Numeracy for Empowerment and Democracy?

ALM8

Proceedings of the 8th International Conference on

Adults Learning Mathematics

ALM

28-30 June 2001
Roskilde University, Denmark

Numeracy for Empowerment and Democracy?

Proceedings of the 8th International Conference of
Adults Learning Mathematics (ALM8)

A Research Forum

28-30 June 2001
Roskilde University, Denmark

Edited by
Lene Østergaard Johansen and Tine Wedege

Centre for Research in Learning Mathematics in association with
Adults Learning Mathematics – A Research Forum.

Numeracy for Empowerment and Democracy?
Proceedings of the 8th International Conference of
Adults Learning Mathematics (ALM8) – A Research Forum

Edited by Lene Østergaard Johansen and Tine Wedege

First published in Denmark in 2002 by the Centre for Research in Learning Mathematics, Roskilde University, in association with *Adults Learning Mathematics – A Research Forum* (ALM).

© 2002 Copyright remains with the writers. All rights reserved. No part of this publication may be reproduced in any form or by any means without the permission of the publishers.

Printed by Roskilde University Printing.
Cover – Special-Trykkeriet Viborg.

Additional copies are available from:

Centre for Research in Learning Mathematics
Roskilde University
PO Box 260
4000 Roskilde
Denmark

Avanti Books
8 Parsons Green
Stevenage
SGI 4QG
UK

ISBN 87-7349-540-9

About ALM

Adults Learning Mathematics - A Research Forum (ALM) was formally established at the Inaugural Conference (ALM - 1) in July 1994 as an international research forum with the following aim:

> To *promote the learning of mathematics by adults through an international forum which brings together those engaged and interested in research and developments in the field of adult mathematics learning and teaching.*
> Within ALM the term mathematics is understood to include numeracy.

ALM and Charitable Status
In July 1999, the membership formally agreed that ALM should take all necessary steps to become a Company Limited by Guarantee (in England and Wales), and that the trustees appointed should seek charitable status. These objectives were achieved in the academic year 1999/2000. ALM is now formally a Registered Charity (Charity Number 1079462) and a Company Limited by Guarantee (Company Number 3901346).

The company address is: c/o Prof Sylvia Johnson, School of Education, Sheffield Hallam University, 36, Collegiate Crescent, Sheffield S10 2BP, U.K.

Objects of ALM
The Charity's objects are the advancement of education by the establishment and development of an international research forum in the life-long learning of mathematics and numeracy by adults by:

- encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;
- promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels, for the public benefit.

Working to Achieve These Objects
Specifically, our aim is to improve the learning of mathematics by adults. Several recent reports show that many adults have difficulties with the basic mathematical skills needed as parents, citizens and workers. For example, the Moser Report (UK, Department for Education and Employment, 1999) suggests that in Britain, 40% of adults have some numeracy problems and 20% have very low attainment in numeracy. The results of the International Adult Literacy Survey (Organization for Economic Co-operation and Development, 1997) indicate that there are similar situations in many other countries.

ALM's membership is concerned about this situation and works in a variety of educational environments to improve the learning of mathematics by adults. Currently many of our members are involved in the adult basic education programs. Other members are involved in teaching mathematics at all levels in higher education, including initial teacher training and ongoing professional development of teachers.

The ALM forum provides opportunities for teachers to bring successful practice from their own classrooms to a wider audience. This happens at the annual conferences and throughout the year in the ALM newsletter and through the communication and networking encouraged by the forum. By reflecting on our own practice and the practice of others and by fostering international links between teachers, we are able to encourage the transference of good practice in curriculum design, and in teaching and learning materials and methods, which have evolved in different countries. Innovative ideas are enhanced through critical appraisal by fellow participants, and pass into the public store of educational material when the Proceedings of conferences are published.

There is not enough information available about what mathematics is needed by adults in their daily and working lives, how adults learn mathematics, and what the most effective androgogical practices are. Our aim is to connect research with practice, by bringing the experience of practitioners and students to bear on the formulation of research questions and the conduct of research, and by making academic research accessible to teachers and therefore benefit their students.

The organization does not support a single theoretical framework, or commission or conduct research. The presentation of papers at our annual conferences provides an opportunity for discussion on research methods and findings, which constitutes an active and participative public peer review process and quality enhancement mechanism.

People working in the field who are not members of the organization and/or who are not able to attend the annual conferences, are also able to make use of the activities of participants and therefore benefit their students by access to the published Proceedings of the conferences and to the web-site www.alm-online.org. The dissemination of the results of our work increases the sum of communicable knowledge about the mathematical education of adults. We believe that these collective actions are of direct benefit to the public.

Board of Trustees
The affairs of ALM are managed by an international Board of Trustees elected by at the Annual General Meeting which is held in conjunction with the annual conference.

How to Join ALM
Anyone who is interested in the objects of ALM can apply for membership. The membership rates for 1 August 2001 – 31 July 2002 are: Individual Member £15; Students/unwaged/low-waged £3 (minimum); Institutional Member £30.

Please contact Prof Sylvia Johnson (ALM Membership Secretary) for a membership form and payment details at the School of Education, Sheffield Hallam University, 36 Collegiate Crescent, Sheffield S10 2BP, UK. Alternatively if you live in certain countries you may pay in your own currency through your local ALM agent.

Further information on ALM is available on the web-site www.alm-online.org

Professor John O'Donoghue, University of Limerick, Ireland
Chair of ALM (1997-2001)

Preface

Within ALM we understand the term 'mathematics' to include 'numeracy'.
(The constitution of **A**dults **L**earning **M**athematics - a Research Forum, 1994)

During the last 20 years, numeracy has been a key word in policy reports, international surveys, adult educational programmes and research on adult and mathematics education. Adult numeracy is a complex and much debated area of practice and research. *"Numeracy for empowerment and democracy?" was* the theme of the eighth international conference of *Adults Learning Mathematics - A Research Forum* (ALM8)
The main issue addressed was numeracy as a possible answer to questions of empowerment and democracy in the broadest sense of these terms. In adult education, two different lines of approach are possible: society's requirements of numeracy or adults' need for numeracy. During the conference, we tried to bridge these two approaches. We found no single answer and new questions were raised.

The conference was held at Roskilde University, Denmark, on 28-30 June, 2001. It was attended by more than 70 researchers and practitioners, which are not mutually exclusive categories in ALM, from 11 countries (Australia, Austria, Belgium, Canada, Denmark, Ireland, the Netherlands, New Zealand, Sweden, the United Kingdom and the United States of America).

The conference was organised by the Centre for Research in Learning Mathematics, Denmark. The programme committee were Jeff Evans, UK, Lena Lindenskov and Tine Wedege, DK. The local organising team were Lena Lindenskov, the Danish University of Education, Lene Ø. Johansen, Aalborg University, Eigil Peter Hansen, Adult Educational Centre, Tine Wedege and Lene Riberholdt, Roskilde University.

This year the ALM conference programme included three invited plenary lectures, a plenary panel, and seven parallel sessions (paper presentations, posters and exhibition, topic groups, workshops). This book includes the proceedings of the conference.

We are grateful to the presenters who wanted to share their research and to join in discussions at ALM8. A special thank you to Mieke van Groenestijn, NL, who managed the ALM8 web-site so well, and to David Kaye, UK, and other ALM trustees who gave linguistic support to the 'foreign speakers'.

Finally we wish to record our special thanks to Roskilde University for hosting the conference and providing administrative support and to the Danish Ministry of Education for sponsoring the conference.

Lene Østergaard Johansen, Aalborg University
Tine Wedege, Roskilde University

Contents

About ALM 3

Preface 5

Plenary Lectures **9**

Lifelong Learning - a political agenda! Also a Research Agenda? 10
Henning Salling Olesen

What Counts as Mathematics in Adult and Vocational Education? 21
Numeracy for empowerment and democracy?
Gail FitzSimons

(Dis)empowering forces in everyday mathematics. Challenges to democracy 33
Lena Lindenskov & Paola Valero

Paper Presentations **43**

Maths and Measurement: Developing measurement skills in adult learners 44
of mathematics. (An evaluation of the efficacy of critical mathematics
methods in bridging education)
Anne Abbott

Secret knowledge: Indigenous Australians and learning mathematics 50
Roseann Benn

Mathematics for Parents: Issues of Pedagogy and Content 60
Marta Civil

Evaluating an educational programme for enhancing adults' 68
quantitative problem-solving and decision-making
Noel Colleran, John O'Donoghue & Eamonn Murphy

What Numeracy Skills do Adults need for Life? 81
Dhamma Colwell

Developing the Ideas of Affect and Emotion among Adult Learners 88
Jeff Evans

Mathematics 97
- The STEPS for Empowerment in a University Foundation Program
Milton Fuller

Goals of numeracy teaching 103
Lene Østergaard Johansen

Calculating people: measurement as a social process 112
Betty Johnston

A grounded approach to practitioner training in Ireland: some findings from a national survey of practitioners in Adult Basic Education 120
Terry Maguire & John O'Donoghue

Adult Ways of Knowing: a Summary of Perspectives from Five Research Groups 133
Katherine Safford

Mathematics and Society – Must all People Learn Mathematics? 139
Wolfgang Schlöglmann

Basic Skills Strategy – a practitioner perspective 145
Valerie Seabright

Mathematics Teaching for the Prevention and Reduction of Problem Gambling 146
Donald Smith

'Real life' in everyday and academic maths 156
Alison Tomlin

Poster Session **165**

Teaching Adult Students Numeracy and Mathematics 166
Richards O. Angiama

Topic Groups **171**

Topic Group A - Developing a Theoretical Framework for Adults Learning Mathematics

Developing a Theoretical Framework for Adults Learning Mathematics: The Case of Numeracy 172
Tine Wedege & Roseanne Benn

Exploring A Hypothesis 173
Mark Schwartz

Topic Group B – Mathematics education for the workplace

A changed perception of mathematics for the workplace 178
Lisbeth Lindberg

Mathematics education for the workplace 179
Jaine Chisholm Caunt

Mathematical Literacy; What's in the name? 180
Henk van der Kooij

Topic Group C – Affective Factors in Adult Mathematics Learning

Affective Factors in Adult Mathematics Learning 181
Jeff Evans

Towards a Holistic Model of Numeracy Competence 183
Beth Marr

Mathematics Anxiety and Perceived Competency to Teach Among Student Teachers 193
Alan Bowd & Patrick Brady

Breaking the Barrier - Student perception on how the necessary maths support has facilitated entry into higher education 200
Patricia Alexander & Poppy Pickard

Workshops **207**

Multimedia Maths - Multiplying Opportunities for Formal and informal Numeracy Learning 208
Laura Carroll & Sarah Kowall

Numeracy Skills for Life: Numeracy in the New Adult Basic Skills Strategy in England 209
Diana Coben

Personal methods as a means of achieving empowerment and democracy 217
Janet Duffin

Gender in ALM - Women and Men Learning Mathematics 223
Inge Henningsen

Practitioners, Questions and Research 233
David Kaye & Eigil P. Hansen

Competence based math in modules 234
Harrie Sormani & Ben Hermeler

Authors **237**

Plenary Lectures

Lifelong Learning - a political agenda! Also a Research Agenda?

Henning Salling Olesen
Roskilde University, Denmark

A personal note: I was invited to the conference in my capacity of being researcher of adult education and learning - in order to present a broader framework for the discussion within that field. However, I do have an affiliation with mathematics: It was my first academic study - not carried very far - and I had the opportunity to teach mathematics at advanced high school level before I let it go in order to do something else. I guess it provides a specific engagement, and maybe also some blind spots in my perception of the field. I have not carried out any research on mathematics learning myself, but I have read some of it on and off. I find the field a very good illustration of some of the problems that educational research is facing - and also of great significance in its own right.

'Lifelong Learning' instead of 'Adult Education'

In adult and continuing education there seems to be at the same time two convergent processes going on: an institutionalizing process, adding schools for adults to the schools for children and adolescents, which is a continuation of a basic trend in modernization, institution building. And also a de-institutionalizing process, broadening the scope of interventions across the boundaries of school, and focussing on learning processes *in and outside schools* (Alheit et al, 2000). Lifelong Learning is a set agenda. Paradoxically lifelong learning achieved its position as a key theme partly by the fact that 'human resources' appear more and more essential in terms of economic growth and structural innovation. What was some decades ago idealistic, wishful thinking, that was slowly worn down by the absence of practical implementation, now seems to be a concern of power elites in the capitalist world (Rubenson, 1996).

As a discourse of Education, Lifelong Learning has a radical built-in assumption, which is also fed by the economic concerns: It assumes that learning takes place in all spheres of life, not only in schools and institutions. '....and Life wide..', sometimes added into 'Lifelong learning', completes the topical metaphor. It relativizes the importance of Schools and intended education, on the one hand emphasizing the limits of the modern dis-embedding of learning from social practices. On the other hand also opening our eyes to an immense potential of self-directed learning outside schools.

It may seem ironic, but is also logical: Only the fact that economy and work need human resources, and the fact that qualification demands include subjective involvement makes lifelong learning *a societal programme*. The change in the rationales from democratic access to human resources, and the fact that these rationales are still separate, reflect the contradictory role of education in capital driven modernization. On the one hand a vehicle of a humanistic political programme for social autonomy and 'empowerment', and on the other hand a necessary adaptation of human beings to their part as 'commodity labour'. Consequently the conceptual common denominator has become 'learning' rather than 'education'. This emphasis on learning rather than education has

lately sometimes been seen as an educational drawback - and sometimes it actually is part of neo-conservative dismantling of welfare policies. But it may also be integrated in a critique of the illusionary expectations that are put on institutional education, in terms of efficiency and in terms of their emancipation potential.

In my opinion this is altogether not too bad: a new political awareness of learning and education - but also a challenge to reinvent the critical and emancipative agenda of life-long learning in a new form, and a need for radical changes in educational thought. For a discussion on research it means that we have to move our focus of attention from education to learning, and we have to study learning in all its contexts, not only in institutionalized forms.

Education → (Lifelong) Learning

Schools → Learning spaces -
workplace, everyday life, cultural activities

Maybe most difficult to handle, but also potentially productive, is the challenge to our identifications with the educational institution. We have moved beyond the happy formula of the educational optimism:

Modernization = Educational institutions = Equality and Emancipation

The criticism of institutions points out that the existing educational institutions are unable or inadequate to fulfil their purpose and promises. Sometimes they might be an obstacle to learning. The post modernists take the critique one step further, as a critique of the basic humanist educational programme. Referring to Foucault, among others they point out the inner relation between institutions, knowledge and power. Educational institutions, by means of knowledge, exercise control and restriction on the experience potential of the protagonists, allowing for some organizations of knowledge and excluding others.

However a similar critical attention is no less relevant in the new and more multiple field of learning. The political Agenda Lifelong Learning calls for critical research, which distinguishes the learning potentials in a number of life worlds, in a historical and societal context. In many of the political discourses of lifelong learning, that discredit educational settings the alternative has very often taken the shape of inserting a new abstraction filled up with an ideological investment, e.g. organizational learning taking models and rationales of the private firm. A conception of learning drawing on managerial ideas about work organisations is probably the most influential discourse of lifelong learning. But also other ‘models’ are drawn on, e.g. naturalistic work process ideas, in which learning is just related to the concrete process of a specific work process, and political and cultural conceptions re-installing the original culture and experience of a community. It would not take us any further to just re-install an educational optimism on behalf of e.g. work-place based learning. There is a strong need for some theoretical sanitation.

Is Lifelong Learning then the end for the educational ideas of modernization - or is it the framework for a new idea of democratization and learning? The answer depends on the way of theorizing - learning, the learner and the learning spaces.

Researching Lifelong Learning

The implications are quite heavy in general in terms of research objects, relevant theoretical approaches and methodologies. Instead of philosophy defining the goals and objectives, pedagogical research on the inputs - the educational means and arrangements - and maybe a technically oriented psychology of learning and motivation we need research which takes a broader perspective on learners, learning processes and the learning environment - learning people in their whole life context. Knowledge and learning can-not any longer be studied in the neat and nice form of school subjects. A whole range of observation defined by important *life spheres* (work, family, leisure and cultural activities, citizenship) and *knowledge and competence domains* (professions, skills, arts) defined by societal division of labour - which are all historically given, must be reconstructed conceptually in the perspective of learning. Lifelong Learning has the advantage of emphasizing *somebody who is learning*, in an experiental lifelong cumulative process, bridging across education, training as well as everyday life. Conceptualizing learning spaces or learning environments depends on a subjective perspective - we have to see them as *lifeworlds, everyday life for specific people.* Likewise the learning subject cannot be seen as abstract pupils, not even when attributed age, gender, class or ethnicity or other culturally distinctive features. We need research approaches that study learners in their own dynamic subjective perspective, which is exactly not an abstract individual, but deeply embedded in a historical and cultural situation, with individual and collective orientations, and with a specific life history experience of this context.

Researching subjectivity of learning and social structural and historical dynamics requires an interdisciplinary research strategy - we will need to draw on disciplines from social science as well as humanities - going across the Gulf which is still very deep. On the other side we can build on many developments from inside existing disciplines - theoretical themes, methodological developments, studies that deal with learning without using this term. (Unfortunately there is also the opposite phenomenon: using the term learning without really dealing with it). Many contexts cannot be understood properly without quite specific knowledge that is produced in different contexts and for different purposes, so knowledge has to be de- and reconstructed for the actual purpose.

Talking on this general level anything belongs to the field of interest. But I do think it would be possible to point out even on this general level a few exemplary themes, which are important in most practical contexts, and which are integrative in theoretical respect, and hence also generative for focussing the research field. Without entering into a discussion I shall just to mention a few:

Exemplary Themes:

- Gender and Wage Labour
- Work life: Self Regulation and Sustainability
- Knowledge and Democracy: Professional Learning, Profession Identity

Instead of discussing these themes or suggesting others I will suggest some implications of the context outlined for the research field and the researchers gathering under the heading of 'Adults Learning Mathematics' - being well aware that I am not familiar with more than fringes of ongoing research and debates. I will reflect on an understanding of the field which seems quite usual, and also very plausible, namely as a necessary *cultural technique*, a tool to access knowledge and to participate in social life and political processes.

Cultural techniques: Literacy, Numeracy, Citizenship.....

Literacy has the status of indispensable *cultural technique*, a tool to access knowledge and other cultural treasures, a tool that you may have or not have, and the possession of which has strong functional and distributive effects. Literacy - command of written language in reading and to some extend also writing - is centrally related to modernization. Modernization - functional differentiation, market mediated relations, gradual dissolution of space dimensions, bureaucracy - would not be possible without written language. Paolo Freire - seeing language as a necessary tool for emancipation at risk of giving up cultural experience and identity - launched a strategy of alphabetization related to 'the oppressed', i.e. poor peoples of the Third World, which takes alphabetization as a way of knowing (Freire, 1972, orig 1970). So in order for alphabetization to be the development of a language and categories of thinking to express yourself, your identity and interests, language learning must be closely integrated with social change and political self-consciousness building. This view has gained a broad influence in Brazil and many other countries in the Third World, exactly because it articulates the contradictions in modernization on the level of language.

Some seem to assume that literacy in its traditional sense - access to reading and writing - is a residual issue, which will disappear as soon as modernization processes have been completed. However - it is still highly relevant. Not all need to read and write from the beginning, but this is becoming necessary - and this is where adult education comes in. The relevance and location of this problem - and indeed of the attempts to solve it - has been an issue defined by the frontiers of modernisation, a matter of integration of populations in the modernizing societies and, seen from the other side, of access to literary knowledge - from the bible onwards. This literacy problem has proved extremely resistant, as one among other indicators that global capitalism does not necessarily imply cultural homogenisation of the world overnight. Today literacy is becoming a political agenda (again) in the highly developed countries - the reservoir of reserve labour must be literate, and social cohesion is threatened by the existence of culturally marginalized segments of the population (or rather: the social integration of individuals is not any longer secured by marginalized communities). And still literacy is an important indicator of the asynchronous nature of the modernisation process on a world scale.

Numbers and mathematical cognition seem to follow the same pattern. Modernization would also not have been possible without the languages of mathematics, including numbers. Some elementary aspects of mathematics have been widely spread and integrated in social practices (market exchange based on simple calculation, construction work embodying geometric models). But mostly the division of labour has been very sharp, and the specialisation of mathematical competences high. Apart from the school

system, active application of mathematical competences are embedded in specialist professions of science, technology and increasingly social technology.

This is shifting: a new agenda is taking shape with the key term Numeracy which was constructed back in the 1950s in Britain, clearly connotating to Literacy, and which has now gained substantial practical interest in adult learning and education as well as in work life. Discourses include a skills need problem, a schooling and education access question, as well as a democracy perspective.

Numeracy is defined as the ability to handle numbers and to understand information in other mathematical terms like percentages, graphs, models etc. (Wedege, 2000, Coben et al 2000). Narrow institutional interests are of course picking up the agenda of numeracy in developed countries: Mathematics seeking legitimacy and new business domains. The overall changes in the entire societal knowledge formation and division of labour are defined as the need for a wider distribution of a new cultural technique, which is more or less the same as mathematics (for example, as a part of a 'back-to-basics' emphasis, and through the continuous use of mathematics for selective purposes in higher education). However, this is not just a question about learning Mathematics as a 'cultural technique', thereby gaining access to certain cultural domains, improving work qualification, and strengthening your democratic competence.

Schools produce very differentiated mathematical skills, often correlating with class and gender. Very often schooling at the same time seems to produce emotional relations to mathematics which are strongly correlated with success and failure. The mainstream interpretation is still concentrated on mathematics as a specific technical mode of thought, and individualizing the successes and failures. We did use to crack jokes about the assignments in arithmetic that were class biased in a way not corresponding with the children (stock trading, financial calculation). Feminists have attacked the myths and misunderstanding of the gendered records of mathematics (Walkerdine et al 1989).

So much for the actual repressive functions of mathematics teaching. But like literacy, people do have numeracy competences. Research into vocational skills and training prove that people with learning difficulties in school may have mathematical skills embedded in practical competences (e.g. Mellin-Olsen 1976, 1987). Likewise research has shown differentiation in abilities to solve mathematical tasks in school and in out-of-school practical settings respectively (e.g. Lave, 1988). The problem seems similar to the one Freire raised in relation to literacy: Mathematics is an abstract language, which does not necessarily encompass or enable the expression of the people's everyday life experiences. Hence learning mathematics is more or less difficult and alienating, and produces negative emotional relations. And the competences actually already available and embodied in existing social practices cannot be abolished by a 'mathematization' of these practices.

Looking into the literature I found a conception in line with and inspired by the Freirean position: 'Ethno-mathematics' (formulated in Brazil by e.g. D'Ambrosio, quoted from Gelsa Knijnik in Coben et al 2000): Numeracy competences are embedded in basic technologies and modes of social regulation (trade and organisation of production). The technological transfers and coercive changes caused by capitalist modernisation deval-

ues old numeracy competences and – primarily - introduces new ones. Abstract calculation and modelling are part of the division of labour which is taking place with modernization.

In the capitalist centres industrialisation and professionalisation of crafts and habitual work processes transfer knowledge to new modalities, and most often in the end into a computer (J. Weissenbaum has talked about 'stolen knowledge' somewhere, quoted from Edgar Weick) - and consequently a demand is returned to people to become 'numerate', i.e. to handle abstract numbers and ways of presenting knowledge in mathematical forms or even worse. This defines the skills needs as well as the need for democratic tools to access and participate in cultural processes.

It is necessary, but not sufficient to understand this - real - challenge in its contradictory quality, it is also necessary to develop a new conception of numeracy and numeracy learning in the context of experience and learning - a new conception of mathematics so to speak.

I wonder if it would not be productive to see learning for *(active) citizenship* in the same context as *literacy* and *numeracy*. Obviously it deals with a number of basic competences which are (becoming) necessary with modernisation process, as a consequence of the building of political machinery as well as the social dissolution of premodern family and communitarian structures. In the last few years the perception of civil society has been emphatically positive, though perceived in multiple ways. Sometimes (active) citizenship in the context of adult education and learning appears to be a cultural technique for political action and self regulation, but actually as a competence it could be related to the realization of the bourgeois democracy, with all the contradictions embedded in it. This becomes very clear when contextualised across Europe (Bron et al, 1998): it can be seen as the vitalisation of civil society in Eastern and South East Europe, which is a learning process of political, as well as entrepreneurial and wage labour, rights and responsibilities, and of market relations - whereas in post-Thatcher Britain, you could call it 'repair work' on the liberal welfare state - and in the Nordic countries, a challenge to the professionalized social democrat welfare state. The *cultural technique*s of political participation and active citizenship have entirely different subjective meanings, and they also relate to historically different processes, they are all related to modernisation processes, but in different ways.

literacy

numeracy

citizenship

One might add *informatics* or *computer literacy* and something like *iconological* or *image-literacy* as basic techniques that you need to master in order to have access to culture, but that at the same time shape or mediate your experience in a specific way. Could these - altogether relevant and interesting - enhancements of the literacy notion in different directions anticipate a new concept of competences for a developed moder-

nity, anticipating a knowledge democracy? Could we order different modalities in a hierarchy - maybe 'literacy', 'numeracy' and 'citizenship' could be privileged as 'the three cultural techniques of modernity'? Apparently there are some similarities among these cultural techniques - though they also reflect modernisation processes in different phases and contexts, and relate to different functions in the social formation. What is more important: they represent different aspects of subjective orientation and social practice. They represent - qualitatively different - aspects of the subject-object-dialectic of cultural development and learning.

Interestingly the recent OECD International Adult Literacy Survey also includes certain aspects of 'quantitative literacy' as well as mathematical types of 'document literacy' (graphs etc) as a part of the notion of 'functional literacy' (OECD, 1997). The survey basically reflects the new shared agenda of highly developed as well as developing countries, that literacy is important for economy and development. It monitors inequalities and surprising lags in adult literacy even in countries with a well-established educational provision - all demonstrating the need to be concerned about this field.

However, most interesting in this context is a remarkable development in methodology. Instead of simply testing reading abilities with standardized tests a complex instrumentation is applied to measure competences in context, intending to produce an index of 'functional literacy'. In a comparative study this is delicate balance in defining tasks ('contexts') which are to some extent comparable and yet relative in either (socially and culturally different) context. However, this methodological difficulty is one of real life. In spite of the fact that the OECD survey is still just resulting in comparisons on a one-dimensional scale (or rather a number of combined scales), related to a rather traditional literacy policy, the methodology opens a space for discussion (deconstruction, if so wished) of the literacy concept. The reason is of course the influence of economic thought: what is interesting is not how people achieve in an abstracted reading skill, but to what extent they can use their literate competences in everyday life. The critical implications for education seem obvious, however the critical awareness does not go very far in relation to the underlying economic thought: we are learning about 'human resources', not about identities and specific experiences and meanings embodied in language.

As different as Freire and the OECD may appear, they both point to the question about language in societal context, and 'society in language' - both reflecting the contradictions of modernization processes. The OECD study representing modernization itself - globalise capitalism - and the necessity to make sure that individuals acquire certain basic cultural techniques, by its methodology situating literacy in a tension between standardized resource and cultural specificity and related resistances. Freire insisting on a not yet broken connection between language, consciousness and social liberation.

Experience in a modernized life world in a global capitalism

I think many western - i.e. modernized, professional - pedagogues have appreciated the motivational aspect of Freire's pedagogy as well as it's social justice perspective: an efficient way of bringing people to the same level as 'we' are, giving them access to language and hence to knowledge and/or power, without really grasping the way in which language is political. But in the context where Freire originated his thinking it

was really a matter of 'functional literacy' in the precise critical sense of linguistic competences that encompass the experience and needs of the language user/learner. The Freirean emancipation vision is located in the experiences and actions of those who are invited by or threatened by this modernization. In a different historical setting, within the capitalist centre in Europe, the German sociologist and political philosopher Oskar Negt developed a new conception for the worker's education from a point of departure very similar to Freire's (Negt 1972, Salling Olesen 1989). Negt pointed out a tension between a language which was constrained within an everyday social situation which seems natural and unchangeable, and a 'petrified' language of theoretical concepts and political (socialist) ideas - both useless for political learning. So he developed the idea about 'exemplaric learning', i.e. first of all to work with the experiences of everyday life and their contradictions. Critique of society and authentic ideas of change must develop out of lived experience, they must create their own language, or revive stiffened languages. Books and written language may serve a political imagination that insists on universal communication and cultural exchange - but only if they are rooted in personal experience of everyday life.

Today the demand for cultural techniques of a universal culture - as they appear at the frontier of capitalist modernisation - is ubiquitous, as it is reflected in the new literacy problem of the capitalist centre. In each case the acquisition of the 'cultural techniques' also represents a potential reification of existing competences that are historically relevant. The micro-studies of everyday life competences in work procedures which have to be given up - sometimes kicked out together with the people who embody them - is just a small example. The concern about motivation in learning mathematics is another one.

The construction of new cultural techniques claim that these other modalities of thinking, communicating and acting are having or will enhance the action and participation range of the learners. I have tried to follow this argument one or two steps, by assuming that 'modernity', in order to integrate social processes, imposes on its members new competence needs. The fact of modernity is that we have established a world where the universal reflexivity is mediated by written language, a technology and coordination which is dependant on 'numeracy', and a political machinery which is dependant on the recognition and practical mastering of 'citizenship'.

Central seems to be to pursue the contradictions in the universalisation - appearing as contradictions in the learning of the literacies as they were observed by Freire, echoed in the discussion of numeracy - and 'unconsciously' taken into account by OECD study. These contradictions point back to the process initiated by a capitalism driven modernization of life worlds. The universalisation of cultural media as well as meanings ('content') become compulsory for the orientation in the modern world, for handling the technological artefacts and organisation, for organisation, and for social action. The question is whether it is possible to reconcile these 'modernizing' competences on the one side with the authentic experience of life world and a self regulated social practice.

Global capitalism has dramatically changed the horizon of the individual as well as the conditions for constitution of any individual or collective subjectivity. It has established the situation of asynchronous development, where transitions from pre-modern communities to modernized societies take place at the same time as nation state boundaries, as

well as other well-defined structures of the modern world, are dissolved or relativized. The gap between the strong causal interdependence, and multiple and local horizons for cultural orientation and possible action is what postmodernism programmatically responds to, with an (in practice: ideological) insistence on the latter (Anderson, 1998).

Oskar Negt's point of departure is the already existing modernized reality, included the modern ideas of liberation, i.e. of the socialist labour movement, but also that the potential for emancipation from capitalist exploitation and reification can only grow out of a political learning process at the centre of the contradictions in capitalist development, including the welfare state which is an important local precondition (Negt/Kluge 1981; Salling Olesen 1999). The critical or emancipatory understanding of the individual learning needs imposed by this development are located in the relation between 'living work', the concrete workers in a specific situation, and capital. But unlike traditional socialist views of workers' education and the working class at large, the focus on experience and learning in everyday life gives way to the changes and to the rethinking of collectivity and the status of (working) class as a monolithic societal agent. In this sense his framework is open to local and multiple developments under the conditions of global capitalism, at the same time as it identifies the dialectic of workers' learning in context in the capitalist centres.

Updating and generalizing this principle, which had only been implied in the conception of exemplaric learning Oskar Negt has proposed 'six competences that a worker in the future will need' (Negt 1989, pp256ff). I think it is an interesting bid for a *social literacy*. In brief they read as follows (my translation):

1. Competence to create cohesion

2. Competence for a conscientious interaction with people and things (ecological competence)

3. Competence for balancing a threatened or broken identity

4. Memory and utopian imagination (historical competence)

5. Sensibility to experience of expropriation

6. Technological competence (refined ability for distinguishing)

This is the order of headlines. In this context we could see the 4th and 5th as immediately corresponding to Freire's ideas. The 6th indicates the acceptance of the modernized man - the necessity of embodying technology. In contrast, the 1st , the 2nd and the 3rd also presuppose the 'effects' of modernization at the same time as they indicate a critical perspective beyond the industrial civilization (the ecological competence) as well as the traditional labour movement collectivism (1st as well as 3rd).

Implications for Lifelong Learning Research

Without going deep into explication of these headlines it is important to state them as dimensions of prospective learning processes, located within modernisation processes but at the same time dissolving or reconfiguring agents and structures of modernity. The dilemma of this learning process seems to go between the universalizing culture, and its tendency to individualize the competence adaptation, and the ideas of a self-regulated, collective experience building. The consequence of the modernisation is the development of a new, societal external regulation of peoples' lives, Brazilian Indians as well as workers in Europe. The question about self-regulation must remain an orientation mark, however historically relative and conceived as a utopian vision. Collective experience is what is dismantled by modernisation, by the de-traditionalisation and the market interlinking of communities far beyond existing collective consciousness. The literacy question could be set as the question about access to the media for collective experience building in a world where the gap between the life world and the scope of societal interrelation is evident. If we state that global capitalism or the world market is the societal relation that enforces this interrelation, then the political struggle in relation to learning and communication goes about the learning to invade the cultural media (language, etc) with the experiences of life world, and to vitalize these media in creating collective experience.

This is meant to indicate a context - very general indeed - in which the research field 'adults learning mathematics' could be situated, and which would lend some kind of orientation to a reinterpretation and reconfiguration of the research field. Whereas the 'exemplary themes' were meant to be strategic ways of opening the interdisciplinary and synthesizing way of taking in knowledge, I would like to end up outlining two axes along which future research could develop. Maybe some will see them as more fancy names for 'the learner' and 'the subject':

Subjectivity and Collective Experience:
Emotional and cultural meanings of 'mathematics' for specific people

Sociology of Knowledge:
Societal forms of knowing and learning 'Mathematics'

Final remark

I think most researchers in this field, though coming from quite different backgrounds, are personally situated at the crossroads of the contradictions we have to study. I guess that most of you have critical but ambivalent relations to mathematics - and maybe also to formalized education. Take it as an important resource. There is a substantial risk that powerful institutions will remain immune to the criticism, implied in Lifelong Learning, but you cannot articulate this critique, not to mention developing alternative routes, without a touch of love and belief in education. And the ambivalence is a productive experience in order to understand the subjective perspective of adults learning mathematics.

References

Alheit, P. et al (2000). *Lifelong Learning inside and outside Schools*. Bremen/ Leeds/Roskilde Universities.

Anderson, P. (1998). *The origins of Postmodernity*. London: Verso.

Bron, A., J. Field and E. Kurantowicz (eds) (1998). *Adult Education and democratic Citizenship II*. Krakow: Impuls and ESREA.

Coben, D., G. FitzSimons and J. O'Donoghue (eds) (2000). *Perspectives on Adults Learning Mathematics. Research and Practice*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Freire, P. (1972). *Cultural Action for Freedom*. Harmondsworth: Penguin (orig. publ. 1970).

Freire, P. (1970). *Pedagogy of the Oppressed*. Seabury (orig. publ. 1968).

Lave, J. (1988). *Cognition in Practice: Mind, mathematics and culture in everyday life.* Cambridge University Press.

Mellin-Olsen, S. and R. Rasmussen (1976). *Skolens Vold*. Oslo: Pax.

Mellin-Olsen, S. (1987). *The politics of Mathematical Education*. Dordrecht, Reidel

Negt, O. (1972). *Arbeiterbildung und exemplarisches Lernen*. Frankfurt: EVA.

Negt, O./A. Kluge (1981).*Geschichte und Eigensinn*. Offenbach, Zweitausendeins.

Negt.O (1989). *Die Herausforderung der Gewerkschaften*. Frankfurt/N.Y.: Campus Verlag.

OECD (1997). *Literacy skills for the knowledge society. Further results from the International Adult Literacy Survey*. Paris: OECD.

Rubenson, Kjell (1996). 'Lifelong Learning: Between Utopia and Economic Imperatives.' in: J. Gaskell and D. Kelly (eds). *Dropping In, Dropping Out*. NY: Teachers College Press.

Salling Olesen, H. (1989). *Adult Education and Everyday Life*. Roskilde: Adult Education Research Group.

Salling Olesen, H. (1999). 'Political Economy of Labour.' in: Salling Olesen and K. Forrester (eds). *Adult Education and the Labour Market V*. Roskilde: Roskilde University Press and ESREA.

Salling Olesen, H. (1996). 'Experience, Biography, and Life History'. in Salling Olesen and P. Rasmussen (eds) *Theoretical Isssues in Adult Education*. Copenhagen: Roskilde University Press.

Walkerdine, V et al (1989) *Counting girls out*. London: Virago Press.

Weber, K. (1998). *Life History, Gender, and Experience.* Copenhagen: Roskilde University Press.

Wedege, T. (2000). 'Technology, Competences and Mathematics.' in Coben et al. (2000), pp191-208.

What Counts as Mathematics in Adult and Vocational Education? Numeracy for empowerment and democracy?

Gail E. FitzSimons
Monash University, Australia

Abstract. This paper will outline Engeström's five principles of *Expansive Learning* and use these as a framework for analysis of what counts as mathematics in adult and vocational education. To illustrate these I will present examples drawn from the work of practitioners and researchers in this subfield of education. I will link these to concepts of democracy influenced by the work of Derrida, as well as to issues of power.

Introduction

Democracy is an idealised concept. Supreme power is said to rest with the people. However, in many if not all countries, power is unequally distributed in terms of Bourdieu's (1991) notions of economic, symbolic, cultural, and social capital. Mathematics is said to be empowering, but questions arise: What mathematics? How much mathematics? For whom? Who decides? These are in addition to the regular questions of how, where, when, and why. So, the question "What counts as mathematics in adult and vocational education?" is a consciously political one. The work of Yrjö Engeström will be utilised to provide a framework for interrogation.

Theoretical Framework

In a keynote address to an Australian vocational education conference in 1999, Yrjö Engeström outlined a framework for *expansive learning* — working towards a reconceptualisation of Activity theory. Building on the work of Lev Vygotsky, he generated a structure for a human activity system, and then a model for two or more interacting activity systems, in order to "develop conceptual tools to understand dialogue, multiple perspectives, and networks of interacting activity systems" (Engeström, 1999:3). He elaborated five principles to summarise Activity theory and cross-tabulated these with four questions central to any theory of learning (figure 1). The concept of mathematics in adult and vocational education is complex and may be viewed and construed from many valid perspectives, according to the interests of the persons or groups concerned. In this context the concept of learner is taken broadly, to include *all* participants in the dynamic process — not just the students.

Activity System as a Unit of Analysis

Firstly, within Activity theory, the prime *unit of analysis* is taken to be "a collective, artifact-mediated and object-oriented system" (Engeström, 1999:4), subsuming individual and group goals, and generating actions and operations. Here, the systems of interest include primarily: (a) vocational and further educational institutions, (b) workplaces, (c) local communities, and (d) individuals. Their interests are assumed to be self-serving as well as mutually related (FitzSimons, 2000); and their respective objectives to accumulate some or all of economic, symbolic, cultural, and/or social capital (Bourdieu, 1991).

	Activity system as a unit of analysis	Multi-voicedness	Historicity	Contradictions	Expansive cycles
Who are learning?					
Why do they learn?					
What do they learn?					
How do they learn					

Figure 1. Expansive learning model (Engeström, 1999:6)

Multi-Voicedness

Secondly, account needs to be taken of the principle of *multi-voicedness* — that is, the different positions for participants created by the division of labour within the Activity system. In this case these include the multiple views, traditions and interests of students, teachers, managers, employers, and government bodies. On the part of individual students, reasons for learning mathematics could include: (a) self-realisation, (b) democratic empowerment, and (c) workplace promotion. In acquiring knowledge of the discipline of mathematics, together with incidental knowledge of mathematics education and knowledge of education-institutional structures, the bigger picture for students could include the possibility gaining workplace or lifeskills know-how and certification. Teachers may be assumed to learn about the needs of their students and relevant industries, for example; employers to learn of the qualifications, and even perhaps of the underpinning theory; while governments need to be able to guarantee some form of public accountability for funding spent on adult and vocational education. From a pedagogic perspective, there is — or ought to be — dialogue and debate between positions and voices, focused (in this case) on mathematics education. From a social and economic perspective, interests of employers and government authorities may lie in the values of control over (potential) workers as individuals or groups. It is assumed finally that, overall, interests are focused on progress in the local and/or global economic and/or social spheres.

The questions arise: Whose voices are generally heard in adult and vocational education? Whose voices should be heard?

Voices of students. Based on data from two separate projects, Marta Civil (2001) notes that participants give reasons along the lines of "being able to help their children" as their main reason for involvement.

> I do the best I can to help my children in math. That's why I'm here. I want to learn different ways to help my children. (p. 208)

But, notes Civil, these women (most of the participants are women) soon become engaged in the learning of mathematics for themselves. By contrast, a younger student is clear about her own goals for herself (FitzSimons, 1994).

> Now that I am 20 years old I really do regret never trying to do maths. I am so scared of maths mainly because everyone makes it sound so hard. So I have come back to school to prove to myself that maths is not hard and no matter how old or young you are you will always need some sort of mathematics knowledge; also I want to improve myself and my mind. (p. 17)

In this quotation, learning mathematics is seen as a personal challenge, a means of improving one's self, and of gaining a form of cultural capital (or legitimate knowledge) (Bourdieu, 1991). However, it is well recognised that adult learners have many other interests and responsibilities in the social and economic roles they adopt. Teachers are often aware of the apparent 'skill deficits' of their students, and frequently blame is laid with the students themselves or with the compulsory school sector (FitzSimons, 2000). Other approaches, informed by listening to the voices of students, are likely to be more appropriate.

Voices of teachers. Many teachers of adult and vocational education students are isolated by institutional structures which locate them singly or in tiny groups within other academic, community, trade or vocational departments. These communities of practice may be valuable in assisting with realistic contextual settings, but the loss of disciplinary support-base in professional development terms may well outweigh the gains. Pragmatically, it may mean the termination of any aspirations for promotion from such marginal positioning (FitzSimons, 2000). Having non-mathematics trained instructors (e.g., vocational experts, tradespersons, literacy teachers) teaching mathematics components can also be problematic. In particular, these people are often unable to address from a well-founded research base the difficulties of adult and vocational education students for whom the traditional school system has already proven unsuccessful. This is especially the case if their own prior experience as a learner is the only model available. Here again, the voices of teachers need to be heard.

Voices of workplace researchers. There is a burgeoning literature on workplace education, and on that mathematics which is actually used in the workplace (e.g., Bessot & Ridgway, 2000; Noss, Hoyles, & Pozzi, 1998). Christian Helms Jørgensen and Niels Warring (2000) stress the importance of taking into account the biographical information about workers, as well as the real opportunities for learning, within the multiple communities of practice which exist in any organisation. From the perspective of developing broad occupational competence, Jeroen Onstenk (1998) identifies six inter-related areas of competence (accompanied by a seventh, the competence to learn) (figure 2).

According to Onstenk (1998),

> Broad professional skill . . . is defined as a multi-dimensional, structured and internally connected set of occupational technical, methodical, organisational, strategic, co-operative and socio-communicative competencies, geared to an adequate approach to the core problems of the occupation. (p. 126)

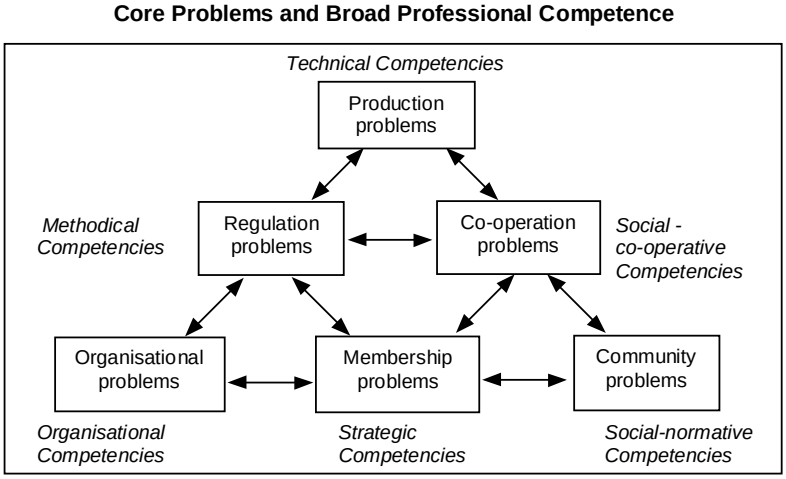


Figure 2. Core problems and broad professional competence. (Onstenk, 1998:100)

Production problems stem from the object of work activity, from the tools and equipment, and depend upon the nature of problem solving permitted by an organisation (Onstenk, 1999). However work activities are carried out within the regulatory organisational and power framework (in keeping with relevant legislative frameworks), and so need also methodical and organisational competencies to deal with planning and solving of routine and non-routine problems arising in production — increasingly so with changing business policies such as flatter management structures. Onstenk (1998: 100-101) describes socio-cultural problems as stemming from the fact that "the work is done within a specific community of practice at group, company and/or professional level, . . . characterized by specific and sometimes competing cultural norms, practices and espoused theories." Membership problems refer to organisational structures and rules, such as labour conditions and relations. They also apply within the local community of practice, where there are formal and informal rules governing the specific work-team, as well as relations with other external individuals and groups. Strategic competencies are needed here and demand a situationally adequate choice. Core problems occur regularly as part of occupational practice (specific to each task and workplace in their balance and intensity), and choices and decisions must be made involving the application of knowledge and skills with the appropriate actions at the optimal time and speed.

Personal experience suggests that vocational mathematics curricula, in an algorithm-driven approach, focus mostly upon one competence — the technological competence associated with production — while the others are generally ignored. However, within the workplace itself, Elaine Butler (1998) reminds us that learning is not neutral or apolitical. Rather, it is about the production, ownership, valuing (or not) and use (or abuse) of the knowledges produced by workers and others.

How might it be possible to take account of the voices of students, teachers, workers, and workplace researchers?

Historicity

The third of Engeström's (1999) principles is *historicity,* "studied as local history of the activity and as history of the theoretical ideas and tools that have shaped the activity"

(pp. 4-5), in order to understand the problems and potentials. Particularly in relation to adult and vocational mathematics education, it is important to be aware of the history of the students, their teachers, educational managers, employers, and government groups. In my opinion, this is pivotal to understanding what *counts* as mathematics (or numeracy, in some countries) in our sector of education. At the macro-level, government policies — for example, economic rationalism, lifelong education, 'learning for work' and 'work for dole' — impact directly on our work as educators and on the conditions of students' learning. At the same time, to the extent that they implicate mathematics, these policies are underpinned by certain, historically constituted, understandings of what mathematics is, to whom mathematical knowledge should be distributed, and how much. The same can be said, at the meso-level, for curricula and courses designed in accordance with these policies, as for the texts (both written and verbal) offered to students, face-to-face or via distance education means. At the micro-level, teachers and students enter the learning arena with unique histories of experience with mathematics and with mathematics education. Each of these needs to be taken into consideration. What is at issue are historicised, necessarily partial, accounts of: (a) mathematics as a discipline, (b) the role of mathematics in vocations and society generally, and (c) the crucial role of mathematics education. I am sure that my colleagues in ALM are aware of — or even ahead of — recent developments in mathematics pedagogy. Many of the changes have been hotly contested within the education sector as well as outside of it — for example, by traditionalists reluctant to see change occur (the 'back-to-basics' movements are a prime example), by academics who feel that there is insufficient research upon which to base the changes (e.g., Clements & Ellerton, 1996), or by mathematicians who are wary of possible threats to their discipline (e.g., Wu, 1997). Here, of course, the impact of new teaching technologies cannot be overlooked — as vectors for learning (e.g., Herremans, 1995), as mediating instruments for learning (Wake, Williams, & Haighton, 2000), as objects of learning (in the sense of tool manipulation), and possibly even as the subject (e.g., Keitel, Kotzmann, & Skovsmose, 1993). Clearly, historicity exposes many problems in what is taken by various stakeholders to count as mathematics, but it also offers great potential to enhanced understanding.

Students. Adult and vocational education students in industrialised countries generally have a well-established history of mathematics education together with established beliefs and attitudes towards the discipline of mathematics itself. Dorothy Buerk (2000) provides examples of insightful student writing which describe the discipline in rich metaphors, such as experiencing an earthquake, driving through a city once well known, using a problematic tool box, or managing to run a used-car. With regard to mathematics education, FitzSimons (1993) documents the story of one student who had to learn to manage the panic sown by practices of 'mental mathematics' which stressed speed and accuracy in an emotionally damaging manner. She also had to overcome public humiliation of her status as a female, and as a student from a rural area, in relation to the possibility of studying mathematics at university. Prior experiences in the cognitive and affective domains cannot but affect the current study situation of adult and vocational education students. (See also Schlöglmann, 2001.)

Teachers. Similarly, teachers' personal backgrounds, as mathematics learners themselves, cannot but help influence their attitude towards both the discipline and the field of mathematics education. The current tenuous status of many is neither unique nor

novel. Gert Schubring (1998) noted, in a study of the history of mathematics teaching in Germany, the fragile social status associated with mathematics instruction — caused by general public disparagement linked to the limited success enjoyed by the few, and the impression of failure experienced by the great majority. Biographies of first-generation mathematics teachers in 19th century Prussia indicate their isolation from colleagues in the (then) high-status classical languages, and resistance of all kinds from the director, colleagues, students and their parents.

Other historical influences on adult and vocational education teachers include changing work conditions (their students' and their own) as a result of pressures arising from government policies such as those mentioned above. This is in addition to the impact of historical developments in the production and distribution of mathematics and mathematics education knowledge.

Governments. There are international agreements to which governments must conform, as well as historical precedents within each country concerning adult and vocational education. There are also certain local historical understandings with business and industry as to the financial and other contributions made by each. In Australia, for example, there have been huge policy shifts since the 1980s concerning the nature, purpose, provision, and cost burden associated with adult and vocational education. Any attempt at expansive learning must take historicity into account.

What kinds of historical contextual factors might be contributing to the current situation of adult and vocational mathematics (or numeracy) education in your country?

Contradictions

Engeström's fourth principle claims a central role for *contradictions*. These are a source of change and development, and may be considered as an indication of historically accumulating structural tensions within and between Activity systems. Contradictions arising from, for example, the adoption of new technologies may generate disturbances and conflicts, but may also lead to innovation, according to Engeström (1999). Within mathematics education, as King Beach (1999) points out, similar contradictions exist between the use-value and exchange-value of mathematical knowledge, creating tensions within students as to their reasons for learning it. In terms of pedagogical practice, in many places adult and vocational education in mathematics is rife with contradictions between intended curricula and actual workplace and community practice, between teaching resources available (including adequately trained staff and new technologies) and intended outcomes, and so forth.

What are some of the contradictions practitoners and students face in adult and vocational mathematics education?

Students. Our students are likely to have agency in various other arenas — as workers, parents, members of the community, and so forth. But, on entering adult and vocational education it is possible that they become (once again?) positioned as relative inferiors. Tine Wedege (1999) studied the blocks and resistance of adults to the study of mathematics, and found that the *habitus* (Bourdieu, 1980) of adults who perceive themselves competent in life without the formal study of mathematics is likely to cause them to resist learning. Mathematics has not been perceived as relevant to their life project.

> This perception stems from their experience in various communities of practice (work, family, leisure), where basic arithmetical skills have perhaps been sufficient to manage the challenges, or where mathematics has been hidden in techniques and technologies. In other words, in communities with different practices — as contexts for knowing and learning mathematics. (p. 223)

The seminal work of Jeff Evans (e.g., Evans, 1999, 2000) is highly regarded concerning the positioning of adult learners of mathematics, and their ability to transfer what they have learned. David Clarke and Sue Helme (1993) gave a graphic illustration of a male subject who was asked to design a five-room flat with a certain area, and presented the researchers with a long rectangle divided into five. The subsequent interview revealed that he had interpreted the task as a purely 'school mathematics' exercise, and would never dream of such a design for personal use!

Teachers. Teachers who are forced to work under situations of overt or implicit control via mechanisms such as competency-based training or International Standards Organisation (ISO) criteria, for example, often face conflicts between what their experience suggests and what their institution demands by way of 'public accountability' (FitzSimons, 2000). Even assessment-driven processes of moderation, while offering novices a form of professional development, can also be problematic in their subtle forms of control over more experienced teachers (Waterhouse, 1995).

Governments. Governments are prone to projecting contradictory messages. For example, they frequently echo the calls from business and industry for graduates who are creative problem-solvers, while simultaneously instituting curricula designed to encourage conformity and docility (e.g., Coben, 2001; FitzSimons, 2001). The rhetoric of lifelong learning (e.g., Delors, 1996) as a social good appears to have been adopted universally, but the reality is often that the economic imperative wins out. Richard Bagnall (2000) suggests the there are three progressive sentiments underpinning adult education: individual, democratic, and adaptive. He claims that each of these has been subverted in some way in countries focused on economic goals, with only the third, adaptive sentiment having a semblance of relevance to the discourses of lifelong learning in its individualisation and the creation of "a mindset of ephemerality, changeability and fragmentation" (p. 30).

Thus far, I have addressed the first four of Engeström's (1999) principles: the Activity system as a unit if analysis, multi-voicedness, historicity, and contradictions. Now, I turn to address the theme of the conference: *Numeracy for democracy and empowerment?*

Democracy and Empowerment

Where do democracy and empowerment come into all of this? Following the work of Jacques Derrida[1] (1994, 1997), democracy is not the present, lived reality of Western 'liberal' systems of government. Neither is it a regulative framework, a source of deduction or determinate judgement. Nor is it a Utopia — an idealised concept that we should aim for and might achieve some time in the future. Rather, it is an ethical demand or injunction, concerning concepts of friendship, community, and so forth. It must be understood in terms of democracy to come; not in the future, but in the sense of maintain-

[1] I am indebted to Evan Kritikakos, from Monash University, for discussion on this issue.

ing now "l'ici maintenant," without presence. As I understand it, democracy is not a thing to be grasped, nor an absolute standard of moral judgement.

In its pragmatic realisation through 'representative democracy,' power is distributed unequally. However, Barry Barnes (1988) comments that although the commonsense view of power is as an entity or attribute of things, processes, or agents, he sees power as a theoretical concept referring to capacity, potential, or capability. More strictly, it should be taken as referring to distributions of these.

> Any specific distribution of knowledge confers a generalized capacity for action upon those individuals who carry and constitute it, and that capacity for action is their social power, the power of the society they constitute by bearing and sharing the knowledge in question. (p. 57)

In other words, "social power *is* the capacity for action [embedded] in a society, and . . . is *possessed* by those with discretion in the direction of social action" (p. 58). Taking knowledge to be accepted, generally held belief, routinely implicated in social action and, consonant with Pierre Bourdieu's (1991) notion of cultural capital, Barnes asserts that the distribution of knowledge in society defines the distribution of power. In a similar vein, Mary Klein (2000a, 2000b) considers numeracy not as a thing to be possessed, but as a capacity for action.

Thus, democratic power depends upon access to knowledge — information selectively derived from a range of possibilities and which is capable of being interpreted and understood — access to which is also unequally distributed. As mentioned in the introduction, mathematical knowledge is said to be empowering. I return to the questions: What mathematics? How much mathematics? For whom? Who decides? Who should decide? These are in addition to the pedagogically- and administratively-oriented questions concerning how, where, when, and why.

The Concept of Technology

Related to power in diverse and complex ways, the concept of *technology* is central to mathematics in adult and vocational education. Firstly, from an industrial perspective it is integrally linked with mathematics in production (in the sense of the totality of effort, physical, intellectual, symbolic, and so forth) in manufacturing, service, and symbolic-analytic sectors. Secondly, technology is utilised as a tool of management, both in industrial settings and throughout the adult and vocational education sector — although here the education sector itself is being transformed in many countries from a public good to a competitive industry (FitzSimons, 2000).

Andrew Feenberg (1995) argues that technology is one of the major sources of power in modern societies; the power wielded by masters of technical systems largely overshadows political democracy in the control they exert over, inter alia, experiences of employees and consumers. However, rather than accepting a thesis of technological determinism, he asserts that technology is but one important social variable. Feenberg continues that modern technology can only be understood against the background of the traditional technical world from which it developed. However, rather than a generic shift, he claims that there has been a significant shift in emphasis of features such as the use of precise measurements and plans, and the technical control over some people by others. Wolfgang Schlöglmann and Jürgen Maasz (Maaß & Schlöglmann, 1988; Maaß, 1998 have drawn our attention to the complex links between technology, mathematics, and society, particularly in adult and vocational education.

Although each one of us can chose to operate in the sense of democracy, as I have outlined it above, within our own classrooms or sites of learning — that is, taking account of the multi-voicedness and the historicity of our students and ourselves, as well as the contradictions which inevitably accompany the work of adult and vocational education, within an ethical framework[2] — it is always within a range of constraints.

Expansive Learning

Engeström's fifth principle proclaims the possibility of *expansive cycles* — "transformations in activity systems" (p. 5). In these relatively long cycles of qualitative transformations, questioning and deviation from established norms sometimes escalates into a deliberate collective change effort. According to Engeström (1999:5) "a full cycle of expansive transformation may be understood as a collaborative journey through *zone of proximal development* [ZPD] of the activity." Or, as Ros Brennan (2000) expresses it, expansion from isolation to collaboration; learning from conversations and research.

In the case of numeracy for empowerment and democracy, is it possible that *Critical Mathematics* (Skovsmose, 1994) may be a serious part of the intended and implemented curriculum for adult and vocational education? What might be a role for aesthetics? Is it possible to value both cognitive and affective development? How might alternative, research-based, curricular and pedagogical practices be incorporated into curriculum and pedagogy? How might alternative, more positive, public perceptions of mathematics/ mathematics education be generated and enhanced to encourage the uptake of lifelong education in ways that benefit *all* stakeholders?

More broadly, is it or might it be possible to involve some or all of the stakeholder groups in a collective change effort to serve their mutual needs? In order to benefit all stakeholders, I believe that an expansive cycle, as described by Engeström, is a necessary component. That is, there needs to be open, respectful dialogue between all Activity groups.

Conclusion

In this paper I have attempted to paint the bigger picture of what is, or what might be, in adult and vocational mathematics education — but not in an overly deterministic or pessimistic manner. Many practitioners are well aware of the power struggles associated with constant change. Engeström's model of expansive learning allows for creativity and interaction between Activity Systems operating in relative isolation. Contradictions, which may be exploited for or against the interests of democracy, could then be negotiated. The ideal would be expansion from isolation to collaboration in design and implementation of curriculum (taken in its broadest sense) and engagement by all stakeholders with this curriculum — not forgetting the importance of members of each group being able to operate within their individual Zones of Proximal Development. Expansive Learning would emanate from conversations, analyses, and genuinely open research; and by stakeholders reflecting on alternative models of implementation.

I believe that in order to support *numeracy for empowerment and democracy*, we actually need *democracy for numeracy and empowerment*.

References

Bagnall, R. G. (2000). 'Lifelong learning and the limitations of economic determinism.' *International Journal of Lifelong Education, 19*(1), 20-35.

[2] See also, Longoni, Riva, & Rottoli (2001).

Barnes, B. (1988). *The nature of power*. Cambridge: Polity Press.

Beach, K. (1999). 'Consequential transitions: A sociocultural expedition beyond transfer in education.' In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education, 24* (pp. 101-139). Washington, DC: The American Educational Research Association.

Bessot, A., & Ridgway, J. (Eds.) (2000). *Education for mathematics in the workplace*. Dordrecht: Kluwer Academic Publishers.

Bourdieu, P. (1980). *Le sens pratique*. Paris: Les éditions de minuit.

Bourdieu, P. (1991). *Language and symbolic power* (G. Raymond & M. Adamson, Trans.). Cambridge, UK: Polity.

Brennan, R. (2000). 'Competing views on online delivery of education and training.' In F. Beven, C. Kanes, & D. Roebuck (Eds.), *Learning together: Working together: Building communities for the 21st century. Proceedings of the 8th Annual International Conference on Post-Compulsory Education and Training* (Vol. 1) (pp. 200-208). Brisbane: Centre for Learning and Work Research, Griffith University.

Buerk, D. (2000). 'What we say, what our students hear: A case for active listening.' *Humanistic Mathematics Journal, 22*, 1-11.

Butler, E. (1998). 'Persuasive discourses: Learning and the production of working subjects in a post-industrial era.' In J. Holford, P. Jarvis, & C. Griffin (Eds.), *International perspectives on lifelong learning* (pp. 69-80). London: Kogan Page.

Civil, M. (2001). 'Adult learners of mathematics: Working with parents.' In G. E. FitzSimons, J. O'Donoghue, & D. Coben. (Eds.). *Adult and life-long education in mathematics: Papers from Working Group for Action 6, 9th International Congress on Mathematical Education, ICME 9* (pp. 201-210). Melbourne: Language Australia in association with Adults Learning Mathematics – A Research Forum (ALM).

Clarke, D., & Helme, S. (1993, July). *Context as construct.* Paper presented at the 16th Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane.

Clements, M. A. (Ken), & Ellerton, N. F. (1996). *Mathematics education research: Past, present and future*. Bangkok: UNESCO Principal Regional Office for Asia and the Pacific.

Coben, D. (2001). 'Fact, fiction and moral panic: The changing adult numeracy curriculum in England.' In G. E. FitzSimons, J. O'Donoghue, & D. Coben. (Eds.). *Adult and life-long education in mathematics: Papers from Working Group for Action 6, 9th International Congress on Mathematical Education, ICME 9* (pp. 125-153). Melbourne: Language Australia in association with Adults Learning Mathematics – A Research Forum (ALM).

Delors, J. (Chair). (1996). *Learning: The treasure within*. Report to UNESCO of the International Commission on Education for the Twenty-first Century. Paris: United Nations Scientific, Cultural and Scientific Organization (UNESCO).

Derrida, J. (1994). *Spectres of Marx : The state of the debt, the work of mourning, and the new international* (transl. P. Kamuf). New York: Routledge.

Derrida, J. (1997). *Politics of friendship* (trans. G. Collins). London: Verso.

Engeström, Y. (1999). 'Expansive learning at work: Toward an activity-theoretical reconceptualization'. Keynote address in *Changing practice through research: Changing research through practice. Proceedings of the 7th Annual International Conference on Post-Compulsory Education and Training.* Brisbane: Centre for Learning and Work Research, Griffith University.

Evans, J. (1999). 'Building bridges: Reflections on the problem of transfer of learning in mathematics.' *Educational Studies in Mathematics, 39*(1-3), 23-44.

Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practices*. London: Routledge Falmer.

FitzSimons, G. E. (1993, March). *Constructivism and the adult learner: Jane's story.* Paper presented at the Constructivism: The Intersection of Disciplines Symposium, Deakin University, Geelong, Vic.

FitzSimons, G. E. (2000). *Mathematics in the Australian VET Sector: Technologies of power in practice*. Unpublished doctoral dissertation, Monash University, Victoria.

FitzSimons, G. E. (2001). 'Mathematics and Lifelong Learning: In whose interests?' In G. E. FitzSimons, J. O'Donoghue, & D. Coben. (Eds.). *Adult and life-long education in mathematics: Papers from Working Group for Action 6, 9th International Congress on Mathematical Education, ICME 9* (pp. 11-20). Melbourne: Language Australia in association with Adults Learning Mathematics – A Research Forum (ALM).

Herremans, A. (1995). *New training technologies*. [Studies on Technical and Vocational Education No. 2.] Turin: International Labour Organisation in association with UNESCO [UNEVOC].

Jørgensen, C. H., & Warring, N. (2001, March). *Learning in the workplace — The interplay between learning environments and biography*. Paper presented at 29th Congress of the Nordic Educational Research Association (NFPF), Stockholm.

