Whose Numeracy?

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Abstract

The three presenters' experience of teaching numeracy varies: Janet and Sue both work with undergraduates; Janet also works with university employees; Dhamma's experience is of teaching adults in Adult and Further Education provision (i.e. not university provision), mainly at the basic level, where many students also have low levels of written or spoken English.
However, although our students are different, our approaches to teaching are not. Our common approach is to discover and validate students' existing knowledge, skills, attitudes to maths and current goals, which are very diverse.
By considering the attitudes and needs of students at different stages of life, might we not further our work amongst adults learning mathematics by gaining greater insights into their different needs and attitudes? We invited colleagues to a session at ALM5 to discuss these issues with us.

Introduction

This discussion session arose from the workshop that Janet lead at ALM4, where she described a course she was running for university employees (Duffin, 1998). Dhamma felt that her approach could not be used in adult basic education, partly because the course demanded quite a high level of literacy and partly because it seemed very abstract in nature: the opposite of Dhamma's practice. When she fed this back, Janet invited her to co-lead a session at ALM5 to open up a discussion about the needs of different kinds of students and what we provide for them. Sue was present when we were discussing this and as she is also deeply concerned with these issues and her teaching approach, although also in a university, is very different from Janet's, we invited her to join us.

While planning the session, we discovered that although we share a common philosophy of discovering and validating students' existing knowledge, skills, attitudes to maths and current goals, we were using the same language to describe different things and different language to describe the same thing.

We were eventually able to agree on the following list of issues in provision for adults learning maths:
gender age / maturity language / literacy
disability / dyslexia cultural differences
student goals / expectations perceptions of mathematics / numeracy
social factors / experience perceptions of teaching / learning.

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This was not intended to be an exhaustive list, but a basis for the discussion to which others could make additions.

We also developed a model, with which we all agreed, of three approaches to teaching of maths to adults (Fig 1). It is the combination of these three approaches which is different in each of our teaching practices. We agreed on the names for two of the vertices: personal constructs of maths, and procedural school / college maths. We could not agree on the title for the third vertex. This exercise reinforced our experience that trying to define the maths we teach is very complex, but this is sometimes hidden by the language we use.

Three ways of teaching maths to adults

At the session at ALM5, after introducing the list of issues and the model of approaches to maths teaching, we each briefly described our own practice. The participants in the session

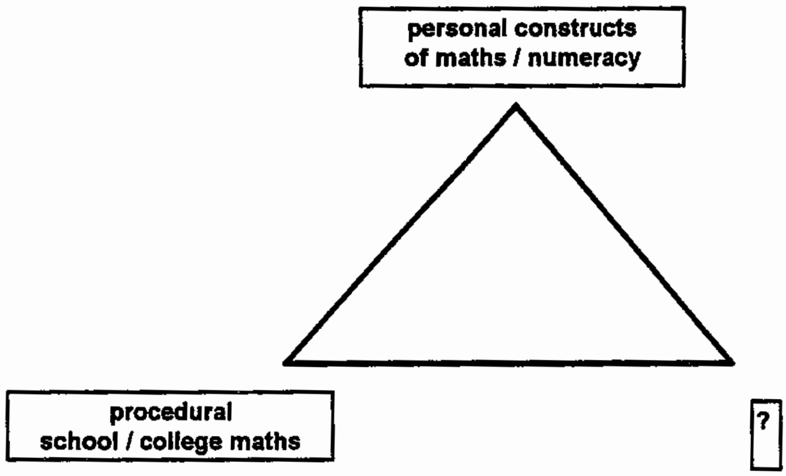

Fig 1. Three approaches to numeracy / maths teaching

were then invited to form small groups and discuss their own practices and how these compared with ours.

There follows a brief account by each of us of our experience in teaching, followed by a summary of the discussion that followed in the session. Inevitably, as these accounts have been written after the conference, they reflect the thinking that has developed after the workshop, rather than being accounts of what was said at the time.

Dhamma's story

This is a retrospective account in two ways: I have not done any teaching for four years, but have been concentrating on research; and the account was written after the conference and therefore represents the evolution of my views through the processes of discussion and the writing itself. For example, I have expanded the model I presented at the conference (Fig. 2) to incorporate meta-cognition (Fig.3), which I have always seen as an important part of adults' learning.

