Introduction to Algebra for Adult Students

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Mathematics Education Reform in the United States

In 1983, the National Commission on Excellence in Education released to the nation a report on the quality of education in the United States. The report, A Nation at Risk, issued a call to arms for reform at all levels of education, through all delivery systems (public, private, and parochial) and across all disciplines. According to the Commission, "The teaching of mathematics in high school should equip graduates to: (a) understand geometric and algebraic concepts; (b) understand elementary probability and statistics; (c) apply mathematics in everyday situations; and (d) estimate, approximate, measure, and test the accuracy of their calculations." (NCEE 1983)

In response to the call to revitalize mathematics education, the National Research Council (NRC) undertook an examination of United States mathematics education from kindergarten through graduate study. The NRC produced a trilogy of reports addressing different aspects of the effort: Everybody Counts, A Challenge of Numbers, and Moving Beyond Myths, Revitalizing Undergraduate Mathematics. This third report defines a sweeping national action plan to address the challenges and improve the delivery systems of Mathematical Sciences education on the post-secondary level. The goals of the reform are:

-Effective undergraduate mathematics instruction for all students.

-Full utilization of the mathematical potential of women, minorities, and the disabled.

-Active engagement of college and university mathematicians with school mathematics, especially in the preparation of teachers.

-A culture for mathematicians that respects and rewards teaching, research, and scholarship. (NRC, 1991)

Moving Beyond Myths describes specific tasks within its' action plan for each category of shareholder including faculty, departments, colleges and universities, professional societies, and government.

Developmental Mathematics Education at University College, Rutgers

University College, a college within Rutgers University, was founded in 1934 to meet the needs of adults who were seeking a college education while trying to juggle the needs of family and employment. The college offered a second chance to students who had not pursued a degree at the traditional age and attracted students with a broad variety of backgrounds, frequently deficient in traditional scholastic admission requirements. Over the years, various attempts had been made to provide appropriate remediation for this population, none of which was totally satisfactory.

In 1992, a collaborative effort between the university Mathematics department, the Mathematics Education faculty within the Graduate School of Education, and the Dean of University College resulted in the design of a two-semester course entitled "Introduction to Algebra for Adult Students". A pilot version of the course was offered during the 1993-1994 academic year by two doctoral candidates from Mathematics Education. The results of that pilot year are the basis of this paper.

Educational Theory Behind Introduction to Algebra for Adult Students

Mathematics Education

The driving education theory incorporated in the course was Constructivism. While a single definition of the term has yet to be achieved, the guiding principles of constructivism hold that knowledge must be actively constructed by learners and that coming to know is a process of organizing and adapting to the world as experienced by the learner (Kilpatrick). This theory is rooted in the work of Jean Piaget and influenced the instructional methodology in two primary ways. First, the organization of the class period was problem-centered and student-driven. Secondly, instructors assumed that many of the students had not reached the formal operations stage of knowledge. Initial activities met the student at the concrete

operational level and attempted to draw them to the higher level, formal operational, where abstraction is possible. Perhaps the best way to understand the role of constructivism is to consider how it works in practice. Later in this paper, in sections titled "A Typical Class Period" and "Student Perception" the manifestation of theory in practice is illustrated.

Adult Education

Several aspects of the adult as learner were incorporated into the course. One was the need for immediacy of application which typifies adult education. Adult learners are present-directed rather than future-directed. Material in the course was topical whenever possible and concerned with issues of interest to adults.

Adult learners are a rich resource for learning. They not only found examples of mathematics in life, but demonstrated successful strategies for solving problems, even when they felt that theirs was not the "right" way to solve them. By sharing multiple strategies, contrasting the approaches, and recognizing similarities between them, students gained confidence in their ability to do Mathematics.

The adult learner is typically self-directed. The fact that s/he has undertaken an educational experience in adulthood reflects a motivation for change and growth which may not be present in a pedagogical situation. Regrettably, the adult developmental math student is frequently "math-anxious" and afraid of appearing foolish. A caring, compassionate atmosphere must be established from the outset.

Mathematical Content

The Nature of Mathematics

The availability of low-cost technological tools for computation and notation manipulation has sparked investigation into the question of what it means to do mathematics and what is the essence of algebra. For the adult learner, functioning in an automated workplace, the importance of higher level thinking skills is particularly germane. Introduction to Algebra for Adult Students constructed a vision of algebra based primarily, but not exclusively, on ideas suggested in two books, Algebra for Adult Students and On the Shoulders of Giants. The main themes of the syllabus, namely, variables, functions and relationships,

equality and inequality, and graphs, were taken directly from the first. The perception of mathematics as a problem-solving enterprise strongly rooted in the search for patterns and relationships was derived from the second.

The Syllabus

Introduction to Algebra for Adult Students paralleled an existing university course which reviewed computation skills as well as teaching algebra. This content proved a fertile milieu for presenting algebra as a generalized arithmetic. In fact, the definition by which we worked was one offered by Dr. Robert Davis, namely, "Algebra is a way of talking about how numbers behave when we don't know the numbers." (Class transcripts) The problem-solving strategy of "trying some numbers" was encouraged for the development of algorithms by the students as well as verification of resulting rules or equations.