Keitel, C., Kotzmann, E., & Skovsmose, O. (1993). 'Beyond the tunnel vision: Analysing the relationships between mathematics, society and technology.' In C. Keitel & K. Ruthven (Eds.), *Learning from computers: Mathematics education and technology* (pp. 243-279). Berlin: Springer Verlag.

Klein, M. (2000a). 'Is there more to numeracy than meets the eye? Stories of socialisation and subjectification in school mathematics.' In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000. Proceedings of the 23rd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 1) (pp. 72-78). Perth: Mathematics Education Research Group of Australasia.

Klein, M. (2000b). 'Numeracy for preservice teachers: Focusing on the mathematics and its discursive powers in teacher education.' In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000. Proceedings of the 23rd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 1) (pp. 347-354). Perth: Mathematics Education Research Group of Australasia.

Longoni, P., Riva, G., & Rottoli, E. (2001). 'Ethics of "delicatezza"'. *Mathematical literacy in the digital era. CIEAEM 53, Verbania – Italy* (pp. 105-106). Verbania: Ghisetti e Corvi Editori.

Maaß, J. (1998). *Technology transfer —A useful metaphor for university level mathematics courses for engineers and scientists*. Keynote address. In ALM-4. *Proceedings of the Fourth Annual Conference of Adults Learning Mathematics —A Research Forum* (pp. 58-62). London: Goldsmiths College, University of London.

Maaß, J., & Schlöglmann, W. (1988). 'The mathematical world in the black box-significance of the black box as a medium of mathematizing'. *Cybernetics and Systems: An International Journal, 19*, 295-309.

Noss, R., Hoyles, C., & Pozzi, S. (1998). *ESRC end of award report: Towards a mathematical orientation through computational modelling project*. London: Mathematical Sciences Group, Institute of Education, University of London.

Onstenk, J. (1998). 'New structures and new contents in Dutch vocational education.' In W. J. Nijhof & J. N. Streumer (Eds.), *Key qualifications in work and education* (pp. 117-132). Dordrecht: Kluwer Academic Publishers.

Onstenk, J. (1999, August). *Competence development and learning at work*. Paper contributed to the European Association for Research in Learning and Instruction (EARLI 1999) symposium, Learning and Working, Göteborg, Sweden.

Schlöglmann, W. (2001, June). *Affect and cognition: Two poles of a learning process.* Paper presented at the 3rd Nordic Conference on Mathematics Education, Kristianstad, Sweden.

Schubring, G. (1998). *Contribution to the ICMI discussion document: The role of the history of mathematics in the teaching and learning mathematics (1997-2000).* Unpublished manuscript.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

Wake, G. D., Williams, J. S., & Haighton, J. (2000). 'Spreadsheet mathematics in college and in the workplace: A mediating instrument?' In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th conference of the International Group for the Psychology of Mathematics Education* (Vol. 4) (pp. 265-272). Hiroshima: Hiroshima University.

Waterhouse, P. (1995, September). 'Exploring the tensions in assessment and the CGEA.' *Adult Literacy Research Network Node (ALRNN) Newsletter, 2.*

Wedege, T. (1999). 'To know — or not to know — mathematics, that is a question of context.' *Educational Studies in Mathematics, 39*(1-3), 205-227.

Wu, H. (1997). 'The mathematics education reform: Why you should be concerned and what you can do.' *American Mathematical Monthly, 104*, 946-954.

(Dis)empowering forces in everyday mathematics. Challenges to democracy

Lena Lindenskov and Paola Valero
The Danish University of Education, Denmark

Abstract. Adult mathematics education has been discussed and analysed in relation to its potential to contribute to citizens' everyday working and whole life practices (Wedege, 2000). The connection between mathematics education and people's lives is seen as a contribution to democracy. "Empowerment" has become a key term in the current discourse in adult mathematics education. However, we find that educational practices intending to bring "empowerment" can easily end up generating "disempowerment". In this sense, empowerment and disempowerment are linked together and constitute a fragile duality that characterises adult mathematics education practices. We will explore this fragile duality using as a theoretical framework four different interpretations of what "powerful mathematical ideas" may mean (Skovsmose & Valero, in press). Logical, psychological, cultural and sociological interpretations allow us to pose questions about the focus of an adult mathematics education that really intends to support a citizen's whole life. We will provide examples from the current Danish reform of mathematics adult education and adult mathematics teacher education. These reforms are based on ideas such as "numeracy", as an everyday competence (Lindenskov & Wedege, 2001).

Bringing two perspectives together

Lena writes…

I am Associate Professor at the Department of Curriculum Research at the Danish University of Education in Copenhagen. My Ph.D. focused on the relations between everyday knowledge and mathematics, and on high school adolescents' interpretations of school mathematics (Lindenskov,1994). In the last six years, I have carried out research on adults' mathematical learning, including adults' learning problems and pleasures. More recently, I have been engaged in educational policy making for adults. Tine Wedege and I were asked by the Danish Ministry of Education to develop a new National Adult Numeracy Curriculum, and to develop a new special teacher education programme for it. We made use of our previous experience: Tine's, mine, and our collaborative research, as well as our experiences as ALM members. The preparation of this plenary session provided me with an opportunity to look at policy making from a distance, and to put on critical glasses – there are only limited possibilities to do this whilst engaged in practice. I enjoyed my co-operation with a good friend and colleague: Paola Valero. Firstly Paola's research on democracy and mathematics education in times of reform poses serious questions to educational planning. Secondly Paola's international perspectives and experiences enable her to view Danish educational settings with a critical eye. Paola sees and interprets phenomena in ways that we, Danes, usually don't see. This paper is a result of our conversation and continuous questioning.

Paola writes…

I am Colombian citizen and have been living in Denmark during the last four years while working on my Ph.D. dissertation on the political dimensions of mathematics education changes in the organisation of secondary schools (Valero, forthcoming). Part

of my research has concentrated on theoretical explorations of the relation between mathematics education and democracy, in particular, from the perspective of developing countries (e.g., Valero, 1999). When invited as a plenary speaker to ALM I wondered whether my reflections, focused on school mathematics, could shed light on adult mathematics education. As Lena and I started working together we realised the strength of re-thinking common concerns in a dialogue that brought two different life-experiences together.

Our meeting around this plenary address has indeed been a challenge, and we are not at all finished with our discussion. We do not present research results in this paper. Rather, we put forward some discussion statements with the intent to question some of the dominant ideas in adult learning mathematics. In order to do so, we discuss the concept of democracy. We present two definitions of democracy that we find relevant in analysing adult mathematics learning. We highlight the concept of empowerment, and argue for its fragile duality, expressed in the fact that empowerment may easily lead to disempowerment. As a tool for analysing this fragile duality, we explore four different interpretations of the relationship between mathematics education, power and democracy as suggested by Skovsmose and Valero (in press). We exemplify these interpretations with episodes from adult mathematics education in order to challenge and raise critical questions to some dominant ideas[1] in research on adult mathematical learning.

Examining the concept of democracy

Denmark is a constitutional democracy. In such a regime, every Danish adult at the age of 18 or more has one vote in the election of the 179 members of our Parliament. This specific electoral method sounds fair to us, fairer than methods used in other democracies such as the United States of America and Great Britain. It sounds fair to us that the electoral system provides a party distribution in Parliament, which equals the distribution of individual votes given to the parties. In between elections, individual citizens have the responsibility of keeping themselves informed on what the politicians stand for and how they put their promises in practice. In this way each individual citizen can fulfil the obligation of choosing the right representative in the following election according to her own convictions, interests and ideology. Keeping informed and taking a political stance require that represented citizens are aware of and capable of understanding and reflecting critically on a variety of debates and crucial decisions which are very often summarised publicly by means of all types of numerical representations. Democracy becomes more solid and stable if there is an increase of citizens' rational participation in the election of their representatives.

The previous description of the meaning of "democracy" is what we will call a classical, liberal conception, which we name "Democracy 1". This conception has been critiqued from perspectives that define democracy in relation to citizenship and not in relation to a political regime (e.g., Mouffe, 1992). Democracy 1, from these perspectives, is seen

[1] Some of these "dominant ideas" in adult numeracy teaching from the 1970s on, are presented in Coben & Chanda (2000:311) as "start from where the student is; enable students to work at an individual pace; base work in contexts relevant to the adults concerned; base content on what the students need or want to know in the short term; an absence of examinations".

as a limited construct that gets reduced to a method of election, and to a prescription of a relation between representatives and represented. It is also conceived as an individual issue, in which personal freedom and choice are paramount. Critical perspectives on classical notions of democracy, instead, place the centre of democracy in people's everyday lives, and in how people engage with others in the transformation of their life conditions. Democracy refers then to the way people interact and get collectively involved in responding, changing and understanding their social, political and economic environment. This conception is what we call "Democracy 2"[2].

Let us now look at the two conceptions of democracy regarding their implications for mathematics education. Democracy 1 needs enlightened citizens who can make sense of the wide variety of numerical information in which public debates are based. Without this capacity citizens cannot follow the main political decisions made by representatives, and cannot take a stance towards representatives' defence of citizens' interests. Hence, mathematics education has the obligation of providing citizens with this important capacity. Mathematics education empowers people through mathematics, in other words, mathematics empowers people.

We find this formulation quite problematic. When one says that mathematics *empowers* people, one has in mind a classical definition of power. Power is the capacity of a social agent *A* to influence the behaviour of another social agent *B*. This would mean that "mathematics" has the capacity of influencing the behaviour of "people". If we say so – and actually think that this is possible and true– we are indirectly giving mathematics the status of a social agent, with its own life, independently from the human beings who had constructed it. We would, therefore, be fundamentally trapped in a Platonist conception of mathematics which we, in the community of mathematics education researchers, have discarded some time ago. Furthermore, this conception of mathematical empowerment is also problematic in the sense that empowerment is conceived in absolute terms: people are empowered or not. They can, or cannot deal with mathematical representations. We know, from our experience, that people's capacities to deal with mathematics cannot be seen simply in these terms.

Democracy 2, in turn, needs aware citizens who can make use of mathematical knowledge and competencies as complementary resources for action in particular situations in their everyday life. Here it is also important that people can cope with the lack of that knowledge and capacities, and that they can figure out strategies to engage in social action requiring these tools. In this view, mathematics education provides opportunities for people to get acquainted with the possibilities of using mathematics as a resource of power. Behind this formulation there is a relational conception of power, that is, power is not an intrinsic characteristic of a person or a social institution, but the manifestation of a relation in which people position themselves in order to influence the outcomes of a situation using diverse tools (Foucault, 1972). Empowerment, then, is also relational. Mathematical empowerment does not emerge from the possession of mathematics, but from the position that citizens adopt to influence the outcomes of social practices where

[2] For more details about the differences between these two conceptions of democracy see, e.g., Valero (1999) and Skovsmose and Valero (2001).

the use of mathematical tools is required (Valero, forthcoming). The role of mathematics education in a democracy of the second type cannot be described in absolute terms. It is clearly bounded to social relations and people's positioning in diverse situations.

Examining (dis)empowerment

One of the consequences of the acceptance of Democracy 2 and of a relational view of power in mathematics education is the recognition that empowerment always goes hand in hand with disempowerment. If power is relational, then there are always positions of "advantage" and "disadvantage" in social relations. This fact is what we refer to as the *duality of empowerment and disempowerment.*

Particularly in educational relations, we find that this duality is fragile, that is, it can easily go in either direction. Traditionally, we have thought that what we do in our classrooms as teachers – and in our task as researchers – is empowering for our students and for the people we work with. However, we see that we need to question this assumption, and be open to the possibility that what we do with the best of the intentions, in fact brings disempowerment to others who adopt different positions and diverse perspectives. The hard statement we all have to face is that whatever is done for the sake of empowerment – from one point of view – may easily be conducive to disempowerment –from another point of view.

In order to provide some evidence to our claim, we have decided to use part of a framework that Skovsmose and Valero (in press) have developed in order to analyse different interpretations of power in relation to mathematics in mathematics education. This frame allows us to identify four possible ways in which mathematical ideas can be seen as related to power:

1. From a *logical* point of view, mathematical ideas are "powerful" if they provide an insight into the conceptual structure of mathematical knowledge. This point of view emphasises abstraction as the essential characteristic that gives mathematics its privileged role as an important form of knowledge. This perspective associates power with the internal characteristic of mathematics as a discipline.

2. From a *psychological* point of view, mathematics ideas are "powerful" in relation to the meaningfulness they have for individuals within their learning experience. The emphasis on meaningfulness highlights the cognitive subject who engages in rebuilding mathematical notions, and who makes sense of them in relation to her emotions, beliefs and attitudes towards mathematics.

3. From a *cultural* point of view, mathematics ideas are "powerful" in relation to the people's engagement in the dilemmas and problems of their own social situation. This view emphasises the role of the community within which people act in the world.

4. From a *sociological* point of view, mathematics ideas are "powerful" in relation to people's engagement in a critique of the use of mathematics as tools for social action. This view stresses society as the space of technological action.

Our intention here is not to give a detailed account of this frame (for details see Skovsmose & Valero, in press). We have selected some of its key elements that allow us to present critical questions to some dominant ideas in ALM research and recommendations for practice. These elements allow us to examine some situations that Lena faced when involved in diverse activities from which some of the ideas for the new national adult numeracy curriculum were developed (see Lindenskov & Wedege, 2000). In what follows, we explore four situations which made us reflect about the fragile duality of (dis)empowerment in adult mathematics education.

I won't do it my way!

Birgitte is a highly motivated, female participant in a ten-month course in adult basic education. Eigil Hansen, the teacher, and Lena Lindenskov, a visiting researcher, stressed in class the importance of participants finding their own informal methods to perform calculations. This recommendation was based on the idea that in adult education it is fundamental to help to become aware of their own methods of calculating, approaching, and solving problems. An appropriate teaching should provide students with time to remember and discuss their methods, and enable them to build further ideas on their own methods (Lindenskov & Hansen, 2000). Students' own methods provide a solid base for dealing with more formal mathematical procedures. These formulations were based on dominant ideas in research on mathematics adult education, which privilege informal over formal mathematics. Informal mathematics and students' own methods are more friendly and attractive than abstract, formal, decontextualized mathematics. Once procedures make sense for the person, then more "detached" forms of mathematics can enter the educational scene. In fact Eigil and Lena also provided students with a variety of formal mathematical methods for calculation.

Observing and interviewing the adult learners during this course challenged these ideas. Three groups of learners were identified according to their calculation methods. The first group were adults who did not have any methods of their own. The second group had their own methods and actually did not care about the teacher's or other learners' methods. The third group had some methods of their own, but improved them or replaced them during the course when formal procedures were introduced. Birgitte was in the third group. She was encouraged to use her own methods, which she found efficient in practice. During the course, she made her own methods evolve into others that integrated formal procedures. One year after the course ended, we invited the participants to an informal meeting. Birgitte related that one day at her job, she explained about the new percentage method she became acquainted with during the course. She felt proud of this method, but at the same time was worried about it being "school mathematics". Probably it was far from everyday use. One of her colleagues confirmed that such a method was in fact well known and had a widespread use in the company. After that day Birgitte decided not to use her own efficient method, but adopted the more formalised one.

This story brings us to the centre of the tension between formal and informal mathematics, and the emphasis that we have given to the necessity of going from informal to formal in adult education. This emphasis has been the reaction in adult mathematics education to the interpretation of the "power of mathematics" from a logical point of view. We intend to "empower" students by making them feel confident with their numeracy

practices, most of which actually happen out of school, that is, in people's daily activities. However, here we see that this assumption may be questioned. In "Democracy 1" informal methods may suffice for fulfilling the obligation of being informed. But in "Democracy 2", where power relations among citizens are salient, we must challenge the view of locating informal mathematics in daily activities and formal mathematics in education. Birgitte found reasons, in practice, to privilege formal mathematics. In this case, we could say that teacher's intentions in giving her confidence may have opened the door to disempowerment.

Were you serious?

This story took place in an experimental course about the new national adult numeracy curriculum, in January 2001. In this course, adult educators had to put in practice with their students some of the basic ideas of the new curriculum. A group of teacher educators based their project on the idea that it was important to use students' own problems in class. The theoretical reason behind this being that people can easily build meaningful mathematical ideas if these ideas relate to elements that people find in their "real" world. So the teacher educators invited students to bring to class their own problems. A student came with a proposal: he drew a bottle indicating its height, width, and the depth of the indentation at the bottom of the bottle. The problem read: does this make one and a half litres? The teachers were not satisfied. Together with the students, they discussed whether this was really an everyday-life problem. It resembled a typical "school" problem. They wanted something real. So another student came with another proposal: "I have to calculate the taxes that I have to pay for my summer house and I can't figure it out" He brought the matriculation registry and the tax invoice. The problem was not that easy… The teachers did not know how to do the calculation. Neither did the other students. The solution was postponed for the following session. In the meanwhile the teachers contacted the local authorities to find out how to calculate the tax. After spending lots of time talking to many people in the local tax authorities, they gave up. It was not possible to precisely know how to do the calculation. The teachers told the students the bits and pieces of information they have collected in their talks with the authorities. The results were inconclusive… students were shocked! Not because of the lack of information about the problem, but because the teachers really did want them to bring their own real problems! They were serious! Talking back to the group of teacher educators when reporting their project, one of the teachers in the group mentioned how the class changed after realising that teachers' intentions of involving the students' context was not a simple motivational device. After that day no more "bottle problems" were proposed. Students' concerns entered the classroom.

This episode invites us to question some of the dominant ideas within psychological interpretations of "powerful mathematical ideas". In mathematics education research there has been a trend to link cognition and affectivity. Students' feelings towards their learning experience and towards mathematics are essential in successful learning. One way of reducing the negative effect of affective issues and even the "traumas" of adults concerning mathematical learning has been to connect school mathematics with real life. This connection, in theory, provides a base for meaning and understanding. When "reality" enters the classroom, one can, however, wonder about the "seriousness of reality". Sometimes neither teachers nor students take seriously the invitation to bring part of their lives into the classroom. After all, some may think, an educational situation in a

school setting differs from actual out-of-school life. Both teachers and students may end up considering "reality" a motivational device that contributes little to learning. In the previous episode students questioned whether the teachers wanted to go further in inviting "reality" into the classroom. Fortunately there was a turning point. But one can also imagine that, in many classrooms, that turning point never manifests. It is clear that in this case, the strategy for involving students emotionally and cognitively with mathematics may lead to disappointment and disempowerment.

How can we know?

Let us consider the following problem, which may be a typical exercise in adult –and school– mathematics[3]:

> It costs 13 Dkr to travel each way on the city bus. The monthly pass costs 235 Dkr. Which is the more economical way to get to work, the daily fare or the monthly pass?

This type of problem invites us to examine some of the assumptions of a cultural interpretation of "powerful mathematical ideas". The problem uses real data – it uses the actual price of bus tickets and monthly passes in Copenhagen. The problem involves "reality" in a different way to that in which psychological approaches do. This problem invites thought about people acting in the social space in which they live. Making a decision about transportation costs is a frequent situation for many of the adults who participate in our courses. As formulated, the task seems to rely on a series of implicit assumptions: the person who is going to answer has a fulltime job, and needs transportation for all the working days in a month. In order to answer, the problem solver needs to compare the daily fare with the fare paid for the monthly card according to its use. Following the rules of "school mathematics" the problem solver may be invited to give an answer that favours the monthly rate. But it is fairly reasonable to imagine that many adults could not reach the desired, expected answer to the problem if solved based on the assumptions of the person who proposed it. Many adults would probably answer that choosing one or the other fare depends on who is going to buy it. It is clear that, for a person with a part time job as, for example, a substitute worker, it may be cheaper to buy daily passes. Transportation in this case may not be needed so often and will probably include longer trips that are not covered by the monthly pass.

When we propose tasks that intend to address students' contexts, we risk misrepresenting students' lives and imposing, unconsciously, our assumptions about their context. We enter here in a discussion of whose contexts in fact come to be integrated into mathematics education. We also can question the extent to which students are allowed to bring forward their own agendas of learning and how the latter enter negotiation with the teacher's instructional intentions.

Another critique of the incorporation of cultural justifications for "powerful mathematical ideas" can be raised here. Let us imagine that, in fact, teachers and students can negotiate a curriculum and engage in an educational experience that fully recognises stu-

[3] A similar task and students' reactions are analysed in Ladson-Billings (1995) in a U.S.A. context.

dents' context. And let us assume that students' situated, practical mathematical knowledge is privileged over other types of more formal knowledge, as some ethnomathematics-based approaches may suggest. Students may wonder whether their own mathematical knowledge is enough to cope with a multiplicity of activities demanding other mathematical tools. It may be possible that a mathematics education intending to bridge school and practical, culturally rooted knowledge generates disempowerment.

We all are concerned, but…

Finally we enter the sociological interpretations of power in relation to mathematics education. Let us suppose that, inspired by some of the grounding ideas of a critical mathematics education (e.g., Skovsmose, 1994), teachers and students engage in project work about current affairs. Learning activities can be organised around real problems, with real figures and data concerning topics such as the boosting of private health insurance in Denmark given the decay of the welfare state, or salmonella poisoning in meat products. While the adult learners may differ logically, psychologically and culturally from the teachers, they all have citizenship in common. Bringing current affairs issues to mathematics education may be an opportunity to meet as citizens and in a more balanced relation of power. Through project work on issues concerning citizens and their lives, students as well as teachers can develop the capacity of seeing how mathematical tools are used in diverse spaces of political, economic and social action. A critical stance towards the situation and towards mathematics can emerge from this educational experience.

But this proposal raises many questions. First of all, is it enough to concentrate on giving meaning to mathematics in relation to a social situation and leave aside completely the logical and psychological views of "powerful mathematical ideas"? In other words, one can wonder whether this sociological perspective, in any case, misses an emphasis on the acquisition of basic skills and the development of a solid conceptual and procedural, meaningful mathematical understanding. Furthermore, it can also be that the intention of contributing to the development of a critical citizenship through mathematics education may create yet another category of "disability": the lack of critique! Once more, we face the fragile duality of (dis)empowerment in mathematics education, this time in relation to a sociological perspective.

Challenging adult mathematics education

The four previous examples shed light on some of the dominant ideas in adult mathematics education. These dominant ideas covered logical, psychological, cultural and sociological interpretations of how mathematics education can be associated with "power". With these examples we intended to show that, as adult educators, we need to pay careful attention to how easily we can generate disempowerment when searching for empowerment. Our general claim is that an adult mathematics education concerned with democracy for citizenship needs to examine in serious ways the conceptions of democracy which support both our theoretical and practical work. We also need to be aware of the multiplicity of points of view and positions that teachers and students may take in an educational situation, and recognise the potential risks and disadvantages that participants –researchers, teachers and students– face in such a situation. Finally, we also need to examine the conceptions about mathematics education that we express and

defend in our work. Each perspective offers some possibilities as well as closes down some others. Maybe a mathematics education that is fully committed to democracy is that one that constantly challenges and questions what we accept as taken-for-granted in our educational and scientific endeavour.

Acknowledgement
We would like to thank Sue Elliott for her comments to previous versions of this paper.

References

Coben, D., & Chanda, N. (2000). Teaching ”not less than math, but more”: an overview of recent development in England – with a sidelong glance at Australia. In D. Coben, J. O’Donoghue, & G. FitzSimons (Eds), *Perspectives on Adults Learning Mathematics* (pp. 307-327). Dordrecht: Kluwer Academic Publishers.

Foucault, M. (1972). *The archaeology of knowledge*. New York: Pantheon.

Ladson-Billings, G (1995). Making mathematics meaningful in multicultural contexts. In W. Secada, E. Fennema, & L. Adajian (Eds), *New Directions for Equity in Mathematics Education* (pp. 126-145). Cambridge: Cambridge University Press.

Lindenskov, L. (1994). *Everyday knowledge and Mathematics: Learning Processes in School.* Roskilde, Denmark, Roskilde University, Ph.D. dissertation. (Translation of the Danish original “Hverdagsviden og Matematik – læreprocesser i skolen” (1993), Roskilde: IMFUFA, No. 246)

Lindenskov, L. & Hansen, E. P. (2001). Two Dilemmas in Communicating Mathematics in Adult Basic Courses: ‘How to meet pre-knowledge of adult learners’ and ‘How to support democratic classroom decisions. In M.J. Schmitt, & K. Safford-Rasmus (Comp.), *A Conversation between Researchers and Practioners* (pp. 29-33) Cambridge, MA: Harvard University and ALM.

Lindenskov, L., & Wedege, T. (2001). *Numeracy as an analytical tool in adult* mathematics *education and research.* Copenhagen: Centre for Research in Learning Mathematics, Publication no. 31, June 2001.

Mouffe, C. (Ed.) (1992). *Dimensions of radical democracy.* London: Verso.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

Skovsmose, O., Valero, P. (2001). Breaking political neutrality. The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, B. Nebres (Eds.), *Socio-cultural aspects of mathematics education: An international research perspective* (pp. 37-56) Mahwah, NJ: Lawrence Erlbaum.

Skovsmose, O., & Valero, P. (in press). Democratic access to powerful mathematical ideas. In L. D. English (Ed.), *Handbook of international research in mathematics education: Directions for the 21st century*. Mahwah, NJ: Lawrence Erlbaum.

Valero, P. (forthcoming). *Reform, democracy, and secondary school mathematics*. Copenhagen, The Danish University of Education, Ph.D. dissertation.

Valero, P. (1999). Deliberative mathematics education for social democratization in Latin America. *Zentralblatt für Didaktik der Mathematik*, 98(6), 20-26.

Wedege, T. (2000). Matematikviden og teknologiske kompetencer for kortuddannede voksne. Roskilde, Denmark: IMFUFA No. 381, Roskilde University, Ph.D. Dissertation.

Paper Presentations

Maths and Measurement:
Developing measurement skills in adult learners of mathematics.

(An evaluation of the efficacy of critical mathematics methods in bridging education)

Anne Abbott
Whitireia Community Polytechnic, Porirua, New Zealand

Introduction

Whitireia Community Polytechnic is a small government funded tertiary institution situated in Porirua, some 15km north of the capital city, Wellington, in New Zealand.

New Zealand Polytechnics were originally set up by the government to provide vocational training and education, but most have gradually diversified and many now teach undergraduate degree programmes. Whitireia has a Bachelor of Nursing degree programme, as well as some degree programmes in business.

Patrick(1999) found that using a critical mathematics education approach with a small self-selected group of adult learners in Wellington, enabled them to overcome barriers to learning mathematics. This small group of learners comprised women, all of whom had tertiary qualifications, and were successful in their own fields.

The students in this course are adults returning to education and hoping to continue into further tertiary training at the polytechnic. None have had any previous tertiary education, and most have left school without adequate secondary qualifications.

The topic of measurement was chosen for this research project partly because I became aware that students were not as good at measuring as they thought they were, and partly because a laboratory technician said that she had observed that some students using the laboratory did not know how to read a scale.

Most adult students take courses in mathematics as part of a more general education programme and in order to gain qualifications, thus becoming participants in “Second Chance Education”. The students vary in age and gender each year.

There were 26 students participating in the research this year.
19 were women, aged between 17 and 55, 7 of them were men aged between 18 and 25
14 women and 1 man were studying Health Science, 5 women were studying Social Science and 6 men were studying Business and Computing.
3 students did not complete the work for the unit due to absence.

The qualifications that adult students seek to acquire almost invariably have prescribed courses of study. Each subject area has a defined “syllabus”, and the Introduction to Higher Learning Course is no different. 19 of the students who agreed to participate in the research filled in an information form giving details of “where they thought they were at”, with regard to their ability to make measurements. Of those, only ¼ of them were confident with the measurements they had to make in their everyday lives. One

person was not at all confident with making those measurements. Just over half were not confident with metric conversion, and the same proportion was not confident about calculating area or volume. Only a quarter of the students were not confident with either conversion or area/volume calculations.

The course of study includes assessments that test a set of standards based criteria called, by the New Zealand Qualifications Authority, "Unit Standards". There are two measurement ones, the first of which expects a student to be able to make measurements, use given measurements to solve problems and to make estimates of measurements. In the past, only between 10% and 15% of the students have gained credit for this Unit Standard.

The second one expects the student to decide what measurements to make and then use those measurements to solve problems. In past years, approximately 30% of the students were credited with this Unit Standard.

Methodology

In her work with adults in New York, Marilyn Frankenstein (1989) used a problem-based approach to encourage her students to become critically engaged in their learning.

In an effort to use this kind of approach with these students, I gave the students a "syllabus" of the topics in which they would need to show competence. These included:

- A list of the units they would need to become familiar with
- Conversions between units, including centimetres
- Choice of an appropriate measuring instrument
- Choice of an appropriate unit, estimation and appropriate accuracy
- Formulae for areas of rectangles, triangles and circles
- Basic formula for volume of a prism

In spite of the fact that most students in ITHL seemed to THINK that they had a reasonably good understanding of measurement, it was interesting to see how many of them did not know how to read a non-unit scale. Many of them are doing health science, with the intention of beginning a degree in nursing next year, so a selection of different sized hypodermic syringes sparked a lively discussion as to the magnitude of the smallest division on those scales! I was able to mix with the students quite freely, and at the same time, check and make suggestions if their scale decisions were questionable.

A series of worksheets, designed to encourage the students to work in pairs or in threes, was devised, but limited to the measurement sets in which they needed to attain competency (*examples of these are available by e-mail for those interested*).

Working together in groups and discussion of which unit to use in the measurements, were both important aspects of the conceptualisation process, as were the short "feedback sessions" at the end of each class, where the students made suggestions as to how the material could be improved to enhance understanding.

The requirements of a syllabus placed time limitations on the students. The knowledge that there would be a competency test at the end of the unit did put some stress on them,

because they wanted to do well. However, with all "second chance" students, absenteeism is quite high, especially if the work is perceived to be difficult, and they find it hard to keep up.

The same material was presented to each of the three classes, but the attitudes to the work and the work effort varied considerably between the classes.

Class 1
This is a mixed class with 4 males and 5 females. All of the males are in their 20's, one has English as a second language, and one has cerebral palsy and writes all his work on a laptop computer. Two of the females are under 20, and the other three are all mothers. One of the women is Russian, and is there to learn English; she is very competent in the Mathematics at this level. The men work together, and the women have split into their two age groups.
This group were all very involved in the work, and very ready to request changes and offer suggestions for improvement.

Class 2
The students in this group were all women who preferred to work in pairs. In the preliminary survey, no student said that they were more comfortable with imperial units, but all except 2 referred to their height in feet and inches!

Class 3
One student in this class was new. He was a male student in his early 20's, who started in the ITHL course in the same week as we started this project. Of all the classes, this one was, with one exception, the least confident and the least competent in their abilities to measure, even length, with a ruler. Some members of this group were also very reluctant to change to a new teaching style. During the 4 weeks that the project was running, the class dynamics changed considerably. The most competent student had a sick child and was absent for 4 of the 12 course hours. This meant that there was no "leader" and the class needed to re-group. One older student, who had demanded that the class be taught in the conventional way (because she perceived that it was the only way to learn), was also absent for most of the course hours (7/12 hours). The young man, who had been working with one of the older women, started to work with the woman in her thirties, and they developed a very good working relationship that has since spread to the other subjects that they do. The 20 year-old was absent for one of the class times, but when she returned, joined in with this duo, to make a group of three. The older woman began working by herself, with frequent requests for help from myself and the other group. She is not a woman who is able to work well in a group situation, tending to "take" rather than contribute.

Introductory Tasks and Observations
The measurement topic was studied over a 4 week period from May 21st to June 15th There are 2 two hour timetabled classes per week, which should have been a total of 16 teaching hours and then 3 hours for assessments in the week June 18th to 22nd. However, during this period of time, each class "lost" at least four classroom hours, due to the June 4th public holiday, and a compulsory visit to a local Maori Meeting House.

In previous years, the topic has been 4 – 5 weeks, with up to 20 teaching hours and then 3 hours of assessment tasks, 2 practical and 1 theory.

The topic of measurement is part of the school curriculum, so I tried an introduction based on a discussion of the everyday considerations of measurement. The students in the first timetabled class were not really sure what they were supposed to do, because there weren't any "answers".

Because this was an action research project, based on the Cardno/Piggott-Irvine model, I was able to '*monitor'* this lesson. A '*review'* showed that the students had not found this approach useful, so another introductory lesson was '*planned*', involving a study of scale in measurement instruments. This was '*actioned'*, and this time, the students found the lesson more stimulating, and a better learning exercise.

Various types of measurement instruments were displayed around the room. The students had a worksheet to fill in, and we started to work on the purpose of each instrument, the unit of measurement and the size of the smallest unit that could be measured on each of the scales. The students were more positive about this approach. There were recognisable "answers", but at the same time, the students could also discuss their decisions with the others in the class.

It was very surprising, given the amount of school time spent on measurement in both science and mathematics, that most of the students were not able to read intuitively, any non-unit scale presented on these instruments.

For example very few students recognised immediately that the smallest divisions in a 1000ml measuring cylinder were 10ml, not 1 ml. One of the syringes had 0.2ml divisions, and frighteningly, most students did not realise that. Even more surprising was the inability of at least half the students to recognise that 1cm and 10mm are the same measurement, and that 1000mm = 100cm = 1m, because one of the tape measures was scaled in centimetres, and the other in millimetres. Maybe next time a metre rule should be included in the selection of measurement instruments.

Most students were able to read the protractor, since we had already used one, and the scale is in 1° divisions. They also had no difficulty with the timers or the 12-hour clock. There is, of course, a notation problem with 24-hour clock time, because most electronic devices, including alarm clocks, have a colon between the hours and minutes, which is NOT standard notation in a timetable. Perhaps next time, a 24-hour transport timetable should be introduced, so that they see the correct written notation.

Some of the students, especially the older women, were very excited about learning to read instruments they had never seen before, and being confident, now, to measure with the instruments they had seen, but had previously not comprehended how to read.

Practical Worksheet Tasks

The capacity worksheet was designed to appeal to the intending nursing students, and considerable dialogue between the members of the group was used in the actual

measurement of "a cup of tea" and "a glass of water" and discussion as to whether the differences in measurement of capacity were acceptable. This type of discourse was similar in all of the groups in all of the classes.

In classes 1 and 2, only one group asked for an explanation as to how the calculations for total input should be done, but the other groups managed to reach a consensus, after several attempts at different methods had been used. This was interesting, because in general, these students prefer to be TOLD how to do a calculation, and then use that as a model for future calculations. Once they had checked that they had "done it right" they continued with the worksheet to the end, and compared their answers with other groups in the class.

Class 3 worked differently. For this worksheet, the group with the competent mathematics student followed her lead and did as she suggested. The pair were very anxious to make sure that they checked with me each time they did a measurement, and they needed to be led through the calculations (this has parallels with Freire's "Banking Model" which was NOT what I wanted from the work-sheet). It was the older woman who insisted on this "teaching".

In the Length and Area work-sheet, it was requested by several of the groups that I show them what formula to use to work out the area of the table-top, and how to tell what unit to use. We discussed how many dimensions were needed to find an area, and settled on 2 dimensions means units^2. The formula for area of a rectangle was put on the board. After this discussion, there were no problems when the students came to find the area of carpet needed for the floor.

For the area and volume work-sheet, the formula for the volume of a cuboid was put on the board, by request. The students wanted CONFIRMATION of three facts, the unit for volume was units^3, because there were 3 dimensions to multiply, there were 1000mm in a metre, and the area of a triangle was half a rectangle. It was obvious, watching them, that the groups worked together on finding and checking with each other, the measurements they had found.

One of the students in Class 1 suggested that the formulae for area and volume should be added to the worksheets, instead of being written on the whiteboard, and the other students in her class agreed with this suggestion.

By this time, in Class 3, the competent mathematics student was absent with a sick child, and the young male started to work with the other two women, thus making a better working group.

The students have calculators that will convert minutes to hours, but none had realised that km per hour meant km ÷ hours. It still concerns me that at this level, the commutative principle is not fully understood – some students convert minutes to hours and then divide by the km.

In most cases, the conversions between g, kg and tonnes had been relatively well understood, and the students managed this work-sheet relatively easily.

Neither the use of string nor trying to use a dressmakers' tape measure gave the students any insight as to the relationship between the diameter and the circumference. Once they had the measurements, they didn't know what to do with them. Either more direction or leave this question out next time. The formulae for circumference and area were put on the white board, and the units for each of them seemed not to pose a problem.

There was one practical workshop that we set up between us, to test our ability to estimate weight, length area and capacity. We ran it as a game, and the students had quite a lot of enjoyment, as well as learning and practising some estimation techniques.

The practical work appeared to have helped the students to comprehend the measurements and the calculations better than in previous years, so it was interesting that most of them were able to manage the theory question in their groups, with relative ease.

In the week beginning June 19th, the students began their assessment tasks for this unit. In the past, very few have achieved Unit Standard 8492 – in 2000, only 4 out of 40 students. This year, of the 23 who sat the unit assessment, 20 achieved it.

The results for the second assessment task were not significantly different from those in previous years. This was partly due to the reduced time available to finish the syllabus for the second unit. There was insufficient time to practise scale drawing or to determine a sensible level of rounding, both of which were part of the unit assessment task.

With these results being as dramatically different from those achieved in previous years, I will need to repeat this research next year to check the validity of the results. I intend also to make sure that there is sufficient time to finish the second unit, so that a comparison of the results from the second unit can also be made with the results from previous years.

Literature

Frankenstein, M. (1989). *Relearning mathematics.* London: Free Association Books
Freire, P. (1970). *Pedagogy of the oppressed.* London: Penguin.
Kemmis, S., and McTaggart, R. (1982, revised 1988). *The action research planner*. Geelong: Deakin University.
Patrick, R. (1999). 'Not your usual maths course: Critical mathematics education for adults.' *Higher Education Research and Development*, **18** (1), 85–98.
Skovmose, O. (1994a). 'Towards a critical mathematics education'. *Educational Studies in Mathematics*, **27**, 35–57.
Tout, D., & Johnston, B. (1995). *Adult numeracy teaching.* Australia: National Staff Development Committee.

Secret knowledge: Indigenous Australians and learning mathematics

Roseanne Benn
University of Exeter, UK and University of South Australia, Australia

I was invited by the Aboriginal Research Institute (ARI) to visit the University of South Australia in February and March 2001 to design and run a project to explore Indigenous Australians' ways of 'becoming numerate'. As a white, non-Australian, I was apprehensive of my qualifications for this task but was persuaded that my research and writings in this area made me a suitable person. In truth, I was delighted to return to a country that seems to me one of the most beautiful and varied in the world and in particular to work with Indigenous Australians. This paper will be the first of a series which come out of this project. It was decided to entitle the project *Numeracy as a social-cultural construct: an investigation of how Aboriginal learners engage with and make sense of mathematics curricula* with aims, objectives and outcomes as given below.

Stated project aims:

- To develop and implement research work that will improve opportunities for indigenous learners.
- To explore the connections, contradictions and gaps between current curriculum constructions of mathematical knowledge and indigenous experience and knowledge of numeracy.

Stated project objectives:

1. To analyse and document how the concept of 'numerate' is defined/constructed within and through current state curriculum documents and assessment criteria.
2. To explore, with a small cohort of Aboriginal teachers and learners, curriculum and assessment practices that will account for and make sense of different constructs of numeracy in order to compare and contrast such understandings with those identified in the formal curriculum documents.
3. To identify where there are gaps, commanalities and/or contradictions in the teaching and learning of mathematical concepts
4. To recommend, in a preliminary way, how numeracy and the formal mathematics curriculum might be taught and assessed in ways that better account for different constructs of knowledge between indigenous and non-indigenous students and also among different groups of Aboriginal students.

Stated proposed outcomes:

- A proposal for further ARI research into the area of numeracy education for indigenous people through an extensive collaborative joint research project extending to other States and including the Torres Strait Islands.
- The foundations of strategic research partnerships through the development of close working relationships with school-based staff, the Aboriginal Education Unit and other key institutions and individuals.

First, some frameworking comments on my beliefs about mathematics, mathematics education and numeracy:

- Mathematics is seen as a gateway subject by many societies. I see mathematics as very important and useful but nevertheless it is arguable that it holds a higher esteem than its actual usefulness justifies.
- The mathematics classroom sits in a wider social, political and economic context. It does not operate independently of other forces in Australian society.
- It is not the intention of this paper to either pathologise Indigenous Australians or to endorse a deficit model of education.

The project is grounded in practice and the belief that individuals learn best when mathematics teaching builds on positive attitudes, is interactive and co-operative, practical and relevant, set in a social, historical and cultural context and enjoyable and fun. It rejects the approach of deficit and disadvantage where any problem with mathematics is located in the learner rather than the system. It moves away from an individual skills-based approach to one of a critical analysis of social and economic factors, a cultural critique of the elitist assumptions about mathematics and mathematics education and a critique of the educational system with its tendency to create deficit models. It is based on the passionate belief that mathematics is a crucial way of knowing that can make a real difference in people's lives but only if it can be seen in the wider context of society, structural inequality and cultural difference. Our modern society has been dominated by certain cultures and ways of thinking. In the new post-modern world, this is being questioned at all levels. There is a growing recognition that there are alternative worldviews, truths, realities and cultures, many of which are nor recognised or valued by society. This is true in mathematics and mathematics education and this paper is concerned with the discovery and recognition of these 'silent' voices. Such an approach built on social, economic, political and cultural awareness, a value system of social justice and equity and a collaborative, co-operative approach to learning might transform mathematics education for some into education for all (Benn, 1997).

Mathematics and Indigenous People

- There have been a large number of reviews and reports on Aboriginal Affairs and Aboriginal education over the past years. For, example the year 2000 saw major reports on both school attendance (Bourke *et al,* 2000) and generating positive self-identities for Aboriginal children (Purdie *et al,* 2000).
- There has been less work done on mathematics but 2000 saw the publication of *Numeracy: A Priority for All* (DETYA, 2000) and *The National Indigenous English Literacy and Numeracy Strategy for 2000-2004* (DETYA, 2000).
- The 2001 South Australian Curriculum, Standards and Accountability (SACSA) framework makes explicit reference to the importance of both Indigenous perspectives and relevant contextualised mathematics.
- South Australian universities provide impressive options on Indigenous awareness issues and compulsory modules for trainee teachers on aspects of teaching Indigenous children.
- There are some excellent curriculum materials around including
  - Contextualising Mathematics R-7 (DECS, 1996)

- The various versions of Aboriginal Perspectives Across the Curriculum (though there is a variable amount of mathematics-related work in these publications) (DECS, 1995)

- A useful but as yet unpublished book from the Aboriginal Education Unit at Enfield, South Australia entitled *Aboriginal Culture and Mathematics*
- There is some good theoretical material around including work by Pam Harris (eg 1991), Michael Cooke (eg 1995), Michael Christie (eg 1991) and Helen Watson (eg 1989).

But, and unfortunately it is a large BUT, the statistics of school achievement by Aboriginal students tells a different story. As yet, I have only informal data but this indicates a worrying position. The following figures are only approximates, as is often the case when individuals have to specify their own backgrounds. In 2000 53 Aboriginal students completed the South Australian leaving certificate SACE, which is approximately 11 per cent of all the Aboriginal children who started secondary school in 1996. Of all the approximately 100 Aboriginal children who tried Year 12, about 25 had a go at Business Mathematics with a low pass rate; about 3 tried Mathematics 1 with one or two passing and none of the three who tried Mathematics 2 passed. This accords with the feeling that Aboriginal students attempt the less 'academic' subjects. Only a small proportion take publicly examined subjects and most take school-assessed subjects. These have lower status and are of less value for university entrance. Aboriginal students do gain access to higher education but often through special schemes. However, their schooling experience may well not have prepared them so well for university study and so the cycle goes on.

The second BUT is again a large one. Visits to schools show very real problems in the teaching and learning of mathematics for all children including, if not particularly, for Aboriginal students. This will be considered in more detail in a later paper.

Discussing the discourses

The issues in and around this project are very complex. The schema outlined in Diagram 1 below gives a framework within which the discussions can be located.

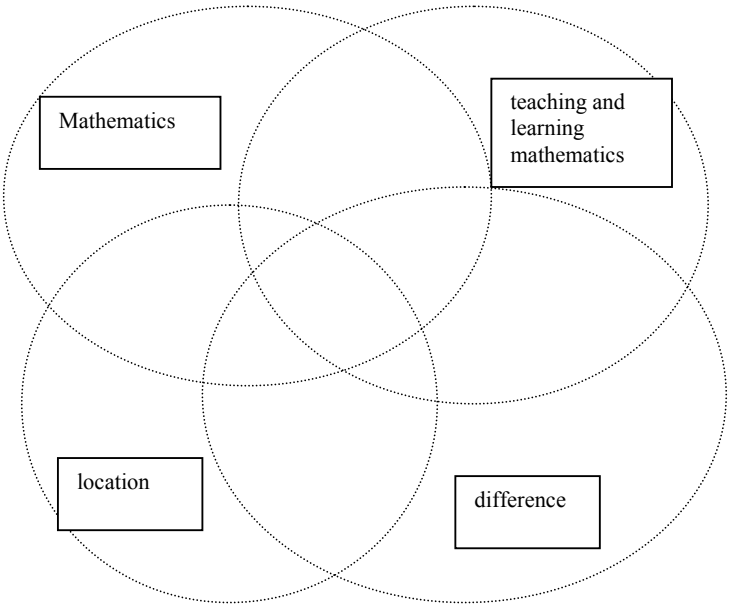
Notions around mathematics; teaching and learning mathematics; location and difference all operate on understandings of Aboriginal people and their numeracy. I will bring in discussion on location and difference but will concentrate in this paper on mathematics and teaching and learning mathematics.

For over 2000 years, mathematics has been dominated by the belief that it is a body of infallible and objective truth, far removed from the affairs and values of humanity. This body of truth is seen as existing in its own right independently of whether anyone believes or even knows about it.

Within the wider context of human thought and experience, the development of mathematics can be seen as the 'grand narrative' of academic Western mathematics which pathologises inability to relate to this mathematics and ignores or marginalises alternative or 'other' mathematics. Difference is repressed, the central narrative is held as certain and the workings of power are concealed. This characterisation of

mathematics has provided an elaborate rationale and legitimisation for the pre-eminence of academic Western mathematics and has contributed to the dominance of certain cultural groups in society. The mathematical narratives of subordinate groups have been denigrated or ignored.

Diagram 1



The certainty of mathematics has been under question. A growing number of mathematicians and philosophers are arguing that mathematics is fallible, changing and the product of human inventiveness. Others (Bloor, 1973; Wittgenstein, 1956) argue that absolute truth is located in utility and the enduring character of social practice rather than a calculation.

> And of course there is such a thing as right and wrong...but what is the reality that 'right' accords with here? Presumably a convention, or a use, and perhaps our practical requirements. (Wittgenstein, 1956).

The importance of this thinking is fundamental. It suggests that:

- Mathematics is a social construct
- It did not develop in a cultural or social vacuum
- It is not a body of truth existing outside human experience
- It is a construct or invention rather than a discovery
- It is social in nature
- It is value laden not value free
- There are different mathematics in different societies reflecting the different needs of those societies.

(Benn, 1997)

It is important to find a way of discussing the issues above in light of the findings of the project without adopting a deficit model and without being patronising or 'colonial'. I have adopted the notion of discourse to enable me to do this.

One aspect of the term discourse is that of the social process through which collective understandings are constructed by groups of people with a common interest. A specific discourse consists of a loose knit collection of concepts, terms, assumptions, explanatory principles, rules of argument and background knowledge which are shared amongst the members of that discourse community (Northledge, 1994). We all belong to a multiplicity of discourses, some informal such as gardening or tennis, some more specialised and elaborate such as law or medicine. There is, for example, a discourse of Aboriginal ceremonial business (a secret discourse) and a discourse of academic or western mathematics (seemingly not secret but in practice effectively so to many).

The discourse of academic mathematics is determined by the dominant culture and it becomes the norm against which other forms of mathematics are judged and found wanting. There are other alternative mathematics discourses which are not recognised by the dominant culture and this contributes to the marginalisation of certain groups in society. Within the construction of discourse lie power and the ownership of knowledge. The discourse of academic mathematics is about more than just the rules of mathematics and the language in which it is expressed. It is about values and valuing. This constructed mathematics is very different from the range of 'lived' mathematics such as the mathematics that Aboriginal (and of course many other people) encounter and use in their everyday lives. This difference arguably results in taking the life out of mathematics and the mathematics out of life.

This raises the issue as to whether there really are different kinds of mathematics, alternative mathematics and in particular alternative urban Indigenous mathematics. I would argue that academic mathematics is different from most people's lived mathematics. To gain an insight into Indigenous mathematics I would recommend *Mathematics in a Cultural Context* by Pam Harris (1991). This book, based on extensive research and experience, sets out to present a body of information related to school mathematics which illustrates some of the differing points of view of Aboriginal and other Australians. She acknowledges that the mathematics outlined is particularly relevant to distinctively Aboriginal communities where traditional values, languages and customs are strong. But Aboriginal people in urban areas are arguably also strongly influenced by these mathematical worldviews that are part of their heritage. One has only to note the strong influence of the British heritage on Australian society to see that ways of thinking (and this includes mathematical ways of thinking) are tenacious. Pam Harris quotes the South Australian Aboriginal Education consultative Committee, which in supporting the establishment of the Kaurna Plains Aboriginal School north of Adelaide, said:

> We contend that the theories and methodologies used in education are designed by and for middle class Australians of Anglo-Saxon extraction. We assert that education has failed to recognize that Aboriginal society is significantly different to western non-Aboriginal society… Therefore, these theories and

> methodologies are largely inappropriate and this also contributes to lack of success for Aboriginal students

Pam Harris examines space, time and money from an Aboriginal perspective. Her work counteracts various stereotypes about Aboriginal people and mathematics but does also illuminate and illustrate differences. Aboriginal people are often assumed to be concrete thinkers, only understanding the practical. Harris suggests that this is not the case but rather different abstractions are used. For example, in western maps there is considerable use of abstraction such as symbols and contour lines. Aboriginal maps often expressed as highly stylised art use symbols and keys such as sacred sites.

This last point illustrates issues around the value systems implicit in any (and each) mathematics. Academic mathematics is often seen as value and context free, 'a peek into the mind of God'. A glance at any examples or questions set will show that this is not so. Think of questions on money. Do they tend to be on mortgages or benefits? Examples on time. Being on time. The use of cars rather than public transport. An example of this was given in one class I visited which was very well prepared and resourced but the exercise was located in buying plants and gardening which was out of the lived experience of many of these children. An other teacher recounted how a student teacher who visited her school had asked for advice on a lesson plan which involved counting, sorting and colouring macaroni and spaghetti. The children in the class were very ill fed and it might have been more appropriate to consider cooking and eating the food. Mathematics in the classroom is not value or context free.

Academic mathematics values the abstract over the concrete, the formal over the informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, and general over particular. It is truly value-laden.

It was clear from my study that many Indigenous parents wish their children to study academic mathematics to gain access to the powerful and prestigious discourse of western mathematics. This helps us to understand why so many are not interested in building on their own everyday mathematics discourse which they feel, probably rightly, is not valued by the wider society. They hope that through gaining the discourse of formal mathematics, their children will have more control in their worlds, gain social power and financial advantage. This way of looking at mathematics links the different kinds of mathematics to power structures in society.

The link between mathematics and politics/power is illustrated by the number of comments made to me that 'we need to learn mathematics in order not to get ripped off'; 'we want what you have got'; 'if we have no mathematics we are vulnerable and exploitable'. In one Aboriginal remote community, trainee teachers saw mathematics as important by in the clinic although only the (white) head nurse used the mathematics. The (Aboriginal) Health Worker did not. Similarly mathematics was seen as important in the store and in the garage, although again it was only used by the (white) storekeeper and the (white) garage owner. Academic mathematics is both a gateway to more control as a user of services but also to more control as a provider of services. Mathematics is tied to notions of power in society.

There would seem to be no concern as to which mathematics should be taught in the urban and rural areas that I visited. It seemed that both the communities and the educationalists saw western mathematics as essential but did argue for greater contextualisation in the students' lives. However, this is far from being the case in one remote Aboriginal community in Central Australia who are considering the development of an new mathematics curriculum based on their traditional rather than western mathematics. It could be argued that they have been forced to do this because the western mathematics approach has failed (and alienated) so many of their people. The issues are very complex but, as I suggested earlier in this paper, people's lack of competency in western mathematics leaves individuals and communities vulnerable to exploitation and with limited life choices.

Implications for teaching Mathematics

The notion of discourse helps us to understand the difficulties so many Aboriginal people have in learning academic mathematics. This approach sees studying as the process of acquiring the usage of an unfamiliar discourse. Studying mathematics is the introduction to the discourse of mathematics not just learning a set of skills hence teaching mathematics needs to concentrate on conveying insights as to how the discourse works rather than just the content. Learners need pathways from the discourse of their everyday mathematics to the unfamiliar terrain of the academic discourse. Teaching becomes narrative developing from the familiar, perhaps through metaphors, to the unfamiliar. The metaphor allows the task to be located within the framework of the familiar mathematics, the familiar discursive practice. If used consciously, metaphors can illuminate and facilitate learning. They invite the use of intuition and imagination, developing new individual pathways from what is familiar and known to what is unknown. The use of metaphor allows the learner to position themselves with regard to the new knowledge. However, this approach can place some quite sophisticated linguistic demands on the learner in terms of communicative competence. Techniques are required to direct the learner's attention to the nature of the discourse while still retaining a normal level of communication. This requires the use of both commenting and meta-commenting by the tutor whereby the comments deal with the mathematics under discussion and the meta-comments draw attention to issues of the discourse itself. This produces active and reflective learners but is very demanding on a probably already hard-pressed teacher.

This analysis of the teaching process helps to identify and understand some of the problem areas in teaching Aboriginal children mathematics. Unless the teacher is familiar with the students' locale and everyday mathematics, they may draw on their own background (white Australian and its heritage) for metaphors. I found myself doing this in one remote Aboriginal Community by referring to candles on a birthday cake.

There are issues around language. For many Aboriginal students their first language may be their Aboriginal language or Aboriginal English. Their second language (or third or fourth) may be Standard Australian English, their third language mathematics. Many people from many backgrounds experience difficulties with literacy in their first language let alone in other languages. So the level of English of students learning mathematics may vary from little to fluency.

Seeing the teaching of mathematics as the introduction to a discourse also requires considerable knowledge and confidence on the part of the teacher:

- To see through the 'commonsenseness' of their own mathematics to the recognition that other mathematics exist and that they are valuable.
- To have at least some awareness of the Aboriginal locale (including language, interests and customs) and everyday mathematics.
- To understand and be confident in western academic mathematics
- To be able to construct a pathway from the everyday mathematics to academic mathematics by use of **appropriate** language and metaphors
- All of these may be problematic. My research indicated that despite much excellent theoretical work and curriculum construction, practical reality sometimes fails to meet these standards.

All these matters raise issues for initial teacher educators, professional development providers, Principals and governments. Teacher education programmes often provide compulsory modules on teaching Aboriginal children. However, it is not clear that these modules make explicit the implications for the mathematics classroom or that they are seen as relevant by many trainee teachers. Young or new teachers are also strongly influenced by their own school experiences and the 'chameleon factor' of the prevalent attitudes in their first staff rooms. Professional development requires perception that a need for development exists, time, money and the commitment of the school. Change in this area requires awareness and commitment (read appropriate resourcing) by principals and governments through their funding bodies.

Some early conclusions

I hold two quotes in my mind as I write this

"I recall no rewarding or exciting experience as related to mathematics" (an Indigenous adult reflecting on their mathematics learning).

"Teaching mathematics should help to prepare children for a future that is not yet known" (a worker in Aboriginal education)

It is crucial to find a way of discussing Indigenous people and mathematics without entering the them/us dichotomy. The concept of discourse helps me to do this. My assessment is that worldwide there are real issues about the learning and teaching of academic mathematics. Australia is no exception to this. Within Australian society, Aboriginal people have particular problems. This is not due to any inherent deficit but a wide range of social, economic and cultural issues. Those who do wish to conquer the discourse of academic mathematics may face a range of hurdles including the ignoring or denigration of their own lived mathematics, mathematics being a third, fourth etc language with attendant problems, and the general lack of awareness and valuing of their own culture exposed through the use of inappropriate or irrelevant metaphors and example.

Much of this is not new. We have known for years what constitutes best practice. I can list it here but to what advantage when it is listed elsewhere. The ideas are known (give or take), the present policies show awareness, and good practice exists. The real problem is how to make it system-wide. We desperately need to break the chain of

mathsphobia that lead so many of my interviewees to express their relationship to mathematics in such strong emotional terms. "I hated mathematics at school" is still a common response even among teachers. I feel anger and sadness when I hear and see the cycle repeating itself. I do not see mathematics and the mathematics classroom as in some way outside of everyday real concerns. I see exclusion from mathematics as exclusion from parts of life and hence as anti-democratic. There needs to be high expectations of Aboriginal students by their teachers. Aboriginality (and the often-associated poverty and socio-economic deprivation) is not an excuse for failure. There should be no compromise standards but an expectation of hard work The Aboriginal communities have a fundamental role to play. If it is their wish that their children achieve the academic mathematics discourse, issues around attendance must be dealt with. But as other research has shown, apart from this factor, parental/family involvement is not mandatory for good results. Schools have to, and of course often do, work out ways so that the school replaces the parent/family.

The mathematics curriculum must be based on notions of respect for all learners, social justice, democracy and the passionate belief that to be in control of mathematics is a step towards being in control of one's life. These beliefs should inform and direct the curriculum whatever mathematics topic is under consideration. However, the expressed needs of students and their parents should not be ignored and any widening of the curriculum should not replace the instrumental goals and self-development requirements of the learner but enhance these. Students can be encouraged to recognise and value the mathematics learning that takes place in all facets of their everyday life from individual to community member to worker to citizen.

The result of such a curriculum is that the learner will acquire the skills and qualifications in mathematics that they need for social and economic advancement but also the critical awareness required to change or attempt to change restrictions acting upon them. The informal lived mathematics of the various groups in society including Aboriginal people exists within society alongside the professional skills of privileged groups. Which of these is valued is a political decision which involves all of society. The choice of which knowledge should be canonised in the curriculum and how this curricular knowledge relates back to society is a political decision which in a democratic society should seek to provide for the needs of all citizens.

Literature

Aboriginal Education Unit (unpublished). *Aboriginal Culture and Mathematics.*

Benn. R. (1997). *Adult Count Too: Mathematics for Empowerment*. Leicester: NIACE.

Bloor. D. (1976). *Knowledge and Social Imagery*. London: Routledge and Kegan Paul Ltd.

Bourke. C., Rigby. K. and Burden. J. (2000). Better *practice in School Attendance: Improving Attendance of Indigenous Students*. Canberra: Commonwealth Department of Education. Training and Youth Affairs.

Christie. M. (1991). 'Aboriginal Science for an Ecologically Sustainable Future' *Australian Science Teachers Journal*. 37(1).

Cooke. M. (1995). *Seeing Yolgu, seeing mathematics*. Batchelor: Batchelor College

Department for Education and Children's Services (1996). *Contextualising Mathematics R-7*. Adelaide: Department for Education and Children's Services.

Department of Education, Training and Employment (1999). *Aboriginal perspectives on the Early Years of Learning*. Adelaide: Department of Education, Training and Employment.

Department of Education, Training and Youth Affairs (2000). *The National Indigenous English Literacy and Numeracy Strategy 2000-2001*. Canberra: Commonwealth Department of Education, Training and Youth Affairs.

Department of Employment, Education, Training and Youth Affairs (1995). *Aboriginal Perspectives across the Curriculum.* Adelaide: Department of Employment, Education, Training and Youth Affairs.

Department of Employment, Education, Training and Youth Affairs (2000). *Numeracy: A Priority for All*. Canberra: Department of Employment, Education, Training and Youth Affairs.

Harris. P. (1991). *Mathematics in a Cultural Context: Aboriginal Perspectives on Space, Time and money*. Victoria: Deakin University Press.

Northledge. A. (1994). 'Access as initiation into academic discourse' in Lemelin. R. (comp) *Issues in Access to Higher Education*. Portland: University of Southern Maine, pp.145-151.

Purdie. N *et al* (2000). *Positive Self-identity for indigenous Students and its Relationship to School Outcomes.* Queensland University of Technology.

Watson. H. and Chambers. D. (1989). *Singing the land, signing the land*. Geelong: Deakin University Press.

Wignall. P. (1999). *Double power: English Literacy and Indigenous Education.* Melbourne: Language Australia.

Wittgenstein. L. (1956). *Remarks on the Foundations of Mathematics*. Oxford: Blackwell.

Mathematics for Parents: Issues of Pedagogy and Content

Marta Civil
The University of Arizona, USA

Abstract. This paper focuses on three "Math for Parents" courses that have been implemented as part of a large project on parental involvement in mathematics. The three courses were very different in terms of content (patterns/algebra; geometry; numbers/ arithmetic), and in terms of pedagogical approach (from social constructivism to somewhat teacher-centered). Using the lens of research on adult learning, issues of content and pedagogy are explored. What do the participants (parents) seem to favor in terms of instructional approach? What content do they seem most interested in learning and why? These questions lead to implications for mathematics learning experiences for adults.

Context

As part of a large project on parental involvement in mathematics (MAPPS)[1], we are developing a series of courses that we call "Math for Parents" (MFP). The audience so far has been parents (mostly mothers) from a working-class, largely Hispanic community. Some are recent Mexican immigrants; others are Mexican American, and others are Anglo. Some speak only Spanish, others are bilingual-- English/Spanish and others speak only English. A main goal behind these courses was to engage parents in the exploration of mathematical topics that their children are likely to be experiencing in school. The parents attending these courses are members of the MAPPS Leadership Teams (some guests parents have also attended these courses) and as such they will be leading mathematics workshops for other parents in the community. Thus, through these courses we wanted not only to expose them to topics their children may be learning but to do so from an adult learner point of view to help them strengthen their understanding of mathematics. So far, three courses have been designed and implemented. Each course meets eight times for 2 hours per session. For the first implementation, the author of each course was also the instructor.

The three courses

Thinking in Patterns: this was the first course taught. Twenty-four parents took the course; there were three teachers from the Leadership Team who assisted the instructor as well as project evaluation staff and myself who were in attendance and provided assistance as needed. The course started with typical pattern problems that lead to variables and expressions. Graphing and using a Calculator Based Laboratory was one of the culminating points of the course that ended with an introduction to solving equations. The materials partially drew on current "reform-based" curriculum materials for school age children. Samples from the local school district curriculum were brought in to help connect the adults' learning experiences to what their children were experiencing in school. The instructor, a Hispanic woman, is also part of the project staff and thus the parents already knew her and had other opportunities to see her outside the course (through other Project events).

[1] Project MAPPS (Math and Parents Partnerships in the Southwest) is funded by the National Science Foundation (NSF) under grant – ESI-99-01275. The views expressed here are those of the author and do not necessarily reflect the views of NSF.

