An investigation to “mathematical literacy” of adults using PISA – items

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During the last decades we can see an increasing number of studies which test the performance of students beside other subjects in mathematics. These studies are a consequence of the meaning of education for the economic position of a country. To get more attention (and perhaps also more money) the studies are presented with a ranking of participating countries based in the results of students' performance. One of the most discussed studies are the PISA – Studies. These studies use as basic concept the concept of "mathematical literacy" in research of adult education of mathematics a very well known concept. PISA claims to test with these studies curriculum independent mathematical competences those are necessary basis competences to handle mathematical problems in jobs and society. If we accept this claim it must be of interest what are the mathematical competence of adults because they work and live in our society. Now exists international studies to the performance of adults in mathematics (OECD 1995) and in a study to the adult education in mathematics in Austria we have investigated the mathematical knowledge of adults in mathematics (Schlöglmann 1998). In the PISA – Elternstudie (Ehmke 2004, Ehmke, Wild and Müller-Kalhoff 2005) have done the first step to use PISA – items to test parents of students who have participated in the PISA – Study in Germany (Ehmke, Wild and Müller-Kalhoff used in their study 7 items of the international PISA – test and some more from the German test). In his Master – thesis F. Raber (Raber 2007) used the same 7 international items to test the performance of adults in Upperaustria.

The PISA concept

The “Programme for International Student Assessment (PISA)” is an internationally standardised assessment that was jointly developed by participating countries and administered to 15-year-olds in schools. It covers the domains of reading, mathematical and scientific literacy not so much in terms of mastery of the school curriculum, but in terms of important knowledge and skills needed in adult life.
For the assessment are paper-and-pencil tests used and the test items are a mixture of multiple-choice items and questions requiring their own responses. The items are organised in groups based on a passage setting out a real life situation (OECD 2003, p.11).

Let us consider the description of mathematical literacy (OECD/PISA uses the word definition):

„The mathematical literacy definition for OECD/PISA is: Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."
(OECD 2003, p.24)

In a more extensive description writes OECD/PISA that mathematical literacy is concerned with the capacities of students to analyse, reason, and communicate ideas effectively as pose, formulate, solve, and interpret mathematical problems in a variety of situations and emphasises that the focus of the assessment lies on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classroom.

„In schools, the mathematics curriculum has been logically organized around content strands (e.g. arithmetic, algebra, geometry) and their detailed topics that reflect historically well-established branches of mathematical thinking – […] However, in the real world the phenomena that lend themselves to mathematical treatment do not come so logically organized.[…] Since the goal of OECD/PISA is to assess students' capacity to solve real problems, our strategy has been to define the range of content that will be assessed using a phenomenological approach to describing the mathematical concepts, structures or ideas.[…] A phenomenological organization for mathematical content is not new. […] Various ways of labelling the approach have been used. In the mathematics framework for OECD/PISA 2003, the label 'overarching ideas' will be used." (OECD 2003, p.34)

These overarching ideas which should meet the requirements of historical development of mathematics, coverage of the domain and reflection the major threads of school curriculum are *quantity – space and shape – change and relationship – uncertainty*.

The German discussion to mathematical literacy use the term „mathematische Grundbildung" to put he concept in the tradition of the German discourse of „Bildung " (Klieme, Neubrand, &Lüdtke, 2001).

We are not discussing in this paper if PISA-items are suitable to fulfil the description of their "definition" of mathematical literacy but especially for the multiple-choice items, basis for the international comparison of the education systems, are doubts appropriate (for a deeper critique see (Jahnke and Meyerhöfer, 2006).

