Adults Learning Mathematics

An International Journal

Chief Editor
Gail FitzSimons

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Objectives

Adults Learning Mathematics – an International Research Forum (see http://www.alm-online.org) has been established since 1994, with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/ numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – an International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Editorial

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This is the second number in the first volume of *Adults Learning Mathematics — An International Journal*. After a lengthy gestation period the first copy of this ground-breaking electronic journal was produced in June 2005. Continuing the strong international flavour of the Adults Learning Mathematics [ALM] movement, this edition is comprised of three articles from authors working in Australia, England, Spain, and Sweden. These articles have been developed from conference presentations at the Topic Study Group [TSG 6], *Adult and Lifelong Mathematics Education*, at the 10th International Congress on Mathematics Education [ICME 10] held in Copenhagen, July 2004. [For further information see: <http://www.icme-10.dk>]

Introduction to the articles

Dubravka Viskic and Peter Petocz open with *Adult Students' Views of Mathematics: Reflections on Projects*, an investigation of students' ideas about mathematics and learning, based on their written reflections on the process of carrying out projects. These projects formed part of a preparatory mathematics course at an Australian university. Using a hierarchical classification previously developed in the context of undergraduate university study in statistics, they categorise the students' conceptions and identify three qualitatively different ways in which students understand mathematics, ranging from limiting to expansive views. They discuss the various aspects of the course that contribute towards the adoption of broader conceptions of mathematics with the potential to enrich the future professional and personal lives of the students. Many students reflected on their increased self-awareness, confidence and time-management skills, and on their positive experiences of working in a group. Many also talked about their excitement at discovering the details and applications of mathematics, carrying out the process of research and learning, and deriving intellectual enjoyment from their studies. The authors claim that the kinds of comments presented in this article demonstrate students developing their affective and social relationship with mathematics. In adult mathematics education, if nowhere else, this relationship is critical if learning in the cognitive domain is to flourish.

From Spain, Javier Díez-Palomar, Joaquim Giménez Rodríguez, and Paloma García Wehrle present the results of a case study about the learning of proportional situations in a school for adults in their article: *Cognitive Trajectories in Response to Proportional Situations in Adult Education.* Pursuing their objective of finding ways of overcoming the forms of exclusion that occur in daily mathematics situations — in this case, ones that involve the use of proportions for decision making — they study how dialogue intervenes in the resolution of the problems that are posed. Following the work of Leontiev and Searle respectively, they analyse the classroom activity from the perspective of the development of content, and the interaction and discourse as a speech act. Based on the doctoral dissertation of Díez-Palomar, they propose the use of cognitive learning trajectories as a methodological tool to support the analysis of the discourse, and present two contrasting examples. These lead to the conclusions (among others) that perlocutionary speech acts can encourage learning, but can also create barriers when the speaker uses a position of power that breaks with egalitarian dialogue.

Since its inauguration, the nature of the ALM community has been such that it is truly supportive of scholars from all backgrounds and at all levels. It is worthy of note that, since completing his doctoral dissertation, Javier Díez-Palomar has been awarded a Fulbright Scholarship to work in the USA with Marta Civil, a long-standing ALM member.

The final article, *Adults' Resistance to Learning in School versus Adults' Competences in Work: The Case of Mathematics*, by Tine Wedege and Jeff Evans, further underlines the international co-operation and collaboration among ALM members. Working in Sweden and England respectively, these well-known and highly respected authors build on and extend their previous work around the critical area of the cognitive-affective relationship for adults and the phenomenon of resistance to learning in formal adult education. This resistance is contrasted with the increasing recognition being given to adults' actual mathematical competences. The arguments and evidence put forward by Wedege and Evans are predicated on the need to take account of the conflicts between the needs and constraints in adults' lives: seen in its interrelationship with motivation and competence, resistance contains the potential to be a crucial factor in all types of learning.

It must be acknowledged that Tine Wedege and Jeff Evans were the Chairs for the TSG 6, and the articles presented here — and others still to come — are testament to the conscientious work and attention to detail carried out by these two long-term ALM members. Once again, the generosity of support given to others by members of this community has been rewarded by the second publication of this journal. I urge readers to consider seriously joining the organisation, attending its annual conferences, and even contributing to further editions of this journal.

Finally, I must thank the reviewers for volume 1, number 1 & number 2, for their careful, honest, timely and constructive reviewing of manuscripts. Without their tireless efforts there would be no refereed journal continuing the important task developing the research field of adults learning mathematics. [See p. 44 for the complete list of reviewers.] I also extend my heartfelt gratitude to the Editorial Team of Juergen Maasz and Mieke van Groenestijn, as well as Kathy Safford-Ramus, current Chair of ALM, for their ongoing support in this challenging but exciting phase of the journal's development.

Gail E. FitzSimons
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Adult Students' Views of Mathematics: Reflections on Projects

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Abstract

In this article, we investigate adult mathematics students' ideas about mathematics and learning, based on their written reflections on the process of carrying out projects as part of a preparatory mathematics course at an Australian university. On the basis of these reflections, we categorise their conceptions of mathematics using a hierarchical classification previously developed in the context of undergraduate university study in mathematics. We discuss the various aspects of the projects and their studies that help at least some of them to take broader conceptions of mathematics with them into their future professional and personal lives.

Keywords: preparatory (mathematics courses), projects, reflections, conceptions (of mathematics), personal growth, professional preparation.

Introduction

What views do adults returning to study have about mathematics? Does a problem- and project-based approach enhance their learning and their approach to mathematics? And what might adults take with them from their mathematics studies into the wider contexts of their professional, intellectual and social lives? This paper looks for answers to these and similar questions on the basis of our experiences teaching adults, and in the context of our (and others') research in various aspects of mathematics education. Wedege (2002, p. 63) states that "the overall purpose of mathematics education research can be described as investigating and forming people's relationship with mathematics." Here, we look at the developing relationship with mathematics shown by a group of adults who have decided to return to study the discipline for a variety of reasons, and many of whom are aiming to continue into mainstream tertiary study.

For many years, one of us (Dubravka) has run a Preparatory Mathematics course at a large university. The assessment for the course includes opportunities for students to carry out extensive group or solo projects in a variety of areas of application, at various mathematical levels, and utilising various types of mathematical technology. Students are asked to reflect on the work that they have done and to carry out a self-assessment of the success of the projects in developing their mathematical background.

With our students' permission, we have kept copies of many of their projects over the last decade, and we use these as a basis for our study. Their reflections provide a window onto their views of mathematics and their learning. Previously, we have presented analyses of their reflections on the process of learning using projects (Viskic & Petocz, 2004) and the different approaches used by groups of different sizes and gender composition (Wood, Viskic, & Petocz, 2003). Here we extend our investigations to an analysis of our students' views of the nature of mathematics, categorised using results from a previous investigation of undergraduate mathematics students' conceptions of their discipline (Reid, Petocz, Smith, Wood, & Dortins, 2003).

Although our students' mathematical abilities are varied, many of them show a maturing understanding of mathematics and its place in their own self development, and various aspects of their lives, including their working life, intellectual life and social life. Students write perceptive commentaries on their mathematical work, their mental development as learners, and the ways in which they incorporate their learning into other aspects of their lives. While many of our students seem to learn a lot about mathematics and their own approach to learning by doing this course, we too learn much about the ways in which adults approach mathematical studies.

Background to the course

The students on which this study is based are a diverse group enrolled for a *Preparatory Mathematics* course at a large university. They are "mature-age" students (aged from 21 up, but sometimes up to 70). They have come from different countries, educational systems and language backgrounds, have generally left school many years before, and have not obtained tertiary qualifications. Although some have recently finished secondary school (within the last two or three years), most have been out of school for many years and have forgotten much of the mathematics they studied there. Some want to study further and are using this course as preparation for entry into various courses at university, some want to change their work situation, some want to review their mathematics to help their children at school; others are doing the course simply for interest or intellectual challenge. One student was a writer who had a mathematician as a character in a novel, and was gathering background information! The class meets once a week for 26 meetings spread over a year. The pedagogic approach is varied, in response to the diversity of the students and their aims, using investigations and discussions, group work and computer laboratories, guest lecturers and videos, as well as more traditional methods.

Although the preparatory mathematics course discussed in this paper has many positive aspects, we must acknowledge that about half of the students who enrol do not finish the course, most of whom drop out early on. There are various reasons for this: some have changes in their circumstances, such as moving to another job in another place, others find it too difficult to balance the demands of study with their work and family life, and yet others simply find it too hard to come to grips with the mathematics. Sometimes they discuss these problems with their teachers, who can occasionally suggest solutions, but for the most part they just leave without comment.

The course focuses on mathematics at the level of senior high school, including algebra, coordinate geometry and trigonometry, followed by functions and calculus with applications. We wrote the textbook (Petocz, Petocz, & Wood, 1992) especially for this course (and others similar to it). There are relatively few books at the preparatory level between secondary school and university, possibly because there is only a small commercial market, but it is an area that is of growing importance as more students arrive at university without the traditional school background in mathematics. We aimed to present mathematics in its historical context, applied to contemporary situations, in order that our students see mathematics as a human activity. Positive reviews of the book (e.g., Mansfield, 1992) have commented on these features. Other users have complained that it contains too much discussion and explanation, and not enough practice exercises for students to become completely familiar with mathematical techniques.

One particular feature of the book is the two chapters devoted to projects (one chapter pre-calculus, the other calculus based). The topics are selected to cover a range of applications and social concerns that might appeal to adult students, including topics in (pure) mathematics that might be useful to them in their university courses. In the first chapter there are projects on *Managing your mortgage*, *Women in mathematics*, *Mathematics of alcohol*, *Geometry of fractals*, *Aboriginal and other counting systems* and *Complex numbers*. In the second, there are projects on *Radiocarbon dating*, *The Greenhouse effect*, *Elasticity in economics*, *Newton's method* and *Infinite series*. In each project, a overall problem is posed and some background information and initial references are given. For instance, one project in the earlier chapter asks: *In what ways have different cultures, in particular, the various Australian Aboriginal cultures, developed and used number systems?* It follows this question with a brief discussion of decimal and other base number systems, and then gives a table of counting numbers from the Gomileroi language (an indigenous group from Northern New South Wales, Australia). This is followed by a series of questions, at first straightforward and well-defined, moving through more extended applied questions that require further information to solve, up to some that could constitute mini research projects. For instance, here are five questions from the project on number systems:

- What is a number system and what is a base?
- What are the advantages of having a base for a number system? What advantages would there be in the Gurindji system with no base? What problems could there be?
- Common bases in other cultures and civilisations include 2, 3, 4, 5, 10, 20 and 60. What are the advantages and disadvantages of each? Investigate the number system of one of these cultures or civilisations.
- The probability that there is intelligent life on some other planet has been calculated to be 0.1. Imagine that you have landed on another planet where the inhabitants have three 'hands', each with four 'fingers'. What sort of number system or systems do you think they would use? Give some examples of their calculations.
- How could you convert numbers from any base into any other base? Give some examples of the method. Can you write a computer program to carry out the conversions?
- Each project states explicitly that "you do not need to answer all the questions and you may want to discuss other discoveries that you have made."

Question 10, the last in each project, invites students to: *"Give a brief account of your investigations, describing the problems you faced and the successes you achieved."* Discussion in class points out the importance of such reflection for students' learning. To encourage students to write a response, the marking criteria explicitly state that there will be 1 mark (out of 10) if a reflection is present: this requirement could be satisfied by a minimal response of a sentence or two, although most students need no more encouragement to write detailed reflections that can be up to two pages in length. Reading through such reflections can also be a very enlightening way for a teacher to get feedback on their teaching and their students' learning, as other teachers have reported (Varsavsky, 2003).

Theoretical position and previous results

We have set up the total learning environment of the course (as discussed in Petocz & Reid, 2002) to encourage the students to take a deep approach to their learning, and to develop holistic conceptions of learning and of mathematics (Marton & Saljö, 1979; Marton & Booth, 1997). The approach taken throughout the whole course is to encourage students to active and participatory learning (see, e.g., Dubinsky, 1999) and to make connections between mathematics and their own life experiences (for instance, in the selection of topics for the projects). The common approach of school mathematics in many countries – a defined sequence of mathematical techniques reinforced with copious exercises in an abstract or pseudo-applied context (referred to as 'situation context' by Wedege, 2002) – is largely replaced with a project- and problem-based approach that aims to foster the development of mathematical literacy, intellectual autonomy and democracy: Skovsmose (2001) refers to the former approach as the 'exercise paradigm' and its replacement as 'landscapes of investigation'. As adults returning to study, our students are also faced with the problem of connecting the mathematics that they have previously studied at school with the informal and implicit mathematical experiences that they have in their working and personal lives: Wedege (2002) points out that many adults do not consider such experiences to be proper or real mathematics.

