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Adults Learning Maths - A Research Forum

ALM-1

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of the Inaugural Conference
of

Adults Learning Maths - A Research Forum

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at
Fircroft College Birmingham UK

compiled by
Diana Coben Chair of ALM

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Disclaimer

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Preface

Diana Coben, Goldsmiths' College, University of London, London, UK
ALM Chair

Adults Learning Maths - A Research Forum (ALM) is a new international research forum bringing together researchers, practitioners and academics concerned with adults learning mathematics. ALM puts people in touch with each other, providing a framework for collaboration and helping to stimulate and develop research plans.

ALM was formally established at the Inaugural Conference in July 1994, attended by researchers and practitioners from the Republic of Ireland, The Netherlands, France, the USA as well as the UK, as a research forum with the following aim:

> *to promote the learning of mathematics by adults through an international forum which brings together those engaged and interested in research and developments in the field of adult mathematics* teaching and learning.*
> ** Within ALM we understand the term 'mathematics' to include numeracy'.*

ALM stems from an article (published in *Adults Learning* 4, 1, 15-16, September 1992) in which I argued the case for more research into adults learning mathematics - by practitioners as well as academics - and for greater access to relevant research in a variety of disciplines. I called for the establishment of a research forum to bring people together to share research findings and generate new research agendas.

There was an overwhelming response from all over the world. There was clearly both a need and a demand for such a forum, and after consultations with colleagues, an initial one-day conference was called in November 1993, attended by approximately fifty people from the UK and elsewhere in Europe. Many others, who could not attend, wrote asking to be kept informed of developments and some offered to become 'foreign correspondents' for the new forum.

To quote from the *Adults Learning* article:

> *We are long past the pioneering era, but numeracy is still the poor relation of adult literacy. What we need is the recognition of numeracy as a field worthy of serious study.*

These papers and the conference from which they stem, are intended as a step towards achieving that long-overdue recognition.

Membership of ALM is open to individuals and institutions. For details, please write to me at Department of Educational Studies, Goldsmiths College, University of London, New Cross, London SE14 6NW,UK

Adults Learning Maths - A Research Forum ALM-1
Conference Papers

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MULTIPLE METHODOLOGIES IN RESEARCH ON ADULTS LEARNING MATHS: RIVALRY OR COOPERATION?

Jeff Evans
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Drawing on a study of cognition and affect in the context of numerate activity among adult students, I show the strengths of each of three research strategies - quantitative, qualitative cross-sectional, and qualitative case study - for critically considering claims about gender differences in mathematical performance.

Introduction

Perhaps the most obvious way to distinguish types of research is by *data type*: quantitative research produces numerical data, analysed statistically, and qualitative produces textual data, analysed non-statistically. But almost immediately we find dilemmas: for example, how should we classify data based on classification, analysed by counts and cross-tabulations (e.g. Table 2 below)?

What about *method type*? Thus experiments, tests and surveys can be seen as "structured" (by the researcher), and many types of interview (e.g. life history), case studies and document use are seen as semi-structured or "unstructured". But this breaks down too: case studies may use test and questionnaire results, and experimental de-briefing may often be semi-structured.

Can we distinguish research by its *aims*, or its *epistemology*? Cook and Reichardt (1979) claim that quantitative research aims at the *testing* of theory, whereas qualitative emphasises *discovery*. Many would agree, arguing the need for research to be based on subjects' perspectives, but the researcher's "avoidance of preconceptions" may lead to a minimal acknowledgement of the role of theory.

The most promising basis, in my view, is the distinction between *types of explanation*: research using *causal* or *covering law* explanation can be distinguished from that using *purposes* or *reasons*. Put roughly, the former tends towards determinism - based on forces (e.g. socialization, instincts, characteristics and/or attitudes) - and aims for generality, whereas the latter emphasises subjects' freedom and often celebrates particularity.

Illustrative material comes from a study of thinking and affect in the context of "practical" activity among adults (Evans, 1993; Evans and Tsatsaroni, 1994). The

subjects were students on a social science degree at a London Polytechnic in the mid-1980s. Most were studying a compulsory maths (pre-calculus) / statistics course, and most were familiar with a range of numerate practices outside of school. Here I focus on methods and findings related to gender differences in mathematical performance.

The Quantitative Research: Theory and Methods

At the start of the course, subjects ($n > 900$) completed the questionnaire at one of their lectures. It included questions on gender, age, qualifications in school maths, and (in later versions) social class measured by occupation; performance; and maths anxiety. Here performance was considered to be explained (causally) by the above characteristics, and by measures of anxiety or affect.

The maths anxiety items were selected from the Mathematics Anxiety Rating Scale (MARS) analysed by Rounds and Hendel (1980) into Maths Course / Test Anxiety (MTA) and Numerical Anxiety (NA). MTA and NA were considered to relate to school and everyday contexts respectively, parallel to the distinction between school maths and practical maths; the latter measure included questions used in the survey of 3000 adults done for the Cockcroft Report (ACACE, 1982).Thus, in the quantitative phase of this resesarch, *context* was specified by the *wording* of performance or anxiety questions.

Results from the Quantitative Research

We can scrutinise the sometimes mythic claims about gender differences in maths performance (Walkerdine *et al.*, 1989), using the quantitative data. First, the men's and women's average score on School Maths performance (10 items) were compared, yielding an uncontrolled difference of 3/4 of a question (statistically significant; $p<.001$) in favour of the male students; see Table 1.

Qualification in maths (i.e. maths course-taking), age (since more of the women were mature students), maths test anxiety and confidence (self-rating) in maths, were controlled for (using statistical modelling). Now the difference for younger students (aged 18-20), about 1/6 of a question, was no longer statistically significant, and that for mature students (21+), just over half a question, was borderline; see Table 1.

Table 1. Gender Differences in School Maths Performance: Uncontrolled Group Means and Estimates Controlled for Qualification in Maths, Age, etc.

	Number of questions correct		
	Men's Average	Women's Average	Difference
Uncontrolled	8.78	8.07	0.71 (Std. Error 0.11)
Controlled			
Young (18-20)			0.17 (S.E. 0.17)
Mature (21+)			0.57 (S.E. 0.24)

Thus the gender differences in performance, which are fairly substantial, become much less so after controlling not only for maths qualification, but also for age, affective variables, and relevant interactions, and the quantitative analysis allows us to do this. (Quantitative research with similar aims is presented in Chipman, Brush and Wilson (1985)).

Qualitative Research: Theory and Methods

We can think of *discourses* as sets of ideas, goals, values and techniques, "competing ways of giving meaning to the world and of organising social institutions and processes." (Weedon, 1987, p.35). Since all our activities are based in discourses, we sometimes speak of *discursive practices*. Basing our analysis on discourses allows us to develop a much fuller idea of the context of mathematical thinking or affect, to include: the crucial role of *language;* the *goals* of the activity, (Newman Griffin and Cole, 1989); the *relations of power* exercised (e.g. Weedon, 1987) and the *material and institutional resources* (e.g. training, equipment, professional associations) made available for the activity.

In my approach, the context is understood as *positioning in practice(s)*. Each practice produces positions which subjects take up; in some contexts / practices, the availability of a particular position may depend on the subject's gender, age, etc. (cf. Hollway, 1989). For example, in the activity of feeding children, the child is positioned as demanding ("more"); the parent is positioned as having to regulate the child's consumption; the form of this regulation is likely to depend on the parent's income and social class. For a discussion of how positioning in such practices might provide the context for children's thinking in one topic of primary school mathematics (the idea of *more*), see Walkerdine and Girls and Mathematics Unit (1989, pp.52-53). Thus the specific practice (or mix of practices) in which a particular subject has his/her positioning will provide the context for that subject's thinking and

affect in that setting.

Semi-structured interviews were conducted in my office with a randomly designed sample (n=25) of students at the end of their first year. The interviews had *life history* and *problem solving* phases; in the latter, subjects were presented with a number of "practical" problems - e.g. deciding how much (if at all) they would tip after a restaurant meal (see below), deciding which bottle of tomato sauce they would buy, etc. (cf. Sewell, 1981). But this interview differed in its use of *contexting questions*: when the student was first shown the "props" for the problem - e.g. a facsimile of a restaurant menu in Qu.4 - before being asked anything "mathematical" - s/he was asked: "Does this remind you of anything you currently do?" And after discussing the question: "Does this remind you of any earlier experiences?" Subjects' responses to these questions were to help me judge the context of their thinking about the problem, etc.

My analysis of positioning in the interview setting was that all subjects would tend to be positioned in two main practices:
academic maths (AM), with positions teacher / student; and
research interviewing (RI), with positions researcher / respondent. To the extent that RI, rather than AM, was called up by the subject, s/he would be able to call up ways of thinking, emotions, etc. from further non-academic practices, with numerate aspects I called "practical maths" (PM). For Qu.4, I expected these to be practices of "eating out", with several configurations of related positions possible:
host / guest;
friends each paying their own share; or perhaps
customer / wait(e)r(ess).
Each position in practice will support different ways of thinking and feeling, including different kinds of numerate or mathematical thinking.

In order to judge which practice the subject called up, I drew on various indicators (Walkerdine, 1988, Ch.3; Evans, 1993):
(i) the "script": e.g. how the interview, problems, etc. were introduced - as "research", "views", "numbers", rather than "test", "maths", etc.;
(ii) unscripted aspects of the researcher's performance;
(iii) the subject's talk, especially the responses to the contexting questions; and
(iv) reflexive accounts: e.g. whether I had been in the position of maths teacher to each student, before the interview.

Two "qualitative" approaches were used. First, a *cross-subject* approach, based on that of Miles and Huberman (1984), aimed to consider, in a comparative way, the results from all of the interviews. Second, each interview was considered as a *case study* of a particular subject's thinking and affect.

Results from the Qualitative Cross-sectional Research

Let us consider the results for all interviewees for problem 4, which presented a restaurant menu; see Appendix. For this analysis, I aimed to record the practice that

predominated in each subject's positioning. Indicators for a predominant academic maths (AM) positioning were considered to be:
- the use of written calculations; or
- the giving of an answer which involved a fraction of 1p.

Indicators for a predominant positioning in "practical maths" (PM) were: - the use of mental calculations; or
- the formulating of an answer in practical, i.e. money, units.

The results from the cross-subject analysis of the relationship between gender, predominant positioning and performance judged as correct for problem 4 (a 10% tip when eating out) are in Table 2.

Table 2. Gender, Positioning and Performance: Cross-tabulation of number of questions judged correct by gender and predominant positioning for problem 4

	Number (and Percentage) Correct		
Predominant Positioning	Women	Men	Total
Practical Maths (eating out)	6 / 7 (84%)	10/11 (91%)	16/18 (89%)
Academic Maths	3 / 5 (60%)	-	3 / 5 (60%)
Total	9 /12 (75%)	10/11 (91%)	19/23 (83%)

We must first stress the small numbers involved - 23 students in all. (Two students did not reach Qu.4, because other life history material intervened.) Also, the level of performance, among both women and men, was rather high. And the gender differences were *very* small (especially remembering the small numbers). However, there are two findings, which - if they can be replicated - would be interesting. First, a slightly lower level of performance among those judged to have a predominant positioning in academic maths than among those positioned in practical maths / eating out (60% - 3 out of 5 - compared with 89%). Second, almost half of the women - but no men - called up academic maths. Taken together, these very tentative findings pose the question as to whether some of the gender differences in performance could be explained by differences in *positioning*. And whether this might be true in other settings, e.g. in large scale testing?

Results from the Qualitative Case Study Research

One case study will show the potential of this approach. "Ellen" (not her real name) was aged 19 at entry, middle class (by parent's occupation), with A-level in maths (which was rare among students in this cohort). A student of Town Planning, she was working part-time as an electronics assembler, and had worked previously in a shop.

In the interview, Ellen expresses overwhelming confidence after almost every question - except for problem 4 (on tipping after a restaurant meal)! When I ask what a 15% service charge would be for her "chosen" meal (seafood platter, costing £3.53), she says "Well, I'd have to use pencil and paper". Then -

> **S**: [7 sec. gap / inaudible / coughs / 6 sec.] Well, 23 1/2 pou- no, that's wrong ... [12 sec.] ... what I've done wrong, oh - (**JE**: Is it wrong?)
> Yeah, umm [laughs nervously] ... I don't know what I'm doing...
> [She realises she has divided 15% into £3.53, and begins to redo the calculation by multiplying the two numbers.]
> **JE**: You might not be used to doing these questions in such a setting as this, so (**S**: Yeah) take - there's lots of time, don't worry.... S: [15 sec.] ... 52.95, 53 pence.
>
> (interview transcript, p.7 - Appendix in Evans, 1993)

But she recovers from her slip, and explains that she rejected the answer produced by dividing because "I just saw that it was obviously not right... it was far too small".

Thus, my **1st Reading** of the episode: The "slip" may be just that, an aberration, a response perhaps to the relative difficulty of calculating 15%, as compared with the first three questions. For the first time in the interview, she momentarily expresses confusion - surprising for someone who has A-level in Maths. A reasonable conjecture is that she is feeling some anxiety, and it appears to be "maths anxiety" (or perhaps "interview anxiety"). This is the sort of analysis that a "clinical interview" would produce. But let us consider this episode more closely.

What is the context of her performance, and of her apparent anxiety? In response to the first contexting question, whether she ever went to a restaurant with a menu like that shown, she had seemed to reply very quietly and hesitantly - and this was *before* any numerical problem was posed. After she "chose" the seafood platter [£3.53], I ask how much she would tip for a restaurant meal: she had replied, again somewhat hesitantly, "...well, 15%, I suppose...".

> After she has performed the calculation with the result described above, I ask
> **JE**: Does this sort of situation here remind you of anything, any earlier experiences, a restaturant, a meal, thinking about service, and so on ...?
> **S**: No ... I don't usually pay ... I mean, I usually look at prices and things, ... add them up in my head...
> **JE**: Even if you're not paying?
> **S**: I don't want to be an expense.
>
> (interview transcript, p.8)

We can offer a second interpretation based on the ideas to do with positioning

in practices. At the beginning of this episode, she seems to have called up a practice (or cluster of practices), which might be called "eating out at restaurants": she chooses a dish and produces a rule for tipping. However, it is not clear how *familiar* she is with practices which require tipping, as she reverts to using pencil and paper - which in this context is likely to be indicative of academic maths (AM) - to calculate a 15% tip. Thus I would argue that her positioning is "inter-discursive", i.e. in a mix of more than one discourse, since she calls up "eating out" as well as being positioned as student in AM, and as interviewee.

Using her responses to the contexting questions and other indicators, we can also understand her anxiety as specific to one, or more, of these practices. Further support for her positioning, and hence her anxiety, being related to eating out comes from her hesitation, etc. at the presentation of the restaurant menu; i.e. the anxiety seems to be exhibited *before* the 15% calculation has been mentioned, and to interfere with her doing it. And her final comment in this episode is also indicative of anxiety related to eating out: "I don't want to be an expense".

Now, calling the anxiety "mathematical", as in the 1st reading, would be accurate only if we were to assume that she was positioned *solely* in an academic maths discourse. It would not be appropriate if, as I argue, her positioning is based, at least to some extent, in eating out as well.

We can note the specificity of her positioning in gender, and other, terms. She "doesn't usually pay" in restaurants. The position of not paying when you eat out is one which is (in the British culture) relatively more available to women, and/or to people younger than their hosts. Also, her specified rule of tipping 15% would have been very unusual for most people, especially students, in the mid-1980s in London. Therefore, the restaurants where she ate seem likely to have been for middle-class patrons, or at least for those not constrained financially.

She mentions that she doesn't want to be "an expense". The term "expense" signifies in different ways - as an amount which could be arithmetically calculated within a mathematical or related discourse, *and* as being a burden within a relationship with other(s) more wealthy or powerful, and/or on whom the subject is dependent (a parent or partner). This "key signifier" thus functions at the intersection of these two discourses. Its negative connotations in the latter context also suggest anxiety - anxiety associated with the activity of eating out, with the related social relationship(s), and with the operations involved in the activity, e.g. choosing a dish, calculating the total cost of her meal.

Thus, my *2nd reading*: It is reasonable to suppose that Ellen's positioning is *interdiscursive* - in eating out, in academic maths, and in being interviewed. Her "performance" - her thinking, the methods used, and the possibilities for critical reflection - need to be understood in this context. The same holds for the anxiety that she seems to be exhibiting.

Could there be other readings? There still seems to be a conflict or

contradiction between the picture Ellen gives of overwhelming confidence about maths and the use of numbers, and the indications of anxiety in this (and other) episodes. Is she perhaps "protesting too much"? This question, which (with others) suggests a need for insights from psychoanalysis for the interpretation of my interviews, cannot be pursued here, but see Evans (1993) and Evans and Tsatsaroni (1994).

Conclusions

In this paper, I have argued the need, when considering the context of cognition, to move beyond the mere wording of problems, to consider the aspects of language as discourse, the goals of the activity, power relations, the wider social and institutional setting. A subject's thinking, the methods used, the possibilities for "critical reflection" need to be understood in the context of her positioning in discursive practices; in Ellen's case, her interdiscursive positioning included being a "guest" in eating out practices. The same holds for affect. For example, this analysis questions whether anxiety, which might at first seem "mathematical", might be considered instead as specific to a positioning in other practices; in Ellen's case, this may have been related to "being an expense" as a guest.

In this research, both quantitative and (two types of) qualitative methodology have been used. Rather than polarising the discussion by asking which method is "best", we can note the relative strengths of each, and attempt to combine the different approaches in a way that is effective for the problem at hand. The *quantitative* approach is useful when we wish to make comparisons across subjects, or groups of subjects, and we aim for some degree of generality: we have seen the importance, and the power, of the controls used above. For example, this approach is useful for studying gender differences in participation or performance, and outcomes of policy interest more generally. The *qualitative case study* approach is useful when we wish to explore the richness, coherence (i.e. not being separated into variables) and process of development of a limited number of cases. We can display episodes of problem solving, in order to understand the process - for research purposes or to improve teaching and learning. The sort of semi-structured life-history / problem solving interview with contexting questions used here allows the tracing of multiple signification (as with "expense"), and the making of judgments about the positioning of subjects. In order to produce different readings, it is advantageous that (some of) the research method(s) should produce "semi-structured" talk which can be reread in the manner illustrated above. The *qualitative cross-subject* approach provides an intermediate approach, for cases where it may be challenging to produce comparability across subjects (as in judgments about "predominant positioning"), but where some generality in findings is sought.

Note

An earlier version of this paper was published as Evans (1994a). The case study is from Evans (1994b).

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Appendix Details of problem 4 in the interview

[Show the "menu" (see Fig. 1) to the subject.]
(CA) Do you ever go to a restaurant with a menu anything like this? ....
(CB) Would you please choose a dish from this menu? ....
(CC) Suppose the amount of "service" that you leave is up to the customer: what would you do? ....
Could you tell me what a 10% service charge would be ? ....
(CR) Does this remind you of any earlier experiences? ....

Note: Questions CA, CB, CC and CR were considered as *contexting questions* (see above).

Fig. 1 The "menu" for problem 4 in the interview

CHICKEN MARYLAND

Served with sweet corn, banana fritter, bacon, fresh tomato, whole French beans, jacket baked potatoes with sour cream and chives or French fried potatoes.

Roll and butter.

Ice cream, or a selection from our cheese board, biscuits and butter.

£3.75

SEA FOOD PLATTER

Served with tartare sauce, whole French beans, jacket baked potatoes with sour cream and chives or French fried potatoes.

Roll and butter.

Ice cream, or a selection from our cheese board, biscuits and butter.

£3.53

GRILLED TROUT 10 OZ

Served with tartare sauce, whole French beans, Jacket baked potatoes with sour cream and chives or French fried potatoes.

Roll and butter.

Ice cream, or a selection from our cheese board, biscuits and butter.

£3.81

Coffee

Special blend black or with cream **27p**

Connoisseur Coffees

Served in large goblet glass with cream: Irish (*Irish Whiskey*), Caribbean (*Rum*), Russian (*Vodka*), Parisienne (*Brandy*), Calypso (*Tia Maria*), Highland (*Scotch Whisky*), Mine Hosts (*Cointreau*)
Connoisseur coffees include sugar unless otherwise requested **67p**

Towards a Definition of Numeracy

Alexandra Withnall
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Introduction

The term 'numeracy' is widely used in adult basic education, but there is little agreement among practitioners as to what it actually means. This paper examines some attempts at defining numeracy and also discusses what it means to be 'innumerate' in today's society.

