Managing Change

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Unlike my contributions to ALM1, ALM2 and ALM3, this contribution took the form of a workshop rather than an account of ongoing research and related work. This one, as did the others, builds on the earlier ones so that it comes as a definition of practice in my teaching of undergraduates doing degrees other than in mathematics who, because of lack of use since leaving school, have tended to find that their knowledge of early number work has slipped away from them. Some of these students have serious mental blocks about mathematics but all of them have to face employers' numeracy tests when, on graduation, they start to look for employment.

My courses have a twofold purpose: to change students' attitudes to early number work in order to give them a different perspective on it and, because they will be in employment when they use the material they have been meeting, to help them to make the mathematics their own so that they will no longer be looking for the teacher's tick to tell them whether what they have done is correct. Instead they should be confident that they can tackle any kind of mathematical problem which might arise in the course of their work in whatever sphere that may be.

To effect the first purpose, I try to give them a mathematical framework within which the material fits so that, instead of practising and perfecting a series of disconnected techniques, they will learn to see connections between topics by means of the few general mathematical principles on which the work rests.

For the second purpose, I aim to put them in charge of their own learning and through this to gain a confidence that is independent of 'having their work marked'. So I never mark any work they do. Instead I try to show ways in which they themselves can check, and hence be confident about, what they have done.

The course starts with them being given a test compiled by a colleague in another university's Careers Department on the basis of current tests in a variety of types of graduate employment. I call it a 'test with a difference' because it is not meant to be done in the normal way of tests. Instead they are to go through it, item by item, recording two things about each: how they feel when they read it (panic, easy etc) and whether they think they can do it (tick), are sure they can't (cross) or are not sure (?).

This way, at the end of the test, they should have a good idea of which topics cause them anxiety as well as which they think they can or cannot do, or are uncertain about. From this they should have a good idea of what they need from the course. I suggest they repeat the test at the end of the course to judge for themselves what it has done for them.

(The workshop gives the test I use for this but there is not room to reproduce it in this account of the session and in any case, colleagues who might want to adopt this form of test would probably prefer to create a test more appropriate to their own particular students.)

My next course of action is to examine their knowledge of the process of subtraction, first by examining their school taught method (many are surprised to discover there is more than one acceptable method), then by asking them to do the same calculation in their head. The second part of the exercise brings out a wide variety of 'own methods', many of them well known alternative subtraction methods, some of them personal and unique to the student concerned. What is significant is that all students reveal that they thought these methods were unacceptable. They did them secretly at school and then tried to fit what they did in their head to the standard way the school had taught them. It was largely this feeling that their own methods were 'the wrong way to do it', combined with their having forgotten the taught procedures, that had led to their very common feelings of inadequacy associated with mathematics. So my first job is to stress the importance of these personal methods, showing them the principles behind their own individual method and emphasising their authenticity both within mathematics and for themselves.

Because the development of mental methods for doing mathematical calculating is to be a major feature of the course, this provides the first opportunity for them to start to move towards the self-confidence that they will need in employment.

Following this exercise I develop the crucial principle on which the whole of the course rests: the idea that **inverse operations undo each other.** Contained within this principle is the fundamental characteristic of our number system, its place value property, and the consequent 'rule' about what happens when we multiply or divide by ten. I question them about what happens when you multiply by ten and more often than not I get the primitive answer that 'you add a nought'. I ask them about multiplying a number containing a decimal by ten and usually get the answer that 'you move the decimal point'. I suggest that it might be better to have a rule that applied to all numbers, both whole and decimal.

We examine the processes of multiplying and dividing by ten, producing a zig-zag down a page thus:

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   7   x 10
  70   x 10
 700   x 10
7000   ÷ 10
 700   ÷ 10
  70   ÷ 10
   7   ÷ 10
     7 ÷ 10
     07 x 10
     7 x 10
    70 x 10
   700
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On being asked to describe what has been happening, they tell me that the 7 has been moving to the left when you multiply by ten and to the right when you divide by ten. I represent this in diagrammatic form because it is easier to remember the visual image involved rather than remembering its rather lengthy verbal form.

x 10 <--
÷ 10 -->

I point out that this now gives us a rule, not only for multiplying both whole numbers and decimals by 10, but also one for dividing by ten, thus, in one simple diagram giving double the information originally sought. I emphasise that this principle will feature prominently in all we do on the course.

At this stage we discuss other mathematical operations and arrive at varying numbers of pairs of inverse operations, depending upon the level of mathematics they have previously reached.

