Adult numeracy: What is it, and who teaches it?

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This position paper is an attempt to broaden the debate on what constitutes adult numeracy and who should teach it. In it I argue that numeracy means more than basic number skills, and consider some possible aspects of numeracy tutor development.

During 2004 I was involved in an adult numeracy project and found myself dismayed by what some colleagues believed should be in such programmes, and about who might teach such courses. Some colleagues thought that adult numeracy was little more than basic number work, and that literacy or second language tutors could adequately teach such a course. These views seem to me to be interrelated and link with what I see as the unfortunate use of the word numeracy. While literacy relates to the ability to read and write, it also has connotations with respect to literature, but numeracy is seen by many as relating only to basic numerical skills. This view of numeracy continues to be the popular view in spite of the dictionary definition that links numeracy with being numerate which means being *'acquainted with basic principles of mathematics and science'*; but perhaps the populace believes that the basic principles of mathematics are all numerical.

In discussing these issues I acknowledge that I am a New Zealand mathematics educator with little experience in the field of adult numeracy—but sometimes fresh eyes see different things.

Adult numeracy course participants

It seems relevant to first think about the range of people involved or likely to be involved in adult numeracy courses, and to think about their prior learning and their needs. In New Zealand I would suggest that whether participants were born in the country or are recent immigrants, they are likely to have had considerable primary school experience and often some opportunity to participate in mathematics in high schools, and I accept that the situation is likely to be different in countries where basic education is not available to all or where more relaxed immigration policies exist.

In terms of the prior learning of most New Zealand participants I would suggest that they have nearly all learnt 'successfully', unfortunately their learning was not as intended—they have learnt that they could not do basic number work or more advanced mathematics. In such a situation I would assume that to repeat a course emphasizing basic number skills will successfully reinforce their 'successful' learning and will do little to help them satisfy their needs.

Next I want to consider the needs of people enrolling for these courses. While they may want basic numerical skills do they want more or need more? And to what extent should such work with number skills be supported by conceptual notions and by basic calculator skills? This suggests a need to consider the aspects of mathematics that impinge on basic citizenship. I see these been drawn from a list of topics that might include at least:

- arithmetic (number skills and applications of these),
- simple geometrical notions (measurement, shapes, scale drawings, geometric patterns),
- algebra (mainly tables, simple graphs, some formulae), and
- statistics (all aspects of data handling, understanding variation, statistical graphs, chance).

Before considering a list of topics, which was how the participants' prior learning was probably organized, it seems appropriate to reconsider what mathematics is, and some alternative approaches to the teaching of mathematics. Note that I am now assuming that the word numeracy is a synonym for basic mathematics although I have not yet defined what the basic elements are.

Mathematics

School mathematics is often broken into arithmetic, algebra, geometry and statistics, but this is only one way to think about the subject. More recently there has been an attempt to look at content (basic facts and procedures) and processes (reasoning, problem solving, communicating, and making connections). But, both content and processes are often interpreted in procedural ways.

Content can be thought of as ‘what one knows’, and processes as ‘what one does’, but these are underpinned by ‘what one thinks’. This leads to a way of thinking about mathematics as three inextricably intertwined aspects — knowing, doing, and thinking, as shown in figure 1.

This thinking dimension is useful in that it provides an alternative way of introducing mathematics. However, before we can do this there is a need to consider what aspects of thinking underpin mathematics and how learning tasks can be designed to emphasize thinking.

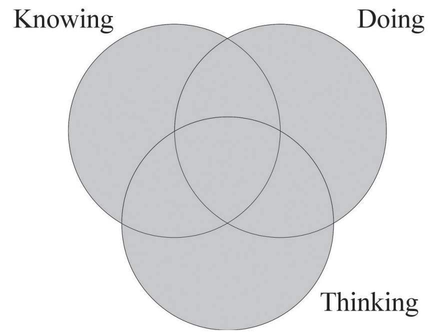

Figure 1. Three aspects of mathematics

Thinking

Thinking seems to be a current curriculum trend nationally and internationally. There are many forms of thinking. Here are some to consider:

- critical, creative, and meta-cognitive
- logical—deductive (general to particular) and inductive (particular to general)
- deterministic and non-deterministic (thinking in causal and complex/chaotic ways, thinking in certain and uncertain situations)
- analytic (breaking into parts) and synthetic (building into wholes)
- visual (diagrammatic, graphical, spatial), symbolical, and verbal
- relational (making connections, conjecturing, multiple representations, modeling, connecting within mathematics, between subjects, and the world)

And this is just the beginning, other forms might be described with adjectives such as: lateral or divergent, intuitive, aesthetic, entrepreneurial, ecological.