Numeracy as Mathematics

The term 'numeracy' seems to have first been used publicly in a report of the Central Advisory Council for Education known as the Crowther Report, which dealt with the education of young people in the 15-18 age group (HMSO, 1959). The term occurred in a section of the report concerned with the curriculum of the sixth form. Here to be 'numerate' was defined as the mirror image of being 'literate'. Numeracy was seen as 'the minimum knowledge of mathematics and scientific subjects which any person should possess in order to be considered educated'. From later paragraphs, it appears that quite a sophisticated level of mathematical and scientific understanding was implied, including the ability to think quantitatively, avoiding statistical fallacies.

Regarding numeracy as a term to describe a discrete set of competences in mathematics assumes that mastery of numeracy equates to a specific length and level of formal education and that there are standard approaches to teaching mathematics which will automatically lead to numeracy. Such an approach also ascribes to teachers of mathematics a role as gatekeepers of mathematical knowledge and culture. The implication is that unless this knowledge has been accessed and acquired, a person cannot be considered fully educated even though they may have demonstrated high levels of ability in other subjects. And who is to judge what 'fully educated' really means? In addition, this kind of definition takes no account of the diversity of contexts in which a whole range of mathematical skills are used by both children and adults, how these may vary across different lifestyles and occupations and how they may need to change over the individual life span.

Mathematics as an Aptitude

Somewhat different from the idea of mathematics as a set of skills to be learnt is the idea that possession of numerical skills is a special gift or talent, a belief explored in some details by Castle (1992). Those who lack this particular gift may develop a complete mental block where numbers are concerned even though they may be otherwise high achievers. This may lead to what has been variously described as 'sudden death' (Tobias, 1978) or 'panic' (Buxton, 1981). However, this phenomenon might be traced back to faulty learning experiences in the past rather than to any actual mental incapacity to deal with abstractions.

Castle (1992) demonstrates how this apparent confusion between aptitude and attitude has extended beyond the classroom in that gender and cultural biases have come to influence attitudes towards mathematics in industrialised societies.

She also comments on the underlying implication that there is equal opportunity for everyone to learn and to make use of individual intelligence or predisposition. The various forces which affect access to education - at all levels - and the relevance and quality of educational experiences are not taken into account. The assumption is that if an adult lacks numeracy skills, then s/he is seen as some way at fault - the 'blaming the victim' syndrome also identified in the early days of the adult literacy campaign in the United Kingdom (Withnall, 1994).

Functional Numeracy

The persuasive idea underpinning notions of functional numeracy is that there is a certain level of competence in numerical skills which is an essential pre-requisite both for effective individual participation in society and for the economic survival of that society as a whole. Thus, there is currently an increasing emphasis on a thorough mastery of basic mathematical concepts as an essential ingredient of most vocational training and for the development of a competent workforce. The implications are that achievement in these basic skills will result in improved personal circumstances in both work and leisure contexts and in family life; and economic growth and increasing competitiveness in relation to other countries in world markets. Conversely, those who fail to master basic numerical concepts will be unable either to operate effectively in their personal lives or to make a contribution to the economic regeneration of society.

The functional approach does acknowledge that there are different kinds of numeracy, that different societies require different types of skills from their members and that the level of personal numerical skills needed to function effectively in a given society is subject to constant change in the light of technological development and socio-economic change. Accordingly, we see attempts to create specifications for the kinds of numerical skills identified as necessary to prepare people for work and to create opportunities for personal development in the United Kingdom today. These appear within the programmes of study prescribed for the mathematics National Curriculum; the Numberpower standards devised by ALBSU which are based upon a competence model; and in the National Council for Vocational Qualifications (NCVQ) Care Skills Units for number in both NVQs and GNVQs which are also based upon demonstrated levels of competence. However, the Numberpower model is very much developed from the use of numeracy in both work and non-work activities, whilst the NCVQ specification is a generic unit model closely tied to the National Curriculum (Barr, 1993).

The notion of functional numeracy enjoys considerable popularity. Currently, it forms the basis for a great deal of practice in the United Kingdom and elsewhere. However, it is not without its critics. Even a decade ago, Riley (1984), himself a numeracy practitioner, considered that the term had been distorted and over-simplified. Writing at the same time, Penny (1984) questioned whether numeracy could ever be reduced to a checklist of coping skills. Rather, she felt it should reflect the ability to understand and use mathematical skills as

a means of communication closely linked to individual needs. More recently, in a wide-ranging critical analysis of the whole concept of functional numeracy, Castle (1992) observes that this approach with its emphasis on coping and survival skills and on training needs merely helps adults to adjust to or to accommodate prevailing social and economic circumstances rather than pursuing social change through questioning and challenging structural inequalities. It must also be pointed out that the assertion that improved levels of personal and societal functioning in respect of numerical competences - or indeed, improvements in overall levels of basic skills in any given society - will automatically lead to a range of desirable economic and social benefits is fundamentally flawed. Causation would be difficult to prove.

Numeracy in Everyday Life

The concept of numeracy in relation to the everyday needs of adult life was considered in more detail within the major government enquiry into the teaching of mathematics in schools announced in 1978. The ensuing Cockcroft Report (1982) used the word 'numerate' to imply the possession of two particular attributes:

- an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of everyday life;

- an appreciation and understanding of information presented in mathematical terms, eg in graphs, charts or tables or by reference to percentage increase or decrease. (HMSO, 1982)

Although the emphasis on being able to 'cope' still implies a functional approach, the implication of this definition is that a numerate person should have sufficient confidence to be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication, a position also taken by Penny (1984). Moss (1984) agreed that numeracy is part of people's daily routine and therefore each individual determines his/her own needs. Changes in lifestyles may therefore cause changes in needs, so that numeracy is something more than 'basic maths' or 'everyday maths'. Moss also pointed to the necessity to pay attention to the **language** of numeracy and to the process of transferring the written word into a mathematical code. In this way, numeracy, literacy and language are inextricably linked and must be closely related to individuals' personal life circumstances, self-perceptions and expectations. Together they form important skills which enable adults not just to 'function' effectively, but to be in a position to take a measure of control over the circumstances of their own lives.

Numeracy as Context-Specific

Evans (1991) points out that in talking about numeracy as a set of skills which can be applied in specific practical contexts, it is also crucial to consider the following:

- the goals (and values) of the activity within which it even makes sense to pose the problem
- the social relations (including the exercise of power) in the setting within which the problem is posed
- the material resources which form the basis for the activity.

It is these aspects, he argues, which make an activity **meaningful** and the meanings are expressed by the language specific to the activity. He is then led to acknowledge that, in principle, there is a discontinuity between eg school maths and the numerate aspects and skills of other different practical activities such as shopping or playing bridge. It then follows that we need to learn and develop numeracy within specific contexts, using the languages appropriate to that activity. This approach must raise a new series of questions about the traditional ideas of what constitutes numeracy and how it can be taught and learned at different levels. In particular, there may be difficulties about people's ability to **transfer** abstract skills to other settings.

Evans moves on to examine the possible numerate aspects of everyday life for adults in industrial societies. He particularly accentuates the importance of what he calls 'critical citizenship', ie:

> engagement with discussions and debates about individual, family and public well-being and about describing, appreciating, evaluating and deciding on future directions.
>
> (Evans, 1991:23)

Being enabled to practise critical citizenship assumes the availability of appropriate **resources** including access to and understanding of official statistics, mostly those produced by government and interpreted through the media. Issues relating to the role of numeracy in the practice of critical citizenship have been further investigated through a research project based in the Department of Mathematics at the University of Essex. The project focused mainly on the kind of numerical information needed by school governors in order to fulfil their roles successfully (Thorstadt, 1991, 1992).

The Effects of Innumeracy

In a sense, 'innumeracy' may be an inappropriate term since it is unlikely that there are many people in industrialised societies who, having received at least a primary education, are completely unable to cope with very simple arithmetical operations. Evans (1989) prefers to talk about 'low levels of numeracy' among adults, noting that the consequences of this may be apparent at several levels in society. For example, an individual's freedom of access to further education and training may be restricted and job performance, may be impaired - a functional approach. At a societal level, low levels of numeracy may result in loss of production, waste of resources, even threats to life if vital mathematically-based information is misinterpreted or misunderstood. Evans also shows how low levels of numeracy can have ideological consequences at individual level in that lack of both competence and confidence in their constructive skills and critical insights can lead adults to become dependent on the views of experts or professionals and leave them open and susceptible to what he calls an alien mathematical mystique. At societal level, lack of numeracy may result in the growth of myths - ideas which are partially or completely false, but which gain currency and influence the beliefs and actions of different groups and organisations in society as well as affecting the practices of society's various institutions.

Further examples of the ideological consequences of low levels of numeracy have been explored in the USA. Paulos (1988), writing about innumeracy as 'mathematical illiteracy' demonstrates how lack of ability and confidence in dealing comfortably with the fundamental notions of number and chance can lead to an uncritical acceptance of meaningless co-incidence and pseudoscientific theory as well as a misinterpretation of simple statistical information. Dewdney (1993) takes these arguments a stage further by showing how politicians, advertisers, stockbrokers, salespersons and others who make use of numbers, charts and graphs can manipulate individuals by playing upon their innumeracy. This he describes as the common inability to **understand** the rules of percentages, ratios, statistics and basic mathematical logic. In this way, it is easy for even highly educated adults to be left open to the chicanery of those he calls 'maths abusers'.

Emergent Issues

The foregoing exploration of competing approaches to the concept of numeracy demonstrates clearly that different writers have based their definitions on a range of different assumptions. In one sense, we have come full circle. The Crowther report put a certain emphasis on the ability to think quantitatively and on the importance of avoiding statistical fallacies. More recent writers have stressed the need for individuals to be more aware of ways in which statistics can be used to manipulate their thinking. They show how this awareness might be achieved through an emphasis on understanding how mathematical rules can be subverted together with a healthy degree of scepticism and the ability to think critically rather than through knowledge of any advanced mathematical or statistical techniques.

In exploring the use of numeracy in everyday life, it is suggested that numeracy is something other than a set of mechanistic, discrete mathematical skills which can be acquired in an artificial context and then transferred to real-life situations. Neither is it merely a set of various skills to be used to enable individuals to function more effectively in a given context which may, in any case, be subject to dislocation and change. Rather, it appears that numeracy consists of being able to make an appropriate response to a much wider range of personal needs occurring in individual lives to different extents. These needs call for the ability to understand broader contexts in which numerical demands are located, to make use of appropriate communication skills, to be able to interpret information presented in a variety of mathematical codes, to exercise judgements according to the nature of the activity and the desired outcome. Such a stance obviously has profound implications for the way in which numerical skills are taught and learnt in adult basic education programmes.

The Lancaster Research

Some of the above issues are currently being explored through a research project underway in the Department of Continuing Education at Lancaster University. Although the research is concerned with an examination of the numerical skills used in everyday life, by people in retirement, preliminary fieldwork with two small groups of older people has already revealed a range of ways in which numerical skills might be required in their daily lives, eg:

1. Domestic numerical skills: perceived as necessary by the individual for personal activities and domestic tasks.

2. Consumer numerical skills: mainly concerned with aspects of managing financial affairs, making purchases, etc.

3. Technological numerical skills: required to understand and operate modern appliances, usually domestic; may also involve computer-based skills.

4. Leisure numerical skills: concerned with a whole range of leisure activities whether personal hobbies or more complex pursuits involving others.

5. Numerical skills for citizenship: understanding and interpreting statistical and other information available within the public domain.

6. Numerical 'mental notes': remembering and visualising numbers and number sequences for a range of purposes, or for curiosity.

This table is by no means exclusive since many of the categories of activities are inextricably linked. For example, purchasing and learning to operate a washing machine requires domestic, consumer and technological skills together with a grasp of statistical information as an informed citizen capable of making choices. A further point relates to the range of skills utilised within each category. For example, respondents in this exercise pursued a variety of leisure activities ranging from knitting - itself requiring the ability to follow complex algebraic sequences and to visualize geometric shapes - to acting as treasurer of a voluntary organisation. The skills required for this latter activity are also complex, but of a different order. These observations go some way to illustrating the points made by Evans (1991) - that it is the goals and values, the social relations and material resources available which are crucial to understanding a particular activity. In particular, the goals of the activity may influence the degree of precision required. These issues will be explored in more depth during the Lancaster project's main fieldwork phase. It may be that the outcomes of the research will have implications for adults generally, not just those in later life.

Conclusion

At present, there is a tendency for policy-makers, practitioners and researchers alike to talk about 'numeracy' as though they shared a common understanding of its meaning. In examining some of the attempts to grapple with a definition already made, it has to be concluded that we do not, as yet, have an all-embracing operational definition with which to work. Nor, it seems, is it necessarily desirable to do so. Numeracy must remain a fluid term capable of re-conceptualisation according to the contexts in which it is used and by whom.

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Finding Common Threads: Researching the Mathematics in Traditionally Female Work.

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Throughout the world, the largest single group of people who attain lowly in institutional mathematics is women. Throughout the world, the traditional work of women involves a very large amount of unacknowledged mathematical thinking and skill. How can we give recognition to the mathematics women actually do?

Gender and Low Attainment

The past thirty years has seen a large amount of research into differences in performance in school mathematics between boys and girls. Though many may claim (and though many would still dispute) that, so far as the school population is concerned, there is no longer a problem, the same cannot possibly be said for the adult population. The fact is that throughout the world, the general public reaction to women doing mathematics is surprise, scorn or disbelief.

Research into the mathematical performance of females is admirably reviewed by Willis (1989). A hundred years ago it was accepted, even amongst educated women that girls were physically, emotionally and intellectually ill-equipped for the serious study of mathematics. Until about thirty years ago it was widely and complacently believed that girls were less *able* than boys. The question was seen in genetic terms as some sort of female disability and there was much research on visual and numeric differences in performance. Competing psychological theories and the various political movements of the 1960s however finally exposed the flaws in the biological arguments that justified class and race as well as gender differences. By the 1970s the question had changed from 'Why *can't* girls do maths?' to 'Why *don't* girls do maths as well as boys?', in other words there was a shift in emphasis from ability to achievement. The lower achievement of girls was seen as a problem, for them, and research centred on affective issues, achievement motivation, success avoidance and so on. In fact, there was no real evidence that girls did achieve less. The evidence is that they achieve as well but that they participate less and the research question of the 1980s was 'Why *won't* girls do as well as boys in mathematics?' or in other words 'Why do girls drop out?' The answer is not genetic, it is social.

Mathematics in Work: Women at Work.

During the 1980s I ran a project called Maths in Work which looked at the mathematics used by young, unskilled workers in the first job in the Inner

London area. This work has been reported elsewhere (for example Harris, 1991a and 1991b) so only the briefest of reviews will be made here. The Project started with research questionnaires listing a set of mathematical and other skills which had been administered to young people at work.

There are many kinds of problems with this type of research not the least of which is the widespread public and professional perception of workplace mathematics as just arithmetic of a certain level. Throughout its chapter on mathematics in the workplace, even the prestigious Cockcroft Report (1982) uses the word 'mathematics' while referring mainly to calculations. Following back the lists of 'basic skills' that now form part of National Vocational Qualifications, through the demands of industry that prompted the Cockcroft enquiry in the first place, and comparing them with those of the seventeenth century in Howson (1982) for example, one can see that the perception of workplace mathematics as sums has hardly changed. Inventions such as logarithms or the pocket calculator seem to make little difference to the public view of working mathematics as the four rules, ratio and percentages.

If the questionnaires (or interviews) of workplace research confine themselves to this limited perception, then they will limit the answers they provoke. The research merely becomes part of the self-fulfilling prophecy that maintains the status quo. Responses to questions <u>not</u> about mathematics however can be very revealing. A finding of the Maths in Work Project was that when the young workers were asked for example about the communications or practical skills they used at work, they often revealed a use of mathematics they had previously denied when asked about it specifically from a mathematics questionnaire. Hairdressers, for example, denied that they used ratio and proportion when asked about them as mathematical skills, yet used the word 'proportion' when they explained the necessity for mixing hair tints accurately, in responses to questions about their practical skills.

Such responses make the scores of responses to mathematics questionnaires meaningless. They also reveal some of the subtleties of workplace research which are not usually analysed. The widespread negativity towards mathematics supported by the public acceptance that people cannot do it allows it to be normal to answer questions about mathematics negatively. The research responses also seemed to suggest that when mathematical skills caused the workers no problem, then they were regarded as common sense. They only became 'mathematics' when the worker could not do them. This widespread response was just one measure of the separation of mathematics skills from the context in which they are used by both workplace research questions and by educational institutions where mathematics is taught. By keeping the mathematics skills as recognisably the mathematics of schools and exams, learned, practiced and assessed out of contexts that give them any meaning, the research ensured that the workers, no longer in a school situation, did not recognise them once they were embedded in the chores of daily life.

The mathematics of educational institutions does enormous damage by this separation. The approach of first teaching the isolated skills and then their applications, as if mathematics was some sort of ointment to be rubbed onto various sore places, is ineffective. The alternative, what might be called the vitamin view of mathematics is to learn to recognise and develop mathematical thinking in its natural habitat, embedded organically in the systematic solution of daily problems. The following is an example of some domestic work I did recently. The reader is invited to analyse the mathematics in it.

> I have just moved house and one of the many jobs I have been doing is making curtains. The house has rather large windows and the whole business of making curtains is therefore expensive. For my bathroom window I found an economic solution in the material made for making duvets which is about seven feet wide. In deciding how much of it I had to buy, I measured the window across and down, allowing a few centimetres at the top and bottom for hems and headings. A properly made curtain has to be at lease double the width of the window if it is to hang and cover well, and because the duvet cloth was so wide I would only need two 'drops' of it, one for each side of the window. The cloth I chose from the range had a small repeat design on it and I bore it home in triumph as a bargain. In the evening I spread it out on floor ready to cut it in half, only to find to my dismay that the all-over pattern had a repeat of about twenty centimetres that I had not noticed in the shop. I had not bought enough to match the design across both curtains. For some time I worked at various solutions: cutting both pieces in half vertically to break up the pattern for example. In the end, I found that by splitting the pattern in half, that is pretending it had a ten centimetre repeat when it hadn't, being mean with the hem and deciding that I was not going to lie in bath and worry about my curtains anyway so long as the room was cosy, I went ahead and made them. I had to line them of course and lining material does not come seven feet wide, but that is another story.

My point is that I had not consciously applied mathematics to the problems of my curtains, windows and budget. I had never thought "What mathematics do I need to solve this?" What I had done was to find myself measuring and calculating in both imperial and metric units at once, depending on what I was measuring and in analysing and manipulating symmetries - an aspect of mathematics that rarely gets a mention in workplace research.

My experience of the research I had been working on, backed up my own experience as a practical householder caused me to reject the skills-and-applications approach in both my research and publications. Instead I did my research in the workplace, working alongside workers and learning their thinking instead of interviewing them and using my own perceptions of their work to analyse what they were doing. For the learning materials I was

contracted to produce, I tried to bring mathematically rich, practical problems from the various places where I had worked into formal classroom situations through learning materials which were honest and realistic and did not prescribe a particular answer.

The first pack of learning materials to emerge from the Project was concerned with the design and making of cardboard boxes. Both learners and teachers enjoyed them and found them effective, so the Project went on to look for more everyday activity with materials as ordinary as cardboard. As an experienced home dressmaker and knitter, cloth was an obvious choice.

Research for a pack of learning materials using cloth as the main resource continued in both the industrial and domestic spheres, (Harris in press). Domestic research included analysing the way a cross-stitch embroiderer in former Yugoslavia and a weaver from Bangladesh worked up the symmetries of their designs, and the handling of different symmetries in a Scottish Tam O'Shanter. Industrial research included a study of how socks, umbrellas and ties are designed and manufactured in bulk. As the Maths in Work office began to fill with samples of cloth the suggestion arose of making them into an exhibition. This was the origin of the exhibition Common Threads set up originally at the University of London Institute of Education for about ten days, but which went on to tour England for two years and the world for the following four. Its exhibits were entirely cloth and its captions entirely mathematics and it demonstrated incontrovertibly the enormous wealth of mathematical skill deployed when women from all over the world get on with their traditional domestic work of knitting, sewing, embroidery, weaving and basket making.

Crediting the Mathematics in Traditional Women's Work

The effects of the exhibition were immediate, indeed they were responsible for its long life and tours that had never been part of the plan. Common Threads stimulated a different perception of working mathematics from the traditional model of the applications of school-based arithmetic. It stimulated cross-curricular work in schools and colleges: it gave confidence to both women learners and teachers who saw their own work credited for its intellectual as well as practical or aesthetic content: it asked pertinent questions about why such work is not traditionally regarded as mathematical and it demonstrated, once again, the gross bias in mathematics education towards male interests, male methods and male achievement.

The international response was also remarkable. Gender bias in mathematics education is a world wide problem and many countries felt that they could use Common Threads as a resource in their own work. It was this expressed need that caused the British Council to take over Common Threads and re-design it for overseas travel. The saga of how the re-design process drew the exhibition back into the rejected public perception of mathematics as a set of low level skills is discussed elsewhere (see Harris in press). The version that toured overseas in two copies was radically different from the original in philosophy and content

but it did speak to mathematics educators in a number of countries when I could be there to demonstrate the now invisible research behind it.

