Getting Unstuck in Maths: Building Mathematical Memory with Rapid Reconstruction

Jeff Simpson
Mastery Learning Systems, Ukiah, California, U.S.A

Haven't we all had students who understand multiplication, but just can't remember their times tables? And others who have accurate rote memory, but can't apply the facts they know -- or don't even understand the concepts behind them? Students taught with pedagogy that relies mainly upon explanations, rules, and mnemonic devices often lack understanding, and are unable to deal with the vast amount of memorizing required. Solving problems with rote-memorized processes and facts tends to make less intuitive learners experience maths as a grocery list of what somebody else knows -- that they have to remember. To them, maths becomes a stockpile of tricks and shortcuts to be used on command, and has little to do with actual mathematical perception and reasoning.

Words do not always get the idea across to the hard-to-teach. That is why adult basic maths books -- which sometimes distill several weeks worth of elementary instruction into a couple of pages of succinct explanations and examples -- leave many adults feeling confused. For the same reason, even our best verbal explanations and illustrations are still insufficient for some learners. In these exasperating situations, we may well think, "How can this person *not* get it?!" At least, that's how we feel until we remember that we, too, seem clueless in certain areas of our lives. Some of us simply don't function well in foreign languages, or are amazingly clumsy in sports, or just can't figure out how to program a VCR -- the list goes on. But we know that, even though we may seem stupid in certain areas, we really are smart. When we're struggling with our sensitive subjects, we don't like how it feels when someone tells us how easy it is, you just ...bla, bla bla... We don't want a pithy *explanation*; we want to be *showed*, we want someone to *do* it with us -- then we'll get it. And we want to be treated with patience, tact, gentleness, and respect for our intelligence. Most of us are, in some ways, "don't-explain-it-to-me" learners; we don't respond well to explanations, but do respond to guided doing. Our less intuitive maths learners also respond well to guided doing, and need to be taught with the same deference that we ourselves expect when working in our tricky areas.

Guided discovery is what people experience in foreign language immersion situations. The guy across the table from you points at the salt, and motions for you to give it to him while saying something you've never heard before -- which you naturally assume to mean, "pass the salt". Your host motions you to a chair, saying something which must mean, "sit here". They are creating contexts of meaning with words and actions. After experiencing identical situations a few times, you understand the words without the gestures, and respond with appropriate action. No translations are needed, no dictionaries, no definitions, and no explanations. Communication occurs without benefit of conjugating imperative verb forms, learning grammar rules for gender, or memorizing endless lists of vocabulary. Meaning is established through doing: brief words and brief actions go together to make you understand both what is to be done, and the words themselves.

For centuries, instruction has followed a basic formula: the teacher expounds a new principle, then the students practice applying the principle by solving problems. Guided discovery reverses this familiar process, and puts doing first, as a way of discovering the principle. To accomplish this, of course, discovery tasks must be so narrowly focused that the learners are bound to notice what needs to be noticed, and then generalize from their own observations. This style of learning is very engaging, and feels very safe. And it teaches learners to think for themselves, because the teacher is only telling them what to do and doing it with them, not summarizing meaning for them by telling answers or giving rules, shortcuts, and tricks. Meaning is established through brief words and brief actions.

Counting is the most basic problem-solving technique. Given problems which can be solved by counting, students notice what happens when they count groups of objects in certain ways. Purposeful repetition of similar discovery tasks enables them to remember what they have noticed, and to discern the principle involved in a way that is open to future application. This process takes more time in the initial stages, but saves time in the long run. Shortcuts, while they are intended to save time, actually confuse people who are unfamiliar with the mathematical territory, and leave them disoriented and incapable of making reasoned application. Guided discovery allows less-intuitive learners to take the long way first, only taking shortcuts later when they are ready.