> I would characterize the pedagogical approach as social constructivist. Parents worked in small groups in a very relaxed atmosphere. Sharing of different approaches was particularly encouraged. The instructor would readily abandon her agenda to follow the learners' suggestions and questions. Dialogue and making sure that everybody had a voice characterizes much of what took place in this course. In a sense I think it bears many similarities with Flecha's (2000) work on dialogic learning:
>
> [dialogic learning] leads to the transformation of education centers into learning communities where all the people and groups involved enter into relationships with each other. In this way, the environment is transformed, creating new cognitive development and greater social and educational equality. (p. 24)

Thinking Visually: this course had a large number of parents in attendance (32); there were no teachers assisting the instructor, but project evaluation staff and myself were there in case assistance was required. The course drew on activities from geometry courses for elementary teachers that the instructor usually teaches. It was grounded on a hands-on approach to geometry, and the activities ranged over working with different shapes, angles, measurement concepts, exploring circles, ending with a discussion of the theorem of Pythagoras. Spread throughout the course were lots of information and activities related to how children learn and to school content issues (e.g., school standards for geometry). The instructor was a Mexican male mathematics educator. He was fully bilingual.

I would characterize the pedagogical approach as teacher centered in that the teacher was in control of the flow. He started on time (parents tended to arrive late); he gave them a break and he finished on time. He had many handouts with very carefully planned activities. The parents worked on them in groups and he then discussed the main ideas, sometimes encouraging participants to share their ideas with the rest of the group. This happened to become more frequent as the course went on. He emphasized meaning-construction and making sense of things.

Thinking with numbers: this course had 15 parents in attendance. There was one teacher assisting whose main role was to provide translation into Spanish since the instructor speaks only English. As usual, project evaluation staff and myself attended each session and gave assistance as requested. The instructor wrote lessons for each session--a total of eight such lessons, ranging over different ways to add, multiply and divide; exploration of multiples, factors, prime numbers; dealing with "big" numbers. The course ended with some fun/ intriguing number explorations. Each lesson was translated into Spanish by an outside translator. Parents got copies of the lessons at each session. The instructor was a male Anglo mathematician.

I would characterize the pedagogical approach as student centered in that the instructor was very attentive to how the parents were responding to the material. The small class size allowed for a very intimate and friendly exchange of ideas. This instructor used humor very effectively and this helped the parents as they tackled what sometimes was rather abstract material. (He tended to end each lesson with formal looking mathematics, for example, "True or False: If $A < B$ then $A + A > B$." He did this to relate part of the course to the

content of a high stakes test that high school students have to take.). Much of his approach could be seen as sharing his enjoyment for mathematics for the sake of mathematics. He involved the parents in this enjoyment and the parents regularly presented their ideas to the rest of the group in a very relaxed atmosphere. He would often refer to them as "teachers" and asked them for their opinion on certain topics in relation to what their children were learning.

Listening to the Parents: What They Value in Learning

My approach to research is qualitative particularly grounded on an interpretive paradigm. I seek to understand the parents' lived experiences in these courses with an emphasis on interactions and of their views of the experience. I rely on several sources of data that include field notes, parents' evaluation comments, video segments and instructors' reflection on the course.

- What do parents seem to favor in terms of instructional approach?

If I had to select one characteristic that most parents seemed to value in the pedagogical approach, I think I would pick something along the lines of "friendliness." In fact, this is what one participant wrote in reply to "what did you enjoy about the session?"

> *The gathering of many friends in a learning environment.*

I certainly concur with FitzSimons (1994) when she points out "the need to establish an atmosphere of mutual respect and a feeling of community in which adult learners are encouraged to be independent learners and to share their expertise" (pp. 24-25). Parents repeatedly wrote about how important it was for them to work in groups, for two main reasons, one cognitive and one affective. They enjoyed the support that they provided each other in a group setting; they also enjoyed listening to each other's ideas about a problem.

> *The groups work so well together; our group really cares that we all understand each step and we all find different ways to explain it if for some reason someone in the group doesn't understand.*
>
> *It's surprising how enjoyable it is to work together on a problem with adults. I wonder if it is the same with children.*

Although there were some men in the classes, most of the participants were women. All my in-depth interviews were with women only. They commented on the importance for them of a supportive environment, where they were encouraged to explore and ask questions:

> *The teacher, [he was] easy going, not intimidating always had something new that was interesting, I thought he was fantastic, I wished he could have been one my teachers in high school.*
>
> *Nice environment, we were comfortable there, we talked to everybody, we got along well.*
>
> *You never felt you were going to be put down.*

We weren't afraid to ask questions.

No one said "this question is stupid."

All the instructors encouraged sharing of ideas but one particularly pointed out how he had stressed this more than in his regular university teaching and that he thought that this had been an important strategy:

One thing that I did pedagogically which I don't usually do was that I had them go to the front of class and show what they have done. When I teach at the university I don't do that; I pretty much talk to the students, they stay in their chairs and I am up in the front.

When asked about what advice he would give to other instructors of these courses, he said:

[It] is very important that you involve the parents, make sure that parents are with you always and don't teach to the ones that seem to be getting it quickly, make sure that everybody there is taking part.

Establishing rapport with the instructor and with anyone else from the project staff was very important to them. With one of the instructors, at the beginning, some of the mothers expressed some concern that he had not really introduced himself and that he did not seem to smile much. In another course, some of the mothers felt uncomfortable with the translator. They felt that she was distant, that they could not relate to her:

I was frustrated at that because I try to learn; we carry a lot of garbage with us, I'm trying to learn, I'm trying to figure this out….. Nosotras estamos aquí y se le olvida que ella es la maestra, si nos están enseñando a nosotros se tiene que bajar a nuestro nivel para enseñarnos, no dejarnos, porque nos dejaba. [We are here (motions with her hand to indicate a level) and she forgets that she is the teacher and if they are teaching us, she needs to come down to our level, not to leave us there, because she did]

"*You meet other people and you get to learn with other grown-ups*" This statement captures what I think was very important with many of these women: the value of friendship in an intellectual environment; spending time with friends (women friends) outside the house learning mathematics. I wish I could portray the sense of pride that some of these women conveyed when they shared how happy they were to be, in a sense, going back to school. One of the women described how she looked forward to each Tuesday when they had class and her friend (also in the program) would come by to pick her up and they grabbed their notebook and off they went to class. And this pride is shared by their children too:

I share my homework with my kids and neighbors' kids. They get all excited. And they think they know more and know what they do. They are proud of us.

- What do parents seem to value in terms of content?

This question is harder to address in that in general these parents seem to be eager to learn everything. Their comments on the feedback forms always ask for more of the topic under discussion. In my paper for ALM-6 (Civil, 2000a), I discussed the dilemma from my point of view of parents as adult learners and parents as parents (that is as learners for their children). That dilemma was in terms of what the content of Math for Parents courses should be. Should it build on the socio-cultural experiences of the participants? Should it reflect the current school mathematics their children are experiencing? The parents' mathematical autobiographies as well as their comments throughout the courses depict an array of largely negative experiences in their prior learning of mathematics. Issues of lack of confidence, of feeling not good at math, of feeling alienated are quite common among our participants. To me it is crucial that there be an open dialogue in which different forms of mathematics are discussed, in which cultural values about mathematics are brought to light, a dialogue in which everybody has a voice (Benn, 1997; Flecha, 2000). The three courses described have accomplished (some to a greater extent than others) the opening up of a dialogue about mathematics. None of the courses, though, have really build on the parents' socio-cultural experiences (for an attempt in this direction, see Civil, 2000b). I am intrigued by how the participants would react to courses that would place a heavier emphasis on their knowledge and uses of mathematics. In my experience working with Hispanic, working-class mothers, they want to learn the "academic" mathematics. But I certainly would like to explore further the combination of the different forms of mathematics in the context of a course for adults. I agree with Benn (1997) when she writes:

> An emancipatory mathematics curriculum could validate each ethnomathematics whilst still acknowledging that many adults return to formal education to acquire the discursive practice and consequent rewards of academic mathematics. (p. 175)

In our work, the parents express an interest in learning "academic" mathematics because of their children but it soon becomes evident that they also want this for themselves. I will next share two snippets from the courses to illustrate participants' thinking about mathematics.

Finding a pattern: In the algebra for parents course many of the initial tasks centered on describing a pattern. In one of the problems the situation was "number of birds in a bird formation." The first formation had 3 birds, the second had 5, the third had 7, and so on. The task was just to describe the pattern but one of the groups became intrigued by how to find how many birds would be in a formation later in the sequence, for example the 50^{th} formation (see Civil, 2000c for more on this problem). With some guiding on my part the group eventually solved the problem. They then presented it to the rest of the class. What I want to highlight from this episode is:

- This was their problem; they made it their own and worked on it while the other groups were following the class "agenda."
- Their presentation to the class was engaging; they walked their peers through the process with questions, allowing them to participate in the co-construction of the solution.
- Their metacognitive process throughout was particularly powerful as they reflected on what had stumped them. In order to find the 50^{th} term they ended up having to use the "51" from the 51^{st} term (the way they solved it was by doing 50 + 51). Yet, they did not think they could go beyond 50 because that is what they were trying to find:

B.: We were stuck; we didn't think we could go beyond [50]; why did we think we couldn't go beyond? That was us.
J.: We came really close but we didn't think…
B.: We could see but we didn't… the stigma of you cannot go beyond.

Even to this day I am intrigued by what B. mean by the "stigma of you cannot go beyond." But her choice of words (stigma) is a humble reminder of how strong their emotions are when it comes to their doing mathematics.

Measuring Angles: In the geometry course, one of the activities had them finding the angles of the different pattern blocks. The parents worked in their groups and came up with different approaches to finding the different angles. The instructor closed this episode with the question of "what did you learn today?" The conversation that followed took only five minutes and yet it was very rich in issues brought up. It involved children's learning (this instructor made a point throughout the course of connecting to how children learn); a discussion around theoretical versus applied learning; and finally a call (by one of the mothers) to the need to have parents more involved in this project to become aware of what should be happening in their children's education. I have reconstructed excerpts from the dialogue here (I'll use I for Instructor and P for participant):

I: What did you learn today?
P: Different ways to find the same answer
I: Why is this important for children?
P: Not everybody learns the same way
I: Exactly, to respect the diversity of children's ways of thinking
….
I: We have learned that there are many ways to solve the same problem. This is a very important point.
[then a mother says:]
P: But isn't there a difference between applied and theoretical? What you showed us is very applied, I can do it , it's easy to do because you know, we have the pieces here, but to find out why, that's
I: The argument goes that because we have concrete pieces, it's easier, but I want to point out that much of the reasoning you did was theoretical. For example, here (he puts a parallelogram next to a square) you said, this is 90º and this is 60º so the angle here is 150º;this is a very valid mathematical reasoning. We used concrete materials but we are doing theoretical reasoning.
[then another mother jumps in, all in Spanish]
P: We knew about 360º because of the circle and we were able to use this, we've known this from basic math, but we didn't remember. But as she (the woman who had just inquired about theoretical and applied) said, now we have these pedagogical materials [e.g., the pattern blocks], it is very important that they use these in schools, that's why it is important that we have more parents coming to the MAPPS meetings so that they can see what is being used nowadays.

This brief episode captures much of what these courses are about. Participants not only learn specific mathematical content, such as the concept of angle, but they are encouraged

to relate it to how children learn, what takes place in schools and how they, as adults, learn. The following reflections on "proving" that the sum of the measures of the angles of a triangle add up to 180º captures this again:

> *I really enjoyed the example of how you prove that there are 180 degrees in a triangle. When you tear out the angles of the triangle and put them together so that the angles meet at the vertices, it forms a line, which is 180 degrees (1/2 or a circle). Wow! What a great and very illustrative proof! Now I know why!!!*
>
> *In high school, I memorized the rules / theorems of geometry, but never really knew how to definitely prove that a triangle has 180 degrees. It's very difficult for kids to understand/enjoy doing geometry when they don't / can't explain why something is. When they aren't sure, there will be doubt in their minds about its relevance. They get discouraged. Teachers need to provide more simple examples like this!*

Reflection

In a sense these courses are different from typical courses for adults. For one thing, many of the parents are not seeking a degree. Their original motivation is in relation to their children's schooling. But, on the other hand, these parents share many of the characteristics of adult learners as described by Benn (1997) and FitzSimons (1994). Many of them are women who did not have very positive experiences as mathematics learners when they were in school. Providing a safe environment in which their questions and ideas are encouraged and honored is crucial to their development as adult learners of mathematics (*I learned how to find the area for a parallelogram without the fear of making mistakes*); as parents (*I learned that I can start teaching my children geometry and it is something they don't have to wait until high school to learn that they are learning geometry*) and as advocates for their children's education (*How can we form questions or what questions can we ask when meeting with our children's teachers?*).

For the many of us in this field who have had the opportunity to pursue higher education, it is humbling, I think, to see how important a course of Mathematics for Parents can be for those taking it. In a focus group interview with a group of women who has taken the three courses (one of them only took two), they talked about the need for us to remember how hard it was for parents to be in these courses, the juggling that it involved with family, job, and other obligations; they talked about how important it was for them, for their personal growth to be in these courses and then one of them said,

> *Es lo bonito que se siente el decir "voy a al escuela"y para mí es importante, yo me siento bonito decir "yo estoy agarrando una clase de MAPPS", es un orgullo decirlo. [it feels so good / beautiful to say "I am going to school" and for me it is important, I feel good, to say "I am taking a MAPPS class", I feel proud saying it.]*

References

Benn, R. (1997). *Adults count too: Mathematics for empowerment*. Leicester: NIACE.

Civil, M. (2000a). 'Parents as learners of mathematics.' In S. Johnson & D. Coben (Eds.), *Proceedings of the sixth international conference of Adults Learning Mathematics - A Research Forum.* Sheffield Hallam University, England, July 1999 (pp. 141-147). Nottingham, UK: CEP, University of Nottingham.

Civil, M. (2000b, July). 'Parents as Learners and Teachers of Mathematics: Towards a two-way dialogue.' In Schmitt, M.J; Safford-Ramus, K. (Eds. 2001) *Proceedings of the seventh international conference of Adults Learning Mathematics - A Research Forum.* July 2000. (pp. 173-177) Tufts University, Medford, MA, USA.

Civil, M. (2000c, July). 'Adult Learners of Mathematics: Working with Parents.' In FitzSimons, G. et al. (Eds. 2001) *Adult and Lifelong Education in Mathematics. Papers from the Working Group for Action (WGA) 6, 9th International Congress on Mathematics Education, ICME9.* (pp. 201-210) Melbourne: Language Australia.

FitzSimons, G. (1994). *Teaching mathematics to adults returning to study*. Deakin University Press.

Flecha, R. (2000). *Sharing words: Theory and practice of dialogic learning*. Lanham, MD: Rowman & Littlefield.

Evaluating an educational programme for enhancing adults' quantitative problem solving and decision-making

Noel Colleran
City of Limerick Vocational Education Committee, Ireland.

John O'Donoghue
University of Limerick, Ireland

Eamonn Murphy
National Centre for Quality Management, University of Limerick, Ireland.

1. Introduction

Recent research (Resnick, 1987, and Soden, 1994) confirms that higher order thinking skills are amenable to instruction and therefore can be taught and learned. It was therefore feasible and realistic to investigate the possibility of devising an educational programme that would enable adult learners improve their quantitative problem-solving skills. However, while a review of the mathematics education literature and mathematical problem solving in particular identified a number of key components that must be addressed if learners' mathematical problem-solving skills are to be improved, the literature was predominantly focused on children and younger students. This unidimensional view was deficient regarding an adult-learner perspective in general but particularly in relation to the subset of the adult-learner population in which the researcher was interested i.e. long-term unemployed adults who had returned to education. The literature did not provide an appropriate theoretical framework for an adult-orientated programme. It was in the field of philosophy that an appropriate theoretical framework was discovered which proved central to the development of an appropriate adult-orientated educational programme.

In 1957 Bernard Lonergan published a book entitled, "*Insight: A study of human understanding"*. Lonergan was a Canadian theologian and philosopher who died in 1984. In his book he describes how 'catching on' or 'getting the point' is a frequent event in the course of our daily lives. This act, the act of insight, provides the foundation for a whole new philosophy on human understanding. Lonergan's *Insight* develops this foundational view and also provides a number of cogent reasons why his philosophy is suitable in the context of adults solving problems:

1. his problem-solving 'programme' is adult-orientated,

2. he believed that a good starting point for the development of problem-solving skills is with the natural thinking process of the adult,

3. he provides a cognitional structure which identified the thinking processes used by adults when they solve problems.

Lonergan's cognitional structure stands at the heart of this educational programme and provides the direction and the substance of the intellectual activities addressed

throughout the programme. However, other elements such as Action Learning and the uses of 'realistic' quantitative situations are synthesised in this original programme to achieve a transformation from theory to a practical process for improving adults' problem-solving skills (Colleran et al. 2000).

1.1 The educational programme

Lonergan's (1957) 'programme for life' is transformed into a concrete educational programme through six carefully designed worksheets and problem-solving activities (Colleran et al., 2000). Throughout the programme learners reflect on the resolutions they generate to quantitative problems and complete worksheets, each with a particular focus on an individual stage of Lonergan's cognitional structure. The following issues are attended to in a sequential and developmental manner:

- experience,
- common sense understanding,
- questions for intelligence,
- insight,
- formulation of insight,
- questions for reflection,
- scientific understanding,
- evaluation,
- decision,
- review.

Worksheet 6 acts as an integrator: pulling all preceding worksheets into one coherent whole. Having completed this final worksheet learners should have incrementally discovered the thinking processes they use when they resolve non-routine problems. The aim of the worksheet activity is to encourage and enable adult learners discover the manner in which they think and to practice reflective thinking which is at the heart of problem solving.

2. Research process

Action research was selected as an appropriate methodological vehicle. The programme evaluation process constituted two action research cycles – the first trial was carried out in two locations, (one group in each location) from January to June 1999 (Colleran et al, 2001). This was a convenient sample of adult learners available to the researcher. The second trial (September to December 2000) was carried out with one group specially selected to address and rectify problems that were identified during the first trial. The second evaluation employed an ethnographic approach to the collection of data and these data were then analysed using the constant comparative method (Glaser and Strauss, 1967). While there may be some references to Phase 1 evaluation this paper concentrates on Phase 2 evaluation of the educational programme.

The data used to evaluate the programme are drawn from the researcher's personal research journal and post-implementation interviews with the tutors and learners as well as primary documentary evidence from learners in the form of completed worksheets and written and spoken comments. The evaluation is reported in case study format.

Anonymity and ethical issues are addressed by referencing the sources of data as follows throughout this paper:

- [RJ] Researcher's research journal (which includes non-participant observation notes, tutor debriefing notes, videotape analysis and evaluation notes and the case study journal).
- [TI] Tutor's transcribed interview
- [LI] Learners' transcribed interview .
- [LJ] Learners' journal/workbook.

All other data referred to throughout this paper are described accordingly.

3. General implementation issues

3.1 Research setting, adult learners and tutor profile

The group of learners selected were participating in Adult Basic Education and had joined the Youthreach[2] programme in September 2000. The age profile and Male/Female comparative experiences detailed in Figure 1.1 and Figure 1.2 illustrate learners' relevant details.

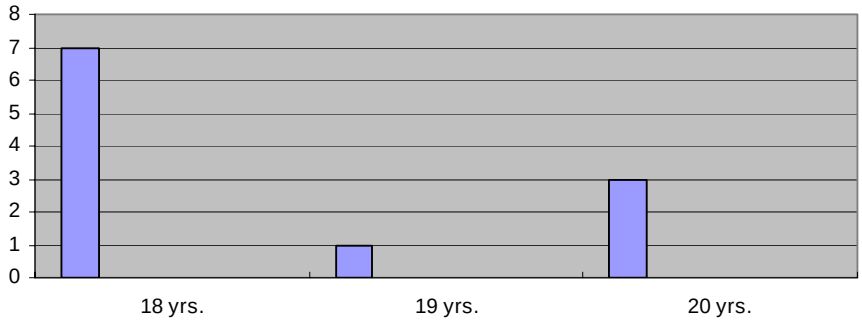


Figure 1.1. Age distribution of participating learners

The tutor who carried out the implementation had been a teacher of mathematics for ten years. His teaching style was 'traditional' in nature - repetition and practice was carried out routinely while discussion and group work were not employed as part of his teaching strategy.

[2] The Youthreach programme was initiated in 1989 through a joint initiative by the National Government Departments of Education and Labour. The programme is managed at regional level by Vocational Education Committees (VEC's). Initially the programme was targeted at young people between the ages of fifteen and eighteen years who had left formal education without any recognised qualification. However, in recent years the programme's client base has broadened out to include adults over eighteen and under twenty-one years of age with no formal educational qualifications.

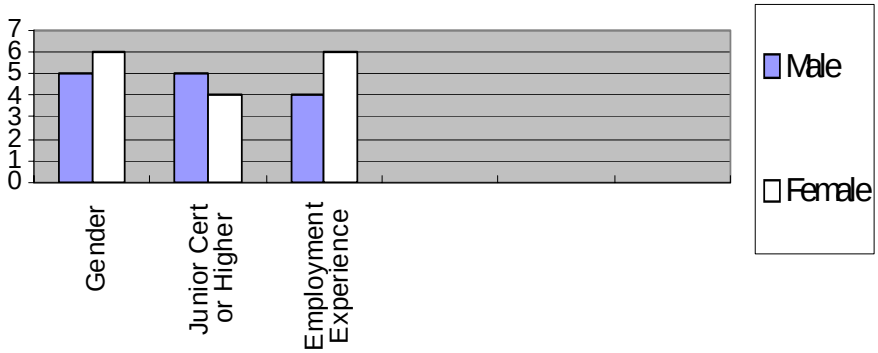


Figure 1.2 Male/female comparative chart.

4. Case study analysis

4.1 Chronology of events

The case study was carried out over a twelve-week period between mid-September and mid December 2000. There were twenty-two one-hour sessions throughout this period of which three were used to prepare the learners for the programme. A number of major quantitative problems were addressed throughout the case study. Figure 1.3 illustrates the point at which each quantitative problem was addressed and the amount of time spent on that problem during the programme. It also illustrates that Action Learning, Journal writing and Worksheet activity continued throughout the case study.

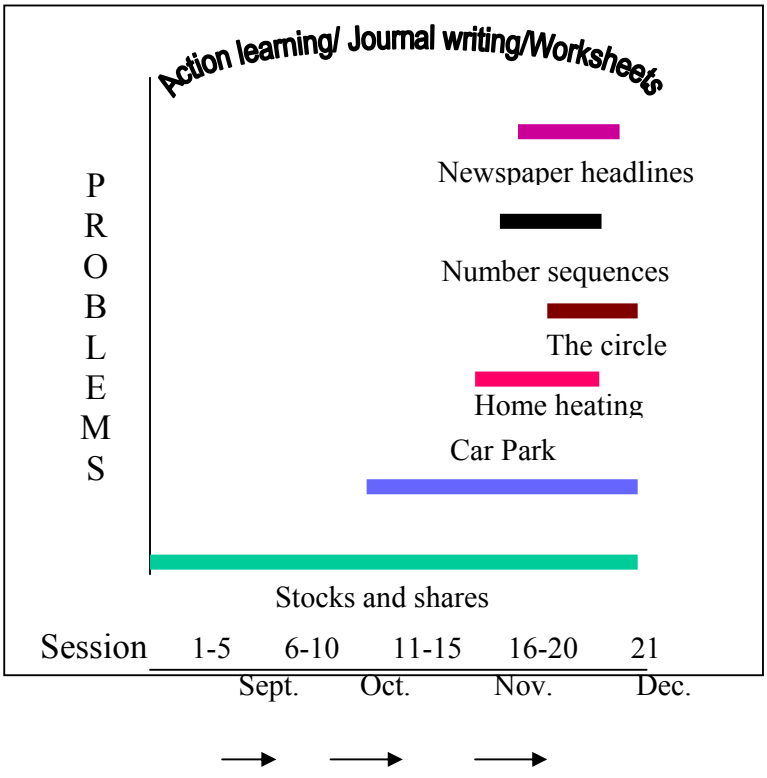


Figure 1.3 Programme timeline illustrating the time given to problems throughout the programme.

5. Quantitative problems

5.1 Stocks and shares

The pilot study suggested that 'realistic' quantitative problems were an effective means for discovering and developing mathematical skills. For this reason the first problem selected by the tutor from the compilation of quantitative problems developed by the researcher was the 'Stocks-and-shares' problem. Learners were given a hypothetical £IR1000.00 allocation to buy stocks and shares over the course of the programme.

This quantitative problem continued throughout the case study, however it was put aside at times to deal with other problems such as:

- designing a car park,
- number and letter sequences,
- the circle,
- newspaper headlines,
- economical ways of heating a domestic house,
- purchasing a mobile phone (see Figure 1.3).

There were many mathematical skills addressed and improved during the stocks and shares problem such as:

- adding, subtracting, dividing and multiplying of whole numbers and decimals,
- calculator work,
- data tables,
- percentages,
- time,
- estimation,
- predictions.

All learners completed a number of data tables representing the names and prices of shares and the amount of money spent on each share. Totals for spending and money remaining were calculated. See Table 1.1 for example of data table produced by Learner 9.

Company	Share Price	No. of Shares	Cost
First Active	£1.69	200	£338
Tullow Oil	£.92p	500	£460
AIB Group	££9.56	21	£200.76
			£1.04 left

Table 1.1. Data table completed by L9 for the 'Stock and shares' problem.

The table above is typical of the manner in which the problem was presented and calculations carried out by all learners.

5.2 Designing a car park

In an effort to address mathematical skills such as measurement and geometry learners were asked to design a car park at the rear of the school/centre to accommodate as many vehicles as possible (see Figure 1.4).

Designing a car park
Explore the enclosed area to the rear of the centre and design a car park to accommodate as many vehicles as possible.

Figure 1.4. 'Designing a car park' problem statement.

In completing this project, which took four sessions, the following mathematical skills were discovered and used by learners:

- linear measurements,
- areas,
- averages,
- scales.

5.3 The circle

The tutor used the image of a circle to enable learners discover the difference between commonsense thinking and scientific thinking and to learn about the characteristics of an important mathematical shape (see Figure 1.5). Firstly the tutor displayed the shape for a few moments and asked learners what they had seen.

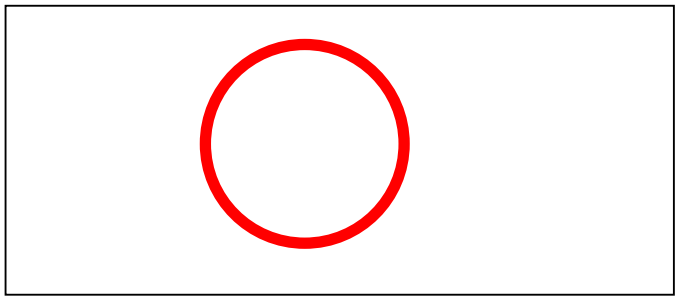


Figure 1.5 "Looking at the shape' problem statement.

Immediately learners said it looked like a round shape, a red ring and so on. The shape was displayed again and learners were asked to write down what they see or what it is.

This was an excellent and enjoyable exploration for many learners. In his post-implementation interview L8 was astounded and satisfied with this session:

I thought it was interesting because at the start of it was just, oh, a circle right. But we kinda started talking about it and got more into it and we managed to get half an hour of talk out of it … from a circle? which I thought was amazing… I thought well, how am I here thinking and talking about a circle for so long. I found it very interesting. **[L8:LI:Dec 14th 2000]**

This was an important session for many learners because they were enabled, through the gentle probing of the tutor, to uncover what they knew about the circle and develop understanding. They also discovered that new understanding is achieved by taking time to think. The researcher's journal entries about these sessions points to the importance of discussion within the classroom and the expert manner in which the tutor moved learners from commonsense thinking to scientific thinking by shifting the questioning into a mathematical and therefore scientific context:

There would be little discussion about the circle if the tutor introduced the definition first. … It is important to emphasise the shift in context employed by the tutor to engage this form (scientific) of thinking. **[RJ: Dec. 11th 2000]**

Other quantitative situations such 'Heating your home' 'Buying as CD player', 'Taking a day trip', Newspaper Headlines' and 'Buying a car', were also addressed throughout the programme.

5.4 Developing mathematical language

The following extracts are indicative of the mathematical language used by learners during their discussions on the stocks and shares problem:

I sold my shares in Riverdeep which stood at £248.84 and invested it into three different companies. I only made a profit of £14.22 on top of my £1000.00 so I have also invested it on top of my £248.84 of Riverdeep shares. **[L11:LJ: Nov. 13th 2000]**

Stocks went up over the week but dropped today. I think we should do stocks more often. **[L3:LJ: Nov. 21st 2000]**

Clearly the mathematical development of learners was enhanced by this learning episode. This progress and development was continued as learners solved other quantitative problems. The overt use of mathematical language in discussions related to all quantitative problems is indicative of an increased use of mathematical thinking by learners throughout the programme.

5.5 Realistic Problems

The programme endeavoured to enable learners engage and resolve quantitative problems that mimicked realistic situations relevant for learners. These situations provided opportunities for learners to use their common sense, their reasonableness and their mathematical skills. Learners discovered the mathematical skills that needed improvement as well as learning new mathematical skills they had not used before. Learners were also more motivated and more participative. In his post-implementation interview the tutor was satisfied that 'ordinary' or 'textbook' problems:

Do not stimulate as much participation. Learners prefer realistic problems and a lot more thought processes occur during the problem solving... it made me aware that you have to give people a lot more time to think out problems…. [Realistic problems] motivate learners more and they are more interesting… I found the realistic problems very effective throughout. Some of the examples I wouldn't have thought of or used before, like for example, the most economical means for heating a house. I found the actual example worked itself with very little effort. **[TI:Dec 18th 2000]**

During observation sessions as well as videotaped sessions there was an obvious increase in the frequency and length of silences. The tutor and the learners were becoming comfortable with a thinking environment. It is clear from the journal entries of Learners 2 and 11 that they were comfortable in a thinking environment where they were expected to exercise their minds:

I thought the class was interesting. We asked more questions and there was no real meaning only what we thought about it ourselves. …It was a thinking class. **[L2:LJ: Nov. 30th 2000]**

The purpose of this class I feel was to stimulate thought, ask questions, find a meaning… I exercise my mind, expanded my thoughts. It gave me a better understanding. **[L11:LJ: Dec. 7th 2000]**

The realistic situations encountered reinforced the need to think things out and make reasonable and deliberate decisions.

Realistic problems have many positive aspects in relation to learning and development. However they did create difficulties for the tutor. In the post-implementation interview the tutor suggested that realistic problems were not susceptible to the same type of control and planning as textbook problems:

The [realistic] problems went on for much longer than I had planned but this seems to be the nature of realistic problems. **TI:Dec 18th 2000]**

5.6 Worksheet activity

The six programme worksheets are designed to firstly, enable learners discover in a gradual and progressive manner the thinking processes they encounter as they solve problems, secondly to provide practice with 'realistic' problems in sample worksheets and thirdly to facilitate reflection on the part of learners through the completion of worksheets (Colleran et al. 2000). Each worksheet focuses on a particular phase of the problem-solving cycle beginning with some reflection on the relevant information already available to the learners. This incremental uncovering and development of the learners' thinking skills and mathematical skills is reflected in learners' comments throughout the programme. Learners also confirmed that they use these thinking skills and mathematical skills not only in the context of mathematics but also in many real-life situations:

…everyday problems there's maths in it without realising … all the maths I didn't realise I was doing …And I actually was doing, doing the maths like and you know myself I think I've improved so I'd say I'd have a lot of confidence. **[L5:LI:Dec. 13th 2000]**

Yeah, in everyday life like paying bills and stuff like that, questioning where did that come in and knowing myself where I stand within money problems or mathematical problems in doing a room or a house or you know all the things I've learned in maths will help me with measurements there and paying money here and there. It will give me some sense of direction I suppose. **[L11:LI:Dec. 14th 2000]**

6. Summary

This summary addresses a number of significant developments such as improved thinking skills and mathematical skills, developing autonomy and the importance of context.

6.1 Improved thinking skills

Learners' improved thinking skills are corroborated from a number of sources throughout the programme. All learners agree that the cognitional cycle was an effective thinking process and was the process they themselves used everyday when they solve problems in all contexts. Many learners confirmed that they had experienced this process in contexts such as paying bills, shopping and having difficulties with relationships. This clearly supports Lonergan's view that the cognitional structure is natural and invariant.

Many learners had been insecure when it came to asking questions before they participated in this programme. They felt that questioning was an indication of stupidity and ignorance and would therefore rarely ask questions. However, as a result of this programme learners had a positive rationale for questioning and had developed sufficient confidence to ask questions when they engaged a problem situation. This altered perspective improved learners thinking skills and gave them the confidence to ask questions when they felt the need.

The classroom environment created by this programme encouraged learners to make sense of the problems they tackled. Learners brought their out-of-school or commonsense experiences into the mathematics classroom as a matter of course and this enabled learners to value their commonsense as a resource and a basis on which a solution or scientific understanding was built (Lonergan, 1957). Valuing and using one's experiences and knowledge is another indication of improved thinking skills.

6.2 Improved mathematical skills

Clearly learners who participated in this programme improved their mathematical skills. They had numerous opportunities to use mathematics in various situations. They became aware of the pervasiveness of mathematics in everyday life and how mathematics can be used to make informed decisions. They discovered the mathematical skills that needed improvement as well as learning new mathematical skills they had not used before. The overt use of mathematical language in discussions related to all quantitative problems is indicative of an increased use of mathematical thinking by learners and a growing confidence within learners of their ability to engage and resolve quantitative situations. The self-evaluation by learners, which was facilitated by the post-implementation interview questions and journal writing, confirmed that they themselves felt they had improved their mathematical skills.

6.3 Developing autonomy and confidence

There are several indications throughout the case study to suggest that learners were developing autonomy. Learners became comfortable quite quickly with the reduced activity and contributions of the tutor in the classroom. They made decisions about the ground rules and the responsibilities of individual learners in the group. Learners were allocated or volunteered to carry out certain activities between sessions. There was clear communication between the tutor and the learners so that each knew their areas of responsibility. Many instances of Lonergan's commonsense learning occurred and learners used their relevant commonsense within each 'realistic' problem situation. This was particularly apparent in the 'home heating' and the 'car park' problems – learners began to value their out-of-school experience. Learners began to feel confident enough to contribute what they thought was relevant in a particular discussion and were willing to take help from other learners or from the tutor if other learners could not help.

As the programme progressed learners wrote more extensive and reflective notes into their journals. The following journal entries by L1 trace a growing confidence over time:

I don't think the group can function without the basic understanding of what we are doing… first class = chaos. **[L1:LJ: Oct 17th 2000]**

After [the tutor] has explained the mechanics of the group I feel better about the group…. Yesterday's class was good. **[L1:LJ:Oct. 20th 2000]**

I feel more calmer and confident in situations… I am more confident within myself and within the group... It [the cognitional cycle] is as [L11] says 'a coping mechanism'. **[L1:LJ: Nov. 6th 2000]**

Learners' growing confidence is also corroborated by the following:

- setting ground rules,
- taking more responsibility for the learning process,
- using the commonsense they already have about realistic problems at hand,
- using the mathematics they already have,
- willing to act on the problems,
- using the cognitional cycle in many contexts,
- explaining or helping other learners,
- calmly approaching quantitative problems as well as other problems,
- writing journals at the end of each session – giving their honest opinions even though they had been told that the researcher would read them later,
- extended thinking times or silences,
- making decisions.

While individually each of the above may not be convincing, however when taken together there is compelling evidence that learners' confidence and therefore their ability to deal effectively with quantitative situations had improved.

6.4 Engaging 'realistic' problems

'Realistic' problems were not susceptible to the same type of control and planning as textbook problems however they did have many positive aspects regarding learning and development. Learners found that they could readily address and engage the quantitative problems presented to them throughout the programme. Evidence that learners were committed and found the problems engaging is corroborated by a number of sources:

- learners found the quantitative situations relevant and realistic,
- many learners were sourcing information outside the classroom,
- learners spent time struggling to identify or name the problem or sub-problem,
- there was serious and extended mathematical discussions throughout the programme,
- many times the group asked for more time to complete discussions,
- learners took it upon themselves to allocate and carry out particular actions between sessions,
- learners employed their reasonableness and decision-making skills particularly with the 'Stocks-and-shares', the 'Car park' and 'Heating the home' problems.

That learners were sufficiently motivated to engage the quantitative situations presented throughout the programme is confirmed by the following mutually supporting statements:

I think today's class was a lot harder than yesterday's because we had a decision to make whether to sell our shares or to buy more and to work out how much profit we made on other shares… I was going to put last week's profits in with this week's but what I didn't know is when those stocks go down so does my profit from last week. **[L4:LJ: Nov 7th 2000]**

I feel very confident in this class because I was trying to solve a real problem and because if I made a mistake I knew it would affect my money situation. **[L9:LJ: Nov 6th 2000]**

I'm still unsure about the measurement of the car. So I am going to measure a car park space today out in the Parkway [shopping centre]. **[L10:LJ: Nov. 30th 2000]**

Making the decisions and taking the actions mentioned above indicate learners' engagement and commitment to the 'real' quantitative situation at hand. Solving 'realistic' problems also convinced the tutor to change his approach from one that valued the amount of problems solved to one that emphasised an improvement in learners' mathematical thinking skills.

6.5 The importance of context

Realistic problems created a context in which learners were enabled and encouraged to draw on relevant personal experiences. The Action Learning process created a context in which learners were encouraged and required to participate in discussions which led to the construction of solutions to particular problems. The problem-solving environment created by both the Action Learning process and the quantitative situations

enabled learners to engage, own and resolve quantitative situations, and take appropriate actions.

The importance of a thinking context i.e. the angle or stance from which one approaches a problem, was convincingly illustrated when the tutor gently changed learners approach to the exploration of a round shape by asking: "If this shape was put on the board in a maths class what would you be looking for?" Immediately learners began talking about circles, radius, circumference and so on. This is very significant for three reasons:

1. Learners, who may have been conditioned to see and think through a 'mathematics-class lens' before they participated in this programme, had broadened out their scope beyond an exclusively mathematical perspective. While this was not helpful in the context of an abstract mathematics problem like discovering why a particular shape is a circle it was illustrative of a mathematics class that encouraged contributions from many angles including mathematics,

2. The change in context provided the catalyst for the insights that led to an explanation,

3. This is an indication of the discovery-constructivist approach employed by the tutor throughout the programme. His gentle probing and questioning enabled learners to discover and construct their own meaning.

6.6 Transfer' of skills

Learners reported the use of the cognitional structure in many real-life contexts. They also believed that the mathematical skills they had used during the programme could also be used to tackle problems outside the classroom. 'Transfer' of learning was not directly addressed throughout the programme, however it seems that the use of many 'realistic' situations coupled with the cognitional structure provided a bridge for many learners to translate (Evans, 2000) their mathematical skills and their thinking skills outside the classroom.

6.7 Tutor's changing attitude

The tutor's objective, in the early stages, was to address and resolve a number of quantitative problems – the number of problems resolved being the criterion for success. However, towards the end of the programme the criterion for success had shifted to the improvement of learners' thinking skills and mathematical skills. The achievement of these criteria was much slower and less obvious than the completion of lots of problems. This fundamental shift in focus meant that the tutor now used the quantitative problems as a means to incrementally develop learners' thinking and mathematical skills and not as an end in themselves. This changing attitude was reinforced through the joint evaluation sessions. This marked change in the tutor's objectives signalled the transition from a teaching approach that valued the number of problems solved to one that emphasised an improvement in learners mathematical thinking skills.

7. Conclusions

The overall conclusions from this evaluation are as follows:

- Lonergan's view that the cognitional structure is natural and invariant is confirmed,
- learners improved their quantitative problem-solving skills,
- learners improved their mathematical skills,
- the importance of 'realistic' problems for the process was confirmed,
- the Action Learning process provided a context in which learners developed a social learning environment that was comfortable, satisfying and enabling.

Literature

Colleran, N., J. O'Donoghue and E. Murphy (2001). 'An Adult Educational Programme for Enhancing Adults' Quantitative Problem Solving and Decision Making.' In: FitzSimons, G. E., J. O'Donoghue and D. Coben (Eds.) *Adult and Lifelong Education in Mathematics: Papers form Working Group for Action 6, 9th International Congress of Mathematical Education, ICME9.* Sydney: ALM/Language Australia.

Colleran, N., J. O'Donoghue and E. Murphy (2000). 'Improving Adults' Quantitative Problem-Solving Skills: A new Approach.' In: Johnson, S. and D. Coben, (Eds.) *Proceedings of the sixth international conference of 'Adults Learning Maths – a Research Forum'*. London: Goldsmiths College,

Evans, J. (2000). 'Adult Mathematics and Everyday Life: Building Bridges and Facilitating Learning 'Transfer''. In: Coben, D., O'Donoghue, J. and Fitzsimons, G. (Eds*) Perspectives on Adults Learning Mathematics.* Dordrecht: Kluwer Academic Publishers.

Glaser, B. G., and A. L. Strauss (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. Chicago: Aldine Publishing Company.

Lonergan, B. J. F. (1957). *Insight - A Study Of Human Understanding*. London: Longmans, Green, and Co.

Resnick, L. B. (1987). *Education and Learning to Think*. National Research Council. Washington, D.C.: National Academy Press.

Soden, R. (1994). *Teaching Problem Solving in Vocational Education,* London: Routhledge.

What Numeracy Skills do Adults need for Life?

Dhamma Colwell
King's College London, United Kingdom

Abstract. The first compulsory Adult Numeracy Core Curriculum in England is currently being introduced by the Department for Education and Employment. The rationale for the curriculum has a deficiency perspective of adults' abilities: those who do not perform well on a selection of 'school mathematics' 'problems', dressed up as everyday tasks, are deemed to be incapable in their everyday lives. The model of numeracy and teaching that the curriculum uses is the traditional one of the hierarchy of computational skills found in the school curriculum as a set of 'building blocks', which can be applied to everyday life. The curriculum also requires that the context of their everyday lives be brought into the classroom by students to be used by tutors for teaching numeracy. This model conflicts with much recent research on everyday cognition which sees learning as participation in practice and knowledge as inextricably embedded in activity and therefore problematises the notion of abstraction and transfer of knowledge from one situation to another.

Introduction

In this paper, the interpretation of the evidence used by the Moser Working Party's enquiry into the level of adult numeracy in England is questioned (DfEE, 1999). While the commitment by the government to widening access to education is welcomed, the effect of having a single compulsory curriculum is examined. The epistemology of the new Adult Numeracy Core Curriculum is interrogated (BSA, 2001) and the assumption that underlies the structure and content of the curriculum is examined: that adults use 'school mathematics' in their everyday lives (Nunes, Schliemann and Carraher, 1993). Methods of assessing adult's everyday numeracy skills are considered.

The assessment of adults' numeracy levels

In 1999, adults' levels of literacy and numeracy in England were considered by the Working Party under the chairmanship of Sir Claus Moser (DfEE 1999). They concluded from the results of part of the National Child Development Survey (NCDS) (Bynner and Parsons, 1997) that seven million adults (about 20% of the adult population) lacked the basic skills required in everyday life. This survey purported to measure adults' use of mathematics in their lives: the respondents were given nine numeracy questions which were intended to reflect everyday life. I shall examine one of these questions as an exemplar of all of them. It typifies not only the numeracy questions in that survey, but questions used in many surveys of adult numeracy, in learning materials for adults and young people and in tests and examinations.

The question postulates an evening's home entertainment for six friends who hire two video tapes at £2.50 each and order a take-away pizza that costs £19.66. Respondents were asked 'What is the total cost? How much does each person have to pay?' (Bynner and Parsons, 1997:116).

In real life, take-away pizzas are not made big enough for six people to share: they could not be safely carried on motor-bikes. According to leaflets put through my door,

the largest take-away pizza is the 'Super 15', which costs between £7.50 and £11.50, depending on the toppings and serves 2-4 persons. A group of six friends would therefore need to order at least two of these pizzas.

In their study of numeracy in practice, Johnston, Baynham, Kelly, Barlow and Marks (1997) also asked a question about sharing a pizza. But they asked their participants how they would actually share the cost between three friends in real life.

> *Say you went out with some friends, and you had a pizza and you're going to share the costs. When people do that they are going to work out how much they are going to pay in different ways. So say it was you and you went with two friends, and the pizza cost 16.90 dollars, how do you think you would pay for it?* (p. 93).

The participants came up with a wide range of responses: one participant said he worked in a pizza shop and could make free pizzas whenever he liked; others said they would take it in turn with their friends to pay; some said they would pay for the group if they were the one with money that day; some did an approximate calculation; some did a rougher estimate to a convenient currency note and said one person would pay the extra; but others did not trust their friends to reciprocate and tried to work out the shares precisely.

The responses they gave were more dependent on social structures than mathematical knowledge: types of friendship, the amount of money they earned and family responsibilities. These varied responses show that in real life situations there is no one correct solution to such a problem. Whether a solution is satisfactory depends on the point of view of the participant.

There is a fundamental difference between the two enquiries. Johnston et al wanted to know how their participants actually worked out a problem in real life. Bynner and Parsons assumed that they knew how such a problem would be worked out and were trying to find out how many respondents could reach their ideal solution.

The pizza in the NCDS test question is not a real-life pizza but a mathematics problem pizza. Adults recognise these kinds of problems as disguised 'school mathematics' problems and perform accordingly (Lave, 1988, Nunes et al, 1993). One participant in the Johnston et al enquiry asked if he was being asked a mathematics question or asked to say how he would really do it. He said he would approach the question in two different ways: for a mathematics question he would provide an exact answer, but in real life, everyone would 'chuck in' whatever money they had and it would even out over time.

Participants are likely to make far more mistakes doing such calculations in a test or interview than when they are faced with an authentic problem in a real situation (Lave, 1988, Nunes et al, 1993). Then they can choose from a whole range of strategies, try them out and assess whether they will give a satisfactory answer. There is no one correct answer:

> ... a dilemma has no factual solution, no general, in principle, correct answer. It is a matter of conflicting values and viable alternatives, which are neither right nor wrong, and none of which is entirely satisfactory. (Lave, 1988:139)

They can use resources such as currency notes and coins to help them. They would not feel self-conscious about trying to solve their problem because they would have posed it themselves. None of these strategies are possible in the test situation, where the problem belongs to the tester, not the person being tested.

There are similar problems with the other eight questions in the NCDS: they are over-structured in order to ensure that there is only one possible correct answer for each part of each question. None of them allow the respondents to solve the problems in the range of ways that would be available to them in their everyday lives. By structuring the questions in the way that they have, the authors have jeopardised what they purport to be finding out: whether adults can solve these kinds of problems in their everyday lives. What they are discovering is whether their respondents can do 'school mathematics' problems. What the survey shows is that seven million adults have some difficulty in performing 'school mathematics'. It does not show what happens in their everyday lives.

'Skills for Life'

In response to the Moser Report, the Department for Education and Employment (DfEE), is developing a national strategy for improving the literacy and numeracy skills of adults in England, 'Skills for Life' (DfEE, 2000). This includes: a new Adult Basic Skills Strategy Unit (ABSSU) within the Department for Education and Skills; National Standards for Adult Literacy and Numeracy (which correlate with National Standards for other groups of learners); Adult Literacy and Numeracy Core Curricula; new assessment procedures and qualifications for learners; National Standards for teachers; and a National Research and Development Centre for adult basic skills.

The Adult National Core Curriculum

The epistemology of the curriculum

The Moser Report recommended that the Adult Numeracy Core Curriculum should be 'context free', 'as far as possible.' (1999:66). The result is a curriculum consisting of lists of arbitrary, unconnected pieces of mathematical knowledge, very similar to the school curriculum (DfEE and QCA, 1999). There is an underlying assumption that there is a self-evident hierarchy of mathematical concepts which learners can progress through (Noss 1990), revealing the 'nostalgic epistemology' on which it is based (Brown, 1993). Alternative epistemologies developed through research on cognition during the last century see knowledge as constructed by the learner through activity, not received as pre-formed packages (Brown, 1993).

The language of the curriculum uses the acquisition metaphor of knowledge (Sfard, 1998): it treats knowledge as if it had a material existence and can be acquired or built up. 'The skills and knowledge elements in the adult numeracy core curriculum are generic. They are the basic building blocks that everyone needs in order to use numeracy skills effectively in everyday life.' (2001:8). The contexts in which

mathematics is used in everyday life are also treated as if they are material: they are described as being 'brought' and 'provided' by the learner.

An alternative to the acquisition metaphor is the participation metaphor (Sfard, 1998): the idea that knowing results from participating in the activities of 'communities of practice' (Lave and Wenger, 1991): the workplace, the family, the college, the numeracy classroom. According to this metaphor, the actors (the learners, the teachers, the managers, the fellow workers) construct their learning between themselves, other people and the environment. This problematises the idea that knowledge, learnt in one situation (the numeracy classroom), can be carried like a set of tools and applied in a different situation (the workplace, the home, the shop), (or the other way round).

The curriculum states that, 'a learner might be able to understand the concept of area and how it is calculated, but success in solving area problems also requires the ability to multiply numbers efficiently.' (p. 7) The participation metaphor envisages people learning to multiply numbers and developing an understanding of area through participating with more experienced others in activities that use this knowledge, like tiling a bathroom. 'What motivates problem-solving in everyday situations appears to be dilemmas that require resolution.' (Lave, 1988:139). However, the curriculum is based on the assumption that it is 'school mathematics' which is used in everyday life.

Contextualisation
Having written lists of mathematical packages that are required to be delivered to learners, the authors of the curriculum enjoin providers to make learning numeracy relevant to adults' lives. They give a list of areas where numeracy might be used in everyday life: citizen and community; economic activity, including paid and unpaid work; domestic and everyday life; leisure; education and training; using ICT in social roles. (BSA, 2000:3) The expectation is that learners will 'bring' their 'context' to the learning environment, so that it can be used by the tutor to contextualise the elements of the curriculum.

> ... the learner brings the context which will be the ultimate proving ground for their improved skills. ... What is different is how adults use these skills and the widely differing past experiences that they bring to their learning. This is the *context* that the learner provides (BSA, 2001:8).

The authors of the curriculum have tried to reconcile two very different things: the hierarchy of mathematical concepts which form the curriculum and their list of everyday contexts in which numeracy might be used. The result is a mixture of 'school mathematics' and real contexts, where the 'school mathematics' forms the structure of the curriculum, while the real contexts are there to teach the 'school mathematics'. For example, 'Enter some simple text on a word processor and experiment with different font sizes' (p. 25) is an activity in a real context where numbers are used to indicate relative sizes. But in the curriculum this is given as a sample activity for teaching adults to 'read, write, order and compare numbers up to 100' (p. 24) rather than as an end in itself.

The authors present the idea that the learner can 'bring' their 'context' to the learning situation as if this were entirely straightforward. But I think it is extremely problematic. For example in the Common Measures sub-section of the Measures, Shape and Space section, Element 5 of Level 2 says, 'Adults should be taught to calculate with units of measure within the same system' (p. 70). The example given is to 'Work out the best value of products of different weights or capacities.'

Lave (1988) found that adults used many different strategies for comparing the values of different items in their grocery shopping: by dividing and multiplying; by comparing the difference in weight with the difference in price and deciding whether one was worth the other; by finding two portions with the same weight and comparing their prices; by considering the storage space required for a larger item; by deciding whether they wanted larger or smaller portions; by considering whether they preferred one item over the other. Often participants made several attempts before resolving their dilemmas. They were able to check whether partial or interim solutions were consistent with reality and whether they were likely to reach a satisfactory answer using their chosen method. Then they were able to make more attempts until they were satisfied, or to abandon the problem as not being worth spending more time on. During this process the participant maintained control of the situation: they had generated the problem themselves and they decided how to resolve it. They did not necessarily require a precise answer: an idea of something being larger or smaller was often enough.

In the classroom, the learner will know that what is valued in the world of curricula, tests and examinations, is knowing how to do the calculation and get an exact answer, rather than what they actually do in real life. If they don't find a problem difficult to solve, they may be able to ignore its manufactured context. If they do find it difficult, the context, especially when it is inappropriate, is likely to distract and confuse them.

Assessment for accreditation

In the end, the assessment procedures for the accreditation of learners will determine how the new curriculum is actually taught. At the time of writing the assessment procedures have not yet been published, but there is a sample examination paper on the Qualifications and Curriculum Agency (QCA) web-site (2001). The paper contains questions which are 'school mathematics' problems, contextualised as everyday mathematics problems, with multiple choice answers. The questions are open to the same criticisms as the NCDS questions: they test 'school mathematics' knowledge and skill, not whether the candidates can solve real problems in their everyday lives.

If QCA wanted to test candidates' knowledge of everyday life, they would have to ask questions similar to Johnstone et al's pizza question: open questions with an unlimited number of possible answers. This is unlikely to happen because of the difficulty, and therefore expense, of marking such answers. But there are precedents for such assessment, for example the Graded Assessment In Mathematics scheme, where grades are based on the results of systematic research (Brown, 1992).

Conclusion

The commitment by the Government to greatly expand numeracy and literacy provision for adults, to widen the range of venues where it can be accessed and to fund more

research into adults' needs and how to meet them, is very welcome. There are many adults who have not been able to benefit from the existing education system, for many different reasons. It is important to offer adults opportunities to raise their educational achievement and to make these opportunities as attractive and accessible as possible.

The necessity of having a national numeracy curriculum is questionable. Adults who present themselves for numeracy education have a range of reasons for doing so: helping their children with homework, passing job or course entry tests, having particular everyday problems that they want to solve, or wanting to tackle what they felt they failed at school. Most of them will have had experience of trying to learn mathematics at school and failing. Offering a re-run of the education that failed them in the past does not seem to be very sensible.

There are two advantages to having one imposed national curriculum for adult numeracy: the qualification will be easily recognised by employers and other course providers; and it will fit in with the National Qualification Framework so that those students who want to progress on to further qualifications will be able to do so without difficulty.

The disadvantage of one curriculum is that neither teachers nor students will have any choice over what they teach or learn. The student who wants to help her children with their homework, the student who wants to work out where to buy her electricity from, the student who wants to learn mathematics out of general interest and the student who wants to pass a job test will all be obliged to follow the same curriculum.

It is disappointing that the idea of a context-free curriculum was adopted by the government. Their motivation for this was presumably to make the Adult Numeracy Core Curriculum fit with other school and vocational curricula and qualifications. However they have made unfounded assumptions about the relationship between school mathematics and the mathematics people use in their everyday lives. By adopting a context-free curriculum they have jeopardised what they purport to be doing: improving adults' proficiency in their everyday lives.

The underlying model of learning, as something to be acquired in small unconnected packages and built up, contradicts much recent research which sees knowledge as participation in activity. The given list of contexts where mathematics is used in everyday life would have made a better structure for the curriculum.

The result of using the curriculum will probably be that some learners will succeed in learning enough of the required curriculum to gain accreditation. It will not necessarily have any effect on the mathematics they use in their everyday lives. Others may fail and this will be their second or subsequent failure, undermining their sense of what they are able to do and making them less likely to try again. The experience may not have any effect on the mathematics they use in the rest of their lives, because they may make no connection between 'school mathematics' and what they do in their everyday lives (Lave, 1988, Nunes, 1993).

The Adult Numeracy Core Curriculum will empower some learners in the sense that by achieving accreditation they will be enabled to gain access to jobs or further learning. Others will gain personal prestige through gaining accreditation. The mathematics learnt will have 'exchange value' as opposed to 'use value' for the student (Lave and Wenger, 1991:112).

Literature

The Basic Skills Agency, (2001). *Adult Numeracy Core Curriculum*. London.

Brown, M. (1993). 'Clashing Epistemologies: the Battle for Control of the National Curriculum and its Assessment.' *Teaching Mathematics and its Applications*, 12:3, pp 97-112, The Institute for Mathematics and its Applications.

Brown, M. (1992). *Graded Assessment in Mathematics.* UK, Thomas Nelson and Sons Ltd.

Bynner, J., and Parsons, S. (1997). *It doesn't get any better. The impact of poor basic skills on the lives of 37 year olds*. London: The Basic Skills Agency.

DfEE (1999). *A Fresh Start - Improving Literacy and Numeracy.* (The Moser Report. - The working group chaired by Sir Claus Moser). London: The Department for Education and Employment.

DfEE (2000). *Skills for Life. The national strategy for improving adult literacy and numeracy skills.* London: The Department for Education and Employment.

DfEE and QCA (1999). *Mathematics, the National Curriculum for England*. London: The Department for Education and Employment.

Johnston, B., Baynham, M., Kelly, S., Barlow, K., and Marks, G. (1997). *Numeracy in Practice: Effective Pedagogy in Numeracy for Unemployed Young People*. Commonwealth of Australia, Australian National Training Authority, Department of Employment, Education, Training and Youth Affairs.

Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life.* Cambridge: Cambridge University Press.

Lave, J. and Wenger, E. (1991). *Situated learning, legitimate peripheral participation.* Cambridge: Cambridge University Press.

Noss, R. (1990). *The National Curriculum and Mathematics: A Case of Divide and Rule?* in Dowling, P. and Noss, R., (Eds.) Mathematics *versus* the National Curriculum. Bsingstoke: The Falmer Press.

Nunes, T., Schliemann, A. D, Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press.

QCA. (www.qca.org.uk) Accessed 17.10.01.

Sfard, A. (1998). 'On two metaphors for learning and the dangers of choosing just one.' *Educational researcher*, 27(2), 4-13.

Developing the Ideas of Affect and Emotion among Adult Learners

Jeff Evans
Middlesex University, United Kingdom

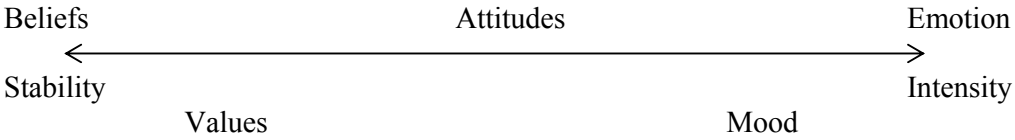
Abstract. The growing emphasis on affect and emotion in studies of adults learning mathematics reflects and is supported by increasing attention from social science and educational researchers to the influence of culture and society, language, and the body. The work reported here uses a discursive perspective to study the role of emotion in adults' mathematical thinking. I consider emotion to be distinct from thinking, but not separable from it. Further, given that much of our evidence about emotions comes from the use of language and discourse in transcripts of interviews and group interaction, I adopt a provisional characterisation of affect and emotion as 'charges' on ideas (or on the terms in which they are expressed). In this study, I use the idea of 'positioning' in social practices to describe the context of a person's thinking and emotional experience during problem solving, and I suggest several types of indicator for emotion to be found in transcripts (or videotapes). An example is given from an interview with a mature student studying mathematics at the beginning of her social science degree.

Introduction

Much recent work reported to ALM emphasises the importance of affect, emotion and feelings among adult learners of mathematics and users of quantitative ideas. This reinforces the attempts of researchers elsewhere to take account of the fact that thinking and learning are emotional activities. I find substantial agreement among researchers in a wide range of fields, ranging from neurology (e.g. Damasio, 1996), through psychology (e.g. Strongman, 1996) and sociology (e.g. Kemper, 1990), that we need to clarify and deepen our understandings of affect.

McLeod (1992) has argued persuasively that affect can be understood as comprising *beliefs*, *attitudes* and *emotions*, and we can position values (DeBellis and Goldin, 1997) and moods on a spectrum that runs from stability and "cool" on the left to fluidity and intensity on the right; see Figure 1.

Figure 1. Range of Types of Affect



Sources: McLeod (1992), DeBellis and Goldin (1997)

Conceptualising emotion

My approach to emotion has two fundamental aspects. First, it is important to have a unified approach to cognition and affect, to thinking and emotion, in the sense that

emotion is seen as *in principle distinct* from ideas, or signifiers (the cognitive), but as nevertheless *attached to* them, though not in a fixed or permanent way. Further, emotion is seen as a 'charge' attached to (or infusing) ideas or (chains of) signifiers; for example, Freud [1916-17], (1974, pp. 443-48) sees anxiety as involving 'motor innervations' or 'discharges'. At one level, this idea of 'charge' is a metaphor, appropriate in that it captures something of the *intensity* and energy of emotion[3].

However**,** there are several additional features. Emotion should be seen as *socially organised*, not as an individual characteristic or essence. This has several consequences. First, emotional experience takes place in a *context*, in the same way as learning or using mathematics. There are a number of ways to think about a context. We could just define it in a 'naturalistic' way, as it is named and described in everyday terms; but that risks basing our thinking on an a-social, under-theorised approach. We could develop the useful idea of *structuring resources* (Lave, 1988), but we need to beware of aligning our thinking with a strong form of situated cognition which under-estimates the possibilities of *transfer* or 'translation' of mathematics learning (see Evans, 2000b: 290-2). The position I argue for here and elsewhere is that the context is formed or 'constituted' within pedagogic or other practices, which make available certain 'positions' (Evans, 2000a; Morgan, Evans and Tsatsaroni, 2002[4]).

Emotion can often be seen as an outcome of social *interaction rituals* (e.g. routine methods of greeting, and also established methods of celebrating changes of status, such as marriage and graduation) leading to positive feelings of solidarity and emotional energy (e.g. Collins, 1990). And emotion must also be understood as based on a person's *history* of involvement in practices; this history is itself structured by the social class of the learner's family and, in case of learning mathematics (or other school or college subjects), by the form of pedagogic practices.

Further, emotional experience is *culturally grounded* in - expressed in, constituted by, learned through - *language*. Thus past experience is *not completely structured* - it is *textualised* (Evans and Tsatsaroni, 1998) - because of the properties of language, and its capacity for forming unexpected, and not consciously intended (by any person), linkages between practices via *intertextuality*. The bases of intertextuality, in turn, are found, both in the cultural linking of practices (e.g. through interaction rituals or through advertisements), and in (the contingencies of) the individual's history of experience; examples are given below (see also Fairclough, 1995). Thus, in including poststructuralist / non-structuralist elements in this analysis, we allow that language may 'have its own dynamics', in this sense.

---

[3] At another level, it seems to me very likely that the charge of emotion may be in principle detectable as a chemical charge (or even an electrical one) in neurophysiological terms (see Damasio, 1996).

[4] Some of the work related to this paper has been done within the project Mathematical Thinking - Teaching and Learning, supported by the Portuguese Foundation for Science and Technology, grant no. PRAXIS/P/CED/130135/98, and has benefited from cooperation, especially with Candia Morgan, Anna Tsatsaroni, and Joao Filipe Matos.

There are several ways to account for the nature of the *charge* or the energy. We could see it as coming from the *goals* of the activity or practice (e.g. Saxe, 1991), say learning mathematics[5]. Another approach is to attempt to analyse individual or group 'motivations'. One way to do this is using a *psychoanalytic* approach emphasising *desire* (Lacan, 1977; Henriques et al., 1984). Here the main concepts are:

- desire: but also the possibility of pain, loss, distress, contradiction; leading to
- the operation of various *defence* mechanisms, effecting *repression*;
- *the unconscious*, which can usefully be seen as 'structured like a language';
- the psychic processes of *displacement* and *condensation* (represented in discourse as metonymy and metaphor, respectively).

Displacement, in particular, provides the basis for (much) fluidity of language and emotion. Hence we have here a basis for an additional 'dynamic' of language.

