Adults' dialogic productive mathematical interactions in the classroom*

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In this paper we analyze two case studies by means of observing a learning situation in which various adults work on the concept of proportions in a dialogic way. Our main aims are to reveal how differences influence mathematics learning, and which positive interactions producing meaningful knowledge can be considered by means of dialogic learning. We based our analysis upon both socio-cultural and cognitive perspectives of mathematics education. From a methodological point of view, we use cognitive trajectories in two in-depth case studies to control the influences of interactions in proportional reasoning. We analyze some aspests in order to improve the numeracy teaching, learning and assessment practices into the mathematics classroom. In this paper we conclude that each person tries to justify the solution they provide, seeking arguments that allow them to sustain their point of view. Through this dialogue, learners could overcome their difficulties and improve their mathematical knowledge.

Introduction

Last year (2004) Spain received 686 224 immigrants more than the year before[6]. In such a context, issues like interculturalism, respect for difference, and guaranteeing the same opportunities for everyone, stand out in today's educational debate. One of the concerns that people who teach in Adult Education have is how to address classes with increasingly diverse people. We find men and women of different ages and backgrounds in the classroom, who have different cultures, languages and ways of understanding the world. We present two case studies in which we have worked with two groups of people in the School for Adults La Verneda – Sant Martí (Barcelona, Spain). The focus of this paper is the meaning production in Adult Education related to Mathematics in proportional situations. The contents related to proportionality are used to illuminate the role of language and interaction in building abstract meaningful mathematical knowledge appropiation. Our main aim is to see how sociocultural differences imply different roles of language in mathematics reasoning in an adult dialogic perspective, but do not imply difficulties for productive interactions.

Theoretical framework

Adults' learning is different from regular school education[7] because adults have many prior experiences that we must not forget in our pedagogical practices.[8] Previous studies demonstrate that the way of using mathematics depends on prior socialization (Berger & Luckmann, 1988). In Spanish adult schools we have groups with extremely different previous experiences and backgrounds. Therefore, in analyzing cognitive data, we must consider many non-academic sociocultural elements influencing adults reasoning and arguments. In such a perspective, diverse authors have centered their research on the study of non-academic mathematical skills and they affirm the existence of a gap between the mathematics learnt at school and that used in other contexts and the need for a framework of equity (Civil 2004). We all have non-academic ways and procedures for solving problematic situations that imply the use of mathematics, and everyone uses these procedures that they have acquired throughout their life according to the cultural context in which they have grown up. In fact, research has moved on to show that people participate in more

[6] INE. 2004. Cifras INE. Informative Bulletin of the *Instituto Nacional de Estadística*. Madrid. Available in digital versión at http://www.ine.es.
[7] We refer to education devoted to compulsory education until the age of 16.
[8] In Europe, we have made many efforts in this field. EU financed a set of research projects about "previous experiences as an important concept and presented results by means of APEL (Assessment of Prior Educational Learning policies), applied successfully in the UK or Sweden. For information on CREA – Center of Research in Theories and Practices that Overcome Inequalities, see http://www.pcb.ub.es/crea.

than one context. So, the same person may indeed draw on different mathematical resources/knowledge depending on the context/practice in which they have participated.

In this paper, our hypothesis is that sociocultural differences do not negatively affect mathematics learning through discourse in a dialogic learning framework (Flecha 2000) which uses informal proportional perspectives (Kaput & West 1994) and legitimates participants' different prior experiences. Our perspective starts from Zevenbergen's (2000) analysis of the mathematical code of the discourses that are produced in the classroom from the cultural perspective of Bourdieu (1979). As we can see in these studies we also assume that some practices can turn into excluding factors for some students because the language that they use does not coincide with the formal language that is used in the classroom. We also accept according to Wedege (2002) and FitzSimons (2004) the need for re-reading the theory of codes and control by Bernstein (1977, 1996), and applying it to the teaching of mathematics. We consider dialogue in a constructive critical way (AlrØ & Skovsmose 2003), and we try to understand how speech acts (Searle 1980) can improve adult learning reasoning even in a diverse "dialogic" egalitarian group (Flecha 2000) to reveal the role of informal proportional reasoning in adults' interactions for improving meaning productions.