Since academic verbiage is not always the most efficient vehicle for instilling an understanding of numerical relationships, current best practice often depends upon constructive experiences to communicate mathematical meaning. Touching and moving things, counting real objects, making projects involving measurement, putting together arrays of three-dimensional manipulatives -- all of these things help pupils find out for themselves what numbers are all about. Guided exploration, when it is well done, helps "don't-explain-it-to-me" learners explain to themselves what the teacher and books are unable to get across in words. But there is still another problem that must be addressed: understanding and remembering are not the same thing. Supposing that guided discovery does establish understanding, what process can be employed to ensure that students *remember* the mathematical relationships that they understand?

While words are admittedly not the most effective tool for leading hard-to-teach students to understanding, rote language mnemonic devices are certainly not the most effective instruments for implanting fluent *memory* of number facts, either. Music students can understand how the lines and spaces are used on a treble staff to represent notes, and can be thoroughly familiar with mnemonic devices such as **FACE** and Every **G**ood **B**oy **D**oes **F**ine -- and still not be able to recognize notes instantly, or even sight-read music at all. That is because memory and speed are usually developed in relation to words related to the notes -- rather than to the appearance and function of the notes themselves. So it is with maths. Even with a firm foundation of understanding bolstered by a bagful of memory tricks, many pupils still have great difficulty remembering maths facts. That is because, even in current best practice, memory is not usually developed in relation to tactile and visual mathematical contexts. Teaching understanding and building memory are conceived of as two consecutive and isolated operations, rather than as two aspects of a continuum of growing mathematical perception.

Conventional maths memory strategies are mainly language-based activities, and operate in a linguistic domain divorced from mathematical contexts. They assume previous understanding, but do not partake of the processes that built that understanding. Choral counting of number families, for example, assumes an understanding of periodicity, or assumes that understanding will be established through repetitive practice. But for the hard-to-teach, counting by fours with the whole class is just an exercise in instant imitation of their classmates' voices -- they may not even know where all those numbers came from, and they can't reconstruct the series of numbers by themselves! To them, it's a list of words to remember by rote, unrelated to groups of things that they have seen or touched.

The traditional use of flash cards assumes that understanding is already in place, and that the student's job is to practice remembering the words for what he has already perceived -- or that repetitious practice will somehow implant a functional understanding. Visualizing answers, rhyming ("six times *eight* is forty-*eight*"), repeated writing, repetitive speed-recall tests, verbal quizzing, and other traditional memory-building devices focus on words, and do not necessarily stimulate a recall of tactile and visual perceptions of mathematical relationships.

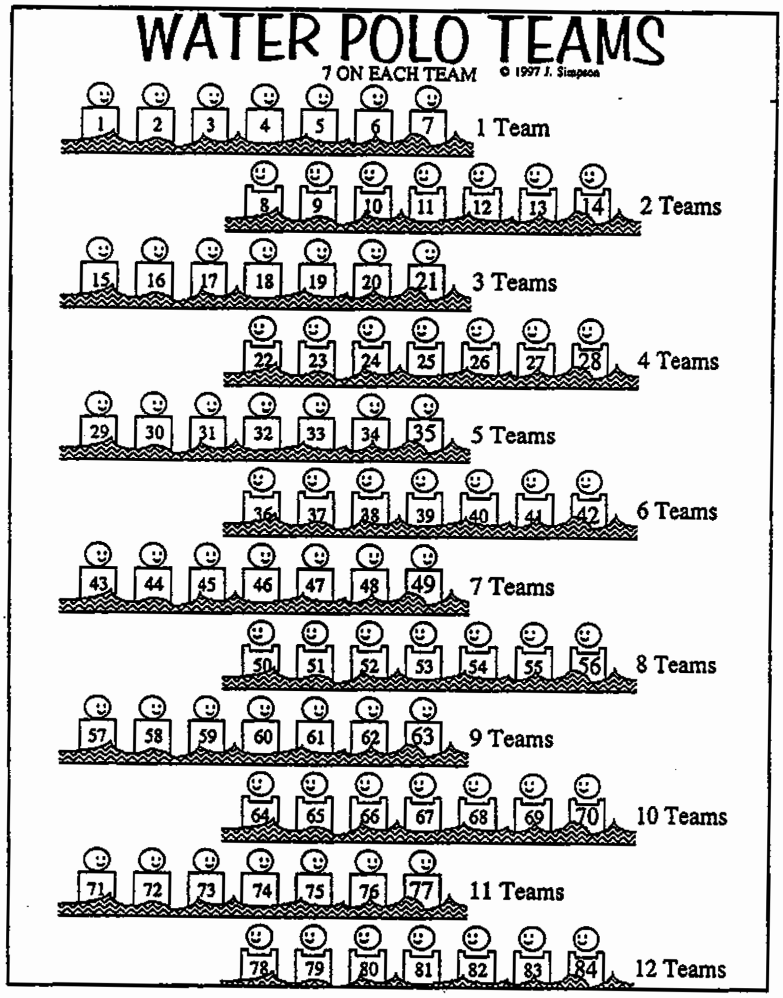
Those devices do not communicate mathematical meaning for "don't-explain-it-to-me" learners -- neither do they store memories that are context-related. Many learners simply can't remember lots of maths answer-words that are not clearly linked to immediate contexts which give them functional meaning. And they are unable to apply what little they remember to solve problems in thoughtful ways, because their memories have not been acquired in functional contexts.

