The 22nd Adults Learning Mathematics – A Research Forum (ALM) international conference was hosted by the American Institutes for Research (AIR) in Alexandria, Virginia. The conference showcased international perspectives related to the teaching and learning of numeracy and mathematics. International experts in numeracy and mathematics convened at the conference with a focus on five key discussion topics:

1. adult numeracy concepts, theories and practice;
2. science technology engineering and mathematics (STEM);
3. numeracy and diversity;
4. numeracy and literacy; and
5. numeracy and the workforce.

As these proceedings show there was a very wide range of presentations made by the participants in the parallel sessions over three days of discussion. Participants came from 16 states within the US and from seven other countries.

Each day of the conference was introduced by inspirational plenary addresses making us aware of how much we can miss if we do not use our mathematical eyes.

Using Maths Eyes to Give Numeracy the Profile it Deserves
Terry Maguire Director, National forum for the enhancement of teaching and learning in higher education

As you will see from the submissions, presentations and abstracts in these ALM 22 Conference Proceedings, the strengths and needs of mathematical teaching, learning and even policy can be viewed from many different angles and, in particular, through exploiting our mathematical eyes. At this conference there was the added experience of AIR focusing on the ways in which teaching and learning can improve people’s lives and how to transform the learning of numeracy and mathematics.

AIR is one of the world’s largest behavioral and social science research and evaluation organizations. AIR’s mission is to conduct and apply the best behavioral and social science research and evaluation towards improving peoples’ lives, with a special emphasis on the disadvantaged. For more information visit www.air.org.
Opening our Mathematical Eyes

Proceedings of the 22nd International Conference of Adults Learning Mathematics – A research Forum

Hosted by American Institutes for Research
Washington DC

July 12 to 15, 2015

Editors
Anestine Hector-Mason
Linda Jarlskog
David Kaye

Local Organisers
Anestine Hector-Mason
AIR Support Staff
Conference Convenor

Local Conference Host
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These proceedings include both refereed and non-refereed papers. People writing papers for the conference were able to choose whether or not they wanted to go through the conference review process. Peer-reviewed papers are marked with an asterisk (*) next to the title in the table of contents.


This PDF version of the proceedings is available from the ALM website at http://www.alm-online.net/

A series of images of the conference and slides from the presentations of the speakers are also available on the same website.
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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established in July 1994 as an international research forum:

To promote the learning of mathematics by adults through an international forum that brings together those engaged and interested in research and development in the field of mathematics learning and teaching.

Charitable Status
ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number: 3901346). The company address is: 26 Tennyson Road, London NW6 7SA, UK.

Aims of ALM
ALM’s aims are promote the advancement of education by supporting the establishment and development of an international research forum for adult mathematics and numeracy by:

- Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit.
- Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.

ALM’s vision is to be a catalyst for the development and dissemination of theory, research and best practices in the learning of mathematics by adults, and to provide an international identity for the profession through an international conference that helps to promote and share knowledge of adults’ mathematics teaching and learning for the public benefit.

ALM Activities
ALM members work in a variety of educational settings, as practitioners and researchers, to improve the teaching and learning of mathematics at all levels. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers, and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world. ALM does not foster one particular theoretical framework, but encourages discussion on research methods and findings from multiple frameworks.

ALM holds an international conference each year at which members and delegates share their work, meet each other, and network. ALM produces and disseminates Conference Proceedings and a multi-series online Adults Learning Mathematics – International Journal (ALMJ).

On the ALM website http://www.alm-online.net/ you will find pages of interest for teachers, experienced researchers, new researchers, PhD students and policy makers.

TEACHERS
The work of teacher members includes ideas for the development of practice which is documented in the proceedings of ALM conferences. We have started work on producing a searchable database for members to share resources and teaching ideas.
EXPERIENCED RESEARCHERS
ALM brings together international academics, enabling the sharing of ideas, publication of specialist papers and dissemination of new projects via the publications available online including our academic refereed International Journal. Here you will also find occasional requests for help from researchers.

NEW RESEARCHERS AND PHD STUDENTS
The international conferences and some regional events encourage a friendly yet challenging environment to test out ideas, re-evaluate assumptions and develop new projects. Further contact via the website is being developed.

POLICYMAKERS
The work of ALM members has a history of analysing policies on mathematics education for adults and providing an opportunity for cross-national comparisons. The annual conferences are held in different countries encouraging a membership from various countries around the world.

ALM members
Our members live and work all over the world. Visit the Join ALM page to contact your regional representative; or volunteer to become one. We welcome new members. Please contact our Membership Secretary for additional information, and to join via the website visit http://www.alm-online.net/become-a-member/

ALM is managed by a Board of Trustees which is elected by the membership attending the Annual General Meeting (AGM) which is held during each annual international conference.
2014-2015 ALM Board of Trustees

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Preface: About ALM 22

Opening our mathematical eyes: seeing math in everything we do.

The 22nd Adults Learning Mathematics – A Research Forum (ALM) international conference was hosted by the American Institutes for Research (AIR) in Alexandria, Virginia. The conference showcased international perspectives related to the teaching and learning of numeracy/mathematics.

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International experts ('thought leaders') in numeracy/mathematics convened at the conference with a focus on five key discussion topics: (1) adult numeracy concepts, theories and practice; (2) science technology engineering and mathematics (STEM); (3) numeracy and diversity; (4) numeracy and literacy; and (5) numeracy and the workforce.

As these proceedings show there was a very wide range of presentations made by the participants in the parallel sessions over 3 days of discussion. Participants came from 16 states within the US and from seven other countries.

Each day of the conference was introduced by inspirational plenary addresses making us aware of how much we can miss if we do not use our mathematical eyes.

Day 1

Using Maths Eyes to Give Numeracy the Profile it Deserves
Terry Maguire
Director, National forum for the enhancement of teaching and learning in higher education
Dublin, Ireland

Aspiring through Education
Dr. Roosevelt Johnson,
Deputy Associate Administrator for Education, The National Aeronautics and Space Administration (NASA)
Washington, District of Columbia.

Day 2

Mathematics in Our Lives
Dr. Freeman A. Hrabowski,
President, University of Maryland, Baltimore County, Maryland
Numeracy: a Prerequisite for Sustainability
Ms. Marilyn Waite,
Environmental Sustainability Engineer;
Owner, Sustainable Visit, Paris, France and Beijing, China.

Day 3

Real-World Problem Solving Through M in STEM
Dr. Padmanabhan Seshaiyer,
Director, Center for Mathematics Professional Outreach and Educational Technology,
George Mason University, Fairfax, Virginia

As you the reader will see from the submissions, presentations and abstracts in these ALM 22 Conference Proceedings, the strengths and needs of mathematical teaching, learning and even policy can be seen from many different viewpoints and particularly exploiting our mathematical eye. At this conference there was the added experience of AIR focusing on how important it is to recognise how these teaching and learning experiences improves people’s lives and recognising how these approaches transforms learning mathematics.

Note:
All session presenters were invited to submit an article for publication and all submissions meeting the guidance on style and format given by the editors were reviewed and are published in these proceedings.

Each presenter at the ALM 22 conference was able to choose whether or not to submit their article for a special edition of the ALM International Journal (ALMIJ). If they chose the ALMIJ route the paper went through the formal, blind copy, peer review process. Those papers that went through this review process are marked with an asterisk (*) in this volume.

When no article has been submitted this conference session is represented by the programme abstract and in a small number of cases by a copy of the presentation slides. The slides of many of the conference sessions are available on the ALM website.
Acknowledgements

ALM is grateful to the many people who have contributed to the ALM 22 conference and the publication of these proceedings:

- The participants without whom there would have been no conference nor these proceedings
- ALM officers and Trustees for ensuring the continuity of the organisation between the conferences
- American Institutes for Research (AIR) for hosting the conference
- Dr Anestine Hector Mason (AIR) as both organiser of the conference and supervising editor of ALM conference proceedings
- ALM Trustees for completing the publication of the proceedings
- Javier Palomar-Diaz, Chief Editor of the ALMIJ for organising the review process
- AIR staff: Claire Bocage, Jasmen Cheese, Emma Hinkens, Allyssa Lyons, Briton Park, Marie Perrot, Deez-Mae Smith, Ruth Sugar and Channet Williams for administrative support throughout the conference
KEYNOTE SPEAKER ABSTRACTS
Improving What We Do for Struggling Adult Numeracy Students

Mr. Steve Hinds

Abstract
Many adults without high school or college degrees in the United States have struggled to learn mathematics. These students who are underprepared in math may enter basic education programs, high school equivalency programs, or remedial community college programs where they hope to improve their math skills and understanding. The speaker will argue that the policies, funding, and operation of these programs unfortunately rest on a set of false hopes about how teachers teach and how students learn. The speaker will outline opportunities, reforms, and fresh thinking that can lead U.S. educational institutions do a better job for this incredibly important group of adult numeracy learners.

Mathematics in Our Lives

Dr. Freeman Hrabowski

Abstract
As a 12-year old, Freeman Hrabowski sat in the back of his church in Birmingham, Alabama, trying to ignore the meeting and doing what he loved: math problems. He didn’t want to be there, but he heard the speaker say that if the children marched, the world would know they wanted a better education. As a student who wanted to excel, that got his attention. The speaker was Dr. Martin Luther King and Dr. Hrabowski joined in the pivotal children’s march for civil rights in 1963. Since then, Dr. Hrabowski, president of the University of Maryland, Baltimore County, has pursued his lifelong passions for both mathematics and increasing the participation of people of all backgrounds in math and science. Dr. Hrabowski will discuss these passions. Seeing mathematics in every aspect of our lives, he will discuss how deep encounters with mathematics provide people with richer experiences in both their everyday activities and their work. He will address how universities can innovate to improve teaching and learning in STEM disciplines – a step that is now essential as we seek to increase the numbers going into these fields that are so central to innovation in our economy.

Aspiring through Education

Dr. Roosevelt Johnson

(The abstract is not available.)

