On the Relationship between Cognitive and Affective Components of Learning Mathematics

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In adult education in mathematics there is a long running discussion about "math - anxiety" (Tobias (1994)), mental blocks of adult learners in connection with particular parts of mathematics (Lindenskov (1996), Wedege (1998)), resistance against mathematical forms of argumentation especially a too formal argumentation (Schlöglmann (1997)).
To elucidate this problems I will give an example which was reported to the Internet discussion group "adult numeracy" by Bonnie Fortini. The title of this case was "Variable - Phobia":

Has anyone run into a case like the student I have had who seems unable to do any math that has unknowns or variables in it? She is mid 40s, very bright, English major going on to a Masters program. She can do all sorts of computations including fractions, percentages, ratios, and word problems are some of her favorite things to do. But as soon as you give her something like 4 + 2x - 6 + 5x= 95, she is totally frozen. She can't get past go when trying to combine like terms, and reacts physically (anxiety, tears, etc.).

I think this example demonstrate very clearly the problematic situation in further education in mathematics. It also gives us a strong hint of a closed connection between the affective and cognitive components of learning. Therefore it seems to be necessary to search for a theoretical concept of learning which combines affective and cognitive aspects.

1. The concept of "affect logic" by L. Ciompi

The concept of "affect logic" (Ciompi (1991)) is a reaction to the situation that facts about the common ground of the cognitive psychology and psycho-dynamics are ignored. The findings show that emotion and cognition (or feeling and thinking, affect and logic) never occur in isolation but always are closely combined. Hence we consider a concept which based on the one hand on the theory of Piaget on the development of cognitive structures and on the other hand on the psycho-analyze of Freud.
Piaget discovered that cognitive structures originate in innate sensomotoric "schemata" which gradually differentiate during childhood through assimilatory / accomodatory interactions with reality and thus constitute a "condensation of action" or of concrete experience. Consequently the relationship of cognitive structures to the environment (and in particular to all important interpersonal transactions) necessarily becomes part of the slowly forming intrapsychic structures. Simultaneously, these become increasingly automatic, internal and mental.

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Since every action contains emotional elements and is ultimately regulated by the Freudian pleasure principle, specific positive or negative affects are assigned to specific cognitive contents. which can be persons, objects, places, situations or concepts. The presence of particularly strong or often repeated affects in similar circumstances lead to a remarkably stable connection between certain cognitions and certain emotions. This "affective constancy" forms a basis not only for simple conditioned everyday patterns of behavior but also for complex "transference" phenomena in the psychoanalytical sense.
As a consequence of these considerations in the conceptualization of "affect logic" the "psyche" is understood as a complex hierarchical structure, comprising not only cognitive but necessarily "affective cognitive schemata" or "systems of reference". On all hierarchical levels these systems of reference correspond to a synchronic condensation of repeated past actions; they are effectively operational programs of feeling, thinking and behaving. These programs store past experience in their structure and therefore they represent probably the most important form of memory. At the same time they provide a functional matrix for all perception and communication.
These basic concepts imply that close relationships exist between the psychological, social and neuro-physiological domains, as mental structures are conditioned by interpersonal processes and the biological substrate of the resulting operational "programs". At the theoretical level, it is important to realize that effective cognitive systems of reference are systems in the sense used in systems theory (systems are complex entities formed by mutually interacting and equilibrating elements in the sense used by Piaget for purely cognitive schemata).Therefore the general principles of system theory can applied to mental processes and of special interests are the homeostatic (structure - maintaining) effect of negative feedback and the morphogenetic (structure - forming) effects of positive feedback. Many psychological observations show that certain kinds of effects (shame, fear, joy, anger) are combined with the same context over an entire life. Furthermore Ciompi was able to show that affects play a role which goes far beyond mere accompaniment of seemingly purely cognitive processes such as abstraction. In the "extraction of invariance", which is the core of solving scientific and mathematical problems by abstraction, common feelings of pleasure or displeasure connect elements that "fit" or "don't fit". Therefore these affects function literally as pointers to solution. Functional cognitive concepts and theories are invested with positive feelings and dysfunctional ones with negative feelings.
(L. Ciompi verifies his concept by many findings in psychology, biology and neurology, but it is impossible to repeat all his arguments).