Methodology used

We have carried out two case studies in the School for Adults La Verneda - Sant Martí, in Barcelona (Spain). Six women formed our first group, in 2003. With this group we created our methodology, as well as a booklet of mathematical activities (in paper and digital format). These adults worked on the theme of proportionality during various sessions.[9] Six adults also constituted the second group. This group of people had very different backgrounds: there were two Spanish people, one person from Senegal, two from Morocco and one from the Dominican Republic. The two Spanish people were two women who did not study in their early life but have decided to study now. The person from Senegal was a man whose business degree from his country is not recognised in Spain, hence his studying to obtain a degree in Spain. The two Moroccan individuals were also men: one of them only attended primary school, the other attended college but he was not able to finish his degree. Finally, the person from the Dominican Republic was a woman who, like the Spanish women, did not attend formal schooling when she was young. In order to collect information for our purposes, we analyzed one of the sessions using a digital video-tape (approximately 70 minutes) from our first group. In our second group we conducted in depth interviews that we also recorded.

Table 1. Examples of mathematics activities.

Task 1	Task 6
In this stand at the market they have elaborated a board on which they calculated the price of various weights, relieving them from having to calculate the price each time they make a sale. Complete the following table:	If we approach or move away from things that surround us, they look larger or smaller. Try it out: join someone and together try to measure the height of the classroom door. What is bigger, your colleague or the door? Do you think if you begin to walk forward or backwards that there will be a point at which your colleague is the "same or equal" height as the door? What distance were they from the door in order for this to happen?

Weight (kg)	1	2	3	4	5	6	7	8
Price€	3	6						

Task 1: Example of a task from the mathematics book AA.VV. 2002. *Matemáticas. Educación de personas adultas*. Barcelona: El Roure. Page 152

Task 6: Example of one of the tasks on the mathematics web site <http://www.neskes.net/mates>

In order to analyze the speech acts and the way that adults argue about a proportional situation, we used several categories (Díez-Palomar, 2004): PC: particular case; DP: diverse particular cases; CI: comprehensive interpretation; GR: generalised recognition. These four categories allow us to distinguish the arguments that correspond to concrete examples or questions from arguments that move to more abstract ideas. On the other hand, in order to understand how interactions between people within the classroom affect them, we have used the following categories of analysis of the functions of language: p: provocation; a: agreement; er: explicative response; de: doubting enunciation; ae:

[9] In those sessions adults worked on 10 activities about proportions, in which elements of daily life (like the activities that were proposed in the context of a market place) were combined with more theoretical elements (such as questions, as for instance, about the meaning of the proportionality constant). The first five questions are "problems with text", that are characterized by presenting an explicit context, with a single formulation and a single possible solution. On the other hand, in the last five, we find "problems of daily life" (in which the context appears only partially, and the solutions can be varied).

assertive enunciation; cc: clarified correction (Diez-Palomar, 2004). Using these categories for the analysis, we have built what we call "cognitive learning trajectories" (CLT). The developments are in a graphic representation where we locate the different sentences that appear in the dialogue among the people who intervene in a class or in the in depth interviews. The sentences were categorized using the categories presented above. CLT have served as an innovative methodological tool to study the dialogues that are produced within the classroom, while people in the mathematics group solve different situations. We also analyzed interactions in terms of illocutionary and perlocutionary speech acts (Searle, 1980). A perlocutionary speech act is one that convinces or produces an intended or unintended action or feelings, while an illocutionary speech act is one performed in the saying, not explicitly intending to bring about change. In the case of teaching, perlocution carries the authority so conviction through authority, while illocution puts the information on the table, so to speak, and allows the receiver to decide on how to interact with it.

Findings

Let us begin with an example to see how we develop the analysis, comprehensive interpretation and findings. In our first case study, persons 4, 5 and 6 were discussing an activity that consists in discovering the proportional relationship that exists between the height of a door and that of the teacher, while the latter walks towards or backs away from the door (See Task 6 in Table 1).

> P6.- If she is in front of the door (the teacher), from far she looks bigger.
>
> P5.- If you walk further away, you see the same or…
>
> P4.- No, I don't think so, I think she will remain smaller (case study 1).

In this piece we can see different illocutionary interpretations: on the one hand person 6, who has a clear idea of the functional idea of proportionality (that is to say, he/she sees that it is a relationship in which one measure depends on the other); and on the other hand, person 4, who does not really see that relationship. The dialogue established leads to the conclusion that there is a relationship of proportionality between the distance of people in the group in relation to the teacher and the door.

