

# Mind and gesture: the numeracy of a vocational student\*

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*In The Netherlands (pre-) vocational education for a majority of the students starts at the age of 15. Students in the basic stream of pre-vocational education are considered weak learners with very little success on pen-and-paper assignments in mathematics. In an increasing number of schools these students are prepared for later vocational education by conducting projects and real-life assignments. They are treated as almost-adults. In my research I try to find an answer to the question which numerical competences these students actually show in working on real-life assignments and in which way they talk about it. In my research I show that these students' situated cognition of numeracy concepts and practices are quite elaborate, if they are allowed to use their own words and make their own gestures. This way of looking at these students can serve as a starting point to develop numeracy education for these students within a setting of project-based learning with considerably more transfer than is the case in the traditional setting of separated vocational and mathematics courses.*

In The Netherlands 55% of the students in secondary education are in Pre-Vocational Secondary Education. Pre-Vocational Secondary Education consists of a theoretical stream, a mixed stream and a basic stream. The mixed and basic stream consists from the 3<sup>rd</sup> year on of general courses such as Dutch, English, mathematics, science, history, economics, combined with vocational courses from the sectors: Technology; Personal and Social Services/Health Care; Economics and Agriculture.

Some 16% of all the 15 and 16 years old students are in the basic pre-vocational training (In Dutch: basisberoepsgerichte leerweg). They proceed to the labor market, or to adult and vocational education. In the vocational courses the goal is preparation for future jobs. In the general courses the goal is to work on general education and to support the vocational preparation. Which educational design suits these goals the best is a topic of many recent discussions. Schools in The Netherlands are in an transition to more autonomy for the schools to design and organize education in a way they see fit. An increasing number of schools are making a choice for a quite fundamental redesign.

## Competency based teaching in education

Recently in The Netherlands there has been an on-going discussion on with regard to the necessary competencies of young people entering the labor market or further vocational education. A growing number of schools share the view that competency based teaching and learning gives the students the most meaningful preparation for the demands from the modern labor market. Competency based teaching and learning is organized around meaningful integrated tasks and problems, where students to certain extent formulate their own learning needs and the attitudes they want to develop further. In the Netherlands this view on learning is most of the time referred to as Authentic Learning, Competency Based Learning or "Natuurlijk leren". In the next section there is a short summary of the key elements of "Natuurlijk leren"

### "Natuurlijk leren" in Vocational Education

APS-National Centre for School Improvement has developed in The Netherlands a design format for a form of authentic learning in Vocational Education. In Dutch it is called "Natuurlijk leren", because of the very natural way the learning takes place.

Meaningful learning tasks are the heart of "Natuurlijk leren". Meaningful learning tasks in "Natuurlijk leren", nearly always involve realistic practical situations and are often developed in conjunction with the business community. For students this leads to a learning process that is meaningful, with the likelihood of transfer and broad development.

To optimize the learning process of students as they work on an a task, two specific teacher roles have been developed to support students: the teacher/coach and the vocational tutor. The teacher/coach guides the students in their personal growth. This involves the development of the personal qualities that are relevant for life such as

cognitive skills, social and communication skills, metacognitive skills such as planning and reflection, and general attitude. This support takes place both within and outside the task. The vocational tutor guides students in their professional development: the instrumental and vocational skills within the professional tasks. The portfolio that a student builds up plays a major role in the guidance offered to the student.

The relevance of this concept for classroom practice is great. The concept of “Natuurlijk leren” is a way of learning in school in which the social-constructivist vision on learning is put into practice in a consistent manner. It requires the school to adopt a way of thinking about teaching and learning that is different from what is generally the case. It requires teachers to consult with each other on a regular basis. It requires the formulation of proper tasks, often in conjunction with business entities. It requires a good balance between guiding students and letting them go on their own. It requires coaching on the job. From students it requires a considerably higher awareness of what they are doing than when working on non-meaningful learning tasks. After all, they must choose, collaborate, reflect on what they are doing, prepare a task, leave the school to fulfill the tasks, and so on. All this leads to broad personal development.