My teaching of basic maths to adults developed within the culture of adult literacy work: my aims have been the empowerment of students, which I have tried to achieve by helping them recognise the mathematical skills and knowledge they have acquired outside the educational system, as well as trying to promote their understanding of mathematics and its applications. These two aspects of maths are represented by the right hand and top vertices of the triangle in the model in Fig. 2.

It is important for adult students to have at least some control over their own learning. I have tried to achieve this by helping them to identify the knowledge and skills they want or need to progress further in formal education, work or for personal development. Students identify very diverse goals: passing job tests, gaining academic or vocational qualifications, helping their children with homework, transferring their knowledge of maths from another language into English, or redressing their perceived failure at school.

But students' expectations of basic maths education are based on their previous experience of learning maths in school, which often seems to have consisted of being required to learn sets of procedures. They usually do not recognise their everyday knowledge as mathematical. It is not just students who have this attitude: they are also reflecting the view of the wider society.

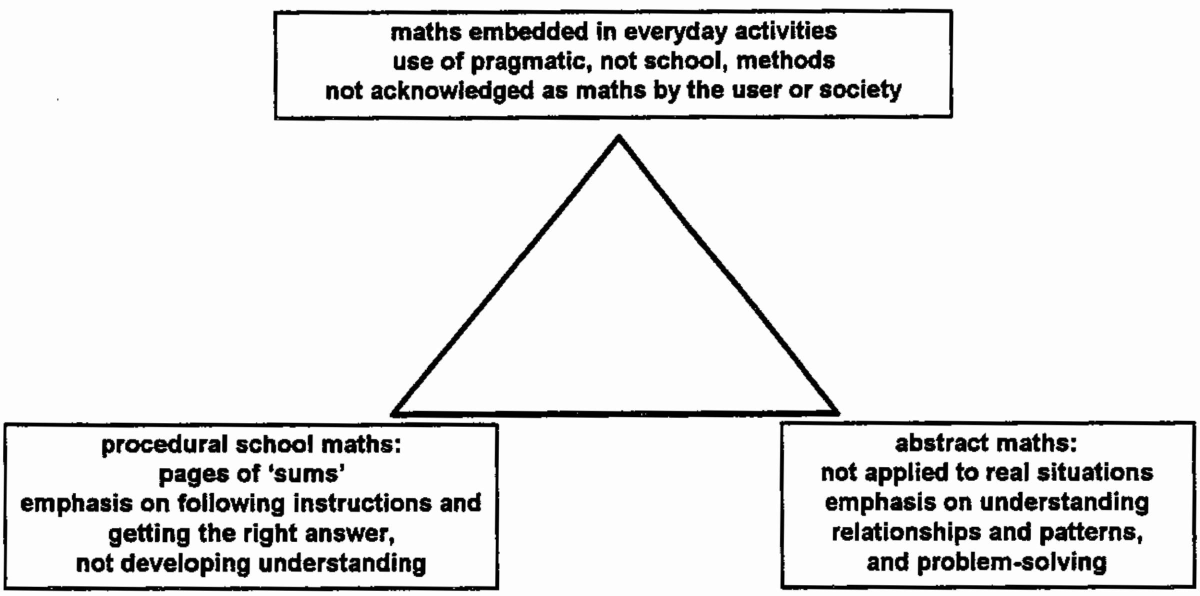

Fig 2. Dhamma's version of three approaches to teaching basic maths

Examinations like those offered by the City and Guilds (a widely used, low level vocational qualification) and specific job tests tend to examine procedural skills, sometimes posed as everyday problems. These exams and tests are used by gatekeepers to jobs and higher level courses and therefore cannot be ignored by those who want access to these. This idea of maths
as a set of procedures is so pervading that students can feel cheated if they are offered different kinds of maths teaching. The left hand vertex of the triangle in Fig. 2 represents this approach to maths.