In some developing countries the potential was particularly inspiring. There are many development projects for women which teach textile skills for commercial independence. For such, Common Threads raised the possibility of learning mathematics while learning textiles crafts and obtaining both a commercial skill and an educational qualification at the same time. Such a project demands commitment, which was never a problem but it demands funds and funds demand access to formal funding bodies. By the very nature of things such bodies are not controlled by radicals ready to abandon the accepted view of workplace mathematics particularly for a bunch of village women weavers.

Another extension of the Maths in Work Project's work with textiles has been more imaginatively supported. For the year 1993-4 I worked as Scientist in Residence at the Women's Institute College, under the COPUS Project. The Committee on the Public Understanding of Science is a project of The Royal Society, The Royal Institution of Great Britain and The British Association for the Advancement of Science. My work at Denman College was to identify and reveal the mathematics in some textiles courses and help plan future projects to developing mathematics. The three courses I worked through and analysed were in knitting, the making of curtains, cushions and blinds, and strip patchwork. The common feature of my participant observations on all three courses was that all on-task talk was all mathematical. The moment the work 'mathematics' was mentioned however, there was an immediate display of attitude problems. The problem for the future is how to handle the attitude: there is no problem with the mathematics potential.

Once again it is revealed that there is nothing wrong with the women's mathematical skills and competence. They do rather more mathematics in their daily work than a lot of people. Yet research into gender and mathematics still pathologises women. Women have been accused of being genetically incapable of doing mathematics, of being unwilling to compete, of not knowing what they are doing even when they do it. It would be possible to go on doing the sort of research that reveals what mathematics women do, that explores way of crediting it, that builds confidence that helps them compete, but the problem is not theirs. The problem lies in a society that sees workplace mathematics as a limited and impoverished version of the real thing and women as incapable of even that. The research perhaps should become more political. Maybe I should address myself in future to exploring and publishing the ways in which public ignorance and male prejudice maintain a pernicious dual handicap on half the world's population.

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Mathematics: Breaking Down The Barriers

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Introduction

This paper summarises the results of a UFC funded project run at the University of Exeter which investigated whether the mathematics GCSE requirement for courses such as teacher education and some social sciences is a barrier to higher education for adult returners on Access courses. Access students are not a homogeneous group so we also questioned whether, if mathematics was a barrier, this was for all Access students or just for particular, identifiable sub-groups. Further and more detailed results of the project have been published in two papers by Roseanne Benn and Rob Burton in 'Access mathematics: a bridge over troubled waters' Journal for Access studies 8(2) 1993 and 'Access and the mathematics deterrent', Studies in the Education of Adults, 26(2).1994.

Research design and methodology

Two questionnaires on attitudes to mathematics, one for students and one for staff, were tested in a pilot survey then sent to 109 institutions offering Access to Social Science and Humanities courses (20% national sample) and Access to Teacher Education courses (100% national sample) in England, Wales and Northern Ireland.(Jones 1992) The sample returns were: Access to Social Science and Humanities courses, 70 courses sampled, 91 per cent (64) returned; Access to Teacher Education, 39 courses, 72 per cent (28) returned. The overall return was 84 per cent. Individual returns from the students were: Access to Social Science and Humanities, 53 per cent; Access to Teacher Education, 54 per cent. The return was 54 per cent with a total of 1471 returns. Forty five tutors replied, giving a response rate of 41 per cent. This lower rate of return reflects the fact that not all Access courses have mathematics tutors.

The questionnaire had three distinct sections. Firstly, we were interested in the level of mathematics of Access students on leaving school and their general level of education. Secondly, using simple attitude scales, students confidence in specific areas of Mathematics was tested. A Likert attitude scale[1] was developed to test whether Maths was a barrier to Higher Education. The attitude scale consisted of ten questions which were piloted and the internal reliability tested using the Cronbach alpha.[2] For the total population studied the Likert attitude scale has a Cronbach alpha coefficient of 0.8552 and hence is statistically valid and has internal reliability. Finally we asked our students a number of questions to establish their social characteristics. The questionnaires were completed in the second half of the Autumn term, 1992.

1 The Likert scale is a unidimensional scale. Equal numbers of negative and positive questions are used to arrive at a 'summated' score.(Likert 1932) In this case 0 suggests mathematics presents no barrier and 40 suggests Mathematics is seen as a high barrier to higher education.

2 For further information concerning the Cronbach alpha, see De Vaus, (1990) pp.88-89.

Analysis and findings

Student attitudes

First, we were interested in Access students' levels of mathematics confidence. When asked their feelings upon finding out that they needed the mathematics qualification, students were uninhibited in their negative responses. We established the outcome of eleven plus years of compulsory mathematics schooling by asking for the level of mathematical qualification on leaving school and whether the student had encountered any particular problems with school mathematics. Low levels of mathematics achievement seem to be a shared experience for those who responded to our questionnaire. Approximately 35% of our sample reported that they had left school with no mathematical qualifications and a further 49% less than O level equivalence. When asked to comment whether they had particular problems with their school mathematics, a worrying 80% of our sample responded in the affirmative. Many of these students were of the opinion that they had been let down by the school system and reported problems of both lack of understanding and communication with the teacher. A significant 84 per cent of our cohort had a low or no mathematics qualification and many showed a negative attitude to school mathematics.

The overall context in which an Access student's attitude to mathematics must be set is a very strong motivation to get into higher education. Over 80% agreed that, whatever the requirement for entry, they would have attempted to get it. But another important factor to emerge amongst at least some of the group was a positive attitude to mathematics itself and a growing sense of confidence. Almost half the students agreed that they enjoyed mathematics, with over half feeling reasonably confident in tackling mathematical problems. Again half perceived no problems in understanding mathematics. Although the subject brings 30% out in a cold sweat, it does not have that effect on a sizable 60%. This shows approximately half the cohort enjoying mathematics and feeling confident with no major difficulties or fears. There is an undecided or wavering group of 10%-20%, with 20%-30% with negative feelings in this area.

The authors' experience in teaching mathematics on Access indicates that there is an appreciable number of students who wish to study mathematics for their own personal satisfaction and development. This is borne out by the 44% of the sample who agreed that they were taking mathematics for their own satisfaction and the 60% who saw mathematics as an important part of their development. A further indication of confidence in mathematics was given when 60% agreed that they would apply for a higher education course with a mathematics entry requirement and only 17% saying that they would not. It must be remembered that this result is probably skewed by the respondents on Access to Teacher Education courses. However a perhaps surprising but similar 60% would even apply for a higher education course that had a mathematics component. If offered another route into higher education without a mathematics requirement, 42% would take it but a significant 36% would not.

The results of our study seem to suggest that once people have made up their mind to enter higher education, perhaps to study for a particular career such as teaching, then the psychological barriers concerning mathematics seem to have been broken. The results above seem to suggest there is a widespread mathematics anxiety that is linked to a lack of understanding. This lack of understanding or bewilderment is quite common in mathematics (Buxton 1981 p109), but for mature students on Access a

combination of motivation and positive teaching methods work to reduce these earlier fears.

Tutor perceptions

When asked whether the mathematics GCSE requirement is a barrier, two thirds of tutors thought it was. Of these, three quarters identified it as a high barrier. Clearly most tutors perceive this requirement as a serious obstacle to progression. This opinion however does not seem justified by an examination of the previous year's results. In over half the replies, no student was identified as having not progressed to their chosen exit or course because they had not obtained the appropriate mathematics pre-requisite. In two thirds of the replies only 5% or less of the students had not progressed due to their mathematics. These very positive results were echoed by the feeling of many tutors that the student group as a whole had made considerable strides in their mathematics over the Access year. When asked how many students in the previous year had a serious problem with mathematics, early in the course, over three quarters of the tutors gave a figure of 20% or more, late in the course half the tutors thought that the number with serious problems was 10% or less.

The care and commitment of tutors shines through their comments. They report that the key quality of their teaching is encouragement and understanding; that tutors need to be patient and remove the often difficult and disabling pressures of time and that Access mathematics needs to be taught in context and have a relevancy to real life and other parts of the course. They note that the involvement of students and tutors in free discussion and dialogue in a supportive atmosphere helps students develop confidence. Students on Access mathematics are clearly coming on to courses with long memories of bad experiences at school. This has many causes - bad teaching, lack of understanding and a lack of relevance to their own lives. Access tutors in recognising this have geared their approaches to reduce much of the anxiety and panic students feel.

This new found confidence in mathematics has wider effects. One tutor noted that by making progress with a subject they had thought they were unable to do, Access students gained an enormous sense of achievement and increased confidence which informed their whole approach to study.

We hence concluded that students are arriving on Access with considerable anxieties about mathematics often based on bad school experiences; that the approach taken by tutors is critical in how students perceive mathematics and how they accomplish what is for some a difficult subject, both emotionally and practically; and that the provision of a mathematics component in an Access course has a number of enabling roles for the students.

We now extend this analysis by considering the mathematics requirement with particular reference to the constituent groups in Access. The social classifications considered were gender, class, age, race, those with little in the way of formal qualifications, and those with no mathematics qualification. The first and most obvious conclusion to draw from our results was that there is a sizable proportion of each group that considered mathematics to be a high barrier for themselves and this is true whether the question is asked directly or indirectly. Earlier we showed that these attitudes were being changed for those on Access courses. Nevertheless these figures indicate an area that must concern all Access providers. It is also important to note that respondents express more concern when asked directly if mathematics is a barrier. This is perhaps

not surprising as the very question may cause them to recall not only their own experiences and fears of mathematics but also the negative views of mathematics within our society. We shall now consider each subgroup separately.

No Mathematical Qualification

It comes as no surprise that people with no mathematics qualification upon leaving school see mathematics as a barrier to higher education and exhibit the highest levels of anxiety. Again this poses a dilemma for access providers. Access is premised on the notion of open entry but it is clear from both our statistics and an analysis of individual comments that some people are entering or considering entering Access with very low levels of mathematical knowledge. Some of these people wish to progress to teacher education or the social sciences and so need the mathematics qualification. It is noted from sources such as the Cockcroft Report (1982) that learning mathematics takes time. Indeed Cockcroft notes that 'A frequent complaint [from students] has been that insufficient time is given to curriculum and method work'(ibid p.209). Tutors on Access also recognise the constraints time imposed upon their students' mathematics development. It may be that people in this group should be counselled to take a mathematics course prior to Access or seriously consider alternative routes that do not require mathematics.

Low Level of Non-mathematical Qualifications

The authors had anticipated that this group might see mathematics as a high barrier but they showed no exceptional problem.

Gender

It is clear from our results that women on Access, whether asked directly or indirectly, show more anxiety about mathematics than men. There are many cultural explanations for this. Women and girls are discouraged from studying mathematics because of its masculine image (Astin and Myint 1971; Rose and Elton 1971; Trecker 1971; Walkerdine 1989). Ironically girls at high school fail to view mathematics as important to them for their later careers (Pedro et al 1981; Wilson and Boldizar 1990). This, as Fennema and Sherman (1977) and Sherman and Fennema (1977) have shown, has a major effect on females' confidence with mathematics both at school and in later life.

It would seem that both at Open Days and in early mathematics sessions on the Access course, tutors need to discuss these cultural experiences and expectations with both men and women to raise awareness in the students that earlier failure in mathematics may be due to this socio-cultural reality rather than their own inadequacy.

Race

In the Access group, 7.8% were Black which compares well with the 5.5% for the whole population in the 1991 Census. As a group, Black people showed the same score as White people on the Likert Scale, with Africans and Carribeans showing higher anxiety when the group is broken down.The results of the direct question about mathematics as a barrier indicated greater problems for the Black group. Each sub-group of Black minorities had a higher percentage stating that mathematics was, for them, a high barrier than for the cohort. Although some of the group sizes are small, this indicates a worrying trend.

Age

Our results do not indicate a clear correlation between age and anxiety about mathematics.

Class

Empirical research has traditionally used the Registrar General's (RG) classification of occupations as the method of classifying people in the social structure. However, past theoretical and research debates concerning social classification and women (Stanworth 1984; Middleton 1974; Ritter and Hargens 1975) would suggest that perceptions of women on Access gained through use of this classification may not bear close relation to reality. Work on this project published elsewhere (Benn and Burton 1993b) agrees with this contention and hence we do not feel able to draw any conclusions on whether links exist between class and anxiety with mathematics.

Implications for Access.

Bad experiences at school for many students raise grave personal doubts about their ability not only to do mathematics but to succeed in post-secondary education at all. We showed, however, that the commitment and considerable skill of Access mathematics tutors work to reduce these anxieties and help students to fulfill their goals. We have identified the particular social groups who find the mathematics requirements of Access and of their future chosen careers a barrier or deterrent but note that there is a sizable proportion of every subgroup that considers mathematics to be a high barrier for themselves. To reduce this deterrent factor, practitioners need to constantly consider admissions procedures and processes and monitor their approach to the mathematics component. Certain groups of students are particularly vulnerable and these may need special attention. It is crucial to widen access but not to have Access students fail yet again because of the pressure of the mathematics component of the course.

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Getting Personal: Research into Adults' Maths Life Histories

by
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Introduction

The idea for our research into adults' maths life histories arose at the first meeting of ALM in November 1993. For many years Diana had recorded interviews with friends and colleagues about their experiences of mathematics, but this had remained an essentially private activity - serious but unsystematic - and without what Mrs Thatcher called the "oxygen of publicity" it had not developed. We discovered that this was an interest we shared, and so began the glimmerings of a joint research project. After several discussions we began to see how we could establish a project that would enable us to develop our interest in a more rigorous and systematic way.

The first step was to identify possible sources of funding. It is important to establish a track record when bidding for other people's money, especially as this is a new area of research, so we put in a bid to the College for a small pilot project to be run in conjunction with the University Careers Service, while continuing to develop other bids. Our bid to the College was successful and the pilot begins in September when we shall be interviewing mature students about their maths life histories and their perceptions of the effect of their mathematical experiences on their career and life chances. In the meantime we have been refining our methodology and undertaking trial interviews with volunteers of both sexes and from as wide a variety of ages and backgrounds (educational, linguistic, cultural etc.) as possible.

Our methodology is qualitative and non-observational (McKernan 1991) and our main research tool is the semi-structured interview augmented by subjects' own narrative accounts (Spradley 1979, Densin 1978, Schatzman and Strauss 1973). We chose this because it enables us to maintain points of comparison between subjects while engaging in in-depth open-ended conversations.

Our interviews so far have thrown up some unexpected and fascinating insights into adults' experiences of maths - like the woman who was given a calculator as a leaving present by her (adult) student because he did not approve of her working out weaving calculations in her head, or the man who never progressed beyond adding and subtracting at school, and yet is clearly using more sophisticated mathematics in his work as a harness maker and as a labourer in the building trade.

We have been agreeably surprised that people have been so willing to talk about their maths life histories. Nevertheless, we have found it difficult to get people to talk about their more recent mathematical experiences.

The Activity

We wanted the seminar group to gain some insights into our project and to help us to take our research further so we asked the participants to interview each other, using an extract from our draft interview schedule (see below). They worked in pairs and we allowed 20 minutes for this - 10 minutes per person. Each pair then had 5 minutes to jot down points, issues and comments to bring back to the whole group. We wanted to keep the interview schedules and the participants' notes on them, so unless they wished to identify their contribution, individuals were not required to do so.

Outcomes

There was a general feeling that the group had been 'thrown into the activity' and some people were initially 'stumped' by some of the questions. By contrast, our interview subjects have all had the opportunity to talk to one of us about the project and everyone has received a letter prior to the interview describing the research and giving ground rules and guidelines about the interview. This is because we wish the interviewee to be as well informed as possible and to feel able to respond only to those questions he or she wishes to answer. We agreed that it might have been helpful had it been possible for the ALM conference participants to have seen the extract of the interview schedule before and that, had they done so, they might have produced different responses. We also recognised that the first interviewee's responses might influence the second interviewee.

Some members of the group had not thought about questions of this sort before and said they would have liked more time to talk and think about the questions. It was considered that the interview schedule was cognitively demanding but generated new and interesting ideas. It also assumed that each participant had a good memory!

Several issues concerning language were raised in the lively discussion after the interviews. How do interviewees translate the language of researchers (for example `space and shape') into the context of their own lives - for example, that of gardens? There was discussion of the difficulty of finding the vocabulary to describe feelings that were evoked when talking about past mathematical experiences. It is interesting that maths is a subject which does evoke much emotion - but has traditionally been seen as not involving the affective domain. The question of fluency of language was raised and concern expressed that some interviewees might have difficulty in communicating their thoughts. So far we have not found this to be the case - in fact quite the opposite.

Members of the group perceived themselves responding in dual roles, one as `professional mathematician' or `mathematics educator' and the other as `in everyday life'. There was also consideration of the influence of economic and personal circumstances affecting the view of estimating and approximating, for example, if you have little money, estimation of bills is not an option. The circumstances in which calculations are performed may affect how they are done

- under pressure, for example, you might use a calculator when in other circumstances you might use your head (Buxton 1981, Maxwell 1989).

Some of the categories used in the interview schedule which are taken from school mathematics tended to focus the interviewee's attention on the school context and it was suggested that we could avoid this domination by contexting the questions; thus grounding the interview more effectively in everyday life.

The layout of the interview schedule and the format of the questions were discussed and many helpful comments made which we have incorporated into the next version of the interview schedule. We know we are never going to have the perfect interview schedule but developing our work in the light of such comments is an important part of the research process.

Some of the participants' comments were predicated on the difficulty they expected we would find in getting people to talk to us. On the contrary, we have had no problem getting people to talk - rather we have had a queue of people wanting to talk to us. We have also had to devise strategies for keeping to our one hour time allowance as once people start to talk about heir maths life history they seem to want to carry on. Many interviewees have commented that they are saying things they have never said before: a comment echoed by several ALM participants.

Certainly the ability to transfer knowledge from one context to another seems to be a key issue (Zaslavsky 1994). We are concerned about the issue of labelling 'mathematical' what the interviewee sees as maths-free. This could be seen as patronising, our definition carrying more weight than the interviewee's.

Conclusion

Our work so far has confirmed for us the importance of adults' maths life histories and we are keen to continue to investigate these, particularly with respect to those areas of adults' lives which they do not immediately identify as mathematical.

Working together has been a thoroughly positive, synergetic experience for us both. It is a great pleasure to discuss the latest interviews, reflect on the process and consider ways of improving it, pass on interesting articles and other pieces of information to each other. The fact that there are two of us has also enabled us to encourage each other and remain motivated through the (at times) hard slog of preparing research bids. Nevertheless, we have enjoyed focussing on different aspects of the field in order to tailor our research bids to the requirements of the various funding bodies and we are currently exploring the possibility of joining with colleagues on a large bid which would include aspects of our research into adults' maths life histories.

We have been delighted with the interest aroused amongst our colleagues at Goldsmiths, elsewhere in the UK and further afield. As a result we are giving several conference papers on various aspects of our research and plan to write about it for publication. All this has given us the chance to discuss our ideas with others and to respond to challenges and insights from a range of different perspectives, none of which would have happened without ALM.

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Mathematical Voyages: The factors which influence students' involvement in mathematics

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The focus of this conference session was a small scale qualitative study involving thirteen mathematics students at the start of a four year Bachelor of Education degree (Secondary). The students were invited to describe their experiences of mathematics using a mathematical autobiography. These writings were then used as the basis for subsequent individual interviews.

Working in three small groups, conference participants were invited to read interview transcripts (with the student identities concealed), to focus on high and low points in the students' mathematical careers, and to put forward ideas for analysing the data.

Comparisons were drawn between these approaches and those used by the researchers who had used three main headings to categorise student experience: Influences, Critical Points and Constraints.

Influences are those factors (positive or negative) which are important to the student in forming their view of mathematics and their attitude to the subject. They can be likened to the wind or a current - it is hard to work against them. Examples of positive and negative influences are given below:

Positive influences	**Negative influences**
Teacher/lecturer enjoyment of the subject	Teacher expectation of student failure
Public recognition of success	Conflict with friends' life style
Mathematics in context	Pressure from family and friends to do well

Critical points are moments of crisis or sudden change when positive and negative influences become more extreme, e.g. a lower than expected examination result; a change of school or college. Both of these examples could be positive or negative critical points. For example, a low examination result could have a motivating or de-motivating effect. What determines the direction of change are the other influences acting on the student at the critical point.

Constraints are those personal factors which override counter influences and critical points. These act as a barrier against success or failure. An example of a positive constraint would be a clear career goal which enables the student to overcome any setback.

