University pedagogy - how social scientists make mathematical meanings: 'Before I went into the exam my friend said do the brackets first..'

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Introduction

These are some findings of a small study of a group of social science degree students, and the ways in which they dealt with the need to demonstrate, in an examination, some basic mathematical and statistical skills. The theoretical context of the study is in the branch of maths education known as "situated cognition", which derives from the works of the social anthropologist Jean Lave. These have attracted much attention recently in the UK, following seminars in Oxford, and represent a shift from the purely psychological analysis of skill acquisition - which separates the learner from the learned - towards studies of learners in a broader social and, I would argue, institutional context.

Lave's pathbreaking book, Cognition in Practice (1990) analysed in detail the mathematical practices of adults faced with "best buy" problems in supermarkets and in a formal test of "school maths". The discontinuities in success are striking: writing on the difficulties adults had with multi-stage calculations in tests, she states: "The rules for transforming problems in school lessons, learned as formulae, mainly by rote, seem very different from the ubiquitous and successful transformation of problems in the supermarket. The latter do not appear to involve formulaic rules at all" (page 62) and "that they struggled to produce half-forgotten algorithms for fraction and decimal transformations during the math test (suggests) that they may not have integrated school-taught algorithms as the "method of choice" into a number of situations in their adult lives."

Lave demonstrates in a variety of ways that investigations of the transferability of mathematics across situations are consistently negative. If this view is correct, it has massive pedagogical implications, since it calls into question our (teachers') natural expectations that we are effective. As Lave and Wenger say: "What gets learned is problematic with respect to what gets taught." (page 40). My first focus is then on students' assessments of what they needed to do to succeed at maths. Findings about the success of informal mathematical methods are repeatedly reported in the literature, and not just among adults. Baker's (1996) findings on children's formal and informal numeracy practices, again carried out from within the ethnographic tradition, are summarised thus:

> "The research explored the part that contexts and culture (values and beliefs) play in numeracy practices, and showed that learners' concern with their performance, in order to get praise, dominated their practices: yet as a motivating factor performance does not drive curiosity or fuel the desire to solve a problem" (page 80).

This raises the further issue, crucial in the present study, of the motivation and reward system for learning mathematics at University, especially for students who may have experienced sustained educational failure in the past, and the analysis of motivations forms a second strand to this paper. These all call into question the very nature of the mathematics taught in our classrooms, which, for my purposes, include university classrooms. It seems, on the basis of anecdotal evidence of generations of adult students, that, for a very large number, the practices of school and college maths classrooms are rather ineffective at giving students skills and knowledge which they can adapt, apply and use in the myriad of situations in everyday, work and further learning contexts in which they are needed. Indeed this is reflected in repeated debates, one currently initiated by our new government, drawing attention to low standards of numeracy in young people.

Furthermore, classroom maths practices may also have the effect of negating the value of the successful mathematical strategies which learners use in real situations. These are informal, often oral, sometimes practical (using fingers), mediated and making sense within specific contexts, particularly work contexts in which mathematical practices are strongly influenced by technology. These have been referred to as "mathematical craft knowledge of concrete particulars and instances" (Ernest, 1997). This is connected to the incontestable fact that learning mathematics, for many students, is clouded by high levels of anxiety and lack of confidence. There is a considerable literature on the affective aspects of learning mathematics, summarised usefully by Evans and Tsatsaroni (1996), starting in the UK with Buxton's book. (1981)

The University context

Most students at UNL take a mathematical component as part of their programme. The present study is of a particular group - students on the Applied Social Science degree. These were chosen because there is no formal mathematical prerequisite for entry into the programme, so they represent the maximum diversity of mathematical attainment and confidence. The unit is known as Data Analysis and its content and outcomes could be summarised as functional numeracy; an ability of display data in a range of forms, and an ability to calculate descriptive statistics, including averages and the standard deviation in a variety of social contexts, most of them invented. The unit is taken by 350 students in total, and a very high proportion of these are mature returners to education. The teaching and learning of this unit has, it seems, always been problematic. There has been considerable resistance from students following pathways in, for example, Cultural Studies, to engaging with Data Analysis at all, and this was strongly borne out in the interviews. However, in practice the real cause for concern is that the institution, for very practical reasons, measures outcomes by a single and very crude criterion - exam passes, and that these exams or other assessment instruments seem to provide the major or only motivation for learning maths for many students.