One important feature of the projects, supported by much research in learning, is that students are encouraged to work in groups (and most of them do). Jacques (2000) gives practical details and discussion of the benefits of working in groups. Booth, Wistedt, Hallden, Martinsson, and Marton (1999) maintain that there are multiple paths to learning and that encouraging students to work in groups enables them to jointly constitute meaning about mathematics. Keitel (1999) points out that all learning interactions are between *persons* and that the focus of this interaction is the development and sharing of meaning. Another important point, especially for some students who have not participated in learning for many years, is the use and integration of current technology and the beneficial effects that this can have on learning (discussed in this context in Wood et al., 2003; also by Dubinsky, 1999). The question at the end of each project is designed to encourage students to reflect on their learning experiences and to carry out the self-assessment that is an important component of successful learning at all levels, but particularly in tertiary courses and in further lifelong learning (Boud, 1995). This self-assessment can take a wide variety of forms, as shown, for example, by Reid and Leigh (1999) in the context of studies in management. Their hierarchy of conceptions suggested an initial basis for the organisation of our students' reflections.

At a conference in 2002 we presented introductory analyses of students' reflections on the process of *learning using projects* (Viskic & Petocz, 2004). We found that we could group their reflections into four categories. From narrowest to broadest they were: *Basic* (writing the minimum to satisfy assessment requirements), *Descriptive* (discussing or summarising mathematical material studied), *Experiential* (reflecting on practical and personal difficulties and successes) and *Integrated* (combining personal understanding of the mathematics with an appreciation of development as learners). The categories reflect an increasing depth of learning experiences during the process of carrying out the project. They show a hierarchical nature, as reflections at the broader, more holistic levels can contain elements from the lower levels. We have also investigated the different approaches used in these projects by groups of different sizes and different gender composition (Wood et al., 2003). We were particularly interested in whether there were systematic gender differences in the way students formed their groups, worked together and utilised the available technology (including computer packages, e-mail and internet) in their learning of mathematics. From a thematic analysis of a large number and variety of reflective comments we concluded that many individuals and groups were extending their familiarity with a range of technologies and that there was no evidence of any gender-based differences.

Here we extend our investigations to an analysis of our students' views of the *nature of mathematics*, testing a framework based on an earlier investigation of university mathematics students' conceptions of their subject (Reid et al., 2003). In that study, a series of interviews was carried out with students majoring in mathematical sciences. Students were asked questions such as: *What do you think mathematics is about? How do you go about learning mathematics? What do you aim to achieve when you are learning in mathematics? What do you think it will be like to work as a qualified mathematician?*. The transcripts were analysed using a phenomenographic approach (Marton & Saljö, 1979; Marton & Booth, 1997). This analysis led to the identification of three qualitatively different ways in which students understand mathematics, ranging from limiting to expansive views.

1. *Components:* Students see mathematics as made up of individual components; they focus their attention on disparate mathematical activities or aspects of mathematics, including the

notion of calculation, interpreted in the widest sense; nevertheless, these components are viewed as part of a coherent mathematical investigation.

2. *Models:* Students see mathematics as being about building and using models, translating some aspect of reality into mathematical form; in some cases, such models are representations of specific situations, such as a production line or a financial process; in other cases, the models are universal principles, such as the law of gravity.

3. *Life:* Students view mathematics as an approach to life and a way of thinking; they believe that reality can be represented in mathematical terms and their way of thinking about reality is mediated by mathematics; they make a strong personal connection between mathematics and their own lives.

A previous study (also using a phenomenographic approach) of students' conceptions of *statistics* (Reid & Petocz, 2002), a component of the same mathematics degree and a possible area of specialisation for them, found a narrower conception of statistics (and hence of an area of mathematics). This would be placed before the first conception (*Components*) on the list, and we have assigned the level 0 to this conception.

0. *Techniques:* Students focus on mathematics or statistics as consisting of isolated mathematical or statistical techniques, without any connection between the techniques and their meaningful use in mathematical or statistical problems.

Further evidence from a larger, international group of students has confirmed the existence of this narrower conception (Petocz, Smith, Wood, & Reid, 2004).

Again, and in common with other phenomenographic outcomes (see Marton & Booth, 1997, for instance), these conceptions seem to be hierarchical and inclusive. Students holding the broader, expansive views are also aware of aspects of the narrower, more limited views, but the converse does not hold: students who view mathematics as techniques or components seem unaware of the broader views. This is the reason why we as educators value the broader conceptions and try to set up learning situations to encourage students towards them.

An aim of the present investigation was to see whether the same framework found using interviews with university mathematics students would be appropriate for discussing our preparatory mathematics students' ideas about mathematics. We wanted to examine whether adults returning to mathematics study showed the same range of ideas about the discipline as students who were studying mathematics at university. In this case, the information that we had – reflections ranging from a few sentences to a couple of pages – was more limited than half- to one-hour interviews. Nevertheless, it seemed that the views of our preparatory students were able to be described using the same framework.

Students' conceptions of mathematics

In this section, we examine some of our students' reflections on their projects in order to investigate their understanding of mathematics itself, and make comparisons with the hierarchy of conceptions that have been found previously in undergraduate mathematics courses. Some of the reflections were written jointly with group members, others were written individually, sometimes even when students had worked together in a group. We start with a reflection from a project carried out on *Aboriginal and other counting systems* illustrating the *Components* conception. The focus is on separate mathematical activities in the context of counting systems (remember, the "Aliens" have three hands with four fingers on each hand).

> We started by working out how to change between bases, particularly 5 and 10, and initially found it so difficult it almost hurt. A big breakthrough occurred when we were working on the Alien question, doing a lot of trial-and-error work and having some bizarre discussions! After that, the notion of base and power fell into place, and the ease of calculation between bases increased considerably. We became quite adept at addition and subtraction in various bases without resorting to tables, including our Alien 4/12 system. Multiplication and division remained challenging, and quite difficult

> in the Alien system. Finding reference material was quite difficult for this project. Information on bases in general, and changing bases, was either very introductory or in journal articles, way beyond our understanding. From this project, we believe we have a better understanding of the working of powers and bases, and that this will help us in our understanding and use of indices, exponentials and logarithmic functions, and in seeing maths as a representational system, rather than a series of absolutes.

Another example from a project on *Infinite Series* also illustrates the *Components* conception. It shows a similar focus on separate but coherent mathematical activities – arithmetic and geometric series, harmonic series, series expansions for π, using a calculator, writing a computer program.

> The concept of an infinite series actually converging to something finite was sort of difficult to grasp at first, but now it seems quite natural. The geometric and arithmetic series were a good place to get into the topic because they were relatively easy to understand. The book by Fitzpatrick was quite helpful for those easier series. Also one of us had studied them overseas in Iran, which was a real big help! Some of the other books we used were difficult, particularly Arfken's and Protter & Murray's. Nevertheless, it was good to see where the subject was going and the first one was quite a help in understanding the harmonic series and getting some other nice results, like Euler's identity. The internet was also helpful, we got various sums and products for pi from there. A big problem was using the calculator to evaluate series: other than being boring, it was easy to get confused and to make a mistake. We felt that it would be best to write some computer programs although we weren't so confident about how to do so. It was quite a challenge. One of us did have some practice with the Basic and Matlab computing language but even so it was difficult to write the programs. After a lot of error messages we finally got them all running which was very exciting!

The next reflection from a project on *Radiocarbon dating* shows the *Models* conception, and gives a clear example of a model that is an application of calculus methods.

> This project has shed light into the process of better understanding how relics and antiques are dated and catalogued. I have also learnt about the inconveniences of carbon dating and why sometimes it is better to use other methods. It was interesting for me to see the calculus methods learnt in class applied to everyday use. I can now understand that rates of change are applicable, and not just useless formulas and complicated symbols. Another fascinating thing I learnt in this project was how the calculus theory can be applied to forensic examinations in detecting, for example, how long a body has been dead. Now when I watch *Crime Scene Investigation* (the TV show) and they are determining how long ago the person died, I know that with Newton's law of cooling I will be able to work it out too!

Another quote showing the *Models* conception is this reflection from a project on *Mathematics of alcohol*. A particular aspect of reality – the effect of alcohol on the body – has been translated into a mathematical model encapsulated by the formula for Blood Alcohol Concentration (BAC).

> In this project I investigated the effects of alcohol in the body and how we could use mathematics to model items such as absorption and the processing of alcohol in the human body. I examined the BAC in different scenarios, and found that males and females have completely different absorption rates and that females are generally disadvantaged when it comes to drinking. Also I found out that the BAC is a little more complicated than I first expected, with so many unknown factors and estimates, such as the amount of alcohol that was left in the stomach hence not absorbed. This project allowed me to study something that was interesting to me, and I really enjoyed the learning process and took a new perspective of how mathematics was related to a real life situation.

The next reflection is from a male student's project on *Women in mathematics*, and shows the strong personal connection that is characteristic of the *Life* conception.

> I guess part of the reason that I chose Women in Mathematics as the topic for my project was curiosity. Who were these women, why haven't I heard of any of these women mathematicians? It's safe to say that I embarked on this project from a position of complete ignorance. I thought back to my high school years and realised that from years 7 to 12 all my maths teachers were male. I remember hearing on more than one occasion the statement, which often goes unquestioned in people's minds, that 'Girls are good at English and boys are good at maths.' I have five younger

> sisters, whom I have all heard proclaim at one stage or another with great conviction 'I hate maths, I'm hopeless at it.' However, after completing this project and hence rendering myself in a position of slightly less ignorance, I now know that there has been many great female mathematicians. I'm left with a great deal of respect for these women, and the courage and determination they each possessed. After learning about the many barriers that these women faced, in particular the extreme negative attitudes towards women in mathematics, I'm brought back to the question of my sisters. Do they really hate maths? They are all intelligent young ladies, are they really hopeless at mathematics? Or even today does there still exist some kind of subconscious stereotype in our society that conditions some young female students to falsely believe that because they are female they are not supposed to like or be good at mathematics, thus encouraging them to focus their energies on the more traditionally 'female' subjects such as English? This is one of the theories that I came across whilst researching the first question of the project.

And finally a reflection from a project on *The Greenhouse effect* illustrating the *Life* conception, showing the student's growing appreciation of the place of mathematics and mathematical thinking in their life, and its possible role in solving environmental problems.

> The reason I selected this project initially was simply due to my genuine interests and concern in our changing environment. I was further drawn to this option as I was most curious to know how mathematics came into the Greenhouse equation. I must certainly say that I got a lot more out of the project than I bargained for. As interesting as it was to learn more about the greenhouse gases that I'd known, the most satisfaction came from learning so much more about the importance and the value of maths. … During my research, I was quite fascinated about the concept of GCMs [Global Climate Models], and will anticipate any improvements being made in the years to come with these models. As expensive as they are, the models aren't quite enough to make accurate predictions on the Greenhouse. But we must bear in mind that there are many other determining factors as mentioned earlier. It's also interesting to learn that without global warming we would freeze. But obviously, the bottom line is moderation, and this does rely quite a lot on the human race. Educating people seems the best solution. Perhaps showing society these mathematical figures achieved in the project may open their eyes. They've certainly opened mine!