Methodology for the study of emotions using discourse analysis

The tools to be used include a combination of a *structural* and a *textual* analysis in reading texts. The **structural phase** produces an analysis of the discourses likely to be 'at play' in the research setting(s). We then attempt to determine the range of possible *positions* made available within these discourses, and the values associated with each position. Here we can use an initial 'logical' approach (see below), which can then be strengthened using theoretical approaches like Critical Discourse Analysis (CDA) (Fairclough, 1995), and pedagogic discourse theory (Bernstein, 2000); see Morgan, Evans and Tsatsaroni (2002). The resources available include documents describing the college course (or other activity), and field notes from the researcher (see below).

The **textual phase** aims at identifying:

(1a) the positionings actually taken up in *interaction*;
(1b) any possible instances of *intertextuality*; and
(2) possible *indicators of emotional experience* (such as emotional expression), including those suggested by insights from psychoanalysis.

The first stage focuses on *interactional aspects* of the text. We read the text, so as to establish - or attempt to establish - each student in one or more particular discursive '*positionings'* (which may also change over the course of a classroom or interview episode). Thus we read the interview (or classroom) transcript for

- instances of a person making claims to valued statuses, e.g. 'knowing' in educational settings; and
- the 'modalities' or degrees of certainty of statements[6].

We can allow for intertextuality, by looking for:

- 'key signifiers' meaningful within more than one discourse.

In the second stage of textual analysis, we look for overt indicators for the experiencing of emotions, such as:

---

[5] Goals can be included as part of the elaboration of a discourse (Evans, 2000a: Ch.6).

[6] In discussion of this paper at the ALM-8 conference, one participant pointed out that men are generally more likely than women to utter statements with certainty.

- verbal expression of feeling;
- emphasis or repetition of certain terms;
- 'body language';
- metaphors; and so on.

Further, another contribution of psychoanalytic insights is to point to certain *themes* of emotional experience. In studies of adults learning mathematics, these might include: *belonging and isolation*, leading us to look for:

- reports of liking and loss in interviews (or expressions of solidarity or competitiveness in classroom interactions); or

anxiety: originally seen as 'the primary affect' by Freud, and, in its form as 'mathematics anxiety' recently emphasised strongly in mathematics education (Evans, 2000a), leading us to look for:

- verbal expression of anxiety, fear, etc.; *but also*
- evidence of the operation of *defenses* against strong emotion, such as anxiety (or other intrapsychic conflicts; see Hunt, 1989), namely,
  - 'Freudian *slips*', or
  - *denial* (of anxiety): e.g. 'protesting too much', making an assertive, exceedingly confident 'statement' about mathematics

The data from the interview

As part of a study of adult numeracy at a higher education institution, I did semi-structured interviews with a (partly randomly selected, partly voluntary) sample of 25 first year social science students (Evans, 2000a). As part of the study, I set down a general 'reflexive account', that is, an account of the ways in which I was part of the social world I was studying (cf. Hammersley and Atkinson, 1985: 14ff.). The passage below is constructed from the methodological chapter of the full report of the study. It can be read as data for analysing the discourses at play in the setting of my research interview, and the positions available to those involved.

> I was an experienced lecturer in statistics at the Polytechnic. Most of my teaching was with the BA Social Science students, and I was very involved with the First Year 'Maths' course - giving some of the lectures, also as the coordinator. Further, about a third of each student cohort would have had me as a tutor (for Maths or Social Policy, or as personal tutor)....
>
> The interviews were done at the end of their first year with students from cohorts 2 and 3 of the study. They were all conducted in my office. At the beginning of the interview, I offered coffee or tea. I described my work as 'doing research on people's experience with numbers, and on what sorts of things help people feel comfortable with numbers, and what stands in their way [...] So what I would like to do in this interview is to give you some space to talk about your experience with numbers, and your feelings about them'....
>
> I asked the student's agreement to record the interview. I emphasised to the student that he/she did not have to answer any question if they did not want to. I began with the 'life history' questions, and then moved on to the problems to be

> solved, each preceded by the first *contexting question*, and followed by the second....
>
> The student was given at most only neutral feedback while attempting the problems. Towards the end of the interview, I gave further feedback, if I felt the student needed it, or discussed 'the answers' to the problems, if requested.
>
> (Evans, 2000, Ch.8)

Structural Analysis of Available Positions

First, we can ask what discourses are at play in this setting? The main practice or activity that is shared by the tutor and the students is teaching and learning mathematics. This practice is 'regulated' or organised by a set of ideas, rules, values, standards, and so on, which we might call the 'official' *discourse* of college mathematics. The positions available in this discourse are 'teacher' and 'student'. It is of course the former who is positioned to assess, and the latter to be assessed. Further value is usually attached to individual students according to performance on academic criteria: thus, we have the positions of '*good / normal student*' and *'bad / failing student'*. In the Polytechnic at that time, this pair was moderated, for Year 1 students only to: *'passing student*' and '*student needing extra work*'.

At the same time, the fact that I 'invited' selected (see above) students to a 'research interview' superimposes a second sort of practice on the first. Here the positions are of 'researcher' and 'interviewee'; it is noteworthy that the former is in some ways now positioned as a learner, and the latter as an authority, on her/his own activities at least. The interviewee is also generally much freer - to not agree to attend the interview, to refuse to answer (or to evade) questions, and so on. We can see that both student-interviewees and teacher-researcher are *multiply positioned*, and in ways that are very possibly contradictory, so this itself may generate emotion.

Textual analysis of positioning and emotion: the case of Fiona[7]

Fiona was a mature student in her mid-20s, from a middle class family. She previously had worked as an unqualified social worker, and was now aiming to qualify, via the Psychology track. She was a member of a group to whom I had taught 'Maths' in the earlier part of their first year; that group had worked well, and had included me in some of their socialising.

After some discussion of her 'life history', especially concerning the study of mathematics, I asked her to solve a few problems. She responded with what I later called 'mock-anxiety' (Evans, 2000: 192). In response to Question 1 (about reading a pie-chart):

> S: I always had difficulty with that, I didn't enjoy it at all. School wasn't a particularly happy time for me anyway, so you might well find that a lot of my answers are negative ... [4 lines] ...

[7] For a fuller discussion of this, and other similar interviews, see Evans (2000: Ch. 10).

> *I was never explained how to work through it step by step* so it certainly makes me feel very *anxious* [...] I don't actually trust my own perception to actually give the correct answer, because I don't feel I [..] know how to work it out properly, so therefore I don't think I would give the right answer - if that makes sense....
>
> (Interview transcript; my emphasis)

Here we can see the possible importance of her 'not particularly happy' history of positionings in mathematics, and in school. We can also see that her feeling of having missed a 'step by step' explanation, and her uncertainty about 'trusting her perception' tend to position her as less competent. And she expresses emotion - anxiety.

When I pose Question 3 (about reading a graph depicting changes in the price of gold):

> JE: ... which part of the graph shows where the price was rising fastest?
> S: Maybe it's me being ignorant..., but there doesn't actually seem to be any time specification along the bottom [axis of the graph] - which I find quite *confusing* [...] my father's a stockbroker, so I *do understand* a little about opening and closing.... ... [6 lines] ...
> I mean there actually appear to be two peaks here, but I should say maybe when gold is at 650, it seems to rise very rapidly in the afternoon until close, and afternoon business, you know, afternoon trading ....
>
> (Interview transcript; her emphasis)

Here we might say that she uses the possible differences in the positions available in the two discourses of college mathematics and research interviewing to attempt to shift the discourse from college maths, where she seems to be uncomfortable, to that of stockbroking. This allows her to establish a claim to knowing. But it does not avoid the possibility of uncomfortable emotion. She takes up the story:

> S:...my father dealt with money all the time, um, because he was a stockbroker, and therefore it was the essence to him and his making a living, but *it wasn't anything that we were allowed to sit down and discuss*, or even talk about, or offer advice [...] we were always told we wouldn't understand [...] - because *time is money, money is time,* and *he hasn't got time to explain to me* the information that he thinks is going to be relevant to me at a later date because I'm a woman and I don't understand...
> JE: Is it - a woman, or you're a child?....
> S: I think it's very much both....
> JE: What about your mother? Does she, is she allowed to ask questions?
> S: Well, no, no, just the same. *Family and business should never mix* [...] my mother wasn't ever allowed to ask and it certainly affected her far more than it did us because as a stockbroker, your home and your material valuables are on the line all the time [...] on a couple of occasions the family home was under great threat [...] It *wasn't something that family and children discuss* ... [2 lines] ... he was the man of the household and he could deal with it [...] ... most of the time, it was *like living under a time bomb* (JE: mmm, mmm, I can

> appreciate that) especially if you don't quite know how the time bomb's made up or when it's going to explode....
>
> (Interview transcript; my emphasis)

Here we find a recurrent theme of *being positioned* as *not understanding* her father's work, and thus of *exclusion*, from knowing, form his work, perhaps from his love; this is reinforced by the injunction in the text (repeated from her father) that 'family and business should never mix'. Elsewhere we see a quasi-repetition 'time is money, money is time', which may *exhibit* (though not express) anger; retrospectively, we can see that this 'reverberates' with her earlier 'complaint' that her teachers would not give 'step by step' explanations. And we see the powerful metaphor of 'living under a time-bomb', where the time bomb is an unpredictable financial disaster, and such a way of living would certainly include *anxiety*.

Is there a link between her father, his work, and her feelings about and way of engaging with mathematics? I ask how she saw her father's work, to pick words, adjectives to describe it:

> S: capitalist, corrupt, business-like, ... mathematical, calculating, devious, unemotional...
>
> (Interview transcript)

Here, we have a 'chain of signifiers', where 'calculating' is a *key signifier*, which links three discourses: (working backwards) family discourses, school mathematics, and stockbroking. The anger that she feels towards her 'unemotional' father, may be displaced onto his 'corrupt' work, and onto school mathematics, leading to ambivalence or resistance to engaging with mathematics.

Conclusion

This study aims to show how one might study emotional experience in the learning and use of mathematics by adults, using discursive perspectives. The structural phase analyses the positions available to all subjects in this specific setting; here I argue that both a college mathematics and a research interviewing discourse are at play. The textual analysis investigates what other discourses may be called up by a particular subject, as a result of her history of positionings in various other discourses. We can summarise these findings; see Figure 2.

When we continue with the textual analysis, of this particular transcript, we find some evidence that the term 'calculating' is a key signifier, providing links between the interviewee's family discourses, those relating to her father's work, and school or college mathematics. We also find indications of a range of emotions: exclusion, anxiety, anger.

Figure 2. Various Positionings available to Fiona from Structural and Textual Analysis

Structural Analysis	*Discourse*	*Positions*
	College Mathematics	Teacher (Assessor) Student (Assessed)
	Research Interviewing	Researcher Interviewee
Textual Analysis	*Discourse*	*Positions*
	Stockbroking	Stockbroker Client
	Family Discourses	Father Daughter

The interesting and suggestive findings invite us to reflect on the methodology. Would we get similar findings from other such interviews? Does the fact that the interviewer and interviewee had known each other as teacher and student affect the interview material produced? For an answer to the first question, the reader may see accounts of some of the other interviews produced in this study (Evans, 2000a: Ch.10). The second question is addressed, in ethnographic work generally, by the use of 'reflexive accounts'; in this study I produced both a general reflexive account for the research setting and group of informants as a whole, and also an individual reflexive account for Fiona (Evans, 2000a: 195-6) and the others. (Such reflexive accounts are of course produced subject to processes of selection and interpretation, as are any data.) Moreover, there is scope for further discussion, as to whether one could have produced similar findings, or even broadly similar research material, if we had been using videos and transcripts of small-group problem solving sessions. For further discussion of such methodological issues, see Morgan et al. (2002).

References

Bernstein, B. (2000). *Pedagogy, Symbolic Control and Identity: Theory, research, critique*, revised edition. New York: Rowman & Littlefield publishers. (Original edn. (1996), London: Taylor & Francis)

Collins, R. (1990). 'Stratification, Emotional Energy, and the Transient Emotions'. In Kemper, T. D. (Ed.) *Research Agendas in the Sociology of Emotions* (pp. 27-57). Albany NY: SUNY Press.

Damasio, A. (1996). *Descartes' Error: Emotion, Reason and the Human Brain*. Basingstoke: Papermac.

DeBellis V. A. & Goldin, G. A. (1997). 'The Affective Domain in Mathematical Problem-Solving'. In E. Pehkonen (Ed.) (1997), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 209-216), Lahti, Finland.

Evans, J. (2000a). *Adults' Mathematical Thinking and Emotions: A Study of Numerate Practices*, London: RoutledgeFalmer.

Evans, J. (2000b). 'Adult Mathematics and Everyday Life: Building Bridges and Facilitating Learning Transfer', in Coben, D., O'Donoghue, J. and FitzSimons, G. E. (eds), *Perspectives on Adults Learning Mathematics: Research and Practice,* Dordrecht: Kluwer.

Evans, J. and Tsatsaroni, A. (1998). 'You Are as You Read: the role of texts in the production of subjectivity'. In P. Gates (Ed.) *Mathematics Education and Society, Proceedings of the First International Mathematics Education and Society Conference* (pp.168-179*)*, Centre for the Study of Mathematics Education, Nottingham University.

Fairclough, N. (1995). *Critical Discourse Analysis*. London: Longman.

Hammersley, M. and Atkinson, P. (1985). *Ethnography: Principles in Practice*. London: Tavistock.

Hunt, J. (1989). *Psychoanalytic Aspects of Fieldwork*. London: Sage.

Kemper, T. D. (Ed.) *Research Agendas in the Sociology of Emotions.* Albany NY: SUNY Press.

McLeod, D. (1992). 'Research on Affect in Mathematics Education: a Reconceptualisation'. In Grouws, D.A. (ed.) *Handbook of Research in Mathematics Education Teaching and Learning* (pp.575-596), New York: Macmillan.

Morgan, C., Evans, J., and Tsatsaroni, A. (2002) 'Emotion in School Mathematics Practices: A contribution from discursive perspectives'. Submitted for Mathematics, Education and Society conference (MES-3), Copenhagen, April 2002.

Morgan, C. et al. 'Methodological Implications of Researching Mathematical Thinking as a Socially Organised Phenomenon'. Symposium submitted for Mathematics, Education and Society conference (MES-3), Copenhagen, April 2002.

Saxe, G. (1991). *Culture and Cognitive Development: Studies in Mathematical Understanding*. Hillsdale NJ: Lawrence Erlbaum Associates.

Strongman, K. T. (1996). *The Psychology of Emotion (4th edn.): Theories of Emotion in Perspective*. Chichester: Wiley.

Mathematics – The STEPS for Empowerment in a University Foundation Program

Milton Fuller
Mathematics Learning Centre, Central Queensland University, Australia

Abstract. At Central Queensland University a foundation program to prepare adults for tertiary studies is offered. The program, Skills for Tertiary Education Preparat**o**ry Studies (STEPS) has been in place for 15 years. The mathematics component of STEPS, Transition Mathematics (TM), has been designed to enable students to enter a range of university programs. In addition, TM is presented so that it makes an essential contribution to empowering the student to build confidence as well as competence in using mathematical skills as part of the lifelong learning process.

This paper will outline the development of Transition Mathematics in terms of the research behind its progressive improvement over the lifetime of the STEPS program. In particular the paper will address:

- The nurturing of confidence in learning based on a background of mathematics being initially perceived as "difficult, dull and boring".
- The challenge of empowering students to fully utilise their own learning styles to become a successful lifelong learner.
- The need to relate mathematical skills to the life experiences of the learner.

Introduction

"Lifelong learning depends for success on the learner's motivation which should result in self directed learning, and promote self-fulfilment". This comment was made in a paper (Coombes et al., 2000:155) presented to a Lifelong Learning Conference in Queensland. The authors were referring to students' learning in the one semester foundation program, Skills for Tertiary Education Preparatory Studies (STEPS) available at Central Queensland University's regional campuses.

This paper is concerned with the development of the mathematics component of the STEPS program. The Mathematics Learning Centre, established in 1984, has been responsible for the mathematics education of the STEPS students since the first intake in 1986. Motivation and self directed learning do play a vital role in the success rate of the students. The achievement of these two attributes is addressed in the description of the development of the first course, Transition Mathematics I (TMI). Mathematics rather than numeracy has been chosen as the title of the course for two reasons: one to give it university status and the other to give the potential undergraduate students a feel that mathematics is a real part of STEPS. The paper will also address the degree of success achieved through learning in TMI to empower the students to progress from backgrounds where mathematics has a history of negative perception, even fear, to confident, positive thinking learners of mathematics. The mathematical content of TMI covers the development of basic numeracy to applications of elementary algebra and introductory statistics. Students aspiring to a higher level of preparation in mathematics may proceed, on successful completion of TMI, to the advanced level course Transition Mathematics II (TMII).

The Initial Stages of TMI

A challenge arose initially with the problems associated with designing a course which not only laid solid foundations for further studies in mathematics but also provided the opportunity to develop a positive attitude to mathematics. The students come from a diversity of backgrounds, most are over 25 years of age and many have had unpleasant experiences with previous exposure to the study of mathematics. This was not surprising as the STEPS program had been devised for this cohort of potential undergraduate students who were disadvantaged in terms of intellectual opportunity.

The very first task was one of reassurance - reassurance that the mathematics component, TMI, was included in the STEPS program because students would really need these skills. The approach is pretty well described by Wlodkowski (1993) with his Time Continuum Model of Motivation, which is based on three critical periods in the learning process – **Beginning** – **During** and **Ending**. A fourth period of **Future** or **Continuing** should be added. "Each period has two major factors of motivation that serve as categories for strategies that can be applied with maximum impact during these periods" (p.61). For the Beginning period the factors are Attitude and Needs. In TMI the Beginning consists of gentle nurturing, illustrating the mathematical component of undergraduate programs and discussions on learning styles, addressing background experiences and reinforcing the lighter side of mathematical processes. Illustrations from publications such as Eastaway and Wyndham (1999) who discuss such topics as Why do Buses Come in Threes? and Why Do Clever People get Things Wrong? do assist in giving mathematics a "human side", something which the majority of the students had not previously experienced.

During the initial (Beginning) period the emphasis is on what could be defined as "numeracy" skills. The context in which numeracy is defined in TMI is what Robyn Zevenbergen (2001) referred to as "those skills associated with using basic mathematics to be effective and competent in making informed decisions as people go about their daily lives". Anne McRae (1995) refers to this initial stage as aiming to provide students "with the competence and confidence to use mathematics for social communication, personal expression and can also lead to further education and training or work". There is concentration on arithmetic operations as preparation for algebra, money calculations involving percentages with reference to newspaper and business advertisements. This stage is devoted to addressing the perception of "difficult, dull and boring".

The Middle Stage

This stage follows Wlodkowski (1993:61). The major factors are stimulation and the affective experience. In this period the learner begins to clarify their own learning style. At this stage the course moves to self paced learning mode. The approach is "guided" self paced with the result that students now are working at different sections of the resource materials. This approach does test the skills of the tutors. Reflection on their learning is crucial as the students meet up with algebra (many for the first time). Dianne Siemon (1986) gives examples of how reflection can aid motivation and confidence. The self paced feature of TMI does allow for student/tutor interaction with students being urged to reflect on, and discuss from their own perspective, their progress or difficulties with the concepts being learned. However as Siemon stresses "Reflection or thinking is seen as productive if *they have the time*" (p.77). The eight hours per week

allocated to TMI over a 13 week period does allow at least some time for the reflective process. Class groups of about 20 students also helps.

Bonding between various individuals develop at this stage of the course. Mature aged students do tend to support each other emotionally. In general unofficial mentors begin to emerge in the groups. Working in groups is encouraged, not only in the mathematics component, but in all parts of the STEPS program. Paul Halmos (1983,p196) begins his paper on the teaching of problem solving with the statement "The best way to learn is to do; the worst way to teach is to talk". He goes on; "For a student of mathematics to hear someone talk about mathematics does hardly any more good than for a student of swimming to hear someone talk about swimming" (p.196). Lecturing is kept to a minimum with short talks to aid the reading of the resource materials is the norm. Group discussions and mentoring plays a crucial role in the learning process.

Technology does play a role in introducing algebraic manipulation and providing opportunity to explore and investigate the concepts being introduced. A kit of graphics calculators is available for classroom use. Numbers are stored into variables and operations, as illustrated in Figure 1, can be used to consolidate the concept of variable.

Figure 1 Introducing Variables

The Final Stage (The Ending Period)

Technology is also used as a learning enhancement in sections dealing with the coordinate geometry of the straight line (Figure 2), compound growth in financial transactions (Figure 3) and descriptive statistics (Figure 4). The versatility of the graphics calculator in developing algebraic concepts is described by Graham and Thomas (2000:269) in their classroom based research project "Tapping into Algebra". Not only is the calculator used to consolidate the concept of variable but also to present graphs of linear functions to encourage exploration of slope and intercept and the numerical representation of each graph. Examples follow.

Figure 2 Exploring features of the straight line

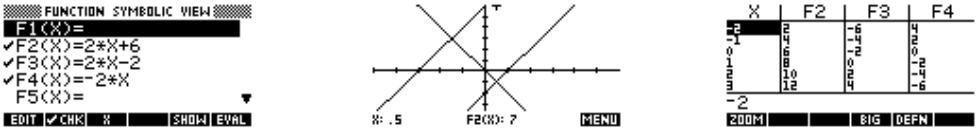


Figure 3 Investigating the growth of money

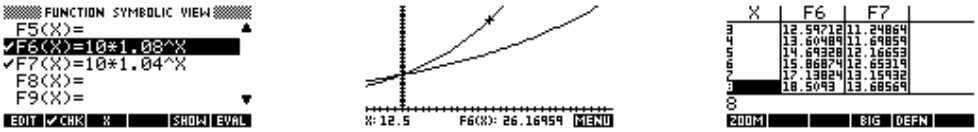
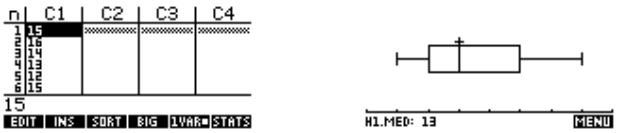


Figure 4 Making sense of data



The emphasis in the final stage is on competence and reinforcement (again following Wlodkowski) and further reflection on the knowledge gained in the previous eight to ten weeks. This is the stage for application of the concepts. Susan Pirie and Lyndon Martin (2000) refer to the Pirie-Kiernen Dynamical Theory for the Growth of Mathematics Understanding and the "folding back" process in learning. They describe this process in terms of problem solving.

> "When faced with a problem that is not immediately solvable at any level, an individual tends to return to an inner layer of understanding. The result of this folding back is that the individual is able to extend their current inadequate and incomplete understanding by reflecting on and reorganising their earlier constructs for the concept" (p.131).

As part of the final assessment the students are expected to attempt simple modelling problems. The first aim is get them to reinforce their writing skills which are being developed currently in other courses of the program. The second is to follow Pirie and Martin in what they define as collecting (a special form of folding back). This is where previous knowledge has to be retrieved for a specific purpose. The modelling exercises require the student to collect the previous knowledge as relationships between variables and then apply that knowledge to a word problem. The outcome is a written report consisting of the problem, the variables defined, assumptions stated and then the solution. This part of the assessment is proving to be quite demanding on the students despite the distribution of specimen questions and associated written reports. It is evident that a lot of effort is required here if success is to be achieved.

Humour and recreational applications of mathematics also play a key role at this stage in building up competence and consolidation. Richard and Francis (2000) describes, recreational mathematics as that branch of the discipline that refreshes ones spirit but it is admittedly a difficult area to define. It is agreed that rigid dividing lines between the theoretical, applications and recreational aspects of mathematics are interconnected especially at the level of this course.

Conclusion

The mathematics developed in TMI is really at an elementary level but it is the attitude, insight into one's own learning style and the confidence and desire to continue learning which are the important goals. In a research project designed to improve the

mathematical education of business undergraduates Flanders and Fuller (1997) found that the content of the course was not the issue. The real issues emerging from surveys of employer groups and university staff in business faculties are:

- Presentation
- The learning environment
- The generation of motivation to continue learning
- Relevance of the material to the life of the learner and to the workplace
- Cooperation between the learner and the facilitator of learning.
- The empowerment of the learner to achieve maximum benefit for their own learning style.

We still have much to do to improve TMI as an initial adventure into the learning of mathematics for mature aged persons, people who have a wealth of experience in living and working but often have a poor perception of mathematics. There is a large pool of potential learners out there. The Australian Vice-Chancellors' Committee in a recent report (2001) outlines the elements which governments should support in funding to meet the need for Australia to be an intelligent, or knowledge-based society. One of those elements is support for enrolments of students from under represented groups. There is already strong evidence (in Australia) that the profile of the new students entering universities is rapidly changing from the school leaver to those with a diversity of backgrounds. The STEPS program provides evidence that students with diverse backgrounds but with life experience and motivation can succeed in contributing to the knowledge society. Unfortunately detailed records have not been kept of the numbers, and which students, have gone onto postgraduate studies. It is known that many have succeeded at degree, honours, masters and doctorate level in business, psychology, health science, education, information technology and other programs. Those who completed the advanced preparatory course, Transition Mathematics II, were also successful in the engineering and physical sciences – even mathematics degree programs!

Acknowledgement
At this stage it is important to acknowledge the effort, research and dedication of the staff of the Mathematics Learning Centre in continuously striving to improve learning in mathematics for STEPS students. This striving is maintained by input from students like the following:

"Self paced learning is a great idea because students learn at different levels"
"Other subjects could learn a lot from maths. Plenty of revision. Difficult tasks are methodically tackled – excellent"
"Keep up the excellent work".

Literature

Australian Vice-Chancellors Committee (2001). 'Our Universities: Our Future'. *An Australian Vice-Chancellors Committee* Discussion Paper (Ian Chubb president).
Coombes, P., et al (2000). 'Educating university bridging students and occupational travellers: interrogating the generic skills approach to lifelong learning'. In Appleton,

K., McPherson, C. and Orr, D. (eds.) *Selected papers, Lifelong Learning Conference*. Rockhampton: Central Queensland University.

Flanders, M., and Fuller, M. (1997). 'Mathematics in Business Education: Understanding through Relevance'. *Mathematics Today* **33**, 5, 148 – 152.

Francis, R.L. (2000). 'Mathematics from Recreation to Application'. *Newsletter of the Consortium for Mathematics and Its Applications*, **76**, 5-7.

Graham, A.T., and Thomas, M.O.J. (2000). 'Building a Versatile Understanding of Algebraic Variables with a Graphics Calculator'. *Educational Studies in Mathematics* **41**, 3, 265-282.

Halmos, P.R., (1983). *P.R. Halmos Selecta Expository Writing*. (Edited D.Sarason and L. Gillman) NY: Springer Verlag.

McRae, A. (1995). 'Assessing Students in Numeracy Programs'. *Numeracy in Focus*, **1**, Joint publication of Adult Literacy Information Office NSW and Adult Basic Education Resource and Information Services, Victoria,15-19.

Pirie, S. and Martin, L. (2000). 'The Role of Collecting in the Growth of Mathematical Understanding'. *Mathematics Education Research Journal*, **12**, 2,127-146.

Siemon, D.E. (1986). 'Reflection can transform mathematics teaching and learning in Mathematics: Who Needs What?' (Editor Ellerton, N.) *Mathematics Association of Victoria*.

Wlodkowski, R.J. (1993). *Enhancing Adult Motivation to Learn*. San Francisco: Jossey –Bass Publications.

Zevenbergen, R. (2001). 'Numeracy: Youth and New Times'. (Paper presented to *Annual Conference, Middle Years of Schooling Assn*, Brisbane Australia).

Goals of numeracy teaching

Lene Østergaard Johansen
Aalborg University, Denmark

Introduction

Denmark is a democratic society built on democratic values, everybody has the right to vote and to participate in the decisions taken in their country. As in many other countries we discuss lifelong learning, and it seems, everybody should have the duty, as well as the right, to participate in further education. The latest news in Denmark is a reform of the adult and further education system. A new element of this reform is to strengthen basic skills such as reading, writing and mathematics, and the reform comprises an additional education level, "Forberedende Voksenundervisning" named FVU or in English "Preparatory Adult Education". FVU will be offered to all persons over 18 who wish to improve their general skills to become better equipped for the labour market and as citizens in a democratic society.

I have had the opportunity to observe and follow a teacher-training course for the teachers to be involved in the FVU-programme. During the teacher-training course the participants were introduced to the term/concept numeracy. And it became obviously to me how difficult it is to define and explain what and how much this term "numeracy" includes. During the course there was a lot of debate about which subjects and mathematical concepts should be part of the new curriculum. Big and important questions appeared in the debate. What are the goals of the numeracy teaching? What kinds of skills are important and necessary for humans to participate in a democratic society? Some of the participants introduced the old classical German and Scandinavian concept of "Bildung" as a goal and an argument for subjects in the new curriculum. And perhaps the concept of "Bildung" can be used as a strong tool for identifying goals of the adult numeracy education.

In this paper I will discuss the question

- In what way is it possible to use the concept of *Bildung* as a tool for identifying goals for teaching mathematics (numeracy) to adults with lack of basic mathematical skills?

To discuss the answers to this question it is essential to define the key entities and concepts.

What is the concept of *Bildung*?

The Danish term *dannelse* comes from the German term *Bildung* but the term has roots back to the Greek term *Paideia*. Through history the concept has been developed through two different aspects. On the one hand *Bildung* can be associated with ‘form’ or ‘shape’; on the other hand it can be associated with ‘picture’ or, more specifically, with ‘ideal’. The first aspect leads to the process wherein people develop and change personality through life. This process can be individual and active or social and more passive; for example parents form their children or schooling forms the pupils. The other aspect refers to the aim or goal of forming or shaping; through history there have

been different kinds of ideals for humankind. For example, in times of Christianity the goal of human development was to be like Jesus Christ; or in times of enlightenment the goal of human development was to be like the ancient Greeks.

Through history there have been different kinds of theories for the concept of *Bildung*. In 1974 the Danish Professor Carl Aage Larsen wrote:

> A theory for the concept of Bildung is founded on an understanding of individuals and Society and includes an ideal of Bildung and the means/ways to reach this Bildung. (Larsen, 1974:63)[8]

In the following I will use his definition of a theory for the concept of *Bildung* as a tool and a direction indicator in a discussion of how the goals of mathematics teaching are closely related to ideals of human development. And as a concrete example of an ideal I will use active citizenship and show how numeracy education can be a way of forming Adults to be active citizens.

Goals of mathematical teaching

The term goal is closely related to some other notions such as: 'reason' and 'argument'. Niss (1996) defines reason as:

> By a (real) reason for providing mathematics education to students within some segment of the educational system we understand a driving force, typically of a general nature, which in actual fact has motivated and given rise to existence (i.e. the origination or the continuation) of mathematics teaching within that segment, as determined by the bodies which make the decisions (including non-decisions) in the system at issue. (p. 12)

Niss points out that reasons more often exist implicitly in society than explicitly defined and articulated in public; through analysing the history finds only three different fundamental reasons for mathematics education (Niss, 1996:13):

1. Contributing to the technological and socio-economic development of society at large, either as such or in competition with other societies/countries;
2. Contributing to society's political, ideological and cultural maintenance and development, again either as such or in competition with other societies/ countries
3. Providing individuals with prerequisites which help them to cope with life in the various spheres in which they live: education or occupation; private life; social life; life as a citizen.

For a reason to make sense it is normally accompanied by a corresponding claim for example

> Mathematics education can indeed contribute to provide individuals with prerequisites which help them to cope with life in the various spheres in which the live.

[8] Translated by me from:
"En dannelsesteori har som grundlag en menneske- og samfundsopfattelse og omfatter et dannelsesbegreb(-ideal) og de midler, hvormed dannelsen opnås."

Goals are discussed first when it is agreed to offer mathematical education to a particular group of students. For me it is interesting to discuss the goals in relation to adults who lack basic mathematical skills. Why teach maths to adults?

As mentioned earlier the participants on the teacher-training course introduced the concept of Bildung in the discussion of the content in the New National Numeracy curriculum. By doing so, the participants at the teacher-training course claimed that there are one or more ideals for humankind to day and that teaching numeracy could be a way to reach this ideal.

Ideals for human development

What kinds of ideals for human development are being discussed? There is not just one ideal that pops up in the discussion! If the discussion of ideals appears in a context of critical education words like *emancipation, critical citizens, democratic competence and Mündigkeit*[9] (Skovsmose, 1994) are often used. Another example is *empowerment*. In Denmark a word like *Allgemeinbildung*[10] has lately appeared in discussion about the goals and ideals for mathematical teaching (Blomhøj, 2000). But in this paper I choose to focus on one particular example. In the discussion in the Western nations (and in other nations as well) one ideal pops up. It is the ideal of **"active citizenship**". The goal of human development and of education is to produce 'active citizens'. What does that mean? There are different definitions depending on the context of the discussion.

When The European Council meets and discuss lifelong – lifewide - learning they define 'active citizenship' as:

> *Active citizenship focuses on whether and how people participate in all spheres of social and economic life, the chances and risks they face in trying to do so, and the extent to which they therefore feel that they belong to and have fair say in the society in which they live. For much of most people's lives, having paid work underpins independence, self-respect and well-being, and is therefore a key to people's overall quality of life. Employability – the capacity to secure and keep employment – is not only the core dimension of active citizenship, but it is equally a decisive condition for reaching full employment and for improving European competitiveness and prosperity in the 'new economy'. Both employment and active citizenship are dependent upon having adequate and up-to-date knowledge and skills to take part in and make a contribution to economic and social life. (Commission of the European Communities, 2000:5)*

The European Council links active citizenship with employability. In that definition they only include the labour force, and the unemployed part of the population who have a real possibility to be employed. The definition excludes senior citizens and others without jobs, such as individuals who are long term sick, pensioned etc.

---

[9] The term *Mündigkeit* relates to a person of full age who has the legal right to speak for oneself and to vote, but the term also have an informal meaning: that of having the capacity to speak for oneself.

[10] The term *Allgemeinbildung* is closely related to the English term general education or liberal education.

If we look among ALM members, we find that last year Roseanne Benn at ALM7 defined 'active citizenship' this way:

> Crewe and Searing (1996) suggests that good citizenship involves two factors: civic engagement and public discourse. 'Civic activity', 'being an active citizen' or 'civic engagement' refers to participation in any significant way in community or social activities and/or involvement in community or social organisations. 'Public discourse' refers to discussion in private and public settings ranging from casual conversation to serious deliberations on public affairs topics from community concerns to part political matters. (Benn, 2000:155)

This definition focuses on communication and participation in society, locally as well as nationally. Both definitions, from the European Council and Roseanne Benn, refer to a citizen living in a democratic society.

Active citizens?

How to form active citizens? Education is, and has always been, a way to 'form' people in a certain way:

> The traditional link between adult education and participatory democracy has been 'education for citizenship' and the equipping of individuals and/or groups to take more active and effective part in the running or changing of society. (Benn, 1997:84).

But education is not the only way to form people into "active citizens":

> Citizenship has to be learned like any other skill but it will be learned not through formal curriculum but through positive experience of participation. (Benn, 1997:85)

In Denmark we have a long tradition for adult education. Starting with the 'folk high schools' where the villagers – peasants sent their young people to get further educated. Later came 'folk high schools' for the working class as well. During the same period a lot of different societies, clubs and civic organisations were born. Initially these societies were for specials groups with the same background or the same jobs, later came other societies grounded in the local area, with members from different job-categories and different social classes. Resent research (Andersen et al., 2000) has revealed a link between the involvement of a large part of the people in these different societies and a well-founded democracy. The Danish people learnt to act as democratic citizens through involvement in the local societies, clubs and organisations. Nowadays we still have a lot of these societies, clubs and organisations but people are no longer involved in the same way, and the link between the societies, club, organisations and the politicians are getting weaker and weaker. The societies have no longer the same influence on the decisions taken by the politicians/government and the politicians no longer have the same influence on the societies and their members' opinions. It is now a question whether membership of a local society still has any influence at all on the development of ones ability to act as an active citizen? There are some indications that it works the other way now…

The European Council claims that education is necessary in order to produce active citizens and that several skills and knowledge have to be present for people to manage modern Society.

> Today's Europeans live in a complex social and political world. More than ever before, individuals want to plan their own lives, are expected to contribute actively to society, and must learn to live positively with cultural, ethnic and linguistic diversity. **Education, in its broadest sense, is the key to learning and understanding** how to meet these challenges. (The European Council, 2000:5)

From this view it is clear that education is a way of forming the people as 'active citizens'. The next question is what kind of education may provide active citizenship?

Formal education in numeracy

Is formal education in numeracy or mathematics a way to form active citizens? Through the years there have been different national and international surveys with the purpose to test the basic skills of a nations population. SIALS 'Second International Adult Literacy Survey' (OECD, 1995; OECD 2000; Jensen 2000) was publish last year and one is soon coming namely ALL 'Adult Literacy and Lifeskills" Testing of the 'numeracy skills' was in SIALS included in the problems of the quantitative literacy and document literacy. But in the ALL-survey there will be specific problems to solve to test the populations numeracy skills. The ALL committee has defined numeracy this way:

> *Numeracy: The knowledge and skills required to effectively manage the mathematical demands of diverse situations. (Gal et al., 1999:10)*

The ALL committee claims that the underlying definition of numerate behaviour is:

> *Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, behaviours, and processes. (Gal et al., 1999:11)*

Behind these different surveys there must be a kind of ideal for human kind. The survey describes the level of knowledge and skills necessary to cope with life today for an adult and thereby a life for an active citizen. For example SIALS showed that one third of the adult population in Denmark do not have the basic mathematical skills required by society today.

But is formal education a way?

> The connection between mathematics education and democracy is neither obvious nor clear. Nevertheless, current curricular reform documents seem to agree on the fact that mathematics education can contribute to the achievement of the democratic ideals of society. (Skovsmose & Valero, 1999:1)

Valero and Skovsmose find their example in South Africa, but as a result of a reform of the adult education system in Denmark, I can use this as an example. The new education

is as mentioned earlier called "Forberedende Voksenundervisning (FVU). An argument for FVU given by the Danish government was (my translation)[11]:

> FVU contains a democratic aspect, to maintain and promote the development of active citizenship, and a socio-economic perspective, linked to the demands and needs of the Labour market. (Undervisningsministeriet, 2000:7)

The Danish politicians see education in numeracy/mathematics as a way to develop active citizens in an adult education context.

The question now is how can numeracy teaching form active citizens? There are several aspects involved in the answer to this particular question, as there are different things in teaching that have influence on the 'matter learnt' in a 'numeracy classroom':

- The curriculum, the subjects and the mathematical concepts included.
- The guideline for the numeracy teaching given by the policy makers (for example the number of lessons alluded, the number of participant on each course etc.).
- The numeracy textbooks and other aids to help with the learning/teaching
- The teachers background and aim for teaching adults
- The learners background and their expectation of the teaching
- The didactical relationship between the teacher and the learners

If we start with the content of the curriculum, is the content pure maths? Is it everyday maths? Is it about reading bus schedules, weather reports, telephone bills etc? The matter learnt in the classroom is dependent on the content in the curriculum. Una O'Rourke and John O'Donoghue had a presentation at ALM 4 where they discussed a new curriculum and they claimed that

> An overemphasis on mathematics may result in perceived lack of relevance among the learners. Similarly an over-emphasis on everyday mathematics may limit the programme to mathematics which is deemed to be 'easy' by the learner and thus the challenge and resulting motivation may be eroded over time. Focusing exclusively on communications or citizenship may result in the development of a programme of learning which fails to meet the learners' expectations of what is 'real' mathematics. (O'Rouke & O'Donoghue, 1997)

They link the expectations of the learner and the chosen content. And it is a crucial decision what content the new curriculum should have, and a difficult one too. When the anticipated participants on the numeracy course are adults who lack basic mathematical skills, some would argue, that these adults already have the skills; they just don't know it yet. The aim of the teaching is to find the skills in the adults.

As a curriculum planner you can sit down and decide this, that and the other subject are important for the learners to learn. However when the ministry of education then decides that you only have 40 hours to teach them all that stuff, or decides that you have to have 20 participants at your course, you have to change your mind

[11] Translated from: "FVU rummer dels et demokratisk aspekt, nemlig at fastholde og fremme udviklingen af aktivt medborgerskab, dels et samfundsøkonomisk perspektiv, som knytter an til arbejdsmarkedets krav og behov."

Another big influence is the role and the content of textbooks and other aids. If a teacher follows a textbook where all the problems are to solve pure maths, or unrealistic examples of everyday maths, then the participants maybe well be confirmed in their view – that knowing maths has no use.

I think that the most difficult part of changing numeracy education is to change teachers' understanding of what is 'real maths teaching'. It is a long tradition there has to be changed. Even though you may change teachers' attitudes and ways of teaching you then have to fight with participants expectations of 'what real maths teaching has to be'.

A very important part of forming or shaping people is what happens when individuals meet, for example, in the classroom. The relationship between the teacher and the participant and the participants' relationships with each other, the way they talk to each other, and the way they work together on mathematical or everyday problems. Is it possible to discuss the answer to a problem or is there only one solution etc?

Goals of numeracy teaching

I started this presentation by stating the core question of my thesis:

- In what way is it possible to use the concept of *Bildung* as a tool for identifying goals for teaching mathematics (numeracy) to adults who lack basic mathematical skills?

First of all a discussion of the concept of *Bildung;* leads to a discussion of ideals in the society. We have to define, what we value and how we look at humans, how we understand our society and how we understand the kind of democracy we are living in. When the ideals are discussed in relation to mathematical or numeracy teaching it also appears how we understand and value the different kinds of mathematical knowledge.

There are a least two different discussions hidden in the literature. One focuses on *Allgemeinbildung* or 'liberal education' (Blomhøj, 2001). In this discussion 'someone' decides what is common-knowledge in society. By this I mean the amount and kind of knowledge that everybody in the society at least should be in possession of, it is a kind of 'canon'- textbooks and already known knowledge are the focus. I think that the international surveys are examples of this discussion. The other discussion focuses on 'general education' (Valero, 1999). By this I mean, education where the 'people' learn about their society, learn to read, write and do maths, so as to be equipped to participate in society, to be included in, not excluded by, society, and indeed to be a part of the transformation of society. In this kind of education it is the people's environment that is the subjects of the teaching, not the textbooks. We talk about everyday maths, ethno maths, functional maths etc.

I have argued that the concept of *Bildung* can contain the goals of numeracy teaching for adults who lack basic mathematical skills! As an example I have shown that active citizenship can be an ideal; active citizens are "the good people", who maintain and develop t society. Depending on the context there will be different understanding of the term 'active citizen'. There will be a difference between what the government means

when it talks about 'active citizens' and what for example the curriculum planner means. Another question to be raised is whether it is a right or a duty for people to be active citizens and/or to join numeracy classes.

The other part of the *Bildung* discussion is about the ways to reach the ideal of for example 'active citizenship'. I think that formal education and formal numeracy education can be a way of forming people to become active citizens. However, there are several different aspects to be taken in account in this discussion. For example 'traditional school mathematical education' is not a good choice if we want to form active citizens in a democratic society. But the result of the education – the forming – is dependent on a lot of different choices.

In the current situation in Denmark with the new national numeracy curriculum, there has been a lot of discussion on the teacher-training course about the way of teaching. Most of the teachers want to teach subjects there are relevant to the participants' everyday life – they want the participants' to bring their everyday lives into the classroom. But the problem is that the mathematics hidden in the everyday context is often very complex and difficult to understand. On the other side there is the intentions from the ministry of education. The ministry wants formal tests at the end of every course, and that means that everybody should, at least have been working on this, and this and this subject. This is just one example of the contrasts between an understanding of the ideals and the ways to reach the ideals.

I see this paper as a beginning to defining the concept of *Bildung* in a mathematical educational context. I know that I still have a lot of work left to do before I have a more satisfying answer to my core question. In Denmark we have a special need to find a good way to justify the new numeracy curriculum and education for the people who lack basic mathematical skills, and to get them to join classes.

Acknowledgements
I would like to thank Ole Skovsmose, David Kaye and Tine Wedege for their comments to the previous version of this manuscript.

References

Andersen, J. G et al Eds. (1999). *Den demokratiske udfordring*. København: Hans Reitzels Forlag.

Benn, R. (1997). *Adults Count too*. London: NIACE

Benn, R. (2000). 'All for One and One for All: Citizenship and Math Education.' In Schmitt, M.J. & Safford-Ramus, K. (Eds.) (2001). *A Conversation Between Researchers and Practitioners – Proceedings of ALM-7, 6-8 July 2000, Boston.* Cambridge: NCSALL

Blomhøj, M. (2000). 'Hvorfor matematikundervisning? – matematik og almendannelse i et højteknologisk samfund.' *Centre for Reasearch in Learning Mathematics, Publication* no **24**. Roskilde: CRLM, Danish University of Education, Roskilde University, Aalborg University.

Commission of the European Communities (2000). *A Memorandum on Lifelong Learning*. Brussels; Commission of the European Communities.

Gal, I.; van Groenestijn, M.; Manly, M.; Schmitt, M.J.; Tout, D. (1999). *Numeracy Assessment Framework for the International Life Skills Survey.* Statistics Canada. (See ALM Newsletter, no. 6, 1999, & http://www.nces.ed.gov/surveys/all/)

Jensen, T and Holm, A. (2000). *Danskernes læse-regne-færdigheder – i et internationalt lys*. København: AKF forlaget.

Larsen, C.A. (1974). 'Den kategoriale dannelsesteori og det eksemplariske undervisningsprincip.' In Jensen, E. (Ed.) (1990). *Didaktiske Emner – belyst gennem 12 artikler af Carl Aage Larsen & C.A. Høegh Larsen.* (pp. 63-69) København: Danmarks Pædagogiske Bibliotek.

Niss, M. (1996). 'Goals of Mathematics Teaching' in Bishop, A. Clements, K. Keitel, C. Kilpatrick, J. and Laborde, C. (Eds.) *International Handbook of Mathematics Education,* (pp. 11-47) Dortrecht: Kluwer Academic Publishers.

O`Rouke, U. and O`Donoghue, J. (1997). 'Guidelines for the Development of Adult Numeracy Materials'. In Coben, D. and O`Donoghue, J. (Eds.) *Adults Learning Mathematics – 4. Proceedings of ALM-4 the Fourth Conference of Adults Learning Maths – A Research Forum,* (pp. 173-191) London: Goldsmiths University of London.

OECD (2000). *Literacy in the Information Age.* OECD, Paris

OECD (1995). *Literacy, Economy and Society.* OECD, Paris

Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education.* Dortrecht: Kluwer Academic Publishers.

Skovsmose, O. and Valero, P. (1999). 'Breaking Political Neutrality: The Critical Engagement of Mathematics Education with Democracy.' *Centre for Research in Learning Mathematics, Publication* no **11**. Roskilde: CRLM, Danish University of Education, Roskilde University, Aalborg University.

Skovsmose, O. and Valero, P. (2001). 'Democratic Access to Powerful Mathematical Ideas.' *Centre for Research in Learning Mathematics*, *Publication* no. **30**. Roskilde: CRLM, Danish University of Education, Roskilde University, Aalborg University.

Undervisningsministeriet (2000). *Lov om Forberedende Voksenundervisning (FVU-loven).* København: The Ministry of Education.

Calculating people: measurement as a social process

Betty Johnston
University of Technology, Sydney, Australia

Numeracy in practice: a research project

> What is special about the language of quantity? My summary answer to this crucial question is that quantification is a technology of distance. ...Since the rules for collecting and manipulating numbers are widely shared, they can easily be transported across oceans and continents and used to coordinate activities or settle disputes. Perhaps most crucially, reliance on numbers and quantitative manipulation minimizes the need for intimate knowledge and personal trust. Quantification is well suited for communication that goes beyond the boundaries of locality and community. (Porter, 1995).

Quantification permeates our society. We quantify everything, or try to. We measure everything, including, problematically, literacy standards. In the adult education community, we haven't got around to measuring numeracy standards in any detail yet, but I am sure we will in time. Measurement of anything involves a lot of questions: what are we measuring? what does it mean? who benefits? In this article, however, I want to discuss a prior question, about the *process* of measurement and how we understand it. I want to provide some evidence for the argument that measurement is not some disembodied process, but a process that shapes us, as we shape it.

My interest in this question has been fuelled by a project I was recently involved in. I was working with four colleagues on project looking at the numeracy practices of young unemployed people[12]. We were interested in practice, not just as 'what people do', but also in how this doing is shaped by broader social structures - a view of practice paralleled in the literature on literacy practices, and drawing on a related notion of practice developed by Connell (1987). Connell describes practice as 'what people do by way of constituting the social relations they live in', seeing human action as involving free invention within structural constraints ("invention within limits", to use Bourdieu's phrase (1977:95)). Such a position presupposes both the person as agent, and the formative role of structure in shaping and constraining possible agency. Practice in this sense is specific, historical and constrained by structure.

Our focus on the project combined two interests: the pedagogical, and the theoretical. We assumed that young people work to make sense of their worlds, including the world as it intersects with mathematics, and that they bring significant skills and experience to the numeracy learning situation. We wanted to explore how numeracy practices are 'organised as a going concern' with young unemployed people in both urban and more

[12] The team working on the project consisted of Kerry Barlow, Mike Baynham, Sheilagh Kelly, Genée Marks and myself. The bibliographical details of both the full report on the project are included in the References. The project could not have been undertaken and completed without the funding and support of the Australian National Training Authority (ANTA) .

rural sites, to tease out particular instances of how different social interests might result in different practices, and to help teachers use the awareness gained to value and extend the practices that students have already 'invented'. Many strands have emerged in the process of the research, but the one I am focussing on here is measurement.

How tall are you?

In our earlier overview article in Literacy & Numeracy Studies (Baynham & Johnston, 1998), we described the different ways in which the young people responded to the question of how tall they were. One or two of the young people said they didn't know, a few answered using metric units, but the overwhelming majority answered using feet and inches with varying degrees of precision - 'six foot something', 'five foot six inches', 'five foot eight and three-quarters'. Were they confirmed and unrepentant imperialists? No, Ashley had reasons for preferring feet and inches:

> *I usually do work in centimetres. It's just that in height I'd actually just like to reach six foot, so I'd rather say I'm six foot than - yeah, it sounds - more - impressive.*

Sam hadn't learnt either system of measurement at school. What she knew she had taught herself:

> *I know the centimetres and millimetres because I learnt that with [knitting]. I taught myself how to do that ... I've never had to do anything by metres or anything very long like that. But I'm starting to learn how to use metres more driving. Got to learn how far a metre is, & 50 metres.... got to judge distances.*

Angie straddled the worlds: she gave her own height as 5 feet 2 inches but her small daughter Kelly's as 'about 65 cm'.

All of them, however, used the metric system for almost all other measurements of length eg wood, steel, material, knitting, driving, as well as for most other measurements - weight, capacity etc - which raises the question of what influenced the choice made. Why were the choices not consistently in one system or the other?

Measurement: a cognitive or social process?

When Piaget and his colleagues studied the progress of children's performance in measurement tasks, they argued that these could be seen as a developing cognitive process (Piaget, Inhelder et al. 1960). When children were asked to complete a task such as building a tower on the floor the same height as one on the table, the stages could be characterised in the following way:

- visual estimation, of the two objects
- direct comparison side by side
- comparison with some body part
- the use of an introduced 'symbolic object' as unit

This process of measurement can however be understood as a social as well as a cognitive process. Denny (1986) in his study of mathematical thought in Inuit society

looks at counting and measurement, and at how they emerge and are used in a society very different from our own.

Counting, says Denny, serves as a way of 'apprehending objects which cannot be perceptually or conceptually identified'. He describes a court case about land rights where an Inuit hunter was unable to say how many rivers were in the disputed area, a failure which was taken by the opposition as clear evidence that the man was unfamiliar with the region. In fact, the man probably knew the actuality of each river, of each bend of each river, of the different plants and animals that lived in the different environments. What *use* , he asks, would there have been in knowing the number of them? Numbers that *are* used are low numbers and often very context specific; some numbers for example, are used for counting concrete objects, slightly different words indicate events or sets.

Likewise measurement in such a society is not used if perceptual judgement alone is thought to be adequate. When it is used it is very context sensitive, with units that vary from task to task and person to person. The maker of a canoe will use his or her own body measurements to make an appropriately proportioned craft. Such flexibility, Denny argues, cannot persist in a more complex society where an increased alteration of the environment leads to increased dependence on the specialised work of others and where some people work as specialised managers, directing the efforts of specialised others.

In the hunter societies, the designer and maker of an object is the same person, and there is no need for standardised units of measurement, or standardised shapes. But in industrial society, where many people are involved in the production of a single item, coordination of their efforts is more efficient if measurements are standard and if shapes are not more or less circular, but precisely circular.

Thus, Denny argues, the material base of our lives generates our particular mathematics and the value we attach to it. In our highly industrialised society, the division of labour has led to a progressive decontextualising of knowledge. At the institutional level, we can see this in our metric system, a system that has given us as our standard - almost global - measures of length, millimetres, centimetres and metres: the first two too small, and the last too large, for comfortable measurement of many everyday objects. Feet and inches may have been less elegant, or less useful to science, but a few inches or a few feet measured most things that ordinary people needed.

Appropriate measuring

At a more individual level, what are the implications of understanding measurement as both a social and a cognitive process? The decision about what kind of measurement is used in any particular situation, is partly a cognitive one, relying on an understanding of the use of increasingly less concrete units. But it also relies on the particular context, including, as Nunes and her colleagues (1995) argue, understanding of the need for intersubjective reliability.

If, for example, you were measuring the length of a table, what would be the most appropriate way to do it? Should you do it

- in metres?

- in feet and inches?
- by comparing it directly with the space against the wall?
- in hand lengths?

The answer of course, is that any of these strategies might be appropriate depending on whether, for instance,

- you had to give the information to someone else, a stranger
- you were out shopping and were more comfortable with imperial units
- you were at home shifting furniture round
- you had no measuring tape and you wanted to do a rough check before moving the table

It is not accidental that the word 'appropriate' emerges here: like the 'mode continuum' in the linguistic context (Martin, 1984, cited in Gibbons, 1995), we might suggest a 'measurement continuum', along which particular practices could be arranged, in such a way that certain features of practice changed in predictable ways, as the contexts became increasingly context-reduced. With increasing distance between the participants, and increasing need for communication, these features would include:

- portability: whether you want to transport the measurement from one location to another
- communicability: whether you want to tell someone else the result of your measurement
- precision: what degree of precision you want in the measurement
- meaningfulness: how meaningful the measurement is - whether it is useful, conceptually interesting, critically relevant, valid, reliable

Measurement in practice

These features are intrinsic to Porter's portrayal of quantification as a 'technology of distance' in the earlier quote. They also begin to give a framework to address my earlier curiosity about what influenced the choice of measurement system made by the young people in our study.

Communication, and meaning, were important for Ashley: he wanted to be able to tell others about his impressive height and six feet was more meaningful than a mere 180 cms. Angie also gave her height in feet and inches, but for quite different reasons. She didn't need to know her own metric height. She was a product of her time, and her family, and their system of units; she was at home, for her own needs, with the imperial system. She did not compare Kelly with her own height, however, but with the official heights of other small children, as given from the moment of birth. Hospital measurements, clothing sizes, baby health centres all use metric truths, and Angie needed to relate to them. Kelly was becoming a product of the metric world.

Meaning, communicability and precision were also factors in such everyday negotiations as buying cheese or ham, estimating distances, or measuring out doses of medicine. The unit of measurement for some of these common objects and qualities was not always what was expected.

In answer to the question: 'How do you ask for the amount of cheese or ham you want?', we initially expected an answer in weight, something like '200 gm'. But the answers were far more varied, including: '5 slices' - if the person was catering for five people; '$2 worth' - if it was the budget that was constraining; 'that piece', or even, 'a handful', if the estimation was easier as a visual one.

Similarly, a question concerning the distance from here to Ashfield, to Armidale, to Adelaide, to South Africa could elicit an answer in kilometres, but it was more commonly given in minutes or hours. One young woman was trying to work out a dose of medicine: 15 mls for an adult, and half that for a child. 'If it was money', she said, 'it would be $7.50.'

In all these examples, communicability was an essential element: the people involved had to read instructions from others, or make known their needs to others. In some form or another, sometimes an unexpected one, they were successful. Precision of weight, on the other hand, had lower priority in buying ham: the constraint might be amount (number of slices) or budget, and it was these constraints that decided the choice of unit and the degree of precision appropriate in the context. In all the situations, meaning was crucial. Why would anyone want to know how far it is to Ashfield? Probably, so that they could go there, and the important aspect of the distance becomes time. Long distances are for travelling. Short ones may be for ordering fence lengths, or planting rows of trees - in which case it would be metres, or yards, that would be more appropriate.

A comparable shifting of the unit of measurement to give a different and arresting meaning can be seen in the advertising hoardings in Sydney trying to warn motorists of the penalties associated with speeding. They show a picture of a speedometer with the needle pointing to 90 km/h, and a message to the effect that 'You are travelling at $179'. It is the disjunction that is arresting: speed is being measured in terms of money. The similar flexible use of measuring units was a familiar strategy for the young people in the study.

Pedagogical implications

Thus we began to see how context determined the kind of measurement that was appropriate. That measurement is a social process means that factors such as communicability, portability and precision influence our students' choice of unit, system or process. If communication, for instance, is not important, then a personal and idiosyncratic system is fine; otherwise some common standard is needed.

Like the concept of genre in writing, measurement may be usefully understood as a range of strategies from which more or less appropriate choices can be made. As with genre also, it could be argued that the more decontextualised the strategy, the more highly valued it is by our society.

But what we as teachers should be valuing is not only the skill of dealing with abstractions, but also the skill involved in relating that abstract knowledge back to concrete experience. Denny (1986) argues that, for the hunter, 'inclusive knowledge of the whole pattern of natural processes will be imperative'. Surely this is true also for

industrialised societies, with their proliferating global problems, which are increasingly complex and inter-related. If the maths that we teach our students is to help them become critically numerate, then it must not consist of abstracted bits and pieces. It must start from the concerns of the real world, and return to them.

Matt: located and passionate knowledge

One of the young men interviewed was Matt, aged 17, who had been largely home educated. He was one of nine children, whose parents managed a farm near a small country town in NSW. He had a passion for the weather growing out of his location: both the physical and social space of his life, the particularities of the place and the social networks - largely familial - in which he lived.

As eldest son, he was responsible for many of the outdoor tasks, including place: one of these tasks was recording the rain. Why did he need to do that, we asked, and his answer included the following:

> *I've always had a fascination with the weather. If I was ever going to research something it would be lightning storms. ...Why? Like it does have its fascination, it's quite spectacular when it comes down full force...the weird thing is like sometimes - I don't know why, but the rain comes right up to our house...And like it will be raining there and it will rain there for 15 minutes non stop, like full pelt, and the house up from us which is probably 800 metres away won't get a drop and I'm thinking well, it's weird, like it will come right up to our house, stop and nobody on the other side of the road will get it!*

He had installed a rainfall recording device, he followed the weather reports in the paper and on television, he hypothesised about weather trends and causes, and over the dinner table he fantasised with his parents about floods:

> *Me and me dad and me mum, like we were working out if we had 100 inches like which places would get flooded and which wouldn't. We reckon if we did get 100 inches Broadfields would be covered right up here, near the Barracks. The railway - you wouldn't be able to come on the railway because it would be flooded right out to - until the ground starts going up into the mountains. You know where Eversleigh, you know where Eversleigh is?... That would be gone. Rowntree would be cut off in several different places and Milson, there'd be only a little bit of Milson on top of the hill because it would be ...an island. And Benton would be completely under water.*

Matt's knowledge was highly situated in local knowledge of the weather he observed, in scientific media derived knowledge of weather reports, in scientific and localised recording on the chart from the farmer's newspaper, in the wider significance of rain for farming in Australia. It was a passionate combination of local knowledge and a knowledge with potential for distancing and abstraction, a knowledge of depths and sudden gaps - as in his slippery understanding of average rainfall as a balancing of measurements over two consecutive years. It was a powerful weaving of the concrete and the particular, with the increasingly abstract and general.

Conclusion

From the stories of these young people, we can begin to see how it enriches our understanding of measurement, to conceive of it not only as the logical development of a measuring tool, but also as a practice growing out of the complexities of social situations.

Of course, such an understanding is only the beginning of an understanding of the social dimensions of measurement. What we choose to measure, who the measurement is for, its meaningfulness, its effect, also involve social processes worth exploring[13]. And while measuring physical quantities is one thing, measuring qualities, personal or social, compounds the complexities. How was the IQ constructed? What does the Tertiary Entrance Ranking (TER) mean[14]? Who does the Research Quantum benefit[15]? Why do we need literacy benchmarks? One of the important effects of the measurement process is that it reduces complexity by focussing on a single element, powerfully clearing away the clutter that is humanity, and presenting an apparently objective picture. We get into trouble when we try to account for quality by measuring quantity, by breaking up the whole picture and missing the connections between categories. We are in danger of simply constructing a 'human sorting house' (Meadmore 1993:61), as we set up norms and deviations from the norms. We are in danger of reducing ourselves to Foucault's 'calculable man' (and woman) (1977:193)

In our irredeemably quantified society, a lack of facility with numbers puts us at the mercy of those who are at home with numbers and use them to describe and prescribe our world. One use of numeracy is to be able to engage with such arguments in their own terms. To remain within the discourse of number however is to risk blindness to the limits of its use. Learning a craft involves not only skill with the tools, but knowledge of when to use them. Yes, let's teach people how to measure (and to count and to calculate), but let us ask also about the appropriateness of the measure, let us ask why, who, and what we are measuring.

Literature

Baker, D. A. (1995). 'Numeracy as a social practice: implications for concerns about numeracy in schools.' In *The Third International Conference on the Political Dimensions of Mathematics Education*. Bergen: Caspar Forlag A/S.

Barton D. & Hamilton M. (1998). *Local Literacies: Reading and writing in one community*. London: Routledge.

Baynham, M. & Johnston, B. (1998). 'Invention within Limits: numeracy practices of young unemployed people'. *Literacy & Numeracy Studies*, **8**, 1, 37.

Bourdieu, P. (1977). *Outline of a Theory of Practice*. Cambridge: Cambridge University Press.

[13] I have begun to explore some of these ideas in Does value count? (Johnston, 1999).

[14] The number that a student in NSW achieves after 13 years of schooling and as a result of multiple exams in the final year… it is the number that determines which, if any, university course you can get into.

[15] The Research Quantum is another index, another single number, this time the one achieved by each university or staff member, a 'measure' of their contribution to research over the previous year.

Connell, R. W. (1987). *Gender and Power: Society, the Person and Sexual Politics*. Cambridge: Polity Press.
Denny, J. P. (1986). Cultural ecology of mathematics: Ojibway and Inuit hunters. *Native American Mathematics*. M. Closs. Austin, University of Texas.
Fasheh, M. (1990). 'Community education: to reclaim and transform what has been made invisible'. *Harvard Educational Review*, **60**(19-35), 2-8.
Foucault, M. (1977). *Discipline and Punish*. London: Allen Lane.
Gibbons, P. (1995). 'Learning a new register in a second language: the role of teacher/student talk.' *Working Papers in Language and Literacy No **1***. Sydney: Centre for Language and Literacy, University of Technology.
Johnston, B., Baynham, M., Kelly, S., Barlow, K. & Marks, G. (1997). *Numeracy in Practice: Effective Pedagogy in Numeracy for Young Unemployed People. A Research Report*. DEETYA: Canberra .
Johnston, B. (1998). 'Adult numeracy.' In Street, B. (Ed.), *International Literacy Handbook*. New York: Garland.
Johnston, B. (1999). 'Does value count?' *Education Links*, **58**:14-15.
Martin, J. (1984). Language, register and genre. *Children Writing: a reader*. Geelong: Deakin University Press.
Meadmore, D. (1993). 'The production of individuality through examination' *British Journal of the sociology of Education*, **14**(1): 59-73.
Nunes, T.; P. Light, et al. (1995). 'Measurement as a social process.' *Cognition and Instruction*, **13**(4): 585-587.
Piaget, J.; B. Inhelder, et al. (1960). *The child's conception of geometry*. London: Routledge & Kegan Paul.
Porter, T. (1995). *Trust in Numbers: the Pursuit in Objectivity in Science and Public Life*. New Jersey: Princeton University Press.