Using Maths Eyes to Give Numeracy the Profile it Deserves

Dr. Terry Maguire

(The abstract is not available.)
Real-World Problem Solving Through M in STEM

Dr. Padmanabhan Seshaiyer

Abstract
In this session, participants will learn about inquiry-based approaches to mathematical problem solving that employs critical thinking strategies, open-ended exploration, and creativity. The participants will be exposed to benchmark problems with real-world connections that will include scientific and engineering exploration. We will also discuss how such connections can help create related rich tasks with varying cognitive demand and 21st century skills to help enhance student learning. The session will also share important pedagogical practices and problem-solving strategies that can help promote the much needed awareness of the M (mathematics) in science, technology, engineering and mathematics (STEM). Come and learn about how meaningful and impactful mathematics education would be if we, as educators, enhance our teaching practices to go from “Here is the Mathematics, Go Solve the Problem” to “Here is the Problem, Let us find the Mathematics to Solve it!”

Unlikely Minorities and STEM Subjects

Ms. Rinske Stelwagen

Abstract
The persistent homogeneity of the group that opts for STEM-related education and jobs has been a concerning issue for years. Despite of all efforts, the subject is still dominated by White males. Even among this group, STEM subjects are not very popular. Why is that? What does math identity have to do with it? What have we done that actually works? And what else can we try? As an “unlikely minority” myself, being one of the four females in a total of 210 persons in my university year, my opinion is that there cannot NOT be a way. We simply have an unacceptable loss of human capital, and we should make an effort to do something about that. Something realistic and effective, probably out of the box, must be done to shift the equilibrium. In this meeting, I will reflect on personal experiences in my own background and university, on efforts made in the Netherlands to increase the popularity of STEM subjects, and on the image problem it seems to have in society in general. All this is done in an effort to draw to the surface what might work and what does not. No answers will be given. Further research and pilot projects are needed. In the meeting, you will be invited to actively join the discussion on effective actions that could be the next step.

Numeracy: A Prerequisite for Sustainability

Ms. Marilyn Waite

Abstract
Sustainable development and sustainability have emerged as defining policy and business objectives of the 21st century. This year marks an important milestone in international climate change negotiations with the December 2015 Paris Climate Change Conference. But what does it take to achieve sustainability? Marilyn Waite will provide concrete examples of how numeracy is the foundation for reaching the goals of sustainable development – a foundation too often neglected in sustainability discussions. Using the PIAAC definition for numeracy (the ability to access, use, interpret, and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life) and Marilyn’s definition for sustainable development (meeting the needs of all generations, present and future, while improving their well-being through social, economic, environmental, and intergenerational efforts), Marilyn will explain how numeracy is a prerequisite for sustainable development. From understanding utility bills to understanding waste, Marilyn will examine how we can bring about an ecolate (numerate + literate) society.
Community Among Adult Learners in a Maths Support Centre Setting

Dr Anthony Brown  
Dr Anthony Cronin  
School of Mathematics & Statistics  
University College Dublin (UCD), Ireland  
Anthony.Brown@ucd.ie Anthony.Cronin@ucd.ie

Abstract: The UCD Access to Science, Engineering and Agriculture programme is a one-year course targeted at mature students (students aged 23 years or older on January 1st of their year of registration) with the aim of equipping them with the academic skills to successfully transition to full-time mainstream higher education. Due to recent curriculum changes to the programme and the emphasis on mathematics as an indicator of progression through higher level education, we discuss the suite of supports offered to this cohort of adults learning mathematics by the Maths Support Centre.

Keywords: Access to education, mathematical confidence, adults learning mathematics, mathematics learning support

Introduction

At the Inaugural European Society for the Scholarship of Teaching and Learning (SOTL) conference in May 2015, Professor Kathy Takayama mentioned that a longitudinal study based on 10 years of student data collected at Brown University in the United States showed that the two most common reasons for dropping out of a degree course among mature students were:

1. Lack of community
2. Poor experience with an introductory course.

With this in mind the authors aimed to make the transition to university for part time Access to Science, Engineering and Agriculture students (henceforth to be described as Access students) as manageable as possible. In this paper we document the suite of supports put in place for this mature student cohort by the Maths Support Centre (MSC) at University College Dublin (UCD), Ireland’s largest university.

The Access Programme

The UCD Adult Education office has been offering the Access to Science, Engineering and Agriculture programme since 2001. Until 2013, when a new mathematics curriculum and a
scientific enquiry module were introduced, the programme remained largely unchanged. From a pool of up to 100 prospective candidates seeking a return to education, approximately 30 students are accepted on to the Access course.

The criteria for acceptance include:

1. No formal 3rd level education (exceptions for interrupted learning – life-changing circumstance etc.);
2. Evidence of some research and knowledge of the Access course;
3. Display self-motivation and progression plan following the courses’ completion;
4. An interest in reading popular science books;
5. English language competence etc.

Evidence of learning in a formal setting in the previous 3-5 years is a strong indicator of success on the Access programme. In 2013 it was also identified by the College of Science in conjunction with the Adult Education Office, that proficiency in mathematics was also a good indicator of retention and completion of the Access course.

**Maths Support Centre & Access Students**

Prospective students are invited to attend three pre-entry ‘Hot Topics in Mathematics’ workshops in the MSC, and are also asked to submit a piece of writing, based on a lecture they attend in UCD on some aspect of Science.

All candidates are obliged to take a diagnostic test in mathematics (without the use of a calculator), which assesses their knowledge of the basics of arithmetic and algebra. This first diagnostic test consisting of 16 multiple-choice questions is taken online via the MSC website. One of the answer options is Not Sure and we encourage students to choose this option rather than guess at an answer so that we can get the fullest picture of their prior knowledge. The diagnostic test can be viewed from the link given here [http://www.ucd.ie/t4cms/Diagnostic_test1.pdf](http://www.ucd.ie/t4cms/Diagnostic_test1.pdf).

The second diagnostic test is then completed with pen and paper on the third and final night of the Hot Topic sessions. This is then corrected and the results disseminated to the students. It should be emphasized that the final mark on the second diagnostic test is not what is been evaluated but rather the progression the student has shown from the initial diagnostic test. This second test covers the same material as the first test and it follows the Hot Topic sessions that concentrate on these fundamental areas of mathematics. Typically the two diagnostic tests and three Hot Topic sessions are completed within one week.

Students are also invited to interview where the course requirements and acceptance criteria are discussed. Acceptance on to the course is based on the student displaying evidence of how they meet the above criteria, and whilst the mathematics score is not a complete disqualifier, the student may be cautioned against taking the course and advised to take additional tuition in advance of taking a place on the programme.
Changes to the Curriculum

After gaining entry on to the programme students study modules in Biology, Chemistry, Physics, Study Skills, Mathematics 1 and Mathematics 2 with both the mathematics modules being mandatory. In total, students will receive 72 hours of mathematics tuition. During the summer of 2013, following consultation with the College of Science, the School of Mathematical Sciences and the School of Electrical, Electronic and Communications Engineering at UCD, the Adult Education Centre decided to re-design the existing Access to Science and Engineering course with a particular emphasis on mathematics. In the previous 12 years only those students wishing to pursue entry on to an Engineering degree were obliged to do both of the mathematics modules (one in each semester) while those wishing to pursue a Science (or Agriculture) degree could choose to study mathematics in semester 1 only. From 2013/14 this condition was changed to the current one where all Access students must do both mathematics modules.

Syllabus changes to the mathematics content meant that two chapters on statistics and probability were introduced for the first time while none of the previous content was removed. Changes to the exam were also made to exclude the option of exam question choice, thus precluding a student from omitting to study certain sections of the course. The questions on the exam were also to be of a more applied nature and less theoretical. These changes are in line with developments made to the end-of-secondary school state examinations in mathematics in Ireland under a new initiative called Project Maths [3]. For more on the impact of Project Maths on the learning of a data analysis module at third level, see Cronin and Carroll (2013).

Maths Support Workshops

A vital component of the suite of supports offered by the MSC to these adults learning mathematics are the (twice) weekly workshops. These are 90-minute, student-led workshops held on a Monday and Wednesday evening (to coincide with the evenings the adults are already on campus for lectures) in the MSC which is located within the main UCD campus library, the James Joyce Library.

In September 2014 there were 30 students registered to the course and 26 students completed semester one. The four students that dropped out all dropped out in the early weeks and none of them attended any of the pre-semester (or during semester) MSC Hot Topic sessions. It is noteworthy that these four students were the only students that didn't attend any MSC workshop. The remaining 26 students all completed the second semester also.

Figures 1 and 2 below show the regular engagement of the adult students with the weekly workshops. It is of interest that the attendance at workshops slowly decreased as semester one progressed and increased as semester two progressed. This may be linked to the timing of the presentation of various topics within the course.
Figure 1: Semester one attendance at weekly MSC workshops

Figure 2: Semester two attendance at weekly MSC workshops
The Access students’ attendance at the MSC sessions can be broken into three cohorts (see Figure 3 below). Cohort A are those that attended three or less sessions, Cohort B attended between four and ten sessions (inclusive) and Cohort C attended more than ten sessions. The following tables show the average final percentages of each of the three Cohorts and the number of students in each.

**Semester 1:**

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<tr>
<th>Cohort</th>
<th>Number of Students</th>
<th>Average Mark</th>
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<tr>
<td>A</td>
<td>7</td>
<td>84.58%</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>90.69%</td>
</tr>
<tr>
<td>C</td>
<td>9</td>
<td>78.51%</td>
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</table>

**Semester 2:**

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Number of Students</th>
<th>Average Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>89.38%</td>
</tr>
<tr>
<td>B</td>
<td>12</td>
<td>79.98%</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>84.50%</td>
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</tbody>
</table>

Figure 3: The final marks by MSC session cohorts

Note that cohort X for semester one does not contain the same students as cohort X for semester two as the attendance patterns were not constant from term to term.

