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Janet Taylor

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Objectives

Adults Learning Mathematics – an International Research Forum

has been established since 1994 (see http://www.alm-online.net), with an anual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/ numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – an International Journal

is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Adults Learning Mathematics – An International Journal

Guest editor
Dr. Javier Díez-Palomar

Special Issue

Parents' involvement in mathematics education: looking for connections between family and school

In this Volume 3(2a)

Editorial

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Welcome to the second of the special issues of Adults Learning Mathematics – An International Journal. This issue is indeed special for led by guest editor Dr. Javier Díez-Palomar, we present eight new and exciting pieces of research over two volumes.

Javier Díez-Palomar is well known to the ALM community through his extensive publications on topics of mathematics learning and teaching including a previous publication in this journal. Dr Diez-Palomar has worked and taken leadership roles with the University of Barcelona in the Centre of Research in Theories and Practices that Overcome Inequalities for many years and is renown internationally for his work with minority groups. The research pieces he has attracted for this special issue exemplify this focus and the respect with which he is held within this international research community.

Also I would like to acknowledge that this special issue would not have been achieved without the dedicated work and inspiration of our ALM colleague, Prof. Dr. Juergen Maasz.

The release of this special issue occurs during changes in the Editorial team of ALM. The journal says goodbye to Prof. Dr. Juergen Maasz and Dr. Mieke van Groenestijn who have been with the journal since its inception. The life, focus and continuation of the journal would not have been possible without their dedication and professionalism. I would like to take this opportunity to formally thank them for this major contribution over an extended period of time.

And so with the departure of two ALM editors I would like to welcome to the Editorial team two interim members, Dr. Chris Klinger from University of South Australia, Adelaide, Australia and Kees Hoogland, from APS -National Center for School Improvement, Utrecht, the Netherlands. With the inclusion of these colleagues in our team the future of ALM-IJ indeed looks promising.

Associate Professor Janet A. Taylor
Chief Editor

Introduction to the special issue *Parents' involvement in mathematics education: looking for connections between family and school*

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The aim of this special issue is to develop the perspective in the field of adults learning mathematics from the point of view of the work with parents who are learning mathematics. Authors were asked to address issues such as identity, culture, community of practice, interactions among parents, teachers and children, which are components of "parents (families)" as community of practices in terms of teaching and learning mathematics. Research in this field has discussed how parents become helpers of their children in doing mathematics. Hoover-Dempsey and Sandler (1995) explain that parents participate in their children's education because this is inherent to their role as parents: they want to help their children to succeed in school. They found that many parents reflected a primary interest in understanding their children and working to help them, to do well with schoolwork. Looking after this interest of helping children to achieve good scores in mathematics, a plethora of research has been completed in this field. The result has been the analysis of topics such as policies on parental involvement (Peressini, 1998), mathematical curriculum analysis (Jackson & Epstein, 2006), issues on equity in mathematics education and parental involvement (Allexshat-Snider, 2006, Martin, 2006), parents' engagement in mathematics teaching and learning (Hoover-Dempsey & Sandler, 1995; Civil & Bernier, 2006; Ginsburg & Rashid, 2006), funds of knowledge (González, Andrade, Civil & Moll, 2001), minorities (Civil & Bernier, 2006; Civil & Quintos, 2002; Martin, 2006; Gutstein, 2006), parents' roles in teaching mathematics and home-school interactions in mathematics (Civil, 2002; Civil, Quintos, & Bernier, 2003), among others. In the last decade an interest in challenging the deficit perspective and traditional assumptions about parental involvement have been noticed (Valencia & Solórzano, 1997; Calabrese Barton, Drake, St. Louis, & George, 2004; Allexshat-Snider, 2006).

The call for papers for this special issue had a great response of many authors working within this field. This reflects the importance that parental engagement and family involvement has in terms of education, and more specifically in students' performances. Previous research confirms that there is a connection between students' scores and parental involvement. Also there is evidence illustrating that low-income families usually have fewer opportunities to engage in their children's education, than middle or upper class families. In this issue some examples are provided. However the ways in which parents would be involved in their children's education may be different depending on their context, needs, and opportunities of involvement. Consequently, more research is needed in this field to clarify additional situations and to provide into actions that will further bridge home and school. We hope this special issue would be one more contribution to this aim.

References

Allexshat-Snider, M. (2006). Editorial: Urban parents' perspectives on children's mathematics learning and issues of equity in mathematics education. *Mathematical Thinking and Learning, 8*(3), 187-195.

Calabrese Barton, A., Drake, C., Perez, J. G., St. Louis, K., & George, M. (2004). Ecologies of parental engagement in urban education. *Educational Researcher, 33*(4), 3-12.

Civil, M. (2002). Everyday Mathematics, Mathematicians' Mathematics, and School Mathematics: Can We Bring Them Together? In M. Brenner and J. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom. Journal of Research in Mathematics Education Monograph, 11* (pp. 40-62). Reston, VA: NCTM.

Civil, M., & Bernier, E. (2006). Exploring images of parental participation in mathematics education: Challenges and possibilities. *Mathematical Thinking and Learning, 8*(3), 309-330.

Civil, M., & Quintos, B. (2002). *Uncovering Mothers' Perceptions about the Teaching and Learning of Mathematics.* Paper presented at the American Educational Research Association, New Orleans, LA.

Civil, M., Quintos, B., & Bernier, E. (2003). *Parents as Observers in the Mathematics Classroom: Establishing a Dialogue Between School and Community*. Paper presented at the NCTM research pre-session, San Antonio, TX.

Ginsburg, L., & Rashid, H. (2006). *Children's other teachers – their parents.* Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, TBA, Mérida, Yucatán, Mexico Online <PDF> Retrieved August 20, 2008 from http://www.allacademic.com/meta/p115649_index.html

González, N., Andrade, R., Civil, M., & Moll, L.C. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk, 6*, 115-132.

Gutstein, E. (2006). "The real world as we have seen it": Latino/a parents' voices on teaching mathematics for social justice. *Mathematical Thinking and Learning, 8*(3), 331-358.

Hoover-Dempsey, K.V., & Sandler, H.M. (1995). Parent involvement in children's education: Why does it make a difference? *Teachers College Record, 97*(2), 310-331.

Jackson, K., & Epstein, Y. (2006). *How parents are framed in reform-oriented elementary mathematics curricular materials*. Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, TBA, Mérida, Yucatán, Mexico Online <PDF> Retrieved 2008-08-20 from http://www.allacademic.com/meta/p115385_index.html

Martin, D.B. (2006). Mathematics learning and participation as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. *Mathematical Thinking and Learning, 8*(3), 197-229.

Peressini, D.D. (1998). The portrayal of parents in the school mathematics reform literature: Locating the context for parental involvement. *Journal for Research in Mathematics Education, 29*(5), 555-582.

Valencia, R. R., & Solórzano, D. G. (1997). Contemporary deficit thinking. In R. R. Valencia (Ed.), *The evolution of deficit thinking: educational thought and practice* (pp. 160-210). London: Falmer Press.

Linking community, families and school: opportunities for the mathematics education of children from excluded communities

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Parents are children's first and most important teachers. This fact is recognized within the public and private sectors of education in the United States today where parents play a valuable and multifaceted role in the school community. For the vast majority of schools, the culture of the classroom reflects the culture of homes located in English-speaking, middle class communities. Parents from these communities move comfortably into the schools, partnering with educators in such roles as PTA membership, serving on school boards, and participating on curriculum advisory committees. Even if parents do not fill these specific roles, all parents are expected to engage fully in parent/teacher conferences and provide support to the classroom teacher in homework assignments.

While middle class, English-speaking parents move easily in and out of formal school settings, the picture is quite different for parents outside the mainstream culture. Parents from diverse cultural and ethnic groups, lower socioeconomic levels, and home languages other than English are limited in their opportunities to engage as partners in their children's education. Reasons behind this limitation lie in social factors external to the parents' communities and in internal conflicts the parents experience themselves regarding their own value and ability to support their children's formal learning. In the area of mathematics education in particular, there is a pervasive social perception that these parents are deficient in their ability to help their children due to their lack for formal education and/or lack of English. Rarely are parents' informal and cultural mathematical practices valued in formal mathematics learning. Parents from excluded communities are acutely aware of a deficit perspective imposed on them and react with feelings of inadequacy, devaluing their own mathematical experiences and their ability to help their children. These factors result in a very real power imbalance between mainstream middle class culture and diverse communities.

The four papers included in this special issue tackle head on the power imbalance that limits parental participation within excluded communities. They provide valuable theoretical frameworks that expand the notion of formal mathematics education to include parental and community knowledge. These papers describe valuable models for ways that the educational community can create networks of participation among teachers, children, and parents that enrich children's formal education and equalize the power equation for diverse communities. Key issues in all of these papers focus on the need to transform parents' own perceptions about themselves as learners and doers of mathematics. Positive mathematics identities are critical in this transformation and give parents the sense of agency needed to confront social attitudes and demand equal educational opportunities for themselves and their children.

Jackson and Ginsburg explore personal transformation with African American mothers learning algebra. They document the evolution of a group of women who previously felt they would never have access to this important gatekeeper branch of mathematics. As the mothers begin to engage with algebraic concepts, they begin to develop the sociomathematical norms of discourse needed to examine efficient, sophisticated and better methods for problem solutions. This legitimate engagement transforms these women's identities in powerful ways and helps them develop a sense of their own capabilities as learners and their sense of efficacy as advocates for their children's formal education.

Latina mothers developing a sense of worth and efficacy is also described by Willey in a project that engaged mothers of elementary age children with University researchers to design an after school mathematics program that connected with the local community. Mothers used their connections with local businesses to uncover mathematical practices and then, working with researchers, created narratives of these practices students could access to learn formal mathematics. The dynamics of mutual parental support that arose from this project strengthened community bonds and transformed the mother's sense of identity to capable doers of mathematics.

Parents' attitudes and perceptions about mathematics education are revealed by Civil, Diez-Palomar, Menéndez and Acosta in the context of a community mathematics program designed to provide an opportunity for parents and children to learn together. Within the venue of this program, Mexican immigrant parents' voices revealed the importance of their own experiences as learners and how these experiences shape their perceptions of mathematics education in the U.S. Parental attitudes reveal the types of mathematical knowledge and practices they value and how the valorization of practices varies greatly across contexts. For these parents, whose formal mathematical experiences in Mexico valued memorization and the application of algorithms, conversations with researchers showed the importance of mutual respect and a shared understanding in building bridges between formal and informal networks of support. Language in this setting played a crucial role in parents' access to opportunities for personal growth, transformation, and legitimate participation in network building between home and school.

Finally, Quintos develops a framework of legitimate peripheral participation in mathematics learning within communities of practice to show how a network generated and promoted by the teacher, that includes the children, the parents, and the community, can provide the crucial bridge necessary between excluded communities and classroom learning. As highlighted in the work by Civil et al., home language once again is a critical component in this bridge. The research in this paper describes how a bilingual teacher formed partnerships with the children's parents and community, and through these relationships was able to uncover rich sources of mathematical practice through personal and community narratives told in the home language.

The idea of legitimate peripheral participation holds great promise for understanding how links can be created between formal educational institutions and excluded communities. These links are critical to enhance both children's learning and address the power imbalances that limit parents' opportunities as advocates for themselves and their children. The papers presented in this issue provide valuable examples of how these links can be created. A great deal of work still needs to be done, however, in connecting community and parents' informal knowledge to formal settings, and in a broader sense, challenging the ideologies that exclude certain populations from the benefits of full participation in the work of educating our children.

Algebra for all? The meanings that mothers assign to participation in an algebra class

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Abstract

In this paper, we report on a series of algebra classes with a group of low-income, African American mothers of elementary-aged children who had limited and negative formal experiences with algebra. We drew from United States reform-oriented elementary mathematics curricular materials in the classes. The women initially arrived to the class out of a desire to support their children's learning but over time also engaged in the class for intellectual purposes. We show how their questions and observations, rooted in their experiences with algebra in secondary education and in their children's elementary mathematics, drove our instruction, and how the women shifted their understandings of who can "do" algebra and of algebraic content. We suggest that the shifts they experienced were supported by three sources of meaning-making specific to algebra: "from within the mathematics, from the problem context, and from that which was exterior to the mathematics/problem context" (Kieran, 2007, p. 711). Our analysis suggests the importance of understanding parents as learners and the potential of reform-oriented elementary curriculum for supporting the learning of adults who had negative experiences with mathematics.

Key words: adult education; algebra; parents.

The significance of algebra

Algebra is often referred to as a "gatekeeper" in U.S. society (Katz, 2007; Moses & Cobb, 2001). Typically, this statement is made in the context of economic prosperity. It is not clear why succeeding in a high school algebra course relates positively to economic prosperity. Rather, it is likely that succeeding in high school algebra is a proxy for other social and cultural factors that are related to occupational and financial success.

High school algebra courses are often used as a sorting mechanism, explicitly and de-facto, for "college-bound" students and those who are not (Chazan, 1996). Historically, this has meant that non-white and low-income students were "tracked" out of algebra in higher proportions than white, middle-class students (Oakes, 1990). States in the U.S. are increasingly making policies that mandate that all students will have access to algebra in ninth grade, and in some cases, eighth grade. Chazan (1996) offers a reflection on the "Algebra for All" movement based on his experiences teaching an Algebra I course for a "low-track" group of high school students. He argues that in addition to providing algebra to all students, it is necessary to reconfigure what is understood as "algebra" in classes and, subsequently, the curriculum as well as the typical ways in which algebra is taught. Otherwise, Chazan warns, it is unlikely that the provision of traditional algebra to those students who historically have been excluded from algebra will result in success for all students.

Even with recent efforts to democratize access to algebra, algebra maintains an air of exclusivity. In this paper, we show how three African American women, who arrived to algebra classes with well-formed views of what algebra was — a disconnected body of knowledge that they did not understand — and corresponding views of who could "do" algebra — "smart," "college-prep" people, people different from themselves — changed their views of what algebra was and who could "do" algebra. Although the women initially came to classes to learn how to help their children and grandchildren with elementary mathematics, over time they came to engage in algebra much as a person studying mathematics for its own sake might do. We use this case of mothers studying algebra to question traditional notions of appropriate sequences of mathematics as well as the role of "real world" contexts in adult education classes. As we show below, these women changed their views of themselves and of algebra through genuine, intellectual inquiry into the mathematics of algebra. Furthermore, their intellectual curiosity drove the pedagogy and content of the courses.

First, we describe Kieran's (2007) framework for characterizing three sources of meaning for developing algebraic understandings. We use this framework to organize our findings about the meanings that the women in this case study made in the context of algebra classes. Second, we describe how algebra is typically positioned in adult education and in United States K-12 education. This is important because we used elementary curriculum as the point of access for an adult education course in algebra. Third, we describe the research context and our methods of data collection and analysis. Fourth, we describe the pedagogy of the courses, with a focus on the type of questions that the women raised. Fifth, we describe the shifts in the meaning that algebra held for these women as well as the shifts they noted in themselves as learners of algebra. Finally, we raise implications for research and practice in adult education mathematics.

Making meaning of algebra

In the mathematics education community, there are different opinions as to what constitutes algebra. Algebra has been described as the following:

1. "a means to express generalizations, relations and formulas; problems; denote unknowns; and solve equations" (Bell, 1996 as cited in Kieran, 2007, p. 713).
2. "generalized arithmetic, the set of procedures used for solving certain problems, the study of relationships among quantities, and the study of structures" (Usiskin, 1998, as cited in Kieran, 2007, p. 713).
3. "generalization and formalization; syntactically guided manipulations; the study of structure; the study of functions, relations, and joint variation; and a modelling language" (Kaput, 1995, as cited in Kieran, 2007, p. 713).

Although these definitions vary slightly, Kieran (2007) argues that a unifying theme across these definitions is that algebra is essentially an "activity" (p. 713). Algebra involves acting on objects such that one object is transformed into another.

If we take algebra as an activity that involves the various elements listed above, how do people come to make algebraic meaning? Kieran (2007) recently reviewed the work of Radford (2004) and Noss and Hoyles (1996) in relation to how students make meaning of algebra. Based on her review of their schemas for meaning-making specific to algebra, Kieran offers that there are at least three sources of making meaning in algebra:

1. from within the mathematics (e.g., from the algebraic structure itself, involving the letter-symbolic form, from other mathematical representations, including multiple representations)
2. from the problem context
3. from that which is exterior to the mathematics/problem context (e.g., linguistic activity, gestures and body language, metaphors, lived experience, image building).

(adapted from Radford, 2004, as cited in Kieran, 2007, p. 711)

Making meaning *from within the mathematics* refers to making meaning from algebraic symbols and representations, including equations, tables, and graphs, and linking symbolic forms to their “numerical foundations” (p. 711). According to Kieran, the use of multiple representations, which allows for students to “coordinate objects and actions within two different representations,” is critical to making meaning in algebra (p. 711). Making meaning *from the problem context* refers to how individuals connect given information about problem situations to symbols and notations. Modelling is included in this category. Making meaning *from that which is exterior to the mathematics/problem context* is meant to capture those aspects of making algebraic meaning which are not embedded in the symbols, representations, or the given problem context. This category “focuses on students’ processes of meaning production in terms of the way diverse resources such as gestures, bodily movements, words, metaphors, and artifacts become interwoven during mathematical activity” (p. 712). This category also includes experiences that students bring to algebraic work from other content domains. As we argue below, the women with whom we worked arrived to algebra class with distinct conceptions of the content of algebra and who could “do” algebra. They also arrived with histories regarding their participation, or lack of participation, in algebra courses. For this reason, it was critical that we understand and acknowledge the meanings that the women brought to and took from the class, what Kieran and Radford consider *exterior to the mathematics/problem context*.

How algebra is positioned in adult education

Adults who return to study mathematics bring with them their own mathematics histories and experiences, in and out of school, and their own near and far term goals. They return to the study of formal mathematics for a variety of reasons, and outcomes vary. Self reported gains in response to participation in adult mathematics learning include self-confidence (Civil, 2000; Evans, 2000), employment, preparation and entry into further study, and in parents’ ability to help their children (Brew, 2000; Civil, 2000).

Adult education theorists argue that because adults have limited time and are deeply engaged in real world activity, they are more likely to persist and learn most efficiently and effectively through instruction that builds on their experience and situates content in contexts that are meaningful to them (Knowles, 1984). However, adult mathematics curricula are often based on learner needs as defined by external organizations or frameworks. For example, in the United States, curricula are often driven by preparation for taking and passing the General Educational Development Test (GED), particularly the mathematics test, which has the highest failure rate among the five tests. This test, primarily a multiple-choice test, includes content from arithmetic through beginning algebra and geometry. Teachers frequently focus on particular problem types that have historically appeared on the test, de-emphasizing opportunities to study any topic deeply. The primary context within which instruction is couched is the test itself. From the perspective of adult education programs, receipt of federal funding requires reporting student progress within the National Reporting System (NRS) and using acceptable standardized assessments such as Test of Adult Basic Education (TABE) and the Comprehensive Adult Student Assessment Systems (CASAS). The six Educational Functioning Levels of the NRS are defined through a traditional ladder-like acquisition of computation skills, beginning with addition and subtraction, then multiplication and division, first with whole numbers, then with rational numbers, and finally reaching algebra and geometry. Algebra is seen as a capstone content area, to be addressed only after all of the more basic content has been mastered.

Alternatively, the requirements for entry into a workforce training program or for gaining a workplace certification include, in many countries, key skills as specified in national qualification frameworks, mathematics units that are geared to the particular context of the workplace, and/or content units that can be made vocationally relevant (Coben, Colwell, Macrae, Boaler, et al., 2003; FitzSimons, 1997; Wedege, 2002). The idea of embedding learning in contexts that are relevant and meaningful has also been embraced within a vision of learning mathematics for social justice, empowerment, and as a mechanism for developing adults’ critical consciousness (Benn, 1997; Frankenstein, 1990; Knijnik, 2007). In both these cases, mathematics learning is practical, functional, and goal oriented and might be expected to engage learners because it is closely tied to their identities as workers or as citizens seeking to challenge and improve their society.

On the one hand, then, in adult education, algebra has often been framed as a fixed body of knowledge to be mastered only after a learner has progressed through a sequence of mathematics courses. On the other hand, in the context of workplace training, algebra has been framed as unnecessary or irrelevant, perhaps because it is associated with abstract mathematics as opposed to "grounded, real-life" mathematics. Importantly, the classes we describe below do not fit neatly into either of these categories. Rather, in our classes, we drew from children's elementary curricula, and in response to the participants in the course, we embarked on what would be considered abstract contexts, but contexts that held meaning for the participants. The abstract contexts held meaning precisely because these women had been denied access to understanding the content in their earlier years of schooling.

How algebra is positioned in elementary mathematics education

Over the last two decades, there have been repeated calls within the United States K-12 mathematics education community to shift how algebraic content is positioned across the curriculum (National Council of Teachers of Mathematics, 1989, 2000). Historically, algebraic content was introduced in the middle grades and formally taught in high school. However, in response to studies that show that elementary students are capable of algebraic reasoning as well as international assessments that show that other industrialized nations outperform the United States in the context of problem solving, mathematics educators have argued for the importance of embedding algebraic work across the grades, beginning in kindergarten (Katz, 2007).

Reform-oriented elementary mathematics curricula that have been supported by the National Science Foundation, such as *Everyday Mathematics* (EM) (University of Chicago School Mathematics Project, 2001) and *Investigations in Number, Data, and Space* (TERC, 1998), provide examples of how algebraic reasoning, in the form of patterns, functions, and variables, has been integrated across the K-6 curriculum. "Early algebra," or algebra in the elementary years, typically includes two main features:

1. generalizing, or identifying, expressing and justifying mathematical structure, properties, and relationships; and
2. reasoning and actions based on the forms of generalizations.

(Lins & Kaput, 2004; Kaput, 2007, as cited in Katz, 2007, p. 7)

Reform-oriented elementary curricula use slightly different conventions and formats to address these two features of early algebra. For the purposes of this paper, we will briefly describe EM's approach, as we drew from these materials as a basis for our work with adults in algebra.

EM weaves two content strands related to algebraic thinking throughout their K-6 curriculum: "patterns, functions, and sequences" and "algebra and uses of variables". We generally drew from the patterns, functions, and sequences work. EM uses several curricular conventions across the grade levels, and increasingly varies the difficulty of the content associated with those conventions as the children advance in grade level. Two such conventions that we drew from include "Frames-and-Arrows" and "What's My Rule?".

"Frames-and-Arrows" are sequences of numbers that follow a particular pattern. "Frames" refer to the boxes in which each number in the sequence is placed, and the "arrows" show the direction in which the operation(s) are to be applied to the numbers. The pattern, or operation(s), is identified as a "rule". For example, if the sequence were 3, 7, 11, 15, …, the rule would be "+4" and there would be arrows from the 3 to the 7, from the 7 to the 11, and so forth, indicating that you were to add 4 to 3 to result in 7, etc. Frames-and-Arrows are initially introduced in first grade and are a staple convention of the EM curriculum through the sixth grade. They increase in difficulty across the grades. For example, older grades include the use of composite Frames-and-Arrows, where there are two or more rules as well as a composite rule, which is the sum of the rules.

"What's My Rule?" are function machines and are also introduced in first grade. They take the form of an "in-out" table, whereby "in" refers to input and "out" refers to output. As with Frames-and-Arrows, the pattern is referred to as a "rule." What's My Rule? also increases in difficulty across the grades. Children have to identify inputs, outputs, and rules, and the "rules," or relationships between the input and output increase in difficulty. As we describe below, we used Frames-and-Arrows and What's My Rule? as points of access into algebra for the participants in the classes.

Research context

The data we report on in this paper come from Parent-Child Numeracy Connections (PCNC), a project intended to support a group of parents'[1] understandings of their children's reform-oriented mathematics instruction and curriculum (for a description of the full study, see Jackson & Remillard, 2005; Remillard & Jackson, 2006). The project lasted for four years, and began when the majority of a cohort of 42 children were in grade 3. The children were all African American or Afro-Caribbean and lived in a low-income neighbourhood in a large city in the United States. Approximately half of their parents had a high school diploma. The cohort were recipients of an Educational Scholarship Program (ESP); if the children graduated from high school, they would receive a last-bottom-dollar scholarship to attend the post-secondary education institution of their choice. ESP provided academic and social supports throughout the children's K-12 schooling to increase the chances that they would graduate from high school and be able to access the college scholarship. In grade two, the children's elementary school adopted EM as its elementary mathematics curriculum, which prompted ESP to approach a local university to work with the parents of these children in regards to the new curriculum. EM was decidedly different from the elementary mathematics the parents had experienced in their elementary education.

