Mapping the 'mathematics problem' in Ireland: A longitudinal study of diagnostic test results*

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There was widespread interest in the mathematical education of students transferring from secondary education to university education throughout the 1990's in the UK (Hunt and Lawson, 1996; IMA, 1996; LMS, 1995; The Engineering Council, 2000, Smith, 2004). This debate centred on the inadequacy of the school preparation in mathematics which left students deficient or under-prepared for their university mathematics courses. Similar concerns arose in the Irish context (Brennan, 1997; Department of Mathematics and Computing, Cork RTC, 1985; Hurley & Stynes, 1986; O'Donoghue, 1999) and wider afield e.g. Australia (Barry & Davis, 1999), regarding the preparation of students entering university and their ability to successfully negotiate their mathematics courses, in particular, service mathematics courses. While the surface features of the so-called transition problem are consistent across many countries, nevertheless, there are significant country differences.

Diagnostic testing was introduced as part of a pilot study carried out in the University of Limerick (U.L.) in the academic year 1997/98 in an attempt to better comprehend the problems faced by students embarking on service mathematics courses and to inspire different short and long term methods of dealing with these difficulties. A database, constructed on the results of the diagnostic tests, was initiated in U.L. in the academic year 1997/98 and has been maintained up to and including the present year. There are 2121 students recorded in the database over the six years from 1997/98 to 2002/03.

The authors have carried out statistical analysis on the database to investigate if Higher/ Ordinary Leaving Certificate Mathematics sufficiently prepares students for the further study of mathematics in U.L.

Service mathematics in U.L.

The Department of Mathematics and Statistics in U.L. is responsible for all service mathematics teaching in the university. Service streams run over 1, 2 or 3 years. In 1993, the Department decided to reappraise their service teaching offerings. This reappraisal arose out of anecdotal concerns regarding the implementation of more effective teaching offerings, addressing the issue of poor attendance at tutorials and the perception that students were less prepared mathematically than they perhaps should have been. The idea was proposed that two equivalent mathematics groups be set up to cater for the varying levels of abilities within service mathematics programmes. Technological Mathematics 1 was set up to cater for those students who were really struggling with Science Mathematics 1 and students (within different programmes) were allocated as regards their mathematical abilities (their attainment at Leaving Certificate Level). Science Mathematics 1 is, therefore, of a higher standard mathematically than Technological Mathematics 1.

Service mathematics is currently organised into cognate service streams by entry standard (mathematics) and degree programme as follows:

- Technological Mathematics
- Science Mathematics
- Engineering Mathematics
- Computer Mathematics
- Business Mathematics.

Students in these service mathematics courses are typical of students making the transition from secondary to higher education in Ireland.

The Leaving Certificate - the transition from secondary to higher education in Ireland

Students who attend second level education in Ireland spend 5 (or 6 if they take the Transition Year option) years at this level. Mathematics is one of three subjects that are obligatory in the second level (secondary school) curriculum. At the end of the third year, students sit the Junior Certificate examination. They then progress to senior cycle, at the end of which they sit the Leaving Certificate examination. Students take 6 or more subjects at this level. Points are allocated according to what grade they achieve in each subject (see Table 1).

Table 1 The Points System

Grade	Points	Grade	Points
HA1 =	100	OA1 =	60
HA2 =	90	OA2 =	50
HB1 =	85	OB1 =	45
HB2 =	80	OB2 =	40
HB3 =	75	OB3 =	35
HC1 =	70	OC1 =	30
HC2 =	65	OC2 =	25
HC3 =	60	OC3 =	20
HD1 =	55	OD1 =	15
HD2 =	50	OD2 =	10
HD3 =	45	OD3 =	5

Key: HA1   Grade A1 in Higher Level Mathematics

   OB1   Grade B1 in Ordinary Level Mathematics

The best 6 grades count towards their total 'points' score, therefore 600 is the maximum numbers of points a student could possibly attain. No points are awarded for grades E or F, which are regarded as 'fails'. Mathematics may be taken at one of three levels: Foundation, Ordinary (O) or Higher (H) level, with Higher Level having the highest level of difficulty. Courses/programmes in third level (tertiary) institutions are offered to students who attain the requisite number of points in this School Leaving examination.

