The evolution of mathematics discourse in high stakes assessment

ALM July 2018

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Outline

- Introduction to the EDSM project, its aims, theoretical and methodological orientation
- The role of 'real world' contextualisation of mathematics – some findings and issues
- Discussion of implications for teachers and for curriculum and assessment development

Setting the scene

- Apparent changes in “standards” over time is a constant topic of debate and drives policy decisions.
- Clearly changes have happened in the mathematics curriculum, assessment and classroom practices but what have these changes meant for the mathematics that students are expected to engage in?
- How may change be studied?
- Focus on school mathematics – but implications beyond for other formal educational settings

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Project: *The Evolution of the Discourse of School Mathematics*

- project funded by ESRC 2010-2014
  - in collaboration with Anna Sfard and Sarah Tang
- aim: to characterise changes in school mathematics in England over the past 3 decades
  - using high stakes GCSE examinations as a window onto school mathematics
  - analysing textual characteristics of the discourse as indicators of mathematical activity

Theoretical assumptions

- High stakes examinations have a strong impact on classroom practice and thus serve as a window onto the mathematics that students experience and are expected to engage in
- We can investigate expectations of student experience by studying the discourse in which they are expected to engage
  - The language we use shapes and is shaped by the way we experience the world
  - There is no dualist distinction between doing/thinking mathematically and communicating mathematically
- Drawing on:
  - Sfard’s communicational theory
  - social semiotics and systemic functional linguistics

Changes in the textual form of an examination question:

- do ***not*** simply make the mathematics more or less difficult

  but

- change the ways in which a student may engage with the text

  and

- change the possible ways of mathematical thinking

- Phase 1: Selection of data-base of exam papers
  - 8 years from 1980 (GCE ‘O’ Level) to 2011 (GCSE ‘Higher Tier’)
- Phase 2: Development of analytical scheme
  - developed iteratively in interaction with data, drawing on Sfard’s mathematics discourse features and SFL
- Phase 3: Analysis of examination papers
  - nVivo-supported coding; quantitative and qualitative comparisons across time
- Phases 4 & 5: Student testing & task-based interviews
  - comparing success rates and strategies for parallel questions with different discursive features

Specialisation as a characteristic of mathematics communication/activity

To what extent is specialised mathematical language used?

- lexical items used in accordance with mathematical definitions, considered at the level of:
  - vocabulary
  - sentence
  - text unit
- **extra-mathematical context**
- **depth of engagement with context**

‘Real World’ Context - policy

The extent of contextualisation in mathematics curriculum and assessment has been an on-going issue in policy debates and curriculum development in England.

- official policy discourse of utility
- critique of ‘over-complicated’ assessment
- calls for return to ‘basic skills’ of arithmetic and algebraic manipulation

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‘Real World’ Context – education community

- to increase motivation and engagement in mathematics
- to facilitate access to the specialised mathematics discourse by using familiar contexts as a ‘bridge’
- to provide students with insight into and skills in the application of mathematics in other disciplines, in employment, citizenship and everyday life
- to develop generic problem solving and modelling skills

‘Real World’ Context - assessment

- to increase accessibility of an assessment item by reducing abstraction or enabling students to relate to a familiar concrete experience
- to assess use of specific mathematical knowledge and skills and to relate these to their utility outside the classroom
- to assess ability to solve problems in which the relevant mathematical knowledge and skills are not specified by the assessment rubric

Problematising use of ‘Real World’ context

- the activity of solving school mathematics tasks is not the same as the ‘real world’ activity represented in the task
- formulaic ‘word problems’ are often dealt with in instrumental ways, e.g. using linguistic cues rather than engaging with context
- students have to know exactly which contextual knowledge to use and which to ignore
- there are gender and social class differences in how students engage with contextualised tasks

We asked

- How has the use of extra-mathematical contexts varied over time in GCSE examinations?
- How may this affect student participation in mathematical discourse?

Analytic Scheme

- developed in interaction with the data, drawing on components of Sfard's communicative theory of mathematical discourse and Halliday's social semiotics
- coding supported by use of nVivo
- all questions coded as:
  - specialised mathematics
  - non-specialised mathematics
  - mixed

Specialised mathematics

2011

10. (a) Simplify $6e + 5f + e - 3f$

....

(2)

(b) Solve $4(2x - 1) = 3x - 19$

$x =$ ....

(3)

(c) Solve $\dfrac{y+4}{5} = 30$

$y =$ ....

(2)

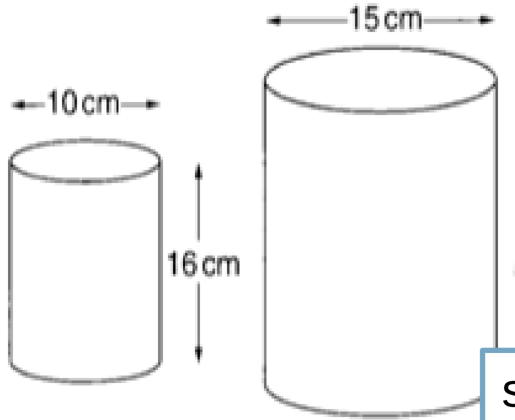
only mathematical objects and processes throughout

Non-specialised mathematics

2004

(a) These tins are similar shapes.
The diameter and height of the small tin are 10 cm and 16 cm respectively.
The diameter of the large tin is 15 cm.