Modern education no longer depends upon rote language devices alone to foster understanding, but does still depend upon them for developing memory! This is a little like driving around in your car *one day* to familiarize yourself with the streets of a new city, and then quizzing yourself on street names with 3x5 cards on subsequent days. Wouldn't it be more effective to keep driving every day until you were thoroughly familiar with all the streets? Of course it would -- but who has the time? And that is the problem with three-dimensional construction activities. It takes a lot of time to lay out a row of eight plastic blocks, another row of eight blocks, another row of eight blocks, and so on until there are seven rows; and then more time to count how many blocks there are altogether in 7 x 8. This time-intensive activity is great for establishing understanding, but it is not convenient for quickly retrieving specific facts. Constructed activities such as this are typically done *once* in the classroom, and abandoned the next day in favor of new activities. Students cannot usually take the objects home, so they only get to "drive around town" once in class. Thus, they get the general lay of the land, but don't know exactly where particular streets are. When a student doesn't remember the next day what 7 x 8 is, reconstructing rows of plastic blocks doesn't usually seem to be a practical option. So the calculators and rote memory devices are trotted out. And the students still can't remember.

But there is a solution to this dilemma. Until now, teachers have had two choices in resolving the memory issue: ignore it, and use calculators; or use rote language mnemonic devices that are inefficient, stressful, and unrelated to the process of mathematical exploration. However, there is now a third choice. A system of rapid mathematical reconstruction has been devised for developing memory that uses strategies that are directly related to constructive exploration: a maths equivalent of driving quickly around town until you know the locations of all the streets, *and* know their names. This simple method uses "manipulatives on paper" to establish fluent, context-derived memory in a cooperative classroom setting. The approach is tactile, visual, and oral; fast, accurate, applicable, highly motivating, and easy to take home for extra practice.

Here is how it works. The students are given a page that pictures numbered objects in a pre-constructed mathematical context. The *Water Polo Teams* page (Simpson: 1994: 10-17), for example, is used for teaching multiplication and division in groups of seven (see Example A). The pupils initially use the page for constructing an orderly understanding of the grouping process, and then re-use it for rapidly reconstructing the groups out of order as a way of developing a memory of specific relationships and facts.