The historically low pass rates for this unit led to an attempt to redesign the course to begin to take into account some of the diversity of the student body, which has not been well served by the didactic, teacher-centred model of teaching used previously. In addition it was felt that degree level students should be able to learn in an autonomous manner, taking responsibility for their learning. A considerable block of staff time was devoted to writing up the course content into a user-friendly self-learning package which included exposition, context, self-assessment activities with detailed feedback and examination-type questions, also with feedback. Students were exhorted to work their way through the materials, at a suggested pace, to cover the 12 week semester. The booklet, contains many suggestions for group activities and study skill hints. Drop-in staffed sessions were provided but these were designated as "optional" in the expectation that students whose previous competence allowed them to learn the skills and techniques without difficulty would not need to use them, but others with greater difficulty would be able to benefit from sustained individual and small group tutor support. It was hoped that the course redesign would enable tutors to be able to pay attention to the needs of individuals who were experiencing difficulty and to actively shift their understandings of success at mathematics towards a more holistic view. This context provides a very severe constraint on what is learned and how the learning can be expressed.

A further issue for University students recalls the general debates on transferability. Particularly in new Universities, the curriculum is shifting markedly away from the transmission of a body of knowledge towards a notion of preparing students for a generic state of graduateness and employability. Much attention is paid to attempting to define these qualities but inherent in all of them is the idea of personal transferable skills, such as communication, problem solving, information gathering being somehow embedded in the curriculum. The recent Dearing report on Higher Education (1997)identifies Numeracy as one of four 'key skills'.

A model for university maths learning

Studies of real life and workplace maths practices have, as we have seen, emphasised the discontinuities between the explicit mathematical knowledge of the classroom and the tacit maths knowledge held and deployed by the individual in her functioning social context. I have summarised this polarisation in the following table, following Ernest (1997a) and others.

explicit maths	tacit maths
consists of:	
definitions abstract knowledge standard tasks, exercises, algorithms and symbolic manipulations, provable, assessable, warranted knowledge surface learning	methods, approaches, strategies knowledge of exemplars from past experiences, situated in context of acquisition and use functional numeracy, estimation strategies, confident, assimilated knowledge, deep learning
motivation is	
extrinsic, provided by exams and other formal assessment	intrinsic, provided by interest, usefulness, enjoyment, etc.
is also referred to as :	
instrumental understanding i.e.: knowledge of rules and how to apply them, memory-dependent tasks, rote learning	relational understanding i.e.: understanding what to do and why, assimilate into appropriate schema (Skemp)(1971)
is characteristic of::	
most university curricula which teach maths as a set of (easily measurable) skills mathematical skills/abilities generally assumed to be transferable between contexts unique subject separate from knower	increasingly of school maths, GCSE and A level, as syllabuses become more contextualised, technology based, etc. approaches, problem solving strategies, confidence create the *possibility* of transfer. known and knower related, situated knowledge

The student interviews

After the examination, groups of students were interviewed. Students who had failed the examination were invited to attend group sessions to look at the reasons for their failure and (and this provided the motivation for their inclusion in the study) to receive some help and guidance and strategy for passing the resit examination. These sessions (3 in total involving about 20 students) were taped and transcribed.

Analysis of results

Firstly I want to look at the type of maths knowledge talked about by the students in the interviews. To what extent did they indicate a desire or an intention to acquire either explicit or tacit mathematical understanding? Secondly I want to examine the major motivational factors which circumscribed their learning. It can be argued, and studies of workplace mathematics have demonstrate this, that mathematical techniques can be learned effectively when learners are engaged in socially meaningful tasks which provide motivation, and regulation for the task. In a University, the motivational context is the narrow one presented by the end-of-semester exam. In a school, the perspective might be rather longer, and in other contexts such as shopping or work calculations, might be quite open ended. I was interested to see how the two groups saw their motivations to learn maths - did it exist outside the exam? Had the motivation been generated at some earlier stage?

(a) Interview evidence: explicit or tacit?