One of the conference groups identified key points and categorised these as first and second order points. For example, a first order point of 'good, enthusiastic teachers' could produce the second order effect of a liking of mathematics. The group recognised an ambiguity in certain of their categories. For example, persistence, tentatively labelled as a second order effect could , in certain circumstances be seen as a first order factor.

The second group saw level of confidence as central to any analyses of student transcripts. Both positive and negative factors were closely related to a student's level of confidence; for example, supportive, approachable teachers could increase student confidence, whereas a lack of perceived relevance could reduce motivation and hence confidence.

The third group identified positive and negative factors for each of the students. Relationships between students and their maths teachers could be either positive or negative. Other factors were interpreted as entirely positive or entirely negative.

It can be seen from this limited discussion that there were similarities between the groups' approaches to transcript analysis and that of the researchers. All groups saw the teacher's role as central. Teachers were strong positive and negative influences. Teachers had made a difference.

Doing what comes naturally

Janet Duffin
University of Hull

My plan for this session was to try to bring together, in an exploratory way, a number of ideas which have come to me over the years from my experiences as a teacher and teacher trainer, as the evaluator of a national project which introduced a calculator to six- year-olds to examine its effects on the traditional primary number curriculum and, more recently, as a researcher developing a theory of learning with a colleague.

I wanted to use the session as a forum for discussion with participants about their own learning and that of the students they work with. My hope was that all of us would gain from the experience and further our understanding of the learning process and the learners we encounter. In consequence, the session was part workshop, part discussion and part research report and I see all participants as vital contributors to all those processes.

The first thing I attempted was an exploration of a small piece of mathematics learned by all of us at school: subtraction. When I undertake this kind of exploration with students, as yet usually unaware of the insights the teacher has to have into the elementary processes of arithmetic, I usually start by identifying through them the subtraction method they learned at school. I follow this up by asking them to do a subtraction mentally.

In the case of experienced teachers, as were the participants at this conference, we miss out the preliminary exercise and concentrate on their own inner methods, which I tend to assume come naturally to them rather than having come from a teacher.

As always happens in such an exercise, I found a variety of methods emerging, many of them similar to those I have met elsewhere, one or two quite different. The methods which come out most frequently are either some kind of counting on or counting back method or an idiosyncratic combination of the two. A method using negatives is particularly interesting. 532 - 286: 200 from 500 is 300, 80 from 30 is -50, 6 from 2 is -4 giving 246.

In this workshop two new methods emerged, one immediately translating the numbers into money, £5.32 - £2.86, indicating a recognition of the power of money and the fact that many less able learners can often handle money yet be unable to work with numbers out of context. The other method the user found quite difficult to articulate while being very confident in its execution.

This reinforces earlier conclusions that we all have personal methods for calculating which seem natural to us and sometimes the methods of others can seem alien and cause conflict for us. I use these words deliberately because they relate to research into the learning process I have been doing with a colleague, developing a theory of learning which we call Natural, Conflicting and Alien.

In this theory we postulate that we all have learning experiences to

which we respond according to how we interpret those experiences. A natural experience is one which feels right and fits with the way we already think; a conflicting experience is one which jars, feels wrong, causes us to feel a sense of confusion; an alien experience is one which seems to have no connection with any part of our current way of thinking.

Our response to a natural experience is to add to earlier experiences and reinforce our current way of thinking; a conflicting experience makes us feel the need for some kind of resolution to get rid of the jarring, the confusion. In this case we may find we have to modify our previous way of thinking to take the new experience into account but sometimes this can be difficult to do so conflict can cause a problem in the progress of our learning. The response to an alien experience can be to avoid or ignore it, to evade it by recourse to something familiar, or to take it on board as a completely new but disconnected way of thinking.

Initially, we only identified natural and conflicting but later found that some of the incidents we encountered, in developing and testing our theory, were not explained by the concept of conflicting. Sometimes what seemed like an obvious contradiction to us as observers did not appear to be noticed by the learner. This led us to introduce the third concept and, in doing so, we realised that a strong element in our theory is the need to try to see the experiences we are concerned with from the point of view of the learner involved rather than merely seeing it from our position as observer. It also helped to draw our attention to the value of being aware of the ways in which we ourselves learn and the fact that in any teaching incident the teacher too is a learner.

Having outlined the theory, the session moved to a consideration of the two other aspects of my current work: trying to help undergraduates overcome personal inadequacies in elementary number work and observing children in the CAN (Calculator Aware Number) project who, as well as having free access to a calculator at all times, were not taught the standard algorithms we all learned at school but were, instead, encouraged to develop their own personal methods of calculating for addition, subtraction, multiplication and division.

These two experiences occurring side by side over a period of years enabled me to gain insights from both groups which I might otherwise not have had. I observed striking similarities between the two groups and one significant difference. Both groups showed that they were equally able to devise their own calculating methods but one group, the children, were supremely confident in what they did, the other, the students, were identifiable by their lack of confidence and a strong sense of inadequacy about their competence with number.

So why are the children confident and the students lacking in it? Backed up by the research I have been doing I am convinced that the confidence comes from 'own methods' being encouraged in the project whereas the students all felt that their own methods had to be done in secret and were 'not the right way to do it'. I have many stories of the undermining of confidence brought about from being forced to a standard way of working which inhibited the internal methods many of them were already developing secretly.

It is my belief that, even for able pupils, the methods taught at school were often alien and, if learned, did not become part of their natural ways of thinking but were learned as something totally disconnected, sometimes with disastrous effects on competence as well as confidence.

In the final part of the session the group looked at and discussed four incidents selected from my own experience which I believe demonstrate the ideas and concepts the session had been designed to bring out. The incidents dealt with concerned a journalist, an undergraduate on a numeracy course, a graduate with a PhD and a less able boy from the CAN project.

The journalist

He was questioning my belief in people's self-devised methods, saying that he himself always used the method he was taught at school. He asked me to give him a specific example of a personal method. When I did so, he said "Oh yes, I often do that myself" thus both refuting what he had earlier told me and demonstrating that, sometimes unconsciously, people do devise their own methods and use them appropriately.

The undergraduate

A group of students were trying to reproduce the method they had been taught at school. I overheard a conversation between two students which eventually I felt compelled to investigate. The conversation was as follows:

A. Why are you doing that?

B Well, I have to put 2 there because 1 won't be enough ..... oh dear, I wish she would let us do it in our heads.

When I investigated this very curious snippet of conversation I discovered that the student was trying to recreate the equal additions method for subtraction. She remembered that she had to add a one to the unit column at the top and the tens column at the bottom but she neglected to remember that, whereas the 1 added in the units column represents a ten, in the tens column it is only one. This meant that she made the 8, say, in the tens column into 18 instead of 9.

Then when she wanted to add her 1 to the top of the tens column and the bottom of the hundreds column she needed a 2 to make the 3, say, into 23 because 18 cannot be taken from 13. Doing this she not only modified the method but it brought her to an impasse when she came to the hundreds column where she found she had to take 22 from 6.

(The question was 632 - 286. I suggest you try it for yourself to see the predicament she got into.)

The student's attempt on the face of it suggests complete confusion but, on reflection, it is possible to see that she had some understanding about place value but failed to use it successfully in trying to recreate a method which probably seemed alien to her at school. That she wanted the chance to do the subtraction in her head shows that she did have an effective personal method.

The PhD seeking numeracy help

She told me that she was hopeless with numbers and always got her family to do any calculations she needed to do but she wanted to try teaching and felt she must do something about her inadequacy.

She did simple additions by counting on her fingers and, after seeking an explanation of a million, she asked if £1000 added to another £1000 would make her a millionaire.

I probed further and discovered that she added two consecutive numbers without using her fingers. For 6 + 7 she told me the answer immediately and explained that two sixes were twelve so the answer was thirteen. Later I discovered her using this idea in her attempts at subtraction by, for example if the top number was 2 and the bottom one 7, saying that if the bottom number had been 6 that would have given 6 but with 7 it gave 5.

It seemed to me that, having had her own method commended, she was able to build on it in new circumstances.

The less able boy in the CAN project

I have followed this boy through school from the age of six to his present position in his third year in secondary school. At his primary school the teachers managed to keep him out of the then 'remedial classes'; in his secondary school he has been relegated to the bottom group, seen as one of 'special needs'.

At six he showed that he could add up a column of figures by finding tens, using, for 6 add 8, the expedient of 'taking 2 from the six and putting it with the 8 so the answer is 14". At 11 he was able to pick out odd numbers from a set of answers and explain why they could not be even, something I have not always found average twelve or thirteen year olds able to do. At 13 I looked with him at a task he was doing about a boy saving up from his paper round to buy a bike, compared and discussed with him our two ways of doing the calculation. There were ways in which his approach bettered mine but each reflected our individual ways of working.

In discussing these incidents and the ideas contained within the session, a number of pertinent questions were raised, I hope to the advantage of all of us. For you the reader, I hope that the ideas I was trying to bring together in the session itself will be discernible from what I have written and, in particular, I hope that the ideas behind my title of Doing what comes naturally will come through in this report.

Further details about the CAN project, the numeracy courses and the research described can be obtained from the author, School of Mathematics, University of Hull, Hull HU6 7RX.

Introduction to Algebra for Adult Students

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Mathematics Education Reform in the United States

In 1983, the National Commission on Excellence in Education released to the nation a report on the quality of education in the United States. The report, A Nation at Risk, issued a call to arms for reform at all levels of education, through all delivery systems (public, private, and parochial) and across all disciplines. According to the Commission, "The teaching of mathematics in high school should equip graduates to: (a) understand geometric and algebraic concepts; (b) understand elementary probability and statistics; (c) apply mathematics in everyday situations; and (d) estimate, approximate, measure, and test the accuracy of their calculations." (NCEE 1983)

In response to the call to revitalize mathematics education, the National Research Council (NRC) undertook an examination of United States mathematics education from kindergarten through graduate study. The NRC produced a trilogy of reports addressing different aspects of the effort: Everybody Counts, A Challenge of Numbers, and Moving Beyond Myths, Revitalizing Undergraduate Mathematics. This third report defines a sweeping national action plan to address the challenges and improve the delivery systems of Mathematical Sciences education on the post-secondary level. The goals of the reform are:

-Effective undergraduate mathematics instruction for all students.

-Full utilization of the mathematical potential of women, minorities, and the disabled.

-Active engagement of college and university mathematicians with school mathematics, especially in the preparation of teachers.

-A culture for mathematicians that respects and rewards teaching, research, and scholarship. (NRC, 1991)

Moving Beyond Myths describes specific tasks within its' action plan for each category of shareholder including faculty, departments, colleges and universities, professional societies, and government.

Developmental Mathematics Education at University College, Rutgers

University College, a college within Rutgers University, was founded in 1934 to meet the needs of adults who were seeking a college education while trying to juggle the needs of family and employment. The college offered a second chance to students who had not pursued a degree at the traditional age and attracted students with a broad variety of backgrounds, frequently deficient in traditional scholastic admission requirements. Over the years, various attempts had been made to provide appropriate remediation for this population, none of which was totally satisfactory.

In 1992, a collaborative effort between the university Mathematics department, the Mathematics Education faculty within the Graduate School of Education, and the Dean of University College resulted in the design of a two-semester course entitled "Introduction to Algebra for Adult Students". A pilot version of the course was offered during the 1993-1994 academic year by two doctoral candidates from Mathematics Education. The results of that pilot year are the basis of this paper.

Educational Theory Behind Introduction to Algebra for Adult Students

Mathematics Education

The driving education theory incorporated in the course was Constructivism. While a single definition of the term has yet to be achieved, the guiding principles of constructivism hold that knowledge must be actively constructed by learners and that coming to know is a process of organizing and adapting to the world as experienced by the learner (Kilpatrick). This theory is rooted in the work of Jean Piaget and influenced the instructional methodology in two primary ways. First, the organization of the class period was problem-centered and student-driven. Secondly, instructors assumed that many of the students had not reached the formal operations stage of knowledge. Initial activities met the student at the concrete

operational level and attempted to draw them to the higher level, formal operational, where abstraction is possible. Perhaps the best way to understand the role of constructivism is to consider how it works in practice. Later in this paper, in sections titled "A Typical Class Period" and "Student Perception" the manifestation of theory in practice is illustrated.

Adult Education

Several aspects of the adult as learner were incorporated into the course. One was the need for immediacy of application which typifies adult education. Adult learners are present-directed rather than future-directed. Material in the course was topical whenever possible and concerned with issues of interest to adults.

Adult learners are a rich resource for learning. They not only found examples of mathematics in life, but demonstrated successful strategies for solving problems, even when they felt that theirs was not the "right" way to solve them. By sharing multiple strategies, contrasting the approaches, and recognizing similarities between them, students gained confidence in their ability to do Mathematics.

The adult learner is typically self-directed. The fact that s/he has undertaken an educational experience in adulthood reflects a motivation for change and growth which may not be present in a pedagogical situation. Regrettably, the adult developmental math student is frequently "math-anxious" and afraid of appearing foolish. A caring, compassionate atmosphere must be established from the outset.

Mathematical Content

The Nature of Mathematics

The availability of low-cost technological tools for computation and notation manipulation has sparked investigation into the question of what it means to do mathematics and what is the essence of algebra. For the adult learner, functioning in an automated workplace, the importance of higher level thinking skills is particularly germane. Introduction to Algebra for Adult Students constructed a vision of algebra based primarily, but not exclusively, on ideas suggested in two books, Algebra for Adult Students and On the Shoulders of Giants. The main themes of the syllabus, namely, variables, functions and relationships,

equality and inequality, and graphs, were taken directly from the first. The perception of mathematics as a problem-solving enterprise strongly rooted in the search for patterns and relationships was derived from the second.

The Syllabus

Introduction to Algebra for Adult Students paralleled an existing university course which reviewed computation skills as well as teaching algebra. This content proved a fertile milieu for presenting algebra as a generalized arithmetic. In fact, the definition by which we worked was one offered by Dr. Robert Davis, namely, "Algebra is a way of talking about how numbers behave when we don't know the numbers." (Class transcripts) The problem-solving strategy of "trying some numbers" was encouraged for the development of algorithms by the students as well as verification of resulting rules or equations.

The course opened with a brief unit on the place of statistics as a means of gathering and organizing quantitative data. This allowed the introduction of graphing early in the course. The next step was to delineate between descriptive data and causal data and to proceed to the concept of functions and dependancy.

By the third full week of class, functions were being represented and tables, graphs, and equations. Rooted in a problem-centered environment, students developed algebraic expressions and equations, evaluated and simplified those entities, and developed their own "rules" for the ways that equations should be solved. Care was taken to use problems which involved positive integers. This allowed students to explore the four basic operations and algorithms for dealing with variables without the added layer of negativity complicating that exploration. Rational expressions involving fractions, decimals, percents, and variables rounded out the first semester work. Ratios and proportions had, in fact, intertwined the material from the early weeks of the semester.

The second semester of the course expanded on themes introduced in the first in light of signed number implications. Work on exponents and polynomials culminated in problems involving quadratic functions. Material covered in the first semester spiraled throughout the second as the richer field of the real number system was explored. To the very end of the

course. class began with a problem which was explored by the students and findings were established based on their solutions.

The Class Itself

The Environment

Substantial time and effort was put into established an open, friendly environment. At the first meeting, students and instructor introduced themselves and shared whatever "hopes and dreams" they had for the course. Data was gathered concerning major areas of interest, county of residence, and type of employment in the hope that people of like interests might seek each other out either in class or outside of class for cooperative efforts. Students were asked to exchange phone numbers with a "buddy" so that notes or assignments missed could be acquired. This last is particularly important for the adult learner since home and work priorities sometimes interfere with attendance.

Students were strongly encouraged, though not required, to work in small groups. A great deal of emphasis is placed on developing this spirit of cooperation and teamwork in students in K-12 instruction. While we felt that problem-solving is enhanced by groups, "two heads are better than one," it was not a goal of the course to impose changes in work habits upon our students. This is one area where andragogy (the education of adults) differs subtly from pedagogy. The privacy of the adult student was respected, and no one was forced to join a group if they chose to work independently. After a short time, everyone participated in a group regularly.

Substantial research has been devoted to the problem of "math anxiety" in adult students and to effective techniques for abating this anxiety. Exercises which build student self-confidence have shown to be one way to reduce this anxiety. Problems were structured to provide successful experiences. Suggested solutions were accepted in a non-judgmental fashion, conflicting solutions were examined by the students, and inappropriate solutions were, in general, discarded by group consensus. There is nothing new in the use of these techniques in the mathematics classroom. The methodology has been common in the K-12 curriculum for some time. It was new to the majority of MAAS students, and is uncommon in the undergraduate classroom.

A Typical Class Period

The typical class period broke down into roughly three stages: introduction, exploration, and summary. An alternate model, when exploratory homework had been assigned, used the bulk of the period to examine the problems individually and then distill pertinent content implications from the solutions.

In the primary model, the instructor introduced the topic of the day by reviewing relevant earlier content. Whenever possible, a real world situation was used to illustrate the type of problem situation to be explored.

The exploration phase consisted of the students determining a reasonable equation for the problem, dependent and independent variables, and the design of a graph showing the data. During the exploration phase, the instructor and practicum graduate students circulated to observe and participate in the student discussions. At no time were the observers to suggest solutions. Their remarks were limited to questions phrased "What do you think?" or "Tell me what you have done so far."

A typical summary involved solicitation of all solutions, the display of these in parallel on the board, and determination of acceptable alternates. Derived concepts or "rules" were listed in a "What we have discovered" section. Additional problems utilizing and reinforcing the discoveries were assigned as homework.

Occasionally, the exploration took too long and summary had to wait for the next class meeting. On those occasions, it was not uncommon for students to stay on, discussing the work among themselves or with the observing students.

Assessment

Assessment was both formal and informal. Observers circulating the class monitored students to determine mastery. Students submitted specific assignments for review by the instructors. Several times during the year, essays were assigned to determine the student perception of the success of the course. Portfolios of the student work were maintained by the instructor.

Formal assessment consisted of a midterm and final in both semesters. The material in the midterm was drawn from

traditional assessment tools such as the Math 025 course final which stressed notational mastery. Additional problems were drawn from the High School Proficiency Test administered to high school students in New Jersey. This test reflects current thinking on the nature of mathematics mastery and seemed a reasonable tool, as IAAS was intended to produce students proficient in high school mathematics. A copy of the Spring final was reviewed and approved by the Mathematics department professor responsible for basic skills courses at the university.

Pilot Year Results

Instructor Perception

The instructors were pleased, and at times astounded, by the success of the course. Both instructors had taught the equivalent traditional algebra course on the undergraduate level for several years. Neither had attempted to used constructivist techniques for instruction in those courses. They were accustomed to the lecture format with specific notational objectives for each class. Their previous use of small group work for algebra instruction was negligible.

The shift from teacher as carefully scripted sage demonstrating appropriate solution strategies to fellow traveller on the adventure path to learning was both fun and frightening. It became quickly apparent that each class resembled a pinball game where the instructor released the ball (problem) and responded to the trajectory (questions) with appropriate levers (suggestions or answers) as needed.

The pace of the course was unpredictable. At times it seemed that so little progress was made that the syllabus was an impossible dream. On other occasions, giant stretches of ground were covered while exploring one problem. In the end, only one topic from the original syllabus was not covered and that could be attributed to the cancellation of three classes by the university due to severe winter weather.

Student Perception

When asked to comment on the difference between this course and other math courses taken in their past, students focused on two differences. The validity and acceptance of multiple

solutions was the most frequently stated difference. Many students attributed failure on earlier assessment tests to a lack of remembrance of "the correct way" to solve a problem resulting in the abandonment of any attempt at solution. The willingness of the instructor to accept alternative solutions both in method and appearance empowered students to attempt tasks which were completely or vaguely unfamiliar to them.

The link to real world applications was also cited as a strong feature of IAAS. In both an early survey as well as an essay in the final, students commented on the emphasis of applications for the concepts and notation demonstrated in the course. This reflected one underlying principle of the course design, namely, that a strong understanding of the fundamental concepts of algebra and applications of the abstractions which algebra sanctions are within the reach of most students. The instructors were accustomed to students remarking, even after successful completion of a course, that they knew how to "do algebra" but had no idea when they would ever use it. By contrast, that question never occurred after Week 3 of IAAS.

The support and assistance of peers was welcomed by the students. Several commented on having come to understand a concept better after a fellow student explained it or showed his/her solution. Small group work facilitated learning by sounding out theories with someone at the same ability and understanding level. Early in the course, a few students were upset when the entire group took a wrong tack and developed an inappropriate solution. That problem seemed to dissipate after the first month and did not occur. One explanation of that might be improved communication of problem statements and goals between instructor and students as they worked together.

Retention

Student retention is one measure of the success of a course. Thirty-one students started the course in September. Two dropped before the midterm, it is believed because of illness which interfered with attendance in the early weeks. Two other students did not return for the second semester. One had moved out of state, the other reportedly had a scheduling conflict. Only one student left during the second semester citing difficulty of mastering the material.