**Survey of Access Students**

A survey of the 26 students was taken in June 2015 to gather the students’ opinions on various aspects of the MSC service in relation to both their mathematics modules. There was an impressive 65% response rate to this survey (17 of 26 students). One such question asked the students to rate, what they felt, were the most difficult topics in the each of the two mathematics courses. For semester one (Mathematics I) an overwhelming 76% of students ranked *Integral Calculus* as the most difficult topic, with *Differential Calculus* (71%) coming a close second. *Functions* (41%) and *Statistics* (24%) were the next two most difficult topics for respondents. Interestingly 71% of students stated that *Arithmetic and Algebra* was one of the easier topics. These topics were the focus of the three pre-semester MSC Hot Topics.

The fact that so many students found Integral and Differential Calculus so difficult may explain the spike in the MSC workshop attendance in weeks 8 to 10 of semester one. However upon deeper analysis of the feedback provided by the MSC tutor of the workshops we see that *Functions* and *Trigonometry* (see the table below) were the central focus of the workshops around this three-week period. Perhaps those students who found Integral and Differential Calculus so hard were among those students in Cohort A but unfortunately the survey was anonymous and so we cannot tell which of the survey respondents were also regular/non-regular MSC workshop attendees.
Table of MSC workshop feedback during weeks 8-11 of semester one

<table>
<thead>
<tr>
<th>Date</th>
<th>Workshop Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 November 2014</td>
<td>Access workshop mainly on functions. Does a definition give a function? Is a function injective, surjective or bijective? Finding inverse functions. I think that the students are doing well but the students are commenting that they think this is the hardest chapter so far.</td>
</tr>
<tr>
<td>10 November 2014</td>
<td>Access workshop on trigonometric formulae and the sine and cosine rules. The students seem to be finding this chapter much easier than the abstract work on functions. There are still some students struggling with domains and codomains. One interesting problem was one student who thought that the rule should be applied to the codomain rather than the domain to get the image. Working on problems related to domain and range of functions and whether definitions of certain functions were valid given the range and co-domain definitions.</td>
</tr>
<tr>
<td>12 November 2014</td>
<td>Access workshop on trigonometric formulae, sine and cosine rules. The students are working well on this section with no real problems.</td>
</tr>
<tr>
<td>17 November 2014</td>
<td>Access workshop. This was originally going to be on differentiation but most students seemed to do well with this, so some were working on trigonometry and others were starting to revise the whole course. There were no real problems.</td>
</tr>
<tr>
<td>19 November 2014</td>
<td>Access workshop. Most of the students were comfortable with differentiation so started revision - mainly on rules of indices. There were no real problems.</td>
</tr>
<tr>
<td>24 November 2014</td>
<td>Access workshop - students working on integration, no real problems.</td>
</tr>
</tbody>
</table>

For semester two, 76% of respondents again ranked Integral Calculus as the most difficult topic while there was a drop to 41% who felt that Differential Calculus was the most difficult. 47% felt that Probability was one of the more difficult topics while Matrices & Vectors (35%) and Complex Numbers (12%) were deemed easier sections of the semester two course. For how these topics are assessed the interested reader may want to consult the final exam papers from both semesters of the 2014/15 academic year via the following two links:


When asked “If you didn't attend the Maths Support Centre sessions during (and after) term time can you state the reasons why?” 64% said they could not attend due to work commitments and 18% cited family commitments as the reason they did not attend. Interestingly 27% of respondents said they “did not need to attend them”.

When asked “How can the MSC sessions be improved?” it is interesting to note that there were no responses relating to the content or style of the sessions but 53% said they’d like “More MSC sessions on different nights but later times” and 41% said they’d like “More online support via forum” and finally 24% said they’d like “More online support via a virtual drop-in centre”. For more on a virtual mathematics learning centre see Golding (2015).
In relation to the issue of the two most common reasons among adult students for dropping out of a degree course, we asked the following question:

*If you found the MSC sessions useful can you state the reasons why?*

41% said they “helped me to meet fellow classmates”, while 29% said they “helped my sense of belonging to the university” and “helped me to learn how to work with others”. Not surprisingly the majority of respondents (65%) said they “helped me solve (mathematical) problems” with a further 59% stating that they “helped me digest material from the course”. Also 47% and 41% respectively, said they “helped me to prepare for the assignments” and they “helped me to prepare for the final exam”. These responses bode well for the initial objective of offering a sense of community and confidence for adult learners within a Maths Support Centre setting.

One student summarised the general positive reaction the adult students had towards the MSC sessions by stating:

“*honestly, I couldn't have asked for more from the support centre*”

**Conclusions and Future Work**

In future work the authors will perform further analysis of the attendance, exam results, exam topics and survey feedback to see if we can identify the students that would have benefited from the MSC sessions but did not attend them and attempt to determine the reasons for this. Is it that they thought they did not need them, or did they know that they needed the sessions, but there was some reason they did not attend them, for example, work or family commitments? Additionally, we will examine if there is any way we can overcome this, for example by scheduling an MSC session for a later time or on a different evening (when the students are not already on campus for lecturers) or through online support etc.

**References**


Video: [https://www.youtube.com/watch?v=KJxWbItOe6c](https://www.youtube.com/watch?v=KJxWbItOe6c)

Slides: [http://supportcentre.maths.nuim.ie/mathsnetwork/imlsn9abstracts](http://supportcentre.maths.nuim.ie/mathsnetwork/imlsn9abstracts)
Making the most of PIAAC: Preliminary investigation of adults’ numeracy practices through secondary analysis of the PIAAC dataset

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Abstract
The Programme for the International Assessment of Adult Competencies (PIAAC) assesses key information processing skills and collects information on how often people undertake a range of activities at work and in everyday life. We are exploring what secondary analysis of online anonymised PIAAC data can tell us about adults’ numeracy practices. In the process we review the accessibility and user-friendliness of the data for novice researchers and practitioners in the hope of encouraging them to explore this rich resource and give a brief account of our experience of the process of accessing publicly-available PIAAC data for secondary analysis.

Key words: assessment, numeracy, adults, mathematics

In this paper we explore what secondary analysis of data from the latest international Survey of Adult Skills in the Programme for the International Assessment of Adult Skills (PIAAC), can tell us about adults’ numeracy practices. In the process we are also reviewing the accessibility and user-friendliness of the dataset. We hope to encourage the exploration of this rich resource by practitioners and researchers, including those with little previous experience of working with large datasets. This is important because, as Hansen and Vignoles (2007, p. 1) point out “In the last few decades, there has been an unprecedented increase in the availability and quality of large-scale data sets that are suitable for use in education research. Analyses of these data have the potential to radically improve the robustness and generalisability of educational research”. In a still young but growing field such as adults learning mathematics, this is especially important.

We focus on secondary analysis of anonymised publicly-available PIAAC data. We draw our understanding of secondary analysis from Dale, Watham and Higgins (2008, p. 520): “Secondary analysis is generally understood as the analysis of data originally collected and analysed for another purpose”. In addition, Heaton (1998) says “Secondary analysis involves the use of existing data, collected for the purposes of a prior study, in order to pursue a research

interest which is distinct from that of the original work; this may be a new research question or an alternative perspective on the original question”. She adds that secondary analysis may be undertaken either by the original researchers or others.

The OECD has published two analytical reports on PIAAC (OECD, 2013a, 2016a). They relate to the first and second rounds of the PIAAC survey in participating countries. The New Zealand Ministries of Education, and Business, Innovation and Employment have published three initial reports: Ministry of Education & Ministry of Business, Innovation and Employment (2016a, 2016b, 2016c). Secondary PIAAC analysis will tackle research questions and topics beyond what is covered in the initial reporting, or take different or more in-depth approaches. Examples include:

- integrating different parts of the PIAAC dataset so as to generate new knowledge and understanding of associations and relationships
- building or refining conceptual or statistical models.
- exploring specific themes rather than taking a broad focus.

This paper explores an example of secondary analysis of numeracy practices in everyday life and work that takes a different perspective from the analysis of OECD (OECD, 2016a, pp. 97-113) which constructs indices that group together tasks involving similar activities.

What is PIAAC?

PIAAC is an international survey that assesses key information processing skills of adults of working age in Literacy (reading) and Numeracy and collects information on how often they undertake a range of related activities in work and everyday life (OECD, 2013c). Two additional assessment components are optional for participating countries: Reading Components (Sabatini & Bruce, 2009); and Problem Solving in Technology-rich Environments (OECD, nd-b). PIAAC builds on previous international surveys: the 1994-1998 International Adult Literacy Survey (IALS) (OECD & Statistics Canada, 2000); and the 2003-2006 Adult Literacy and Life Skills Survey (ALL) (Satherley, Lawes, & Sok, 2008; Statistics Canada & OECD, 2005).

PIAAC is planned as a repeating survey with a ten-year cycle. The first cycle was undertaken in two rounds, with a third round scheduled for 2016-2019. Round One included 24 countries with data collected in 2011/12 and findings published in 2013 (OECD, 2013a). New Zealand, together with eight other countries, participated in Round Two, with data collection April 2014 – February 2015. Results were released in June 2016 (OECD, 2016a).

Some key New Zealand findings are:

- New Zealand adults’ literacy and problem solving skills are on average among the highest in the OECD
- New Zealand adults’ numeracy skills are on average higher than the OECD average
- Although there are significant differences in skills between ethnic groups, average literacy and numeracy skills have been rising faster among Māori and Pasifika than in the total New Zealand population

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2 Normally 16-65 years but this can vary, e.g., Australia extended the range to 15-74 years.
3 Round 1 PIAAC countries were: Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, The Netherlands, Norway, Poland, Russian Federation, Slovak Republic, Spain, Sweden, United Kingdom, the United States.
4 Round 2 PIAAC countries were: Chile, Greece, Indonesia, Israel, Lithuania, New Zealand, Singapore, Slovenia, and Turkey. Round 3 PIAAC countries will be: Argentina, Colombia, Hungary, Kazakhstan, Mexico, Peru and the United States of America (the latter repeating PIAAC in subpopulations).
- Overseas-born New Zealanders have on average higher literacy and numeracy scores than overseas-born people in any other country.
- While there are no differences in average literacy and problem solving skills between men and women, men have higher numeracy skills on average than women.

