PROPORTIONAL REASONING OF ADULT STUDENTS IN A SECOND CHANCE SCHOOL

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RATIONALE OF THE STUDY

- Proportional reasoning is a foundation for the understanding of algebra and the transition from informal to formal mathematical thought. (Doyle et al., 2016)
- Many adults and students fail to reason proportionally (Lamon, 2007)
- Importance of the teaching of fractions: a precursor of the teaching of rational numbers.
- Many studies on children’s understanding of fractions and proportional reasoning VS few studies about adults. (Baker et al., 2012; Doyle et al., 2016).
OBJECTIVE OF THE STUDY

- Better perceive how adults with low mathematical knowledge understand, process, and represent fractions.
- Examine how these adults use proportional reasoning during problem solving.
- Our research questions:
  - To what extend do adult students understand the different sub-constructs of fractions?
  - How adult students use sub-constructs of fractions in solving problems involving proportions and percentages?
SETTING OF THE STUDY

- Second Chance School: main public institution related to Adults’ Basic Education.
- City in North-West Greece.

- Target group: Adults who have not completed the compulsory secondary education and do not have the appropriate qualifications and skills to adapt to modern vocational requirements.

- Duration: Two academic years (25 teaching hours).
- Mathematics taught for 3 hours per week.
PARTICIPANTS

- 30 adult students.
- They had a primary school leaving certificate and attended the second year of a SCS.
- Their mathematical skill levels varied from basic elementary through secondary level.
- They were highly motivated and they wanted to learn.
- Most of them were women, 30 to 44 years old and unemployed or unskilled workers.
METHODOLOGY

- Task sheet with 13 open-ended tasks
  - completed in 1.5 hour.
- Interviews (40 min) with five adults (three transcribed)
  - to get more data about their ways of thinking.
- During the previous 3-4 weeks taught:
  - fractions and their representations
  - fraction equivalence
  - percentages
  - fraction addition
  - problems that required proportional reasoning
METHODOLOGY

- Tasks: fraction sub-constructs’ definitions
  (Charalambous & Pitta-Pantazi, 2007; Doyle et al., 2016)
FRACTION SUB-CONSTRUCTS

- Part-whole: the symbol notation $p/q$ to represent the partitioning of a whole entity into $q$ equal shares and then taking $p$ out of the $q$ shares.

- Measure: the placement of a fraction on the number line.

- Quotient: $p/q$ is perceived as the amount obtained when $p$ quantities are divided into $q$ equal shares.

- Ratio: $p/q$ involves a comparison between the two quantities $p$ and $q$ (nominator and denominator).

- Operator: process of taking the fraction $p/q$ of some quantity by the multiplication of $p$ with this quantity and then the division of the product by $q$. 
RESULTS

- The research took place during May 2018 and the data analysis is still ongoing.

- We present preliminary results for five tasks.
John and Maria are making lemonade. Given the following recipes whose lemonade is going to be sweeter? Justify your answer.

- John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 1 spoon of sugar for every 7 glasses of lemonade.
- John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 4 spoons of sugar for every 8 glasses of lemonade.

(Doyle et al., 2016)
RESULTS-TASK1: RATIO

- Most students computed fractions, used the notion of ratio, and then tried to compare them.
- Some of them converted the fractions to other equivalent ones with the same denominator and then compared them.
- Others divided the nominator with the denominator.
- Some of them tried to find how many spoons of sugar are used for every glass of lemonade.
RESULTS-TASK 1: TRANSCRIPTS

- **Maria:** I get the largest denominator and I put it in the first fraction and the second denominator in the second fraction. I do it in reverse. And then I multiply $2 \times 7 = 14$ and $5 \times 7 = 35$. I continue the next and compare $14/35$ with $5/35$. Of course $14/35$ is larger, since it has the bigger nominator. The sweeter lemonade is John’s.

- **Yanna:** It is 2 spoons of sugar in 5 glasses of lemonade and 4 spoons of sugar for every 8 glasses of lemonade so the 8 glasses have 4 tablespoons of sugar and the 5 glasses... Because in the 8 glasses it is about 1 spoon in every 2 glasses while here at 5 comes less than half of a spoon for every 1 glass of lemonade. Isn’t it?
Three pizzas are shared equally among four students. What fraction of a pizza will each student eat? (Charalambous & Pitta-Pantazi, 2007)

All students answered correctly. Many of them constructed three rectangles representing the three pizzas and divided them into four pieces. These students used the representations of the fractions.
**RESULTS-TASK3:QUOTIENT**

**Ασκηση 3η**
Τρεις πίτες μοιράζονται ίσα σε 4 παιδιά, τι μέρος-κλάσμα της πίτας θα λάβει το κάθε παιδί; Δικαιολογήστε την απάντησή σας.

