Adults Learning Mathematics: Transcending Boundaries and Barriers in an Uncertain World

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In this presentation I will:

• address the possible interests that adults might have in learning mathematics in a fractured and fragmented world with constantly changing horizons in terms of politics, economics, technology, the environment, etc.

• discuss different aspects of mathematics and numeracy in relation to democracy and work

• draw on Bernstein’s theories to stress the importance of understanding its underlying structures and relationships, in order to support numeracy curriculum, teaching, & assessment
Outline [2]

- critique the use of outcomes-based frameworks
- use music theory and performance to offer some illustrations
- draw some conclusions and implications from the above
- emphasise the importance of keeping adult mathematics and numeracy practitioners and researchers professionally informed through having access to high quality research related to their interests, such as ALM’s own journal.

My focus will be on linking people, practices and research
What are Adults’ Interests in Learning Mathematics? [1]
PAST: Unfinished Business

... arising from restricted access to mathematics education due to:

socio-economic reasons
– relative poverty in their formative years leading to missing school, changing schools frequently, leaving school early ...
– social & cultural traditions & expectations [especially women & girls]
– being part of a social or cultural minority group
– family immigration choices
– unexpected pregnancy in school years
– ...
What are Adults’ Interests in Learning Mathematics? [1]

PAST: Unfinished Business ctd.

- **political views**
  - of individual worth of students vs. political expediency in decision making [at all levels of education]
  - outdated conservative beliefs in education [curriculum, pedagogy, assessment]
  - preference for funding favoured social classes/cultures of voters

- **acts of human aggression & destruction**
  - war, ethnic cleansing, forced relocations, ... environmental disasters
  - individual teachers’ negative discrimination, mental & physical abuse [racial abuse, intentional bullying, etc.]
What are Adults’ Interests in Learning Mathematics? [2]

PRESENT: More contemporary reasons

Curiosity:

• the joy of wanting to learn more mathematics [techniques, big ideas, history, etc.] for its own sake and finding it making sense!!!

• Helping others:
  • friends & family members [e.g., children at school, partner’s work, social or political groups, ...]

Compulsory requirement:

• condition of receiving government or other benefits
• preparing for further study or employment: entry test or hurdle requirement
Finding a new direction, often after significant life change:

- relationship breakup, children all left school/home, release from carer’s duties, ...
- realising & attempting to overcome limited options in work/life
- needing to keep up with technology, especially new mathematical & learning technologies
- [re] learning mathematics in English [or other language]
- seeking alternative, meaningful & respectful adult teaching approaches
- seeking personal enrichment & new friendships
Contemporary Issues: GLOBALISATION

The phenomenon of globalisation has been characterised as a series of inter-related flows including knowledge, capital, jobs, people and cultures.

It is also associated with increasingly rapid developments in ICTs, along with transport — facilitating cultural and economic exchange.

One consequence of globalisation is that the historical model of full-time education followed by full-time work until a dignified retirement no longer applies to the vast majority of people.
The likelihood is that there will be

• several changes of jobs and even occupations, in different geographical locations
• periods of unemployment or under-employment
• casualised / contract labour or relatively more secure employment
The 1\textsuperscript{st} Industrial Revolution used \textit{water and steam power} to mechanize production.

The 2\textsuperscript{nd} used \textit{electric power} to create mass production.

The 3\textsuperscript{rd} used \textit{electronics and information technology} to automate production.

The 4\textsuperscript{th} Industrial Revolution, builds on the 3\textsuperscript{rd}, the digital revolution that has been occurring since the middle of the last century.

It will bring \textit{advanced robotics and autonomous transport, artificial intelligence and machine learning}, advanced materials, biotechnology, etc. (Gray, 2016)
Automation and the technologically mediated global movement of people, knowledge, and capital, is profoundly changing work and working lives.

The demand for highly skilled workers has increased while the demand for workers with less education and lower skills has decreased.

**By 2020**

- 35% of skills that are considered important in today’s workforce will have changed.

Some jobs will disappear, others will grow and jobs that don’t even exist today will become commonplace.
People working in sales and manufacturing will need new skills, such as technological literacy. Workers will have to become more creative.