These six quotes were selected from a large number of reflections that could be used to illustrate each of the three conceptions described in Reid et al. (2003). Interestingly, there did not seem to be any evidence of the *Techniques* conception, although there were some reflections focusing only on practical problems such as getting to the library, or finding convenient times for group meetings (giving no clues about conception of mathematics): for instance, *"The project was very time consuming, as a lot of resources that are easy to read do not go into enough detail, and those that go into enough detail are too difficult to read."* We believe that the construction of the projects and the various applications featured in each of them (even those focusing on "pure" topics) make it difficult for students to write reflections consisting of isolated mathematical techniques without any meaningful use. In this sense, all of the projects *"provide activities and assessment that encourage students towards the broadest levels of understanding of mathematics, and away from the narrowest levels,"* as recommended in Reid et al. (2003). Further evidence for this comes from several reflections that seem to show students broadening their view of mathematics. There was no evidence that some projects were more likely than others to give opportunities to expand their views to the widest level. As they work through a project, the mathematics that they are using gets connected to their own lives, even if the topic is a "pure mathematics" topic such as infinite series. The following reflection from a project on *Mathematics of alcohol* seems to show a student moving from a *Models* to a *Life* conception:

> Mathematics helps in a number of ways to formulate sensible and legal drinking patterns. Firstly, it's by being able to measure the amount of alcohol in a given drink we are able to ascertain the effect that this will have on blood alcohol concentration. By being able to calculate the BAC we can then formulate or predict the relationship between certain BACs and accident and mortality rates on our roads. These figures can be entered statistically on graphs. It is the interpolation and extrapolation of such mathematical figures here that allows us to deem what is sensible, thereby enacting those values as legal limits for the benefit of all.

Another reflection on the topic of Managing your mortgage shows an initial focus on mathematics as Components before broadening into the Life conception:

> After all of us separately deciding that mortgage formulas were helpful and extremely fascinating (!) we formed a study group in order to complete the project. After calculating and re-calculating most questions in order to achieve five of the same or at least similar results we gave up and decided to go with the majority… At present none of us own our own home and certainly none of us are looking forward to parting with the extreme amounts of interest (hard cash!) necessary to take out a mortgage. We have come to the conclusion that yes, mortgage formulas are extremely helpful (unless you are trying to derive them) and it is fascinating to discover how the bank can end up with so much of our hard-earned money!

Although it is sometimes not straightforward to categorise each student's individual reflections, the previous framework (from Reid et al., 2003) seems to be adequate to describe the full range of views represented in the overall collection of reflections. On the more practical level, the complete collection of reflections, and even the small set of quotations that we have selected for this paper, provides a useful window into our students' thinking about mathematics.

Discussion and conclusions

As well as illustrating conceptions of mathematics, these quotations from our students' reflections also throw light on what adults may take with them from their studies into the wider contexts of their lives. Many of the project topics are concerned with application of mathematics to various social, environmental and civic problems (alcohol, greenhouse effect, mortgages). Some students' reflections show that they are initially surprised by the social connection with mathematics, in the same way as the adults described by Wedege (2002) who do not consider that what they are doing in their workplaces is actually mathematics: Finally, though, at least some of them are appreciative of the growth in awareness that results from their study.

Of course, we cannot claim that this broader conception of mathematics is necessarily taken unchanged into students' future personal, academic and professional lives. Yet there is evidence from previous studies (Reid & Petocz, 2003) that students who have developed broader conceptions of mathematics or statistics will use these broader conceptions if they perceive them to be appropriate for their future learning; for instance, in interpreting the meaning of a mathematical model for clients or other people involved in the situation. On the other hand, they can use a narrower approach if they feel that it will be rewarded; for instance, learning formulas, techniques and theorems by rote for a traditional 'closed-book' examination (Reid & Petocz, 2002). Nor can we claim that all adults will develop and benefit from such broader conceptions of mathematics. The students who enrol in this preparatory course are generally motivated to undertake further study of mathematics: Yet about half of them do not complete the course for various reasons, and some of those who do complete it seem to remain with the narrower conceptions of the subject.

Nevertheless, many students reflect on the increase in their self-awareness, confidence and time-management skills, and their positive experiences of working in a group. From many such reflections, here are three short comments, one from a solo project, the other two from larger groups of students:

> Looking back, I actually appreciate what I have picked up with time management. I don't know whether I had made many discoveries, rather than what I have been learning about myself. I seem to have a lot more confidence in myself, knowing that I can do the work that is required. Also learning that it will take time, I won't get it straight away but if I stick to it then it will eventually all unfold. I loved that feeling.

> With some material on the table, we were able to start in a somewhat slow fashion. The slow start can possibly be due to the four different cultural backgrounds. One of the over-riding aspects of the

> group was that everyone displayed an overwhelming level of courtesy in order to accommodate the values of each of our divergent backgrounds.
>
> Finally, a brief summary of the group's experience in working on this project. We all found the topic enthralling and could easily have devoted the entire project to any one aspect or individual [mathematician]. The challenge was working cohesively given the constraints of individual schedules and expectations: a learning experience as much as the project itself, albeit an enjoyable one.

A final and important point was that many students talked about their excitement at discovering the details and applications of mathematics, carrying out the process of research and learning, and deriving intellectual enjoyment from their studies:

> I feel that I have gained much in the general understanding of fractals, satisfying an old curiosity in a field I had previously assumed as being beyond my reach. Learning about fractals has been an enjoyable and fascinating experience.
>
> The research I have contributed to this project changed my views about the "out of Africa" theory, that modern humans evolved in Africa about 100,000 years ago and then spread around the globe. How radiocarbon dating and the role of mathematics in obtaining age estimates on organic materials as old as 50,000 years is just amazing.
>
> After reading a small portion of Newton's achievements, I decided that this was an interesting topic. … I found the topic stimulating, so much so that I am now reading anything remotely connected with mathematical science in that period, and I am considering studying mathematics as a degree rather than finance and economics as I was intending.

Comments such as those presented in this section demonstrate students developing their "affective and social relationship with mathematics" (Wedege, 2002, p. 76) and illustrating its importance in their increasing levels of numeracy and their broadening notions of the nature of mathematics. Our students' reflections on and self-assessment of their projects yield fascinating and useful insights into their learning. Many students show a maturing understanding of mathematics being integrated into their own self development. For those students who successfully complete the course, these reflections attest to the enhancement of learning that has resulted from their work on projects in *Preparatory Mathematics*. To our future students we would commend such reflection as a powerful component of learning, and to colleagues running similar courses we would recommend reading and considering their students' reflections.

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Cognitive Trajectories in Response to Proportional Situations in Adult Education

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Abstract

We present the results of a case study about the learning of proportional situations in a school for adults. The objective is to find ways of overcoming the forms of exclusion that occur in daily mathematics situations that involve the use of proportions for decision making. We study how dialogue intervenes in the resolution of the problems that are posed. To this end, we analyse the activity (in line with Leontiev, 1981) from the perspective of the development of content. We analyse the interaction and discourse as a speech act (in line with Searle, 1980). We propose the use of cognitive learning trajectories as a methodological tool to support the analysis of the discourse. Among other conclusions we found that perlocutionary speech acts can encourage learning, but can also create barriers when the speaker uses a position of power that breaks with egalitarian dialogue.

Key words: numeracy, teacher training, adults' beliefs, learning in everyday life, proportional situations, cognitive trajectories, dialogic learning.

Introduction

Proportional reasoning is a person's ability to effectively use proportional schemata. This ability plays a central role in the development of mathematical thinking and it is often presented as the cornerstone of higher mathematics as well as the most important of the basic mathematical concepts.

As adults, we function in a world in which proportions constantly arise and it is important for us to know how to use them in order to not feel excluded and to be able to make appropriate decisions, in accord with a correct evaluation of the information provided us. Using the concept of proportion incorrectly can lead to making erroneous decisions, with greater or lesser harmful consequences, depending on the circumstances. For example, putting an additional cup of flour in a culinary recipe can ruin a cake and embarrass us in front of our guests. But making a mistake in the proportion of medication that must be administered to a sick person at home can have fatal consequences for that individual. Furthermore, adults' understanding of proportional reasoning (following Carraher, Carraher, & Schliemann, 1982; Noss, 2002; Noss & Hoyles, 1996) is a key area in adult education due to the implications that this understanding (or lack thereof) has for everyday life activities.

Writers on proportionality in adult education point out the importance of contextual factors in the learning process. See, for example, Hoyles, Noss, and Pozzi's (2001) work on proportional reasoning in nursing practice.

Research shows that previous experience is an element that intervenes in proportional reasoning. As Schliemann and Magalhaes (1990) point out, when adults find themselves with proportionality problems that are known to them, they solve them correctly. But when the problems fall out of their prior experience, then they do not recognise that the relation of proportionality is the same one, and tend to make mistakes.

We know from previous research (Post et al., 1998) that instruction with proportional reasoning should begin with situations that can be visualized or modelled. Instruction should build on adults' intuitive understanding and use objects or contexts that help them make sense. Adults need to understand key ideas in order to have something to connect with procedural rules. To support the development of key aspects of proportional reasoning, adults should explore proportional and non-proportional situations in a variety of contexts using concrete materials, collecting data, building tables, and determining the relationships between the numbers.

In the more extensive research investigation in which this study is framed, we propose to see how dialogic learning functions in a mathematics class for adults where the concept of proportionality is studied. But in this article we study the dialogue that takes place within the group in order to answer questions as to what kind of interactions produce effective learning (or not). What are these interactions? And what implications are there from a practical point of view with regard to pedagogical strategies? In order to achieve this goal, we undertook an in-depth case study. The following sections present the theoretical framework and methodological tools that we used to conduct our research and some findings for discussion.

Theoretical Framework

Our theoretical framework is *dialogic learning*. *Egalitarian dialogue* is one of the seven principles of the dialogic learning, developed by CREA – Centro Especial de Investigación en Teorías y Prácticas Superadoras de Desigualdades [Centre of Research in Theories and Practices that Overcome Inequalities] (Flecha, 2000). It is defined as the dialogue that is produced between two or more people, when the value of their contributions is considered as a function of the value of the respective arguments and not according to the position of power or authority within the group. Through dialogue, individuals have the opportunity to make sense of different concepts. They can use different strategies, such as negotiation of meaning (Clarke, 2001), recognition of concepts within the zone of proximal development (Vygotsky, 1979), accommodation of new structures of meaning by the equilibration process (Piaget, 1970), and so forth. But any of these strategies have to be based on an egalitarian process of interaction in order to be effective (because most times unidirectional interactions from teacher to students produce mechanical answers, but not effective understanding). So, in this sense, egalitarian dialogue is a condition for developing effective learning.

In order to attempt to interpret the dialogue produced in the classroom (Alrø & Skovsmose, 2003), by working with young students, Dreyfus, Hershkowitz and Schwarz (2001) refer to three elements: construction, recognition, and *building with*. They are interested in language as a tool to build cognitive structures related to mathematical understanding. In our case, we consider that people use language as a vehicle for learning, and are able to face difficulties and learn to overcome these through social interaction (Vygotsky, 1979). Therefore, for us, *language* has two clearly differentiated uses: on the one hand, it is a *vehicle for communication;* but on the other hand, it is also a *constructor of reality* (and, in this sense, a tool to learn with). Language is what allows us to build and express abstract ideas, such that words are symbols that are separated from the objects they refer to. Considering that learning is produced when a person is able to move from the concrete sphere of *practical intelligence* to the very abstraction of thought means that language occupies a central role in this process, in accordance with what Luria (1979) and Piaget (1970) contend.

At the same time, from the point of view of interaction, we can talk about *functions of language.* Language can be a tool to communicate ideas, but also to impose our point of view on others. As everybody knows, there are different styles of teaching: some more dialogic and others more unidirectional. To analyze the educational consequences of these we include in our study of the theories of Austin (1971) and Searle (1980). As they claim, there are illocutionary speech acts when we want to transmit information and perlocutionary speech acts when we want to influence the conduct or thinking of others. Searle defines speech acts as the collection of linguistic emissions. He distinguishes two characteristics of the linguistic sounds: on the one hand, the fact that they have meaning (illocutionary acts); on the other hand, the fact that they want to say something (perlocutionary acts). We use this classification because it allows us to see the ways in which power claims affect the dialogue that is produced within the classroom.

Methodology

We carried out an in-depth case study in the School for Adults, la Verneda – Sant Martí, in Barcelona (Spain) in 2003, within the framework of a larger research investigation about dialogic learning. The learning environment was structured so that there were face-to-face meetings between students and the teacher. In addition, there was a website designed for learning mathematical topics of interest to adults. The main objective of this article is to share some insights into adult learning within dialogical frameworks. We want to compare egalitarian strategies to teach and learn mathematics (focused on the concept of proportion) with non-egalitarian strategies. For this reason, we will focus our analysis in concepts such illocutionary and perlocutionary speech acts (Austin, 1971; Searle, 1980). We assume that individuals who are capable of providing concrete explanations of proportionality from the general (abstract) idea of proportionality understand the meaning of this idea (Piaget, 1970). We are interested to see if there is a uni-directional interaction between teacher and students, or bi-directional (i.e., when students use concrete examples, general explanations, and so on).

