach to it, which was useful for two of the three questions.

In comparison to the British studies in the early 1980s, 56% of these 12 and 13 year old students correctly substituted a value for c in the formula $d = 20 - 2c$, while in the British study only 44% of 13 year olds could complete correctly the simpler substitution in $m = 3n + 1$. Similarly on a simplification task, while there was one class whose results were very poor, generally more than 30% of the students could simplify linear type expressions with a number of terms in one or more variables and, when there was only one variable, 55% completed the simplification. This compares to 27% in the UK study.

The students spoke positively about the experience of using the computer approach and, with the exception of the one teacher, the teachers are continuing to use the approach.

The interviews clearly illustrated the development of student understanding and the need for experiences over a longer period of time to allow students sufficient interaction with the concepts and associated language. A number of students during the interview, when asked to give explanations, altered their solutions as they found their errors while they were explaining. Some were also unsure and only attempted the problems when they were encouraged to have a go, although their attempts were often reasonable.

Most of the students were limited in solving equations by their perception that the answer had to be a whole number and there were only a few who realised that decimals could be used. This reflects the observation in many classes that in spite of the encouragement to use decimals and negative numbers the teachers tended to just use whole numbers. Analysis of the interviews shows that there are some students still with a perception of the letter as a particular whole number, a small number who still need the multiplication sign included to make sense of the algebraic notation and a small group who link the value of the letters in some way to the position in the alphabet. Many of these errors tend to show in the harder problems. Even though the student may have been operating efficiently and effectively on simpler problems of the same type showing partial understanding, when faced with a more difficult task they often revert to making the same errors they made earlier.

Some other considerations

From the studies which have been done it seems that the spreadsheet use enhances algebra learning but there are some warnings. One difficulty raised in the initial discussion was the reading of 3*a* when *a* = 2 as the number 32. The spreadsheet will not eliminate this problem as it requires the operation sign to always be included. It does mean that while the students are learning they do not have the shorthand 3*x* to cause confusion but must always write it as 3*x*. Other researchers, not using a computer environment, have suggested that the operation sign should be included for all students in the early stages of algebra and only dropped when students have developed sound initial concepts. Our studies so far would support this. Interestingly the students have little difficulty changing the A1 to the more general *a* or *x* but unless care is taken still sometimes read 3*a* with *a* = 2 as 32.

The discussion with the students sharing their learning is a critical aspect of the approach. It has also been useful for the classes to have some lessons without the computer where they discuss how they would tackle the same type of problems and imagine they are the computer spreadsheet. The teacher's role becomes one of supporter, questioner and discussion leader trying to enable the students from the discussion to link ideas together, thus developing their understanding.

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