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κοβούμε τις πίτες σε 4 κομμάτια και το κάθε παιδί παίρνει από τρεις κομμάτια.
Does the shaded part of the rectangle correspond to the fraction $\frac{2}{3}$? Justify your answer.

All students except one answered negatively. During the interviews they said that they relied on the fact that the three pieces were not equal. Some students drew the correct divided rectangle.
RESULTS-TASK 8: FORMAL PROPORTIONAL REASONING

Which of the following fractions is closest to 1?

a) \( \frac{2}{3} \)  

b) \( \frac{3}{4} \)  

c) \( \frac{4}{5} \)  

d) \( \frac{5}{6} \)

(Doyle et al., 2016)

This task was a little bit difficult for the students. Most of them answered that closest to 1 is the \( \frac{5}{6} \) but did not justify their answer.

Few of them justified their answer and they used either the equivalence of fractions or the sub-construct of quotient. One student represented the fractions and she relied her answer on the fact that the piece left for the was the smallest.
Ασκηση 8ο

Ποιο από τα παρακάτω κλάσματα είναι πιο κοντά στο 1; Δικαιολογήστε την απάντησή σας.

a) $\frac{2}{3}$  
b) $\frac{3}{4}$  
c) $\frac{4}{5}$  
d) $\frac{5}{6}$  

(Α) $\frac{2}{3} = 0.66$  
(Β) $\frac{3}{4} = 0.75$  
(Δ) $\frac{5}{6} = 0.83$

Το υπόλογιση το οποίο είναι ισομετρικά στη μονάδα είναι το $\frac{5}{6} = 0.83$
RESULTS-TASK 8: FORMAL PROPORTIONAL REASONING

Άσκηση 8η

Ποιο από τα παρακάτω κλάσματα είναι πιο κοντά στο 1; Δικαιολογήστε την απάντησή σας.

a) \( \frac{2}{3} \)    b) \( \frac{3}{4} \)    c) \( \frac{4}{5} \)    d) \( \frac{5}{6} \)

Πιο κοντά είναι τα \( \frac{5}{6} \)
Joanna: 5/6 is closer to 1, then 4/5, then ¾ and then 2/3.
Teacher: The answer you gave is correct, but how did you think about it?
Joanna: Because we make a pie and divide it into 6 pieces if I eat the 5 I get the most. At 4/5 you will get less. The other one who gets the ¾ will get less and the other who gets the 2/3 even less. So when I get the most I'm closer.
Teacher: Closer, where?
Joanna: At 1.
Teacher: What represents the 1 for you?
Joanna: 1 is the whole.
RESULTS-TASK 11: MEASURE

- Locate the following numbers on the number line and justify your answer.

(Charalambous and Pitta-Pantazi, 2007)

This was the most difficult task for the students since only four of them answered it correctly. These students based their answers on the comparison of the fractions and the division of the number line.
Ασκηση 11η
Εντοπίστε τους παρακάτω αριθμούς στην αριθμογραμμή:

#1) \( \frac{1}{6} \)

#2) \( \frac{4}{3} \)

#3) \( \frac{5}{6} \)

\( \frac{1}{6} \) \( \frac{4}{3} \) \( \frac{5}{6} \)

Δικαιολογήστε την απάντησή σας.

Η αριθμογραμμή είναι σωστή καθώς από το 0 επέκτη το 1 σε 3 σειρά μέρη,
για να γίνει την δόκηση:

Ωστόσο, την αριθμογραμμή από το 0 επέκτη το 1 σε 3 σειρά μέρη και
τα τοποθετώ τα χάσκατα.
Άσκηση 11η
Εντοπίστε τους παρακάτω αριθμούς στην αριθμογραμμή:
#1) $\frac{1}{6}$
#2) $\frac{4}{6}$
#3) $\frac{5}{6}$

$\frac{1}{3} \quad 1 \quad \frac{4}{6}$
$\frac{1}{3} \quad 1 \quad \frac{4}{6}$

Δικαιολογήστε την απάντησή σας.

$\frac{2}{3}$ είναι $\frac{2}{3}$ στη συγκεκριμένη διάφανη $\frac{1}{6}$

$\frac{2}{3}$
CONCLUSIONS

- Our ongoing study focuses on proportional reasoning during problem solving of adult students.
- Most students easily solved tasks that required the formulation of a fraction either from a sentence or from a pictorial representation.
- The students understood the different subconstructs of fractions since they used them in their answers.
- They represented the fractions and it seemed that it was very helpful for them.
CONCLUSIONS

- At the same time, they had difficulty with the tasks that referred to the measure sub-construct.

- They found it difficult to compare fractions using the number line.

- They couldn’t extend their knowledge about part-whole to the measure.
Thank you for your attention!
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