> P1. Can my height be the same as the door?
>
> P5. Well, I don't…it depends on the distance you look at the door from.
>
> P1. Look to see if at any point you see me the same as the door …
>
> P5. From here, no…
>
> P1. What about if I stand here, in front of you all?
>
> P4. Ah, yes, if you stand there…
>
> P6. You look taller, of course, if you see it like a prism… (…)
>
> P6. Then of course, if you get really close to us, then it does look like you were the same height as the door. Because we are...but if you would stand here like it says, then of course, we would see everything as being very far in proportion (case study 1).

On the basis of video tapings we have identified different CLTs (see figures 1-3) revealing what happens in the interaction by adapting the type of graphs proposed by Dreyfus, Hershkowitz and Schwarz (2001). In fact, the lines reveal how the adults achieve more abstract language and higher levels of proportional reasoning (inscribed in successive circles) according to time. Thus, different people's interventions appeared in the dialogue (p1, p2, and so on), revealing an increasing trajectory (See Diez-Palomar, Giminez & Garcia, 2004, for a detailed explanation of such trajectories).

In the fragment of the transcription that we reproduce, we see the dynamic of one person who is the protagonist of the cognitive learning trajectory in an activity: towards the end of the fragment she enters into a dialogue with herself (a kind of monologue) in which she goes through the particular cases to recognise what is implicit in the rules of proportionality.

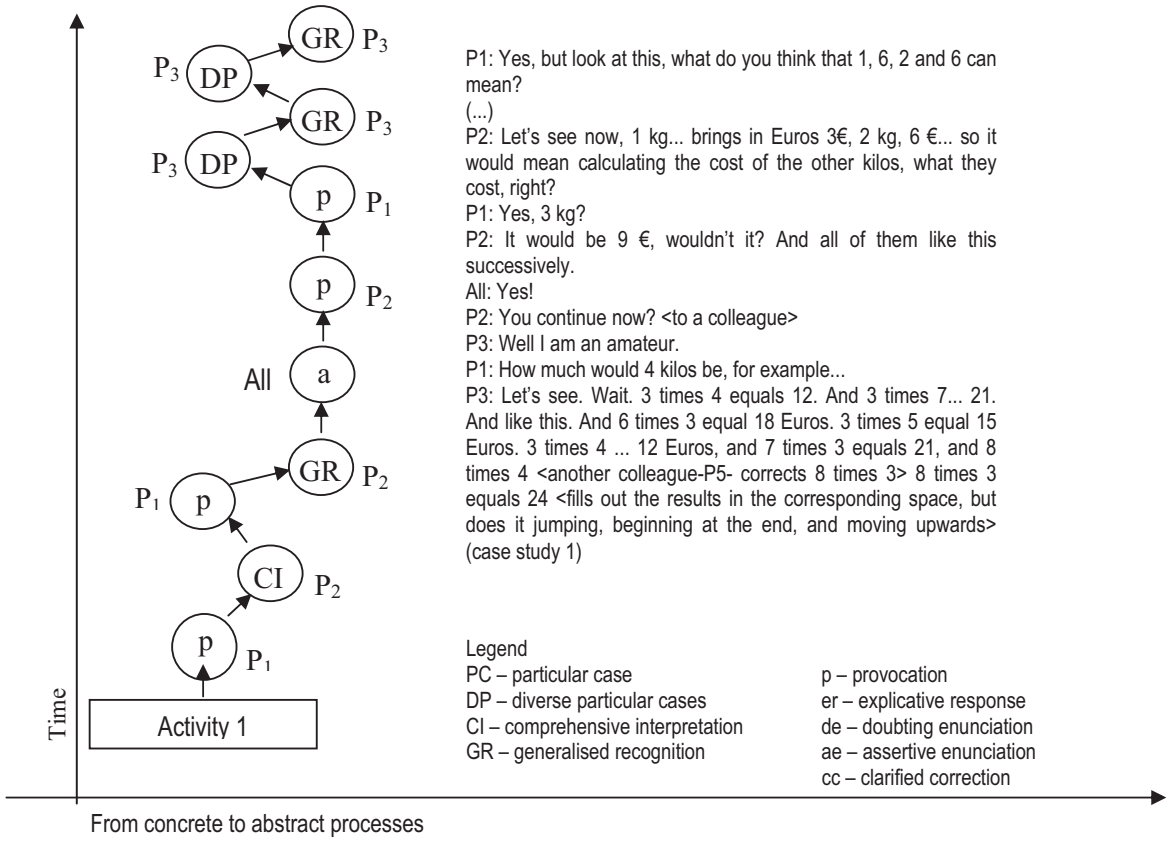

Figure 1. Cognitive learning trajectory from activity 1(a) (case study 1).