As my teaching was within the organisational structure of adult literacy work, some, but not all, of the students I have taught have low levels of literacy skills. Others are native speakers of languages other than English, and may have learnt maths at school in another language, or may have had very little schooling. Some students require more help with the language required to solve problems than the mathematics. I have also taught students with disabilities which affect their learning, for example visual impairment or dyslexia. Disabled students may require intensive help to discover their optimum ways of working.

In teaching, I have always found myself trying to balance the different strands of the work: eliciting from students their use of maths in everyday life, and trying to develop their understanding of maths, without dismissing their expectations of learning maths procedures. If a student has a job-test in a few weeks time, then it is only possible for them to learn some useful procedures. With other students I have been able to negotiate a curriculum which contains elements of all three approaches, but which favours maths in everyday contexts.

I have formulated problems, using materials such as real train timetables, gas bills, advertisements, newspaper articles and maps, and tools like tape-measures, kitchen scales, thermometers, clocks and calendars. During the problem-solving process, we have discussed the range of possible useful maths procedures, both formal and informal, and students have practised their preferred methods, while becoming aware that they are not definitive.

I have incorporated some abstract maths into the work where it is appropriate, with the intention of developing students' understanding. For example, when students are solving a problem that involves percentages of sums of money, we have looked at how and why the place value system works when multiplying and dividing by a hundred.

I feel it is important to help students recognise that their previous lack of achievement is due more to the social structure of the education system than their real abilities. Along with other adult numeracy tutors, I have made time for students to discuss and write about their experiences of learning and using maths. We produced a broadsheet of students' writing, the Take Away Times, hoping that seeing their experiences and views in print was an empowering experience for the contributing students.

I was also a member of a team of tutors who developed accreditation of numeracy for adults for the London Open College Federation as an alternative to the more traditional examinations. The accreditation units focus on everyday maths topics like measurement and time and can be combined with units on the language of maths or study skills in maths. Being able to give credit for language and study skills enables the provider to allocate extra teaching time for this part of the work.

In our triangular model, there is no place for students' reflections on their own learning. I have represented this by the apex of a tetrahedron in Fig. 3, with the original triangle as the base.

I cannot claim to have achieved the aims of my teaching. Some students have passed job tests or entrance tests to higher level courses. Others have participated fully in the activities of the class and some have expressed their enjoyment and satisfaction. On the other hand, many students drop out of adult numeracy provision and it is not always possible to find out why they have done so.

From the research I am doing (Colwell, in this volume), I am beginning to question whether the teaching approach I used, where I formulated problems from everyday materials, may actually disempower students. People have their own ways of formulating and solving problems in their everyday lives, which may dispense with calculation or the use of formal tools. Do policy-makers or teachers have the right to make judgements about what adults 'ought' to be able to do? And if we do teach students more mathematical ways of solving problems, will they actually use them in real life? And might these undermine the knowledge and experience they already have: will they feel that the strategies they have successfully used in the past are deficient?

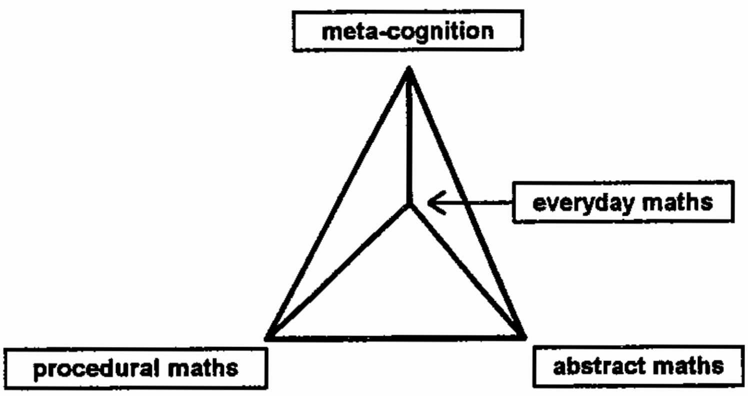

Fig 3. Reflecting on the personal process of learning maths

A better approach to teaching would seem to be to enter into a dialogue with students about the real situations in all our lives and how we deal with them, and to share strategies and techniques. The point at which a student identifies a need or desire to learn a mathematical way of doing something, is the appropriate time to teach it. We need to persuade gatekeepers to courses and jobs to use methods of assessment which test real problem-solving skills, rather than the recall of calculation procedures learnt in school.