The “Natuurlijk leren” approach differs by school. Each school must find its own modus that matches with its objectives and characteristics. Authentic learning (APS-style) was developed in the adult and vocational training sector. Other educational sectors are currently showing great interest. (Van Emst , 2000)

### **Mathematics in the basic stream**

Most of the students in the basic stream of Pre-Vocational Secondary Education have to participate in a final examination in Mathematics. The exam consists of mathematics questions in context, so called Realistic Mathematics Education (De Lange, 1996). The test is a written test. The students score low on these written tests. In other cases there are questions that are made so easy that they only have to write down yes to gain the points for that question. These are called “nodding-questions”. In some schools it was noticed that in the basic stream of pre-vocational education the scores of the students on this kind of written tests at the end of the second year are roughly the same as their scores at the end of the fourth year. The mathematics courses do not seem to yield much added value. I find in schools the same situation as Gillespie found in schools in England and Wales:

One response was to provide “stand-alone” maths techniques courses often outside specific vocational contexts. But many students saw such courses as not relevant to their main courses, and there was little evidence of their being able to transfer skills from them to their vocational work. (Gillespie, 2000, p.55)

I observe that in the mathematics classroom motivation problems outweigh any development of numerical competences (for my work definition of numerical competency, see further on).

To remedy this problem there are two contesting views in The Netherlands. One view states that training the students rigorously in basic literacy and arithmetic in combination with practical vocational education is the most sensible way to proceed.

The other view claims that a paradigm shift to project based authentic learning will end the problems.

A fruitful discussion about the fundamentals of numeracy (or mathematics or arithmetic) within the education for these students is overshadowed by the conflict between the ideologies behind the points of view. The discussions between the supporters of the different views are quite fierce and emotional. This kind of discussions are reported from other countries as well, for instances Australia (FitzSimons, 2002; Zevenbergen, 2004) and the USA (Wilson, 2003).

### **Mathematics in competency based education**

In the authentic learning view there is until now very limited attention for (possible growth in) numerical competences. The growth in attitude, dispositions and practical skills seems to be more in focus. Sometimes there is a kind of noticeable aversion to numeracy or mathematics, because it is seen as a “subject matter approach” which was just abolished. Many sources report that little is known about the numeracy demands of contemporary workplaces and the way in which young people are working in these environments, and that the issues have been under-theorized. (Zevenbergen, 2004)

My personal stance is to investigate and theorize further on the fundamental role of “coping with the quantitative aspects of the world around us”, or “coping quantitatively with aspects of the world around us”. My research is conducted in schools who have decided to work in pre-vocational education according to the principals of authentic learning or “Natuurlijk leren”.

My interest is in the mathematics or the numeracy that is embodied in the students and embedded in the tasks they are performing. Comparable lines of approach are followed by Forman and Steen (Forman & Steen 2000) and Packer (Packer, 1997). This approach is also advocated by Zevenbergen. She argues for the use of ethnography for studying the workplace practices from the perspectives of the participants rather than as the mathematics educator attempting to uncover the hidden mathematics of the context. (Zevenbergen, 2000)

### **Definition of numerical competency**

The word competence can lead to misunderstandings. In my vision a competence is a personally constructed or developed ability. I do not use the word competence to describe the qualifications you need in all kind of vocations. For numerical competency I have used the following definition:

Numerical competency is the intertwined knowledge, skills and dispositions (attitudes) necessary to adequately and autonomously cope with the quantitative aspects of the world around us or to cope quantitatively with aspects of the world around us.

The definition seems to leave out all kind of spatial competences and reasoning competences. Calling it mathematical competences could solve that. That however gives connotations to the scientific, paradigmatic and rigorous approach of mathematics, which is not fruitful for my goals.

I think my definition is close in spirit to the definition of Evans in *Adults' Mathematical Thinking and Emotions*:

Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical and even mathematical information in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value. (Evans, 2000, p. 257).

My definition is not close to the now popular definition from the OECD:

Mathematical literacy is the capacity to identify, to understand, and to engage in mathematics and make well-founded judgments about the role that mathematics plays, as needed for individuals' current and future private life, occupational life, social life with peers and relatives and as a constructive, concerned and reflective citizen. (OECD, 1999, p.24).

Evans' definition appeals to me the most. It is very broad and covers more than merely the quantitative side. This definition fits well with what I am interested in, namely how people interpret and deal with the quantitative and mathematical side of the world in their own activities.

The OECD definition is used in many places. Part of the reason is that this definition is at the foundation of the global PISA study. This definition assumes that mathematics is something beyond us, something that is independent of culture and free of values. The definition argues that we can make a judgment about the role that mathematics plays, as if mathematics were a real existing phenomenon of its own. This approach is not helpful for my research and certainly not helpful for the students in my research.