One component of PCNC was parent math classes. Parent math class sessions lasted 6-8 weeks at a time, met 2 hours per week, and were held three times a year over the course of 4 years. Initially, the topics of the classes focused on measurement and percent, however, at the request of the parents, the topic of three of the 6-8 week sessions was algebra. The parents requested algebra in part because of the format of the classes, in which parents were asked to bring in questions they had about their children's mathematics. Many of their initial questions had to do with patterning activities, such as Frames-and-Arrows and What's My Rule? In the course of the discussions about why the children were given these activities and what they might be learning from them, we (the instructors) told the women that the patterning activities were the beginnings of algebraic thinking and were included to gradually build knowledge and skills that would help the students be successful with algebra. After a while, the women began to have fewer questions about their children's work and asked if they could study algebra during the sessions. In response, we designed tasks that grew out of EM conventions.

Although we had long-term goals and developed lesson plans prior to each algebra class, the actual content of each meeting and the activities that took place were inevitably modified or pre-empted by the learners' questions or observations. Occasionally the women brought in questions from their children's mathematics homework, and the ensuing discussions went in unplanned directions, but generally connections were made between these discussions and aspects of algebra. We drew from the EM materials to structure the algebra classes in an effort to connect to the work the participants' children were doing.

As we show below, classes were discussion-based, and tended to follow a structure of 1) instructors posed an algebraic task to the group; 2) participants worked on the task with individual assistance from the instructors; 3) participants shared solutions; 4) participants were prompted to make observations about the various solutions and to justify their solution paths; and 5) participants revised their solutions if necessary. Because we designed the tasks to connect with the children's work, we did not follow a typical algebra sequence, in which work with variables and solving linear equations precedes graphical representations of linear

[1] We use the term "parents" to include parents, grandparents and other caregivers.

functions. Rather, over the course of the three algebra sessions, we addressed the following topics in this order: pattern work using Frames-and-Arrows, with an emphasis on inverse operations and composite functions; function work using What's My Rule?, with an emphasis on determining "rules" that were generalizeable; graphing lines using the in-out tables in the context of What's My Rule?; naming linear equations from in-out tables; introduction to negative integers in the context of the coordinate plane and graphing; determining the equations of lines from graphical representations (slope-intercept form).

Twelve parents attended parent math classes over the four years. The data for this paper come from a case study of three mothers (Dionne, Lucille, and Betty) who attended all three algebra sessions. All three women had limited experience with formal algebra and formal higher education.

At the time of the last adult algebra class, Dionne was in her early forties and was a single parent of 3 children. Dionne's eldest daughter was completing her first year of college, her second daughter graduated from high school and was planning to attend the local community college, and Dionne's son was in fifth grade. Dionne seems to have been tracked into a non-academic course of study in high school and may not have graduated. She did not remember studying mathematics in high school. She recently completed an 8-week nursing assistant course and passed a certification test. She spoke about one day becoming a community counsellor.

Lucille was in her early fifties, married and had five children. Her two eldest lived on their own, and her middle daughter had just graduated from high school and was attending the local community college. She had two younger children, a son in fifth grade and a daughter in third grade. Lucille did not complete high school and worked part time as an assistant in an after-school program located in her children's neighbourhood elementary school.

Betty was in her early fifties, the mother of five children between the ages of 21 and 31, and the grandmother of six grandchildren. She often functioned as primary caretaker of her fifth grade granddaughter. Betty graduated from high school and was currently working in an insurance office doing accounting-related work. Through the ESP's educational support program for parents and caregivers, Betty enrolled in a proprietary school to become a medical assistant, fulfilling a long-standing dream. She attended classes after work and was among the highest achievers in the program.

The two authors co-facilitated the parent math classes. Kara previously taught high school mathematics and was a graduate student at a local university. She worked closely with the ESP program, teaching in their after school program, overseeing the mathematics program of their summer program, tutoring children as needed, and providing educational support to parents who returned to college or technical studies. She had many formal and informal opportunities in multiple settings to engage with family members of the women featured in the case study. Lynda previously taught math in high school, in developmental classes at community colleges, as well as in adult education programs and was a researcher at a local university.

Data sources and methods of analysis

There are two main sources of data for this paper: video-recordings of the sessions and audio-recorded interviews with the participants. All classes were video-recorded (24 algebra classes over three eight week sessions), and an outside observer took detailed field notes during each session. We also conducted three or four audio-recorded interviews with each woman, including a task-based interview, each lasting approximately an hour. With the exception of the task-based interview, the interviews were semi-structured. For the purpose of this paper, we focused on the participants' responses to questions about participating in the parent math classes, views of algebra, views of themselves as learners of mathematics, and their purposes for attending the classes.

Our analysis focused on the meanings that the women assigned to their participation and to algebra, how these meanings shifted over time, and shifts in how the women participated in the classes. We began by viewing the videos and reading the corresponding field notes for evidence of the practices, social norms, and sociomathematical norms of the classes (Cobb, Stephan, McClain, & Gravemeijer, 2001). Classroom social norms are “characteristics of the classroom community and … regularities in classroom activity that are jointly established by the teacher and students” (Cobb et al., 2001, p. 123). Examples include “explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement or disagreement, and questioning alternatives when a conflict in interpretations had become apparent” (p. 123). Cobb et al. argue that these norms are not specific to learning mathematics; rather, this set of norms cut across discipline-specific learning situations.

To complement discipline-neutral social norms, Cobb et al. argue that it is important to establish and identify socio-mathematical norms in mathematics classroom practices, in other words, norms that are particular to learning mathematics. Cobb et al. suggest that examples include “what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical solution” (p. 124). Identifying socio-mathematical norms in the classroom was especially useful to our analysis because we were interested in what activities in the class supported participants to shift their identities of themselves as learners of algebra in relationship to their understandings of algebraic content. Cobb et al. state, “We conjecture that, in guiding the establishment of particular socio-mathematical norms, teachers are simultaneously supporting their students’ reorganization of the beliefs and values that constitute what may be called their mathematical dispositions” (p. 124).

Through coding for norms and socio-mathematical norms across the classes, it became apparent that questioning and making observations were classroom practices that both the instructors and participants engaged in, and that there were particular socio-mathematical norms established around questioning and making observations that supported the shifts we were interested in. We then coded each class for particular types of questioning and observations; the various kinds of questioning and observations shifted over time, and we conjectured that these shifts corresponded with a shift in their reasons for continuing to engage in algebra and how the women were viewing algebra and themselves in relation to algebra. Simultaneously, we read through the interview transcripts and coded for instances where they described algebra, themselves in relation to algebra, their participation in the classes, and the shifts they experienced in relation to the classes.

Initial understandings of who can do algebra and what algebra is

“I’m a mother and I don’t know algebra.”

Each of the three women had been “tracked out” of algebra in high school to varying degrees, and each of the women was eager to learn algebra. In doing so, they challenged their initial models of “who could do algebra” as well as their own historical identities as people for whom algebra was not possible.

For example, Lucille indicated that in high school, she was in the track that studied basic mathematics. She was not offered an opportunity to take algebra and wasn’t one of “the algebra people.”

> We had basic [math]. Yeah, so it wasn’t, like you got introduced to it a little bit. We didn’t really, you didn’t really hit on it like we doing it now. Like you had to be … in another section, to be into that mindset. So we didn’t do, we didn’t do algebra like that. Algebra was basically a college thing, so like we didn’t really touch on that.… They were the algebra people.

Betty took algebra in high school, but she did not feel successful at it and did not see herself as capable of mastering the content. She indicated that she “dropped out of the academic” track after struggling with algebra and switched to the “clerical” track.

> The teacher that we had was one of the ones, like he wanted you to catch on so he always went at a fast pace. So the ones that didn’t catch too good, kind of got like lost. … And I had to get extra help and all that kind of [thing]. Uh, I wasn’t interested in it at all. I just couldn’t wait to get out of

> that class with a passing grade so my mom wouldn't kill me, but, after that, I didn't want no more courses in math. I dropped out of academics and went to clerical. You know, the easy math.

Dionne never had algebra in high school but did attempt to take a Developmental Algebra class at the local community college later in life. After a short time, she dropped the class because she did not understand the content and did not know how to get help. She described the experience as "devastating"; it confirmed to her that she was not one of the "smart people" who could learn algebra.

> Well, when I think of algebra, I think of algebra as something hard. (laugh) Hard. Something, something only smart people can do. Somebody, you know, uh, got glasses on, with a lot of books in their hands.

The women's feelings of inadequacy were reinforced when they encountered algebra as they were trying to help their older children with algebra homework and were unable to understand the work. Dionne describes such feelings as follows:

> My oldest daughter is 17. She knew how to do [algebra] but I didn't. And I would look at it and I would say, "Teach me this." And she would say, "It's easy, it's easy. Do these little numbers here, and then you do it with this number here, what the sign says." It looks like a puzzle that you cannot fix. I mean it. It looks a puzzle, like a real hard puzzle and the piece you can't put [it] together, and this piece here. I'm just used to saying 2 plus 3 is 5. Then, and the 'a' and the 'b' and the '=.' I said, "No, No, I don't know this at all." I was so hurt. I was very hurt. I was hurting.

At the same time, their desire to help their children was one of the reasons they were interested in learning algebra. All three women were responsible for children who were in the same grade encountering the same early algebra content, and they felt a parental responsibility to be able to help them. Lucille described recognizing that in order to provide help to her younger children, she needed to understand the algebra content they were receiving

> [I needed help so I could] more or less get updated with the way they're making the changes because … [my son is] getting it in elementary school and [my older daughter] didn't get [algebra] to almost high school. It's like time is really changing.

And, although Dionne described a desire to help her youngest child, Jerome, with his elementary algebra, she recognized her limitations in content knowledge.

> Interviewer: So [did] you help Jerome with that [assignment focused on algebraic reasoning]?
> Dionne: No, uh ah. I don't know algebra. I'm a mother and I don't know algebra.

The mystique of algebra

Each of the women had substantial views of what "algebra" was, rooted in their formal and/or peripheral experiences with algebra. There was an air of mystique that surrounded "algebra," and this mystique was reinforced by their limited access to the content. Most of the mystery seemed to centre on the idea of variables, *x'*s and *y*'s that were decidedly confusing. Dionne spoke about her experience in a Developmental Algebra class.

> I came into class and the teacher had numbers as long as the board. I look[ed] at it and I didn't understand it. Not one bit. $4x$ here and 7, and I was like, "What is that?"

Lucille, who had never studied algebra before but watched her older children work on algebra homework, remarked,

> I just, I think what made it so complicated looking to me was I had to understand the *x* and the *y*. And I think that always seemed so confusing. … It just seemed like, "Oh my god, what the heck is this?"

Betty had experienced algebra in high school, but never felt she understood what she was doing,

> I used to always wonder, well, how do you add letters? How do letters come in? But, I didn't really, no one really explained…

Mechanisms for change: how participants' questions and observations provided access to algebra

Questioning is recognized as a critical instructional tool in the teaching of mathematics for understanding, particularly in the K-12 mathematics education literature (Chapin, O'Connor, & Anderson, 2003). Strategic questioning can provide access into students' mathematical thinking and understanding and therefore can be used as a tool for on-going formative assessment. Questioning can also serve as a tool for scaffolding students' understanding of the content at hand. Particular questions like, "Why? How do you know that's true? Could you have solved the problem in a different way?" push learners to justify their solutions and to make their solutions public.

Within the algebra class, as instructors, we used these types of questions to push the participants to justify their solutions, to make their solutions public, and to further their understanding of the content at hand. However, what emerged from our analysis of the video data was that participants developed an understanding of the content at hand because *they* asked questions and made observations (solicited and un-solicited) throughout the classes. And, their questions and observations often pushed us as instructors to change the direction we had intended to take during a class period, and instead to engage the learners in sequences of activities that were more challenging than we had initially intended. Below, we offer two illustrative examples of typical sequences of instruction, followed by a discussion of the types of questions and observations that emerged across the series of classes and the instructional work they helped support.

What's My Rule?

A significant source of meaning for the women was the problem context of their children's work. However, as the example below illustrates, although we often began with the children's work, the women's questions and observations launched instruction that went beyond the context of their children's work. On April 23, 2005, we were in the midst of the third series of algebra sessions. The theme of this particular class was What's My Rule? based on the EM convention described above. In the previous sessions, we identified patterns in Frames-and-Arrows, both single and composite functions, and focused on processes of informal proof, namely how many examples were necessary to try before one could feel confident with a pattern. We began this class by looking at an EM What's My Rule? worksheet. We used the "in" and "out" terminology provided in the worksheet, and solved for "outs" given a "rule," solved for "ins" given a "rule," and determined "rules" given a set of "ins" and "outs." The idea of "inverse operations" had been developed since the first class of the session, and the women made use of it to complete What's My Rule? tables. Figure 1 provides an example of the first What's My Rule? problem we completed.

IN	OUT
100	*50*
120	*70*
70	*20*
150	*100*
200	*150*
50	*0*

IN
⇓
RULE
Subtract 50

⇓
OUT

Figure 1. Completed "What's My Rule?" problem, $x - 50 = y$. The given information is in plain font; the information the participants filled in is in *italics*.

About half-way into the ninety-minute class, we took the first "What's My Rule?" table from the worksheet ("subtract 50" or $x - 50 = y$) (see Figure 1) and introduced the idea of graphing the relationship on large, chart-sized graph paper. At this point, we did not algebraically name the line. Rather, we worked through how to plot a coordinate point. We explained to the women that the "in" is typically named the variable, x, and the "out" is typically named the variable, y. Kara began by graphing the point (100, 50), modelling for the women how to plot a point on graph paper and how to write the coordinate of a point. The following discussion ensued.

42:00 **Lynda:** Where would the point be if the x was 75?
Lucille: Halfway between. [Lucille is referring to the fact that 75 would be between 70 and 80 on the x-axis given that we used a scale of 10.]
Dionne: What would the y be?
Betty: (softly) 25
Lynda: Can you say that again?
Betty repeats her answer.
Lucille: Why do you all make it so easy and the book makes it seem so hard?

We ask Dionne to graph the point (150, 100) and Betty to graph (70, 20) from the What's My Rule? Table.
…
Lucille: You all make algebra seem fun, but I'm sure it gets more complicated.
…
Kara: What do you all notice?
Dionne: The numbers are even.
Lucille: (stands up) If you connect the dots, they all line up in a straight line.
Dionne: They all point to 50 [on the x-axis].

We ask them to connect the dots using a ruler.

Lynda: So what do you notice?
Lucille: It almost seems like half the graph.
Lynda to Dionne: And you noticed that it hit 50.
Lucille: And that's what we started out as.
Lynda: So it hits 50 when x is 50. So what's y going to be?

Lynda adds 50 as an "in" to the What's My Rule? Table (see the last row in Figure 1).

Dionne: Zero.
Betty: Zero.
Lucille: Okay, so where did I get confused at?
Kara: So if x is 50, put 50 [as an "in" on your personal What's My Rule? tables.]

Lucille: Zero.
Lynda: Why? How come?
Lucille: Because it would be minus 50.
Lynda: What if *x* was 40?
Dionne: Minus
Betty: Minus 10.

Lucille shakes her head in agreement. Dionne looks quizzically at the paper. We have a discussion of negative numbers in the context of temperature. The women work on finding where y = -10 lies on the graph. Eventually, they plot the point (40, -10). Lucille and Betty then plot (200, 150) together.

Lucille: Hmm, so you're still staying on the line, staying on the border.
Lynda: I think that's a really important thing to notice, that all of these things that we're figuring out land on the same line. Why is that?
Dionne: As long as the rule is the same, they'll all be on the same line.
Lynda: So you know what they do? They give this line a name. From what we're doing, what would you say the name of this line is?
Lucille: The rule.
Lynda: And what's our rule?
Lucille: The rule is subtract.
Lynda: Subtract what?
Lucille: 50

Lynda then writes x – 50 = y, describing it as the "in" minus 50 equals the "out."

Dionne: This is really something. (Everyone laughs.)
Lucille: It's a little mind-blowing, but it seems so simple once you get to working it out.

As illustrated above, at lines 12 and 17, we ask the women what they notice about the points they have plotted. Dionne observes that the inputs and outputs are all even, and Lucille notices that the points, if connected, would make a straight line. Dionne then adds that the "line" points to 50, meaning that the *x*-intercept is at 50. These observations are unsolicited. This is the first time Lucille and Dionne have graphed on a coordinate plane, or so they remember. After we have them connect the dots, building on Lucille's observation, Lynda capitalizes on Dionne's observation about the *x*-intercept and has the women figure out the *y* for $x = 50$. After plotting a few more points (see line 36), Lucille observes that all of the points are on the same line. We then briefly discuss why this is so (Dionne offers that they all follow the same "rule"), and then we name the line using the form of an equation with variables.

The women's observations drove the trajectory of our instruction, and therefore of their learning of algebra, that day. Our initial goals for the day were to learn how to graph coordinate points, generated from a table, and to connect them in a line. However, their curiosities and observations led us to explore algebraic ideas that typically might have been considered "beyond" a first-time experience with graphing coordinate points. Importantly, we recognize that we were able to flexibly respond to their observations and questions because we were not subject to an external curriculum or assessment. However, as we argue in the Discussion section, the fact that this was possible raises questions for how we typically frame adult learners in the context of K-12 and Adult Education.

Drawing a line with one point

Whereas the previous example illustrated a case where we began with the problem context of the women's children's work, this example illustrates that another source of meaning for the women was rooted in the algebra itself. In addition, this example is further evidence of how the women's questions and observations shaped the path our instruction took, and in particular, into unanticipated territory. A week after the class described above, we had the women generate their own rules and ins (x-values) on which they would operate. Then, we had each woman graph her relationship given the set of points (in, out), or (x, y), that she generated. Although we did not design this purposefully, all of the "rules" that the women chose were additive and were linear. Mathematically, this meant that all of the lines they graphed were parallel (i.e., they had the same slope of 1). Dionne generated and graphed the lines $x + 8 = y$ and $x - 10 = y$, and Lucille generated and graphed line $x + 12 = y$. (Betty did not attend class this day.)

Once the women had graphed the lines, Lynda asked, "Anybody notice anything?" Her question led to a series of observations, including Lucille's observation that the lines were parallel, to a discussion of how one knows that lines are parallel, which then led to an innovative technique on the part of Lucille for drawing a line. After a brief discussion about Lucille's observation that the lines were "parallel," Dionne made an observation that Lucille's graph $x + 8 = y$ was "more" than $x - 10 = y$. After some probing, Lynda and I understood that Dionne's use of "more" referred to the following—both Lucille and Dionne used 70 as one of their x-values. So, for the same x, ($x = 70$), Lucille generated 78 as her y ($70 + 8 = y$) while Dionne generated 60 for her y ($70 - 10 = y$). We then engaged in a group discussion about the difference between the y-values for any given x when comparing these two linear equations. Dionne and Lucille showed the y-values they found for an x of 70 (60 and 78, respectively) on the graph paper. Lynda revoiced what they said, and then asked, "You (Dionne) were subtracting 10, and you (Lucille) were adding 8. So how far apart are they?"

Dionne: You, well from her I'm 8. Gotta be 8.
Lucille: Mm mm *(meaning she doesn't agree; Lucille stands up and runs her fingers along the difference between Dionne's line and her line.)*
Lynda: You're 8 from there, and yours is
Dionne: Mine's is—
Lucille: 10
Dionne: 10, no, no, no not 10. Let me see. What is it? 12. No, no, can't be 12. If she's, hers is 8 more than mine's so mine's will be um, 8 less from her. Cause 78 and 70.
Lynda: No, but yours isn't 70, remember what—
Dionne: Mine's is 10 from 70.
Lynda: Yeah, cause you had to subtract 10.
Dionne: Yeah, this is 10 and this is 8. Then it's a 60.
Lucille: Wait a minute. 18. 18?
Dionne: So what was your question again?
Lynda: How far apart are they?
Lucille: 18? 18 inches?
Lynda: How do you figure? Why is it 18?
Lucille: Cause if you were to take and add the difference between that, it would make it 18.
Lynda: Show me.
Dionne: From 60 she said.
Lucille: *(Lucille shows with her hand the difference between (70, 60) and (70, 78).)* From 60 to 78 would be your 18 inches more. Difference.
Lynda: Could you know that by looking at your equations? Could you get that 18?
Dionne: Yes.
Lynda: How do you figure?
Dionne: With the plus 8 and the minus 8? 8 plus

Lucille: well if you
Dionne: 10
Lucille: wasn't looking at the minus and the plus, it would be 10 and 8 which is 18.
Lynda: Yeah, cause what happened, you went down 10 and she went up 8.
Lucille, Dionne: Yeah, yeah, 18.
Dionne: *(laughs)*
Lucille: That's crazy. *(laughs)*

In the excerpt above, the women determine that the difference between the y-values for the same x-value for the two lines, $x + 8 = y$ and $x – 10 = y$, is 18. In what follows, Lynda asks the women what the difference would be for an x of 40. This then leads to the important question of whether the difference will always remain 18 for any x. Over a few minutes, Lucille and Dionne establish that the difference between the y-values for any given x with respect to their two different lines remained constant, no matter which x-value they chose. At one point, Lynda asked them, "What if your in was 500? What would the difference be between the y's on your lines?" Dionne and Lucille agreed that it would be 18. Dionne then offered, "It [the x-value] could be 1000, and it would still be 18 inches [because of our rules]." Lynda responded, "That's why the in is called a variable, cause it really doesn't matter what it is. You can always find out what the y is because of the rule."

Lynda then asked again, "What else do you notice about the lines?" Dionne responded that she noticed that she used more points to construct her line than Lucille did. This then sparked a conversation about how many points one needs to draw a line. Initially, Lucille and Dionne disagree about the number of points needed to draw a line (see lines 84-95). They eventually both agree that they could have drawn the same line with three points. However, at line 97, Lynda asked the women if they could draw a line with one point. At line 100, Lucille argues that she can.

Kara: To draw this line, how many points do you use?
Dionne: I did 6.
Lucille: And I did 5.
Lynda: Does it matter for drawing the line?
Dionne: Yes.
Lucille: No.
Dionne: She said no and I said yes.
Kara: If you had only gotten 3 points, would you still have drawn the same line?
Lucille: Cause if you're saying it varies with it going off the chart, no.
Lynda: It wouldn't matter?
Lucille: No, it wouldn't matter if you only had 3 points. Cause you still gonna draw the line. (She extends her arms in both directions.)
Lynda: The line would be the same.
Lucille: Yeah.
Lynda: What do you think [Dionne]?
Dionne: It doesn't matter.
Lucille: You got the dots, but you still going to extend the lines.
…
Lynda: Okay, how about if you had only one point?
Lucille: It's still going to, it's still going to (extends arm).
Kara: So if you only had the point (40, 48), could you have drawn that line?
Lucille: Off of just that one dot? Mm hmm (indicating yes).

Mathematically, a line is constructed of at least two points. At this point in our sessions, we had only introduced the method of plotting points to construct a line. Assuming that the women would find the task of drawing a line with one point impossible, we asked Lucille to generate one point for $x – 5 = y$, and for Dionne to generate two points. We then asked both to graph their lines using only the points they generated.

Lucille quickly generated a new point (55, 50). She volunteered to plot it and construct a line. Lucille stood up, ran her right index finger along $x = 55$ and her left index finger along $y = 50$ and placed a dot where they met. She then picked up a ruler and carefully aligned it so that it went through the point (55,50) and was parallel to the other lines drawn on the paper. Indeed, Lucille drew a line using one and only one point. Lynda asked her, "How do you know where to draw it?" Lucille responded, "Cause I have to stay on the dot." Lynda then took the ruler, placed it on (55, 50), and tilted the ruler so that it was no longer parallel to the other lines. She asked Lucille, "How do you know it's not like this?" Lucille responded:

> Because it's supposed to be parallel! And I was also thinking that when we was doing it last week, it was like we weren't going like a (Lucille moves her hand so that it traces a spiky line graph), so I'm thinking okay, we talked about the parallel, and the dots connecting, so I'm figuring they have to be parallel. Was I right? Was I right? (laughs) So that's why I figured it didn't matter if we didn't have any more dots because it's still going to go off the graph whichever way that it does.