By 2020 the top three skills needed by workers will be:

- Complex problem solving
- Critical thinking
- Creativity

*What does this mean for ALM? For us as teachers in vocational and further education? For us as researchers? For our students in this changing world?
Mathematical thinking develops through three worlds of mathematics:

• the *practical mathematics* of shape and number, with experiences in shape, space, and arithmetic (embodied operations);

• *theoretical mathematics* with a focus on properties, leading to Euclidean proof and algebra, and definitions based on known objects and operations; &

• *formal mathematics*, working with properties proved from given axioms and definitions, and formal objects based on formal definitions.

Each world is interlinked with the others and valuable in its own right.

(David Tall, 2013)
Anna Sfard (2017), in her conceptualisation of mathematics “as a certain well-defined form of communication,” asserts that:

“to think mathematically means communicating — with others or with oneself” — in the special way generally endorsed by the mathematical community.
The duality of mathematics as a means and a system:

Mathematics provides a *means* for individuals to explain and control *complex situations* of the natural and of the artificial environment and to communicate about those situations.

Mathematics is also a *system of concepts, algorithms and rules, embodied in us*, in our thinking and doing; we are subject to this system, it determines parts of our identity. This system runs from everyday quantifications to elaborated patterns of natural phenomena to complex mechanisms of the modern economy.

(Roland Fischer, 1993)
Bernstein (2000) advocates three pedagogic rights which enable democracy to function: enhancement, inclusion, and participation.

- At an individual level, the right to enhancement is the right to the means of critical understanding and to new possibilities and a condition for confidence
- At the social level, the right to be included, socially, intellectually, culturally, and personally, is a condition for communitas
- At the political level, the right to participate must be about practice which has outcomes; it is the condition for civic practice or discourse
In formal mathematical pedagogic situations:

• arbitrary selections are made from the discipline of mathematics which provide different forms of consciousness to different social groups, with

• differential access to ‘unthinkable’ knowledge in the possibility of new knowledge creation [e.g., curiosity, creativity, innovation]  

• as distinct from ‘thinkable’ knowledge which takes the form of official knowledge [e.g., rule following]. (Bernstein, 2000)  

*
In society & across a wide range of industries and occupations people are required to **use, develop, and communicate mathematical ideas and techniques in a diversity of ways with others** who have differing expertise, experience, and interests, including in mathematics itself.

Problems requiring mathematical reasoning and calculations are usually
- **embedded in physical and/or intellectual tasks, rich in context,**
- **with a range of constraints that are oftentimes mutually contradictory,**
- **but always need a workable answer & within a short space of time.**
In many jobs problem solving is an expected and routine part of the day’s work: Every new request or order requires an original or customised solution within given parameters.

Whether using mathematics explicitly or implicitly in these processes, no matter how trivial, the worker must also take into account all of the relevant contextual knowledge in their decision making.

Crucially, the kinds and complexity of problems that occur at work contrast sharply with those found in formal mathematics education texts and assessment tasks.
Doing mathematics at school and doing mathematics at work are two very different activities, epistemologically and socioculturally.

(FitzSimons, 2013)

Most mathematics teachers and students who visit work sites, and even workers themselves, find it very difficult to recognise any activity they are able to judge as being mathematical beyond number and measurement. (Williams & Wake, 2007; Nicol 2002)
Practice-based innovation as a cyclical process of learning (Ellström, 2010)

The explicit work process:
- work as officially prescribed and formally organized
- on the basis of explicit knowledge (ideas, theories, models)

The implicit work process:
- work as subjectively interpreted and performed in practice
- on the basis of implicit (tacit) knowledge
- variation and improvisation in interpretation and performance

The logic of development:
- Variation/improvisation
- Developmental learning
- Transformation

The logic of production:
- Standardization
- Implementation
- Adaptive learning
- Reproduction
Many people do not consciously recognise that learning takes place at work:

• **learning via non-formal education** may occur via specific activities
• **informal learning at work** is an ongoing process that **encompasses** personal, social, and cultural knowledges and skills

Importantly, there is **an ongoing need for communication of an educative kind** between stakeholders where information, often **mathematical**, is sought and shared.

**Pedagogic relations play an important role!**
Mathematics-related knowledge is, or can be, transformed from the academic discipline of mathematics into the specific context of the problem at hand.