The maths required to understand other subjects are probably better taught as part of those subjects: the maths required for bricklaying integrated with the skills training. Because teachers of other subjects can lack confidence about their maths skills and knowledge, they sometimes prefer to allocate anything mathematical to maths teachers. Creating teams of teachers with complementary knowledge might improve this situation.

Abstract mathematics, on the other hand, could be learnt as an enjoyable activity, like literature, art or music. Some maths, for example various kinds of number patterns or set theory, are accessible to people without knowledge of other branches of mathematics and could be enjoyed by many more people.

Janet's story

Before I talk about my own experience of adults learning mathematics I should like to say that I share totally Dhamma' 5 commitment to trying to build on students' own knowledge and experience, and the strengths and insights these can bring to the learning experience. From what Dhamma tells us her experience has been wide and varied. Mine on the other hand has been restricted to undergraduates following non-mathematics degree courses and, more recently, with university employees such as librarians, secretaries and other similar categories: a small and very specialised subset of the whole set of adults learning mathematics.

My courses originated in the university when it was realised that many non-mathematical undergraduates were likely to experience difficulty when approaching graduation and seeking employment. Many were fearful about the numeracy tests they would face when applying for jobs so the original aim on setting up these courses was to try to prepare students for these tests, by helping them with the mathematics for whatever employment they might enter.

My courses, therefore, have two main aims. The first is to help such students towards independence in their mathematical thinking rather than feeling the need for somebody else to tell them whether their answer is correct or not. I see this as essential in employment. With this thought in mind a slogan for the course came to be 'Independence not tick dependence'.

The second aim is to base their understanding of their early arithmetic on mathematical principles rather than, as so often in school, learning isolated, apparently unconnected arithmetical techniques. This aim I call 'Principles not unconnected techniques'.

My courses are therefore radical in two ways from the students' viewpoint: that I want to change attitudes and to give them a mathematical framework within which to build their competence in number. The content of the course is geared to the latter, the managing of the course to the former. I offer the opportunity to consider a wide variety of methods and strategies, based on the principles developed, and I try to use these to help them to become responsible for their own learning. I set work but I do not mark it though I am always willing to discuss any difficulties encountered by participants as the course progresses. From time to time I ask them to take a new look at the specific techniques they learned in school both to enhance their understanding and develop the confidence they initially lack.

Since my course is strictly limited in terms of time available (normally ten one-hour sessions with, until recently, the opportunity to extend this to around twelve or fifteen sessions for those who need more time) it almost inevitably has to be planned on conventional lines. It is based on lectures rather than on other more informal teaching styles. It is, therefore, to a large extent tutor directed rather than being wholly in the hands of the student and is hence more prescriptive than I would ideally like it to be, albeit with a large element of student choice of calculation methods.

Nevertheless, my aim is to be responsive to students and to encourage them to participate and question in order that they might begin to be able to build for themselves the mathematics they need. In the undergraduate classes I have always been able to identify at least two kinds of student: those who ran into early difficulties with mathematics, and who have serious emotional problems associated with it, and those who appear merely to want a revision of what they learned at school. The former have, in general, been more responsive to the changes demanded of them while, though not always, the latter tend to be impatient of what I do, often feeling that I am wanting to go back to things they really think they already know.

In their course-evaluation comments some students indicate their position by such comments as, on the one hand, 'I liked the idea of stepping behind school work to discover why things work' and, on the other, 'I wanted to refresh techniques, not examine concepts and have my approach changed'.

Indeed I would say that, for those with serious problems, usually stemming from very early school experience, the extra time taken on the course is largely because of a need - and the opportunity I try to give to bring out their school problems so as to help resolve them. More than that, I am often able to discover something about their personal ways of calculating and to reassure them that these are valuable and to be built upon rather than, as they tend to see it, 'not the right way to do it'.

For some, this in itself can prove to be a crucial discovery from which they can then advance towards a new confidence. I had one such (mature) student who subsequently followed a PGCE course and, when in a school as a preliminary to that course, was delighted to have the teacher say 'Aren't you good at mathematics; I wish I was as good.'