A few minutes later in the class, Lucille looked carefully at where $x – 5 = y$ crossed the y-axis and announced that her line was a bit off; it should have crossed at the y-axis at exactly $y = -5$. (It crossed just a few hairs above $y = -5$.) We had not discussed the relationship between y-intercepts and the slope-intercept form of an equation at this point; however, Lucille noticed that the numbers given in the other equations were the same as where those lines crossed the y-axis (i.e., the y-intercepts).

Types of questions and observations

Over time, the basis for participants' questions and observations changed. In the first sessions, the majority of the questions stemmed from the participants' experiences with their children's EM curriculum, including questions about Frames-and-Arrows and In-Out Tables. They generally did not understand the purpose of the conventions or how to solve problems using them. However, once they developed familiarity with the EM conventions, we were able to use the conventions to explore algebraic content, as illustrated above.

Their understanding of the EM conventions, then, supported them to ask questions that were rooted in a desire to understand algebra, as they recognized it. For example, on May 14, 2005, we solved linear equations and did not initially do so in the context of graphing. We decided to solve linear equations in response to a section that Lucille found in the children's curriculum in which they were to solve such equations. About an hour into our session on solving linear equations, Lucille asked, "This equation problem ($12 = 5n + 2$), does that still play a part on the graph paper?" Lucille was interested in the relationship between solving a linear equation and graphing a linear relationship. We decided to graph the equation $y = 5x + 2$, and then we explored what the x-value was when $y = 12$. In order to graph the equation, we distributed blank What's My Rule? tables, and the women chose a variety of inputs. Lucille exclaimed as they each plotted their points to graph the equation, "Oh ladies, I didn't mean to start something! [But this] gets my curiosity piquing!" These types of questions, which we argue were rooted in intellectual curiosity, were a staple of the instruction of the classes. Sometimes the questions were in response to us asking the participants what they noticed, but oftentimes, they were unsolicited.

The women also asked questions about bits and pieces of "formal algebra" that they either had seen in the context of their high-school aged children or had seen when they attended school. For example, they asked about "the little 2 next to the x", meaning exponents. During a class focused on determining the slope of lines and comparing slopes, one woman asked, "Now tell me something. I've seen something with a U shape," meaning a graph of a quadratic equation. We also used these types of questions to shape our instruction.

The freedom with which the women asked questions and made observations was supported by the ease with which they worked together. This may be because they arrived at the class with a common interest in supporting their children, and they knew each other from the neighbourhood, although they did not regularly socialize with one another prior to the classes.

They developed a relationship based on trust and common purpose. This was critical because the women arrived with different strengths and challenges, mathematics and communication-wise—Lucille struggled with simple computation and relied heavily on a calculator, Dionne sometimes struggled to articulate her reasoning, and Betty was reserved about sharing her ideas publicly. They also were comfortable with and came to celebrate the idea that there were often multiple methods and approaches that led to the same conclusion. Lucille said,

> Yeah, that's the part I enjoyed … I think we did it in a different way, some of us did it in a different way, but we still wind up with the answer. And you know, just to see that, she did her's one way, and I did mine one way, and another one did. And we still come up with the same – and it was amazing how we all did it differently but we came up with the same answer. So…you know, that goes to show you that not everybody sorta thinks alike. You know? Cuz I had my way of working it out, she did her figures in another manner and you know, it goes to show you, okay well, you know, it was something different. …But when we sat down, we could talk about it and discuss how we did our problem and they did their problem but we all wind up with the same answer.

Shifting identities as learners of algebra and understandings of algebra

By the end of the sessions, each woman had redefined her identity as an algebra learner. This redefinition seemed to be tied closely to the fact that what they had learned resonated with their previous models of algebra—namely, that there were variables and equations. Each woman initially perceived algebraic content as unattainable and difficult. However, by challenging themselves to engage in learning "real" algebra, and finding that they were able to understand the content, they were able to see themselves as accomplished and competent, and found the experience personally rewarding.

> Dionne: I feel more adult. Yeah! Adult math. Something I should have, something I didn't but I should know. Kids use it, but it's more adult, it makes you feel more adult. I feel like it's more adult math to me, maybe not to you and Jerome [her son], but it really feels like adult math … Like it makes you feel like you can do something more harder. I feel more accomplished. I can accomplish more. Like I accomplished something. … Before this class, I felt like a 5, but with this class, I feel like a 10. I do. That's how I felt all through the time after the time I got out of high school. Because I did not know how to do the algebra. It was the algebra. For all those many years, I did not know.

> Betty: When I was getting the right answers, …then you feel good, like oh, okay. I always felt bad about not being good at math, but then I said, "Hey, I can do this." So that made me feel better.

Similarly, the reasons why the women attended class shifted, albeit subtly, over the course of the sessions. Initially we provided parent math classes in the context of parents providing support for their children's learning of mathematics, and the women attended so that they would be better equipped to support their children. An additional conjecture of ours regarding parent math classes was that parents would benefit from the chance to re-experience mathematics, given that so many of them had negative experiences in school with math. This, we believed, might support them in developing a greater appreciation for the reform-oriented mathematics that their children were receiving in elementary school. However, as Lucille articulates below, the women developed an additional purpose for attending class—to intellectually engage in mathematics for the sake of doing mathematics.

> Lucille: [Algebra class] was my moment for me. …It was still a moment for me to do something with myself so I kinda enjoyed that part about it. You know? Because it was something for me, and at the same token, it was something that I was learning and can pass on to my child.

Importantly, the shifts in the women's identities as learners of algebra were supported by their increasing understandings of the content. It was important that the content we engaged in during class was compatible with their previous notions of what "algebra" looked like. In other words, in order to support shifts in their understandings of who could do algebra, the model of algebra in the classes had to resonate with what they remembered as algebra. For example, during the class sessions, the idea of a variable, exemplified by "x's and y's," emerged from discussions of the graphs of the What's My Rule? activities. So although the What's My Rule? did not look familiar from their previous encounters with algebra, the connections we and they made to variables did and substantiated for the women that they were indeed learning algebra. Over time, Lucille found the algebra not only accessible, but fun.

> I enjoy my lessons, you know um… I thought it was going to be more complicated than that and it wasn't — [it was] more like, Okay, now this starting to tick, tick, tick, tick, tick, and I'm like, "Okay, it's not as bad as I thought"…. And I just found it was very interesting and I enjoyed it. I don't know — I had fun. I had fun. [laugh]

In fact, she recruited other parents to the class because she believed that algebra could be accessible to everyone.

> I would tell them about um, how we started the basics of algebra. How…you'd be surprised how, once you get the swing of it, you know, the flow of it becomes more easier than you think. You know, some people look at algebra, they paint the picture of it being real hard, you know and I've said it to a couple of parents and I'm like, "No, but it's not as bad as you think" 'cause they already have that wall put there – Algebra is hard, it's hard, it's hard, it's hard. You know? And I'm like, "No, no, no, once you start with the basics of it, and you learn it from the ground up, you know… it's not as bad as it seems."

Betty was the only woman of the three who made "real world" connections to learning algebra. However, she was enrolled in a medical assistant program and occasionally brought in proportion problems related to dosages from her classes, which we solved using algebraic methods. She came to understand algebra as useful and practical.

> Now, it seems to be just another way of doing math. …As a method for solving. I see like, um, I don't know, but like if you was like buying a floor from a store, and you know how you have to measure the room, you need to know how to cut the corners and all that, I think it's, I think people use it. But I couldn't understand back then why anyone would use it, but I see people do use it, like at their jobs.

Discussion

Three sources of meaning

We began this article by offering Kieran's (2007) characterization of three sources of meaning-making regarding algebra. The women we describe derived meaning in algebra from all three sources suggested by Kieran — from within the mathematics, from the problem context, and from that which is exterior to the mathematics/problem context. This is important, we argue, given that the varied sources of meaning supported the women's shifts as learners of algebra (and mathematics, more broadly) and their understandings of algebra. Importantly, all three sources of meaning described below supported the women's shifts in their understandings of the content and of themselves as learners and doers of mathematics. The boundaries between the mathematics, the problem contexts, and that which was exterior to either of those contexts were blurred within the actual second-to-second interactions that constituted the algebra classes.

From within the mathematics

As illustrated in the excerpts above, the women often made observations or asked questions about the algebra they were doing. Sometimes the questions were about different algebraic representations (e.g., equations and lines). Other times, the questions and observations were in relation to the mathematics they had investigated in previous classes, as illustrated in Lucille's observation about parallel lines in the April 30 class. She drew on understandings she developed in the previous class about what parallel lines looked like to support her conjecture about the lines the women had constructed in the subsequent class.

The meaning making in the realm of "within the mathematics" that we observed supports our characterization of the women as being intellectually curious about the mathematics, which, we argue, supported the shifts they identified in themselves as learners of mathematics. Civil and Bernier (2004) argue for the importance of framing parents, particularly low-income parents of color, as "intellectual resources" in relation to mathematics. Although we have not provided evidence that the women acted as intellectual resources for their children as a result of the classes, we have provided evidence that indeed, these women acted in ways within the context of the class that could potentially support similar interactions with their children with regards to mathematics.

From the problem context

Meaning in algebra can also be derived from the problem context, which includes the "events and situations" (Kieran, 2007, p. 712) onto which algebraic symbols and conventions can be mapped and/or connected to mathematics content of the outside world. We typically did not embed algebraic content within what would be considered "real world" contexts. Rather, we found that for these women, the problem context which held significance for them was a mixture of symbolic notations that they had seen in their previous schooling and the EM work (e.g., Frames-and-Arrows, In-Out Tables) that their children were doing, which they came to understand as "early algebra" in parent math classes.

The women's role as parents striving to support their children's mathematics learning shaped the primary problem context in which they made meaning of algebra. Their children's EM home- and school-work functioned as a bridge between the meanings of algebra that they initially brought with them to the classes and the meanings they came to assign to algebra through participation in the classes. They perceived that the algebra they explored grew out of their children's assignments and in turn would feed back into the children's work as they felt better able to support that work. All three women originally participated in the parent math classes because they wanted to be better prepared to help their children with schoolwork. While they came to want to study algebra for themselves as well, the references to their children's homework and the curriculum materials used in the children's classes were frequent.

From that which is exterior to the mathematics/problem context

In addition to wanting to understand and assist their children with mathematics, the women asked to focus on algebra because they had been alienated from or denied algebra in the past. Given their histories, the class represented a context in which they could challenge themselves and their perceptions of algebra as frustrating, difficult and mysterious. As they persisted, algebra became intellectually challenging, accessible and pleasurable. This historical and personal context to their participation, that which was exterior to the content and the problem contexts of the class, was a significant source of meaning for the women. Furthermore, we contend that the social aspect of the class — women coming together who shared, to an extent, a similar past and present with algebra — constituted another important source of meaning. Saturday afternoons were a social event that supported academic and social shifts in the women's understandings of mathematics.

Implications for research and practice

We have provided an illustration of women re-engaging in algebra. These women experienced shifts in their identities as learners of mathematics and in their understanding of mathematical content that they had previously symbolized as reserved for "smart" people. Below, we offer two implications for research and practice from this work.

First, the role of the elementary mathematics curriculum was central to the academic and social accomplishments of the algebra sessions. We used the children's materials as an entrée into algebraic activity. The women had experiences with the children's materials, from which we could base instruction. They simultaneously learned about how to approach the unfamiliar conventions of their children's work as well as the algebraic content embedded in their children's work. We began by using the materials as the problem context, and over time, used the materials as tools for developing further, more sophisticated understandings of algebra. We suggest that children's materials can serve as useful bridges for parents between their children's mathematics education reality and the parents' own understandings of mathematics.

Second, a related point is the importance of framing parents as learners, or as intellectual beings, in addition to identifying them as supports for their children. Typical interventions that involve parents are solely concerned with the child. The parents are framed as a vehicle for improving the child. However, in the case of the algebra sessions, although the women first came to the parent math classes so they could better support their children, they engaged in the mathematics as learners. Interventions that focus only on the outcome of the child ignore the potential for impact on adults' experiences for themselves.

In conclusion, we would like to return to the current popular sentiment in the United States of "algebra for all." Although that statement is typically made in reference to high school students, our data is evidence of the potential for adults who were denied access to algebra in high school to re-experience and re-engage with algebraic content in meaningful ways. These women were written off in the context of secondary education and, subsequently, had written themselves off as capable of learning and doing algebra. Although algebra is typically framed as a capstone in adult education, this project showed that it might also serve as a point of entry for adults for whom it has been historically understood as a barrier.

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References

Benn, R. (1997). *Adults count too: Mathematics for empowerment*. Leicester: NIACE.

Brew, C.R. (2000). Implications for women and children when mothers return to study mathematics. In M.J. Schmitt & K. Safford-Ramos (Eds.) Proceedings of the Seventh International Conference of Adults Learning Mathematics-A Research Forum (pp. 167-172). Massachusetts, USA: Tufts University.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). Classroom discussions: Using math talk to help students learn. Sausalito, CA: Math Solutions Publications.

Chazan, D. (1996). Algebra for all students? *Journal of Mathematical Behavior, 15,* 455-477.

Civil, M. (2000). Parent as learners and teacher of mathematics: Toward a two-way dialogue. In M.J. Schmitt & K. Safford-Ramus (Eds.), Proceedings of the Seventh International Conference of Adults Learning Mathematics—A Research Forum (ALM-7) (pp. 173-177). Massachusetts, USA: Tufts University.

Civil, M., & Bernier, E. (2004). Parents as intellectual resources in mathematics education: Challenges and possibilities. Paper presented at the Research Presession of the National Council of Teachers of Mathematics, Philadelphia, PA.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in mathematical practices. *The Journal of the Learning Sciences, 10(*1 & 2), 113-163.

Coben, D. with Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V. (2003). *Adult numeracy: Review of research and related literature. London:* National Research and Development Centre for Adult Literacy and Numeracy.

Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practices*. New York: Routledge/Falmer.

FitzSimons, G. (1997). Mathematics and vocational education in the training sector: Professional development implications for teachers. In D. Coben (Ed.), Proceedings of the Third International Conference of Adults Learning Mathematics—A Research Forum (ALM-3), (pp. 183-192). London: Goldsmith College, University of London.

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *The Journal of Negro Education, 59*(3), 336-347.

Jackson, K., & Remillard, J. T. (2005). Rethinking parent involvement: African American mothers construct their roles in the mathematics education of their children. *The School Community Journal, 15(*1), 51-73.

Katz, V. (2007). *Algebra: Gateway to a technological future.* Washington, D.C.: Mathematical Association of America.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. K. Lester (Ed.), Second Handbook on Research in Mathematics Teaching and Learning (pp. 707-761). Greenwich, CT: Information Age Publishing.

Knijnik, G. (2007). Brazilian peasant mathematics, school mathematics and adult education. *Adults Learning Mathematics--An International Journal, 2*(2), 54-62.

Knowles, M. (1984). *The adult learner: A neglected species.* Houston, TX: Gulf Publishing.

Moses, R. P., & Cobb, C. E. (2001). *Math literacy and civil rights.* Boston: Beacon Press.

National Council of Teachers of Mathematics. (1989). *The curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings.* Dordrecht, The Netherlands: Kluwer.

Oakes, J. (1990). Opportunities, achievement and choice: Women and minority students in science and mathematics. In C.B. Cazden (Ed.), Review of Research in Education, 16 (pp. 153-222). Washington, DC: American Educational Research Association.

Radford, L. (2004). Syntax and meaning. Paper presented at the International Group for the Psychology of the Mathematics Education, Bergen, Norway.

Remillard, J. T., & Jackson, K. (2006). Old math, new math: Parents' experiences with Standards-based reform. *Mathematical Thinking and Learning, 8*(3), 231-259.

Smith, E. (2003). Stasis and change: Integrating patterns, functions, and algebra throughout the K-12 curriculum. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), A research companion to Principles and School Standards for School Mathematics (pp. 136-150). Reston, VA: National Council of Teachers of Mathematics.

TERC. (1998). *Investigations in number, data, and space*. Menlo Park, CA: Dale Seymour.

Wedege, T. (2002). Numeracy as a basic qualification in semi-skilled jobs. *For the Learning of Mathematics, 22*(3), 23-28.

University of Chicago School Mathematics Project. (2001). *Everyday mathematics* (2nd ed.). Chicago: SRA/McGraw-Hill.

Immigrant Latina mothers' participation in a community mathematization project

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Abstract

This paper shows how Latina, immigrant mothers expanded their mathematical boundaries by participating in a community mathematization project. These mothers were not only participants, but also curriculum designers in collaboration with university researchers. By involving the mothers in the design of the project, researchers improved the likelihood that the project would reflect Latina/o *funds of knowledge* and capitalize on Latina/o epistemology. This contrasts with typical approaches to mathematics teaching and learning that ignore the knowledge of parents or do not define them as collaborators. The data collected by children and parents from community sites were collaboratively analyzed by teams of undergraduate student facilitators, elementary students, parents, and researchers. The final products were digital stories that illustrated the mathematics in the daily operations of a particular community site as well as posed a potential mathematical problem that the worker might encounter. Our findings reveal the mathematics that is evident to the mothers, as well as how social networks function to support Latina parents' development of mathematical knowledge and agency.

Key words: mathematization, collaboration, Latina parents, mathematical knowledge, agency

Introduction

In the United States, current federal and local discourse around education, and in particular mathematics education, is dominated by proclamations of an achievement gap between White students on the one hand and African-American or Latina/o students on the other (Martin, 2007; see National Centre for Education Statistics, 2007). As a result of such discourse, images are created that portray non-White students as less capable than their White counterparts and a huge thorn in society's side that needs to be dealt with by more stringent methods of accountability. It is quite easy to see how the public can become influenced by the rhetoric and fall victim to notions of a seriously deficient population of minority learners. Embedded within this context, there is a large and growing population of Latinas/os, one that includes a foreign-born, adult population that has fewer years of formal education than other ethnic groups (United States Census Bureau, 2007). Implicitly and almost by default, Latinas/os are being subsumed under a category of mathematically incompetent.

It has been the position of many critical mathematics educators to reject the notion and language of deficiency as it pertains to certain minority groups set against the standard of high-achieving, White students (e.g. Khisty & Willey, in press; Martin, 2007, in press; Martin & McGee, in press). This is not an effort to ignore the reality that Black and Latina/o students consistently have lower levels of educational attainment than White students, but rather an effort not to contribute to a discourse that manifests into a dangerously powerful ideology that inhibits the production of the necessary social structures to reverse this trend and move towards a more equitable educational system. With this in mind, and as we stretch our minds to understand the role of mathematics in Latina/o parents' lives, we are grounded in notions such as funds of knowledge (Moll, Amanti, Neff, & Gonzalez, 1992; Gonzalez, Andrade, Civil & Moll, 2001), which puts forth an alternative theoretical framework and highlights empirical evidence that non-dominant groups are bearers of vast amounts of social and cultural capital that is not effectively utilized – but could be – in the US school system as we currently know it.

The issues surrounding the mathematics education of Latinas/os are numerous and multi-faceted. They include macro-socioeconomic forces and ideologies such as racism and education to serve the evolving workforce. Also, they include practical obstacles like language proficiency and the role of parents in the school system, and thus, their child's academic development. This article addresses one aspect of mathematical association among Latinas/os: the mathematical base, processes, and development of immigrant, Latina mothers in an urban context.

In particular, this paper reports the results of a community mathematization project. Before the details of the project are discussed, a few words need to be said about mathematization and the assumptions and foundations from which the research team worked. First, it is acknowledged that mathematization is not something that has historically been included in the formal, U.S. mathematics curriculum. Therefore, the research team does not claim to have formal experiences in mathematization. We assume the same is true for the formal Mexican curriculum, the one that was learned by the mothers of this study. From this standpoint, the researchers and the mothers began from equivalent positions. This idea is nuanced, however, because our initial interviews with the mothers revealed that the mothers, in general, had limited opportunities to engage in whichever curriculum the Mexican schools presented for an assortment of reasons. Regardless of this fact, none of the participants have participated in an activity of this nature, and no one knew precisely what to anticipate. One thing was clear from the beginning, however: we could not realistically expect the students and mothers to mathematize community practices automatically. It has been documented that even mathematicians have struggled to understand the mathematical practices embedded in work and those that reflect individuals' unique way of knowing (e.g. González et al., 2001).

The basis for the creation of this project is two-fold. First, the research team collectively agreed that mathematics was used to varying degrees in the majority of complex work practices. – even if it was not recognized. More importantly, the team hypothesized that if parents and their children could experience mathematics together in the community, it would promote and facilitate informal mathematical discourse that would both support students' formal mathematical development, as well as transform each actor's mathematical identity (internalized and perceived) into a mathematics "knower" and a mathematics "doer."

In the sections that follow, I will expand upon the concept of funds of knowledge, in addition to other perspectives that showcase the assets of non-dominant groups. Then, I will describe a community mathematization project, collaboratively designed with Latina mothers, that aimed to expose Latina/o epistemology and the expansive mathematical knowledge bases of five immigrant mothers. This mathematization project is part of a larger study of Latinas/os' mathematics learning that I will also elaborate. Next, I will describe the ways in which the mothers translated the information provided by the community members into 'real' mathematical substance. This is reflected in their final products, digital stories. Finally, I will conclude with the implications of this study and question to probe in the future.

Theoretical positioning

Moll and an interdisciplinary team of researchers at the University of Arizona has constructed, refined, and empirically supported the concept of *funds of knowledge* for over 15 years (Moll et al., 1992; González, Moll, Floyd-Tenery, Rivera, et al., 1995; González et al., 2001). Funds of knowledge began as a tool to concretely illustrate to pre-service teachers the social and cultural capital found in perceptively deficient homes that could potentially be utilized in literacy instruction. Since then, it has crossed disciplinary borders and been applied to such fields as mathematics (e.g. Civil, 2002; Civil & Andrade, 2002; González et al., 2001; Díez-Palomar, Simic & Varley, 2006).

A derivative perspective of funds of knowledge places Latinas/os' cultural and social capital (e.g. epistemology, experiences) at the centre of the learning experience (Díez-Palomar, Simic, & Varley, 2006). This is a striking contrast to many U.S. classrooms that frequently neglect to acknowledge the fact that Latinas/os are not like their White counterparts in many ways, in terms of language, culture, and socio-historical experiences. It has been documented that non-dominant group members have proven competent in academic tasks when they view them as relevant (Saxe, 1991; González et al., 2001), which speaks to the importance of moving away from curricula and activities architected by and for the majority population if we are to better-serve subjugated groups in schools.

The study

Our research is conducted in a large, urban school district in a very large, post-industrial city in the Midwestern United States. Within this school system, there are approximately 421,000 students enrolled in the more than 620 schools. Of these students, nearly half (48%) are African-American and nearly an additional two-fifths (39%) are Latina/o. This leaves only 8% White and 5% Asian/Pacific Islander, Multi-Racial and Native American students. Furthermore, 85.6% of students are classified as coming from low-income families and 14.4% are categorized as “Limited English Proficient” (School District Data, 2007).

In the middle of this huge school bureaucracy lays Wilson Dual Language Academy, our research site. The Academy is made up of approximately 425 students in Pre-Kindergarten through 6^{th} grade. Demographically, the school is 99.4% Latina/o. The school has a relatively large transient population reflected by the 25.6% mobility rating. Additionally, 98.3% of students are eligible for the government’s free or reduced lunch program, and 68% are categorized as English language learners (ELLs) (School District Data, 2007). The Academy advertises itself as a “World Language Magnet Cluster School, offering a Dual Language Program in Spanish and English. The school’s goal is for all students to be bilinguals by the end of 6th grade (School District Data, 2007).”

The research reported here is based on data gathered in an after-school project loosely modelled after the work of *The Fifth Dimension* (Cole et al., 2006) and *La Clase Mágica* (Vásquez, 2003). The after-school is designed to give Latinas/os experiences doing non-remedial mathematics in curriculum topics beyond the students’ grade level (4^{th}). Such topics include probability, algebraic thinking/patterns, and complex problem solving. Students are encouraged to be self-directed, to work collaboratively, to verbalize their thinking, and to ask questions. Playfulness between adults and children is a critical part of interactions in the after-school.

Fourteen to twenty third and fourth grade students have voluntarily attended *Los Rayos de CEMELA* for nearly two years. They meet twice a week for one and a half hour sessions. Within the sessions, students are allowed many freedoms to choose their activities and to dictate the course a project will follow. This design is deliberately chosen by researchers to create a drastically different environment than the one which is typically found in the traditional classroom. The activities are designed to foster high-order thinking and reasoning skills. All sessions of *Los Rayos* are captured by multiple video cameras and are later analysed both by individual researchers and through collective group analysis.