Thinking about thinking does not mean that content and processes (knowing and doing) can be ignored. Just as a content-oriented course and a process-oriented one suggest different teaching approaches, so thinking provides another way to approach the subject.

From my perspective every subject in the curriculum provides different ways of thinking and making sense of the world, and thus an adult numeracy course should contain basic elements of content, process and mathematical thinking.

While others may not disagree with my reasoning, the following ‘shaded area’ task illustrates some different ways of thinking.

Example: *Shaded Areas*

Look at figure 2. Assume that the shading continues starting with the square being split into four, the bottom LH square shaded, the top RH square split into four, and so on.

What fraction of the square is shaded?

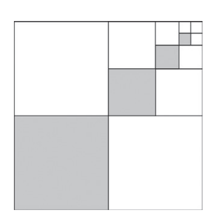

Figure 2. Shaded Quarters.

Some approaches:

(i) Most high school mathematics teachers think symbolically, their approach is: Aha! a geometric progression, so $(^1/_4) + (^1/_4)^2 + (^1/_4)^3 \ldots$ and (of course then) $S_\infty = a/(1 - r)$.

(ii) One alternative is to think visually, to look at the diagram, and break it into a series of L-shapes, and think what fraction of each L is shaded.

(iii) A third approach is to think verbally: imagine a square cake being cut into four, three of us take a piece, the remainder is cut into four, again three of us take a piece, and so on. How much of the cake will each of us eat?

Aspects of mathematical thinking for emphasis

If one assumes that mathematical thinking needs more emphasis in adult numeracy (and in school mathematics), then one needs to consider what might be done. I believe that a teacher cannot change all their practice at once so there is a need to focus on a small number of changes. I would suggest three aspects need urgent consideration and I am indebted to a colleague, John Mason of the Open University in England, for two of these.

Visualization

The shaded area task suggests the need to think visually and to look at diagrams differently. This emphasis on the use of diagrams is not new—whenever learners are given a task they should be encouraged to draw a diagram. Many problems can be solved simply by drawing diagrams, and diagrams provide different representations that help students see things differently.

Dimensions of variation

John Mason has worked for some time on the construction of mathematical tasks for classroom use. One focus he uses he calls exploring ‘dimensions of variation’. This involves asking questions about the ways and the extent to which a task can be varied and extended. (Note: variation is used here not in the statistical sense, but to indicate changes that might be made to a task.) For example, when we learn to add whole numbers, how might this be extended to other numbers (fractions, decimals, negative numbers, irrational numbers) and to addition in algebra. This focus on dimensions of variation means that classes do not work through pages of exercises, but have tasks that they can extend and discuss, and approach in numerous ways.

Generalizing

John’s other focus was on generalization. This is related to ‘dimensions of variation’ and involves taking any idea, conjecturing about how it might be applied more generally, and then verifying (or disproving) the conjecture. In his words (Mason, 2003):

> Any lesson without an opportunity to generalize is not a lesson in mathematics.

It is easy to think of familiar examples and consider how they might be varied. For example, Pythagoras’s theorem, variations include the converse, using > 90° or < 90° instead of = 90° (and converses for these), shapes other than squares on the sides, the general result in symbols (the cosine rule), and visually for triangles that are not right-angled (showing squares and the triangles altitudes extended to divide each square into corresponding rectangles of equal area).

Olympic Rings or the W-problem

A simple numerical example, the Olympic rings problem (or the W problem) can illustrate how these three forms of thinking might be used.

Example: Imagine the five Olympic rings which form nine regions. Imagine a dot in each region and a W passing through them (with dots on each vertex of the W and at the midpoint of each arm). Is it possible to put the numbers 1 to 9 on the dots, so that each arm of the W adds to 19. If so, how? If not, prove it.

Firstly, the visualization requires a diagram, and as many people are not quite sure how the Olympic rings overlap, expressing the question as the W-problem can help.

After experimenting (something not always encouraged in traditional approaches to mathematics, but often necessary) most people find it not difficult to prove the impossibility— the 1 has to be used, whichever arm the 1 is on the largest possible total is 18 (1 + 8 + 9), so 19 is impossible.