The U. L. test

The U.L. diagnostic test is a paper-based test consisting of 40 questions. This test was developed by the author (JO'D) in 1997. A panel of six lecturers in the Department of Mathematics and Statistics, who had extensive experience in service mathematics teaching and course design, was presented with a provisional panel of 70 questions drawn up by O'Donoghue. This list was then reduced to the 40 questions considered most appropriate for the level of the targeted groups. An important design consideration was that the test could be completed in one class period without rushing. The questions were also selected in order to examine the fundamentals required for successful completion of the Technological Mathematics 1 and Science Mathematics 1 courses. The test is subdivided into nine sections: arithmetic, algebra, geometry, trigonometry, coordinate geometry, complex numbers, differentiation, integration and modelling. The level of difficulty of the questions is set at the Irish School Leaving Certificate (Ordinary Level) with the exception of the integration question, which is only covered in the Higher-Level syllabus. The test was checked against the SEFI Core Level Zero Syllabus for engineers (Barry & Steele, 1993), the Junior Certificate Mathematics Syllabus, the Leaving Certificate Ordinary Level Mathematics Syllabus and compared to other diagnostic tests to make sure it was fit for purpose (O'Donoghue, 1999).

The questions themselves are open ended with a rough work column provided on each page. A 'Don't Know' box is included after each question and students are encouraged to use this rather than guessing. In this way lecturers know that a topic has either been forgotten or not learned at all. The questions are marked on an all or nothing basis; 1 mark for a correct answer and 0 for an incomplete or incorrect answer. The test was written so that the use of calculators and log tables are not required. The tests are corrected by hand with results posted within 2-3 days (for information on implementation of the U.L. test, see Murphy, 2002). Students who score less than 20 out of a total of 40 are categorised as 'at risk'.

The Database

The database of diagnostic test results was initiated in the academic year 1997/98 and has been maintained up to and including the present year. It contains data for the Science Mathematics 1 and Technological Mathematics 1 classes. The diagnostic test itself was designed with an engineering/ technology bias which is why these groups are tested and not other service mathematics courses such as the Business Studies group (O'Donoghue, 1999). In the first year of testing (1997/98) only the Technological Mathematics I were tested so for that year there is no data on the Science Mathematics I group.

The database contains over 60 items of information for each student which includes student data, scores on each question and other comparative data e.g. results on next university examination. When the data was first collected, there were 2726 students in the database. For a number of reasons information was missing for quite a number of students. One of the authors (OG) cleaned out the database of those students for whom information was lacking. As a result, at present, there are 2121 students recorded in the database over the six years from 1997/98 to 2002/03. In Table 2, the number and percentage of students per group (i.e. Technological Mathematics 1 and Science Mathematic 1) are shown (e.g. in the academic year 1998/99, 58.1% of the students who were tested and recorded in the database were in the Technological Mathematics 1 group. In the previous year Science Mathematics 1 students were not tested so all the students (100%) recorded that year were in the Technological Mathematics 1 group).

Table 2. Numbers/Percentages of students within each group, each year.

	1997/98	1998/99	1999/2000	2000/01	2001/02	2002/03	Total
Tech Maths	236	270	245	274	116	184	1325
% within year entering U.L.	100%	58.1%	58.9%	60.9%	48.3%	58.6%	62.5%
Science Maths		195	171	176	124	130	796
% within year entering U.L.		41.9%	41.1%	39.1%	51.7%	41.4%	37.5%
Total	236	465	416	450	240	314	2121
% within year entering U.L.	100%	100%	100%	100%	100%	100%	100%

Analysis of Leaving Certificate grades and levels

The purpose of the research at this stage was to investigate if Higher or Ordinary Leaving Certificate Mathematics sufficiently prepare students for the further study of mathematics in U.L. Measures used include:

- School Leaving Certificate Mathematics Levels (Higher, Ordinary)

- Mean Diagnostic Score by Leaving Certificate Level

- Leaving Certificate Grade (Higher, Ordinary).