There are pedagogic relations at work in communications:

• between co-workers
• between managers or team leaders & workers
• between client & supplier of services or goods, etc.

Workplace mathematics is not often formally recognised as a pedagogic activity, unlike school mathematics.
Recontextualisation offers a powerful means of “bridging the gap”
between theoretical and experiential knowledge.

- Students in formal education need to learn these skills, for themselves, and to communicate effectively with other stakeholders at work, etc.

- Recontextualisation combines the disciplinary knowledge of mathematics with contextual knowledge of the specific situation, including personal & sociocultural knowledges, in the form of communication.

This is what all teachers do!
In Summary

Doing mathematics at work [& elsewhere] involves:

**Learning** at work &

**Communicating at work** (through symbols, diagrams, speech, gestures, etc.)

via **Recontextualisation**

— *i.e.*, **transforming** what you know mathematically and contextually to find the most appropriate decision or action
Bernstein’s Map of Knowledge Structures [1]

**Discourse**

*Vertical* within

- Hierarchical knowledge structures

- D.R.

*Horizontal* within

- Horizontal knowledge structures

- Repertoires

- Reservoir D.R.

**Power relations** between

*Vertical* within

- Grammars

  *strong* 

  *weak*

*Horizontal* within

- Transmission

  *Explicit* (apprenticeships)

  *Implicit* (crafts)

Note: D.R. = Distributive Rules
Bernstein’s Map of Knowledge Structures [2]

**Discourse**

*Vertical discourse*

Disciplinary mathematical knowledge

Power relations between

*Horizontal discourse*

Contextual knowledge

Hierarchical knowledge structures

D.R

Horizontal knowledge structures

algebra, geometry etc.

Grammars

Strong ‘school-math’

Weak ‘math-containing’

Transmission

Explicit (apprenticeships) Implicit (crafts)

Reservoir (group)

Repertoires (individual)

personal, social, & cultural skills & knowledges

Note the reciprocal [horizontal] relations between vertical discourse and horizontal discourse
The Importance of Both Vertical Discourse & Horizontal Discourse

While the a working understanding of the **vertical discourse of mathematics** [at whatever level the person has reached in each of its sub-disciplines] is essential

- knowledge of the **horizontal discourse within the particular context** is absolutely necessary to make sense of the task at hand.

**When employers complain** that new graduates cannot use their mathematical knowledge and skills, this is because they have not yet comprehended the horizontal discourse at the worksite.

- There are specific ways of being a research scientist, a mathematics teacher, a plumber, etc.
Outcomes-based frameworks & qualifications rest on a notion of knowledge as information that can be divided into little bits that can be selected and combined at will.

This ignores:

• that scientific knowledge is necessarily organized in bodies of hierarchical conceptual relationships,

• the value of such bodies of knowledge, and

• the necessary conditions for their acquisition [i.e., sequencing]
In qualifications frameworks, knowledge must be selected:

- [only] because it leads to the required learning outcome
- and not for any other reason, such as its intrinsic value and interest,
- or to provide a foundation for further knowledge development.

*Knowledge does not need to be assessed but can be inferred from competent performance, implying that ...*

- everyday knowledge is equivalent to structured bodies of knowledge.

i.e., vertical and horizontal discourses are interchangeable!
Structured bodies of knowledge are important. They allow us to:

account for and explain the natural and social world in systematic ways, participate in, and reflect on key human experiences [cf. Bernstein & Democracy] such as the literary, visual, or musical domains.

Disciplines such as mathematics [& music] enable abstraction, reflection, prediction, and application across time and local contexts.

Music performance: Like mathematics, playing music requires mastery of simple pieces, gradually building up to a reflective and more original practice, offering insight into musical conventions & boundaries, even if only in order to violate them!
Music & Mathematics [1]

Jubal, Pythagoras and Philolaus engaged in theoretical investigations, in a woodcut from Franchinus Gaffurius, Theorica musicæ (1492).
They are both pancultural, with historical & cultural variations, including ethno-mathematics or ethno-music, & something available to all of us [in theory at least e.g., rhythmic clapping, singing]

Mathematics and music each have:

• a **structured body of knowledge** which is ever evolving

• a **codified set of symbols** with universally accepted meanings within the discipline

• **several different sub-fields** [instrumental and vocal specialisations, genres etc in music], **each with their own rules**
the possibility of knowledge and skills leading to performance being passed on through an oral tradition, often culturally specific, and displayed/performed in contextually appropriate settings.

