CROSSING BORDERS – RESEARCH, REFLECTION & PRACTICE IN ADULTS LEARNING MATHEMATICS

Proceedings of the 13th International Conference of Adult Learning Mathematics (ALM)

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About ALM

Adults Learning Mathematics – A research Forum (ALM) was formally established in July 1994 as an international research forum with the following aim:

To promote the learning of mathematics by adults through an international forum, which brings together those engaged and interested in research and developments in the field of adult mathematics learning and teaching.

Charitable status

ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number 3901346). The company address is 26, Tennyson Road, London NW6 7SA

Objectives of ALM

The Charity's objectives are the advancement of education by the establishment of an international research forum in the lifelong learning of mathematics and numeracy by adults by:

- Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;
- Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.

ALM activities

ALM members work in a variety of educational settings improving the learning of mathematics at al levels both as practitioners and researchers. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world.

ALM does not foster one particular theoretical framework but encourages discussion on research methods and findings.

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Preface

ALM13 INTERNATIONAL CONFERENCE, QUEEN'S UNIVERSITY, Belfast July 16th – 20th 2006

The papers that follow are the Proceedings of the Thirteenth International Conference of *Adults Learning Maths*. This year's conference of ALM (Adults Learning Maths, an international researcher and practitioner forum), was hosted by the School of Education at Queen's University, Belfast. Over 65 researchers and practitioners in the fields of adult numeracy and mathematics met to share research, practice, and innovation under the theme of "Crossing Borders –Research, Reflection and practice in Adult Numeracy". During the conference Dr Diana Coben was honoured for her sterling work in adult numeracy which has led to her being the first Chair of Adult Numeracy from September based at King's College, London.

The conference provided a platform for exchange of research and ideas for practice reflecting on the contribution ALM has made to the field of adult numeracy internationally. Professor Gelsa Knijnik, Brazil, presented a plenary session developing the theme of cultural influences on mathematics, presenting examples from her work in ethno mathematics.

Papers and workshops covered issues from professional development, classroom practice, practitioner research, researcher reports and practical, hands on workshops.

Researchers from eleven countries participated in the conference and shared their current research in Adults Learning Mathematics.

United Kingdom
Dr Diana Coben led a number of sessions with colleagues from Learning Connections, Scotland; NRDC and King's College. An action research project using ICT in developing numeracy was presented. Diana alongside Dr Alison Tomlin presented findings from a project which examined the crossing of borders within curriculum boundaries. Finally she reflected on the NRDC project into what constitutes effective practice in numeracy and how can it be measured.

Clive Kanes, King's College, London reviewed the notion of "mathematics" problematising the idea of content in terms of alternative context, epistemological status, and boundaries identifying the notion of knowledge itself. He also offered a paper which applied a radical consideration of the boundaries of discursively in mathematics. Dr Alison Tomlin, King's College, and Jay Derrick, Institute of Education, presented an innovative workshop on the theme of Utopia – with no restrictions or boundaries in our approach to adult mathematics education. Jay presented a workshop using two ongoing research projects on formative assessment and improving assessment.

South Bank University, LLU+ participant Beth Kelly gave a critical overview of numeracy skills within the vocational context since 1997 and presented implications for future developments. Graham Griffiths and Rachel Stone from LLU+ discussed

the issues involved in what should be the pedagogy for adult numeracy tutor education. David Kaye also from LLU+ offered a workshop which incorporated the use of the history of mathematics into teaching of adult numeracy.

Sarah Richards, Abingdon & Witney College, presented a paper which promoted the facilitation of development of self efficacy and the support of cognitive re-framing and behavioural experimentation. Lynne Jenkins, Swansea University, reported on results from a quantitative study on pre-nursing students which identified nursing student's mathematical ability in order to develop an online tool to assist in drug dispensing.

Professor Fred McBride, Queen's University, and colleagues presented their ICT based work (ALTA) which is a development to enhance formative assessment, at this conference, specifically related to adult numeracy. They reported on a small scale project using their work at Belfast Institute of Further and Higher Education.

Kelly Watson and Fiona Watters, both numeracy practitioners in Belfast, presented their work in embedding numeracy within a vocational area, engineering, reflecting on issues of best practice in numeracy teaching.

Valerie Seabright and Fred Greer, Queen's University, offered a workshop investigating context based activities in the numeracy classroom challenging the concept of context, whose context and purpose of context. Valerie also offered a paper as part of her research on development of critical thinking skills in adult numeracy professional development arguing for positive implementation of thinking skills development as an essential requirement to enable reflective practice which is useful.

Shelley Tracey, Queen's University, offered a workshop which reflected on a creativity workshop used with a numeracy tutor group. She discussed the identification of qualities of creative practitioners and its impact on curriculum design.

Dr Joe Allen, Queen's University, presented practical examples of teaching mathematics using ICT providing an opportunity for hands on experience.

U.S.A.

Kathy Safford Ramus and Anestine Hector Mason both from USA presented, on different occasions their experience of the research undertaken by the U.S. Office of Vocational Education which is researching mathematics and numeracy pedagogy and is to be used to support curriculum design, professional development and assessment in numeracy. Lynda Ginsburg, Rutgers University, examined the interactions and affect of parents' mathematics on school learning and homework in mathematics.

Fred Peskoff and Leonid Khazanov, from Manhattan, further developed the concept of maths anxiety and strategies to overcome the issues. Peggy Moch, Valdosa State University, offered the CANE (Commitment & Necessary Effort) model as a basis for understanding and implementing interventions to improve adult's attitudes to mathematics.

Australia and New Zealand

Gail Fitzsimons presented on two occasions, firstly on the discourses of mathematics and numeracy using a vertical and horizontal dimension and stressing the need to cross borders between the two discourses. She also presented an evaluative framework aimed at assisting development of materials to support adult numeracy using new technologies. Chris Klinger, Flinders University, developed the theme of early learning experiences and their impact on perceptions of mathematics. A statistical study was reported using a measurement tool devised by the author.

Donald Smith, Victoria University, linked the development of probability teaching with teaching about gambling and stressed the importance of preventative issues in teaching in the area of gambling. Dr Barbara Miller Reilly, Auckland University, presented her research which explored teaching approaches used in mathematics for second chance learners investigating teaching approached and affective changes.

Sweden, Denmark, Holland, Germany and Austria

Lene Johansen, Aalborg University, discussed the question of whether it is possible to empower adults through numeracy teaching. She referred to the research she has undertaken and to the contribution ALM has made to the debate over the last 13 years.

Tine Wedege, Malmo University, opened the debate about what represents quality in research in the field of adults learning mathematics. She presented quality criteria to consider for research papers within the field. This session offered a follow up of the discussion to be continued on the ALM website and at next year's conference.

Margarita Fries, Vuxenufbildningen, demonstrated the use of alternative methods of learning using written pieces suggesting this facilitated learning where students are able to record everything that is important and are able to use the "green Bible" in internal assessments. Inge Henningsen, University of Copenhagen, discussed the international studies of adult literacy and numeracy, which have major influences on policy, but which are not always analysed and interpreted professionally or correctly.

Eigil Hansen, VUC Syd, Copenhagen, a practitioner in art and maths presented new ideas and examples of mathematics potential in art following on from previous contributions to ALM conferences.

Kees Hoogland, Utrecht, presented his research into the effect of using a numeracy toolkit on students, the toolkit being a "just in time toolkit" available when students run into difficulties with their mathematics. Juergen Maasz, University of Linz, offered a paper on the role of mathematics in politics and the economy, following from a request from the editor of "Der Standard" to examine the role as it could be conceived as a manipulating tool or just another ideology. Jens Langpaap, University of Hamburg, developed the theme of problem solving using two distinct approaches, solving real world problems by using knowledge about the mathematical world and solving mathematical problems by using knowledge about the real world.

Ireland

Eabhnat Ni Fhloinn, Dublin Institute of technology discussed the DIT maths learning centre, a new initiative to provide mathematical support to any student with a mathematics component linking activities with use of e-learning. Olivia Gill, University of Limerick, presented her research on the effectiveness of a maths learning centre for at risk students and those wishing to improve grades at a higher

level. She outlined the work undertaken over a number of years which has produced a database of information on aspects of levels of mathematics prior to university providing a standard comparative measure for incoming students. Gerard Golding, University of Limerick, presented his research findings into factors influencing learning in mathematics for adults returning to third level study, specifically linked to advanced mathematics.

The conference provided a platform for exchange of research and ideas for practice reflecting on the contribution ALM has made to the field of adult numeracy internationally. Professor Gelsa Knijnik, Brazil, presented a plenary session developing the theme of cultural influences on mathematics, presenting examples from her work in ethno mathematics.

Cultural differences, adult education and mathematics

Gelsa Knijnik

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It is a pleasure to be at this conference and to have the chance to enjoy this lovely city and its friendly people. I would like to thank Valerie and her ALM colleagues for inviting me to speak in this plenary session. The theme of my talk is "Cultural differences, adult education and mathematics". The ideas I will present on this topic are based on what I have learned as a researcher and a practitioner in maths education, working for the last 15 years with the Brazilian Landless Movement. It is from this very specific place that I organized my presentation. Moreover, the ideas that I would like to discuss here are rooted in what I have been learning with landless peasants who participate in adult education projects as students or as teachers. They inspire my academic life and provide the guidelines for the ideas I have been developing based on my readings about the theme. These are the reasons why the examples I will present during the talk, that allow me to establish close connections between theory and practice, come from the Brazilian Landless Movement.

The key idea I intend to discuss in the talk is the following: Adult mathematics education must incorporate different mathematics. Developing the argument requires presenting theoretical and empirical evidence, which allows me to say that there are different mathematics. Thus, it is necessary to show the social, cultural and even mathematical reasons which support the use of the strong expression "must incorporate" in the argument: Adult mathematics education must incorporate different mathematics.

In order to achieve these aims I organized the presentation into four parts. The first I called "Setting the scene: Brazilian Landless Movement and their struggle for land and education". Here I will provide a glimpse of some aspects of this peasant social movement, especially the educational work they are improving. In the second part I outline the theoretical background that supports the idea that there are different mathematics. In the third part of my talk I will give an example of two different mathematics: one mathematics produced by a *form of life* found in Landless peasant culture and mathematics, produced by a *form of life* found in school culture. Here I will highlight the social, cultural and mathematical reasons that allow me to argue that adult education must incorporate different mathematical practices. The talk ends with some final remarks.

Setting the scene: Adult Education and the Brazilian Landless Movement and their struggle for land and education

Michel Hardt and Antonio Negri (2001) begin their well-known book "Empire", saying that "it is materializing before our very eyes (...) [since] we have witnessed an irresistible and irreversible globalization of economic and cultural exchanges" (Ibid, p.11) which instituted "a global order, a new logic and structure of rule – in short, a new form of sovereignty. Empire is the political object that effectively regulates these global exchanges, the sovereign power that governs the world." (Ibid, p.11).

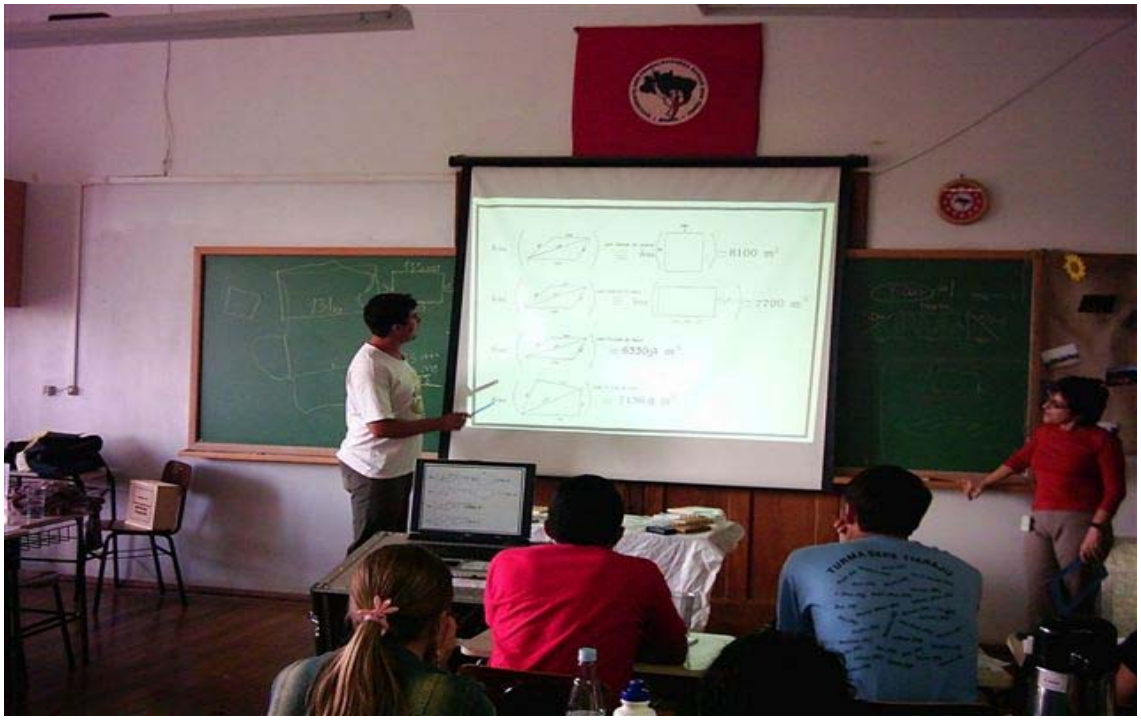
This is the new Imperial Order that I take as a background to discuss adult education and to present a brief overview of the Brazilian Landless Movement and their struggle for education. I do this because I consider it important to attempt to understand adult education as a field of knowledge as well as the contemporary social movements and their educational processes within this new world configuration characterized by the "absence of boundaries", in which "the rule of the Empire operates on all registers of the social order, extending down to the depths of the social world". (Ibid, p. 15).

Hardt and Negri, on examining the potentials for constructing alternatives that counter the imperial power, highlight the role taken on by the struggles of "all those exploited and subjected to capitalist domination", (Ibid, p.72) which includes the new social movements.

Among the many international struggles of social movements that have close relationships with education, especially mathematics education, we can situate the Brazilian Landless Movement's struggles for land reform.

Quoting from its site, "Landless Movement, in Portuguese, Movimento Sem Terra (MST) is the largest social movement in Latin America with an estimated 1.5 million landless members organized in 23 out of 27 states. The Landless movement carries out long-overdue land reform in a country where less than 3% of the population owns two-thirds of the land on which crops could be grown. Since 1985, the MST has occupied unused land where they have established cooperative farms, constructed houses, schools for children and adults and clinics, promoted indigenous cultures and a healthy and sustainable environment and gender equality. The MST has won land titles for more than 250,000 families in 1,600 settlements as a result of MST actions, and 200,000 encamped families currently await government recognition. Land occupations are rooted in the Brazilian Constitution, which says land that remains unproductive should be used for a "larger social function."(http://www.mstbrazil.org).

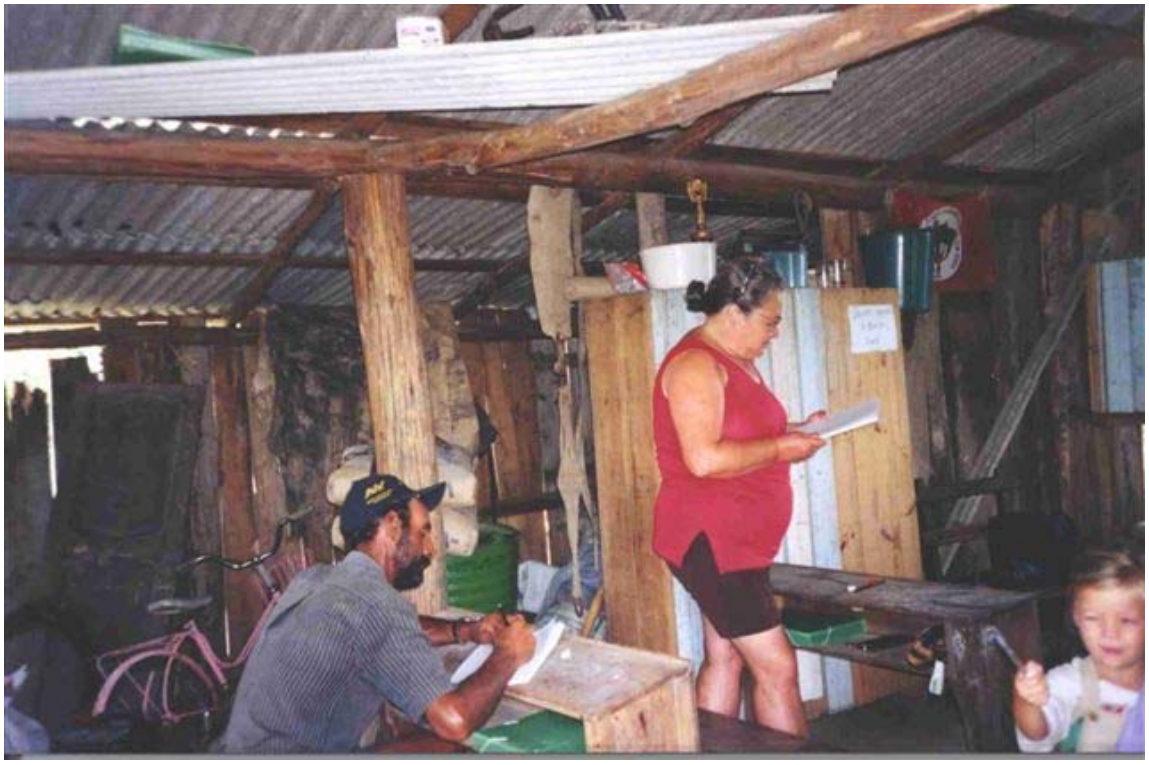
The educational process that has been developed by the MST over its 20-year history must be understood beyond schooling, since each landless subject educates her/himself through her/his participation in the everyday life of their communities and also through the wide range of political activities developed by the Movement. This means that the children, youth and adult peasants are educated by the multiple facets of the struggle for land which produce very specific social identities. Nevertheless, these social identities do not form something compact, uniform, in which hundreds of families from different social strata would ultimately become a unified whole, homogenized by the struggle for land.



To look at this social movement with such lenses implies considering that if there is some kind of intention of establishing a "landless identity", this intention is never completely fulfilled. In summary, the landless educate themselves in the struggle – in the occupations, the marches, in their ways of organizing the settlements, through their cultural artefacts – learning the many possible meanings of "being landless". But in this educational process there is a sort of rebellion against fixing *one* social identity. There are many axes – such as those of gender, sexuality, ethnicity – which in their crossovers ultimately shape multiple landless identities, multiple ways of giving meaning to the struggle for land. In summary, I would say that the peasant culture of the Brazilian Landless Movement is marked by difference.

The schooling activities developed by the Landless Movement cover Child Education, Elementary and High School Education, Teacher Training Courses and projects of Education of Youths and Adults. As shown in the MST website (www.mst.org), the Landless Movement Schooling project involves: 1800 schools in camps or settlements (grade 1 to 8), with 160 thousand students and 3900 teachers; 250 educators who work with children up to 6 years; 3000 educators working with 30 thousand peasants of Literacy and Numeracy projects of Adult Education; and Teacher Training Courses implemented in partnership with public and private universities around the country.

This schooling project, according to one of the Landless Movement official documents, sees the need for "two articulated struggles: to extend the right to education and schooling in the rural area; and to construct a school that is in the rural area, but that also belongs to the rural area: a school that is politically and pedagogically connected to the history, culture, social and human causes of the subjects of the rural area (...)" ((Kolling et al, 2002 p.19). The movement has dedicated itself to conceiving the schooling of its children, youths and adults paying attention to these two struggles.



The theoretical background and empirical elements that give support to the idea that there is different mathematics.

Now, as I said before, I will outline the theoretical background and present empirical elements that give support to the idea that there are different mathematics. First of all I would like to make explicit the theoretical toolbox with which I operate. It is the Ethnomathematics field, this field of knowledge, which was born in the south of the planet, but whose repercussions reached not only peripheral countries but also the central ones of the north. What are the meanings I am assigning to this toolbox called Ethnomathematics? Lately I have been saying that it studies the Eurocentric discourses which constitute academic mathematics and school mathematics; analyses the effects of truth produced by the discourses of academic mathematics and school mathematics; discusses issues of difference in mathematics education, considering the centrality of culture and the power relations that institute it; and it problematizes the dichotomy between "high" culture and "low" culture in mathematics education (Knijnik, 2004).

What is at stake here is to problematize what is taken as "given", as "natural", in school mathematics, in particular, in adult mathematics education. For example, how is it established what kind of knowledge counts as "mathematics" and what knowledges are not allowed to be considered as "mathematics"? How were our "truths" about learning and teaching mathematics instituted? In summary: How we became what we are as educators?

To think about these questions using Ethnomathematics as a toolbox implies consideration of the centrality it assigns to the notion of culture. Culture, from this perspective, is seen as a human production, which is not fixed, determined, closed in its meanings for once and for all. This way of conceptualising culture implies considering it a conflictive, unstable and tense terrain, undermined by a permanent dispute to impose meanings through power relations. Culture is not considered as a

body of "traditional" knowledges, as an inert set of knowledges that is transmitted from generation to generation.

Moreover, it is assumed that there is a close connection between mathematics and culture: mathematics produces culture, but it is also produced by culture.

In summary, we can build the following rationale

1. Mathematics and culture are strongly connected.

2. What we usually call "mathematics" is not a social production resulting from all our efforts; it does not incorporate mathematical contributions of all cultures, from the west and the east, from the north and the south. In fact, if we take a brief look at History, we will see that the *mathematical heritage of humankind* is identified only with Western academic mathematics, the mathematics produced by the Western mathematicians.

3. Identifying only part of the world's mathematical knowledge as "the" mathematics masks power relations that legitimise a very specific way of producing meaning – the Western, white, male, urban and heterosexual one.

What we usually call mathematics is a male science for two reasons. First, many studies show that the world of mathematicians is inhabited mostly by men. Throughout History, women were put in a position that informed us that mathematics was not "for us". As Walkerdine (1998) very well explains, we learn at school, at home, through the media and other social artefacts that we do not have the "rational mind" required by the formal, abstract language of mathematics. In these different social settings we learn that mathematics is "for them", for men. But there is a second reason which allows us to say that mathematics is a male science: the objectivity, the abstraction and the formalism that mark this field of knowledge called mathematics, in our Western culture is identified with the way that men give meaning to life, the way men learn to think, to behave in our society. When we problematize the male characteristics of mathematics we are taking these two aspects of the field of mathematics into account.

In summary, we can say that what we call mathematics is a very specific way of interpreting the world, a way constituted by a very specific language, marked by a very specific grammar, closely connected to its uses, to a form of life. We, mathematics educators, are aware of this form of life, of this language, this grammar… And we know how often its marks can be seen in teaching-learning school pedagogical processes, including those addressed to adult education.

These ideas are at the centre of the Ethnomathematics taught. Using them as a toolbox, and following the German philosopher Wittgenstein (2004) precisely what he established in his work ""Philosophical investigations", we can analyse adult mathematics education from an unusual perspective. In fact, Wittgenstein's theorizations about notions such as games of language, uses and forms of life allow us to consider as mathematics other mathematical knowledges besides the one usually identified by "the" mathematics. His theoretical approach allows us to consider as mathematics different adult mathematical practices, produced by different adult forms of life. As we observe in our educational work, the language, which constitutes adult mathematical knowledge, is different from the abstract and formal language of what is usually called "the" mathematics. It is clear that both languages are different.

Again, according to Wittgenstein, we can say that these different languages, associated with different forms of life, different grammars produce different mathematics. Here comes an important issue: I am assuming that there is more than 'a single' mathematics. This implies a denial of the idea that the adult mathematical practices we found in everyday life of diverse cultural groups are mere "applications" of "the" mathematics. Based on Wittgenstein, I am assuming another philosophical position: there are many mathematics, all of them having *family similarities*, as Wittgenstein highlighted. Of course, we are not saying that all mathematics have the same social value. From sociology we learn that there is one that is legitimized in our west culture: the one produced by the mathematicians at the academy. From social movements like the Brazilian Landless Movement we learn that they need – and want – to acquire this socially more valuable mathematics. But, again, it is important to reaffirm the position I am assuming: there is more than a single mathematics; there are many mathematics. In the next part of my talk I will provide an example of these ideas, based on my work with the Landless Movement.

Two different mathematics

One of the research projects I implemented with the Landless had as its goal: to examine oral mathematics practiced by adults of that peasant culture. I was interested in knowing more about their oral mathematics, specifically that involving addition, subtraction, multiplication and division. As I have observed in my fieldwork, oral mathematics practices are present in the everyday life of the peasants who participate in this social movement, in their forms of life. The low levels of schooling that did not allow them to be aware of written algorithms require the constant use of oral mathematics, with its specific grammar. However, at least in Brazil, in the context of adult education, there is a sort of "forgetfulness" about this world outside school, about this mathematics which is different in its uses, in its grammar from the written mathematics taught at school, the written algorithms included in adult mathematics curriculum. In curricular terms, it is useful to investigate the meanings produced by this "forgetfulness", by the dichotomization and antagonism of these two mathematics. The investigation of such meanings may lead to a localized and partial achievement of a "curricular justice", which Connell defines as curriculum

organization that takes as one of its principles consideration of "the interests of those who are at a disadvantage" (Connell, 1995, p. 12).

I would like to show you three aspects of the oral mathematics produced by the landless peasants. The first concerns the close ties between oral calculation strategies and the contingencies in which they are situated. Thus, for instance, a peasant explained that, on estimating the total value of what he would spend to purchase inputs for production, he rounded figures "upwards", ignoring the cents, since he did not want "to be shamed and be short of money when time comes to pay". However, if the situation involved the sale of some product, the strategy used was precisely the opposite. In this case, the rounding was done "downwards", because "I did not want to fool myself and think that I would have more [money] than I really had."

What was observed is that, different from the school mathematics that emphasizes the uses of written processes and the "forgetfulness" of the context, discussed by Walkerdine (1988), the oral mathematics of the peasant culture is strongly contextualized and involves complex reasoning.

A second aspect refers to the strategy of adding, based on a decomposition of the values to be orally calculated. This is what happened with one of the students in the workshop given by the students, when faced with a situation in which he had to calculate 148+239. He explained that, "first one separates everything [100+40+8 and 200+30+9] and then adds up first the numbers that are worth more [100+200, 40+30, 8+9]. (…) This is what really counts". This strategy was found among almost all adults who said that they "were good" at mental calculation. Differently from the addition algorithm taught at school, in oral procedures the peasants considered above all the values of each parcel that was involved and how much difference it would make if it were hundreds, tens or units, i.e., they prioritized the values that contributed more significantly to the final result.

This priority also emerged when the numbers involved in the calculation are decimals. It is observed that recurrently, the peasants use decomposition "to make up integers". This strategy was employed by *Dona Nair*, a retired settler, who, as a child, attended school for only one year, and did not learn to read or write. On explaining the way she uses mental calculation in her daily activities, she referred to a situation in which two products are purchased, one of them costing R$2.70 and the other R$2.90. She said that to find the amount to be spent, she first of all adds up the integers and then the cents, as follows: "2+2 makes 4. I complete the 90 [cents] with 10[cents] of the 70[cents] to make another 1 *real*. So 4+1 completes 5 *reais*, plus the 60[cents], and I have 5 and 60." Like those previously mentioned, also in situations involving decimals, what is prioritized in the calculation process are integer values that, according to the peasants, are "more relevant" to the final sum, a relevance which is marked by their culture.

A third aspect I would like to mention concerns the duplication strategy present in the oral multiplications, a process similar to that used in ancient Egypt, as indicated by Peet (1970). *Seu* Nerci, an illiterate landless man, whose interview was filmed and later used as pedagogical material in a teaching training course of Landless pre-service adult education (Knijnik et al, 2005), when multiplying 92 x R$0.32 (corresponding to 92 litres of milk produced and sold at 32 cents of real[7] [R$0.32] a litre), first doubled the value of R$0.32, and obtained R$0.64; then he repeated the "doubling" operation twice, finding the amount of R$2.56 (corresponding to 8 litres). He added to this the value of 2 litres calculated previously, and thus found the value

of 10 litres of milk: R$3.20. The next procedure was to successively double the values found, i.e., he obtained the result of 20, 40 and 80 litres. Keeping “in his head” all the values reckoned throughout the process, *Seu* Nerci ended the operation adding to the value of the 80 litres, those corresponding to 10 litres and 2 litres (calculated previously), and thus found the result of 92 x R$0.32.

Seu Nerci never went to school. When he was a child, the closest school to his home was 20 miles away and there was no public transportation in the rural zone where his family lived. Since early childhood, boys and girls were introduced into agricultural labour and no children went to school. He did not use pencil and paper to write down the sums as he multiplied them. When the video was made he suddenly withdrew to another room at the back of his house to perform the multiplication, only reappearing after he had come to the final result. Here other regularities about oral mathematics should be presented. The first concerns the need, explicitly mentioned by the adults, “to concentrate to think”. Like *Seu* Nerci, most of the adults observed at mental calculation activities became deeply involved in the act of reckoning, in an attitude of isolation and introspection. But, unlike *Seu* Nerci, many of the literate adults observed usually took notes during their mental calculations. The notes were used as “markers” throughout the process, especially in those involving greater complexity.

In summary, we can observe the high level of reasoning involved in the landless oral mathematics. Even from the perspective of what we consider “the” mathematics, there is a broad, important set of subjects operating in this oral mathematics. It is true that it is neither formal nor abstract as written mathematics. It is true that it has another grammar, that oral mathematics and written mathematics are not the same. But we cannot say that from an epistemological standpoint one is more valuable than the other.

Moreover, observing adults practicing their oral mathematics and hearing their comments about their mathematics I understood the importance of analysing oral mathematics from a cultural perspective. It has been shown that the peasant oral mathematics is produced by the Landless culture and at the same time, such a culture is produced by this specific mathematics. From this perspective, one can see the cultural relevance of incorporating their oral mathematics in peasant adult education, since this mathematics is part of their way of living, of giving meaning to life. It cannot be assumed that at school they could leave “part of themselves” outside. When they come to adult education projects, their peasant culture comes with them, even when the school curriculum tries to impose a sort of “forgetfulness” about who they are, the music they enjoy, the food they appreciate, the grammar they use when talking, the grammar they use when adding, subtracting, multiplying and dividing. When this subtle imposition of denying their culture occurs, it is not surprising to see that it brings with it a resistance process. This resistance can be can be expressed by adult peasants through rejection of school, no-learning attitudes can be expressed by pretending that they accept such an imposition, simply pretending, because when they go outside school their oral mathematics is revived, showing that it can survive school conservative practices that considers only one kind of rationality, one kind of language and grammar as mathematics. Maybe it will be possible to enlarge our adult mathematics education world, including other mathematics, other forms of life… also those mathematics belonging to other cultures. Maybe this enlargement can produce broader repercussions, opening possibilities for a better relationship among people from different parts of the planet, from different cultures. Maybe our dreams of a more solidarity society can be fulfilled.

Some final remarks

I will end my talk by referring to work done by Alison Tomlin, an English colleague from whom I always learn about how to use the mathematics education field to contribute to the construction of this new society, with more justice and peace. Many years ago, Alison sent me the following three worksheets of a work in progress on the 'measures' project developed by NRDC, from 2002 to 2004. In this material Alison gave visibility to the “cubação” process used by landless peasants.

Alison, I remember very well an occasion on which I, as I usually do, showed these worksheets to a group of landless in-service teachers. One of the teachers said, proudly, “Now they [English students] will see that we also produce mathematical knowledge”. Thank you.

Alison Tomlin

National Research and Development Centre for adult literacy and numeracy

Alison Tomlin
Department of Education and Professional Studies
King's College

Name _____ Date _____

Politics and land measurement

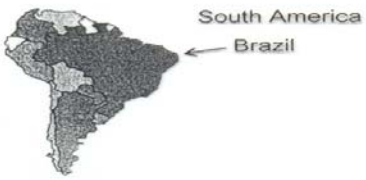
Here are two ways of measuring area.
They come from maths work
by the Landless People's Movement in Brazil.
The landless people are winning some court cases
against landlords and the government,
and they need maths for land measurement.
Jorge and Adão are two members of the movement
who showed other people
these traditional ways of measuring land.

1. Jorge's method of estimating area

Here is a land with four walls.
First, we add all the walls.
Second, we divide the sum by four.
Third, we multiply the obtained number by itself.
This is the 'cubãçao' of this land.

2. Adão's method of estimating area

This is a land with four walls.
First, we add two of the opposite walls and divide them by two.
Second, we add the other two walls
and also divide them by two.
Third, we multiply the first obtained number by the second one.
This is the 'cubãçao' of this land.



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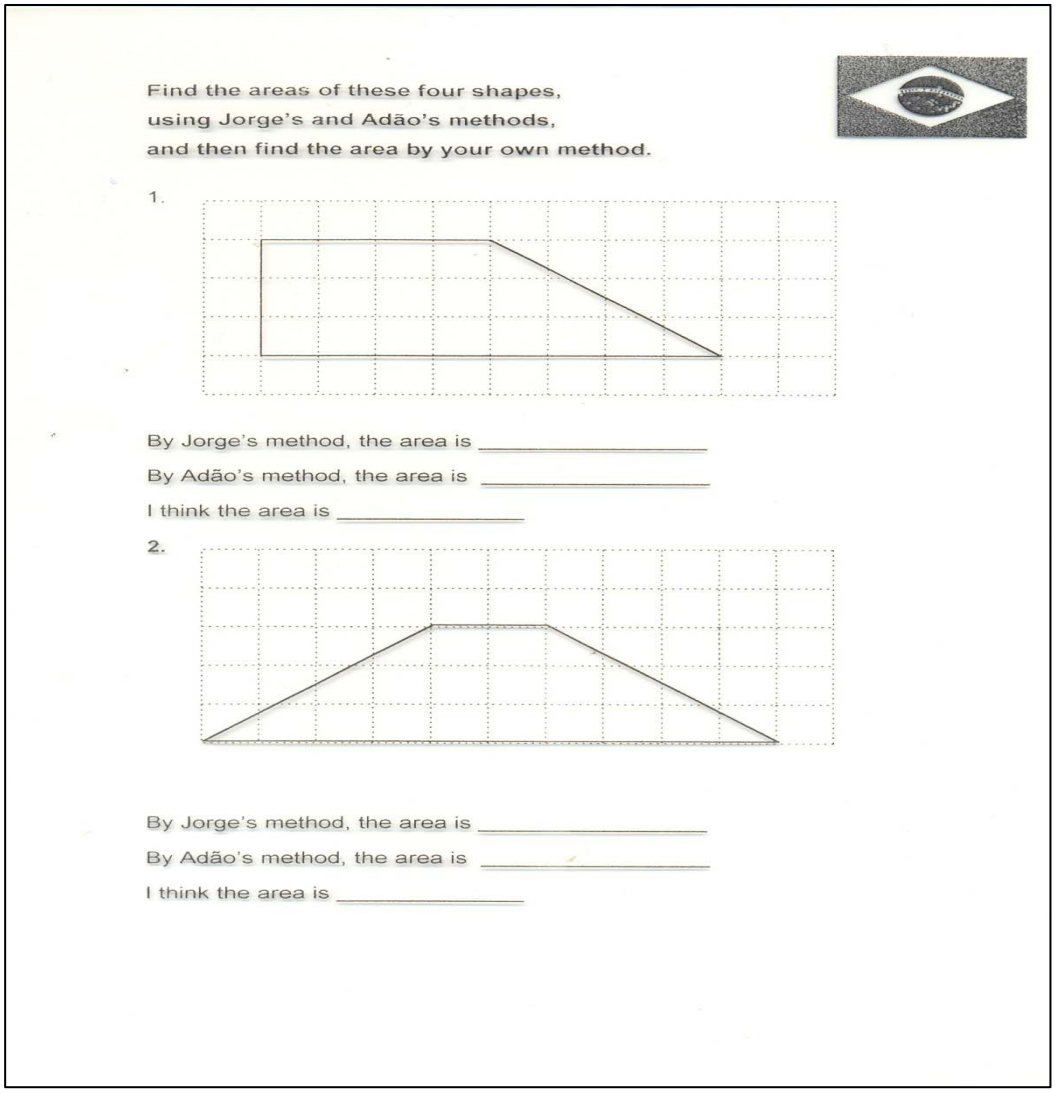

Find the areas of these four shapes,
using Jorge's and Adão's methods,
and then find the area by your own method.

1.

By Jorge's method, the area is _____

By Adão's method, the area is _____

I think the area is _____

2.

By Jorge's method, the area is _____

By Adão's method, the area is _____

I think the area is _____

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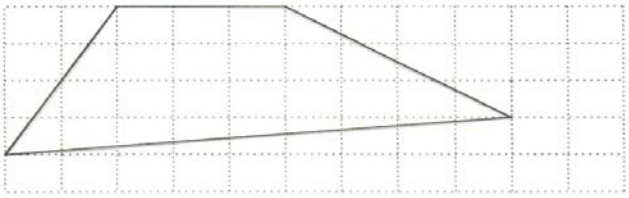
3.