The PIAAC survey is carried out by: interviewing a sample of at least 5000 adults in each participating country\(^5\) in their homes; collecting a broad range of information through a Background Questionnaire\(^6\); and assessing skills in the PIAAC domains. The language of assessment is normally the official language or languages of each participating country\(^7\). Depending on their computer skills\(^8\), participants either enter their responses to the assessment items of the main skill domains on the assessor’s laptop computer or complete a paper version using printed test booklets. The Background Questionnaire is administered face-to-face in the respondent’s home by an interviewer who enters the answers into a laptop computer. All aspects of countries’ implementation of PIAAC is strictly monitored and quality-assured by the PIAAC Consortium. PIAAC governance is provided by a Board of Participating Countries.

The Background Questionnaire collects data on participants’ educational background, skills used at work (for those currently or recently in employment) and in other contexts such as the home and the community. For example, people are asked about their voting habits, volunteering, languages spoken, political efficacy and health. The following variables are covered: demographic characteristics; other personal characteristics (including learning disposition and self-assessed health status); education and training characteristics; work characteristics; self-assessed mathematics skills for work; self-assessed reading and writing skills for work; and skill use in everyday life.

Respondents with very low literacy skills (as assessed by some initial questions) are not assessed in the main skill domains, but instead go directly to the Reading Components assessment. This covers “the basic set of decoding skills that enable individuals to extract meaning from written texts: knowledge of vocabulary, ability to process meaning at the level of the sentence, and fluency in reading passages of text” (OECD, nd-a, p. 1).

In PIAAC, proficiency is considered as a continuum of ability involving information-processing tasks of increasing complexity defined as ‘proficiency levels’ (OECD, 2013b, p. 64). Six proficiency levels are described for Literacy and Numeracy (Levels 1 to 5, and below Level 1) and four for Problem Solving in Technology-rich Environments (Levels 1 to 3, and below Level 1). These summarise what adults with proficiency scores in each skill domain can do. The ‘average’ individual with a proficiency score in the range defining a level will successfully complete items located at that level approximately two-thirds of the time. PIAAC measures cognitive skills through test items that have a range of contexts: work-related, personal, society, and education and training. The OECD has released a number of sample test items (OECD, 2016b, pp. 21-23; 26-28; 31-32). The work context of PIAAC’s cognitive skill assessment is complemented by self-reports on generic skills required in work, including interpersonal skills and physical skills.

As for earlier international skill surveys, PIAAC published results include: average scores on each skill domain for countries and population subgroups; comparisons over time and with other countries; and proportions of different sub-populations reaching different benchmarks for each skill domain.

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\(^5\)Participating countries can increase the sample size to provide better information on specific sub-populations; e.g., New Zealand oversampled 16-25 year olds and Māori.

\(^6\) The PIAAC Background Questionnaire is at http://www.oecd.org/site/piaac/BQ_MASTER.HTM.

\(^7\) In some countries, PIAAC assessment has also been conducted in widely-spoken minority or regional languages.

\(^8\) Problem Solving in Technology-rich Environments is assessed only on a computer, so only participants who have sufficient computer skills and choose to use a computer are assessed on this domain.
PIAAC breaks new ground by: expanding the range of skill domains measured; including information on the skills of adults with levels of proficiency below Level 1; expanding the self-reported measures of the use of skills at work; introducing self-reported measures of qualifications matched to work; using computers to administer this kind of international assessment; making data publicly available for review and secondary analysis on an unprecedented scale via the PIAAC website⁹; having an online version of the assessment publicly available¹⁰; and not specifying Level 3 as a benchmark (OECD, 2010, p. 4). In IALS, Level 3 was considered to be the minimum skill level required to cope with the demands of modern society (OECD & Statistics Canada, 2000, p. xi). However, while many higher-level jobs require Level 3 or above, there is no evidence that everyone needs to be at Level 3. This change is significant because, as Black and Yasukawa (2014) point out, the Level 3 criterion has been used by powerful institutions to promote a crisis discourse in adult literacy and numeracy.

For more detail about PIAAC constructs, methods and how the survey was undertaken in participating countries, see the PIAAC Reader’s Companion (OECD, 2016b).

**Numeracy in PIAAC**

Numeracy¹¹ is defined in PIAAC as:

> the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life … Numerate behaviour involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways. (OECD, 2012, p. 34)

PIAAC directly measures numeracy proficiency through the Numeracy assessment. This provides average scores on the PIAAC Numeracy scale for the whole population, or for subgroups.

PIAAC also collects numeracy-related information via the Background Questionnaire. The interviewer asks respondents how often they undertake seven¹² numeracy practices in work (if appropriate)¹³ and everyday life. For example:

The following questions are about activities that you undertake as part of your job and that involve numbers, quantities, numerical information, statistics or mathematics.

- In your job, how often do you usually calculate prices, costs or budgets?
- Use or calculate fractions, decimals or percentages?
- Use a calculator – either hand-held or computer based?
- Prepare charts, graphs or tables?
- Use simple algebra or formulas?
- Use more advanced mathematics or statistics such as calculus, complex algebra, trigonometry, or use of regression techniques?

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⁹http://www.oecd.org/site/piaac/publicdataandanalysis.htm
¹⁰A public online version of the PIAAC survey is at http://www.oecd.org/skills/ESonline-assessment/.
¹¹The Numeracy domain in the ALL survey is comparable with the Numeracy domain in PIAAC but not with the Quantitative Literacy domain in IALS.
¹²The international version of the PIAAC Background Questionnaire includes the first six of these numeracy activities. New Zealand asked for the seventh activity, ‘measure or estimate the size or weight of objects’, to be added as a national extension. This activity was in the 2006 ALL survey and it was found to be analytically useful as it helps identify groups of people whose numeracy activities have this simple practical purpose.
¹³The same questions are asked in the past tense for people who are not currently working, but who worked in the last 12 months.
• Measure or estimate the size or weight of objects?

The frequency options are: Never; Less than once a month; Less than once a week but at least once a month; At least once a week but not every day; Every day. These data provide measures of the frequency and diversity of participants’ numeracy activities. The activities can be analysed separately, or a range of options can be developed for deriving an index of numeracy activity by combining the data across the activities.

The Adult Literacy and Life Skills Survey (ALL) asked similar questions, and since analysis of these questions, together with occupational characteristics, provided a coherent picture, we have some good assurance of the validity of these measures (Satherley, Lawes, & Sok, 2009).

**Analysing adults’ numeracy practices: issues and types of analysis**

A range of types of analysis may be used to explore questions about adults’ numeracy practices. Simple univariate tabulations can provide a ‘big picture’ view while multivariate analysis such as multiple regression can provide measures of the contributions of different factors to associations with numeracy practice. For example, frequent practice of numeracy activities at work may be associated with: higher levels of education; numeracy-related fields of study; specific groups of occupations; and higher measured numeracy skill. Such analysis can show which factors are most strongly associated with frequent numeracy practice and how much increase in frequency of numeracy practice is associated with one unit of measured numeracy skill, whilst holding other factors constant.

PIAAC data cannot tell us to what extent frequent numeracy practice causes high numeracy skill, so researchers should not use ambiguous language such as ‘leads to’, ‘brings about’, ‘influences’, or ‘is linked with’. In any case, it may be that numeracy practice, opportunity or requirement to undertake numeracy practice, and numeracy skill are all mutually reinforcing. Even where data show a strong association between two factors (for example, numeracy practice and numeracy skill) whilst controlling for other factors, we cannot infer that a change in one factor will result in a change in the other, either on an individual or a group level. Another limitation is that PIAAC does not provide a measure of either the intensity or complexity of numeracy activity. For example, finance analysts doing nothing but calculating costs and budgets would report the same way as someone who worked on costs and budgets for 10 minutes every day. Researchers also need to be aware of issues relating to continuous and categorical variables. For example, PIAAC frequency options are five separate categories, although for some analytical purposes it may be legitimate and useful to derive a continuous frequency variable from the discrete categories.

Some other methodological or technical issues users should be aware of include:

• The 2016 PIAAC report (OECD, 2016a) presents OECD averages based on the 28 OECD countries participating in Round 1 or Round 2. This differs from the averages presented in (OECD, 2013a) based only on Round 1 countries.

• Numeracy skill is directly measured, whereas participation in numeracy activities is self-reported.

• The PIAAC data are rich enough to provide scope for different indicators of a construct, such as job-skill mismatch. What indicator is the most useful will depend on the purpose of the analysis, or the need for comparability with published findings across countries.

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14We recommend novice researchers should consult a textbook covering quantitative research methods in the social sciences (e.g., De Vaus, 2014; Frankfort-Nachmias, Nachmias, & DeWaard, 2015).
• Where a user analyses PIAAC data across all or many participating countries, the large sample may mean that nearly all differences are statistically significant. This will not apply for an analysis for one or a few countries, and very fine analysis with many variables for a single country may entail large sampling errors and therefore few significant differences.

Some questions about numeracy activities that PIAAC secondary analysis can help answer

In this section we pose some questions to demonstrate the process we went through in our exploration of the PIAAC dataset and comment on how these questions could be answered through secondary analysis of the PIAAC data. The OECD has published an initial analysis of skill use at work and in everyday life (OECD, 2016a, pp. 98-100). This is a country comparison that aggregates activities and summarises frequencies into an index. Our paper aims to suggest more in-depth and more detailed analysis focused on more specific research questions beyond that of the OECD report.