We observed a group of six women who were studying to obtain the equivalent of the compulsory secondary education certificate (i.e., the level for 16 years of age). These adults worked on the theme of proportionality over several sessions. We illustrate here the first of these sessions where ten tasks were proposed: in the first five, the dialogue is based on written questions, and in the last five on computer support. All activities were about proportions, in which elements of daily life (e.g., activities set in the context of the market place) were combined with more theoretical elements (e.g., questions about the meaning of the constant of proportionality). The first five questions are “textbook problems” that are characterized by presenting an explicit context, with a single formulation and a single possible solution. On the other hand, in the last five, we find problems of daily life in which the context only appears partially, and the solutions can be various. Table 1 shows two examples of activities that were carried out.

Table 1. *Examples of Activities*[1]

Task 1

In this stand at the market they have a board on which they have calculated the price of various weights, relieving the sellers from having to calculate the price each time they make a sale.

Complete the following table:

weight (kg)	1	2	3	4	5	6	7	8
Price €	3	6						

Task 1: Example of a task from the mathematics book AA.VV. (2002, p. 152).

Task 6

If we approach or move away from things that surround us, they look larger or smaller. Try it out: join someone and together try to measure the height of the classroom door. Which is bigger, your colleague or the door? Do you think if you begin to walk forwards or backwards that there will be a point at which your colleague is the same or equal height as the door? What distance were they from the door in order for this to happen?

Task 2: Example of one of the tasks in the mathematics web.
<http://www.neskes.net/mates>

The session that was analyzed (approximately 70 minutes) was taped with a digital video, and subsequently, transcribed. In order to analyze data collected we used *cognitive learning trajectories* (CLT). In this, we considered, at the same time the contents of the discourse as well as the functions of the language in the interaction (Díez-Palomar, 2004), in order to understand how interactions through dialogue can allow (or not) effective learning about the concept of proportion.

With respect to the *contents of the discourse*, following Piaget (1970) we have developed four categories of analysis. These categories are the following: particular case (PC); diverse particular cases (DP); comprehensive interpretation (CI) and generalized recognition (GR). These four categories allow us to distinguish the arguments that correspond to concrete examples or questions from arguments that relate to more abstract ideas. In addition, we have used an extra category, evocation of the constant (Ek), related with the idea of *constant of proportionality*, as a part of the mathematical content of our research.

In order to understand how interactions *between* people within the classroom affect them, we have used some categories focused on the *functions of language*, following de Saussure (1974; see Introduction). These categories are as follows: provocation (p); agreement (a); explicative response (er); doubting enunciation (de); assertive enunciation (ae) and clarified correction (cc).

In order to analyze dialogues utilizing both kinds of categories, we used CLT which served to graphically represent the data collected and to show different types of interaction between individuals. Each CLT represents a sequence of teaching and learning comprised of the involvement of different participants who are identified with a code, such as P1 (person 1). We analyzed the data in order to identify some evidence of concrete reasoning or abstract reasoning in relation to the idea of proportion. In addition, we looked at the meaning of interactions in analyzing the functions of language which individuals used to learn. For this reason, we also identified whether people used language as a tool to explain something related to mathematical concepts, or as a way to express doubt. We wanted to see if all participants were involved in the process of teaching and learning, or only a few. Moreover, we looked to

[1] All translations were carried out by the authors.

see whether or not the teacher (P1) participated more, or less, than students, and in which particular situations this occurred. For example, if the participant explained something, prompted additional dialogue, or gave a clarified correction this would be noted in the analysis. This process was repeated with students. We used codes (as described above) to identify each dialogue, and we drew little circles on a chart where the X-axis symbolized the degree of abstraction and the Y-axis symbolized the time of the sequence. The chart was used to explain who was involved in the interaction and what kind of interaction it was. This methodological tool allowed us to better understand if the interactions were perlocutionary or illocutionary, and if there was an egalitarian situation, or not, in order to achieve our objective.

Results and Discussion

The videotape shows a group of women who were working together on several activities about proportion. They had different backgrounds and prior experiences. Dialogues show that some women were more participative than others. Most of the time the teacher was involved in the action in order to encourage more dialogue through provocations. The dialogues were fluent.

The nature of dialogues

From the analysis of the discourse, one of the first aspects to point out is the difference between the frequency with which the categories of particular cases and generalized recognition appear. We were very interested to find some evidence about the learning process. For this reason our focus of attention was on the nature of dialogues (concrete vs. abstract). What we denote as generalised recognitions are examples of dialogue where it can be seen that the person who is talking uses the general mathematical idea (in this case, the idea of proportionality) in its mathematical sense. The following fragment is an example. The context is an activity in which a person is asked to complete a table in which there appear two variables: the quantity of kilos sold of a product and the total amount of money. Between both variables there exists a relation of proportionality that is functional (one depends on the other). In the comment that P4 makes, it is made clear that she recognizes this mathematical idea of functional relation.

> P4: ...that is to say, giving distinct values to K of the first quantity, mass, we obtain the values of the second quantity …

During the session, we observed that the women usually made reference to concrete examples in order to illustrate their arguments. These concrete cases always referred to the concrete implementation of the general norm; in other words, it was about specific examples from the general collection that is the mathematical norm.

In the following example, the teacher uses a specific case to explain the general mathematical norm, which refers to the conversion of a quantity expressed in base 10, to base 60.

> P1: ... What we are saying now, the fraction part of 75 is a decimal. To convert it to minutes we have to multiply or divide by 60. If we have 3.75, as a decimal, to convert it to minutes we multiply by sixty ...

While all participants almost always use particular cases, generalized recognitions are used basically by the teacher and one of the women in the group (person 4, who adopts the main role in the classroom, and is more participative). This brings to light the difficulties that adults have to demonstrate the same type of intervention, even within an egalitarian dialogue. It is evident, however, that there is a fluid dialogue within the classroom, in which more or less everyone in the group participates. In addition to the capacity for abstraction that we all have, such as generalized recognition or evocation of the constant, the teacher does not monopolize them. In this way, we could observe how person 4 intervenes 33% of the times, which is way over the 30% of the teacher. In the category of evocation of the constant the same thing happens:

person 4 intervenes 40% of the time, while the teacher 20%. In Table 2 we can see the global distribution by percentage of the interventions in the different categories of the discourse.

Table 2. *Percentages by Categories of Analysis from the Total of Interventions*

	PC	DP	CI	GR	E*k*	p	a	De	er	ae	cc	Total
P1 (teacher)	11	14	23	30	20	79	18	7	33	23	50	33
P2	0	0	8	4	0	6	18	7	0	8	0	5
P3	0	14	8	15	0	3	9	22	9	8	25	9
P4	21	58	8	33	40	3	18	14	0	0	25	17
P5	26	14	22	7	20	3	10	7	0	8	0	10
P6	31	0	8	4	0	0	18	29	33	30	0	14
P7	11	0	23	7	20	6	9	14	25	23	0	12
Total	100	100	100	100	100	100	100	100	100	100	100	100

But not everyone intervenes in the same way. Table 3 shows the different types of interventions of each person in the group. Person 2, for instance, tends to agree with the comments that are introduced during class, which suggests that she is following a strategy of economizing her efforts and is simply dedicated to agreeing, or she finds difficulty with regard to the content and is therefore not able to reach the abstract content in most situations. Person 3 also seems to encounter difficulties when resolving the situations, as indicated by the quantity of the doubtful enunciations they make (21%).

Table 3. *Percentages by Person from the Total of Interventions*

	PC	DP	CI	GR	E*k*	p	a	de	er	ae	cc	Total
P1 (teacher)	4	2	6	14	2	48	4	2	8	6	4	100
P2	0	0	13	13	0	24	24	13	0	13	0	100
P3	0	7	7	30	0	7	7	21	7	7	7	100
P4	15	15	4	34	8	4	8	8	0	0	4	100
P5	31	6	19	14	6	6	6	6	0	6	0	100
P6	27	0	5	5	0	0	9	18	18	18	0	100
P7	11	0	15	11	5	11	5	11	15	16	0	100
Total	12	4	8	17	3	21	7	9	8	8	3	100

The opposite happens in the case of person 4, who makes many generalized recognitions during the dialogue (34%). Person 5, on the other hand, prefers to make comprehensive interpretations (19%), which is a way of giving meaning to the different mathematical concepts that appear in the activities. In this sense, it is a clear example of creation of meaning, because it means understanding the mathematical content through the use language (in line with Luria, 1979).

Although not everyone participated in the same way, this table shows that all the women were involved in the session. But this first analysis does not allow us to affirm whether these kinds of dialogue were egalitarian (or not), because we have no information about the nature of interactions. For this reason, we need to analyze the dialogues from use of language perspective.

Analysis of the functions (perlocutionary to illocutionary speech acts).

By analyzing the dialogues in depth (from the use of language perspective), we observe that the provocations have a clearly positive role during the egalitarian dialogue, given that they are a form of creating incentives and inviting everyone's participation in the classroom. Provocations are perlocutionary speech acts, but with a positive effect on the learning process. The following example is illustrates how the teacher (who makes quite a few provocations, as shown in table 3) takes an active role by inviting people in the group to reflect on the meaning of the constant of proportionality from an arithmetic table. It is important to stress the egalitarian character of the provocations. It is this egalitarian character that encourages collective communication and participation in the classroom, as in the following quote:

> Teacher: Yes, that's good, very good. But, how does this table go? Then.
> P3: Good.
> Teacher: OK, well, but how does this table go?
> P4: ... by three ... The price of a kilo, by the kilos it is … if it costs three Euros, then every kilo is three Euros. (Fragment of the transcription of the videotape)

On the other hand, the rest of the people do not tend to coincide with this characteristic: even though they also use some provocation, they tend to make other types of interventions (to resolve the situation, to ask a question, or simply to agree on what is being said in the classroom). Here, the context is resolving an activity about the time that shop assistants take in tidying up one of the market stalls...

> P3: The more assistants there are, the less, ... that is the more people, the less it takes ...
> P7: One takes 24 minutes ... 2, 12
> P3: Two, in less, in half. (Fragment of the transcription of the videotape)

In this fragment we see how two of the people involved in the group construct an argument through dialogue to explain a case of inverse proportionality. In order to help people distinguish direct proportion from inverse proportion, it is usual (in the Spanish tradition) to talk about the rule of more and less. It becomes a part of a ritualized traditional argument from the 1960s. In fact, it results in a wrong version of defining proportion as an increasing or decreasing function. What we observe as a new process is that P1 introduced such a rule just to give new status to a generalized recognition. The rule is not an imposition, but a result of an egalitarian dialogue. In this sense, P1 accepts that the women can use the generalized rule they observed by themselves for such problems, instead of being worried about the theoretical mathematical content.

Sometimes the women didn't recognize the general rule implied in a specific activity, but tried to understand the problem. In these situations, we talk about comprehensive interpretation. The following fragment is an example of a comprehensive interpretation: participants 4, 5, and 6 are discussing an activity that consists in discovering the proportional relationship that exists between the height of a door and that of the teacher, when the latter walks towards them or backwards from them (see Task 6, Table 1). The activity states: if we get closer or farther from the things that surround us, they become bigger or smaller to our sight. Do a test: try with someone else to measure the height of the door of the classroom between the two. What is bigger your class mate or the door? Do you think that if she starts walking forwards and backwards there will be a moment that your classmate will be as high as the door? (...)

> P6: If she (the teacher) is in front of the door, from far away she looks bigger.
> P5: If you walk further away, you see the same or…
> P4: No, I don't think so, I think she will remain being smaller.

In this transcript we can see different illocutionary interpretations: on the one hand Person 6 has a clear idea of the functional idea of proportionality (i.e., she sees that it is a relation in which one measure depends on the other one); and on the other hand, Person 4 does not really see that relation. The dialogue established leads to the conclusion that there exists a relation of proportionality between the distance at which the persons of the group are located, the teacher, and the door. But they didn't enunciate a general rule like "the size of something is directly proportional to the distance to our eyes." They only observed that something happens when the teacher walks back and forwards. So, they try to understand through a comprehensive interpretation what is happening.