By Jorge's method, the area is _____

By Adão's method, the area is _____

I think the area is _____

4.



By Jorge's method, the area is _____

By Adão's method, the area is _____

I think the area is _____

Any comments? For example, what method did you use to find the areas yourself? Do you think one method is better than the others? Can you think of any situation in Britain where groups of people use different mathematical methods?

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Crossing borders between numeracy and mathematics: Developing curriculum pathways in mathematics

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This paper reports on a research project on 'Developing Curriculum Pathways in Mathematics Education Post-14', undertaken in England for the Qualifications and Curriculum Authority (QCA) by a team from King's College London and Edexcel. The project aimed to construct pathways in numeracy/mathematics through which adults and young people will be able to progress – crossing the borders between different courses and qualifications such as Adult Numeracy, Key Skills Application of Number, Mathematics GCSE and A/AS level GCE. We outline the key features of our overall model, focusing in particular on adult numeracy and mathematics at 'basic' levels, i.e., from Entry Level to Level 2 in the National Qualifications Framework, including the new Functional Mathematics.

The 'Developing Curriculum Pathways in Mathematics Education Post-14' project in England was undertaken for the Qualifications and Curriculum Authority (QCA) by a team of researchers from King's College London and Edexcel (an Awarding Body)[1]. The project aimed to construct pathways for progression in numeracy and mathematics for 14-19 year olds and adults (Brown, Coben, Hodgen, Stevenson, & Venkatakrishnan, 2006; Edexcel & King's College London, 2005, 2006). In this paper we are focusing mainly on Functional Mathematics in the context of the *Skills for Life* Adult Numeracy curriculum and qualifications (DfES, 2001); further detail on the project overall is available in an online paper (Brown, Coben, Hodgen, Stevenson, & Venkatakrishnan, 2006)[2].

Context

We developed our 'Pathways' model (see *Figure 1*, below) against the background of recent government reports which have shone a spotlight on mathematics and numeracy, especially for young people and adults. These include the recent Smith Report on mathematics education post-14, *Making Mathematics Count* (Smith, 2004),

[1] The team consisted of: Margaret Brown (Principal Investigator King's College London), Diana Coben, Jeremy Hodgen, Clive Kanes, Ian Stevenson, Alison Tomlin, Hamsa Venkatakrishnan of King's College London; the team members from Edexcel were Jeffrey Goodwin (Programme Director), Graham Cumming and Kate Halliwell. This paper draws on reports to QCA by the team as a whole (Edexcel & King's College London, 2005, 2006).

[2] This paper has been updated to reflect developments since our presentation at the ALM13 conference; for example, Functional Skills Standards have been agreed and are available at http://www.qca.org.uk/15895.html - see, (ACME, 2007a, 2007b; QCA, 2007).

the Tomlinson Report (Tomlinson, 2004), the government's White Paper on 14-19 year olds' education and skills (DfES, 2005) and the Skills White Paper (DfES/DWP/HMT, 2005), which all focus on this area and age group. In the wake of these reports, the government's Department for Education and Skills (DfES) gave QCA the remit to develop functional skills in English, ICT (information and communications technology) and mathematics.

The 14-19 White Paper recommended the retention of GCSEs (General Certificate of Secondary Education, normally taken at age 16) and A level GCEs (General Certificate of Education, normally taken at age 18) and the introduction of new specialised diplomas at Levels 1, 2 and 3. The new General (GCSE) Diploma will require the achievement of five A* to C grade GCSEs, including English and mathematics. A pass at grade C or above will be impossible without the achievement of a pass in the new Functional Mathematics qualification. Students will be required to show 'mastery' of the functional elements of mathematics essential for life, learning and work, with a pass/fail assessment. The new specialised diplomas require a core of functional skills in mathematics and GCSEs, *Skills for Life* qualifications and Key Skills qualifications will have the same functional core. Currently Adult Numeracy and Key Skills Application of Number tests at Levels 1 and 2 share the same item bank so they are effectively the same qualification. Functional Mathematics will also be available as stand-alone qualifications from Entry level to Level 3 or 4; this may be particularly relevant for vocational diploma students and adult learners. Functional Mathematics is thus at the centre of the reform of the Mathematics curriculum and the wider 14-19 and adult curricula. This is particularly important for adult learners because the Adult Numeracy/Key Skills Application of Number tests are not well articulated with GCSE Mathematics – and GCSE Mathematics at grade A* to C is the target to which students aspire and which many employers and training providers require.

QCA define Functional Mathematics as follows:

> Each individual has sufficient understanding of a range of mathematical concepts and is able to know how and when to use them. For example, they will have the confidence and capability to use math's to solve problems embedded in increasingly complex settings and to use a range of tools, including ICT as appropriate.
>
> In life and work, each individual will develop the analytical and reasoning skills to draw conclusions, justify how they are reached and identify errors or inconsistencies. They will also be able to validate and interpret results, to judge the limits of their validity and use them effectively and efficiently. http://www.qca.org.uk/15710.html

In spring 2006 QCA ran consultation events on functional skills and conducted an online consultation with interested parties. The timetable for the introduction of the new Functional Mathematics standards and qualifications is set out on the QCA website at http://www.qca.org.uk/functionalskills/ (accessed 1 September, 2007; last updated: 22 May, 2007) as follows:

- from September 2006: a small-scale trial of the draft functional skills standards and approaches to their assessment, with awarding bodies.
- from September 2007: piloting of functional skills qualifications.

- from September 2010: final qualifications for English, mathematics and ICT to be ready for full use.

Further detail is given as follows:

- September 2008 Diploma joins pilot;
- September 2010 Functional skills qualifications in English, mathematics and ICT available;
- May–June 2007 Functional skills pilot qualifications submissions begin;
- Pilot qualifications accredited;
- Piloting arrangements approved;
- September 2007 Pilots begin: standalone and GCSE.

The Functional Mathematics Standards are at http://www.qca.org.uk/qca_6069.aspx.

Against this background we were tasked with the development of a curriculum and assessment model for mathematics provision setting out clear pathways from NQF Entry level to Level 3[3].

Scope and outline of our project

Phase one of the project ran from January to March 2005. We produced a report critiquing the curriculum models set out in the Smith Report (2004) and outlined our pathways model. Phase two ran from April to September 2005 and included consultation on our proposed model with focus groups. We produced a proposal for a reformed curriculum and assessment structure taking into account our consultations, the Smith Report and the White Paper. In Phase three, from October 2005 to July 2006, we developed a fully worked up model for small-scale trialling in the field.

Our 'Pathways' model

Key features of oour 'Pathways' model include: a stronger emphasis on modelling and ICT at all level; increasing the accessibility of mathematics for a range of learners; including Functional Mathematics as stand-alone qualifications; providing greater mathematical challenge for more able students; improving the coherence of mathematics qualifications to enhance public and employer understanding. At Entry Levels, Level 1 and Level 2 (the levels covered by the current Adult Numeracy, Key Skills Application of Number and GCSE Mathematics qualifications), we proposed: a double GCSE award at both Foundation and Higher tier levels, replacing the present

[3] A group from the University of Leeds undertook a parallel project for the QCA (University of Leeds School of Education, nd); in Phase 2 the two groups worked together. Leeds and King's College London/Edexcel reports were submitted to the QCA in July 2006.

single award (this has been superseded by a subsequent government announcement)[4]; Functional Mathematics as a key route for adult returners; clear progression routes from Key Stage 3 (i.e., the schooling of children aged 11 to 14); with extension material incorporated into the new double GCSE in order to challenge able students.

Our approach to Functional Mathematics was of a piece with our approach to the development of pathways in mathematics education more generally. We proposed a modelling approach, informed by FSMQ (Free-Standing Mathematics Qualifications: http://www.fsmq.org/site-map,1353,NA.html) and PISA (OECD, 2003). Our conception of modelling in this context has two elements: exploration: learning about a model that someone else has made by exploring it; and expression: building a model in which learners can express their own understanding of a situation.

Assessment

We proposed that the assessment of Functional Mathematics at Entry Levels should be by portfolio. At Levels 1 and 2 a broad range of skills, including long sustained questions should assess students' modelling skills (i.e., their ability to work out the salient features of a problem and apply mathematics appropriately and effectively in solving it). On-screen assessment, initially with a paper-based alternative for 'terminal task' style questions, and/or a portfolio of similar specified classroom tasks should complete the assessment. At Level 2, Functional Mathematics will be a Pass/Fail hurdle to the achievement of the new double GCSE grade CC, the threshold grade at which students are regarded as proficient in the subject.

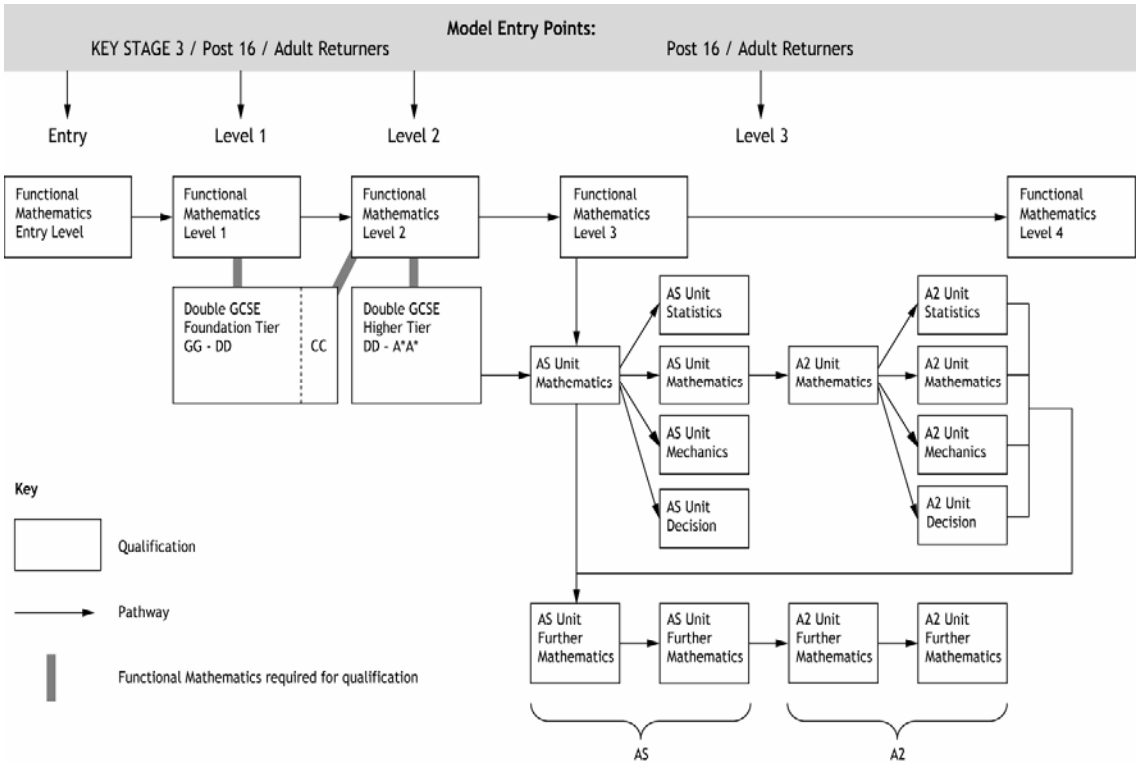

Figure 1: Our 'Pathways' Model of Numeracy and Mathematics Qualifications (Edexcel & King's College London, 2006)

[4] The Secretary of State for Education, Ruth Kelly, announced on 8th March 2006 that there will be two separate Mathematics GCSEs, including a Further Mathematics GCSE for the most able students, so this proposal has fallen.

Reference: Edexcel, & King's College London. (2006). ***Developing Curriculum Pathways in Mathematics: Phase One. Interim Report.*** **London: Edexcel/King's College London.**

The proposed model provides, in line with the DfES White Papers, a clear progression through a Functional Mathematics strand from Entry Levels, through Level 1, Level 2, Level 3 and Level 4. As noted in the last section, students, whether Key Stage 4 or adult will be able to start at the appropriate level and proceed over time as far and as fast as they can progress.

For some, mainly post-16, students, Functional Mathematics will be the only mathematics they will do. Hence Functional Mathematics qualifications need to be independent of others and highly regarded, both as courses and qualifications by students, teachers and user communities.

The Functional Mathematics pathway may be particularly appropriate for many students taking the new vocational diplomas. Our model is designed to promote Functional Mathematics Level 2 in the medium term as the default required qualification for further education and employment, whilst a Grade CC or Functional Mathematics Level 3 would be the requirement for qualification and employment routes requiring a more abstract mathematical content.

Content

Functional Mathematics must be challenging at the appropriate level and yet require a high standard of mastery for a pass. It must also be sufficiently broad in scope in terms of content, process and context of application to satisfy all groups of students and potential employers. This means that in addition to financial mathematics and other applications of number, it must include substantial components of ICT, handling data, and some applications of shape and space e.g. use of plans, computer designs, maps etc.

We believe that Functional Mathematics should meet the following criteria:

- relevance
- thinking skills
- conceptual understanding
- use of technology
- mastery
- research-base
- improving classroom practice
- improving take-up of mathematics
- student independence.

The first four criteria are drawn directly from the QCA/ACME principles for Functional Mathematics drawn up at the ACME workshop on 14–19 Mathematics Pathways held on 13 October 2005 (ACME, 2005). The final two criteria relate to improving classroom practice so as to make students' experiences more interesting and successful, and to enable them to work more independently of teachers. Research on student attitudes suggests that they prefer lessons where they can be active in decision-making and work in groups. They are then better able to transfer the mathematics of the classroom into real world functional contexts. This means that although the mathematical skills content of Functional Mathematics may not be very different from that of Key Skills Application of Number or Adult Numeracy, both the way that the standards are expressed and the nature of the assessment need to make it clear that active problem-solving in small groups is a necessary way of working. Since many students will have been tackling the same mathematical ideas and procedures (e.g., percentages, mental calculation) over many years with limited success, it will be particularly important to break the pattern of earlier lessons and try some new approaches which engage students and respect their wider abilities.

In most classes some time will also be spent improving more routine skills; but even here we would expect learners to be actively involved in self-assessment and encouraged to take responsibility for their own improvement. This does not necessarily imply individual working, although some students may prefer that. It could equally involve a group of students with similar levels of attainment, each sharing responsibility for the learning of the whole group, or peer teachings, or whole class activity, provided this can be done at a level appropriate for all students. It is important to note that interesting and engaging problems can be designed which encourage routine skill practice alongside strategic thinking.

Standards

We believe that in order to influence classroom practice the standards to be given most prominence should be those concerned with process and applications rather than those expressed in terms of mathematical content and skills.

Assessment structure

We believe that assessment of Functional Mathematics can be organised to be valid, reliable and efficient whilst also encouraging improvements in classroom practice. However the tensions in this area have to be faced.

To maximise validity, assessment of Functional Mathematics capability should be carried out on a continuing basis by teachers in situations which are as close as possible to being authentic. However, this is likely to lead to some reliability problems in ensuring comparability of judgements. Whilst we believe that these are not insuperable, it would be costly and time-consuming to achieve an acceptable level of comparability if this formed 100% of the assessment. In addition, such a high reliance on teacher assessment would not currently be acceptable to many politicians, parents, or teachers themselves.

Teacher assessment would also be costly in terms of teachers' time. This may be possible in smaller classes which are more likely to be found in FE or workplaces, or with groups of lower attaining students. However, to require a substantial load of teacher assessment in all GCSE classes would add unreasonably to mathematics

teachers' workloads in a situation where they already feel under considerable pressure.

At the other extreme, if all the assessment is based on multiple choice or short answer test items, the fifth QCA/ACME requirement, relating to appropriate assessment methods, will not be met and validity is likely to be compromised. This is in the sense of both construct validity, i.e., whether capability to tackle sustained problems is actually being assessed, and consequential validity, i.e., in its narrowing effect on classroom practice to focus only on practising answers to short questions.

To try to resolve these tensions within acceptable financial and staffing constraints, we recommend that the assessment of Functional Mathematics at levels above Entry Level should include two parts, in the traditions of Key Skills and Free Standing Mathematics Qualifications (FSMQs): a short answer component and a longer, more sustained component.

Dependent on level and learner, assessment may be more appropriate as a portfolio or as an examination. We recognise that offering alternatives creates comparability problems and additional work will be required to narrow the differences between the alternatives to minimise this. We propose that, at least until the end of this project, both options are explored further. Either method will need to be assessed by teachers or professionally trained markers. At Entry Levels we recommend only portfolio assessment to avoid the problems of communication, which are likely to arise in the on-screen tests discussed below.

For the short answer component, we would encourage variety in the items, with some being complex and cognitively demanding, e.g., to include redundant data, decision-making across two steps, etc... The World Class Test papers http://www.worldclassarena.org/v5/support_materials.htm# demonstrates that such items are possible. We recommend the type of modelling item used in PISA[5] in which a single context is set up and forms the background for several questions. There probably needs to be a corresponding reduction in the number of items to allow students additional thinking time. Because of the difficulties of communicating the context in writing, especially with many students who are relatively new to English or whose language skills are poor, we recommend the use of oral speech and DVD clips. Considerable investment needs to be made into the development of suitable items. We predict these items will be machine-marked and therefore they will need to be either multiple-choice or to have easily identifiable short answers. Until a random selection from a large item-bank becomes feasible it will be necessary to offer these tests between two and four times a year. For a short period this may require additional resources in terms of appropriate computer supply, but we believe that this is anyway necessary in order to teach Functional Mathematics satisfactorily.

In addition to the above, the examination would contain a small number (two or three) of long questions. These would require sustained work, including communication and presentation skills, selection of strategies, and justification of both the results and the choice of methods used. At first this section may have to be paper-based although our intention is that it would become on-screen as technology allows. Even initially, this might be presented on-screen and some parts might be answered on-screen using e.g., spreadsheets and graph-plotters. However, currently it would be difficult to expect students to do certain types of work on-screen, e.g.,

[5] PISA: Programme for International Student Assessment (OECD, 2003).

algebra, so some of the work could be either done on a tablet or scanned in to a computer.

We are proposing at all levels that a portfolio can be used as the method of assessment. This would include a number of tasks which would be carried out in the classroom under controlled conditions, and would have a clear assessment scheme (e.g., GAIM[6] tasks) to enable comparable marking standards. These may take the form of tasks set by an Awarding Body from which a selection could be made. At Entry Level we believe there should only be a portfolio approach with a freer choice, allowing students, especially adults, to work in contexts they find engaging. We also believe that for those studying for vocational diplomas a portfolio of tasks would be most appropriate, as this can relate to their vocational specialism. However, this needs to be well defined, possibly even to the extent of allowing only selection from a set of tasks specific to each diploma.

Conclusion

Ken Boston, QCA Chief Executive, hopes that the 14-19 White Paper will stimulate development at Entry level, where…

> there is clearly a need for a structure to enable the most needy learners to stretch their potential to the utmost. (Boston, 2005)

We hope our project will contribute to this development so that for the first time, there will be joined-up pathways between Numeracy and Mathematics curricula and qualifications so that adults working at Entry level will be able to progress.

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Adult Numeracy: Crossing Borders of Discourse

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In a presentation to TSG 6, ICME 10 (FitzSimons, 2004), I drew upon the work of Basil Bernstein to distinguish analytically between mathematics and numeracy as vertical and horizontal discourses respectively. It is commonplace that adults feel that they 'cannot do maths' and that people in all kinds of trades and professions believe that they 'never use any of the maths they learned at school'. I propose that this is because they have only encountered the vertical discourse of mathematics, recontextualised by educators into 'school mathematics', with generally unproblematic assumptions about transfer through so-called 'real-life' applications. In this paper I elaborate further on this theoretical foundation to argue that it is necessary to cross the borders between the two discourse types in order to successfully teach mathematics to adult learners.

Numeracy, whether for adults or children, is a social construct. It draws upon mathematical skills and knowledges developed over a lifetime. These may be learned informally from — and even taught by — family, friends, and other significant others. In the case of pre-school children and unschooled youth and adults, learning is dependent on the social and cultural settings available to the learner in their various communities of participation. In countries where formal education is the norm, funded to a greater or lesser extent from the public purse, decisions are made by governments (advised by bureaucrats) about the quantity and quality of education for various groups of learners. Decisions are made based on, for example, perceptions of the good of the nation and, where governments are elected, and perceptions of the likelihood of re-election. Accordingly, the voices of certain groups of stakeholders are privileged over others, depending on the political complexion of the government of the time. The major focus may be on, for example, improving business or national economic outcomes or on democratic citizenship, or some combination of these. Whatever the focus, numeracy is necessarily related to mathematical skills and knowledges.

The provision of formal mathematics education in school as well as adult and vocational education (even if it is labelled as numeracy) is necessarily political in that certain selections are made to meet the interests and needs of certain social and economic groups above others. Curricula are determined — whether at the state level or even the individual school or classroom, and again this is a political decision — drawing on an arbitrary selection from the discipline of mathematics. Official and local pedagogic practices then combine to recontextualise that arbitrary selection in differentiated ways. Evaluation of the learner's performance — whether via international or state-wide testing, or teacher-designed assessment or individual classroom/online feedback — may also be heavily influenced, if not determined, by

political decisions: for example, mandated testing for accountability purposes, ongoing funding, and local, national, or even international comparisons.

In different countries different terms are used to apply to the use and even construction of mathematics outside of the classroom. *Numeracy* is very commonly used, often with the pejorative tag 'basic' attached in the case of adult education. One perspective is that this term is used in an attempt to disassociate from the cold, hard, judgemental image of the discipline of mathematics in an attempt to popularise it with school children and their parents, as well as adult learners. Other terms used are *quantitative literacy*, *mathematical literacy*, *democratic numeracy*, and also *functional mathematics*. However they are labelled, the curricular content, pedagogy, and assessment still remain political phenomena. The intended outcome of mathematics and numeracy teaching is *numerate activity* but a focus on the discourse of (school) mathematics alone will not guarantee such activity.

Discourses of Mathematics and of Numeracy

Bernstein (2000) describes mathematics as being a *vertical discourse* due to its coherent, explicit, and systematically principled structure. It takes the form of a series of specialised, codified languages, with many sub-disciplines (e.g., algebra, geometry, trigonometry). In formal education, the discipline of mathematics is recontextualised for the purpose of enculturation. Just as the school subject of woodwork is qualitatively different from the trade of carpentry, so school or formal adult mathematics education is different from professional mathematics or statistics; also from workplace numeracy.

Following Bernstein (2000), I argue that the construct of numeracy is an example of a *horizontal discourse*. This is due to the strong affinity between the burgeoning corpus of research reports on workplace and everyday activities involving the use and re/construction of mathematical knowledges (e.g., Eraut, 2004; Hoyles et al., 2002; Kent et al., 2004; Wake & Williams, 2001) and Bernstein's description of a horizontal discourse as "a set of strategies which are local, segmentally organised, context specific and dependent, for maximising encounters with persons and habitats" (p.157). He continues that the knowledges of horizontal discourses are "embedded in on-going practices, usually with strong affective loading, and directed towards specific, immediate goals, highly relevant to the acquirer in the context of his/her life" (p.159). This description bears close resemblance to the characterisation of arithmetic in Lave's (1988) research on shoppers and weight-watchers.

Compared to the discipline of mathematics, numeracy is weakly classified in terms of its necessary integration with context. Whereas in mathematics there is a well-known hierarchy from common sense up to so-called uncommon sense, in numeracy common sense is of the essence. High level abstractions alone are insufficient and may even prove counter-productive. Numeracy cannot be said to have a specialised language, except at the most local level of use in context. For example, the use of the term "thou" (i.e., thousandths) is widely used in the building and automotive industries, but may not have meaning elsewhere. Numeracy is not necessarily explicit or precise (but can be if required), and its capacity for generating formal models may be limited to the context at hand rather than generalisable.

In essence, then, numeracy is a *horizontal discourse* which draws upon foundations of mathematical knowledge developed by individuals over a lifetime of personal experience and enculturation but which, unlike the *vertical discourse* of the discipline

of mathematics, relies on common sense and is context-specific and -dependent, directed towards the achievement of specific, immediate, and highly relevant goals.

Vertical discourses such as mathematics consist of specialised symbolic structures of explicit knowledge; its procedures are linked hierarchically. The formal pedagogy is directed towards some unspecified projected application and is an on-going process, generally continuing over an extended period of time. By contrast, according to Bernstein (2000), the pedagogy of horizontal discourses is usually carried out through personal relations, with a strong affective component. It may be tacitly transmitted by modelling or showing, or by explicit means. The pedagogy may be completed in the context of its enactment, or else it is repeated until the particular competence is acquired. From an individual's perspective, "there is not necessarily one and only one correct strategy relevant to a particular context" (p.160). Bernstein concludes that horizontal discourse "facilitates the development of a repertoire of strategies of operational 'knowledges' activated in contexts whose reading is unproblematic" (p.160).

Whereas the transmission of formal mathematics knowledge is likely to progress from the concrete to the mastery of simple operations, to more abstract general principles, the teaching of numeracy to adults may have more in common with the reverse processes which take place in workplace learning. In other words, general principles are understood but need to be made concrete in order to be realised. Ultimately, the learner will be expected to develop a repertoire of context-dependent strategies, based on experiential learning from a more 'knowledgeable' person (or persons) in a given situation, where achieving the task itself is the priority — not the learning of mathematics per se. As discussed above, context-specific and localised models may also be developed and practical knowledge/expertise, together with common sense, is highly valued.

Pedagogies of Numeracy

It is well recognised that the mathematics classroom is a community of practice distinct from that of professional mathematicians; also from the workplace. In my opinion, then, numeracy is composed of mathematical knowledges and skills, however derived (i.e., formally & informally), in combination with reflective knowing in context — knowing which draws upon a lifetime of experience. Numerate activity is concerned with acting within the discourse practices *and* socio-cultural practices appropriate to the task at hand. It entails both explicit, codified knowledge and implicit or tacit knowledge. That is, the borders of vertical discourse and horizontal discourse must be crossed and re-crossed in order to develop the capacity for numerate activity, and they must be continually crossed and re-crossed in its pedagogic processes. In short, the teaching of the discourse of mathematics alone (even with 'applications') can never guarantee a numerate person.

Keitel, Kotzmann, and Skovsmose (1993, p.275) suggest a process which reverses the traditional order of pedagogy — that is, to start "from powerful technological constructions or identifying rich contexts in which serious social problems are posed" (see also Borba & Villarreal, 2005, particularly with respect to technology-based programs). Mathematical concepts and modelling activities should then be used to enable understanding of the problem, formulation of alternative solutions, and negotiation with others about their acceptability. In this way, students and teachers

should become directly involved in issues such as improving the local environment. In the case of adults, even at the most 'basic' levels, any contexts must be realistic and/or engaging, rather than the trivialised and demeaning pseudo-contextualisations which appear when the focus is on the mathematical process and the context is used as a camouflage. As I have demonstrated in FitzSimons (2000), it is possible to satisfy externally driven curricular demands with actual needs of the learners through a deeply respectful and ethical investigation and sharing of some of the realities of their everyday work and life, conducted as a partnership, and concluding with a reflection on how things might be done differently. As Keitel, Kotzmann, and Skovsmose observe, the processes of learning, knowledge generation, and possible interventive action need to be combined. Self-confidence will be enhanced when learners feel as though they are being listened to or consulted. The prime focus is on connecting mathematical knowledge to other types of insight and activities. Thus, they say, "students could obtain *knowledge about knowledge* which we have called reflective knowledge, and the combination of reflective knowledge and social activities forms a part of what we have called democratic competence" (p.277). Interestingly, democratic competence is included in Wedege's (1995) definition of technological competence in the workplace, which also included professional and social qualifications. In her words democratic competence is "to evaluate and take part in decision-making processes regarding new technology in the workplace" (p.58).

However, evaluating new technology is not the sole justification. Wedege (2000) identified disempowering consequences of workers not being able to participate in workplace mathematical discourses that management might use to control workers — technologies of management (FitzSimons, 2002) — and I have witnessed the same phenomenon in my workplace research in the pharmaceuticals manufacturing industry. These typically take the form of discourses which employ, for example, graphs representing change over time, statistical charts monitoring production output or wastage, days lost due to various causes ascribed to workers (and not to management failure or lack of adequate and timely maintenance of plant or machinery!), charts presenting percentages, percentage change, or production quantities involving very large numbers. Many of these topics appear at quite advanced levels in school curricula, and tend to be overlooked in worker education. Yet, my own experience shows that people are able to understand the concepts given time and contextualised situations well known to learners.

It is obvious that being able to answer pen and paper or computer generated assessment questions in a classroom situation may result in certification — and this is to be commended if it engenders a sense of achievement, even pride, in the learner and contributes towards enhancing their employment prospects and/or helping children with their homework — but it does not guarantee the creative adaptation of existing mathematical knowledge and possible innovation in the immediate context of a problem to be solved at work or in life generally. Moreover, there is a slippage between what a person is capable of doing and their disposition to do so within the particular work/life context in which they are situated at any given time. Buckingham (1998) identified compelling reasons why workers might stay silent and not participate in workplace mathematical discourses even when they are capable of doing so: keeping one's job or not drawing undue attention to oneself are serious reasons.

Issues at Stake in Adult Numeracy

There are four major questions to be addressed when addressing the complex task of crossing boundaries between mathematics and numeracy:

- Who are learning?
- Why are they learning?
- What are they learning?
- How do/might they learn?

The answers to these will frame the definition of numeracy and will necessarily be local rather than universal in its orientation. However, the answers to these four questions may even be conflicting in themselves — as any practitioner knows. The following brief set of questions is drawn from my current work-in-progress research framework and gives some indication of the complexity of the issue.

What are the motives for adult students to take on learning mathematics/ numeracy supported and delivered (wholly or in part) by new learning technologies?

- Is it to achieve a credential?
- Is it to achieve develop new and/or deeper understandings?
- Is it to prove something to one's self?
- Is it to be able to help significant others to learn mathematics?
- Is it to support their own or their family's business/financial interests?
- Is it to learn more about technology?
- Is it to learn more through technology?

Of course, the answer could be any combination of these, or something else all together.

Numeracy is logically connected with mathematics just as literacy is logically connected with language (Lee, Chapman, & Roe, 1996). However, numeracy is also connected with language. Beyond the semiotic resources of the symbolic system and visual displays of mathematics, the communication and sharing of meanings is an integral part of numeracy.

In the UK the definition of *Functional Skills* is given as:

> Functional skills are those core elements of English, maths and ICT that provide an individual with the essential knowledge, skills and understanding that will enable them to operate confidently, effectively and independently in life and at work. Individuals of whatever age who possess these skills will be able to participate and progress in education, training and employment as well as

> develop and secure the broader range of aptitudes, attitudes and behaviours that will enable them to make a positive contribution to the communities in which they live and work

The definition of *Functional Maths* is given as:

> Each individual has sufficient understanding of a range of mathematical concepts and is able to know how and when to use them. For example, they will have the confidence and capability to use maths to solve problems embedded in increasingly complex settings and to use a range of tools, including ICT as appropriate.
>
> In life and work, each individual will develop the analytical and reasoning skills to draw conclusions, justify how they are reached and identify errors or inconsistencies. They will also be able to validate and interpret results, to judge the limits of their validity and use them effectively and efficiently.
>
> *http://www.qca.org.uk/15895.html*

My personal view about the naming of competencies and other 'desired' knowledges and skills by governments is that they are done for good political reasons — rather than on educational grounds. Clearly this 'functional mathematics' privileges the views of some stakeholders over others. It has obviously left out any of the aesthetic side of mathematics, to begin with. Although it supports analytic and reasoning skills, I imagine that it excludes employing these skills to critique poor quality of management where it exists, and even to critique the government itself. It does not take any account of the hard-to-fill jobs which are so boring that no-one will apply for them, except illegal immigrants (personal communication from Ewart Keep, SKOPE, Warwick Business School — now Cardiff).

As Corinne Hahn has remarked on the concept of *numeracie* in France: "for most of the people numeracy is supposed to concern only a special part of the population (immigrants, people with no qualification)" (see http://www.statvoks.no/discuseng/ C. Hahn, 14th March 2006). This resonates with my reading of Bernstein (2000). That is, there is a classification of knowledge so that only some people have access to 'unthinkable knowledge', where [mathematical] knowledge is impermeable (p. 11). I believe that this is also the case whenever politicians and bureaucrats talk about 'basic numeracy' — only ever intending to encompass calculations of whole numbers and fractions — decimal and vulgar/common. I have written about this previously (e.g., FitzSimons, 2002): This practice is inherently undemocratic.

Secondly, Corinne's question: "Does it mean that other mathematics are not 'functional'?" suggests further questions to me:

- Who decides what mathematics *is* functional?
- Does it exclude Islamic geometry, for example? Euclidean geometry? Etc. etc. When? Why? For whom?
- On what basis are these decisions to be made and by whom?
- Is it always the voices of big business and industry, attempting to minimise their financial contributions and maximise opportunities to receive government funding, as is the case in some neoliberal economies? Or, is there a social contract between employers and educators working to support the best interests of the students and social cohesion, as in the Scandinavian countries?

Further to my previous discussion about vertical discourses (e.g., mathematics) and horizontal discourses (e.g., numeracy), Bernstein draws attention to the fact that vertical discourses "have their origin and development in official institutions of the state and economy", and horizontal discourses in "everyday or life world" (p.207). Clearly these two intersect — whatever we choose to name them — and maybe the UK definition of functional mathematics is an attempt to do so — but I believe that we need to be clear that they are different discourses with different practices, and not assume, as many people have in the past, that the teaching of (official) mathematics will ensure (locally) numerate activity. We need to strive for the integration of the two discourses in curriculum and pedagogy, including assessment. As has often been pointed out, word problems can never be a substitute for the messy realities of actual constraint-filled practice. This is what makes numeracy as defined here very difficult to assess 'officially'.

What can we as numeracy researchers and educators do?

Following the UK definition of functional skills as: "essential knowledge, skills and understanding that will enable them to operate confidently, effectively and independently in life and at work," I support the goal of enabling such behaviours. It is the use of the term 'essential' that concerns me. This is where the politics comes in — who decides what is essential and on what grounds? Although it may be possible to describe numerate activity in universal terms to cover the range of possible levels of mathematics and contexts of application in very general terms, the construct of numeracy is, in my opinion, provisional and contingent, and defies a unique definition.

The most important issue is to have our voices heard among the stakeholders in political decision-making. Our task as mathematics/numeracy researchers and educators is to argue for the inclusion of the broadest possible curriculum which balances the needs of the various stakeholders — especially those of the learners whose voices tend to be ignored (or who are not really in a position to articulate their own needs), while at the same time working from a deep understanding of numeracy, based on informed research into workplace and other social- and community-participation needs of adults. This means, for example, as well as responding to invitations to comment on proposed changes to curricula, also including a focus on the learners' actual needs and interests in research reports, and in research proposals. Baxter et al. (2006) are to be commended on their measurement project report which includes the broader picture that frames the constraints on what teachers and learners are allowed to do under increasingly restrictive accountability demands in the form of assessment-driven curricula combined with very limited teaching time. The research team of two university-based researchers and four teacher-researchers also argue the case for the future acceptance of adult numeracy students as (co)researchers in their own right.

One way that learners' needs could be supported is through the development of a teaching force valued for its professionality, allowed appropriate time for reflection and creative lesson planning, and enabled through ongoing and appropriate professional development to make judgements about the best outcomes for each particular group of students, situated in their unique contexts. In Denmark I recently visited a vocational education college where highly committed teachers were valued

and encouraged by their managers to personally interact with students, along with a dedicated support team. For me, and probably for many others of my teaching generation, this approach represents a return to the halcyon days of the pre-neoliberal era. Denmark has a strong economy as well as strong commitment to the social contract. Perhaps other nations might draw some lessons from this.

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Mathematics Support for Adult Learners

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In 1997 a scheme to help " at risk" or mathematically under prepared first year undergraduates in the University of Limerick was undertaken by O'Donoghue to assess the extent of the 'Mathematics Problem' and try to ascertain the best means of dealing with it. Diagnostic testing was introduced in a bid to diagnose those students who were most prone to failure and/or dropping out. This procedure also served to make students themselves aware of their level of expertise and helped to classify major areas of weakness in the group of first years as a whole. The main intention, however, of diagnostic testing is to identify the support that will be needed to help remedy the situation. It was discovered that 30% of first year service mathematics undergraduates required extra support. This was one of a number of contributing factors leading to the establishment of a Mathematics Learning Centre. In October 2001, the University of Limerick's Mathematics Learning Centre opened. One of the aims of the Mathematics Learning Centre is to address the mathematics needs of mature students and adult returners. In this paper, the authors discuss the facilities/resources provided by the Mathematics Learning Centre for Adult learners of mathematics in the University of Limerick.