However, proving that 19 is not possible is only the beginning—in terms of dimensions of variation or generalizing one can ask, what numbers other than 19 could be used, and can each only be achieved in one way? This will involve conjecturing and verifying and while the results are surprising they also give considerable practice with basic number work and logical thinking.

Other aspects of thinking

Thinking in problem solving

I still remember the 4-step problem-solving presented by my mathematics teacher (Towers, 1954) when I started high school (see figure 2).

He referred to:

step 1 as a translation from a real problem to a standard mathematical one,

step 2 as procedural,

step 3 as translating the solution back to the language of the problem, and

step 4 as checking the reasonableness of the solution.

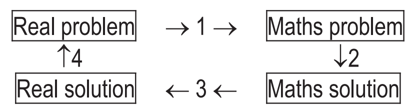

Figure 3. The problem solving cycle

He said that school mathematics has for too long emphasized step 2—the step that anyone can be trained to do, and that steps 1, 3, and 4 require mathematical thinking. Now step 2 can often be done with a computer or calculator and we need much more emphasis on steps 1, 3, and 4.

Thinking about proof

In many cultures mathematics is seen as useful for solving problems, in some it has a recreational dimension, and in western cultures it is often seen as important because of the aspect of proof — although, being a human construction this proof is relative to axioms (assumptions) and has nothing to do with truth. But what do we accept as proof in class?

- Is a visual proof enough? For example, was the diagram enough in the 'shaded area task' to prove that $(^1/_4) + (^1/_4)^2 + (^1/_4)^3 + \ldots = (^1/_3)$?
- And what about computer proofs; think of the four-colour theorem — is proof changing with technology? Is 'dragging' in Cabrí providing a geometric proof? (Dragging is all one needs to prove Pythagoras's theorem!) Or. is 'convincing a peer' what is meant by proof at school level?

Again, an example might illustrate this.

Example*:* Imagine a right-angled triangle with a square on each side as shown in figure 4. Join the vertices of the squares to form three triangles between the squares (as in the diagram).

What might you conjecture about the four triangles?

Can you generalize this?

Does the triangle have to be right-angled?

Can you prove your conjecture in more than one way?

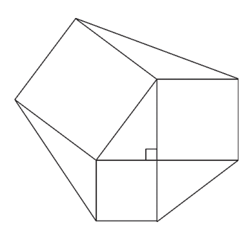

Figure 4. Pythagoras plus.

Yes, all four triangles have the same area, and you can prove this by 'dragging', by using trigonometry, or coordinates, or with a rotation, and it is true for all triangles.

An important issue for mathematical thinking in adult numeracy concerns proof. Should we accept informal proofs and proofs without words? How does the 'audience' influence the rigour of proof? And, are some adults in numeracy courses parents who wish to help their children?

Statistical thinking

Statistical thinking is important because for some people, statistical literacy and making sense of newspaper articles is a new way of introducing much numerical work and the numerical work can be learnt *incidentally* which may be less threatening than direct instruction.

Wild and Pfannkuch (1999) identified four general types of thinking that relate to mathematics and five types of thinking that are fundamental within statistics. Their general thinking included strategic thinking, seeking explanations, modeling, and applying techniques; and their foundational thinking included recognizing the need for data, 'transnumeration' (changing representations), considering variation, reasoning with statistical models, and integrating the statistical and the contextual—and all of these help us expand our views of thinking.

Contextual thinking

Using statistics, problem solving, and rich learning activities (see appendix), usually involve contextual thinking and one aspect here is to ensure that the thinking about the context is not so new or engaging that it detracts from the mathematics model being employed. If the context is a reasonably familiar one then this is not so likely to occur and the context can provide a bridge between the more concrete contextual thinking and the more abstract mathematical thinking.

Approaches to teaching

In considering approaches to adult-numeracy teaching I recall a comment made by a language school director about staff. He said that the most important criterion when hiring staff was that the tutors were not high school teachers as they "talk too much". That made me think about what we do as teachers. I have also commented how many adult numeracy students have learnt that they cannot do mathematics, and I assume that the approach to teaching adult numeracy should be different from that experienced in schools. A third notion is that adult learners are in a somewhat different power relationship with their tutors and that treating them as adults is important.

At the same time, there are some commonalities in teaching all age groups including:

- the need to start where the learners are (including acknowledging their tacit understanding)
- the need to use meaningful (authentic) tasks that the learner wants to work on,
- the responsibility for learning is with the learners,
- the teacher is important and can influence whether the learner turns 'off' or 'on', and
- learners construct their own 'schemas', the difference with older learners is that their initial schemas are more robust, having lasted longer, and are often harder to change.