Leaving Certificate mathematics levels

Table 3 shows the breakdown of Leaving Certificate levels within the Technological Mathematics 1 and Science Mathematics 1 groups. It is clear from Table 3 that the Science Mathematics 1 students have a stronger background in Mathematics than those in the Technological Mathematics 1 group. 42.3% of the Technological Mathematics 1 students take Higher Level Mathematics at Leaving Certificate while 57.2% of the Science Mathematics 1 students have passed Higher Level Leaving Certificate Mathematics. This is expected since Science Mathematics I was designed for the more mathematically able students. The percentages of students in the Technological Mathematics I groups with Higher Level mathematics in the Leaving Certificate are more or less consistent over the years analysed whereas the Science Mathematics I groups' percentages appear to vary more. This may be explained by the percentage of students who took Higher Level mathematics in the Leaving Certificate within each of the programmes in the Science Mathematics groups. In 1998/99 56.4% of these students had Higher Level mathematics and this percentage grew to 68.4% the following year.

Within every programme of study in this group there was an increase in participation in Leaving Certificate Higher Level mathematics with the exception of the Environmental Science group who had a decrease of only 0.1%. This is strange as the points for some of the Science courses in U.L. dropped in that year (e.g. the CAO points for Environmental Science dropped from 385 to 370, Industrial Biochemistry points dropped from 430 to 420 etc

so entry standards were lowered but for that year there were more students with Higher Level Leaving Certificate mathematics.

Table 3. Leaving Certificate Levels

	L.Cert Level		1997/98	1998/99	1999/00	2000/01	2001/02	2002/03	Total
Tech Maths	Higher level	Count	95	117	108	111	53	76	560
		% within year entering U.L.	40.3%	43.3%	44.1%	40.5%	45.7%	41.3%	42.3%
	Ordinary level	Count	141	153	137	163	63	108	765
		% within year entering U.L.	59.7%	56.7%	55.9%	59.5%	54.3%	58.7%	57.7%
	Total	Count	236	270	245	274	116	184	1325
		% within year entering U.L.	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Science Maths	Higher level	Count		110	117	88	73	67	455
		% within year entering U.L.		56.4%	68.4%	50.0%	58.9%	51.5%	57.2%
	Ordinary level	Count		85	54	88	51	63	341
		% within year entering U.L.		43.6%	31.6%	50.0%	41.1%	48.5%	42.8%
	Total	Count		195	171	176	124	130	796
		% within year entering U.L.		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Mean diagnostic scores by Leaving Certificate levels

In Figure 1 we see that students with Higher Level Leaving Certificate Mathematics score much higher on average than those with Ordinary Level Mathematics, which is expected. Both trends appear to be declining. One worrying feature of the Ordinary Level trend is that for three years it is below the pass mark of 20 and for two years the bars are at roughly the pass mark. This suggests that perhaps the Leaving Certificate Ordinary Level Mathematics syllabus is not adequate preparation for service mathematics courses in U.L.
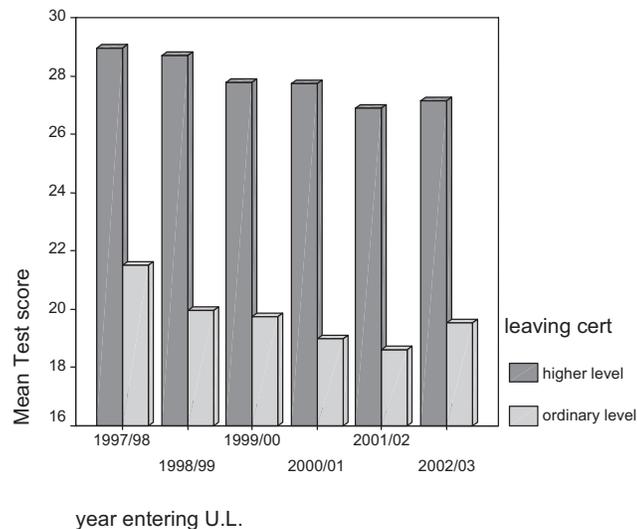

Figure 1. Combined mean diagnostic scores (Leaving Certificate mathematics level)

Looking at Technological Mathematics 1 (Figure 2) the trends are the same as above. Added to this is the fact that the highest average for Higher Level Leaving Certificate Mathematics (1997/98) is around 29 marks out of 40. This is alarming when one considers that 39 out of 40 questions on the diagnostic test are pitched at Ordinary Level or lower.