In general, our research goal is to find intersections between mathematics learning, language, and culture. More specifically, CEMELA seeks to describe patterns that emerge regarding how Latino children use Spanish and English in mathematics, the cultural resources they draw on, and how social networks develop because of and around language and culture. This is part of understanding the processes in which students develop identities as mathematics learners through their interactions with university students and researchers and participation in the after-school mathematics program. We wish to better understand and map how networks (learning communities, for example) form around language, culture, and mathematics activities, and influence children's identity as having math competency. In other words, we recognize language and cultural resources and redefine them as learning capital. The focus is on the nature of learning as one of interdependence between social networks, resources, and contexts.

The community mathematization project

As previously indicated, this article reports on one project within the larger study just described. The focus is primarily on the Latina mothers, however, in order to capture the full context, there will be references to the other participants, namely, the elementary school children, the undergraduate group facilitators, and university researchers. The project is rooted in the paradigm of interactive research (Gitlin, 1990; Pizarro, 1998), which necessarily requires activating and utilizing the voice of research subjects in collaboration with researchers with the ultimate goal of benefiting all constituents. Parents were asked to collaborate with researchers to develop a design for a project in which students would venture into the community and interview local community members regarding the mathematics they used in the work environments. A main objective of the project was to find and understand mathematical practices in the community that people likely overlook in their day-to-day lives, even though these practices are within sight. First, since the parents are already networked within the community, each parent identified at least one business or service provider that would be willing to work with students and explain the daily operations. A flower shop, mechanic, beauty salon, fire station, travel agency, physical therapy office and produce store were examples of such places. Then, the parents escorted the students to these sites to become acquainted with the tasks of the workers. Eventually, the list of sites was reduced to four that were sure to accommodate students on subsequent visits and be fruitful for the benefit of mathematization.

In order for the parents and researchers to create common objectives for the project, many discussions took place around the idea of mathematization (Hoyles, Noss, & Pozzi, 1998; see also Freudenthal, 1991; Gravemeijer, 1997; Gravemeijer & Doorman, 1999; Greer, 1997; Streefland, 1991); that is, the exploration and raising awareness of the mathematics in the surrounding environment (in this case, the sites mentioned above). The objective was to demonstrate to all participants that mathematics is reality as well as abstract. Furthermore, we aimed to scaffold this experience so participants would realize that mathematics is more inclusive than just arithmetic operations.

Because mathematization is a somewhat unfamiliar practice, researchers and parents took part in a series of preliminary activities that moved them to a shared understanding of mathematization. For example, we collectively examined snapshots of a butcher and convenience store worker, made a list of all the tasks constituted in their respective work, and hypothesized potential problems they might encounter, all with the objective of getting to the hidden and embedded mathematics involved in various business and social practices. Another motive for these preliminary activities was the initial intention for the parents to become versed in mathematization in order to guide the children through the extensive process. By working through hypothetical situations, the parents indirectly learned how to isolate mathematical practices done by a typical worker that largely are unrecognized as mathematical practices. The culmination of this project was the creation of a digital story that illustrated the business practice and posed a potential mathematical situation that the worker might encounter. Our research questions for this study are:

- What role do cross-generational networks and interactions with elementary school students, university student facilitators, parents, and other community members play in the development of Latina/o parents' mathematical identity?
- How do children and parents support each other's reasoning and learning of mathematics, language, and literacy during the digital story project?
- How do children and parents translate stories told by community members into more mathematical language?
- How might social networks function in supporting Latina parents' development of mathematical knowledge and agency?

These questions are too involved to simultaneously elaborate and satisfy the scope of this paper. Although how children and their learning may be positively influenced by the funds of knowledge their parents bring forth and related issues are major considerations in the larger study, children are not the focus here. Therefore, I will focus primarily on the fourth question, while incorporating aspects of the first three questions in the Results and Discussion sections.

In order to answer these questions, we used the data embedded in the digital stories developed by parents and students as well as video documentation of the entire process. Focus group interviews with the parents also informed researchers as to what phenomena were occurring. The data were analyzed by multiple members of the project, and emergent themes were noted and discussed. Then, all data were reviewed at least once more with special attention paid to the particular instances of mathematical sense-making, agency, formation and utilization of networks, and identity transformations.

Mathematical sense-making

We defined mathematical sense-making as any stage in the process of understanding, visualizing, abstracting, or applying a particular mathematical idea. These stages can take a variety of forms, some of which are indexed by acts of mathematical struggle, collaboration, or success.

Agency

Agency was defined as any act that indicated a participant was undergoing mathematical empowerment. That is, the participant was not being acted upon by the activity, but rather was a critical actor in the process of moving from one level of mathematical schema to a more advanced level.

Formation and utilization of networks

Networks, in this sense, are the social interactions that occur within a space in which actors are connected by a particular activity system. In this case, the activity system is the after-school mathematics club, and more specifically, the activity is the mathematization project. This network has been forming for nearly two years. We are interested in marking instances in which the network is evolving and in which the participants are relying on various network members.

Mathematical identity transformations

At the beginning of the after-school program, participants assumed a certain place (both physical and intellectual), or role, within the activity system. This is a reflection of their internalized identity. With respect to the scope of this paper, we have isolated a portion of the activity system to include only the mathematization project. Therefore, we are more specifically concerned with participants' mathematical identity; that is, the ways in which a person views themselves as a mathematics "doer" or "knower." Inevitably, these identities have changed based on a series of interactions around mathematical tasks and activities. We are particularly interested in the transformations that have occurred through the mathematization process in terms of the active role participants assumed as mathematical "knowers" and "doers."

Results

In order to capture the depth and breadth of this project, as well as an example of a transformation that took place, I will present the case of one mother, Carmen. Carmen visited the mechanic with one group of children, a university researcher, and an undergraduate pre-service teacher. After her first two visits, she struggled to identify an area of mathematics in which she was interested. Carmen expressed concern that she wanted to "talk" about something important in her digital story. Eventually, she realized that it was the exhaust system – specifically, the pipes – that she was interested in, and she went to visit the mechanic again on her own to obtain more information. At the next after-school session, Carmen shared the information that she had learned, but noted that some of the ideas still were not clear. She began to formulate and ask questions of the other participants in the after-school program. They discussed terms like diameter, angles, inches, and degrees. The parents seemed unsure about the specific applications of these terms to the pipes.

Chaley, a university researcher who serves as the coordinator of the after-school program, was requested to join the group of parents and other participants when they were having conversations about these issues. When he came into the room, he inquired about the progress of their digital stories. The mothers mentioned that they wanted to work on the mathematics that they saw at the different places in order to create a problem for their stories, but at that moment, they were trying to make sense of the pipes. They had questions like, how does one measure the diameter of a pipe? What does the symbol " " " mean (referring to the symbol commonly used to represent inches)? Chaley reported that he felt as if they wanted him to tell them, but he tried not to do so. Instead, they had a lengthy discussion surrounding these topics.

Chaley, who is from Central America, reported that he led the mothers to what he thought they might already know and helped them connect this knowledge to these concepts. Regarding the diameter, he started by giving them examples of objects from which the diameter could be found rather concretely, like a glass and a plate. Then, he found other examples of geometric shapes, like a circle, and asked them, "Where do you think the diameter is in these objects?" As a group, the mothers began sharing their ideas about what it might be. One mother said, "If I'm not wrong, I think it is the space…distance from one edge of the circle to the other, across it all."

The conversation soon evolved into a question-posing session. The question, "How do we measure things?" was the basis of the discussion. Based on their observations and their experiences, the group talked about how one might go about measuring the length, diameter, and angle of a pipe in order to appropriately and accurately bend it to fit underneath the car. The participants used protractors and rulers and measured some local objects and angles. This led them to the units of measurement in each case, and they talked about centimetres and degrees, respectively. However, the conversation also brought up the difference in the metric system used in Latin America compared to the standard system used in the United States. They were well-informed about the conversion, or equivalence, of unit parts to whole units (i.e., 12 inches in one foot), however, the mothers still did not know about their symbolization. Chaley thought of using the height measurement on their identification cards as a resource to give the group more contextualized information, and they noticed that on their cards, the symbols: " " " and " ' " represent inches and feet, respectively. As a result, the group successfully figured out the unique mathematics symbol system for this measurement situation. After this, Carmen was still unsure about what to use in her problem, so she offered an explanation to the group about the different pipe diameters used for different cars, and she wondered whether that would be a good problem to include in her digital story. The researchers responded that that was a great idea and that it just had to make sense to her. At this point, Carmen needed only to develop her explanation a little more so that it would be clear to the rest of the participants and children in the digital story.

To be sure of her understanding and explanation, Carmen independently created three different-sized replicas of exhaust pipes made of construction paper. Later, she mounted the hand-made exhaust pipes on a backdrop and included this representation in her digital story. Figure 1 below shows the result.

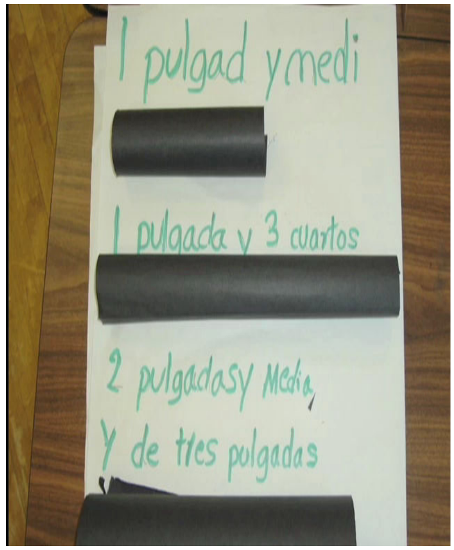

Figure 1. Carmen's representation of three different-sized exhaust pipes.

Carmen's digital story – a compilation of digital photos, strategically organized, narrated, and choreographed to background music – focuses on a combination of two topics: the interviews conducted by the students and the questions she raised about the exhaust pipes, which occurred independently of the first three visits with the children. The students' questions focused primarily on how much the mechanics charge in order to repair a car. The mechanic responded that it depends on the time spent on the repairs.

Carmen's interactions with the mechanic pertained to the exhaust system on cars. He informed her that there are four sizes of exhaust pipes, and the different sizes are based on the diameter of the pipe. The four diameters are 1 ½ inches, 1 ¾ inches, 2 ½ inches, and 3 inches. The mechanic told Carmen that if she knew the size of her exhaust pipe, she could tell the mechanic, and they could more efficiently repair the car. She was intrigued by the practicality of this insight, and it spurred the following questions for the conclusion of her digital story (speaking to her audience at *Los Rayos*): What is the diameter of your exhaust pipe? How can you measure it? Would you measure it with a ruler or a protractor?

Analysis and discussion

The crux of this study lies in the process through which the mothers moved from one level of mathematical understanding to a more advanced level that could be communicated mathematically. In the case of Carmen, there were three critical components that facilitated this process to varying degrees: social networks, agency, and mathematical sense-making. Of course, these factors are not mutually exclusive. Certainly, social networks contributed to mathematical sense-making, and agency was driven by a desire to make mathematical sense, to show merely two examples of the interplay of these ideas. Also, while mathematical identity transformation may or may not have facilitated the mathematization process, it undoubtedly was an end result.

Agency

Whether in formal schooling or in informal contexts, it is not uncommon to struggle with a new or unfamiliar concept. This is especially true if one is deliberately trying to advance their current knowledge base, as in the case of Carmen. Surely, she could have selected a mathematical topic that she was comfortable explaining to others, but she took seriously her responsibility to expose mathematical practices of the community that are beneath the surface, or beyond simple arithmetic. Her conscious and intentional efforts to choose a worthwhile topic to explore reflected her role as a parent, community member, *Los Rayos* participant, and an individual who simply desires to develop mathematically.

Perhaps most important to note is the fact that Carmen returned to the autoshop independently and without a prompt. This act embodies the objectives of the project: to find and understand a mathematical practice in the community that people likely overlook in their day-to-day lives, even though the practices are within sight. Furthermore, this individual sense of agency that Carmen demonstrated indicates that this mathematization project will not end when the semester is finished. Rather, it will likely become embedded in casual conversations and activities within which Carmen's family participates, one of the precise outcomes the project had in mind when it was conceptualized.

It is our position, based on our ethnographic presence, that Carmen very well could have completed the mathematization and production of the digital story with the initial three visits to the mechanic. But this additional act of agency is evidence that she has moved beyond the narrow instructions and design parameters of the project; she understands that her roles as a parent and project-design collaborator are to aim to meet the intended objectives; and, she sees the value in portraying a unique mathematical situation that is not likely to be thought about in our regular interactions with cars. Therefore, Carmen epitomizes the idea that the activity did not dictate how she manoeuvred through the series of steps in the project, but rather, how she took ownership of the project and consequently shaped it in order to maximize its value for all participants.

Mathematical sense-making and social networks

As alluded to, these factors are not mutually exclusive. Carmen's sense of agency surely contributed to her struggle to make sense of mathematical topics that were slightly beyond her current schema, and as a result, she actively searched for more knowledgeable others to assist in her meaning-making process. Vygotsky (1978) called this the zone of proximal development.

When Carmen realized that she might have a topic to explore (the exhaust pipes), she searched for any available adult that could potentially provide clarity or suggestions as to how to develop her ideas. Chaley and Carlos, a researcher who worked with the parents regularly, were two examples of people Carmen approached. While their guidance was primarily logistical, they still suggested pathways by which she could accomplish her goals.

More importantly, it was the mothers that provided the bulk of support and suggestions that made the mathematization process possible. Over the past seven months, the mothers had grown increasingly more cohesive and comfortable talking amongst the group about a variety of issues, some very personal. The trust and respect they placed on the group served as the foundation from which they could partake in mathematical discussions, ones that likely would have been more intimidating among strangers, especially given their limited opportunities for formal schooling. The group acts as a single unit, most likely the result of sharing similar social and ethnic backgrounds. A demonstrative example of the mathematical conversations that could take place was when the mothers could discuss measurement and units, concepts that could be interpreted by some as elementary skills and knowledge. Nonetheless, the mothers spoke freely, partially because all of the mothers were raised utilizing the metric system and had similar difficulties adapting to a foreign system, and it was clear that the aim of the conversation was to arrive at distinct conclusions about these topics in order to be able to pose an appropriate problem situation.

Mathematical identity transformation

It is easy for researchers or an observer to speculate that the participants' mathematical identities were transformed. After all, the research team maintains that this was a substantial, multi-faceted project that incorporated a lot of previous knowledge and opened up opportunities for individual and collective mathematical development. As a result, each participant's perception of mathematics in the community, including our own, has likely changed. Certainly, mathematical situations emerged that had not previously been considered. Furthermore, participants were challenged to articulate a particular mathematical situation in depth, as well as attempt to make sense of other participants' mathematical scenarios. Shifts in mathematical identity are difficult to detect from the dialogue and work that took place during the project. Therefore, we rely on follow-up focus group discussions in order to determine what exactly occurred as a result of this project, in terms of mathematical identity, if anything at all.

Carmen reported that as a result of this experience, she now sees mathematics everywhere. Below is an excerpt from the focus group:

1. Chaley: Y, siempre ha pensado así? Siempre pensaba que todo llevaba matemática, o cambió de pensamiento por alguna razón? *And, have you always thought like that? Have you always thought that everything involved mathematics, or did you change your mind for any reason?*
2. Carmen: No, ahora sí lleva más matemáticas. *No, now I see more math.*
3. Chaley: Pero, cómo se dio cuenta usted de que las matemáticas están en todos lados? *But, how did you come to realize that math is everywhere?*
4. Carmen: Porque para todo estamos ocupándolo. *Because we use it for everything.*
5. Chaley: Pero entonces, yo le creo eso a usted y estoy de acuerdo con usted, pero siempre ha pensado así? O hasta ahora piensa diferente? *But then…I believe you, and I agree with you, but have you always thought that way, or now do you think differently?*
6. Carmen: No, hasta ahora pienso diferente, que pues para todo necesitamos matemáticas. *No, now I think differently, that we need math for everything.*

This exchange clearly shows a transformation from a person who was not inclined or prepared to extract the mathematics from a given situation to a person who cannot help but notice the mathematics embedded in a given situation. It also should be noted that Carmen's mathematical disposition has changed over the course of the year. At the beginning of the year, Carmen freely offered her opinion and insights about the general topic at hand, but it appeared as though she lacked confidence when discussing mathematics (with the exception of arithmetic). During and after this project, as described above, she embraced the mathematical challenge of the project, and her confidence in working with a variety of mathematics topics was virtually visible.

Collaborative processes

Not exclusive to the mathematical context, another finding revealed that the collaborative process is not automatic. This is particularly true of inter-generational collaboration, as our study shows. The interactions between the parents and the children, and between the parents and the undergraduate facilitators, did not occur as naturally as we had anticipated. Recall that we intended to first work with the parents as mathematizers so that they would, in turn, facilitate the mathematization process with the children. However, collaboration is not as simple as "plugging" someone into a pre-existing group and expecting it to be a natural fit. We underestimated the existing dynamics of the groups, some of which had been working together for a year.

As we are currently learning in subsequent studies, certain measures can be taken in order to make clear the roles of participants. For example, more emphasis should be put on grounding participants in the notion that all collaborating members have a significant amount of knowledge and experiences upon which to capitalize (*funds of knowledge*). All participants should know that not only does this knowledge exist, but also that it has substantial value in the context of the learning process. Finally, the notion of funds of knowledge should not only be discussed with participants, particularly pre-service teachers, but also complementarily demonstrated through explicit examples. The objective is to transform dominant perceptions of Latina/o parents from non-participatory bystanders in the educational process to critical partners, ones who maintain an invaluable epistemological basis that educators need to access and capitalize upon (Moll et al., 1992; Pizarro, 1998). This is an important aspect to consider when planning activities with Latina/o parents and students, or for any multi-generational groups engaged in a network to promote learning in mathematics through collaboration.

Implications

Undeniably, there is much more to learn about the resources and learning capital that Latinas/os bring to the mathematical context. In the big picture and considering the relatively new demographic shifts in the United States, the mathematics education field has just begun work that aims to systematically and thoroughly understand the inter-sectionality of mathematical development, language, and social practices as they occur in Latina/o communities. In a way, any assertions made at this point could potentially have the counterproductive effect of reducing the complexity of the situation. We are cautious as to not contribute to society's desire to obtain "quick fixes" to the teaching and learning of mathematics to Latinas/os in the United States. With that in mind, there are still a few implications we wish to reiterate.

First, this study implies that all adults, regardless of race, ethnicity, socioeconomic status, gender, or educational achievement, have the capacity to understand existing and conceptualize new mathematical situations of various genres (i.e. measurement, algebraic thinking, geometric reasoning, number systems) based on community practices. Not only this, but they draw upon an assortment of resources, including innumerable life experiences, in order to make sense of these practices. Witnessing the mothers' development is a key instrument in challenging the dominant perception that Latina/o families are a hindrance, rather than a support, for the educational achievement of Latina/o youth. While this is an important consideration for changing research paradigms with respect to Latina/o communities (Pizzaro, 1998), it is a critical component in the development of pre-service teachers (undergraduate facilitators) who will soon be responsible for the formal mathematics instruction of Latina/o children. Certainly, this experience is a more powerful demonstration of the potential of Latinas/os than any lecture or literature can convey.

Secondly, while there were many successes and valuable insights gained from the community mathematization project, the researchers realize that more efforts are needed to promote effective collaborative processes. It cannot be assumed that all parents – especially immigrant, Latina/o parents, many of whom have limited, formal educational opportunities – fit comfortably into the school environment, regardless of the intellectual task. Moreover, if there are already pre-existing norms and group dynamics, the context is confounded even more. While we advocate valuing the knowledge and experiences of Latina/o parents, we simultaneously call for heightened sensitivities to the historical and social backgrounds of parents on behalf of school personnel.

Concluding remarks

In the United States, there is a great need for a paradigm shift in how we view the knowledge bases and capacities of minority families. The current perceptions of Latinas/os from dominant groups have serious effects on both the children in schools, as well as their parents. Our research agenda aims to chip away at the harmful ideologies that have permeated the classrooms and homes of non-White populations. Certainly, this study is a counter example to those who maintain that the problem lies within the family or the home, as if there is some kind of deficiency prohibiting Latina/o youth from achieving at high levels. Instead, we view this study as evidence of the willingness and potential of Latina/o parents to engage in change in their communities, starting with fostering mathematical development in themselves so that they can readily support their children's learning and aspirations.

It is our intention to build upon the theoretical position presented here and advance our research agenda in an effort to support this community and maximize the effects of the parental efforts we have witnessed here, as well as those that are quietly taking place nation-wide. While we have touched upon the complex nature of collaboration between parents and the community, we insist that this collaboration has tremendous promise and that it is the only way to instigate impacting change on communities that have historically suffered from a lack of equal educational opportunities.

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References

Civil, M. & Andrade, R. (2002). Transitions between home and school mathematics: Rays of hope amidst the passing clouds. In G. de Abreu, A. J. Bishop, and N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 149-169). Dordrecht, The Netherlands: Kluwer.

Civil, M. (2002). Culture and mathematics: A community approach. *Journal of Intercultural Studies, 23*(2), 133-148.

Cole, M. & The Distributed Literacy Consortium. (2006). *The fifth dimension: An after-school program built on diversity*. New York: Russell Sage Foundation.

Diez-Palomar, J., Simic, K., & Varley, M. (2006). Mathematics is everywhere: Connecting mathematics to students' lives. *The Journal of Mathematics and Culture, 1*(2), 20-36.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht, The Netherlands: Kluwer Academic Press.

Gitlin, A. (1990). Educative research, voice, and school change. *Harvard Educational Review, 60*, 443-466.

Gonzalez, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging Funds of Distributed Knowledge: Creating Zones of Practices in Mathematics. *Journal of Education for Students Placed at Risk, 6*(1&2), 115-132.

Gonzalez, N., Moll, L., Floyd-Tenery, M., Rivera, A., Rendon, P., Gonzalez, R. & Amanti, C. (1995). Funds of knowledge for teaching in Latino households. *Urban Education, 29*(4), 444-471.

Gravemeijer, K. (1997). Commentary: Solving word problems: A case of modeling? *Learning and Instruction, 7*(4), 389-397.

Gravemeijer, K. & Doorman, M. (1999). Context problems in Realistic Mathematics Education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111-129.

Greer, B. (1997). Modeling reality in mathematics classrooms: The case of word problems. *Learning and Instruction*, *7*(4), 293-307.

Hoyles, C., Noss, R., & Pozzi, S. (1999). Mathematizing in practice. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum*. London: Falmer Press.

Khisty, L. L. & Willey, C. (in press). After-school: An innovative model to better understand the mathematics learning of Latinas/os. To appear in B. Bronwyn, A. Razfar, P. Bell, R. Stevens, and J. Remillard (Eds.), *Learning out-of-school time.*

Martin, D. B. (2007). Beyond missionaries or cannibals: Who should teach mathematics to African American children? *The High School Journal,* October/November, 6-28. Chapel Hill, NC: University of North Carolina Press.

Martin, D. (2009). Researching race in mathematics education. To appear in *Teachers College Record, 111*(2).

Martin, D., & McGee, E. (in press). Mathematics Literacy and Liberation: Reframing Mathematics Education for African American Children. To appear in B. Greer, S. Mukhophadhay, S. Nelson-Barber, & A. Powell (Eds.), *Culturally responsive mathematics education.*

Moll, L., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: A qualitative approach to developing strategic connections between homes and classrooms. *Theory Into Practice, 31*, 132-141.

National Centre for Education Statistics. (2007). Status and trends in the education of racial and ethnic minorities. Retrieved from the Internet March 4, 2008, from http://nces.ed.gov/pubs2007/minoritytrends/ind_3_9.asp.

Pizarro, M. (1998). Chicana/o power!: Epistemology and methodology for social justice and empowerment in Chicana/o communities. *Qualitative Studies in Education, 11*, 57-80.

Saxe, G. (1991). *Culture and cognitive development: Studies in mathematical understanding.* Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

School District Data. (2007). Retrieved from the Internet, August 12, 2007, from: http://www.cps.k12.il.us/AtAGlance.html.

Streefland, L. (1991). *Fractions in realistic mathematics education: A paradigm of developmental research*. The Netherlands: Springer.

United States Census Bureau. (2007). Current population survey for Illinois, 2004-2006. Cited from B. Alejo & S. Puente. (2007). Forging the tools for unity: A report on Metro Chicago's Mayors Roundtables on Latino integration. Chicago: University of Notre Dame.