To guide discovery, the pupils are given simple tasks that cause them to discover the number facts on the graphics page, in order. Using brief, polite commands, the teacher instructs the class to take a blank sheet of paper and cover all but the first team. The teacher's job is then to watch the students and make sure they do what they were instructed to do. One of the great pitfalls of teaching is, the "I taught them, but they didn't learn" syndrome. We teachers are often fooled into thinking that simply because we have expressed ourselves well on a given topic, the pupils have actually heard and understood what we have said. In every stage of a guided discovery lesson, the teacher gives directions for the students to *do* something. And then he watches them do it. The teacher does not give any explanations, rules, shortcuts, tricks, or answers, but does show the learners how to count to discover facts, and how to correct their own mistakes. The situation causes the learners to draw the correct conclusions.



Example A

When the teacher sees that everyone has one team showing on their page, he asks the students to silently count how many swimmers are on the team. Even though the swimmers are all numbered, some pupils will count them by touching each one with their finger. They must be allowed to do this; many adults are missing surprisingly basic entry-level concepts and skills. When everyone's fingers have stopped counting, the teacher asks everyone to tell how many swimmers there are. Then he tells the class to move their blank papers ("cover cards") to show two teams, and asks them to count how many are on the second team by itself; then how many are on both teams altogether. If anyone answers incorrectly, the teacher says, "No, count again." Students must find their own answers and correct themselves, not merely parrot the findings of others; and they must not be allowed to answer for each other. Requiring individual response is a way of promoting a needed sense of self-reliance, which restores self-confidence and erases fear.

Multiplication and division both deal with three basic questions: How many groups? How many in each group? and How many altogether? The teams page helps students formulate mental imagery that represents these three interrelated ways of counting. The page also gives the teacher immediate visible and audible feedback on the state of the students' thinking, which makes immediate correction possible. To a careful observer, the way in which students perform the tasks shows what they know, what they don't know, and what needs work. Intervening unobtrusively, the teacher adjusts tasks when necessary, guided by this all-important question: "What does the student need to *do*, so he can get it by himself?" No more "I taught them, but they didn't learn it". A guided discovery lesson not only enables students to learn new concepts and facts -- it requires them to!

The learners continue down the page one team at a time, discovering the three "how many's" for each multiple, moving at a pace appropriate for the group. The teacher might ask pupils to think about how many players will be on the next number of teams *before* moving the cover card, only moving it to check the accuracy of their thinking. After all the teams and players have been enumerated in order, the directions are changed so that the students rediscover the facts out of order. They are asked to move their cover cards as quickly as possible to show three teams (for example), and tell how many players; then eight teams, zero teams, and so on. This is also a tactile way of establishing a sense of estimate, and promotes familiarity with the number facts in a non-pressured way.

So far, all the directions have been verbal. Now the learners are given a page of problems (Simpson: 1994: 10-18) that cause them to go back to the graphics page and rediscover the number facts out of order again (see Example B). The students have just finished working

WATER POLO TEAMS

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name:

date: date due:

final time:

corrected by:

Progress Chart
1:00 :45 :30 :25 :20 :15 :13

Water polo teams have seven swimmers. Practice until you can say all the answers in 13 seconds. Use the *Water Polo Teams* chart to help yourself.

1 team = ? swimmers	8 teams = ? swimmers	5 teams = ? swimmers
9 teams = ? swimmers	0 teams = ? swimmers	7 teams = ? swimmers
3 teams = ? swimmers	11 teams = ? swimmers	2 teams = ? swimmers
10 teams = ? swimmers	4 teams = ? swimmers	12 teams = ? swimmers
	6 teams = ? swimmers	

Answer Key:

1 team = 7 swimmers	8 teams = 56 swimmers	5 teams = 35 swimmers
9 teams = 63 swimmers	0 teams = 0 swimmers	7 teams = 49 swimmers
3 teams = 21 swimmers	11 teams = 77 swimmers	2 teams = 14 swimmers
10 teams = 70 swimmers	4 teams = 28 swimmers	12 teams = 84 swimmers
	6 teams = 42 swimmers	