The students in Science Mathematics 1 with Ordinary Level Leaving Certificate Mathematics do not fare any better than those in Technological Mathematics 1 (see figure 3). For four of the five years their average score is below 20. The Higher Level students score slightly higher than those in Technological Mathematics 1.
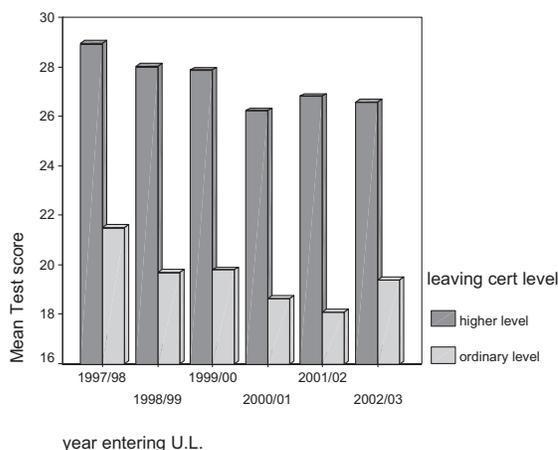

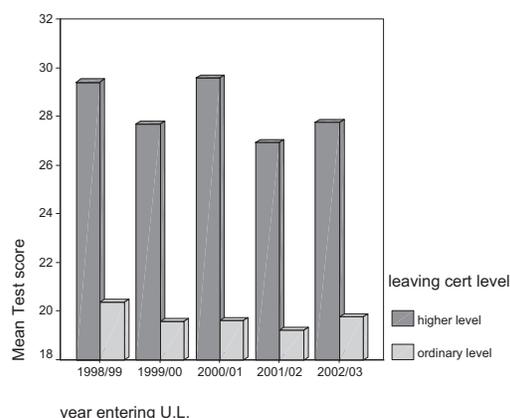**Figure 2. Technological mathematics 1 mean diagnostic scores**

Figure 3. Science mathematics 1 mean diagnostic scores

For one year, Science Mathematics 1 students with Higher Level Leaving Certificate Mathematics score slightly lower than those in Technological Mathematics 1. There is quite a difference in 2000/01 between these two groups. The Science Mathematics 1 trend is quite jagged whereas the one for Technological Mathematics 1 is distinctly declining (see figure 4). This increase in the Science Mathematics group performance in 2000/01 may in part be attributable to a higher percentage of Higher Level students within the group in that year.
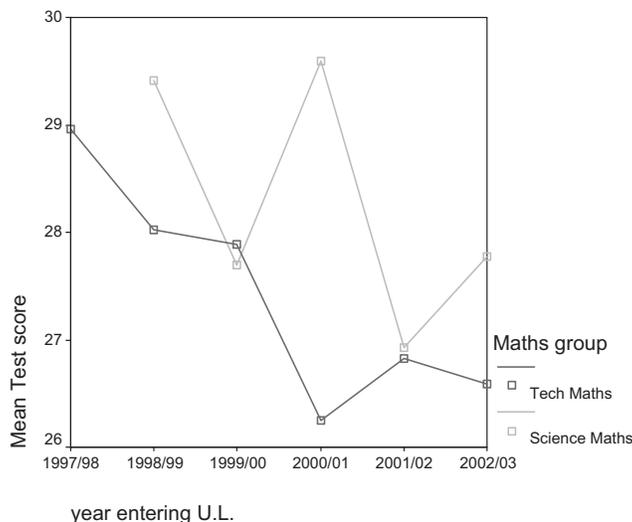