A grounded approach to practitioner training in Ireland: some findings from a national survey of practitioners in Adult Basic Education.

Terry Maguire and John O'Donoghue
Department of Mathematics and Statistics, University of Limerick, Ireland

Introduction
The potential for empowering individuals through adult mathematics education is currently a topic of international significance. Research is currently centred on:

- training practitioners and identifying the skills, knowledge and competences that they need to enable them to create an environment that engenders empowerment on the ground
- policy that engenders empowerment in the individual learner and how policy percolates through the system
- motivating adult learners to access and participate in Adult Basic Education (ABE).

This paper reports on the preliminary findings of a national survey of practitioners in the field of adult mathematics education in Ireland. This survey provides a profile of the practitioner in Ireland, identifying their training needs and providing a contribution to the international debate on practitioner training. This paper also addresses issues of policy in the Irish context and provides a framework for examining empowerment through adult mathematics education at three levels: practitioner level, policy level, and the level of the learner.

Background

The International Adult Literacy Survey (OECD, 1997) highlighted the poor level of adult literacy in Ireland and was pivotal in initiating policy development in this area. There is no national data of the numeracy levels of adults in Ireland outside the results of this survey.

Over the three domains of literacy, 25% of the Irish population were found to score at the lowest level, indicating that a very significant percentage have problems with all but the very simplest literacy tasks. The Irish performance in a comparative context is poor; the percentage of participants who are at the lowest level literacy is higher in Ireland than anywhere else except Poland (Morgan et al, 1997).

Since the publication of these results the adult literacy problem has been elevated to centre stage in educational policy. Provision for adult literacy in the education sector increased from £0.86m in 1997 to £8.8m in 2000. An investment of £73.5m has been committed to this area in the National Development Plan (NDP), 2000-2006 (DF, 2000).

The plethora of national policy initiatives now emerging in Ireland, for the first time clearly demonstrate a growing conviction at all levels of society, including Government

that adult education is a vital component in a continuum of lifelong learning (Department of Finance (DF), 2000; Department of Education and Science (DES), 2000; Department of the Taoiseach (DT), 2000; National Adult Literacy Agency (NALA), 1999; Qualifications (Education and Training) Act, 1999.

Despite these initiatives a number of lacunae are emerging; Ireland has no clear policy on adult mathematics/numeracy education. The Department of Education and Science has adopted a narrow definition of numeracy that views numeracy as being restricted to number (Farrell, 2001, pers com.). The National Adult Literacy Agency, which has responsibility for the implementation of policy in adult basic education, defines literacy as encompassing numeracy. There is similarity between the situation in Ireland and that outlined for Australia by Cummings(1995). '*That the inclusion of numeracy as a component of literacy: sometimes explicitly included in literacy agendas, sometimes implicitly, sometimes omitted: is not sufficient'*

To date there has been no systematic research into the numeracy needs of various stakeholders including employers, practitioners and the learners themselves.

Building learner demand is one of the most pressing challenges in the broad field of adult education today (Wagner, 2000). Government policy alone will not motivate ABE learners nor initiate the process of empowerment; an effective service provision is required.

Bailey and Coleman (1997) highlighted the existence four types of barriers to participation in ABE: situational barriers; informational barriers; institutional barriers and dispositional barriers. The removal of these barriers will require an effective, coordinated policy development across a number of government departments and agencies charged with delivery. Learners should be able to direct their own learning and have an identified structure to enable them to influence programme content, pedagogy and policy.

In an effective ABE system, practitioners play a central role in the process of empowerment. If the ABE sector is to make the quantum leap envisaged for it in the White Paper on Adult Education (DES, 2000), it must have a highly trained corps of practitioners who are dynamic and equipped to lead change. They must play a key role in policy debate and must reflect the distinctive identity of the sector in the field of professional practice and research. This research provides baseline data on the practitioners currently involved in delivering mathematics in this ABE.

National Survey of Practitioners of Mathematics to Adult Learners in Ireland

In the period February to May 2001, a national survey of practitioners involved in teaching mathematics to adult learners was implemented. The survey gathered information on various aspects of practitioner experiences, perceptions and needs, both in their classroom practice and professional development.

Five hundred questionnaires were distributed to practitioners of mathematics/numeracy nationally. The sample included practitioners that represent the spectrum of adult mathematics education delivered in the further education sector in Ireland. 312 (62%)

valid questionnaires were returned. For the purposes of this paper the discussion of the results of the research work in progress will be limited to those that pertain to ABE (n=175).

Methodology

The research incorporated the use of qualitative and quantitative methodologies (in-depth interviews, literature review, questionnaire survey). The research instrument was constructed using the following process;

- Draft questionnaire was drawn up based on the literature
- Validation and revision of draft using qualitative methodology
- Revised draft drawn up and piloted
- final research instrument

Recorded in-depth interviews were carried out and recorded with a number of practitioners, working with different agencies involved in ABE. Each interview lasted about one and a half hours and focused on; previous school experience; career pathways taken; the importance of teaching qualifications; teaching practices; numeracy as a concept and support systems.

The research instrument was revised and piloted (six times), to ensure clarity, and coherence of question flow. For the purposes of the survey the term 'mathematics' was defined to incorporate all levels of mathematics from the most basic level including what might be called numeracy; an 'adult' was defined as anyone who has left fulltime mainstream education.

Numeracy practitioners were targeted through area adult literacy coordinators and through the National Council for Vocational Awards (NCVA) who provide accreditation for ABE.

Adult Literacy Coordinators were contacted by telephone to secure the participation of their practitioners in the survey. Although 3656 practitioners are involved in adult literacy centres nationally, coordinators estimated that only 5%(approx) actually deliver numeracy. A total of 225 questionnaires were sent out to coordinators for distribution to the numeracy practitioners associated with their centre.

The NCVA as part of its in-service provision invited all practitioners of NCVA Foundation and Level 1 mathematics to attend an in-service seminar on mathematics. An additional 82 questionnaires were issued to the practitioners who attended this programme.

Details of the practitioners represented by the research sample are outlined in Table 1. Data was analysed using SPSS for Windows (version 10.0.5.)

Table 1 A breakdown of the proportion of practitioners represented by the sample.

	Accredited ABE (NCVA*)	Non Accredited ABE **(Adult Literacy Centre)**
No. of practitioners delivering adult mathematics/numeracy	156+	183■
No. questionnaires circulated	82	225
No. questionnaires returned	80	146
% valid returns	96%	76% (114)○
% of total sample represented	51%	62%

* ***National Council for Vocational Awards***
\+ ***figure based on 1999/00 figures from NCVA***
■ ***Approx. no. based on information provided by Adult Education Co-ordinators***
○ ***Once Area Coordinators had circulated questionnaires to each numeracy practitioner, those not required were returned.***

Results

The results are discussed under the following headings:

- General teaching experience
- Specific experience in teaching adult mathematics
- Teaching practices
- Training
- Attitudes towards mathematics and teaching mathematics
- General classification details

General Teaching Experience

The majority of practitioners work part-time (62%) or as a volunteer (35%). Only 18% were involved on a fulltime basis (Figure 1). 6% of those working fulltime (n=2) and 38% (n=30) of those working part-time also worked as a volunteer.

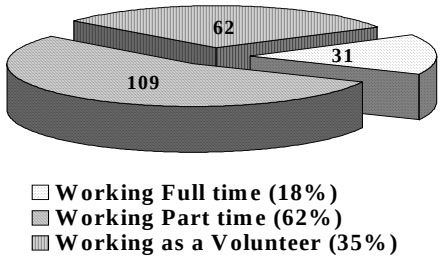


Figure 1 Type of employment of practitioners

More than half the practitioners had no experience of teaching students in mainstream education, however there was evidence of experience of teaching adults. 37% of those sampled had more then 10 years experience, 21% with 6-10 years experience; 41% been involved in teaching adults for 5 or less years (Figure 2).

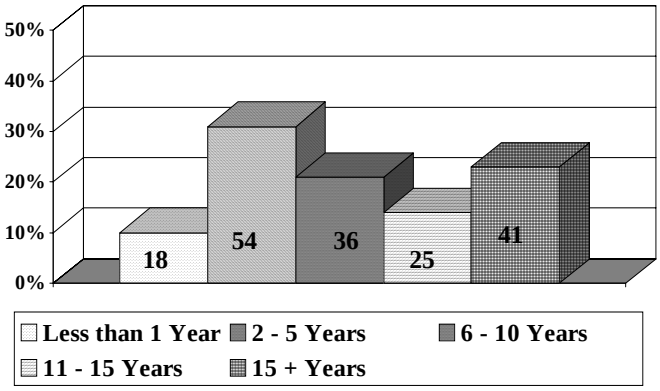


Figure 2 Number of years teaching experience

Specific Experience Teaching Adult Mathematics

Approximately 60% of practitioners have been involved in adult education for more than 5 years; however, practitioners had less experience in delivering mathematics (Figure 3 & 4). 33% of practitioners were delivering mathematics/numeracy for the first time. 46% had less than 5 years experience, only 12% had greater then 10 years experience.

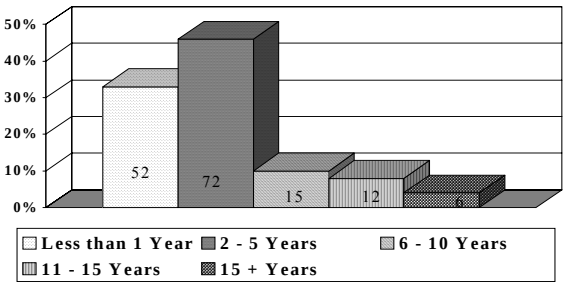


Figure 3 Number of years teaching adults mathematics/numeracy

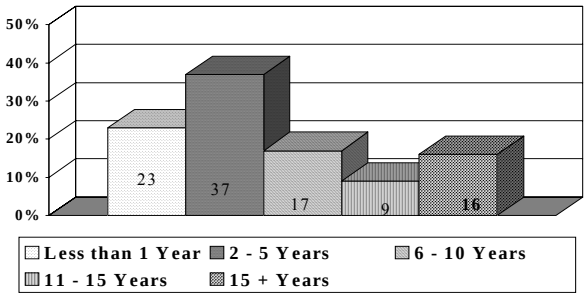


Figure 4 Number of years teaching mathematics/numeracy

Teaching Practices

The views practitioners have on teaching adult learners are outlined in Table 2. Results indicate some differences of opinion on whether adults are more demanding (39% agree, 34% disagree) than mainstream second level students and on whether a chalk, talk and practice approach is effective in the adult classroom (45% agree, 32% disagree).

Table 2 Practitioner views on the adult learner

Practitioner Views on the Adult Learner	**Agree with statement (%)**
Adults learn best when they are treated as equals•	97%
A different approach is required in the classroom when teaching adults	96%
Adults are more demanding than other students	39%
Adults have a complex about mathematics	72%
A chalk, talk and practice approach in the classroom works well with adults	45%

The teaching practices used by practitioners in the classroom are outlined in Table 3. Over 90% of practitioners indicated that they always/usually use problem solving (94%) and practical work (90%) as a teaching method. 68% of practitioners always/usually use group work, 59% used blackboard, chalk and talk, 36% use project work. Only about a quarter of those sampled use technology and investigational work.

Table 3 Teaching practices used by practitioners in their classroom.

Teaching Practice	Usually/Always	Never/Rarely
Problem Solving*	94% (150)	6% (10)
Practical work	90% (140)	10% (16)
Consolidation and practice	87% (130)	13% (20)
Group work	68% (95)	33% (46)
Blackboard Chalk and Talk	59% (91)	41% (64)

Teaching Practice	Usually/Always	Never/Rarely
Project Work	36% (36)	64% (81)
Technology	28% (34)	73% (91)
Investigational Work	24% (28)	76% (87)
Rote Learning	13% (16)	87% (107)
Invited Guest Speaker/Role Play	<3%	

***** *Interpreted as exercises of mathematics problems.*

Training

68% of practitioners considered that they have average /sufficient training in teaching adults in general. This contrasts with 82% of practitioners indicating that they have had none or insufficient training in teaching adults mathematics specifically

93% of practitioners felt there was a need to develop a training programme for those involved in adult mathematics/numeracy. The vast majority (93%) would attend such a course if it were available.

Practitioners were also asked to indicate the specific training they felt would help them to be more effective in teaching mathematics/numeracy to adults; their priorities are outlined in Table 4.

Table 4 Specific training required by practitioners (ranked)

Specific Training Practitioners Feel Would Help Them To Be More Effective In Teaching Mathematics To Adults	**Valid %**	**n**
Designing Resources	67	(117)
Ways of applying mathematics	64	(112)
How adults learn mathematics in particular	63	(111)
Coping with math's anxiety	60	(105)
Using technology	43	(75)
How adults learn in general	40	(71)
Profound understanding of elementary mathematics	36	(63)
Problem solving	34	(60)

Practitioners gave the highest priority (range 60% - 67%) to: designing resources and activities to suit all levels of learners; ways of applying mathematics in different contexts; the way adults learn mathematics in particular; coping with mathematics anxiety. Lower priorities included (range 43% - 34%); using technology; how adults learn in general; more profound understanding of elementary mathematics; problem solving.

Over 90% of practitioners considered that all the elements listed above, except using technology, should be included in a training programme for mathematics/numeracy practitioners (range 98% - 90%). Only 69% thought it important to include the use of technology in delivering mathematics.

Attitudes Towards Mathematics and Teaching Mathematics

60% of practitioners acknowledged that mathematics was their favourite subject in school and feel they have a natural ability with numbers. 44% felt that they never understood all the mathematics they were taught in school. 24% indicated that mathematics made them anxious at school. 19% really struggled with mathematics in school (Table 5)

Table 5 Practitioners feelings about mathematics

Statement About Mathematics	Disagree %	Neither Agree nor Disagree %
Mathematics was my favourite subject when I went to school	28 (47)	12 (21)
When it comes to numbers I seem to have natural ability	25 (43)	18 (30)
Mathematics made me anxious when I was at school	59 (97)	18 (30)
I never felt I understood all the mathematics I was taught at school	57 (79)	9 (15)
I have middle of the road mathematics ability	31 (53)	10 (17)
I have really struggled with mathematics when I was at school	68 (112)	13 (21)

General Details

ABE practitioners are predominately female (80%). Approximately 60% of the practitioners are aged between 35 and 54 years. Approximately one fifth are over 55. 17% are between 25 – 34 years. A tiny minority (1%) are less than 25 years of age.

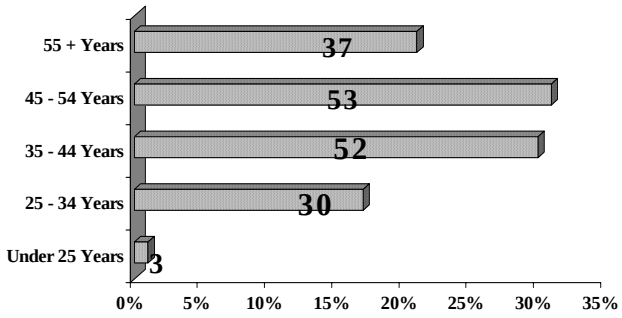


Figure 5 Age profile of practitioners sampled

Qualifications held by Practitioners

In terms of qualifications, only 7% of practitioners have a degree in mathematics. 46% of practitioners have a degree or post graduate qualification that is not in

mathematics. A further 47% do not have a degree. The highest mathematics qualification of over half the respondents (55%) is Leaving Certificate ordinary level.

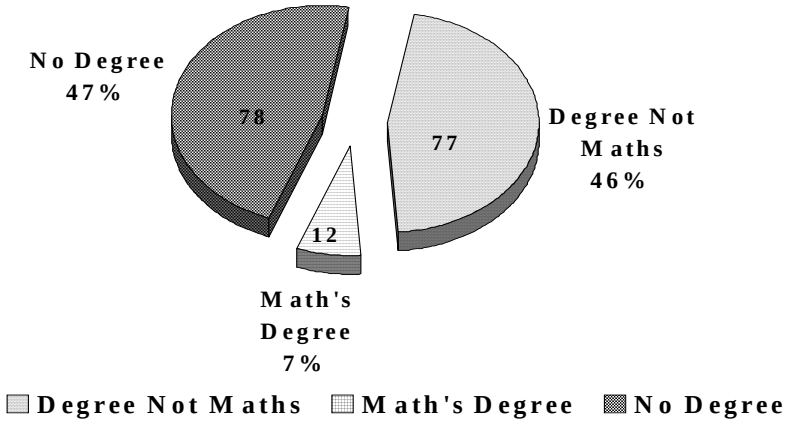


Figure 6 Highest Overall Qualification

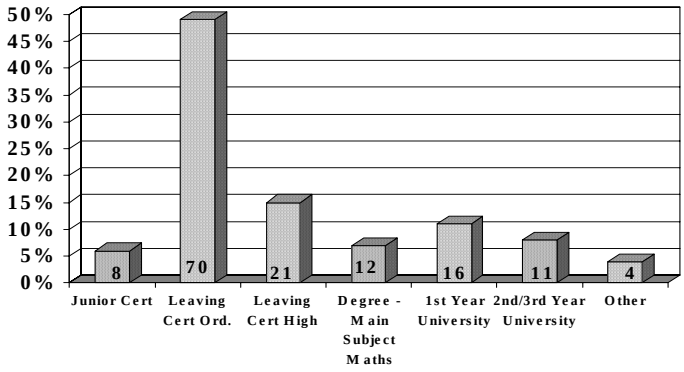


Figure 7 Highest mathematics qualifications held by practitioners

Discussion

The results of this research point to a part-time, volunteer work force in ABE, who have had some training in teaching adults but little or no training in teaching mathematics to adults.

To paraphrase Cooney (1999) who speaks about mathematics teaching in general; effective practitioners of adult numeracy need to know mathematics, know their students, have knowledge of the pedagogy of mathematics and a commitment to their own lifelong learning.

Practitioners in ABE in Ireland know some mathematics. More than half have studied mathematics to Leaving Certificate Ordinary Level, (State Terminal Examination, taken at around 18years). Of these practitioners 46% indicate that they did not understand all the mathematics that they were taught and 19% of the practitioners sampled involved in delivering adults mathematics education struggled with mathematics themselves. Anecdotal evidence to date is that effective adult numeracy practitioners can develop renewed processes of mathematical thinking and engender positive attitudes in adult numeracy students. Ineffective adult numeracy teachers unfortunately can disempower and reinforce the low status of the adult learner. (Benn, 1997; Coben and Chandra, 2000; Cummings, 1995; Bailey &Coleman, 1997). This research demonstrates that practitioners do not have a strong mathematical base to effect competent and confidant delivery to adult learners that have acknowledged anxiety about mathematics.

It can be argued on both empirical and philosophical grounds that what teachers learn is framed in the context in which that knowledge is acquired A teacher's view of mathematics is more or less consistent with the way they experienced learning mathematics themselves. (Cooney, 1999). The paucity of training to teach adult mathematics evidenced by this survey, coupled with an exam-orientated education system at second level, means that the only experience, of mathematics teaching of the practitioners in Ireland is very limited and one focused on getting the right answer and memorising the formula; rather than on generating an understanding of mathematics.

The support that practitioners have shown for a training programme that would effectively equip them for delivering mathematics to adult learners, and the willingness of practitioners to participate in such a programme were it available, demonstrates a clear commitment by practitioners to their own life long learning. Quality training that will empower the practitioner to engage in good practice for teaching adults mathematics and at the same time facilitate empowering mathematics in their own students is essential.

It is clear that empowerment is becoming a central thesis of this paper. Since that is the case it is necessary to look closely at the process of empowerment in ABE in Ireland. For the purposes of this paper empowerment is defined as 'the act of taking away demotivators and barriers in the system', (Persico, 1991:61). This definition has its roots in workplace organisations but offers an approach to examine in a holistic way, the current structure of the sector.

There are three levels in the ABE system through which the process of empowerment of the individual, through adult mathematics/numeracy education, is initiated: policy level, practitioner level, and the level of the learner.

Using this framework, Ireland has a top down policy of implementation (linear model- see Figure 8) driven by a perspective that prioritizes literacy above a combined literacy and numeracy agenda. A fragmented ABE provision is rapidly trying to respond to reactive, albeit well-funded policy initiatives in the absence of a theoretical and research framework on which to base their approach.

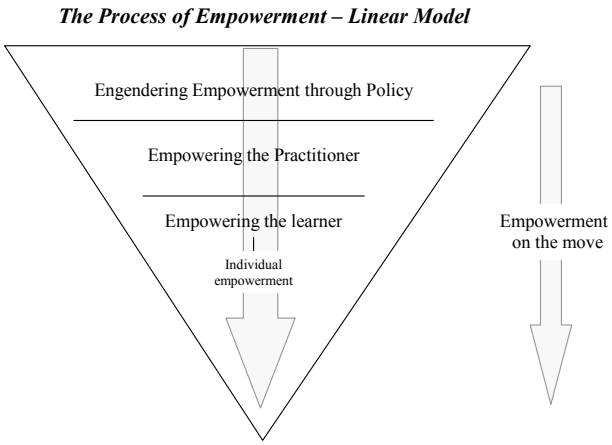


Figure 8 **Top down approach to the process of empowerment**

Moving from this linear model at one end of a scale, it is suggested that a better approach to the initiation of individual empowerment in adult learners is to have a more integrated process (Integrated model –see Figure 9)

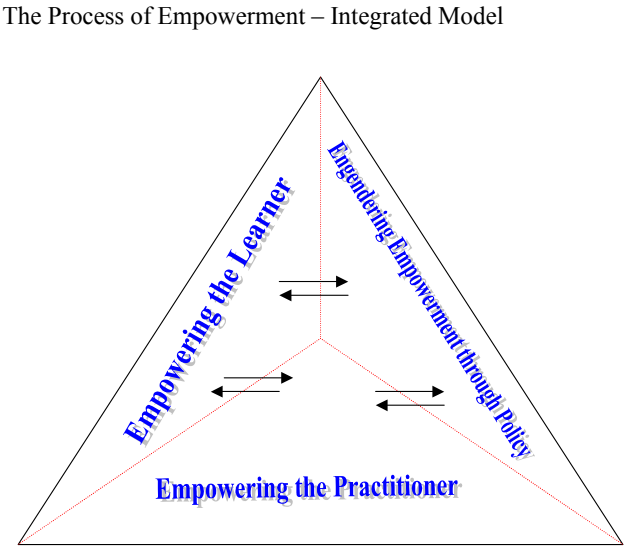


Figure 9 **Integrated approach to the process of empowerment**

In this integrated model both learners and practitioners should have an identified structure to enable them to influence programme content, pedagogy and policy. The findings emerging from this study support the conviction that the practitioner plays the pivotal role in the process of empowerment in ABE. Highly trained, confident practitioners could, for example, through a recognized forum, inform policy, moving policy development to being proactive rather than reactive and ensure policy is

implemented at centre level. Practitioners can facilitate the initiation of empowerment in individual learners allowing them to address their own needs and encouraging them to take responsibility for their own learning.

In reality the process will include a hybrid of the two models. A dynamic interaction between individual stakeholders will shift the balance one way or another, depending on for example; new revised policy initiatives, policy prioritisation and funding; practitioner experience, practitioner training, and the stage of individual empowerment of the learner.

Conclusion

The current national policy emphasis on ABE, coupled with the funding that has been allocated to this sector provides an opportunity in Ireland for addressing and removing barriers to the process of empowerment. In order to progress the ABE system to a more integrated model of empowerment; the authors assertion based on their research is that:

- All the elements of an effective ABE system must individually and collectively engender empowerment,
- There is a need for a systematic and democratic exploration of the nature of numeracy. Government policy and priorities must recognise that numeracy is not a single concept that can be incorporated within literacy or be strongly guided by the school mathematics curriculum,
- Practitioners need to be provided with the tools they require to become *'a highly trained corps of practitioners who are dynamic and equipped to lead change, to play a key role in policy debate and to reflect the distinctive identity of the sector in the field of professional practice and research.'*
- There must be a proactive approach to targeting specific groups of adult learners who must be encouraged to engage and develop confidence in the ABE system, that is an integral part of a national framework of lifelong learning.

Acknowledgements

The authors would like to thank Dr. Judy Ward, University of Arkansas, for her advice and comments on the design of the research instrument.

References

Bailey, I. and U. Coleman (1997). *Access and Participation in Adult Literacy Schemes*. Dublin: National Adult Literacy Agency.

Benn, R. (1997). *Adults Count Too: Mathematics for Empowerment*, Leicester: The National Organization for Adult Learning (NIACE).

Coben and N. Chanda (eds.) (2000).'Teaching 'Not less than mathematics, but more'; an overview of recent developments in adult numeracy teacher development in England - with a sidelong glance at Australia'. In: *Perspectives on Adults Learning Mathematics: Research and Practice*. Dordrecht: Kluwer Academic Publishers, 307-327.

Cooney, T. (1999). 'Conceptualizing Teachers' Way of Knowing'. *Educational Studies in Mathematics, Vol. 38*, 163-187.
Cumming, J. (1995). *Adult Numeracy Policy and Research in Australia - the Present Context & Future Directions*. Department of Employment, Education, Training and Youth Affairs, Australia
Department of Education and Science (1999). *Qualifications (Education and Training) Bill 1999.* Dublin: Stationery Office.
Department of Education and Science (2000). *Learning For Life - White Paper on Adult Education*. Dublin: Stationery Office.
Department of Finance (2000). *National Development Plan 2000-2006*. Dublin: Stationery Office.
Department of the Taoiseach (2000). *Programme for Prosperity and Fairness*. Dublin: Stationery Office.
Farrell, J. (2000). E-mail communication, Department of Education and Science, Subject: Numeracy; received 23rd May 2001.
Morgan, M., B. Hickey and R. Kellaghan (1997). International Adult Literacy Survey: Results for Ireland. Dublin: Educational Research Centre.
National Adult Literacy Agency (NALA) (1999). *The National Adult Literacy Agency, Strategic Plan 1999-2001.* Dublin: National Adult Literacy Agency.
OECD (1997). *Literacy Skills for the Knowledge Society*. Paris: OCED.
Persico, J. (1991). *Leadership and Empowerment in a Total Quality Organization, Total Quality Management, Vol. 2(1)*, 57-61.
Wagner, D. (2000). *EFA 2000 Thematics Study on Literacy and Adult Education.* International Literacy Institute, University of Pennsylvania-UNESCO.

Adult Ways of Knowing: a Summary of Perspectives from Five Research Groups

Katherine Safford
St. Peter's College, New Jersey, USA

Introduction

It has been ten years since I began my journey from directive to constructivist mathematics professor. While that journey is by no means completed, it is probably safe to claim that student-teacher dialogue in my classes now resembles the latter more than the former style. When I think about my role within the classroom, I consider it to be that of facilitator of student learning rather than transmitter of knowledge. Despite this personal shift, I continue to struggle with the dilemma of students who expect, who wait for, direction and demonstration of problem solutions that can be copied into notebooks and memorized for reproduction on examinations.

The majority of my students at St. Peter's College are borderline adults, eighteen or nineteen years old. In the United States, many in the adult education research community would not consider them adults at all, the threshold being 25 years of age. However, they are adult when viewed through legal, sociological, and psychological lenses. The search for explanations of the strong student resistance to building and owning a personal mathematics knowledge base led me to investigate research on student, and teacher, perspectives on learning. This paper presents some of the results of that search. Most of the studies presented are not specific to mathematics but do offer insight into the transformation of perspective that occurs during the college years. Some critics may argue that generalization from a small sample gathered from diverse populations over forty years is incoherent and worthless. In response, I suggest that the common themes that emerge when the five studies are compared provide explanations for dynamics within my classroom and, perhaps, yours.

William G. Perry, Jr.

In the late 1950's and early 1960's, William Perry headed a research project into the intellectual and ethical development of college students at Harvard University. The study followed two cohorts of students through their four years at Harvard. While Perry presents the subjects as culturally diverse, they were in fact white males from middle or upper class homes (Perry, 1968:6). The students were selected from two random samples of freshmen who had completed *A Checklist of Educational Views (CLEV)*, an instrument designed by the Perry research team. A total of 140 students volunteered to be interviewed, 84 participated for the full four years.

Analysis of the interviews identified a general progression from a strong belief in a duality of "right versus wrong" in the freshmen year to acceptance of relativism and the effect of multiple responsibilities by the end of senior year. For some of the participants this change had been fitful and included a certain amount of regression but, in general,

by graduation time the interviewees were at or near the position termed "Position 9" by Perry. Table 1 details the evolution that emerged from Perry's study.

Major Parts	Positions
The modifying Of dualism	Basic duality
	Diversity of opinion seen as unwarranted confusion
	Diversity and uncertainty are legitimate but temporary
The realizing Of relativism	Legitimate uncertainty accepted as extensive
	All knowledge and values are contextual and relativistic
	Need for personal commitment within a relativistic world
The evolving of commitments	Initial commitment in some area
	Explores issues of responsibility
	Affirmation of identity among multiple responsibilities

Table 1. Perry's Development Scheme

Carol Gilligan

In the 1970's, Carol Gilligan worked with Lawrence Kohlberg on questions of moral development. In the course of that work, she came to question the appropriateness of generalizing schemas of human development from studies conducted with male subjects.

She argues that "within the context of U.S. society, the values of separation, independence, and autonomy are so historically grounded …and so deeply rooted in the natural rights tradition that they are often taken as facts (Gilligan, 1993:xiv-xv)." When development is measured by male standards, "the quality of embeddedness in social interaction and personal relationships that characterizes women's lives in contrast to men's, however, becomes not only a descriptive difference but also a developmental liability when the milestones of…development …are markers of increasing separation (Gilligan, 1993:9)."

In her pivotal book *In a Different Voice,* Carol Gilligan synthesizes the results of interviews from three research studies that queried women about their "conceptions of self and morality (Gilligan, 1993:2)." Repeatedly, the subjects speak of morality embedded in relationships regardless of their level of maturity. She reports that "in the different voice of women lies the truth of an ethic of care, the tie between relationship and responsibility, and the origins of aggression in the failure of connection…by positing…two different modes, we arrive at a more complex rendition of human experience which sees the truth of separation and attachment in the lives of women and

men and recognizes how these truths are carried by different modes of language and thought (Gilligan, 1993:173-174)."

While Gilligan did not address intellectual development, her work opened the door to later studies that examined female perceptions of knowledge. The issue of "voice", raised by Gilligan, surfaces in later research on knowing and reasoning by Belenky, Clinchy, Goldberger, and Tarule and by Magolda.

Mary Field Belenky, Blythe McVicker Clinchy, Nancy Rule Goldberger, and Jill Mattuck Tarule

Belenky, Clinchy, Goldberger, and Tarule sought to utilize Perry's open interviews to explore the experiences and problems of women learners as they acquired knowledge. The research they report in *Women's Ways of Knowing: the Development of Self, Voice, and Mind* is based on interviews, repeated in many cases, with 135 women from formal academic institutions and from what they termed "invisible colleges," human services agencies that advise women about problems encountered in parenting children (1986, Belenky, Clinchy, Goldberger, & Tarule, 1986:12). They were diverse in age, ethnicity, vocation, and locale.

The analysis of the interviews resulted in a five-stage progression of knowledge perspectives. At the lowest, saddest level were the women who saw themselves as mindless and voiceless. At the highest category, the women interviewed saw all knowledge as contextual and themselves as creators of knowledge, not receptors of wisdom from some external source (Belenky, et al., 1986:15).

Category	Description
Silence	Mindless, voiceless, subject to whims of external authority
Received knowledge	Capable of receiving and reproducing knowledge from external authority but not creating knowledge on own
Subjective knowledge	Truth and knowledge are conceived of as personal, private, and subjectively known or intuited
Procedural knowledge	Women are invested in learning and applying objective procedures for obtaining and communicating knowledge
Constructed knowledge	Women view all knowledge as contextual, experience themselves as creators of knowledge, and value both subjective and objective strategies for knowing

Table 2. Categories of Knowledge according to Belenky, Clinchy, Goldberger, and Tarule

Marcia Baxter Magolda

Marcia Baxter Magolda encountered the dilemma of fitting Perry's positions to female experience when she attempted to develop a ratings manual for identifying students' ways of knowing (Baxter Magolda, 1992:6). When studied through the gender lens, she realized that there were indeed similarities across gender as well as important differences. This realization sparked a five-year longitudinal study with hopes to clarify

the distinctions. Like Perry and Belenky, Clinchy, Goldberger, and Tarule, she chose to use open-ended interviews to gather data that would then be analyzed qualitatively to identify categories and themes (Baxter Magolda, 1992:9). One hundred and one traditional age students, half of them males, participated in the study that was conducted at a public university. Seventy remained in the study for the five years. The structure that evolved from the study consists of four patterns of knowing analogous to those of Perry and Belenky except that the lowest Belenky level, Silence, does not appear. This is perhaps due to the educational level of the subjects. The silent, voiceless individuals in the earlier study were not college students. Table 3 summarizes the patterns that emerged from the study.

Pattern of knowing	**Characteristics of learner and knowledge**
Absolute	Learner obtains knowledge from instructor Knowledge is certain or absolute Women: Knowledge is received Men: Knowledge must be mastered
Transitional	Learner understands knowledge Knowledge is partially certain and partially uncertain Women: Inter-personal acquisition of knowledge Men: Impersonal formation of knowledge through debate
Independent	Learner thinks for self, shares views with others, creates own perspective Knowledge is uncertain and personal Women: Inter-individual focus on thinking for oneself while considering the views of others Men: Individual focus on thinking independently, forming one's own learning goals
Contextual	Learner exchanges and compares perspectives, thinks through problems, integrates and applies knowledge Knowledge is contextual Gender differences converge

Table 3. Baxter Magolda Patterns of Knowing

Daniel D. Pratt

The previous four studies looked at knowledge acquisition from the perspective of the student. Daniel Pratt offers a theory that looks at the perspectives of teachers of adults, the individuals who are in the power position in the classroom setting. His study is not clearly defined but consisted of studying "253 teachers of adults, trying to understand what teaching means across vastly different settings. Each teacher was asked what it meant 'to teach' (Pratt, 1998:xii)." In the analysis of the findings, Pratt describes beliefs about knowledge and learning.

Pratt found that the teachers held two differing views of knowledge. The first, which he termed "objective," was the belief that knowledge existed externally. The task of the student is discovery or mastery of a body of knowledge that is waiting to be studied.

The learner and the subject remain distinct from each other; in fact, the ability to speak rationally and objectively about subject matter is commended. "A statement is true when it corresponds to reality as empirically validated, and false when it does not. In other words, truth is a matter of the accuracy of reproduction…of reality as judged by some authority (Pratt, 1998:23)."

Subjective knowledge, on the other hand, is intimately connected to learner experiences and beliefs. Learners need not, indeed cannot, separate the new information from prior knowledge, beliefs, or values. Pratt describes subjectivism in this way, "There can be no value-free observations; what counts as data, that is, what we then report from our observations, is influenced by the interests, purposes, and social practices of those doing the observing (Pratt, 1998:25)." All knowledge results from new experiences being perceived, modified, and absorbed into previous knowledge thus altering or extending the personal knowledge base. Pratt asserts that, while these are polar positions, most teachers fall somewhere on the continuum between the extreme views.

Perspective on teaching	**Description**
Transmission	A relatively stable body of knowledge and/or procedures can be transmitted to the learner
Apprenticeship	Teaching is the process of enculturating learners into a specific community
Developmental	Learning involves revising or replacing existing cognitive structures to incorporate new knowledge
Nurturing	Teaching effects a change in learner self-concept and self-efficacy
Social reform	Learners and content are secondary to a political, social, moral, or religious ideal. Emphasis is on the collective rather than the individual.

Table 4. Pratt's Perspectives on Teaching

In addition to a view of knowledge, Pratt found that the teachers held one of five perspectives on the act of teaching itself. He defined perspective as "an interrelated set of beliefs and intentions which give meaning and justification for our actions (Pratt, 1998: 33)." Every teacher could be described as a cross product of his or her view of knowledge and perspective on teaching, although one could hold more than one perspective. Speaking in terms of mathematics, a teacher could believe that the subject matter was an objective set of principles and procedures that could be transmitted to a student by a skilled teacher. At the same time, that teacher might envision her students as newcomers to mathematics, immigrants who need to be enculturated in the content and procedures of the mathematics community. Table 4 describes the five perspectives on teaching that emerged from his interviews.

Conclusion

The findings of the five studies offered answers to some of my questions. The behavior of the eighteen to nineteen year old students in my mathematics classes can be

explained as typical of peers interviewed by Perry and Baxter Magolda. Teachers who perceived the subject matter as objective material whose knowledge can be transmitted to students may have reinforced this developmental stage. Teachers and students may view knowledge as objective. This is a particular danger in mathematics, a discipline that has a history of valuing rational inquiry and logical proof.

New questions arose, however, to take the place of the old ones. Is it possible for older students to perform at multiple levels? For instance, might an otherwise contextual adult persist in viewing mathematics as an absolute discipline? Can adult students be invited to join the mathematics community by an instructor who takes an apprenticeship or developmental perspective in their teaching? What are the implications of this research for numeracy instructors? And finally, how can I structure my mathematics classes so that I can support students as they begin the transition from a dualist to a relativist position? These questions will influence the agenda for my research, and I hope yours, over the coming years.

Literature

Belenky, M.F., Clinchy, B.M., Goldberger, N.R., and Tarule, J.M. (1986). *Women's ways of knowing: the development of self, voice, and mind.* New York: Basic Books, Inc.

Gilligan, C. (1993). *In a different voice.* Cambridge, MA: Harvard University Press.

Magolda, M.B. (1992). *Knowing and reasoning in college: gender-related patterns in students' intellectual development.* San Francisco: Jossey-Bass.

Perry, W.G., Jr. (1970). *Forms of intellectual and ethical development in the college years.* New York: Holt, Rinehart, Winston.

Pratt, D.D. and Associates (1998). *Five perspectives on teaching in adult and higher education.* Malabar, FL: Krieger Publishing.

Mathematics and Society - Must all People Learn Mathematics?

Wolfgang Schlöglmann
Universität Linz, Austria

1. Mathematics as a tool – origins

Numerical symbols were used more than 30,000 years ago (early stone age = Paläolitikum). But for thousands of years, the sophistication of these signs did not increase. The nomadic way of life during that era was closely bound with nature, and there was no need for more mathematics (Struik, 1967). The development of mathematics took a great leap forward following the emergence of settlements in Mesopotamia at about 10,000 BC. Agriculture demanded planning and documentation. Trade between villages contributed to this demand. Small artifacts made of clay, so-called "tokens", were used to count and document goods. The system of tokens evolved into a hierarchical system with various subsystems according to local commercial demands. Arithmetic appeared in a primitive form (Schmandt-Besserat, 1977).

The next leap occurred after the formation of cities. These bigger societal units created a demand for new and better organizational methods. Division of labour and the demand for a growing economy required new planning methods as well as the standardization of quantitative measurements for the different kinds of goods. The first systems of writing and numerical symbols emerged in Mesopotamia and Egypt. It is important to note that a single, unique number system did not exist at that time.

According to the system of tokens, there were different systems with different bases for different objects (people, animals, grain, areas, time and calendar, milk products etc.). We can deduce that an abstract conception of numbers did not exist, nor was there a well developed arithmetic. Arithmetical operations were strongly related to operating with clay tokens (Nissen et al., 1991). The development of a unique number system and arithmetic was the result of a longer process. At the end of this process, mathematics was a tool for

* documentation
* planning
* ordering
* organisation of trade
* many professions
* astronomical calculations

With its emergence as a tool for practical problems, mathematics had an impact on society. In various ways, mathematical thinking penetrated society. The validity of formulas and procedures was proven by passing the test of real-world practice.

2. What kind of society needs the tool of mathematics?

Demands created by changing forms of society required new means for documentation, planning, trade etc. The solution in all cases was the development of some kind of mathematical objects, such as a system of tokens, or, later, systems of written signs for numbers.

Is the tool of mathematics necessary in a special kind of society? To discuss this problem, I use a concept introduced by the philosopher, Peter Heintel (Heintel, 1992), according to which societies are classified depending on the kind of communication occurring within them.

Small groups can be organized by direct communication. All information is transmitted from person to person. No other mediation channels are necessary. Learning processes are also organized directly. Such societies try to live by adapting to nature, need few artifacts and have no demand for planning and documentation. They react to the demands of nature and don't plan for the future. Many such groups have few words for numbers, and natural objects like stones suffice to document a counting process.

In larger societies, such direct communication between all members of the community is impossible. The society changes from a society organized by direct communication to one organized by indirect communication. Such an organizational form demands common standards, values etc. Such societies therefore need a more sophisticated language because the standards and values must be formulated in new, more abstract concepts. In larger societies, labour processes are mostly specialized and often organized in a hierarchical way. Specialisation and division of labour demand more planning and more documentation to organize labour in a rational way. To fulfill these demands, there is a need for a means of communicating advice, regulations, as well as various skills, in a more abstract way. Written documents are able to reach more people. If not all members of a society are able to read written documents, the people who can read hold higher positions.

In summary, mathematics is a tool that fulfills the demands of societies organized by indirect communication. Mathematical terms allow documentation through mathematical procedures, and reliable calculations in the context of labour and trade processes become possible. All processes in such societies evolve from the particular to the general. Mathematics, too, has developed like this. It first appeared in forms whose terms and procedures depended strongly on context, and evolved into a general tool for many problems.

3. Mathematics and culture

Many important developments in mathematics occurred in ancient Greece. While mathematics was developed as a tool for solving practical problems in the Orient, it was the Pythagoreans - the first group of mathematicians – who first used mathematics as a means for understanding the laws of the world (van der Waerden, 1979). In their philosophy, the laws of the world were written in mathematical language. Just as musical harmony is based on proportions of numbers, the harmony of the universe is based on mathematics. To them, studying mathematics was a way of discovering the eternal laws of the universe. Therefore, mathematical objects like numbers and geometrical figures became objects of interest. The aim of mathematical development

amongst the Pythagoreans was not to solve practical problems but to discover mathematical laws.

New, systematic ways of defining mathematical objects and of justification were necessary. Real-world applicability was not the main justification for the correctness of a mathematical algorithm: only proofs that followed the laws of logic were acceptable. Only human reason was able to discover eternal truth. Mathematics was a means for applying reason. The result of this new philosophy was a close relationship between mathematics, logic and truth. The eternal laws of the world are written in mathematical language, and studying mathematics was a way of discovering those eternal laws. Mathematical thinking and rational thinking were closely related (Pichot, 1995). Mathematical, systematic representation became a model for scientific representation.

In my opinion, the most important impact of the work of the Pythagoreans on the further development of mathematics was the separation of mathematical terms and their real representation. This separation alone led to the development of mathematics as a means for rational thinking and a general means of communication. As a basis for rationality, mathematics became a part of the culture of societies that have rationality as their societal foundation.

4. Mathematics and democracy

The roots of modern democracy lie in the ideas of the Enlightenment. Rationality is fundamentally important for the structure of society. Mathematics is an important background for rationality. Democratic societies are organized by indirect communication. Therefore, democratic societies have a need for, on the one hand, values and principles, and on the other hand, procedures that implement values and principles, that facilitate democratic practice by communicating the principles in an unequivocal and reliable way.

Mathematics fulfills these conditions. For example, consider the democratic election procedure. The election procedure has to implement the principle of same value per vote. This basic principle is often combined with other principles, such as the principle that the result of the election should lead to a stable majority in parliament. Taking into account particular historical features of a society, the democratic system has many instantiations – many different election procedures are possible. But every procedure is formulated as a mathematical algorithm.

In recent political discussion, a new interpretation of equality has been introduced, according to which equality should not only be a principle that is valid on the individual level, it should also be valid for social groups. This means that within a democracy, each social group should be represented as an important subsystem of society, with its own appropriate societal share. This principle, too, is based on mathematics: on statistical arguments. Statistical methods are important for analyzing societies (Frankenstein, 1990; Evans/Rappaport, 1998).

In discussing the relationship of mathematics to democracy, one should say something about the status of mathematical models that underlie various aspects of democracy. These models are based on norms, and these norms are the consequence of a social

process. Norm-based models, which are used in many applications too (e.g. in business), are not justifiable by mathematics. Mathematics is only used to guarantee a reliable implementation of a norm; but mathematics is not able to guarantee the correctness of the norm.

These short examples demonstrate that a democracy needs mathematical methods to fulfill its principles.

5. Mathematics and new technologies - new function of the tool of mathematics

In the above sections, it was pointed out that societal demands led to the development of mathematical methods and procedures. Mathematics affected structures and conditions in society. Trade and administration, in particular – with the extensive use of money - stimulated the search for better methods, and mathematics was applied extensively in these areas. Later, mathematical methods were used in astronomical calculations, and more and more professions began to use mathematics.

The new status of mathematics in Antiquity led to a relationship between mathematics and rationality, as well as to research in "pure" mathematics. In particular, the separation of mathematics and its applications opened the way to the development of mathematics as a "general tool" that could be used in the sciences and numerous other fields. The use of mathematics in more and more areas of social, economic and professional life increased. In many cases, mathematics became integrated into a field to such an extent that it became a standard fixture in the background, while losing its visibility in the foreground.

There have been great changes in recent years. With the dawn of new technologies, mathematics began to play a new role within the society. I would like to begin a description of this new role with a quotation by Edward E. David, from a report for the National Research Council of USA on the resource "Mathematics".

> "When we entered the era of high technology, we entered the era of the mathematical technology." (David, 1984: 435)

That quotation expresses the following facts:

* Mathematics is the basis of all new technologies, since algorithms are the basis of software, and materialized mathematical logic is the basis of computer hardware (microprocessors).

* Mathematical theories and models are becoming increasingly important as the basis of a variety of forward-looking alternatives, in simulating planning in economic and technical fields, for example in control, automation and construction; or in political and social life.

* Mathematics has long been established as the scientific core of the natural sciences and, to an increasing extent, also of social science (Maaß/Schöglmann, 1988).

Highly industrialized countries are characterized by the use of their technologies because technology determines the structure of society (the philosopher Heinz Hülsmann uses the term "technological formation" (Hülsmann, 1985)). Many sociologists call our age "the era of information technology" - and this technology could not exist without mathematics (Schlöglmann, 1992). An important necessity of technology is that it be able to facilitate the distinction between the development of knowledge and the application of knowledge - briefly put, the "why" from the "how". The user of knowledge needs an understanding not only of how to apply the knowledge, but also of what the knowledge really is in the first place. We all use computer programs, but we don't know what goes on behind the scenes in the computer box. Mathematical "black boxes" have a long tradition (Maaß/Schlöglmann, 1988), but with the rapid ascent of the computer, their use has increased super-exponentially. The increased use of mathematical black-boxes has contributed to the "disappearance of mathematics"; on the other hand, these black boxes determine our work processes, economic and social life.

6. Why are mathematical methods so credible?

The position of mathematics in our society is rather paradoxical. On the one hand, many people see mathematics as abstract, remote from the life, incomprehensible. On the other hand, the same people have full confidence in mathematical methods - they pay invoices, accept calculation of election results, accept the use of complex mathematics in technology and the economy. Roland Fischer (1999) notes that: "Mathematics is the materialization of the abstract."

That means mathematics puts abstract thought into a concrete form. Mathematical objects are abstract; but their concrete form, consisting of numerical symbols, formulas, graphs, makes manipulation possible. For example, we use graphs to present in a concrete way complex, abstract relationships. This specific characteristic of mathematics – construction of abstract concepts on the basis of concrete representations - could be the reason why many people have confidence in results obtainded through mathematical methods.

7. Must all people learn mathematics?

I hope my lecture has given an insight into the various functions of mathematics within our society. Let me summarize why mathematics ought to be part of all school curricula. These two arguments resonate with the conference theme, "Numeracy for Empowerment and Democracy", and with my two main points: mathematics is a tool; and mathematics is an integral part of culture.

1. Mathematics in our society is a tool used to organize our everyday life. Mathematics is also used as a tool in many occupations. The use of mathematical black boxes in the form of computer programs gives them a new quality and a new challange for didactics. "Numeracy for Empowerment" calls our attention to these functions of mathematics and to the problem of mathematics education.

2. Mathematics is a part of our culture. "Numeracy for Democracy" stresses this function of mathematics. Democratic principles such as equality, righteousness and so on need an operational concretization. On the one hand, democracy demands a means

for communicating and discussing principles in a rational way. Mathematics, with its close relationship to rationality, is our concept to do this. On the other hand, democracy demands operational procedures for its concrete implementation. Mathematics is again the tool that facilitates this.

The concept of the "responsible citizen", a citizen who is able to participate in societal processes in a rational way, is part of numerous educational philosophies. At the University of Roskilde, in particular, much has been done – both in research and in teaching - to implement this principle (Mogens Niss, Bernhelm Booß-Bavnbeck, Morten Blomhøj and many others).

Literature

David, E.E. (1984). ´Renewing U.S. Mathematics: Critical Resource for the Future.` *Notices of the Americam Mathematical Society*, **31(5)**: 435-466.

Evans, J./Rappaport, I. (1998). ´Using Statistics in Everyday Life: From Barefoot Statisticians to Critical Citizenship`. In D. Dorling/S. Simpson (Eds.), *Statistics in Society: The Arithmetic of Politics* (pp 71-77). London: Arnold.

Fischer, R. (1999). ´Technologie, Mathematik und Bewußtsein der Gesellschaft`. In G.Kadunz u.a. (Hrsg.), *Mathematische Bildung und neue Technologien* (pp. 85-102). Stuttgart-Leipzig: B.G.Teubner.

Frankenstein, M. (1990). ´Incorporating Race, Gender, and Class Issue into a Critical Mathematics Literacy Curriculum.` *Journal of Negro Education*, **59(3)**: 336-347.

Heintel, P. (1992). ´Skizzen zur "Technologischen Formation"`. In W.Blumberger/ D.Nemeth (Hrg.), *Der Technologische Imperativ* (pp. 267-308).München-Wien: Profil.

Hülsmann, H. (1985). *Die technologische Formation.* Berlin: Verlag Europäische Perspektiven.

Maaß, J./Schlöglmann, W. (1988). ´The Mathematical World in the Black Box-Significance of the Black Box as a Medium of Mathematizing.` *Cybernetics and Systems: An International Journal,* **19**, 295-309.

Nissen, H.J. et al. (1991). *Frühe Schrift und Techniken der Wirtschaftsverwaltung im alten Vorderen Orient.* Bad Salzdetfurth: Franzbecker.

Pichot, A. (1995). *Die Geburt der Wissenschaft.* Darmstadt: Wissenschaftliche Buchgesellschaft.

Schlöglmann, W. (1992). ´Mathematik als Technologie`. In W.Blumberger/D.Nemeth (Hrg.), *Der Technologische Imperativ* (pp. 189-198). München-Wien: Profil.

Schmandt-Besserat, D. (1977). ´An Archaic Recording System and he Origin of Writing.` *Syro-Mesopotamian Studies,* **1**: 31-70.

Struik, D.J. (1967). *Abriss der Geschichte der Mathematik*.. Berlin: VEB Deutscher Verlag der Wissenschaften.

Van der Waerden, B. (1979). *Die Pythagoreer*. Zürich-München: Artemis.

Basic Skills Strategy – a practitioner perspective

Valerie Seabright
Uxbridge College, United Kingdom

As recently employed Basic Skills and learning Support manager at a large Further Education College on the outskirts of London I have the remit to develop with colleagues, the new Basic Skills Initiative introduced this year in the UK

The Moser report in the UK, discussed at last year's ALM conference has been used as a basis to develop a new curriculum to improve the numeracy and literacy levels of adults in the UK. It has been introduced with intentions of developing both the standards of literacy and numeracy and incorporates the improvement in training of teachers to deliver and include basic skills in many vocational programmes.

My presentation would outline the Moser Report background and political implications of the strategy and from a practitioner's point of view the impact in my own particular workplace. The rigid imposition of testing, insufficient training of staff, limited funding in an already over complex adult and further education system and limitations of the curriculum itself will adversely affect the ability of individuals to improve levels of literacy and numeracy to be empowered in today's technological society.

I would envisage some of the discussion during the presentation to include study of the actual numeracy curriculum, assessment and delivery.

Mathematics Teaching for the Prevention and Reduction of Problem Gambling

Donald Smith
School of Education, Victoria University of Technology, Australia.

A Personal Note: After some years as an adult educator working in language, literacy and numeracy, I have recently become a research student and part-time educator of trainee teachers. The encouragement of colleagues from many lands at international mathematics education conferences such as ALM, ANN, NCTM or ICME, has been important in stimulating my contribution to the broad field of mathematics education. Thank you for that continuing encouragement.

Introduction

These are some questions I'd like us to address together:

- *Is gambling an increasing social problem?*
- *What might be a role for maths educators in relation to problem gambling?*

This paper will identify issues that require further work. It is a first attempt to outline the beginnings of responses. The aim is to discuss the role of mathematics teaching for understanding the pitfalls of commercial gambling, particularly electronic gaming machines (EGMs). After giving a general context, research on chronic gamblers will be considered briefly, before turning to some details of appropriate teaching programs.

Chronic gamblers, particularly players of electronic gaming machines, may have poor mathematical understanding (Griffiths, 1999). Treatment of problem gamblers has concentrated on psychological aspects other than mathematical understandings. Typically, mathematics teaching about statistical distributions relevant to understanding long term gaming outcomes is accessed a long way up the hierarchy of mathematical knowledge, by specialised students, not by general learners (Begg, 1997). Specifically designed mathematics education programs are needed to communicate understanding of the long-term outcomes of gambling on unfair games. This paper continues the broader theme presented at ALM7 (Smith, 2000), that we need to develop teaching of the consequences or significance, of some more advanced mathematical knowledge, to lower level learners.

Gambling: An increasing social problem?

In many countries gambling has become an important social problem; certainly in Australia, where there has long been a strong gambling culture. The recent explosion of gambling as a social problem in the region I am from, Victoria, came about with the opening of our first casino almost a decade ago, but more significantly with the introduction of electronic gaming machines, which we call 'pokies', in bars and clubs spread throughout the suburbs and country towns. However the spread is uneven with greater concentration of machines occurring, apart from the central city, in less wealthy areas (Doughney & Kelleher, 1999).

In some other countries, particularly the United States and Canada, electronic gaming machines are also well established and problem gambling is a growing social concern. Australian gaming venues differ from ones in England, where the size of prizes is quite limited, and where the machines accept coins rather than larger currency notes. However, the British Government's review of the Gambling Act (1968) may lead to a liberalisation of gambling restrictions. Perhaps one of the most significant would be the introduction of linked jackpots, in which a very large jackpot is available right across the network of machines, allowing big-win incentives.

Gambling in Australia. (A humorous interlude?)
In recent years Australians have enjoyed thinking of themselves as world champions, particularly at sports. Traditionally gambling has had strong associations with sport and alcohol. Australians tried for a long time, but failed, to outdrink the Germans in beer per capita, and at the Olympics couldn't outswim the Dutch champions. Australia does have more sheep than New Zealand, but the NZ cousins have no other claim to fame so we'll leave them that. Unfortunately, Australians can claim to be the world's biggest (and hence most unsuccessful) commercial gamblers (Costello & Millar, 2000; Peard, 1995). Indeed, that's a very Aussie sort of national achievement. Something to make a larrikin proud.

Bettors in Australia lost a net AUD$14.5 billion in the last financial year (Ellicott, 2001), up from $12.4 billion in 1999 (Costello & Millar, 2000). About 80% of Australians participate in gambling activity each year, half of them in raffles and 30% play the pokies (Roy Morgan Research, 2000). The Government's Productivity Commission (1999) found that around 2% of adults have gambling problems. Problem gamblers make up at least one third of the expenditure on EGMs, and about 10% of Aussie pokie machine players are problem gamblers. Australia, with a population of around 20 million, is said to have about one-fifth of the world's modern EGMs (Costello & Millar, 2000).

Gambling: A role for maths educators?
Being mathematics educators we may be predisposed to believe that mathematics education is very relevant to this problem, but problem gambling can be resistant to the glory of mathematical revelation of its futility. Besides the many non-mathematical factors contributing to individual gambling behaviour (including physiological dependencies), mistaken intuitions about probability often survive considerable contradictory evidence (Tversky & Kahneman, 1982). Probability concepts have been difficult to teach successfully (Garfield & Ahlgren, 1988). The task for maths educators is to develop clear, succinct and convincing explanations.

Without understating the significance of factors other than mathematics knowledge, affecting gambling behaviour, a comparison may be made with cigarette smoking, where physiological dependency is strong. When people give up smoking, ultimately they do so as a result of cognitive choice. Cognitive comprehension of their situation precedes action to overcome the addiction. To the extent that problem gamblers do not possess appropriate cognitive information about gambling they are inhibited from making informed choices. Problem gamblers do not comprehend the extent to which

EGMs are a long-run losing game. Illusions of control of random processes (Langer, 1975), a key correlate of problem gambling (Moore & Ohtsuka, 1999), suggest a basic lack of genuine understanding of randomness and independence of random events.

Psychological research on problem gambling

If we inquire as to the nature of pathological, chronic or problem gambling, (terms that overlap), we find that psychologists have done a lot of research over the past quarter century. Characterisation of problem gamblers has concentrated on psychological aspects other than mathematical understandings. Typically, the research has not sought explanations, nor treatment, in terms of mathematical understandings. A substantial study in Texas (Wallisch, 1996) typifies the non-mathematical perspectives which have been brought to the gambling problem. Rugle & Melamed (1993) report that

> "the characteristics of pathological gamblers that seem to recur conspicuously in the research are high energy level, impulsivity, low stress and frustration tolerance, poor judgment, difficulty learning from experience, labile effect, poor planning and goal setting, opposition to authority, underpinnings of inadequacy and insecurity, and shallow relationships." (1993:107)

That mathematical understanding might be more relevant to mathematically analysable games than are other psychological constructs, such as opposition to authority, may have escaped the attention of many researchers.

Fallacies

Within a cognitive approach, Rogers (1998) outlines the main theories of gambling as related to non-pathological lottery play. Various biases and irrational thinking patterns are found in lottery gambling. They include misunderstanding of lottery odds, a susceptibility to the gambler's fallacy and cognitive entrapment, a belief in hot and cold numbers, unrealistic optimism, a belief in personal luck, superstitious thinking, the illusion of control, the erroneous perception of near misses, a susceptibility to prize size and rollover effects, the framing of gambling outcomes and finally, the influence of social factors on lottery play. He suggests that the psychology of lottery play needs a more unified theory, largely cognitive in emphasis. Drawing on the available research literature about cognitive distortions present during gambling, Toneatto (1999) has also presented a typology of gambling-relevant cognitive distortions. These include

> "magnification of gambling skills, minimization of other gambler's skills, superstitious beliefs (including talismanic, behavioral, and cognitive superstitions), interpretive biases (including internal attributions, external attributions, gambler's fallacy, chasing, anthropomorphism, reframed losses, hindsight bias), temporal telescoping, selective memory, predictive skill, illusion of control over luck…and illusory correlation." (1999:1593)

There has also been a considerable amount of research into subjective perceptions of randomness (see Falk (1992), for a survey). Systematic biases have consistently been found. People tend to reject sequences with long runs of the same result (such as a long sequence of heads), and they consider sequences with an excess of alternation of different results to be random. In the case of two-dimensional tasks, clusters of points

seem to prevent a distribution from being perceived as random (Batanero & Serrano, 1999).

Fischbein and Schnarch (1997) used a questionnaire with seven probability problems to examine probabilistic misconceptions. The questions related to representativeness, negative and positive recency effects, compound and simple events, the conjunction fallacy, effect of sample size, the heuristic of availability, and the effect of the time axis.

In the spirit of the conference (empowerment), I suggest that research which just identifies fallacies would not be a contribution to empowerment unless it was accompanied by, or leads to, teaching to improve understandings. Research into negative circumstances, which does not contribute directly to changing those circumstances, is unworthy, and probably an offence against the subjects. Error analysis of the above fallacies suggests that clear understanding of a limited number of key mathematical concepts cuts through them all. These concepts include fair game, net loss game, average expected result or long run effect of house margin, effect of the law of large numbers on the average result, randomness, independence, and variability (causes of Variance). These key concepts are explained below.

Attempts at teaching about gambling

In recent decades there has been increasing attention in Anglophone nations to the teaching of probability and statistics. Statistics education should be more important than it is (Reading & Pegg, 1995). "Probability and statistics have not historically held a significant place in the high school mathematics curriculum" (Watson, 1995:120). They are relatively recent additions to the primary and secondary curricula, becoming common during the 1960's, but "there is not a well-established received wisdom about what constitutes good classroom practice." (Truran & Truran, 1995:346). Paulos (1988) espoused this developing importance:

> "Statistical tests and confidence intervals, the difference between cause and correlation, conditional probability, independence, and the multiplication principle, the art of estimating and the design of experiments, the notion of expected value and of a probability distribution, as well as the most common examples and counter-examples of all of the above, should be much more widely known. Probability, like logic, is not just for the mathematicians anymore." (1988:134)

Very basic gambling games have long been used instructionally in classrooms. Biased dice have been used to informally introduce sophisticated mathematical understanding of concepts of chance (Smith, 1997). Others have used horse racing and roulette activities (Mangan, 1994), sometimes without regard to whether introduction of standard gambling games might increase propensity for commercial gaming by increasing familiarity.