NOT TO SCALE



Work out the height, *h*, of the large tin.

(b) Here are the labels for the tins.
They are also similar shapes.

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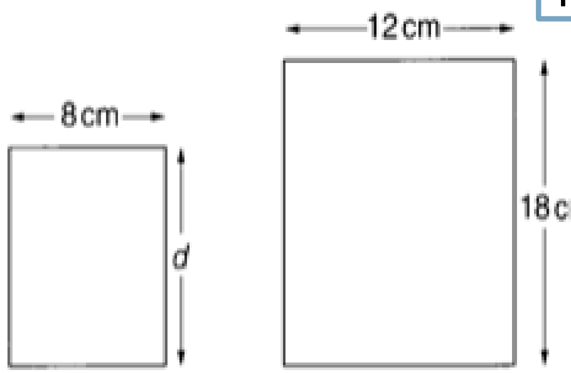


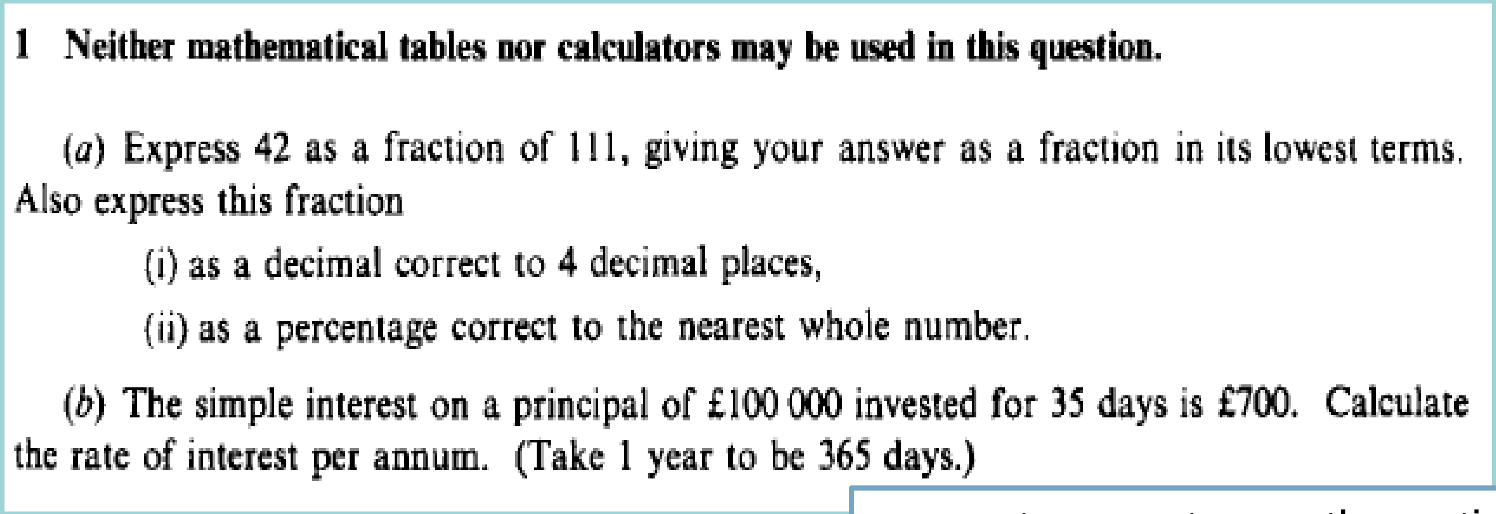
Calculate *d*.

(c) The tins are filled with baked beans.
The mass of the beans in the small tin is 1000 grams.
Calculate the mass of the beans in the large tin.

some extra-mathematical objects or processes in every part of the question

Mixed

1980



1 Neither mathematical tables nor calculators may be used in this question.

(*a*) Express 42 as a fraction of 111, giving your answer as a fraction in its lowest terms. Also express this fraction

(i) as a decimal correct to 4 decimal places,

(ii) as a percentage correct to the nearest whole number.

(*b*) The simple interest on a principal of £100 000 invested for 35 days is £700. Calculate the rate of interest per annum. (Take 1 year to be 365 days.)

presents an extra-mathematical context but some parts of the question are not related to this context

	specialised	non-specialised	mixed
1980	64%	28%	8%
1987	36%	56%	7%
1991	38%	61%	2%
1995	40%	51%	9%
1999	49%	42%	8%
2004	64%	28%	8%
2010	64%	36%	0%
2011	60%	37%	2%

1982 Cockcroft Report triggered major reforms to make mathematics more accessible and an entitlement for all students.

1988 National Curriculum "Using and Applying Mathematics" 20% of assessment

1995 QCA review calls for more algebra

1999 QCA review notes "trivial or distracting" context

2010 QCA AO2: "*Select and apply mathematical methods in a range of contexts*" 25-35%

2011 Ofqual subject criteria: 20-30% for "functional elements of mathematics"

Going deeper …

- Even where there is an element of extra-mathematical context in every part of a question, the relationship between the context and the expected mathematical activity varies.