The so-called 'Mathematics problem' as it is styled in the UK research on mathematics education encompasses issues in the transition from school mathematics to university mathematics. Similar issues and concerns arise in the Irish context. A collection of descriptions of the 'Mathematics problem' has been assembled in Ireland by O'Donoghue (2004) and includes the following:

- Mathematical shortcomings of entering students
- Mathematical deficiencies of entering students
- Pre-requisite mathematical knowledge and skills
- Mathematical preparedness/under-preparedness
- Mathematics at the school/university interface
- Issues in Service mathematics teaching
- Numeracy/Mathematical literacy.

With such a diverse number of career prospects available to students on exit from Higher Education nowadays and the escalating importance of mathematics in such fields as Business, Engineering, Education or Pure Mathematics, there is an obligation on universities to provide a suitable mathematics education for each student.

At present there is widespread concern among third level academics in many countries (e.g. Australia, the U.K.) about the poor level of mathematical preparedness of first year undergraduates in mathematics intensive courses. Research shows also, that the problem is not just that some students are "under-prepared'' but that even students with good Leaving Certificate/ A-Level grades struggle with even the most basic aspects of mathematics (LMS, 1995; NCCA, 2005).

Added to this problem is the fact that many believe that not only are students under-prepared, but that there is also a decline in standards and there is evidence in the U.K. to suggest that there has been some grade dilution over the years (Hunt and Lawson, 1996). There are concerns that this under-preparedness will have serious short and long-term consequences not only for individual students (i.e. failure and dropping out (O'Donoghue, 1999)) but also for the professional reputation of various universities and for the economic progress of a country (LMS, 1995; Flynn, 2005). There are fears in the U.K. that a drop in the level of the mathematical proficiency of undergraduates will lead to them falling behind their peers in other countries and, as a result, the country itself will have to rely on others for inventions and developments (LMS, 1995).

The problem of mathematical under-preparedness has also been reported throughout universities in Ireland (Hurley and Stynes, 1986, O'Donoghue, 1999). There is evidence of this drop in standards in Ireland as far back as 1984 when research carried out in Cork Regional Technical College (Cork RTC) drew attention to the problem of the poor mathematical grounding of their first year students. The authors concluded that the incoming undergraduates were deficient in basic mathematics. In the following year, Hurley and Stynes (1986) carried out a similar investigation in University College Cork (UCC) with similar results, their first year students demonstrated poor articulation of basic prerequisite mathematical knowledge. In the late 1980's in the National University of Ireland, Maynooth (NUIM), and more recently in Dublin City University (DCU), it became apparent that students were having the same troubles as students elsewhere in Ireland. Academic staff initiated diagnostic testing to establish where the weaknesses lay and continue this process to the present day.

Due to mounting concern across the board in the Department of Mathematics and Statistics in the University of Limerick (UL), a study entitled "An Intervention to Assist 'At Risk' Students in Service Mathematics at the University of Limerick" was undertaken to gauge the degree of mathematical under-preparedness of first year undergraduate students participating in mathematics intensive courses. Mathematics lecturers complained that students displayed:

- lack of fluency in basic arithmetic and algebraic skills,
- gaps (or in some cases absence of) in basic prerequisite knowledge in important areas of the school syllabus e.g. trigonometry, complex numbers, differential calculus,

- an inability to use or apply mathematics except in the simplest or most practised way (O'Donoghue, 1999, p.3).

A pilot study carried out in the academic year 1997/98 suggested that up to 30% of incoming students were 'at risk' and would need supplementary help to complete first year successfully. Evidence from this study and a similar study carried out the following year convinced the author(s) that the problem would persist and take on a permanent disposition (O'Donoghue, 1999).

The UL Study

In the past decade there has been increasing unease amongst lecturers in the University of Limerick (UL) as regards the mathematical competence of first year students participating in mathematics intensive courses. Lecturers observed that students were by no means articulate in certain topics which are not only present on the Leaving Certificate syllabus, but provide a foundation for third level studies in mathematics e.g. Trigonometry, Calculus, Complex Numbers to name but a few. Students also displayed unsophisticated approaches to basic arithmetic and algebraic problems and became perturbed when presented with problems posed in a dissimilar way to what they were used to.

It was (is) felt that lack of competence in the field of mathematics

- Increases student failure rates and deflates self esteem,
- Hinders progress in other areas of the degree they are pursuing i.e. Engineering, Physics, Chemistry, Business etc.,
- Diminishes standards of degrees.

A study to help " at risk" or mathematically under-prepared first year undergraduates in the University of Limerick was undertaken by O'Donoghue (1999) to assess the extent of the problem and try to ascertain the best means of disentangling, or at least dealing with it. A pilot study was carried out in the academic year 1997/98 on first years undertaking Technological Mathematics 1 & 2. In the subsequent year, the project was extended to students taking Science Mathematics 1 & 2 also.

Diagnostic testing was introduced in the University of Limerick in a bid to (as the name suggests) diagnose those students who were most likely to fail. This procedure also served to make students themselves aware of their level of expertise (or lack of it as was often the case!) and classify major areas of weakness in the group of first years as a whole. The main intention of diagnostic testing is to identify the support that will be needed to help remedy the situation.

For the pilot study (1997/98) 257 (out of 308) Technological Mathematics students sat a 40 question diagnostic test in the third week of the first semester. Students who scored less than 20 were deemed to be "at risk". There were 69 (27%) in this category. A two-pronged strategy was put into action: a combination of support tutorials and a front-end skills package. It was not obligatory but it was strongly advised that the "at-risk" students attend these additional (support) tutorials (1 per

week) for the duration of each semester. In these tutorials, students worked with their tutor on significant difficulty areas highlighted by the diagnostic test. These tutorials, taught by qualified secondary school teachers, were habitually student-led. A front-end skills package in arithmetic and algebra was formulated and there were four tutorials (two each week) held based on this package. This was completed early in the process. The project team kept records of students' results and attendance and developed a database of this data. Statistics showed that students who availed of the services provided attained good results in the end of term exams.

The aims and objectives of the ensuing year (1998/99) were analogous to those of the preceding year with a supplementary one apropos identification of required facilities to comprise computing/ Internet resources. This particular goal was achieved by providing extra MAPLE classes (supervised by post graduate students) and CALMAT tutorials for students. One other divergence from the previous year was that the diagnostic test was administered in week 1 of term instead of week 3 because it was felt that there would be a superior turn out at this time, which there was.

Each term, in addition to two lectures and one problem session (given by a lecturer), there was one tutorial per week (usually taught by a post graduate student) for everyone although attendance was not mandatory. Again, a database of statistics was maintained right the way through both semesters and results were comparable to those in the previous year. The project team observed that while some of the "at risk'' students still failed, the results of those who did not attend special tutorials were inferior to those who did.

It was discovered that 30% of first year service mathematics under graduates required extra support. While the support tutorials showed improvements for the most part, tutors and students alike felt that a more individualised structure was essential to reach those who were, perhaps, lost in larger group situations. Analysis also showed that certain mathematical topics were not given as much attention as they should have received and in some cases, no attention at all (O'Donoghue, 1999).

In the academic year (2000/01) in addition to other services, students received access to three timetabled hours of supervised CALMAT sessions on a "self-help" basis. This did not prove successful as students did not feel confident working alone and were often lacking relevant information necessary for using this package. This was one of a number of contributing factors leading to the establishment of a Mathematics Learning Centre. In October 2001, the University of Limerick's Mathematics Learning Centre opened.

The UL study was unprecedented in Ireland. It was a very important study in that it provided empirical evidence which exposed the extent of the problems of mathematical under-preparedness in third level service mathematics courses. It identified many of the factors (internal and external) that contribute to this problem and paved the way for the delivery of effective support services including the development of a Mathematics Learning Centre, the first one in Ireland. In addition, a database of diagnostic test and end of semester results was initiated in the academic year 1997/98 and is updated annually for research purposes.

Ethos of the Mathematics Learning Centre

The Mathematics Learning Centre (MLC) is a special initiative of the Department of Mathematics and Statistics at the University of Limerick (UL). The purpose of the MLC is to support students' learning across all programmes in UL by:

- Delivering appropriate support for students on service mathematics courses,
- Providing a dedicated area with supervised access to help and resources,
- Addressing the needs of special groups (e.g. mature students, adult returners, transfer students etc.),
- Researching the needs of learners in terms of materials, pedagogy, delivery systems and other supports.

All services are based on a supervised self-help model that integrates faculty members, students, media and Information and Communication (ICT) inputs and approaches.

Management and Staffing

The management of the MLC is entrusted to the full time manager whose duties include administration of diagnostic testing, maintenance of the UL database, timetabling and staffing issues and organisation of support tutorials to name but a few. At present there are 12 PhD students who work in collaboration with the manager in the MLC.

Resources and Facilities

The Drop-In Centre

The MLC is open from 10am to 12pm and 2pm to 4pm Monday to Friday and 6pm to 8pm on Thursdays. The centre operates a drop-in facility so students do not have to make appointments and provides free one-to-one consultations. The centre is fully supervised with the manager and two postgraduate students on duty at all times. The timetable of teaching assistants/tutors is displayed on the notice board as different postgraduates specialise in different areas e.g. pure mathematics, applied mathematics, statistics etc. so students are aware when the best times to attend for their own individual needs are.

Diagnostic Testing

The Centre continues to carry out diagnostic testing (approximately 450 students each year) and uses the results to identify and inform those students who would need supplementary help to complete first year successfully.

Support Tutorials

Support tutorials are set up and taught on a weekly basis in addition to regular tutorials. Due to the high number of mature students present, a special tutorial is also run for mature students in each group in addition to the classes for the other students. Support tutorials take place in the evenings so as not to interfere with regular lectures and tutorials. These are taught postgraduate students or the manager. These classes last for approximately one hour and are student led. The numbers in these groups are quite small (less than 10) for maximum benefit.

Textbooks

The MLC has multiple copies of all the required textbooks for the various mathematics courses in UL. Students are invited to use the textbooks while in the centre but are not permitted to take them away. Other resources include Loughborough University's 'Engineering Maths First Aid Kit' and HELM (Helping Engineers Learn Mathematics).

Computer Assisted Learning (CAL)

There are 5 computers in the MLC which provide access to CALMAT tutorials. The authors have discovered that students prefer one-to-one consultations to these tutorials so they have not proved very successful but remain available.

Examination Revision Programmes

For the week prior to examinations, due to the high numbers in attendance, the manager organises revision programmes for all the main service mathematics courses. Students are divided up by surname and are informed several weeks prior to these sessions of times to attend and to which particular room they must go. These revision programmes are taught by the postgraduate students and focus on the previous year's examination paper. These have proved incredibly popular with 1134 attendances in this particular week alone in the second semester of 2005/06.

Peer Tutoring

One of the new developments in the MLC was the introduction of peer tutoring. Student teachers who do Physical Education (PE) degrees in UL have the option of taking mathematics as an elective. These students have teaching practice in primary and second level classrooms throughout their degree programmes. The MLC decided to tap into this resource and ask for volunteers to teach mathematics to mature students on Access courses. This has proved very successful with two rewarding outcomes: the mature students get one-to-one help from qualified mathematics teachers and the PE students get extra teaching experience.

Online Support

The MLC website provides online support for students on service mathematics courses in UL (Figure 1).

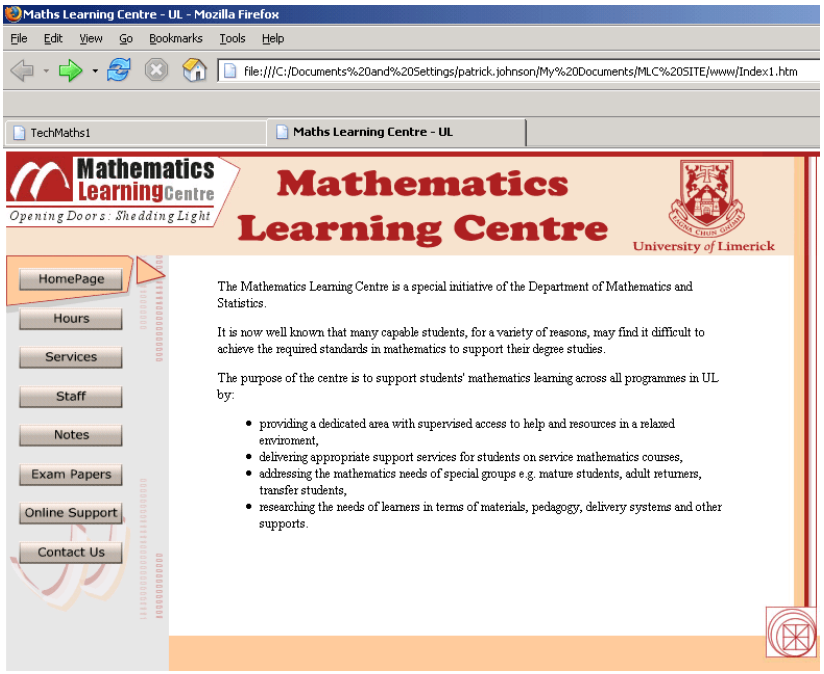

Figure 1 MLC Website Template

On this website, students have access to past examination papers and sample solutions for their specific courses (Figure 2).

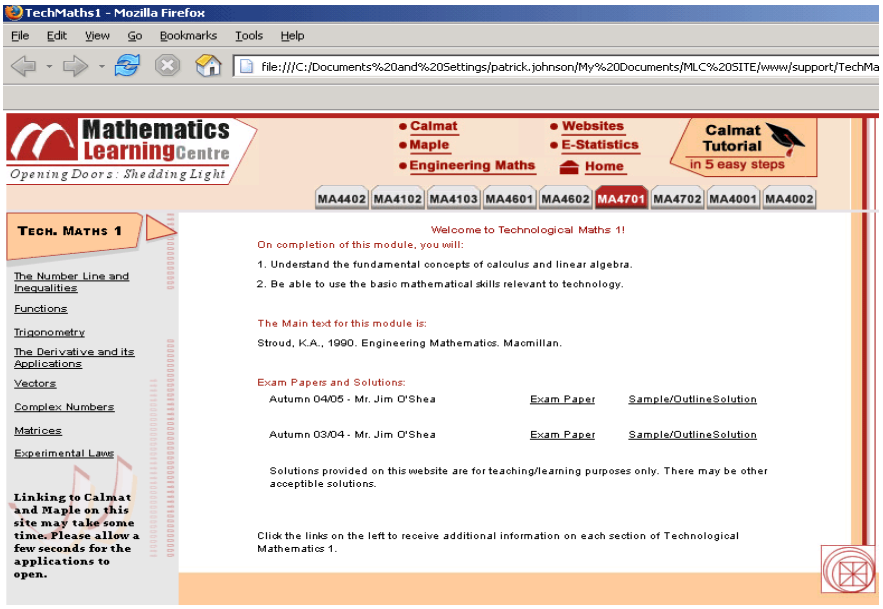

Figure 2 MLC Website

At present the website is being developed to improve the appearance of the website itself and to provide help specifically designed for each service mathematics course (Figures 3 & 4).

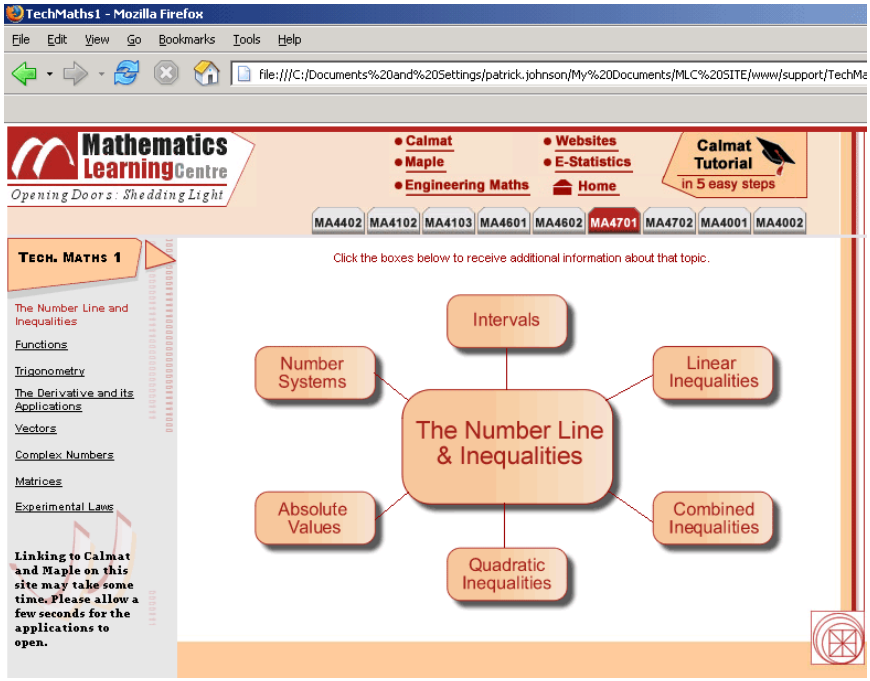

Figure 3 Development of Website

Students are invited to print off notes on any area of their mathematics course that they are having difficulty with.

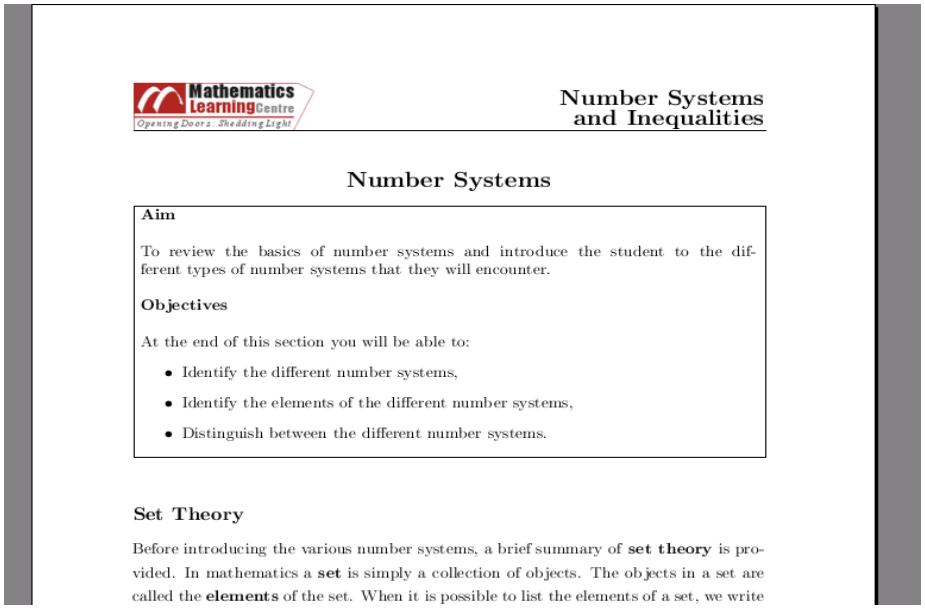

Figure 4 Samples of Notes Available on MLC Website

Measuring the Effectiveness of the MLC

It is hard to gauge the success of such an establishment but it has proved very popular with students from all different faculties e.g. Colleges of Business, Education, Engineering, Science and Informatics and Electronics. In the academic year 2001/02 there was a total of approximately 3353 service contacts (1516 to drop in centre and 1837 to support tutorials) by 431 individual students with the Mathematics Learning Centre (MacMullen, 2002). By the academic year 2004/05 this had increased significantly as there were over 5,200 contacts (Table 1) between the drop-in centre

and support tutorials (Gill, 2005). This shows the growing need for such an innovation to be made permanent in universities today.

Table 1 Statistics for U.L. MLC and support tutorials, 2004/05

	Semester 1	**Semester 2**	**Whole Year**
Number of Support Tutorials Held	134	132	266
Number of Individuals at Support Tutorials	339	385	607
Number of Attendances at Support Tutorials	1062	1212	**2274**
Number of Attendances at Drop-In Centre	1395	1545	**2940**
Number of Individual Students at Drop-In Centre	921	1019	1082
Total Number of Individuals Using Drop-In Centre and Support Tutorials in 2004/2005	**1011**	**1308**	**1337**
Total Attendances at Drop-In Centre and Support Tutorials in 2004/2005	**2457**	**2757**	**5214**

'At risk' students are defined as students who are at risk of failing their mathematics module as judged from scores on the diagnostic test (less than 20 out of 40), in which case they will have to re-sit the examination or repeat the whole year if they fail it a second time. The worst-case scenario is that these students may end up dropping out of their studies altogether. It is these students that the Mathematics Learning Centre wishes to prioritise in terms of services supplied. All students are welcome but those who are at risk are one of the top priorities. Most of these students need extra tuition to pass their mathematics courses. If help is given, students can then spend more time keeping abreast of their other studies.

Gill (2006) used the UL database to compare the results of the end of term examinations of those students who are characterised as at risk and participated in the MLC support tutorials with those who did not attend to see if there was a difference in examination performance. It was shown that students who do participate in these support tutorials do tend to outperform those who do not attend.

Table 2: Mean scores on semester 1 examination for students in at risk category

	1997/98	**1998/99**	**1999/00**	**2000/01**	**2001/02**	**2002/03**
Did attend support tutorials	68.4%	58.6%	41.2%	37.0%	46.1%	48.3%
Did not attend support tutorials	48.9%	49.0%	25.2%	32.1%	46.9%	51.7%

Table 3: Mean scores on semester 2 examination for students in at risk category

	1997/98	1998/99	1999/00	2000/01	2001/02	2002/03
Did attend support tutorials	66.0%	63.4%	52.3%	49.3%	65.8%	58.2%
Did not attend support tutorials	47.3%	53.7%	44.9%	40.2%	51.5%	52.2%

There is a distinct, if not decisive, advantage for those students in the 'at-risk' category who attend support tutorials over those who do not attend as measured by the results on the next and subsequent university examinations (mathematics).

Unfortunately, these attendees are in the minority so some action needs to be instigated to reach all those who need help but are not availing of it. Analysis of the UL database showed that 78.3% of those deemed to be in need of them, failed to attend the support tutorials in the first semesters and 78.5% in the second semesters. These are very high percentages. It is difficult to surmise why they are not attending. The classes are held outside their regular timetable hours (usually in the evenings) so as not to interfere with their ongoing studies. While on the surface this looks like a good course of action, many of these students have heavy timetables and may not relish the thought of 'extra hours'. Many, particularly the mature students have work and family commitments and so are unable to attend but this is not the case for everyone. It seems that some intervention must be implemented to reach those who require help but are not currently availing of it.

Research in the MLC

Research was viewed as an important support activity for the Mathematics Learning Centre from the outset. A number of projects were undertaken. A short selection of completed projects is listed below and currently a number of projects are in progress.

Gill, O. (2006) *What Counts as Service Mathematics? An Investigation into the 'Mathematics Problem' in Ireland*, unpublished thesis (PhD), University of Limerick.

Golding, G. (2006) *Learning Support for Adults Learning Advanced Mathematics: The Case of Adults Learning Vectors*, unpublished thesis (PhD), University of Limerick.

Lehane, J. (2006) *Active Learning in Secondary School Mathematics: Focus on Algebra*, unpublished thesis (M.Sc), University of Limerick.

Vilkomer, T. (2005) *An Analysis of the Theoretical and Practical Aspects of the Identification of Mathematically Gifted Students (with special reference to Krutetskii and Kolmogorov)*, unpublished thesis (M.Sc), University of Limerick.

Murphy, M. (2002) *Mathematical Underpreparedness Among Third Level Entrants: The Possible contribution of the Second Level Mathematics Experience*, unpublished thesis (M.Sc), University of Limerick.

Future Developments

Experience has shown that students value personal interaction most highly. However it is not always possible to provide this so there is a need to invest in online services and to customise these for student use out of hours and off campus. We have identified a need to address pedagogical issues such as teaching and learning approaches for various groups e.g. tradition age students, mature students and students on different degree programmes. Allied to this is a need to improve our tutor training.

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Adult learners go home to their children's math homework: What happens when the parent is unsure of the content?

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This paper draws on data from a research study examining the interactions between adult learners who are parents and their children as they work together on school mathematics homework tasks. The parents are committed to helping their children and engage in practices that are more reminiscent of peer/peer collaborations than of teacher/student dynamics. In addition, the children's homework presents an additional context within which adults learn mathematics.

Recently, many large urban school districts in the United States have adopted new mathematics curricula for elementary school. The curricula were developed in response to the NCTM standards (1989, 2000) and build upon the body of cognitive psychology research on children's mathematics learning. The materials from these curricula often use notation, computational procedures and problem-solving strategies that may be unfamiliar to parents because they differ from the notation, procedures and strategies they experienced during their own schooling. The homework papers that children bring home generally reflect the language, procedures and strategies used by these curricula and used in classroom math activities.

Among the roles that parents may take on as they engage with their children around homework are modelling behaviors and attitudes, reinforcement of successful school behaviors, and direct instruction (Hoover-Dempsey and Sandler, 1995, 1997). Shumow (1998) studied parents' work with their children on mathematics homework in conjunction with classroom practice based on Cognitively Guided Instruction (Fennema & Carpenter, 1989). Although the parents were initially unfamiliar with the school-based instructional strategies, through an intervention, they learned to "collaboratively guide" their children in problem solving and increase their knowledge of their children's reasoning. Similarly, parents in a study by Hyde, Else-Quest, Alibali, Knuth, & Romberg (2006) were coached on the mathematics content, and were provided with a procedural strategy and a conceptual approach for solving pre-algebra problems. In both studies, the subjects were suburban, middle class parents whose own educational attainment ranged from secondary school completion through graduate degrees. The second study found that mothers with higher educational levels "performed better in conveying mathematical content and in scaffolding" (p.136).

However, many adults were not successful in school, having left schooling before completing secondary school and/or demonstrate limited numeracy skills (Kirsch, Jungeblut, Jenkins, & Kolstad (NALS), 1993; National Assessment of Adult Literacy (NAAL), 2005). Some of these adults return to schooling in adult education programs. When asked about their motivation to return to learning, people often report that they have found that they are unable to help their children with school-related tasks such as homework. They believe that if they learn the school content they missed or do not remember, they will be better able to support their children's school learning (Brew, 2000). These parents believe they have a responsibility to help their children with homework and report that they do so.

The present study approaches parent involvement in homework by acknowledging that virtually all parents try to be and see themselves as resources for their children's learning. The study seeks to make visible how parents, who have made a commitment to address their own needs regarding further mathematics learning, work with their children on mathematics. These parents have not been part of an intervention designed to improve or modify their ways of working with their children. This perspective challenges the prevailing assumption that views parents who did not complete secondary school as limited or deficient in their ability to help their children.

That said, how *do* such parents cope with their children's math homework? In particular, how do parents respond and interact with their children when they, the parents, are not sure about the mathematics content of the tasks or familiar with their children's strategies?

To begin to answer these questions, a pilot study with a small group of parents was undertaken to determine the range of parent reactions and responses when the mathematical demands of a homework assignment are unfamiliar to them. This paper will describe some examples of parental strategies for managing unfamiliar mathematics homework content.

Methodology

Subjects

The subjects were eight women (7 African-American and 1 Caucasian) from the inner-city of a large city on the east coast of the U.S. They ranged in age from 34 to 61, all mothers except the oldest who is a grandmother raising four grandchildren. For convenience, I will call them all "parents;" the grandmother's charges frequently refer to her as "Mommy" as well as "Grandmommy." Only one of the women completed secondary school, and she reports that she only studied fractions and decimals in secondary school, never algebra. One parent was married, but she said that she is the adult in the family who generally works with the children on homework. The women were recruited through two adult basic education programs.

Each parent was raising a child who was in 3rd or 4th grade during data collection but was also parenting between 1 and 4 other children, with most currently also living in the home. The children attend public elementary schools and all are eligible for free or reduced lunches based on parents' low income. All participating children are in schools that use the *Everyday Mathematics* curriculum.

The mothers who participated in this study all said they hoped to learn more mathematics so they could be in a better position to help their children. All but one also reported that they hoped to improve their math skills so they would be able to pass the mathematics portion of the GED test (secondary school equivalency certificate), and ultimately get a job, a better job, or go on to further study or credentialing.

Data Sources

Data reported here was gathered through hour-long semi-structured interviews with each parent and hour-long, parent-child task-based interviews that were video-taped. Both parent interviews and parent/child sessions were held at times that were convenient for the parents and in their choice of locations. All parent interviews were held in adult education classrooms, and were audio-taped. Parent/child sessions were mostly held in homes after school or on weekends, though two sessions were held in school buildings at the request of the parents. During sessions that were held in homes, siblings were always present and periodically interacted with the mother and/or the target child, were sometimes arguing and often a television was playing — in short, these sessions were probably quite similar to typical homework events.

During the interviews, parents talked about their experiences learning math in school, their experiences learning math as adults, and the ways they help their children with homework. During the parent/child session, each parent spent an hour working together with her child on tasks from the third, fourth and fifth grade *Everyday Mathematics* curriculum materials. All parents and children agreed that the tasks were similar to the math homework assignments the children bring home and that the homework sessions were pretty much like what normally happens around homework. All the children said that when they don't understand something on their math homework, they first ask their parent, and then sometimes, if necessary, they or their parent asks an old sibling or other family member for help.

The videotaped parent/child sessions were watched multiple times to identify instances when parents were unsure of the mathematics content but engaged in interactions that were deemed supportive or productive in helping the child make progress, complete assigned tasks, or add to the value of the child's problem solving experience. Patterns of interactions were seen within and across individuals. This first analysis will contribute to the development of a typology of ways in which adult basic education learners/parents engage with their children over mathematics learning.

Tasks for parent/child sessions

The tasks were taken from the curriculum materials but were slightly more difficult than the children had encountered in their classrooms.

One task for the fourth graders and their parents was a multiplication "word problem," a school-based task that was familiar to the parents from their own schooling. The children have been learning non-traditional, alternative computational strategies for multiplication, strategies that parents did not encounter either during

their own schooling or in their adult education classes. The 4th grade children were given the following problem:

> 129 people came to the school's spaghetti supper. Tickets cost $7.50 per person. How much money did the spaghetti supper make?
>
> (*Everyday Mathematics*, *Practice Sets*, p. 123)

Another type of task (which varied somewhat for 3rd or 4th graders) was a patterning exercise that would be familiar to the children from the *Everyday Mathematics* materials being used in their schools. These tasks might also be familiar to the parents from homework assignments. Of particular difficulty for many parent/child dyads were exercises that required determining the rule that would enable completion of a sequence.

A 3rd grade example of the "What's my rule?" task is (the actual paper included a graphic representation of the sequence using squares, circles and arrows):

RULE: _24_ , ____ , ____ , _42_ , _48_ , ____
(Everyday Mathematics, *HomeLinks*, 3rd grade, p.27)

A 4th grade example:

RULE: ____ , ____ , ____ , _61_ , ____ , _75_ , ____ , _____ , _____"

created from blank "Frames and Arrows" form, *Everyday Mathematics*

Thus, the multiplication task described above would probably be a familiar task to parents, but the children's strategies would likely be different from parents' own learned strategies. The "What's my rule?" tasks were unfamiliar to the parents except in the context of their children's homework, but the children have worked on similar tasks in school. In each case, it was likely that some portion of the task would present a challenge to parents.

Findings

In some instances, while children and parents were working on the above and other tasks, a child asked for help and received enough help from the parent to be able to complete a task. Other times, a parent indicated that she was unsure of what to do but the parent and child worked together to come to some resolution. However, sometimes, a parent suggested a solution strategy that was misconceived and the two floundered together, sometimes neither ever realizing that there was a problem. In addition, there were instances in which a parent assumed (wrongly) that she knew what to do and in a very directive way, led the child through steps and procedures regardless of whether or not the child understood what was being done.

Clearly many unproductive interactions resulted from parents' limited understanding of the mathematics and may have caused additional confusion and frustration for the child during homework sessions and also upon returning to class. That said, there were also fruitful interactions that were common across multiple parents. I will focus on some of those *productive interactions* that occurred when the parent struggled to find a strategy or procedure that could help the child. There were three categories of such interactions that emerged when parents were unsure of the mathematics needed for the task: *parallel work, positioning the child as teacher, and collaborative work with justifying conjectures*.

Parallel work

As expected, one issue that emerged was the use of alternative algorithms that children are learning in school as part of the *Everyday Mathematics* curriculum. Some of the parents say they have tried to learn the multiplication algorithms but "cannot keep them straight." The adult education classes do not spend time on alternative algorithms, especially because these parents are all able to multiply fairly efficiently with the algorithm that has traditionally been taught in U.S. schools. Many of the parents and children have negotiated a practice of parallel work, during which parents use the traditional algorithm to check the children's answer that was derived with an alternative algorithm:

Makita describes how she deals with her daughter's use of the "lattice method" for multiplication:

> She get her answer her way and I get it mine. I tell her as long as we get the same answer. See her way, I don't understand it. But she gets the right answer every time. I don't understand it. It's too criss-crossed and messed up, but she gets the answer.

The negotiated parallel work is evident when Makita and her daughter work to solve the 3 digit multiplication problem during the parent/child session:

> Makita: You do it on here (pushing over a piece of scrap paper) and I'll do it on mine (using a different piece of scrap paper). You can do your lattice across if you want to. (Both work on their papers. As Makita writes on scrap paper, she hides her work from her daughter by lifting the corner of the paper. Makita finishes quickly and then watches over her daughter's shoulder, commenting to her about making sure she writes down all the numbers to fill the lattice. When her daughter finishes, Makita checks the answer.)
>
> Makita: That's the right answer!! (Both laughing, they give each other a high-5 slap.)

Similarly, Dionne and her son, Jerome, are comfortable working together on homework while using different computational strategies, without really understanding each other's work. Although Dionne says she does not understand what her son is doing (he uses a "partial product" multiplication strategy in which numbers are decomposed and each multiplied by each of the others, summing the results) and is confused about all the zeros, she apparently does see that the initial digits are developed from multiplying digits from the numbers in the problem. She tries to help by catching errors and providing helpful hints, although Dionne and Jerome sometimes seem to be talking past each other:

> Jerome: The number's big, 1000.
>
> And 50 times 100 equals five thousand
>
> 700 times 9, six thousand one hundred
>
> 700 times 20.
>
> Dionne: This one's incorrect (pointing to 6100). Seven times nine. You're close.
>
> Jerome: (pause) 3 (changing 6100 to 6300).
>
> Dionne: Nines is very tricky. It's a big number but the bottom number is small, so don't look at that.
>
> Jerome: Yeah, it's closer to 10.
>
> Dionne: Reverse it too. If it looks too scary. Put the 5 in front of the nine, or even the 6 or the 7, and then it looks much better. Or at least, not as hard.

Once Jerome is finished with his computation, he asks his mother to "do it the old fashioned way" so he can see if his answer is right. The two have clearly worked together like this before, as they both know exactly what is meant by "the old fashioned way."

During the interview, Dionne described her experience working with her son on his mathematics homework:

> I don't know how they do the new style of anything they do. But I checks with the old way and it's right and I say, 'How did he get that answer?'

Positioning the child as teacher

Some parents use homework time as an opportunity to learn from the child about what s/he is learning in school. This also serves as a learning opportunity for the parent if she, herself, is unsure of the content. The parent puts the child in the role of the "explainer," answering the parent's questions and explaining how s/he is approaching and solving the problems. In this example, the parent is following the child's activity, confirms observations that are already obvious to the child, and otherwise provides encouragement and support. Working on the 3rd grade "What's My Rule?" problem,

Alice: A rule, hmm.

Brandon: That adds up to 42. (Counting on fingers) 24, 29, 33, 37,

Alice: That can't be right.

Brandon: I'm trying to add up to 42. See if it's 37, 37.

Alice: Hmmmm

Brandon: Hey counting by 5s. 'cause I just counted to 37, but I have to add 5 more.

Alice: So that number must not be right.

Brandon: Hey, we can use blocks.

Alice: You want to try the blocks?

Brandon: We have to get 24. (He starts counting out 24 blocks, Alice helps.)

Alice: And after we get 24? So then what do we do?

Brandon: Try to get 42. See if it's 5. Because if it's 5, we can get 42.

Alice: 24 and 5 is 29 and 5 more?

Brandon: (counting out groups of 5 blocks and counting up from 24) 39…Nope, that can't be it. So keep the 24.

Alice: Hmmm

Brandon: Have to be less. Let's try 7. (counts out 7 blocks) 25, 26, ….31. (Counts out another 7 blocks) 32, 33, 34 … 38. 39, 40, 41, 42. That's more.

Alice: That's what I think.

Brandon: You can't use 7. Let's try with scrap paper. I think I can add it up. 24. (writes and speaks) 24 +6 +6 +6. (counts on fingers) Yep. that's 6, equals 42. Add it up yourself.

Alice: So you put 6 in there (rule box). What number goes in here then?

Brandon: You have to add it up. 24, 25, 26, … 30 Wait (puzzled).

Alice: You right. OK, go ahead.

Brandon: And 6 more. 36 and 6 more is 42. And 6 more is 48.

Alice: Ohhhhhhhhh. So that's how you do it

The parent reported afterward that she had not known how to go about solving the problem before her son showed her what he was doing. However, she used the same techniques of asking a few questions and confirming his work when she did know the appropriate mathematical strategies or procedures. She consistently expected him to

articulate his reasoning and take responsibility for the work. In fact, she made similar nondirective comments when her son made errors in reading the directions for the activities.