There has always been a big falling away in attendance at these courses and it has been this that, with the reduced resources now endemic in educational establishments, has caused my classes to be questioned. Cuts in available finance required me to try to condense them into half the time hitherto allocated to them - an impossible task in view of their dual purpose. This itself has been a source of concern to me because the curtailed course did not allow me to achieve either of its two objectives. Nor was it satisfactory to the students as post-course evaluation sheets showed: 'There wasn't enough time to take in the ideas'.

In contrast, there has been little fall off in attendance in the staff courses and participants have been much more willing to enter into discussion than are the students. Prior to doing these staff courses I had always associated the difference between the students with a willingness, or lack of it, to respond to demands for change and the fact that the changes involved very early work in which some feel they are or should be competent already. So I tended to attribute the greater responsiveness of those with genuine and frightening problems to the course being an opportunity for a new start.

The staff courses introduced another variable into the problem for these were all people who were already in work and therefore knew what it required of them in terms of mathematics. This was not, however, an entirely new phenomenon because it also made me aware that some of the undergraduates are also people who have come late to a degree course and who may themselves have already had work experience.
So the problem of fallout is now more complex. Is it because of lack of general maturity or is it directly related to work experience? Or is it simply a question of the difference between student expectation arising from their prior school experience of mathematics; that their inner perception of mathematics is coloured by that experience so that there is a mismatch between my perception and theirs of what they need? Or is it that, because the course is not part of their eventual degree, students are unwilling, and indeed unable, to put in the effort required to complete the course as it necessitates work on it in their own time.

In view of the difference between the undergraduate and staff classes, it could be concluded that staff find it easier to adapt to a new approach and are more prepared to be participatory and less suspicious of being taken back to very early number work because they are secure in their perception of themselves while the undergraduates, particularly the ones straight from school, do not yet have that confidence in themselves as individuals.

But there is another element in the problem of the undergraduate fallout which stems directly from their motive in coming to such a class. It seemed to me that it could be the difference between those students who merely want to pass the employers' tests they will inevitably encounter and those who are more farseeing and want to make themselves competent to cope with any mathematical demands they may meet in employment.

In discussing this with a careers officer I was told a story about some naval recruits who were taken onto a ship and, after a few minutes of explanation about the workings of the ship, were told that they were going to have to pilot it out of the harbour. This experience brought home to them the extent of the knowledge they were going to need for the job they were training for and their tutors were consequently able to start their training with the simplest possible information which might otherwise have been unacceptable to them as far too easy and therefore not worth doing.

Clearly, in the environment of adults learning mathematics, where a much more informal and equal partnership is generally the aim between students and tutors, the expedient described above would not be appropriate or even feasible. Indeed, the whole question of who is to determine whose mathematics, or numeracy, is to be the one which pertains in the adult classroom appears to be denied by this expedient, where the assumption is that the mathematics taught is undoubtedly that of the tutor rather than the student.

But if courses are meant to empower students a prime element in them must be student involvement but, in the wide variety of courses available, this is sometimes not a prime focus. Where a tutor has student-empowering perspectives these may be unacceptable to students whose only experience of mathematics has been of the procedural, or technique based kind.

There is no doubt that such mismatches of expectations can occur between tutor and students, and sometimes in the same class between different students in it. How do we resolve this very real dilemma? It is my hope that, from this workshop where three of us are describing and trying to account for our personal experience within the sphere of adults learning mathematics, we shall all get a little nearer to resolving the problem of whose mathematics and how we foster its development.

Sue's story

This story is about the recent development of an accredited numeracy course at Sheffield Hallam University. The course is offered as an option to first and second year students. It is designed to equip students with what we have termed advanced numeracy skills and the confidence they need to support their current studies and in their future employment. The course recognises that many adults express panic and lack of confidence with mathematics and numeracy and aims to help students overcome their anxiety and develop confidence.