Figure 4. Leaving Certificate higher level mathematics diagnostic scores

The trends for students with Ordinary Leaving Certificate mathematics (see figure 5) are more worrying for two reasons. Firstly, both lines show a decline in mathematical standards (the Technological Mathematics 1 mean score was between 21 and 22 in 1997/98 and by 2002/03 it was below 20. The Science Mathematics 1 score was above 20 in 1998/99 and below in 2002/03). Secondly, for the last four years both lines have remained below the pass mark of 20 (with the Technological Mathematics 1 mean score as low as 18 in 2001/02). This leads the authors to believe that, conceivably, Ordinary Leaving Certificate mathematics may not be sufficient preparation for these service mathematics courses in U.L. as measured by the diagnostic test.
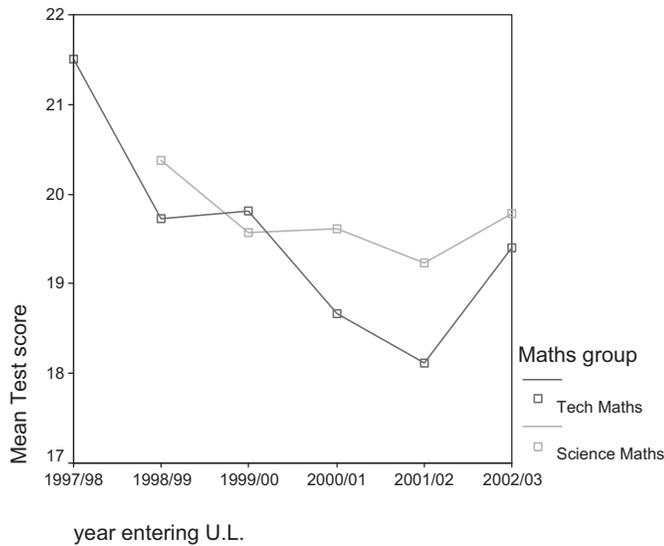
Figure 5. Leaving Certificate Ordinary Level mathematics diagnostic scores

Leaving Certificate mathematics grades

Having looked at the levels of Leaving Certificate Mathematics, one of the authors (OG) was curious to see if it was possible to define a specific grade (which students have attained in the Leaving Certificate Examination) which may indicate that they will experience difficulties in these courses. In other words, is the minimum entry requirement of a C3 in Ordinary Level Mathematics really sufficient for entry to Technological Mathematics (B3 for Science Mathematics) or should it be raised to avoid problems of mathematical under preparedness in U.L.?

Higher A1 is the highest grade that students who do the Leaving Certificate Examination can get. As shown in Figure 6, it is no surprise that the median score for this group has consistently remained above 30 between 1998/99 and 2002/03 (only one student in 1997/98 had a HA1). Figure 7 demonstrates how students who achieve a Higher C3 in the Leaving Certificate examination should have sufficient knowledge to pass the diagnostic test in U.L. The median score remains close to 30 each year.
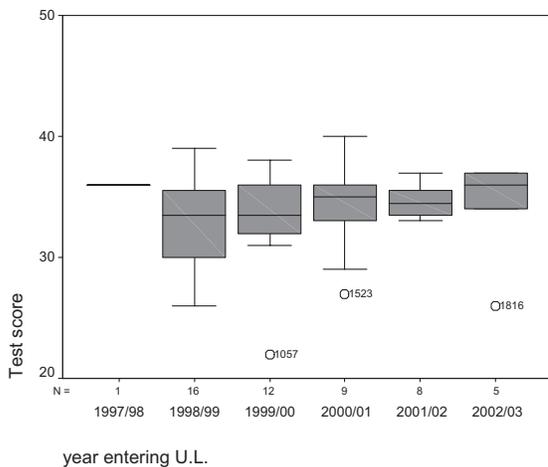

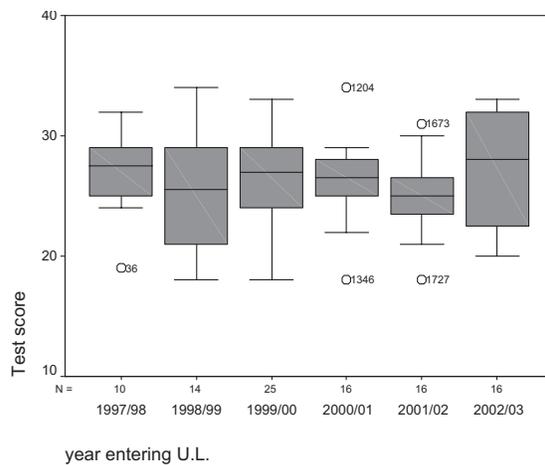**Figure 6. Higher A1 Mean Diagnostic Scores**

Figure 7. Higher C3 mean diagnostic scores

Students who score an A1 in Ordinary Level Leaving Certificate Mathematics appear to have sufficient knowledge for passing the diagnostic test, but just about! Each year the median score for these students has remained at just above the pass mark of 20.