Collaborating and justifying conjectures

Sometimes, when both parent and child are unsure of how to solve a problem, they work together to come to an agreed upon solution, with one or both putting forth suggestions and trying to justify the methodology and conclusion. For example, in the "What's My Rule" problem, Renee is trying to help her son, who has already decided what to do. Renee is struggling to make sense of the situation for herself and uses various representations to grasp the situation.

> Renee: 61, ___, 75 (Both Renee and Wakim quickly count up from 61 on their fingers.) So it's about 14?
>
> Wakim: (counting again on his fingers) No, about 7.
>
> Renee: Can't be 7. Aw right, Look we can do it like this. (Making hash marks while counting up from 61.) 61, 62, 63…70, 1, 2, 3, 4, 5. (Counting the hash marks.) 1, 2, 3… 13, 14. 14.
>
> Wakim: Nah, ah. About 7.
>
> Renee: Here, 14 (Counting up from 61, counting each hash mark.). Wait, you right. Wait a minute, you might be right son, (counting hash marks again) 61, 62, 63, … 67, 68. So that's 68. You right. I'm sorry, you right. So that's 68, and that's 75.

Dionne and Jerome also discussed this problem. Because Dionne was unsure of a strategy to solve the problem, she did not immediately understand what her son did. She questioned his conclusion so he explained his reasoning to her, convincing her of its logic.

> Dionne: We got to make our rule.
>
> Jerome: 75 minus 61 should give us the answer.
>
> Dionne: hmm.
>
> Jerome: 14.
>
> Dionne: OK.
>
> Jerome: And 61 plus 14 is 75. It's 7.
>
> Dionne: Hmm?
>
> Jerome: It's 7, you gotta cut 14 in half.
>
> Dionne: OK.

Jerome: 14. So that would be 68.

Dionne: Wait a minute (taking the paper). It's 14. You have to cut it in half?

Jerome: Yeah. 61 and 14 gave us 75 and that's already here (pointing across the space between the 61 and the 75. So we have to cut it in half.

Dionne: So it would be 7. OK, OK. So we got, our rule is plus 7.

Jerome: Yep.

Dionne: Write that down, it's very important.

In both these cases, the parent could have just gone along with the child's solution, but in each case, it was important to the mother to understand the problem and the proposed solution so as to be able to confirm its accuracy and monitor her child's activity.

Discussion

Surely, all of the mothers in this study would benefit from additional mathematics learning and they would likely then feel more comfortable and confident in helping their children with math homework. But even though all participating parents report wanting to feel more confident helping their children, their commitment to their children and to their perceived responsibility to encourage and support their children's mathematics learning and success in school also cannot be doubted.

The parent/child interactions described above are reflective of the types of interactions that are promoted for the classroom by the NCTM Standards and the authors of the *Everyday Mathematics* curriculum. These include making and investigating mathematical conjectures, communicating mathematical thinking, recognizing the comparability of multiple strategies, flexibly shifting among various representations, cooperative learning and sharing ideas through discussion (NCTM, 2000; Teacher's Reference Manual, *Everyday Mathematics*, 2004). As the parent and child frequently engage as equals in these practices, their interactions are more reflective of peer/peer collaborations than of teacher/student, or even parent/child, power positions.

In an environment in which both participants are struggling to understand a mathematical idea, it is perhaps more likely that interactions as described above would emerge naturally. While the power hierarchy of the parent and child roles have been well established in most families, the hierarchy between the knower and the learner is perhaps less rigid, especially when the parent is not always the "knower." The parent also identifies herself as a learner of mathematics, both in the adult education program setting and also in the realm of children's mathematics homework. When parents and child see themselves as two learners working together as partners, they seem to develop patterns of interactions that promote the learning of both parties, and interact in the ways similar to those that teachers promote within the classroom. Such strategies were most apparent in the "What's My Rule?" tasks, when the parent

was not very familiar with the task itself and with the reasoning and strategies that could be used productively to complete the task.

On the other hand, the multiplication word problem was very familiar to the parents from their own schooling and from their adult education classes. Every parent (but one) recognized that to find an appropriate answer they needed to multiply, and they felt confident in their ability to find the solution. The difficulty for the parents was that their children used different procedures and they were unable to understand or use their children's procedure. The parents and children came to resolve this situation by recognizing the utility and value of both procedures and used one to check the other. Mothers' early attempts to learn the children's procedures had been confusing and frustrating, so the mothers and children "agreed to disagree" and each solved the problem relatively independently. Some mothers still tried to be helpful by watching for small multiplication errors while others just positioned themselves on the side.

Apparently, one unexpected "benefit" of parents' limited mathematics knowledge that could be applied directly toward the completion of the children's homework is the shared opportunity to have a learning experience. Most of these parents seem to be less directive than they might have been had they been more confident of their own mathematics knowledge. They were less directive than the better-educated parents in Shumow's (1998) study. The parents' eagerness to learn and the interactions that evolved from the parents' inquisitiveness may in fact provide an additional, rich learning opportunity for their children.

Finally, one answer to the question, "Who is learning from children's mathematics homework?" is that the parents are learning. Parents are learning about their children's schooling in mathematics; they are learning some of the mathematics content they did not learn when they were in school; they are learning to solve problems, explain their reasoning, and focus on mathematical thinking; and they are learning that there are alternate strategies and procedures that can be used to solve problems.

Implications for teachers of adults and children and for further research

Adult education teachers often see themselves and their programs as the primary setting within which adult learners acquire the mathematical knowledge they missed during prior schooling. However, the context of children's homework provides another venue within which parents learn and around which adult education can provide additional learning support. Adult educators should acquire and examine samples of the materials that children bring home from school so they and their learners can become familiar with the materials, the conceptual ideas that underlie the mathematics needed, and strategies that will support both parents' and children's learning.

As adult educators acknowledge that one of the important goals for many of their adult learners is to better help children with mathematics homework, they will be better able to target their mathematics instruction to help learners develop the skills and strategies that promote productive interactions with children.

As elementary teachers better understand the needs and strategies of parents as they try to help their children, they will be better able to provide appropriate supports to

enable parents to be effective partners in children's successful learning. Further, it is clear from the data presented, that parents can be, and often are, powerful resources for their children. Appreciating and nurturing these resources will only enhance children's opportunities to learn.

Further research into how adults' continuing mathematics education may impact their work with their children is in progress. Intervention studies with this parent population are also needed to determine the kinds of supports or programming for parents that will be effective in enhancing the parents' own mathematical development and also produce a positive impact on their children's learning.

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Adults just don't know how stupid they are. Dubious statistics in studies of adult literacy and numeracy

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International studies of adult literacy and numeracy play a major role in the public debate on education and labour market policies and have consequences for decisions regarding the educational system. It is therefore important that data from such studies are analyzed and interpreted professionally and correctly. It is my contention that this is not always the case. This discussion paper will focus on some recent studies (IALS and ALL).

The last 15 years have seen a proliferation of international studies of literacy and numeracy and the results from these have had decisive influence on decisions regarding education and labour market policies. Adults were the target group for the OECD initiated *International Adult Literacy Surveys (IALS)* conducted in the period 1994-98 in 21 countries. The results were published in a series of international reports, the final one being OECD (2000) and a number of national reports, in Denmark, Jensen et al. (2000). IALS was followed in 2003 by *Adult Literacy and Life Skills (ALL)* with 6 participating countries: Bermuda, Canada, Italy, Norway, Switzerland, and US comprising 10,000 adults from each country. In IALS the testing was done on three scales: Prose, document and quantitative scales and scores were derived by Item Response Methods (IRM). ALL had four scales: Prose, document, numeracy, problem solving, and scores were again derived by IRM. The results were published in an international report (Statistics Canada & OECD, 2005) and in a number of national reports.

Since the international surveys have vast influence it is important that data from such studies are analyzed and interpreted professionally and correctly. In this discussion paper it is argued that this is not always the case. Examples are primarily taken from the Norwegian report on ALL (Gabrielsen et al., 2005). Similar examples are found in other national and international reports.

My areas of interest are

- Quality of the quantitative analyses,
- Quality of survey items,
- Consequences of the constraints that the Item Response Models pose on the investigations?

- Validity of results and ethical questions related to “the narrative of the deficient population”.

Quantitative analysis

> Assessing basic adult skills is expected to provide important information to policy-makers on different levels. Firstly such information is useful when planning and streamlining children and young people’s basic education. It is equally important for determining which policies (to) adopt for adult and continuing education of different adult population groups. (Gabrielsen et al., 2005, p.20)

The international literacy and numeracy surveys, IALS and ALL, include a number of external variables (age, gender, education etc.) used to study special aspects of literacy and numeracy proficiency in the participating countries. To avoid false or nonsensical conclusions analysis of the relationship between scores and the external variables in most cases call for multidimensional modelling. In this section, I will consider the following issues from a statistical point of view:

- Confounding and selection,
- Multiple hypothesis testing,
- Irrelevant information.

Confounding and selection

Confounding by for instance gender means that an effect (a difference in some outcome of interest) that appears to be attributable to some other variable will disappear if the analysis is carried out separately for men and women. Controlling for confounders is one way of eliminating (or at least reducing) “spurious” or false correlation.

As an example, consider the following (fictive) findings.

> “Higher breast cancer rates for mascara users. Mascara protects against testis cancer.”

The first statement is literally speaking “true”, since it only reports about the simultaneous appearance of two events: mascara-use and breast cancer. The second is (most plausibly) false since it implies a causal (negative) connection between mascara-use and testis cancer. However, from a text analytic point of view the first one is false too. Presenting something as a “finding” in a report implies that the author expects it to carry “meaning”. In the example the meaning is that mascara is causing breast cancer. The effect is, however, partly or totally attributable to the fact that both breast cancer and mascara are more common among women than among men. One would expect the effects to disappear if the analysis is carried out separately for men and women and in this case the correlation is due to confounding and should not be reported as a “finding”.

In Henningsen (2004) gender is studied as a possible confounder in three reports on IALS and a number of examples are given of reported associations that must be expected partly or totally to be a result of gender as a confounding factor.

Similarly, many results from ALL are reported without controlling for confounders. In all participating countries literacy and numeracy proficiency show a high positive association with education. Hence, every factor associated with education can be expected to show association with literacy and numeracy scores without necessarily being of any consequence in the sense that controlling for education would make the association disappear. The following table gives examples from Gabrielsen (2005) with education and other background variables as (unnoticed) confounders. The list is not intended to be complete, it only points to results where one would have expected the authors to investigate the influence of possible confounders before reporting associations as "findings", even if the direct effects in some instances could be of interest, too. All findings listed as "reported effects" are from the summary in Gabrielsen et al. (2005).

Table 1: Education as confounding factor in results from Gabrielsen et al. (2005)

Factor	Confounder	Reported effect
Age	Education	"Norwegian adults in the age group 21 to 45 are clearly better with regard to numeracy than the age groups over 46."
Age and gender	Education	"The difference (in numeracy) between men and women is smallest in the younger age groups and increases with age."
Job	Education	"People employed in occupations which require higher education; usually have noticeably better reading and life skills competence than those employed in jobs requiring less education."
Ethnicity	Education, age	"Non-western immigrants are a group with poor literacy skills, [….] Immigrants from Denmark/Sweden and from other western countries on the other hand, obtain a better average result on both literacy scales than what is found for Norwegian-born adults."
Labour market participation	Education, age, gender, health	"Adults who for all kind of reasons are not included in the labour market show a significantly weaker average result on the numeracy scale compared with those who work or are in a training situation."
Region	Education	"Adults in the Oslo- and Akershus region [the capital] obtain the best literacy results both on the prose and the document scale."
TV	Education, age, health	"Adults who spend up to 2 hours daily watching TV, video and DVD show better average scores on all four skills as compared with people who say they spend more than 2 hours a day watching these media."

Associations due to confounding do not give meaningful or useful information. In all the above mentioned cases the interesting issue is if there is an effect above the effects caused by the confounding factors, and the data are rich enough to answer this by use of appropriate multidimensional statistical methods.

Multiple hypothesis testing

In ALL (2005) we find an extensive number of statistical comparisons. Investigating correlations between background variables and literacy/numeracy scores means that literally many hundred comparisons can be made. Statistical tests performed on a 5% significance level will on the average find a non-existing difference in 1 out 20 cases. Hence, even if effects due to confounding or other inhomogeneities were eradicated, we would by definition expect to find "false" significant differences in 5 out of every 100 comparisons. This is the background for the well-known activity "a fishing expedition" or the phenomenon "mass significance".

How to conduct a "fishing expedition":

- Observe a large number of background variables,
- Perform a large number of significance tests,
- Present all significant results as research findings.

Listing significant results in the report on a survey signifies that the authors expect these to carry "meaning", as in the following "finding":

> Adults in the Oslo-Akershus region obtain the best average score both on the prose and the document scale, in front of Trøndelag and Western Norway. The differences between these three regions are, however, not statistically significant. On the other hand adults in the Oslo and Akershus region have significantly higher average scores than the other three regions. (Gabrielsen et al., 2005, p.57)

As indicated in table 1 the reported regional differences are probably confounded with education, since the level of education is higher in the capital (Oslo-Akershus region), and the relevant question is whether there is regional variation apart from regional differences in education, age gender and so on. This could in this case have been established by conventional (multivariate) statistical procedures.

Irrelevant information

In surveys background variables of no immediate relevance for the research questions are often included, because they could be possible confounders. Hence, it could be appropriate to control for them in the analysis, while inclusion as primary independent variables usually does not make sense. Other information included for instance in the ALL questionnaire just appear to be irrelevant. Two examples from ALL (2005):

> Participants in religious groups have on the average better scores on the numeracy scale compared to non-participants. (Ibid. p.128)

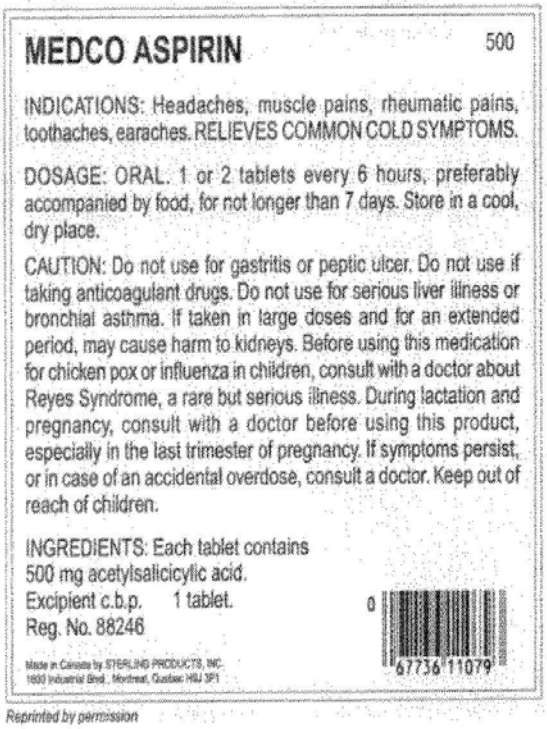
> […..] find, on the other hand, no such differences in scores pertaining to the categories "Collecting money" or "Collecting food or other items for charitable purposes". (Ibid. p.128)

Quality of test items

The international surveys have a large accompanying body of material discussing the theoretical foundations for the test items. (See, e.g. Statistics Canada & OECD (2005) and Statistics Canada (2005)). Hence, it is natural to ask if the items (the problems) in the adult literacy and numeracy studies come up to the intentions of the theoretical framework.

In IALS and ALL only a few test items are made public. This is in itself both a scientific and a "democracy" problem" since it prevents "outside" researchers from making independent evaluations of the test results. The published items suggest, however, that item quality in itself could be a serious problem. Because of space limitations I have chosen to make a detailed investigation of only three items. (More items are discussed on http://www.math.ku.dk/~inge/all.) In Statistics Canada & OECD (2005) two of the items (Medicine label and Fuel Gauge) are presented and I have taken this as a starting point for the following discussion.

Item 1: Medicine label


MEDCO ASPIRIN 500

INDICATIONS: Headaches, muscle pains, rheumatic pains, toothaches, earaches. RELIEVES COMMON COLD SYMPTOMS.

DOSAGE: ORAL. 1 or 2 tablets every 6 hours, preferably accompanied by food, for not longer than 7 days. Store in a cool, dry place.

CAUTION: Do not use for gastritis or peptic ulcer. Do not use if taking anticoagulant drugs. Do not use for serious liver illness or bronchial asthma. If taken in large doses and for an extended period, may cause harm to kidneys. Before using this medication for chicken pox or influenza in children, consult with a doctor about Reyes Syndrome, a rare but serious illness. During lactation and pregnancy, consult with a doctor before using this product, especially in the last trimester of pregnancy. If symptoms persist, or in case of an accidental overdose, consult a doctor. Keep out of reach of children.

INGREDIENTS: Each tablet contains
500 mg acetylsalicicylic acid.
Excipient c.b.p. 1 tablet.
Reg. No. 88246

0 67736 11079

Made in Canada by STERLING PRODUCTS, INC. 1600 Industrial Blvd., Montreal, Quebec H6J 3P1

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Example 1 (*Prose scale level 1)*

Use the medicine label to answer the following question

What is the maksimum number of days that you should take this medicine?

(Gabrielsen et al. 2005:213)

This first item is from the prose scale. The item shows a medicine label and the reader is directed to use the medicine label to answer the following question

What is the maximum number of days that you should take this medicine?

The comments from Statistics Canada & OECD (2005, p.284)) are:

> One of the easiest tasks [….] because the reader was required to locate a single piece of information that was literally stated in the medicine label. The label contained only one reference to number of days [….] there is no other reference to days in the medicine label.

The label has, however, strictly speaking no information on the maximum number of days you should take the medicine. It only says "for not longer than seven days", a statement signifying "seven days in a row". The strategy envisaged by Statistics Canada & OECD (2005) is that the reader will reason that with only one number in the text this number must be the correct answer to the question. This is typical "school mathematics" (compare Wedege (2006)). Persons with "real life" experience have hopefully developed a more reflexive attitude, checking whether the number in the text really gives the correct information. This is a test item, where the reflective adult is definitely worse off than adults using the superficial strategy suggested by Statistics Canada & OECD (2005). In this case it is even trivial to remedy this problem. Would it have been a less satisfactory problem had the question been:

What is the maximum number of days in a row that you should take this medicine?

Item 2: Throwing dice

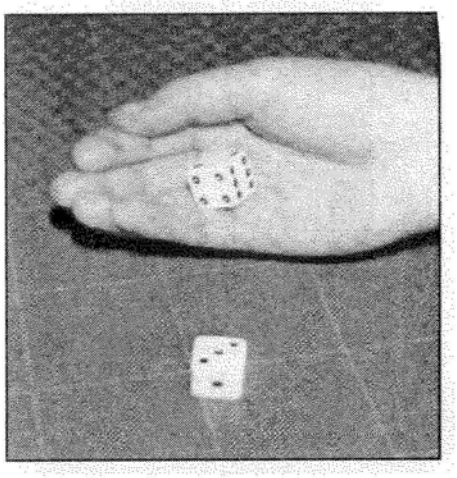
The next problem shows a hand rolling a dice.

Example 2 *(Numeracy)*

Use the photograph to answer the following question:

You roll two dice – one after the other. You get a three with the first dice. What are the chances that you also get a three with the second?

(Gabrielsen et al. 2005:213)



This problem is presented as straight forward but is actually venturing into the difficult relationship between probability modelling and real life. The following invented dialogue between an interviewer and a test person illustrates the situation.

Dialogue on dice

Interviewer: Look at the photograph. The first dice showed a three. What is the probability of this dice showing three?
Test person: I don't know. Maybe 1/8. Probably less.
Interviewer: Why do you think so?
Test person: Look at the position of the dice. It will most likely roll with three on the side, so the probability of a three must be less than 1/6. .
Interviewer: But what about the information that the first dice showed three?
Test person: Isn't that irrelevant? The result of the first roll won't influence the result of the second.
Interviewer: Exactly, that is why the right answer is 1/6, .
Test person: But that is nonsense.
Interviewer: In school a throw of two dice is used to model stochastic independence. You show you recognize that by giving the answer 1/6.

The experienced teacher would probably recognize that the intention with the problem is to test familiarity with the notion of "stochastic independence". In a school setting this is often illustrated by throws with dice or coins and it is taken as an (implicit) convention that throws are independent and dice are "fair". In this frame the answer 1/6 can be taken to signify that the reader knows how to construct probability of independent events. The item, however, breaks with this theoretical frame by directing the reader to look at and use a picture containing all kinds of empirical information. If the dice is thrown by an open hand as in the photograph, there is no reason to think that the probability of a three is 1/6 since you have information on the throw. Again the reflective student will probably be worse off than somebody just seeing the item as a conventional textbook problem and answering without thinking too much. This gives a definite advantage to adults just out of school compared to people with some experience. The problem can be seen an illustration of incomplete modelling. The independence of the two throws is not stated neither is the "fairness" of the dice. One could contend that this "goes without saying" and I would tend to agree, were it not that "fairness" of the dice was explicitly stated in the next part of the problem.

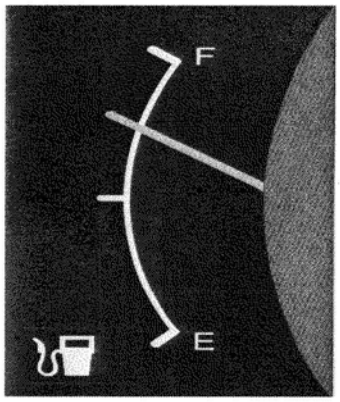
Problem 3: Fuel gauge

Example 3 *(Numeracy)*

Use the fuel gauge in the photograph to answer the following question:

The tank in this car holds 48 liter of gas. Approximately how many liters remain in the tank (Assuming the fuel gauge is accurate.)

(Gabrielsen et al. 2005:217)



The item gets the following presentation in Statistics Canada & OECD (2005, p.299):

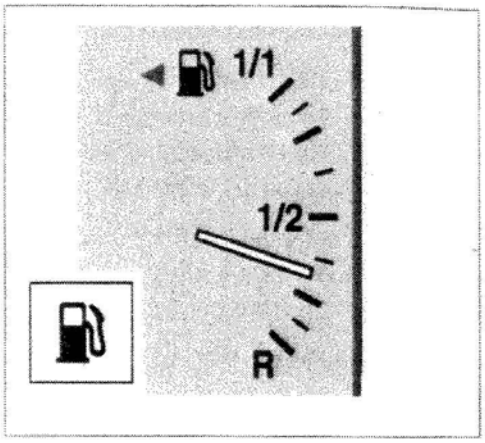
> This task is drawn from an everyday context and requires an adult to interpret a display that conveys quantitative information but carries virtually no text or numbers. No mathematical information is present other that what is given in the question.

This is on the surface a problem from a context that is well known to many adults (but could be expected to discriminate against adults not having a car). The test constructors concentrate in their comments on the quantitative side of the task, disregarding the modelling side.

> […] adults must first estimate the level of gas remaining in the tank, by converting the placement of the needle to a fraction. Then they need to determine how many gallons this represents from the 48 gallon capacity stated in the question or directive. (Ibid.)

Contrary to the notion in the text above it is not obvious how to convert "the placement of the needle to a fraction". The item can be seen as a model of a fuel gauge that is in itself a model of what is going on in the fuel tank and a very crude one, too. Again, if you are an experienced teacher or student you will probably recognize the intent of the problem and mentally fill out the gaps, but as a model of a real life fuel gauge it is an incompletely described model. Let me indicate the flaws. Real life fuel gauges are meant to convey information, hence (almost) all have numbers on the gauge and drivers are not supposed to guess on a linear relationship between the length of an arc on the gauge and the corresponding amount of gas in the tank. A lot of (most?) fuel gauges are actually not made that way (See for instance figure 4 showing a fuel gauge from a Toyota Corolla). Moreover experience tells you that (for good reasons) a fuel gauge on *Empty* does not mean that the fuel tank is empty. Consider this paragraph from the netpage *Understanding your dashboard gauges*

> **Fuel gauge**: Deliberately designed to be inaccurate! After you fill up the tank, the gauge will stay on full for a long time, then slowly drop until it reads ¾ full. […] When the needle drops below E, there is usually 1 or 2 gallons left in reserve.



Fuel gauge from Toyota Coralla. Note that the distance from ½ to ¾ is longer than the distance from ¾ to full.

So, if one assumes the fuel gauge to be accurate as stated in the item text the gauge cannot be expected to have a linear relationship between the length of an arc on the gauge and the corresponding amount of gas in the tank and the deviations from linearity will depend on the make of the car. Hence the question cannot be answered without further information/assumptions. It is seen from the comment, that the absence of text and numbers are a point to test constructors. This is however what makes the modeling incomplete, since the implicit assumptions are not borne out in the real life situation that they are modeling.

To sum up the three items show:

- Lack of care in formulating the items (medicine label),
- Problems are school problems not adult life problems (throw of dice),
- Lack of modelling experience (fuel gauge).

Item response models

Analysis of all recent international literacy and numeracy studies are based on item response models (IRM). This has some advantages but it severely constrains the test battery because it rules out "differential item functioning" (DIF). This is expressed by Murray (1995, pp.11-12):

> The fundamental assumption built into the IALS design is that proficiency is related in a regular way to item difficulty, a way that is invariant across language, culture and subculture.

In most surveys this assumption of non-differential item function (DIF) is combined with the demand that persons can be characterized by a single score on each scale, giving raise to the following constraints on the items in a test battery:

- The measured proficiency can be represented by a single latent scale,
- No differential item function, i.e. item difficulty is invariant across all subgroups of persons,
- All subsets of questions give (in principle) the same ordering of person proficiency.

Both IALS and ALL operate with multiple scales. Both surveys have two literacy scales (prose and document literacy) and the empirical results show that the two scales measure different proficiencies since they, for instance, function differently for

men and women. In contrast there is only one numeracy scale indicating that proficiency in numeracy is described by a single latent variable.

An extensive body of research seems, however, to contradict this. We expect various groups of adults for social or educational reasons to have different skills' profiles. It could be

- National variation due to different priorities in education,
- Gender-related differences,
- Age-related differences,
- Different adult mathematics knowledge produced by different adult's forms of life.

On the surface this contradicts "the fundamental assumption in IRM", that proficiency is related to item difficulty in a way that is invariant across language, culture and subculture. But this invariance is not a general statement on "numeracy". It only points to constraints on a test battery where results can be modelled by IRM. In practice the one-dimensionality of the test battery is brought about by extensive pilot-testing where "misbehaving" items are deleted. In this context "misbehaving items" are not poor problems, they are just items that function differentially, i.e. are easy for girls or for boys, are easy for students that otherwise do badly, are especially difficult or easy for persons from certain countries or social groups and so on. Such trimmed or thinned out test batteries can be modelled by IRM. Since the items are secret it is, however, not possible to ascertain which notion of "numeracy" is being tested by the "trimmed" test battery. For a more detailed discussion see Blum et al, (2001, 2006) and Sticht (2004). Figure 5 illustrates the piloting procedure.

IRM can be a powerful tool in construction of scales and exploration of dimensions in proficiency. This demands, however, that the assumption of one-dimensionality is abandoned and with this the ambition of creating league tables as final results. IALS and ALL, however, make different claims. The scores are being presented as assessments of skills "required in today's labour market and everyday life"? This raises the following questions

- Are the "necessary" skills the same for all persons regardless of age, gender, occupation, and country?
- Can the "necessary" skills be represented by one latent dimension?
- Is it possible to use the same instrument to measure the "necessary" skills for all persons?

And last but not least:

- Is there any empirical validation of the claim that the skills measured by for instance IALS and ALL are indeed the skills "required in today's labour market and everyday life"?

Self-assessed proficiency

In the previous section we questioned the tenability of the claim that IALS and ALL really measure the skills necessary to live and work in today's world. In the literacy and numeracy surveys the adults, however, consistently state that they perform better in the workplace and in everyday life than their test results are made out to show.

In the Norwegian report on ALL (2003), Gabrielsen et al. (2005, p.20) and co-authors claim

> A total of one third of the Norwegian adult population's literacy skills are defined by international experts as inadequate when compared with what is required in today's labour market and everyday life.

However, when asked if they have the proficiency in numeracy that is necessary to meet the demands of their main job more than 95% of the adults in the surveys agree with the statement. This is illustrated in table 2 based on tables from Gabrielsen et al. (2005).

Table 2: Evaluation of own proficiency in numeracy compared to demands of main job

Proficiency in numeracy necessary to meet the demands of main job	Agree	Disagree	Total
Adults on ALL level 1+2	94,4% 1828	5,6% 108	1936
Adults on ALL level 3+4+5	98,0% 3486	2,0% 72	3558
Total	5314	180	5494

Source: Gabrielsen et al. (2005) table 6.8 (authors translation)

Table 2 shows that most Norwegian adults in the labour market feel that they have the skills that they need to perform their job in contrast to the results of ALL. Similar results are found in the Danish reports on IALS (Jensen et al. (2000)). From a different perspective Payne (2006) performs a critical investigation of the corresponding English numbers and concludes that illiteracy in England is grossly overstated. This raises a number of questions.

- Are people mistaken when they say that their proficiency in numeracy is good enough to meet the demands of their main job?
- The discrepancy between the self-assessed proficiency and the conclusions is not treated seriously in the reports.

- Is it ethically defensible to disregard the opinions and statements of the adults regarding their own skills and ”narrate” big groups of adults in the labour market as excluded from society and lacking in basic skills?

Internationally for the last 10-15 years major efforts have been devoted to the task of creating courses in numeracy and literacy that match the demands that adults with little and no education meet on the job and in every day life. At the same time there is a growing body of research into “numeracy in context”, the numeracy skills needed in specific job functions (Wedege, 2002) and these demonstrate why it is difficult to speak in general terms about the skills “required in today’s labour market and everyday life”. Moreover, tests do not automatically examine what they intend to investigate. In IALS and in ALL this is exemplified by difference between skills assessed by the test score and the test persons’ own assessments. I find it disturbing that the reports send the message that the experiences and assessments of the test persons themselves have no validity compared to the test results. Is it a viable for the adult education community to let surveys convey the impression that “adults just don’t know how stupid they are”?

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Diagnosing the Problem: an investigation into pre-registration nursing students' mathematical ability

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Numeracy is a key skill that is deemed to be fundamental within the clinical environment in order to safely administer medicines and fluids to patients (Elliott & Joyce 2005, Wright 2004, Wilson 2003, Hutton 1998). The United Kingdom (UK) Nursing and Midwifery Council (NMC) clearly states that nurses are required to "accurately interpret numerical data and their significance for the safe delivery of care" (NMC 2004). Likewise the Quality Assurance Agency for Higher Education (QAA) who sets standards for students seeking a nursing award states that students must be able to understand and carry out drug calculations.

This paper reports on the results obtained from a quantitative study carried out on pre-nursing students. The study sought to identify pre-registration nursing students' mathematical ability in order to develop an effective on-line teaching tool for numeracy in general and drug calculations in particular and to identify where possible correlations between the students' demographic data and their results in an on-line diagnostic quiz.

Errors in drug calculations are a problem on a national and international scale. The actual scale of the problem is difficult to quantify with some contradictory evidence appearing in the literature. One study by Hoyles et al (2001) has identified that nurses are quite confident in carrying out drug calculations using proportional reasoning. It could be argued that problems occur when proportional reasoning methods fail i.e. when the numbers do not work out easily. Nurses themselves are worried about the issue with a plethora of guides and books available for nursing students to help them with drug calculations (Coben, 2005). An Australian study suggested that student nurses' perceptions of their mathematical ability are not always borne out in results with some students underestimating and others overestimating their ability to do drug calculations (Weller, 2000). In other studies nursing students have been identified as students who have problems with mathematics and experience a great deal of ‘maths anxiety’ (Sabin, 2002; Rice and Bell, 2005). Entry to nursing courses requires a GCSE or equivalent qualification but research suggests that there is no absolute link

between level of qualification and a student's basic numeracy level (Elliott & Joyce, 2005; Batchelor, 2004; Tariq, 2004).

This is not a problem exclusive to nursing students, students applying to other disciplines including those containing mathematics elements are currently given tests in order for their mathematical ability to be assessed (Batchelor, 2004; Tariq, 2004).

What is our research about?

There was an increasing realization that there was a problem in teaching the drug calculation element of the course to pre-registration student nurses. The problem was that students did not appear to have the basic numeracy skills which should have been acquired prior to nursing course. A team of interested academics was drawn together - three members from the School of Health Sciences undergraduate nursing team including two nursing tutors who taught drug calculations to nursing students, a mathematics lecturer from the Department of Adult Education and two members of the Health Science e-learning team.

It was decided that the only way feasible to overcome the basic numeracy problems was to provide students with an on-line teaching tool to cover all the basic material students would need to perform drug calculations. It was felt that a baseline study of the pre-registration nursing students' ability was required and the best way to gather this information in addition to basic demographic data would be to give the students an on-line diagnostic quiz. It is the development of the on-line diagnostic quiz and the results from this quiz that this paper reports on.

The Diagnostic Quiz

The test consisted of 25 questions of GCSE level covering all topics needed in nursing concentrating particularly on those numeracy skills needed for drug calculations - fractions, SI units, decimals, percentages, formulae, ratio, rounding, area & perimeter, basic stats, graphs and charts. It was decided to make the questions strictly mathematical and not in a nursing context as the test was to be given to students at the start of the course. The test which was on-line and would take 30 minutes to complete was to be administered in small tutorial groups where individual access to PCs was possible. Students remained anonymous although demographic data was collected. The test was initially optional and students also had the option of doing the test but opting out of the research. At the end of the test students were told which questions they had wrong and advised to take their results with them when they went to see their personal tutor. The data was collected in a form which was then imported into SPSS for analysis.

Example questions

10. **What is the smallest number in the list?**
☐ *0.031*
☐ *3/10*
☐ *3/100*
☐ *35/100*
☐ *Don't know*

15. **0.05 of 100 ml is:**
☐ *0.5ml*
☐ *51ml*
☐ *5ml*
☐ *0.005ml*
☐ *Don't know*

18. **A journey of 60 miles took from 11.45a.m. until 1.15p.m. Using the formula *S = D/T* the average speed of the journey can be shown to be:**
☐ *40 m.p.h.*
☐ *50 m.p.h.*
☐ *60 m.p.h.*
☐ *30 m.p.h.*

Results from the First Cohort (September 2005)

The diagnostic test was originally administered to the September cohort of pre-registration nursing students and at that time was optional. Only 45% of the cohort did the test (82 students). The distribution of results was slightly skewed as can be seen from the graph in figure 1.

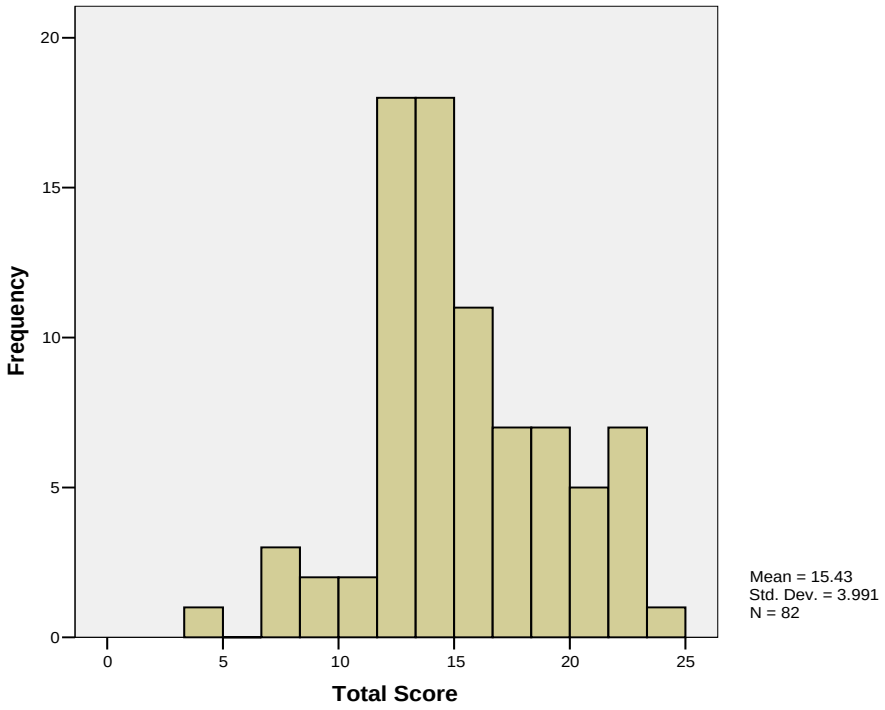

Figure 1. Distribution of results for September cohort

Since the sample could not be treated as random the data was not suitable for further analysis. It was therefore decided to ask permission of the School Ethics committee to make the test compulsory for next cohort- March 2006.

Demographic Results for the March Cohort

(a) **Age**

The modal age group was 18-25 with another smaller peak at the 31-35 age group. The age range was quite large with the majority of students being over 31.