**How often do people perform numeracy activities at work and in everyday life?**

PIAAC numeracy activities at work and in everyday life and frequency categories can be shown in tables as simple frequency tables, i.e., activity categories can denote rows with frequency categories, denoting columns.¹⁵

**What are the patterns for different groups in numeracy activities at work?**

The PIAAC Background Questionnaire sheds light on the part numeracy activities play in different kinds of work for different groups of people. PIAAC classifies **Occupations** as: Managers; Professionals; Technicians; Clerical; Service & sales; Agriculture & fisheries workers; Trades; Machine workers; and Labourers. Even at this broad level, analysis of the 2006 New Zealand ALL data show distinct profiles of numeracy activity for different occupations, for example, managers, trades workers, technicians, professionals, and agriculture and fisheries workers engaged in relatively frequent numeracy practice compared to clerical, labourers, machine and service and sales workers (Satherley et al., 2009). Broad PIAAC **Industry sector** groups include: Agriculture & fisheries; Manufacturing; Construction; Trade; Transport & communications; Finance & real estate; and Health & education. Analysis of PIAAC data will enable researchers to identify sectors with unexpected pockets of high or low numeracy activity. We can also look at the association between numeracy practice and measured numeracy skill to answer questions such as:

• How likely are people with strong numeracy skill to have jobs that entail frequent numeracy activity?

• Where in the economy do we see areas of job-skill mismatch? One way of tackling this question is developing an indicator of job-skill match that compares actual numeracy skill with the frequency of undertaking numeracy activities. A possibility is grouping respondents into (a) high, medium and low numeracy skill and (b) high, medium and low frequency of undertaking numeracy activities at work; then derive a third grouping: over-skilled, under-skilled and matched; then we could investigate industry or occupation patterns for this indicator (see OECD, 2016a, pp. 129-143).

Analysis by socio-demographic characteristics can also show different numeracy activity patterns for different groups by: gender; age group; education level; and field of study. New Zealand’s oversampling of 16-25 year olds and Māori will allow these groups’ numeracy activities to be examined in detail.

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¹⁶ PIAAC dataset categories and sub-categories are italicised for ease of reference.
What are the patterns for different groups for numeracy activities in everyday life?
We can analyse numeracy activities in everyday life similarly to work contexts. In this case analysis is likely to focus on socio-demographic characteristics and measured numeracy skill.

What similarities or differences exist for numeracy activity at work and in everyday life?
By looking at participants’ numeracy activities at work and in everyday life, we may discern patterns of global differences between these two contexts for individuals. For example, if we were interested in calculator use in work compared to in everyday life, we could cross-tabulate these variables.

What similarities or differences exist for numeracy and literacy activity at work?
Literacy activities at work included in the Background Questionnaire cover eight reading and four writing activities: Read directions or instructions; Read letters, memos or emails; Read articles in newspapers, magazines or newsletters; Read articles in professional journals or scholarly publications; Read books; Read manuals or reference materials; Read bills, invoices, bank statements or other financial statements; Read diagrams, maps or schematics; Write letters, memos or emails; Write articles for newspapers, magazines or newsletters; Write reports; and Fill in forms. One approach to making sense of this level of detail is to summarise the numeracy activities by creating a frequency index. For example, we could assign a numerical code for the frequency options and add the codes to obtain a total score, which would be a measure of the frequency and diversity of numeracy activities undertaken. Someone with a high score often engages in several different numeracy activities. We could generate similar indices for reading and/or writing. This would allow us to study the characteristics of, for example, people who score highly on both numeracy and literacy activities at work.

What similarities or differences exist for numeracy and literacy activity in everyday life?
We could take a similar approach to explore numeracy and literacy activity in everyday life. For example, we could look at questions about whether numeracy and literacy activities in everyday life go together for many people or not. What are the characteristics associated with undertaking a lot of different numeracy and literacy activities? Can we identify characteristics that are associated with seldom undertaking few numeracy and literacy activities?

What can PIAAC tell us about adults’ financial capability?
PIAAC can shed light on financial capability, for example, through a study of the relationships between directly measured numeracy skill and the self-reported numeracy activity of calculating prices, costs or budgets. Similarly we could study literacy skill and the literacy activity of reading bills, invoices, bank statements or other financial statements. Acknowledging that numeracy activity is self-reported, inferences about financial literacy skill may still be possible.

What are the patterns of work-related numeracy activity together with ICT activities?
The PIAAC Background Questionnaire asks how often people participate in the following ICT activities: use email; use the internet in order to better understand issues related to your work; conduct transactions on the internet; use spreadsheet software; use a word processor; use a programming language to program or write computer code; participate in real-time discussions on the internet. These could be analysed using a similar approach to the comparisons between numeracy and literacy activities.

What are the characteristics of people who seem to be matched or mismatched on numeracy skill at work and numeracy activity? Are Field of study of highest qualification, or Occupation related to the match or mismatch?
To what extent people’s numeracy skill aligns with their numeracy activity is an important issue. A working-age population might contain significant proportions who could easily manage more
(or a higher level of) numeracy activity than they actually engage in, or significant proportions who are attempting numeracy activity that their skills do not support. In these cases, a work skills policy issue seems to emerge about whether policy levers should be applied to support a more efficient allocation of numeracy skills and numeracy activity.

One approach to better understanding numeracy skill and activity mismatch would be a multivariate analysis of Numeracy activity, Measured numeracy skill, and Main field of study of highest qualification, and Occupation. This could build on an OECD report on the first round of PIAAC entitled The System-level Causes and Consequences of Field-of-study Mismatch (Montt, 2015).

**What changing patterns for young people’s numeracy activities can we see on their pathways from school to tertiary education to work?**

Here we could identify subgroups of 16-25 year olds who are: at school; in formal tertiary education; in full-time work; or combining work with study. We could identify what characteristics are strongly associated with participating in numeracy activities in everyday life (including study). These could include respondents': age; gender; level of highest qualification; the level of qualification they are studying for; the main subject of qualification they are studying for; the type of educational institution they are studying at; whether they are also undertaking non-formal or informal study; and their occupation and numeracy activities at work (for those currently or recently working).

**How have patterns of numeracy activity at work changed over time?**

The ALL survey asked how often people participated in six numeracy activities as part of their job. The ALL activities were: Measure or estimate the size or weight of objects; Calculate prices, costs or budgets; Count or read numbers to keep track of things; Manage time or prepare timetables; Give or follow directions or use maps or street directories; and Use statistical data to reach conclusions. The first two on the above list of activities were re-asked, the first in New Zealand’s PIAAC survey only, and the second internationally. Using these data, we could look at how often workers undertook the numeracy activities in common between ALL and PIAAC, and investigate whether changes over time are associated, for example, with changing work, with changing occupational composition, or changing levels of education.

We turn now to a brief account of our experience of the process of accessing publicly-available PIAAC data for secondary analysis.

**Using PIAAC Data Explorer to access PIAAC data**

Information on PIAAC Public Data & Analysis is available online. The OECD PIAAC Gateway website Data Tools › Datasets and Tools is designed to give users the tools needed to analyse the PIAAC dataset. The Data Tools section provides information on how to analyse a chosen dataset and Data Files are available, categorised as National, International, and Trend. Users are advised to read the guide What You Need to Consider(AIR PIAAC Team, nd) and watch online Distance Learning Data Training (DLDT) modules to learn about appropriate statistical procedures and methods of analysis before accessing the PIAAC datasets. Users may also wish to consult a survey statistician or a psychometric expert for technical advice.

The PIAAC International Data Explorer (IDE) is a web-based application for accessing PIAAC data that does not require any advanced statistical knowledge or specialist software. It is a point-and-click interface for creating statistical tables and charts and exploring levels of adult skills and demographics.

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17http://www.oecd.org/site/piaac/publicdataandanalysis.htm
18http://piaacgateway.com/datasets/
19http://nces.ed.gov/training/datauser/#PIAAC
There are four steps to progress through when using the PIAAC Data Explorer, supported by the DLDT modules. We outline our journey through the process as follows.

**Step 1**

In the first step we chose a Subject (*Numeracy*) from three options: *Literacy; Numeracy; and Problem Solving in Technology-rich Environments*. Note that *Adults (16-65)* is the only option for *Age* at this point. After the choice of *Numeracy*, a fuller screen appears. Initially there is a choice of which Background Questionnaire dataset to use: *PIAAC 2012, ALL 2003 or All years/studies*; we chose *PIAAC 2012*.

Next we selected a dependent variable from the 72 Categories and Groups available. Then we selected a jurisdiction or group from the 20 national and 17 sub-national entities available, as well as the OECD average. Under the Category *Scale Scores*, and Sub-Category *Skills*, the Measure *PIAAC Numeric: Numeracy* is already selected. Under the Group *International*, we chose the jurisdiction *OECD average*. We clicked on *Select Variables* to move to Step 2.

**Step 2**

At Step 2, we selected independent variables (from a choice of 334) in order to examine the strength of associations between these and the dependent variable chosen above. Under the Category (and Sub-Category) *Major reporting group* we chose the variable *All Adults*. In addition, under the Category *International Background Questionnaire*, and Sub-Category *Skill use – literacy & numeracy*, we chose the following six variables:

- *Skill use work - Numeracy - How often - Calculating costs or budgets*
- *Skill use work - Numeracy - How often - Use or calculate fractions or percentages*
- *Skill use work - Numeracy - How often - Use a calculator*
- *Skill use work - Numeracy - How often - Prepare charts graphs or tables*
- *Skill use work - Numeracy - How often - Use simple algebra or formulas*
- *Skill use work - Numeracy - How often - Use advanced math or statistics*

We then clicked on *Edit Reports* to move to Step 3.

**Step 3**

Step 3 allowed us to choose the types of statistics we wanted to report, collapse any variable response categories, adjust table layouts and refine the formatting of the reports to be generated, using the variables chosen above. To summarize, we trialled the use of the PIAAC International Data Explorer (PIAAC IDE)\(^{21}\) by exploring information about the percentage of adults currently or recently in work who self-report the frequency of their engagement in given numeracy practices in their work contexts, and their average Numeracy scale scores for each category. We focused on the PIAAC jurisdiction *OECD average*, then chose as variables *Skill use work* for each of the six specified numeracy skills in order to generate the average PIAAC Numeracy scale scores and the percentages of adults aged 16-65 in each usage response (frequency) category.