In music, popular social and cultural songs are sung on appropriate occasions, & are a valuable means of social cohesion:

– Happy Birthday
– Auld Lang Syne
– Danny Boy [Londonderry Air]
– Will Ye Go Lassie Go?

Both vertical & horizontal discourses exist in music & mathematics.
<table>
<thead>
<tr>
<th>Note</th>
<th>Rest</th>
<th>Relative value</th>
<th>*Dotted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole note</td>
<td>semibreve</td>
<td>1 (=) 1 1/2</td>
<td></td>
</tr>
<tr>
<td>half note</td>
<td>minim</td>
<td>1/2 (=) 3/4</td>
<td></td>
</tr>
<tr>
<td>quarter note</td>
<td>crotchet</td>
<td>1/4 (=) 3/8</td>
<td></td>
</tr>
<tr>
<td>eighth note</td>
<td>quaver</td>
<td>1/8 (=) 3/16</td>
<td></td>
</tr>
<tr>
<td>sixteenth note</td>
<td>semiquaver</td>
<td>1/16 (=) 3/32</td>
<td></td>
</tr>
</tbody>
</table>
‘Equivalent’ note values cf. equivalent fractions

*A dotted note* is a note with a small dot written after it. The dot increases the duration of the basic note by half of its original value.
Music & Mathematics [5]
Patterns & Relations - Circle of Fifths
As in mathematics, there are obvious differences between professional, amateur, & novice musicians.

- The more skilled and knowledgeable the musician, the easier it is for them to confront and solve unexpected problems that may arise in the form of being asked sight-read and perform an unseen piece of music, or even to perform it in a different key [transpose it up or down].

A Musical Example

What meaning you make depends on where you are on the continuum.

Look at this short piece of sheet music. What do you see? Does it have any meaning for you?
What meaning did the previous slide, hold for you?

- You might see little other than hieroglyphics, you might see some familiar patterns, or you might immediately recognise what it is.
- If you are highly skilled, and can immediately see the underlying structures, you could even perform this on a keyboard or sing the song after a minute or less.
- If you are semi-skilled, you may be able to put something together after a longer, even lengthy period of time.
- If you are only able to join in with community singing, there is little likelihood that the music script will help you at all, and you are powerless in this situation.
If you felt a sense of incomprehension:

• Is it too much of a stretch to imagine that this is how mathematics appears to many adults?
• What are the consequences for people with a limited, interrupted, or discontinuous mathematics education?
• How will learning only “the maths you need” for a certain job at a certain moment in time help?

[contextualised curricula vs. coherent curricula]
[Horizontal discourse vs. vertical discourse]
Towards developing structural knowledge in numeracy

Teachers and students need access to high quality mathematics education materials to help support & develop **more secure and connected content knowledge** aimed at

- Complex problem solving
- Critical thinking
- Creativity

towards **Confidence**

**Inclusion**

**Participation**
Mystic Roses: Looking for patterns and relationships
*try simple cases *make a table *alternative methods
Exploring the relationship between graphs & events
Combining language with visual representations

Here are 6 bottles and 9 graphs.
Choose the correct graph for each bottle.
Explain your reasoning clearly.
For the remaining 3 graphs, sketch what the bottle should look like.

(a) (b) (c) (d) (e) (f) (g) (h) (i)
Height Height Height Height Height Height Height Height Height
Volume Volume Volume Volume Volume Volume Volume Volume Volume

©MathCentre for Mathematics Education, University of Nottingham, 1981.
Conclusion [1]

As Bernstein (2000) argues:

- **Abstract theoretical knowledge** enables society to conduct a conversation about itself, and to imagine alternative futures.

- **Access to ‘unthinkable’ knowledge** enables the possibility of new knowledge creation [e.g., curiosity, creativity, innovation].

With access to the underlying structural knowledge of mathematics, adults will have the possibility

- to make the transition to address new mathematical & numeracy demands &

- to recognise the same mathematical structures in novel contexts?
Conclusion [2]

Learning outcomes in adult and vocational education tend to:

• abstract the lived experiences of adults from the power relations of work and life &

• negate the possibilities of understanding and criticism.