In 1997/98 an Ordinary A2 was enough to pass the diagnostic test. In 1999/2000 and 2001/02, it was not! For 1998/99 and 2002/03 the median score is borderline.
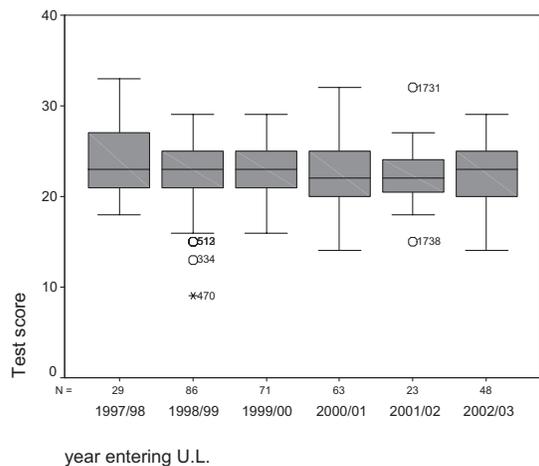

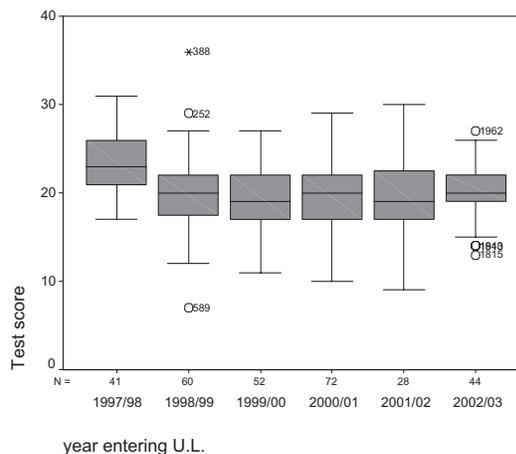**Figure 8 Ordinary A1 mean diagnostic scores**

Figure 9 Ordinary A2 mean diagnostic scores

Again, in 1997/98 students with an Ordinary B1 had a median score of around 20. For every year after that the median is below 20.
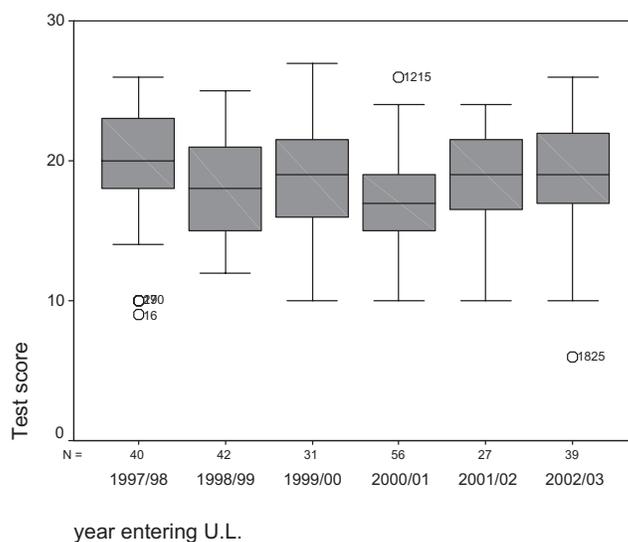