Of the twenty-six who stayed the course, seventeen passed the final with a grade of B or higher. It was not clear who was

pursuing further math studies at the university. Four students expressed some kind of plans to do so. Further research will attempt to follow their experiences. It is known that one student took the Intermediate Algebra course, Math 026, over the summer and passed. He is now taking pre-calculus.

Research Methodologies

The major research tools utilized during the first pilot year were portfolios and videotapes. The portfolios contained traditional assessment tools, midterms and finals from both semesters. Selected homework was included in the portfolios. At various points in the year, attitudinal essays were assigned which were used to gauge the general success of the course in the students' minds as well as recording progress of student self-concept concerning mathematics. These were also included.

The last five weeks of the course were videotaped. These tapes are useful in capturing the environment of the class, but came too late to provide a mechanism for measuring growth and change in the students. They are useful in demonstrating the typical class period, but fall short even there, as most of the taped material was review, so little exploration by the students has been captured from the first year.

Implications for Research

The research conducted during Introduction to Algebra for Adult Students has implications for undergraduate and adult mathematics instruction in three directions which for lack of a better representation will be termed downward, outward, and upward.

The population addressed by the class is heavily represented in Adult Basic Education. Many students pursuing ABE and General Education degrees lack basic math skills and do not see themselves as capable of doing mathematics. The gains of students in IAAS in both content and confidence suggest that the techniques used might also benefit the adult in basic education courses.

In the classroom setting on the developmental undergraduate level, which I have termed outward, the research from this course can be expanded to include the traditional college students. It

is likely that constructivist techniques applied in a caring, non-judgmental environment would prove successful with this population also. One premise of IAAS was the thought that making the course look as different as possible from previous math class experiences might free students to recognize their ability to successfully tackle mathematics.

In what has been termed "upward", the research has applications for courses both in the calculus pipeline and out of it. There is already some research being conducted at Rutgers on the use of exploration in the calculus course (Author's note--I saw something about this in Rutgers Focus last Spring but am not sure of the specifics.) Certainly the issue of re-examining content in light of technology has implications for pipeline courses. The use of packages like Mathematica and Mathcad take pressure off the need to teach notation manipulation and free some time for concept exploration. It is sad that so many students take calculus without ever seeing the beauty and logic of this essential field of mathematics.

For students in humanities programs, the recognition of the place of mathematics in their areas and the application of math theory to the practice of their discipline is crucial to their future success. Few jobs, if in fact any, function today without mathematics skills. A solid knowledge of what math is, how it came to be, and where it may lead should be part of the intellectual baggage of anyone who calls themselves a college graduate and who will function in the workplace of the twenty-first century.

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" ADULTS RE - LEARNING MATHEMATICS: MATHEMATICAL INVESTIGATIONS."

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Abstract:

Mathematical Investigation is a source designed to provide effective practice in some basic concepts in mathematical education and the development of adults' learning skills in a form that generates interest and enthusiasm. This, hopefully. will lead the way to the AH!, HA! experience -

> I thought I had forgotton forever, but it has encouraged a new found interest in mathematics and finally dispelled the mist of difficulty surrounding this subject.

'Adults relearning Mathematics' is some preliminary study of students' attitudes to the learning of Mathematics and teachers who taught them and it is an essential study to activate this awareness.

Introduction:

Mathematics learning in adults Continuing and Community Education has, by implication, three areas of concern: An overview

(1) **Community Education** Certainly, over the last decades, a revolution has occured in adults learning Mathematics and in particular, the concept of "Adults in Community Education", as a process whereby human populations adjust their environment. For a start (Saunders. P. 1981), theoretical orthodoxies have been challenged and the application of such a theory to the analysis of cities have given rise to new questions about the role of the urban system in a capitalist economy in which competition have le d naturally to one form of human organisation by forcing increased functional and spatial differentiation thereby creating utilitarian ties of natural interdependence. The development of "Adult Learning Mathematics- a research forum" is such a revolution.

Analysis of this way of thinking leads me on to Mathematics curriculum principles and practice - educational objectives (see below). My view is that it requires a critical evaluation of the term "Community Education" as well as the re-examination of the more radical traditional approaches to Mathematics in Adult Continuing and Community Education. This requires too raising issues and questions such as "Who are the community?" "How does the concept 'community' come to be used so widely in social theory by sociologists, psychologists and philosophers of Community Education?" (See McKenzie. R. (1967), Jones D. et al (1974), Plant R. (1974), Harvey D. (1973) and Rissman L. (1964))

I shall try to shake the dust off some of these ideas:-

CURRICULUM PRINCIPLES AND PRACTICE.

(ii)

Closely associated with the concept of Community Education concerns the development of curriculum principles and practice as well as the concept of educational objectives. Here the educational implications are fundamental. The increased incidence of planning ,preparation in development and the evaluation in mathematics in adult continuing and community education, means that one needs to include and assume and consider in the discussion curriculum theory, the most essential item in the armoury of modern innovators of continuing education.

What is meant by an educational objective? The need to clarify what is meant by an educational objective empirically has led to a considerable amount of research work, in particular in the USA. One of the most useful and plausible classifications of objectives is that proposed by Bloom's Taxonomy of Educational Objectives et al [1956].The generic term Taxonomy was carefully chosen by Bloom and his co-workers to indicate to us that their classification possessed certain characteristics. These are the cognitive defined as comprising objectives which emphasise reproducing something which has been learnt, the affective comprises objective which emphasise a degree of acceptance or rejection and the psychomotor-which consists of objectives which emphasise some act which requires a neuromuscular co-ordinations. Behaviour of the student is broken into roughly corresponding to thinking, feeling and acting.

What then is the significance of such classifications for Mathematics in Adult Continuing and Community Education teaching and learning? Bloom and his co-workers concerned themselves to compile a taxonomy of educational objectives regards knowledge as the "recall specific and universals, the recall of methods and processes or the recall of patterns, structures or setting" (Bloom el al 1956 pp 62-78 for classification, (Skemp. R. 1971) which according to Stenhouse (1975 p.17) is primarily a psychologist's classification, the logic of being related to the measurements of learning. As such, the significance of such classifications for Mathematics teaching is clear,

For example, if a student learns Pythagoras' theorem, he has achieved an objective in the cognitive domain, if eventually he comes to enjoy logical proofs and to appreciate what constitutes a "beautiful proof" then s/he has achieved objectives in the affective domain. Similarly, geometric drawing and say, the use of slide rule or the computer terminal, do provide us with some objectives in the psychomotor domain.

Fig ROA/1

TOPICS		BEHAVIOURS A	B	C
	1	*	*	*
	2	*	*	
	3	*		
	4	*	*	

Certainly, with Mathematical operations, where most behaviours as in fig ROA./1 appear to have cognitive origins, and Bloom's taxonomy is particularly relevant. For example, if behaviour A forms a basis for evaluation of one class, the behaviours of type AB will form another, while another type of ABC will form yet another class. All of these reveal some pattern that would be of some value in the development of theory and theory of learning Mathematics and clarify the language of Educational Objectives which might help to remove one source of confusion amongst educators.

(111) **Adults Learning Mathematics.** The third area of concern in this presentation is Adults Re-learning Mathematics through Mathematical Investigations. This has profound effects on the teaching of Mathematics since it gives students the power to perform numerical calculations with ease, (as can be seen in the work of the case study), which, for some, would be almost impossibly difficult.

Most adult students, defined as people over 18 years of age, share a fairly uniform experience of Mathematical education in the education system. Through their primary and secondary education, they learned how to add, subtract, divide and multiply. Perhaps many did not like it that much, but most seemed convinced that it was both useful and somehow good for them. Yet many adult students went through this experience of being enumerate will tend to shudder at the word "Mathematics". This no doubt provides us with a clue to the first prong of the attack that lead to revolutionary changes required in what was previously one of the most static of school subjects. With the new trends in education like the development of the National Curriculum as a whole which derived from the belief

that learning could actually be enjoyed, change had to be considered along these lines, particularly in Mathematics' education.

Why do so many people have problems with Mathematics? (see Frankenstein et al 1989). How can teachers of Mathematics, (my emphasis), help students deal with Mathematics' anxiety? What is Mathematics? Mathematics studies the relationships and calculations from quantity, shape, statistical information and time. None the less it is a practical activity with inference logic, and not merely a theoretical study. Although Mathematics has enormous power to solve practical problems, it is still regarded, justifiable, as an abstract subject. Yet Mathematics is a culminative subject and the analogy of constructing a house is appropriate here. My research into Adults Re-learning Mathematics shows that if the foundation of the house is weak, the resulting structure is bound to be shaky. A good foundation coupled with a good method of teaching Mathematics will surely produce good results.

It is therefore, appropriate to mention the Foundation Course in Mathematics that I teach at Goldsmiths' College, University of London which started in 1990. In addition to the questions posed above, I ask myself the following set of questions: (Angiama R. O. 1992).

1.1	What am I trying to do?
1.2	Why am I trying to do it?
1,3	How am I doing it?
1,4	Why am I doing it that way?
1.5	Why do I think this is the best way?
1.6	How do I know that it works?

and show how they can be brought to bear on Mathematics in Adult Continuing and Community Education. I shall also want to address each of these questions very briefly and in particular to relate them to the experiences of Adults Re-Learning Mathematics through Mathematical Investigations.

1.1 What am I trying to do?

The foundation course in Mathematics is designed to give those without the conventional qualifications the opportunity to study Mathematics to a standard which will allow entry to the Intermediate Certificate in Mathematics' course offered by the Department of Mathematical Sciences. The Intermediate Certificate in Mathematics is a one year course requiring attendance on one full day and two half days a week whilst the Foundation Course in Mathematics is a Saturday per week one year course to study Mathematics to A level standard. Completion of the Certificate Course leads to a place on one of the Mathematics' Honours Degree courses here at Goldsmiths' College.

1.2 Why am I trying to do it?

The reason why I am trying to do it is because it will help fill the gap in the shortage subject areas like Mathematics, within the education system. My goal then in developing the Maths' Foundation Course is that we should engage many people in the discipline in order to keep abreast with changes in society brought about by increased industrial and commercial activity which involved people in more complex Mathematical calculations. Both in the twentieth and twenty first centuries, many industrialists began to express concern about the mathematical inabilities of young people who came to them from schools and colleges of higher education for employment. As Elaine Williams notes in the Independent in July 23rd 1992:,

> Behind the bright demeanour of our graduates and under graduate students lie some fearfully innumerate individuals.

1.3 How am I doing it?

The focus is the development of modules of unit booklets and the value for students is greatly enhanced which included an element of student self-assessment including a self-diagnosis of strengths and weaknesses. Such self-assessment by students of their own learning has been very effective and each unit has its explicit aims and objectives, outlining the knowledge, skills and processes inherent in the unit booklet and these are shared with the students who have been asked to complete an exercise at the end of most pages. The unit scheme also offers the possibility of negotiation between the Course Co-ordinator and students on certain key issues, such as the method of teaching and learning Mathematics. I argue that Mathematics is a cumulative subject and it is like when building a house, if the foundation is very weak then it follows that the structure is bound to collapse. I also argue that a good foundation coupled with a good method of teaching is sure to produce good results, and we can be sure to turn out many good Mathematicians to meet the needs of our challenging and ever increasing demands of the schooling system and the demands of the technological world. The great acceleration of technological development here and around the world since the second world war has shown no sign of abating, rather increasing and the demands of the future are tremendous. Yet the teaching methods of teaching Mathematics remain very much one of teaching particular techniques formally in Mathematical Investigations, Mathematical Videos, Visiting Lecturers, Audio-Visual Demonstrations and Practice on the computers.

1.4 Why am I doing it that way?

The reason why I am doing it that way is because I believe that most adult students share a fairly uniform experience of Mathematics' education as well as young people in the education system. I have argued that teaching methods of teaching Mathematics remain very much one of teaching particular techniques formally and practice on the computers. Why computers? It is because the use of computers has helped adult students Re-Learning Mathematics better and gain more confidence in the subject. Throughout the long history of mathematical education, men and women have always wished they could calculate more quickly. Through the centuries, as each mathematical discovery was made and knowledge advanced, the calculations facing mathematicians became more and more complicated and demanded an ever greater amount of time. The use of the computer in the course has helped many adult students Re-Learning Mathematics and needs further investigation especially for very weak students but also to add variety for all students. At the same time many adult students who went through this experience of Re-Learning Mathematics confess in their innumeracy and lack of proper guidance.

A student of mine on the Foundation course in Mathematics (in 1991/2) said this,

> Leaving school over thirteen years ago with no guidance to prepare myself with the outside world has had an effect upon me even up to now. Gaining 4 CSE's I was unlucky not to find employment straight away from school. It took me two years to land my first job. My first encounter with work was a challenging one. But after settling down as a booking clerk, it wasn't enough. I wanted to do more but was unsure what it was I wanted to do. (MW1992)

1.5
WHY DO I THINK THIS IS THE BEST WAY OF DOING IT

The mathematics Foundation Course is now gaining momentum among the students of Electronics from the University of Greenwich, English and Psycology graduates and particularly among primary school teachers who have the responsibility to identify childrens mathem atical achievements not only in relation to the National Curriculum, but also in relation to their social, emotional and personal development.

The children in our schools today are growing up in an age rapidly been altered by science and technology. Keeping pace with accelerated advances and being unaware of what the future will expect of these children makes the task of the primary school teacher and those who produce the theories to guide these children more exciting and challenging than ever before.

It is essential that these children should have some understanding of mathematics and it's allied subjects in order to cope with modern living and realise that most of the scientific advances are based on simple concepts of mathematics that they can comprehend. I would argue also that much of A- level mathematics is concerned with mastering techniques.

1.6
HOW DO I KNOW IT WORKS

Mathematics Education is our saving 'Grace' to our living on this earth. A former student who is now reading a four year BSc mathematics degree here at Goldsmith's College said:

> The Foundation Course in Mathematics is the ideal course for recent school leavers and adults who are interested in updating their skills in the field of mathematics. By devotion, time and interest for just one year, you can achieve a level of mathematics to GCSE and A-level standard. After completing this course with no doubt you will be so motivated that one will more than anxious to continue to a higher level mathematics. I have done this course during 1990-1991 and at present I am doing a four year extension degree in mathematics. Thanks to the Mathematics Foundation Course (Ronald Josiah 1993).

Case Study 1
Investigation (M. Soudans.)

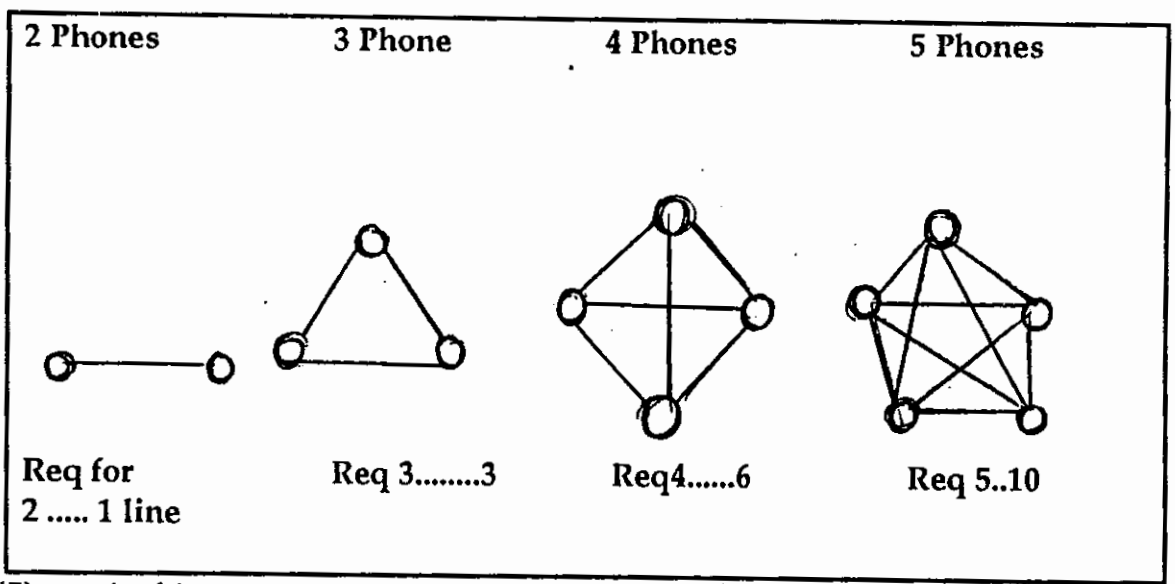
(1) Introduce the problem:

We have a set of telephones and we can't find out how many lines do we need to connect them because every telephone has to be connected to every other telephone.

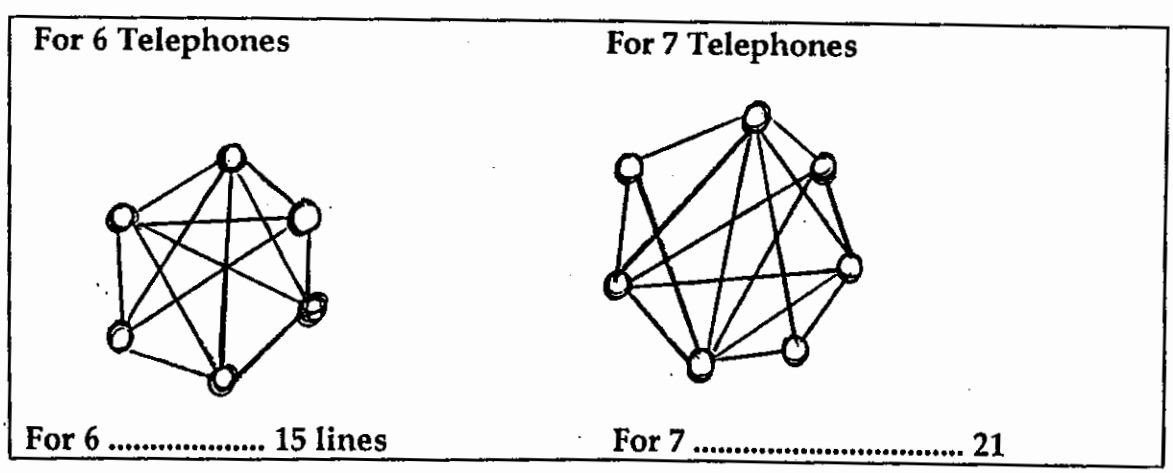
(2) To solve our problem we need very simple examples.

(3) Let's try 2/5 telephones by drawing lines between them.

(4)



(5) At this stage we can't say anything. We need to continue with more examples to see if any clear rule comes up.



Now let's collect all our results, from above, and show them in a table:

Number of Telephones	2	3	4	5	6	7
Number of Lines	1	3	6	10	15	21

Having done this table it seems to me that the number of lines of any number is the sum of the last number of telephones and of lines.

THE RULE FOR THIS INVESTIGATION.

Let us have n number of telephones and n' number of lines and P number of lines for (n - 1) telephones.

So $n' = (n - 1) + p$

Let us try to work out for 8, 9, 10 and 11 telephones.

For 8, n = 8 and p = 21
so n' = (8 - 1) + 21
= 28
So for 8 telephones we need 28 lines.

For 9, n = 9 and p = 28
n' = (9 - 1) + 28
= 36
So for 9 telephones we need 36 lines.

For 10, n = 10 and p = 36
n' = (10 - 1) + 36
= 45
So for 10 telephones we need 45 lines.

For 11, n = 11 and p = 45
n' = (11 -1) + 45
= 55
So for 11 telephones we need 55 lines.

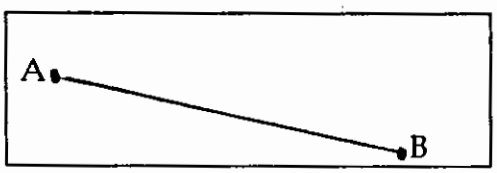
Case Study 2
Investigation 11 (m. Soudans)

Introduce the problem:-

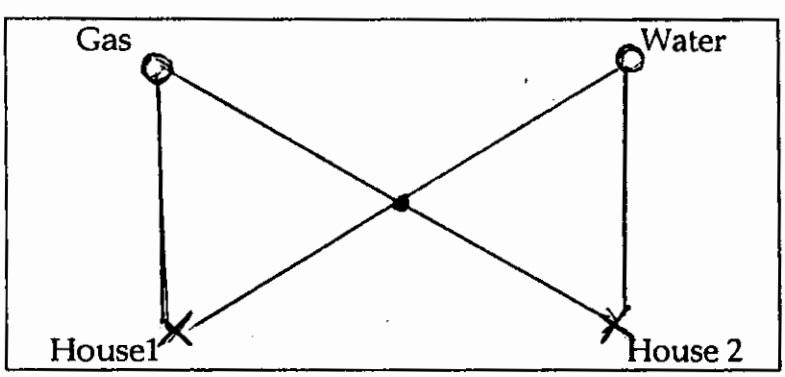
Each house gets water and gas from different stations i.e water station and gas station.