Some possible approaches to adult numeracy are:

- changing from a content/procedures approach to a process/thinking approach,
- using inquiry-based learning focusing on problem types that the class wish to solve,
- having broad curriculum and negotiating the details with the class,
- making use of technology wherever it is applicable,
- employing 'rich learning activities' (Ahmed, 1987; Cox, 1998, see appendix), and
- using more group work and discussion so that learning and doing mathematics is seen as a communal activity rather than an individual enterprise.

One aspect of the curriculum relates to teaching particular algorithms, or using multiple approaches. I have been surprised when working in teacher education when a class of 15 was given the problem 9 x 17, all the class obtained the correct answer, but nine different approaches were used. In the same way, the farmyard problem (53 heads, 150 legs, how many hens and how many sheep) that might be used in the context of simultaneous equations can also be solved with a simple equation, by guess and check, by logic, by drawing, and by acting out a situation. It does seem to me to be important that adult numeracy tutors acknowledge that many approaches to rich tasks are appropriate and course participants should be encouraged to solve problems in more than one way.

Curriculum and assessment

Assuming that numeracy means being *'acquainted with basic principles of mathematics',* then we need to ask what are the basic principles, what is mathematics, and what is meant by acquaint? For me basic principles include processes and thinking, mathematics is more than number and the number operations, and 'acquaint', in this context means more than 'know of', it includes 'know how', 'know when' and 'know why'.

I think there is a role for group work and group assessment, and that assessment tasks (and learning tasks) need to be constructed so that assessment is a learning activity and that early attempts at tasks can be edited and resubmitted. Assessment needs to focus on what the student knows and thinks, not on what they do not know. Perhaps a work portfolio might be suitable for assessment.

Tutors

In terms of who teaches adults to become numerate, I assume that most educated people are familiar with basic numerical skills, and when pressured, feel that they can help others develop these skills. However, if I was to suggest that most mathematicians are basically literate and could help others develop literacy skills I would not expect a positive response from literacy tutors! I do understand how adult numeracy courses have evolved as a part of literacy courses but at some stage there is a need to break this tie and acknowledge that adult numeracy, or basic mathematics, or perhaps 'mathematical thinking for adults' needs to stand alongside literacy as another subject that helps learners develop further ways of making sense of their worlds. If this is not always possible then there is a need for literacy tutors to have professional development to extend their thinking about numeracy to include the basic principles of mathematics.

Numeracy tutors' professional development

Whether adult numeracy tutors are literacy tutors, former mathematics teachers, specialists, or have come through some other career path, I see a need for professional development that considers many issues including those outlined in this position paper. I see a need for tertiary institutions to offer relevant papers (courses, subjects) and for groups of tutors to initiate their own professional development. Because the needs of the client group in adult numeracy courses are somewhat different from the needs of children in schools, adult numeracy tutors should be leading the way rather than lagging behind colleagues who teach mathematics in schools.

The future

Much needs to be done and it is unlikely that governments or tertiary institutions will take the lead, and if they did it may not be in what we might regard as the right direction. I have previously argued (Begg, Davis & Bramald, 2003) that the development of teachers, curriculum, resources, assessment, and policy, along with theorizing, research and reflections are all interrelated. This implies that there are numerous ways one might start to work to make changes, but the lack of likely assistance leaves it to tutors as professionals to take responsibility for any development — a big ask, but at least adult numeracy tutors will have more control of their own futures.

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Appendix

Criteria for rich learning activities (modified from Ahmed 1987 & Cox 1998)

Dimension	Criteria
Approach	– approach the unknown through what is known to the students – be accessible to all students at the start – allow further challenges and be extendible – challenge the better students without overwhelming the weaker ones
Properties	– be interesting to the students, and to achieve this, to the teacher – have an element of surprise – be enjoyable (that is, engaging) – should not trivialize the subject
Appropriateness	– introduce material within the programme at a time relative to its use – provide opportunities for constant review
Possibilities	– invite students to make decisions – involve students in speculating, hypothesis making and testing, proving or explaining, reflecting, interpreting – do not restrict students from searching in other directions – promote discussion and communication – encourage originality/invention – encourage 'what if' and 'what if not' question
Focus	– emphasize key general principles more than technical details – provide specific illustrations of general principles – be seen both as an end and as a basis for subsequent work and study – avoid the temptation to teach too much material

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