Figure 10 Ordinary B1 mean diagnostic scores

Grade dilution

The results above have various implications. They suggest that there has been some grade dilution in Leaving Certificate grades i.e. an Ordinary A2 nowadays is equivalent to an Ordinary C3 (the minimum entry requirement) ten years ago. If we examine the mean scores over the 6 years for OA1, OA2 and OB1 (shown in Figure 11), there is more concrete evidence to back up this suggestion of grade dilution
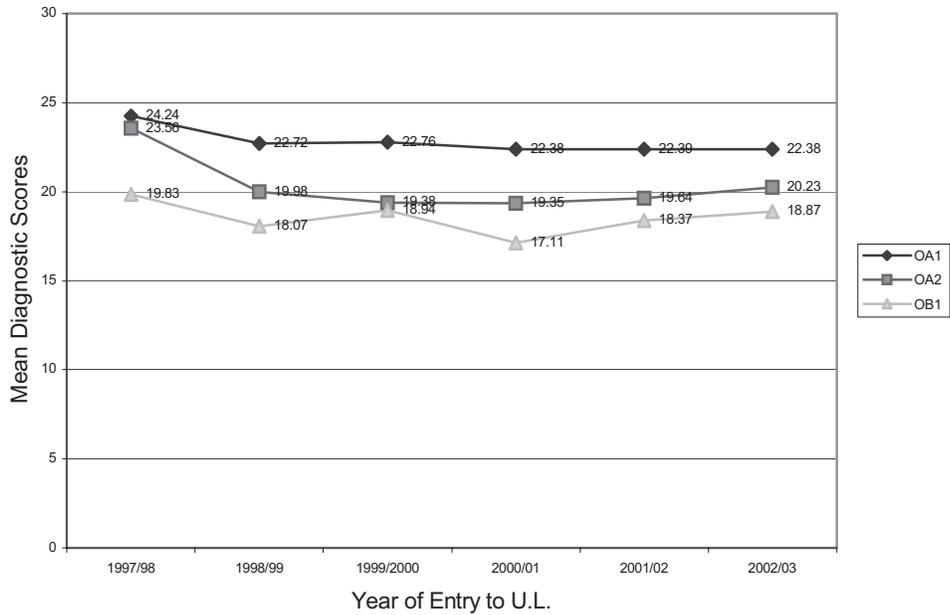
Figure 11. Mean diagnostic scores for OA1, OA2 and OB1

For three years (1999/2000, 2000/01 and 2001/01) the mean diagnostic score for an OA2 was lower than that achieved by an OB1 in 1997/98. For the last three years in the study the mean diagnostic score for an OA1 was lower than that for an OA2 in 1997/98 also. A comparison between HC1, HC2 and HC3 in Figure 12 shows the same story.
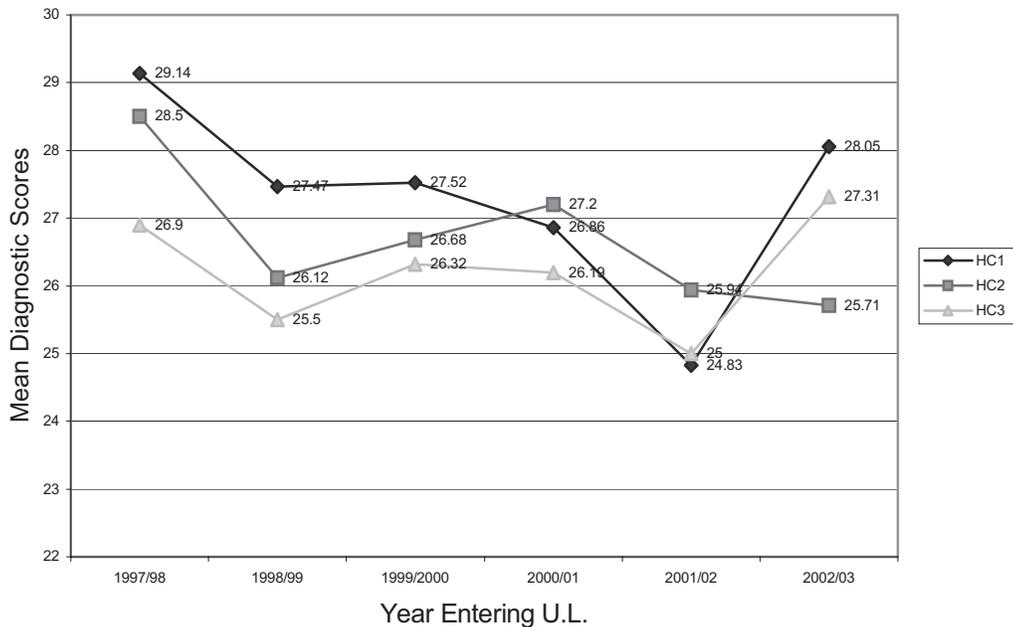

Figure 12. Mean diagnostic scores for HC1, HC2 and HC3

The mean score for HC1 in 2002/03 is lower than that achieved by HC2 in 1997/98 and the mean score for HC2 in 2002/03 lower than that for HC3 in 1997/98. This observation in turn has implications for other issues such as entry admission standards, university curriculum and support mechanisms.