(b) **Gender**

90% of the cohort was female and 10% was male.

(c) **Previous mathematical qualifications**

The modal group for previous qualifications is the Access to Health group of students (31%), with the next largest group having GCSE A* - C grade (28%) as their highest previous mathematical qualification.

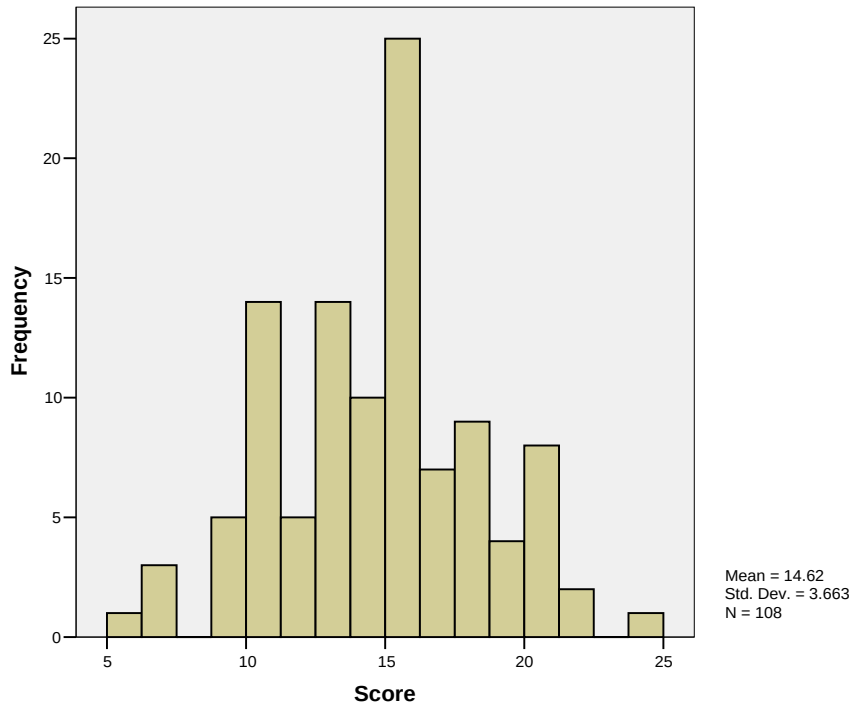

Figure 2. Scores obtained by the March Cohort

The distribution of the scores students obtained in the diagnostic quiz was quite symmetrical with students obtaining a mean score of 14.62 and a standard deviation of 3.663. The modal score is 15 out of 25, which equates to 60%.

If 18 out of 25 (72%) is taken as the pass mark for the diagnostic quiz only 24 students i.e. 22% pass the test. In other nursing diagnostic quiz tests 80% has been used as the mark to be obtained to indicate proficiency in the subject area.

Table1. Results for individual questions

	Qu	Topic	% correct		Qu	Topic	% correct
	1	Money	95%	*	14	%	80%
*	2	Units	93%	*	15	Dec/Unit	33%
*	3	Units	76%	*	16	Rounding	83%
*	4	Units	31%	*	17	Rounding	73%
*	5	Decimals	85%	*	18	Formulae	40%
*	6	Decimals	33%	*	19	Formulae	32%
*	7	Decimals	28%	*	20	Formulae	20%
*	8	Fractions	61%		21	Area &Perim	65%
*	9	Frac & %	66%		22	Stats	74%
*	10	Frac & Dec	30%		23	Timetable	95%
*	11	Fractions	65%		24	Charts	84%
*	12	Ratio	83%		25	Piechart	7%
*	13	Dec & %	17%				

The students as a group did not perform well in many basic numeracy areas such as decimals (questions 6 and 7), units (question 4), formulae (questions 18, 19 and 20),

and to a lesser extent fractions (questions 8, 9 and 11). This is even more apparent when questions contain a mixture of topics such as questions 10, 13 and 15.

The first and sixth columns in the table indicate whether the questions are key subject areas which will need to be understood in order for students to be able to perform drug calculations. For some particular key questions very small numbers of students are obtaining the correct answers e.g. only 17% obtained the correct answer for question 13.

It is also worth noting that the ratio question was answered well with 83% of students obtaining the correct answer. This would seem to corroborate the findings of Hoyles et al (2001) where they found the preferred methods for nurses was to use proportional reasoning methods.

Re-classification of data

In order to carry out further statistical analysis of the subgroups the demographic data was further grouped as follows:

Age was re-classified into two groups 'Young' and 'Mature' where 'Young' = under 25 and 'Mature' = 26 and over. Previous qualifications was re- classified into four groups - Access, GCSE, 'Other' and A level where Access = students who have studied an Access to Health Studies course, GCSE = students with GCSE or 'O' level grades A* - C, 'Other' = students with GCSE or 'O' level grades D - F, Certificate of Education, or CSE and A level = students who had studies mathematics at A-level. These students were separated out of the data and analysed individually because of the small numbers involved in this category. Finally, scores were re-classified into two groups - Pass or Fail where Pass = a score greater than or equal to 18/25 (72%) and Fail = a score less than 18/25 (72%).

Cross tabulations of the demographic data with Pass/Fail

No relationship was found between the gender of the student and whether they passed or failed the diagnostic quiz. Since the number of male students was small this result has to be viewed with caution. No relationship was also found between the age of the student and whether they passed or failed the test. However, a relationship was found between previous qualifications and whether a student passed or failed the test with less access students than expected and less students in the 'other' category than expected passing, and more students in the GCSE category than expected passing.

Cross tabulations of the demographic data with individual questions

A relationship was found between previous qualifications and whether students achieved a correct answer to questions 3 (units), 7 (Decimals), 12 (ratio), 13 (percentage and decimal), 17 (rounding), 19 (formulae), 21 (area and perimeter) with less Access students than expected obtaining the correct answer.

There was a relationship between age and whether students answered question 9 (percentage and fraction) and 19 (formulae) correctly with less young people than

expected obtaining the correct answer for question 9 and more young people than expected obtaining the correct answer for question 19.

Two possible explanations may exist for the result for question 9. Either this topic area was covered more rigorously in 'older' style of syllabi or perhaps more mature students have more experience of this type of 'every day maths' calculation.

The explanation for the differing performance may be that because it is a strictly 'school maths' topic it will have been covered more recently for younger students. It must be noted, however, that these explanations are just conjectures.

Further analysis of the means obtained by various subgroups

Tables 2, 3 and 4 give the mean scores obtained by various demographic subgroups.

Table 2. Gender analysis

Gender	Mean Score	Standard deviation of score
Male	15.9	4.07
Female	14.3	3.45

Table 3. Young and mature groups

Age	Mean Score	Standard deviation of score
Young	14.4	3.91
Mature	14.5	3.60

Table 4. Previous qualifications

Previous qualifications	Mean Score	Standard deviation of score
Access	13.1	3.19
GCSE/'O' level grades A* - C	16.6	3.15
Other	13.4	3.20
'A' Level	17.4	2.42

T-tests were performed on the mean scores obtained by students in the various subgroups in the tables above and the following results were found:

- There was no significant difference between the scores obtained by male and female students,

- There was no significant difference between the scores obtained by young and mature students.

There was a significant difference between the mean scores obtained by:

- Access students and GCSE/ 'O' level grades A* -C students;
- Access students and 'A' level students;
- 'Other' students and GCSE/'O' level grades A* -C students and
- 'Other' students and 'A' level students.

(Access and 'Other' students obtaining a lower mean score in each case)

There was no significant difference between the mean scores obtained by:

- 'A' level and GCSE students,
- Access students and 'Other' students.

Conclusions

Key Challenges

Overall the pre-registration nursing students' performance in the diagnostic quiz indicates key areas where the students in general do not have the basic mathematical skills which are required to enable them to master drug calculations proficiently. The findings have verified nursing tutors' opinions that there are weaknesses in the basic numeracy skills amongst the pre-registration nursing students which need to be addressed. Students are having particular problems in questions involving decimals, SI units, formulae, and to a lesser extent, fractions. These deficiencies are particularly marked when these topics are combined as they are when performing drug calculations.

Age and Gender

In general there is no difference in the performance of students because of gender or age although for two isolated questions (question 9 and question 19) age appears to be a factor with more mature students faring better in question 9 and younger students performing better for question 19. An explanation may be that the Mathematics in question 9 could have been acquired through life experiences by adults or perhaps the topic area was more prominent in the older mathematics syllabi studied by more mature students. On the other hand the Mathematics in question 19

is strictly 'school maths' in nature and hence will have been covered more recently by younger students.

A-level students

Perhaps surprisingly the 'A' level students in this survey do not appear to be performing as well as expected. The 'A' level students in the survey do not appear to be performing significantly better than students with GCSE/'O' level grades A* - C. One school of thought is that ' A' level maths should be the minimum entry requirement for student nurses in order that students will have the required numeracy skills to perform drug calculations. It has been shown in this survey that the basic numeracy techniques required to do drug calculations may not necessarily have been acquired by 'A' level students. This is borne out by other surveys which have found that students applying to numerate disciplines do not have the basic numeracy (and algebra) skills for their degree choice.

Access Students

Perhaps the most important finding from this research is that the students who have come through the Access route are faring worse against 'A' level students and GCSE grades A*-C students and do not perform better than those students in the 'other' category i.e. those students who have obtained a lower grade GCSE /'O' level or have a Certificate of Education or CSE. They also have particular weaknesses in certain key areas and are performing worse than other students in these areas. This is a worrying finding as the largest percentage of students (31%) in UWS, come through this route.

One explanation for this worse performance by students coming through the Access route could be that the Mathematics taught on Access courses is taught in a nursing context whereas the diagnostic quiz is strictly numeracy/mathematics based. Hopefully further investigation into the syllabi of the Mathematics taught on the Access courses from which UWS pre-registration nursing students come will shed light on this very important finding.

On-going research

This research was carried out on the March 2006 cohort of pre-registration nursing students. The next stage of the research involves testing the September 2006 cohort of pre-registration nursing students before and after they have access to the on-line Mathematics teaching tool. This tool has been designed with particular reference to those areas in which the pre-registration nursing students are having the most difficulty. It is hoped that the research will show that after having had access to the on-line Mathematics teaching tool there will be a positive improvement in students' performance in these key numeracy mathematical areas in order that they will be able to go on to perform drug calculations with confidence and proficiency.

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On mathematics attitudes, self-efficacy beliefs, and math-anxiety in commencing undergraduate students

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The attitudes, self-efficacy beliefs, and math-anxiety of commencing undergraduate students are examined with the aim of identifying differences between Science students and Arts/Humanities students in their perceptions of mathematics, including their own capabilities. The IMAES instrument developed by the author was administered to students during an academic transition programme conducted in Week 0 of the academic year. Data analysis revealed that participants who were beginning arts and humanities programmes tended to share the pervasive negative attitudes, low mathematics self-efficacy beliefs, and anxiety of mathematics attributed to the broader population by many previous studies, whereas their counter-parts enrolled in science-based programmes had attitudes and beliefs that were significantly more positive. The differences were found to be strongly linked to gender and, consistent with earlier reports by this researcher, also to early mathematics learning experiences. The implications of these findings are discussed in the contexts of providing mathematics support to students beginning tertiary studies and of fostering improvements in quantitative literacy in society.

Almost universally, pupils, parents, and teachers appear to regard mathematics as both important and useful (Coben, 2003:93) and yet it is well known to practitioners and researchers in the field of adult mathematics learning that, despite this regard, many adults (perhaps even in the majority) exhibit at least some degree of anxiety when confronted with overtly mathematical tasks, that they have only a vague concept of what mathematics is really all about, lack confidence in their own mathematical abilities, and often fail to appreciate the extent to which they actually and routinely engage in essentially mathematical thinking as they go about their daily activities. Twenty-five years ago, Sewell (1981) reported that there were negative math attitudes in at least half of the adult population and it appears that the situation is no better now – in a recently published report, the Confederation of British Industry found that nearly half of the adults (equivalent to some 15 million adults in the 16 to 65 age group in England) participating in a study of functional skills had numeracy attainment levels that were at or below the standard expected of 11 year-olds finishing primary school (CBI, 2006). Many otherwise responsible adults in our communities are essentially disenfranchised by their mathematical naiveté, lacking sufficient quantitative literacy to confidently and critically evaluate the full range of information they encounter and on which they need to rely to make informed and well-considered decisions, whether this be in the home, in the workplace, or in the wider community as consumers, parents, or voters.

The seeming paradoxical nature of these contrary positions is somewhat resolved by recognizing that mathematics is well regarded in an abstract, conceptual sense – when

it is, perhaps, other than personal. The negative aspects that often attach to accounts of experiences with mathematics invariably arise when the subject is placed on a personal level, and these aspects encompass not only math attitudes, but also math anxiety (math phobia in extreme cases), and low math self-efficacy beliefs. Since the 1960s, research has been conducted into attitudes towards mathematics but this has tended to be in a very general context. Literature that relates specifically to the math attitudes of adults is less prolific by far and "the knowledge-base is as yet insecure" (Coben, 2003:54). Nevertheless, several observations are now well established, including those of Singh (1993):

- Students commonly identify mathematical abstraction and lack of relevance as causative factors for their dislike of and failure in mathematics,
- Fear of failure induced by the nature of some mathematics teaching and assessment practices as a cause of anxiety in adults,
- The significant influence of teachers to motivate or estrange math students,
- Negative attitudes towards mathematics may be more pervasive in the female population.

Reinforcing the last point, the IEA's Third International Mathematics and Science Study (TIMSS) found that gender differences favoured boys rather than girls and other studies have found that children's attitudes towards mathematics decline between primary and secondary education (McLeod, 1994). Traumatic early math learning experiences, particularly those arising from primary school experiences and the attitudes of parents, are capable of exerting a long-term effect (Relich, 1996). Pupils' conceptions suffer from the destructive effects of "unimaginative instruction and non-positive teacher attitudes", a phenomenon with recursive features since it is known that many candidate teachers enter university with negative math feelings, often taking these attitudes into their teaching careers, influencing students and yet another generation of prospective teachers to perpetuate the same traits (Philippou and Christou, 1998).

Extensive literature demonstrates that most modern cultures exhibit signs of anxiety, stress, lack of confidence, and phobic reactions in the face of mathematical problems (Macrae, 2003). Math anxiety is commonly characterized by feelings of tension, apprehension, or fear that impacts on math performance (Ashcraft, 2002). It is associated with loss of self-esteem, negative reactions to mathematical concepts and evaluation procedures, and with many constructs including working memory, age, gender, self-efficacy, and mathematics attitudes (Cates and Rhymer, 2003). Indicated as a factor in attrition rates, regardless of actual ability (Jones, 1996), math anxiety is also more closely associated with females than with males (Macrae, 2003; Ashcraft, 2002).

When students with math anxiety encounter intractable math content in their learning environment, they tend to have weak or negative mathematics self-efficacy beliefs. Bandura (1986) defined self-efficacy beliefs as "people's judgements of their capabilities to organize and execute courses of action required to attain designated types of performances" (Bandura 1986:391). Self-efficacy beliefs are a better predictor of success than an inventory of skills or prior achievements, and relationships have been found between self-efficacy for solving math problems and math anxiety, math attitudes, general mental ability, math self-concept, and math

experience (Finney and Schraw, 2003). For the mathematically challenged and math anxious, there is often a history of a complex interplay of influences that cause many students to seek to avoid math instruction and practice, leaving perceptions that mathematics is uninspiring, uninteresting, and irrelevant (Klinger, 2004). Philippou and Christou (1998) cite numerous researchers typically reporting that male students express higher mathematics self-efficacy than do female students and it seems that the divergence begins during middle school and increases with age. At all levels of schooling, gender differences favour boys, even when girls' mathematics achievement indexes match or exceed those of boys. While there are few studies that report on the mathematics self-efficacy of adults in a general context, Lussier (1996) investigated gender and mathematics background and found that men report lower anxiety scores and higher self-efficacy scores than women.

If one accepts that it is desirable to seek to improve the status of mathematics on a personal level, that is, to pursue significant and long-term improvements in quantitative literacy, then this endeavour must, ultimately, be community driven. At that level, it cannot be expected to occur without the guidance of those who are best informed and most influential in society, which surely must include its most highly educated members. But what is known of their views of mathematics? One approach is to examine the quantitative literacy of undergraduate students. An earlier study (Klinger, 2006) considered pre-tertiary adult learners participating in an alternative, non-traditional, entry program for admission to higher education; the present work continues by focussing on commencing undergraduate students. Because the learning of mathematics is affected by the confidence of learners in their mathematical abilities and the attitudes, beliefs, and feelings they harbour towards mathematics (Coben, 2003), the study examines students' conceptions of the subject and their perceptions of themselves and of their relationship to mathematics, being attributes that lie at the heart of math learning behaviour (Philippou and Christou, 1998). In particular, a profile of math attitudes, math anxiety, and mathematics self-efficacy beliefs is established and examined to determine (a) whether it is generally consistent with the wider population; and (b) whether the profile for science students differs significantly from that of arts/humanities students and, if so, what are the key differences. The latter aspect stems from a desire to examine the validity of the widely held stereotype that the Sciences favour the mathematically-inclined, while the Arts and Humanities favour other academic strengths.

Methodology

The *Inventory of maths attitude, experience, and self-awareness* (IMAES) instrument (Klinger, 2006) was administered to 167 volunteers who were participants in a 4-day academic transition programme for commencing undergraduate students. The programme is conducted annually in Week 0 of the academic year and comprises two distinct strands, one for students about to begin science-based courses (including those in engineering and the health sciences), and the other for those about to begin arts/humanities courses (encompassing social sciences, education, and law).

The IMAES instrument is a multi-part questionnaire that uses (mostly) 5-point Likert scales for responses to 95 statements about math attitude, math anxiety, math self-efficacy beliefs, and past/early math learning experiences, with the attitudinal statements involving the affective, behavioural, and cognitive domains. The questionnaire also provides for the collection of information about the respondent's

gender, age-group, ethnicity, last year of completed schooling, and last year of completed math education, as well as the area of study identified by the name of the degree in which the respondent is enrolled. Students were advised that participation was entirely voluntary and independent from any topic, course, or assessment activity. Incomplete questionnaires were set aside and not used in the study.

Factor analysis confirmed that the survey items were validly partitioned into their respective domains and these were then further segregated into sets of positive and negative statements (since these are not necessarily mutually exclusive). The reliability of each subset construct was tested by computing the internal consistency coefficients (Cronbach's alphas) for each and these ranged from 0.75 to 0.94 (mostly in the top of the range) – more than satisfactory since they exceed the 0.70 level desired of alpha coefficients for the purposes of a psychometric instrument (Nunally, 1978).

To obtain aggregate scores for each primary construct except 'Experiences', each item was first means-weighted according to its relative severity, with each weight computed as the quotient of the subset mean and the item mean. Aggregate measures were then obtained by summing the weighted scores and re-scaling the results to range from 0–10. A further aggregation joined the negative and positive subsets on a single scale, such that an overall score of zero indicates neutrality. The final data were then analysed using standard statistical procedures.

Results

Correlations: all respondents

First, all respondents were considered in aggregate – that is, without separating them into their respective academic streams. Spearman rank correlations were calculated to identify cross-correlations in the demographic factors of gender, age, schooling, and math education, and likewise with both the negative and positive scales for attitude, anxiety and self-efficacy.

Negative factors

Significant ($p < .05$) correlations were found between gender and math education, anxiety, self-efficacy, and early learning experiences; with a *p*-value just over 5%, a relationship between gender and math attitude was also considered to be established. While these correlations are not large (typically from 0.2–0.3), they are all in a direction that supports the well-established findings that females fare worse than males. Significant negative relationships were observed between age and level of schooling completed as well as age and math education, indicative of the stereotypical non-traditional background frequently associated with mature student profiles. Greater math education was associated with more schooling and those with more schooling reported significantly fewer negative early math learning experiences. Negative perceptions of things mathematical showed a decline as the level of math education rose, consistent with the observation that those continuing with high school math into their senior years have usually chosen to do so.

Cross-correlations between negative attitude, anxiety and self-efficacy were strong (0.8–0.9) and overwhelmingly significant. Not so strong (0.4–0.5), but still very important, was the relationship between these factors and negative early math

learning experiences – in fact, the early learning issues had a stronger relationship with attitude, anxiety, and self-efficacy beliefs than any of the other factors, with the single exception that math education had a marginally stronger influence on self-efficacy.

Positive factors

The patterns of significant correlations for the positive scales revealed similar relationships but with some differences, too. Gender discrimination with respect to positive attitude and self-efficacy was again apparent, but not so with respect to anxiety and positive early math learning experiences. Age-group showed a modest negative relationship (-0.2) with good early learning experiences (in contrast, there was no corresponding significant link in the negative factors), and the level of schooling appeared not to be a related factor in any of the positive scales, whereas it figured significantly in negative learning experiences

Cross-correlations between attitude, anxiety, and self-efficacy in the positive scales were very similar to those in the negative scales; however, the relationships with positive learning experiences appeared only half as strong (approximately) as those seen in the negative scales.

Early learning factors

An examination of correlations between early learning factors and attitude, anxiety, and self-efficacy in both the positive and negative scales (see Table 1 below) showed that there was a tendency of mutual exclusion for negative and positive early math learning experiences. The data also revealed a stronger relationship between negative experiences and other negative factors than was the case for positive early math learning experiences and the remaining positive factors. Moreover, *negative* early learning experiences were more strongly associated with the *positive* scales than were the positive early learning experiences with the same scales.

Table 1 Correlations between early learning factors and attitude, anxiety, and self-efficacy

Spearman Rank Correlations, N=158									
		Early learning - positive	Early learning - negative	Attitude - negative	Anxiety - negative	Self-efficacy - negative	Attitude -positive	Anxiety - positive	Self-efficacy - positive
Early learning +ve	Corr	1.000	-.369**	-.175*	-.217**	-.190**	.206**	.273**	.258**
	Sig. (1-tail)	.	.000	.014	.003	.008	.005	.000	.001
Early learning -ve	Corr	-.369**	1.000	.448**	.469**	.401**	-.221**	-.332**	-.323**
	Sig. (1-tail)	.000	.	.000	.000	.000	.003	.000	.000

**. Correlation is significant at the 0.01 level (1-tailed).

*. Correlation is significant at the 0.05 level (1-tailed).

Comparative attitude, anxiety, and self-efficacy scales

Here, the data were split into the two academic streams of Sciences and Arts/Humanities so as to consider differences in the means of aggregated scores in each primary construct. Figure 1 illustrates the results, showing the means of aggregated scores for each construct by academic strand, with Sciences on the left in each chart; error bars indicate ± two standard errors. The differences between the two strands, while not vast, are clear and substantial in each instance, indicating that positive attitudes prevail among students in the Sciences strand, while students in the Arts/Humanities strand have attitudes that are at best ambivalent and, more likely,

negative; the results are similarly so for the math anxiety and self-efficacy scales. Incidence rates were counted for positive and negative scores in each construct for each strand, giving percentage ratios (positive:negative) of 64:36, 60:40, and 67:33 respectively for attitude, anxiety, and self-efficacy in the Sciences and the corresponding ratios in the Arts/Humanities were 46:54, 46:54, and 49:51, consistent with the observed trends in differences in the means.

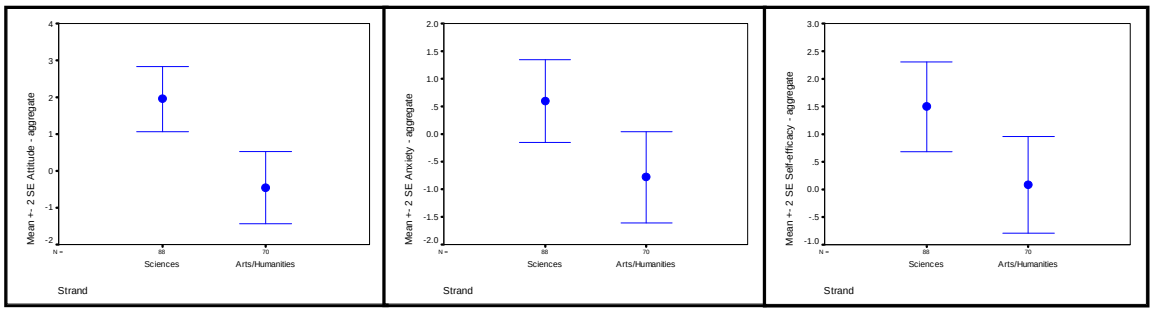

Figure 1 Comparative math attitude, anxiety, and self-efficacy scales (left to right)

One-tailed *t*-tests for significance in mean differences were conducted to confirm the significance of the differences illustrated by the charts, resulting in extremely low *p*-values ($p << .01$) that provided overwhelming evidence to infer that the observed differences were real effects with some systematic cause.

Further *t*-tests were applied to the semi-aggregated positive and negative scales for each attribute, with results consistent with the last, and yet again at the level of individual statements in the IMAES instrument to identify the greatest and most significant differences between the two student groups. In the affective attitude domain, these distinguished between attitudes of engagement and attitudes of aversion – that is, interest, confidence and appeal *vs* distaste, discomfort, and fear. In the behavioural domain, the results contrasted attributes such as patience, perseverance, care, and willingness to help others, with hesitant behaviour and lack of concentration. Key attributes showing significant differences in the cognitive attitude domain largely involved items involving an appreciation of the utility and value of mathematics.

The results for statements associated with the math anxiety construct revealed that the customary emotional responses to stereotypical math class situations were most evident: of sixteen significant differences, fourteen were from items on the negative scale and these were of the form that often precedes accounts of bad early learning situations – for example, “I worry about being called on in maths class” ($p = .003$); “I used to cringe when I had to go to a maths class” ($p = .009$); “I fear maths more than any other subject” ($p = .02$).

Significant differences between the groups for the math self-efficacy construct translated mostly into significant differences (*p*-values between .001 and .012) in *beliefs* about mathematics ability, illustrating the divide between those who believe they can (e.g. “I was one of the best students in maths at school”; “I can usually solve any mathematical problem”) and those who believe they cannot (e.g. “Mathematics is not one of my strengths”; “I believe that I have a lot of weaknesses in maths”).

Early mathematics learning experiences

In the Arts/Humanities strand, the incidence rate for negative early mathematics learning experiences was 41%, one and a half times that of the Sciences, where the rate was 28%. For positive experiences, in the Sciences the incidence rate, at 86%, was a fifth higher than that of the Arts/Humanities, at 73%. That is, negative early math learning experiences were considerably more prevalent among arts/humanities students, while the science students had typically enjoyed a greater number of positive encounters with mathematics in their early learning environments. Both groups reported considerably more good experiences than bad – although, again, the difference very much favoured the science students, being a little more than three-fold compared to a two-fold difference for their counterparts. The significance of the observed differences was verified using the non-parametric Mann-Whitney Test for significant differences (this was chosen because of the nature of the distribution of these data). The test results provided $p = .054$ for the negative experiences, a relatively strong outcome for this inherently weaker test, and $p = .08$ for the positive experiences, a lesser result but still significant at the 90% confidence level.

Gender-based differences

From the foregoing, it was apparent that differences between the strands indeed existed and a clear relationship was established between negative and positive early math learning experiences and the various facets of perceptions students have about mathematics. Moreover, that relationship appeared to be linked to the differences in those perceptions that characterize the two main academic strands. In view of these findings and because gender issues figure prominently in the literature on math anxiety, further analysis was undertaken to examine any gender influences that could be discerned from the survey data. Considering, first, the gender mix between all respondents, it was found that females outnumbered males almost 2:1 (actually, about 64% female). In the Sciences – once considered a male-dominated field – the ratio was about 5:4 in favour of female students, whereas in the Arts/Humanities strand the ratio was 5:2. These gender ratios are essentially identical to those found in university statistics for all commencement enrolments in 2005.

Incidence rates were counted for positive and negative scores in each construct for each gender, giving percentage ratios (positive:negative) of 68:32, 65:35, and 68:32 respectively for attitude, anxiety, and self-efficacy in males and the corresponding ratios for females were 49:51, 55:45, and 57:43. Because these ratios were very similar to those observed with respect to the academic strands, a set of one-sample *t*-tests was applied to each subset of data grouped by gender within academic strand, using the aggregated scores in each construct. The tests examined how each sub-group mean in each scale differed from zero and the results are shown in Table 2 below. For males in the Sciences, the mean differences are all positive, with suitably small *p*-values and the 90% confidence intervals of the difference all to the right of zero, confirming the generally positive regard for mathematics traditionally expected of male science students.

Table 2 *t*-tests for differences from zero in aggregate scales by strand and gender

One-Sample Test: by strand and gender

Strand	Gender		Test Value = 0 t	df	Sig. (1-tailed)	Mean Difference	90% Confidence Interval of the Difference Lower	Upper
Sciences	male	Attitude - aggregate	3.541	36	.000	2.051	1.073	3.030
		Anxiety - aggregate	1.777	36	.042	.835	.042	1.629
		Self-efficacy - aggregate	3.757	36	.000	2.062	1.135	2.989
	female	Attitude - aggregate	2.877	50	.003	1.875	.782	2.967
		Anxiety - aggregate	.765	50	.224	.420	-.500	1.339
		Self-efficacy - aggregate	1.880	50	.033	1.084	.118	2.051
Arts/Humanities	male	Attitude - aggregate	1.859	19	.039	1.330	.093	2.567
		Anxiety - aggregate	.916	19	.185	.580	-.514	1.674
		Self-efficacy - aggregate	2.332	19	.015	1.580	.408	2.752
	female	Attitude - aggregate	-1.960	49	.028	-1.168	-2.167	-.169
		Anxiety - aggregate	-2.618	49	.006	-1.330	-2.182	-.478
		Self-efficacy - aggregate	-.969	49	.169	-.514	-1.404	.376

By contrast, whilst males in the Arts/Humanities strand showed positive and significant mean differences for attitude and self-efficacy (though to a lesser extent than their peers in the Sciences), their anxiety score was not significantly non-zero, suggesting that these students, as a group, may be less comfortable when confronted with quantitative material and tasks. The corresponding results for female students in the Sciences reveals a very similar picture – attitudes and self-efficacy beliefs were significantly positive (again, to a lesser extent than males in this strand) but the mean anxiety level was not, rating very close to the mean difference for males in the Arts. The last subgroup shown in Table 2 – females in the Arts/Humanities strand – is revealing: all mean differences are below zero, with high significance for the attitude and anxiety scales. Whilst the self-efficacy result was not significantly non-zero, the 90% confidence interval demonstrates that self-efficacy beliefs were weak at best.

Recalling the small but very significant correlations between gender and each aggregated attribute for all students, the relationship was again examined by both strand and gender. As Table 3 shows, no significant correlations were found in the Sciences; rather, the relationships were located entirely in the Arts/Humanities strand.

Table 3 Correlations between gender and aggregate scores, by strand

Correlations

Strand			Attitude - aggregate	Anxiety - aggregate	Self-efficacy - aggregate
Sciences	Gender	Pearson Correlation	-.021	-.059	-.127
		Sig. (1-tailed)	.423	.293	.119
		N	88	88	88
Arts/Humanities	Gender	Pearson Correlation	-.278**	-.250*	-.260*
		Sig. (1-tailed)	.010	.019	.015
		N	70	70	70

**. Correlation is significant at the 0.01 level (1-tailed).

*. Correlation is significant at the 0.05 level (1-tailed).

Discussion and conclusion

The IMAES scores and cross-correlations reported here for all respondents establish a profile of math attitudes, math anxiety, and math self-efficacy beliefs that displays characteristics that are in good general agreement with literature on these attributes in the broader population, particularly with regard to the effect of prior mathematics learning experiences and the evident gender influences. While there are no directly equivalent data yet available, it appears a reasonable speculation, however, that the scores obtained in this study are less profound in the negative scales and more favourable in the positive scales than might be expected for the greater population, given the often reported prevalence of negative attitudes and low math ability. That is, as a group, these commencing undergraduate students may be more comfortable with things mathematical than expected of the social norm.

A clear and definite result is the evidence that significant differences exist between the two academic strands of Sciences and Arts/Humanities, apparently lending support to the common notion that the former favours the mathematically-inclined. When subjected to closer scrutiny, it became evident that a major contributing factor to the observed differences was the substantially poorer views of mathematics held by female students in arts and humanities degree programmes. Arguably, the expectation from a more traditional, stereotypical viewpoint might have been that any differences between the strands should be explained by a bias of stronger views in what was once male-dominated science. While there were indications that some such bias exists, it is insufficient to account for the observations since the mean scores for males in the sciences were not sufficiently high in the positive scales nor sufficiently low in the negative.

Students identified in this study as having predominantly negative perceptions of mathematics comprise about one third of all respondents and, of course, those who are merely ambivalent add appreciably to that proportion. This has implications for the provision of mathematics and numeracy support to students beginning tertiary studies since it may be inferred that transitioning students in need of support are most likely to be female students undertaking degrees in the Arts and Humanities and, in these, it is apparent from this study (and elsewhere in the literature) that it is more likely than not that at least some level of math anxiety will be a factor, as will low self-efficacy beliefs. Moreover, it should be expected that many of these students will present with math histories that include accounts of mistreatment, even abuse, during prior math learning experiences, with adverse affect on their perceptions of mathematics and on their attitudes towards the process of learning math. As reported previously (Klinger, 2004), effective support strategies *must* initially address this aspect ahead of specific content by appreciating the need to revisit, deconstruct, and overcome the impact of past poor encounters with math, thereby to change students' relationships with mathematics. In this manner, self-efficacy beliefs can be re-defined so that negative attitudes yield to increased motivation and diminishing anxiety.

This study also flags implications for the much broader issue of fostering improvements in quantitative literacy in society. As indicated in the introduction, this must be guided by the community's best-informed and most influential members, which must include those with the greatest education. Yet this study indicates that a very substantial proportion of entrants to higher education are afflicted with

predominantly negative views of mathematics. Unless the university experience challenges and radically improves their perceptions, it appears unlikely that, as future graduates, they will go on to promote the need for change in math teaching and learning at any level, let alone the sort of profound changes that will be necessary to make a real difference. The validity, or otherwise, of this scenario is an open question that warrants investigation.

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A number story as subjective area of experience– a case study with a maths avoider

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In several definitions of numeracy mathematics has the function to solve individual everyday life problems. However, the other way round it is important too. In learning processes, everyday life knowledge and experience can be useful to improve the understanding of mathematics. For example, learners (especially maths avoiders) can construct number stories to get an inner understanding of mathematical terms. The construction process is complex and means more than just the simple translation of a mathematical content into a story. We find complex connections between the learner's mathematical competences, his individual view of the real world and his individual preference for constructing number stories. The theory about subjective areas of experiences provides a frame to describe these areas and their connections.

Maths avoiders are defined as adults with just rudimentary mathematical knowledge that is not functional in everyday life. The tendency to avoid mathematical action and situations is not only caused by the lack of mathematical competence but also by emotional aspects such as maths anxiety, a negative view of mathematics or a lack of self -confidence.

In my actual project, I teach math avoiders who are women, aged from 26 to 52 years (see Langpaap, 2005 and 2006). Following the major ideas of numeracy that have been discussed in previous years, I use the learners individual everyday life experiences as an anchor for instruction, problem solving and learning processes in general. At the beginning of every lesson the student talks about a mathematical problem or phenomenon she was confronted with during the previous week. On that we construct a mathematical problem to use it as one main task of the lesson.

Schlöglmann (2002) noticed that everyday life experience is a suitable starting point for reducing learning obstacles and for the development of mathematical comprehension. The idea of focusing on the 'real world maths' is part of various definitions of numeracy. The OECD (2001, p.22) for example defines mathematical literacy with three essential aspects:

Mathematical literacy …

> 'revolves around the wider uses of mathematics in peoples lives rather than being limited to mechanical operations',
>
> **includes** "the ability to put mathematical knowledge and skills to functional use rather than mastering them within a school curriculum" and

> **implies** the ability "to pose and solve mathematical problems in a variety of situations as well as the inclination to do so, which often relies on personal traits such as selfconfidence and curiosity"

The definition especially deals with the idea that mathematical knowledge and skills are functional "tools" to cope with individual real life problems. The user of this "tool" translates the real world problem into a maths problem. The solving process takes place inside the "maths world" then the results are interpreted in the context of the real world situation. Problem solving in general is seen as a circulating process of formalising and reinterpretation (an example is the model from Blum (1996)).

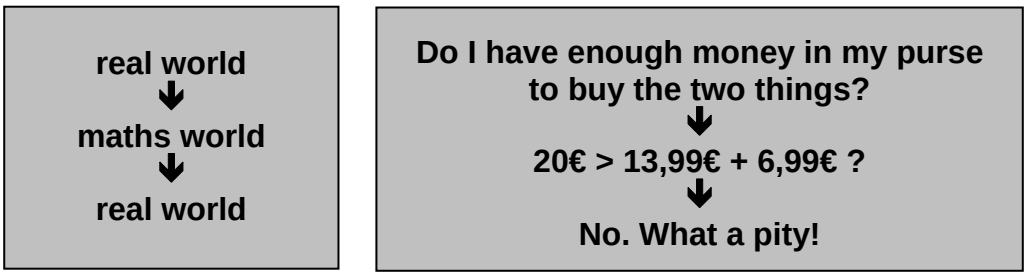


Figure 1

While the definition of numeracy describes "the ability to put mathematical knowledge and skills to functional use" as a target of learning maths, the learning process itself contains also the opposite direction: Real world knowledge is used or can be used to understand mathematical ideas and structures.