The next fragment is another example arising from the effect that the previous experience has in the comprehensive interpretations of the women of the mathematics group. Here, person number 3 is mistaken when she says that the k means kilo (when actually, in the context of the activity, it can be seen that it means constant of proportionality). This is example shows the importance of dialogue in an egalitarian context, because through the contributions of each person in the group the doubts are resolved and the mathematical knowledge is constructed.

> P1 (teacher): The k is three kilos. The k is what we call the constant … in this case it is a k, which is always the same; it is 3, isn't it? This is the constant, that we always call k.
> P3: k is three euros, k is k, from kilo… <shakes her head doubtingly>
> Teacher: k is constant, it means constant.
> P4: Or, by giving k distinct values of the first quantity, weight, we get values of the second quantity... (Fragment of the transcription of the videotape)

This second analysis shows that learning is a social process (as Vygotsky, 1979, noted) where people participate through dialogue in order to understand the different activities. But we are interested in showing not only how dialogue can allow effective learning, but how interactions in egalitarian frameworks can produce effective learning. For this reason we analyze the dialogue through interactions, to see if perlocutionary or illocutionary speech acts are useful (or not) to produce learning (i.e., to see whether individuals can move to concrete examples from abstract ideas). This is the role of cognitive learning trajectories.

The Cognitive Learning Trajectories (CLT) Analysis

We have recognized different cognitive learning trajectories by adapting the type of graphs proposed by Dreyfus, Hershkowitz, and Schwarz (2001). In this way, there are cases in which the participants of the mathematics group use diverse particular cases to guide the dialogue towards the resolution of a proposed problem. In other cases, nevertheless, dialogue is at first located in the fringes of high abstraction, and immediately afterwards shifts towards the pole of the concrete. And, in other cases, through interaction with others in the group and with the provocation of the teacher, the trajectory of a concrete activity ebbs and flows from the concrete to the general, in order to continue the same process and end up in the emission of a generalized recognition. We have found each of these situations in the analysis of the videotape[2].

[2] About half-way through P3 makes a comment about being an amateur. This is not recorded in the CLT because it is in the affective domain and this model is only addressing the cognitive domain.

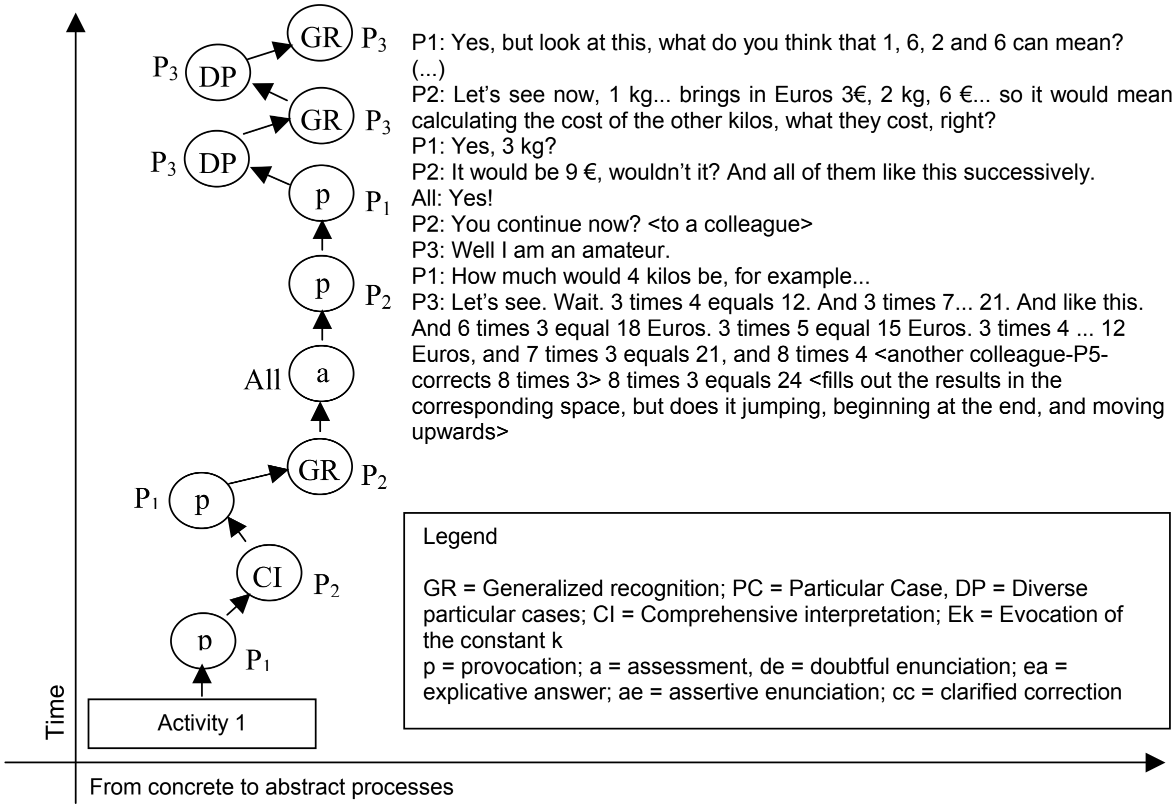

Figure 1. *Cognitive learning trajectory from activity 1(a)*

In the fragment of the transcript that we reproduce, the dynamic is of one person [P3] who is the protagonist of the cognitive learning trajectory: towards the end of the fragment she enters into a dialogue with herself (a type of a monologue) in which she goes through the particular cases to recognizing the implicit in the rules of proportionality. The teacher opens with a provocation.

Let us see, however, what happens in a lineal trajectory. When it occurs, we observe that on the two occasions it is when the teacher uses dialogue with a *perlocutionary* aim. This results in influencing in the response of the participant. This type of action occurs fundamentally where the task implies a technique of calculation associated with the content and in which the teacher understands that she must give a secondary role to dialogue in order to prioritize the reasoning that leads to a process of resolution. In these cases, provocations attain mechanical responses, because the teacher wants to break the traditional contract that the importance lies in the result (given that she does not want to insist at this moment that to provide an answer there must be a conversion of measures), but it is not understood well by the adults who are still tied to this contract.

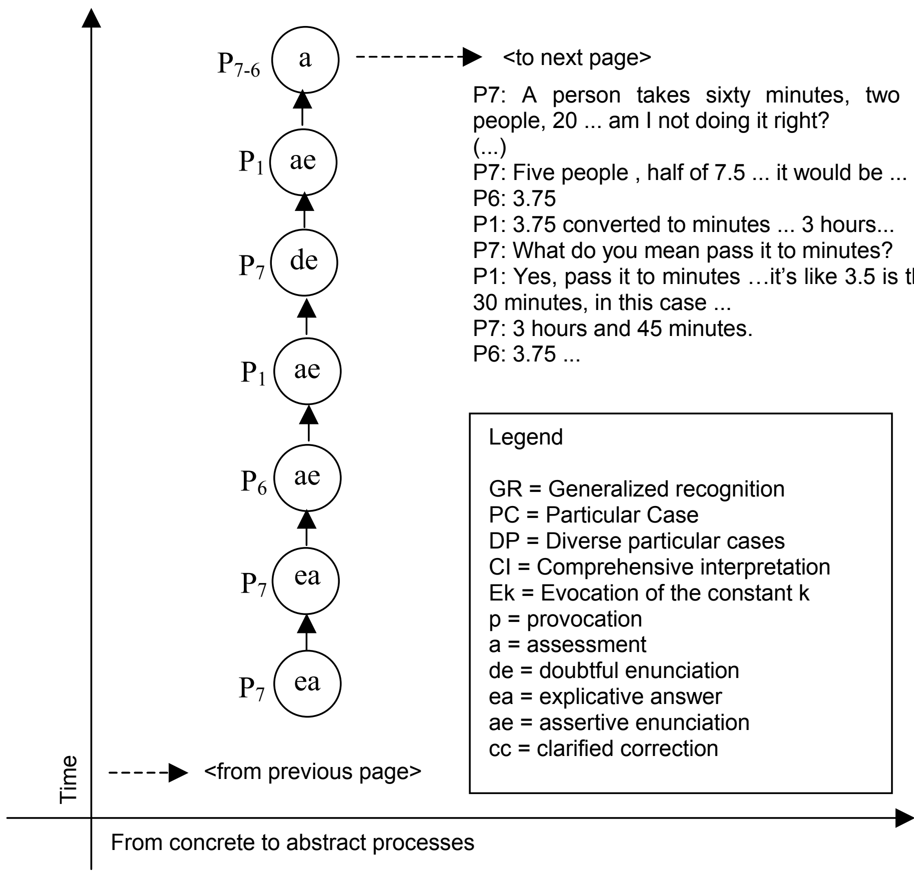

Figure 2. *Cognitive learning trajectory from activity 3.*

These two examples allow us to understand the importance of egalitarian dialogue in learning. When this dialogue exists in a perlocutionary manner based on the position of power of the teacher, there is an exchange of questions and responses which do not leave room for reflection, or comprehension of the mathematical concepts. On the other hand, when dialogue takes place from the illocutionary point of view, or in the case of perlocutionary, based on claims of knowledge (in line with Habermas, 1987), a phenomenon occurs that is termed creation of meaning. That is to say: the person is capable of using the concept upon which they reflect, and endow it with meaning. When someone is capable of using an abstract concept to make concrete examples (e.g., in order to explain their meaning), this indicates that they really understand the meaning of this concept (given that they are capable of using it in diverse concrete situations). And this is what occurs when a person tries to help someone understand (through arguments) the meaning of proportionality.

Conclusion

After studying the totality of the data, and not only the previous examples, we have been able to observe two types of conclusions:

On the one hand, with respect to the mathematical content from the observation of the cognitive trajectories analysed, we have empirically verified that adults who understand the abstract meaning of a mathematical concept tend to explain it through successive concatenations of concrete examples that appear immediately after the generalization.

Examining the examples as described above, we have not been able to assume that adults develop the knowledge of finding the theoretical value of the constant of proportionality. As a consequence, we argue that it should not be among the required competencies in learning, as is usually indicated for younger students. An absolutely clear idea of the activity with respect to the proportion is not attained here either. Thus, the influence of the “rule of three” to resolve proportional problems appears to be a barrier for recognizing the activity characteristic of the proportional function (f(x+y)=f(x)+f(y)). But what has been attained is that they have a very clear knowledge of the practical value of proportionality. With practical knowledge participants are able to consolidate a procedural strategy in order to know the value that is being asked of them. Some people are able to remember the rule of three, but they reconceptualize it, and do not recognize it as a unique tool for resolving problems.

On the basis of what we have observed and analysed, we consider the interpretive development of the proportion phenomenon important; also the different registers with which the participants express themselves (tables, calculations, symbols, techniques of mental calculation, etc.) as well as the use of adequate language.

Regarding the dialogue which was established, we recognize that illocutionary approaches encourage dialogue, which constitutes the fundamental path taken by adults for learning and jointly constructing the meaning of the abstract mathematical concepts. We also found that perlocutionary speech acts can encourage learning, but can also create barriers when the speaker uses a position of power that breaks with egalitarian dialogue.

In addition, egalitarian dialogue animates participants’ learning, because they have to seek the correct arguments to justify the solutions that are proposed in the different problems. On the other hand, when the teacher directly gives the answer to the problem, without leaving room for dialogue within the classroom, adults are limited to agreeing without understanding the meaning of these reasonings. This is clear in the lineal cognitive trajectories.

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Adults' Resistance to Learning in School versus Adults' Competences in Work: The Case of Mathematics

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Abstract

In adult education, resistance to learning is a well known phenomenon. There is an apparent contradiction between many adults' problematical relationships with mathematics in formal settings and their noteworthy "mathematics-containing" competences in everyday life. However, there is very little research done on the subject, and resistance is often explained purely as a lack of motivation and the symptom described as non-learning. In order to investigate adults' resistance to learning, we must take into account the set of conflicts between the needs and constraints in adults' lives. In this paper people's resistance is seen as interrelated with their motivation and their competence and thus as containing the potential to be a crucial factor in all types of learning.

Keywords. Adults' resistance to learning; school vs. workplace; mathematics; cognitive-affective relationship; affect; emotion; motivation.