Vásquez, O. A. (2003). *La clase mágica: Imagining optimal possibilities in a bilingual community*. Mahwah, NJ: Lawrence Erlbaum Associates.

Velez-Ibanez, C., & Greenberg, J. (1992). Formation and transformation of funds of knowledge among U.S. Mexican households. *Anthropology and Education Quarterly, 23*, 313-335.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Parents' interactions with their children when doing mathematics

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Abstract

In this paper we discuss a study that looks at the kinds of practices that Latino parents use to support their children's learning of mathematics. Some of our findings reveal that parents react to their children's mathematics education looking back to their own experiences as students. Thus, in some cases, they are faced with pedagogical strategies unfamiliar to them. This may lead to conflict but also to possibilities for exchange of strategies and learning for both parties, children and parents. Our study shows the resourcefulness among parents who not only had different learning experiences but also may encounter a language barrier. Parents turn to the community for help, such as taking their children to the community centre, or asking other family members or neighbors for help with homework. There are also parents who express some concern about the level of the mathematics that their children are studying. Some parents deliberately engage with their children in activities that promote and require explicit display of mathematical reasoning and knowledge. Our findings point to the different ways in which community, family and school are connected.

Key words: parents' perceptions, Latino families, interactions, community.

Introduction

This paper contributes to the research on parents' engagement in mathematics teaching and learning (Civil & Andrade, 2002; Civil & Bernier, 2006; Epstein & Sheldon, 2005; Lehrer & Shumow, 1997; Remillard & Jackson, 2006). Research with low-income, minoritized and immigrant families reveals that there is often a cultural and social gap between their expectations for their children's education and their experiences with the actual schooling their children are receiving (Abreu, Cline, & Shamsi, 2002; Calabrese Barton, Drake, Perez, St. Louis, & George, 2004; Civil & Bernier, 2006; Civil, Bernier, & Quintos, 2003; Horvat, Weininger, & Lareau, 2003). Research also points to the difficulties that parents face when trying to help their children with mathematics homework (Ginsburg, 2006; Remillard & Jackson, 2006). Some authors tend to attribute this situation to the current approaches to the teaching and learning of mathematics. These approaches are largely based on a socio-constructivist view of how people learn (NCTM, 2000). Because now mathematics is taught in a different way from years ago, parents may find it difficult to help their children (Peressini, 1997, 1998). Other researchers affirm that there is a socio-cultural reason embedded in these kinds of situations (Civil & Bernier, 2006; Civil, Bernier, & Quintos, 2003). The difference between mathematics instruction in the USA and in other countries (in our case, Mexico) may lead to a "gap" that is difficult to overcome for some families. Some parents leave the responsibility of education to the teacher, while others become active in looking for resources in the community to help their children. Our research centres on parents' engagement on teaching and learning, with a particular focus on mathematics.

Theoretical framework

This approach to human activity is rooted in Vygotsky's (1978) perspective of cultural practices where learning is a social process that is situated in the culture. That means that we cannot understand learning exclusively as a cognitive process, but as a socio-cultural one in which people internalize cognitive structures (such as mathematical reasoning, for instance) by using mediating artifacts. Interactions among individuals are crucial to produce learning: People learn from each other. Furthermore, learning always occurs within a socio-cultural context that also mediates the whole process of learning. According to Nasir and Hand (2006), social others and interactions are one of the most prominent characteristics of socio-cultural approaches. These variables play a crucial role in learning and development, hence learning is a product of these social relationships. González and colleagues (González, Moll, & Amanti, 2005; González, Andrade, Civil, & Moll, 2001) propose the term *funds-of-knowledge* (FoK) to analyze this learning process pointing out that social and cultural aspects also intervenes in learning through variables such as the identity. According to them, knowledge also has a socio-cultural basis that should be take into account. Rogoff (1995) points out that learning involves a personal plane (individual cognition, emotion, behaviour, values and beliefs), as well as a interpersonal or social plane (including communication, dialogue, cooperation, conflict, and interactions with others). From a pedagogical approach we draw on ideas from Flecha (2000) and Freire (1970, 1998), who define learning as a process in which individuals are able to transform critically their world using dialogue (and dialogical action) as a means to acquire that learning. In this article we will not elaborate on concrete aspects related to how this approach works in terms of learning. This theory is useful to us since it points out the importance of interaction and context (which impacts on interactions) in learning.

In this article we focus on three research questions:

1. What kind of practices do parents use to help their children with their mathematics?
2. How do parents' different learning experiences affect their perceptions of their children's mathematics teaching and learning?
3. What are the most prominent cultural elements that mediate parents' perceptions of and reactions to practices related to their children's mathematics education?

We understand these three questions in terms of our assumption that learning is a social process and hence it is mediated by interactions among individuals. Prior research in the area of mathematics education and parents suggests that there is some sort of connection between how parents help their children doing mathematics and their cultural and social milieu. Within a sociocultural framework, we know that interactions are an important factor to explain how learning works. In addition, an analytical perspective grounded on the impact of cultural practices over the learning process (of mathematics) bring to us a theoretical framework to considerer the importance of cultural identity aspects of Latino families when parents help their children doing mathematics. For this reason we would like to explore how socio-cultural elements impact Latino children's educational experiences, as expressed by our research questions.

Method

As Flecha and Gómez (2004) claim, phenomena in social sciences are social constructions; that is, the subject of study is a social construction based on people's interactions. If we want to get answers for our research questions, we need to include the participants' voices in the research. Critical communicative methodology –CCM– (Gómez, Latorre, Sánchez, & Flecha, 2006) is a way to achieve that goal. This methodological approach draws on an intersubjective epistemology; that is, social phenomena are produced by individuals through interactions. Our society is a social construction, as claimed by Berger and Luckmann (1988). For this reason a methodology that includes everybody's voices is needed to analyze social processes, such as education. CCM uses discourse analysis as an analytical tool, drawing on Habermas's *Theory of Communicative Action* (1987). Therefore, we look at arguments provided by the participants in the research through dialogues (collected using qualitative techniques).

Our study took place in a borderland city of the Southwestern United States. Twenty-five Latino families were recruited for the study. Most of these families were originally from the northern part of Mexico. Many of the parents had attended school in that country and thus had schooling experiences different from those of their children. We want to point out that the schooling experiences of this group of parents were not homogeneous. Several of them went to primary school in rural areas and *ejidos* (common lands), where sometimes one teacher taught students from first to sixth grade all in one classroom at once. Attending secondary school for these parents usually meant having to walk long distances, or moving to live with relatives in a larger town. For some others, those options were not there (i.e., it was not possible to move to a larger town, or being a girl and considering that it was unnecessary to pursue more education or being the older brother who had to help out the parents to provide for the younger siblings). On the other hand, in our group of parents we also had some who have a college degree or at least a few years of college education.

We carried out interviews, focus groups, workshops, classrooms observations, and debriefing with parents. Data were analyzed using the CCM approach, as well as grounded theory (Glaser & Strauss, 1967). CCM directs our attention to dialogues and how every participant in the research constructs their own understanding, and thus their argumentation, to justify their interactions in terms of "learning mathematics." Grounded theory provides us with a method to get an inductive approach to the data in terms of categories. All of them emerged from the data collected, which validates our analysis since those categories correspond to participants' insights, rather than researchers' interpretations. Furthermore, we discussed some of our interpretations with some participants in order to validate them, as CCM recommends. To include participants' voices as co-responsible for the research process becomes a critical way to research *with* Latino parents (and children), rather than *on* Latino parents, which makes the conclusions drawn more reliable (Flecha & Gómez, 2004).

Discussion

We start this section by providing some "vignettes" from one family to illustrate our first question, "what kinds of practices do parents use to help their children with mathematics?" Then, we use data from parents' interviews to address the second question, " how do parents' different learning experiences affect their perceptions of their children's mathematics teaching and learning? Our third question (what are the most prominent cultural elements that mediate parents' perceptions of and reaction to practices related to their children's mathematics education?) somehow encompasses the first two and is addressed last. There are examples to illustrate how parents' prior experiences, as well as cultural identities, impact how they interact with their children doing/solving mathematics. Items such as "country of origin," "cultural identity," and "age/generation" are important variables to understand how parents and children interact around mathematics.

Analysis of interactions between parents and children

Margarita, an immigrant mother from Mexico, started attending the Math for Parents course in Spring 2005, when we first offered one at her child's school. At that time the course (eight sessions, 2 hours per session) took place in the morning and was geared to parents only; the topic was "exploring fractions, decimals, and percents." The following year, we switched to an evening format in which parents came with their children. Margarita then attended with her husband Sergio and their daughter Berta, a third grader. In this paper we focus on the interactions between the parents and the child during these workshops. We also draw on an interview in their home during that same time (Spring 2006) in which the daughter sought her parents' help with homework, as the interview took place.

Vignette 1: Berta and her mother (Margarita) on the paper clip task

In this particular activity, parents were working with their children on subtraction by grabbing a handful of paperclips from a box of 100 clips, counting how many they took and then they had to figure out how many were left. Berta picked up 25 and figured out that there were 75 left by using the 100-number chart and counting from 25 to 100. Her mother had also figured out that there were 75 left, but she had done that independently. The facilitator wanted to engage mother and daughter in a dialogue about how the mother had done it.

1. Bety: Pero ella supo que era setenta y cinco, ¿cómo supo tu mama que era setenta y cinco?
[But she knew it was seventy five, how did your mom know that it was seventy five?]
2. Berta: We counted by tens
3. Bety: Tu mamá, pregúntale a tu mamá cómo supo que eran setenta y cinco.
[Your mom, ask your mom how she knew it's seventy-five.]
4. Berta: (to her mother) ¿Cómo supiste que eran setenta y cinco?
[How did you know it was seventy-five?]
5. Bety- ¿Cómo le hiciste en tu cabeza Margarita?
[How did you figure it out in your head, Margarita?]
6. Margarita- Yo lo hice en mi cabeza.
[I figured it out in my head.]
7. Bety- Pero ¿cómo?
[But how?]
8. Margarita- ¿Cómo? que tenemos cien en la caja y el resto veinte y cinco queda setenta y cinco.
[How? well we have one-hundred in the box I take away twenty-five, seventy-five remain.]
9. Bety- Pero ¿cómo supiste que era setenta y cinco?
[But how did you know it was seventy-five?]
10. Margarita- Por lógica...
[Through logic.]
11. Berta-(raising her hand) setenta y cinco. Bety, (hitting her paper with her pencil and counting) ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, eighty one, eighty two, eighty three, eighty four, eighty five, eighty six.
[seventy five]
12. **Margarita**: Porque es de 100 menos 25.
[*Because it's 100 minus 25*]
13. **Bety**: Pero ¿cómo lo hiciste, lo sabes automático?
[*But how did you do it, do you know it automatic?*]
14. **Margarita**: Me lo sé automáticamente.
[*I know it automatically.*]
15. **Bety**- Sí, ya sabes que…
[*Yes, you already know that*…]
16. Bety- ¿Cómo le hizo tu mamá?
[How did your mom do it?]

In this excerpt we notice the efforts of the facilitator to involve the daughter in talking to her mother about how she had approached this task (lines 1, 3, 16). The daughter seems more interested in talking about how she did it (line 2, 11). The facilitator, Bety, turns to the mother to try to get her to explain how she did it. This dynamic was quite common between Margarita and her daughter. Margarita seemed to enjoy coming to these workshops as a learner herself. Her behavior is different from that of some other parents, including her husband, in that they come to help their children in a very direct way. Margarita, on the other hand, seemed to want to solve the activities for her own enjoyment and learning. This may have been due to her original participation in the course that was for parents only, in which parents were basically adult learners. This is not to say that Margarita did not scaffold her child's learning, as we will see in a later vignette.

Bety continues to probe Margarita on how she did it and Margarita shares that she thought of money:

1. Bety-Ella pensó en el dinero.
[She thought about money.]
2. Berta- Ella no hizo, yo lo conté.
[She didn't do it: I counted.]
3. Bety- Ella también le hizo en su cabeza más rápido.
[She also did it in her head faster.]
4. Berta- (looking at her mother) A ver, hazlo... (returning to her paper) no pero yo lo hice más rápido porque yo ya sé
[Let me see, do it,... no but I did it faster because I already know]
5. Bety- Diferent[e]
[*Different*]
6. Berta- que allí están los números y es twenty five.
[that the numbers are right there and it's twenty five]
7. Bety- Eso es lo más importante que hay formas diferentes.
[That's the most important thing, that there are different ways]
8. Margarita- (looking at her daughter) hay formas diferentes
[*there are different forms*]
9. Berta-(asking her mother) A ver, ¿cómo lo hiciste?
[*Let me see, how did you do it?*]

In this excerpt we see that Berta challenges that her mother did it (line 2 "she didn't do it, I counted"). Berta followed what in her view was an approved method to solve the problem, while she did not see how her mother did it. Her mother did it in her head, "using logic." We see from other interactions with Berta that she has a concern for the approach, for making sure that she using the expected approach. The facilitator, while teasing her about her mother's method being faster, introduces the idea to there being different approaches (lines 5, 7), to which the mother agrees (line 8). After that Berta, somewhat reluctantly asks her mother to share her method.

Vignette 2: Berta and her father on addition and subtraction

At another evening workshop, parents and children were supposed to practice strategies for addition and subtraction that are different from the "standard" ones. For example, the worksheet showed "23 + 46 + 7 ="; the idea was for the families to solve this addition sentence horizontally by breaking apart and combining numbers. The parents had a sheet showing different approaches and teachers and facilitators were going around the room explaining those approaches. During this scene, one of the teachers is explaining the worksheet with the different approaches to Margarita. Sergio, Berta's father, did not seem to pay attention to that and right away took the pencil and set it up vertically for his daughter to solve, using the traditional addition algorithm ("put it here, look"). The daughter was trying to solve it horizontally, but the father insists ("do it like this, look"), and Berta switches to her father's paper and does the vertical addition. Their interaction is very procedural with the father telling her what to do. After Berta finishes that first sum, she proceeds to the next one, which she immediately re-writes vertically and goes through the same procedure than in the previous one. She does the same thing with the third sum (23 + 26 + 27 =). Berta makes a mistake and the father intervenes:

1. Sergio (father)- dieciséis, está mal ahí. *[Sixteen, it's wrong here.]*
2. Berta – ¿Cómo?, a ver, a ver, a ver [*How? Let's see, let's see, let's see.*]
3. Sergio- Mira, siete y tres son... *[Look, seven and three is…]*
4. Berta- Yo, yo lo quiero hacer. *[I, I want to do it!]*
(Berta is counting on her fingers)
5. Berta- [laughs] sí está bien la tuya Pa. No me tienes que decir porque yo quiero aprender.
[Yeah dad you were right. You don't have to tell me because I want to learn.]
6. Sergio- Siete. Está mal ahí. Y la una que llevabas, mira pon el uno aquí arriba.
[*Seven. It's wrong there. And the one that you were carrying, look put the one here on top*]
7. Berta- ohhhhhhh con razón! *[Ooooh, no wonder…]*
8. Sergio- Mira está bien fácil. [*Look, it's really easy*]

(Berta does one more on that sheet -which happens to be a subtraction problem but she does it as an addition- and then goes back to her own worksheet to work again on the first one, 23 + 46 + 7, which her father had originally set up for her; she does her own setup vertically.)

9. Berta- más cuarenta y seis más siete
[plus forty-six plus seven]
10. Sergio- Ponle cero para que no te confundas.
[Put the zero so that you won't get confused]

(The father is referring to the 7, when setting it up vertically he had written it as 07 and he wants his daughter to do the same thing; Berta does that and proceeds to solve the addition problem.)

In this interaction the father is watching everything his daughter does and intervenes as soon as she makes a mistake. His style is direct and somewhat authoritarian, but the daughter does not just follow what her father tells her. She does not want to be told, she wants to do it herself and learn (lines 4 and 5). This may explain why on line 9 she goes back to the first problem (the father does not ask her to do that; she just goes back to it). Her father had set it up for her and she wants to do it all by herself.

After that, they switch to subtraction. The idea for this workshop was to look at subtraction as adding too; for example, to do 51 – 22, one could do from 22 to 30 (8), then 30 to 40 (10), 40 to 50 (10) and then one more to 51, so the answer is 29. The worksheet the father and daughter were working on shows some indication that they (Berta?) had attempted this method (but incorrectly as there was 2 from 22 to 30 and a 1 from 50 to 60). The father proceeds to explain to her the standard algorithm for subtraction. Once again, it is very procedural ("this one—pointing to the 5—gives 1 so this becomes 11"). What is interesting to point out, however, is that although the father did not seem very interested in other approaches to subtraction, a few days later during an interaction in the home around a homework task (discussed in the next vignette), he actually brings up this other approach to do subtraction (based on adding up) when he is trying to help his daughter. We move now to this third and last vignette.

Vignette 3: Margarita, Sergio and Berta on a subtraction problem

This interaction took place in their house, while one of the researchers was conducting an interview. Berta requested help on a task that basically said "Jack has 73 pennies; how many more pennies does he need to be able to change them for $1?" The mother right away made a connection to the paper clip task (see Vignette 1):

Margarita: Setenta y tres; setenta y tres; es lo que hicimos con los clips. Pero estos son pennies. ¿Sí? M'hijita? Y sacamos cuántos pennies nos quedaron adentro.

[Seventy-three; seventy-three; it's what we did with the clips. But these are pennies, right, "my dear"? And we found how many pennies were left?]

But Berta does not seem to see (or hear) the connection to that activity. After a few seconds she turns to her father for help:

1. **Berta**: Papá, necesito ayudaaaaaa [*Dad, I need heeeeeelp*]
2. **Sergio (father):** Mira m'hija. Está muy fácil. Mira. Fíjate nomás. Te pones… [*Look m'hija. It's very easy; look, just pay attention. You put…*]
3. **Berta**: Yo lo tengo que hacer [*I have to do it!]* (raising her voice)
4. **Sergio**: ¡Ya sé! Ya sé. Mira. No te lo voy a hacer. Si tienes setenta, ochenta, noventa, cien. ¿cuánto? ¿cuánto es? Diez, veinte, treinta. Pero como ya tienes tres, no serían treinta. Serían ¿cuánto? Si tienes treinta [*I know, I know; look; I'm not going to do it for you; if you have seventy, eighty, ninety, one hundred, how much; how much is it? Ten, twenty, thirty. But since you already have three, it wouldn't be thirty. It would be, how much? If you have thirty*]
5. **Berta**: Mi… mi… ¡Mil! [*One, one… one thousand!]*
6. **Sergio**: No. Tienes tres aquí... [*No. You have three here…]*
7. **Berta**: ¡Noventa! [*Ninety!]*
8. **Sergio**: No, un momeeeento. [*No, just a mooooment*]
9. **Berta**: ¡Ochenta! ¡Setenta! [*Eighty! Seventy!]*
10. **Sergio**: Espérate, espérate. Si tienes setenta y tres aquí, setenta y tres para ochenta, hay siete. De ochenta a noventa, hay diez. De ésta acá, va otros diez. ¿Cuántos serían? Siete, diez, y diez, ¿cuántos serían? [*Wait, wait. If you have seventy-three here, seventy-three to eighty, it's seven. From eighty to ninety, it's ten. From here to there, another then. How much would it be? Seven, ten, and ten, how much?*]
11. **Berta**: ¡¡¡Veintisietee!!! [*Twenty-seven!!!]* (yelling)
12. **Sergio**: ¡Pues sí! A ver… pero bórrale, bórrale. Y hazlo bien. [*That's it! Let's see…so, erase, erase. And do it well.*]
13. **Berta:** (at the same time and screaming)) Pá, pá, ¡pero lo tengo que hacer asíiiii! [*Dad, dad but I have to do it this way!]*
14. **Sergio**: Y ¿cómo te lo enseñaron? [*And how did they teach you?]*
15. **Berta:** Así. Le tengo que hacer así. Así como uno, así, y cuánto hace, y luego esto plus cero, seis ceros... [*Like this, I have to do it like this; like one, one, and how much, and then this plus zero, six zeros…]*

In this interaction, once again, the father takes over the task, but the daughter resists that, as she wants to do it herself (line 3). Her father says that he is not going to do it for her (line 4) and asks her questions to get her involved but as we can see, the daughter seems to be yelling out numbers that have no bearing on the problem (line 5) or that it is not clear how she is relating them to the problem (lines 7 and 9). The father uses an approach similar to what was expected in Vignette 2 (but which he did not do that for that one), by going from 73 to 100 in steps (line 4 and again, line 10). In line 10, he basically tells his daughter what to do "seven, ten, and ten, how much?" so that she "gets" the correct answer. Yet, Berta does not like this approach and is concerned that this is not the way she is expected to do it. Berta seems concerned that her father's method is not going to be accepted, in part because she is not going to know how to explain it:

1. **Berta**: pero si tú me lo hiciste a mí, para mí, y luego me dicen: ¿qué era? Porque la maestra me dice: "¿cómo lo hiciste tú?" Yo no voy a saber, y entonces me va a decir la maestra: "¿quién te lo hizo?" Entonces. Yo no voy a saber cómo funcionan todas esas cosas en la cabeza.]
2. [*But if you did it for me, to me, and then they tell me, what was it? Because the teacher is going to say, "how did you do it?" And I'm not going to know, and then she's going to say, "who did it for you?" I'm not going to know how all these things work in the head.*]

As in Vignette 2 we see that Berta wants to know how to do things herself. She does not want to just be told how to do it. The interaction goes on for quite a while until finally Margarita and the researcher bring up another tool that they had seen at the workshops, the 100 chart and it seems that this is what Berta thought she had to do. In the paper clip problem Berta had used the 100 chart and it is possible that since this interview took place not very long after that workshop, that Berta felt that that was the way she had to do it.

Analysis of parents' perceptions of their children mathematics teaching and learning

The case of Margarita, Sergio and their daughter Berta illustrates different kinds of interactions between parents and children. Margarita seemed to see herself as learner in the workshops and in general tended to engage in the activities within that frame of mind, that of an adult learner. So, in the paperclip task (Vignette 1), Berta and her mother had worked on it independently from each other and the facilitator wanted to get them to talk about their approaches. Sergio came to the workshops to work with his daughter and in general his approach was direct teaching, based on how he had learned (Vignette 2). Vignette 3, which captures an interaction in the home, offers a different glimpse of these interactions. Here (in the home), Margarita is helping her daughter and she does that by making a connection to one of the mathematics workshops (the one with the paper clip problem); Margarita does not quite tell her daughter how to solve the problem but mostly tries to remind her of this prior experience. Sergio also helps his daughter but in a more direct way. However, in this particular example, he does use one of the alternative approaches to subtraction instead of the "traditional" one he had learned. In fact, he uses the approach that was expected in Vignette 2.

This case is not representative in the sense that we can say that all parents in our study interacted along similar ways with their children. We have parents who use direct instruction (along the lines of Sergio) but we also have parents who mostly ask questions, and others who "just" watch their children. But this case does reflect the influence of parents' prior learning experiences. Margarita was eager to keep on learning (we have evidence of that from interviews with her); she was not only attending our mathematics courses since the beginning but was also attending English classes in a community centre. Hence her identity as "learner" is what predominates in the workshops. Sergio saw his role more as teaching his daughter and teaching her as he had learned.

Previous research suggests that parents' prior experiences with mathematics affect how they perceive it (Abreu, 2004; O'Toole & Abreu, 2005). Data obtained in our study confirms this idea. Parents in our study comment on how their own experiences as learners of mathematics influence their approach to helping their children. Many of the parents in our study perceive schooling in Mexico as being quite different from what they see their children experiencing now in the United States of America. Their memories of learning mathematics as children are full of references to school practices that emphasized memorization of facts, computation, and sometimes problem solving. This, to them, looks very different from the type of activities that their children are doing in the U.S.A. schools. All these differences produce both positive and negative effects regarding children's mathematics education.

Experiences parents have had as students filter their perceptions on the education their children receive and shape the type of interactions parents have with their children when it comes to mathematics education. A generalized perception is that the level of mathematics education in Mexico is higher than that of the United States; this assessment is made with reference to the curriculum and the teaching practices. Based on their experiences, parents favor some school practices more than others.