\(^{21}\)http://piaaccdataexplorer.oecd.org
The PIAAC International Data Explorer generated seven draft reports. These seven reports can be reviewed, edited, deleted or copied. New reports can be created, new formats and statistics chosen. We selected Averages and Percentages from the drop-down menu Statistics Options.

We clicked on Build Reports to move to Step 4 to view the completed reports.

### Step 4

At the fourth step PIAAC IDE generated a report for each of the six variables of Numeracy skill use at work. Each report lists Averages of the Numeracy scale scores and Percentages of the population (each with Standard Errors) for the response categories: Never; Less than once a month; Less than once a week but at least once a month; At least once a week but not every day; Every day. Each report table can be viewed by selecting the report name from the drop-down menu. At this stage, we could preview and select information displayed in these tables or charts.

In addition, PIAAC IDE will generate (on request) comparisons between frequencies of skill use with significance tests for each of the six variables (Numeracy skill use at work) for the statistic specified. This allowed us to compare, for example, the means of two frequency groups to see if the difference is statistically significant. For this trial run we requested these significance tests be generated for each numeracy skill for both percentages and averages. Our use of large samples will mean that almost all differences will be statistically significant. (This issue is discussed in the section above entitled “Analysing adult numeracy practices: issues and types of analyses”). Our purpose in this instance was to examine the tables that would be generated by PIAAC IDE.

We then chose Export reports and downloaded our selected reports as Excel worksheets, allowing us to manipulate the data generated. Tables 1, 2 and 3, below, were produced through such a reorganisation of these Excel original reports/tables. At this stage we could have chosen instead to download the reports as HTML or Word documents.

The first six reports, one for each of the six variables of Numeracy skill use at work, are summarised in Table 1 (below).

#### Table 1

**Average Numeracy scale scores and percentage of adults for frequency options for Numeracy skill use at work variables**

<table>
<thead>
<tr>
<th></th>
<th>Never</th>
<th>Less than once a month</th>
<th>Less than once a week but at least once a month</th>
<th>At least once a week but not every day</th>
<th>Every day</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Calculating costs or budgets in a work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>264 (0.3)</td>
<td>49 (0.2)</td>
<td>289 (0.6)</td>
<td>10 (0.1)</td>
<td>290 (0.7)</td>
</tr>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Use, or calculations of, fractions or percentages in work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>257 (0.3)</td>
<td>46 (0.2)</td>
<td>283 (0.6)</td>
<td>9 (0.1)</td>
<td>291 (0.6)</td>
</tr>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Use of a calculator in a work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>254 (0.4)</td>
<td>30 (0.2)</td>
<td>278 (0.7)</td>
<td>8 (0.1)</td>
<td>284 (0.7)</td>
</tr>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Preparation of charts, graphs or tables in a work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>261 (0.3)</td>
<td>60 (0.2)</td>
<td>299 (0.5)</td>
<td>13 (0.1)</td>
<td>298 (0.5)</td>
</tr>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Use of simple algebra or formulas in a work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>260 (0.3)</td>
<td>55 (0.2)</td>
<td>298 (0.6)</td>
<td>9 (0.1)</td>
<td>296 (0.7)</td>
</tr>
<tr>
<td><strong>Average of Numeracy scale scores and Percentage of adults by Frequency of Use of advanced math or statistics in a work context</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012 OECD Average</td>
<td>270 (0.3)</td>
<td>86 (0.2)</td>
<td>303 (0.7)</td>
<td>7 (0.1)</td>
<td>306 (1.0)</td>
</tr>
</tbody>
</table>

NOTE: The Numeracy scale ranges from 0 to 500. Some apparent differences between estimates may not be statistically significant.

Average (Avg); Percentage (Percent); Standard Error (std error).

SOURCE: Organization for Economic Cooperation and Development (OECD), Program for the International Assessment of Adult Competencies (PIAAC), 2012.

Reports used for this table were generated using the PIAAC International Data Explorer. http://piaacdataexplorer.oecd.org

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22 The data used were the OECD Average for PIAAC 2012.
Looking at percentages in Table 1 across the six Numeracy skill use at work variables (rather than within each skill), we found that the percentages of adults responding Never with regard to the category, Numeracy skill use at work - Use a calculator, was the lowest (30%) (of all the six numeracy skills) and the percentages of adults in the response category Every day was the highest (37%) (of all six skills). The use of calculators in the workplace therefore emerges as very common, since less than one third of adults in, or recently in, work indicate that they never use them at work.

The next similar frequency response pattern is Use or calculate fractions or percentages with 46% indicating Never and 23% indicating Every day, although Calculating costs or budgets is similar, with 49% and 21% respectively in these response categories. In other words, almost one half of adults in, or recently in, work reported that they never used these two numeracy skills at work. Focusing on averages across the six Numeracy skill use at work variables (rather than within each skill), we found that the averages of Numeracy scale scores within the frequency option Never are the lowest (of all frequency categories) for all six skills. These percentages and averages for the less complex Numeracy skills may indicate that adults with the lowest numeracy skill are choosing to work in jobs that entail infrequent numeracy activity. Equally it may indicate the converse – employers may recruit staff in a way that closely matches skills to the job, or infrequent numeracy practice may lead to skill loss. If the latter, this would support the findings of research on the British Cohort Studies that showed adults’ skills diminishing with lack of use (Bynner & Parsons, 1998, 2000).

For the more complex numeracy skills, Numeracy skill use at work - Use advanced math or statistics shows percentages of adults in the response category Never as the highest (86%) (of all six skills) and percentages of adults in the response category Every day as the lowest (2%) (of all six skills). The next similar (but less striking) skill response pattern is Prepare charts graphs or tables, with 60% indicating Never and 6% indicating Every day, although Use simple algebra or formulas is similar, with 55% and 17% in these response categories respectively. Averages of numeracy scale scores for Numeracy skill use at work - Use advanced math or statistics are somewhat higher within most response categories. These percentages and averages may suggest that people with strong numeracy skills have jobs that entail frequent numeracy activity.

Table 2
Frequency of use of a calculator in a work context. Significance tests of the differences between either percentages or averages, calculated for each of the five frequency options

<table>
<thead>
<tr>
<th>Difference in percentages between variables</th>
<th>Difference in averages between variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>Every day</td>
</tr>
<tr>
<td>Less than once a month</td>
<td>Less than once a week but at least once a month</td>
</tr>
<tr>
<td>Difference:</td>
<td>Difference:</td>
</tr>
<tr>
<td>P-value:</td>
<td>P-value:</td>
</tr>
<tr>
<td>&lt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>&lt;</td>
<td>&lt;</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
</tr>
</tbody>
</table>

LEGEND:
< Significantly lower.
> Significantly higher.
x No significant difference.

NOTE: Within country comparisons on any given year are dependent with an alpha level of 0.05.

OECD Average, PIAAC 2012
Organization for Economic Cooperation and Development (OECD)
Program for the International Assessment of Adult Competencies (PIAAC)
Generated using the PIAAC International Data Explorer.
Examples of the types of reports generated by PIAAC IDE in the form of tables of differences between percentages and averages, with associated p-values are shown in Tables 2 and 3, below, for two numeracy skills (percentages and averages are combined).

To see how one percentage (or average) compares with those for other frequencies, users should read across the row for that value in Tables 2 and 3. The displayed symbols indicate whether that value is significantly higher, significantly lower, or not significantly different from the value associated with that column. The p-value indicates the probability with which a difference in percentages (or averages) between frequency groups as large as observed here could occur by chance, if there were actually no difference. The customary significance level is 5% (0.05). The p-value must fall under this significance level for the results to be deemed statistically significant. As expected, most differences were statistically significant with very low p-values.

Table 3

Frequency of preparation of charts, graphs or tables in a work context. Significance tests of the differences between either percentages or averages, calculated for each of the five frequency options

<table>
<thead>
<tr>
<th>Difference in percentages between variables</th>
<th>Difference in averages between variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>Less than once a month</td>
</tr>
<tr>
<td>Never</td>
<td>Diff = 48</td>
</tr>
<tr>
<td>Less than once a week but at least once a month</td>
<td>Diff = -48</td>
</tr>
<tr>
<td>At least once a week but not every day</td>
<td>Diff = -50</td>
</tr>
<tr>
<td>Every day</td>
<td>Diff = -54</td>
</tr>
</tbody>
</table>

LEGEND:

< Significantly lower.

> Significantly higher.

x No significant difference.

NOTE: Within country comparisons on any given year are dependent with an alpha level of 0.05.

OECD Average, PIAAC 2012
Organization for Economic Cooperation and Development (OECD)
Program for the International Assessment of Adult Competencies (PIAAC)

Generated using the PIAAC International Data Explorer.

Conclusion

Overall, we found our exploration of the publicly-available PIAAC dataset stimulating and challenging in equal measure. While the scale of the dataset may appear daunting to novice researchers and practitioners, we would encourage readers to undertake their own exploration, using the range of support and tools for analysis available online. Our focus here has been on adults’ numeracy practices: just one of the many areas of interest on which data are available in this rich resource. Ultimately, the choice of focus lies with the reader. Our advice is: start with a simple question – something that intrigues you - and take it from there.

See below for an explanation of p-value.
References


Changing images of mathematics in contrasting learning situations for students

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The mathematical performance of adults with similar processes in different situations can vary, suggesting that their views of the mathematics are affected by the context in which the activity is taking place. Results from a multi-method study of post-16 students on vocational training courses in England, involving seventeen case studies of student groups, show how students’ views of mathematics, engagement and performance were sometimes transformed in the transition from school to vocational training. Using the concept of an image of mathematics, students’ perceptions are explained in this study by a dynamic rather than a static model. Personal images of mathematics were grounded in prior experiences but still significantly affected by changes in the learning situation such as curriculum, pedagogy and classroom culture. These changing images of mathematics provided opportunities for students to reposition themselves in relation to mathematics and stimulated a re-engagement with learning. When new positive images were formed they became strong influences, leading to the development of mathematical skills and knowledge during their vocational training that students had failed to achieve in a school classroom.