For maths avoiders the term 19x1,5 could be hard to understand and impossible to solve. The term may be easier to understand if the learner thinks about the numbers and operations in the view of everyday life practice and experience. One method to do this is the use of number stories (see Langpaap, 2006). The learner translates the formal problem into a real world context. Led by a number story he solves the problem there and finally he interprets the result as formal solution.

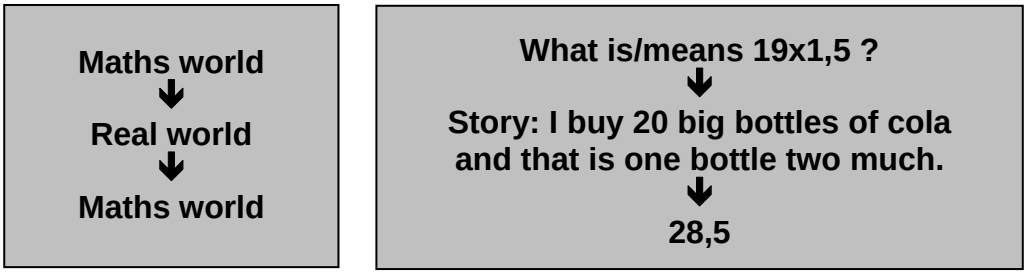


Figure 2

Subjective areas of experience:

I analysed the construction process of some of my students' number stories. I found some characteristic aspects of these processes. While constructing their own number stories

- the students identify themselves with characters, objects and actions of the stories,
- they discuss discrepancies of the setting and try to resolve them,
- they reflect on the implications of characters, objects and actions for the stories' credibility
- and they try to legitimate the stories' elements. (See Langpaap, 2006)

Thus, the process of construction seems to be more than a simple translation of mathematical elements into real world elements and the other way round. The whole process is chararcterized by aspects of identification and legitimization. For some maths avoiders it seems to be an important concern that their number stories get a realistic setting. The construction process does not only happen in the rational dimension. Also emotional aspects often play an important part.

To describe the connections between the maths worlds, the real world and the number story, I use the theory of *subjective areas of experiences* (*SAE*) from Bauersfeld (1982). Bauersfeld describes individual experience and knowledge as

- stored in a non-hierarchical and cumulative way,
- connected to situations,
- and organised in distinct areas.

One of the benefits of this model is the fact that it contains the different dimensions of human experiences. Every SAE contains "the totality of the individuals' emotions, cognitions, physical perceptions, so every dimension of the individual experience and perception" (Bauersfeld 1982, p.2, translation by JL).

Using number stories as a tool is not a simple and self-explaining act. We know from the discussion about illustrative materials and audiovisual aids that the sense and the usability of these objects have to be learned and experienced by the user in an active process (Lorenz, 1992). Number stories have to be seen in the same way. In the theory of SAEs number stories are not regarded as mediators or bridges between the maths world and the real world. The concept of number stories is a world of its own and has to be learned and experienced. It exists as a third world beside the maths world and the real world and is connected to them by its special function of comparison.

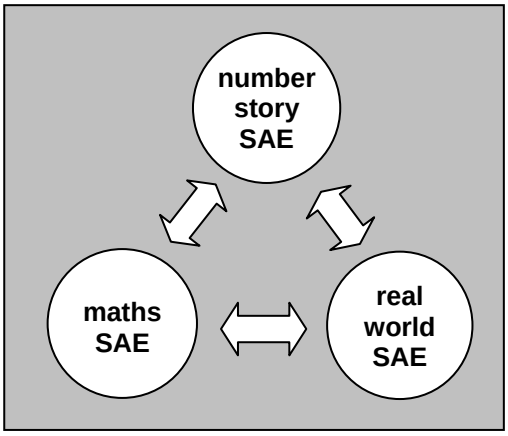
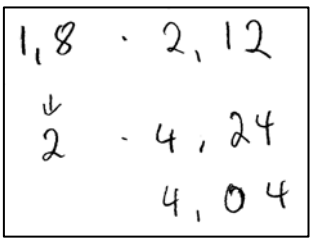


Figure 3. A case study: Ines solves 18x2, 12

Ines, an administration employee, is 40 years old. “Mathematics” is a “horrible” word for her. Her fear of failing in mathematical situations leads to blockades in thinking. On the other hand, she has some arithmetical competences of primary school level. For example, Ines is able to solve 55+27 by 55+20+7 or 17x12 by 10x12 + 7x12. For some mathematical aspects Ines has a lack of inner understanding. For example, she counts the problem 7x80 by 7x8 and “adding a zero”. “Adding a zero” means for her something like “going a level higher”. The inner understanding of a multiplication by 10 is felt as difficult by her because “10 is too big and it is easier to multiply by 5 or 2”.

In one lesson the multiplication of decimals was discussed and practiced. An aim was to get an inner understanding of the mathematical procedure of multiplication. The main strategy was to change a term with decimals into a term with natural numbers. Ines was able to solve problems like 3,3x1,7 by changing it step by step into 33x1,7 and 33x17. To understand this strategy, number stories were used. After that Ines had an inner understanding of that procedure and she was able to solve similar problems.

The following scene happens one week later. Ines works on the term 1,8x2,12. She tries to solve the problem by a “neighbourhood strategy”. She changes the term into a similar but simpler one: 2x2,12. To compensate the difference she substracts 0,2. By that she ignores the distributive characteristic of multiplication:



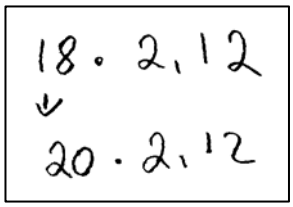
2,12

problem:	*1,8 x 2,12*
Ines calculates:	*2 x 2,12 – 0,2*
correct is:	*2 x 2,12 – 0,2 x*

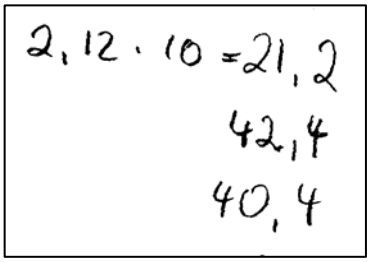
Figure 4

This mistake shows a deficiency in the concept of multiplication. As Ines draws an arrow to symbolize the change of the first factor, one explanation of this behaviour could be that Ines focuses especially on this part of the term. Her view does not include the connection of this change to the second factor and the multiplicative aspect of the whole term.

In the case of Ines this mistake is a structural one that frequently happens. Ines does the same mistake analogously when she solves the term 18x2,12:



Ines: [...] 2,12 multiplied with 10. (She writes "2,12x10=21,2".) Two, that I take two times and than it is 42,4. (She writes "42,4".) So, now I have two too many (she points at the left picture). Then I have 40,4. (She writes "40,4".)



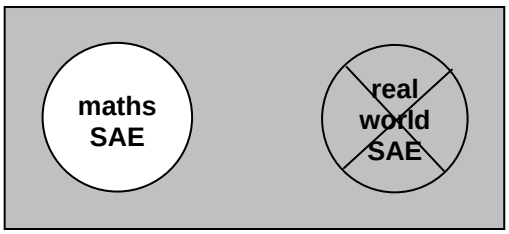
In short her way is also here:

problem:	*18 x 2,12*
Ines calculates:	*20 x 2,12 – 2*
correct is:	*20 x 2,12 – 2 x 2,12*

Figure 5

Ines' argumentation "*now I have two too many*" shows that she doesn't realize the distributive characteristic of the multiplication. Although her procedure is wrong we can find several aspects of mathematical competence. Ines knows that the "neighbourhood strategy" is adequate for the problem. Ines uses the commutative and associative characteristics of multiplication when she calculates 20x2,12 as 10x2,12x2. For that she gets the correct result. In earlier lessons she shows that calculating 10x2,12 means "10 times" for her and so more than just moving the decimal comma. We can say that Ines has a lot of competence that is related to the given mathematical problem.

In the view of SAEs she activates formal mathematical knowledge and uses procedures to solve the problem. Her thinking takes place inside the borders of her maths SAE. Also the procedure and its mistake is part of this SAE.



As Ines is convinced that her result is correct she is asked to construct a number story for the problem.

L: That you would do so?

Ines: Hm/

L: Ok. Please tell me a number story for this problem (he points to 18x2). Something where both numbers can be found. (11 seconds pause.) Let's

suppose you go shopping, buying something for a party. A story where both numbers occur.

L's intention is that Ines could probably find the mistake on her own through the construction of a number story. A story about shopping could give a vivid comparison between two quantities (18 times and 20 times). L does not prescribe whether the story should be real and based on individual experience or fictional. Ines associates spontaneously a real experience:

Ines: I have looked that there is a lot of cola (she laughs), that there is a lot of cola (she laughs again), that there have been 0,33[spoken: zero- comma-thirtythree] -litre-cans. But

L: Tell me one. Only to this [problem]. Forget what you have already written down.

Ines: Hm/

L: And give this problem (points at it), give this problem a sense. (19 seconds pause.)

Ines: That's hard to me.

L: To create a number story?

Ines: Yes, because I don't know [such] numbers that appear somehow.

Ines associates spontaneously objects of her real life. The tins of cola are connected to an individual real life experience. Ines says "*zero comma thirtythree*" instead of "*zero comma three three*". So the quantity is expressed in an everyday life speech and not a mathematical speech. Ines activates here a real world SAE with its special focus and language.

The number 0,33 Ines remembers is structurally similar with the number 2,12. Both numbers have three digits and two places behind the comma. In the sense of transfer Ines searches for comparable patterns of different SAEs. The formal number 2,12 exists in the maths SAE and its equivalent she wants to identify in a real world SAE.

Ines is unsatisfied with her own suggestion of this context. This is initially indicated by the word "*but*" and finally expressed by her statement "*I don't know [such] numbers that appear somehow*". For her a number story seems to be acceptable if the relevant pattern of numbers is realistic for the chosen real world context. Although theoretically a number story could be fictional and L hasn't asked for a realistic setting Ines is searching for a real world context experienced by herself.

To sum up, it can be said that Ines' way of construction is characterized by two perspectives: a perspective of structural comparison of patterns and a perspective of likeness to reality. The suggested context (the buying situation) fails because the numbers of the formal term do not pass Ines' reality check. The mathematical SAE and the real world SAE are kept isolated from each other.

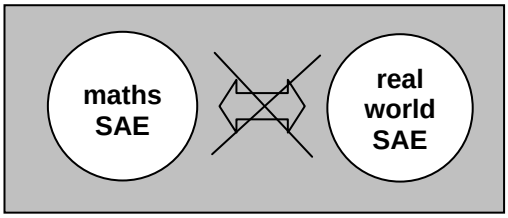


Figure 7

Although Ines doubts that an adequate context exists, L asks again:

> *L: Where does 2 1 2 exist?2,12 - what could that be?*
>
> *Ines: I would say someone has watched too much sports and counts the milliseconds. That is the only thing where so little units exist.*

Now Ines associates with the number 2,12 a time quantity in a specific sports context. As in the buying situation the scenery is connected to involved persons. In the buying context Ines herself is an actor, in the sports context a third person is introduced. She characterizes the extent of the actors TV consumption as "*too much*". So Ines doesn't reduce the setting to the mathematical core but involves known persons or gives a social comment to the actor of her story.

Her description of the context as "*the only*" gives evidence again to the interpretation that it is difficult for Ines to associate adequate contexts. But unlike the buying situation, now the mathematical SAE and the real world SAE are connected to each other.

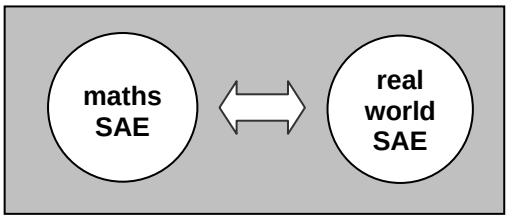


Figure 8

Since the connection of both SAEs exists, the sports context might serve as base for a number story. But in the following L ignores this and goes back to the buying context as counterproposal. He believes that this context is easier to use:

> *L: I make it a little bit easier for you.*
>
> *[…]*
>
> *L: […] You go to Aldi to buy something. You buy 18 bottles of cola. 18 bottles of cola for each 2,12€ (he points to the term 18x2,12). Ok? (Ines: Hmhm/) That's the story.*
>
> *Ines: I think I know what the mistake is.*
>
> *L: Ok/*

Ines: I have bought two times more. (L: Hmhm/) Two units. Then I must not substract two but this (she points to "2,12") two times, so 4,24.

L: Ok. Could you explain this to me. How do you get this idea?

Ines: Because of this "I buy two times more".

Ines explains her idea not in a formal way but gives a citation of her own narration: "*I buy two times more"*. This expression describes the distributive structure of multiplication. So by the help of the story Ines now has got an inner understanding of the mathematical problem.

Ines' way of acting indicates that she focuses on the maths SAE and the real world SAE simultaneously. The story's elements are part of the real world setting but at the same time Ines points with her fingers to the formal terms. This is accompanied by a change in speech. The more narrative statement "I *have bought two times more*" and the more formal statement "*Then I must not subtract two but this two times*" are closely interrelated.

The transfer becomes possible through an argumentation that is embodied by a number story as a third SAE. The impulse of the story's action is legitimated by the logic of real world experience: surplus goods must be given back. Schütz (2003) describes the fact that individual everyday life acts are connected to aims and intentions as *pragmatic motive.* In this view, individual everyday life procedures are pragmatic and make sense if they work. Likewise the pragmatic motive of the number story makes Ines understand the distributive structure. Ines changes the focus from the formal difference of 18 and 20 to a perspective one which is driven by the pragmatic view of the stories context. Gray, Pitta and Tall (1997) found that the connection of procedural thinking with declarative knowledge is characteristic of high achievers in mathematics. In this case Ines connects a mathematical procedure with knowledge about a specific context. In short: Here mathematical procedures become meaningful in combination with the learners real world experience and embodied by the structure of a story.

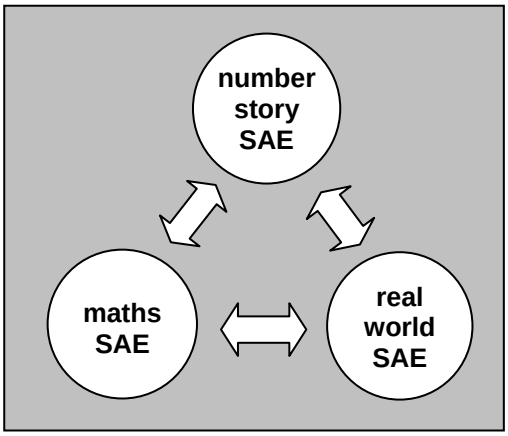


Figure 9

Conclusion

In the scene we found four different constellations of SAEs:

- In the first constellation Ines argues only within the mathematical SAE. Although some aspects of mathematical competence have been identified Ines was not able to solve the problem correctly. Her mathematical concept of multiplication is insufficient regarding the distributive characteristic.

- The second constellation is characterized by the search for an adequate real world context. This process is dominated by comparison of patterns and by checking of likeness to reality. Ines searches for realistic contexts related to her own experience. The context should include numbers similar to the given one. The check failed and the compared SAEs stay isolated to each other.

- In the third constellation a connection between the maths world and an everyday life context is made. The sports context passes the check of patterns and the check of likeness to reality.

- In the forth constellation the context is prescribed by the teacher. Ines is able to focus on both SAEs: the maths SAE and the real world SAE. The number story as a third SAE embodies the comparability of both SAEs. Secondly it is *the pragmatic motive* of the story that makes Ines understand the mathematical problem.

The construction of a number story is not a trivial act but the result of various processes. The success of such a construction depends on experience with number stories. This is a fact and probably natural for all learners. In the case of Ines we find some special aspects that may be typical for low achievers or maths avoiders. Her construction of the number story is characterized by the search for realistic and vital settings which are connected to her own real life. The results of this case study supports the results of previous studies (see Langpaap 2005, 2006), where an intensive dealing with aspects of the own number story has been found. Identification with and legitimation of persons, objects and actions of the own story seem to be typical for this group of learners.

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Adults Returning to Mathematics: Key Themes Emerging in Responses to Three Different Teaching Approaches

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This document is published in full in the International Journal of Mathematics Education, Science & Technology. The full reference from the paper is given below:

A case study approach was used to explore second-chance mathematics courses offered at the University of Auckland: two larger courses and one individual study program. Most of the students' previous experiences learning mathematics had been less than ideal. A different teaching approach, by committed experienced teachers, was used in each course. In evaluating their effectiveness, I focused on affective change in the students, relating this to their achievement. Qualitative and quantitative data were collected over several years. This paper discusses some of the main findings of this large study. Important themes which emerged from the data analysis were the importance of teachers' acknowledgement of students' past experiences and current feelings about mathematics, dealing effectively with students' differing background knowledge of mathematics, as well as the effective use of contexts in non-routine problems. Students' reactions to each of the three approaches were compared and found to reflect the nature of the approach. The two approaches which presented mathematics in a new way had the effect of changing students' views of the nature of mathematics.

Miller-Reilly, BJ (2007) International Journal of Mathematical Education in Science and Technology, 38 (7), 891-905 *Three different teaching approaches in pre-calculus bridging mathematics.*

Is it possible to empower adults through numeracy teaching?

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All over the world politicians talk about lifelong or ' lifewide' learning and they set special programs to make sure that adults have the possibility to participate in education. All over the Western world Numeracy teaching has been a part of this educational program. The politicians and part of the educational system have a very limited goal for numeracy teaching. In the community of ALM, numeracy teaching is not just seen as a means to reach and maintain the welfare society but in fact as a means to empower adults. In this article, I shall introduce two different models and understandings of numeracy teaching which I have found by reading all the proceedings from earlier ALM conferences, and I shall raise the question: is it possible to empower adults through numeracy teaching?

Two stories about adults and mathematics

The field of adults and mathematics is characterised as having great heterogeneity (FitzSimons et al, 1996) and according to Wedege (2001) this is due to the lack of a "grand narrative" and the great complexity of the subject area as a research domain.

Perhaps the field lacks a "grand narrative" but reading through all the conference papers from the ALM conferences plus several books written by members of ALM, I shall argue that at least two "grand stories" about adults and mathematics live and are told in the research community of ALM (Johansen, 2006). The two stories appeared to me when I tried to fit my readings into a specific curriculum development model (Hiim and Hippe, 1997). I chose to use this model as a frame to categorise my readings even though it was made to help the teachers prepare for their lessons. Out of my categorising two different models appeared. I have chosen to name them "The school subject curriculum model" and "The ethno curriculum model". The first story based on "The school subject curriculum model" is often told as a bogey - a story about what "we" do not want, or as a model for adult independent learning where adults alone and independently perhaps at home, learn school mathematics. The other story based on "The ethno curriculum model" is presented as "an ideal model" – what "we" really want.

The school subject curriculum model

Figure 1 represents "The school subject curriculum model". And I will shortly go through each theme of the model.

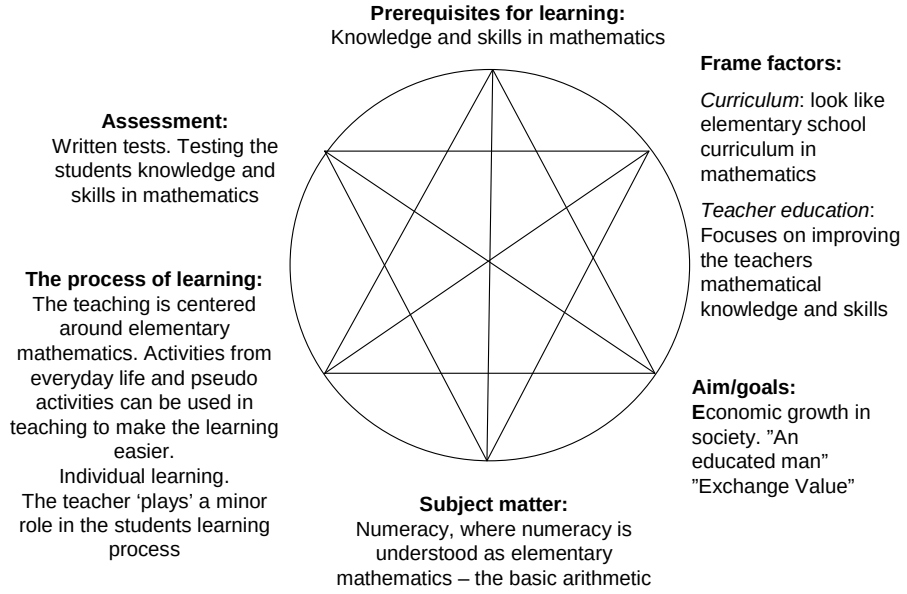

Figure 1: **"The school subject curriculum model"**

The focus and starting point, in the school subject curriculum model for teaching adults numeracy, is the lack of formal and basic skills in mathematics of adults. The goal of teaching numeracy is to equip the adults with testable basic mathematical skills. The curriculum is developed to make it look like the curriculum in mathematics from elementary school. The professional development of the teachers is centred on improving the teacher's mathematical skills or, if that already is advanced, improving their competence in teaching adults. The teaching of numeracy focuses on elementary mathematics. It is possible for the teacher to bring in activities or "look a like activities" from everyday life of adults to ease the learning of mathematics. The teaching can be organised differently according to individual as well as group work, it is however always the elements of school mathematics which structure the teaching. The teacher himself plays a minor role in the learning process of the participants – and as I have mentioned earlier it is a model where it is possible for the participant to learn mathematics all by her/himself. Written tests are the preferred means to test the participants' prerequisites, the progression of the participant's mathematical skills as well as the end results of the numeracy course. The aim is economic growth in society through equipping adults with formal and basic mathematical skills that have "exchange value" (Coben, 2002) and therefore open the doors to the labour market and further education.

The ethno curriculum model

The focus and starting point in "The ethno curriculum model" is that adults are already in possession of math-containing knowledge and competencies before they enter the numeracy course even though they may not be aware of their own competencies; even though they may think that "math is what I can not do". In "The

ethno curriculum model" numeracy is understood as "not less than math but more" and the focus is not on school mathematics.

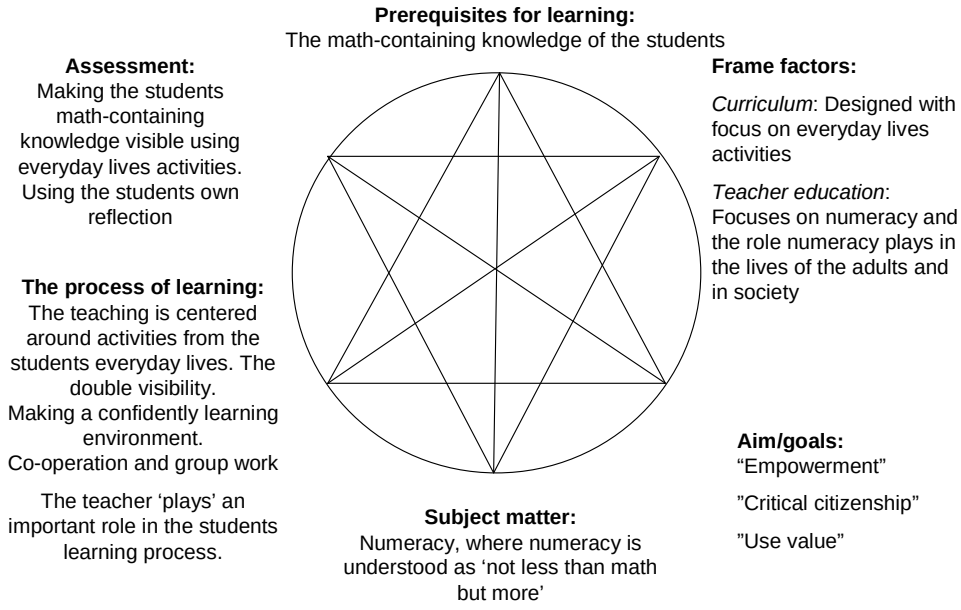

Figure 2: **"The ethno curriculum model"**

The aim of teaching numeracy to adults is to empower the adults and equip them with knowledge and skills that have "use value" (Coben, 2002) for them. The learning process of the participants and therefore the teaching has to be planned in such a way that the math-containing skills and knowledge of the participants' are made visible as well as the math-containing part of the participants' everyday activities are made visible. A way to do so is to bring everyday activities into the classroom and let the activities determine which mathematical skills and knowledge the adults will meet and work with in that lesson or alternatively to bring the participants and the teacher into everyday life performing everyday activities. In the curriculum the subject matter can/shall be described as a range of activities from everyday life instead of the elements of school mathematics.

If the participants are to be tested before they enter the numeracy course new types of testing material must be developed. It is a good idea to use everyday activities to test the math-containing skills and knowledge of the participants instead of written math tests. The professional development of the numeracy teachers focuses on numeracy in adults' everyday and working life – the teachers have to learn to open their "numeracy eyes" to see numeracy and math-containing skills and knowledge in normal everyday activities as well as realize which math-containing skills and knowledge the participants possess. The teacher plays an important role in the participants' learning process especially in the way he/she is responsible of creating a confident learning environment.

Comparing the two "grand stories" about adults and mathematics, it appears that they possess a lot of different ideas and understandings of, for example what numeracy is, what is good practice in adult numeracy, what is the role of the teacher, what is good professional development, how to test adult numeracy etc. and there are different ideas of goals of numeracy teaching. In the school subject curriculum model, economic growth is the goal whereas in the ethno curriculum model empowerment is the goal; in the same way exchange value versus use value are different goals of numeracy teaching in the two stories. Setting up these kinds of enormous or beautiful aims for an often very short numeracy course is something which is seen in almost all countries however, they are rarely contested. In the rest of this article I dare to challenge the concept of *Empowerment* as a result of adult numeracy teaching in a Western country.

Empowerment and education

According to Gal (1999:12) empowerment is a multi-faced construct that has gained the attention of professionals and researchers in several fields.

> A key assumption underlying the notion of empowerment is that people have the potential to cope well with many life challenges but have found themselves in a position of (relative) powerlessness over their life course due to situational, social, organizational, or other circumstances and barriers. Another assumption is that the process of becoming more empowered involves the acquisition of essential skills and knowledge, but just as much the acquisition of self-perceptions and a sense of self-efficacy.

When or how do adults, who want to join a numeracy course, feel powerlessness? A lot of adults feel the lack of power over mathematics – "mathematics is what I can not do". Perhaps they experience the lack of power over their job situation as well, they can be unemployed or unconnected to the labour market or they can experience that their competences are a barrier to changing jobs. Many researchers bring in the idea that innumerate adults lack power of their economic situation, and perhaps some adults feel that too. Many adults experience lack of power over decisions taken by the government and the local politicians, I feel that lack too, and I do not think that it has anything to do with my level of numeracy.

In the context of mathematics education Ernest (2002, p.249) has discussed empowerment.

> Empowerment through mathematics necessitates a consideration of the development of the identity of learners and their potential through the development of mathematical and related capacities.

Ernest (2002) finds it useful to distinguish between three different domains of empowerment concerning mathematics and its use. Ernest distinguishes between:

Mathematical empowerment –gaining power over the language, skills and practices using and applying mathematics; Social empowerment – gaining the ability to use mathematics to better one's life chances in study and work and to participate more fully in society through critical mathematical citizenship; Epistemological empowerment – gaining a growth of confidence not only in using mathematics, but also a personal sense of power over the creation and validation of knowledge.

> The aim of empowering learners as epistemological agents is a radical and summative one, as it brings together and integrates all three of the different types of empowerment discussed above. Only when all of these powers are developed will they feel they are entitled to be confident in applying mathematical reasoning, judging the correctness of such applications themselves, and critically appreciating (including rejecting, in some cases) the applications and uses of mathematics by others, across all types of contexts, in school and society. Thus epistemological empowerment is the culmination of all the other types of empowerment discussed here (Ernest, 2002, p.258).

Trying to understand this idea of empowerment raises the question: "Is numeracy teaching a way to facilitate empowerment?"

I wonder…

First of all, I wonder what kind of empowerment we are talking about when we set up empowerment as an aim and a goal for numeracy teaching. Is it the radical epistemological empowerment Ernest introduces above, which brings together all three domains of empowerment i.e. mathematical, societal and epistemological, or is it something less, or nothing like that at all?

I wonder if it is possible to gain power over numeracy as long as it is defined in so "fluffy" terms. Reading through the conference papers from ALM conferences it appears that several authors have tried to catch and describe the concept of numeracy, and they have formulated different definitions of numeracy. Numeracy is, however, still a deeply contested and notoriously slippery concept (Coben, 2003). One definition is the one we meet in the school subject curriculum model where numeracy is understood as just basic mathematics. Another definition is the one we find in the ethno curriculum model where numeracy is understood as "not less than math but more". An example of this kind of definition is formulated by Tout (1997, p.13):

> We believe that numeracy is about making meaning in mathematics and being critical about maths. This view of numeracy is very different from numeracy just being about numbers, and it is a big step from numeracy or everyday maths that means doing some functional maths. It is about using mathematics in all its guises – space and shape, measurement, data and statistics, algebra, and of course number – to make sense of the real world, and using maths critically and being critical of maths itself. It acknowledges mathematics but more. It is why we don't need to call it critical numeracy – being numerate is being critical.

This definition of numeracy provides the opportunity to reach the radical empowerment as a goal. However, I still wonder if numeracy teaching can change the identity of the learner? Can one be empowered without changing one's identity and self-conception?

Furthermore I wonder if math-containing skills and knowledge made visible through everyday activities in numeracy teaching will automatically make the adult more mathematically empowered.

Everyday activities are in the ethno curriculum model seen as the main means to make the participants' math-containing skills and knowledge visible. The reason for making the adults' abilities visible to themselves is to make them more confident with mathematics and make them understand that they are able - so to say. However, working with adults returning to school it appears that once adults have succeeded in using and applying a piece of mathematics, it becomes "non-mathematics" (Coben and Thumpston, 1995) and as Harris (1995) has realised that mathematical skills that caused the adult workers no problem in their working life were regarded as common sense – and it only became mathematics when the adult worker was not able to cope with it. So I ask the question if the adult will become mathematically empowered or will he/she automatically exchange the now visible abilities with "non-mathematics"?

I also wonder if it is possible to gain power over numeracy without first getting power over mathematics - that is mathematical empowerment as Ernest (2002) has named it. Is it possible to be "numeracy" empowered without gaining power over the mathematical language, signs and symbols?

I still wonder if it is possible to become socially empowered through numeracy teaching without first getting mathematically empowered and without some teaching in social science subjects. Is it possible to choose enough everyday life activities to become socially empowered and able to act as a critical citizen?

Finally, I can not stop wondering if I was correct in raising the question: Is it possible to empower adults through numeracy teaching?

Acknowledgements

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From Research to Relief: An Examination of Case Studies Illustrating Math Anxiety

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A number of strategies designed to alleviate math anxiety have been proposed. Three main categories include approach strategies, avoidance strategies and social support strategies. While all of the categories are helpful, approach strategies appear to be the most beneficial. "Real life" case studies can illustrate the usefulness of all of these strategies.

A number of strategies have been proposed to alleviate mathematics anxiety including attending a mathematics tutoring laboratory, engaging in relaxation training, learning to improve study habits, and even adopting a "Math Anxiety Bill of Rights" (Davidson & Levitov, 1993; Hackworth, 1982; Tobias, 1991). Peskoff (2001) mentioned that although many self-help manuals, videotapes, pamphlets, and handbooks exist that attempt to help students overcome mathematics anxiety, there is nonetheless a lack of empirical research on student and faculty assessment of their comparative effectiveness. He designed a survey in which both community college mathematics faculty and students were asked to analyse various coping strategies. All of them were designated as either approach strategies, avoidance strategies, or social support strategies. The survey completed by the students is presented in the Appendix.

Approach strategies involve the active learning of mathematics and include behaviours such as asking questions in class, setting aside extra study time before examinations, letting your instructor know when you are having difficulty following a lesson, and using extra books, websites, or other resources for learning (Peskoff, 2001; Blair, 2006).

Avoidance strategies essentially involve taking a temporary break from studying mathematics in order to "feel better", and then returning in an enhanced coping mode. Unlike approach strategies which involve the direct learning of mathematics, avoidance strategies do not. However, they can provide the emotional replenishment necessary for subsequent successful learning. Examples include systematic relaxation, exercise or sports activities, and positive self-talk – i.e. reminding yourself when you feel distressed, that you are a good student who can learn mathematics (Peskoff, 2001; Blair, 2006).

Social support strategies involve sharing experiences with others. The underlying premise is that students will feel better and have less anxiety if they realize that their experiences are common and that they are not alone. By talking to other students as well as counsellors and tutors, they can diminish anxiety by forming a common bond with peers (Peskoff, 2001; Blair, 2006).

In his study, Peskoff (2001) asked students and faculty to rate a series of strategies in terms of frequency of use and helpfulness. He found that both groups deemed approach strategies to be the most beneficial. In addition, students who were less anxious tended to use approach strategies more than students with high math anxiety.

Case Studies

The case study approach is commonly used in the fields of law, medicine, and business administration to help apply theory to practice. This approach is equally useful in the teaching and learning of mathematics since various categories of coping strategies become far more meaningful when they are used to assist "real" students. Each case represents a student who is confronted with a potentially stressful situation when attempting to study mathematics (Peskoff, 2007).

Questions to Consider

For each scenario, consider the following questions:

1. How do you think the following students would most likely respond to each situation?

2. What coping strategies (listed in the Appendix) do you think should be used to resolve each problem? What outcomes would result if these strategies were implemented successfully?

Case I

Mario is taking an Introductory Statistics course. For the first four weeks of the semester, he was able to complete his weekly homework assignments with little difficulty. He is now attempting his fifth homework assignment on probability. After successfully answering the first three questions, he gets stuck on several problems in a row. His answers "don't match those in the back of the book" and he can't figure out why.

Case II

Ida is sitting in her Elementary Algebra class. The teacher is reviewing homework problems on the board. Ida successfully completed the assignment the night before, but nonetheless listens attentively. She is a serious student who attends class regularly and prides herself on taking careful, clear notes. After the review, the instructor introduces subtraction of polynomials. Ida feels "completely lost and confused." She copies the rules and problems from the blackboard but has no idea what she is writing.

Case III

Diana is enrolled in a precalculus course. She hasn't studied mathematics since she graduated from high school ten years ago. Although the work seems somewhat familiar at the beginning of the semester (she at least "remembers having learned it"), Diana decides to go to the mathematics laboratory for tutoring. She asks the tutor to review graphing straight lines. The tutor's response is: "you should already know this before taking the course."

Case IV

Miranda is taking a course in medical dosage calculations in order to prepare for the nursing program at her community college. If she does not get an A or B in the course, she will not be admitted to the program. She is already a nurse's aide and has learned on the job to perform calculations using ratios and proportions. The course instructor is teaching a new method called dimensional analysis which he says is often used in chemistry courses when converting units of measure. Miranda does not understand this method and "finds it confusing." When she discusses this with her instructor, he tells her that dimensional analysis is the most logical method and insists that it is really the easiest to learn.

Conclusion and Request for Feedback

It should be noted that no "answers" will be provided in this discussion. In fact, it can be asserted that there really are no correct or incorrect answers. Readers are encouraged to submit responses to the above questions to the authors. They hope to present a future paper which collectively compares the responses of different readers. It is expected that similar to the diverse background of the audience (and of course, their students), the responses and comments submitted will be equally diverse.

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Appendix

Coping Strategies Survey for Students

Math Course (Circle One): ALGEBRA PRECALCULUS

Sex (Circle One): MALE FEMALE

Directions:

The following is a list of strategies that students may use in order to learn mathematics effectively and do well in their mathematics courses. Please respond to both questions listed below each of the following behaviors by circling **any** number from 1 to 5 where:

1 = not at all 3 = somewhat 5 = very much.

All responses will be kept confidential and used for research purposes only.

1. Using the school's tutoring centre or a private tutor.
   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

2. Practicing systematic relaxation, physical activities, or exercise.
   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

3. Discussing experiences or difficulties related to your mathematics course with other students in your class.

   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

4. Discussing experiences or difficulties related to your mathematics course with your school counsellor.

   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

5. Using additional textbooks or review books other than the required text.

   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

6. Asking your instructor mathematics questions in class.

   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

7. Completing homework assignments on time so that you don't fall behind.

   a. How often have you tried this? **1 2 3 4 5**
   b. How helpful has it been?
   OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

8. Reminding yourself that you are a good student if you start to feel incompetent.

a. How often have you tried this? **1 2 3 4 5**
b. How helpful has it been?
OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

9. Setting aside extra study time for review before class exams.

a. How often have you tried this? **1 2 3 4 5**
b. How helpful has it been?
OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

10. Letting your instructor know if you don't understand the course material.

a. How often have you tried this? **1 2 3 4 5**
b. How helpful has it been?
OR how helpful do you think it would be if you tried it? **1 2 3 4 5**

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Professional Development in the *Adult Numeracy Initiative:* A Project of the United States of America Department of Education

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In October, 2005, the United States Department of Education (USED) funded the first large-scale federal numeracy project, the *Adult Numeracy Initiative (ANI).* It was envisioned as the first of a series of numeracy projects that will be based on the findings and needs identified by the *ANI*. This paper will introduce the overall purposes of the initiative and focus on one of the six tasks, a survey of professional development currently offered across the United States.