About the mathematics curriculum. As noted above, almost all parents interviewed for this study reported that they thought that the teaching of mathematics was more advanced in the Mexican schools than those in the United States. This is a common perception that we have documented elsewhere (Bratton, Quintos, & Civil, 2004). We will not discuss this much here, though here is a typical example of a comment from one of the mothers in this study:

Lucrecia: Más elevadas, son más elevadas, porque el niño aquí está con una división de una matemática por fuera, y allá ya están casi entrando con tres afuera, el niño, entonces ya es más baja. Es más baja. Sí, porque aquí están haciendo las tablas, repasándolas y ellos las tablas pues en segundo año, por eso se me hace un poquito más bajo, o será al niño también se le hacen más fácil aquí porque como viene más adelantado de allá, a lo mejor por eso al niño se le hacen más fácil a él por eso, yo digo...
[More advanced, they are more advanced, because the boy [here] is doing a division with one [digit] *mathematics on the outside, and over there they are starting almost with three outside, the boy; then it is lower. It is lower. Yes, because they are doing the multiplication facts here, reviewing them and they, the facts, well in second grade. That's why I think it is a little bit lower* [here] *or it might be because it is easier for the boy here, since he comes more advanced* [from there]*, or maybe it is just because it is easier for the child here, and that is why I tell him…]*
(Lucrecia, Interview 1, Feb. 2005)

As we have argued elsewhere (Civil, Quintos, & Bratton, 2004), this issue of comparison of levels is complex. In fact, one of the mothers in our current study has a different perception, as she sees that what her child is currently learning in kindergarten is more advanced than what she remembers learning at that age (but of course, we need to be cautious as to how much one remembers from being in kindergarten):

1. **Javier (interviewer)**: Esta otra pregunta dice que si las matemáticas que tú aprendiste en la escuela piensas que son diferentes a las que tu hijo está aprendiendo ahora.
[This other question asks did you think that the mathematics that you learned in your school is different from the math that your son is learning now?]
2. **Dorotea (mother):** . . . hasta ahorita como él está en kínder pues no he visto mucho. Pero lo que me he dado cuenta que, que más rápido le empezaron a poner matemáticas que a mí. Él ahorita esta en kínder y ya está sumando, y a mí en kínder, pues yo no me acuerdo que me hayan puesto sumas.
[… until now, since he is in Kindergarten, well, I have not seen much. But what I have noticed is that they started to teach him mathematics earlier than to me. He is in Kindergarten right now and he does additions already, and in Kinder, I don't remember that I had learned addition.]
3. **Javier**.- Y tú de qué, de kínder ¿qué?, ¿guardas algún trabajo, alguna?
[And you, from Kinder, what? Do you keep any work from kinder? Anything?
4. **Dorotea:** No.
[No.]
5. **Javier**.- ¿Y qué recuerdos te vienen de kínder?
And, what memories do you have from Kinder?
6. **Dorotea** : De kínder yo recuerdo nomás que era pintar, dibujar, recortar, pegar; más ese tipo de cosas manuales que letras y números no me acuerdo yo. Y ahora pues el está en kínder y ya se sabe las letras y ya tiene que aprender los números, y pues sumar.
[From Kinder I only remember painting, drawing, cutting and pasting, more of that kind of manual skill than of writing or number, I don't remember about that. And now, well he is in Kinder and he knows the alphabet already, and he already has to learn the numbers and to learn how to add.]

(Dorotea, Interview 1, Jan. 2006)

About the teaching practices. A recurrent theme among the parents in our study has to do with the approaches to teaching and learning that they recall being used when they went to school in Mexico. Learning (memorizing) the multiplication facts is one of the topics that comes up the most often, as the parents feel that this is not emphasised at their children's school. In the quotes below Victoria comments on how in Mexico she had to learn the multiplication tables by heart and know them in any order while her children in their current schools do not know them and refer to the back of their notebook to look at the multiplication tables. She then comments on how she is trying to teach them the multiplication facts but she implies that it is a struggle. Similarly, Mónica also talks about her having to memorize the multiplication facts when she went to school in Mexico. It is interesting to note that she wonders if things are still like that in Mexican schools. Mónica does not comment on her trying to teach how she learned to her child.

She was actually attending the "Mathematics for Parents" workshops to learn how they were teaching multiplication now to be able to help her child.

Victoria:.- . . . y hago mi lucha para.. y ocupo las matemáticas, y luego pues a veces los niños por ejemplo los que están en cuarto, es muy difícil para ellos porque aquí no les exigen que se aprendan las tablas y en México sí. A nosotros, cuando estábamos en la escuela, nos exigían que nos aprendiéramos la tabla del 1 al 10 y todas y revueltas y como sea, porque nos las preguntaban. Y aquí no, aquí los niños batallan mucho para poder hacer una cuenta sacan el cuadernito; aquí les dan un cuaderno y atrás traen las tablas y ahí están. Y les digo: no, tienen que aprendérselas. Y a veces que los agarro y sí, en ese ratito se aprenden las tablas pero ya otro día ya no se acuerdan. *[and I fight for… and I use mathematics, and then, well, sometimes the children for example, the ones in fourth grade, it is very difficult for them because here they are not asked to learn the multiplication facts, but in Mexico, yes. We, when we went to school, were asked to learn the multiplication facts from 1 to 10, and all of them, and mixed and in any way, because they quizzed us on them. But not here, here the kids struggle too much to make a calculation; they take out a little notebook. Here they get a notebook and on the back cover they have a chart with the multiplications facts, and there they are. And I tell them: No, you have to memorize them. And sometimes when I get a hold on them, then they learn the multiplication facts by heart but by the next day they don't remember any longer.]*

(Victoria, Interview 1, Nov. 2005)

Mónica- Pero no es como aquí, no es. Y creo que allá también está cambiando, no sé, no he ido a la escuela ahora en México, pero era demasiado estricto, you know, vamos a la escuela a aprender. Y es lo que yo te digo, yo me acuerdo que por ejemplo en matemáticas, especialmente en la multiplicación. Aprendíamos porque nos las, las hacíamos en una tonada . . . yeah...
[But it is not like here, it's not. And I think that it is changing over there as well, I don't know. I have not gone to school in Mexico now, but it used to be too strict, you know, we go to school to learn. And that's what I tell you, I remember that, for example in mathematics, especially in the multiplications. We learned because we turned them into a song . . . yeah…]

(Mónica, Interview 1, Nov. 2006)

The quote below captures this same idea of memorizing the multiplication tables, but comes from a classroom observation that we conducted with Margarita. Hence, she is drawing not only on what she may have perceived from what her daughter brings home, but she is also building on what she sees happening in the classroom (in bold in the quote):

Margarita: En tercero nos daban, unos palitos como paletas, como paletas y canicas. Canicas y no había material de escuela, y aprendimos en el pizarrón. Teníamos que ver al maestro, que hacía 1+1 , en vertical. Vertical, ¿se llama así? [ella hace un gesto con la mano haciendo una línea vertical] y las restas también. Y en segundo llegamos a hacer, hasta, así también hasta el 100, y hacíamos las sumas y las restas, y en tercero, nosotros sí estudiamos las tablas, desde el, hasta el 3. Hasta el 3 nos las sabíamos, y nos las teníamos que aprender de memoria, **y aquí no: va a ver en el tercero de la niña, no va a ver nada de memoria. Y sí hay materiales. Y voy a seguir yendo a la clase de matemáticas de la niña**.
[In third grade they gave us little sticks like lollipops, like lollipops and marbles. Marbles, and there was no school material, and we learned on the blackboard. We had to watch the teacher do 1+1, vertically. Vertically, is that what it's called? (she gestures pointing with her finger a vertical line) and also the subtractions. And in second grade we got to do up to, up to, that way up to 100, and we added and subtracted, and in third grade, we did study the multiplication facts, from the, to the 3. We learned them up to 3, and we had to learn them by heart. ***Not here: you would see in third grade, se is not learning anything by heart. And yes, there are materials. And I will continue going to girl's mathematics class***.]

(Margarita, Classroom Observation, Oct. 2005)

Most parents comment that learned their multiplication facts by heart and they try to use this approach with their children, as Norberto tells us here about how he works with his nephew:

1. **Norberto (uncle)**: Y eso es una, unas, es una...lo más básico de las matemáticas. "Y es cuestión de práctica" le digo, ¿no? "Es cuestión de práctica. Te las vas a aprender, pero tienes que seguirlas usando." Y sí, ya le preguntaba. Sí, sí se las sabía. Y ya se las preguntaba al revés, y ya se, ya se confundía. Pero ya hasta que si se...
   [And this is one, one , it's one… the most basic thing in mathematics. "And it is a matter of practice" I tell him, isn't it? "It is a matter of practice. You are going to learn them [by heart], *but you must continue using them." And yes, I quizzed him on them. Yes, he did know them. And I asked them to him backwards, and got confused. But you already did…]*
2. **José María (interviewer)**: ¿Cómo que se las preguntabas al revés?
   [What do you mean by asking them backwards?]
3. **Norberto**: Pues, en lugar de empezar del cuatro por uno, empezaba cuatro por nueve, o le decía nueve por cuatro.
   [Well, instead of beginning four times one, I started by four times nine, or I would say nine times four.]

(Norberto, Interview 1, Nov. 2007)

In this section we have illustrated how parents' own learning experiences colour their perceptions of how their children are currently learning mathematics. Their perceived differences in content (e.g., learning arithmetic) and in approaches (e.g., learning through memorization) between their experiences as school children and what they see their children doing may lead to conflict between children and parents (Civil & Quintos, 2006). Some parents in our study said that they reinforce their children's mathematical skills at home (e.g., practicing their multiplication facts). Some of these home practices could place children in an uncomfortable position because they are in the middle of two different teaching "cultures" (the one from the school and the one from the home). One example of this is in the interaction between Berta and her father, in Vignette 3, when Berta is concerned that the approach her father is showing her is not the way she is expected to use. This "gap" between parents' and children's mathematical practices appears as an element that explains conflictive situations between them in terms of interactions, when parents try to help their children in mathematics.

Analysis of cultural elements that mediate parents' perceptions

In this section we look at cultural elements (in particular as the result of parents' prior schooling) that seem to mediate parents' perceptions of and actions in their children's mathematics education. A clear influence of prior schooling is at the level of practices and approaches to doing mathematics. When Latino children ask their parents to help them with their mathematics homework, parents apply methods that they learned at school in Mexico. This is particularly the case in arithmetic, where parents often feel more comfortable helping their children. In Civil (2006), we present a case in which the author is trying to explain to one mother one of the subtraction methods in the reform-based curriculum in place at that school. But the mother thinks that her approach (the "standard" algorithm of regrouping) is easier and that is the one she proceeds to teach her son. This is similar to what another mother, Tamara, describes in the next excerpt:

Tamara: Pues yo pienso que es diferente porque cuando yo le enseñé la, la... resta, ella no sabía. "¿Qué hago?" me decía, "¿qué hago?" pues tuve que enseñarle: pídele prestado uno al que está enseguida y luego ya le pones, si es un nueve al que le pediste uno, pon un ocho arriba, le digo. Va a cambiar le... tu, tu... ecuación allí, tu cuenta pero así te va salir. Y para que estés segura si está bien, nomás súmale, le digo, el resultado y lo que le restaste. Y así lo empezó a hacer ella.
[Well, I think that it is different because when I taught her subtraction, she did not know. "What do I do?" she told me, "what do I do?" so I had to teach her: Borrow one from the one next to it, and then you write, if it is nine the one you borrow from, put down eight on top, I tell her. You are going to change the.. your, your… equation there, your calculation but if you are going to solve it that way. And so that you are sure about your answer is good, just add it up, I tell her, your result and the amount you subtracted. And she started doing it that way.]

(Tamara, Interview 1, Nov. 2005)

Tamara and the mother in Civil (2006) try to use their prior knowledge (which is grounded in their cultural experience with their own schooling) to help their children at home. They try to make meaningful connections between the children's current homework and their knowledge grounded in practices coming from another cultural system of reference. But these methods are often different from the ones used by the teacher in their children's school, and this may lead to conflict, as we have noted above; sometimes, however, this exchange may actually be an opportunity for children and parents to learn from each other. For example, in the excerpt below, we see how one of the mothers, Selena, is telling the interviewer about an issue that she has encountered when trying to teach multiplication to her son. She teaches him as she was taught and the child points out that her method is different from how they are teaching nowadays. Here we cannot infer that the child or the mother necessarily see it as a conflict:

1. **Selena**: No, él me pregunta, él me pregunta, yo le digo nada más la, que la diferencia, que quince... que igual que en la multiplicación se las enseñan de una manera y luego le dicen de otra.
   [No, he asks me, I only tell him that the difference is fifteen… that it is the same as with multiplication, that they teach him one way and later they tell him in a different way.]
2. **Javier**: ¿y tú se las enseñas igual que aquí?
   [And you, what do you teach him here?]
3. **Selena**: Yo se las enseño como a mí me la enseñaron, pero hay veces que él me dice, me entendió como se las dije, pero me dice: mira mami, está bien, pero a mí me la enseñan así.
   [I teach him they way I was taught, but sometimes he tells me, he understood the way I told him, but he tells me: look mommy, this is fine, but this is how they teach me.]

(Selena, Interview 2, Feb. 2006)

But it is the case that most often this difference in approaches does lead to confusion or conflict. This confusion is not only with the parents or with the children, as we have had examples of teachers being confused by the methods the parents use –directly or through their children. Sometimes these situations could interfere in children's learning, because the argument "My teacher taught me in a different way, so yours is wrong" could be used by individuals to justify an answer that is wrong, independently from the way used to get the result, as the following quote points to:

> **Victoria**: El niño lo está haciendo mal. El niño dice que la maestra empieza al revés, y le digo "no, las cuentas se empiezan de aquí para allá", "no" dice, "la maestra empieza de allá para acá". "No", le digo. Entonces hace él, hace la resta y la hace mi esposo, y le dice mi esposo: "Mira, ésta está bien y la tuya está mal". La saca en la calculadora y le sale como la hace mi esposo pero a él no le salen los números, entonces ya le hace la prueba, no, la prueba que hacemos nosotros. Le dice: "mira haz la prueba". Y le enseña mi esposo a hacer la prueba y todo y le dice: "¿Ves? la tuya está mal y la mía está bien". "Pero no, es que la maestra me enseña así". "No", le digo; "es que no te enseña así la maestra; es que tú estás mal".
> *[The boy is doing it wrong. The boy says that the teacher begins at the other end and I tell him, "no, you begin the calculations from here to there." "No," he says, "my teacher begins from there to here." "No," I tell him. So he does, he does the subtraction and my husband does it as well, and my husband tells him, "look, this is right and yours is wrong." He takes out the calculator and he gets what my husband tells him but he does not get those numbers, and then he performs the test, right? The test we do. He says, "look, do the test." And my husband teaches him to do the test and all and he tells him, "do you see? Yours is wrong and mine is right." "But no, this is the way my teacher teaches me." "No," I tell him, "your teacher doesn't teach you that way. You are wrong."]*
>
> (Victoria, Interview 1, Nov. 2005)

Another effect of parents' prior schooling experiences is at the affective and confidence level. Several parents (mostly mothers) have shared with us that they did not have very good experiences with mathematics when they went to school. This may affect their interaction with their children, as in the case of Lucrecia who told us that she stays away from mathematics and that she lets her husband help their child with mathematics homework because, "he knows a lot of mathematics, since he uses it everyday at work."

> **Lucrecia**: Pues yo no me meto con las matemáticas con mis hijos. A mi esposo sí le gustan las matemáticas y se los dejo a él (risas) yo no! En México teníamos exámenes con varias materias pues en la escuela, yo me encargaba de todas menos matemáticas, porque yo no tengo paciencia es lo que te digo, como no me gustan soy muy desesperada y yo no sé qué me pasa es cuando yo… (risas) siento que me enojo con mucha facilidad y por eso prefiero no encargarme yo de las matemáticas para que los niños no perturbarles eso por eso prefiero yo dejárselo a mi esposo, y como mi esposo pues le gustan, él tiene facilidad para explicar y todo..., él, pues el se encarga, el se encarga.
> *[Well, I don't deal with mathematics with my children. My husband likes math, and I leave it to him (laughter) not me! In Mexico we used to take exam in several subjects at school, and I was in charge of all of them but mathematics, because I have no patience, I'm telling you, since I don't like it, I get desperate and I don't know what happens to me when I… (laughter). I think I get upset too easily and that is why I rather not take responsibility of math so that I don't make the kids upset, that is why I prefer to leave it to my husband, and since my husband likes it, he has an ability to explain it and all… he, well, he takes care of it. He takes care of it.]*
> (Lucrecia, Interview 1, Feb. 2005)

How parents see themselves as doers of mathematics has direct implications in terms of helping their children with their homework in mathematics, as we can see in the previous quote. This identity is defined by prior experiences. Lucrecia reports in the interview how she disliked mathematics when she was a child:

1. **Javier**: Mira, si quieres empezamos ya. La primera pregunta es: ¿Cómo describirías tu experiencia como estudiante de matemáticas, cuando ibas a la escuela, cómo la describes?
   [Look, if you want we can start now. The first question is, How would you describe your experience as a mathematics learner when you used to go to school? How do you describe it?]
2. **Lucrecia:** Pues regular.
   [Well, so, so.]
3. **Javier:** ¿Regular?
   [So, so?]
4. **Lucrecia:** Sí. No creas que era muy buena ni me gustaba mucho.
   [Yes, don't you think I was good or that I liked it a lot.]
5. **Javier:** ¿No, por qué?
   [No? Why not?]
6. **Lucrecia:** Como que, no sé si yo era cerrada de cabeza o los maestros no se daban para entender.
   [It's like, I don't know whether my brain was closed or my teachers didn't explain themselves.]

(Lucrecia, Interview 1, Feb. 2005)

Not only do we see how prior school experiences in mathematics affect parents' perceptions of their children's mathematics education and the way parents interact with children doing mathematics, but also how parents' identities (in this case Lucrecia identifies herself as not being good at mathematics), get molded and this in turn shapes their interactions with their children.

A final cultural element that we look into, and that is of key importance in our context with immigrant parents, is language. Vygotsky (1978) reported the importance of language as a cultural tool. He found that human beings use language to solve everyday problems and suggested that there is a connection between cognitive development and language as a cultural artifact. In our context, English is the language of instruction in the schools (for a more detailed discussion of the impact of different language policies on parental engagement in mathematics education see Acosta-Iriqui, Civil, Díez-Palomar, Marshall, & Quintos-Alonso, 2008). We agree with Moschkovich (2002) in that a view that focuses on English Language Learners (ELLs) learning mathematics as essentially learning vocabulary is a limiting view of what learning mathematics entails. When learning practices are bilingual and students are allowed to use their first language as well as the main language in the school, then their results improve significantly (Cummins, 2000). Teaching and learning mathematics in a context in which the home language and the language of instruction are different is certainly complex and it is not "just" about translating terms. The parents in our study are certainly aware of some of this complexity and often refer to their limited knowledge of English as a barrier. We want to point out, however, that some parents do act on this concept of barrier by, for example, as in the case of Selena and Jacinta (below) or Margarita (mentioned earlier) who say that they are taking English classes.

> **Selena**: porque hay cosas que verdaderamente no le puedo ayudar… hay cosas que me las, me dice "mami yo te voy a leer aquí a ver dime tú," y él me lo traduce en español; hay veces que le entiendo lo que me está diciendo en inglés, hay veces que definitivamente no le entiendo nada, por eso yo estoy yendo a clases de inglés, entonces, este, hay cosas, que le digo yo, "m'hijo, no pues no le entiendo aquí."
> *[Because there are things that I really cannot help him with… there are things that, he tells me, "mom, I'm going to read it here, let's see, you tell me" and he translates it into Spanish; sometimes I understand what he's telling me in English, but others, definitely I don't understand anything, that's why I'm going to English classes, so, hmm, there are things that I tell him, "m'hijo, no I don't understand it here."]*
>
> (Selena, Interview 1, Nov. 2005)

Sometimes parents comment on the effect of their children's limited knowledge of English on the children themselves. In the exchange below it is interesting to note that while the researcher / interviewer suggests possible alternative explanations for the situation ("he does not pay attention"; "he reads too fast"), the mother focuses on the child's limited command of English as the main reason:

1. **Jacinta**: ... y dice, que... él batalla en lo de inglés, porque los problemas se los saca mal, ¿por qué? Porque no lee bien.
   […and he says that… he struggles with English, because he gets the problems wrong, Why? Because he does not read well.]
2. **Javier:** Ya, no pone atención...
   [Right, he does not pay attention…]
3. **Jacinta:** No pone atención...
   [He does not pay attention…]
4. **Javier**: ¿O lee muy rápido?
   [Or, does he read too fast?]
5. **Jacinta:** No... o como no sabe mucho inglés todavía...
   [No,… or since he does not know much English yet…]
6. **Javier**: Ah... es más problema del [idioma]
   [Oh… it's more a [language] *problem]*
7. **Jacinta:** Sí, ése es el problema de él.
   [Yes, that is his problem.]

(Jacinta, Interview 1, Nov. 2005)

Jacinta goes on to elaborate on the affective impact that their limited knowledge of English had on her children:

> **Jacinta:** Les está costando mucho por la [lengua]. Primero se deprimieron bien feo: se querían ir. "No, yo no quiero estar aquí... es más, no voy a estudiar, no quiero hacer nada." Por él... sí yo, que voy a una escuela de inglés...
> *[It's being difficult for them because of the* [language]. *First they got depressed very bad: They wanted to leave." No, I don't want to be here"… even worse, I won't study, I don't want to do anything." It's because of him … yes, that I go to English classes…]*
>
> (Jacinta, Interview 1, Nov. 2005)

Emilia, a mother who already knew some English when they arrived to the United States, comments on how in her case she is the one who translates for her sons who did not know any English when they first arrived. In the excerpt below she is talking about her oldest son who was in 6th grade when they first arrived.

> **Emilia:** yo lo que les digo a ellos para que no se me desesperen, porque estamos hablando ahorita que el niño entró en enero, ¿eh? Estamos hablando de tres meses, a lo sumo, el niño, el niño va a tener. Y es un cambio, son 11 años de su vida, bueno, más bien vamos a decir, 5 años casi de su vida, que ha estado escuchando español, aprendiendo en español, de repente tú le dices que lo mismo lo va a hacer ahora en otro idioma, sí se frustra un poquito, ... porque dice: "ay! Mami", dice, "y cosas que preguntan que son bien fáciles, y yo desesperado porque yo quiero contestar porque yo sí entendí. Y hay otras cosas que no entiendo, y después puso la respuesta, y yo ya me lo sabía pero no entendí la pregunta. Si no entendí la pregunta, no puedo dar la respuesta, porque no les entendí." Eso es lo que lo desespera.
> *[what I tell them so they don't get anxious, because we are saying that the child came in January, huh? We are talking about three months, at most, the child, that's how long the child has been here. And it is a change, it is 11 years of his life, well, let's say, almost 5 years of his life, that he has been listening to Spanish, learning in Spanish, all of a sudden you tell him he will be doing the same but now in another language, yes, he does get a little frustrated…because he says, "Oh! Mommy," he says, "and things they ask that are so easy, and I get anxious because I want to answer since I did understand. And there are other things that I do not understand, and then he/she gave the answer, and I already knew it, but I did not understand the question. If I did not understand the question, I cannot give the answer, because I did not understand them." That is what gets to him.]*
>
> (Emilia, Interview 1, March 2006)

As the comments from these mothers show, language plays a role both at the cognitive and affective levels. Language is a key component of an individual's cultural identity. In the case of the families with whom we work, Spanish is their shared language and hence an important part of their identity. Yet at school, children are using primarily English. Besides the to-be-expected challenges when parents and children interact around homework with the two languages at play, there is a potential source of conflict as children and parents have to navigate two different languages with different valorizations.

Conclusion

In this article we have addressed these three questions:

1. What kind of practices do parents use to help their children with their mathematics?
2. How do parents' different learning experiences affect their perceptions of their children's mathematics teaching and learning?
3. What are the most prominent cultural elements that mediate parents' perceptions of and reaction to practices related to their children's mathematics education?

In terms of our first question, we have found that parents interact with their children in different ways and intensity, as the case of Margarita and Sergio with their daughter Berta illustrates. They are both interested in their daughter's learning of mathematics. They come to the workshops to support her in her learning, and in the case of Margarita to be a learner herself. We see that Sergio's style of interaction is very direct; he wants to show her how to do it but also respects his daughter's request to have to / want to do it by herself (Vignette 3, lines 3 & 4). When parents do not know or do not understand the school methods, they rely on their own knowledge from school to promote mathematics learning, as Norberto's quizzing his nephew shows. Using a different interaction style, Margarita acts as a model of using mathematics by doing mental computations and using logic (Vignette 1, line 10). Although we have no evidence to say that Berta recognizes her mother's capability and knowledge as yet, we believe that

Margarita's role as a model of a mathematics doer may help Berta's growth in mathematics. All this leads us to state that

- Parents engage in a dialogue with their children as they do mathematics, varying in the degree of how directive the parents are.
- Parents are intellectual resources as brokers of the knowledge coming home from school and as models of mathematics users/doers.
- Parents use everyday situations to help their children learn mathematics.