- Depth of engagement with context required:
  - *ritual* – a contextualised question posed in a way so familiar within school mathematics that it may be considered an 'exercise'
  - *mundane* – a superficial context, most of which can be ignored; the mathematics demanded is obvious
  - *deep* – significant engagement with the context is necessary in order to identify the mathematics demanded

(based on Nyabanyaga, 2002)

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Ritual context

- a familiar 'exercise'

16. In a sale the normal price of a book is reduced by 10%.

The sale price of the book is £4.86

Calculate the normal price of the book.

2011

Mundane context

- mathematics demanded is obvious

2010

21. A field is in the shape of a rectangle.
The width of the field is 28 metres, measured to the nearest metre.

(a) Work out the upper bound of the width of the field.

.... metres
(1)

The length of the field is 145 metres, measured to the nearest 5 metres.

(b) Work out the upper bound for the perimeter of the field.

.... metres
(3)

(Total 4 marks)

Deep context

- engagement with context is essential

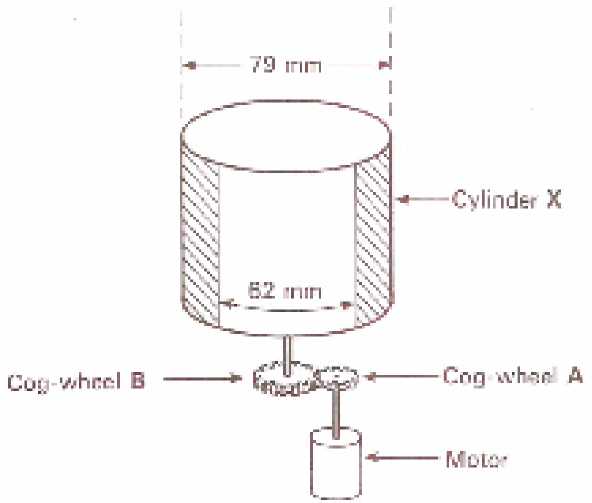


Fig. 5

Figure 5 shows part of a test mechanism to produce the flashes of light in a model lighthouse. The cylinder X is made of metal with a slit cut out of it. The motor drives the cog-wheel A, so that A rotates at a constant rate of 4 revolutions per minute. Cog-wheel A has 12 teeth and the rotation of A is transmitted to X through another cog-wheel B which has 32 teeth.

a) Calculate the time taken for one revolution of X.

The diameter of X is 79 mm and the arc length of the slit is 62 mm.

b) Calculate the time, in seconds to the nearest one-tenth of a second, of the flash of light in this test.

The final version of the model is designed to produce the following sequence during one rotation of X

ON (8 seconds) OFF (4 seconds) ON (8 seconds) OFF (25 seconds)

This is achieved by using the same cog-wheel A rotating at 4 revolutions per minute and with a different cog-wheel B.