The Adult Numeracy Initiative was charged with four basic objectives:

- Define the state of the adult numeracy discipline in the United States
- Identify major issues and topics critical to the development of math skills in adults
- Conduct an environmental scan of existing programs and practice
- Assist in the development of a research agenda in adult numeracy.

A series of six different tasks were defined to accomplish these objectives. These are discussed elsewhere in these proceedings in a paper by Anestine Hector-Mason of the American Institutes of Research, the research organization that was awarded the *ANI* grant. This paper concentrates on one task, the environmental scan of existing numeracy professional development (PD) in the United States.

The ED request for proposal (RFP) defined seven research questions for the initiative to investigate and answer. Three of those questions are directly related to the environmental scan task:

What types of programs have been implemented at the state and local levels through federal funding that incorporate or focus on adult mathematics instruction?

What practices exist in professional development and certification requirements for teachers of adult mathematics education that are worthy of replication?

What types of programs have been implemented at the state and local levels through federal funding that focus on adult mathematics instruction related to adult English language acquisition learners?

Of all the tasks in the initiative this one presented the greatest challenge because of a mismatch between the question being asked, "What programs have been implemented…" and the Education Department request that the project team focus on professional development. Nevertheless, we proceeded on the ED assumption that programs with good PD would contain quality content and materials.

The project team met in late 2005 and developed a set of tasks to be undertaken in order to conduct the environmental scan. These were:

- Develop Criteria for Program Selection
- Identify Potential Programs
- Review Existing Materials
- Develop Guided Questions
- Collect Data From

- Program Directors
- Participants

- Analyze Findings
- Prepare Report.

Develop Criteria for Program Selection

The first task facing the project team was the definition of a "program." Professional development in the United States can be as brief as a half-day lecture by an expert on some topic or as extended as multi-year programs that meet regularly to discuss topics and materials that they have used or will try in their classrooms. After review by a panel of education and numeracy experts, a set of criteria was defined to be used to determine whether a PD effort qualified as a program and would, therefore, be included in the scan. Some criteria were considered essential while others were determined to be desirable but not necessary. The resultant criteria are as follows:

Essential

- Occurs over time, not "one-shot"
- Built upon activities that advance teacher concept and content knowledge
- Reflect authentic materials
- Based on andragogical principles
- Materials conform to state or national (NCTM, ANN, AMATYC) standards
- Participants are from adult basic or secondary education settings

- Program materials are publicly available and accommodate varying backgrounds of participants.

Desirable

- Evidence exists of an evaluation component
- Program explicitly incorporates an affective factor intervention
- Technology plays a role in the PD administration or materials.

Concurrent with the definition of criteria, the project team began to seek out numeracy projects with PD components. Originally the RFP had limited the scan to federally funded projects but it was apparent early on in the initiative that a more inclusive net would have to be cast, so in the end we included any project that had been grant-funded by either private organizations or public agencies. Publically funded grants are listed on the websites of the granting agency. We searched the sites of the Department of Education, the Department of Labor, and the National Science Foundation. Adult education funds in the United States are locally controlled by the states. Therefore, state adult basic education administrators and their professional development staff, where they exist, were contacted and asked to describe any numeracy work being done through their state agency. A request went out on the Adult Numeracy Network (ANN)-sponsored numeracy listserv as well as the National Association of Developmental Educators (NADE) special interest mathematics listserv. Print requests were included in the Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM) newsletters. Officers of numeracy and developmental organizations were contacted personally. No stone was left unturned.

Review Existing Materials

The project team attempted to collect and review commercially available materials whether or not they were being used currently by numeracy projects. Vendors at the NADE, MAA, and NCTM conferences were approached and asked to send material suitable for adult numeracy instruction or professional development to the project team. Programs identified for inclusion in the environmental scan were asked to send copies of program-specific materials to AIR. A template was developed for recording data about the program and materials used. This template was expanded in the later interview phase in order to capture all the data about a program in one place to facilitate analysis. The template recorded data that identified the program, the source of support, the instructional setting, the nature of the participants, the mathematical content, the materials used, and the program assessment and findings.

Conduct Interviews

To assure a high degree of uniformity among interviews, an interview protocol was constructed to gather information about each program. Eight areas of inquiry were addressed with specific questions and prompts within each area. Interviewers were research assistants at AIR as well as project staff. Training in using the protocol and reactions to possible interviewee responses took place before the interviews began. The interviews have actually only begun as we meet here at ALM so I cannot report any findings, even initial ones. Interviewees are notified of the impending interview in advance of the telephone call. In one case, that of the New York City Math Exchange Group (NYC MEG), the interview was conducted in person with six people who are members or leaders of the program.

Areas addressed by the interview protocol are:

- Background Information: Verify initiative name, director/designer and contact information
- Costs
- Participants: Recruitment, Characteristics, Incentives
- Type and Duration of Professional Development
- Instructional Content and Materials
- Teaching and Learning Strategies
- Program Assessments and Findings
- Summary and Reflection.

Analysis and Report

Once the individual project templates have been completed, the information will be reviewed by the interviewees for accuracy. The data will be analyzed using the qualitative method of *grounded (emergent) theory.* This will allow the project team to identify any patterns that exist and compare projects, ranking each against the others according to the emergent themes. The findings will be weighed against the counsel provided by the literature review, an early task of the *Adult Numeracy Initiative* (Condelli, *et al.*, 2006). It is expected that models of "good practice" will emerge from the process, and a future research agenda for Education Department numeracy projects can be confidently suggested by the *ANI* project team. A report on *ANI* will be presented next year at ALM-14 in Limerick. As they say on television, "Stay tuned for further developments."

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Workshops

Workshop

What constitutes effective practice in adult numeracy?

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This workshop focused on a project based at King's College London, 'A study of effective practice in inclusive adult numeracy teaching' which aimed to correlate a range of factors with learners' progress. A total of 472 learners in 47 classes participated in the study across the two academic years 2003/04 and 2004/05. The research took place in a range of settings in different geographical areas of England. The workshop focused on the question: what constitutes effective numeracy practice? We conclude that there is no 'one size fits all' approach to teaching adult numeracy – flexibility is the keynote.

Our project, 'A study of effective practice in inclusive adult numeracy teaching', was one of a suite of five projects initiated by the National Research and Development Centre for Adult Literacy and Numeracy (NRDC) and funded by DfES and European Social Fund, covering areas of numeracy, reading, writing, English as a Second Language (ESOL) and Information Communication and Technology (ICT). The projects built on the *What Works Study* for Adult ESL Literacy Students directed by Larry Condelli at the American Institutes for Research in Washington DC (Condelli & Voight, 1999) which investigated the range of practices in adult ESOL classrooms, the progress made by learners, and the correlations between the two.

The numeracy study was based at King's College London (Coben et al., 2007). The project team consisted of Diana Coben and Margaret Brown (joint Principal Investigators), Valerie Rhodes, Jon Swain, Katerina Ananiadou and Peter Brown[9]. The project aimed to investigate: a range of naturally occurring variation in teaching

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[9] This paper draws on the final report of the project (Coben et al, 2007); it was written and the workshop was presented by Diana Coben on behalf of and on the basis of work by the whole team.

numeracy to learners in different settings; any correlations between different practices and learners' progress; and draw out the implications for teaching, teacher training and continuing professional development (CPD). We investigated a range of approaches to the teaching of numeracy to diverse adult learners in different settings, aiming to be as inclusive as possible. We tried to ascertain the progress learners made (or did not make), and the correlation between any progress or regress and the teaching approaches used. The project took place in two phases in the academic years 2003/04 and 2004/05, and involved 472 learners (of whom we have complete data on 250) and 34 teachers in 47 classes (17 in Phase 1, 2003/04, 30 in Phase 2, 2004/05); 30 of these classes were in further education (FE) Colleges (11 in classes for 16-19 year olds and 19 in classes for adults, including ESOL, ICT, etc), 4 in adult or neighbourhood colleges, 4 in workplaces, 2 family numeracy classes, 2 JobCentre Plus classes, 2 prison groups, 1 Army course and 1 class run by a private training provider. The classes were located in clusters around six teacher-researchers based in North Lancashire, London, Gloucester and elsewhere in England. Adult numeracy classes in the study were very diverse in terms of the range of learner ages, abilities, dispositions, purposes and aspirations. Some classes had distinctive characteristics, for example, learners with language difficulties or with poor levels of motivation. The classes took place in different settings over different time intervals, at different times of day, and had different attendance patterns.

We used a mixture of quantitative and qualitative approaches: 250 learners were assessed using a test we developed from selected items drawn from the national *Skills for Life* survey (DfES, 2003) and 243 completed attitude surveys, in both cases at two time-points in order to assess their progress. Background information was collected on teachers and learners, and semi-structured interviews were carried out with 33 teachers and 112 learners.

Findings

Many learners contrasted their (generally) more negative experience of learning mathematics at school with the positive experience of learning numeracy as an adult: over 90% of learners interviewed expressed a high level of satisfaction with their course and their teacher, and there was overall a 61% retention rate. Learners recognised that the relationship between the teacher and effective learning was critical; it was important for teachers to develop good relationships with learners and to treat, and respect them, as adults. Classroom observation indicated a high level of mutual respect.

The teachers were generally experienced and well-qualified, with many having previously taught mathematics in primary and secondary schools. The teachers' subject knowledge was generally found to be adequate. Teachers valued 'flexibility' as a key feature of effective practice. Some believed that the diversity of learners, together with the range of possible types of activity to meet different mathematical aims, meant that no one pattern of lesson activity or organisational method was optimum. Knowledge and careful preparation were felt to be important in dealing with learner diversity. We asked teachers to give us their views about effective teaching and they said that teachers need to: be flexible; plan well, with lots of variety of approach; help learners articulate what they understand; and help learners make connections.

We observed a wide range of different teaching approaches, including the way classes were organised and how resources were used. Whole class and individual work

predominated, with teachers demonstrating procedures and learners working through worksheets. Most teachers gave clear explanations, an asset much valued by learners, and also broke work down into smaller steps and gave feedback to learners about their work. Learners were usually highly engaged and teachers were enthusiastic, and generous in giving praise.

We saw few higher order questions, little pair or group work, and little use of practical resources or ICT, although activities were often varied. Teachers stressed the importance of active learner participation in discussion, in order to develop conceptual links underpinning skills, to develop language skills and to enhance social relationships. There were problems finding an assessment instrument which was both short and appropriate to sensitively and validly measure progress across such a diverse learner group. Consequently, the results do not necessarily do justice to the learning which was observed in classrooms.

Taking all the classes together, learners made significant progress in terms of test score over the duration of the numeracy courses, with an average gain of 9%. However, there was a very wide range of average gains between different classes. While eight of the 45 classes made average gains of over 15%, a few had lower average scores at the end of the course. There was very little association between the size of gains and types of learner, except that those with no previous qualifications and those wanting to become more confident, tended to make larger gains. There was no association between gains and teacher characteristics, and none between the number of teaching hours and learners' progress.

There were very few significant correlations between progress made and the extent of different classroom approaches used. The only significant positive correlations were with procedural teaching to the whole class using examples on a whiteboard (interactive or traditional). The strongest negative correlations with attainment gains tended to be with the use of resources and with a large proportion of individual work. However, none of the relations were strong and there were many counter examples; for example the use of whole class teaching and whiteboards was a distinguishing feature of the classes with the lowest as well as the highest gains. Thus it seems likely that even where there were significant correlations these factors may well not have been causative of high or low performance, but merely associated for other reasons with certain types of class.

Teaching approaches and typologies were also identified with teachers described in terms of their balance between transmission, connectionist and constructivist approaches:

- *Connectionist* teaching is concerned to develop the conceptual understanding of learners and frequently makes connections to other areas of maths including moving between symbolic, visual and verbal representations.

- *Transmission* teaching is principally concerned with mastery of skills. Mathematics is seen as a series of discrete packages to be taught in small steps with an emphasis on procedures rather than conceptual understanding.

- Using the *constructivist/scaffolder* style, the teacher works alongside learners, co-constructing concepts, asking questions. They provide a series of activities to help raise learners' thinking and conceptual understanding to a higher level.

Again, there were no significant correlations with gains made; while the two teachers of classes where learners made the most progress (over 30%) used a combination of constructivist and connectionist approach, there were also transmission-style teachers with very high gains.

Learners' attitudes generally became slightly more positive at the end of the course. The changes tended to be greatest for older people, and related particularly to a perception of numeracy as less difficult. Qualitative data suggested that once learners are able to overcome their initial anxieties, both about the course and about mathematics, and when blocks and barriers are overcome, numeracy courses can have a significant and positive effect on their identities both in general, in terms of improving people's level of confidence and self esteem, and specifically, in terms of their identity as people who can do mathematics. Some learners have been able to develop new aspirations and form new dispositions to learning. No significant correlations were identified between changes in learners' attitudes and the approaches teachers used, or any particular set of classroom characteristics.

The heterogeneous nature of adult numeracy teaching, and the number of variables amongst teachers and learners, make it difficult to produce findings that can be generalised across the whole sector. Factors, such as learners' motivations and purposes for attending the course, their aspirations, their abilities and dispositions towards numeracy, their socio-cultural background and experiences outside the classroom, may be more influential than anything the teacher does. It may also be that approaches which work well with some learners in some settings may not work well in other contexts.

We came to feel that the underlying assumption in the research design - that the greater the rises in attainment between two points in time the better the teaching - was flawed. We found classes where researchers with many years of experience in education thought that the teaching was good but learner progress was weak, and some apparently poor teaching where progress was strong. It was thus difficult in many cases to relate the observed quality of the teaching and the measured increase in learning.

How should we define effective numeracy practice?

The question of what counts as effective practice in adult numeracy education is both complex and straightforward. It is straightforward insofar as adult numeracy provision is inspected according to standards set out in the Common Inspection Framework (ALI/OFSTED, 2001) by the Office for Standards in Education (OFSTED) and the Adult Learning Inspectorate (ALI)[10]. However, little is known about effective practice in adult numeracy education from a research perspective and the tendency has been for numeracy to be overshadowed by literacy in official reports, including Inspection reports, so that information about adult numeracy is often impossible to disaggregate from that on adult literacy. The relationship between

[10] A new single organisation has been created by merging the activities of OFSTED with the children's social care remit of the Commission for Social Care Inspection (CSCI), the Children and Family Court Advisory and Support Service (CAFCASS) inspection remit of HM Inspectorate of Court Administration (HMICA) and the Adult Learning Inspectorate (ALI) (http://www.ali.gov.uk/News/Talisman/issue_48/Strategy+board+appointed.htm, accessed 17 February 2006).

effective teaching and successful learning in adult numeracy has yet to be established; this study represents a step towards this goal.

An example of effective practice

In this section we present a detailed description of one teacher's numeracy class as an example of what the research team considered to be 'effective' practice.

The class took place in a FE college in London, and ran for two hours on one evening each week. It consisted of about 14 learners working at Entry level 3 to Level 1, and the age range was 18–60 plus. The teacher was an experienced numeracy tutor; she has a PGCE in secondary education; her highest mathematics qualification is at 'A' level; and, at the time of the observation, she had been teaching numeracy for 21 years.

In terms of assessment results, this class achieved an average gain of over 30% with many learners making exceptional progress. In addition, learners' enthusiasm towards numeracy was noticeable both from the attitude surveys and the class observations. This was achieved by using a predominantly connectionist and constructivist approach which emphasised conceptual understanding rather than routine procedures. Mathematics was conceived as a network where the teachers and learners construct concepts together.

The teacher created a non-threatening atmosphere and learners' misconceptions were used as examples to discuss with the whole group. Learners were encouraged to discuss problems and concepts both between themselves and with the teacher, building a strong collaborative culture. Numeracy learning was viewed as social activity in which people took ownership of what they were doing, and where understanding was formed through discussion. A variety of group, individual and whole class teaching was used; however, even when learning was organised on an individual basis the learners were still encouraged to discuss problems and help each other, which helped to develop a greater understanding. The class was taught in an open style, which allowed higher order, diagnostic questioning that uncovered learners' thinking.

A range of materials and teaching resources were used, which ranged from worksheets to games, and activities including whole-class role-play. Calculators were freely available. The teacher used problem-solving activities which challenged the learners. She was also flexible and able to change direction to respond to the learners' needs.

Figure 7.5 below is an extract from a researcher's observation sheet. It is a narrative account that was filled in contemporaneously, and attempts to describe what was going on. For the purposes of this example, and in the interests of space, we are only taking the first hour of the session. The comments that appear in blue italics are retrospective and were not included on the original sheet; they are not intended to be exhaustive, but provide characteristics of what we believe constitute 'effective' practice. The narrative also shows how complex teaching is; how many decisions teachers have to make; and how hard they often have to work. The names of the teacher and learners have been changed.

Time	Content/focus of the session
7.00	Topic: Percentages Becky (BH) held up an individual mini-whiteboard (A4 white laminated card) with '%' hand-drawn on it. She asked the learners to tell her what it was and what it meant. In response to one learner saying it looked like a division sign she drew a division sign [÷] on the main (fixed) whiteboard and initiated a discussion about the relationship between percentages, fractions and decimals. She asks learners to call out different percentages and she writes them up on the main whiteboard . BH: "10% means divide by 10"
	The teacher asks open questions; does not give answers; initiates discussion, looks at relationships and connections and assesses learners' prior knowledge. The teaching is interactive and the teacher reinforces understanding.
7.15 8.00	BH gave learners small cards with statements on 2 lines (e.g., I have 76. Who has 10% of £6500?) Learners have to read out their questions and answer if they have the right answer, otherwise keep quiet. BH: "If your neighbour is quiet they may be asleep, so you can look at your neighbour's card." At the end Becky confirmed to the class that they knew 10%. BH (having drawn on small whiteboard): "10% of 30. So what's 5%?" "So what's 30%?" "If I wanted 90% of 500? Greg says 'take off 10%'". BH asks for a number and Greg says "300": "50% of 300? What's 75%? Half is 50%, then halve that and add it to the 150. Notice we're talking about a half and a quarter". Learners call out the answers; Becky writes on large whiteboard. BH: "Can you see a pattern? What's 55% of 300? You can do it however you like." Learners hold up their whiteboard cards as they do it. They ask each other what they've got. Becky helps one man (Moji). She asks (re 55% of 300) "What would be an easy percentage?" Moji: "50%". BH "Sandra: tell Moji what to do" (she does)."One way is to use what you know here and here" (shows examples on main whiteboard). BH points out there are many different ways of doing percentages. In some situations one method is good, in others, another method might be better. "17 and a half percent. If you think you know what to do, write it down on your board. 10%; 5%; 2 and a half%: What have they done here? Can you work out 17 and a half% of 300?" (shows it written on mini whiteboard with figures above each other) Learners work out each element and then add them together. BH asks why they've added them. Learners explain. BH: "That's VAT. It's not too bad. Now try it with my nice number (400). Just to see how comfortable you are with it, I'll give you an even nicer number (800)." Sandra gives the right answer BH: "Did you do that in your head: that's impressive. So 17 and a half% doesn't hold any threats for you. How about 63%? How will I break that down?]" (learners call out different ways of breaking down 63%). BH: "Distinguish between ones you can do in your head and more tricky ones – you'd use a calculator for those." BH: "Let's try 63% of £800". She goes around the room (using the space in the middle) helping learners as appropriate, e.g., not lining numbers up: BH: "There's a terribly dangerous thing happening to everyone in the room and it's all my fault! Karen, let me show what you did": she writes 400 wrongly aligned with the other numbers to be added. BH: "Be careful that you always find percentages of the same number (800). Always refer back to the number you're finding the percentage of." BH: "Will 63% be more than half or less than half? Always think about doing a check. There are different ways of checking. We can learn some of those as we go along." BH: (writing on whiteboard) when you see 25% what does it mean? 'a quarter', 75% three quarters; 33 and a third 'a third'.
	The teacher uses interactive games and asks questions. She builds on, and uses, learners' strategies, points out that there are many different strategies that can be used, highlights that some may be better than others, and shows learners which ones to use. The teacher is, again, getting learners to look for patterns. The learners work collaboratively; some assume a teaching role and explain strategies to each other. The teacher breaks maths down and works through examples. She points out that there are different ways of solving problems. The teacher assesses different ways of working and asks learners to justify what they've done. She breaks maths down using learners' own methods, and encourages mental calculation. She gives praise and there is appropriate use of technology. The teacher monitors learning and identifies learners' misconceptions. She emphasises need for checking and reinforces concepts learned with whole group.

Figure 7.4: A narrative account of a numeracy session from a researcher's observations sheet

Conclusions

It has proved very difficult to find clear associations between either teaching approaches or classroom characteristics and changes in learning or attitudes. The clearest positive correlations are with procedural teaching to the whole class using examples on a whiteboard (interactive or traditional). The strongest negative correlations with attainment gains tended to be with the use of resources, and with a large amount of individual work.

However, these characteristics do not necessarily define effective, or ineffective, teaching. Most of the factors observed to a greater extent in the classes with the highest gains were also observed to a greater extent in the classes with the lowest gains, which shows that they do not necessarily have a causal effect on class gains. Conversely, there is considerable variety in teaching approaches and classroom characteristics among both the highest and lowest performing classes. An example is given above of part of a lesson with one of the highest attaining groups in the study. Although this style of teaching would meet current perceptions of good practice, it does not exemplify all those characteristics which correlate best with high average gains and it includes some features which correlate with low gains. It was difficult in many cases to relate the observed quality of the teaching and the measured increase in learning and we are unable on the basis of correlative data to recommend practice. Learner, rather than teacher factors seemed to be crucial in the progress made. The effect of teaching practice was dominated by other factors. Teachers' subject-specific pedagogical knowledge was found to be important in enabling them to be flexible in their teaching.

There would not seem to be clear implications for practice. However, for teacher education and CPD teachers not only need to have a firm grasp of subject and pedagogical knowledge, but also of subject-specific pedagogical knowledge (in this case of numeracy). This will enable them to be flexible in their approaches and to cater for the wide range of diversity of learners and provision in adult numeracy. The focus for research might be towards more work on learners, since learner factors seemed to be crucial in the progress made. This might include work on initial and changing identities with respect to mathematics and education. Above all, policy makers must accommodate the diversity and complexity of adult numeracy education – there is no 'one size fits all' of effective practice in adult numeracy teaching.

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Workshop

Crossing borders between numeracy and ICT in adult numeracy teaching in Scotland

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This was an interactive workshop in which we presented our work in progress in Phase 2 of 'The use of information and communication technology (ICT) in adult numeracy teaching in Scotland', a project undertaken for Learning Connections [11](work in Phase 1 was presented at ALM12 in Melbourne in 2005). We are working with adult numeracy practitioners throughout Scotland, supporting them in developing action research projects that involve the use of a range of ICT with adult learners in their numeracy groups. The project 'meets' online on WebCT, where we share resources, hold discussions, post work in progress, etc., as well as face-to-face in periodic workshops. We presented examples of work in progress and explored issues in using ICT in adult numeracy teaching in a range of contexts.

Adult Literacies – the Scottish approach

The International Adult Literacy Survey (IALS) found that: 23% of adults in Scotland may have low skills; 30% may find their skills inadequate to meet the demands of the 'knowledge society' and 'information age'.

In response, the *Adult Literacy and Numeracy in Scotland* (ALNIS) report (Scottish Executive, 2001) was published to coincide with the launch of the Scottish Adult Literacies Strategy in June 2001. The strategy aimed to double learning opportunities within three years, working through community planning partnerships, with a budget of £65m from 2001–08 and a target of attracting 150,000 learners by 2006 (Learning Connections, 2005b). 'Learning Connections', the body which is funding this project, is the 'development engine' established to support and develop literacies practice in the Scottish Executive.

The ALNIS report sets out the Scottish approach to adult literacy and numeracy as one which appreciates the plurality of literacies, seeing them as rooted in social contexts, valuing and building on people's own literacies and relating to their lives. It promotes the development of knowledge, skills and understanding of literacies. The Scottish approach aims to: attract new learners; motivate learners; encourage self direction in learners; offer new ways of teaching and learning; increase access and extend learning hours; offer a range of learning styles; and engage with the ways that

[11] When this project was begun, Learning Connections was part of Communities Scotland, the Scottish Executive's housing and regeneration agency. From 1st August 2007, Learning Connections has been part of the Lifelong Learning Directorate of the Scottish Government's Directorate General for Education.

ICT is changing literacies. The ALNIS report championed the use of ICT in adult literacies work: "Maximising the potential of ICT" was one of four critical factors in the strategy's success. The report called for: "A quality learning experience with strong emphasis on the role of ICT". The ALNIS report also encouraged the integration of ICT into adult numeracy teaching, for example through the use of spreadsheets and proposed intervention studies in the use of ICT in teaching numeracy and the development of innovative tools to move closer to some of the fun of ICT in action.

Despite the plural notion of "literacies" in Scotland, by 2004 it was clear that numeracy was being overshadowed by literacy, a situation which the report *Adult Numeracy: Shifting the focus* (Coben, 2005) sought to address. The report noted "The growing need and demand for numeracy skills, often involving ICT, in the workplace and elsewhere" (p.xx) and recommended: guidance for tutors about adult numeracy resources (including ICT); encouragement of integration of ICT into adult numeracy teaching, for example through the use of spreadsheets; intervention studies in the use of ICT in teaching numeracy; development of innovative tools to move closer to some of the fun of ICT in action. Our project is one response to these recommendations.

The 'Use of ICT in adult numeracy teaching in Scotland' project

The project is running in two phases. Work in Phase 1 was presented at ALM12 in Melbourne in 2005 (Pullen et al., 2006); the report of Phase 1 is on the learning Connections website at www.scotland.gov.uk/learningconnections (Coben, Stevenson, Mellar, Kambouri, & Mogey, 2005). We are currently in Phase 2, which runs from November 2005-December 2006. We are supporting adult numeracy practitioners throughout Scotland in developing their action research projects. These involve the use of a range of ICTs with adult learners in their numeracy groups. The project 'meets' online on WebCT, where we share resources, hold discussions and post work in progress, as well as face-to-face in periodic workshops. The project team comprises:

- Diana Coben and Ian Stevenson, King's College London
- Jim Crowther and Nora Mogey, University of Edinburgh
- Maria Kambouri and Harvey Mellar, Institute of Education, University of London
- Sheena Morrison, Sheerface Ltd. (Development Worker).

The aim of the project is to explore, extend and improve the use of ICT in adult numeracy teaching in Scotland. We are doing this by: supporting tutors to develop action research projects with and for adult learners, exploring the use of a wide variety of ICT; promoting self reflection; developing the sharing of ideas and expertise through face to face meetings and online; providing staff development in the use of a range of technologies and their application to adult numeracy; and

developing online and electronic media-based teaching and learning materials and guidance for tutors on using ICT in adult numeracy, based on their action research projects, for use in the wider adult numeracy field.

New developments in Scotland that have supported this project include the launch of the curriculum framework for adult literacy and numeracy (Learning Connections, 2005a), together with the roll-out of numeracy and ICT training for practitioners. The launch of the dedicated website, www.adultliteraciesonline.com, www.aloscotland.com and the development of the Numeracy Network and online forum mean that the project is able to make work in progress available to tutors across Scotland and beyond.

Practitioners' action research projects

The tutors in the project have volunteered their own time to participate in the project, often on top of very busy working lives. They work in a wide range of community and education settings across Scotland for a range of funders who have different target audiences, supporting different learners' needs and with requirements to demonstrate different learning outcomes (some accredited, others not). They share an enthusiasm for using ICT and they are prepared to try out new ways of doing things, reflecting, evaluating, sharing, learning and adapting their approach. Some are supported by a team, while others are working in isolation. They have different levels of experience, expertise and confidence in teaching numeracy, doing research and in using, manipulating and teaching with ICT. The context of adult numeracy teaching in a city such as Edinburgh or Glasgow is very different from that in the rural Shetland Isles.

The tutors' action research projects, which are still under development, reflect the differences and similarities noted above (a list of the tutors and their institutions is given as an Appendix to this paper. For example, Owen Smith of Inverness College is investigating the teaching of numeracy at a distance using video conferencing, while Neil Sutherland of Fife Council is aiming to give community adult learners both an understanding of all aspects of fuel bills for checking purposes and confidence in selecting or changing suppliers, meanwhile, Trisha Tilly is working with prisoners at Cortonvale Prison on handling money in real life situations, while introducing as much ICT input as possible into the course in order to provide variation, challenge, and support motivation. Zoe Kennedy and Joe Lennon of the British Trust for Conservation Volunteers (BTCV) Scotland are using ICT and numeracy to design and create a show garden for public display.

Innovative uses of ICT in numeracy teaching on the project include incorporating sound into worksheets (particularly valuable for ESOL and literacy learners), WebQuests, mind mapping and blogging, with imaginative use of software such as word processing and spreadsheet programs to make numeracy more meaningful, accessible and attractive to learners; personal USB sticks for learners are giving learners a sense of personal control and enabling them to share ideas easily with their fellow-students and to transfer their work onto other computers if they wish.

The support strategy we have implemented to address some of these challenges comprises: the project team; regular workshops at Edinburgh University; WebCT supported by Edinburgh University MALTS, including online Chat sessions; visits to tutors by the Development Worker; and raising the profile of adult numeracy and ICT at national and international conferences, including ALM.

Throughout the project we have struggled with a tendency for ICT concerns to overshadow those of numeracy – it is all too easy to focus on the technology for its own sake, rather than on the teaching of numeracy to adults *using* technology. We are bearing in mind Lynda Ginsburg's typology of approaches to integrating technology into adult basic education, developed in the U.S. National Center on Adult Literacy (NCAL). She differentiates between:

1. technology as curriculum;

2. technology as a delivery mechanism;

3. technology as complement to instruction; and

4. technology as an instructional tool. (Ginsburg, 1998)

As in Phase 1, we are finding some evidence of tutors moving from using ICT primarily as a complement to instruction, as a delivery mechanism and a means of achieving language learning and communicative goals, towards also using technology as an instructional tool. For example, Suzi Gibb, of Bethany Christian Trust, decided to create an activity on finding out about the costs of local tourist attractions in Edinburgh and whether these were affordable to learners receiving welfare benefits. She initially put together a word processed document with a list of websites but wanted to energise the material and try to make it more interactive and engaging for her learners. The Development Officer, Sheena Morrison, conducted a one-hour telephone coaching session and introduced Suzi to using spreadsheets with hyperlinks and boxes to complete with automatic feedback and to insert images with links to video. Suzi and her manager Louise Clark then worked together to create a vibrant WebQuest which included tasks, a clear process, guidance material and a specific conclusion. The learning and teaching material was emailed back and forth between Suzi and Sheena until the final product was produced and ready for use. Once used in practice, the activity needed further adjustments. For example, initially, when learners completed answer boxes, automated feedback told them if they were wrong before they had finished the calculation; this stopped them in their tracks. Suzi and Louise told Sheena they needed a formula which led the learner forward, something which 'said' "you need to do this, then this and then this, before you know if it is right". This entailed creating nested 'if' and 'and' statements on a spreadsheet[12], which Sheena trained the tutors to do. Suzi and Louise then updated the WebQuest and tried it out with other tutors before using it with learners.

We want to ensure that both the tutors and the team are as broad-minded, creative and responsive to learners as possible about the possible uses of ICT to teach adult numeracy, so we have devised a checklist with the tutors, as follows:

[12] [12] See for example, 'Using If Statements in Excel'. URL: http://www.homeandlearn.co.uk/ME/mes7p1.html, accessed 10 February, 2007.

- What is the learning problem that's being addressed?
- How does ICT relate to that learning problem?
- Is ICT the problem?
- Is ICT being used to make numeracy more attractive/palatable, etc.?
- How does numeracy relate to the learning problem?
- How are you addressing ICT and numeracy?
- How will you know if any learning has happened as a result of your project?

This is proving useful in keeping everyone's 'eye on the ball' and we commend the checklist to all those developing the use of ICT in their adult numeracy teaching. Our final report on the project will be given at the ALM14 conference to be held at the University of Limerick in 2007 and the project report will be published by NRDC in association with Communities Scotland[13].

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[13] The final report has since been published (Coben et al., 2007).

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Appendix:

Tutors participating in the project and their institutions

Tutor	Organisation
Dot Butler	West Fife Enterprise Ltd.
*Ana Calixto	Edinburgh University Settlement Community Learning Centre
John Cameron and Darragh Hare	Adult Literacy and Numeracy Team, South Lanarkshire
*Nancy Craig	Dundee Council
*Marjorie Drew	Midlothian Adult Literacy and Numeracy Initiative (MALANI) in partnership with Jewel and Esk Valley College
Mark Frith	Inverclyde Adult Literacies Partnership
Suzi Gibb, Louise Clark* and Ruth Burton	Bethany Christian Trust, Edinburgh
Carol Gibbons	Clydebank College
Shirley Jones	Shetland College
Zoe Kennedy and Joe Lennon	British Trust for Conservation Volunteers (BTCV) Scotland, Ayr
Sheila Maher	Workers' Educational Association (WEA), Inverness
Kirsty Paterson	Angus College, Arbroath
Angela Smith	Dumfries and Galloway College
Owen Smith	Inverness College
Donna Stafford	Community Learning – Adult Literacies, Barrhead, East Renfrewshire
Neil Sutherland	Fife Council, Education Services
Trisha Tilly	Stirling Council/Cortonvale Prison
David Watson	Anniesland College, Glasgow

* These tutors also participated in Phase 1 of the project.

Workshop

Using the History of Mathematics in Teaching Adult Numeracy

David Kaye

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The workshop was an interactive session which was designed to stimulate discussion about the historical origins of a wide variety of topics from mathematics that are relevant to adult numeracy. Following this a number of activities were introduced that were based on old numeral systems and the use of tables in solving calculations. This paper will describe the session that was presented at ALM 13 followed by a brief evaluation of the event. A final section will give a brief overview of the rationale for using the history of mathematics in teaching mathematics and numeracy to adults.

The session (Part 1) – Time Line

About 15 people attended the session. The first thing they were asked to do was to leave the room and gather in the corridor outside. This gave room for the first activity, which was to form a time line. Each participant was given a card with a brief description of an event in the history of mathematics such as “Counting in 60s” or “Discovery of Zero”. (See Appendix A) The participants were asked to arrange themselves in chronological order according to the event card they were holding, thus forming a time line. This encouraged a lot of discussion and sharing knowledge to try to decide which came before which.

The session (Part 2) – Resolving the time line

In the presentation that followed each event was given an approximate date and brief explanation. This encouraged further discussion about how mathematical ideas have developed (Appendix A).

The session (Part 3) – The History & Geography of Mathematics

The presentation continued with an outline of the origins of key mathematical events, and how these are investigated by historians of science. Two points were stressed. The first was to warn against what may be called the ‘great man’ theory of history, though out of date in historical studies is still common in popular views of history. This is the view that attributes a discovery to a particularly gifted individual who produces the new idea out of nothing. It was stressed that analyses of discoveries now place much more importance on the political, social and economic context of the time, than the genius of a particular individual.

The second point was to counter the view that all of the most important mathematical developments have a European origin. This part used the work of G. Joseph on the non-European roots of mathematics (Joseph, 1991). This argues that many of the earlier discoveries in mathematics were not European, and presents in detail this early

history of mathematics. The most obvious examples include the way we count time, using base 60, is the 5000 year old mathematics of Ancient Babylon (now Iraq) and the numeral system we now use had its origins in India about 1500 years ago.

The session (Part 4) – History of Mathematics Activities

The activities centred on introducing unfamiliar numeral systems and providing opportunities to use these systems to perform simple calculations. Those selected were an alphabet based system which was originally in use 2000 years ago with the ancient Greek and Hebrew alphabets, but was in this activity adapted to this alphabet (see Appendix B). There were also opportunities to use the more familiar Roman numerals and in contrast the cuneiform numerals of ancient Babylon. There were some simple calculating exercises and addition and multiplication squares were provided as calculating aids. The historical point that tables themselves have an important place in the history of mathematics was also made.

Evaluation

Those who joined the session at ALM13 were very enthusiastic and added much to the success of the workshop. There was a very wide range of knowledge of, and experience with, the history of mathematics; the workshop encouraged questions to be asked and knowledge shared. One participant from USA was already committed to using the history of mathematics in her classes and described how her students made short plays about specific events from the history of mathematics. Another participant from Israel showed how the dates of the days in her diary were written in Hebrew characters following the ancient alphabet numeral system.