These principles (the general and the specific aspects of inverse operations) are then applied to looking at the four basic operations of arithmetic but, before we embark on this exercise properly, I ask them to estimate the sum of two two- or three-digit numbers and we establish that the traditional written method for such examples does not correspond with the way that is natural for finding an estimate.

For an estimate it is normal and natural to look first at the left hand digits to arrive at an approximate answer whereas, in the standard written method for addition and subtraction, it is normal to work from the right.

For addition I recommend always working from the left and tell them this is the only positively prescriptive recommendation I will make, for the other three operations the choice of method will be theirs. Some accept this easily, others find it more difficult to discard the traditional right to left addition convention.

Indeed, in one case, a student said that addition was the only thing she felt she could do and to have to discard it would take away from her the only thing about mathematics in which she had any confidence at all. She couldn't do that she said. I didn't insist and later she came to me and said she had finally plucked up courage to try it and had found that it was more efficient than the traditional way.

(It is impossible in this account to go into the details of the suggested procedures for all the operations but they all relate to inverse operations and, in particular, multiplying by 10, 100..., 5, 50 .... and 2.5, 25 etc.)

We proceed from the four operations to establishing the connection between fractions, decimals and percentages, topics they have learned at different ages at school without seeing their close association and the power this gives them if they can change quickly between them when they encounter problems relating to them.

My method of procedure is to encourage the use of mental methods, both for an initial estimate for any required calculation and for carrying out the calculation itself. As the course proceeds, we discuss the ways in which the estimate can be used: to check any calculation however done (especially if it is with a calculator) and, eventually also, to consider the reason why the estimate was either more or less than the exact answer. We also discuss ways in which we might use the estimate to get the exact answer in some circumstances. All this is intended to enhance their number awareness and to increase confidence in their answers when reached.

The general conduct of the course is through interaction and, often, by challenging their school methods, sometimes before establishing more efficient methods through the use of the inverse principle, sometimes to try to encourage awareness of the sources of those school methods in order to produce a generally more reflective and critical attitude based on a realisation of the structure of the standard methods instead of a blind acceptance of them from the teacher.

The second part of the course is devoted to a close examination of percentages, and in particular to investigating the many ways in which the general public, and sometimes official bodies, reveal an inadequate understanding of the principles on which procedures for working with percentages rest. The inverse principle is especially useful at this stage of the course because of the many misconceptions about percentages in the community. Here are two examples of such misconceptions:

- Arthur Scargill, during the 1984 miners' strike spoke of the increased efficiency achieved by the miners. He said 'Our productivity is now 350% more than it was before, showing that we were 350% less efficient then'.

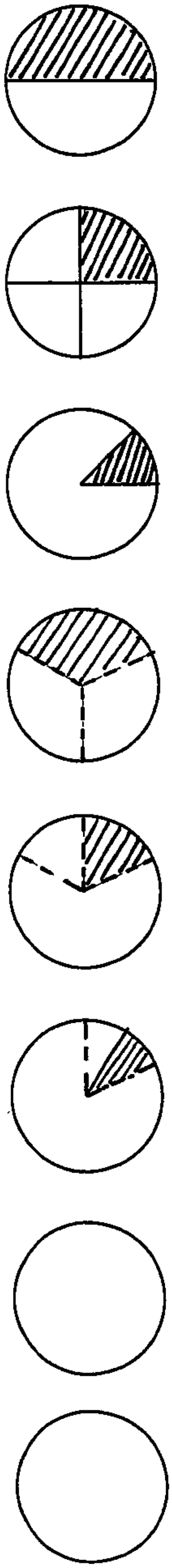
- In a report about a shortage of consultants in the NHS it was said that 'there are now twenty times fewer consultants than there were a few years ago'.

In addition there is some examination of diagrammatic representations of these ideas to give them confidence about looking at the many such representations as occur in a variety of different contexts. For example, besides creating a table of equivalences between fractions, decimals and percentages, we establish that, in the pie-chart types of graph common in both professional and everyday circumstances, there is also a visual relationship to be established which can aid estimates about relative values in a variety of settings.

For example, half a circle or 180 degrees represents a half, 0.5 and 50%, a quarter circle or 90 degrees represents a quarter, 0.25 or 25% etc. The ability to assess the fractions, decimals or percentages represented by a specific sector of a circle can positively influence the ease with which such diagrams can be interpreted (See Figure 1).