2. Consequences for adults' learning processes in mathematics

Adults have many experiences concerning mathematics, especially school mathematics, but most of them have also contact with mathematics in their job and in the everyday life. All these experiences are combined with positive or negative affects and these affects influence their learning processes. Because the learning situation in further education courses is often similar to that in school, many affects connecting with school learning influence the learning of mathematics in further education courses. The affects can be positive - the participants enjoy learning mathematics and this positive feeling stimulates further learning processes. But many adults have had difficulties with mathematics in school and their experience with school mathematics is combined with failure in examinations. The reactions need not be as extremely as the reported case of "Variable phobia" or lead to blockade of all learning processes connected with mathematics. In my opinion it is a widespread phenomenon that many adults think that they are incapable of understanding mathematics. A frequent statement is: "I know I don't understand mathematics. Teach me rules, so that I know how to proceed. Do not waste time with mathematical explanations". Such persons have often decided at an early level of learning mathematics in school that they are unable to understand mathematics and therefore they have to look for other strategies for solving mathematical problems. One strategy is, the belief that each mathematical problem can be solved by the application of the right formula. An other strategy says that one need only apply the latest algorithm. Some people try to combine all numbers in the task by numerical operations, often starting with the first number. In the following I will call these strategies "replacement strategies" because they replace an understanding of mathematics. Their aim is merely to pass examinations. In the next section I will give an example of this kind of "problem solving strategy".

I would like to complete these remarks on the affective components of adult learning with a hint about the fact that, at least in Europe, many participants of mathematics courses are in a difficult personal situation. They have to visit courses because they are unemployed and they have to learn for a new job. In this situation, many of this adults have lost their self - confidence. Therefore the teaching situation in these courses is very difficult and a great deal of experience about the affective components of learning is required to lead such a course to a success.

3. Replacement strategies

This is a part of an interview with a participant of a course in mathematics. The participants of this course had just written a test in mathematics and the questions in the interviews are questions in the test. In this part to percentage calculation. (I = Interviewer, M = Participant)

Task 6a: What is 20% of 500 AS?

I: Task 6a involved percentages: What did you do here? (M looks at her paper).
M: I should calculate 1%, but how?
I: You know 100% and want to know how much 1% is.
M: Times 100, äh, divided by 100, äh no times 100.
I: You know 100% and want to know how much 1% is.
M: 500 divided by 100, I don't know, I am speechless.
I: (Gives a hint from the calculation in the test): *You have chosen the right formulation* (as a rule of three). *What would you have done if you had known that 1% is so many Schillings and you want to know 20%?*
M: Yes, than I would had calculated times 20.
I: And how do you get from 100% to 1%?
M: - - -
I: You divide by 100.
M: Mhm

Task 6b: What is the total amount if 300 AS are 20%?

I: Now we come to task 6b.
M: This is the same, I guess? I suppose it is 100 again.
I: 300 AS are 20%.
M: How much is 1%. Divided by 100.
I: No
M: Times 100
I: No
M: Times 20
I: How do you get from 20% to 1%?
M: 80
I: No
M: This doesn't suit me.
I: Think of task 6a. How did you get from 100% to 1%? Divide by 100.
M: Mhm
I: How do you get from 20% to 1%?
M: Divide by 20%
I: Now you know how much 1% is. How many % do you want to calculate? The total amount. What percentage is the total amount?
M: 20%
I: No
M: 100%, but how?
(The interviewer explains the task)
I: In what do you think lie the difficulties in task 6?
M: If I know the formula how for calculating 1% and 100%, then I would know how to do it, but as it is....