The course opened with a brief unit on the place of statistics as a means of gathering and organizing quantitative data. This allowed the introduction of graphing early in the course. The next step was to delineate between descriptive data and causal data and to proceed to the concept of functions and dependancy.

By the third full week of class, functions were being represented and tables, graphs, and equations. Rooted in a problem-centered environment, students developed algebraic expressions and equations, evaluated and simplified those entities, and developed their own "rules" for the ways that equations should be solved. Care was taken to use problems which involved positive integers. This allowed students to explore the four basic operations and algorithms for dealing with variables without the added layer of negativity complicating that exploration. Rational expressions involving fractions, decimals, percents, and variables rounded out the first semester work. Ratios and proportions had, in fact, intertwined the material from the early weeks of the semester.

The second semester of the course expanded on themes introduced in the first in light of signed number implications. Work on exponents and polynomials culminated in problems involving quadratic functions. Material covered in the first semester spiraled throughout the second as the richer field of the real number system was explored. To the very end of the

course. class began with a problem which was explored by the students and findings were established based on their solutions.

The Class Itself

The Environment

Substantial time and effort was put into established an open, friendly environment. At the first meeting, students and instructor introduced themselves and shared whatever "hopes and dreams" they had for the course. Data was gathered concerning major areas of interest, county of residence, and type of employment in the hope that people of like interests might seek each other out either in class or outside of class for cooperative efforts. Students were asked to exchange phone numbers with a "buddy" so that notes or assignments missed could be acquired. This last is particularly important for the adult learner since home and work priorities sometimes interfere with attendance.

Students were strongly encouraged, though not required, to work in small groups. A great deal of emphasis is placed on developing this spirit of cooperation and teamwork in students in K-12 instruction. While we felt that problem-solving is enhanced by groups, "two heads are better than one," it was not a goal of the course to impose changes in work habits upon our students. This is one area where andragogy (the education of adults) differs subtly from pedagogy. The privacy of the adult student was respected, and no one was forced to join a group if they chose to work independently. After a short time, everyone participated in a group regularly.

Substantial research has been devoted to the problem of "math anxiety" in adult students and to effective techniques for abating this anxiety. Exercises which build student self-confidence have shown to be one way to reduce this anxiety. Problems were structured to provide successful experiences. Suggested solutions were accepted in a non-judgmental fashion, conflicting solutions were examined by the students, and inappropriate solutions were, in general, discarded by group consensus. There is nothing new in the use of these techniques in the mathematics classroom. The methodology has been common in the K-12 curriculum for some time. It was new to the majority of MAAS students, and is uncommon in the undergraduate classroom.

A Typical Class Period

The typical class period broke down into roughly three stages: introduction, exploration, and summary. An alternate model, when exploratory homework had been assigned, used the bulk of the period to examine the problems individually and then distill pertinent content implications from the solutions.

In the primary model, the instructor introduced the topic of the day by reviewing relevant earlier content. Whenever possible, a real world situation was used to illustrate the type of problem situation to be explored.

The exploration phase consisted of the students determining a reasonable equation for the problem, dependent and independent variables, and the design of a graph showing the data. During the exploration phase, the instructor and practicum graduate students circulated to observe and participate in the student discussions. At no time were the observers to suggest solutions. Their remarks were limited to questions phrased "What do you think?" or "Tell me what you have done so far."

A typical summary involved solicitation of all solutions, the display of these in parallel on the board, and determination of acceptable alternates. Derived concepts or "rules" were listed in a "What we have discovered" section. Additional problems utilizing and reinforcing the discoveries were assigned as homework.

Occasionally, the exploration took too long and summary had to wait for the next class meeting. On those occasions, it was not uncommon for students to stay on, discussing the work among themselves or with the observing students.

Assessment

Assessment was both formal and informal. Observers circulating the class monitored students to determine mastery. Students submitted specific assignments for review by the instructors. Several times during the year, essays were assigned to determine the student perception of the success of the course. Portfolios of the student work were maintained by the instructor.

Formal assessment consisted of a midterm and final in both semesters. The material in the midterm was drawn from

traditional assessment tools such as the Math 025 course final which stressed notational mastery. Additional problems were drawn from the High School Proficiency Test administered to high school students in New Jersey. This test reflects current thinking on the nature of mathematics mastery and seemed a reasonable tool, as IAAS was intended to produce students proficient in high school mathematics. A copy of the Spring final was reviewed and approved by the Mathematics department professor responsible for basic skills courses at the university.

Pilot Year Results

Instructor Perception

The instructors were pleased, and at times astounded, by the success of the course. Both instructors had taught the equivalent traditional algebra course on the undergraduate level for several years. Neither had attempted to used constructivist techniques for instruction in those courses. They were accustomed to the lecture format with specific notational objectives for each class. Their previous use of small group work for algebra instruction was negligible.