Regarding our second question, data discussed show that parents' perceptions of their children's mathematics teaching and learning are mediated by their previous experiences with school mathematics. A common perception is that the level of mathematics education in Mexico is higher than the one in the United States. This perception is based on concrete school practices such as the role of memorization and choice of algorithms. We have shown a small sample from a larger set of instances that illustrates this assertion. We brought in the voices of Victoria, Mónica, Margarita, and Norberto to share their memories of how they were taught in school in Mexico: largely based on memorization techniques. They value this skill, in particular knowing the multiplication facts, as fundamental for learning mathematics, as Norberto says explicitly. Parents also bring in algorithms that differ from the ones being taught to their children, as Selena, Tamara, and Victoria report. The difference in valorization of or emphasis on memorization over other skills and the discrepancies in the algorithms are at the same time a potential source of conflict, confusion, and frustration, and an opportunity to complement the mathematics education that children are receiving in the schools. We summarize these findings as

- Parents' school experiences in mathematics, different from those of their children's, can be used to enhance the children's education.

With respect to our third question, data discussed in this article suggest two relevant cultural elements that mediate parents' perceptions of their children's education and the corresponding reactions. The first is schooling in general, as parents identify with the more tangible expression of different content and approaches used for teaching and learning mathematics. These differences tend to become a source of confusion for everyone: children, parents, and teachers. In some cases these differences turn into a learning opportunity, as may be the case with Selena when she implies an exchange of information with her son but, more often than not, they have a negative impact at a cognitive level, creating confusion with different algorithms, as a well as at an affective level, diminishing the confidence of the participants as mathematics doers, and therefore their identities and potentially their self-esteem. The second element is language. We observe that the implications of English being the only language of instruction are also at both levels: cognitive and affective. At the cognitive level we find the following ways in which instruction in English only becomes an obstacle for learning: Children, especially recent immigrants, have more difficulties learning the concepts in a language over which they have no command. They may also feel (or be) excluded from participating in classroom practices and are therefore deprived of learning opportunities. In the case of parents, the children's natural source of help outside school, they are limited in their ability to help because it is much harder for them to understand what is being taught in school in a language other than their own. At the affective level, the most evident frustration comes from not understanding the concepts, but underlying this discomfort is also the sense of not belonging and a reduced confidence in the children's ability to participate and thus show their mathematical competence. Recapitulating, we assert that:

- Differences in schooling (different approaches to doing mathematics) and in language may lead to challenges as parents try to help their children with school mathematics.

- The impact of the aforementioned elements is at both cognitive and affective levels.

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References

Abreu, G. de (2005). Cultural identities in the multiethnic mathematical classroom. In M. Bosch (Ed.), *Proceedings of the fourth Congress of the European Society for Research in Mathematics Education* (pp. 1131- 1140). Sant Feliu de Guíxols, Spain: FUNDEMI IQS, Universitat Ramon Llull.

Abreu, G. de, Cline, T. and Shamsi, T. (2002). Exploring ways parents participate in their children's school mathematical learning: Case studies in multiethnic primary schools. In G. de Abreu, A. J. Bishop and N. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 123-147). Boston, MA: Kluwer.

Acosta-Iriqui, J., Civil, M., Díez-Palomar, J., Marsahll, M., & Quintos-Alonso, B. (2008). *Conversations around mathematics education with Latino parents in two borderland communities: The influence of two contrasting language policies*. Manuscript in preparation.

Berger, P. & Luckmann, T. (1988). *La construcción social de la realidad* [The social construction of reality]. Barcelona: Herder.

Bratton, J., Quintos, B., & Civil, M. (2004, March). Collaboration between researchers and parents for the improvement of mathematics education. *Paper presented at the 1st Annual Binational Symposium of Education Researchers,* Mexico City, Mexico.

Calabrese Barton, A., Drake, C., Perez, J. G., St.Louis, K., & George, M. (2004). Ecologies of parental engagement in urban education. *Educational Researcher, 33*(4), 3-12.

Civil, M. & Andrade, R. (2002). Transitions between home and school mathematics: Rays of hope amidst the passing clouds. In G. de Abreu, A.J. Bishop, & N.C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 149-169). Boston: Kluwer.

Civil, M., Bratton, J., & Quintos, B. (2004). *Latino parents' views on mathematics education: Are we listening?* Paper presented at the National Council of Teachers of Mathematics (NCTM) Research Presession, Philadelphia, PA.

Civil, M. & Quintos, B. (2006). Engaging families in children's mathematical learning: Classroom visits with Latina mothers. *New Horizons for Learning Online Journal*, *XII* (1).

Civil, M. (2006). Working towards equity in mathematics education: A focus on learners, teachers, and parents. In S. Alatorre, J. L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the Twenty Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, 30-50). Mérida, Mexico: Universidad Pedagógica Nacional.

Civil, M., & Bernier, E. (2006). Exploring images of parental participation in mathematics education: Challenges and possibilities. *Mathematical Thinking and Learning, 8*(3), 309-330.

Civil, M., Bernier, E., & Quintos, B. (2003). Parental involvement in mathematics: A focus on parents' voices. Paper presented at the annual meeting of AERA, Chicago, IL.

Civil, M., Planas, N., & Quintos, B. (2005). Immigrant parents' perspectives on their children's mathematics education. *Zentralblatt für Didaktik der Mathematik, 37*(2), 81-89.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Tonawanda, NY: Multilingual Matters.

Epstein, J. L. & Sheldon, S. B. (2005). Involvement counts: Family and community partnerships and mathematics achievement. *The Journal of Educational Research, 98*(4), 196-206.

Flecha, R. (2000). *Sharing words.* Maryland: Rowman & Littlefield.

Flecha, R., & Gómez, J. (2004). Participatory Paradigms: Researching 'with' rather than 'on'. In B. Crossan, J. Gallacher & M. Osborne (eds.) *Researching widening access: Issues and approaches in an international context.* pp. 129-140. London: Routledge.

Freire, P. (1970). *Pedagogy of the oppressed* (M. B. Ramos, Trans.). New York: Herder and Herder.

Freire, P. (1998). *Politics and education* (P. L. Wong, Trans.). Los Angeles: UCLA Latin American Centre Publications.

Ginsburg, L. (2006). *Adult learners go home to their children's math homework: What happens when the parent is unsure of the content?* Paper presented at the 13th Annual Conference of Adults Learning Mathematics, Belfast, UK.

Glaser, B. G. & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* New York: Aldine de Gruyter.

Gómez, J., Latorre, A., Sánchez, M., & Flecha, R. (2006). *Metodología comunicativa crítica* [Critical communicative methodology]. Barcelona: El Roure.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Students Placed at Risk, 6*(1-2), 115-132.

González, N., Moll, L., & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households, communities and classrooms.* Mahwah, NJ: Lawrence Erlbaum.

Habermas, J. (1987). *The theory of communicative action* (Vol. 2). Boston: Beacon Press.

Horvat, E. M., Weininger, E. B., & Lareau, A. (2003). From social ties to social capital: Class differences in the relations between schools and parent networks. *American Educational Research Journal, 40*(2), 319–351.

Lehrer, R., & Shumow, L. (1997). Aligning the construction zones of parents and teachers for mathematics reform. *Cognition & Instruction, 15*(1), 41-83.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*, 189-212.

Nasir, N.S., & Hand, V.M. (2006). Exploring sociocultural perspectives on race, culture, and learning. *Review of Educational Research, 76*(4), 449-475.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: The Council.

O'Toole, S. & Abreu, G. de (2005). Parents' past experiences as a mediational tool for understanding their child's current mathematical learning. *European Journal of Psychology of Education*, XX, 75-89.

Peressini, D. (1997). Parental involvement in the reform of mathematics education. *The Mathematics Teacher*, *90*, 421-427.

Peressini, D. (1998). The portrayal of parents in the school mathematics reform literature: Locating the context for parental involvement. *Journal for Research in Mathematics Education*, *29*, 555-582.

Remillard, J. T., & Jackson, K. (2006). Old math, new math: Parents' experiences with *Standards*-based reform. *Mathematical Thinking and Learning, 8*(3), 231-259.

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. In J.V. Wertsch, P. del Rio, & A. Álvarez (Eds.), *Sociocultural studies of the mind* (pp. 139-164). Cambridge, UK: Cambridge University Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Parental engagement in a classroom community of practice: boundary practices as part of a culturally responsive pedagogy

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Abstract

This study addresses the pressing need to recognize and include disenfranchised students within mathematics education in a way that incorporates the voices of their communities. We use the concept of boundary practices to point to the multiple ways in which the mathematics learning practices in a classroom connect or disconnect, include or exclude, adults as a resource for a culturally responsive pedagogy. Case studies developed through ethnographic methods of data collection allow an in-depth analysis and exploration of emergent themes. Our data suggest that the nature of the community of practice plays a critical role in establishing relationships with parents. Parental involvement is distributed and influenced by the nature and history of the community, as well as by the identities of the participants. Finally, our data indicate that a culturally relevant pedagogy facilitates an egalitarian dialogue with parents and between parents and children.

Key words: parental involvement, culturally responsive education, communities of practice.

Introduction

Many minoritized students in the United States of America experience mathematics as a curricular gatekeeper and an irrelevant subject (Apple, 1992; Gutstein, 2003; Stinson, 2004). Furthermore, mathematics education is portrayed only as a product to address unemployment and to serve economic goals which support a market-driven economy (Moses & Cobb, 2001). This perspective ignores the possibility of transforming current unequal power structures as well as the goal of education to endorse human dignity (Olivares Alonso, 2006). These contradictions have critical implications that need to be addressed in order to improve the educational opportunities for minoritized communities, in particular, in our context of a predominantly low income Mexican American community in the Southwest United States of America.

An inclusive model of parental engagement is central to a culturally responsive mathematics education (Gay, 2000; Ladson-Billings, 1995). In the last decades, research, policies, and pedagogical models have announced the benefits of parental participation. Dominant views, however, often leave involvement or empowerment undefined (Lankshear, Gee, Knobel, & Searle, 1997; Vincent, 1996). This vagueness in the rhetoric of parental involvement tunes into a deficit view of low-income and ethnic- and language-minoritized communities. A deficit-driven model claims educational problems lie in inadequate socialization within families (Taylor, 1997). In addition, this perspective often narrows the definition of parental involvement. It fails to consider the diversity of the funds of knowledge of students and their communities as well as the institutional role in perpetuating unequal opportunities.

A socio-historical view considers parental engagement to be a situated and dynamic activity system (Calabrese Barton, Drake, Perez, St. Louis, & George, 2004; Civil, 2007; Civil, Bratton, & Quintos, 2005). One of its main tenets is that the model of participation needs to be culturally responsive and consider the political and historical contexts that mediate power relations. This research study forwards this perspective through the analysis of the interaction of parents in their children's mathematics education and the school system using the framework of communities of practice (Wenger, 1998). In particular, we use the concept of boundary practices to address the particular moments of inclusion or exclusion of parents. This discussion describes multiple ways in which the mathematics learning practices in a classroom connect or disconnect, include or exclude, parents as a resource for a culturally responsive pedagogy.

Theoretical framework

Cultural-historical frameworks underscore that parental involvement is a process and a product of the historical and socio-political context of schools including its geographical location, culture, race, and income level of students' families; and it is influenced by educational policies, research, and teaching practices (Calabrese Barton, et al., 2004; Lareau & Horvat, 1999; Lareau & Shumar, 1996; Mapp, 2003; Taylor & Dorsey-Gaines, 1988; Valdés, 1996). Two concrete efforts grounded in this perspective are the projects of funds of knowledge and of learning communities.

There are multiple projects that illustrate the first – that is, how schools, teachers, and researchers may tap into households' funds of knowledge (Barton, 1996; González, Andrade, Civil, & Moll, 2001; González, Moll, & Amanti, 2005; Hammond, 2001; Moll & González, 2004; Patterson & Baldwin, 2001). In these projects, the teachers become learners and therefore encounter opportunities to reconceptualize students, their families and communities, as well as the curriculum. The specific resources in the community become evident as the teachers move away from the enclosed area of the classroom. A focus on the strengths and assets of the families and communities implies changing the focus from needs of the communities to the possibilities present within them (Delgado-Gaitan, 2001; Guajardo & Guajardo, 2002; Kincheloe & McLaren, 2002). Thus, the knowledge gained from a community can be used as a tool that further benefits members of that community.

In relation to the second, an effort that includes the transformation of power relations beyond the classroom is the learning communities project led by the research group of Ramón Flecha in Spain and other parts of the world (Elboj, Puigdellívol, Soler, & Valls, 2002). These learning communities are projects of social and cultural transformation of an educational setting and its context. These communal projects' principle is that social change cannot be constrained within the classroom, but must include all the spaces in the community and its diverse participants. The learning communities use dialogic learning, through the active participation of the community, materialized in its different spaces including the classroom. Although parents enter the classroom to participate in school-like activities, their role is of collaboration with the teaching staff. There is a continuous praxis that counters dominant power structures. It is significant to mention that there is a dialectic relation of change between the educational practice and the relation with the community the school serves.

The strength of the two projects described above has been well documented (Elboj, et.al., 2002; González, et.al., 2005). However, there is a need to further understand the connections or disconnections between students' communities and their mathematics learning experiences. Many teachers are not working within schools that support a radical change in their relation to the community, as the project of learning communities requires. The concept of funds of knowledge, on the other hand, is an effort individual teachers can implement. However, establishing the connections between the cultural knowledge of the communities and the mathematics curriculum can be challenging (Civil, 2007). For over ten years mathematics educator Marta Civil has led research efforts on parental engagement and mathematics education in working class, Latino communities in the United States of America (Civil, 2002;

Civil, 2007; Civil & Andrade, 2003; Civil, Bratton, & Quintos, 2005; Civil, Planas, & Quintos, 2005; Civil & Quintos, 2006). This research agenda and outreach efforts are grounded on the Funds of Knowledge research projects, and on the concept of parents as intellectual resources (Civil & Andrade, 2003), as a means to emphasize parents' contributions to their children's mathematics education. In this research we use the concept of communities of practice; in particular, the concept of boundary practices, to describe the interconnections between the members of the classroom community and parents.

Boundary practices: parents' participation in mathematics education

The framework of communities of practice positions social relations at the centre of the learning process. Lave and Wenger (1991) state that communities of practice,

> ….imply participation in an activity system about which participants share understandings concerning what they are doing and what that means in their lives and for their communities…A set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. (p. 98).

These researchers describe ways in which groups of individuals co-participate in a practice, interact with each other, and as a result, learn. It is not their intention to describe necessarily collaborative or democratic settings. Moreover, they suggest that in order to understand these particular communities and the learning that occurs within them, it is necessary to consider the socio-historical, cultural, economic and political factors that shape them. Thus, in order to address the unequal opportunities for quality education of disenfranchised communities, the lenses of analysis must include a socio-historical and political perspective that addresses racism and biases in society that permeate the educational system.

Research that focuses on learning beyond official educational settings challenges a traditional understanding of learning, including the area of mathematics education (Abreu, 2002; Lave, 1988, 1996; Masingila, 1994; Masingila, Davidenko, & Prus-Wisniowska, 1996; Nunes, Schliemann, & Carraher, 1993). Lave & Wenger (1991) redefine learning as legitimate peripheral participation. The term legitimate emphasizes that the access to a practice and to the negotiation of meanings and membership to a community constitute part of the learning. The term peripheral suggests that there are transitions in learning constituted by different ways of participating in a practice, shifting forms of belonging to a community, changing ways of becoming part of a community, and developing ways of making sense of experiences (Wenger, 1998). The learning opportunities, then, are structured through social relations, artifacts and tools, roles and rules of interaction, negotiation of meanings, the activities and its goals, and the practice. In summary, learning as participation involves the individual's evolving trajectories of participation in a community of practice. These transformations necessarily include the identity and perspectives of the individual, the negotiation of meanings, forms of membership in a community, and the practice in itself (Wenger, 1998). In our case, the community of practice is the classroom participants, and mathematics learning is defined as students' trajectories of participation in the particular mathematical practices.

Although a 'community of practice' is the unit of analysis, its members are not isolated from other individuals or communities. In this study we explore the interaction between the classroom community of practice and boundary practices that occur around school mathematics learning involving family members. The term 'boundary practices' (Wenger, 1998) refers to the lines of connection and disconnection, inclusion or exclusion, between members of a community of practice and particular individuals, or between members of two or more communities of practice. Through this concept it is possible to explore the interactions that relate to the classroom community and that include members of other communities, such as the families of students. Wenger (1998) describes two ways in which boundaries are established. First, the nature of the community in itself creates boundaries between members and non-members. Second, there are boundary encounters that bridge a community to the outside world. This concept situates the connections with family members in the practices within the community as well as in those practices in which one of the main goals is to connect students' mathematics learning with family members.

Boundary practices are embedded in activity systems, which are historical and multivoiced (Cole, Engestrom, & Vazquez, 1997). The structures of the school institution as well as the individual members influence these practices through their historical trajectories, beliefs, roles and power relations, instruments, and goals. The educational institution contributes with mediating tools, such as regulations of parental participation, adoption of textbooks, and norms for teachers; means that derive from particular belief systems and interests. As part of these systems, teachers make decisions in the everyday practice. Boundary practices exemplify the dynamic, historical, and situated nature of the interactions between the school system, through a particular community of practice, and students' families.

Fundamental to establishing a culturally responsive education are boundary practices that include the funds of knowledge of diverse communities as well as practices that contest the oppressive relations with minoritized communities (Civil, Bratton, Quintos, 2005; Ladson-Billings, 1995; Martin, 2006). The goal of this article is to contribute to efforts that propose a situated notion of parental engagement using the framework of communities of practice and connect these practices to the implications for adult learners. Parental engagement in mathematics education is a practice that can contribute to transforming current power structures that disenfranchise minoritized students and their communities.

Methods

This study takes place in a fifth-grade classroom at an urban elementary school in the Southwest United States of America, in which ninety percent of the students are labeled as of Latino background[1] and almost all of the students receive free or reduced lunch[2]. All the participants have some understanding of English and Spanish, however several of them predominantly use just one of the two languages in the classroom. The parents of all the children in this classroom signed a waiver requesting bilingual education for their children as established by the state legislation, Proposition 203[3]. Many of the children were previously in English-only classrooms and some recent immigrants from Mexico, in Spanish ones. This classroom was chosen based on the researchers' personal respect for the teacher as well as the teacher's local and national recognition for her teaching practice (Recipient of the Presidential Award of Excellence in Mathematics Teaching at the Elementary level). She is a teacher-researcher and an analysis of her teaching is a prime opportunity to learn from a teacher with a profound understanding of mathematics and a pedagogical vision of transformative learning.

The participants in the study are: eighteen fifth-grade students, the parents of four of these students, and the classroom teacher, Patricia[4]. We developed in-depth case studies for four of the eighteen students. These students are Mexican immigrants or Mexican-Americans. The students were selected based on the teacher's knowledge to include diversity in gender, mathematical proficiency, and language fluency in English and in Spanish. In this paper we focus primarily on the case study of Yessenia, and her mother, Lorena. Yessenia immigrated with her mother and sister when she was an infant.

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[1] Most of the students labeled by the school as Latino are of Mexican descent, a smaller number of students is from other countries in Latin America.

[2] Free or reduced lunch is used as an indicator of poverty level.

[3] Proposition 203 is state legislation that was approved by Arizona voters in 2000 and is now part of the Arizona state statutes. It proposes to replace bilingual education with Structured English Immersion classes for a period of one academic year. It states, "Although teachers may use a minimal amount of the child's native language when necessary, no subject matter can be taught in a language other than English" (A.R.S. Section 15-751 [5]).

[4] All names are pseudonyms.

Data collection and analysis

The year-long qualitative case studies (Dyson & Genishi, 2005) explore in detail the particular engagement of families in connection to their children's mathematics learning experiences. The use of multiple case studies bridge local particulars to the abstract social phenomenon of communities of practice. We used ethnographic tools for our data collection which took place in three sites: the classroom, students' households, and two after-school programs. A detailed review focus on those activities that, from these authors' perspective, are significant boundary practices. This in-depth analysis included the transcripts from the interviews with the teacher, students, and parents; as well as video transcripts and field notes of classroom observations. Through grounded theory (Charmaz, 2001) we explore emergent themes. The different sources were used to triangulate the information and build thick descriptions. The discussion focuses on different boundary practices and the implications for the learning of mathematics, including parents as adult learners.

Boundary practices: parents as legitimate peripheral participants

In this article we discuss some boundary practices of this classroom community towards students' family members. The first practices comprise the mathematics practices in the classroom, while the second set of practices involves those encounters that have the explicit purpose of bridging students' learning and parents.

Mathematics practices in the classroom

The different components of a community of practice establish the boundaries of its membership (Wenger, 1998). In this article we focus on the boundaries established by the identities of the participants in interaction with the negotiation of meanings of a mathematics curriculum based on a culturally responsive pedagogy.

The evolving identities of the members of a community contribute to its boundaries; a guiding member of this community is the teacher. Patricia's cultural identity and belief system guides her teaching practices. She is a confident and experienced mathematics educator who has a sound understanding of the mathematics involved in teaching at the elementary level; she enjoys exploring issues about mathematics education and mathematics; and is devoted to the education of the Latino community in the United States, of which she is a member.

Patricia views mathematics as a sign system that allows her to describe, learn about, and be critical of the world. Her view of mathematics as a human activity implicitly opens the access to mathematics learning rather than assuming it as an innate skill of a few individuals. She explains her view about the importance of mathematics.

> **Beatriz** (researcher): So, in which ways do you now believe math is important?
> **Patricia**: Oh Gosh, [mathematics] is a way of life… If you start to think about it we live in a mathematical landscape and if you don't know the math, you know you can't appreciate the content, so the more we study math in real life, how is a human activity, and if it is a human activity then it has to, you have to understand it cause you are a human being and you function with it, so the more I see this world, the more I see the math in it, the more I want the kids to understand it; math is all around. (Interview, June 29, 2006)

Patricia's belief on the possibility of access by all of her students and its relevance in the everyday life is evident in her teaching practice. The mathematical practices are interconnected to the different content areas such as literacy and science as well as to students' everyday experiences. For instance, Patricia connected their science experiment to the introduction of a mathematics investigation. The students had been working on a science exploration of Owl Pellets. Students dissected this indigestible material regurgitated by an owl and searched for bones such as teeth, skulls, or claws. At this point the class discussed their conclusions based on the types and number of bones they found in each of the pellets. Patricia explained the meaning of the term conclusions, "You are trying to make sense of what you are seeing; they are based on what you know." In one of the pellets children found seven jaws, so the group of children concluded there were four rodents, since jaws come in pairs. Patricia wrote 7/2 while a student spontaneously described it as an improper fraction. Yessenia explained, "It is four, seven divided by 2 is four rodents and one jaw missing." This group's conclusion, voiced by Yessenia's explanation, highlights the marked interconnection of the numbers with their meaning in this classroom. In an isolated context four would be an incorrect answer; the answer expected would be three and one half. However, the students solve 8/2 since the seventh jaw implies the existence of another rodent. This conversation included students' sharing of personal stories that related to rodents. They also shared their theories about the gender and age of the rodents. The children deduced it was a family (a male, a female, and two babies); while Patricia theorized there were two female mice and some babies. In her experience raising mice she learned females are the ones who usually nurse the offspring. She concluded,

> Do you see how experiences help us draw conclusions? Ustedes sacaron la conclusión de que había una hembra y un macho. Yo les digo, que pensé que eran dos hembras. Mi pensamiento fue que quizá la lechuza se comió las dos hembras y los bebés que tienen los ojos cerrados…. [You drew the conclusion that there was one female and one male. I am saying, I thought there were two females. My thought was that maybe the owl ate the two females and the babies who have their eyes closed.] We both used our experiences.
>
> (field notes, January 11, 2006)

In this community children are expected to use their previous experiences. The previous discussion connects students' ideas about conclusions to the upcoming mathematics investigation. Patricia told students they were going to switch to a mathematics investigation to make interpretations, conclusions, and generalizations. This connection to science makes the frontiers of school mathematics porous and contributes to the social-emotional investment of students. The connections with students' previous experiences or others significant to them promote students' participation as subjects in their learning. The identity of the individual includes the communities to which students belong outside of school. Belonging to the classroom community means bringing in their out-of-school life.