(c) Calculate

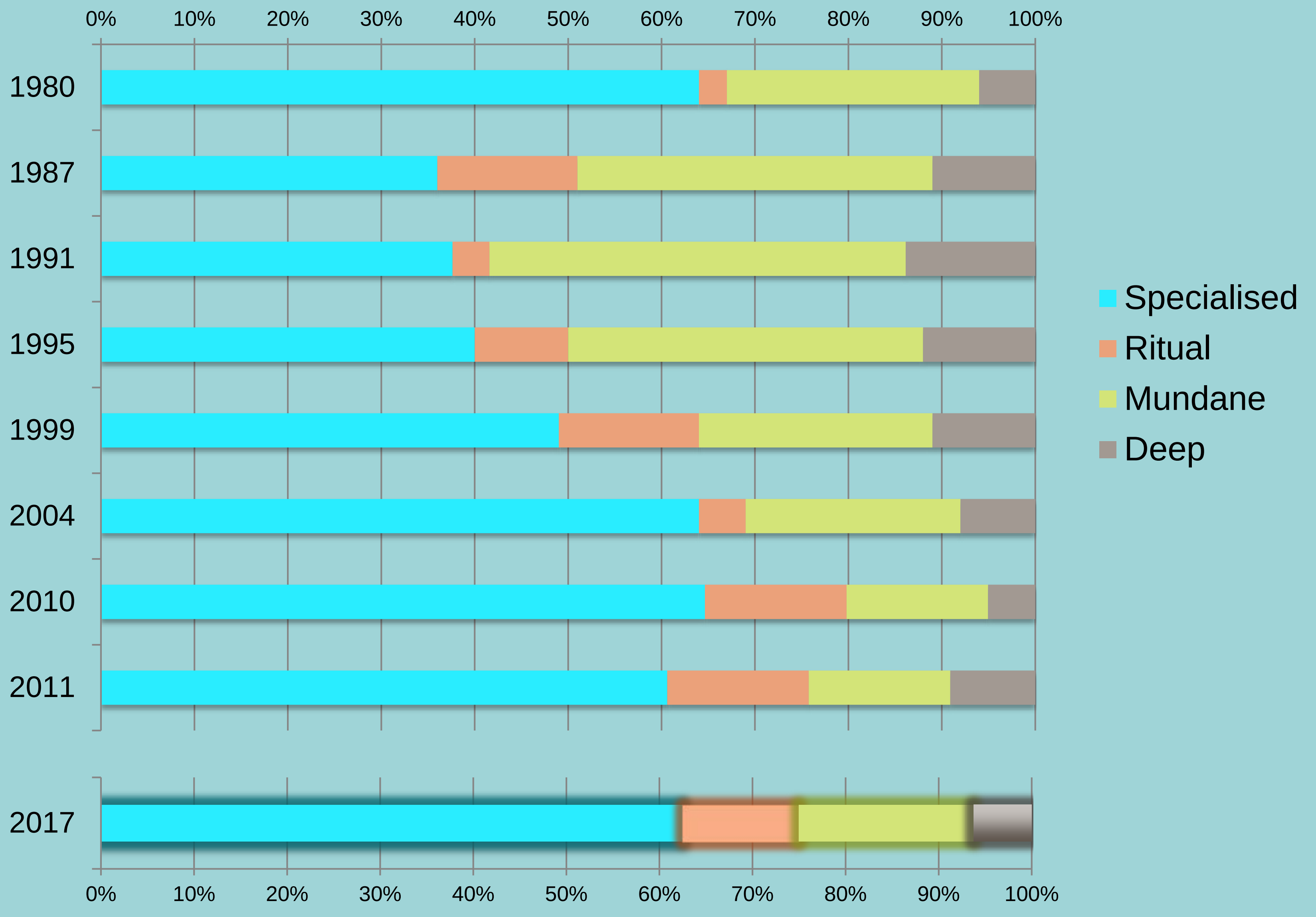
(i) the time taken for one revolution of X in the final version.

(ii) the number of teeth needed in the new cog-wheel B,

(iii) the arc length, in millimetres to the nearest mm, needed for each of the new slits.

1987

More or less ‘real’?



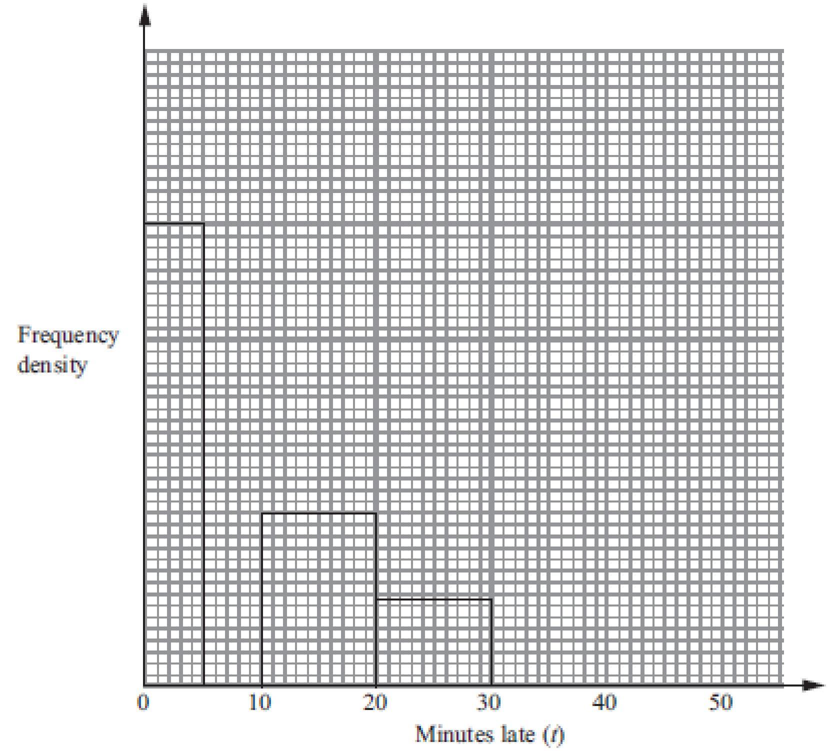
disconnected context

frequent in recent data handling questions where the ‘real life’ origin of the data is stated at the beginning but no reference is made to it in the tasks to be completed.

2010

25. Some trains from Manchester to London were late.
The incomplete table and histogram gives some information about how late the trains were.