For some this course is very demanding, demanding both in its insistence on a radical new look at very early arithmetic and in the style of teaching being used: not of transmission but of suggestions based on principles which it is up to the individual to incorporate into their own natural ways of thinking. But that very act itself can cause problems for some, habituated into expecting everything they learn to come from the teacher so that all they have to do is to absorb what is given them and be able to reproduce it on request.

Instead they are given choices, have their perceptions challenged and need to come to an entirely different perception both of the material itself and how they need to operate with it to become the independent adults employment will expect them to be.



$\frac{1}{2} = 0.5 = 50\% = 180^\circ$ or π

$\frac{1}{4} = 0.25 = 25\% = 90^\circ$ or $\frac{1}{2}\pi$

$\frac{1}{8} = 0.125 = 12\frac{1}{2}\% = 45^\circ$ or $\frac{1}{4}\pi$

$\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\% = 120^\circ$ or $\frac{2}{3}\pi$

$\frac{1}{6} = 0.1\dot{6} = 16\frac{2}{3}\% = 60^\circ$ or $\frac{1}{3}\pi$

$\frac{1}{9} = 0.04\dot{3} = 4\frac{1}{3}\% = 40^\circ$ or $\frac{\pi}{6}$

Figure 1

I often find that those who initially appear to be the least able in terms of mathematics, some having massive 'blocks' and senses of inadequacy, sometimes generated from primary school experience where requests for help towards understanding of the processes being presented to them were met apparently by the injunction 'It's not for you to understand, just do it', often respond much better to the course than do others who are apparently less handicapped mathematically. Such students are so relieved to find that their own ideas and own methods are in future to be valued that they lap up the new diet.

Others, who perhaps did not have so much difficulty with the experience they had in school, and who therefore do not feel so bad about themselves, are often only looking to being reminded of those school methods, only half forgotten. These students often resist the change the course demands of them, perhaps because they feel disinclined to undertake radical change when they feel they only need to be reminded of what they once knew.

The workshop itself presented a lot of material about student attitudes revealing these different approaches to what is on offer and participants were invited to consider these. It also contained multiple examples of the inconsistencies extant in the community generally and posed questions from the course designed to bring out anomalies, misconceptions and significant mathematical examples of these from the press and media.

The course ends with a self-assessment sheet of real life anomalies derived from the media which the students are asked to make comments on in the light of what they have learned on the course.

I will give just two examples of these:

What percentage increase in tax was represented by the increase of VAT from twelve and a half percent to 15%, from 15% to seventeen and a half percent?

Do the following all mean the same, all different or are two the same and one different and, if so, which is the different one?

three times (as much as)
three times more than
increased by 300%

Their responses are accepted at the time of presentation and then the three statements are re-assessed after the relevant work on percentages is completed.

In my research into learning where my colleague and I (Duffin and Simpson 1993) have identified all experiences as of three main types: natural, conflicting and alien, and have given possible responses of learners to each of these three types of experience, we have begun to feel that most learners and teachers (who are of course also learners) fall into two main categories. We identify these as being those who are satisfied to learn things as discrete entities unconnected with each other while others feel the need to learn things in a connected way.

From the evidence I get from the students who come my way, many of them have been taught their early mathematics in the disconnected way while they themselves would prefer to be able to connect their learning. In my teaching I am teaching in a connected way because I feel that, for the purposes of the course and the future of the students, this is the best way forward. But it most assuredly does not suit some students who still want to get their information in a discrete way and feel frustrated at what I have to offer. Student comments indicating these are included in the material. Here are just two such comments:

'Some of the earlier sessions were all about things we already knew and we felt frustrated but you have to go to the end of the course to see how it all hangs together and why we had to do those things.'

'She asked us for our methods and then showed us lots of better ones; it was a waste of time. I learned more from a scientific friend in half an hour than I got from anything she did.'

My research colleague and I puzzle long and hard over this problem as we recognise that our own natural ways of learning mathematics are different. We puzzle too about whether people are born with a tendency towards one or other of the two ways of learning or become that way because of their own early experience of learning.

Certainly this is a problem in my classes. I don't know the answer to this nor how I can even begin to cater for those who do not want to learn in the way that I want to teach. It is an ongoing problem and is why I called the workshop Managing Change. Some of the discussion in the workshop centred round this problem without coming to any firm conclusions.

Reference:

1. Duffin, J.M. and Simpson, A.P. (1993) Natural, Conflicting and Alien Journal of Mathematical Behaviour 12,4.