The name of the course, 'Advanced Numeracy', reflects the need for a 'legitimising label' in order to gain recognition and approval for accreditation from the institution. It also, much more importantly, reflects the complexity and status that we ascribe to the term numeracy. This advanced numeracy is not the utilitarian ability to perform basic arithmetic operations. The term 'advanced numeracy' has at its heart what Marilyn Frankenstein describes as 'The kind of mathematical literacy needed to clarify issues, to understand the structure of society, and to support or refute options (which) is more than the ability to calculate...' (1989). It embodies the ideas of critical mathematics and recognises mathematics both as a powerful tool and a tool of power. This image of advanced numeracy and the overt aim to address feelings of anxiety are reflected in our expectations of what a student should be able to do by the end of the course:

- formulate problems in mathematical or statistical terms read, discuss and write about simple mathematical and quantitative ideas and techniques, and talk confidently about these, explaining own reasoning
- use some strategies to overcome difficulties encountered with the mathematical and statistical tasks tackled collect, record and analyse data sets as a means of proposing answers to some types of problems they pose
- perform with confidence and understanding and, where appropriate, with the aid of IT tools, a range of techniques for problem solving in advanced numeracy
- know the constraints within which the solution to a problem lies verify that a solution is reasonable

- reflect constructively on their experience of learning advanced numeracy and the images of the subject which support or conflict with that experience.

The content of the course is not tightly defined, but is broadly selected according to the needs of the learners. There is an expectation that the students will participate actively and an emphasis on discussion and communication. These ideas are rooted in my earlier experiences in teaching adults on a GCSE equivalence course developed for the South Yorkshire Open College Federation and have been developed through working with students in higher education who express difficulties with mathematics and do not seem at all radical to me. However, some students and colleagues find these ideas challenge their previous experience of numeracy; they have different perceptions of numeracy and of teaching and learning.

Colleagues, outside the mathematics community, bring their own anxiety and life experiences and often think of numeracy as a procedural, mechanistic skill rather than one of thinking and problem solving. This viewpoint can result in difficulties with the image of numeracy as an essential 'core skill' for undergraduates and hence prefer to marginalise it rather than to integrate it with other core skills.

Students who volunteer their dissatisfaction with their own level of numerical or mathematical understanding and ability often have disquieting stories to tell of their own school experiences. Many, nonetheless, have a strong desire to replicate some aspects of these unsatisfactory experiences. One student praised previous teaching which was 'mechanistic' in nature whilst acknowledging that it did not succeed for her. In contrast another student, although technically competent and seemingly successful mathematically, expressed dissatisfaction because of a lack of understanding. His experience on a short course, which shared the aims and approach of the 'Advanced Numeracy' course, resulted in the comment 'It was as if I had blinkers on before ... this course made me look at things in a different way' and 'I got this tunnel vision of looking at things and it needed broadening... that was one of the things you did ... that was handy.'

As a teacher I have a tension between acknowledging and valuing the students' starting points which often are firmly rooted in a desire to improve procedurally and recognising that this can be what limits them. In terms of our triangle model my aim is to encourage and facilitate students' moves from a reliance on procedural school/college maths towards a confidence in their personal constructs of maths/numeracy. For me the third vertex of this triangle could describe mathematics with words like connectedness, pattern, relationships, problem solving, open-endedness etc. But it also needs to be a place-holder for words describing an approach to teaching which empowers and encourages decision making which I try to do through the content and delivery of the course.

As an example, a favourite starting point in algebra is a discussion of formulas for calculating the reading age of text. There are many versions of this task in print, one of which is in SMP 16-19 Mathematics book 'Problem Solving'. Here three reading age formulas are introduced and the student is told that each formula has a limited range of validity and an example of this is given. The student is then invited to choose a type of article or book and design and validate a method of assessing reading age. I choose to adapt this task to allow the students a bigger part of the performance.

(The fact that I am directing the students' performance is a tension for me. The students have a strong voice in negotiating the content of the course, indeed they are described as participants who are expected to bring life, study or work contexts in which they are involved, as a basis for developing personally relevant knowledge and skills. But the preparation, management and support for learning is high on my agenda as is the desire to empower students mathematically.)