Introduction

The perspective in the idea of lifelong learning structuring today's educational system demands a rupture with the limited conception of learning and knowledge. Individual and collective learning processes do not take place only as schooling within formal education, and the focus has to be moved from teaching in institutions to learning in the workplace and in everyday life (Olesen, 2002). This is also the case for the learning of mathematics. However, in the case of mathematics there is a further specific issue. Adults develop to a great extent their mathematics-containing competences and qualifications through participation in communities of everyday practice (see, e.g., FitzSimons & Wedege, 2004). Nevertheless, it appears that their beliefs about mathematics are primarily related to their school experiences, and mathematics is experienced by many adults as something that others can do, but that they themselves cannot do — nor do they need it (Coben, 2000; Wedege, 2002a). People's everyday competences do not count as mathematics (FitzSimons, 2002) and the subjective relevance of the school subject mathematics is questioned by many individuals within and outside of the educational system. The phenomenon of the coexistence of the social significance of mathematics, as widely acknowledged within the culture, with the invisibility and irrelevance felt by many individuals, has been called "the relevance paradox" (Jensen, Niss, & Wedege, 1998). But maybe this is only a paradox for mathematics teachers. When people reply that they don't use and don't need mathematics in everyday life, they may be referring to school mathematics, not to "mathematics at work" (Wedege, 2002a, 2004).

From the perspective of lifelong learning, education is experienced by adults as a field of tension between felt needs concerning what one wants to learn — or has to learn — and various constraints (Illeris, 2003a). This set of conflicts is a background to adults' learning processes in late modern society, which we must take into account if we want to discuss adults' motivation and resistance to learning. The objective of the ongoing research project "Adults learning mathematics in school and everyday life" is to establish an interdisciplinary theoretical framework to describe, analyse and understand the conditions of adults' learning processes — including social and affective aspects (see http://mmf.ruc.dk/~tiw). We conduct this research both within the frame of formal adult mathematics education (school) and as informal mathematics learning in communities of everyday practice (e.g., work or "leisure activity" groups). This paper — reporting on work in progress — is based on international networking in this project and more specifically on the working paper "Motivation and resistance to learning mathematics in a lifelong perspective" where we present and discuss motivation and resistance as interrelated phenomena (Evans & Wedege, 2004).

Our work on adults' resistance to learning in school deals with factors within education, not with enrolment problems or adults' barriers to returning to mathematics education. The work presented is based on the assumption that people's relationship with mathematics has three key aspects: cognitive, affective, and social. The focus is on the relationship between the cognitive and the affective — the latter understood to include beliefs, attitudes, emotions and motivation - and also on the assumption that both of the first two, and their interrelationship, must be understood as not merely individual, but also as socially organized (Evans, Morgan, & Tsatsaroni, in preparation). It is argued that inter-disciplinarity is necessary in the research domain of adult mathematics education in and for the workplace. Three positions in adult mathematics education are presented as interconnected also to the perspective of resistance. Literature concerning resistance in adult education is briefly reviewed. Finally, two accounts of interconnected motivation and resistance to learning mathematics in two adults' lives are presented as illustrations of the framework we are aiming to develop here.

Three dimensions of adults' learning

A general framework for understanding adults' learning has been produced by Knud Illeris (2003b, 2004) by combining a range of learning theories. He makes two fundamental assumptions:

1. All learning includes two basic processes: an external interaction process between the learner and his or her social, cultural and material environment, and an internal psychological process of acquisition and elaboration; and
2. All learning includes three dimensions embedded in a culturally situated context: the cognitive dimension of knowledge and skills, the affective dimension of feelings and motivation, and the social dimension of communication and co-operation.

We find it a useful basis for our work to think of people's relationship to mathematics as able to be analyzed in these three key aspects: cognitive, affective and social, remembering that a specific socio-cultural context always sets the basic conditions for the learning processes. This theoretical choice illustrates the need for interdisciplinary work in our field, as we argue below.

Research in Workplace Mathematics: Inter-Disciplinarity is Necessary

The research domain "'Adults' Mathematics in and for the Workplace" is being cultivated in the borderland between research in mathematics education and in adult education, including vocational training, from where we import and reconstruct concepts, theories, methods and findings (Wedege, 1998, 2004). In the same vein, in *What counts as mathematics?*, Gail FitzSimons (2002) locates her work on the borderlands of the fields of mathematics education, adult education and vocational education — connecting all three. And she continues: "Perhaps it is due to these borderland crossings that my research is often located at the margins of each of these research communities." (p. 2). In this no-man's land, the construction or reconstruction of conceptual frameworks are important tasks in research. Lave's theory of situated learning and Engeström's theory of expansive learning are two examples of general theories that have been used and re-interpreted in studies of adults' mathematics in work (FitzSimons, 2002; Wedege, 1999a).

Research in adult and mathematics education

The subject area of adult education encompasses formal adult mathematics education as well as adults' informal mathematics learning in the communities of everyday practice, for example in the workplace. The development of adult education research into an independent academic field is closely associated with the institutionalisation of adult learning. But, although the development of the field of practice is an important criterion for relevance, this means not just the ability of research to answer the problems in the field of practice, but also to criticise and reformulate these problems (Olesen & Rasmussen, 1996). Within the field of mathematics education research, relevance to the practice of teaching or learning mathematics is also a criterion of quality. The subject field is constituted by the problem field of *mathematics education as a practice in all its complexity*. Which means that the subject area is "always-already" structured and delimited by the concrete forms of practice and knowledge that are currently regarded as mathematics teaching, mathematics learning and mathematics knowing. However, a critical perspective might be opened up when studies concern the functions of mathematics education in society and in people's lives (Wedege, 1999b).

It is an important part of the self-conception in the research field of adult education that it cannot be subordinated in a disciplinary context (for example, as a sub-discipline in pedagogy, psychology, or sociology), but that inter-disciplinarity is a significant feature (Olesen & Rasmussen, 1996). The field of mathematics education research also makes use of concepts, methods and results from other disciplines (psychology, sociology, linguistics, anthropology, philosophy). In the beginning, the studies were *multi-disciplinary*. The original approach in research was to deduce the consequences for mathematics education from findings in the other disciplines, especially psychology. To create *inter-disciplinarity* the imported conceptual frameworks have to be derived and modified (Brousseau, 1986). In adult vocational and further education, the reasons for teaching and learning mathematics are to be found outside mathematics. That is another reason why inter-disciplinarity is essential in both education and research, and reconstruction of conceptual and theoretical frameworks from other disciplines is a central task (Wedege, 1999b). In addition, mathematics education research has a specific relationship to mathematics as a scientific discipline, as a social phenomenon, and as a school subject (Niss, 1994). What is recognized as mathematics, and what is not, is important to research, and is also a political question; a question about mathematics and power (FitzSimons, 2002; Mellin-Olsen, 1987).

In Danish adult education research and development in the late 1980s and the 1990s, the theoretical construction of a general concept of *qualification* was a driving force as adult education is closely connected with work as an individual and a social phenomenon (Olesen, 1994). Today the term *competence* is almost hegemonic in educational discourses. In order to study the relation between vocational mathematics education and mathematics at work, Wedege (1999b) imported and reconstructed the concept of qualification, in which a dualism is incorporated - qualification is seen both as a characteristic of the requirements for skills and abilities of the job function and as a characteristic of the skills and abilities of the (potentially) working person. After a survey of adult mathematics education research, Wedege claimed that two different lines of approach are possible *and* necessary: the *objective approach* (the labour market's requirements with regard to mathematical knowledge), and the *subjective approach* (adults' need for mathematical knowledge in their present and future workplaces). We understand people's *competence* as their "readiness for action and thought and/or an authorisation for action based on knowledge, know-how and attitudes/feelings (dispositions)" (Wedege, 2001, p. 27). Competence is always linked to a specific situation context while qualification is linked to a job function.

Research combining adults, mathematics, and society in workplace studies opens up the possibilities of making the adults' mathematics visible in their competences situated at the workplace, and making mathematics visible in the qualifications demanded from the labour market. This could be the basis of studying adults' motivation and resistance to learning mathematics as interrelated phenomena in a field of tension between constraints and needs.

Motivation and Resistance in Mathematics Education

Using this interdisciplinary approach, how can we study resistance to learning mathematics among adults? Resistance is often explained on the basis of lack of motivation in the individuals affected, and its symptom seen as non-learning. We can locate motivation and resistance — understood as conditions for people's learning and non-learning — as part of the affective dimension. Much relevant work on affect in a broad sense has been done in the field of mathematics education (see, e.g., Evans, 2000). According to McLeod's (1992) review, beliefs, attitudes, and emotions are used to describe a wide range of affective responses to mathematics. These three terms are not easy to distinguish but they can be understood to point to differences in *stability*: beliefs and attitudes are generally relatively stable over time, but emotions may change rapidly. They also vary in levels of *intensity* of the affect that they describe, increasing in intensity from "cold" beliefs about mathematics, to "cool" attitudes related to liking or disliking mathematics, to "hot" emotional reactions to the joys or frustrations of solving non-routine problems. McLeod also distinguishes beliefs, attitudes, and emotions with respect to the degree to which cognition plays a role, and the time they take to develop. They have been positioned along a spectrum that runs from *stability* and "cool" on the left, to fluidity and *intensity* on the right (see Figure 1).

Beliefs Attitudes Emotion
<-->
Stability *Intensity*

Figure 1. Spectrum of types of affect.

In this model beliefs include self-perception (e.g., “Mathematics — that’s what I can’t do.”), aspects of ‘identity’ (e.g., “We ‘semi-skilled workers’ are not using mathematics versus ‘the others’ using mathematics), and confidence (see Leder, Pehkonnen, & Törner, 2002); attitudes (e.g., mathematics anxiety) are more stable than emotions (e.g., panic or joy; see Evans, 2002). (For a recent review of research into this area, see Evans, 2000).

When we turn to motivation and resistance, it might appear that not so much relevant work has been done, to date, within mathematics education research. Neither the term *motivation* nor the term *resistance* is to be found in the subject index of the first and second international handbooks of mathematics education (Bishop, Clements, Keitel, Kilpatrick, & Laborde; 1996; Bishop, Clements, Keitel, Kilpatrick, &Leung, 2003). However, in psychology, motivation related to mathematics is an intensively studied topic; see for example Markku Hannula’s review (2004) and several chapters in Leder et al., (2002). According to Hannula, motivation has been discussed in mathematics education under the terms motivational orientation, interest, and motivational beliefs. He surveyed some important distinctions made in psychological approaches to educational contexts: (a) intrinsic motivation (rewarding in itself) vs. extrinsic motivation (offering an outside reward); and (b) learning (or mastery) goals, performance (self-enhancing) goals (“I will raise my mathematics mark”) and ego-defensive (avoidance) goals (“You don’t need mathematics in life”). Mastery and performance are usually seen as competing motivational orientations but, in his research in mathematics classrooms, Hannula found instead that they can support each other as goals.

From a socio-cultural perspective, Stieg Mellin-Olsen (1987) distinguished concepts of social rationale (*S-rationale*) and instrumental rationale (*I-rationale*) for learning mathematics, which might be seen as somewhat parallel to the psychological concepts intrinsic and extrinsic motivation. Mellin-Olsen claimed that on most occasions both rationales may be present. The S-rationale is evoked in a student by a synthesis of his self-concept, his view of school and schooling, and his concept of what is significant knowledge — according to his beliefs and values — and of possible future value. The I-rationale is related to the way in which the school is viewed as instrumental in providing the pupil with a “future”. Its most important manifestation is in the way in which the (external) examination system can provide certification, and the I-rationale will exist almost independently of the actual content of the mathematics curriculum.

When it comes to resistance, we find very little research has been done so far in mathematics education. Drawing on the work of Vygotsky and Leont’ev among others, Mellin-Olsen (1987) developed a theory of Activity as a basis for mathematics education. His idea of resistance is closely linked to concepts of *culture* and *ideology,* the latter defined as the set of attitudes which the individual takes from the system of groups s/he has as referents for her/his behaviour. The social environment to which the individual reacts is central in his analysis: “… social groups resisting social reproduction, such as those rejecting school, are real. Their existence has to be included in theories about such reproduction” (pp. 196-197).

To locate resistance Mellin-Olsen (1987) needed a concept of culture in which the resistance is produced. With reference to Giroux, he defined culture as “not simply the lived experiences within society, but the lived antagonistic relations situated within a complex of socio-political institutions and social forms that limit as well as enable action” (p. 97). Mellin-Olsen’s political concept of culture sees the individual as both a producer and reproducer of culture. “What we experience and will take advantage of in the context of education is the existence of resisting cultures and counter-cultures which reject the dominant ones” (p. 198). His message to mathematics teachers is about accepting resistance rather than rejecting it, and about how to turn indifference and destructiveness into constructiveness, through activity.