The participant knows that there is a connection between percentages and 100. She attempts a solution by multiplying by 100, discards this solution immediately and divides by 100. But she does not accept this answer and corrects it again by multiplication by 100. The interviewer want to help, she gives the correct answer but she is not sure about the correctness of the answer. Her comments and her answers indicate that she does not have a sufficient concept of percentages. Therefore she tries to connect her knowledge about the connection between percentages and 100 by combining these operations. even when she gives the correct answer she is not sure and discards the answer immediately. All her answers indicate that she is not sure of the concept of percentages and she tries instead to solve the problem by combining numbers and operation and hopes in this way to arrive at the correct solution.

In the second task M takes up the method of solution which was successful in solving the first exercise without thinking about the new structure of this exercise. After the interviewer rejects the answer, she tries a different operation. The interviewer rejects the new answer again and she now changes of the number. The number which is connected with percentages in her concept (100) does not work with either operations. Hence she takes the number of the instruction and attempts a solution with one of the two operation. Because the interviewer does not accept the answer she takes another of her replacement strategies connects two numbers (100 and 20) and tries it again. It seems she takes subtraction as the operation, presumably because multiplication would give a

number which seems to be too large. With the help of the interviewer the exercise is solved but I am not sure that the explanation has improved the understanding of the participant in this subject. A hint that this interpretation is correct is the fact that M supposes that the use of a formula would help her to solve the problems correctly.
I think that this interview gives a good impression of this type of problem solving strategy. First of all in order to solve of mathematical problems learners need the correct formula. If they have the correct formula they have only to put in the correct numbers. If they don't have the right fomula, this means that this strategy doesn't work and so they change to one of their replacement strategies, in order to connect the numbers in the task by means an operation. The choice of the operation is influenced by their knowledge about the quantity involved in the result. It is important for the didactical discussion that the meaning of a problem is irrelevant for the solution strategy. For this kind of strategy it is not necessary to understand the mathematical background and therefore the learners don't try to understand the mathematics. In their opinion the job of a teacher is to give the right formula and to train them to put in numbers in these formulae. In further education courses the participants often reject didactical concepts which function by understanding the mathematical concepts and in applied problems by means of a connection between the mathematical structure and the structure of the real world situation.
If we try to interpret these problem solving strategies from the point of view of affect logic, then we see that these persons to do mathematics is regulated by negative feelings. They are sure that they cannot understand mathematics but they have to be able to tackle mathematical tasks, for instance in order to pass examinations. In this situation they try to decrease their anxiety by looking for a strategy for solving mathematical problems. To have a strategy and to do something in a difficult situation leads to a positive feeling in a negative situation. The replacement strategies used prevent an increase in their anxiety which could lead to blockades.
The replacement strategies are strategies which are applicable in many situations connected with mathematical tasks and often helps to be partially successfully. The fact that they have a strategy which is at least partially successful in solving problems in a diffuse situation leads for many adult learners to a strict binding on their strategies and to a rejection of other learning strategies which may be proposed by a mathematics teacher. This rejection can often be very emotional.

4. Bibliography

Ciompi, L. (1991) 'Affects as Central Organising and Integrating Factors. A New Psychological/Biological Model of the Psyche' in *British Journal of Psychiatry* pp97- 105

Jungwirth, H./ Schlöglmann, W. (1997) *Mathematische und mathematikhaltige Weiterbildung.* Abschlußbericht zum Forschungsbericht. Linz

Lindenskov, L. (1996) 'Kursistundersøgelse på AMU - centre om alment - faglig kompetence i matematik' in T. Wedege (ed.) *Projekt: Faglig Profil i Matematik.* Kopenhagen, Arbedsmarkedstyrelsen pp73-84

Tobias, S. (1994) *Overcoming Math Anxiety.* New York - London: W.W.Norton

Wagneder, P. (1998) *Diplomarbeit*, Universität Linz

Wedege, T. (1998) 'Adults Knowing and Learning Mathematics' in S.Tøsse et al. (eds.) *Corporate and Nonconform Learning. Adult Education Research in Nordic Countries* pp177-197 Trondheim: Tapir