The students were looking at slides of the exam they had failed. They were looking at questions such as:

12 ÷ 4 + 2 =
20 — 4 ÷ 2 =
(16 — 4) X 3 + 1 =

They commented:

"I remember BODMAS; something to do with brackets first".
"couldn't remember , mind went blank, all I could remember was the stupid BODMAS"
"Yeah, I remember, he was going dedadedadeda and it was my first time. I didn't know what was going on."
"Before I went in to the exam my friend said do the brackets first."

These comments all show students searching for a key or formula, something they needed to 'know' or 'remember' to help them solve an unfamiliar problem. The 'rule' which might have helped them is sometimes known as BODMAS, but it did not seem to have made sense to any of these students and their knowledge seemed to be limited to a recall of the name of the acronym.

The second group of questions concerned correcting decimal numbers to a given number of decimal places or significant figures. The comments included:

"I thought you moved the decimal points further up. I remember when I was at school we used to do something like that."
"My problem is what to do with the fractions on the other side. So you look at it in a bigger context - the whole number, before the decimal." (surprised)
"I'm confused - there's so many solutions to a problem."
"Is it the way it's written down you ? Oh...I get a decimal point..." (disappointed).

This second group of comments again focuses on the detail of the symbols while losing the wider meaning of the numbers. The focus is very much on the individual operations needed, they were searching for a recipe which always worked.

"I'm all right with stuff I think I can use day to day in every day living but ... like this positive and negative numbers I could not understand the relevance of it, you know, I still don't" (F2)

This student really made explicit how negative the whole learning experience was!

A further question asked students to calculate mean, median, mode, standard deviation and adjusted frequencies from a data set presented as a frequency table. Their comments included:

"I have a rough idea but there are so many things to do: the mean, the adjusted frequency, which is which in the exam, that's where I get kind of confused."
"It's just knowing which one to apply to which."
"Mean is the average or middle number."
"No, the mean is where all the numbers are laid out in order and then you divide by the total, by 2 that one is the middle."

Here the students are again trying hard to recall formulae, and in particular trying to remember a (non-existent) formula for the median which has lodged itself as easy to remember.

"No, I just need to remember that the mode is on the left hand side not the right."

This comment reflects a conventional layout of frequency tables, and the common confusion between the modal value and the modal frequency.

"Will you get the same answer if you don't use the calculator?"

There was considerable resistance to using a calculator.

> "I can't remember. It's over the total number of members that answer. One bit you take away those who refuse to answer. You have to read the question."

This last comment was made with some hesitation, as if the numbers themselves should produce a correct answer, and reference to the context of the question was somehow inappropriate.

(b) Interview evidence: Motivational

The students' conversation was dominated by the exam they would have to retake. Not one of them was able to give a meaning or a context to what they were doing.

> "These numbers - so many of them just make me panic - I think it's wrong if I miss one - and they carry so many marks, you must get it right."
> "I did the exercises and got them right. When it came to the exam I just panicked."
> "The problem is the standard deviation - I'd just miss that out."
> "Also (another unit) was so hard you had to put so much into it it was unbelievable...people actually let this go."
> "I want to pass it so desperately - I've never failed anything in my life until this."

The comments here demonstrate the extent to which the examination dominated the learning of these student groups. They behaved strategically by leaving out topics, they panicked, they calculated and allocated their time in the exam. They allowed themselves to fail in order to buy themselves time for other parts of their course. The examination encourages this type of behaviour by allowing students to pass with merely 40% of available marks.

Conclusion

It should be stressed that these findings are very tentative, and that the method of enquiry is subject to refinement. However, it seems reasonable to propose:

- That conventional examinations are an inadequate motivator for mature students, for those with low confidence, to learn very basic maths
- That students who fail exams such as these display very serious misunderstandings about the nature of the mathematics they are trying to do, and seem to see it as a series of explicit rules rather than attempting to make their knowledge tacit. These exams actively encourage the acquisition of mathematical understandings in an explicit form
- Successful teaching of numerical ideas such as these to mature students needs a much more explicit emphasis on learning skills and methods of approach to problem solving than is ever provided in a large modular degree scheme
- That this view of mathematics learning has quite serious curricular implications for adult students - given that the HE curriculum is based on assumptions about transfer.

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