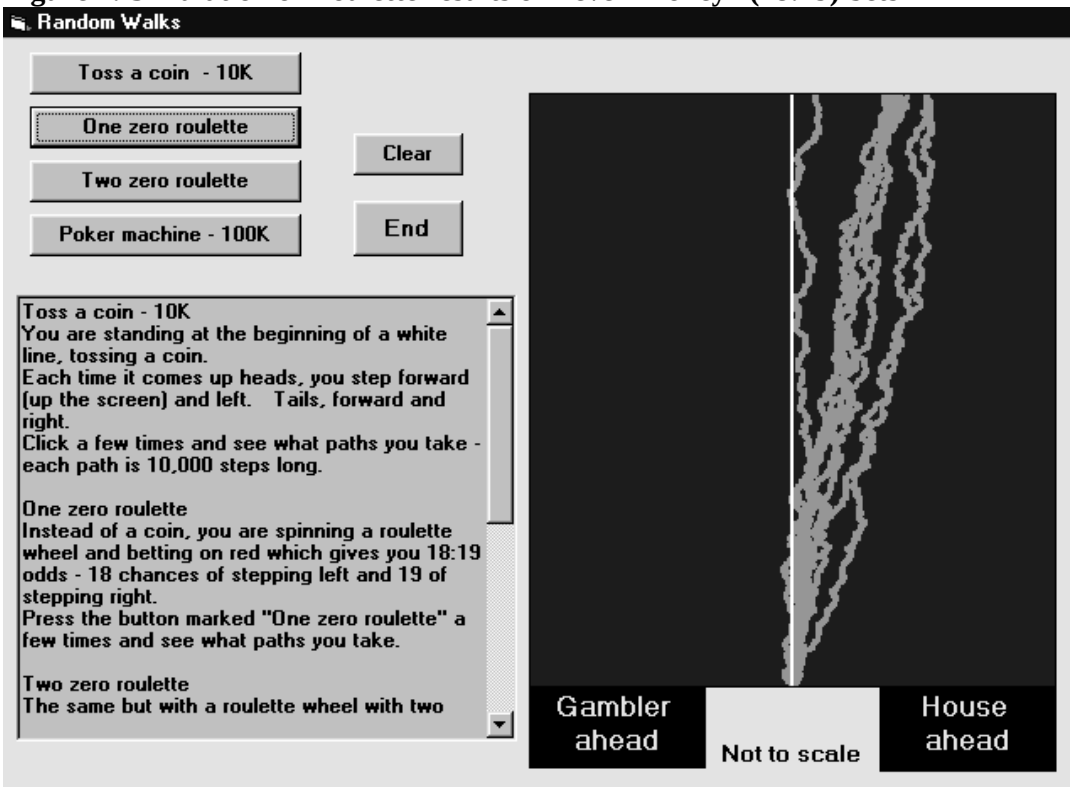
Many teachers have used MSExcel to produce basic gambling simulations. For example, Walpole (2000) has introduced some relevant demonstrations into health education classes. His students see the effect of random walks of 1000 steps, simulating coin tossing. They are then able to generate random payouts at chosen odds and wagers,

while crudely varying the house take ("casino rigging"), and display the individual and cumulative outcomes of 1000 turns. This gives a visually striking message. "Teaching", say Borovcnik and Peard (1997), "has to focus on establishing stable intuitive insights and thus cannot rely only on presentation of the mathematics of the discipline." Recent technology not only allows sophisticated long run simulations, but also their effective presentation. Vincent (2000) has published computer-based die and coin tossing simulations using MicroWorlds for teaching probability.

Striking simulations: Know the Odds

A teaching kit launched in June 2001 by Know the Odds Inc. contains examples of some graphically portrayed outcomes of long term gambling, that are a desirable component of an accessible teaching program. The kit includes a short video which explains why people become problem gamblers and what happens when they do. The video is supplemented by computer software, which explains the role of probability theory in problem gambling, and there are teachers' notes as well as activities and reference to web materials (Falkiner, 2001). The computer software provides a number of pertinent activities. The following figures give some indication:

Figure 1: Simulation of Roulette results on "even-money" (18:19) bets

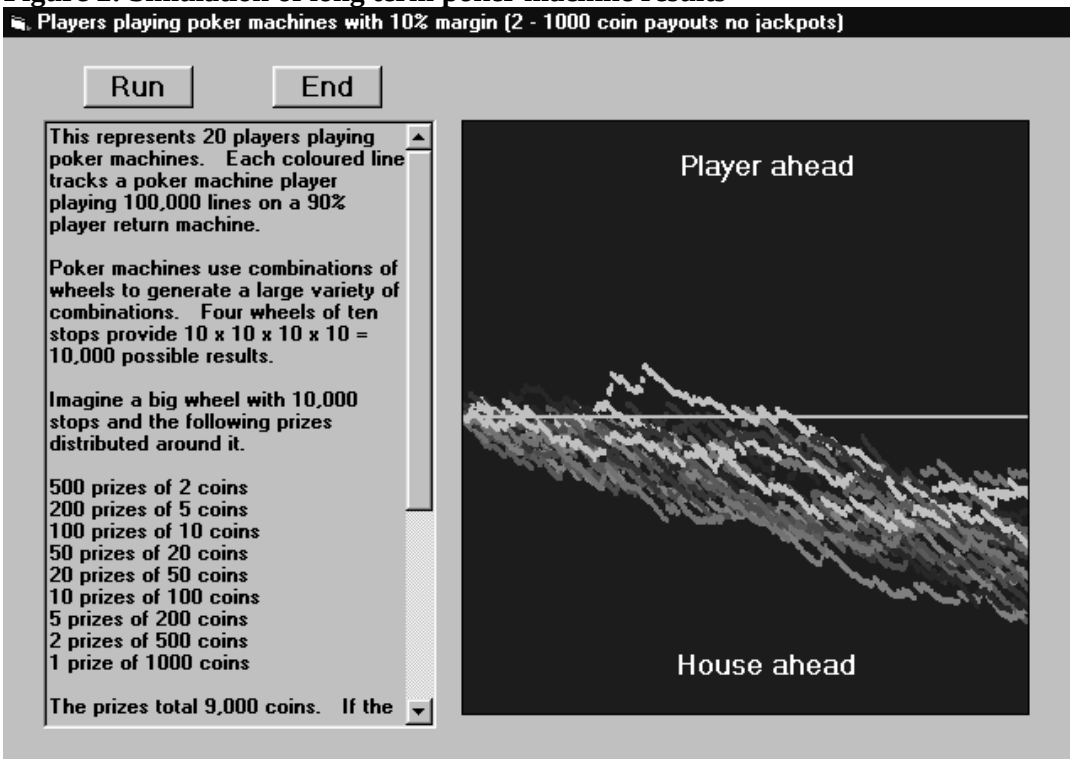


While the Figures presented here are static, the original program display is dynamic. Students are able to watch the results being generated, and see the effects of their iteration. Here is another example:

> "Poker machine: Now instead of a coin or roulette wheel you have a poker machine with overall odds of 9:11 (90% player return), no jackpots, 2 - 1000 coin payouts.
> Each time you put in a coin you step forward and right unless you get a payout. With a pay out you take one step forward and a number of steps to the left, one step for each coin less one (the one you put in). For example, if the machine pays out 100 coins you take one step forward and 99 steps to the left.
> Press the button marked "Poker machine - 100K" a few times to see what paths you take. Each path is 100,000 steps long" (Falkiner, 2001).

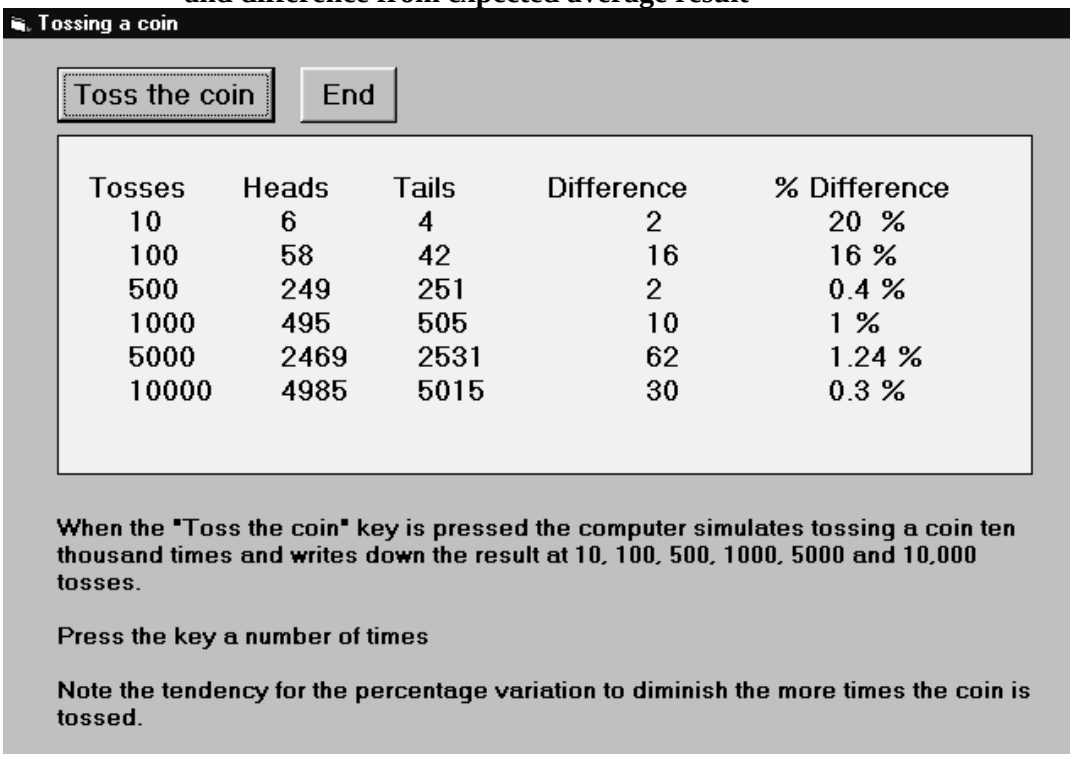
Another feature simulates 20 such outcomes on the one screen. The main feature lacking from these demonstrations is a trend line to indicate average expected result. This excellent program, in Visual Basic, has not been independently examined to verify its performance.

Figure 2: Simulation of long term poker machine results



Law of large numbers:
Another feature on the Know the Odds program is coin tossing 10, 100, 500, 1000, 5000, & 10000 times with % variation indicated, which allows consideration of regression to the mean in very simple terms:

Figure 3: Simulation of results of increasing number of coin tosses, and difference from expected average result



As impressive as these computer simulations may be, Lejoie (1997:181) reminds us that "simply observing something happen on a screen will not guarantee that the learner has made the connection between, say, a graphical representation and the meaning behind [it]. It is often necessary to make the direct connection for the learner." But technology can empower students by letting them engage in doing. Of course, standard teaching methods, including concrete development of concepts, must complement use of new technologies. For gamblers, simulations most closely resembling actual games may be most useful.

Past styles of teaching probability and statistics have been well criticised. There was "a focus on procedures to the neglect of underlying concepts and big ideas" (Metz, 1997:223). The following concepts are suggested for inclusion in any mathematics teaching attempts to counter gambling misunderstandings:

Key concepts

fair game A fair game would be one in which the player had equal chances of winning or losing over time. (i.e. neutral causal variability -see below).

net loss game Commercial gambling is not a fair game. The game is rigged so that players will make on average a net loss over time, (the average expected result is a negative percent of funds turned over, of total wagers).

average expected result: house margin. Understanding the expected impact of the house margin on game play. Total loss of each wagered amount is expected, on average, after the number of games equal to the multiplicative inverse of the house margin. Total loss of all available funds is expected on average after the previous number of games times

the number of wagers available. Total loss of funds is almost certain after twice this many games, for example.
law of large numbers: effect on the average result. Understanding the tightening of expected results around the average result in extended game play. The more you will play, the more certain is the loss that will result.
random Not all events occur following definite patterns. Some events are not individually predictable. With random outcomes no particular result can be predicted.
independence The outcome of each independent random event has no effect on the outcome of any other (though depending on the game they may affect the rate of payment on a subsequent win, but not alter the likelihood of that win occurring).
variance Variability has two elements. It is causal in part, and random in part. While the luck of favourable random outcomes is quite possible in the short term, in the long run the causal variability (ie. average expected result) will dominate, as the random effects dilute each other over time.
Over any short term run, seemingly quite strange patterns of results may occur randomly.

Democracy and empowerment in the context of commercial gambling
Empowerment can take many forms. Common wisdom has told us "a fool and his money are soon parted," but intervention is possible with both the subject, and the instigator, of the parting. To some extent, the idea of teaching appropriate mathematics to problem gamblers is a deficit model, in which the gamblers could be held responsible for their lack of knowledge, and their behaviour. It is they who are asked to give up the habit, not the habit supplier. In Australia, supply of alcohol to a drunk person is illegal, as is supply of heroin to anybody (even weekend users who have no problems). So, there are two examples of restrictions on suppliers.

Moreover, there are consumer protection laws, and criminal laws with regard to fraud. If someone doesn't genuinely understand gambling odds, and their consequences, how valid is the contract they make to play? Do they get what they think they are buying? What if gambling promotion, offering an unqualified chance to win, is actually deceptive conduct? In relation to commercial gambling, it is fair to ask what has happened to the legal protections against unscrupulous conduct.

The current movement for ethics in business also raises interesting perspectives. To take advantage of another, as opposed to engaging in fair trade, is morally objectionable, and often societies penalise some such behaviour.

The notion of financial risk is not confined to gambling games. Currency exposure, investment choices, consumer credit, home loans, mobile phones, tertiary education fees, superannuation, and retirement planning are all areas where costly dangers lurk, and affect most people. Wide social application of mathematically based prudence is desirable.

Banning scams, even if the penalties were heavy, will not eliminate them altogether. So, whatever the societal responses, developing individual decision making practices is important for protecting people. As educators we should be introducing the significance of advanced concepts at lower levels of maths skills. Improved awareness of how

commercial gambling is stacked against the player, and what limits there are to an individual's power to influence events, can be an important part of individual empowerment. Whether the students are problem gamblers or general learners, teaching which focuses on a grasp of the key concepts should function as a counter to uninformed gambling. New capabilities for simulation and display should contribute to effective teaching in this area.

Literature

Batanero, C., & Serrano, L. (1999). 'The Meaning of Randomness for Secondary School Students.' *Journal for research in mathematics education*, 558-567.

Begg, A. (1997). 'Some Emerging Influences Underpinning Assessment in Statistics.' In I. Gal & J. B. Garfield (Eds.), *The Assessment Challenge in Statistics Education* (pp. 17-25): IOS Press.

Borovcnik, M., & Peard, R. (1997). 'Probability.' In A. J. Bishop & K. Clements & C. Keitel & J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education*: Kluwer Academic Publishers.

Costello, T., & Millar, R. (2000). *Wanna Bet? Winners and Losers in Gambling's Luck Myth*. Melbourne: Allen & Unwin.

Doughney, J., & Kelleher, T. (1999). *Preliminary Local Area Gambling Research: Economic Effects*. Melbourne: Workplace Studies Centre, Victoria University.

Ellicott, J. (2001, August 24). 'The real high-rollers.' *The Australian,* pp. 34.

Falk, R., & Konold, C. (1992). 'The psychology of learning probability.' In F. Gordon & S. Gordon (Eds.), *Statistics for the Twenty-First Century* (Vol. MAA Notes Number 26, pp. 151-164.). Washington, D.C: Mathematics Association of America.

Falkiner, T. (2001). Know the Odds. Melbourne: Know the Odds Inc.

Fischbein, E., & Schnarch, D. (1997). 'The evolution with age of probabilistic, intuitively based misconceptions.' *Journal for research in mathematics education, 28*, 96-105.

Garfield, J., & Ahlgren, A. (1988). 'Difficulties in Learning Basic Concepts in Probability and Statistics: Implications for Research.' *Journal for research in mathematics education, 19*(1), 44-63.

Griffiths, M. D. (1999). 'The psychology of the near-miss (revisited): A comment on Delfabro & Winefield' (1999). *British Journal of Psychology, 90*, 441-445.

Langer, E. J. (1975). 'The Illusion of Control.' *Journal of Personality and Social Psychology, 32*, 311-328.

Lejoie, S. P. (1997). 'Technologies for Assessing and Extending Statistical Learning.' In I. Gal & J. B. Garfield (Eds.), *The Assessment Challenge in Statistics Education* (pp. 179-190) IOS Press.

Mangan, S. (1994). 'Taking a gamble with mathematics.' In C. Beesey & D. Rasmussen (Eds.), *Mathematics without limits* (pp. 282-286). Melbourne: Mathematical Association of Victoria.

Metz, K. E. (1997). 'Dimensions in the Assessment of Students' Understanding and Application of Chance.' In I. Gal & J. B. Garfield (Eds.), *The Assessment Challenge in Statistics Education* (pp. 223-238).

Moore, S. M., & Ohtsuka, K. (1999). 'Beliefs About Control Over Gambling Among Young People, and Their Relation to Problem Gambling.' *Psychology of Addictive Behaviors, 13*(4), 339-347.

Paulos, J. A. (1988). *Innumeracy*. NY: Hill & Wang.

Peard, R. F. (1995). 'The effect of social background on the development of probabilistic concepts.' In R. P. Hunting, G. E. FitzSimons, P. C. Clarkson & A. J. Bishop (Eds.) *Regional Collaboration in Mathematics Education* (pp. 561-570). Melbourne: Monash University.

Productivity Commission. (1999). *Australia's Gambling Industries* (10). Canberra: AusInfo.

Reading, C., & Pegg, J. (1995). 'Teaching statistics: Background and implications.' In L. Grimison & J. Pegg (Eds.), *Teaching Secondary School Mathematics* (pp. 120-139). Sydney: Harcourt Brace.

Rogers, P. (1998). 'The cognitive psychology of lottery gambling: A theoretical review.' *Journal of Gambling Studies, 14*(2), 111-134.

Roy Morgan Research. (2000). *Seventh Survey of Community Gambling Patterns and Perceptions*. Melbourne: Victorian Casino Gaming Authority.

Rugle, L. J., & Melamed, L. (1993). 'Neuropsychological Assessment of Attention Deficit Disorder in Pathological Gamblers.' *Journal of Nervous and Mental Disease, 181*(2), 107-112.

Smith, D. (2000). 'Adult Numeracy Teaching - An Australian Focus on Social Contexts.' In M. J. Schmitt & K. Safford (Eds.) *Proceedings of ALM7 - the Seventh International Conference of Adults Learning Mathematics – A Research Forum.* (pp. 87-92). Boston: ALM & NCSALL.

Smith, R. (1997). 'Chance activities for middle primary school.' In N. Scott & H. Hollingsworth. (Eds.) *Mathematics, creating the future* (pp. 430-435). Melbourne: Australian Association of Mathematics Teachers (AAMT).

Toneatto, T. (1999). Substance Use and Misuse. *34*(11), 1593-1604.

Truran, J. M., & Truran, K. M. (1995). 'Research into the Learning and Teaching of Stochastics.' *Review of Mathematics Education in Australasia 1992-1995* (pp. 345-364): Mathematics Education Research Group of Australasia.

Tversky, A., & Kahneman, D. (1982). 'Judgment Under Uncertainty: Heuristics and Biases.' In D. Kahneman, Slovic, P. & Tversky, A. (Ed.), *Judgment Under Uncertainty: Heuristics and Biases.* Cambridge: Cambridge University Press.

Vincent, J. (2000). *Computer Enriched Mathematics for Years 7 and 8* (2nd ed.). Melbourne: The Mathematical Association of Victoria.

Wallisch, L. S. (1996). *Gambling in Texas: 1995 Surveys of Adult and Adolescent Gambling Behavior.* Austin.: Texas State Commission on Alcohol and Drug Abuse.

Walpole, G. (2000). Gambling.xls. Melbourne: Unpublished computer program.

Watson, J. M. (1995). 'Probability and Statistics: An overview.' In L. Grimison & J. Pegg (Eds.), *Teaching Secondary School Mathematics* (pp. 140-163). Sydney: Harcourt Brace.

Real life' in everyday and academic maths

Alison Tomlin
King's College London, UK

1. Introduction

This paper draws on data from a participant action research project with adult basic maths students, in inner-city areas of south London (Tomlin, 2000, 2001). The students were all attending general basic maths courses (that is, they were not linked to, for example, vocational courses or workplace demands) for which, in the period when we did the research, there was no set curriculum in the UK and course content was negotiated between students and tutor. On the other hand, many of the students wanted a formal qualification and all were required, by the funding regulations, to choose from a limited range of exams and course-work-based certificates. The students attended maths courses for two hours a week, and most also attended literacy courses.

The data I will present challenges the view that curricula for basic maths should be determined by tutors' or policy-makers' notions of 'real life' maths. I don't want to argue against using maths from students' 'everyday lives', and all the groups with whom I worked did a lot of that. However, I do want to argue against the idea that we (tutors, researchers and policy-makers) can predict, for students, what 'real life' means, or that we can always help with the maths that students need in their everyday lives.

First, I'll look at examples of the claim that adults returning to basic maths should be offered curricula and learning materials from 'real life' or 'the everyday world', drawn from UK national policy and the work of Marilyn Frankenstein and Roseanne Benn. I will then present some snapshots of data drawn from the students' and my research which, I argue, challenge this dominant perspective. In analysing the data I used tools from critical discourse analysis (e.g. Fairclough, 1992; Gee, 1999), which helped me to recognise my own position, both as a dominant voice in the classroom and as someone who believes, as do many in adult basic education, in the importance of a 'real life' focus.

2. The claim that adults should work on maths from 'real life' or 'the everyday world'

This claim seems common sense – the implied alternatives are 'unreal', or 'abstract', and sound like the kind of maths many of our students have suffered in the past. Indeed, it may have contributed to their being failed in school maths.

The claim that adults should work on maths from 'real life' comes from different perspectives within adult basic education. The examples I give here come from research and policies which I hope will be familiar to ALM members.

2.1 UK official policy

The new UK adult basic skills drive was discussed at ALM8 by Diana Coben, Dhamma Colwell and Valerie Seabright. Here I want only to point to the place of 'real life' in the new curriculum. The teacher's job is to relate maths skills, which are seen as universal,

to the students' 'real life' contexts. The terms *everyday, familiar*, *straightforward* and *practical* are used as descriptors for levels and content. These definitions are from the glossary:

> Everyday: an adjective used to describe numbers, measures, units, instruments, etc. that fall within the daily lived experience of most people in non-specialist contexts. (DfEE and Basic Skills Agency, 2001:89)
>
> Familiar: describes contexts, situations, numbers, measures, instruments, etc. of which the learner has some prior knowledge or experience. (p. 89)
>
> Straightforward: describes information, subjects and materials that learners often meet in their work, studies or other activities. (p. 91)

'Practical' is not defined in the glossary, but we can work out possible meanings from the contexts in which it is used. Here is one example: at Entry Level 2, learners should 'use and interpret +, -, x and = in practical situations for solving problems'.

The sample activities for this section are

- Match words to symbols, using a range of vocabulary
- Match word problems to written calculations
- Translate single-step word problems into symbols and solve. (DfEE and Basic Skills Agency, 2001:26-27)

'Practical' here seems to mean the use of spoken or written word problems, the traditional way to bridge the calculation: real world gap.

2.2 A critical perspective

Marilyn Frankenstein, writing from a Freirean perspective (e.g. Freire, 1972), argues for centring the curriculum on students' contexts:

> Teachers can ask students about the issues that concern them at work, about the non-work activities that interest them, about topics they would like to know in more depth, and so forth. These discussions can indicate the starting point for the curriculum. Then the teacher's contribution can be to link up the students' issues with an investigation of the related hegemonic ideologies. (Frankenstein,1987:195-196)

and

> Hopefully, the more practice students have in creating questions, the more they will become used to asking questions, in school and in their daily lives. (Frankenstein, 1987:210)

UK government policy sees maths skills as free-standing, best learned in the students' contexts, and then transferable to new contexts. Frankenstein similarly sees maths learning as originating in the students' everyday lives but then seeks to use the learning process as a way towards changing students' lives, as part of a critical agenda. The political stances are radically opposed, and that is reflected in the examples of 'real life' contexts: the UK government's are often domestic or vocational, whereas Frankenstein's are typically drawn from social-economic data. However, the

assumptions that maths is best learned in real life contexts and that tutors can come to know students' contexts are common to both.

2.3 A citizenship perspective

Roseanne Benn argues that

> If mathematics is a dynamic, living and cultural product, the contextualisation of problems is essential. (Benn, 1997:38)

Benn here seems to imply that it is possible for maths problems to have no context. I shall suggest that the 'context' may be the group's own discourse, rather than something drawn from outside the classroom.

Benn's suggestions for pedagogy and the curriculum (though she is not prescriptive) are centred on a citizenship agenda:

> Fostering critical awareness and democratic citizenship for adults through mathematics requires questioning and decision-making, discussion, permitted conflict of opinion and views, challenging of authority, and negotiation. Hence the curriculum must include these components, and materials should include socially-relevant projects, authentic social statistics, accommodate social and cultural diversity and use local cultural resources. (Benn, 1997:88)

My difficulty is that 'fostering critical awareness' is presented here as something that students need and tutors can give; 'challenging of authority' is something timid students need to learn from confident teachers. But the authority may be the teacher's; the challenge may be (often is, in my case) *rejection* of this curriculum and a demand for, say, 'straight' fractions, as in the example that follows.

3. Students' work on 'unreal' mathematics

Here I have space to give only one of those challenges to my authority and demands for 'unreal' maths. The story concerns two different groups of students; the link is my own persistence in thinking the adult basic maths curriculum should be based in demands arising from students' 'real' lives. The two discussions from which I shall quote came at the end of the academic year, and were not part of the formal maths work of the two groups. The first discussion followed on from a students' conference, and the second was an end-of-year meeting.

After the students' conference

The students' conference had included discussion of what students characterised as 'everyday' and 'textbook' maths. I called on 'real life' to justify my arguments against work on fractions. I was immediately challenged:

Alison	*The fractions, you just don't use it in real life, at all, ever, I don't think.*
Jeremy	*You do, you do, you do.*
Alison	*When?*
Tracy	*Actually I think you do.*
Jeremy	*That's where you're wrong, because [in] the topic I was studying [accountancy], right, they had to write all the fractions out in the*

	book for you, so you could work out the 8%, the 10%, the 15%, 25%.
Alison	*Oh yeah, percentages, yeah.*
Shazia	*Percentages, but that comes into fractions as well, doesn't it. 25% is a quarter.*
Alison	*Yeah.*
Jeremy	*No, no no, no, but they show you the fractions, like say you've got a 100 pounds, how to work the VAT [tax] at 8% or 25% or 15%. To make sure you got it right, they worked (.) they did the fractions, and they were put beside the percentages.*
Alison	*Right. But you can, you see, you can do percentages by decimals instead of by fractions, and I think it's easier. If you do it by decimals, I think it's easier.*
Shazia	*Quick, write that [i.e. the method] down! [laughing]*
Lorraine	*Show us how you do that.*
Jeremy	*That's what it was in the book, and it was quite clearly explained.*
Antoinette	*We went to a shop, a store, and they have (.) three third -*
Alison	*A third, a third off, or something.*
Antoinette	*Yeah.*
Shazia	*Yeah, you do -*
Alison	*Yeah, it's true. Ok, I'll back down.*

This exchange made me realise that students saw me as being too general in my argument against fractions. I still believed that in everyday life people did not need to use more complicated fractions, but recognised I needed to distinguish between these and Antoinette's 'a third off'.

The following day: an end-of-term discussion

My agenda included discussion of students' options for the following year, and checking aspects of the research project, including how students wanted to be identified in anything I wrote; I did not intend any work on fractions at all. The group discussed topics in maths which they wanted to pursue, and mentioned fractions, at first in the context of the requirements for entry into the next level of maths course. I had the previous day's discussion in my mind, and I commented,

> *things like five sevenths multiplied by four fifths ... you never do it in your normal life, do you?*

Joyce said,

> *You learn it, and then you forget all about it, because it's never necessary.*

Meanwhile, Dave had been working on it:

> *I got 0.5714285 for that*

and then Joyce worked out an answer:

> *Four seventh. And what the hell is four sevenths?*

My suggestion that complicated fractions were unnecessary (except for higher level courses) had backfired on me. I tried to get off the fractions topic, but Joyce didn't want to stop:

> *If somebody ask you that question [5/7 x 4/5] now, you know like you just mention it. It's not a thing you're going to use anyway. Just to know how to do it …*

In responding to Joyce, I said,

> *Where it comes in useful is when you're doing algebra, to know that when you're multiplying fractions you multiply the top by the top and the bottom by the bottom.*[16] *Those rules come in useful later on …*

I was suggesting a context for which I thought work on the standard algorithms for fractions calculations would be 'useful', even though Joyce had implied that usefulness was not the only criterion for deciding whether a topic was worth studying. I had given the multiplication rule as an example only, but Dave interrupted me:

> *What did you say, multiply the top by the top and the bottom by the bottom? But that's only if you've got the same numbers on the bottom, isn't it?*

This question revealed an error, and then Dave gave me an example to work on (4/5 x 4/7). We did that work, and I tried again to get back to my own view:

> But ... this is not useful, I don't think, you can go through the whole of your life without this, but in maths, in later studies in maths, suppose you had half of some number and you don't know what this number is …

… and then we worked on solving equations (e.g. 3x/2 = 6) for about 10 minutes.

I then firmly changed the subject and asked the students about further participation in the research; then the conversation drifted onto holidays. Dave interrupted us:

> Can I just show you something ... I just want to know whether this is correct or not or whether I'm missing something, a stage, out.

This turned out to be division of fractions.

I returned again to my own agenda, asking the students what names they wanted me to use for them in writing up research, and checking with each student individually. As soon as each student had replied, Joyce started up the fractions discussion again. She had meanwhile been working with Violet on fractions problems they set themselves:

> *Well we still want to learn, what we take this thing for. Right, I'm doing this, trying to, this is something I want to remember, as I go along… this one we just done, the multiply. Four sevens twenty-eight, 28 times 2, something six, fifty-six?*

(The calculation was 4/5 x 7/10 x 2/7.)

This story shows me, as tutor, several times being pushed back into a fractions lesson. The fractions were 'meaningless' in that they had no content beyond the numbers themselves: the four-fifths were not four-fifths of the population, of a pizza, or of a budget. Yet the manipulation of fractions seemed to be, for that hour or so, the only subject that had real interest for the students.

Why were the students so interested in the abstract manipulation of 'meaningless' fractions? I didn't ask that question - perhaps I should have, but I was concerned with

[16] 'Top' and 'bottom' are often used in English for 'numerator' and 'denominator'.

my own agenda. I can suggest some possible answers based both on these transcripts and other conversations. Many students associate fractions with being failed in school maths. Some for that reason seek to avoid fractions; others want to overcome that earlier failure. Some of the group wanted to check they were up to the required standard for a higher level course the following year. The abstract maths becomes 'real' through students' engagement with it. In Benn's terms (quoted above) the students 'challenged authority' – but it was the teacher's authority; they used 'local cultural resources' - but their main resource is their solidarity in working together on their own problems.

Although this group pursued traditional fractions algorithms, they didn't do it in a traditional way. The questions were student-generated and expressed orally, and people worked alone or with friends as they chose. Many of the questions were harder than a tutor would set for a course at this level.

Next I turn to an example of a 'real life' maths problem.

4. Paying for electricity

When I asked a group of students to 'bring in some problem to do with maths', Margaret asked the group about her electricity bill. She was moving; she had the choice of a quarterly-read meter (with further choices of payment method, some of which reduced the unit price), pre-paid plastic card (requiring a deposit) or cash meter. She had a low weekly cash income, and wanted to know both which was the cheapest payment method, and which would be easiest to handle. Within the discourse of maths education, working out an answer would require checking the unit rates, standing charges and deposits for all payment methods; if relevant (a question in itself) checking Margaret's previous average bills; estimating her likely consumption in the new flat; applying answers from the first questions to Margaret's probable future consumption; and comparing results. This is technically demanding, even with a calculator; for example, unit charges are expressed in pence to two decimal places.

Margaret understood perfectly well that the maths involved was 'too difficult' for her - that was exactly why she brought the problem to the class. It was solved, but not by mathematical methods, or at least, not following the steps outlined above. Others in the group advised her that the card would suit her best; their reasons included flexibility (you can charge it up as much as you want) and security. I said I thought a quarterly bill, paid by direct debit, would be cheaper. The students agreed, but all, including Margaret, ruled it out because they could not take the risk of direct debits on small and fluctuating bank accounts.

Real problems are *real* – Margaret *had* to have a solution, then and there. Margaret knew the other members of her group well enough to tell them she didn't know what to do about her bills. Her colleagues were able to draw on their own experience to suggest a solution – in Luis Moll's terms, they used the group's 'fund of knowledge':

> *funds of knowledge are manifested through events or activities*. That is, funds of knowledge are not possessions or traits of people in the family but characteristics of people-in-an-activity. (Moll & Greenberg, 1992:326; original emphasis)

This story points to the folly of assuming that tutors, policy-makers or researchers can predict students' mathematical questions, or always answer them. I could have analysed Margaret's question and made elements of it into a manageable mathematical inquiry - but I could never have known enough about the other variables involved to help her towards a decision. It's manipulative, perhaps, of tutors to seize on 'real life', as we judge it, and slow it down by analysing it. Further, such an analysis, or 'specification of practice', as Jean Lave puts it, may misrepresent the situation:

> The problem is that any curriculum intended to be a specification *of* practice, rather than an arrangement of opportunities *for* practice (for fashioning and resolving ownable dilemmas) is bound to result in the teaching of a misanalysis of practice … and the learning of still another … In the settings for which it is intended (in everyday transactions), it will appear out of order and will not in fact reproduce "good" practice. (Lave, 1997:33; original emphasis)

Margaret's problem was 'practical, familiar and everyday', in the terms discussed above, and the group's solution was 'straightforward', but it was beyond the competence of the tutor. Margaret's colleague students shared the knowledge of 'people-in-an-activity' – that is, 'good practice' in this context - in ways I could not.

5. Conclusion

I have had space here for only two stories from many in the research project. Students in the project argued forcefully and consistently against a constricted curriculum. These quotations come from students' notes of discussions at their conference:

> Most students want both text and everyday learning. (Mario; original emphasis)

> Maths helps you get a job, helps with your shopping, measurement, your money, i.e. bills, and if you have children you now can help them. People wanted to improve their maths work, i.e. sit exams and catch up with lost years.
> (Sandra)

Analysis of data from the whole research project showed a consistent demand for an extremely wide range of curricular possibilities. Individual students may choose to work on particular topics (and choices may vary over time), but all the students argued that the available curriculum should be very wide.

Antoinette was angry that a tutor seemed to be defining her maths needs for her:

> When me and Jeremy was talking about what we was doing, the [tutor] was saying that you need maths, um, for measuring and all of that thing what you is doing, and I thought, how can [s/he] ask those questions? Why can't [s/he] just go [i.e. leave the discussion to the students].

Shazia agreed, and added,

> You know the one that got me ... is the one with, which is the best thing, going through textbooks or doing [...] news articles... and I said both, I said both.

One of the ways in which tutors and policy-makers constrain the basic education maths curriculum is through our focus on 'real life' and 'everyday' contexts. My concern is not

to argue that a basic maths curriculum for adults should or should not be based on such contexts, but to raise questions about the basis of the distinction and about the ethics of tutors' being positioned as able to decide what is or is not real life for students.

The data supports an argument that meaning is discursively formed, so that students' engagement with the material (and the potential for learning) depends not on the material itself, but on the discursive contexts of the work, including group relationships, prior experiences and the practices behind the production of the learning materials. 'Academic' maths can become the real world; apparently 'relevant' materials can be meaningless; and some apparently mathematical real world problems may not be soluble through tutors' mediation.

Jean Lave argues that we need to:

> move away from the relation that looms so important because of its theoretical and institutional history - that between the 'everyday', or 'concrete', and the 'theoretical', scholastic abstraction of school maths - towards a different distinction: that between things (real and imaginary) that do and do not engage learners' intentions and attention. (Lave, 1992:88)

In talking as though we (tutors and policy-makers) know or can predict students' everyday lives and maths problems, we risk being profoundly patronising. What is real, everyday or relevant, or has meaning, depends on things far more complicated than tutors can know, including:

- students' personal aims for the course of study – e.g. recovering from previous experiences of being failed, or moving flat
- students' personal maths histories, in and out of education
- the history of relationships in the particular group
- the discourse of texts, including the practices behind its production – a student-written problem is not treated the same as a text-book problem
- the particular discursive contexts at the moment the maths is being done.
- the tutor's role in the group.

If meaning is discursively formed, as I have argued here, then I think a key element in course planning that lies within tutors' influence is the framing of group discourse - what students are able to say, with what authority, in the classroom. We must stop speaking for students, however kindly we do it; unless there is more space for students' voices, we will never be able to explore, in Lave's terms, 'things (real and imaginary) that do and do not engage learners' intentions and attention.'

Literature

Benn, R. (1997). *Adults Count Too: Mathematics for Empowerment*. Leicester: NIACE.

DfEE and Basic Skills Agency. (2001). *Adult Numeracy Core Curriculum*. London: Basic Skills Agency.

Fairclough, N. (1992). *Discourse and Social Change*. Cambridge, UK: Polity Press.

Frankenstein, M. (1987). 'Critical mathematics education: An application of Paulo Freire's epistemology.' In I. Shor (Ed.), *Freire for the classroom* (pp. 180-210). Portsmouth, NH: Boynton/Cook.

Freire, P. (1972). *Pedagogy of the Oppressed* (Myra Bergman Ramos, Trans.). Harmondsworth: Penguin Books.

Gee, J. P. (1999). *An Introduction to Discourse Analysis: Theory and Method*. London: Routledge.

Lave, J. (1992). 'Word problems: A microcosm of theories of learning.' In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing.* (pp. 74-92). New York: Harvester Wheatsheaf.

Lave, J. (1997). 'The Culture of Acquisition and the Practice of Understanding.' In D. Kirshner & J. A. Whitson (Eds.), *Situated Cognition: Social, semiotic, and psychological perspectives* (pp. 17-35). Mahwah, N.J. and London: Lawrence Erlbaum Associates.

Moll, L. C., & Greenberg, J. B. (1992). 'Creating zones of possibilities: Combining social contexts for instruction.' In L. C. Moll (Ed.), *Vygotsky and Education: Instructional implications and applications of sociohistorical psychology* (pp. 319 - 348). Cambridge: Cambridge University Press.

Tomlin, A. (2000). 'A "democratic classroom", but who speaks loudest? Research with basic mathematics students.' In J. F. Matos & M. Santos (Eds.), *Proceedings of the Second International Mathematics Education and Society Conference* (pp. 374-383). Lisbon: Centro de Investigacao em Educacao, Universidade de Lisboa.

Tomlin, A. (2001). *Participatory approaches to work with adult basic mathematics students.* Unpublished PHD thesis, King's College London, London.

Poster Session

Teaching Adult Students Numeracy and Mathematics

Richards O. Angiama
Professional & Community Education (PACE)
Goldsmiths, University of London, United Kingdom

Abstract. The theme of this paper is Teaching Adult Students Numeracy and Mathematics (TASNM). The paper aims to support the teaching and learning of mathematics to adult students. I suggest adult students of mathematics need to be encouraged to explore new horizons of learning to seek out the next level of knowledge and to face the unknown. If mathematics is presented in a non-threatening way, with adequate provision made for those people who want to study the subject, it could lead to a much more numerate society with considerable benefits for industry. But how should we as adult educators teach mathematics so as to support adults to function satisfactorily in their work and everyday lives? How can teachers of mathematics help adult students deal with mathematics anxiety? How do teachers of mathematics transform instructional practice to better serve adult learners' needs? How can a teacher of mathematics facilitate adult students moving from being passive learners to active agents of their learning process? Will learning about statistics and probability enable adult students to view their world more critically and encourage them to ask more questions? What are the most effective ways for adults to acquire important mathematical skills and abilities? These are some of the questions raised in this paper.

Introduction

My theme in this paper is Teaching Adult Students Numeracy and Mathematics (TASNM). I shall look at what is meant by the words, 'numeracy' and to be 'numerate' by referring to relevant literature, and by considering the implications of these ideas for Teaching Adult Students Numeracy and Mathematics in my role as a teacher and researcher in my own teaching situation. I take the following definitions as my starting point:

> "Numeracy […] is a proficiency that involves a confidence and competency with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is graphs, diagrams, charts and tables. (Numeracy Task Force, 1998 para. 15).
>
> We would wish the words 'to be numerate' to imply the possession of two attributes. The first is an 'at homeness' with numbers and an ability to make use of the mathematical skills which enables an individual to cope with practical demands on his/her life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, or tables by reference to percentage increase or decrease. Taken together these imply that a numerate person should be expected to be able to appreciate and understand some of the

> ways in which mathematics can be used as a means of communication. (Numeracy Task Force, 1998, para. 39)
>
> Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much (DES, 1982, para.34).

According to recent surveys, the UK has a problem with numeracy skills, with one in four adults classed as innumerate (DfEE, 1999). Roseanne Benn (1997) has examined the low level of numeracy in our society, the reasons why this is critical and the forces acting on adults which contribute to this state of affairs. These forces include experiences and philosophies of mathematics and mathematics education, social and cultural factors, political imperatives and the aims and aspirations of the adults who, despite all odds against them, wish to return to re-learn mathematics. Certainly, in a democracy, adults need to learn mathematics not only to develop skills to generate and solve their own mathematical problems but also to gain qualifications. Thus, as Benn argues, numeracy for empowerment and democracy requires that adults understand why and how mathematics is generated, used and maintained in society with consequences for democracy and critical citizenship. I believe what we need to do as adult educators of mathematics is to convince adults who may have anxieties about doing mathematics that they will improve their skills and that this is worth doing, as citizens of a democratic society. Adult students of mathematics need to be encouraged to explore new horizons of learning and to seek out the next level of knowledge and to face the unknown.

Adult learners of mathematics need access and success, their past anxieties and feelings of failure must be removed and replaced with positive attitudes. Failure should be replaced with success, competition with co-operation, irrelevance with relevance and clear, concrete, meaningful mathematical explanations. It is clear from the definitions quoted above that numeracy does not refer only to operating with numbers, but refers to a much wider range of skills. These skills are necessary as the "foundation for higher levels of mathematical study […and…] further skills for adult life" (Numeracy Task Force, 1998, para.14). How should we as adult educators teach mathematics so as to support adults to function satisfactorily in their work and everyday lives? How do adults learn to become numerate and how should they be taught?

Implications for Teaching

I suggest that mathematics, perhaps more than any other subject, requires very careful teaching. If mathematics is presented in a non-threatening way with adequate provision made for those people who want to study the subject it could lead to a much more numerate society with considerable benefits for industry. The aim for a teacher teaching adults learning mathematics should be: if I'm going to support these people through their educational and personal development, I should encourage them to master a range of approaches to learning, because then they will get more out of life.

Richard Skemp (1971) wrote extensively about anxiety and the effect of learning on the student. He deduced that one of the causes of students becoming anxious about mathematics is a lack of understanding. The more anxious the student becomes, the harder s/he tries, but the worse s/he is able to understand, and so the more anxious s/he be-

comes. W.W. Sawyer (1959) argued that mathematics was feared in the 1940s, and that this must be because of the way it was taught. He felt that mathematics should not be learnt unless it is to be used and using it would help to dispel students' fear. He concluded that a badly taught subject is an 'imitation' subject. Teaching the real subject involves showing either the background or the meaning of it. He argues that it is far easier to learn the real subject properly than to learn the imitation badly.

Our aim as educators of mathematics to adults should be to raise the low level of the adult students' participation in mathematics education. This can be best achieved through maximising their potential by the development of effective teaching methods and learning strategies which lead to skill acquisition. Many adult students returning to study mathematics arrive at basic mathematics classes announcing that they cannot do mathematics because of fear and that they need to start from the beginning. Diana Coben and Gillian Thumpston (1995) found that people tend to describe the mathematics they can do as 'common sense' and only what they cannot do as mathematics. Anxiety about mathematics, described by Buxton (1981), can also contribute to this situation. At the same time, many other adults, whilst being able to perform mathematical calculations, feel inadequate because they know they are not using the proper methods (Hughes, 1986).

An example of Teaching Adult Students Numeracy and Mathematics based on an approach aiming to alleviate adults' anxieties and relate to their interests is given below, with some illustrative material.

Shopping Task Analysis

A class of 21 adult students of mathematics carried out a survey of supermarkets to find out which supermarkets provided the best advantages; this became known as the 'Shopping Task Analysis'. Through group discussion, a set of questions was generated and collated. The students had to try to find out the ideal supermarket for each family represented in the class, compare prices, and represent the data collected in a simple but mathematical way. They were divided into groups consisting of two to four students, mixed by gender, ethnicity and ability. An example of this work is given below.

Shopping Task Analysis Questions
Where does your family shop for food? Is this a sensible decision?
Which is the cheapest shop?
Which is the easiest to get to?
Which has the best car parking?
Which is the most convenient to shop in?
Which offers the greatest choice?
Which has the most comprehensive stock?
Itemise a typical 'shopping basket'
Decide what data to collect to answer the above questions.
Decide how to measure convenience e.g., aisle width, number of checkouts, and so on.
Find the cost of a 'shopping basket' in each shop.
Find out how long it takes to get to the supermarket, bus routes, cost of getting there (petrol), fares…
Find out if there is always parking space and how far it is from the car park to the shop.
Compare the width of aisles with the width of a trolley.

Find out how long people have to queue.
Find out how many brands of a range of items each shop stocks.
Obtain any other information/measurements.
Do calculations
Draw diagrams recording information and results.
Relate calculations back to questions.
Decide what is important.
Weigh up the evidence.
Decide which shop you think is best and why.
Recognise any other factors not considered.
Write about and discuss your work.
Criticise your work.
Present your solutions.

Questionnaire, with responses

1. What supermarket is most frequently used by your family?

Sainsburys	Tesco	Asda	Gateway	Co-op
II	~~IIII~~ ~~IIII~~	0	~~IIII~~	IIII

2. Why? Because it's

Cheaper?	~~IIII~~	
Nearer?	~~IIII~~ ~~IIII~~ II	
Other?	Sister works there:	Tesco
	Ansederer works there:	Co-op
	Mum has to go into town anyway:	Sainsburys

3. Roughly how much does your family spend on food per week?

£10 - £20	£20 - £30	£30 - £40	£50+
0	~~IIII~~ II	~~IIII~~ ~~IIII~~ I	III

4. How does your family get there?

Bus	Car	Walk
I	~~IIII~~ ~~IIII~~ ~~IIII~~ II	III

5. How many in your family?

2	3	4	5	6
0	III	~~IIII~~ III	IIII	~~IIII~~ I

6. Does your family buy brand names, own-make or both?

Trade (Brand) names	~~IIII~~ II
Supermarkets' own make	IIII
Both	~~IIII~~ ~~IIII~~

7. What does your family reckon is the best thing in the supermarket you chose?

Nothing special	II	Tesco, Gateway
Nearest to home	III	Gateway, Tesco
Car park spacious	~~IIII~~	Tesco, Sainsburys
Size of supermarket	II	Tesco, Co-Op
Friendly service	III	Tesco, Co-op
Café	I	Tesco
Good quality food	II	Tesco, Sainsbury

Conclusion

All in all, I argue that at present mathematics occupies a marginal position within adult education whereas adult educators as action educational researchers should be tackling this 'disadvantage' just as rigorously as others, in order to equip people to be more active citizens. I argue that we as adult educators teach mathematics must support adults functioning satisfactorily in their work and everyday lives. We must help adult students to deal with mathematics anxiety and transform our instructional practice in order to better serve adult learners' needs. We must facilitate our adult students moving from being passive learners to active agents of their learning process. We need to consider what are the most effective ways for adults to acquire important mathematical skills and abilities. Teaching based in students' interests, such as that outlined above, should help teachers to raise the standards of adults learning mathematics.

References

Benn, R. (1997) *Adults Count Too: Mathematics for Empowerment*. Leicester: NIACE.

Buxton, L. (1981) *Do You Panic about Maths?* London: Heinemann

Coben, D. & Thumpston, G. (1995) 'Researching Mathematics Life Histories: A Case Study'. In: D. Coben (comp.) *Mathematics with a Human Face.* London: Goldsmiths, University of London/ALM, pp39-44

DES (1982) *Mathematics Counts*. Report on the Committee of Enquiry into the Teaching of Mathematics (The 'Cockcroft Report') London: HMSO.

DfEE (1999) *A Fresh Start: Improving literacy and numeracy*. The report of the Working Group chaired by Sir Claus Moser (The Moser Report). London: Department for Education and Employment

Hughes, M. (1986) *Children and number: difficulties in learning mathematics*. Oxford: Basil Blackwell

Numeracy Task Force (1998) *Numeracy Matters: the preliminary report of the Numeracy Task force, published for consultation January 1998*. London: Department for Education and Employment

Sawyer, W.W. (1943) *Mathematicians' Delight*. Harmondsworth: Penguin

Skemp, R. R. (1971) *The Psychology of Learning Mathematics.* Harmondsworth: Penguin

Topic Groups

Topic Group A

Developing a Theoretical Framework for Adults Learning Mathematics: The Case of Numeracy

Tine Wedege, Roskilde University, Denmark
Roseanne Benn, University of Exeter, United Kingdom

As stated in the constitution of the research forum and the theme of this conference, 'numeracy' is a central theoretical construction in research, practice and politics within the area of ALM. The term 'numeracy' was introduced in the United Kingdom in the late 1950s as a parallel to the concept of 'literacy'. In one definition, numeracy has two attributes: (1) 'at-homeness' with numbers and functional skills; (2) some appreciation and understanding of information which is presented in mathematical terms. (Cockcroft, 1981). There have been many other definitions (see Benn, 1997) generating a lively debate between educational planners and researchers in the English-speaking countries (the United Kingdom, the USA, Australia etc) about the content and meaning of the concept of 'numeracy'. (Yasukawa & Johnston, 1994; Evans, 2000). In recent years in Denmark, there has been an on-going conceptual and educational construction of numeracy, translated into Danish as 'numeralitet'. (Lindenskov & Wedege, 2001).

Several studies examine numeracy in society. They represent, however, different approaches to the subject area, for example: an *objective perspective* (society's requirements of numeracy) *versus* a *subjective perspective* (adults' individual need for numeracy); or a *numerical skills approach* (numeracy as basic skills) *versus* a *numerate competence approach* (numeracy as an everyday competence).

We propose to focus on concepts of numeracy in the on-going exploration of the theoretical frameworks for adults learning mathematics in this topic group.

References

Benn, R. (1997). *Adults count too. Mathematics for empowerment.* Leicester: NIACE.
Cockroft, W.H. (1982). *Mathematics counts*. London: Her Majesty's Stationery Office.
Evans, J. (2000). *Adults' mathematical thinking and emotions– A study of Numerate Practices.* London: Routledge/Falmer.
Lindenskov, L.; Wedege, T. (2001). *Numeracy as an analytical tool in adult mathematics education and reseearch.* Centre for Research in Learning Mathematics, Publication, no.31.
Yasakawa, K.; Johnston, B. (1994). 'A numeracy manifesto for engineers, primary teachers, historian ... a civil society –can we call it theory? *Proceedings of the Australian Bridging Mathematics Network Conference*, Sydney University, pp.191-199.

Note. The debate about the identity of the new research domain ('Adults Learning Mathematics as a community of practice and research') and about developing theoretical frameworks started at ALM4 in 1997 and continued through ALM5 to ALM7 in 2000. (See the ALM proceedings: Wedege, 1997; Wedege, Benn, Maasz, 1998; Benn & Maasz, 1999; Benn & Wedege, 2000)

Topic Group A

Exploring A Hypothesis

Mark Schwartz
Portland Community College, Oregon, USA

> For I do feel qualified to play mind games, and suggest that in some fields, at least, they are not played enough. The essence of such games is to define and to lay out possibilities, and it is logically possible to lay out all of them, especially if you include a category entitled "Possibilities We Haven't Thought of Yet." (Colin Tudge, 1997:227)

I need to do some exploring, being prompted to do so by images I carried with me from the ALM8 conference. I would hope that this exploration might aid in our beginning to craft some defining questions. At least, we need to accept that any hypothesis we offer is based on some assumptions and although a list of these assumptions might be interesting to explore, more critically we need to examine the hypothesis that emerges from it, since it is a driving force in dialogues about numeracy. At this point, I can't reasonably document my assumptions and you are likely to have different ones. But remarkably, we seem to come to that same position regarding the relationship of the domains of numeracy, empowerment and democracy.

I don't believe I am alone in asserting that there are direct linkages between numeracy, empowerment and democracy. In essence, the hypothesis seems to be: if students demonstrate an increase in numeracy skills then there will be an associated increase in empowerment and democracy. I come to this from my own experiences and also after reflecting on conversations and articles by colleagues in numeracy and basic skills. At this point in the evolution of theories of learning and teaching of numeracy, it seems critical that exploring our assertion about numeracy, empowerment and democracy is as critical as exploring students' actions in and assumptions about math and their beliefs about themselves in the math domain.

The discussions I have with colleagues are typically illustrated with anecdotes of students exhibiting behaviors from these 3 domains. Embedded in these stories is another assumption, that being that the students typically coming through our doors also are likely to not possess the same degree of empowerment and democracy as those not seeking our services. It seems that this perception is part of the anecdotal reporting we share. These students may well be productive, vigorous and mature individuals and all we can state for certain is that they need to change their numeracy capability. But, in order for our hypothesis to bear fruit, we must somehow accept that if we were to measure students' levels of empowerment and democracy, these levels would need to be low enough such that any increase could be measured. Would empowered and democratic people come through our doors seeking a change in numeracy? Further, if a student clearly demonstrates high levels of empowerment and democracy, then any observable increase in numeracy would not have an associated increase in the other domains simply because of a ceiling effect. It therefore seems that the hypothesis we hold of the rela-

tionship between numeracy, empowerment and democracy might contain something more complex than arriving at a common definition for each of the domains.

We seem to believe or at least express a belief that when we teach math we also teach something resulting in empowerment, something that aids students to more effectively participate in their society; to be more democratic, which could be signaled by their being more accepting and deliberative of others' ideas. We believe that their being more numerate provides them also with life skills that enable them to be more expressive and have increased social intercourse. We profess that not only have they increased their numeracy capability but also they have a better sense of self, purpose and identity. They simply function better in society. These are very positive and valued events in a person's life and as noted above, we swap success stories and anecdotes because it is truly a pleasurable experience we share with these students. But on what ground can we anchor our hypothesis? How can we partition the influence an increase in numeracy has on empowerment and democracy from the influence of other simultaneous events? It is likely that as these students are addressing their numeracy skills they are also addressing other areas such as writing, reading, and trade skills. We need to reflect a little on where our hypothesis really takes us.

Let me begin with a mind game and address, as Mr. Tudge proposes "Possibilities We Haven't Thought of Yet." In our group's discussion at ALM8, attention was directed at the image of a xyz axis, with numeracy, empowerment and democracy each being an axis. The issue wasn't the metric on the axes but rather the point (0,0,0). We assumed the intersection of these 3 domains. Let us continue as if indeed this assumption is true.

Because if it is true then other things must follow. Using the xyz axis and given that any point in this space might therefore describe an individual's extent of numeracy, empowerment and democracy we seem to be proposing the following: if a person comes through our door with an observable and measurably low numeracy and exits our door with an observable and measurably increased numeracy, then they must also be exiting with an increased extent of empowerment and democracy, although we don't currently "test" for this. Graphically, they have "moved" from point x(1), y(1), z(1) to point x(2), y(2), z(2), where all the values at point two are greater than the values at point one. The issue is not whether the change from point 1 to point 2 is significant, statistically or behaviorally. The issue is that if we can allow this change to occur, then a change from any point on this graph to any other point is also allowable. Let's explore the possibilities.

In the circumstance of a person coming through the door for the purpose of changing their numeracy, there are three possible outcomes: (1) an increase, (2) a decrease, or (3) no change. We have experienced but don't dwell on these latter two outcomes. For the moment, allow that these 3 outcomes are observable and measurable; that we have a tool for assessing change and that we trust it. Given our hypothesis, I believe that we have to allow that the same possible outcomes can exist for empowerment and democracy as well. And if this is so, then we have 3 possible directions of change in each of the 3 domains and thus, 27 possible outcomes to consider. However, of the 27 possible outcomes, some are outright silly while others are a bit more disturbing since they pre-

sent more immediate challenges to our beliefs about students and changes in their behaviors relative to numeracy, empowerment and democracy .

Look at the following table. The first row simply numbers the 27 possible outcomes. In the most left-hand column, N represents numeracy, E represents empowerment and D represents democracy. The entries in each row for N, E, and D mean that for that domain, the measure has stayed the same (S), increased (I), or decreased (D). Read the table using row N (numeracy) as the key. For example, outcome 14 (our hypothesis) would be read as "when numeracy increases, empowerment and democracy increase." Please recall that this is an exploration and that we could read the table in other ways but it wouldn't address our question.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
N	S	I	D	S	I	D	S	I	D	S	I	D	S	I
E	S	S	S	S	S	S	S	S	S	I	I	I	I	I
D	S	S	S	I	I	I	D	D	D	S	S	S	I	I

	15	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	
N	D	S	I	D	S	I	D	S	I	D	S	I	D	
E	I	I	I	I	D	D	D	D	D	D	D	D	D	
D	I	D	D	D	S	S	S	I	I	I	D	D	D	

Table 1

Look at just a few of the things brought out by this table. First, our hypothesis is only 1 of 27 outcomes. It tends to be the most visible one to us simply because of the accrued interest and attention we've given it over the years. Our experiences tend to favor it. But, I'm not so quick to accept that because of its visibility, outcome 14 is the most likely. Further, I suspect that these 27 outcomes are not equally likely for if they are, then our favored hypothesis has only a $1/27^{th}$ likelihood of success. In other words, of 1000 students, 37 of them are likely to experience an increase in empowerment and democracy if their numeracy increases. What power holds us to an outcome that has such a low prospect of occurrence?

Next, extracted from Table 1 and presented below are only those events where numeracy increased (see Table 2). Notice that there are nine outcomes, only one of which, 14, is our hypothesis. So, as done for the entire table above, if we were to accept that these are the only possible outcomes, then our hypothesis has a 1 in 9 chance of occurring. In other words of 1000 students, 111 of them are likely to experience an increase in empowerment and democracy if their numeracy increases. These are improved odds but still only equal to the other 8 outcomes when numeracy increases. It still begs the question of why this hypothesis holds such power for us.

	2	5	8	11	14	17	20	23	26
N	I	I	I	I	I	I	I	I	I
E	S	S	S	I	I	I	D	D	D
D	S	I	D	S	I	D	S	I	D

Table 2

Still, five of these outcomes (5, 11, 14, 17, 23) including our favored one, should leave us optimistic since with an increase in numeracy an increase in at least one of the other domains occurred. This changes our odds to 5 of 9 or in other words, of 1000 students, 556 are likely to experience an increase in empowerment or democracy or both if their numeracy increases. These are much improved odds but do not forget the assumptions taken to get to this point.

The remaining 4 outcomes (8, 17, 20, 26) present a challenge. All have one or both of the domains of empowerment and democracy decreasing when numeracy increased. These would be outcomes that would clearly force a closer look at the hypothesis.

Returning now to the other 18 outcomes in Table 1 where numeracy did not increase, I leave it to the curiosity and imagination of the reader to play with them. For example, if our hypothesis is correct then outcome 27, where all domains decrease, would be consistent with it. Outcome 13, where numeracy doesn't change while empowerment and democracy increase, is one that clearly seems reasonable, particularly if other life-changing experiences are simultaneously occurring. However, outcomes 15 and 26 are inconsistent with our hypothesis and also perhaps alarming. Are they just silly or could we conceive of an inverse relationship? Regardless, we need to consider it simply because the logic of the table offers it as an outcome.

But the essence of this "mind game" isn't to explore in depth all the possible outcomes and speculate about possible associations. Rather, the purpose is to note what might be plausible if we do indeed intend to experimentally explore our hypothesis about students' behaviors. Please note also that there is no attempt to quantify any measurable thresholds of change, although these would clearly need to be identified in an experimental context. Also, t is conceivable that there might be significant individual differences within or between populations along national, or societal or local norms.

This little exploration has been offered to hopefully provide some context for reflecting on our hypothesis. Collectively, I believe we have significant anecdotal evidence that our hypothesis is true. But again, if it is true, then the other 26 outcomes are theoretically available. The issue then is to try and craft some explorations of students' levels of empowerment and democracy and, at least, try to correlate it with their measurable numeracy skills. I firmly believe that people are empowered and act more democratically as a function of increased numeracy skills. However, this mind game leads me to pause and consider why the numeracy, empowerment and democracy relationship holds such

power for us despite the theoretical odds against such an event. We clearly need further exploration.

Literature

Tudge, Colin (1997). *The Time Before History. 5 Million Years of Human Impact.* New York, Touchstone, Simon and Schuster Inc.

Topic Group B

A changed perception of mathematics for the workplace

Lisbeth Lindberg
Göteborgs University, Sweden

In almost every country in Europe there is a trend to raise the educational level of the population. It is a political issue as well as a demand from the big companies and labour unions. The reasons are sometimes more philosophical than utilitarian.

When it comes mathematics is it said that we need to know more mathematics for the workplace. But it is sometimes hard to find mathematics. Many researchers have tried to observe workplaces and to interview workers about mathematics in their workplace. There seem to be hard to define what specific mathematics workplaces need now and in the future.

It might be time to discuss what kind of mathematics or is it frightening to talk about mathematics per se? It might be better to talk about arithmetical skills and problem posing and problem solving. I will give some examples to demonstrate the complexity of the field.

Topic Group B

Mathematics education for the workplace

Jaine Chisholm Caunt
CSET Lancaster University, United Kingdom

I am the Professional Development Educator for the Workplace Basic Skills Network, based at Lancaster University and I would like to offer a contribution to the topic discussion on "Mathematics Education for the Workplace" This will start with an initial description of my role within the Workplace Basic Skills Network, which is a membership organisation for basic skills practitioners delivering in the workplace across the UK. I have facilitated a workshop on numeracy at our annual conference for practitioners in the workplace. I would like to give an overview of current practice in the UK, of how practitioners integrate numeracy teaching into specialised basic skills programmes and other vocational training. This will consider the impact on current Government Initiatives on Basic Skills, the changes to the qualifications structure and the impact of the new Numeracy curriculum. I also intend to provide case studies of current best practice and showcase materials that have been developed by and for numeracy practitioners.

I would also like to consider ways in which numeracy teaching and learning can be an empowering and transformative process. Numeracy can be considered as socially and culturally constructed, and I would support Kerka's (1995) view that "*Critical numeracy means that learners empowered with functional skills can participate fully in civic life, sceptically interpret advertising and government statistics, and take political and social action.*"

I shall highlight the way that financial literacy and welfare rights awareness programmes can enable learners to develop numeracy skills in a relevant and exciting way, raising awareness of their rights, and encouraging participation in active citizenship. Many employers in negotiation with their employees have established credit unions within the workplace. This venture enables easy access to savings and loan facilities to people who are frequently marginalized by mainstream financial organisations; and the management by its own members, facilitates a democratic process of financial support and encourage financial literacy and numeracy within the workplace will be reviewed.

Topic Group B

Mathematical Literacy; What's in the name?

Henk van der Kooij
Freudenthal Institute, The Netherlands

Mathematical Literacy (ML) is a key issue in modern society. The term ML is used instead of numeracy, because the latter one allows narrow-minded interpretations by policymakers and politicians, like 'the ability to add and make basic calculations' (Noss, 1997).

Although there is agreement on the importance of ML, a description of what it means in terms of knowledge, subjects, skills and attitudes is hardly found. In order to find a suitable description of the required contents of ML it is best to look first at the definition of mathematics as a science. The science of number and shape is the traditional description of mathematics. A description that much better fits the mathematical activities of today is the science of patterns (Devlin, 1997).

In the OECD/PISA project, ML is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (OECD, 1999). Although the PISA project is about assessment of ML for 15-years old students, it provides a good basis for thinking about a definition of ML for citizens and workers in the modern society of today and the future. ML should not be described on the basis of mathematical contents in the first place, but on mathematical big ideas and mathematical competencies. Examples of these big ideas and competencies are found in the PISA-document.

Using these big ideas and competencies as a starting point on the one hand and using the experiences of a curriculum project in The Netherlands in vocational education and the change to competency based vocational education in The Netherlands (Onstenk, 2000) on the other hand, a description of ML will be presented and discussed in terms of the skills and attitudes needed to be a productive, critical worker and citizen in a technological society.