The shift from teacher as carefully scripted sage demonstrating appropriate solution strategies to fellow traveller on the adventure path to learning was both fun and frightening. It became quickly apparent that each class resembled a pinball game where the instructor released the ball (problem) and responded to the trajectory (questions) with appropriate levers (suggestions or answers) as needed.

The pace of the course was unpredictable. At times it seemed that so little progress was made that the syllabus was an impossible dream. On other occasions, giant stretches of ground were covered while exploring one problem. In the end, only one topic from the original syllabus was not covered and that could be attributed to the cancellation of three classes by the university due to severe winter weather.

Student Perception

When asked to comment on the difference between this course and other math courses taken in their past, students focused on two differences. The validity and acceptance of multiple

solutions was the most frequently stated difference. Many students attributed failure on earlier assessment tests to a lack of remembrance of "the correct way" to solve a problem resulting in the abandonment of any attempt at solution. The willingness of the instructor to accept alternative solutions both in method and appearance empowered students to attempt tasks which were completely or vaguely unfamiliar to them.

The link to real world applications was also cited as a strong feature of IAAS. In both an early survey as well as an essay in the final, students commented on the emphasis of applications for the concepts and notation demonstrated in the course. This reflected one underlying principle of the course design, namely, that a strong understanding of the fundamental concepts of algebra and applications of the abstractions which algebra sanctions are within the reach of most students. The instructors were accustomed to students remarking, even after successful completion of a course, that they knew how to "do algebra" but had no idea when they would ever use it. By contrast, that question never occurred after Week 3 of IAAS.

The support and assistance of peers was welcomed by the students. Several commented on having come to understand a concept better after a fellow student explained it or showed his/her solution. Small group work facilitated learning by sounding out theories with someone at the same ability and understanding level. Early in the course, a few students were upset when the entire group took a wrong tack and developed an inappropriate solution. That problem seemed to dissipate after the first month and did not occur. One explanation of that might be improved communication of problem statements and goals between instructor and students as they worked together.

Retention

Student retention is one measure of the success of a course. Thirty-one students started the course in September. Two dropped before the midterm, it is believed because of illness which interfered with attendance in the early weeks. Two other students did not return for the second semester. One had moved out of state, the other reportedly had a scheduling conflict. Only one student left during the second semester citing difficulty of mastering the material.

Of the twenty-six who stayed the course, seventeen passed the final with a grade of B or higher. It was not clear who was

pursuing further math studies at the university. Four students expressed some kind of plans to do so. Further research will attempt to follow their experiences. It is known that one student took the Intermediate Algebra course, Math 026, over the summer and passed. He is now taking pre-calculus.

Research Methodologies

The major research tools utilized during the first pilot year were portfolios and videotapes. The portfolios contained traditional assessment tools, midterms and finals from both semesters. Selected homework was included in the portfolios. At various points in the year, attitudinal essays were assigned which were used to gauge the general success of the course in the students' minds as well as recording progress of student self-concept concerning mathematics. These were also included.

The last five weeks of the course were videotaped. These tapes are useful in capturing the environment of the class, but came too late to provide a mechanism for measuring growth and change in the students. They are useful in demonstrating the typical class period, but fall short even there, as most of the taped material was review, so little exploration by the students has been captured from the first year.

Implications for Research

The research conducted during Introduction to Algebra for Adult Students has implications for undergraduate and adult mathematics instruction in three directions which for lack of a better representation will be termed downward, outward, and upward.

The population addressed by the class is heavily represented in Adult Basic Education. Many students pursuing ABE and General Education degrees lack basic math skills and do not see themselves as capable of doing mathematics. The gains of students in IAAS in both content and confidence suggest that the techniques used might also benefit the adult in basic education courses.

In the classroom setting on the developmental undergraduate level, which I have termed outward, the research from this course can be expanded to include the traditional college students. It

is likely that constructivist techniques applied in a caring, non-judgmental environment would prove successful with this population also. One premise of IAAS was the thought that making the course look as different as possible from previous math class experiences might free students to recognize their ability to successfully tackle mathematics.

In what has been termed "upward", the research has applications for courses both in the calculus pipeline and out of it. There is already some research being conducted at Rutgers on the use of exploration in the calculus course (Author's note--I saw something about this in Rutgers Focus last Spring but am not sure of the specifics.) Certainly the issue of re-examining content in light of technology has implications for pipeline courses. The use of packages like Mathematica and Mathcad take pressure off the need to teach notation manipulation and free some time for concept exploration. It is sad that so many students take calculus without ever seeing the beauty and logic of this essential field of mathematics.

For students in humanities programs, the recognition of the place of mathematics in their areas and the application of math theory to the practice of their discipline is crucial to their future success. Few jobs, if in fact any, function today without mathematics skills. A solid knowledge of what math is, how it came to be, and where it may lead should be part of the intellectual baggage of anyone who calls themselves a college graduate and who will function in the workplace of the twenty-first century.

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