The next CLT comes from our second case study. The person was using a cooking receipt from Senegal to illustrate the concept of proportion. To do it he used a "generalized recognition". In this case, we observe that this person uses a typical didactic strategy that is a specific example for the explanation or exemplification of a general idea. The shape of the CLT is shown in figure 2.

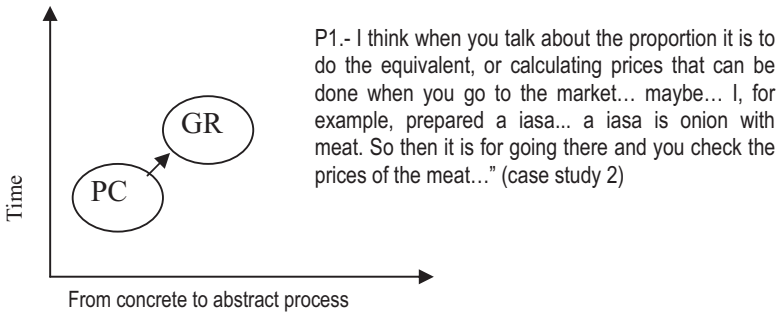

Figure 2: Cognitive learning trajectory from an in-depth interview (case study 2).

From a semantic point of view, we observe that in this situation (figure 2) there was a mistake in lexicon, in the word that this person uses when referring to the proportion. He associates "proportion" with "equivalence". While he is doing this he confuses "equivalence", which is a relation like a = b, with "proportion", that is a relation like a/b = a'/b'. While explaining his point of view, he related how the amount of meat and onions he buys changes depending on the number of guests that will come for dinner. Therefore we are directly confronted with an example of direct proportional relation. This demonstrates that he understands the concept, but he had difficulties with the language: he didn't find the right word to describe it, because Spanish was not his first language. He knew the meaning of the concept. The difficulty was in the code (language), when he explains the word in the context of the "generalized recognition".

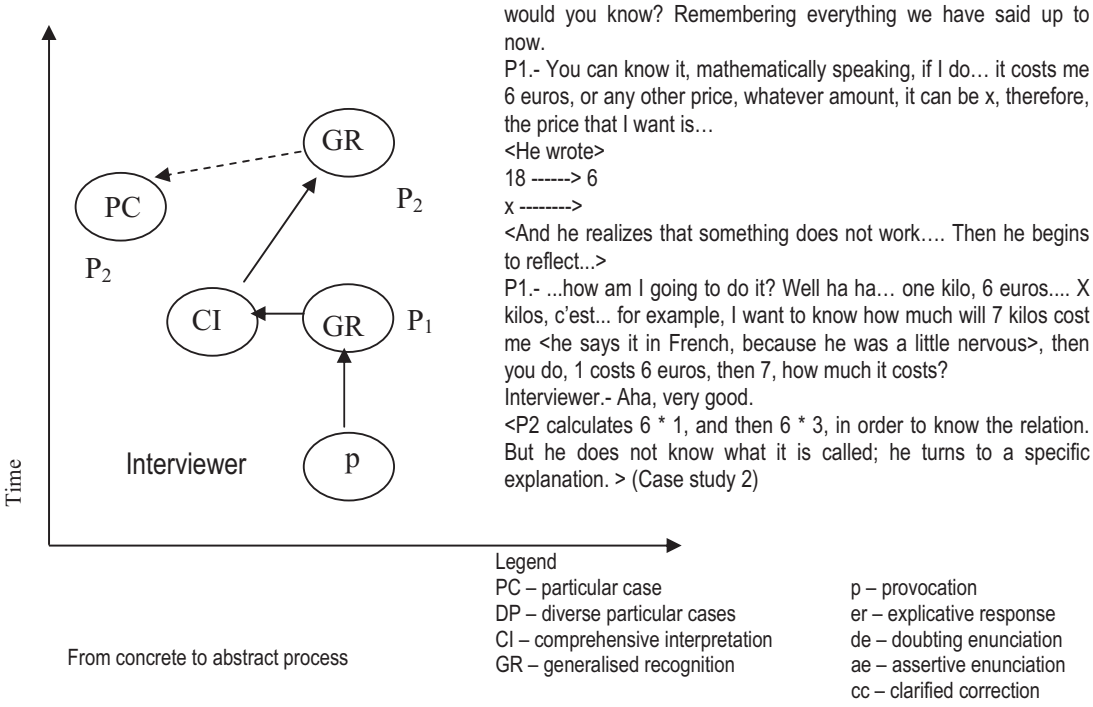

Figure 3: Cognitive learning trajectory from an in-depth interview (case study 2).