Changes in learning situations have often been associated with concerns about continuity (Nicholls & Gardner, 1999) or the transferability of cognitive knowledge (Perkins & Salomon, 1992; Singley & Anderson, 1989) rather than the positive opportunities afforded by new experiences. In this paper students’ perceptions of mathematics are examined in the transition from school to vocational education, leading to a consideration of the impact of new learning experiences on those with a history of failure and disaffection. Based on research findings from a study of students (aged 16-19 years old) in vocational education, the paper takes the form of a report of work in progress. Firstly, a brief explanation of the context for the study and the research background are provided, in which some key concepts are highlighted of relevance to the particular research questions addressed in the paper. These are followed by a presentation and discussion of the research findings, with some interim conclusions indicating areas for further development.

Context

The English education system is characterised by a historically grounded division of academic and vocational pathways. Students take the General Certificate in Secondary Education (GCSE) examinations in a range of subjects, including mathematics, at the
end of lower secondary education (age 16 years) and these largely determine whether they subsequently follow an academic route towards Higher Education or a vocational pathway. Although there is some employer-based training in the form of apprenticeships, the majority of post-16 vocational education is in the form of college-based courses delivered by Further Education (FE) colleges, whilst schools are the main providers of academic courses. Students who follow a vocational pathway would therefore normally be subject to changes in both curriculum and educational institution as they move into post-16 education.

Recent changes in policy require students who fail to achieve a grade C or above in GCSE Mathematics to re-sit the examination or work towards this by taking an alternative qualification such as Functional Mathematics. Functional Mathematics focuses on application and problem solving in ‘real life’ scenarios, both familiar and non-familiar, whilst GCSE Mathematics is founded on an academic knowledge-based curriculum. The effects of this prioritisation of GCSE Mathematics over Functional Mathematics means that many vocational students in colleges experience a repetition of the same mathematics curriculum they had taken and ‘failed’ in school. For students whose future use of mathematics in the workplace may demand a different set of mathematical knowledge and skills from those expected for higher academic study, this policy causes some concern.

The students in this study were on vocational programmes in FE colleges and were studying for a Functional Mathematics qualification but had previously followed a GCSE Mathematics curriculum in school. Students’ own comparisons of their learning experiences in school and in college with these two different curricula are central to the paper and provide the main data source from which their perceptions of mathematics in these contrasting situations are explored. Within these experiences and perceptions of mathematics, however, there are a number of strands that are relevant to the following discussion and these are briefly introduced in the following section.

**Background**

There have long been concerns in England about the mathematics skills of young adults and the adequacy of their preparation for the workplace that are continually revisited by the government (BIS, 2010; DfES, 2005) and employers (Confederation of British Industry, 2015). One problem that has received much attention is the apparent change in mathematical performance from classroom to workplace. Various studies have shown, for example, how children who were competent with certain mathematical calculations when acting as street sellers failed to perform similar calculations accurately in a more formal situation (Carraher, Carraher, & Schliemann, 1985) and how adults have difficulty connecting numeracy learned in a classroom with similar processes when used in a supermarket (Lave, 1988). These inconsistencies are often attributed to difficulties with the transferability of knowledge from one situation to another and suggest that the context in which learning takes place is a significant factor affecting the way students ‘see’ and understand mathematics. This presents challenges in vocational education for subjects such as mathematics where classroom-based teaching is often used to develop knowledge that students will need to apply in workplace situations. The different views of mathematics that students develop in these situations are therefore fundamental to understanding their difficulties.
There is an underlying assumption that knowledge gained in one situation will be of benefit in another and that a change between situations should be fluent. In the transition from primary to secondary education, continuity is often seen as advantageous (Nicholls & Gardner, 1999) and when mathematics is used in different situations then the presence of ‘boundary objects’ between the separate activity systems is viewed as beneficial since it connects the two experiences (FitzSimons, 2013; Kent, Noss, Guile, Hoyles, & Bakker, 2007; Williams, Wake, & Boreham, 2001). In this study, however, with students who have not developed sound mathematical knowledge in school, their misconceptions or lack of understanding may require intervention and change in college rather than continuity.

Perceptions of mathematics have previously been considered in terms of a general public view or an individual position. Within society, mathematics is often seen as a remote and elitist subject but a powerful gatekeeper (Volmink, 1994). Although mathematical knowledge is valued, it is seen as only accessible to a minority so the subject becomes an instrument of social division. In the context of this public view Volmink (1994) suggests that a supportive learning environment, in which students feel free to explore mathematical problems, is one of the factors that will have a positive effect on the perceptions and behaviour of individuals.

The perceptions of individuals regarding mathematics have been examined in several studies, through a variety of methods using, for example, metaphors or visual images to illustrate and explain how children or adults view the subject. These methods suggest that imagery, in visual or verbal forms, may be useful in the analysis and understanding of different perceptions of mathematics. In a study of the dominant perceptions of mathematics amongst the members of the public, Lim and Ernest (2000) use the concept of an image and identify a number of key elements, with suggestions that these mental images may provide some important indicators of behaviour. The elements listed include a range of beliefs and emotions, both about the nature of mathematics and the learning of the subject (Lim & Ernest, 2000), thereby making a link to affective aspects of these perceptions that may well influence engagement with mathematical activity.

In general, an image is defined as a mental representation, idea or conception. Studies of students learning mathematics refer to the use of images as a means of representing both process and concept in visual forms (Tall, 1994). These images are rich sources of information and can communicate a more holistic view of a process or set of concepts than a sequential approach to learning. A contrast is drawn between static and dynamic forms of imagery (Presmeg, 1997) with the dynamic having the additional power of being able to demonstrate the development of mathematical processes or conceptual ideas. Presmeg (1997) also refers to the creation of mental images from experience as a distinct form of visual imagery based on memory. Although the construction and use of such images has been examined in the context of cognitive development, it seems that the power to represent students’ broader perceptions of mathematics in a holistic and dynamic manner has rarely been explored. In particular, Presmeg’s (1997) dynamic and memory forms of imagery appear well suited to represent perceptions based on personal experiences and responsive to changing situations.
Thompson (1996) suggests that images are not just mental pictures or representations but can include an affective element. The incorporation of affect means that fundamental constructs such as beliefs, attitudes and emotions (McLeod, 1992) become important to consider as part more personal, holistic images of mathematics, although other elements such as values (DeBellis & Goldin, 2006), motivation (Zan, Brown, Evans, & Hannula, 2006) and self efficacy (Pajares & Graham, 1999) might also be included.

The influence of certain elements of affect on learning, such as anxiety, has been recognized and evidenced, particularly in problem-solving activity (DeBellis & Goldin, 2006) but the nature of the interaction between affective and cognitive functions and how this influences the learning process is less clear. The interaction may be seen as an intertwining of beliefs and emotions in an ‘engagement system’ that influences behaviour (Goldin, Epstein, Schorr, & Warner, 2011). Alternatively, Di Martino and Zan (2011) suggest a three-dimensional framework of emotional disposition, vision of mathematics and perceived competence that affects the attitudes of students towards learning mathematics. These models that incorporate different beliefs, attitudes, emotions and behaviour into one framework are varied and the diversity indicates the complexity of attempting to combine multiple aspects. As models designed to serve different purposes and derived from research with different aims, these provide some suggestions of affective elements of student perceptions and how they might be combined, without necessarily fully representing the research findings from this particular study.

Research design

The primary aim of the research project was to identify and examine the factors that influenced the experience of vocational students learning mathematics, including college policies, structures, organisational culture, student background, prior experience and the teaching approaches used. Within this paper, however, we focus on the perceptions of mathematics developed by students in school and in college and whether these have any effect on their learning of the subject during their preparation for the workplace. We will consider just two questions:

1. What changes in students’ perceptions of mathematics take place in the transition from school to vocational education?
2. What effects do these changes have on students’ learning of mathematics in college?

Within this paper, only the results of relevance to these particular research questions will be presented but a more detailed account of the study, the results and analysis can be found elsewhere (See Dalby, 2014).

The research took the form of a multi-method study of students on vocational programmes in England within the areas of Hair and Beauty, Construction and Public Services (i.e. preparation for entry to the Emergency Services or Armed Services) who were learning mathematics as an additional subject. Seventeen case studies of student groups and their teachers were developed using data from focus groups, individual student card-sorting activities, lesson observations (of mathematics and vocational sessions), plus teacher questionnaires and interviews, (with both mathematics and vocational teachers). The multi-method design allowed triangulation of both sources...
and methods to be incorporated into a study that was both exploratory and explanatory, with concurrent data collection and analysis. Quantitative data from the individual student activities and questionnaires was summarized and analysed using statistical methods, whilst coding and constant comparison methods were used with qualitative data to identify key themes. These themes were then further explored through within-case and cross-case comparisons.

Results

In the individual activities students placed sets of statements (derived from preliminary work with students in other colleges) on a Likert scale to show their responses to mathematics in school and college. The results produced ordinal data from which differences in student ratings for the same statement, in school and college, were calculated. These differences were then summarised into responses showing negative change, positive change or no change and then tested for significance using the sign test. The numbers of students in each category is shown in Table 1 with the level of significance.