Say the water station is point A and the house is point B.

As we know, from the first investigation, we two points we have one straight line.



Now we test this with **two houses and the two services:**

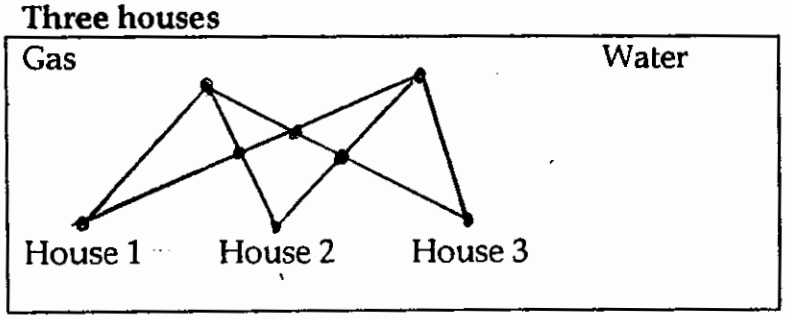


Here's a diagram of the pipes when there are two houses and there's **one** place where the gas and water pipes cross over each other.

Reminder:- Our problem is to find a general rule for n number of houses and the amount of times that the pipes cross over each other.

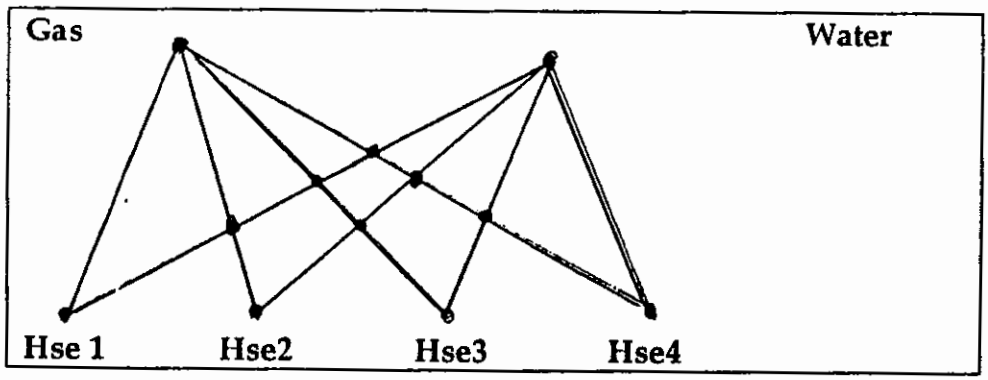
So let's go with more examples:-

Three houses



For 3 houses 3 pipes cross over.

4 Houses



For 4 houses 6 pipes cross over.

In order to save space here I will recount what happened when 5, 6 and 7 houses were investigated in the same way. The results were as follows:

For 5 houses 10 pipes cross over.

For 6 houses 15 pipes cross over.

For 7 houses 21 pipes cross over.

Lets copy all these results on a table so as to see if there is a rule for n houses.

Number of Houses	Number of cross-overs.
2	1
3	3
4	6
5	10
6	15
7	21

If we examine the results very well we will find that each line has a relation to the previous one.

Let us say that n = the number of houses and p = the number of cross overs.

So p = (n - 1) + k (where k is the number of cross overs for (n- 1) houses.

Having reach this conclusion let's try the rule out for 8 and 9 houses.

For 8 houses, **p = (8- 1) + 21**

= 28

For 9 houses, **p = (9 - 1) + 28**
= 36

In the same way, using this formula, you can work out the number of pipe cross overs for any given number of houses.

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Mathematics and Numeracy in the Practice of Critical Citizenship

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This work arose from separate projects which we have discussed together but are presenting in two separate sections. Section 1 is basically the work of Jeff Evans and Section 2 of Ingrid Thorstad.

Section 1
Initial Considerations

In recent discussions across various branches of education, it is striking to note the growing acceptance that mathematical and numerate thinking are "situated"; that is, they have to be observed and understood **in context**. This agreement is evident both in mathematics education (Lave, 1988; Nunes et al., 1993) and in adult education (Riley et al., 1984; Thorstad, 1991).

Indeed, the idea of **numeracy** has been used to emphasise the need for "the maths" to be learned (and often used) **in context**. What distinguishes the context of everday numerate thinking and problem solving from that of academic mathematics is the different activities and practices which form the different contexts. Thus the numeracies used in the work of builders, pharmacists and shop-managers are all different - because they are based in different practices and hence are specific to them. Therefore there is in principle a *discontinuity* between, say, school maths, and the numerate ideas and skills that are used in shopping - or between numerate aspects of two different practical activities, such as shopping and playing bridge. These ideas have been explored and developed in a wide range of recent research projects: those of Nunes, Schliemann and Carraher (e.g. 1993) in Brazil; those of Jean Lave (1988) in the USA (for a summary, see Evans and Harris, 1991); those of Valerie Walkerdine and others in the UK (e.g. Walkerdine and Girls and Maths Unit, 1989;); and those of Nick Taylor and others working within the democratic movements in South Africa (e.g. Taylor, 1989).

Now if numeracy is specific to each meaningful context in which it may be "found", it follows that we need to learn and develop it within specific practices, using the languages specific to that activity. (NOTE 1)

Let us consider what numeracy conceived within this new framework might be like. There is a rich variety of non-school practices which involve "numerate" aspects, and in which almost every adult in industrial (and other) societies participates:
- many work practices, such as engineering, accounting, making machine tools; plus
- a range of "everyday" non-work practices.

Some of the latter include:
- purchase (and/or production) and consumption of food and other necessities;
- involvement with, and raising of, children;
- paying for (and/or building) and maintenance of housing and its surroudings; plus
- engagement with discussions and debates about individual, family and public well-being, and about describing, appreciating, evaluating, deciding on future directions of public policy.

This last activity might, for the moment, be called *critical citizenship.*

It is possible to envisage the learning of what might be recognisable as numeracy or mathematics-type "skills" being taught in a way that is grounded much more explicitly in many of these practices, or perhaps in a combination of several. However, for school level learning, it would seem that the wide variety of *work* practices that might be relevant to the pupils makes the task of choosing a single (or a few) appropriate "grounding(s)" exceedingly difficult. And this difficulty will be heightened by the continuing high level of change in workplace practices that is predicted.

We would argue that, in many settings, there is likely to be more shared experience around contexts related to the learners as *citizens*, present and future, than around those related to them as future *workers.* Thus, we would like to explore more fully the characterisation of a practice of *critical citizenship.* In the UK of the 1990s, it is likely to involve engaging with issues such as (NOTE 2);

> *What have been the increases in income for different groups in society since 1979, and have these been fair?*
> *What would it mean for the increase in unemployment to be slowing down, would it matter much, and is it doing so?*
> *What would it mean for inflation? (see next section)*

We make a start here by focussing on the skills and resources likely to be necessary for the practice of "critical citizenship". The specific skills are likely to include :
- knowing about information sources and how to read them (see e.g. Irvine Miles and Evans, 1979; Frankenstein, 1989, esp. Ch. 17; Marsh, 1988);
- the ferreting out of information produced, but not published (see Irvine et al. (1979, Ch. 10) for some hints);
- methods for the production of information / data at a small-scale level in the community (see e.g. Kane, 1984); and
- interpretation of information either from other sources, or from one's own research.

For other possibly relevant skills, see the discussion of community researchers and "barefoot statisticians" in Evans (1989). (NOTE 3)

Thus, an important resource in the struggle for critical citizenship is access to, and a grasp of, *official statistics*, from which we obtain most of our information about government spending, unemployment, inflation, poverty, and many more (e.g. Irvine et al., 1979). In 1990s Britain, access to these sources of information takes place (largely through the media and) in a context constituted by the following tendencies by government and the state:

- mystification of trends through publishing of claims without reference to full details of the assumptions etc. which are necessary to interpret them; e.g. on the oft-repeated claims that there has been "increased government spending" in certain areas, see e.g. Radical Statistics Health Group (1987) and Radical Statistics Education Group (1987);
- frequent changing of crucial definitions; e.g. unemployment, "new hospital schemes" (a measure of the claimed expansion of the NHS);
- ceasing to publish certain statistics e.g. the number "living in poverty".
For fuller discussion of these last two issues, see e.g. the Channel Four *Dispatches* programme "Cooking the Books", Jan. / Mar. 1989.

These and other examples give grounds for optimism that links can be made between distinct practices, that can promote learning, including learning that may be useful for the practice of critical citizenship. In this enterprise, the concept of *numeracy* - especially in the sense above - can help to point the way ahead through developing numerate thinking which is grounded in practices such as *critical citizenship*.

These ideas prompted us to look for activities where critical citizenship could be practised. In the exploratory research described in the next section, we focus on adults training to be school governors, and their understandings of several numerate topics, including percentages and inflation.

Section 2

Understanding Inflation

I aim in this paper both to trace my journey from one piece of research to another in order to describe some of the processes involved in initiating research, and present some thoughts provoked by my results.

My starting point was 'Adult Numeracy and Responsible Citizenship', a project initiated while funded by the UFC. I was aware that considerable attention had been given to the numeracy requirements of adults in relation to their everyday needs and work but very little in relation to the numeracy needed to understand issues of political and social concern. Marilyn Frankenstein's text book 'Relearning Mathematics' (1981) is one of the few exceptions.

I decided to focus my research on 'Numeracy and School Governors' as school governors were being asked to assume greater responsibility for the management of schools, including the financial details of management. I was concerned that the changes either would completely disenfranchise many parents, especially those who were anxious about their maths, or at least would mean that they would not participate in crucial financial debates. Not everyone I spoke to agreed with my preoccupation: some thought that the 'detail' of financial calculations could be left to sub-committees.

I approached the research from different angles. On the one hand I observed Governing Body Meetings and Governors' Training Sessions and talked to paid officials in order to understand what might be expected of a school governor - and in particular their

numeracy needs. On the other hand I gave out a questionnaire at training sessions to a toal of 47 school governors and I interviewed some of them in greater depth.

It was only after some deliberation that I decided to include some mathematical questions in the questionnaire. I based two of the questions on calculations and concepts that had been required at one of the School Governors' training courses. In one question they had to calculate and add on 5% to an item on the school budget. The second question was designed to assess their understanding of the mechanics of inflation. I was interested in the latter question for several reasons. Not only had it arisen on the training course but it was clearly related to the notion of critical citizenship. Furthermore it was based on a question that Bridget Sewell (1981) had asked it as part of her study for the Cockcroft Committee. Sewell had written:

> Percentages play an ever increasing part in the dissemination of information, both through the news and from central government. Those who lack the skill even to calculate 10% are surely handicapped when attempting to understand the affairs of society.(Sewell, 1981).

Her question was phrased as follows:

> On the news recently it was said that the annual rate of inflation had fallen from 17.4% to 17.2% '
> What effect do you think this will have had on prices?
> (If answer 'none') What do you think ought to happen if it had fallen to, say, 12%?

There were other reasons too for my interest in this question: the replies of her interviewees did not fall into the standard pattern. In answering the other questions it would seem that most people either assessed their capabilities fairly accurately or underestimated them, as when they chose to answer a question they usually got correct. *Inflation was the only question which many people elected to answer and yet gave the 'wrong' answer.* Of the 50 people she asked, 16 gave the correct answer, 22 the wrong answer and 12 did not answer.

I updated the question and phrased it slightly differently as it was part of a questionnaire and not given orally.

> The rate of inflation has fallen a lot in recent years.
> What effect do you think the recent fall to 4½% should have had on prices?
>
> The prices should have continued to go up
> No change
> Prices should have gone down.
> I don't know.

Over 80% of them gave correct answers to the '5% question' but only half of them gave a correct answer for the question on inflation. I then went on to interview 16 of them as part of the broader research which gave me an opportunity to ask them how they had 'worked out' the answer. And, because I was now interested in finding out more about how people explained their choice of answer, whether or not their answer was right or wrong, I also asked some students in an adult numeracy class.

There were two particularly interesting answers from those who had answered

incorrectly in the questionnaire: both had said that prices should fall. One governor, when she came to explain her choice said:

> The prices wouldn't actually go down would they? They wouldn't go up so fast.
> (But you said they would go down.)
> But some things go down, like house prices. They've gone down a lot.

That she understood the concept of 'the rate of inflation' is clear from this answer. It would seem that her first answer, the wrong answer, was as a result of her awareness of the importance of house prices in terms of budgeting. The second used all her 'common sense' to help her come to terms with the concept:

> If the rate of inflation comes down, therefore the mortgage comes down. That's what you look for. But prices do not come down - they should but they don't. I do not bring my prices down. You tend to think, I'll pocket that little bit extra and when prices have gone up again I haven't got to put the price back up.

This governor correctly realised the link between the mortgage rate and the rate of inflation, but she wrongly thought that falling inflation meant that shop prices should fall. However, this mistake was cancelled out by her practical decision not to lower her prices. I was particularly interested in these two answers as they seemed to illustrate the benefits and hazards of learning in informal situations.

These aspect of the research recall a series of studies into the learning mathematics in out-of-school situations, in particular those carried out by a team in Brazil into the mathematics of young street vendors, foremen on building sites and bookies. Their research seemed pertinent to the situation many adults entered, be it school governors' needs to 'learn on the job' or all adults' needs to understand as much as possible about inflation. In all these work situations, the people involved seemed to have a better understanding of the topic than the adults that I interviewed had of inflation. The reason for this may be that anyone acquiring new mathematical concepts in work environments, where incorrect answers can lead to immediate and disastrous consequences, is in a position where the calculations are verified. However, this is not true in the case of isolated adults learning about inflation. Most of them had gained their knowledge from the television and newspapers and so their concepts were rarely challenged and they were reduced to making sense of the concept without the benefit of feedback. The stage of interactive learning, through discussion with others, be it at work or in a class, is missing, as is the opportunity to verify their ideas. On top of all this, most people's cynicism leads them to believe that prices would never go down in practice even if that is what they believed should happen in theory. However, the results of Schliemann and Acioly (1988), who investigated the understanding that bookies had of probability, clearly show that the greater the length of time at school the greater the likelihood of their giving logical answers to questions relating to probability which seems to indicate the need for a variety of approaches to learning and in particular formal input as well as meaningful everyday situations.

So, from the initial question: Do school governors understand 'the' meaning of inflation? I moved on to: What do adults understand by inflation? Further questions such as: How do adults acquire their knowledge? Does the quality of their understanding bear any relation to the different ways they acquired their understanding, their mathematical knowledge and ability, or the number of years that they spent at school?

Notes

1. This raises a whole series of questions about traditional ideas of numeracy and how it is taught and learned. In particular, the position on numeracy put forward here is exceedingly sceptical that transfer of learning from school to other settings such as particular jobs is anywhere near as straightforward as the proponents of other views on numeracy would have it. For further discussion of these issues, see Evans (1995).

2. We should like to acknowledge the contribution of members of the audience attending our presentation of this paper at the ALM Founding Conference, July 1994.

3. Radical Statistics Group (RS), including its thrice-yearly newsletter. RS also receives requests from voluntary and pressure groups for help in questionnaire construction, sampling, etc. in connection with surveys of "needs", self-monitoring, etc. Further information from: RS, c/o London Hazards Centre, 3rd Floor Headland House, 308 Gray's Inn Rd., London WC1X 8DS.

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The Role of Informal Learning in Adult Numeracy

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I want to start by trying to explain what I mean by 'informal' learning, and before I can do that I need to talk about context. The term context can be used in two distinct ways: mathematical problems set in the context of people's lives and the methods employed for solution, and the context in which the learning of mathematical skills/knowledge takes place. This gives me then three areas for discussion: **settings** in which mathematical skills are acquired, the **methods** which are used to solve problems, the nature of the **problems** themselves, in which I can consider informal versus formal. Eventually, with particular reference to my own research, I hope to be able to consider the significance of the informal in learning.

Let us start by looking at the places. Informal settings include the home and workplace while formal settings include the classroom. Are there any differences between them and the type of learning which can take place there? Informal settings are natural, whereas formal settings are specially set aside areas often designed for the purpose. In informal settings there is a fusion between the intellectual and the emotional, the relationship with the teacher, the teacher's personality are key elements in the process. In formal settings the separation of the intellectual from the emotional is encouraged and the latter often denigrated. Because an informal setting is culturally based the learning itself supports traditional beliefs and values: meanwhile formal settings are externally provided and there may be scant regard for the learner's background and little cultural resonance. An informal setting allows for a variety of teaching/learning strategies such as imitation, identification, co-operation and observation. In a formal setting the artificiality of the arrangement of teacher and many pupils means that language must be heavily relied upon.

I want to move on to look at formal and informal methods. My washing machine broke down recently and I called in someone to mend it. The bill I was presented with is shown as fig. 1. I include this not because it seemed a large amount (I might do better to buy a new machine!) but because this is a very good example of someone calculating 17.5% VAT by an informal method. The calculation has been made, not by the school textbook method of

$$\frac{17.5}{100} \times \text{amount} = \text{VAT}$$

but by finding 10%, then 5%, then 2½% and then adding. My workman was busy solving a problem which was generated by his everyday activity. All to often we think that we can reflect the real world in the problems we set our students simply by clothing the same old classroom examples in the language of everyday.

The following problem comes from the National Assessment of Educational Progress which has been used with more than 90 000 Americans.

> A parking lot charges 35 cents for the first hour and 25 cents for each additional hour or fraction of an hour. For a car parked from 10.45 in the morning until 3.05 in the afternoon, how much money should be charged?

COMPACT SERVICES

688 LONDON ROAD
WESTCLIFF-ON-SEA
ESSEX, SS0 9HQ
Telephone: (0702) 712092/3
VAT NO 250 2346 00

SPECIALISTS IN
REPAIRS — SPARES — SALES
TO MOST
DOMESTIC APPLIANCES

INVOICE TO: Name Hilton

Address 5 Dutton Rd

Westcliff

Date 9-2-93. INVOICE Nº 15346

DESCRIPTION	AMOUNT
Cooker Thermostat	97 98
9.70	
4.85	
2.43	

No claim can be considered without production of this invoice.

AUTHORISED SERVICE CENTRE

RETRA

SUB-TOTAL	97 98.
VAT @17½%	16 98
INVOICE TOTAL	113-98

fig. 1 Repair bill showing VAT calculation in left-margin

This question was answered correctly by only 47% of the 17 year olds tested (proving that the USA has a numeracy problem). A simple analysis of what happens in the formal test situation as opposed to real informal parking lot arithmetic challenges the validity of what is supposed to be assessed. This is based on Maier (1991).

TEST	PARKING-LOT ARITHMETIC
(1) use pencil and paper to do arithmetic	(1) mental arithmetic used
(2) complete data provided in the question	(2) gather data from a variety of sources; watch, parking-lot board, ticket, parking attendant
(3) data provided is exact	(3) round figures up/down to ease calculation
(4) use standard, approved algorithms (to gain the method marks)	(4) use non-standard arithmetic to ease mental calculation
(5) arrive at an exact, correct answer	(5) arrive at an estimation of the likely cost
(6) work in isolation	(6) ask for a second opinion if necessary
(7) work to given time constraints	(7) stop figuring it out at some point when reasonable answer reached
(8) previous experience not related to this problem	(8) use previous experience of parking lots to test for reasonableness in (7) above
(9) no further use for the answer calculated	(9) use answer to check have enough money to pay

I am sure people could add to this list without any difficulty, as well as point out that parking lots do not operate like this (even in the USA). It highlights the fact that methods of calculation are different in different situations as is the nature of the task. Note that for the supposed real-life problem no previous everyday experience was relevant, it did not matter how much you knew about cars or parking all you had to do was remember the correct algorithms. In real life adults constantly bring in extraneous information from a wide variety of sources (e.g. school, television, newspapers, work) to help with the mathematics. As Thorstadt (1992) showed, it is this inclusion of a whole variety of experience that may lead to unexpected answers to seemingly straightforward numerical problems.