Example B

these very problems; the only difference is that now they are being cued by print. This is a significant difference for many learners, who are used to being confused and frightened by written problems -- and may even have trouble reading at all. But these thought problems only contain two words, which makes them possible for everyone. There are two other reasons for pre-working the problems with oral directions first. It provides another occasion for rediscovery, which gradually begins to build memory without conscious effort; and it ensures that the learners will succeed with the print activity. Providing this hidden safety-net of guaranteed success helps the students gain trust in their own ability. At this stage, the pupils will not yet remember all the multiplication facts, and may still make some mistakes. But there is no reference to rote-memorized answers or to a standard multiplication chart (which is really a table of somebody else's answers) to make corrections. In order to retrieve unknown answers, students must rapidly reconstruct the whole mathematical context with their eyes or hands. Thus all memories are developed in a conceptual and contextual framework.

The teacher tells the learners to fold the new page (Example B) in half, and set it down so they are looking at the side with the question marks. Someone reads aloud the first problem:
"1 team = ? swimmers". The teacher says, while demonstrating, "That means use your paper to show one team." The students do so. "And how many swimmers do you see?" The pupils answer. "What would you do to answer the next question?" The students move their papers to show eight teams, and tell how many players there are. "Answer the rest of the questions silently by yourself." The teacher watches and helps, if necessary.

"Who would like to answer the first three questions aloud (across the top row)?" Thus the teacher begins the "inviting process" that begins with the students choosing to challenge themselves to do what they know they are capable of doing -- and ends with reliable and rapid recall of all the sevens multiplication facts on command, without reference to the graphics page. "Who will do the second row? It is not cheating to look at the *TEAMS* page; the more you look and touch, the easier it is to know and remember." Seeing how easy it is, someone volunteers for the second row, and someone else for the third, and then the fourth.

The instructor invites a volunteer to try the whole page aloud. The teacher nods or says "Yes" or "Good" for correct answers. When there is an incorrect response, the teacher says, "No, count again." When the student has finished the whole page, the teacher complements him on getting a perfect score. Even if the pupil has made mistakes, he gets a perfect score because he is required by the situation to correct himself on the spot -- and credit is given for self-corrections. The teacher asks for another volunteer to do the whole page, and asks that student to select a partner to confirm his accuracy. The "checker" looks at the answer key on the other side of the paper and says yes or no, but may not give corrected answers. Meanwhile, everyone else in the class is getting repeated exposure to the targeted number facts -- but still in a physical and problem-solving context.

Having modeled the partner activity, the teacher asks everyone to find a partner and take turns doing the same thing. Students who were too shy to volunteer in front of the class are now free to perform individually, shielded by the anonymity of the class noise. Partners may work in languages other than English, if desired, and can use similarly structured pages to work ahead of the class or to catch up. Students do not even have to be working on the same page in order to work together as partners, and those who cannot tolerate working with a partner can use the answer key to check their own work. Reconstruction becomes increasingly rapid, memory more automatic, and 100% scores continue to build self-esteem. Because feedback and self-correction is immediate, whatever students remember is remembered correctly. The partner testing process also confirms for the teacher that the pupils are actually learning what is being taught. There are no written responses, so no one has to spend time correcting papers; and the same paper can be reused for as much individual practice as is needed.

The partners take several turns answering all the questions, going for smoothness and speed, answering the problems in different orders, to defeat the accidental use of rote language pattern memory. Although the graphics are always available for self-correction, the learners gradually and naturally wean themselves from using them, only looking when they need to. Some learners, of course, take more time to go through this process, and some less. The guided-discovery/rapid-reconstruction process is equally effective for special needs-learners, prisoners, students who are not proficient in English, and ordinary school drop-outs. The difference is in the number of steps included and the degree to which they must be made explicit. The directions given in this article address students with the most difficulties, to make the fail-safe nature of the process apparent to the reader. Less needy students can skip certain steps and advance more rapidly. It is the teacher's job to observe the students and make sure that each one proceeds in a way that requires individual mathematical thinking, and feels smooth, easy, and joyful.