Minutes late (t)	**Frequency**
$0 < t \leqslant 5$	16
$5 < t \leqslant 10$	10
$10 < t \leqslant 20$	
$20 < t \leqslant 30$	
$30 < t \leqslant 50$	8



(a) Use the information in the histogram to complete the table.
(2)

(b) Use the information in the table to complete the histogram.
(2)

irrelevant context

frequent in Shape, Space and Measures, where a ‘real life’ object is the object of study but no extra-mathematical knowledge or reasoning is required

Emma visited a stately home.
In one of the rooms there was a semi-circular carpet.
The diameter of the carpet was 8 feet.
Work out the area of the carpet.
Give the units of your answer.

2010

Changes over time in use of non-mathematical contexts

- The proportion of questions involving only specialised mathematical discourse has returned to the 1980 level after a period when it was much reduced.
- The proportion of “deep” contextualisation has remained very low throughout.
- There has been variation in the depth of contexts. By 2010/11 the proportion of contexts categorised as “ritual” had risen. This is most evident in Handling Data questions.
- Use of Algebra in context has fallen to a negligible level.

How do variations in contextualisation affect students' mathematical activity?

- Example 1: Comparison of student test responses to parallel questions with different relationships to context
- Example 2: Task-based interview with one student solving probability questions involving context in different ways

Brazil has an area of 8 500 000 km^2 correct to the nearest 100 000 km^2.

(a) Write down the limits between which the area of Brazil must lie.

.... km^2 and .... km^2
(1)

The population density of a country is the average number of people per km^2 of the country.

Brazil has a population of 144 million correc

(b) Calculate the maximum and minimum v

original
1995

The population density D of a country is given by the formula

$$\boxed{D = \frac{P}{A}}$$

where P is the population

and A is the area in km^2.

Given that, in Brazil,

P = 144 million correct to the nearest million,

A = 8 500 000 km^2 to the nearest 100 000 km^2,

calculate the upper and lower bounds of the population density of Brazil.

"contrived"
based on 2011

Brazil has an area of 8 500 000 km^2 correct to the nearest 100 000 km^2.

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(1)

The population density of a country is the average number of people per km^2 of the country.

Brazil has a population of 144 million correct to the nearest million.

(b) Calculate the maximum and minimum values of the population density of Brazil.

original question 1995

little specialised language

- no algebraic notation
- *maximum and minimum values*

deep context

Students are expected to engage with the context itself and to operate mathematically ("calculate") on the objects of this context.

The population density D of a country is given by the formula

$$\boxed{D = \frac{P}{A}}$$

where P is the population

and A is the area in km^2.

Given that, in Brazil,

$P = 144$ million correct to the nearest million,

$A = 8\,500\,000\ km^2$ to the nearest $100\,000\ km^2$,

calculate the upper and lower bounds of the population density of Brazil.

contrived question
based on 2011

specialised mathematical language

- algebraic notation
- *upper and lower bound*

mundane context

The activity of mathematics involves operating on specialised mathematical objects

Mathematics is construed as a specialised domain, applied to a real world context.

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original 1995

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The population density D of a country is given by the formula

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contrived "2011"

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calculate the upper and lower bounds of the population density of Brazil.

fully correct answers

	all	school A	school B
original version (less specialised)	25%	23%	26%
contrived version (more specialised)	32%	40%	29%

			original version	contrived version
UB/LB of population given			16	0
PD=P/A	no UB/LB given		1	5
	UB/LB based on result		1	15
UB/LB of PD calculated	using P and UB/LB of A		7	0
	using UB/LB of both P and A	UB(P)/LB(A) and/or LB(P)/UB(A)	4	11
		UB(P)/UB(A) and/or LB(P)/LB(A)	2	10
		all combinations calculated	0	1
UB(P)xUB(A) and/or LB(P)xLB(A)			3	0
other			4	2
		n =	*38*	*44*

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UB(P)xUB(A) and/or LB(P)xLB(A)			3	0
other			4	2
		n =	*38*	*44*

Nearly half those answering the original (less specialised) version gave the upper and lower bounds of the population as their answer, making no attempt to work with the definition of population density.

			original version	contrived version
UB/LB of population given			16	0
PD=P/A	no UB/LB given		1	5
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		all combinations calculated	0	1
UB(P)xUB(A) and/or LB(P)xLB(A)			3	0
other			4	2
		n =	*38*	*44*

Of those answering the contrived (more specialised) version, nearly half made the error of calculating the population density using only the given values.

			original version	contrived version
UB/LB of population given			16	0
PD=P/A	no UB/LB given		1	5
	UB/LB based on result		1	15
UB/LB of PD calculated	using P and UB/LB of A		7	0
	using UB/LB of both P and A	UB(P)/LB(A) and/or LB(P)/UB(A)	4	11
		UB(P)/UB(A) and/or LB(P)/LB(A)	2	10
		all combinations calculated	0	1
UB(P)xUB(A) and/or LB(P)xLB(A)			3	0
other			4	2
			38	*44*

Half of those answering the contrived version and a third of those answering the original version made an attempt to take account of the accuracy of population and/or area in order to calculate upper and lower bounds of population density.

Of these, all those answering the contrived (more specialised) version used upper and lower bounds of both variables.