It might be useful to give the following analysis of informal and school mathematics, from Harris and Evans (1991), as a summary of the discussion above:

INFORMAL MATHEMATICS	SCHOOL MATHEMATICS
embedded in task	decontextualised
motivation is functional	motivation is intrinsic
objects of activity are concrete	objects of activity are abstract
processes are not explicit	processes are named and are the object of study
data is ill-defined and noisy	data is well-defined and presented tidily
tasks are particularistic	tasks are aimed at generalisations
accuracy is defined by situation	accuracy is assumed or given
numbers are messy	numbers arranged to work out well
work is collaborative, social	work is individualistic

correctness is negotiable	answers are right or wrong
language is imprecise and responsive to setting	language is precise and carefully differentiated

When I started my research project at Essex University I was charged with finding out about 'innumeracy'. I found there was no concensus on what was meant by innumeracy, but everyone thought that there was a lot of it about. In an attempt to get to grips with the concept I revisited the research that had concentrated on finding out what were the numerical skills needed or used by the adult population. In particular I read Cockcroft (1982), Sewell (1981) and Harris (1991). I thought that if I knew what 'numerate' was then the opposite must be 'innumerate'. Confidence is often a key factor in an adult's ability to use a skill, so research which aimed to find out how people feel about numeracy was also interesting. I read Bovill (1990). I began to be fascinated by this and when the opportunity arose to sample a group of people who used arithmetical skills in their workplace I decided to conduct my own mini-survey.

The methodology employed was a structured interview with 20 employees using questions from a prepared sheet. I essentially asked each interviewee about a particular numerical skill (e.g. adding decimals) whether they used it in their work, in everyday life, how confident they felt about using it, what methods they employed for dealing with the computation both at work and at home. The list of skills used was essentially obtained from Cockcroft and Harris. I have not the space to go into all my results, but I will deal with those that surprised me.

My respondents talked about 'figure work' rather than numeracy or mathematics. 'I like doing figure work' was frequently mentioned. I wondered whether this meant that they were denigrating their own skills. Do they think that what they are doing is at a lower level than mathematics or numeracy?

The survey found that the numerical skills identified as being useful for work (fig. 2) followed very much those found by Harris (fig. 3). The real surprise was the number and range of skills people identified as being used in everyday life (fig. 4). What was revealed was a very rich mixture of mathematics which was brought into such diverse everyday activities as designing counted cross stitch embroidery or sailing a boat at the weekend. While at work respondents were only required to perform routine and mundane calculations. This finding supports Evans (1991) who suggests that the contexts in which mathematics is taught should revolve around citizenship rather than the workplace. There was evidence that the areas in which adults had problems, or were least confident, were those where they had not found the skill useful in everyday life, e.g. area of a circle. There was also an indication that problems occurred where the adult was expected to remember a formula.

There were definite differences between the way that people chose to compute depending on whether they were at work or not (fig. 5). There was more use of mental arithmetic and paper and pencil in everyday life despite the fact that respondents had calculators at home. This supports the view that people use different methods for solutions in different contexts.

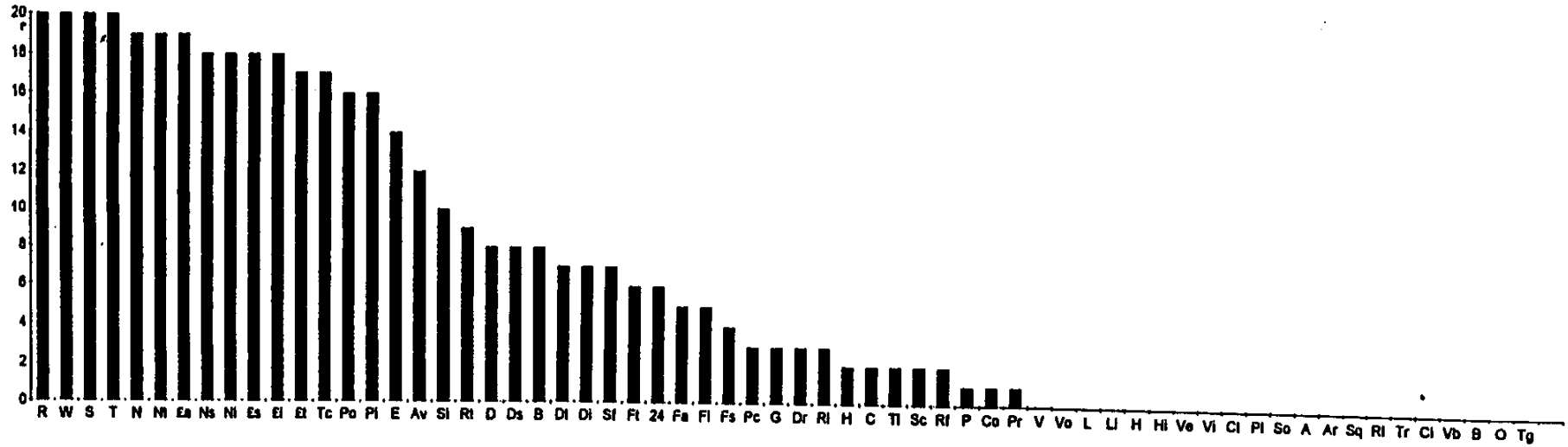
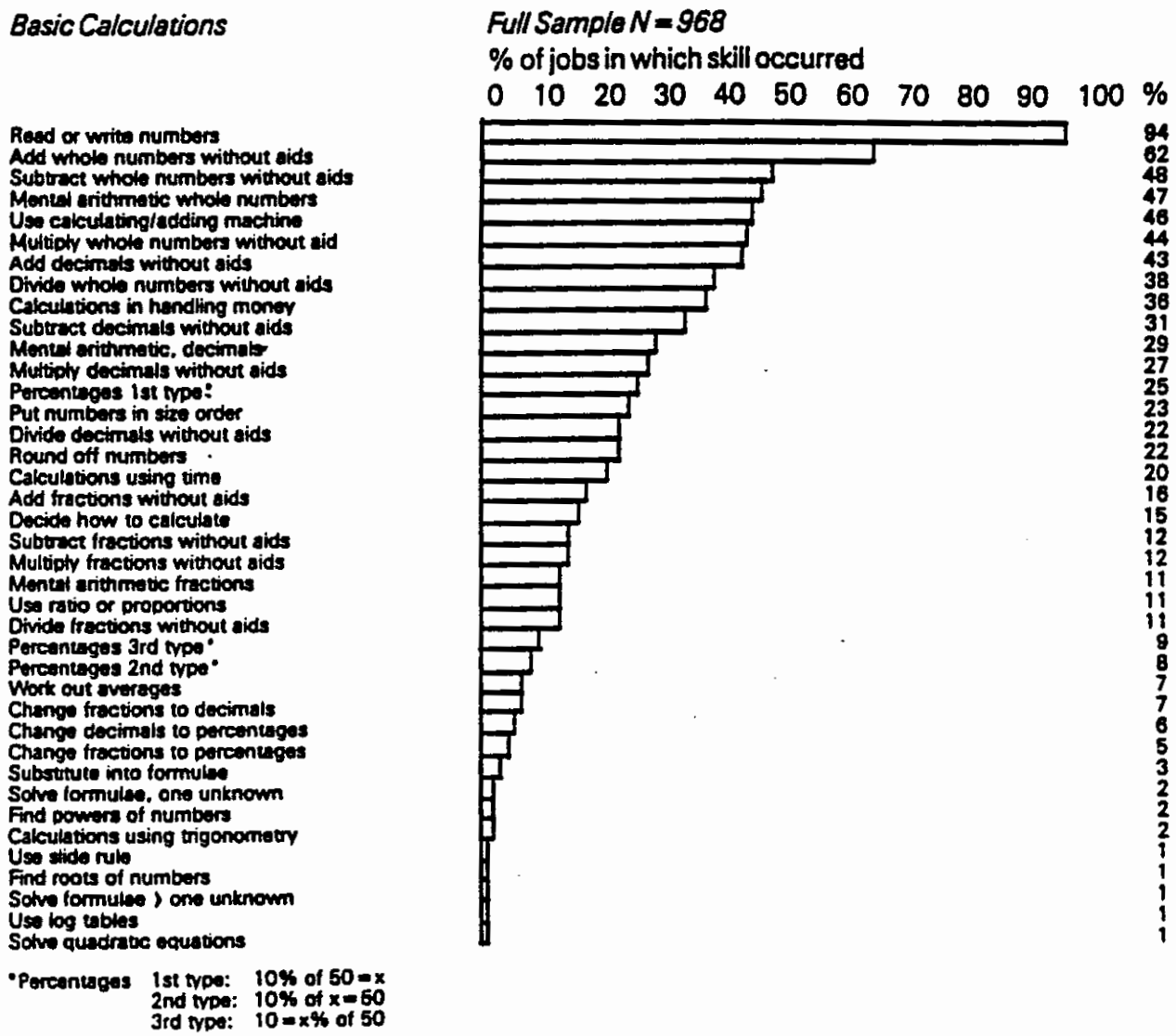


fig. 2 USE OF NUMERACY IN THE WORKPLACE (FREQUENCY ORDER)



*Percentages 1st type: 10% of 50 = x
2nd type: 10% of x = 50
3rd type: 10 = x% of 50

NB. These figures represent the numbers of people who answered specific questions. The same people sometimes show that they are in fact using some of the above skills when they answer questions from the other questionnaires. The figures above should therefore be taken as minima, not absolutes.

fig. 3 Use of numeracy in the workplace (Harris)

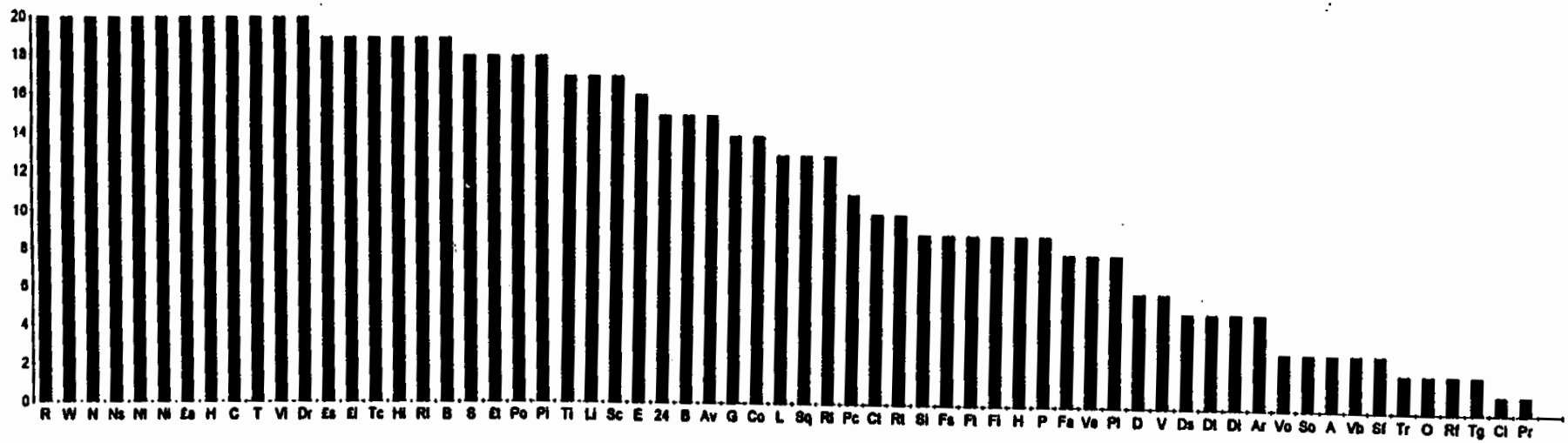
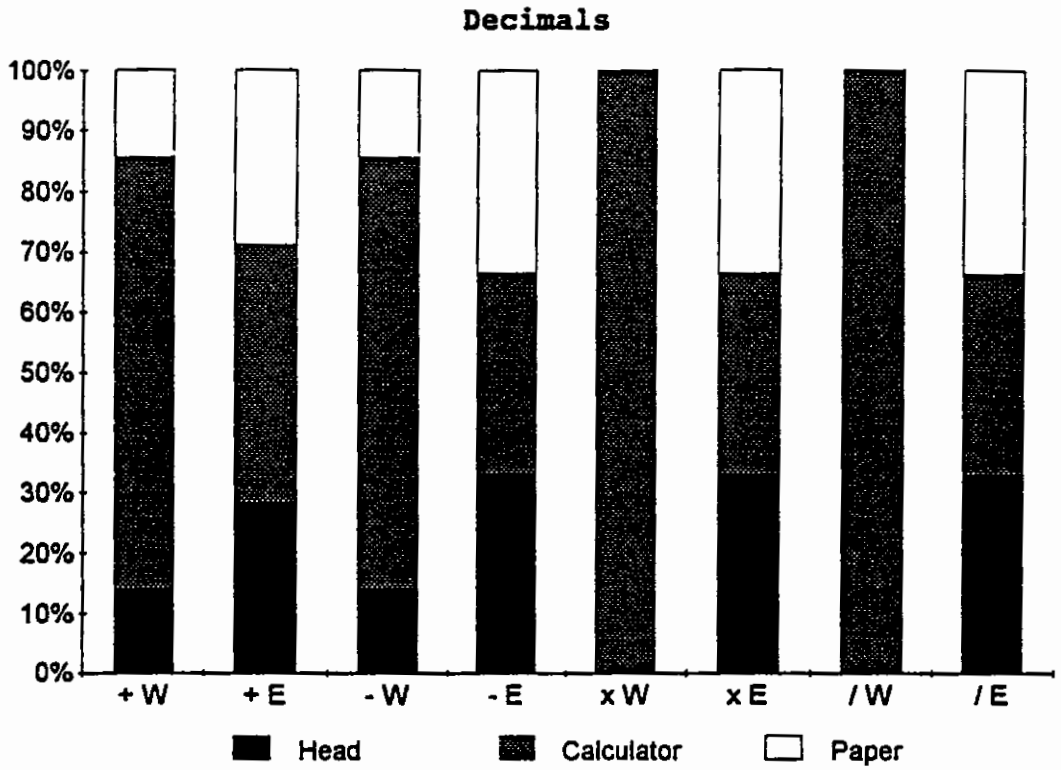


fig. 4 USE OF NUMERACY IN EVERYDAY LIFE (FREQUENCY ORDER)

KEY

E	rounding off	NUMBERS
R	reading	
W	writing	
S	spoken	
Si	size order	
N	addition	WHOLE NUMBERS
Ns	subtraction	
Nt	multiplication	
Ni	division	
£a	addition	MONEY
£s	subtraction	
£t	multiplication	
£i	division	
H	handling	
C	giving change	
D	addition	DECIMALS
Ds	subtraction	
Dt	multiplication	
Di	division	
Fa	addition	FRACTIONS
Fs	subtraction	
Ft	multiplication	
Fi	division	
Po	of an amount	PERCENTAGES
Pi	increase/decrease	
V	VAT on	
Vo	VAT out	
T	digital/analogue	TIME
24	24 hour	
Ti	time table	
Tc	time calculations	

L	length	metric	MEASURE
Li	length	imperial	
H	weight	metric	
Hi	weight	imperial	
Ve	volume	metric	
Vi	volume	imperial	
Ct	convert	metric/imperial	
Rt	read table		GRAPHICAL
P	pictogram		
B	bar chart		
Pc	pie chart		
G	graph		
Pl	plane shapes		GEOMETRY
So	solid shapes		
A	angle with protractor		
Ar	recognise angles		
Sq	square		AREA
Rl	rectangle		
Tr	triangle		
Ci	circle		
Vb	brick		VOLUME
Dr	directions		MAPS
Sc	scale		
Co	co-ordinate		
Ri	adapt		RATIO
B	best buy		
Av	average		STATISTICS
Pr	probability		
O	odds		
Sf	substitute formula		ALGEBRA
Rf	rearrange formula		
Tg	trigonometry		TRIGONOMETRY



METHODS USED TO COMPUTE DECIMALS

(W = WORKPLACE E = EVERYDAY LIFE)

fig. 5

The use of imperial and metric measure is illustrated by fig. 6. Respondents overwhelmingly used imperial measure when given the choice, even though they may have been taught the metric system at school. There appeared to be various reasons for this:

imperial units seem to be a more 'natural' unit, for example, they are related to parts of the body

the size of the imperial unit is more useful than corresponding metric units, for example, 8 oz. of flour is more user friendly than 250 g

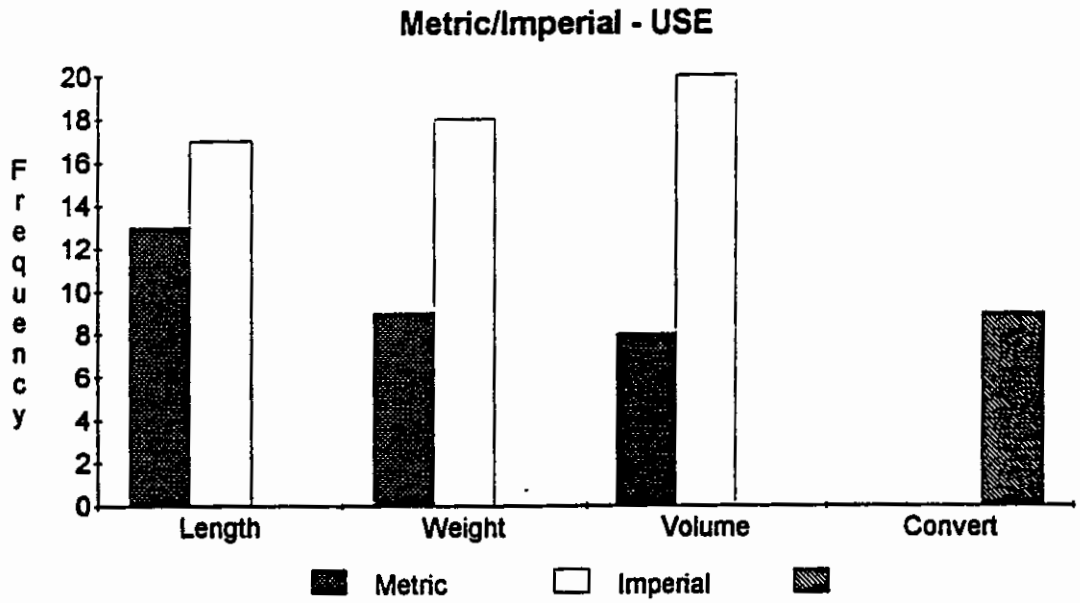
the UK has still not gone metric; try buying a kilo of apples, driving a kilometre and drinking a litre of beer at your local!

imperial measure is used in the home.

If I go back to the original theme of informal learning, I think that this research supports the view that informal settings are more efficacious than formal ones. Further analysis of the use of imperial rather than metric units had respondents saying "I was not taught feet and inches at school only metric - I've used it with my father and my grandfather". "If I measure I use metric - with Dad I use feet and inches". "Metric length wouldn't mean anything to me - at school it was metric, but my parents talked in imperial". It was also clear that respondents were making extensive use of conversions between metric and imperial units, and knew the most common ones. Imperial measure is learned informally. Metric measurement is taught in schools; it relies on language and it challenges common sense ideas about the size of units. Is there little wonder that it is so readily discarded?

Finally I must note the finding that there is a greater breadth of use of mathematics in everyday life and a narrowness in the skills required in the workplace. Respondents were able to identify mathematics in a whole range of hobby and leisure interests. It was fascinating to note from the long list of examples cited by respondents how practical the mathematics was. One respondent said, "I could adapt a recipe practically but not mathematically". This would imply that, for her, physically dividing/multiplying quantities is not mathematics. Perhaps she feels 'real' mathematics only happens on a piece of paper. Doing long division is mathematics, separating objects into piles is not. This is another example of people denigrating their skills, which is sad. A real understanding of a great deal of mathematics comes only through handling physical objects, people need to be given the confidence to recognise this as valuable.

In conclusion, are there any messages here for adult educators working in the field of numeracy? I would suggest that a lot of good practice in adult education has already emerged from a deep understanding of the importance of informal as opposed to formal settings for learning. Although adult classes take place in set aside areas, tutors pay careful attention to the ambience of the classroom. The relationship between tutor and learner is always recognised as key by both parties. This is why my adult students still expected to carry on discussing their essays when they met me in Sainsbury's. Adult tutors recognise the importance of previous learning by their students and will seek to build on both intellectual and emotional experiences. The teaching methods employed often encourage imitation, identification and observation and adults are expected to work in self-supporting groups co-operatively. Due recognition of the importance of the learner's cultural identity is taken when designing learning materials and in classroom practice.