The process as described so far takes about ten to fifteen minutes. When the students feel safe in the knowledge that they know exactly what they're doing and are performing accurately, the instructor invites a volunteer to be timed while answering the questions. At this point, there is no emphasis on speed; the stated goal is just to find out how long it takes a person to get a 100% score. Rather than saying "Go!", which feels like a race and tends to induce tension, the teacher simply begins timing when the pupil gives the first answer. If the student takes thirty-nine seconds to say all the answers, take his page and circle 1:00 with your pencil on his Progress Chart, and say, "You were much faster than one minute." Circle :45 as you say, "And you also beat forty-five seconds. You got two speed levels on your first try." The natural reaction of most pupils is to want to try again so they can go faster. People like doing well what they know they can do. They want to go for the faster times without being pressured to do so, and will work until they can achieve the fastest target time. The "inviting process", applied to rapid rediscovery, leads students to voluntarily develop a fast and accurate mathematical memory.

Seeing that the situation is low-pressure and knowing that they can succeed, other students volunteer to be timed. If the second student goes faster than the first, the first pupil's desire to practice for increased speed grows greater. Thus begins an anxiety-free, infectiously fun game that stimulates the learners to get faster and faster. This process provides the students with pleasure and self-confidence, but its pedagogical purpose is to unobtrusively, naturally, and finally wean the students from the graphics while their imaginations are being stocked with tactile and visual imagery; to go as fast as they want to go, they have to not look at the graphics. At this point, they are ready for another page which reverses the phrasing of the questions: "56 swimmers = ? teams". Neither this page nor the previous one say anything about multiplication or division, but the students are learning the basic concepts before encountering specialized vocabulary and mathematical signs. At the end of this pleasant work-out, students have a good grasp on reasoning with groups; a fast, accurate, and context-embedded recall of targeted number facts; and a logical sense of how to apply them. And they are ready to intelligently deal with more abstract manifestations of the multiplication/division process.

Having modeled the entire procedure in class, students can then replicate it at home or in class and easily adapt it to new settings. A wide variety of graphics can be used to clearly represent the functions of multiplication, division, addition, subtraction, and all four operations with fractions, decimals, and pre-algebra. These functional illustrations can be entirely pre-constructed, like the water polo teams chart, or more loosely constructed in ways which require the learner to organize or construct the elements. They can employ linear, circular, wave-like, and blocked formats -- all of which are useful for picturing problem-solving in suitable ways. These "manipulatives on paper" put function before nomenclature, allowing "don't-explain-it-to-me" learners to explore mathematical relationships and concepts, make meaningful generalizatons, and build a reason-based memory without being confused or frightened by technical jargon and premature abstractions. And they teach computation through problem-solving, reversing the traditional order that has proven to be so difficult for challenged learners.

The system is logical, methodical, and works well with students whose ability to remember maths facts and concepts is not stimulated either by old-fashioned methods or by current best practices. The approach is simple enough that any student with a finger, an eye, and the ability to count can succeed with it. It has already proven to be effective with thousands of poor urban low-achieving children and adults in America, both mainstream and special needs, in schools and in prisons.

Complicated though it seems to be at times, maths is always about one thing: counting. The guided-discovery/rapid-reconstruction process successfully reduces basic maths to its simplest elements, allowing students to quickly *count* groups of objects, *notice* the patterns and relationships that the counting reveals, and *remember* the things that they noticed. The method is not reliant on wordy explanations and language-oriented mnemonic devices, and succeeds where those fail. But it does depend upon meeting the students where they are in their understanding, and proceeding methodically with the learner's perspective clearly in mind. The rapid reconstruction process extends the modern practice of constructionist activities in practical ways, enabling pupils to remember what they have done, rather than forcing them to recall what they have tried to memorize. It is a mathematical way of building mathematical memory, and uses a unified process to integrate memory with understanding and application.

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