But some of those answering the original question used upper and lower bound of area, together with the given approximate value for population.

conjectures based on this analysis

- specialised discourse in the question allows students to recognise necessary mathematical knowledge and techniques and to identify the relevant variables
- the use of everyday language to define population density seems to have been a major stumbling block
- the way information is structured may make a difference to students’ recognition of the role each piece of information plays in the solution

- To get more detailed insight into student thinking we interviewed 12 16 year-old students in 3 schools.
- We asked them to solve 2 questions on each of 3 different topics:
  - probability
  - sequences
  - angles
- The students talked as they wrote to make their thought processes clear to us.

2010

There are 3 orange sweets, 2 red sweets and 5 yellow sweets in a bag.

Sarah takes a sweet at random.

She eats the sweet.

She then takes another sweet at random.

Work out the probability that both sweets are the same colour.

1995

On average, Alice comes to tea on 2 days out of every 5.

If Alice comes to tea, the probability that we have jam tarts is 0.7.

If Alice does not come to tea, the probability that we have jam tarts is 0.4.

In the space below, draw a tree diagram to illustrate this information.
Write the appropriate probability on each branch.

What is the probability that we will have jam tarts for tea tomorrow?

....

....

....

....

2010

1995

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(a) In the space below, draw a tree diagram to illustrate this information.
Write the appropriate probability on each branch.

(b) What is the probability that we will have jam tarts for tea tomorrow?

....

....

context: very familiar "ritual" *vs*. unfamiliar deep

human presence: Sarah acting alone *vs*. Alice and "we" participating in a joint (possibly unfamiliar) social activity

specialised modes: implicit *vs*. quantified probabilities and demand for tree diagram

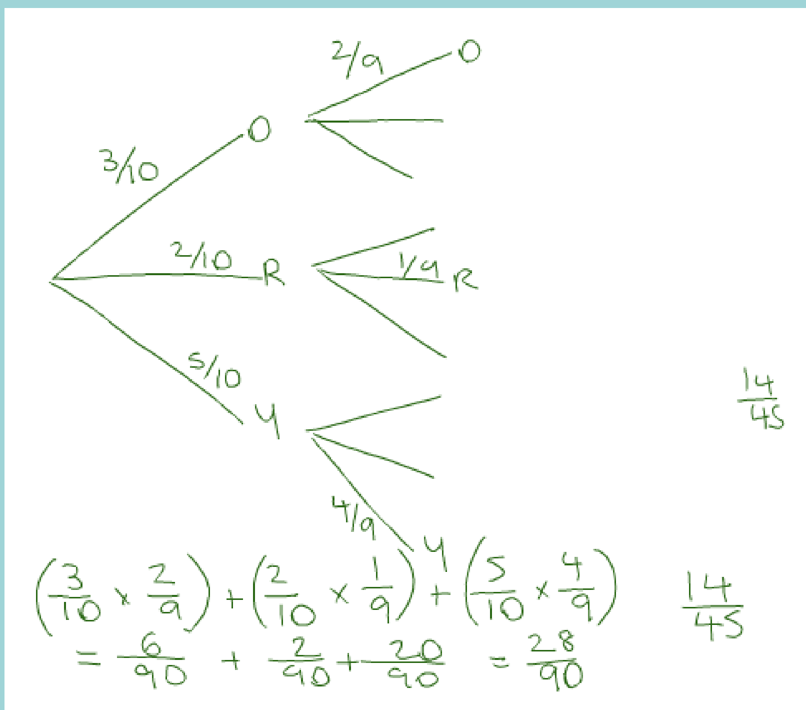
Information structure: Series of simple single clause sentences vs. subordinate conditional clauses.

Logical structure: temporal narrative *vs*. description of general conditions

student responses

- sweets:
  - 6 out of 12 correct probability
  - errors due to incorrect choice of operation or calculation slips
  - 11 out of 12 correct tree diagram (4 of whom omitted irrelevant branches)
  - little reference to context after initial reading of question
- jam tarts:
  - 4 out of 12 correct probability
  - 3 out of 12 answer 7/25, considering only the case when Alice visits
  - 9 out of 12 correct tree diagram
  - frequent reference throughout to Alice, coming to tea, having jam tarts