This CLT (figure 3) belongs to a dialogue between two people who are solving a problem contextualized in a market (case study 2). Person 1 first turns to an academic strategy to solve the problem: "regla de tres" (three rule).[10] In other words, he tries to use a general rule for a specific case that he encounters in this activity. He turns to his prior academic experience, and presents a concept that has nothing to do with directly explaining the proportion: the unknown quantity. On the contrary, person 2 explains the relation between the two numbers that are written in the table that appear in the book that they are using in the session. While proceeding this way, he realizes that in multiplying the number of kilos that appear in the table by 6, he figures out the price. Person 1 attempts to apply the "three rule" to the problem and has difficulties. At one point he switches to French, which is his native academic language.

Now let us see what happens in a lineal trajectory (figure 4).

When change occurs here, we observe that it is when P1, the teacher, uses dialogue with a *perlocutionary* aim, it means influencing the response of the participant. This type of action occurs fundamentally in the part of the task that implies a technique of calculation associated with the content, and in which the teacher understands that she must give a secondary role to student dialogue in this case in order to prioritize the reasoning that leads to a process of resolution. In these cases, provocations attain mechanical responses, because the teacher wants to break with the traditional contract that the importance lies in the result (given that she does not want to insist, in that moment, that to provide an answer there must be a conversion of measures), but it is not understood well by the adults who are still committed to this contract.

These examples allow us to see the importance of egalitarian dialogue in learning. When this dialogue exists in a perlocutionary manner based on the position of power of the teacher, *there is an exchange of questions and responses that does not leave room for re-thinking or comprehension of the mathematical concepts*. On the other hand, when dialogue takes place from the illocutionary point of view, or in the case of perlocutionary, based on claims of knowledge (in terms of Habermas*), it appears to be a source of "creation of meaning", that is to say: the person is capable of using the concept on which he or she reflects, and endows it with meaning*.

[10] *Regla de tres* in Spanish is the concept used to describe the mathematical strategy that is applied to solve situations where there is one unknown element. For example, if one kilo of bread costs 2 euros, how much will 2 kilos of bread cost? It is called *regla de tres*, because 3 of the 4 elements of the equation are known, while the fourth is unknown.