Patricia's teaching follows the suggestions from the National Council of Teachers of Mathematics, Principles and Standards (2000) and a socio-constructivist perspective. In her teaching, she focuses on developing students' communicative competency and mathematical reasoning. She promotes connected investigations that facilitate children's active participation in mathematics discussions, visualizations of patterns, mathematization of their world, and connections of concrete and symbolic representations. The investigations frequently launch from a world context and are based on concrete experiences that promote students' participation in the negotiation of mathematical meanings. Furthermore, they connect students' communicative competency in everyday life to the academic discourses of mathematics. These inquiry projects give time for students to gain experience in the different practices. These expanded zones of development are the result of the nature of the activities, social interactions, and mediating tools.

Mathematics learning in this community, however, goes beyond standards-based mathematics (NCTM, 2000). Patricia's humanist vision of education goes beyond mathematics as a communicative competency or interconnected meanings and representations. It includes the identities of students as subjects of their learnings. A humanist perspective counters a so-called "banking education" that considers individuals as objects in the learning process and knowledge a commodity to be accumulated (Freire, 1998). This identity of members as subjects requires that individuals make sense of the mathematical tools and use them for their own purposes.

Personal stories are a habitual discourse in this community; they are both a way of participation and a means to negotiate meanings. Patricia and students share stories both in planned situations and in spontaneous spaces. The stories are tools to enrich their mathematical knowledge just as their academic investigations have the purpose of enhancing their life experiences. Not only do students learn the mathematics situated in familiar contexts, but participants learn their out of school experiences are a valuable part of their learning in school.

Throughout the year, the students established these connections between school mathematics and mathematics in everyday experiences. In the introduction of the investigation previously described, before Patricia continued her explanation of the mathematics investigation, Yessenia shared, “Miss, at soccer practice the captain made small groups, we were working with math.” Patricia smiled and replied, “We are always dealing with math.” Yessenia connects the activity of making groups from a specific number of players at her soccer practice with mathematics. She recognizes they are making groups and identifies this activity as doing mathematics. The meanings negotiated and practices in the classroom establish multiple boundaries that can be more or less inclusive or exclusionary. The use of students’ funds of knowledge is a boundary practice that opens access to students’ cultural identities.

The connection with children’s cultural experiences in this classroom involves a critical lens. Patricia not only asks students to use their previous experiences but to use a critical lens towards discriminatory and unjust practices. For instance, through the year-long topic of world-mindedness Patricia brings the world population closer to this classroom and promotes a caring and informed attitude towards others in the planet. This stance of world-mindedness means that students need to develop a critical stance to reflect on and act in their world. This attitude is not extraneous to mathematics; on the contrary, it is through mathematics, as well as the other content areas, that Patricia promotes her goal. Patricia encourages children to use the different domains of mathematics to make sense of their world. Therefore, in order to achieve this goal, children also have to make sense of mathematics. For instance, during the first week of classes Patricia read to the class the children’s book If the world were a village (Smith & Armstrong, 2002). The author invites the readers to consider the world as if it were the size of a village and includes topics such as world population, religions, nationalities, food distribution, schooling and literacy, money and possessions, electricity, among other topics. During the reading aloud Patricia highlighted some aspects for the children to consider as citizens of the world situated in their particular contexts. She read, “Sixty percent of the world population is hungry and of those, twenty six percent are severely undernourished and twenty four percent of the population always has enough food” (field notes, August 18, 2005). Then, she stopped and posed the question to the students, “Why does this happen?” Patricia was not searching for answers as much as highlighting a call for critical thinking. She continued reading about the languages most spoken in the world. Some of the languages listed were Chinese, Hindi, English, and Spanish. After Patricia read these data she said to the students, “If you speak Spanish, you are at a great advantage because it is one of the four most spoken languages in the world.” Patricia connects the meanings presented in this book to the identity of students as Latina/o and counters the demeaning view of Spanish –or any other languages different from English — in this particular geographical context which tries to eliminate its use as a linguistic resource in schools. She uses this information to highlight the value of students’ knowledge and identity. Furthermore, Patricia emphasizes the importance of them making informed decisions. She said, “Smith (the author) wants you to know you are the citizens of the future, you need to know the information to make the right decisions” (field notes, August 18, 2005). In this short event, Patricia emphasizes the way mathematics is a tool to make sense of their world and their own identities. This connection of mathematics and the world and a vision of social justice also permeate Patricia’s boundary encounters with parents. Two of them are explained in the following section.

Boundary encounters

The boundary encounters in this community exemplify the dynamic and situated nature of the interactions between the school system, through a particular community of practice, and students’ families. The educational institution contributes with tools that convey a particular belief system, such as regulations governing parental participation, adoption of textbooks, and norms for teachers. Also mediating these relations with families of minoritized communities are asymmetrical power relations. The teacher, then, makes the decisions in their everyday practice within these activity systems.

The perspective of the teacher on parental involvement is a key mediator of her interactions with parents. In the following excerpt Patricia describes her role in supporting parents instead of supplying a list of pre-established tasks that parents need to complete.

> **Beatriz**: What do you expect from parents as a teacher?
> **Patricia**: Is not what I expect from parents, as much as what I expect from myself for parents. I have learned that parents trust a teacher considerably and that's scary. But for me the parents are already doing what they are supposed to do, send their kids to school and my job is to try to teach children in such a way that they [children] talk about what they are learning when they go home, and also to try to have the evidence so parents believe in their children so they can be advocates for their children.
> (teacher interview, June 29, 2006)

Patricia dialogues with parents in ways that they can become informed advocates for their children. In her everyday practice she includes tools such as the students' agenda to inform parents about their mathematics learning. Children record personal statements, based on state standards, about their mathematics learning. She also acknowledges that often parents are not comfortable with mathematics and feel foreign to their children's experiences in school. The following is a description of two boundary encounters, one in the school and one after-school. These encounters exemplify some of the contradictions that abound in parental involvement activity systems.

Immersion: classroom observation

The family members of all the students in this fifth grade classroom were invited to observe and participate in a mathematics lesson. All these interactions were in English and Spanish. During this visit, one of the activities consisted of revisiting an exploration of the surface area of rectangular prisms. The investigation began with a discussion of the importance of the surface area of leaves, with a focus on the vegetation of the desert where they live, and the role of surface area in the packaging industry and therefore the environment. The mothers, in collaboration with their children, then drew three-dimensional prisms, traced the faces, and described their observations. When the children left for lunch, the classroom observation concluded with a discussion guided by the observations of the mothers and one grandmother about their children's mathematical learning experiences. In what follows we discuss Lorena's participation in this classroom observation.

In this context the bilingual community welcomes these adults as legitimate peripheral participants. The teacher and Yessenia use both languages allowing Lorena to participate in the conversations. Throughout the school-year children had the choice of either language, but Yessenia oftentimes chose to do the classroom tasks in both languages. Lorena values English highly and wants to preempt her daughters from the limitations she experiences as an emergent English speaker. Her desire for her daughters to know English, however, does not counter her value of Spanish. Most of their family still lives in Mexico, and even the ones in the United States mostly speak Spanish. Lorena is also proud of their Mexican identity and tells her daughters that they should also take pride in their nationality. In this way, she connects Spanish with their identities and this community of practice supports her goal.

The use of Spanish is not sufficient to include these mothers whose dominant language is Spanish; relevant mathematical concepts also need to be addressed. The negotiation of mathematical meanings departs from concrete experiences that underscore participants' previous knowledge of surface area and supports it through a collaborative community and concrete tools. Lorena and others are invited to connect the concrete prism, geometrical representations in two-dimensions, and the formula of surface area. In this way, Lorena first visualizes the surface area and then connects it to the formula. Mathematics as a human practice that supports sense-making allows her to be a legitimate peripheral participant. The connections between the different representations add to the transparency of the mathematical meanings. In an interview after this experience, Lorena redefines her view of mathematics, making reference to her conversations with Patricia about mathematics as a communicative competence with which to create meanings. Furthermore, she also reconsiders her ability as a learner. For a moment, she shifts from a deficit view of herself to a critical analysis of her learning experiences. She describes herself using a new lens that included the analysis of the educational system in which she participated.

Lorena: *Yo sé que soy inteligente porque ahora me doy cuenta, pero antes, no sé que me pasaba a lo mejor decía, 'no voy a aprender, no voy aprender' entonces eso a lo mejor también por eso no aprendía, a lo mejor podía haber llegado más lejos, más lejos de la primaria. ...Sí, se las enseñan diferente también. Se las enseñan diferentes porque yo pues nomás estudié las tablas... yo estudiaba, no, y ya con eso, pero no te explicaban y ahorita ya te explican, sí no entiendes de una forma, te explican de otra hasta que ya le entiendes.* (parent interview, May 10, 2006).	Lorena: I know that I am intelligent because now I can see it, but before, I don't know what happened to me, maybe I said to myself, 'I'm not going to learn, I'm not going to learn' and maybe because of that I didn't learn, maybe I could have gone further, further than elementary school. …Yes, they teach [mathematics] differently too. They teach them differently because I only studied the times tables… I studied them and that was all, but they didn't explain and now they do explain, if you don't understand in one way they explain you in a different way until you understand. [Authors' translation]

Homework: At home and in an after-school

Homework is a good example of a school-related practice that for the case study families is situated and distributed, that is, depends on multiple agents. The four students in these case studies attend after-school programs. Yessenia's support focuses on mathematics, while the other programs help with homework from all content areas. Lorena relies heavily on this program's capacity to help her children with their homework. While she checks her children's homework completion, other parents trust that their children do their homework. Yessenia, as well as other children, uses her sister's help as an important resource.

The analysis of parents' historical narratives in relation to homework practices also bring to light that school staff often fail to share their assumptions and decision making with parents. When Yessenia was in fourth grade Lorena was concerned that her daughter did not talk about her learning of mathematics and did not receive much homework. Lorena interprets this silence as a lack of emphasis placed on this subject in the classroom. She ignores the underlying reasons of Yessenia's behavior since Lorena did not share her concern with the teacher at the time.

This year Yessenia and Lorena attend an after-school program that focuses on mathematics at the school site. While the overt purpose of the program is to help children with their mathematics homework, Lorena's personal purpose for attending is to reinforce her close relationship with Yessenia. This is one space Lorena creates to spend time with her daughter.

In this context the connection with the classroom curriculum was mainly sustained through Yessenia. She brought questions or shared topics discussed in class. The following segment is a description of a time in which Yessenia asked for help with her mathematics homework. Yessenia brought the following question from her textbook: *"*Can you show 0.02 using only tenths place-value blocks? Explain*."* (Charles, Barnet, Briars, et al., 1999).

For those not familiar with the place value blocks and their uses with decimals, the equivalencies established were the following: one small square unit represents one hundredth, a line of ten square units represents one tenth, and the flat of a hundred square units represents one. In her homework, Yessenia wrote the following response, "No, you can't use tenths place value blocks." Yessenia's answer was correct but the teacher revised it and said her solution was not clear since she did not explain the reason why one could not use tenths place value blocks. The tutor explained to Yessenia the decimals using place value labels, but the tutor was unfamiliar with the use of place-value blocks with decimals and was not using this means to explain the question. Yessenia built on her classroom learning experiences and shared her knowledge of these representations using drawings of the blocks with the tutor. She drew base ten blocks trying to make sense of the decimals. She explained to the tutor that with two lines or two tenths she could not represent two-hundredths because they were smaller. Yessenia was unsure of her statement so she also represented the decimals drawing money. This time she explained to the tutor "I have two pennies and that [line] is two dimes." During this interaction, Lorena's participation consisted of watching Yessenia's efforts and the non-verbal cues of the tutor to evaluate Yessenia's explanation (field notes, February 9, 2006).

In this encounter two central resources set the elements of boundary for Lorena; the language of interaction and the mathematical meanings in negotiation. Lorena becomes an outsider as soon as Yessenia reads the question in English. Although Yessenia is now in a bilingual classroom, her educational history did not support her development of academic Spanish. In this way the history of a practice that included only English situates Lorena as an outsider when she tries to participate in Yessenia's mathematics learning. This position is especially contradictory when Lorena's goal for participating in the after-school program is to reinforce her close relationship with Yessenia. In this manner, the language choice for homework and instruction influences the access of parents to their children's mathematical learning.

The second structuring resource in these interactions connects to the negotiation of mathematical meanings. Lorena's schooling experiences taught her that only some children are innately good in mathematics while others are not born to become members of learning communities in school (parent interview, July 19, 2006). While attending school in Mexico, she was retained (i.e., kept back) in elementary school for several years and, eventually, referred to a Special Education school. Since then, she had defined herself as an outsider to mathematics because she could not memorize facts and algorithms. In the example discussed above, Lorena is situated as an outsider, not only due to language issues but also because she did not remember learning decimals at school and views school mathematics as a subject matter disconnected from her common experience. She evoked her personal history of exclusion in her mathematics education experiences. In contrast, Yessenia's experience with learning mathematics is one that focuses on creating meaning (e.g., from the abstract numbers of two-hundredths and two-tenths to the place value blocks and her experiences with money). In her classroom, mathematics is treated as a language or tool to create meaning. Yessenia turned to these connections with world experiences (e.g., money) and was able to make sense of the decimal numbers. Yessenia, therefore, did not conceive mathematics as a series of procedures or rules to be memorized or practiced. This position is radically different in that it empowers her with respect to the mathematics. It is not the mathematics that tells the individual its rules to be followed blindly, but it is the individual who gives numbers meaning and then manipulates them. Yessenia's approach to mathematics contests the hegemonic relationship towards mathematics existent in many educational settings.

Conclusions and implications

A comprehensive review of the opportunities for culturally and linguistically minoritized students demands an imminent response to the prevalent inequities within and beyond mathematics education (Apple, 1992; Gutstein, 2003; Martin, 2000; Stinson, 2004). As Martin (2000) suggests equity goals call for a vision that includes the socio-historical context of classrooms and schools. Practices of parental engagement that disrupt oppressive power structures can open spaces of egalitarian participation in education for and by disenfranchised communities.

In this article we focus on mathematics education as a practice of particular communities, most importantly, we examine the boundaries around these communities. School mathematics has evolved as an exclusionary practice with boundaries that often leave students and families from marginalized groups outside (Civil, 2007; Martin, 2006; Stinson, 2004). The concept of boundary practices situates the notion of parental involvement; it focuses on the organization and history of the community, roles and power relations, as well as the identities of the participants. It is an analytical tool to examine the ways in which these practices include or exclude students' cultural resources, embracing their connections with their families, as they reproduce or disrupt current power relations.

The inclusion of students' funds of knowledge opens the boundaries of school mathematics for their legitimate participation. These pedagogical practices make explicit that mathematics is a cultural practice. Moreover, the inclusion of students' funds of knowledge from disenfranchised communities validates their knowledge and counters asymmetrical power relations. It promotes their engagement and their identity as subjects in their education (Freire, 1998). In this way their cultural resources, namely their families, are symbolically included. Parents' knowledge becomes a resource. The identity of the individual includes the communities to which they belong outside of school. In this case, belonging to the classroom community means bringing in their out-of-school life and supporting the transformation of those disenfranchising situations. Furthermore, when parents are invited as participants, as in the case of the classroom observation, it renegotiates the boundaries of membership. Parents as agents of transmission of cultural and historical legacy are an indispensable tool for learning.

Students' linguistic competencies are important funds of knowledge. Language use in a community is not only a tool to negotiate meanings but is an element of boundary, which is often marked by the historical discrimination towards minoritized linguistic communities. Thus, the nature of school mathematics practices goes beyond the realm of negotiation of meanings. Pedagogical practices function as boundary practices; therefore, they need to become central to efforts of parental engagement. A culturally relevant pedagogy promotes the inclusion of parents in their children's education and an egalitarian dialogue between parents and teachers.
Adults from minoritized communities are overrepresented in groups with histories of exclusion from school mathematics. Parental engagement efforts have the potential to disrupt these histories. For example, teachers identified Lorena as an outsider to school mathematics, and consequently she identified herself as such. The boundary practice of immersion or classroom observation is significant for Lorena as a parent as well as a learner. In this practice she engaged in an experience that redefined mathematics as a language and a tool for sense making. Surface area was discussed not only as curricular goal but as a way to understand better her everyday context (e.g., the vegetation in the desert). A culturally responsive education requires that learners are subjects of their learning, in other words, that they make sense and stay in control of their experiences (Freire, 1998). Dewey (1938) asserts that, for an experience to be educational, learners need to continue to be curious and see themselves as legitimate members of these communities. These experiences have a significant potential not only in the educational trajectories of children but of adults.

There are significant implications for both teachers and decision makers from Patricia's pedagogy as well as from the concept of boundary practices. A teacher's stance on parental involvement is a key mediator of her interactions with parents. In this case, Patricia's vision of social justice assists children's families in their role as advocates for their children. She hopes to support their effort in countering racist and biased policies, relations, and practices with professionals. This role is particularly critical but hard to assume when parents are not recognized as and they themselves do not feel they are legitimate peripheral members of school mathematics. Educators, policy makers, and researchers need to explore ways to systematically include the funds of knowledge of minoritized families, including bilingual or multilingual mathematical communicative competencies, and dialogues that counter current asymmetrical power relations with minoritized students and their families.

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References

Abreu, G. d. (2002). Towards a cultural psychology perspective on transitions between contexts of mathematical practices. In G. d. Abreu, A. J. Bishop & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 173-192). Great Britain: Kluwer Academic Publishers.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education, 23*(5), 412-431.

Barton, D. (1996). Family literacy programmes and home literacy practices. In D. A. C. Baker, J.; Fox, C. (Ed.), *Challenging ways of knowing: In English, maths and science* (pp. 52-61). London: Farmer Press.

Calabrese Barton, A., Drake, C., Perez, J. G., St. Louis, K., & George, M. (2004). Ecologies of parental engagement in urban education. *Educational Researcher, 33*(4), 3-12.

Charles, R. I., Barnet, C. S., Briars, D. J., Crown, W. D., Johnson, M. L., Leinwand, S. J., et al. (1999). *Math: Grade 5 teacher's edition* (Vol. 1). Glenview, IL: Scott Foresman-Addison Wesley.

Charmaz, K. (2001). The grounded theory method: An explication and interpretation. In R. M. Emerson (Ed.), *Contemporary field research: Perspectives and formulation* (pp. 109-126). IL: Waveland Press Prospect Heights.

Civil, M. (2002). Everyday mathematics, mathematicians' mathematics, and school mathematics: Can we bring them together? In M. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom. Journal of Research in Mathematics Education Monograph #11* (pp. 40-62). Reston, VA: NCTM.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105-117). New York, NY: Teachers College Press.

Civil, M., & Andrade, R. (2003). Collaborative practice with parents: The role of researcher as mediator. In A. Peter-Koop, V. Santos-Wagner, C. Breen & A. Begg (Eds.), *Collaboration in teacher education: Examples from the context of mathematics education* (pp. 153-168). Boston, MA: Kluwer.

Civil, M., Bratton, J., & Quintos, B. (2005). Parents and mathematics education in a Latino community. *Multicultural Education, 13*(2), 60-64.

Civil, M., Planas, N., & Quintos, B. (2005). Immigrant parents' perspectives on their children's mathematics. *Zentralblatt für Didaktik der Mathematik, 37*(2), 81-89.

Civil, M., & Quintos, B. (2006). Engaging families in children's mathematical learning: Classroom visits with latina mothers [Electronic Version]. *New Horizons for Learning Online Journal*, XII.

Cole, M., Engestrom, Y., & Vazquez, O. (1997). Introduction. In M. Cole, Y. Engestrom & O. Vazquez (Eds.), *Mind, culture, and activity* (pp. 1-21): Cambridge University Press.

Delgado-Gaitan, C. (2001). *The power of community: Mobilizing for family and schooling*. Lanham, Maryland: Rowman & Littlefield.

Dewey, J. (1938). *Experience and education*. New York, NY: Simon & Schuster.

Dyson, A. H., & Genishi, C. (2005). *On the case. Approaches to language and literacy research.* New York: Teachers College, Columbia University.

Elboj, C., Puigdellívol, I., Soler, M., Valls, R. (2002). Comunidades de aprendizaje. Transformar la educación. Barcelona: Graó.

Freire, P. (1998). Pedagogy of the oppressed. In A. M. Araujo F. & D. Macedo (Eds.), *The Paulo Freire Reader* (pp. 45-79). New York, NY: Continuum.

Gay, G. (2000). *Culturally responsive teaching: theory, research, and practice.* New York, NY: Teachers College Press.

González, N., Andrade, R., Civil, M., & Moll, L. C. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Student Placed at Risk, 6*(1&2), 115-132.

González, N., Moll, L. C., & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Mahwah, NJ: Lawrence Erlbaum.

Guajardo, M. A., & Guajardo, F. J. (2002). Critical ethnography and community change. In Y. Zou & E. H. Trueba (Eds.), *Ethnography and Schools: Qualitative Approaches to the Study of Education* (pp. 281-304). Oxford, England: Rowman & Littlefield Publishers, Inc. .

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37-73.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York, NY: Routledge.

Hammond, L. (2001). Notes from California: An anthropological approach to urban science education for language minority families. *Journal of Research in Science Teaching, 38*(9), 983-999.

Kincheloe, J. L., & McLaren, P. (2002). Rethinking critical theory and qualitative research. In Y. Zou & E. H. T. Trueba (Eds.), *Ethnography and Schools: Qualitative approaches to the study of education* (pp. 87-138). Oxford, England: Rowman & Littlefield Publishers, Inc. .

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *Culturally Relevant Teaching, 32*(3), 465-491.

Lankshear, C., Gee, P., Knobel, M., & Searle, C. (1997). *Changing literacies*. Buckingham: Open University Press.

Lareau, A., & Horvat, E. M. (1999). Moments of social inclusion and exclusion: Race, class, and cultural capital in family-school relationships. *Sociology of Education, 72*(1), 37-53.

Lareau, A., & Shumar, W. (1996). The problem of individualism in family-school policies. *Sociology of Education, 69*(2), 24-39.

Lave, J. (1988). *Cognition in practice.* Cambridge, United Kingdom: Cambridge University Press.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, culture, and activity 3*(3), 149-164.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, United Kingdom: Cambridge University Press.

Mapp, K. L. (2003). Having their say: Parents describe why and how they are engaged in their children's learning. *School Community Journal, 13*(1), 35-64.

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Martin, D. B. (2006). Mathematics learning and participation as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. *Mathematical Thinking and Learning,* 8(3), 197-229.

Masingila, J. O. (1994). Mathematics practice in carpet laying. *Anthropology and Education Quarterly, 25*(4), 430-462.

Masingila, J. O., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics, 31*, 175-200.

Moll, L. C., & González, N. (2004). Engaging life: A funds-of-knowledge approach to multicultural education. In J. A. Banks (Ed.), *Handbook of research on multicultural education* (2nd ed., pp. 699-715). San Francisco: Jossey-Bass.

Moses, R., & Cobb, C. (2001). Organizing algebra: The need to voice a demand. *Social Policy, 31*(4), 3-12.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). Reflections on street mathematics in hindsight. In *Street mathematics and school mathematics* (pp. 127-154). New York, NY: Cambridge University Press.

Olivares Alonso, E. (2006, November 19th). Lamenta la ONU que la educación se limite a un sentido mercantilista. *La Jornada*.

Patterson, L., & Baldwin, S. (2001). A different spin on parent involvement: Exploring funds of knowledge within a systems perspective. In W. Goodman (Ed.), *Living and Teaching in an Unjust World: New Perspectives on Multicultural Education* (pp. 127 - 139): Westport, CT: Heinemann.

Smith, D. J., & Armstrong, S. (2002). *If the world were a village.* Tonawanda, NY: Kids Can Press Ltd.

Stinson, D. W. (2004). Mathematics as "gate-keeper" (?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The Mathematics Educator, 14*(1), 8-18.

Taylor, D. (Ed.). (1997). *Many families, many literacies: An international declaration of principles*. Portsmouth, New Hampshire: Heinemann.

Taylor, D., & Dorsey-Gaines, C. (1988). *Growing up literate: learning from inner-city families*. Portsmouth, New Hampshire: Heinemann.

Valdés, G. (1996). *Con Respeto: Brindging the distances between culturally diverse families and schools*. New York: Teachers College Press.

Vincent, C. (1996). Parent empowerment? Collective action and inaction in education. *Oxford Review of Education, 22*(4), 465-482.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York, NY: Cambridge University Press.