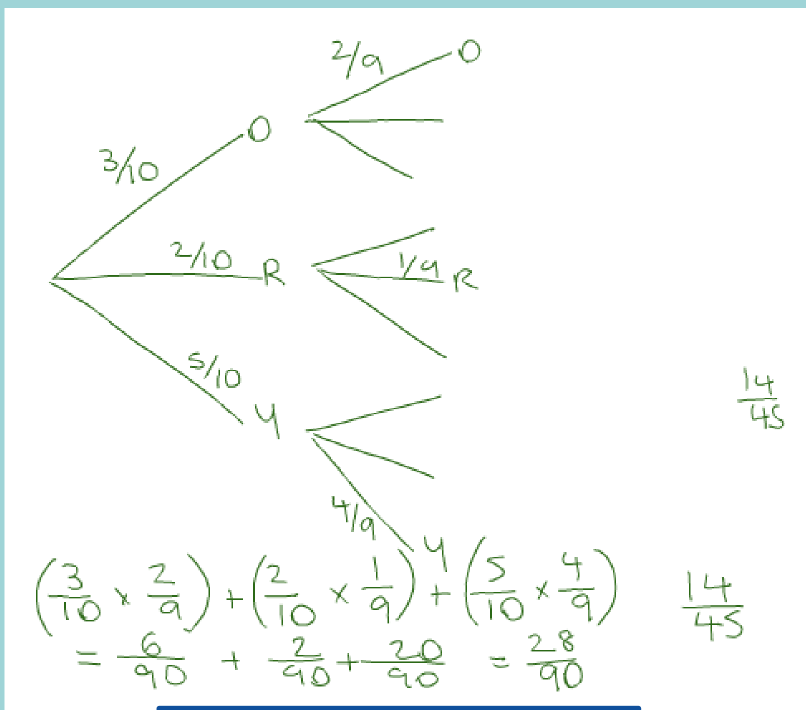
Sweets



*Umm..right. ...yellow... And there’s 3 out of 10 oranges 2 out of 10 and then 5 out of ten yellows So she takes a sweet at random, she then eats the treat – sweet, and then takes another one ...ok…same colour. I don’t need to do 3 but .... …yellow red red... them two don’t really need to be there. OK, orange would be 2 out of 9 because there's now 1 less because she's eaten it. Then this would be 1 out of 9 and this one would be 4 out of 9. So 3 over 10 times 2 over 9 plus 2 over 10 times 1 over 9, plus 5 over 10 times 4 over 9 equals…they’re all over 90 so you can do that**...** And that would be 6, that would be 2, that would be 20. And then that would equal 28 over 90. (1/1A)*

Sweets

reading the question



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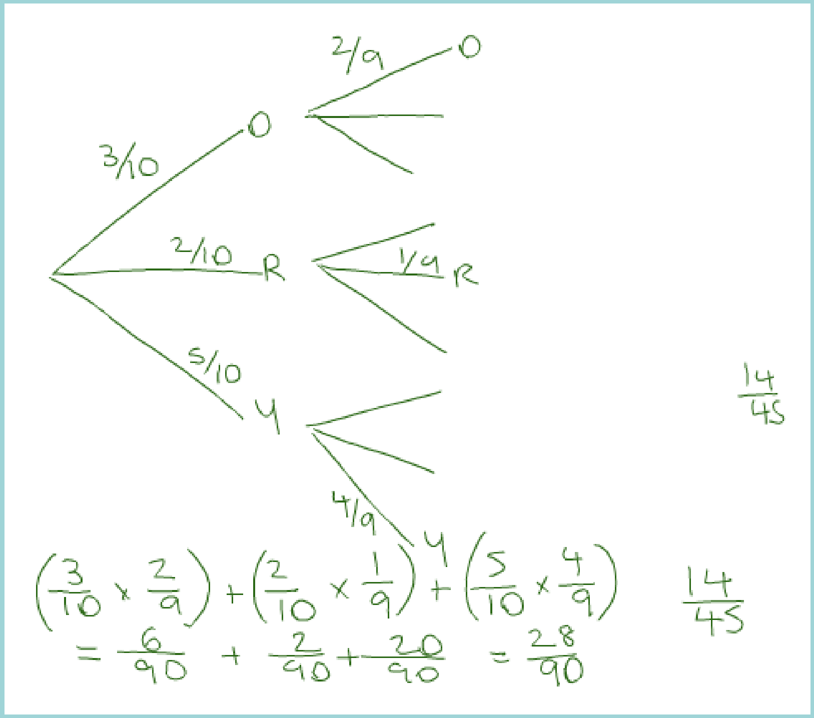
justifying solution method

Sweets



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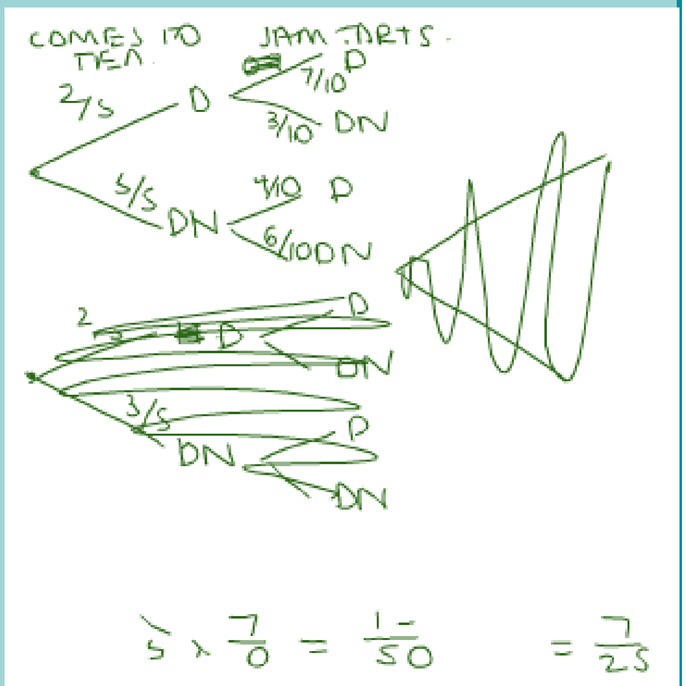
Jam tarts