In the Second International Handbook of Mathematics Education there is a chapter on Mathematical Literacy written by Jablonka (Hoogland & Jablonka, 2003; Jablonka, 2000). There she states that mathematical literacy has been defined in a wide variety of ways over the past few years and one thing everyone more or less has come to agree on is that mathematical literacy cannot be defined in terms of mathematical knowledge. Mathematical literacy is in fact mainly about the functional aspect of mathematical knowledge. It is about individual competencies to use mathematical knowledge in a practical, functional way; mathematical literacy in order to ... or mathematical literacy for ... . Furthermore Jablonka argues very convincingly that the differences in approach to mathematical literacy are directly linked with the (political) goals that are pursued in mathematics education in individual countries.

My definition connects closely to these ideas.

## **Research set up and methodology**

### **Target group and key question**

My target group is students in basic pre-vocational secondary education.

The key question is to identify the numerical competences the students show in conducting tasks in practical vocational situations. In particular I am seeking to identify the embedded mathematics of the task and the embodied mathematics of the students. To do this I observed the students and conducted interviews with them, most of the time directly after completing a task. In all situations the tasks were complex, long-lasting (a few weeks, 20 – 100

hours). The tasks were meaningful for the student. The educational settings were all a variant of Authentic Learning settings.

### **Three major pitfalls**

In the exploratory stages of the research, it became clear that three things were to be considered. I call them pitfalls in discussing the numerical competencies of students that are considered weak learners. The three major pitfalls are:

1. Oversimplified ideas on numeracy. The common sense image many adults have of numerical competency is "Being proficient in formal operations with numbers." Those images are constructed in adult's early childhood school experience. General educationalists, politicians, and even many mathematicians, seem to hold these images predominantly.
2. The formatting power of school mathematics. Most common under math teachers and teachers in vocational courses is to translate the things they see in terms of school mathematics. When asked which mathematics or which numeracy is necessary for the good performance of the students in metal, electro, shop keeping, and animal care for example, the teachers predominantly say: operations, percentages and fractions – a typical example of the formatting power of school-mathematics (Skovsmose, 1997).
3. Deficit thinking. Activities of students are often described in a deficit manner. For example, highlighting all the things they cannot do, or pointing out that they have learning retardation, learning disabilities or even learning disorders. A typical example of research outcome from this way of thinking is: "Only 21% of the ... knew how to calculate ..."

In trying to avoid these pitfalls, I moved to an ethnographical type of approach. It is a challenging thought to use the ideas behind ethnomathematics (D'Ambrosio, 1985; Gerdes, 1986) to study the embedded and embodied mathematics in the activities of young vocational students in the "jungle" of early 21st century urban and vocational life.

I quote Zevenbergen from her chapter *Ethnography and the Situatedness of Workplace Numeracy in Education for Mathematics in the Workplace*:

This growing body of research suggests that a much broader conceptualisation of mathematics may be called for due to the various forms of mathematics which develop in situ and are not part of the formal school curriculum. For many of the students for whom school mathematics has been a disempowering experience, a broader conceptualisation of mathematics may offer different experiences than the current regimes of school mathematics that are on offer in contemporary institutions. This may be particularly the case for those students undertaking vocational oriented courses where it is common for students to have low levels of numeracy and have been alienated from the formal schooling process. (Zevenbergen, 2000, p.222)

The ultimate goal is to design educational settings where numerical competencies can be developed further by reflections, guidance and new experiences. As a basis for that, I really want to understand what the students do, think and how they talk about it when quantitative situations are at hand.

### **The research set up.**

The research is conducted as follows. The students are at work on meaningful tasks in a authentic learning situation. For example they are making: a bicycle trailer for a surfboard; garden lights for an external client; spotlights for the school; garbage bins for the school shop; tomato soup for the school lunch; a flower bed.

After finishing or almost finishing their products, they are asked to come and tell something about the way they manufactured the product. The product is always at hand. This session is videotaped and analyzed.

The methodology is a variant of Stimulated Recall. Stimulated recall is used by different researchers in the field of mathematics education (Lyle, 2003; Zevenbergen, 2004), and was used extensively in learning second language research (Gass, 2000). This kind of retrospective reporting is used to explore learners' thought processes (or strategies) at the time of an activity or task.

The research is still quite exploratory and descriptive. In the forthcoming months I hope to triangulate the findings through studying the students workbooks and portfolio, another source of demonstrated numeracy competencies. In the workbooks the students collect the preparation activities necessary for performing the tasks.