Metric/Imperial - USE
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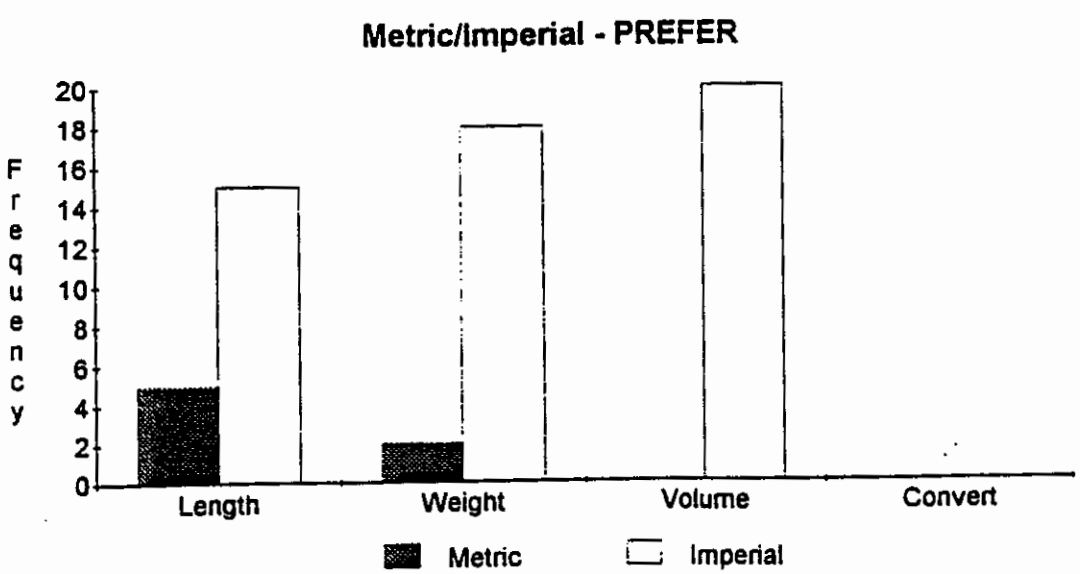
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fig. 6

Adult numeracy tutors still have problems deciding content. I think the key is to base work in everyday uses of mathematics rather than the supposed needs of the workplace. I suggest this partly because there is a greater richness in the mathematics used there, but also because the changing nature of employment and the frequent need for retraining render some computational skills ephemeral. Clearly the best people to set learning agendas are the adult learners themselves. Tutors need to have the confidence to cope both with this and the need to allow learners to develop their own informal methods of computational solution. It is only by learners developing methods which are meaningful to themselves that skills can be improved. There is a further point here about the application of concepts and skills learned in one context being applied to another. Lave (1988) writes

> conventional academic and folk theory assumes that arithmetic is learned in school in the normative fashion in which it is taught and is then literally carried away from school to be applied at will in any situation that calls for calculation

As Harris and Evans (1991) point out transfer is

> an assumption rather than a finding.....the viability of the concept of transfer remains largely unquestioned in England and Wales, particularly so in the sectors of government that determine curriculum content

Hence we need to make sure as educators that our students own their own learning and have a large number of different contexts in which to practise their skills.

I wonder whether there is genuinely a problem with innumeracy. As the discussion on parking-lot arithmetic showed, we must be wary of results which have been obtained by using tests. Perhaps I am wearing rose-tinted spectacles when I see numerate adults everywhere engaged in exciting mathematical activities. It is not that people are innumerate: it is just that they are doing 'figure work' while the people they think of as proper mathematicians struggle with the long divisions.

I would like to thank Prof. Stephen Barnett for his help and encouragement during my period at Essex University.

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Mature Students in HE; Academic Maths Support

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Academic Maths Support at Sheffield Hallam University (SHU)

At SHU there was a demand from staff and students alike to provide some form of support for students who seemed to be struggling in mathematics. SHU's academic maths support facility, Maths Help, was set up in response to this demand. This seems to be a need which is common to many institutions and this need is frequently met by the setting up of some form of support. (Beveridge and Bhanot (1994:13) report that 76% of FHE institutions have some form of maths support.) Maths Help offers a daily open access drop-in session and specially tailored short courses. It serves education students at the Mathematics Education Centre and students who are maths specialists and who are studying other serviced subjects at City Centre sites.

An analysis of who uses Maths Help has revealed that the need for support in mathematics is widespread and not, as might be assumed, only for those with weak or non-traditional maths backgrounds. Students with a range of previous qualifications, on a range of courses with a range of attainment and performance levels come with a range of problems.

Methodology

The initial purpose of our study was to evaluate the effectiveness of the Maths Help system with a view to improving the level and nature of the support offered. We are involved in an ongoing project to review who comes forward for help, why they come, why others don't come, what is causing students' difficulty and to investigate whether Maths Help is supporting those needing support, if it is actually helping students and if it is the most appropriate form of support.

In this research we have focussed our investigations on the students themselves through direct observation, questionnaires and in-depth interviews. We have also undertaken an analysis of Maths Help users and an analysis of assessment performance.

Theoretical Context

We have chosen to set our work in the context of general research

findings on teaching and learning within HE rather than the more specific research on maths learning most of which has been conducted at school level. A predominant concern of research has been to investigate students' experiences of studying and learning in HE with a view to improving that experience. Marton and Saljo (1984:36-55) focussed attention on deep and surface approaches to learning. These concepts are now established within the research literature and embedded in many of the subsequent curriculum developments. (Gibbs 1992:24-161)

More recent work has focussed on a variety of factors which encourage deep or surface learning. A number of studies have investigated the relationship between students' approaches to learning and the contexts in which they are working. Biggs (1989:17) reports that

> Teaching that gave evidence of deep learning contained in sharp form one or more of the following:
> - an appropriate motivational context
> - a high degree of learner activity
> - interaction with others, both peers and teachers
> - a well-structured knowledge base

The student complements these four elements with her/his intrinsic motivation. We are particularly interested in the motivational factors and the acquisition of a well structured knowledge base.

There is an assumption in the literature that deep approaches to learning are always "best". At first we questioned this; for many of the students we help, maths is not the main focus of their study; there is increased availability of IT maths tools which, for example, enable students to solve equations without "understanding". However, preliminary findings suggest that students are keen to understand their maths and that a lack of deep understanding of some "basic" concepts is creating problems for a significant number of students. Staff want their students to have "lasting learning" and "transferable learning". Gibbs (1992:4) examines the notion that a full understanding is not always required and that an ability to memorise without understanding is sometimes enough. He reports that

> Studies have shown that a surface approach does tend to produce marginally higher scores on tests of factual recall immediately after studying. However this small advantage is quickly lost. A surface approach leads to rapid forgetting and as little as a week later students who have taken a deep approach will score far higher than those who have taken a surface approach, even on tests of factual recall.

Maths Help in this Theoretical Context

We have considered our initial quantitative findings (who comes to Maths Help, what they ask for help with etc) within this theoretical context.

By providing the drop-in facility we are sending out signals to students. For some this reinforces the validity of their anxiety and can create an expectation of a "quick fix". For others this provides a non-embarrassing , non-threatening initial opportunity to ask for help.

Does the drop in facility encourage surface approaches to learning? If it does, do we need to be worried? Or, is it important that students have at least the opportunity to admit their difficulties in the belief that this is the first step towards being able to do something about them?

Do we provide the four key elements described earlier in a drop-in situation? Our conjecture is that this is very difficult to do. Is the students' motivation intrinsic? Our evidence suggests that the students' need for Maths Help is assessment driven, that is, their motivation is extrinsic.

Can the specially tailored short courses do better? We can create an environment that encourages learner activity and interaction, we can work towards a well structured knowledge base. Much depends on the students' motivation. The majority of students who request short courses and who come to short courses are mature students. The provision of short courses can provide an opportunity for mature students to bring their wealth of experience to the learning situation; the issue for us then is about **reconstruction** of their knowledge base - not "topping-up" or "filling the gaps".

Our data suggests that significantly large numbers of students, including those with formal mathematics qualifications, feel that they do not have a well structured knowledge base in mathematics. Some students without this nevertheless appear to have a deep approach to learning, often culled from other life or work experience. Others clearly have neither a well-structured knowledge base nor a deep approach to learning. The following excerpts from in-depth semi-structured interviews with two students illustrate this:

Student A says

I got 90% in my algebra assignment.
I don't understand algebra.
You can learn by simply accepting but it does not allow you to put it into other contexts.

At work someone would come to you with a problem. What you saw on the surface wasn't the real problem. You had to look deeper.

Student B says

I only got 20% in the algebra assignment.
I can do basic algebra.
I'm that frightened and I know I can't do it.
I understand the help I've been given but I still don't know how to start.
I don't know what you're supposed to do.
I liked it on my HND course. A topic was introduced, questions were set, there were lots of assignments. I need deadlines. Basic explanations.
I failed the maths on my HND course.

Our interpretation is that Student A appears to be "alright", this student can take a deep approach and more importantly perhaps can identify problems and take action. For A, Maths Help's role is that of facilitating the reconstruction of the student's knowledge base.

Student B, however, insists that a surface approach is right albeit acknowledging that it has not worked. B may be able to do basic algebra but does not have a well structured knowledge base. B does not have the connections and the ability to relate new work to old which would enable a starting point for solving problems to be found.

How can Maths Help help student B? Student B does not have a well structured knowledge base and is not taking a deep approach. Our conjecture is that it is easier to help a student to reconstruct their knowledge base if the student is able to take a deep approach. Can we instruct students in the use of deep approaches? Ramsden, Beswick and Bowden's (1987:168-176) research shows that attempts to do this have resulted in an effect opposite to that desired.

The theoretical context of deep and surface approaches to learning whilst providing a helpful framework for the categorisation of students does not help us to formulate appropriate support strategies for some students. The students we see have already learned, or rather failed to learn, algebra. Students with deep-rooted difficulties think they have learned algebra; this is one of the major problems. How can we help students restructure their knowledge in such a way that they can use it? We need to investigate the meaning of algebra to individuals and so it may be more helpful to interpret our findings in the context of hermeneutics.

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A discussion exploring areas of enquiry into the role of language in learning mathematics

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The participants in the workshop described their work experiences and their interests in the role of language in learning mathematics. Experiences were very wide-ranging, from native speakers of English teaching English-speaking and ESOL students from many countries, in Britain in a wide variety of provision, to participants from Africa and South America teaching in other parts of the world in different languages.

The group reflected that this range of language experience was similar to that of groups of students studying mathematics. In Britain, mathematics students' level of English varies from very little to fluency, and their oral and written skills may be at very different levels. In other countries a similar situation occurs where adult students may be learning maths in a first, a second, or subsequent language.

Many bilingual students have extensive mathematical knowledge in another language than the one in which they are presently studying. They may need to transfer this knowledge into the second language and may wish to develop their mathematical knowledge further in the second language. Many difficulties may be encountered, but this process may also help to develop greater understanding of the mathematics.

Other students may have little formal mathematical education, but much practical experience of calculation and problem-solving. But practical knowledge in one culture and language context may not translate easily into another because the cultural cognition may be different. Examples given were of speakers of Fang in central Africa who refer to time in terms of events rather than dates, and the Inuit language which has many different words for different kinds of snow: both use different cognition from Northern European culture and language.

The group discussed the language used to express mathematical concepts and ask mathematical questions: formal statements of mathematical truths both general and particular: questions, and combinations of statements and questions. Understanding and being able to use this register of language requires a knowledge of extensive vocabulary and the particular syntax used. Examples given were the use of the passive voice in scientific methodology and the conditional form.

A study into the most fundamental language used in mathematics: the structure of the number words, and their relationship to the concept of place value is that carried out by Fuson and Kwon. They compared the number words for the `teen' and tens numbers in European and East Asian languages and found that in Chinese and many Asian languages the words for the `teen' numbers state the ten first and the unit value second, and are identical with the words for ten and the units. In European languages some or all of the `teen' words are

expressed with the unit value first and the ten second. For example the number fourteen is `ten four' when translated literally from many East Asian languages. (Fuson and Kwon in Durkin and Shire, 1991:226)

Fuson and Kwon found that the result of this form of expression was that children in Eastern Asian countries were able to learn the `teen' numbers earlier than American children and that the Eastern Asian children also grasped the concept of place value earlier. American children tended to learn the `teen' words by reciting them as an extension of the unit words without understanding the place value aspect of these numbers: e.g. that fourteen is one ten and four units.

The researchers found a further difficulty for American children in the irregularity of some of the `teen' words: eleven, twelve, thirteen, fifteen; and some of the ten words: twenty, thirty, which have a less obvious relationship to the unit words than in the East Asian languages, where they tend to be completely regular.

They also found that East Asian children were formally taught a method for adding `up over ten' and subtracting `down over ten', i.e. adding and subtracting with two unit numbers whose total exceeds 10, by learning and using the number bonds of 10, e.g. 4+6=10, 3+7=10. American children in the study were not taught these strategies.

This study of children may explain the root of the difficulties some English-speaking adult students have with adding and subtracting `up over ten' and `down over ten', and with understanding place value. This could prove a fruitful area of enquiry. It also highlights potential difficulties some students may face in transferring their mathematical knowledge from one language to another.

The group went on to discuss the comprehension and use of diagrams, mathematical formulae and calculation algorithms (the formats in which calculations are written) and symbols. The visual literacy required to use diagrams is acquired knowledge which can be culturally specific. The syntax of formulae has to be learnt as well as their meaning and application. Calculation algorithms vary from one language to another and also within a language. Symbols are also used differently in different languages and differently within one language in different contexts. For example the dot in English is used as a full-stop and as a decimal point. In Arabic the same symbol means zero and the decimal point is represented by a comma, as it is in most languages, except English.

Where mathematical problems are expressed in a contextual format, an understanding of the context and knowledge of the language used in the context are necessary for comprehension before the mathematical problem can be addressed. Where practical activities are undertaken, the student needs to understand the instructions and be able to record results and state conclusions. The explanation of mathematical concepts and the discussion of mathematical problems can only be expressed through the medium of language.

Two pieces of research into the strategies children use to solve addition and subtraction word problems throw light onto difficulties some adult students have with these

kinds of problems, and provide potential models for research into adults' strategies for problem-solving.

Teubal and Nesher (1991) studied children's responses to word problems which present information in a different order to the natural time sequence. For example, "Now there ar 80 pupils in the auditorium, after the entrance of 20 pupils during the break. How many pupils were there at school before break-time?" (Teubal and Nesher in Durkin and Shire, 1991:135)

They found that younger children employ a strategy of writing down a mathematical equation in the form of the natural time sequence, which can result in the numerical value to be calculated appearing anywhere in the equation. Older children are more likely to proceed straight to an equation which puts the value to be calculated alone on one side of the equation, so that a solution can be obtained directly.

Some adult students may not have progressed to the later stage in this learning process, and therefore still be using the earlier strategy. Teubal and Nesher's model could be used to study adults' strategies for solving these kinds of problems.

De Corte and Verschaffel state that for children, "word problems that can be solved by the same arithmetic operation but differ with respect to their underlying semantic structure have very different degrees of difficulty." (De Corte and Verschaffel in Durkin and Shire, 1991:119)

They describe fourteen different semantic structures for elementary addition and subtraction word problems, in a study by Pauwels. For example, the percentage of correct answers for the following three questions which all require addition of the two given numbers were 97%, 83% and 47% respectively.

Combine 1	Joe has 3 marbles; Tom has 5 marbles; how many marbles do they have altogether?
Change 6	Joe had some marbles; then he gave 5 marbles to Tom; now Joe has 3 marbles; how many marbles did Joe have in the beginning?
Compare 3	Joe has 3 marbles; Tom has 5 more marbles than Joe; how many marbles does Tom have?

(De Corte and Verschaffel in Durkin and Shire, 1991:120-121)

The words `combine', `change' and `compare' refer to classifications of types of addition and subtraction questions developed by Greene and associates.

De Corte and Verschaffel report that in these kinds of elementary addition and subtraction word problems, the order in which the numerical values appear in the problem determines the strategies children choose to attempt solutions: starting with the larger or the smaller number, counting on or counting back, matching sets, etc. They found that children spontaneously use a wide variety of informal solutions to word problems, and they see this as a strength to be built on in teaching:

> ... verbal problems could be attributed a new function in maths education; they could indeed be mobilized in the first grades to promote a thorough understanding of the formal mathematical concepts and operations, instead of being assigned almost exclusively an application function.
>
> (De Corte and Verschaffel in Durkin and Shire, 1991:129)

In their experiment the teaching of word problems is not postponed "until children have learnt the formal operations of addition and subtraction; on the contrary word problems are presented before introducing these formal operations and the related number sentences."

This concurs with good practice in teaching numeracy to adults who have far more experience than children of informally solving real numerical problems. The very detailed analysis of the semantic structure of word problems and the models of solution strategies developed in these studies could be applied to studies of adult students learning at the same mathematical level, both those studying in their mother-tongues and those studying in a second or subsequent language.

The group went on to discuss the practice of asking students to write about their learning and their use of mathematics to facilitate their understanding of their own learning process and to promote their control of their own learning.. These require other registers of language.

The questions arise of how students' levels of language in all these different registers can be assessed, and how their levels of mathematical knowledge can be accurately determined when the language of assessment is different from the language in which the mathematics was learnt, or the student's level of literacy is different from her level of maths. And how can the acquisition of the different registers of language required be promoted in the context of learning mathematics. Participants in the group expressed interest in pursuing inquiries into some of these issues.

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"Starting a PhD - and keeping it going !"

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This was a "how to" session where we talked through some of the joys and difficulties of research work. I'd hoped that we'd get a mixture of people coming along who were at different stages of the research process - and we got just that. In a group of eight one participant had extensive experience of supervising PhD students and also had very lively memories of his own experience as a research student, several of us had embarked fairly recently on major pieces of research including PhD work, some of us had ideas for research projects at various stages of development, and one person already had vast amounts of data collected in an earlier project which she wanted to analyse more systematically. It was a very lively mix, and using as starting points the attached questions we picked each others' brains, aired concerns and shared the excitements in a very open way. I've tried to jot down here the ideas we came up with; of course some contradict others, and we weren't trying to put together a recipe for success; the session's value lay I think more in the sense of shared enterprise than in the specific ideas we came up with. For those at the start of the process it was particularly helpful to hear from those farther along and from one of the group who was an experienced supervisor.

We started by reaffirming that what starts you off and keeps you going is your interest in your research questions. We talked about how important it was to keep a sense of what the central questions were, and we agreed that fading interest in the key issues is a critical sign not to be ignored. We spent very little time thinking about the areas of skill or knowledge we'd need, moving quickly on to talking about how to choose and use a supervisor, how to keep records and notes, and how to monitor your own progress. Our tips were

* Shop around for a supervisor; don't forget that Universities need you as much as you need them.
* Talk to potential supervisors and read some of their work to satisfy yourself that they are knowledgeable about and interested in **your** issues.
* Talk about potential supervisors with friends, colleagues etc to try to sharpen up your own ideas about what kind of supervisor will suit you.
* Ask if your (potential) University runs courses you might need - in research methods etc.
* Find out if research students are encouraged and helped to meet and talk to each other - are there graduate research seminars for example ?
* Check the library facilities; try a mini search of the literature.
* Try to find a supervisor you feel comfortable with; the ideal combination is somebody who doesn't intimidate you but does

stimulate you.

* Remember when you're planning and doing your research that it's **your** timetable which must drive it, not some idealised version.
* You may need to keep reminding yourself and your supervisor that it's your agenda that counts, it's your research, not theirs.
* Be as open as possible - come to an understanding about how much time/energy you really have available.
* If the worst comes to the worst, change your supervisor ! It's your PhD, not theirs, and not the University's !
* Use card indexes rather than computer databases unless you have a lap-top you can use on the move.
* Keep two card indexes, one for references, the other for brief notes on particular topics.
* Keep a research journal as well as notes on what you're reading and field notes - the journal will help you keep a sense of progress.
* When you're writing, try to find a "friendly" reader to comment as you go along - in addition to your supervisor.

My own observations: I hope ALM continues to include in conferences "how to" sessions like this as well as research reports. We need to continue to find time to talk to each other about the research process if we are to meet the needs of teacher/researchers at all stages of the process. It's a very good way of supporting each other.

Starting a PhD - and keeping it going !

What I find really .....

.... interesting ..... stimulating .....

.. infuriating ..... puzzling .....

..... depressing ..... amazing .....

about my own work with adults ?

or

about adults learning maths in general ?

Starting a PhD - and keeping it going !

What skills or areas of knowledge will I need ?

What can I do to acquire them ?

Starting a PhD - and keeping it going !

What really worries me about doing research

Starting a PhD - and keeping it going !

my ideal supervisor .....

Starting a PhD - and keeping it going !

How to handle a supervisor

Starting a PhD - and keeping it going ?

Reasons for giving up...

I haven't got the time !	
I haven't got the energy	
I'm not ready for the field work	
I want to have a relaxed summer	
I want to see **more** of my friends, not **less**	
This whole PhD thing seems terribly earnest; I want some fun !	
I'm actually a bit bored with the practice I'm meant to be action-researching	
People who think this project really should be done are going to be disappointed with what I produce	
The project will show up all the weakness of what I've been doing for years	
Do I really want a PhD anyway ?	
I haven't got the time !	

THINGS THAT KEEP ME GOING.....

There's something I really do want to find out	
I can see some fun ahead	
I can see ways of working with other people on it	
I don't **have** to do this	
I'd have a smart answer to the salesmen who want to call me Mrs or Miss or (sneeringly) Ms.....	