The study

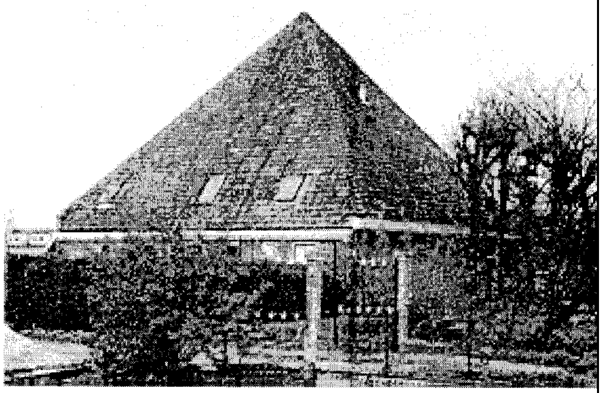
1. The questionnaire

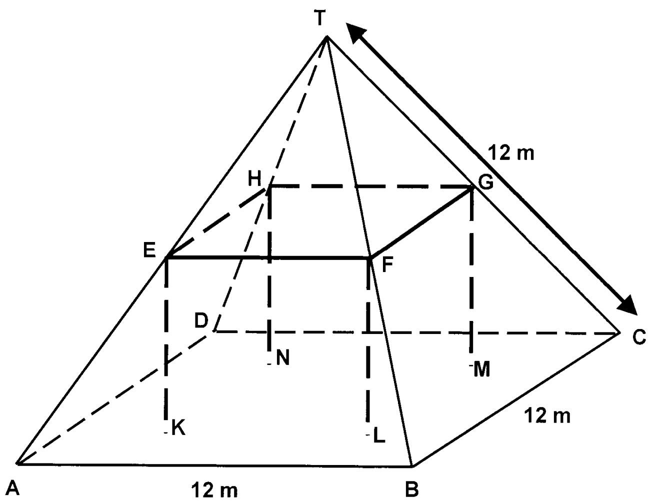
The questionnaire starts with some hints to the test about the period of working, the anonymity of the test and its aim as a scientific study, that the items come from the PISA – Study and how to handle the items.
These general hints about the test are followed by questions to the person: age – sex – highest level of education that the person had finished - profession.

The main part were the 7 PISA items:
Farms (2 questions) – Walking (1 question) – Speed of a racing car (2 questions) – Painted cube (2 questions).

FARMS

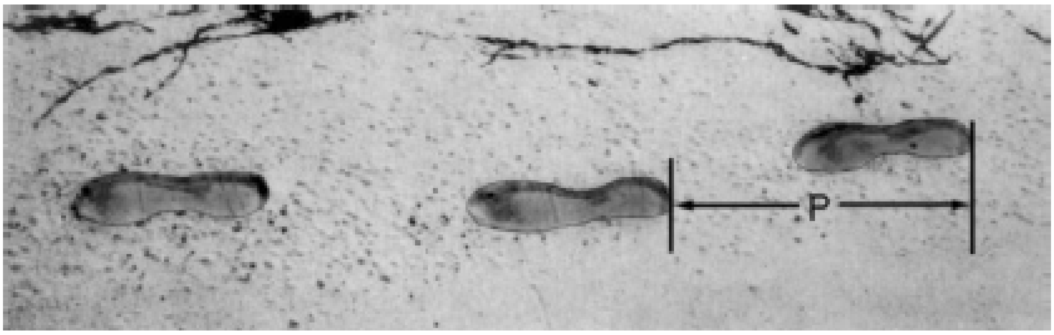
Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.
Below is a student's mathematical model of the farmhouse roof with measurements added
The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a block (rectangular prism) EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT and H is the middle of DT. All the edges of the pyramid in the model have length 12 m.





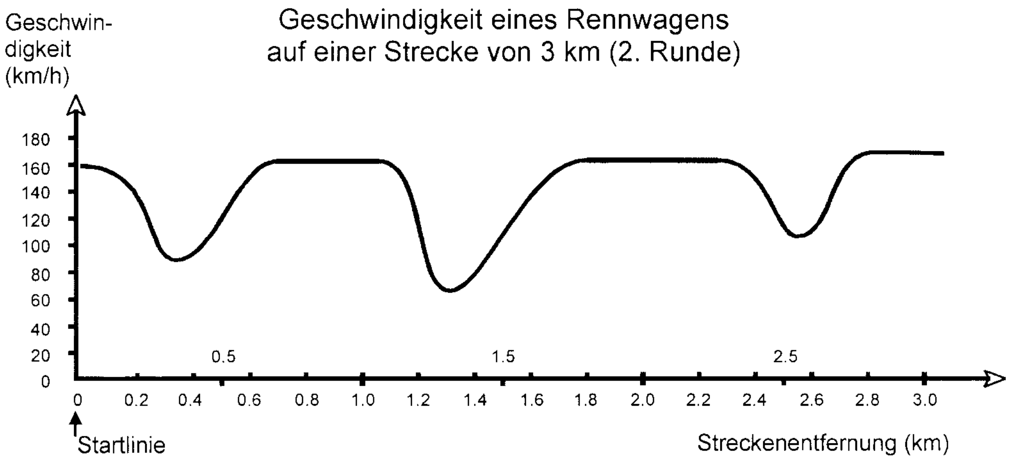
Question 1:
Calculate the area of the attic floor ABCD.
The area of the attic floor ABCD = _______________ m2
.
Question 2:
Calculate the length of EF, one of the horizontal edges of the block.
The length of EF = _____________ m