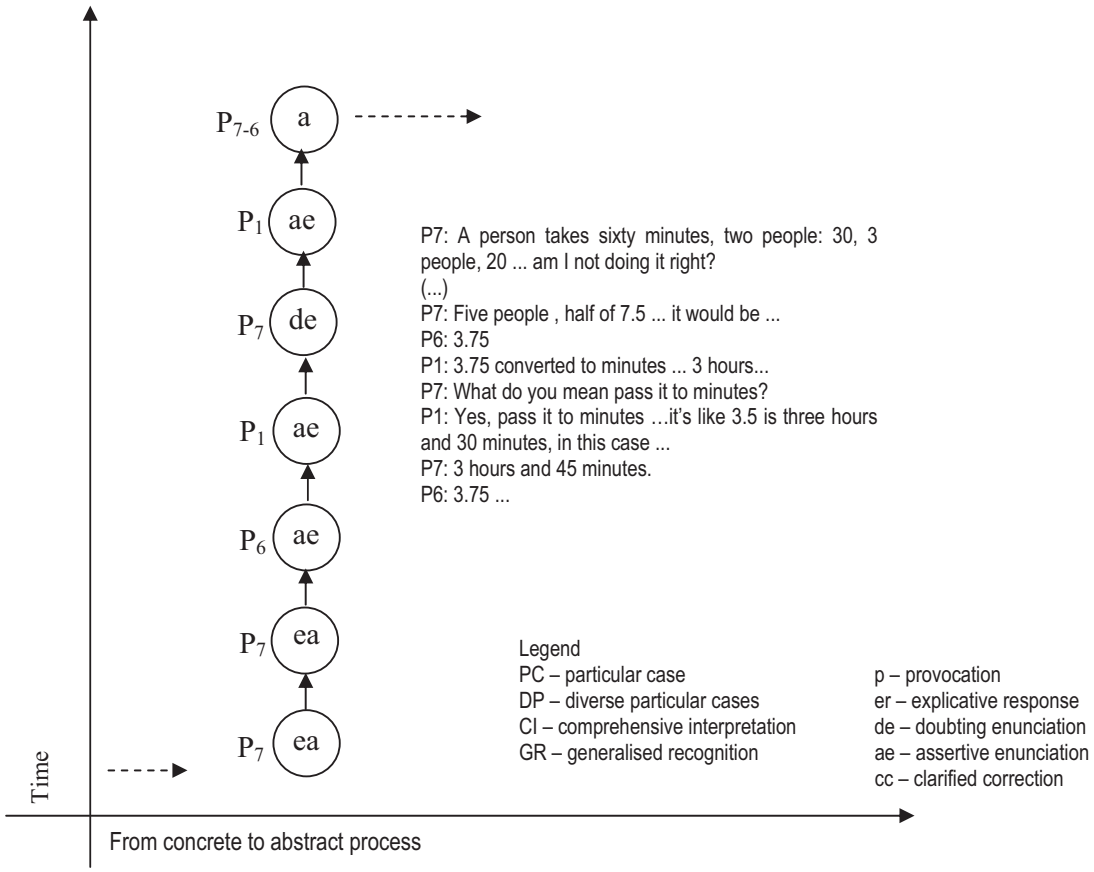

Figure 4. Cognitive learning trajectory from activity 3 (case study 1).

When someone is able to use an abstract concept to make concrete examples (in order to explain their meaning, for example), it indicates that they really produce better proportional meanings (given that they are able to use it in diverse concrete situations). This is what occurs when a person tries to help someone understand (through arguments) the meaning of proportional issues by using their own previous experiences (for example in the case of the shopping in Figure 2 and 3), and in many cases accepting the "other's" perspective. It also means that productive interactions are possible in such a framework.

Conclusions

Through the different examples that we have analyzed in this paper, we can observe how people solve different problematic situations by using dialogue. Each person tries to justify the solution they provide, seeking arguments that allow them to sustain their point of view. Through this dialogue, people's different cultural traditions may be present in the classroom.

We can observe that the fact that each person utilizes a different register to talk about the activity of mathematics does not constitute a difficulty in learning the mathematical concept of proportion. In an environment based on egalitarian learning there is room for every interpretation, as long as they are products of an argument oriented to finding a solution and not a point of view imposed by someone in the group. When a dialogic learning process takes place, everyone in the group can solve the activities "in their own way", making use of methods that they have learned previously. On the other hand, in a learning environment that does not include all voices, where everything depends on teaching, which is considered to be the "normal" way (Gorgorió & Planas, 2004), difficulties arise given the existence of different codes (even languages) between people in the classroom. This becomes clear when we analyze the discourse, bearing in mind its effects, be it illocutionary or perlocutionary (Searle, 1980).

Regarding the dialogue established, we recognize that illocutionary approaches encourage dialogue, which is the fundamental path taken by adults for learning and jointly constructing the meaning of abstract mathematical concepts. We also found that perlocutionary speech acts can encourage learning, but can also create barriers when the speaker uses a position of power that breaks with egalitarian dialogue.

From our experience, we raise some important differences for proportional reasoning:

(i) between ontological and symbolic objects,

(ii) between material properties (what is near and far) of proportional meanings and mathematical (ideal) properties of proportion (mathematical morphism properties);

(iii) between everyday objects (comparing distances, for instance) and mathematical objects (function with a linear equation formula);

(iv) between natural (size of shapes) and naturalized (increasing or decreasing observations) objects, and unfamiliar objects (linear equation).

Egalitarian dialogue encourages participants' learning, because they have to seek the correct arguments to justify the solutions that are proposed in the different problems. On the other hand, when the teacher directly provides the answer to the problem, without leaving room for dialogue in the classroom, adults are limited to agreeing without understanding the meaning of these reasonings. This is clear when linear cognitive trajectories appear. It was also observed that egalitarian dialogue elicited the *differences* in meaning production for mathematics objects, in our case proportion. Such possibilities were successful because of the intentional position of introducing *productive interactions*, avoiding the figure of meanings being somehow worked into people´s minds (Lins, in press).

Therefore, all of this can help us to obtain a practical consequence for orienting the development of the curriculum in classrooms for adults: the inclusion of every voice from an egalitarian point of view contributes to breaking barriers that are encountered by adults from different cultures and educational backgrounds instead of making decisions based on normative, prescriptive, or based on "ended" knowledge.

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