## Results of the trial study

Things we learned in the trial study were:

- Do not mention that the interview is about mathematics or numeracy. We just stated that we are interested in the way that they manufactured their products. On some occasions, the teacher mentioned that the interview was about mathematics. The students in both cases reacted defensively. This is a well-known phenomenon (Gillespie, 2000), usually referred to as math anxiety (Tobias, 1994)
- Talking without the product at hand is much more difficult for the students, then when the product is at hand. The more concrete the situation is, the better the students are able to express their numerical competences.
- Every question to these students, which can be answered with a yes or no, is answered like that. These questions do not invite them to keep talking.
- In many occasions when the students interpreted a question as a school question, they blanked out. With a school question I mean a question where the questioner knows the answer and is testing if the student knows the answer. In a daily life question the questioner does not know the answer and the respondent is supposed to know the answer.

Questions that avoid these obstacles are:

- Please, can you tell me something about how manufactured this product?
- I cannot understand how you how this part is made, can you give me more details?
- How long does it take to make such a product?

In most cases the students are proud of their products and very willing to talk about it.

## Impressions from the research

Up to now 15 students in 6 different schools were observed and interviewed. Each school visit took one day. The interviews were held by three different people and later analysed. This constituted the trial phase of the research.,

The examples given here are not comprehensive. They are merely illustrative and provide an indication of what was noticeable in brief conversations with students on numerical competences.

### Example 1

Reinder made a window frame from a drawing. He is in the final completion phase. He talks openly and with understanding. He can easily switch back and forth between the drawing and the product. He also has insight into the construction. Reinder explains: 'These are fixed like so in the wood and then this separates and these are secure and can never be separated again.'

Reinder clearly gesticulates to show the perpendicular directions and the angle of the force to that. He knows that strength comes from such constructions, where the forces are perpendicular to that. His accompanying gestures are used very precisely to support and visualize.



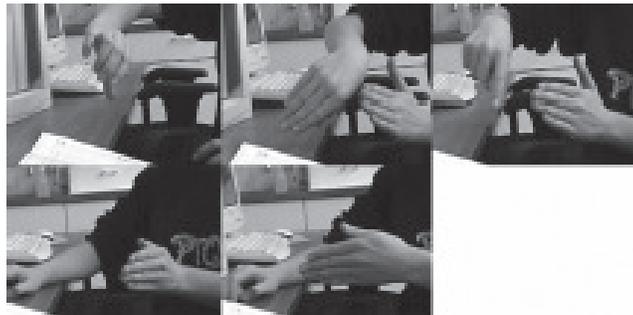
## Example 2

Reinder: 'First you have to write down everything neatly with your set square. This must be 45 degrees. You do this with your set square. You place it like this, which puts it at exactly 45 degrees.' 45 degrees is shown on the table with a flat hand.



The interviewer inquisitively asks: 'Exactly 45 degrees right away?'

Reinder answers self-confidently: 'Yes. It (the set square) has something like that. It goes like this and then there is such a corner in there and you can convert it somewhat, which then makes it exactly 45 degrees'. The workings of the flexible set square are shown with two hands. There is an understanding of angles, how they can be shown and how a device can be adjusted in using it.



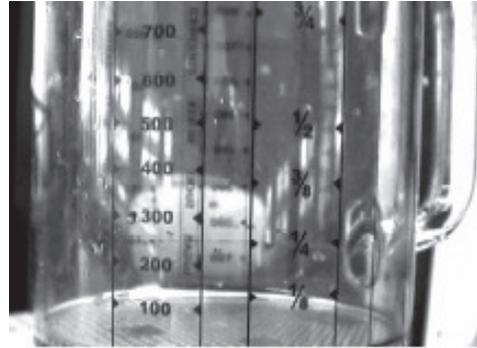
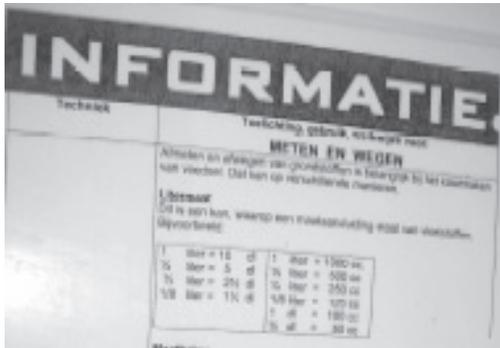
## Example 3

Cooking is also performed in the Technology and Welfare department.