WALKING

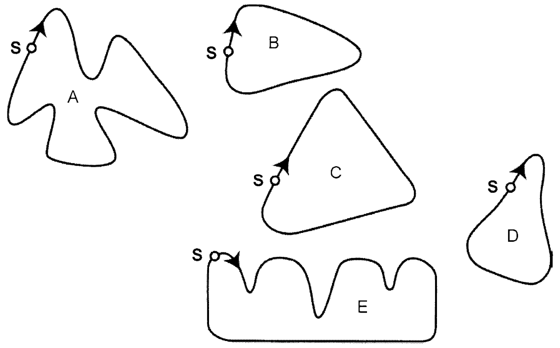


The picture shows the footprints of a man walking. The pacelength *P* is the distance between the rear of two consecutive footprints.
For men, the formula, n/P = 140 gives an approximate relationship between *n* and *P* where,
n = number of steps per minute, and
P = pacelength in metres.
Question 1:
If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

SPEED OF A RACING CAR:
This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.

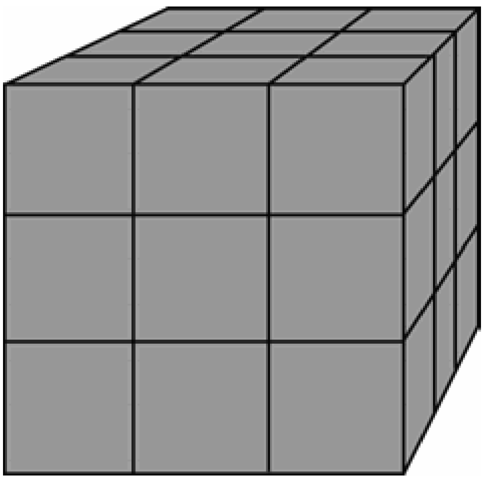


Question 1:
What is the approximate distance from the starting line to the beginning of the longest straight section
of the track?
A 0.5 km.
B 1.5 km.
C 2.3 km.
D 2.6 km



Question 2:
Here are pictures of five tracks: Along which one of these tracks was the car driven to produce the speed graph shown earlier?

PAINTED CUBE
A big cube is painted. This big cube will afterwards cut from three directions in respectively three layers. It evolves by that many smaller cubes as in the following picture.



Question 1:
How many small cubes do exist?
Question 2:
A further big cube is painted. This big cube will afterwards cut from three directions in respectively five layers. It evolves by that many small cubes.
How many of these small cubes do have exactly **three** painted lateral faces?

2. The items from the perspective of mathematics didactics

Farms: Answering question 1 it is not necessary to consider the pictures as well the photo as the mathematical model. If one read the text to the task you find that the area of the attic floor is a square with a lengths of 12 m. To solve the problem it is only necessary to calculate the area of a square.
Question 2 needs for a correct mathematical solution the intercept theorem but in a multiple choice concept it is impossible to see if the intercept theorem is really used. A solution is also possible to measure the length of EF in relation to the length of AB or argue that TE is 1/2TA and TF is 1/2TB therefore must EF be 1/2AB.
Walking: This item should investigate the handling of a formula. The task can be solved through transformation of the formula but it is also possible to ask 70 divided by which number is 140?

This formula is what is often called a rule of thumb. Such formulas are often used in vocational situations but in this situation for the solution of problems professional use mostly tables and do not transform formulas.

Speed of a racing car: Question 1 handles the problem of the speed along a track. The problem solver has to recognize that the change of speed is connected with bends.

Before a curve the speed is decreasing and after the bend on a straight section the speed is increasing until reaching the high speed. Than the high speed is constant before the following bend requires a decreasing of the speed. The problem solver finds 4 suggestions for a right solution but it is for instance impossible to recognize if the problem solver uses only the constant piece of the graph for a straight section or knows that the straight section starts at the end of the bend.

To answer question 2 the problem solver has to take information from the first graph and use this information to decide which picture of the five tracks is the correct one.

Firstly three minima in the graph mean three bends therefore only the tracks B, C and D meet this condition. Secondly the speed is decreasing a short piece after the starting line and this fulfils only the tracks B and D. Thirdly the lowest speed means the most narrow bend and this is bend two and therefore is track B the right answer.
Painted cube:

In question 1 it is only necessary to count the cube on the front face and multiply by three according to the three layers.