References

OECD (1999). *Measuring Student Knowledge and Skills; A New Framework for Assessment.* Paris

Devlin, K. (1997). *Mathematics, The Science of Patterns.* New York: Scientific American Library

Noss, R. (1997). *New Cultures, New Numeracies.* Inaugural Professorial Lecture, Institute of Education, University of London

Onstenk, J. (2000). Op zoek naar een krachtige beroepsgerichte leeromgeving. CINOP, The Netherlands.

Topic Group C

Affective Factors in Adult Mathematics Learning

Convenor: Jeff Evans, Middlesex University, London, UK

Presenters: Beth Marr, Royal Melbourne Institute of Technology University, Australia
Alan Bowd and Patrick Brady, Lakehead University, Canada

Both meetings of this Topic Group were attended by twenty to twenty-five participants. The first meeting began with an introduction of the topic and the participants. Then, in the first of two papers, Beth Marr (2001) presented her model of 'Holistic Numeracy Competence'. This includes not only skills, knowledge, and 'transfer and applications', but also areas like 'autonomy and independence', confidence, and 'personal connections'. Hence this represents a useful attempt to see cognition and affect as a whole. Beth Marr's work is based on semi-structured interviews with a small sample of experienced numeracy teachers, and is one of a number of studies of appropriate assessment methods for Adult Basic Education in Australia.

The second paper, by Alan Bowd and Patrick Brady (2001), addressed the problem that, while all adults undergoing training for primary school teaching will be required to teach mathematics, a large proportion suffer from mathematics anxiety, arguably because of their own earlier school experiences and problems with performance. In a moderately large sample of primary trainee teachers, they found relatively high levels of mathematics anxiety (as measured by the Mathematics Anxiety Rating Scale). They conclude that too little account is taken of student teachers' expressions of anxiety, and this fails to address problems that may affect their pupils later.

We include a third paper on the outcomes from practical experiments in challenging earlier beliefs and attitudes, and providing positive experiences, by Poppy Pickard and Patricia Alexander (2001). Unfortunately, this paper could not be delivered at the conference by the authors.

In the second meeting, the discussion ranged widely. Some of the conclusions articulated by members of the group include:

- Expertise (e.g. in mathematics, numeracy) should be seen as including both capability *and vulnerability*, thereby incorporating an affective aspect.
- Anxiety is not purely negative, and in any case, is not completely avoidable in learning or in life.
- We need *space* in our teaching sessions for students to be able to experience, and to express, their related emotions.
- We need to emphasise the *positive* emotions and achievements that may be experienced in learning and doing mathematics. There is an important role for the teacher in validating these.

- We may be able to use positive media images that become available at particular times in particular cultures; for example, recent British television programmes on the proof of Fermat's Last Theorem, World War II code-breaking, and so on..
- Following the reported initiatives in addressing affective issues, for example in Sweden, we need to consider the value of political interventions.

Participants seemed pleased with the group sessions. There was a willingness to discuss these issues at future meetings of ALM. Two ongoing issues that occur to the convenor at the time of writing are the relating of the work discussed here to overviews of the research in this area (e.g. McLeod, 1992), and further discussion of methodology.

References

Bowd, A. and Brady, P. (2001). 'Mathematics Anxiety and Perceived Competency to Teach', this volume.

Evans, J.(2001). 'Developing the Ideas of Affect and Emotion among Adult Learners', this volume.

Marr, B. (2001). 'Towards a Holistic Model of Numeracy Competence', this volume.

McLeod, D. (1992). 'Research on Affect in Mathematics Education: a Reconceptualisation'. In Grouws, D. A. (ed.) *Handbook of Research in Mathematics Education Teaching and Learning* (pp.575-596), New York: Macmillan.

Pickard, P. and Alexander, P. (2001). 'Breaking the Barrier – Student perception on how the necessary maths support has facilitated entry into higher education', this volume.

Topic Group C

Towards a Holistic Model of Numeracy Competence

Beth Marr
Royal Melbourne Institute of Technology University, Australia

with Sue Helme – RMIT

Background

Recent developments in Australian Adult Basic Education have resulted in a wide diversity of programs to suit many different client groups. Competency based adult literacy and numeracy curriculum frameworks have been developed at national (Coates et al., 1995) and state levels (eg. CGEA, Vic). However, with increasing focus on accreditation and accountability more has been asked of practitioners than ever before in terms of assessment and reporting. At present, practitioners have few assessment resources upon which to draw, and are struggling to meet the needs of their students on one hand, and funding bodies on the other, whilst at the same time do justice to the ABE philosophy of holistic, situated and critical education.

Support related to assessment methods in adult numeracy has been limited, and there has been little opportunity for discussion of broad assessment and competency issues. Studies by Marr & Johnston, (1999) and Marr et al. Adult, Community and Further Education Board (1998) identified unmet need for quality assessment resources and strategies which would meet the philosophies of ABE and relate to the assessment criteria of the various accredited curriculum documents. The *Holistic Adult Numeracy Assessment (HANA)* project was established to research the 'wisdom of practice' (Lovitt & Clarke, 1988) of experienced teachers and to communicate this in a practical manner, in order to guide assessment practices of less experienced teachers.

Method

Researching Current Assessment Practices of Experienced Teachers

Current assessment practices of sixteen experienced teachers were surveyed in one-to-one and focus-group discussions. The teachers taught in a range of pre- vocational, pre-employment and general literacy/numeracy programs in city and country areas. They represented a diversity of community-based providers, TAFE institutions and prisons, across Australia. The conversations were based around an interview proforma developed using issues and concerns arising from past projects and related literature. Aside from questions regarding assessment procedures, one section of the proforma probed participants' ideas on the meaning of competence and how it informed their judgments and assessment processes.

Analysis of the data from the initial survey has led to discussion on many issues, including this paper exploration of what experienced numeracy teachers mean when they judge a person 'competent' at a particular level of numeracy. It was an attempt to tease

out what they value, or take note of, when they make judgments about learners' numeracy competence. Do the existing curriculum documents capture the whole picture of competence, or is there more to it? A draft paper presenting a model of holistic competence was prepared from this data and used for ongoing discussion and consultation.

Action research with an 'Experienced Practitioner Group'
Stage two of the project used developmental, 'Action Research' techniques (Kemmis & McTaggart, 1982), with an 'experienced practitioner group' (EPG), to discuss assessment issues and extend existing methods of assessment. Six EPG meetings were held, commencing with a discussion of the draft 'holistic competence' model below. The meetings were audio-taped and transcribed by the facilitators. Issues arising in each meeting were used as stimulus for subsequent discussion, or for exploratory activities, which assisted the teachers to articulate their practice. For example, mention of 'negotiating assessment', prompted an exploration of various meanings of 'negotiation' and associated strategies which could be shared with other teachers.

As well as responding to pre-selected papers and issues, the participants were asked to select particular aspects of their assessment practices to observe and record in journals between meetings. For example, one participant, who was very interested in ideas from the original paper, began tracking the 'cues' that she used to form impressions of her students. Others experimented with different methods of keeping track of students' progress. They reported on their journal 'highlights' at the meetings and the responses and questioning from other participants provided further informative data.

Building a Model of Holistic Competence
Since it is a fundamental issue to anyone involved in making judgments about the learning of others, it is not surprising that the question *'What do you think it means to be competent in numeracy?* stimulated a great deal of thought. Perhaps more surprising was the feeling that practitioners were not accustomed to discussing this question. However, prompted to articulate their thoughts, their initially hesitant responses crystalised into extensive analysis which acknowledged the complexity of the notion of 'competence'. For example, one practitioner drew many threads together:

> It means having an understanding and the ability to do it now, and maybe in 6 months time you still have the ability to do it then, or if not … the resources to draw on to tackle it again, and the confidence to know that you can … it's just a matter of finding out how (Marg).

In spite of the diversity of teachers interviewed, there was surprising resonance in the characteristics, or 'criteria', mentioned in their descriptions of competence. The characteristics appeared to fall into clusters of essential features, which, when combined, created a portrait of a person with a growth in numeracy competence. None of these characteristics was seen as sufficient alone. They emerged from the data as complementary parts of the whole; the jigsaw pieces which combine to create the picture.

Taken together, the features described a holistic notion of competence which has at its core a change of 'Identity' or alteration of 'self concept'. The majority of teachers interviewed talked of a change from an **'I can't …'** type of person to an **'I can ..'** kind of person. That is, a shift towards an identity as a more numerate individual.

> There is a *belief* that they can't, that is very fixed. In interview they often talk in terms of being that 13 year old. Not that they are now 30 and they can do lots of other things. Its like *'I've never been able to ..'* A lot of people are very stuck with the vision of who they were when they left school. The vision that they *can't* manage or were no good at school. That whole belief system has to be challenged so that people can let in that they *will* know …(Carol).

The components fall loosely into cognitive and affective characteristics. The cognitive components were: *Skills and Knowledge*, using a *Task Process Cycle* and *Application and Transfer*. The more elusive, affective components, which we have called *Confidence, Personal connections, Awareness of learning* and *Independence or Autonomy* were seen as integral to competence by most of the experienced teachers interviewed.

Our attempts to represent these key features in a diagram have undergone several alterations during the consultation stage. The final metaphor, or model, of a jigsaw was chosen because of the interlocking nature of the pieces, and the essential part of each in completing the picture.

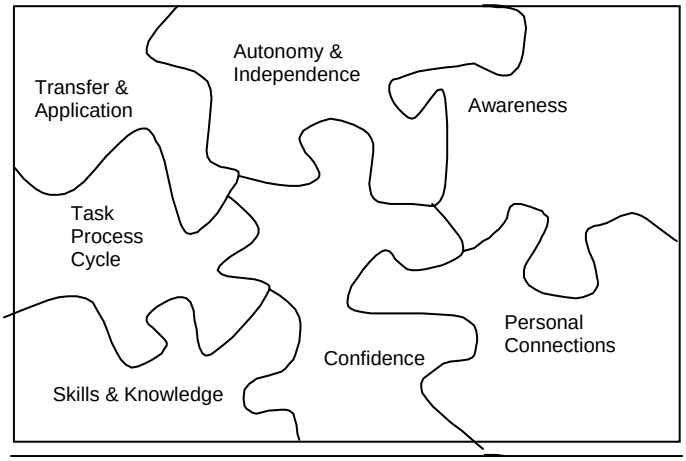


Figure 1. Model of Holistic Numeracy Competence.

The remainder of the paper describes these complementary aspects, expands on the jigsaw metaphor and discusses the implications of the model for assessment practices.

Cognitive Aspects

Using Skills and Knowledge

Achieving the skills and knowledge components listed in the curriculum documents was clearly a basic requirement for competence. Three aspects of this were highlighted by the teachers' comments: *repeated demonstration, understanding, and integration.*

Repeated demonstration: There was concern that students were able to confidently demonstrate the skills on more than one occasion. For example:

> They can do it yesterday, they can do it today, they can still do it tomorrow….I do say to the guys just doing this once won't be enough for me … They need to come back next week and do it, be able to do that again … with relative comfort, with confidence (Monica).

Understanding: Teachers wanted students to demonstrate that they had some understanding of concepts that went beyond the demonstration of skills and processes.

> The questions they ask. Like asking '*how does that work?'*. Or if you give them a formula, like for the area of a triangle, why do you have to halve it? Making those connections. *'Oh yes!, I can see that. The triangle's half of a rectangle'* (Karen).

Integration: It was also important that different aspects of numeracy became integrated, or drawn together by the students. They need to be seen as set of related competencies rather than isolated skills. Teachers said that they look for evidence that students are fitting different pieces of knowledge together and connecting new mathematics skills into their existing repertoire of past knowledge.

> The recognition of what it's about and where it fits, and applying what knowledge they had before into it…when somebody comes back and says, *'Oh, that's about so and so'*, they're recalling the earlier time…they're making the bits fit (Sandra).

Or as Carol reported "Sometimes they say 'this is like those other ones that we did' so they are really looking at that prior experience and making connections."

Task Process Cycle

Teachers stressed the importance of students being able to find a pathway through tasks, not just demonstrating skills.

> So it's that process that's involved. When they actually get down to the maths of it, it's just such and such divided by so and … but it's all this other stuff that comes beforehand that will stop them getting to that … (Ellen)

This 'Task Process Cycle' can be conceived of as four related components as shown in Figure 2.

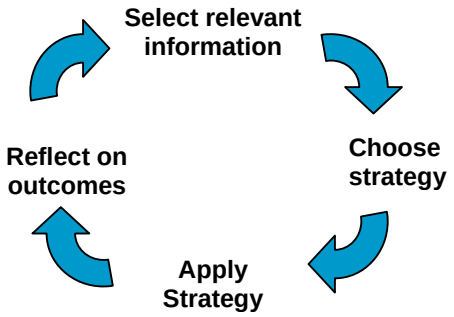


Figure 2. Task Process Cycle

Many teachers referred to the importance of fostering this approach to numeracy at all levels, and very early in the teaching program.

> I value mathematical thinking more highly than being able to do accurate tasks all the time … I have students who just do sums … reams and reams of sums where all the problem solving has been done. All they have to do is provide the answer, and they never start to think about where those numbers have come from. As opposed to students who really get in behind and try to understand the processes (Karen).

Practitioners discussed reflective thinking, or consciousness of the process, as an integral part of the cycle: "Metacognition. Thinking about thinking. *'So to solve that problem, what do I have to do?' '*'(Neil). In elaborating this aspect of competence, Carol described a person who

> worked through a problem, and then he would see there was something missing, and he would go back and change what he had done … so he was actually thinking about the processes he was using (Carol).

The evaluative aspect of the cycle was reiterated by most teachers as a sign of competence "When someone comes up with an answer and says *'it's not right'*. That they know it's not right is great (Sandra). Marg agreed that this indicated a difference in competence: "between getting the wrong answer but *knowing* you've got the wrong answer and getting the wrong answer but *not* knowing.".

Transferring and applying skills to different contexts

> I definitely don't feel that competence is only being able to perform skills. It has to be applying those skills in a variety of situations … a real-life problem that may involve skills from a number of numeracy maths areas as well as the problem solving process of getting from A to Z.

Many practitioners highlighted the importance of students being able to apply numeracy in the world outside the classroom. "It's not a matter of ticking boxes, it's a matter of really working with that student as long as you can, to see if you can make an assessment that this person has the skills (Nicole). In assessing her students, Nicole asks herself: "Would this person, in a shop, be able to deal with the money, would they be able to find their way around the world…and could they recall, if the need is there?"

Transferring and applying skills is the culmination of the cognitive domain, complementing the other aspects, since ideally, numeracy includes a combination of skills and knowledge used within the task process cycle to handle new situations.

> I think it's more knowing the process that's important and it's also knowing how to find the process or how to understand it and then it's being able to apply the skill, or dare I say, transfer the skill and being able to use it or work out how to use it (Kath)

Affective aspects of competence

Comments such as: "When I feel that they've gained the skills, can apply them over a variety of situations and have the self esteem and the confidence …to do more " indicate that the *affective* components of the model are considered essential companions to the cognitive aspects.

Confidence

This component was the most interwoven of all: the word 'confidence' arising constantly in descriptions of all other aspects. The respondents were all aware of the significant effect of mathematics anxiety on students' learning, "Self esteem is extremely important. That has to be established and built up before a great deal of learning will occur" (Ellen). Shifts in students' confidence were, therefore, seen as vital.

> … Confidence is an important part of competence. It can be something that is totally not related to maths, just the way they come and sit in the classroom. You can see confidence in their body language.. ….. It's definitely a big part of assessment even though it is not written down (Karen).

Experienced teachers explained that they look for more positive self-talk and confident body language as indicators of this kind of change. "How you rate the student has a lot more to do with how they go about the task than what the task is" (Karen). "They sort of draw it in closer and they sort of 'get into it' like engaging with the piece of paper. Whereas it's a bit like a distance thing when they are not sure. They sort of look at it and it's not theirs". (Carol)

Personal Connections

This aspect seems to touch on students' emotional relationship to their learning. It might be a connection with students' personal lives, interests and goals that motivates students to learn.

> Competence is inextricably tied up with what the student wants to achieve. They're not going to learn anything unless they have a purpose, and their purpose is more than achieving the Learning Outcomes (Sandra).

Sometimes it is the ability to see their learning as useable afterwards, applicable to their life outside the classroom, that indicates real learning taking place: "making connections between what they do outside and what's happening. Saying 'Oh this is like ..' (Yvonne).

> For example, after we had done the doll's house (practical measuring in class) one of the students said he had helped his brother build a shed … the brother didn't know where to put the end of the tape measure. Like where the nought was and they kept getting this little bit wrong. *'I* (the student) *told him it was because he was measuring from the wrong part'* Something that was real knowledge that had happened in the class (Carol).

Awareness of Themselves as Learners

Another component of competence highlighted by practitioners was students' awareness of the skills and knowledge they had gained, and the ways in which they had gained them.

> Students need to recognise what they know and understand… For somebody else to be telling them they're competent I'm not quite sure whether that helps … to be told you're competent enough to ride a bicycle but you keep falling off, you know you're not (Sandra).

Sandra also discussed the strategies that she used to assist students develop this awareness:

> I encourage students to become more aware of their own competence by pointing it out to them when they explain something to another student … sometimes they make a statement that means they understand something and I highlight it by saying that if they say it, it means they understand it…to be able to put it into words.

Nichole suggested student participation in assessment as a strategy for focusing students' awareness of their learning. "There's a lot of discussion … It's a matter of them telling me how they're feeling and whether they can do it, whether they're happy, and they also get feedback from me".

Carol explained the importance of students having an awareness of their learning style, as well as what they have learned. "Also knowing *how* you learn. This is my big thing about metacognition." She discussed a visual learner who benefited from realising that she could understand better if she drew diagrams or pictures. She also described some male students who were "very active, touchy, sort of 'doing people' - mechanics and the like … That's how they've learned things". Their learning style was validated by encouraging the use of concrete aids like blocks and counters. "They know that's how they need to do it, then they can move on from there. Once people know that it is OK to do it any way that you like, then I think that is very important for them to grow"(Carol).

Growth of Autonomy as a Learner

This dimension of competence describes a growing independence in the learner. As Nina said "Their move from dependence to independence is something I look fairly closely at" she added that "taking some control over their learning" was important to her. This sort of active participation in learning was mentioned often. Yvonne described one student's developing independence: taking class investigations home over the break and extending them. "She had the incentive to do more and more". Similar sentiments were expressed by Karen.

> I really like to see them taking charge of their own learning. It's really good when they come up to you and say *'I really don't know this well enough. What can I do to be able to do it better?'* They have the confidence to ask you questions about their learning. I like them to get involved and see that they can take control of it. They don't need me to tell them everything.

Students' growing autonomy is also evident in their willingness to have opinions and take risks. Getting started on new tasks with less assistance than before was a frequently mentioned sign.

It's the confidence to think things through without saying I don't know I've got to go to somebody else … Some of these people have been so wrong for so long, there is a real risk in putting anything down on paper at first … sort of testing out ideas. … It's a risk they get better at taking as they go on. It's sort of the confidence to have an opinion and to say what they think (Carol).

> The ability to come up with strategies, even if those strategies don't work. Looking at it, saying well that didn't work, we'll try it a different way. Students who scribble on their paper, generating ideas, thinking about what they are doing (Karen).

The Whole Picture: Change of Identity

Shifts in students' self concept, or identity, were almost universally mentioned as a central feature of change in competence. As Yvonne put it: "that whole identity of who you are, and how that changes as you become more competent". The centrality of 'identity', coming from the teachers interviewed, resonates strongly with aspects of James Gee's address at an Australian national literacy conference (Gee, 2000). He likened teaching new literacies, to recruiting someone to take on an identity. From this point of view, literacies (which include numeracy) are seen as social languages. A person 'enacts an identity' whenever they switch social languages, for instance teenagers switching communication styles for parents and for peers. Citing the seemingly miraculous amount of learning displayed by pre-readers who are keen to 'take on the identity' of a Pokemon expert, Gee suggested that if teachers could turn their 'passion for skills' into a 'passion for identity', then learning would be transformed. He contrasted this approach to the approach of school education, which breaks learning up into little bits, suggesting that if Pokemon were taught in school, then the usual children would fail.

There seems to be a link here with the desire, or ability, of numeracy students to see themselves as able to become numerate; the belief that the 'I can ..' identity is attainable.

Further to the Model: The Jigsaw Metaphor and ….

Discussion of the original version of the model centred around whether teachers agreed with the components, how the affective features influenced assessment and reporting, and how the model might encourage teachers to expanded their repertoire of assessment and/or teaching strategies. Debate about the naming, and overlap, of the aspects of the model, and the difficulty of drawing definitive lines between them, reinforced the complementary and interconnected nature of the components, indeed the complexity of competence. However, there was broad agreement that competence embraces both the affective and cognitive domains, and that teachers draw information from both when making their judgments.

There was also strong agreement that students' confidence is an important cue for assessment of competence, but how, and when, that operates is a subject of ongoing conversation. Concerns were expressed about whether *all* of the affective aspects would need to be exhibited before a student could be considered competent. On the other hand, the group also stressed the importance of drawing attention to these affective aspects, by

giving them prominence within a model, rather than allowing them to disappear into the background.

Reference to some of these aspects exists, in differing degrees, within the various accredited curriculum documents, yet the extent to which they are acknowledged or validated in practice, and how this occurs, is unclear. Teachers all 'see' it differently - through the eyes of their experience.

To consider this notion further it is useful to again draw on the jigsaw metaphor. Different people working on a jigsaw give attention to, or 'see' different things, depending on their expertise or experience. There is a temptation for new puzzle solvers to focus only on the obvious, the bright, central, features, like the skills & knowledge of curriculum documents. But more experienced solvers take time searching out the straight edges and the subtle colours of the background, realising that such strategies will pay off in the end. We could liken this to a numeracy teacher paying early attention to affective aspects of their students: taking time setting up an environment in which real learning might take place later; encouraging students to think about their learning styles and the like. Teachers might also concentrate on the Task Process Cycle: whole tasks, not bits and pieces of skills. Just as the efficient puzzle solver remembers to keep their eye on the whole picture they are trying to create, not get totally engrossed in putting together isolated features, the holistic numeracy teacher needs to alternate between the big picture - the changing numeracy identity - and the many complementary pieces which create that picture.

Concluding Remarks and Implications

If we value aspects of the model we have been examining, then the next step is to supply leadership to teachers on how to take the more holistic approach.

In order to draw attention to the affective side of the model, the EPG has been experimenting with strategies to explore students' feelings about numeracy learning and encourage greater independence. Methods to focus on students' articulating their learning, and gaining insight into learning strategies, have also been trialled with participants' students. Feedback has allowed further refinement of the ideas.

Several other strategies, related to the model, have been discussed and developed by the EPG. Open-ended tasks have been favoured, because they enable students to demonstrate their own level of competence in multi-level classes and thus provide *all* students with opportunities for success. There has been a focus on negotiated assessment strategies which encourage independence and allow students to apply numeracy skills to their own areas of interest. Using real artefacts, such as supermarket items, menus and maps, in learning and assessment tasks, was also a preferred technique. This not only validates students' informal knowledge, and facilitates personal connections with the numeracy, but also strengthens the Task Process Cycle, since the relevant information must be found from the real item, and the results of calculation related back to reality.

The project aims to publish a resource that links assessment strategies to the model and illustrates them by exemplary tasks and sample student responses. It is hoped that this

will assist teachers to expand their notions of holistic numeracy competence and the strategies for assessing it.

References

Adult, Community and Further Education Board. (1996). *The certificates in general education for adults.* Melbourne: Adult, Community and Further Education Board.

Coates, S., Fitzpatrick, L., McKenna, A. & Makin, A. (1995). *National reporting system: A mechanism for reporting adult English language, literacy and numeracy indicators of competence.* Melbourne: Office of Training and Further Education.

Gee, J. (2000). Invited Address to the Annual Conference of the Australian Council of Adult Literacy, September 2000, Fremantle.

Kemmis, S. & McTaggart, R. (1988). *The action research planner.* (3rd ed.) Geelong: Deakin University.

Lovitt, C. & Clarke, D. (1988). *The Mathematics Curriculum and Teaching Program Professional Development Package.* Canberra: Curriculum Development Centre.

Marr, B. & Johnston, B. (1999). *Absolutely Worth the Effort: Report of the Adult Numeracy and Mathematics On-line Project* (*ANAMOL*). Melbourne: RMIT & Australian Department of Employment, Training and Youth Affairs.

Marr, B., Wallace-Clancy, L., Smith, P., Halliday, P. & Goddard, R. (1998). *The evaluation of the effectiveness of the mathematics and numeracy stream of the certificates of general education for adults.* (Unpublished research report for the Victorian Adult, Community and Further Education Board.)

Topic Group C

Mathematics Anxiety and Perceived Competency to Teach Among Student Teachers

Alan D. Bowd and Patrick Brady
Lakehead University, Canada

Mathematics anxiety has been used to help explain avoidance of mathematics and low mathematics performance by students at all levels, along with poor mathematics instruction by teachers having difficulty teaching the subject through lack of confidence stemming from anxiety (Buhlman & Young, 1982; Kelly & Tomhave, 1985; Hadfield & McNeil, 1994). Mathematics anxiety, defined as Afeelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations@ (Richardson & Suinn, 1972) has been associated with lower academic achievement among students at all grade levels (Buhlman & Young, 1982; Kelly & Tomhave, 1985; Hadfield & McNeil, 1994), as well as with lower self-esteem, test-taking anxiety and mathematics avoidance (Dew, Galassie & Galassie, 1984).

Numerous studies have demonstrated an association between mathematics anxiety and gender (e.g. Meece, Wigfield & Eccles, 1990; Pajarees & Urdan, 1996). Females have been found consistently to score higher on measures of mathematics anxiety, however, the nature and importance of the correlation is widely disputed (e.g. Felson & Trudeau, 1991; Flessati & Jamieson, 1991). The relationship between gender and mathematics anxiety, evident in studies of adolescent and adult samples, has not been found consistently among elementary students (Pajares & Miller, 1995). This is compatible with the observation that girls regularly outperform boys in elementary school mathematics, implying that increased mathematics anxiety contributes to a decline in their performance at high school and beyond (Hyde,1993). The relationship between mathematics anxiety and gender is a relevant consideration in teacher education because of the significantly greater number of women entering the teaching profession, particularly at the elementary level (Ontario Ministry of Education, 1999).

Mathematics anxiety is a complex construct. Factor analysis of the Mathematics Anxiety Rating Scale MARS (Richardson & Suinn, 1972), perhaps the most widely cited measure, has resulted in varying interpretations. Rounds & Hendel, (1980) suggested that mathematics test anxiety and numerical anxiety are distinctive components, to which Ferguson (1986) has added abstraction anxiety. Factor analyses of several measures of mathematics anxiety have indicated a variety of relationships between components of mathematics anxiety and perceptions of mathematics (Bessant, 1995; Casey, Nuttall & Pezaris, 1997). Hembree (1990) noted that measures of mathematics anxiety likely reflect related constructs such as test-taking anxiety and perceived self-efficacy in mathematics. For the present study mathematics anxiety was conceived of as a combination of Adebilitating test stress, low self-confidence, fear of failure and negative attitudes toward mathematics learning@ (Bessant, 1995:327) exhibited in academic settings and daily life (Richardson & Suinn, 1872).

Mathematics anxiety among preservice and practising teachers

Teacher behaviour has been hypothesized to be a critical factor in the development of mathematics anxiety, particularly in the elementary school where teacher behaviour is assumed to be of greater influence. However, there is little empirical evidence bearing on the extent and pedagogical consequences of mathematics anxiety in both preservice and experienced teachers. Swetman, Munday & Windham (1993) proposed that mathematics anxiety among elementary teachers related to the development of negative attitudes toward mathematics among their students, resulting ultimately in depressed achievement. However their investigation found no relationship between teachers= mathematics anxiety and student attitudes toward mathematics. Mathematics anxiety among preservice elementary student teachers has been found to be unrelated to both teaching performance in schools and achievement in a mathematics pedagogy course (Battista, 1986). Researchers and educators, however, continue to propose that Athe disproportionately large number of mathematically anxious teachers at the elementary school level... may promote the early onset of mathematics anxiety among their students@ (Hadfield & McNeil, 1994:376; Hackworth, 1985).

Purpose of the study

The prevalence of mathematics anxiety among future teachers has been neglected in the literature. This is a significant omission in light of the importance widely attributed to teachers= differential treatment of boys and girls in mathematics classes. The majority of elementary teachers are women and most secondary mathematics teachers are men (Ontario Ministry of Education, 1999); it is important to understand their attitudes toward mathematics and how these might affect their confidence to teach mathematics and subsequently how they might be transferred to their students. The objectives of this study are to describe gender differences in mathematics anxiety among preservice student teachers, the relationship between mathematics anxiety and reported experiences in mathematics learning at school, and expressed confidence to teach.

Method

Participants

Participants were 357 students (242 female, 115 male) enrolled in a compulsory final year teacher education course in special education at a small Canadian university. Because the course was compulsory participants represented both elementary and secondary divisions with a variety of academic majors. Participants ranged in age from 21 to 53 years with means of 25.58 years, SD=5.57 (females) and 26.55 years, SD=5.34 (males). This study formed part of a larger investigation of mathematics anxiety and perceived teaching competence; participation was voluntary and anonymous.

Instruments

Mathematics anxiety was measured using the Mathematics Anxiety Rating Scale, MARS (Richardson and Suinn, 1972). The scale consists of 98 Likert format items designed to assess mathematics anxiety in both daily life and academic situations. Participants also completed a brief questionnaire designed to measure relevant mathematics background experience, perceptions of school experience with mathematics and beliefs about mathematics. The questionnaire further included one five-point Likert format item AI feel anxious when doing mathematical problems@. Background experience items

addressed participants= highest level of formal mathematics study, whether that study was at an advanced or general level and the number of years since formal course-work had been completed. Perceptions of school mathematics experience and beliefs about mathematics were measured using seven statements in a five-point Likert format (1 = strongly disagree through 5 = strongly agree). The items related to participants= enjoyment of mathematics in elementary and high school, whether they experienced teaching by rote in high school, their beliefs about the intrinsic interest and practical uses of mathematics, and their expressed confidence to teach in general and specifically to teach mathematics. An open-ended item permitted participants to write Aany comments related to this survey@. These were later coded by emerging themes.

It was anticipated that higher levels of mathematics anxiety would be associated with negative attitudes toward mathematics and negative school experience. Confidence to teach mathematics among elementary student teachers was expected to be associated with lower mathematics anxiety, while confidence to teach in general was expected to relate negatively with mathematics anxiety for the total group of participants. Females were expected to score higher for mathematics anxiety and to express more negative beliefs about mathematics. Females were also expected to report more negative perceptions of their school experience with mathematics. Gender differences were not predicted for background variables including level of formal study and the length of time since last taking a course in mathematics.

Results

A significant gender difference was found for mathematics anxiety. The mean MARS score for females was 204.3 (*SD* = 68.41) and the mean score for males was 173.41 (*SD* = 54.59), a difference significant at the .001 level (*t* = 4.24). These scores are higher than those reported for statistics students (Hunsley & Flessati, 1988) but lower than those reported for introductory psychology students (Flessati & Jamieson, 1991). Participants also responded to a five-point Likert format item in the questionnaire: AI feel anxious when doing mathematical problems@. A higher score indicates agreement. Mean for females was 3.26 (*SD* = 1.33) and the mean for males was 2.74 (*SD* = 1.21), *t* = 3.56, *p*<.001. The correlation between this item and total MARS score was *r* = .57, *p*<.001.

Perceived competence to teach

Mathematics anxiety as measured by the MARS was found to be associated negatively with confidence to teach. The correlation between total MARS score and expressed confidence to teach during a recent practice teaching session was *r* = -.36 (*p*<.001), and the correlation between total MARS score and expressed confidence to teach in general was *r* = -.44 (*p*<.001).

Expressed confidence in teaching mathematics during practice teaching correlated with two aspects of prior experience with mathematics. These were the level of mathematics education achieved (*r* = .25, *p*<.01) and the years elapsed since last taking a formal mathematics course (*r* = -.11, *p*<.05). Confidence in teaching mathematics during the practicum also correlated with several attitudes and beliefs about the subject. These were enjoyment of studying mathematics in high school (*r* = .31, *p*<.01), enjoyment of studying mathematics in elementary school (*r* = .24, *p*<.01), indicating that mathematics

was the least liked subject at school (r = -.34, p<.01) and indicating that mathematics is intrinsically interesting (r = .27, p<.01).

Gender and attitudes toward and beliefs about mathematics
Significant gender differences emerged with respect to experience with mathematics when in school. Males were more likely to agree that Ain high school my learning of mathematics was mostly by rote@ (t = 2.56, p<.01) and females were more likely to agree that Amathematics was the subject I liked least in school@ (t = 2.26, p<.05). Males were more likely than females to report enjoying studying mathematics at high school (t = 1.70, p<.05), however there was no significant gender difference for perceived enjoyment of mathematics in elementary school. and significant differences were found for both (males: t = -3.28, p<.001; females: t = -6.96, p<.001). The open-ended item requesting any comments related to this survey attracted responses from 62 individuals, of whom 14 referred to differential mathematics learning experiences in elementary school and high school. Eight of these were female elementary student teachers. The following are representative examples:

> AI did not have much encouragement from math teachers in high school. My perception of my teacher=s view of my math difficulties is that I was inferior or >stupid= because I required additional help or explanations. I associated being good in math as a male quality@ (*female, elementary)*.
>
> AI enjoyed math in the primary grades but after that the lessons progressed beyond my abilities and I continued to drop further behind. In high school math was a nightmare and a source of humiliation. I was gifted in English and did very well in other subjects, so I think teachers were unwilling to address my specific problems with math@ (*female, elementary)*.
>
> AI love math and it was my most exciting course in the elementary school@ (*male, elementary*).

The three items assessing beliefs about mathematics resulted in small but significant differences. Males were more likely to perceive mathematics as “useful and practical” (t = 3.41, p<.01)) and to agree with the statement that Amathematics is intrinsically interesting@ (t = 2.74, p<.01). Females were more likely to agree with the item that Amost of the mathematics I have learned has been of little use@ (t = 1.75, p<.05). The following open-ended response helps illustrate this point:

> AIt=s hard to remember how I felt twenty years ago. As well I worked for eleven years in a bank so practical applications of math are no problem. The abstract stuff - algebra, geometry etc. totally confound me. I had to take math over in summer school, all four years of high school, because I kept flunking. But I volunteer to do everyone=s income tax. Go figure!@ (*female, secondary*).

Discussion
It is important to recognize that the retrospective design employed in the present study presents some important limitations in the interpretation of results. Recollection of experiences in the elementary school may be coloured by subsequent experience, for example. Likewise, the fact that the investigation was correlational implies that caution should be exercised in reaching conclusions about causal relationships.

The student teachers participating in this study exhibited levels of mathematics anxiety comparable to student participants reported in the literature (e.g. Flessati & Jamieson, 1991) and gender differences favouring males. The present investigation is not so much concerned with the structure of mathematics anxiety as its functional consequences, particularly for women teachers. As well, a number of factors emerged in this study that bear upon questions regarding practical implications in elementary and secondary classrooms as well as the genesis and development of mathematics anxiety. The gender differences in mathematics anxiety found in this study are consistent with widely reported findings for a variety of populations. While some (e.g. Hyde, Fennema, Ryan, Frost & Hopp, 1990) have speculated that women more readily admit to anxiety or that mathematics anxiety is a form of more general test-taking anxiety, the present investigation was not so much concerned with the structure of mathematics anxiety as its functional consequences. The male and female student teachers who participated in this investigation did not differ in the extent of their formal mathematics education. Most had completed five years of secondary mathematics at an advanced level because of admission requirements for the teacher education program. Therefore gender differences that appeared in this study cannot be attributed to underlying differences in mathematics achievement. Furthermore, since males and females did not differ in the time that had elapsed since last taking formal mathematics course work this variable may also be eliminated as a possible determinant of gender differences in mathematics anxiety for the present samples. It should be noted that more males studied mathematics beyond first year university than females even though their achievement before university was equivalent. It would appear that young women who are considering teaching as a profession, and have consequently been required to complete mathematics in high school at the same level as prospective male teachers, but nevertheless adhere to the stereotype of mathematics as a masculine domain and proceed to avoid the subject at university. Further, female participants reported more negative beliefs about mathematics. Although the differences are not large, they are consistent: women were less likely to believe mathematics is useful, practical and of personal use, or that it is intrinsically interesting.

Since most student teachers will go on to teach mathematics in the schools the implications of these findings are important. Those student teachers experiencing higher levels of mathematics anxiety are likely to avoid teaching mathematics. When they do teach the subject it seems likely that they will be less effective because of a dislike of mathematics and that they may transfer their anxiety to their students. These implications are consistent with the findings that mathematics anxiety is associated with reduced confidence in one's ability to teach and particularly with reduced confidence in one's ability to teach mathematics.

The data presented here suggest that negative school experiences with mathematics are more frequent at the secondary level than during elementary schooling. Participants recalled enjoying mathematics more during their elementary education and there was a tendency for those exhibiting higher levels of mathematics anxiety to state that their learning of mathematics during high school was by rote. This represents cause for concern since it is possible to achieve well in elementary mathematics simply by following algorithms without understanding. However, in high school the amount of material to be learned and its more abstract nature (algebra and calculus) mean that high achievement without understanding is not possible for most.

The organization of the high school may also be a contributing factor in the development of mathematics anxiety. Studies suggest that the departmental structure tends to lead to a subject focus rather than the child-centred orientation that is more characteristic of the elementary school (SOURCE). One consequence of this may be reluctance on the part of students experiencing mathematics anxiety to seek help from teachers as well as less concern with the individual on the part of subject-oriented teachers teaching one hundred or more students.

The importance of experience at high school in the development of gender differences in mathematics anxiety was further supported by moderate to high negative correlations between perceived enjoyment of high school mathematics and anxiety. Among women this relationship was high. However, there were also low (for men) and moderate (for women) negative correlations between mathematics anxiety and perceived enjoyment of elementary mathematics, implying that mathematics anxiety begins in the elementary grades and that high school experience hastens its growth, particularly for females. Negative beliefs about mathematics (for example, its practical value and personal usefulness) are more prevalent among women and are better predictors of mathematics anxiety among them than for men. One possible inference is that these kinds of beliefs develop, along with a marked increase in mathematics anxiety during the high school years, and may provide a rationalization for subsequent mathematics avoidance among young women.

The fact that female student teachers in this study scored higher for mathematics anxiety and expressed more negative feelings about mathematics suggests that teacher educators should consider addressing this issue in preservice training.Teacher education programs need to address the issue of gender differences in mathematics anxiety among future teachers in both the elementary and secondary school. The assumption that specialist teachers are responsible for mathematics instruction in high school and that therefore mathematics avoidance by girls in adolescence is a function of elementary school experience fails to address the fact that mathematics is a part of all school subjects as well as daily living. All teachers need to model positive attitudes and beliefs about mathematical concepts and skills.

References

Battista, M.T. (1986). The relationship of mathematics anxiety and mathematical knowledge to the learning of mathematical pedagogy by preservice elementary teachers. *School Science and Mathematics,* 86, 10-19.

Bessant, K.C. (1995). Factors associated with types of mathematics anxiety in college students. *Journal for Research in Mathematics Education,* 26, 4, 327-345.

Buhlman, B.J. & Young, D.M. (1982). On the transmission of mathematics anxiety. *Arithmetic Teacher*, 30, 3, 55-56.

Casey, M.B., Nuttall, R.L. & Pezaris, E. (1997). Mediators of gender differences in mathematics college entrance test scores: A comparison of spatial skills with internalised beliefs and anxieties. *Developmental Psychology*, 33, 669-680.

Dew, K., Galassi, J.P. & Galassi, M.D. (1983). Mathematics anxiety: Some basic issues. *Journal of Counseling Psychology*, 30, 443-446.

Felson, R.B. & Trudeau, L. (1991). Gender differences in mathematics performance. *Social Psychology Quarterly*, 54, 113-126.

Ferguson, R.D. (1986). Abstraction anxiety: A factor of mathematics anxiety. *Journal for Research in Mathematics Education*, 17, 145-150.

Flessati, S.L. & Jamieson, J. (1991). Gender differences in mathematics anxiety: An artifact of response bias? *Anxiety Research*, 3, 303-312.

Hackworth, R.D. (1985). *Math anxiety reduction.* Clearwater, FL: H & H Publishing.

Hadfield, O.D. & McNeil, K. (1994). The relationship between Myers-Briggs personality type and mathematics anxiety among preservice elementary teachers. *Journal of Instructional Psychology*, 21, 375-384.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21, 33-46.

Hunsley, J. & Flessati, S.L. (1988). Gender and mathematics anxiety: The role of math-related experiences and opinions. *Anxiety Research*, 1, 215-224.

Hyde, J.S. (1993). Gender differences in mathematics ability, anxiety and attitudes: What do meta-analyses tell us? In Penner, L.A., Batsche, G.M., Knoff, H.M. & Nelson, D.L. *The challenge in mathematics and science education: Psychology=s response,* (pp. 237-249) Washingto, DC: American Psychological Association.

Hyde, J.S., Fennema, E., Frost, L.A., & Hopp, C. (1990). Gender comparisons of mathematics attitudes and affect: A meta-analysis. *Psychology of Women Quarterly*, 14, 299-324.

Kelly, W.P. & Tomhave, W.K. (1985). A study of math anxiety/math avoidance in preservice elementary teachers.*Arithmetic Teacher,* 32, 5, 51-53.

Meece, J.L., Wigfield, A. & Eccles, J.C. (1990). Predictors of math anxiety and its influence on young adolescents= course enrollment intentions and performance in mathematics. *Journal of Educational Psychology,* 82, 60-70.

Ontario Ministry of Education (1999). *Quick facts: Ontario schools.* Toronto: Queen=s Printer.

Pajares, F. & Miller, M.D. (1995).Mathematics self-efficacy and mathematics outcomes: The need for specificity of assessment. *Journal of Counseling Psychology*, 42, 190-198.

Pajares, F. & Urdan, T. (1996). Exploratory factor analysis of the Mathematics Anxiety Scale. *Measurement and Evaluation in Counseling and Development*, 29, 35-46.

Richardson, F.C. & Suinn, R.M. (1972). The mathematics anxiety rating scale:sychometric data. *Journal of Counseling Psychology*, 19, 551-554.

Rounds, J.B. & Hendel, D.D. (1980). Measurement and dimensionality of mathematics anxiety. *Journal of Counseling Psychology*, 27, 138-149.

Swetman, D., Munday, R. & Windham, R. (1993). Math-anxious teachers: Breaking the cycle. *College Student Journal*, 27, 421-427.

Topic Group C

Breaking the Barrier – Student perception on how the necessary maths support has facilitated entry into higher education

Poppy Pickard, School of Communications and Mathematical Sciences &
Patricia Alexander, School of education
University of North London, UK

Abstract. Brush Up Your Maths (BUYM) is an intensive short course, which has been pioneered at the University of North London for the last five years. Many mature students do not have the required level of mathematics to enter higher education. BUYM enables students to demonstrate that they can reach this necessary level. It also provides an opportunity to prepare for study after a period of absence. Students work in small groups as well as receiving one-to-one tuition.

However, for some students undergoing a BUYM course meant they had to confront their negative anxiety about mathematics. This raises the issue of empowerment; since one of the functions of a compulsory mathematics entry component is that it acts as a 'filtering mechanism' in terms of access onto their desired course and ultimately their employment potential.
For some students, BUYM was perceived as a means to an end whilst for others an opportunity to 'relearn some mathematics'. Last years cohorts are now nearing the end of their first year of their undergraduate course. This paper discusses the extent to which students' previously held views about mathematics were altered by their BUYM experience.

The student view was canvassed through a questionnaire and a follow-up interview with a smaller group. It should be stated that neither of the authors of this paper contribute to the teaching of BUYM, although one acts as academic organiser. The findings are discussed with particular focus given to the student's reflection on their mathematics competency and their views about the usefulness and purpose of BUYM. Finally the paper reviews the pedagogy of BUYM in the light of these findings.

Introduction

> "Mathematics is not only an impenetrable mystery to many, but has also more than any other subject, been cast in the role as an 'object' judge, in order to decide who in society 'can' and 'cannot '. It therefore served as the gatekeeper to participation in the decision making process of society. To deny some access to participate in mathematics is then to determine, a priori, who will move ahead and who will stay behind." [Volmink, J.]

This paper will discuss student's reflections on their mathematics competency after having completed a short intensive course in order to proceed to a degree, diploma or postgraduate course within the university. Students' previously held views about mathematics are discussed in terms of the learning environment, their engagement with the materials presented, and their expected methods of learning.

Many students are debarred from engaging in further study, or from re-entering education and training because they do not have the required level of mathematics to enter higher education. Reports concerning access provision in HE suggest (Halsey, 1993), the need for institutions to adopt a more flexible approach in their selection criteria, e.g. validating students prior experience. The University of North London (formerly the

Polytechnic of North London) is well known for its commitment to the education of students from disadvantaged groups, in particular mature students. This is reflected in a number of policy statements particularly on Access (University of North London 1990), and in the university's implementation of a 'capability curriculum' which is intended to prepare students for access to a tightening graduate employment market.

The University of North London has long been aware of this barrier and has worked to provide alternatives. Prior to 1996, one-week courses were run within the Business School and Education School to enable students to improve their mathematics, in order to demonstrate that their knowledge of the subject is of, a level sufficient for university entry. Towards the end of 1996 this process was formalised through the validation of Brush Up Your Maths (BUYM) as a university short course. The course is now well established and has recently been revalidated with an additional distant mode using delivery through WebCT, which provides student tracking and management facilities.

There is considerable mathematics education literature to suggest that mathematics operates as a 'critical filter', with respect to access to particular career paths for individuals (Burton, 1990). Indeed many students are barred from entering a career in teaching due to the lack of a grade C in GCSE mathematics. Competency in mathematics is an essential requirement for many undergraduate courses due in part to the interdisciplinary nature of the course design. An example is in the social sciences where students are expected to examine perspectives beyond the previously narrow boundaries of 'traditional' subjects, this can include an ability to generate and interpret statistics. Also, in common with many of the 'new' universities in the UK, there is an increased range of 'vocational orientated' courses, on offer that incorporate 'work-based' mathematics, for example in consumer studies and health sciences. This can be seen as a response to calls for a numerate workforce, particularly by industry and government in order to support the 'advancing technological age'. Although it may not always be apparent at the outset, it is probable that students *will* encounter some mathematical ideas during their university course.

The majority of the students in this study were mature women, many of whom wanted to pursue a career in education. As mentioned earlier, the BUYM course provides an opportunity for students to prepare for study after a period of absence. The student's prior mathematics experience is typically school mathematics curriculum or GNVQ (usually in FE college). For some students in the group, participation and engagement entailed confronting negative anxiety about mathematics per se and about their ability to do the mathematics involved. However, as elective participants, it is not surprising to find that progression on to their desired undergraduate course is a salient motivating factor within the student group. In common with findings (Hahway et al., 1993) concerning adults' motivation to learn, a typical student's personal approach to the BUYM course, can be described as goal orientated self-deterministic in character.

Learning maths in school is different to learning maths in an adult education context (although the subject content is the same). The relationships between the learner, teacher and others in the group have obvious shifts in the power exchange. Hence it would not be appropriate to adopt the same teaching methods or strategies. Adults have a wider range of experiences to draw on (positive and negative); it is not always possi-

ble for the teacher to have access to this information (as previous qualification is not necessarily a good indicator of students understanding of particular concepts). Also, it was the case that there were students who were very confident about their understanding of some areas of maths in comparison to others.

The rationale for the paper is found on the recognition that within the student group most of the students had 'fragmented knowledge' of their previous school mathematics. A recurrent feature of school mathematics is the presentation of a compartmentalised curriculum. Often students are unable to make the connective links, one example is the concept of associativity, which could be developed across a range of topics.

The pedagogical approach therefore sought to help the students make connections between areas of maths and to provide an overview of how different areas of mathematics relate to one another. Within such a short course, emphasis is placed on 'filling the gaps', in the students knowledge of aspects of a topic and to revisit areas which they had identified as being problematic. The teacher thus adopts a more supporting or facilitatory role, to assist the students make sense of anomalies and constraints in their understanding of the subject, rather than merely convey knowledge. The student's self-select areas of concern, and are encouraged making a self-evaluation of their difficulties. In this sense learning is perceived as participatory (Lave and Wenger, 1991). We were interested in the student's reflections on their learning of mathematics and what (if any) were their revised views or altered attitudes to mathematics having completed such an intensive short course.

The concept of 'empowerment' is ambiguous between different conceptions of power (Griffiths, 1998). Here empowerment is considered in terms of personal agency, described as emancipatory action, within an informed decision-making process. The purpose of this study is to gain an insight into the usefulness of BUYM in terms of the students' desired study goals, and also students' views concerning their own maths competency. Empowerment is perceived as the extent to which it enables the individual to participate, and therefore be changed in some way, as a consequence of autonomous action. The issue of empowerment goes beyond that of merely stating that the students have choices about how the learning is organised. Although this may well be the case in their experiences on the BUYM course in contrast to their formal school mathematics. One aspect of empowerment as perceived by this paper concerns 'creative acts' made by the individual (teacher and student) that facilitate learning. Another aspect of empowerment, is that of success on BUYM enabling a student to pass through a mathematical filter that has previously barred their access to HE.

As reported earlier the BUYM course is well established and its pedagogy, influenced by the work of Pickard and Cock (1996) has become well formed. Through the course students have become independent learners, who are encouraged and supported in the seeking of further knowledge. They become skilled at formulating and confident in asking questions. Small group work encourages collaborative learning and the sharing of mutual past experiences. Requiring some study before the course begins, both allows the student to perform a self audit of what they already know and also start the process of independent study. Working in this way encourages decision making and the management of learning activities.

In the early days BUYM used a study guide and an accompanying textbook, however the development of the Internet course stimulated a rewrite of all the teaching materials, which are now both, paper-based and web-based. These were used for the first time with the summer 2000 cohort who are considered in this paper. The one-week course operates on a ratio of a maximum of 15 students to one tutor. The daily start and end times appropriately flexible, making it suitable for students with family commitments. Students are sent a pre-course study guide, which they are expected to work through before joining the course; this contains work on number, calculator use and simple probability. Apart from providing information about some of the content, the pre-reading materials set up expectations about coverage and allow the student to identify areas of mathematics that they wish to focus on. Students are encouraged to reflect on what they already know to make decision concerning personal goals during the course. The remainder of the materials are contained in a second study guide which students are given at the start of the course. This covers mainly algebra, graphs and aspects of data handling.

After an introduction to the course by the tutor, students work in small groups of two, three or four. This is where the majority of the student tutor interaction takes place, thus enabling students to build their confidence and not feel threatened or intimidated. Students are able to ask for help as and when they need it, however tutors take a pro-active approach and will regularly check student's work, offering support and explanations as appropriate. Peer tutoring is encouraged within the small groups. This can be a very positive and affirming experience for a student who may have previously lacked confidence in maths. Also the class size is sufficiently small for the tutor to quickly develop an overview of a student's understanding and encourage them in this direction. Where there are certain aspects of the course that benefit from tutor input, for example understanding the gradient and intercept of a straight line, this is again accomplished by giving explanations to the small groups of two, three or four as and when they are ready.

On the fourth day of the course students are given a mock assessment, this is marked overnight and individual feedback is given the next day. Students who score very highly on the mock are told there is no need for them to take the end assessment. Students have thus been able to take and pass BUYM without the anxiety often associated with taking a 'test that counts'. The remaining students take the end assessment, three days after the end of the course thus providing a space for revision and consolidation. The pass mark is 60%, enabling students to demonstrate what they know rather than what they don't know. Students who have a marginal pass on the mock are still required to take the end assessment. Occasionally after the first mock assessment tutors may feel a student will not be ready to take the end test, in this case he or she is allowed to defer until the next group are taking an assessment (usually one to two weeks later). Last year a total of 95 students took the BUYM end assessment of these 73 passed and 62 of these joined the University of North London

The students canvassed completed the BUYM course prior to the start of their HE course. Towards the end of the second semester of their first year of study questionnaires were e-mailed to all successful BUYM students who had joined the University of North London. This was followed with a short interviews with students who were prepared to offer further information. It is acknowledged that the scope of the study is

comparatively small, and that the results need to be considered within the wider context of HE access courses.

Discussion of findings
Each question on the questionnaire is followed by a summary of the findings. Follow up interviews usually sought greater depth than the answers the respondents gave on the questionnaire itself.

- What course are you currently doing at the University of North London?
  Mainly Post Graduate Certificate in Education (PGCE) or Business students

- Was the course your first choice?
  In the main yes, one or two changed to courses with lower entry requirements.

- Before you applied for the course would you have said that the course contained a mathematical dimension / element?
  PGCE students were very definitely expecting some mathematics, about half of the others were.

- What did you think about the compulsory mathematics requirement as entry on to the HE course?
  Most agreed with the necessity for this. Although there was a frustration expressed by some PGCE students that the TTA had now made an passing an end test in numeracy a compulsory part of gaining Qualified Teacher Status.

- Before entry to the BYUM, what was your previous mathematics experience?
  All the respondents had studied GCSE or GCE Ordinary level, from one year earlier to 25 years earlier.

- What did you find to be the most useful aspect of the BUYM course?
  Respondents were asked for, the three things they liked most. Affective factors played a large part of this. Students liked the welcoming atmosphere and working alongside students who shared common goals. They felt it was intensive and concentrated and that they learnt a lot in a short time. They also appreciated the individual help and being taught in groups of two or three.
  There was a lot of praise for the teaching materials, in particular the Pre-course study guide. Students who passed at the mock found this very positive as well as unexpected. Not least there was considerable praise for the tutors who were described as:' calm with a persevering attitude', 'pro-active', 'encouraged peer support'.

- What did you find to be the least useful aspect of the BUYM course?
  No-one found three things they didn't like, however a couple of students felt the student-tutor ratio of about 1 to 12 did not allow for enough individual help. A couple of people didn't like the algebra. A couple of students found working on their own strange and would have preferred more traditional teacher input.

- Can you identify any application of or use of the mathematical knowledge, skills or concepts in your current course you where introduced to from the BYUM?
  PGCE students recognised this as being useful throughout their course and in particular for the individual audit they had to complete as part of their PGCE assessment. Business students commented that the BUYM had been useful for their Quantitative Methods module that they study in their first semester.
  There was a strong sense in which students were now making connections with their BUYM knowledge.

- Have any of your previous held views / convictions about mathematics been challenged by your experience on the BUYM course?
  PGCE students stated they were:' less worried by mathematics', 'felt better about it', 'could do it better', 'my own maths ability has adjusted, may in part be due to the teaching at school'.
  Business students reported 'a change in a positive sense'.

- What would you do differently? What advice would you give to someone completing the course next year?
  All the advice offered had a sense of reassurance in it, comments like:
  'Go for it', 'stick at it', 'do the yellow book (Pre-Course Study guide) *before you start the course', 'really work hard and it will be OK'..*

- What in your view would have been a successful model for supporting mathematics understanding for your current course?
  No other suggestions were offered, students were happy with BUYM.

Conclusion

The rationale for the paper was to explore how the BUYM experience may have shifted student views about mathematics. Students elected to follow BUYM as if completed successfully it removes the 'maths barrier' from the entrance to Higher Education. All the respondents demonstrated a very positive attitude towards the course, which allowed them to gain access to study in Higher Education.

More interestingly studying as adults enabled a shift in the power relationship between teacher and students. Whilst a connectivist approach allowed students to make links, and recognise mathematical precepts such as associativity in multiple contexts. Students were also able to be selective and focus their learning where their knowledge was patchy. The success of this is evident in the connections made between BUYM and subsequent degree study.

It is hoped to continue this work with future cohorts of BUYM students and in particular contrast the success of students completing the new Web based course with those following the existing course.

References

Burton, L. (1990). Passing through the mathematical filter - implications for students, courses and institutions.

Halsey, A.H (1993). 'Trends in Access and Equity in Higher Education: Britain in International Perspectives.' *Oxford Review of Education. Vol. 19*, No. 2, 129-40.

Hashway, R.M., Duke, L.I., & Farmer, V.F. (1993). 'The Reading Habits of Adult Learners'. *Reading Improvement 30(1)* spring, 21-25.

Lave, J., and Wenger, E. (1991). *Situated Learning : Legitimate Peripheral Participation.* Cambridge.: Cambridge University Press.

Pickard, P. and Cock, S. (1996). 'Flexible Maths at University'. In Coben, C. (Com.) (1997) *Proceedings of ALM-3 the Third International Conference of Adults Learning Maths – A Research Forum.* (pp. 123-134) London: Goldsmiths University of London.

Workshops

Multimedia Maths- Multiplying Opportunities for Formal and informal Numeracy Learning

Laura Carroll & Sarah Kowal
Adult Literacy Media Alliance, USA

How does television and video multiply opportunities for adults to practice math for a range of purposes? How could multimedia materials be used to strengthen teaching practice? We will raise these questions and provide opportunities to discuss such curriculum issues as the pros and cons of an iterative and recursive curriculum versus a sequential one, of informal versus formal learning and the role of mixed media in adult numeracy and literacy learning.

Join us during this interactive session that will demonstrate how television, video and the Internet can be used to reach and teach adults the numeracy and literacy skills they need. You will hear about a nationwide adult learning service available in the United States that harnesses the power of multimedia for use in both formal and informal educational settings. Come critique a magazine-style television show, TV411, that uses popular television genres? comedy, documentary, celebrities, animation - rather than standard educational TV, to engage adults in lifelong learning activities and practices. Also view and discuss how video clips from movies and/or commercials can be useful learning tools. You will see how a multimedia approach to quantitative literacy learning can peak and retain learner interest, build on skills and concepts that adults already know and/or use in everyday life, and reduce the negative attitudes about mathematics that many adults share. Finally, you will hear about the formative and summative research around this kind of innovative educational programming.

Participants will meet TV411's Laverne (a recurring character in the television series) in a video segment on Estimating a Job (estimating the cost of a painting job and figuring out the take home pay) and in a segment on Probability and Breast Cancer (using statistics and charts found in a breast cancer brochure to explain the concept of probability). Participants will also preview a math lesson on our new web site. Participants will engage in a hands-on activity to design a maths lesson plan that incorporates video, web, and print materials. You will have the opportunity to discuss how pop format media can be used to present explicit and embedded instruction of mathematical concepts (e.g., part/whole relationships, direct and inverse variation, ratios and probability), problem-solving strategies and attitudes about mathematics.

Numeracy Skills for Life: Numeracy in the New Adult Basic Skills Strategy in England

Diana Coben
School of Continuing Education, University of Nottingham, United Kingdom

Abstract. In March 2001, the UK government pledged at least £1.5 billion over three years in the biggest ever national drive to tackle poor reading and mathematics skills which affect up to seven million adults in England. The strategy is called *Skills for Life*. It includes:

- free basic education for all adults who want it - to be available by calling a freephone telephone number for details of local courses;
- new measures to improve the basic skills of public sector workers, including the NHS and the Army;
- help for businesses to provide training for their employees with pilots of grants to cover National Insurance for low-skilled workers doing courses;
- a new university based national research centre which will help develop best practice in teaching adults the basics;
- a new national network of up to 2000 fully equipped learning centres at colleges, workplaces and schools, where courses will be provided;
- a new curriculum and standardised tests, with early introduction in nine pathfinder areas;
- improved prison education with a focus on basic skills, and the option that prisoners will have training as one of their licence conditions will be piloted;
- Jobseekers will have a mixture of rewards and penalties to encourage them to improve their basic skills.

This announcement follows the Moser Group's review of adults' basic skills published in 1999. The Moser Report, *A Fresh Start: Improving Literacy and Numeracy*, recommended a national adult basic skills strategy to include the development of Standards and a core curriculum in adult numeracy, along with teacher education to enable practitioners to deliver the new curriculum. With the implementation of the Moser Group's recommendations, for the first time in England there is a national adult basic skills strategy, backed with serious money and coordinated by a new Adult Basic Skills Strategy Unit based in the newly-formed government Department for Education and Skills (DfES) and working across government departments. Against this background, this paper looks at the strengths and weaknesses of the national adult basic skills strategy for England in relation to adult numeracy.

Introduction

Adult numeracy is seriously under-researched – a situation which ALM is doing much to address. In England, and no doubt elsewhere, there is a shortage of experienced adult numeracy teachers and researchers. It is estimated that 40% of adults have some numeracy problems; 20% are reckoned to have 'very low'numeracy (DfEE, 1999a:19). In Nottingham, where I work, the figure is 23.7% (Basic Skills Agency figures cited in Peto, 2000:15). There is conceptual confusion over the nature of adult numeracy and the situation is complex in that adults' difficulties with numeracy may be compounded by difficulties in literacy and/or language. Research and development, including professional development of practitioners, is needed in order to clarify conceptual confusions, identify best practice in adult numeracy teaching and provide professional development opportunities necessary to ensure that it becomes widespread.

Investigation is needed at various levels (including the conceptual) in a range of contexts, including the classroom, the community, the workplace and prisons.

Against this background, in March 2001, the UK government pledged at least £1.5 billion over three years, as part of the biggest ever national drive to tackle poor reading and mathematics skills which affect up to seven million adults in England. The strategy is called *Skills for Life* (DfEE, 2001a). The Government has set a target of improving the literacy and numeracy skills of 750,000 adults with low levels of basic skills by 2004. In this paper I look at the strengths and weaknesses of the national adult basic skills strategy for England in relation to adult numeracy.

Background

The national adult basic skills strategy is part of a stream of recent reports and government policy initiatives in lifelong learning, in particular the 'basic skills' of literacy, numeracy and English for Speakers of Other Languages (ESOL). These are designed to improve the nation's education and skills, and hence increase productivity, while combatting social exclusion. These initiatives include the establishment of the National Advisory Group for Continuing Education and Lifelong Learning (NAGCELL), with a brief to advise the Secretary of State for Education and Employment on matters concerning adult learning (NAGCELL, 1999; NAGCELL, 1997). A National Skills Task Force was also established to advise the Secretary of State on developing a National Skills Agenda to ensure that Britain has the skills needed to sustain high levels of employment, compete in the global market place and provide opportunity for all (DfEE, 2000). The Prime Minister, Tony Blair, has established a Social Exclusion Unit (SEU) aiming to bring about 'joined up government', co-ordinating the activities of government departments on specific projects and initiatives intended to benefit groups at risk of social exclusion. A White Paper *Learning to Succeed* (DfEE 1999b) set out the government's wider agenda on post-16 education and training and presaged the Learning and Skills Act 2000 (Great Britain, 2000), which restructures the whole of what is now known as the 'Learning and Skills sector'.

The Moser Report

The ground-breaking report which dealt specifically with adult basic skills was the Moser Report (DfEE 1999a), which recommended:

- National targets
- An entitlement to learn
- Guidance, assessment and publicity
- Better opportunities for learning
- Quality
- A new curriculum
- A new system of qualifications
- Teacher training and improved inspection
- The benefits of new technology
- Planning of delivery

Skills for Life

Skills for Life: the national strategy for improving adult literacy and numeracy skills (DfEE, 2001a) follows on from the recommendations of the Moser Report (DfEE, 1999a). With the implementation of the Moser Group's recommendations, for the first

time in England there is a national adult basic skills strategy. The strategy is backed with serious money and coordinated by a new Adult Basic Skills Strategy Unit based in the newly formed government Department for Education and Skills (DfES, formed in June 2001) and working across government departments.