After the filled speculaas, it is time to make tomato soup. The recipe calls for 2½ decilitres of water.

Lisanne: "Then the teacher said we should look at that piece of paper. Then I looked at that piece of paper and that's where I saw ¼ of a litre."



'And then you have to look at those arrows and add water to the middle of this arrow.'

This student can translate from recipe to list to measuring jug. What is striking here is that is assumed she cannot do it independently. The school solved the problem by hanging up a kind of crib sheet. Apparently, in this lesson, learning that  $2\frac{1}{2}$  dl is the same as  $\frac{1}{4}$  of a liter or 250 ml is not an objective, while the measuring jug can be a rich source of discussion with this student.

#### Example 4

In the Commercial Technology (ComTech) department Zjosque and a group were assigned to make four garden lamps. They prepared a design after a discussion with the customer. They have seven weeks to design and manufacture. The design phase has just been completed.

Zjosque explains: "You bend the top of the garden lamp from a sheet. This (the triangle) can be sharp but also wide, it always fits in, like this. Because it (the cap) folds along with it.'



Zjosque can explain and visualize this well. The interviewer asks about the size of the sheets that are used to make such a lamp. They set the sizes experimentally so they can be made from a 50 x 50 cm sheet. They work a lot with sketches and they draw, cut and paste in advance.

#### Analyses of the videos

Analyses of these (and other) examples yielded two sorts of insights: insight in the used numeracy competences of the students and insight in the way the students express themselves about these numerical competences. These findings were from the exploratory stages of the research. This will be done more methodically in the later stages of my research.

The competences the students showed in the observed situations were both numerical and spatial. In both cases interpretation of the quantitative or spatial facts was the key activity. Examples of that are:

- Matching numbers with the sizes of the product of parts of the products.
- Interpreting numbers as a clue for the kind of screw, drill or other things.
- Interpreting numbers as a measure instruction
- Using numbers to make a list of needed parts, counting
- Interpreting construction plans.

- Other findings were:
- Hardly any operations with numbers showed up. In cases where additions had to be made, the students used a calculator or Excel in a very natural way.
- Matching complex plans and schemes with the real product was almost never a problem, as long as they were both there.
- The students hardly use any mathematical language to express their numerical or spatial competences.
- All the students use gestures to support their expressions of numerical competences. Those gestures seem to be an integral part of their thinking, they do it in the numerical situations, in the matching situations and of course in the spatial situations.

## Conclusions

To summarise it:

- Students have competences in a large number of areas that we can categorize as numerical or spatial competences and if-then reasoning. In each conversation of several minutes about a concrete task, a number of these competences can be observed and identified
- Students can reason well if they are working in the situation or if they literally have the product in hand (window frame, spotlight, drawing, measuring jug). When it comes to hypothetical situations or if a concrete situation must be constructed mentally, most of them have a lot more difficulty showing their competences.
- For students, gestures and physical visualizations are important components in their numeracy repertoire. Give the students the opportunity to literally use their hands in expressing themselves about the numeracy activities they performed, then they are able to express themselves much better than in other situations.

## Recommendations for the education of these students

If I look at this specific target group, 15 and 16 years old students in the basic vocational stream of the VMBO, working on larger integrated vocational tasks, my recommendations are:

- Stop exercising with operations, they are hardly used and hardly necessary in performing the vocational tasks. Stop doing sums from the book, except when necessary for written tests or exams. Maybe designing all kind of (more playful or competitive) activities where they use the numbers 1 – 20 are a better approach.
- Work on numerical competences with students in direct and practical situations. How? By taking a good look at what they do in those situations, discussing it with them, having the students explain exactly what they are doing, and having students look at it together and learn from one other.
- Try to link mathematical words and skills directly and *in situ* to the things they do and show.
- Have students collect all sorts of materials and their own productions in numeracy. Collect them in a work folder, file or portfolio and make that the topic of the lessons.

This means that in my opinion it is almost impossible to integrate the mathematics courses and the vocational courses if they are separated in place and time. It is hard to integrate that what is not there.

Mathematics teachers must look very close at the activities of the students in doing practical vocational tasks. Vocational tutors must learn to look for the embedded mathematics in the activities. Dialogue between them and with the students is necessary to reveal the embodied mathematics of the students and to enhance the numerical competencies of the students.

Education to enhance the student's numerical competences in pre-vocational education must be redesigned with this as a starting point. That may be the greatest challenge for teachers, developers and textbook writers.

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