Using the History of Mathematics in Teaching Numeracy and Mathematics to Adults

The best summary of exploring the history of mathematics in teaching mathematics is in a special edition of '*For the Learning of Mathematics*' (Fauvel, 1991). In the introductory article John Fauvel summarises reasons for using the history of mathematics in teaching and ways of doing this as follows.

Some reasons that have been advanced for using history in mathematics education:

- Helps to increase motivation of the learner
- Gives mathematics a human face
- Historical development helps to order the presentation of topics in the curriculum
- Showing [learners] how concepts have developed helps their understanding
- Changes [learners'] perceptions of mathematics
- Comparing ancient and modern establishes value of modern techniques
- Helps to develop a multicultural approach
- Provides opportunities for investigations

- Past obstacles to development help to explain what today's [learners] find hard
- [Learners] derive comfort from realising that they are not the only ones with problems
- Encourages quick learners to look further
- Helps to explain the role of mathematics in society
- Makes mathematics less frightening
- Exploring history helps to sustain your own interest and excitement in mathematics
- Provides opportunity for cross-curricular work with other teachers or subjects.

Some ways of using history in the mathematics classroom:

- Mention past mathematics anecdotally
- Provide historical introductions to concepts which are new . . .
- Give "history of mathematics" lessons
- Devise classroom or homework exercises using mathematical texts from the past
- Direct dramatic activity which reflects mathematical interaction
- Encourage the creation of poster displays or other projects with a historical theme
- Setting projects about local mathematical activity in the past
- Using critical examples from the past to illustrate techniques or methods
- Explore past misconceptions/errors/alternative views to help understanding and resolving difficulties for today's learners
- Devise the pedagogical approach to a topic in sympathy with its historical development
- Devise the ordering and structuring of topics within the syllabus on historically informed grounds.

(Fauvel, 1991)

These reasons, though developed 15 years ago are very relevant to the good practice in teaching adults that is currently being promoted. The history of mathematics breaks down many of the barriers that are encountered in teaching adults mathematics. Firstly it encourages discussing the mathematics. The emphasis is no longer on interpreting symbols or accurate calculating techniques but is more about the purpose of the topic. Secondly it identifies mathematics as a cultural subject, which is open to investigation by those with skills in history, literature and art, breaking down the

barriers of the ‘two cultures’ of the humanities and sciences. Thirdly, the history of mathematics provides an opportunity for learners to know **about** mathematics, without necessarily learning how to **do** it. This extends the range of mathematical topics that can be considered accessible to a much wider group of learners.

Finally I want to give credit to all my colleagues in the British Society for the History of Mathematics (BSHM) who have given me support and encouragement in pursuing this topic. For those who want further information please consult the website http://www.dcs.warwick.ac.uk/bshm/

There are also useful sources in the bibliography (Appendix D)

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Appendix A

Using History of Mathematics in teaching Adult Numeracy

DATE	TOPIC	DETAILS
-3000 BC	Counting in 60s (like time)	Babylonian counting system in base 60
-1550 BCE	Word problems – finding missing amount	Problems that may now be solved by algebra in Ahmes papyrus
-1500 BC	12 hour day	Dividing the day into 12 hours – ancient Egypt
-300 BC	Defining a point, a line and	Euclid's elements – classical Greek geometry
600	Discovery of Zero	Zero as place holder 458 Zero as a number 628
700	Finger counting in Britain	Bede's publication explaining finger 'numerals'
825	Origins of the word algebra	The origins of the word algebra in Al Khowarizme's work
1150	Algebra (of Al Khowarizme) into Western Europe	Arabic knowledge reaches western Europe mainly through contact made in the crusades
1200	Current numerals (with zero) introduced to Western Europe	The currently used system of numerals, with zero introduced in Fibonacci's 'Liber abaci'
1478	Lattice method of multiplying	Lattice method (gelosia) first printed description in Treviso
1557	The equals sign	Introduced by Robert Recorde in his *Whetstone of witte*
1580	Introduction of symbolic algebra	Use of symbols (vowels for unknowns, consonants for knowns) in work of Viète
1687	Invention of the calculus	Published by Newton in and Leibnitz in
1800	Metric system of measurements	The metric system designed during the French revolution
1825	Non-Euclidean geometry	Beginning of modern mathematics in 19th century
1937	Definition of the decibel	The measures of the intensity of sound agreed at a meeting
1960	Establishment of the SI units	The international system of units was agreed

Time Line Activity

Appendix B

Alphabet Numeral Grids

This alphabet numeral system is based on several systems in use about 2000 years ago, the most well known being the Greek and Hebrew alphabets.

The basis of the system is this:

the first nine letters represent the numbers 1 to 9 (counting in ones)

the next nine letters represent the numbers 10 to 90 (counting in tens)

the next nine letters represent the numbers 100 to 900 (counting in hundreds).

A similar sort of 'code' can be made using our familiar alphabet.

For example:

D = 4

N = 50

X = 600

and therefore XND = 654

and similarly YE = 705

(Mathematically it is important to notice that there is no zero in this system)

Now try these.

3 is		B is	
14 is		J is	
100 is		JA is	
5 is		OG is	
55 is		T is	
555 is		TF is	
999 is		ULC is	

Appendix C

Addition Grid

	A	B	C	D	E	F	G	H	I	J
A	B	C	D	E	F	G	H	I	J	JA
B	C	D	E	F	G	H	I	J	JA	JB
C	D	E	F	G	H	I	J	JA	JB	JC
D	E	F	G	H	I	J	JA	JB	JC	JD
E	F	G	H	I	J	JA	JB	JC	JD	JE
F	G	H	I	J	JA	JB	JC	JD	JE	JF
G	H	I	J	JA	JB	JC	JD	JE	JF	JG
H	I	J	JA	JB	JC	JD	JE	JF	JG	JH
I	J	JA	JB	JC	JD	JE	JF	JG	JH	JI
J	JA	JB	JC	JD	JE	JF	JG	JH	JI	K

Multiplication Grid

A	B	C	D	E	F	G	H	I	J
B	D	F	H	J	JB	JD	JF	JH	K
C	F	I	JB	JE	JH	KA	KD	KG	L
D	H	JB	JF	K	KD	KH	LB	LF	M
E	J	JE	K	KE	L	LE	M	ME	N
F	JB	JH	KD	L	LF	MB	MH	ND	O
G	JD	KA	KH	LE	MB	MI	NF	OC	P
H	JF	KD	LB	M	MH	NF	OD	PB	Q
I	JH	KG	LF	ME	ND	OC	PB	QA	R
J	K	L	M	N	O	P	Q	R	S

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Workshop

What should pedagogy for adult numeracy consist of? Or it ain't what you do but the way that you do it.

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The current context of professional development of adult numeracy teachers in the UK gives a welcome opportunity to discuss what should a training programme for teachers consist of. We are, at the moment, one of the parties involved in the revision of the Subject Specifications for adult numeracy and the content of an appropriate pedagogy for adult numeracy is on our minds.

In writing the draft documents that we have so far produced we have been developing our thinking of what is appropriate in such a document and how it may be used. Exemplifying general teaching and learning standards, such as 'apply appropriate initial assessment procedures' does not appear to be a positive move forward. In our view it is more sensible to start from what appears to be specific to adult numeracy learning and teaching. Another issue for us is the level of detail of such descriptions which may produce no particular disagreement with the profession over content but may cause problems if it is expected that all descriptors need to be evidenced in some audit process.

We argue that any document which purports to describe a pedagogy for adult numeracy will be situated in a particular time and context and should be thought of as indicative rather than definitive. It is our view that a pedagogy of adult numeracy is not a static object but a dynamic entity that should be constantly revisited and revised, and that any description of it should be understood as a time constrained, best shot that should be tested and subjected to serious critique. We welcome such critique and look forward to discussions that move forward the development of teachers.

In recent years there has been a sea-change in teacher training in England for adult language, literacy and numeracy. The professional development of teachers has come to the fore in many countries including the UK. International perspectives on these have been discussed in a number of places. In mathematics education, the US has seen a special edition of the CBMS / AMS / MAA publication, The Mathematics Education of Teachers (2001), the UK has seen a lot of work in primary school teachers education in issues of the journals of BRSLM (**http://www.bsrlm.org.uk/**) and AMET (**http://www.amet.ac.uk/**). In Australia, the National Staff Development

Committee for Vocational Educational and Training has developed some extremely useful and influential training packages and materials for adult numeracy. In addition Maguire and O'Donoghue (2004) and Maguire et al (2004), have discussed some issues of professional development of adult numeracy teachers in Eire, Denmark and the UK.

In many ways the current UK context offers some optimism for the development of adult numeracy education. The requirement that all new teachers should be appropriately trained, and the existence of targets to encourage all teachers to take part, has created the space for real development in adult numeracy education. Indeed, the majority of participants on programmes run by the LLU+ have expressed some very positive views related to their training (see Griffiths and Kaye (2004)). Nevertheless, the same learners also expressed some concern over the lack of particular elements of content of these programmes, namely, an adult numeracy pedagogy.

Some key questions have occupied our thoughts: what skills and knowledge are required of adult numeracy teachers and how should they be covered in teacher education programmes?

The chart below highlights some key stages in the evolution of the attempt to standardise training and professionalise the post compulsory sector in England and Wales.

Year	Development
Up to 2001	No compulsory teacher training qualification required to teach numeracy in the post-16 sector although a small selection of short specialist programmes are available (e.g., C&G 9281, 9285)
2001	Success for All – government strategy requires all adult/further education teachers to hold or be working towards a full certificate in education (or post-graduate CertEd). Approved qualifications must meet national standards for teaching + supporting learning
2002	Numeracy, literacy teachers now required to demonstrate that they meet set specifications in their subject area, in addition to a generic teaching qualification
2006	Generic teaching standards and subject specifications under review for implementation in September 2007

While efforts to raise the professional status of numeracy teachers are welcome, the attempt to do this via the standardisation of programmes causes some concern. Following the introduction of the Subject Specifications for adult numeracy teachers in 2002 (DfEE (2001)), it soon became apparent that making teachers 'do some hard sums'[14] and giving them some background information on personal and social factors affecting learning was not really equipping them to teach their subject. Aside from the evaluations from our own trainees, this concern has been documented by Office for Standards in Education (Ofsted) (Ofsted (2003)) and by the National Research and Development Centre for adult literacy and numeracy (NRDC) (Lucas et al (2003)).

[14] It was mentioned to the authors that some of the original subject specifications were included to 'satisfy the professors of hard sums' i.e. meaning that some mathematicians would be concerned if a certain level of mathematics was not included.

Clearly, something was missing. At LLU+ the feedback from our own teacher training programmes was that while the course sessions were fun and participants were exposed to imaginative variety of teaching methods, they did not feel they were learning as much as they would have liked that would be useful to them in the numeracy classroom. To this end, we began enriching our programmes on offer with opportunities to explore mathematics and numeracy at a basic level and to discuss and evaluate ways to teach it. Therefore, when a request came from the government Department for Education and Skills (via the London Strategic Unit) for us to draft a paper outlining a pedagogy for adult numeracy, we had lots of ideas to put into it. This then became a working document which has led into our contribution to the current revision of the subject specifications for numeracy teachers.

The recognition that general pedagogical skills and knowledge and subject knowledge need to be accompanied by subject pedagogical knowledge has led to an integration of the generic standards for teaching and the specific skills and knowledge that it is felt numeracy practitioners need to teach their subject. LLU+ is playing a role in this revision and this has led us to consider the following questions:

- What do we mean by a pedagogy for adult numeracy?
- What should be in it?
- What role should it play in teacher training?
- And who decides?

The purpose of this paper is to explore the issues raised by these questions.

Adult Numeracy Pedagogy – what does it mean?

Dictionary definitions of 'pedagogy' include the following:

- pedagogy noun [U] SPECIALIZED the study of the methods and activities of teaching Cambridge http://dictionary.cambridge.org/
- Ped-a-go-gy … [uncountable] formal the practice of teaching or the study of teaching Longman http://www.ldoceonline.com/

The following is a definition from Wikipedia

> Pedagogy is the art or science of teaching. The word comes from the ancient Greek Paidagogos, the slave who took children to and from school. The word "paida" refers to children, which is why some like to make the distinction

> between pedagogy (teaching children) and andragogy (teaching adults). The Latin word for pedagogy, education, is much more widely used, and often the two are used interchangeably.
>
> Wikipedia

Nevertheless to many, pedagogy is a slippery concept, one that maybe always existed but was never fully defined. The use of the word 'art' implies a creative process. 'Science' and 'study' suggest a publicly accepted and researched (but not necessarily static) body of knowledge, while 'practice' indicates a set of skills, with echoes of apprenticeships and mentors.

In the arena of teacher education, Shulman (1987) introduced the term 'pedagogical content knowledge' in a paper that investigated a number of elements that might be described as pedagogy. Shulman was making a distinction between the practical issues of methods and techniques and the theoretical perspectives and rationale for those techniques. The author also emphasised, and continues to do so, the subject specific nature of teaching and learning.

> I think that there is a great deal to be learned from the generic approaches. But at the same time, I've been struck by how incomplete these programs are and how much they leave unexamined that is absolutely essential to improving teaching. Teachers never teach something in general -- they always teach particular things to particular groups … in particular settings.
>
> Shulman in interview
> **http://www.nsdc.org/library/publications/jsd/shulman131.cfm**

The importance of context and how it might affect both what is taught and how it is taught gives rise to the following problem: it is impossible to define a body of knowledge and skills that would address every single teaching context that a numeracy practitioner might find themselves in. Therefore some generalisation is inevitable. But is it possible to distil those skills and knowledge that will equip a teacher for every possible situation?

A further issue is the nature of the difference between a pedagogy for adult numeracy and a curriculum for trainee numeracy teachers. This is particularly challenging if you believe, as we do, that a pedagogy for numeracy should not be an immutable set of standards and content, a 'cure-all' that is applicable to all circumstances. There are parallels here with epistemologies of mathematics itself. If one takes a fallibilist perspective, i.e. that mathematics is invented rather than discovered, and as such is socially constructed, time and place-dependent and subject to change and revision, then the art, science, study or practice of teaching it must be similarly open to critique and revision. Furthermore, if pedagogy, like mathematics, is invented rather than 'out there' in some sense, then the fact that it is created from a particular perspective and a particular culture should be made transparent. In other words, any pedagogy should be itself reflexive, inasmuch as the very critique of it should form a part of it. This gives rise to the second and third questions outlined in our introduction. What exactly is it that should be in a pedagogy for numeracy, and how does it relate to what we want numeracy teachers to do or be able to do during and by the end of their training?

Adult Numeracy Pedagogy – what should be in it?

One possible starting point for a consideration of pedagogy content is the schools teacher training curricula for primary and secondary school mathematics in England. Each curriculum is structured in three sections:

(A) Pedagogical knowledge and understanding required by trainees to secure pupils' progress in mathematics

(B) Effective teaching and assessment methods

(C) Trainees' knowledge and understanding of mathematics.

Some examples from the Secondary Mathematics ITT curriculum follow (see TDA website http://www.tda.gov.uk/upload/resources/doc/a/annexg.doc)

Section A 2. Trainees must be taught that pupils' progress in mathematics depends upon them teaching their pupils: e. to be accurate and rigorous, including the importance of : using mental and written methods to give approximate answers to computations, *e.g. to make a mental approximation prior to computations done on a calculator.*
Section B 7. Trainees should be taught to plan mathematics teaching, identifying the knowledge, skills and understanding which pupils are to acquire, and build on and, where appropriate, relating them to other areas of mathematics, including: i. giving sufficient attention to oral and mental work; ii. making effective use of purposeful enquiry within mathematics; iii. consolidating and practising skills on a regular basis; **etc.**

And the Primary Mathematics ITT curriculum as described in the 4/98 Circular (DfES (1998)):

Section A

3. Key aspects of mathematics underpinning progression

In order to understand how to develop pupils' mathematics, all courses must ensure that trainees know and understand the following key aspects of mathematics. They must be taught how and why the different elements work, how they are connected and how they underpin pupils' progress in developing understanding of, and skills in, mathematics.

a. *Structures and operations*, including:
i. the structure of number *e.g. order and size*
ii. the conceptual links between different aspects of number *e.g. place value, zero, fractions, powers of ten, and how the relationship between these provides a conceptual framework for decimals;etc...*

Section B

6. As part of all courses, trainees must be taught:

a. how to teach accurate and rapid **mental calculation**, through ensuring that pupils:
i. identify and use the properties of number and the relationships between them: size (including estimation and approximation), order and equivalence;
ii. understand the operations of addition, subtraction, multiplication and division;
iii. have instant recall of number facts, including multiplication tables;
iv. use known number facts to derive others

etc.

Section C

13. Subject knowledge and understanding

As part of all courses, trainees must demonstrate that they know and understand:	***To underpin the teaching of the Key Stage 1 and Key Stage 2 programmes of study, including:***
b. mathematical reasoning and proof • the correct use of =, ≡, ⇒, ∴; • the difference between mathematical reasoning and explanation, as well as the proper use of evidence; • following rigorous mathematical argument; • familiarity with methods of proof, including simple deductive proof, proof by exhaustion and disproof by counter-example	***for example:*** -demonstrating and checking a particular case; -the dangers of drawing conclusions after an event has occurred a few times; -recognising the difference between something that happens occasionally and something that will always happen; -using experimental evidence to determine likelihood and to predict; -proving, for example, that numbers divisible by 6 are also divisible by 3 (deduction); -proving, for example, that there are only 11 unique nets of cubes (exhaustion); disproving, for example, that any quadrilateral with sides of equal length is a square (counter-example)

The following points are of interest:

The first is that the division of teaching and assessment methods from pedagogical knowledge relating to mathematics follows the distinction that Shulman makes (1987). In both curricula there is an attempt to identify those aspects of pedagogical knowledge and understanding that are specific to the teaching and learning of mathematics, while in Section C there is an attempt to highlight those aspects of mathematics knowledge and understanding that are relevant to the teaching of it at Key Stages 1 and 2. This feels much more useful than the some of the content of the Subject Specifications for Adult Numeracy Teachers, where it seems there has been little or no attempt to relate mathematical knowledge with numeracy pedagogy. However, in the schools documents there appears to be some lack of clarity in the decisions made regarding what goes in what section. This will be returned to later.

Neither curriculum claims to be comprehensive, and initial teacher training providers are expected to include additional content of their own devising on their courses. Nor is either curriculum claiming to be a course model, so, for example, it is not expected that sections A, B and C above would be taught discretely. In the Secondary School Mathematics document (for those teaching 11-18 year olds), section C is minimal, as it is expected that trainees will have covered a large amount of mathematics during their degree programmes (although it is suggested that an audit of mathematics skills is carried out).

The terminology used causes us some concern: the use of the words ‘required’ and ‘secure’ in the title of section A implies that there is a fixed and defined set of knowledge and understanding that will guarantee the progress of pupils (and as if student progress itself is a linear process that can be ticked or not ticked on a checklist). Similarly, we question the assumption in the title of Section B that teaching and assessment methods can be effective in themselves. Even if a measure of effectiveness could be agreed upon, surely that would depend on where, when and how such teaching and assessment methods were utilised. The skill of the teacher comes in choosing what approaches to employ and when, and the knowledge of a wide range of methods in the first place.

Finally, the use of the words ‘should’ and ‘must be taught’ throughout make us wonder why it is felt that such a level of prescription is necessary. Certainly in the UK, while teacher training qualifications have been required for school teaching, as is noted above, it is only more recently that the government has attempted to describe what should be in such programmes. Indeed, the circular 4/98 which describes the mathematics and pedagogy that primary school teachers should all be aware of, did not produce a particular backlash although there is some evidence that providers were concerned with the workload involved in evidencing skills.

> … the subject knowledge requirements of the Standards are very demanding, but there is a very great latitude in the way that they are being interpreted, particularly in relation to the evidence institutions expect tutors to produce for their own internal purposes or for Ofsted inspectors, who sometimes have a rather narrow content based focus. In both cases we are up against the obsession that the system has at all levels with summative assessment, which runs counter to all the evidence that formative assessment is much more effective in raising standards. We must constantly counter the arguments for more testing by providing evidence that there are other much more effective ways of raising standards, both in teacher education and in school mathematics.
>
> French (2003)

The proceedings of the British Society for the Research in the Learning of Mathematics (BRSLM) have many references to the professional development of primary school teachers (eg Rowland et al (2003)) yet there is no suggestion that the knowledge and skills included in the circular were problematic to teacher educators. We suggest that it is likely that the content of the circular had developed from practice and had little content that was considered controversial. This is in stark contrast to the specifications for adult numeracy which appear not have developed from practice and have an emphasis on higher level personal mathematics skills that do not appear to have much direct relevance to classroom teaching.

We also question the focus in teacher training programmes on formal assessment of trainees' personal mathematics skills, possibly at the expense of exploring the related pedagogical issues.

> It could be argued that one of the most important aspects of an ITE course should be about helping students to gain an understanding of some of the "big ideas" in mathematics. This is difficult to achieve if monitoring the acquisition of a large amount of mathematical content becomes the dominant feature of all courses.
>
> Harries and Barrington (2001)

Kaye (2006) highlights those aspects of mathematics and numeracy that are not focussed on discrete skills, such as historical and cultural perspectives. Such perspectives on the nature of mathematics also need to be reflected in any account of how the subject should or could be taught, and as such, have an important place in numeracy teacher education.

The difference between essential and useful knowledge and skills for trainee teachers is a thorny one, and is illustrated by the example given below. On a course we run at LLU+ for experienced numeracy practitioners training to be teacher trainers, we designed an activity to help them to try to identify and classify the pedagogical and subject knowledge, skills and understanding that it might be useful for trainee teachers to have in the curriculum area of fractions.

We came up with a list of elements, some taken from textbooks on primary school mathematics teaching (Mooney et al. (2002)) and others arising from observations of trainee teachers that we had carried out on previous teacher training programmes, and asked the trainee teacher trainers to classify them according to three headings that were similar to those given in the schools curricula above. Our own classification was as follows:

Teaching Fractions - Examples of underpinning skills and knowledge for trainee teachers

Subject knowledge

- **Definitions of a fraction**, e.g. part of a whole, a comparison between a subset and a whole set, the point on a line between 2 whole numbers, the result of division of whole numbers, comparing the sizes of 2 measurements or sets of objects, as a proportion etc
- **Conceptual understanding of operations of fractions** (e.g. what does $^{3}/_{8} \div {}^{4}/_{5}$ actually mean?) along with the algorithms for these operations
- **Alternative methods for finding fractions of quantities (**e.g. ¾ x $^{100}/_{1}$ or 100 ÷ 4 then x by 3)
- **The history of fractions** and their application in real life
- **The relationship between fractions, decimals and percentages** and different ways of calculating these equivalencies
- **The significance of being able to manipulate fractions in algebra**

Pedagogical knowledge

- Typical learner **errors and misconceptions** relating to fractions and their sources
- Possible **progression** (e.g. different models of what to teach in what order)
- When to use **'deep' or 'surface' approaches** (simply teach the algorithms or focus on conceptual understanding?)
- When to use **holistic or sequential approaches** (e.g. use a fraction wall to compare fraction sizes or find the lowest common denominator and convert to equivalents)
- Factual knowledge of elements and sample activities in the **Core Curriculum** relating to this area
- Possible **contexts** in which to teach fractions
- **Rationale** for teaching fractions
- **Research** related to teaching and learning fractions
- Analyse the **language associated with fractions**
- **Links with other curriculum areas**, e.g. measures, shapes

Teaching methods

- Have a repertoire of teaching and learning strategies relating to fractions and be able to select the most appropriate ones for a given group of learners, with reference to learning theories and other factors
- Identify strategies for responding to common errors and misconceptions in this area
- Be aware of the range of resources available for the teaching and learning of fractions and be able to select, design or adapt ones that are fit for purpose
- Be able to design formal and informal assessments for learning related to fractions
- Analysis of teaching activities related to fractions, including identifying underpinning theories of learning

The identification and classification of these elements was felt by those that tried it to be a useful exercise for preparing for teacher training, inasmuch as it provoked much

discussion and argument over the choice and placing of the content. However, it raised many further issues regarding pedagogy and teacher training.

One was how to classify the elements given. Every time we run this activity, different groups argue over different ways to classify the content, and we ourselves find changes in our views over time. But perhaps the classification itself is not important. In a teacher-training programme, one would hope that such content would be integrated in any case, rather than taught discretely in the sections given. A more important question, perhaps, is the justification of such content in the first place. Most of the practitioners with whom we have used this exercise have agreed from a 'common sense' perspective that the elements listed above capture some of the skills and knowledge that are useful for numeracy teaching. But we wonder if there is a case for more research in this area in order to provide a rationale for such content.

Another issue is that the list is by no means comprehensive (and is in fact a shortened version of a much longer list). Does this matter? Perhaps the aim of a pedagogy/teacher training curriculum should be that it initiates a process rather than attempts to cover everything there is to know about teaching a particular topic or area. However, the very model itself on which the above table is constructed is questionable. Why choose fractions as a discrete entity? Should 'fractions' be taught as an isolated topic? In selecting a numeracy curriculum area in which to exemplify subject and pedagogical knowledge, we have unintentionally prescribed the very thing we wished to avoid, namely that curriculum areas are taught in isolation from each other. There is some attempt to rectify this by the inclusion of elements such as 'links with other curriculum areas' and 'possible contexts in which to teach fractions', but nevertheless, there are still many assumptions apparent in the construction of such a document. Also missing, for example, is the diagnosis of existing learner knowledge, experience and understanding of this area, and how to utilise that in further learning, or teaching by theme or a wider, shared, authentic purpose rather than treating the curriculum as separate sets of skills that are to be acquired in a linear fashion.

This highlights the issue of how to construct an outline of a pedagogy for numeracy. A model that starts with generic skills (e.g plan a lesson) becomes meaningless unless accompanied by exemplification for numeracy, which carries with it the danger that the exemplification itself is seen as prescriptive. But starting with the subject content (in this case, fractions), brings with it its own dangers, such as the implication that this topic be treated as distinct, or the lack of emphasis on the importance of starting from where the students are.

In our own pedagogy document, commissioned by the London Strategic Unit, we used the following section headings:

- Key areas and principles
- Examples of underpinning theory and policy

- Exemplification in numeracy teacher training
- Exemplification in numeracy classroom.

Some examples are given in the table below, with some further examples in appendix 1.

Key Areas (and principles) **Language and numeracy** Mathematical thinking can be developed through the use of language. Teachers therefore need strategies for developing learner use of numeracy language, for moving key terms and phrases from passive to active vocabularies, for promoting mathematical discussion in the classroom and for developing the numeracy communication skills of presentation, justification and explanation Issues related to the learning of mathematics in English as Another Language, and the implications of this for teacher and curriculum
Examples of underpinning theory and policy Mathematics Register (Fullerton 1995) Laborde (1990) Pimm (1991)
Exemplification in numeracy teacher training The use of language and literacy teaching strategies to help learners develop their numeracy vocabulary Exploring strategies for promoting discussion and communication in the numeracy classroom Strategies and resources for using numeracy as a vehicle for literacy and language development
Exemplification in numeracy classroom Writing key terms on the board and asking learners to repeat them (e.g. 'width', 'metres squared') Activities which encourage the active use of numeracy terms such as one learner guessing a shape by asking another learner to describe its properties. Eliciting the informal maths understanding of learners and translating this into formal maths notation.

The aim was to capture the skills that numeracy teachers use when teaching, and the background knowledge needed to support those skills. As with the design of the activity about the skills and knowledge required to teach fractions outlined above, several questions arose during the construction of this document. One was the decision about what to put in and what to leave out. Another was how to justify our choices. Wherever possible, we included relevant rationales and links to relevant theory research policy documentation. However, it is impossible to be truly objective in terms of coverage. We, as much as anyone, have our own ideas about what 'ingredients', in terms of skills and knowledge, a numeracy teacher needs. It was therefore important to be transparent about our standpoint. And because of this, we were at pains to emphasise that the pedagogy document was a 'working' one, not one set in stone.

Some of the content arose from issues experienced during the considerable number of numeracy lesson observations that we as a team have carried out with our course participants, for example, teachers only using one method of calculation and not being open to methods that learners themselves may already use. However, this gave rise to the issue of how to avoid being too prescriptive. As mentioned earlier, we did not want to produce a 'how to teach numeracy' manual, mainly because we believe

that there is no single ‘correct’ way to teach numeracy. Our aim, therefore, was to produce a broad spread of areas of skills and knowledge that it would be useful for numeracy practitioners to have in order to be able to select the most appropriate teaching methods for their particular contexts at this particular time.

What role should a pedagogy for adult numeracy play in teacher training?

Training courses for primary and secondary school mathematics teachers that we have been involved with, either as trainers or participants, have involved spending considerable amounts of time ‘playing’ with numbers and mathematical concepts and exploring the implications of teaching them. Up to now in England there has been very little of this on teacher training programmes for adult numeracy, the focus being on developing trainees’ personal mathematics skills at a higher level, and on ‘generic’ teaching skills. Such ‘generic’ skills include; finding out about institutional support structures for students with specific needs; describing and evaluating initial assessment procedures; discussing summative assessment and so on. This is partly because on most general teacher training programmes in the post-16 sector in England, numeracy teachers are thrown in with, for example, construction teachers, hairdressing teachers, business administration teachers and others, and so the pedagogical skills looked at can only ever be generic ones.

With the revision of the national standards and specifications, the emphasis is now on considering how general teaching skills can be interpreted in a numeracy context, with the idea of increasing the number of teacher training programmes on offer especially for numeracy practitioners. This gives rise to the question of how to plan and run such programmes, and in what ways a pedagogy for adult numeracy can be integrated.

Some approaches that we have taken to integrating numeracy teaching-related issues into our training programmes at LLU+ are as follows:

- Modelling activities and methods that can be used (or adapted for use) in numeracy teaching, with evaluation by trainees.
- Exploring mathematics at a basic level and discussing teaching implications. For example looking at connections between topics and curriculum areas and how this might affect course planning.
- Setting up numeracy teaching planning activities for trainees to carry out (e.g. plan an activity to introduce the topic of volume), with appropriate support, intervention and feedback from the trainer (e.g. helping trainees in selecting a suitable context based upon learner needs and experience).

- Examining examples of numeracy lesson plans and finding ways to enhance or improve them, for example, adding in language based objectives and appropriate activities to cover these.

- Considering case studies of particular learners or groups of learners and how they might be supported in learning numeracy.

- Evaluating resources, including real and ICT-based materials, and discussing how they may be used or adapted. This may include critiquing the supposed 'level' that a resource is pitched at and how it may be scaffolded or changed to accommodate students working at other levels.

- Analysing common learner 'errors' and discussing how to respond to them.

- Role-play, for example peer-teaching.

- Looking at mathematics from historical or cultural perspectives and discussing how this might impact on the teaching and learning of it.

- Engaging with relevant theory and research relating to the teaching and learning of numeracy and mathematics.

- Reflecting on own practice.

Although we try to avoid being too prescriptive, a significant minority of our course participants want just that – an all-purpose ready-made 'how to teach' bundle of ideas for activities that they can take wholesale and use with their students. We try to encourage trainees to evaluate any teaching methods that we use and consider how they would adapt them for their own contexts. It is noticeable, however, and perhaps understandable, that newer teachers simply want to apply any new methods acquired much as they are. Those who are more experienced are more apt to reflect on alternative approaches and their applications in different contexts.

Does this mean that training programmes which are packed full of 'ideas for teaching numeracy' are appropriate for beginner teachers? The danger is that by demonstrating, for example, the use of clock faces to introduce halves and quarters, it may appear that we are advocating that this is the only way to introduce halves and quarters. This suggests then that the modelling of teaching methods and activities should therefore be tempered with critical evaluation and consideration of alternative approaches. However, it is important to recognise that professional development for teachers is a process, not a race towards an end 'product'. The aim of teacher education, we feel, should surely be to enable practitioners to become reflective and critical regarding their own practice, and that of the teaching community as a whole, and to assist them in developing the tools with which to be critical, rather than turning out supposed 'ideal' teachers. This may mean trying out all sorts of 'packaged' methods and resources as a new teacher before being able to analyse and evaluate them in the light of experience.

Another issue is that some of the numeracy-related pedagogical content of our courses has only an indirect link with teaching, and our trainees sometimes find it difficult to see the relevance of such content, for example, a discussion of views on the epistemology of mathematics. Making explicit the rationale for the inclusion of such content (in this case, possible links between people's views of mathematics and the ways they teach it – e.g. Ernest (1989), Lerman (1990)) is therefore essential.

Numeracy pedagogy does not only have implications for input in teacher training sessions. The roles of lesson observations and mentoring play a key part in the professional development of practitioners. By what criteria are we judging the effectiveness of a lesson? There is a danger with observations that we simply want to turn the observee into a version of ourselves. Perhaps the emphasis in observation should be less upon what the observed teacher does per se, but more on what their rationale is for the methods and approaches selected, and how they evaluate these afterwards. Can numeracy pedagogy here play a role in establishing such a rationale, and the criteria against which to (self) evaluate?

Conclusion

As previously mentioned, a focus on the professional development of teachers is a welcome move. While it is not expected that any set of descriptors or training programmes will work with all teachers in all circumstances, it is helpful if such work has the confidence of the profession at large. The version of training that followed the introduction of the original subject specifications for adult numeracy did not have that confidence and necessitated a change of tack. The work so far on the revisions of the subject specification shows that the profession can accept a set of descriptors, although there are concerns about how much is reasonable to evidence. For us the precise content of any specification is not that important. The development of teachers is something that will take time and, as mentioned above, no one course will ever produce the perfect numeracy teacher. Rather the professional development of teachers is a 'lifelong' process and will require ongoing work. Given such circumstances it seems appropriate that whatever the redefined specifications are, we allow institutions some flexibility in interpretation and accept that over time guidance will need to change.

In addition, notions of what constitutes 'good practice' and an appropriate curriculum for numeracy learners will change over time. Part of any curriculum for numeracy teachers should therefore incorporate the critical skills needed to question the very content of such curricula, evaluating it against theory, research, experience and the cultural, social and political climate of the time. Cooney (2001) claims that the aim of teacher education is to develop 'clever' teachers who can envision a different world of teaching but operate within their own classroom constraints. We prefer to replace the word 'clever' with 'critical', but the premise remains the same. The question then remains as to what guidance teacher educators need, and whether a prescribed set of standards can in themselves achieve this aim.

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Appendix 1: Examples of content from the working document describing a pedagogy for adult numeracy

Key Areas (and principles)	Examples of underpinning theory and policy	Exemplification in numeracy teacher training	Exemplification in numeracy classroom
Assessment in numeracy What counts as evidence in numeracy assessment (and for what purpose)? The role of informal assessment in numeracy such as observation of tasks or discussion is essential in terms of subsequent choices made with regard to teaching approaches. Using results to inform planning and using feedback to promote further learning in numeracy are also areas which should be explored in some depth, as there can be missed opportunities for numeracy learning through the way that feedback is given.	M4L Stage 2 Pathfinder on formative assessment Black and Wiliam (1998) Iddo Gal (1998)	Evaluation of written or ICT based numeracy assessments in terms of validity. Exploration of other forms of assessment in numeracy, and of ways to use information gained, e.g. through role play, or watching then pausing videos of lessons to discuss what learning is taking place and what the teacher could do next.	Using mini whiteboards to ascertain learning or understanding – e.g. draw a picture of $^2/_5$ Using feedback from numeracy assessment to promote further learning – e.g. 'Does your answer seem sensible?' 'Explain your method. Can you think of a different way to work this out?' Using open ended questions to ascertain understanding of concepts, e.g. if this □ is ¼ then draw what you think the whole shape might look like.
Understanding of numeracy curriculum areas and the connections between them A thorough 'at homeness' with the Adult Numeracy Core Curriculum areas and the possible progression for each area beyond Level 2 (e.g. how some areas of Number lay the foundations for algebra). A wider understanding of the contexts and applications of curriculum topics, both at Core Curriculum and higher levels.	Maths Explained for Primary School Teachers – D. Haylock (2005) (Paul Chapman) Primary Mathematics – Teaching Theory and Practice – Mooney et al (Learning Matters) Other maths texts at higher levels	Exploring concepts at Core Curriculum Level, e.g. using squared paper (or folding pieces of paper) to explain or interpret what is meant by $^2/_5 + {}^1/_3$, $^2/_5 - {}^1/_3$, $^2/_5 \times {}^1/_3$ and $^2/_5 \div {}^1/_3$,	Being familiar with a range of methods for, say, long multiplication, and thus being able to show flexibility in the teaching of this and start with the methods that learners feel most comfortable with. Being better able to explain/discuss place value and the role of zero with learners as a result of teacher exploration of non-positional number systems
History of mathematics Knowledge of different number systems, systems of measurement and their development over time.	Arguments from the British Society for the History of Mathematics, e.g. critiques related hierarchy Georges Ifrah (1994)	Exploration of different topics	Discussing with learners a brief history of Number, in order to situate current curriculum content.

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