Question 3 needs the reflection that only the small cubes on the edges have exactly three painted lateral faces independent from the number of cuts.

Summarising this considerations to the items we can say that if the goal is to test the competence to solve real world problems with mathematical methods doubts are appropriate. The items are interesting tasks but the real world aspect in farms and walking are artificial, the painted cube is typically for intelligence test and only the speed of a racing car need knowledge from a real world situation.

3. Data collection

To investigate knowledge and competences of adults is much more difficult as of students (Schlöglmann, 1998). For tests with students you find the test persons in schools and school authorities can help to find enough students and to get a sample to fulfil the needs of statistics. But in the case of adults all tests must be done voluntary and there is no institution to find enough test persons. Therefore in nearly all cases of small investigations you will not have a representative sample and you must interpret the data very carefully. In the case of this study the test was done beside parents, relations and adult friends by adults who was waiting in waiting-rooms of an emergency ward of a hospital and a dental ambulatory and we are very gratefully to this persons who helped to make this study.

4. Some results of the study
Characteristics of the sample:
Basis for the study are questionnaires of 140 persons (54% female and 46% men). With respect to the age are 24% persons between 20 and 29 years old, 24% between 30 and 39, 23% between 40 and 49 years, 19% between 50 and 59 years and 10% more than 60 years. If we compare this distribution of the age with the distribution of the adults in Upperaustria we see that especially the group of persons with more than 60 years are strongly underrepresented (10% to 26%) all the other groups are overrepresented.

To get information about education the participants have to sign in a list their highest finished education. 10% have finished the compulsory secondary school (Hauptschule (HS)), 40% part-time vocational school (Berufsschule (BS)), 14% full-time vocational schools (Berufsbildende Mittlere Schule (BMS)), 10% upper secondary general school (Allgemeinbildende Höhere Schule (AHS)), 11% Uppersecondary vocational school (Berufsbildende Höhere Schule (BHS)) and 15% University, College or Teacher-College. If we compare our sample with the distribution in the population we can see that the participants of the study are better educated than the average. In the sample are more persons with higher education especially the group with compulsory secondary school are strongly underrepresented (10% to 39%). This is also in connection with the low representation of elder people because within this group of population are more persons with a low education.

Results of the test:

Whole sample: 62,3% correct answers, expectation 4,36.
Correct answers of the items and in bracket the value on the PISA – competence scale: Farm1 79% (492), Farm2 70% (524), Walking 46% (599), Racing car1 (492) 67%, Racing car2 39%, (655) Cube1 82% (481), Cube2 53% (591).
Correct answers per age group: 20-29 years 70%, 30-39 years 61%, 40-49 years 69 %, 50-59 years 61%, more than 60 years 36%.
Correct answers per education: HS 45%, BS 59%, BMS 47%, AHS 84%, BHS 80%, University and Colleges 76%.

Discussion of the results:

If we compare the results of the study with the PISA – competence scale we find a full accordance. Racing car2 has 655 points on the competence scale and the lowest rate of correct answers followed by Walking and Cube2.

If we consider the situation from the perspective of the age we find that the number of correct answers is highest in the group of 20-29 years old participants is decreasing for the group of 30-39 years old people and we would expect that the rate of correct answers would decrease with the age. But in the group of 40-49 we find nearly the same result as in the best group. We found a similar result also in other studies (Schlöglmann, 1998) and our conjecture is that this group are parents of children and help them to learn mathematics.

If we look at the results of the eldest participant we find on the one hand the lowest number of correct answers and on the other hand no correct answer for the items

Walking and Racing car2. If we consider that in this group we find many people with low education and in the curriculum of the "Hauptschule" was at this time nearly no algebra and no functions we should expect such a result. But we should also take this result very carefully because this group is very small (14 participants).

The relationship between education and results are in accordance with the expectation. More education leads to better results. That the group AHS and BHS have better results as the University/College group is a consequence that in this both groups are students included who have not finished their university are college studies. We will finish this paper with a short comparison between the results of the German parents study and the results of students. The mean value of the Austrian and the German adults are similar (62,3 % to 62,6%). The Austrian adults are better space and shape and the German adults in change and relationship. This result appears also in the comparison of Austrian and German student. Furthermore the adults are better than students 62,3% to 49%).

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