....Umm...So when they do…the probability that they have jam tarts…is…that they do have and then they do not…do not… They do when she does come for tea. I think these are both the same; I need to cross one out. OK. She does…this is comes to tea…and this one is jam tarts. OK, so the probability they do is 0.7 which actually should be 7 out of 10 I think? Change that to 7 out of 10. And then this would mean that was 3 out of 10 and when she does not come to tea it's 4 out of 10. And then that one would be 6 out of 10 I think. And then, what is the probability…and that needs to be ***.(?)****..as well. What is the probability that we will have jam tarts for tea tomorrow? Um so if she comes…OK, she will come to tea 2 out of 5… OK I think it's…them two, so that's 2 out of 5 times 3 out of 10 equals 14 out of 50, which is also 7 out of 25. There. (1/1A)*

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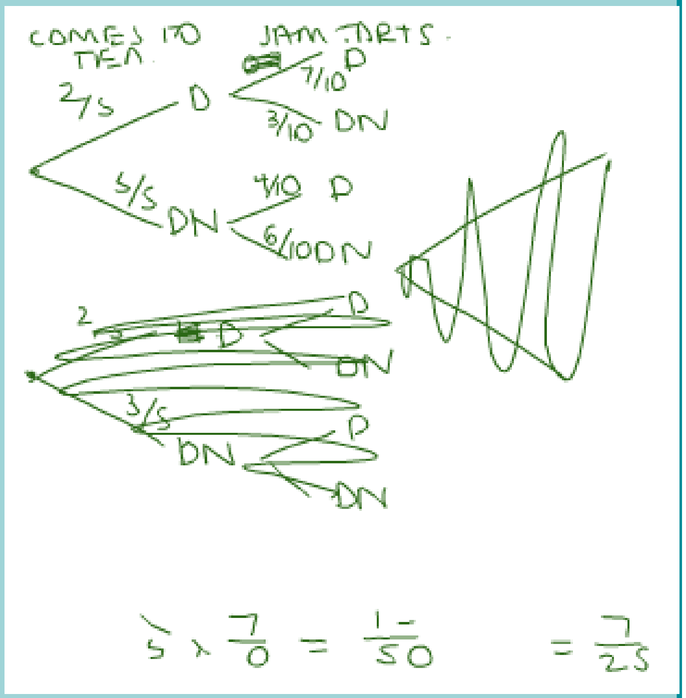

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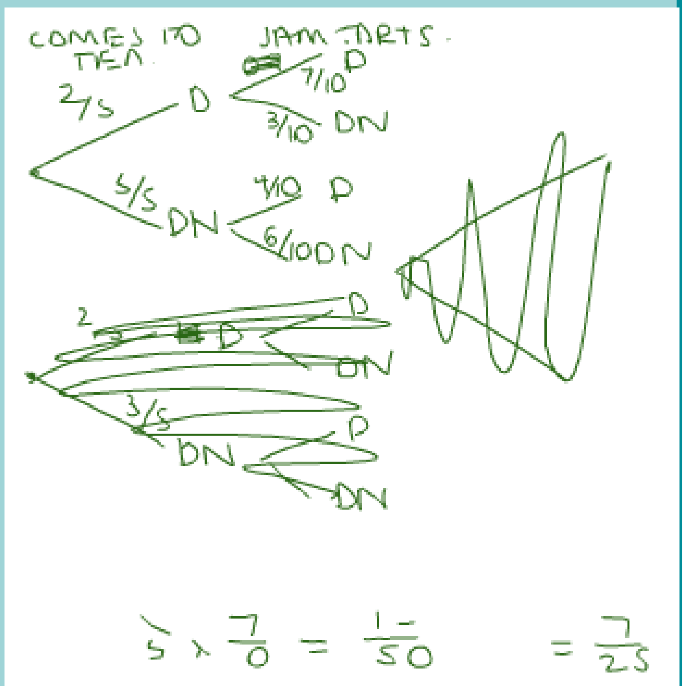

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Conjectures based on this analysis

- references to context play several roles during problem solving
- these roles vary with the depth of the contextualisation
- a task with ritual contextualisation is more likely to be solved fluently
- complexity of task information structure affects problem solving processes

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Implications and discussion points

- What are the objectives of contextualisation?
- How do we want students to engage with mathematics (in context)?
- What roles do tasks with *ritual*, *mundane* or *deep* contextualisation have in teaching, learning and assessment?
- Is it possible to resolve tensions between access and authenticity?

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