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Discussion and conclusion

Almost 30% of students entering service mathematics courses in U.L. are defined as 'at risk' year on year. Hunt and Lawson (1996) carried out a study to investigate if the mathematical standards of first year students in Coventry University had declined between 1991 and 1995. They concluded that, not only had standards declined, but that there was evidence of grade dilution (Hunt and Lawson, 1996). The authors have discovered that the same issues are prevalent in U.L. also. The main difference here, though, is that Irish students take mathematics throughout their primary and second level education. Then why do we have the same problems in Ireland as elsewhere at the transition stage?

Murphy (2002) carried out an investigation into the mathematical under-preparedness present among Irish third level entrants. She looked at the second level experience and issues of transition to see if they were contributing to the problem. She focused on the didactical contract present within typical Irish second level classrooms. She discovered that the teaching in Irish second level classrooms was completely dominated by the Leaving Certificate examination. Her findings are corroborated by Lyons et al (2003) in their study of mathematics classrooms in Ireland.

Problems such as large syllabi, course dilution at Leaving Certificate Level and the 'points system' are, undoubtedly, contributing factors (Murphy, 2002). The Points System has some adverse affects on second and third level education, creating more problems than are immediately apparent. Students endeavour to attain as many points as possible and select subjects that will help them realise this end. For some students this is more important than choosing subjects that would be deemed more apposite for their chosen university course or career. With such an emphasis on points rather than on understanding, students are forced to sacrifice the latter for the former. Teachers are affected too. They are required to help students achieve these goals. In this way they are forced to adopt a "teach- to-the- examination"(O'Donoghue, 1999:6) approach and find it necessary to leave out chunks of the mathematics curriculum in order to do so. Not enough time is spent on teaching–for-understanding so students must rely on rote learning and cramming to help get them through (O'Donoghue, 1999). Students are encouraged to choose the questions they are going to attempt in their Leaving Certificate examination months before they sit it. As a result, students enter college perhaps never having heard of complex numbers through no fault of their own. If we examine the structure of the Higher Level Examination it is easy to see why:

Paper I

Q1, 2	Algebra
Q3	Matrices, Complex Numbers
Q4, 5	Sequences and Series, Induction, Logs and Inequalities
Q6, 7	Differentiation
Q8	Integration

Students have 2½ hours to complete 6 questions from above.

Paper II

Q1	The Circle
Q2	Vectors
Q3	The Line and Transformation Geometry
Q4, 5	Trigonometry
Q6, 7	Permutations, Combinations, Probability, Statistics and Difference Equations
Q8	Further Calculus and Series
Q9	Further Probability and Statistics
Q10	Groups
Q11	Further Geometry

Students must attempt 5 questions from the first 7 options and 1 out of questions 8 to 11.

It is possible that students could arrive at college without having been introduced to, for example, Complex Numbers and Matrices and Trigonometry. As ridiculous as it seems, the reality is that students could quite possibly get an A1 by avoiding them completely.

Students who take Higher Level mathematics for the Leaving Certificate are especially conscious of how time consuming it is and many are prepared to transfer to Ordinary Level so as to spend time on other subjects which will give them the points they desire. So despite the fact that students may achieve very high marks in their Leaving Certificate examination and get accepted onto the course of their preference, it is no indication of how adept they are at mathematics. The emphasis is on managing their effort to achieve maximum points, not on improving their points through better performance.

One of the aims of the Leaving Certificate mathematics syllabus is to prepare students for further study and professional preparation and, in particular, further study of mathematics in its own right (www.ncca.ie/math_sen.htm). The Points System indirectly promotes rote learning making it increasingly difficult for teachers and students to achieve these aims. It results in a very narrowly focused examination pedagogy. This in turn leaves students, even good students, inadequately prepared for a more independent type learning environment in Higher Education.

While this analysis is based on the U.L. sample, there are good grounds for arguing that the findings are indicative of the situation nationally. The mathematics experience of Leaving Certificate students is very similar throughout a highly centralised national secondary school curriculum. Several recent reports and studies have singled out the traditional nature of school pedagogy, the negative influence of 'teaching-to-the-exam' particularly in mathematics, and other influences of the points system (e.g. OECD, 1992; Lyons et al, 2003; Commission on Points, 2000). Another indicator is the growth of mathematics support services for students entering Higher Education. Currently all seven universities in the Republic offer some form of mathematics support. In these ways Ireland has acknowledged its 'Mathematics Problem' and is tackling it.

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