Mellin-Olsen's research interest and empirical data mainly concern children and adolescents. If we undertake research in the problem field of adults learning mathematics, we may find three interrelated positions which have to do with resistance to learning mathematics, from a socio-cultural perspective.

Three Interconnected Positions in Adult Mathematics Education

From a large quantitative data set in the International Adult Literacy Survey [IALS] (Organisation for Economic Co-operation and Development [OECD], 2003), we know that many individuals assessed to be on the two lowest levels of literacy (1 & 2) nevertheless believe their skills are good or excellent and thus do not see any need to engage in learning. In this survey, societal and the labour market's requirements with regard to adults' quantitative literacy have been defined operationally, at an international level, at five proficiency levels, in answer to the question "What does a person need to compete successfully in a marketplace which increasingly requires technological know-how and high-level skill?" (OECD, 1995). For example, Level 3 was defined as the level necessary to cope with the challenges of today's and tomorrow's society. People's functional skills required for daily arithmetic operations were assessed according to the above standards, whereas adults' self-assessment of their skills seems to have been based on their experiences as competent people in everyday life.

In the light of the OECD results, the following three interrelated positions frequently taken by adults have been constructed to illustrate the complexity of the subject field.

"I am not here to learn mathematics."

Many adults' perspective on vocational or further education can be summarized in this statement: "I am here to be a nurse (or painter, etc.), not to learn mathematics." Adult education is in principle post-compulsory, but the subject of mathematics is often sold as a part of the package. It is a well-known phenomenon that students who commence vocational education are often surprised to learn that they must undertake the study of mathematics as part of their courses (Strässer & Zevenbergen, 1996; Wedege, 1999a). Normally, we connect this belief/attitude with people having a poor relationship with mathematics or with people to whom mathematics is not a central or visible part of their professional identity. However, in a recent study, Wedege (in preparation) found examples of this belief/attitude even in people who had studied mathematics at an academic level. A group of unemployed graduate engineers started vocational retraining at university. The course was named "From graduate engineer to grammar school teacher" and the participants were very surprised that they had to study two demanding mathematical subjects in algebra and calculus. They were convinced that they already had enough mathematical knowledge to teach in upper secondary school and had expected that pedagogy would be a central part of their new professional identity, not mathematics.

"Mathematics — that's what I cannot do."

Many adults' beliefs about mathematics as something beyond their reach is summarized in this statement: "Mathematics — that's what I cannot do" (Wedege, 2002a). Diana Coben (2000) and her colleagues have used the term "mathematics life histories" to describe adults' accounts of their mathematical experiences throughout life - both those that are explicitly mathematical (e.g., being taught subtraction at school, or working out a budget as an adult) as well as those in which mathematics can be seen to be implicit (e.g., knitting or judging distances when driving). Almost all the people interviewed remarked on the importance of mathematics and success in mathematics examinations. On the other hand, it appears that once people have succeeded in applying a piece of mathematics, it becomes *non-mathematics* or *common-sense*. Thus, they never perceive themselves as successful: mathematics is always what they cannot do. The mathematics one can do, which one does not think of as mathematics has been called *unrecognised mathematics* (Wedege, 2002a), or *invisible mathematics* (Coben, 2000). Coben claims that this position, may have limiting effects on the individuals concerned and perhaps more widely, on conceptions of mathematics in society in general. First, for the individuals concerned, mathematics is rendered unattainable. It becomes, by definition, what they cannot do. Second, the individuals' negative self-image as unable to do mathematics may affect their confidence as learners, since mathematical ability is widely considered an index of general intellectual ability. Third, in society at large, the image of mathematics as difficult and open only for the selected few, is maintained rather than challenged.

"No, I don't use mathematics at work."

A third position — connecting the first two positions from an analytical perspective — is represented by the well known phenomenon that the answer is "No" when you ask adults (without a clear mathematical profile in their profession) if they use mathematics in their work (Bessot & Ridgway, 2000; Wedege, 1999b). Mathematics is interwoven with technology - in technique, work organisation and qualifications. However, modern computer techniques hide the use of mathematics in the software, and mathematics as a visible tool disappears in many workplace routines (e.g., Noss, 1997; Strässer, 2003). But this objective invisibility isn't the only reason for the negative answer. As stated above adults don't recognise the mathematics in their daily practice. They just don't connect their everyday activities with "mathematics" — which most of them associate with the school subject or the discipline.

Differences between informal mathematics (e.g., street mathematics, folk mathematics) and school mathematics (i.e., that which people learn and practice in formal education) have been investigated in a series of studies (e.g., Bakker et al., 2005; Evans, 2000; FitzSimons, 2002; Lave, 1988; Mellin-Olsen, 1987; Noss & Hoyles, 1996; Nunes, Schliemann, & Carraher, 1993; Schliemann & Acioly, 1989; Wedege, 1999a, 2002b). The well known school activity of solving problems might serve as an example of these differences. In this kind of approach (now traditional in many countries), which might be called *task-directed mathematics education,* instruction is directed towards the students' acquisition of procedures, formulae, and concepts, through the teacher's presentation of theory, examples from the textbook, and the solving of tasks set by the teacher or the book. Reality is a pretext to use mathematical ideas and techniques. The problem solving constitutes a central element and structures the teaching. The problem is primarily used to practise skills and to test skills and understandings. Thus, the problem is often solved individually by the student and it might be conceived as cheating to hand in a joint solution. The problem is formulated by the teacher, the textbook, or the program. The problem has only one correct solution. Precision is often treated in a fetishistic way, for example in keeping a larger number of decimal places in the answer than could be useful in any conceivable practical application. Solving so-called problems has no practical meaning: the results are not used for anything except, maybe, solving more problems. What can be called the task-context Wedege (1999a) is often sketched as a practical one, but the aim is to find the correct result by using the correct algorithm, not to solve the practical problem.

By contrast, in the workplace what could be called the situation-context (Wedege, 1999a) throws up problems which may (or may not) require the use of mathematical ideas and techniques. These problems result from the need to solve a working task where the numbers are to be found or constructed with the relevant units of measurement (e.g., hours, kilograms, millimetres). It is the working requirements and functions, in a given technological context, that control and structure the process, not a narrowly-defined task. Some of these problems may look like school tasks (the procedure is given in the work instruction) but the experienced worker has his/her own routines, and methods of measurement and calculation. Circumstances in the production process might cause deviations from the instruction or might reduce the number of random samples in the quality control process. It is characteristic that problems are solved in different ways and that different procedures and solutions might be acceptable. In the workplace, solving problems is a joint matter: you have to communicate, and to collaborate. Solving problems in the workplace context always has practical consequences: a product, a working plan, distribution of products, or a price (see Wedege, 1999b).

Resistance in Adult Education

Resistance to learning is a well-known phenomenon in adult mathematics education. Often adults' resistance in the learning situation has to do with the fact that they have experienced themselves as competent persons without mathematics, and that mathematics has not been perceived as relevant to their life projects. This kind of resistance seems to be close to the ego-defensive type of motivation. The goal is to defend yourself and your self-perception: "Mathematics is not important in my life." This belief stems from the adult's experience in various communities of practice (work, family, leisure), where basic arithmetical skills have appeared to be sufficient to cope with the challenges, or where mathematics has been hidden in techniques and technologies.

In adult education, too, there has been very little research done on the subject, and resistance is often explained purely as a lack of motivation and the symptom as non-learning. However, in recent research in adult education, we may find understandings of resistance as a response to the learning situation.

Non-learning.

In a broad-ranging discussion of learning from experience, Peter Jarvis (2001) has produced a typology encompassing non-learning, non-reflective learning, and reflective learning. He included three types of non-learning, namely: presumption, non-consideration, and rejection. *Presumption* is an almost thoughtless and mechanical response to everyday experience — the situation is well known and we know precisely what to do[1]. There is a sense of harmony between the biography and the individual's experience. Non-consideration is another common response to potential learning situations. The individuals realise that there is a disharmony between their biography and their experience (disjuncture) but they do not respond by adapting and learning something new. Rejection is when people have an experience but deliberately reject the possibility of learning (disjuncture). They don't wish to change their understanding of things since their whole identity is based upon it.

In Jarvis's terminology, presumption is different from the other two forms of non-learning: there is no disjuncture and no change. In this light, we would call non-consideration and rejection *resistance*.

[1] We are indebted to one reviewer for the suggestion that, rather than being non-learning, *presumption* might be considered as learning if, for example, the level of automaticity of the relevant skill were to increase.

Situational resistance and ulterior resistance.

Some people fail assessments in a course or a training programme: they didn't learn what they were supposed to learn. This is directly observable at a certain level, and the explanation is usually sought in the design or delivery of the programme or in the selection procedures for it. At the micro level, tutors or trainers of adults are familiar with individual learners who continually raise objections to the teaching material, or who fail to perform as might be expected. According to James Atherton (1999), their behaviour is most commonly explained as lack of motivation, ability or aptitude. However, he finds that in many cases these explanations are correct, but not exhaustive. Most failures to learn are products of a number of interacting variables, and Atherton submits that a degree of personal resistance — different in kind from "lack of motivation" — is to be considered along with other factors.

His article "Resistance to learning" is derived from an interview-based study carried out with 124 participants — with established ideas and practices — on in-service training programmes in social services. In his investigation, it was hypothesised that resistance is itself symptomatic of a situation where the learning is experienced as supplantive, that is, the material replaces or threatens knowledge or skills which have already been acquired, rather as additive learning.2

The researcher was a member of the tutorial team in the 25 courses/training programmes being studied. The interviews were recorded as part of the evaluation procedure. One of the questions went like this: "What areas of the course learning have caused you to re-think your previous approach to your practice?" (Atherton, 1999, p. 80) This was the question that most explicitly addressed the issue of supplantive learning. The respondents articulated a loss of certainty, which was provoked or triggered by the experience of the course. The diversity of the triggers was surprising to the researcher and there were very few respondents who reported none.

Atherton opposes *situational resistance* provoked by the immediate situation (e.g., the work is boring or too difficult, insufficient pre-course information etc.) to *ulterior resistance* of supplantive learning provoked by underlying concerns with one's ongoing sense of competence and identity generally. The interaction of the two kinds of resistance is complex; for example, some students' reactions (e.g., complaints), presented as situational resistance, may have a more ulterior basis; these might be called *rationalizations*. This raises issues that may be addressed via psychoanalytic ideas.

Defence versus resistance

In Illeris's general theory of learning, a basic distinction is made between non-learning caused by mental defence and by mental resistance: "the defence mechanisms exist prior to the learning situation and function re-actively, resistance is caused by the learning situation itself as an active response" (Illeris, 2003b, p. 404). We understand him to use the term *resistance* to refer to situational resistance, and to contrast this with psychic defences that also may lead to non-learning. Defence is a barrier to learning (relevant or not). Resistance implies the possibility of learning. According to Illeris, resistance contains a very strong learning potential for accommodative and even transformative learning — what we call supplantive learning following Atherton (see above and footnote 2).

[2] Atherton's two terms, additive learning and supplantive learning, could be related to Piagetian terminology. There we find theoretical frameworks (or terminology) with different levels of learning: cumulative, assimilative, accommodative, transformative or expansive learning (cf. Illeris, 2003b). We understand *additive learning* as cumulative or assimilative learning; and *supplantive learning* as accommodative or transformative.

Thus Illeris echoes Mellin-Olsen (1987), although Mellin-Olsen is also explicitly interested in resistance to oppressive social relations of power in society at large. Evans (2000) acknowledges Mellin-Olsen's characterization of resistance, and also seeks to illustrate situations where processes understandable through psychoanalytic insights may lead to resistance to learning arrangements (see below).

Overall, we argue that adults' feelings, attitudes and beliefs (e.g., self-perception in relation to mathematics) are produced in ways that depend upon the social conditions of their learning processes (Wedege, 2002a) — and, at the same time, are particular to the individual's history of positionings in specific practices (Evans, 2000).