We need to teach adults how to use mathematics to answer back to power at work and in society more generally, going well beyond the mastery of mathematical techniques.

e.g., with the prevalence of quality control statistics, process workers cannot be blamed for faulty machinery or lack of maintenance – but they have to understand the mathematics of the graphs and tables first.
Conclusion [3] ...

• How can we as educators encourage and support adults to transcend boundaries and barriers at work and elsewhere in an uncertain world?

• As members of ALM, do we not have an ethical responsibility to ensure that our students, at whatever level, have the most powerful, structurally supported & connected mathematical knowledge which is at the same time flexible enough to meet contextually specific demands?

[more ...]
Adults Learning Mathematics — An International Journal [ALM-IJ]

Adults Learning Mathematics: An International Journal is the only academic journal covering the field of adults who learn mathematics. This journal was first published in 2005 after many years of work.

3 Research Questions:

• How do teachers and researchers of adults learning mathematics become familiar with the latest research in their field?
• What are important indicators of journal quality?
• What is that a journal has direct control over?
How do teachers and researchers of ALM become familiar with the latest research in their field? (after Nivens & Otten, 2017)

- Ours is a **practice-engaged field**, closely linking practitioners and researchers.
- One **important medium for the dissemination** of scholarship is through **journals**, especially those **freely available and readily accessible**.
- The **impact is likely to come through enactment of the ideas** by adult education teachers, education methods courses or return to study mathematics lecturers, & professional developers; also possible influence on policy makers.
- The accumulating body of knowledge, diverse as it is, can also provide **support for current & future research students** in the field.
What are important indicators of journal quality?
What it is that a journal has direct control over?

Apart from citation indexes, the quality of the editorial and review process is important, through:

- insightful and relevant reviews
- an editorial process that actively assists the authors in the revision process
- minimal time lapses from submission to decision and from acceptance to publication.

Authors can bring visibility to their work via personalized alerts from Google Scholar, ResearchGate, etc. [also upload articles from ALMIJ]
Two important components of research quality

- How useful the research is to other scholars or practitioners [citation-based methods]
- The opinion of scholars in the same field as to the novelty, validity, and importance of the work.

More personal indicators of quality include whether readers considered the articles to be readable, interesting, and relevant to their current work.

The editorial board being leaders in their field is very influential.

(Williams & Leatham, 2017)
Although a journal is, in a sense, the sum total of the articles it publishes, a journal is also the people and processes involved in soliciting, reviewing, and making accessible those articles, and notions of “high quality” should incorporate those factors.

(Nivens & Otten, 2017)

ALM-IJ is a home-grown special purpose journal dedicated to supporting the aims of ALM.

I wish to acknowledge all who have contributed in some way since the idea was first conceived.
ALM-IJ Acknowledgements
[just to name a few]

Original concept and advice: thanks to Diana Coben, John O’Donoghue, & other ALM Trustees, including Jeff Evans, Tine Wedege, and especially to Jürgen Maasz, who as Chair of ALM, worked so hard for ALM-IJ to be established.

Original Editorial Board formulation & invitation: thanks to Alan Bishop for his advice

Obtaining the ISBN & other formal processes: thanks to Jürgen Maasz

Original Cover design & layout: thanks to Mieke van Groenestijn

Thanks to all other editors: Janet Taylor, Kathy Safford-Ramus, & Javier Díez-Palomar

Thanks to all the reviewers since 2004.

Last but not least, thanks to all the authors who have submitted articles — without whom this journal would not exist!
Mystery Song

https://www.youtube.com/watch?v=Sc9Ww3rmT7I
Mystery Music
Will Ye Go Lassie Go?
Lyrics & Chords

Oh the [D] summer [G] time is [D] coming
And the [G] tree's are sweetly [D] blooming,
And the[G] wild [D] mountain [Bm] thyme,
Oh, the summer time has come And the trees are sweetly bloomin',
And the wild mountain thyme Grown around the purple heather.
Will ye go, lassie, go?
And we'll all go together, To pull wild mountain thyme
All around the purple heather, Will ye go, lassie, go?
Chorus