The strategy (some of which, confusingly, pre-dates the launch of *Skills for Life)* includes a Core Curriculum in adult numeracy (BSA, 2001) based on Standards (QCA, 2001), with associated teacher education (see the Basic Skills Agency website at www.basic-skills.co.uk); and tests for learners (see the DfES website at http://www.dfes.gov.uk/pns/pnattach/20010114/1.htm). Basic skills are mapped onto the national qualifications framework as indicated in *Figure 1*, below.

Figure 1. The national qualifications framework (BSA, 2001:4)

<table>
<tr><td></td><td></td><td>Key Skills
Level 5</td><td>National qualifications framework
Level 5</td></tr>
<tr><td></td><td></td><td>Key Skills
Level 4</td><td>National qualifications framework
Level 4</td></tr>
<tr><td></td><td></td><td>Key Skills
Level 3</td><td>National qualifications framework
Level 3</td></tr>
<tr><td>National Curriculum
Level 5</td><td>Literacy/Numeracy
Level 2</td><td>Key Skills
Level 2</td><td>National qualifications framework
Level 2</td></tr>
<tr><td>National Curriculum
Level 4</td><td>Literacy/Numeracy
Level 1</td><td>Key Skills
Level 1</td><td>National qualifications framework
Level 1</td></tr>
<tr><td>National Curriculum
Level 3</td><td>Literacy/Numeracy
Entry 3</td><td rowspan="3"></td><td rowspan="3">Entry Level</td></tr>
<tr><td>National Curriculum
Level 2</td><td>Literacy/Numeracy
Entry 2</td></tr>
<tr><td>National Curriculum
Level 1</td><td>Literacy/Numeracy
Entry 1</td></tr>
</table>

I discussed the new curriculum in a paper presented at the Ninth International Congress on Mathematics Education (ICME9), where I gave it a cautious welcome (Coben, 2001). Here my focus is on the strengths and weaknesses of the *Skills for Life* strategy as a whole, as shown in *Figure 2, Literacy and Numeracy Skills Strategy*, (excerpted from *Skills for Life)* and the ambitious agenda announced by the Secretary of State for Education and Employment in March 2001 and set out below (see next page):

Figure 2: Literacy and Numeracy Skills Strategy - From: *Skills for Life* (DfEE, 2001:14-15)

Our strategy will target those in key priority groups with literacy and numeracy needs:

Those with literacy and numeracy needs in regular contact with government and its agencies, comprising

- 280,000 unemployed people
- 1.5 million other benefit claimants
- Around 250,000 prisoners and people supervised in the community

Around 200,000 public sector employees with literacy and numeracy needs in

- Central government
- Local government
- Armed Forces
- National Health Service

Approximately 1.5 million low-skilled people in employment with literacy and numeracy needs, particularly

- Those in occupations and sectors with low average literacy and numeracy rates
- Young people in employment

Other groups at high risk of exclusion due to poor literacy and numeracy skills, including

- Around 60,000 homeless people with literacy and numeracy needs
- Up to 1 million refugees, (…) and other speakers of English as an additional language
- Parents with poor basic skills, including the 250,000 lone parents with no qualifications
- 1.7 mil. adults with lit. & numeracy needs who live in disadvantaged communities

Learners' needs will be identified, addressed and monitored by government agencies and partners, including:

- Employment Service
- Benefits Agency
- Health services
- Community and voluntary organisations
- Prison Service
- Probation Service
- Public sector employers
- Employers
- Trade unions
- National training organisations
- Small Business Service
- Connexions Service
- Employment Service programmes
- Social Services
- Local Authorities
- Residents' Associations
- Learning partnerships
- Local Learning and Skills Councils
- Voluntary and community organisations
- Religious bodies
- Health services
- Refugee Council and similar bodies
- Age Concern and similar charities
- Football clubs
- Libraries
- Informa., Advice and Guidance Partnerships

Free training will be provided through:

- Dedicated provision e.g. family literacy programmes for parents
- Full-time courses, including intensive 'booster' courses
- Part-time courses
- Self-study, 'mentored' learning and **learndirect**

Our strategy will, by 2004, improve the literacy and numeracy skills of 750,000 adults in England, comprising:

- 130,000 jobseekers
- 40,000 other benefit claimants
- 40,000 prisoners and others supervised in the community
- 10,000 public sector employees
- 50,000 adults in low-skilled jobs
- 110,000 young people
- 210,000 general basic skills learners including those on **learndirect**
- 50,000 refug. and speakers of other languages
- 60,000 parents
- 50,000 people who live in disadvantaged communities

- free basic education for all adults who want it - to be available by calling a freephone telephone number for details of local courses;
- new measures to improve the basic skills of public sector workers, including the NHS and the Army;
- help for businesses to provide training for their employees with pilots of grants to cover National Insurance for low-skilled workers doing courses;
- a new university based national research centre which will help develop best practice in teaching adults the basics;
- a new national network of up to 2000 fully equipped learning centres at colleges, workplaces and schools, where courses will be provided;
- a new curriculum and standardised tests, with early introduction in nine pathfinder areas;
- improved prison education with a focus on basic skills, and the option that prisoners will have training as one of their licence conditions will be piloted;
- Jobseekers will have a mixture of rewards and penalties to encourage them to improve their basic skills. (DfEE, 2001b)

What follows is a personal view, offered at the beginning of what is by any standards a major initiative in a sorely neglected field.

Some Strengths and Weaknesses of *Skills for Life* in Relation to Adult Numeracy

There are forty-four incidences of the word 'numeracy' in the *Skills for Life* Executive Summary, each one linked with literacy and/or English for speakers of other languages. So the first strength of *Skills for Life* in relation to adult numeracy is that numeracy is *in there*; it is recognised as important, and if it is usually mentioned after literacy and ESOL perhaps we should not read too much into that. The unprecedented scale and breadth of the *Skills for Life* strategy is to be applauded. The fact that such a wide range of organisations, from government agencies to football clubs, are to be involved means that the initiative should have a chance to put down roots in society and outlast the three to five years earmarked by the government. The targetting of groups of adults with low basic skills who are at risk of social and economic exclusion as 'key priority groups' should mean that money is spent where it can make most difference to the lives of disadvantaged adults. Given that poor numeracy skills have a more deleterious impact on adults' life chances than poor literacy skills (Rivera-Batiz, 1994; Bynner & Parsons, 1997a; 1997b; 1998; Bynner *et al*, 2001), this has to be good news for actual and potential adult numeracy students. The range of free 'training' in adult basic skills (and the fact that it *is* free) is to be welcomed, including full-time courses and 'family literacy and numeracy' programmes for parents. Increased provision of adult numeracy education, including full-time courses offering the chance of intensive study, can only be a good thing.

The most important strength of the *Skills for Life* strategy, though, for me, is the inclusion on the agenda of "a new university based national research centre which will help develop best practice in teaching adults the basics" (DfEE, 2001b). This is particularly important for adult numeracy, given its hitherto under-researched state (Brooks *et al*, 2001). The relevant paragraphs (131-135) in *Skills for Life* state:

A national research centre for adult literacy and numeracy

131. There are still gaps in our knowledge about what will motivate large numbers of adults to take part in learning and what will help them to achieve rapid and significant improvements in their literacy and numeracy skills. We cannot afford to leave the methods we use to chance. We have therefore embarked on a programme of investigation designed to identify best practice from around the world and to test out a range of new ideas and approaches.

132. In January 2001 we held an international colloquium for some of the world's leading experts in literacy and numeracy skills to share their knowledge and experience. Our strategy was found to be comprehensive and consistent with efforts elsewhere. It is also evident that the projects in our pathfinder areas will make a real contribution to world understanding of these issues. In addition, we have set up a working group with the Governments of Denmark, Ireland and Portugal to look at literacy and numeracy skills within the context of the European Union. The report from this group is being submitted to the European Union summit meeting in Stockholm in March 2001.

133. Our strategy must be supported by a more continuous programme of research to ensure that its implementation and future development are based firmly on evidence. We are commissioning a baseline survey of literacy, language and numeracy need in England to determine in more detail the scale of the problem we have to tackle. We will establish a national research centre for adult literacy and numeracy to lead and co-ordinate this work. Its main function will be to conduct research into pedagogical practice, drawing from relevant international experience and developments at home. Among countries of the English-speaking world, we aim to lead the way in tackling literacy and numeracy problems, and we will want to exploit any knowledge and best practice we have learned in overseas education markets.

134. By identifying best practice and innovative research, the research centre will develop and deliver teacher training and professional development based on quantitative and qualitative evidence of the effectiveness of different approaches to learning, working with and building upon the work of the Basic Skills Agency. Its work will lead to the development of a qualifications structure for teachers of literacy, numeracy and English as an additional language, which will provide a clear career path for those who wish to specialise. And most importantly, its work will enable us continuously to evaluate and to improve our national strategy. We aim to complete the first stage of this evaluation in early 2002.

135. The centre will be based in an existing institution or a consortium of institutions and will be chosen through a competitive tendering process. We are aiming to announce the location of the centre this autumn so that it can be operational by early 2002. It will need to work closely with the Adult Basic Skills Strategy Unit and form a close partnership with the Basic Skills Agency. (DfEE, 2001a).

This offers a real possibility that theory, research and practice may at last come together to inform policy in this area. If adult numeracy can hold its own in this context, it might just be about to come in from the margins of educational provision and public concern.

So what are, or might be, the weaknesses of *Skills for Life* in relation to adult numeracy? Here, I think, the question is more how *Skills for Life* may be interpreted, rather than what is in the document itself. For example, if numeracy remains the 'poor relation' of literacy, despite its equal footing in the *Skills for Life* document, the full

potential for adults to improve their 'basic skills' will not be realised. The new adult numeracy core curriculum is another case in point, and a key one. It remains an enigma at this stage in the implementation of the adult basic skills strategy: much depends on how it is developed by practitioners and the response of adult learners. As I argued in my paper for ICME9, the curriculum is heir to the tensions and contradictions between several pairs of opposing forces. These I characterised as: the need for social inclusion and for economic competitiveness; the urge to maximise efficiency and to develop creativity; the rhetoric of lifelong learning for individual fulfilment, set against the reality of constant re-training in an insecure job market, stressing the importance of adults' contexts, while prescribing content (Coben, 2001:145). It is too early to say how this will work out in practice.

In one area, policy in relation to unemployed people, there is real cause for concern. The document includes the announcement that in two pilot areas a requirement is to be introduced that unemployed people with literacy and numeracy deficiencies must "address their needs" or risk losing benefits (DfEE, 2001, para.25). This fundamentally changes the relationship between adult student and provision and student and teacher from one in which students attend voluntarily to one in which they attend under direct financial duress.

In my conclusion to the ICME9 paper I urged that the curriculum should be seen as the beginning, not the end of a process. The same goes for the *Skills for Life* strategy as a whole:

Those of us who care about adult mathematics/numeracy education should rejoice that adults' basic skills have such high priority, backed by major resources at the heart of government. The title of the government's press release on March 1, 2001, says it all: *Free Basic Education for all who need it as part of biggest ever national basics drive* (DfEE, 2001) and we should welcome this. […] we should welcome the fact that adult basic skills education is regarded as an entitlement in the new adult basic skills strategy. We should rejoice that numeracy/mathematics education has made it onto the adult basic skills agenda at last and that an adult-specific curriculum has been developed, rather than just offering the children's National Curriculum to adults in unadulterated form (pun intended). But this must not be the end of the story. (Coben, 2001:145-6)

Whether adult numeracy can become a real 'skill for life' for those for whom it remains a closed book, will depend to a great extent on how practitioners, researchers and adult learners respond to the government's challenge. It is to be hoped that the Research Centre will play a key role in this process.

Literature

Brooks, G., Giles, K., Harman, J., Kendall, S., Rees, F. & Whittaker, S. (2001). *Assembling the Fragments: A Review of Research on Adult Basic Skills.* RR220 National Foundation for Educational Research. Nottingham: Department for Education and Employment.

BSA (2001). *Adult Numeracy Core Curriculum.* London: Cambridge Training and Development on behalf of The Basic Skills Agency.

Bynner, J., McIntosh, S., Vignoles, A., Dearden, L., Reed, H., Van Reenen, J. (2001). *Improving Adult Basic Skills: benefits to the individual and to society*. DfEE/DfES Research Centre, Wider Benefits of Learning, Centre for Longitudinal Studies, Institute of Education, Centre for Economic Performance, London School of Economics Institute for Fiscal Studies. Research Report No. RR251. Sudbury: DfEE.

Bynner, J. & Parsons, S. (1998). *Use it or Lose it? The impact of time out of work on literacy and numeracy skills*. London: Basic Skills Agency.

Bynner, J. & Parsons, S. (1997a). *It Doesn't Get Any Better. The impact of poor basic skills on the lives of 37 year olds*. London: The Basic Skills Agency.

Bynner, J. & Parsons, S. (1997b). *Does Numeracy Matter? Evidence from the National Child Development Study on the impact of poor numeracy on adult life*. London: The Basic Skills Agency.

Coben, D. (2001). 'Fact, Fiction and Moral Panic: the changing adult numeracy curriculum in England'. In: G.E. FitzSimons, J. O'Donoghue & D. Coben (eds) *Adult and Lifelong Education in Mathematics. Papers from Working Group for Action (WGA) 6, 9th International Congress on Mathematics Education, ICME9.* Melbourne, Australia: Language Australia in association with Adults Learning Mathematics – A Research Forum, pp125-153.

DfEE (2001a). *Skills for Life.* Nottingham: DfEE (www.dfes.gov.uk/readwriteplus).

DfEE (2001b). *Free Basic Education for all who need it as part of biggest ever national basics drive.* Press Notice 2001/0114, 1 March, 2001. London: Department for Education and Employment.

DfEE (2000). *Skills for All: Proposals for a National Skills Agenda*. Final Report of the National Skills Task Force. London: Department for Education and Employment.

DfEE (1999a). *A Fresh Start: Improving literacy and numeracy*. The report of the Working Group chaired by Sir Claus Moser (The Moser Report). London: Department for Education and Employment.

DfEE (1999b). *Learning to Succeed: a new framework for post-16 learning*. White Paper, Cm 4392, June 1999. London: The Stationery Office.

Great Britain (2000). *Learning and Skills Act 2000: Chapter 21*. London: Stationery Office.

NAGCELL (1999). *Creating Learning Cultures: Next Steps in Achieving the Learning Age* - the second report of the National Advisory Group for Continuing Education and Lifelong Learning, Chair: Professor R.H. Fryer. London: NAGCELL.

NAGCELL (1997). *Learning for the Twenty-first Century*. First report of the National Advisory Group for Continuing Education and Lifelong Learning, Chair: Professor R.H. Fryer. London: NAGCELL.

Peto, B. (2000). *Thinking Ahead 2000/2001*. Prepared for Greater Nottingham TEC by The Observatory, November 2000. Nottingham: The Observatory, Nottingham Trent University.

QCA (2001). *National Standards for adult literacy and numeracy*. London: Qualifications and Curriculum Authority.

Rivera-Batiz, F.L. (1994). 'Quantitative skills and economic success in the labor market', *IUME Briefs,* Teachers College, Institute for Urban and Minority Education, Columbia University, New York.

Personal methods as a means of achieving empowerment and democracy

AWorkshop session

led by **Janet Duffin** (University of Hull, United Kingdom)
with **Henk van der Kooij** (facilitator) **Diana Coben, Terry Maguire** and **Anne Simonsen**

Preamble

The development of personal mental calculating methods has always been of interest and concern to me. It has featured in many of my sessions at ALM from its very beginning. At ALM 1 (1994) I did a session called *Doing what comes naturally* where I spoke of the research into learning I was doing with a colleague as well as looking specifically at the calculating methods of students I had encountered in my work with innumerate undergraduates. My work with children in the CAN (Calculator Aware Number) project (Shuard et al., 1991) made me acutely conscious of the distinction between these two groups: the children having supreme confidence in their own mental calculating ability, the students having none.

As a result of these concurrent experiences I have always felt that we should be encouraging inner competence with number in everyone in the community. Consequently, the issue has come up in a variety of guises in my contributions to ALM proceedings over the ensuing years. The general theme of this year's conference, *Numeracy for Empowerment and Democracy?*, therefore, seemed to offer a golden opportunity for pursuing the issue once more, which is why I decided to put forward a proposal for a workshop on it.

Preliminary remarks to participants

What I should first like us to do in this workshop is to investigate a number of personal methods for a particular calculation 17 x 17 which arose spontaneously in conversation with two ordinary members of the public on different occasions. I should then like to try classifying different types of calculation employed by people, and follow this by discussing some of the issues involved in trying to help everyone to develop their own personal mental calculating methods. Finally, I hope to discuss what might be involved in maximising the potential of such methods as well as the use of ‘jottings’ to supplement an individual's short-term memory span while calculating.

Evidence of the disempowerment of many adults through their sense of inadequacy in both mathematics, and that special part of it which we know as numeracy, is amply recorded in the pages of ALM proceedings and elsewhere. Indeed, I would postulate that all of us involved with ALM spend a great deal of our time trying to undo the effects of previous experience which appear to have disempowered so many adults in the community. With this in mind it is not difficult to be aware that, if the disempowerment could be reversed as the population became truly numerate, this could be a satisfactory outcome for adults who feel less adequate as human beings because of their lack of numeracy (Duffin, pending). Moreover, it could, in consequence, make

them better equipped to contribute to the furtherance of democracy in whatever country they currently inhabit.

Plans for the workshop were as follows:

- to record participants' mental methods of calculating 17 x 17, noting the categories of methods produced
- to tell the story of the two people whose methods for this helped to initiate the session
- to attempt to classify possible mental methods for this and other calculations as brought forward
- to discuss individual mental methods and how to enlarge short term memory capacity by using jottings as an aide-memoire
- if time, to outline the steps taken to advance general number competence in the countries represented by the participants.

Record of the actual session
Firstly, there were only three participants besides the facilitator and myself so the session did not go quite according to plan though for me it was interesting and useful. I decided therefore that the final report should be attributed to all the participants not merely to its proposer, hence the inclusion of the names of all participants at the top of this report.

Participants' mental methods for 17 x 17

1. I'd say 17 x 10 is 170; 17 x 2 is 34 (because I already knew that); I need another 7 lots of 17 (on top of the 170) so that's 3 times 34, which is 102, plus another 17. I pictured that (34 x 3) in my head as if it were on paper. Then I'd add 102 to 170 (272) and add the other 17 to that to give me 289. Phew!
2. I saw a picture of it. Went 7 x 7 = 49; 7 x 10 is 70 (giving me) 119; 10 x 17 = 170; 119 + 170 = 289. Exactly as I would do it on paper.
3. First I said it's between 15 x 15, that's 225, and 20 x 20, that's 400, so the result is near to 300. To get the precise answer I said 10 x 17 = 170 and added 5 x 17 (that's half 170 which is 85) and then added 2 x 17 which is 34. It's just a little bit difficult to add those three numbers in your head but 170 + 85 is 255, and 255 + 34 is 289.

Two stories from people in the community
After inviting participants to describe their personal mental method for 17 x 17, I told two stories from my own experience:

While conversing with a colleague, he quite suddenly and spontaneously said: If I was asked to do 17 x 17 in my head, I would say 20 seventeens is 340 so I would have to take away 51 to get the answer 289 but I always thought that was the wrong way to do it.

When telling this story to my daughter a few days later, she said: Oh, I wouldn't do it that way; I'd say 10 seventeens is 170; ten sevens is 70 and 7 sevens is 49 so the answer is 289; I'd do it that way because it's easy; I can't do anything hard.

What was interesting to me about these two stories was that both these people were slightly apologetic about their way of calculating in their head and yet both had found excellent ways of doing the calculation. It is my feeling that this apologetic attitude relates to the fact that, in the UK at least, little attention has generally been paid to the encouragement of children's own mental calculating methods, save through the CAN project which positively cultivated personal methods. The result is that people, whom I believe to be very resourceful in thinking out personal methods, do not enter adult life with any sense of the worth of these methods. Hence the hesitation and the air of apology.

Another interesting feature of the session was that the facilitator had told me that he too had always encouraged the development of mental calculating methods when he had been working in a school so he too was interested to see what the outcome of the workshop would be. So after the introductory parts of the workshop outlined above, he spoke about his experience gained from trying to develop personal mental calculating methods among the pupils he had taught in school.

He described how he put the pupils into groups and encouraged them to devise their own methods for working out the problems he set them. He spoke of how difficult it sometimes was for the tutor to wait for the children's own methods to emerge - a feature also encountered by the teachers in the CAN project (Duffin 1996) who used to speak of how difficult they found it to hold back and not intervene as pupils struggled to find their own way of calculating something.

Discussion of some of the problems he used to give his pupils led to him also speaking of how children proved to be perfectly capable of finding an arithmetical solution to problems for which a mathematician would normally go straight to an algebraic method. This too is something I have often met in my encounters with children both within and outside the CAN project.

The issue of the use of calculators entered the discussion and the way in which, in the UK at least, these are discouraged at an early age, a position reinforced by a lot of media attention and the prejudice of the general public against their use. This in spite of the fact that work in Scandinavian countries* and in Australia** - in contrast to beliefs in Germany and Switzerland (Bierhoff 1996) - has demonstrated that their use can foster number competence and the development of mathematical ways of operating.

Postscript
One participant at least has emailed me to say that the session ‘reinforced my belief that there is more than one way to do things and students, when given the space, can often find their own way of working out problems - not often the traditional way’. Another, a teacher, wrote ‘For me it was a good experience to hear that there are others who think in the same ways about learning mathematics’. The third said that her instinct, on being asked to do a calculation in her head, was to reach for pen and paper and that she tended to picture a calculation in her head as if it were written on paper.

There wasn't time in the actual session to discuss, as planned, the various methods produced by the participants nor to discuss the ways people can use jottings to aid their mental calculation when these become a little too burdensome.

It is interesting that two of the participants mentioned picturing the sum in their head as if it was on paper though they used this picture at different stages of the calculation. This in itself indicates the power of our early learning of the traditional pen and paper method for multiplying which continues to dominate an attempt to do a calculation mentally.

In the first of the three methods recorded above the participant started the calculation with a genuine mental approach to it using the obvious easy way into it of doing 17 x 10 first, followed by a mention of twice 17 before going on to calculate the remaining 7 seventeens required by means of a reorganisation of the numbers making use of the 34 she already knew. It was only at the final stage that she resorted to a pictorial view of the calculation of three 34s before finally adding together the three numbers she had now obtained. She did not tell us how she did the addition which would also have been interesting. While this method appears on the face of it to be a rather tortuous one, as her final exclamation of 'Phew!' indicated, it has within it some of the seeds of competent mental calculation and showed a versatility that is one of the qualities of that competence.

In the second calculation the participant claimed that she saw a picture of the 'sum' in her head and did it exactly as she would have done it on paper. However, there are some quite interesting apparent anomalies here for, in calculating the units line of the traditional multiplication algorithm, she condensed the mental calculation (in her recorded version of her procedure) into 7 x 7 = 49 and 7 x 10 = 70 = 119 before doing the final addition of 119 and 170 (10 x 17).

I have met this practice when recording a mental calculation before. It occurred from time to time amongst children in the CAN project and, when it did so, mathematicians were concerned that the children were getting into bad recording habits which could be detrimental to their later mathematical development. A method of recording for such examples was sought though the project team felt that, as a recording of the sequence of the mental processes the children were trying to represent on paper, it should be accepted without comment until a later date when the formal recording of mathematics was being developed.

At this stage I would like to examine this in terms of the actual process that takes place in the mind of the person doing the calculation. Let us take the example above. The participant was describing what she got from multiplying 7 by 7 followed by doing the same for 7 by 10 before adding the two answers. It is likely that she held the answer 49 in her mind before going on to get the 70 and that her mind almost immediately gave her the 119 she got by adding the two so, in recording it, she merely wrote down the answer using the symbol = to indicate that the result would be the sum of the two. She might have said 'giving' instead of using the equals sign but this seemed a simpler way to record it. Is this, then, an example of the mind doing something but in the articulating of what was being done being led astray mathematically? Or do the words

of a CAN child who, on being asked how he had done a particular calculation said 'I just held (that bit) in the back of my mind while I did (that bit) in the front of my mind', offer a telling, if not neurological, explanation of what occurs in the mind when attempting a multifaceted mental calculation: something akin to the use of the memory keys on a calculator perhaps.

This is speculation on my part but I can think of no other explanation for a mathematically aware adult, when attempting to record her own mental processes, making what can be a common recording error in those not yet familiar with the precision of mathematical recording and the meaning of the equals sign. But it is an explanation that accords well with the experience of the CAN children whose mental calculating facilities became very highly developed.

The third example is interesting in a different way for here the participant started her approach to being asked to do the calculation mentally, by looking at 20 x 20 and 15 x 15 which she knew without having to think about them. Knowing square numbers can be very useful as part of the internal equipment which is helpful for acquiring number competence, and helping with mental calculations, but in this instance their use provides an estimate to give her some initial feel for the range within which her answer must lie. This practice of finding an approximation (estimate) before doing any calculation is one I always recommend - and more especially when a calculator is to be used for the calculation. It is part of the repertoire of my guidelines for becoming numerate and having confidence that the correct answer has been arrived at precisely because of the empowerment that this kind of confidence can engender.

It is interesting to note that this participant, who does not, as do the others, use any form of the traditional multiplication algorithm, merely splitting the calculation into three parts: 10 x 17, 5 x 17 and 2 x 17 (using what might be called a 'partition' of one of the seventeens into 10 + 5 + 2 in order to simplify what she has to do) falls into the same trap as that mentioned above when she records her thinking. Is it legitimate to suggest that these participants provide some evidence of the truth of my speculation about what is happening in the brain in connection with such mental calculations? It is not the first time that I have found examples of similar phenomena occurring amongst both CAN children and the adult population.

I am indebted to the three participants at the workshop on mental calculating methods for their generosity in supplementing my faulty memory by emailing me about the session as they remembered it and to the facilitator for his contribution about how he worked with children encouraging them to develop their own personal calculating methods. Without them I would have been unable to write this report.

Because so few people attended the workshop it has not been possible to provide any more evidence to support the claim that the development of competent mental calculating methods can contribute to empowerment and democracy through numeracy. However, I remain sure that there is a host of exciting mental calculating going on all around us and I hope to return to the idea through an article in the ALM newsletter which will build on what came out of the workshop.

Notes

* Similar projects to the CAN project have been carried out in Denmark and Sweden but I have no direct references for them.

** In Australia the calculator project known as CPM (Calculators in Primary Mathematics) was based on the CAN project and corroborated its findings. Publications (sundry) and reports at international conferences (ICME - International Congress of Mathematics Education - 1988, 1992, 1996) are available and a video of children calculating can be obtained from Susie Groves at Deakin University, Melbourne, Australia.

References

Bierhoff, H. (1996). *Laying the Foundations of Numeracy*, Discussion Paper no. 90. London: NIESR (National Institute of Economic and Social Research).

Duffin, J. (1996). *Calculators in the Classroom*. Liverpool: Manutius Press.

Duffin, J. (pending) 'Numeracy in Higher Education' in *Effective Learning and Teaching in Mathematics and its Applications*. London: Kogan Page Publishing Limited.

Shuard, H. et al. (1991). *Calculators, Children and Mathematics*. London: Simon and Schuster.

Gender in ALM – Women and Men Learning Mathematics.

Inge Henningsen
University of Copenhagen, Denmark

Abstract. Paul Ernest and Mary Harris gave keynote addresses on "Images of Mathematics, Values and Gender" and "Women, Mathematics and Work", at ALM2 and ALM3 respectively, both addresses explicating the need for gender being considered an important, explicit and independent factor in ALM. That women and men to a great extent inhabit different cultures both at work and in civil life points to the need for a gender specific notion of numeracy and a gender conscious teaching of mathematics, since women and men must be expected to approach mathematics from different perspectives, encounter different difficulties and bring different strengths to the classroom. It is argued that the pervasive notion of "Mathematics=male" have unsuspected gender implications. It might cause male mathematics anxiety to be more deep-rooted and identity threatening, and when empowerment and democracy are linked with mathematic ability women are implied to less democratic. Gender is an important agent in adults' relationship with mathematics and numeracy, and this strongly gendered learning of mathematics should imply a strongly gendered teaching of mathematics.

Numeracy is gendered.

Numeracy is destined to be a key concept for adults learning mathematics. (See e.g. Coben et al, 2000; Lindenskov & Wedege, 2001). One strength of the concept of numeracy is that it is lifted out of the eternal, disembodied realm of pure mathematics and grounded e.g. in time and place (Ibid.; Evans, 2000). This, however, points to a gender specific concept of numeracy related to the fact that women and men to a great extent inhabit different cultures both at work and in civil life.

Hence my research question:

- *How should gender enter into a definition of numeracy?*

Gender is of course not the only aspect to be taken into account when adults learn mathematics, class, age and social circumstances being others. On a theoretical level this diversity is in most cases readily conceded – even built into definitions of numeracy. Day to day practices tend, on the other hand, to forget or ignore the differences. Here – as in many other instances - gender seems to act as a useful reminder about the ever present heterogeneities in society, an optic that enables differences to be foregrounded and reinterpreted. In the following pages, I take a social constructivist stance on gender. On the learning of mathematics, I feel most at home with Evans' ideas of "positioning" to soften up the strong ideas of situated learning, opening up the possibility of some measure of transfer (Evans, 2000:26).

Two conflicting meanings of numeracy?

In the literature there is a number of (provisional) definitions of numeracy (Gal et al., Adult Literacy and Lifeskills (ALL), 1999; Evans, 2000; Lindenskov & Wedege 2001). These definitions differ on the question of what one could term "the unity of numeracy". The concept of numeracy has (at least) two sides: numeracy as a theoretical

concept and numeracy as it is defined for instance in government programs for mathematics education and comprehensive assessment activities[1]. This accounts both for the complexity and the attraction of the concept of numeracy, the latter to a great extent stemming from the short distance between theory and practice in current educational activities. This duality is, however, not unproblematic since it has led to numeracy currently being used with two conflicting meanings. One is a research definition of numeracy that can be exemplified by Evans' definition:

> Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or *subculture* (my italics) to participate effectively in activities that they value. (Evans, 2000:236)

This concept of numeracy obviously allows for a plurality of numeracies and gender will almost inevitably enter into the delimitation of numeracy in various subcultures. The other can be exemplified by Organisation for Economic Co-operation and Development (OECD, 1996) stipulating that quantitative literacy on their level 3 is the minimum requirement to manage the complex demands from work and everyday life in the knowledge society. This represents a concept of a unified, common numeracy. Evans' definition opens up for the simultaneous existence of a number of numeracies, whereas OECD operate with a numeracy in principle common to all in a given society. Lindenskov and Wedege (2001) point out that numeracy is dependent on time and geographical localisation as shown in the following quotation:

> Numeracy changes in time and space along with social change and technological development. (Lindenskov and Wedege, 2001:5)

It is my contention that the dissimilarities due to factors like age, social class or gender are of the same magnitude as temporal and geographical dissimilarities implying that these factors should enter into any definition of numeracy.

Evans' definition points to a set of subcultures each with their own numeracy in accordance with the observation that the quantitative requirements from work and everyday life differ with age, gender, social class, job etc. On a theoretical level this is rarely contested. With this framework in mind the unified definitions of numeracy immediately beg the question of operationalisation. What does "minimum requirement to manage the complex demands from work and everyday life" mean? How is the unified numeracy used in assessment and in planning of courses defined? Is it

- the union of numeracies over all possible subcultures?
- the intersection of numeracies over all possible subcultures ?
- the numeracy for one or more special subcultures?

[1] It was remarked at ALM8 by John O'Donoghue that in his opinion numeracy was best left an undefined term, but governments everywhere were preparing training in numeracy, hence decisions should be made.

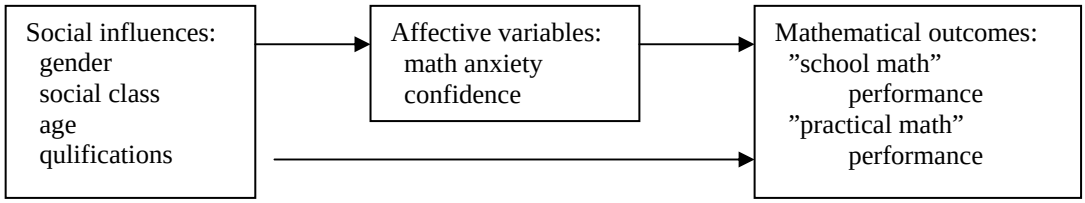
These three possibilities have very different implications: the first pointing to a very comprehensive concept of numeracy and the second to a possibly empty set of quantitative qualifications. Since nobody to my knowledge has made any systematic empirical study of the content of numeracies pertaining to societal subcultures in one or more societies, it is a fair guess that the numeracy used by e.g. OECD is of the third kind[2].

The research question on how gender should enter into a definition of numeracy is thus almost immediately mixed up with a number of practical questions due to the proximity of the theoretical and the practical side of in numeracy. What should courses contain? How should tests be made? Hence a research definition of numeracy that challenges the "unity of numeracy" will in a very fundamental way challenge most of the ongoing activities e.g. in international assessments like ALL and IALS, since the philosophy here builds on a common notion of numeracy: The creation of comprehensive international test batteries is only meaningful if numeracy is similar across borders, social groups, age groups and gender.

Gender and mathematics and….

There is a growing body of scientific literature exploring gender and mathematics learning mainly based on research on students in high schools and universities. (For an overview see Fennema, 1995; Hanna, 1996). This research demonstrates that content, context and ways of instruction are all of importance for girls learning mathematics. Girls/women prefer problems with a people/nature content, women do better in internal/project oriented assessment than in traditional timed exams and women benefit from a teaching style stressing collaboration and open-ended problems. (See e.g. Blithe & Clark, 1995).

Life histories are important for the learning of mathematics[3]. Thus, adult women and men must be expected to approach mathematics from different perspectives, to encounter different difficulties and to bring different potentials to the classroom. With a highly gender segregated labour market, mathematics in the workplace will, on the average, be different for women and for men. Studies of possible interactions between gender, occupation and age could shed interesting light on adults and mathematics. Evans (2000:55) uses the following conceptual map for studying adults' mathematical thinking and emotions.



[2] It would be an interesting exercise to go backwards from test batteries like IALS and deduce which subcultures define the content of the numeracy in the tests.

[3] In Evans' (2000:187) investigations a woman reacts negatively to a problem concerning tipping in a restaurants. It makes her feel dependent, since she is often paid for by others in restaurants. Another example is pay slips being presented to people out of work. Also, for people short of money, with unpaid bills etc. problems based on paying of bills and shopping in supermarkets might arouse anxiety.

In an ALM contexts it might be fruitful to study how these factors interact with work performance and work demands and also study the correlation – if any - between mathematics performance and work performance in a gender perspective. The proceedings of ALM comprise many important studies of mathematics in typical female occupations (nursing, weaving, needle work). Additional knowledge might be gained here by introducing a gender perspective i.e. comparing male and female nurses, male and female textile workers etc. This would be the case both in relation to the choice of mathematics learned and also to a study of the possibly gendered consequences of the introduction of mathematics in predominantly female occupations.

The "necessary" qualifications are relative.
There is a not so subtle interplay between the demands of the work place and the qualifications of the labour force. Often the mathematics demands of the work place are taken for granted (OECD, 1995). This is reflected in the present governmental interests in adult mathematics courses being presented as something necessary for the work force[4]. In reality these demands are negotiable and there is a possibly gender differentiating looping effect from the skills of the population back to the "necessary" demands. The French sociologist Raymond Boudon observes:

> Young persons will act rationally in their choice of education. They will try to get as much education as possible, since the more education they get, the higher social status they can expect. The problem here is that if all young people invest in education, then the social benefits of education will go down. Hence the young people must opt for even more education. The level of education in the population will go up, but the individual young person will not benefit. (Boudon 1973:198-99)

One possible translation in a gender perspective might read like this: "The labour market is gender segregated. Most occupations recruit either women or men. Women in the labour market are in general not expected to know mathematics. Hence the typical female occupations make do with no or little mathematics competence in the work force. This in turn implies that women are employable with less mathematics than men. The mathematics demand of the work place is to a great extent a function of the organisation of the work processes. Learning mathematics might therefore not benefit the individual woman, since her job might not be geared to make use of her new qualifications. A comprehensive effort to upgrade the female part of the work force might in turn upgrade the demands of the work place turning women with no extra qualifications out of the work force."

This is a version of the familiar dilemma of work, on one side, workers being disqualified by technological changes making old qualifications superfluous and, on the other hand, workers being disqualified by higher educational expectations making old workers superfluous. Many operations and much communication requiring literary or quantitative skills are, however, more demanding than is necessary feeding into the above

[4] In Denmark the results from SIALS (Second International Adult Literacy Survey) are claimed to show that 28% of the adult population have poor quantitative literacy.

mentioned looping effect, where perceived demands from the labour market create educational efforts that lead to higher demands in a vicious circle[5].

For a simple illustration I will take one of the problems from the International Adult Literacy Survey, IALS, (OECD, 1995).

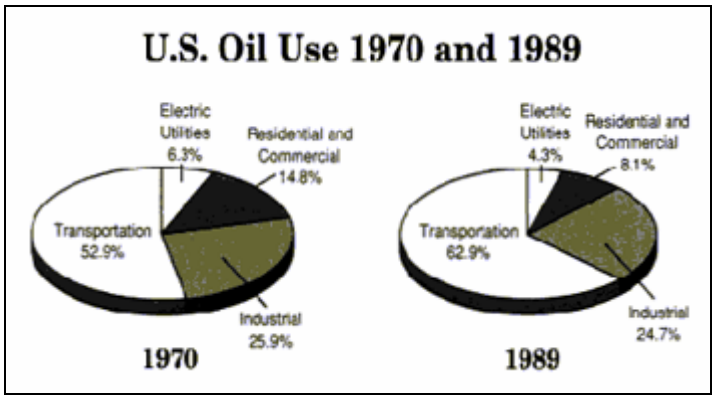


Figure 1. Two pie charts (OECD, 1995:41)

In order to test *document literacy* (see e.g. Lindenskov & Wedege, 2001:26 for an explanation) the reader has to compare two pie charts that show oil consumption in 1970 and 1989 respectively and summarise how the percentage of oil used for different purposes changed in this period of time. The problem as posed in Fig. 1 might be difficult, but this is first and foremost because it in a technical sense is bad graphics. Every professional knows that if you want to illustrate change you don't use pie diagrams. Fig. 2 shows an alternative representation. This problem is not difficult. Some graphs are difficult because they illustrate complex problems. Most graphs are, however, difficult because they are badly made.

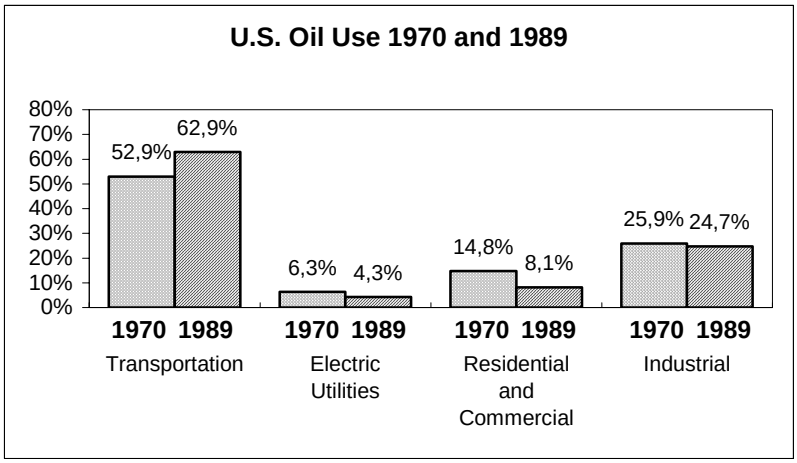


Figure 2. Bar chart for problem in Figure 1.

The same observations pertain to everyday life skills. It is of course important to be able to follow the arguments in the news, where quantitative arguments take up much space.

[5] In a recent Danish government report on employment of immigrants in Danish governmental administration it is pointed out, that many agencies make unreasonably stringent language requirements even in jobs where Danish language skills are partly irrelevant.

It is, however, also the case that the quantitative arguments often don't make much sense. The point here is that numeracy might not be of much use understanding the media debate, since the majority of journalists are innumerate and numbers are not used to convey information but to give texts a sheen of authority and credibility. ALM would do well in taking a stance on the public misuse of numbers. Otherwise one could inadvertently support the disempowerment and frustration in the population in relation to written statements, by appearing just to accept whatever nonsense is published and prescribe more mathematics education of the public as medicine. I think it should be the duty of mathematics teachers to be critically aware of all the unqualified uses of mathematics and try to weed them out for good of their students. In the same vein studies of mathematics content in work and further education could sometimes be judiciously used to reduce the mathematics content in work and further education. Icons on computers and in international airports have made life much easier for the illiterate. The same is the case with take-home videos instead of written instructions in DIY centres. Many researchers study the mathematics content in work. To make full use of the work force it would be useful if some would study ways of organising work with as little mathematics content as possible. As a mathematics teacher one is sometimes too happy to point to all the uses of mathematics in work and everyday life thereby adding importance to ones own contribution to society. From a gender perspective one would expect the male part of the labour force to bear the brunt of any excessive use of mathematics.

Mathematics anxiety and gender

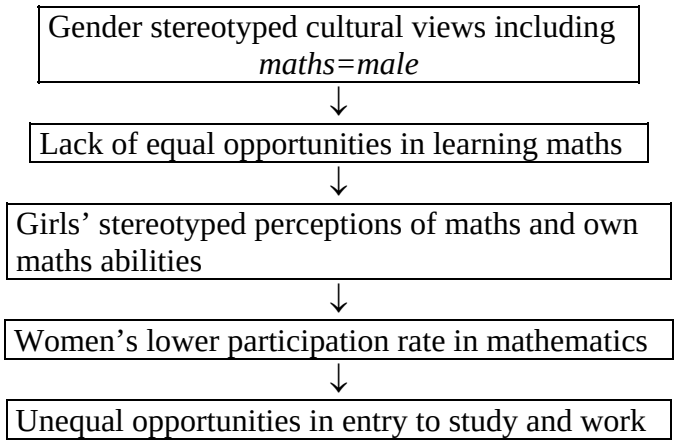
Many authors have explored mathematics anxiety both in school students and in adults. Tobias (1978), one of the pioneers, found in her investigations that women had higher levels of mathematics anxiety than men. Others have made the same findings. Mathematics anxiety is significant for adults learning mathematics. Here one would expect men and women to have different problems if only because of the different expectations society imposes on men and women. Not being able to do mathematics is very identity threatening for a man, while women traditionally have been allowed to shrug their shoulders and do something else. Hence a failure in school mathematics is not so debilitating for women as for men. One would therefore expect male mathematics anxiety to be more pervasive and more deep-rooted than with females. It is my contention that many studies could be interpreted or reinterpreted to point in that direction. Below are some examples.

In a study of medical students in Norway Annfelt (1995) reports on striking differences between men and women, in the sense that some of the male students were very worried that their inferior results in mathematics in high school would impinge on their performance as doctors, while none of the women had similar concerns. Hence the men seem to see qualifications in mathematics as a measure of their general ability, while the women have no such notion.

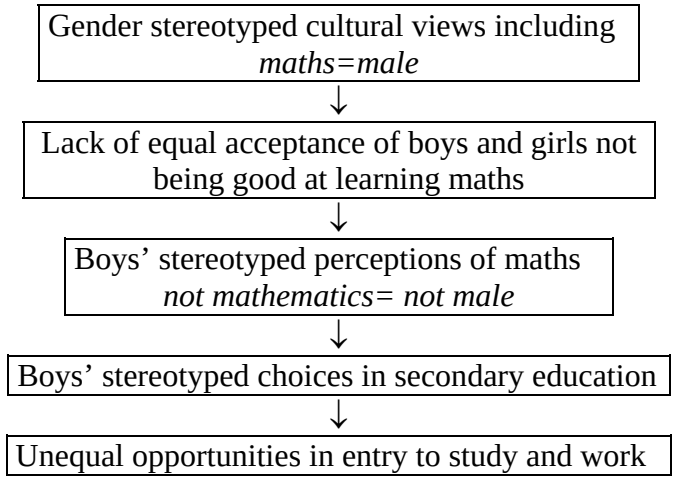
Evans (2000:36-7) studies performance in school and practical mathematics in a population of polytechnic students (taking BA Social Science or studying for a two year Diploma in Higher Education). The women do consistently worse than men in test and many more women than men have low entrance qualifications in mathematics. One way of looking at this is that bad grades or lack of qualifications in mathematics are less of a deterrent to women than to men. (Another explanation might of course be that the women have chosen subjects with less demanding mathematics content.)

These observations points to the understanding that *mathematics represents the danger of failure to men and an opportunity for success for women.* Here it must, however, be noted that that being good at mathematics might in some instances still be a problem for women (see e.g. Cordeau, 1995 and Damarin, 2000).

As a last reinterpretation I will consider what Ernest at ALM2 termed *The Reproductive Cycle of Gender Inequality in Mathematics Education,* concentrating on the school/college side of the cycle (Ernest 1995:10).



In Denmark anyway we see the flip side of that coin, namely men being kept out of education by mathematics. Danish high schools have two tracks: Languages and Science/mathematics. While the girls are divided evenly between the two tracks, almost all boys (75%) opt for the science track or don't go to high school at all. Hence only 40% of high school graduates are boys. So there is another version of *The Reproductive Cycle of Gender Inequality in Mathematics Education* that goes like this



Paradoxes of numeracy, democracy and empowerment

Many authors have identified mathematics as a dominantly male science. (See Ernest 1995 for an overview). At the same time knowledge of mathematics is claimed to be an important feature of democratic competence and also as a means of empowerment (Niss, 1994; Skovsmose, 1998; Benn, 1997). Gal (1998) and Lindenskov & Wedege (2001) have in the same vein linked numeracy with democracy and empowerment. These are very important observations with far-reaching consequences for education and research. They might, however, have unintended gender implications. If empowerment and democracy is linked with numeracy, this might in turn be taken to imply that innumeracy (or lack of mathematics) causes lack of power and lack of democratic competence. Hence with the widespread public belief that women are less numerate compared to men, women are by implication considered less democratic and an objective reason is given for their being less powerful in society.

Here is the paradox: although numeracy does contribute to democratic competence no studies have established a positive correlation between level of numeracy/mathematics competence and democratic competence. We have no proof that other activities or competencies do not contribute the same or more to democratic competence. Speaking about numeracy and democracy we have to acknowledge different and possibly gendered ways to develop as democratic citizens, not privileging any one way.

The same holds true for numeracy and empowerment. In Evans' definition acquiring numeracy so to speak automatically implies empowerment, since it "will enable a typical member of the culture or subculture to participate effectively in activities that they value". On the other hand, a more or less voluntary education in (somebody else's) numeracy might easily be disempowering. Therefore the theoretical definition of numeracy has very practical implications and it is important for the ALM community to distance itself from any unsubstantiated claims about mathematics education in itself leading to empowerment. In the same way claims like "technological development is making mathematics for all ever more important" have, in view of "Math=Male" (Ernest, 1995), the (unspoken) implication that "men are ever more important". No research has however validated such a general claim about mathematics – reality being much more complicated with both higher and lower demands for certain groups. Unsubstantiated claims of the importance of mathematics carry with them the importance of mathematicians but they also - through the connection between maleness and mathematics – imply a devaluation of typical female knowledge.

Discussions of numeracy, democracy and empowerment might fruitfully be embedded in the ongoing debate on and conceptualisation of the complexity of modernity and postmodernity (see e.g. Giddens, 1990) characterised by the disembedding and reembedding of social relations. Giddens points to the fact that lay persons in society deal with these complexities (his "abstract systems") by means of "trust relationships", stressing that abstract systems are by their very nature simply not accessible to the lay person. Many statements on mathematics and democracy might give the impression that some of the complexities of modernity may be circumvented by learning mathematics.

Building blocks for a gendered numeracy
In my abstract for ALM8 I wrote

> I will try to give a number of concrete instances, where I see gender as an important agent in adults relationship with mathematics and numeracy, inviting the participants to reflect with me on the pitfalls and possibilities of gender in ALM. (*Abstract ALM8*)

It turned out, however, that this was very difficult. There is a considerable literature on what make women feel *bad* about mathematics. There is some research on what make women feel *better* about mathematics but very little about what makes women feel *good* about mathematics. It appears that on a practical level gender differences in mathematics are so poorly understood, that most of us (I anyway) cannot even imagine what a gender specific mathematics would look like.

Ernest (1995) describes the widespread public image of mathematics as "difficult, cold, abstract, theoretical, ultra-rational …remote and inaccessible..". Later he notes the similarity to Gilligan's (1982) "separated" stereotyped male values. A science of mathematics consistent with Gilligan's "connected" values should be based on and valorise relationships, connections, empathy, caring, feelings, intuition and tend to be holistic and human-centred in its concerns. Is such a mathematics possible without losing the "unreasonable effectiveness of mathematics"? Is numeracy the solution? Is numeracy just "mathematics with a human face", the sugar-coating of a cold and abstract science, or is numeracy in some fundamental way different from mathematics? To me the construction of a gendered numeracy would here serve as a litmus test on numeracy in this capacity. So my final recommendation is that since we have a strongly gendered learning of mathematics we should also have a *strongly gendered teaching of mathematics.* Hence

- *Gender should enter consciously and concretely into every aspect of mathematics teaching.*

References

Annfelt, T. (1995). *Konflikt mellom profesjonalitet og kvinnelighet: et dilemma for kvinnelige medisinerstudenter*. Lecture at NFPF konference, Århus March 1995.

Benn, R. (1997). *Adults count too. Mathematics for empowerment*. Leicester: NIACE.

Beyer, K. (1995). 'A Gender Perspective on Mathematics and Physics Education: Similarities and Differences'. In Grevholm, B. and Hanna, G. (eds). *Gender and Mathematics Education: an ICMI Study in Stiftsgarden, Akersberg, Hoor, Sweden 1993,* Lund: Lund University Press, 21-38.

Blithe, T. & Clark, M. (1995). 'Choice, Success and Context in Questions in University Statistics Examination for Male and Female students in New Zealand'. In Grevholm, B. and Hanna, G. (eds). *Gender and Mathematics Education: an ICMI Study in Stiftsgarden, Akersberg, Hoor, Sweden 1993,* Lund: Lund University Press, 89-101.

Boudon, R. (1973). *L'inégalité des chances. La mobilité sociale dans les sociétés industrielles.* Paris: Armand Collin.

Coben, D., O'Donoghue, J. & Fitzsimons, G.E. (2000). *Perspectives on Adults Learning Mathematics. Research and Practice*. Dordrecht: Kluwer Academic Publishers.

Cordeau, A. (1995). 'Empowering Young Women in Mathematics'. In Grevholm, B. and Hanna, G. (eds), *Gender and Mathematics Education: an ICMI Study in Stiftsgarden, Akersberg, Hoor, Sweden 1993*, Lund: Lund University Press, 121-28.

Damarin, S.K. (2000). 'The Mathematically Able as a Marked Category'. *Gender and Education* 2000/12.

Ernest, P. (1994). 'Images of Mathematics, Values and Gender' in Coben, D (ed) *Proceedings of ALM-3 the Third Conference of Adults Learning Mathematics a research Forum* London: Goldsmiths University of London, 1-15.

Fennema, E. (1995). 'Mathematics, Gender and Research'. In Grevholm, B. and Hanna, G. (eds.), *Gender and Mathematics Education: an ICMI Study in Stiftsgarden, Akersberg, Hoor, Sweden 1993*, Lund: Lund University Press, 45-64.

Gal, I. (1998). 'Numeracy Education and Empowerment: Research Challenges'. In Groenestijn, M. van & Coben, D. (eds.) *Mathematics as part of Lifelong Learning.* Proceedings of the fifth international conference of Adults Learning Mathematics - A Research Forum (ALM-5). London: Goldsmiths College, University of London, 9-19.

Gilligan, C. (1982). *In a different voice,* Cambridge, Massachusetts: Harvard university Press.

Hanna, G. (ed.) (1996). *Towards Gender Equality in Mathematics Education*: An ICMI Study, Dordrecht: Kluwer.

Harris, M. (1994). 'Women, mathematics and work' in Coben, D. (ed.)Proceedings of ALM-2 the Second Conference of Adults Learning Mathematics- A Research Forum. London: Goldsmiths University of London.

Evans, J. (2000). *Adults' Mathematical Thinking and Emotions*, London: Routledge-Falmer.

Giddens, A. (1990). *The Consequences of Modernity*. Stanford: Stanford University Press.

Lindenskov, L. & Wedege, T. (2001). *Numeracy as an Analytical Tool in Mathematics Education and Research.* Centre for Research in Learning Mathematics 2001/31. Roskilde: Roskilde University .

Niss, M. (1994). 'Mathematics in Society'. In R. Biehler et.al. (eds) Didactics of Mathematics as a Scientific Discipline Dordrecht: Kluwer Academic Publishers 367-78.

OECD (1995). *Literacy, Economy and society. Results of the first International Adults Literacy Survey.* Statistics Canada.

Gal, I.; van Groenestijn, M.; Manly, M.; Schmitt, M.J.; Tout, D. (1999). *Numeracy Assessment Framework for the International Life Skills Survey.* Statistics Canada. (See ALM Newsletter, no. 6, 1999, & http://www.nces.ed.gov/surveys/all/)

Skovsmose, O. (1998). 'Linking Mathematics Education and Democracy: Citizenship, Mathematical archaeology, Mathemacy and Deliberative Interaction.' *Zentralblatt für Didaktik der Mathematik*. 98/6 195-203

Tobias, S. (1978). *Overcoming mathematics anxiety*. New York: Norton.

Practitioners, Questions and Research

David Kaye (UK) & Eigil Peter Hansen (DK)
City of Westminster College, London, United Kingdom
VUC Vest, Albertslund, Denmark

The main aim of this session is to explore and encourage the integration and validation of the work of practitioners in the continuing work of ALM. This session is introduced jointly by:

Eigil Peter Hansen (Denmark)
I work in VUC (Adult Education Centre) in Denmark with a kind of 'second chance' course. At the VUC adults have the opportunity to learn mathematics (and other subjects) in which they did not succeed when they were younger. I introduce my students to simple mathematical models to motivate and encourage them to think in more abstract ways. My students question this, when they know other ways to solve the problems I give them. I will present and discuss the difficulties my students and I experience in this process.

David Kaye (UK)
I work in a further education college (technical and community education) in central London (UK). My main teaching responsibility is additional student support for numeracy and basic mathematics for a variety of college courses. A number of different models of delivery are used including open access workshops, small group support, team teaching and one to one sessions. My introduction will aim to identify the types of difficulties my students have with learning the number skills and mathematical techniques required for their main course.

Following our presentations we will encourage workshop participants to contribute, drawing on their own experience. As the workshop draws towards a conclusion we will ask a number of questions:

- Can any of us identify one single change that would produce a major improvement for our students?
- Can we see a way to study, research, and reflect on, the problems and contradictions we experience in a more structured way?
- Can we identify a major research theme that is not being addressed (as far as we know!)?
- Are any of us aware of any research results that need to be implemented on a much wider scale?
- What are the social, institutional and financial barriers that we face in implementing change?
- Our main questions and conclusions will be shared with the conference.

Competence based maths in modules

Ben Hermeler, Baronie College, Breda, and
Harrie Sormani, CINOP, 's-Hertogenbosch, the Netherlands

From the year 1997 the content of the adult education in Holland has been described in the so called "Kwalificatiestructuur Educatie" (Qualification Structure Education), mathematics being one of the qualifications.

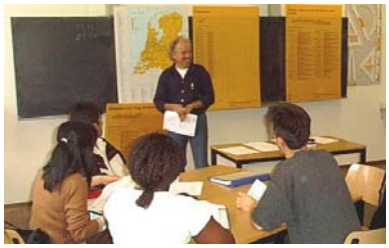
The community colleges (community training centres) had to make their education more flexible by means of modular courses. This flexible system needed a different teaching method than the 'whole class instruction' concept of teaching. In Holland we have set up a flexible system of maths education for adults called 'open learning'. There will be plenty of interaction and communication between teacher and student and between students in this.

An important premise of 'open learning' is that the teaching is guided by demands and needs of the student. Moreover every student is himself responsible for the learning process and must learn to deal with that responsibility. The student is central and decides what to learn. This means that the student decides which objective he or she wants to attain and that, in communication with the teacher, a course is built that matches the training needs of the student. The education has to be demand- centred and not supply-centred. It is not the teacher who decides on the subject of the course, but the adult student himself. That is why the course begins not only with a test, but also with an interview. The student is given the opportunity to say and show both what he knows and what he wants to learn.

The ideas behind our concept of open learning are based on the theory of competence based education. Competence can be defined "as the dynamic developing ability to adequately handle demands, expectations and problems which can occur in labour and real life situations" (Onstenk, 1998). We don't think it's important to define very exactly the final achievement that the student has to accomplish, but to take as a starting point situations which the student expects to meet in future. We want the course to extend the students' ability to solve real problems by mathematical means.

Another premise concerns the teaching method. The person who thinks that adults can learn mathematics only from paper and books is wrong. To learn maths requires interaction which encourages reflective thinking. If one opts for a more independent form of learning and modular training, course interaction will be indispensable.

Interaction takes place by consulting each other, when students cooperate and when they argue about the ways to solve a maths problem.



The role of the teacher is important in guiding this process. By means of interaction students reflect on ways to solve problems and through that their maths abilities are raised qualitatively.

These premises demand new teaching materials and a renewal of the teaching method. We also have to deal with the fact that students can enter the course at different times thoughout the year. After a few years of developing and teaching we have seen an improvement of the results. The progress in the students' maths ability is spectacular. The students achieve on an average the same results as before but now in half of the time.

We want to share and discuss our experiences with others, therefore we participate in the ALMAB-project. In this project Denmark, Norway and Holland work together on the subject of Adults Learn Mathematics Across Borders. The project examines possibilities and limitations of using mathematical tasks developed in other countries. It also examines how mathematical courses might contribute to enhance international awareness and understanding.

Reference

Onstenk, Jeroen (1998): 'New structures and new contents in Dutch vocational education.' In Nijhof, Streumer (ed) *Key qualifications in work and education*. Dordrecht: Kluwer Academic Publishers.

List of Contributors

Anne Abbott
Whitireia Community Polytechnic
New Zealand
a.abbott@whitireia.ac.nz

Patricia Alexander
University of North London
London, United Kingdom
p.alexander@unl.ac.uk

Richards O.Angiama
Goldsmiths College
United Kingdom

Roseanne Benn
University of Exeter
United Kingdom
r.c.benn@exeter.ac.uk

Alan Bowd
Lakehead University
Canada
alan.bowd@lakeheadu.ca

Patrick Brady
Lakehead University
Canada
pbrady@sky.lakeheadu.ca

Laura Carroll
Adult Literacy Media Alliance
USA
lcarroll@edc.org

Jaine Chisholm Caunt
Workplace Basic Skills Network
United Kingdom
j.chisholm-caunt@lancaster.ac.uk

Marta Civil
University of Arizona
USA
civil@math.arizona.edu

Diana Coben
University of Nottingham
United Kingdom
diana.coben@nottingham.ac.uk

Noel Colleran
Adult Education Department, Limerick
Ireland
ncolleran@clvec.iol.ie

Dhamma Colwell
King's College London
United Kingdom
dhamma.colwell@kcl.ac.uk

Janet Duffin
University of Hull
United Kingdom
j.m.duffin@maths.hull.ac.uk

Jeff Evans
Middlesex University
United Kingdom
j.evans@mdx.ac.uk

Gail FitzSimons
Monash University
Australia
gail.fitzsimons@education.monash.edu.au

Milton Fuller
Central Queensland University
Australia
m.fuller@cqu.edu.au

Eigil Peter Hansen
VUC Vest, Albertslund
Denmark
eh-mat@post4.tele.dk

Inge Henningsen
University of Copenhagen
Denmark
inge@math.ku.dk

Ben Hermeler
Cinop
The Netherlands
b.hermeler@baroniecollege.nl

Lene Oestergaard Johansen
Aalborg University
Denmark
ljo@dcn.auc.dk

Betty Johnston
University of Technology, Sydney
Australia
betty.johnston@uts.edu.au

David Kaye
City of Westminster College
United Kingdom
david.kaye@cwc.ac.uk

Sarah Kowal
Adult Literacy Media Alliance
USA
skowal@edc.org

Lisbeth Lindberg
Göteborgs Universitet
Sweden
Lisbeth.Lindberg@ped.gu.se

Lena Lindenskov
The Danish University of Education
Denmark
lenali@dpu.dk

Theresa Maguire
University of Limerick
Ireland
terrymag@iol.ie

Beth Marr
Royal Melbourne Institute of Technology
Australia
beth.marr@rmit.edu.au

Eamonn Murphy
University of Limerick
Ireland

John O'Donoghue
University of Limerick
Ireland
John.ODonoghue@ul.ie

Henning Salling Olesen
Roskilde University
Denmark
hso@ruc.dk

Poppy Pickard
University of North London
London, United Kingdom
p.pickard@unl.ac.uk

Katherine Safford
St. Peter's College
USA
Safford_K@spc.edu

Wolfgang Schlöglmann
Universität Linz
Austria
w.schloeglmann@jk.uni-linz.ac.at

Alison Tomlin
King's College London
United Kingdom
alison.tomlin@kcl.ac.uk

Mark Schwartz
Portland Community College
USA
markdotmath@earthlink.net

Paola Valero
The Danish University of Edcuation
Denmark
paola@dpu.dk
(2002: paola@dcn.auc.dk)

Valerie Seabright
Uxbridge College
United Kingdom
vseabrig@uxbridge.ac.uk

Henk van der Kooij
Freudenthal Institute
The Netherlands
henk@fi.uu.nl

Donald Smith
Victoria University of Technology
Australia
Donald.Smith@vu.edu.au

Tine Wedege
Roskilde University
Denmark
tiw@ruc.dk

Harrie Sormani
Cinop
The Netherlands
hsormani@cinop.nl

Numeracy for Empowerment and Democracy?

The idea of lifelong learning was introduced by UNESCO in the late 1960s and re-appeared in a different context and a different form in the late 1980s. In the meantime lifelong learning as a guiding principle for re-structuring education had changed from a utopian idea (democracy or access to democracy) to an economic imperative.

During the last 20 years, numeracy (or mathematical/ quantitative literacy) has been a key word in policy reports, international surveys, adult educational programmes and research on adult and mathematics education. Adult numeracy is a complex and much debated area of practice and research. In adult education, two different lines of approach are possible: society's requirements of numeracy or adults' need for numeracy.

During the 8th international conference of *Adults Learning Mathematics – A Research Forum*, we tried to bridge these two approaches. The main issue addressed was numeracy as a possible answer to questions of empowerment and democracy in the broadest sense of these terms. We found no single answer and new questions were raised.

Centre for Research in Learning Mathematics
Danish University of Education
Roskilde University
Aalborg University