Typically we begin by considering various texts and discussing why we find some easier to read than others. Many ideas are generated: too many long words, long sentences etc. The notion of formulas for the reading age of text is then introduced and at this point there is usually much debate on ethical and philosophical issues as well as the mathematical appropriateness of such formulas. The students proceed by looking in detail at one of the formulas, the Simplicity Formula, reading age = 25 - 15 p, where p is the proportion of words with one syllable. When this formula is applied to various texts - a children's story, a newspaper article, some mathematical explanation, a classical novel - the limited range of validity is soon noticed. A discussion of the usefulness of this and other formulas ensues. Through this we are able to think about algebraic concepts of variable and structure. There are many other vehicles for such an approach - wind chill formulas, Naismith's rule for calculating the time it would take to walk a planned route taking into account the amount of climbing involved. In essence they all involve a modelling approach in which the student has to think about the problem, decide on what are important factors and essential to the model, use appropriate mathematical analysis and test whether the solution is reasonable.

This is not simply studying mathematics in context thus de-focussing on abstract ideas in favour of more practical and relevant mathematics. In an increasingly technological world mathematics has become less visible for most. Having some ways of thinking about and accessing the underlying models of technological solutions is important. The student plays an active part and has considerable control over both the mathematics and the process.

The discussion

In the discussion period, after participants talked amongst themselves about their own attitudes and experiences, a number of relevant issues were raised which helped us to take our thinking further.

Participants shared our belief that the crucial need, in all sections of adult education, is to take account of students' existing knowledge and experience and their already established competencies, as well as their potential for further learning. They shared our experience that students' expectations can be different from, and sometimes in opposition to, those of their tutor and the discussion began to explore this apparent mismatch. Students' experiences of problem-solving and calculation are from both school and everyday life.

However, it is the school experiences which tend to be evoked by formal educational situations, and students' perceptions of what mathematics is appear to come mainly from their schooling. This undoubtedly has implications for work with adult learners.

The various dichotomies that can arise by considering our triangular model include those which arise from the conflict between our perceptions of mathematics and those created in the minds of students resulting from a procedural kind of mathematics common in schools. This conflict can be exacerbated for students who, in their adult life have developed problem solving and calculation strategies which they may not recognise as authentically mathematical because of the apparent mismatch with what they see as the all-powerful established mathematics.

All of us, and this was shared by participants in the session, see it as our responsibility to build on that mathematical expertise, while recognising that the widely varying experiences of our students and their reasons for coming to learn affect their approach to and attempts to learn mathematics. Moreover, we as tutors want to try and extend our students' views of what mathematics is, to a deeper understanding of the mathematical concepts that underlie its application.

The wide variation between our experiences as tutors also has a bearing on our perceptions of, and our way of coping with, our task of helping our students in their attempts to learn or relearn mathematics. While two of us are working in higher education, the other has worked largely in adult basic education. While much of our work is with classes we also work with individuals.

One of the issues which emerged from the session was that of the necessary distinction that has to be made between those students who are fresh from school and those who have had an intervening period of living and working. In our experience this brings an added, usually beneficial, dimension to the complexity of the adult classroom.

Other factors bearing on that complexity were seen to be those concerned with language and culture as well as other issues mentioned in the introduction. This effect can be compounded in classrooms where the students themselves may have either common cultural backgrounds or different ones but are less likely to share ours.

These inevitable differences and the different expectations and aspirations of students even within the same class can greatly complicate all the apparently more straightforward differences that can be found within a class and between that class and its tutors. All of these experiences can bring out different aspects of the complexities of teaching and learning.

The discussion revealed a consensus view that we need to respect our students' perspectives but that our responsibility is to move them forward in their expectations of mathematics.

We hope to move towards finding ways of operating that will take proper account of the very varied backgrounds of our students and the specific influences that motivate and sometimes cramp their attempts to develop their mathematical knowledge further.

Acknowledging the differences

We have learnt a great deal from the experience of running this workshop and writing it up: it has stimulated discussion which has helped us forward in our thinking. It has left us with a much clearer picture of both the essential samenesses within our work as well as of its differences. We have struggled to identify our shared meanings and to recognise the multiple differences within what we do. We have recognised different language usage as well as some of the subtleties we need to take into account in our pursuit of acceptable practice. The experience has revealed that there is a very wide range of students with very different expectations and different approaches to teaching them. But the effectiveness of these approaches for different kinds of students demands rigorous inquiry. We hope this experience reflects the aims of ALM as a whole.

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