And we'll [G] all go to[D] gether,
To pluck [G] wild [D] mountain [Bm] thyme,
All a[G] round the [Em] blooming [G] heather,

Selected publications [1]


Selected publications [2]


Useful websites [1]:
 Mathematical Processes
 The Shell Centre for Mathematical Education

Problems with Patterns and Numbers (1984)
The Language of Functions and Graphs (1985)

Useful websites [2]
Mathematics Content, Games & Activities

Suitable for and freely available to adult learners & their teachers

https://www.mathsisfun.com
https://nrich.maths.org/
Summary: Vertical discourse & Horizontal discourse

<table>
<thead>
<tr>
<th>Vertical discourse</th>
<th>Horizontal discourse</th>
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<tbody>
<tr>
<td>refers to <strong>disciplinary knowledge</strong></td>
<td>refers to <strong>everyday knowledge</strong></td>
</tr>
<tr>
<td>is described as being <strong>theoretical, conceptual, &amp; generalisable knowledge</strong></td>
<td>refers to <strong>contextual knowledge</strong></td>
</tr>
<tr>
<td>is <strong>coherent, explicit, and systematically principled</strong></td>
<td>is likely to be <strong>oral, tacit, multi-layered, &amp; contradictory across but not within contexts</strong></td>
</tr>
<tr>
<td>with <strong>strong boundaries</strong> between itself and other disciplines.</td>
<td><strong>segmentation of knowledge</strong> is the crucial feature:</td>
</tr>
<tr>
<td>The <strong>procedures are linked hierarchically</strong> &amp; thus allow the <strong>integration of meanings beyond relevance to specific contexts.</strong></td>
<td>o learning to tie one’s own shoe laces</td>
</tr>
<tr>
<td><strong>Context specificity</strong> is achieved through <strong>recontextualisation</strong></td>
<td>o learning to brush one’s teeth</td>
</tr>
<tr>
<td>Mathematical concepts can apply to a range of contexts beyond those available in the classroom.</td>
<td>o learning to count change</td>
</tr>
<tr>
<td><strong>These knowledges are related through the functional relations to everyday life</strong></td>
<td><strong>A person</strong> may build up an extensive <strong>repertoire of strategies</strong> chosen according to the context</td>
</tr>
<tr>
<td>&amp; the <strong>group</strong> may likewise build up a <strong>reservoir of strategies</strong> of operational knowledges.</td>
<td><strong>There is not necessarily one best strategy.</strong></td>
</tr>
</tbody>
</table>
Numeracy can be used to guide actions or decisions, to consider possible hypotheses, or to question assertions of others.

Communication is likely to be multi-modal in form (through symbols, speech, gestures, etc.), and here it differs significantly from much of school-based mathematics activities and their assessment.

Numeracy has two important interlinking aspects:

structural & contextual

[Vertical discourse & Horizontal discourse]
Based on **structural knowledge** derived from the discipline of mathematics:

**Numeracy** assumes a working knowledge of the **decimal system of numbers** for various forms of counting, including

- **probability and statistics**: data handling from collection to data entry to technology assisted calculation to presentation (e.g., tables & graphs) to interpretation

- notions of **change**: percentage change & common **periodic phenomena**

- **measurement**: through developing a practical understanding of the metric system and its links to the number system; & **geometries of space and shape**

- all underpinned by mathematical reasoning.
Numeracy as contextualised communicating with oneself and others

- in response to contextual prompts, real and virtual, both internal and external
- drawing on personal, social, and cultural knowledges which may be tacit, & even acquired tacitly.
- may be embodied through personal experience & includes estimations of numbers and measures, consciously or otherwise

Note: Communicating at work where there are a multiplicity of discourses can require being able to move fluently between formal and informal mathematical discourses as needed. [cf. Barwell, 2016]
In Summary:
Numeracy at any level involves:

• doing and communicating mathematics through interpreting & responding to the mathematics of others, in meaningful contexts

• communicating through a variety of modes: verbal, written, gestures, diagrams, tables, graphs, online, etc., often simultaneously

• developing contextually appropriate skills of estimation and approximation in number, shape, & space

• responding to & drawing on available structuring artefacts & tools

• & socially, culturally etc. informed human interactions in any specific context.