According to Evans (2000), adults' resistance to doing mathematics problems in a specific situation results from a range of social processes and it needs to be understood as an effect of one's cumulative history of positioning in practices related to learning mathematics. Positioning includes both the positions one has been put in, by others (often more powerful), over the course of one's schooling — for example, as "a good student in maths", or as "a lazy student" — and the positions one has succeeded in taking up oneself — for example, as "the joker in the class" (Evans, 2000; Evans et al., in preparation). One's positioning is influenced — but not determined — by class and gender. The subject's positioning "results both from the general social availability of positions in discourse, and from the investment for the particular person to take up a specific position" (Evans, 2000, p.133).

Examples of Motivation and Resistance in Two 'Adults' Lives

In this section analysis of interviews with two adults will serve to illustrate the conceptual framework presented above.

Peter

As part of a study of adult numeracy at a higher education institution, Evans (2000) carried out semi-structured interviews with a (partly randomly selected, partly voluntary) sample of 25 first year social science degree students. One interview, with Peter (a pseudonym), is presented here to illustrate themes relating to resistance and the emotions. Peter was 20 years old at entry to the degree, and from a middle class background. He had passed O-level (age 16+) Mathematics and was specialising in Economics.

At the beginning of the interview, he tells Evans he had to keep taking O-level Mathematics until he passed it on the third try: "So I was […] under a little bit of pressure ... [at grammar school]".

He explains how he came to be taking Mathematics O-level:

> S: See, my father's an engineer and all of my brothers, bar one, are teachers — and I was not pushed, but I was *gently persuaded*, in the area of taking sciences at O-level, then another two sciences at least at A-level, going on and doing some kind of teacher training degree….
> [… 2 lines …] … so, you know, I was *always* pushed towards taking Maths at O-level and A-level, and also Physics at O-level and A-level, and unfortunately I just didn't master either of them....
>
> (Evans, 2000, p. 211, Peter's emphasis)

Being the youngest of five sons in such a "mathematical family" was a mixed blessing — especially when it came to homework — the activity which quintessentially involves *inter-discursive positioning* across family and school practices:

> S: My dad is the sort of person who will, if you ask him a question, instead of giving you straight answers, he says — well, hold on a minute, I'll go and find a book — and there's another book, and then another book, and hopefully a five minute explanation turns into a half hour looking through [books], and you're getting a very complicated explanation. And I found from that I don't think I was ever really interested in mathematics....
>
> (Evans, 2000, p. 211)

Being tutored by his oldest brother - himself a qualified mathematics teacher, was even worse:

> S: ... if I didn't get anything right, then ... it was even more — you know, lecturing and er, you know, sort of, not exactly saying that I was stupid, but getting onto the old intelligence bit — so I suppose I became a bit scared of maths in general as a subject as well as physics, and as I say, it was a relief to take something else as *non*-numerical, or easy to grasp, as Law or Economics or History [subjects he did at A-level] (JE: Right, sure) — there *are* numbers involved, but they're just not *forced* on you in the same way ....
>
> (Evans, 2000, pp. 211-212; Peter's emphasis)

Building on his distinction between Mathematics and "non-numerical" subjects, Peter goes on to assert that the mathematics used in Economics is not "hard to handle" since he can understand the symbols; he contrasts this with algebra where, in his view, there is "no excuse to make up symbols to replace something [which already has] an actual reality". Here the basis for his resistance to mathematics comes from differentiating it from other subjects that are easier to manage (in some sense).

Peter expresses a range of emotions in the following story about school:

> S: I remember [...] in maths lessons being caught day-dreaming [...] And then I would have to [..] admit that I wasn't paying any attention.... [4 lines] ...
>
> I think that was something that was unique to maths lessons, [...] being asked questions and not knowing the answer, and ... being very, very — first of all, you know, embarrassed and ashamed — and then a little bit angry at being asked a question in the first place. What am I doing here, y'know, sitting in front of these useless numbers? — they'll never be any use to me — and why would I want to know how long the side of a triangle is?
>
> (Evans, 2000, pp. 213-214)

This chain of feelings on the part of a student — surprise, embarrassment, humiliation, anger, leading to resistance — is not uncommon in our experience as teachers of adults.

Peter also exhibits some lack of confidence about mathematics, when Evans asks him to try some problems part-way through the interview:

> JE: ... if I give you a few questions to try. Would you be happy about that?
> S: Well, reasonably ...
> JE: Reasonably ...
> S: Yes, I've been to a couple of job interviews where there's been some pathetically easy sums on a piece of paper - but because they've been on a sheet, set out in front of me *with someone looking over me*, I haven't been able to do them (JE: yes). I think that's something that's come from having been taught by my father in that way ...
>
> (Evans, 2000, p. 213; Peter's emphasis)

Here his resistance to doing questions in the interview seems related to anxiety exhibited by the story about his job interview, which he himself links back to earlier experiences with his father.

This discussion shows that Peter experienced pressure in several ways — to do mathematics and physics by his father and brothers, and to work at mathematics in particular ways, stipulated by them. We can understand these particular resistances as related to situational resistance, provoked by an immediate pedagogical situation. Ulterior resistances can be illustrated by the threats to his sense of competence or identity which were posed by his (understanding of his) brother's remarks in the tutorial, or by his difficulties with algebra, or indeed by the process of being "caught napping" by the school teacher and experiencing the chain of negative feelings to deal with.

Now, these emotions and resistances are profoundly social: they depend on Peter's positioning in family (and school) discourses. In this family, being able to do mathematics signifies intelligence, and rationality, for the father and five sons at least. His father and his four brothers have put a lot of effort into knowing about mathematics. Thus, Peter is positioned as lacking success, even incompetent, in mathematics, and more generally. And, if Peter made mistakes in a teaching session, his older brother "would get onto the old intelligence bit". This led to pressure on him, and to his reactions of ambivalence (conflict) and resistance, and almost certainly to a "vicious circle".

Continuing the analysis with the categories introduced above, we can see some of his reactions as defensive. He had clearly experienced much anxiety, from the constant pressure of the early homework sessions, and similar experiences at school. In order to avoid this anxiety (and pressure), he reacted by "mathematics avoidance", by choosing relatively "non-numerical" subjects, such as Economics, his degree subject. There is clearly some overlap between what Atherton calls ulterior resistance and what Illeris calls defensive.

Some of these ulterior/defensive reactions may have been subconscious, in the sense of not fully reflected upon, others unconscious, in the sense of being kept completely outside of the person's conscious awareness, so as to cover over the pain, for example of an unconscious conflict; see Evans (2000, pp. 210-217) for further discussion of Peter's conflicts about "mastery" of mathematics. Briefly, though he fantasises about mastery and success, his apparently resistant actions are likely to militate against success.

The interview thus provides examples of relatively conscious resistance to mathematics — not reading his father's books, not using textbooks for first year mathematics. But there are also examples of what may be unconscious resistance — "losing concentration", not attending lectures on time, not reading the instructions for Question 1 in the interview (Evans, 2000, p. 215). His resistance in all these situations may be seen as an attempt to avoid, or to manage, anxiety.

Ruth

In the account of the mathematics life history of a 75 year old woman[3], Wedege (1999a) identified changing conditions of Ruth's learning process in different contexts through her lifetime, using terms like blocks, resistance, rationale, motivation. In that account, it was argued that habitus (Bourdieu, 1980) was an important generator of resistance and

[3] Ruth is a close relative of, and was interviewed by, TW; one of the effects of this was that the interview could draw on a range of regional allusions and shared meanings.

motivation.4 In order to try out the usability of our developing theoretical framework defined above as an analytic tool, we have read the transcribed interview again.

This woman, Ruth, scored the lowest mathematics grade in secondary school leaving assessment, but has always been very competent in arithmetic. At fifty Ruth went to the technical school where she scored the highest grade in mathematics. However, she has always looked upon herself as someone not knowing and not doing mathematics. Wedege (1999a) remarked in her analysis of the interview that Ruth's motivation to be a draughtsman made her overcome her blocks, but not her resistance against learning mathematics. Her intentions had changed but not her dispositions to mathematics incorporated through her lived life. According to the theory of Bourdieu, these were based on the habitus of a girl born 1922 in a provincial town as a saddler's daughter, that of a pupil in a school where arithmetic and mathematics were two different subjects at a time where it was "OK for a girl not to know mathematics", and the habitus of a wife and mother staying home with her two daughters as a basis of actions (and non-actions) and perceptions. Habitus as a system of dispositions generating practices and representation undergoes transformations, but otherwise durability is the main characteristic. It certainly has affinities with the idea of the *cumulative history of positionings* discussed in the previous section.

With the terminology introduced above, we can analyse the situation like this: Ruth's motivation to learn mathematics (to be a draughtsman) made her overcome her defence but not her resistance against learning mathematics. She overcame the defensive reaction prior to the learning situation, but resistance resulted from the learning situation itself and she rejected the opportunity to learn:

> Ruth: … a slide rule. Yes, we had to learn to use that. But I worked everything, *everything* out manually. I multiplied six figure numbers and there were six rows of them under that. I finished it before the others had finished pushing this ruler around.
>
> (Wedege, 1999a, pp. 215-216).

Her motive in the learning process and her reason for learning mathematics were strictly connected to the wish of being a draughtsman. Her motivation was performance (to pass the examination), rather than mastery (of the slide rule or of mathematics), although there would also be good reasons for the motivation to be mastery.

> Problems to do with figures from tables and statistics that are converted into graphs and curves. This is applied mathematics but as a draughtsman Ruth did not experience it as such. It would almost seem that her perception was: what can be used in practice is not mathematics. In the mathematics lesson it is called "measurements": at the studio it is called "scale" (...) In her perception, mathematics is needed by the architects, not by the draughtsmen.
>
> (Wedege, 1999a, p. 218).

Thus the learning was purely additive, not supplantive. Ruth learned to pass the examination but didn't change her belief about mathematics as something not relevant and mathematics as what she couldn't do.

[4] The main purpose of the article "To know or not to know mathematics — that is a question of context" (Wedege, 1999) was to discuss a possible combination of Lave's theory of situated learning with Bourdieu's theory of habitus and their suitability for analysing adults knowing or not-knowing mathematics in different situation contexts.

Conclusion

In adult mathematics education, not learning and not knowing are important issues as well as learning and knowing. Although adults have mathematics-containing competences developed in work, it appears that their beliefs about mathematics are primarily related to their school experiences, and mathematics is experienced by many adults as something that others can do, but that they themselves can't do — and, furthermore, don't need. Many adults who enrol in vocational education courses are surprised that the programme includes teaching in mathematics. They have experienced themselves as competent persons without mathematics and mathematics has not been perceived as relevant to their life projects. Thus their reactions may be resistance in the learning situation.

Resistance to learning is a well-known phenomenon in adult education, however there is very little research done on the subject and, as mentioned above, resistance is often explained purely as a lack of motivation and the symptom as non-learning. However, from his investigations within in-service professional training programmes, Atherton (1999) identified a phenomenon of personal resistance resulting from the learning situation. In his general theory of learning, Illeris (2000b) distinguished defence that exists prior to the learning situation and functions re-actively, and resistance caused by the learning situation itself as an active response. According to Illeris, resistance contains a very strong learning potential for accommodative and even supplantive learning. On the basis of this assumption, we have begun here to develop a conceptual framework to investigate adults' resistance to learning mathematics, and its possible as interrelation with motivation (see also Evans & Wedege, 2004).

While the conceptions of resistance in education in the work of Atherton (1999), Illeris (2003), and Jarvis (2001) are psychological, the approaches in the work of Mellin-Olsen (1987), Evans (2000), and Giroux (2001) are sociocultural. Concepts of dominant ideology, social reproduction, and resistant cultures are central in these theoretical frameworks. To Mellin-Olsen, the existence of folk mathematics (people's everyday mathematics) demonstrates that mathematics is both produced and reproduced, and the critical question What counts as mathematics? asked by FitzSimons (2002) makes visible the power relations in adult and vocational education.

Wedege (1999a) has tried to make sense of the contradiction between competence and resistance by analytically expanding the context for knowing mathematics from people's experiences and perspectives in the learning situation to also include their habitus as a system of dispositions generating practices and representations. On the basis of these empirical and theoretical studies, we call for further research on adults' resistance to learning mathematics, and further development of a theoretical perspective based on a socio-cultural approach. This will contribute to our ongoing work to develop a framework for understanding the conditions of adults' learning processes in formal education and in everyday life.

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