Table 1. A summary of differences between students’ experiences of mathematics in school and college.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Negative change</th>
<th>No change</th>
<th>Positive change</th>
<th>Z value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I worked hard</td>
<td>25</td>
<td>36</td>
<td>41</td>
<td>-1.85</td>
<td>Not significant</td>
</tr>
<tr>
<td>It was difficult</td>
<td>55</td>
<td>28</td>
<td>20</td>
<td>-3.93</td>
<td>1%</td>
</tr>
<tr>
<td>I got distracted</td>
<td>54</td>
<td>29</td>
<td>20</td>
<td>-3.84</td>
<td>1%</td>
</tr>
<tr>
<td>I liked maths</td>
<td>23</td>
<td>39</td>
<td>41</td>
<td>-2.13</td>
<td>5%</td>
</tr>
<tr>
<td>I felt stressed</td>
<td>55</td>
<td>33</td>
<td>15</td>
<td>-4.66</td>
<td>1%</td>
</tr>
<tr>
<td>I was bored</td>
<td>50</td>
<td>26</td>
<td>27</td>
<td>-2.51</td>
<td>5%</td>
</tr>
<tr>
<td>I liked the teacher</td>
<td>17</td>
<td>22</td>
<td>64</td>
<td>-5.11</td>
<td>1%</td>
</tr>
<tr>
<td>I felt confident</td>
<td>13</td>
<td>47</td>
<td>43</td>
<td>-3.88</td>
<td>1%</td>
</tr>
<tr>
<td>It was interesting</td>
<td>17</td>
<td>33</td>
<td>53</td>
<td>-4.18</td>
<td>1%</td>
</tr>
<tr>
<td>I understood it</td>
<td>14</td>
<td>37</td>
<td>52</td>
<td>-4.55</td>
<td>1%</td>
</tr>
<tr>
<td>It was confusing</td>
<td>42</td>
<td>41</td>
<td>20</td>
<td>-2.67</td>
<td>1%</td>
</tr>
<tr>
<td>I could have done better</td>
<td>51</td>
<td>28</td>
<td>24</td>
<td>-3.00</td>
<td>1%</td>
</tr>
</tbody>
</table>

The results indicate that students found functional mathematics in college easier than mathematics in school and more interesting. They were less distracted in college, less stressed, liked their maths teachers more, were more confident, less confused and understood the mathematics better. Although less significant, there was some evidence that students liked mathematics more in college and were less bored.

These attitude changes were then explored further in the focus groups where students described their learning experiences of mathematics in more detail. For the majority of
the students in the study, their prior attainment was below the GCSE Mathematics grade C that is widely accepted in England as a satisfactory minimum standard. Many students expressed their dislike of mathematics in school, linking this to descriptions of negative learning experiences and outcomes. Disaffection, disengagement and failure were common themes and emotional responses were often strong, with anxiety, frustration or low self-efficacy sometimes leading to deliberate avoidance behaviours.

Many students believed that mathematics was a difficult subject and remote, in the sense that understanding the curriculum content and achieving success in examinations seemed personally inaccessible. This was often linked to perceptions of being alienated in school mathematics classrooms, finding the subject challenging but having limited personal agency and believing they lacked some basic aptitude for the subject compared to their peers. There was also a belief amongst students that much of the mathematics curriculum in school was irrelevant to their lives and learning the subject served no useful purpose, except to pass an examination that was viewed as important by their school and society.

Both qualitative and quantitative data, from the focus group discussions and the card-sorting activity, provided some clear evidence of more positive attitudes in college, particularly in certain case studies. This suggested a categorization of the case study groups into those with positive, negative or mixed attitudes. A comparative analysis of contrasting cases was then carried out, leading to the identification of key contributory features. In the negative groups students’ attitudes, beliefs and emotional responses to learning mathematics remained similar to those in school, whilst in positive groups students began to find the subject more relevant and accessible. The primary elements that emerged from a preliminary analysis are shown in Table 2.

**Table 2. A summary of key elements in students’ perceptions of mathematics.**

<table>
<thead>
<tr>
<th>Positive case study groups</th>
<th>Negative case study groups</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Subject</strong></td>
<td></td>
</tr>
<tr>
<td>Mathematics forms an essential component of everyday life. There are applications within many vocational and personal situations.</td>
<td>Mathematics is an irrelevant academic subject. There are few connections to everyday life or vocational employment.</td>
</tr>
<tr>
<td><strong>Learning process</strong></td>
<td></td>
</tr>
<tr>
<td>Learning mathematics is about developing useful skills.</td>
<td>Learning mathematics is about acquiring knowledge.</td>
</tr>
<tr>
<td><strong>Purpose</strong></td>
<td></td>
</tr>
<tr>
<td>Learning mathematics is essential to achieve vocational competence.</td>
<td>Learning mathematics is about passing an examination and has no vocational purpose.</td>
</tr>
<tr>
<td><strong>Attitudes to learning mathematics</strong></td>
<td>Attitudes are positive, e.g. enjoyment, interest.</td>
</tr>
<tr>
<td></td>
<td>Attitudes are negative, e.g. hate, disinterest.</td>
</tr>
</tbody>
</table>

These key elements suggest four main areas for consideration within students’ perceptions of mathematics:
• Beliefs about the nature of mathematics;
• Beliefs about the process of learning of mathematics;
• Beliefs about the purpose of learning mathematics (in relation to current personal values and goals);
• Emotional responses and attitudes to learning mathematics.

There was also evidence from the lesson observations and focus groups that these more positive attitudes led to increased engagement and better understanding. Better understanding also encouraged students to sustain their engagement with learning, leading to further confidence gains. In contrast, students with on-going negative attitudes to mathematics often returned to passive or active avoidance strategies, continuing an established pattern of disengagement and making little progress.

Although there were some significant factors associated with college policies and structures (that are not reported here), an analysis of the lesson observations and teacher interviews suggested that the teaching approaches, the enacted curriculum and classroom culture were strong influences that enabled students in the positive groups to develop a different view of mathematics. These views also affected their personal positioning in relation to mathematics and, rather than finding the subject remote and inaccessible, students more readily engaged with learning in the classroom and developed a better understanding.

In summary, the findings show how students’ attitudes to learning mathematics, emotional responses and beliefs about the subject are closely inter-linked and influence their behaviour in a learning situation. These attitudes, beliefs and emotions are, however, not static and contrasting learning experiences can lead to some significant changes. In this study students approached mathematics in college with a legacy of past experiences that influenced their initial attitudes and beliefs. These prior experiences had led to a set of well-established personal beliefs about the subject, attitudes to learning and emotional responses but these were not fixed and exposure to a different curriculum, learning environment and teaching approach had a considerable impact.

**Discussion**

Within this study the social context for learning was clearly an influential factor. Rather the changing social situation raising concerns about a lack of continuity (Nicholls & Gardner, 1999) or transferability of knowledge (Perkins & Salomon, 1992; Singley & Anderson, 1989), the transition to college provided an opportunity for positive change in students who were disaffected and unsuccessful with mathematics in school. For students to benefit, however, teachers needed to provide contrasting learning situations in college classrooms to those students had experienced in school. The key conditions for positive change seemed to be discontinuity and difference. This is the antithesis of some of the arguments for school to school (primary to secondary) or school to workplace transitions where the construction of continuity or similarity is considered beneficial, whether through the curriculum, or by connections such as the use of
boundary objects’ to link the situations (FitzSimons, 2013; Kent et al., 2007; Williams et al., 2001). These transitions are, perhaps, viewed from a position where cognitive development is assumed to be the primary driver in learning and takes precedence over affect. For the low-attaining and disaffected students in this study, affective aspects of their experiences were shown to have a strong influence within the learning process and addressing these had a significant impact on their progress with mathematics.

Some of the key elements of affect evidenced in the study show a close alignment to other research findings. There was evidence of changes in attitudes, emotions and beliefs (McLeod, 1992) and also of changing values (DeBellis & Goldin, 2006). As highlighted by Zan et al (2006), anxiety was a common response and did have some effect on students’ behaviour in learning situations. A key finding, however, concerned how these attitudes, beliefs and emotional responses to learning mathematics changed over time. Rather than beliefs and values remaining fairly stable whilst attitudes and emotions were more transient (Evans & Wedege, 2004; McLeod, 1992), there was evidence of concurrent changes in beliefs, attitudes and emotions over a short period of time in the positive groups, suggesting the presence of fast-changing aspects of each element (Goldin, 2003; Hannula & Laakso, 2011).

Different elements of affect were linked together in students’ descriptions of their perceptions and responses to mathematics but it is difficult to define a clear framework to show these connections. Emotions were clearly linked to experiences of learning mathematics (Hannula, 2002) but attitudes and beliefs were also grounded in previous encounters with mathematics as a learner. Attitude was linked to engagement (Goldin et al., 2011) and this clearly had an effect on student progress with learning mathematics but emotions and beliefs were also entwined into patterns of student behaviour within lessons. Students’ perceptions of mathematics in the study, therefore, could be considered as based on a network of attitudes, beliefs and emotional responses to learning mathematics that is linked to cognitive development but representing the complex connections is not easy.

Comparisons to other networks are useful here but show some inadequacy in representing the findings from this study. For example, in the ‘engagement structure’ proposed by Goldin et al (2011) students’ beliefs are intertwined within cognitive and affective structures and explain “in-the-moment mathematical behaviour” (Goldin et al., 2011, p.558) but not the longer term influence of affect on behaviour. In Hannula’s (2002) framework for analysing attitude there is a distinction between emotions stimulated whilst learning mathematics, emotions associated with the subject of mathematics, expected consequences of the mathematical activity and the relation to personal values. This provides a useful separation of affective elements related to attitude but does not capture the way in which beliefs about mathematics were woven into students’ responses. Similarly, Di Martino and Zan (2011) suggest a framework of emotional disposition, vision of mathematics and perceived competence that includes some key elements but does not seem to fully encompass the emerging picture from this study.

In search of a construct that describes a more holistic view of the interaction of students’ beliefs, attitudes and emotions over time the notion of an image of mathematics seems attractive. As explained by Tall (1994) visual images can bring together different
elements to provide an overview of concept and process, with dynamic representations being particularly useful in the understanding of changing situations. The construct of a personal mental image of mathematics, therefore, has some potential in explaining students’ affective responses and the changes that take place. Within this study there was evidence of some similar key elements to those identified by Lim and Ernest (2000) within public images of mathematics, such as attitudes towards learning mathematics, emotions associated with learning mathematics, beliefs about the nature of mathematics, beliefs about the purpose of learning mathematics and the value of mathematics to the individual. Noticeably, though, students frequently linked their opinions to personal learning experiences and the emotions associated with these encounters rather than making direct statements about mathematics.

The personal emphasis within students’ perceptions, grounded in their experiences, was dominant. Although there was some reference to the importance of mathematics within society many students seemed unconvinced and more concerned about how mathematics related to their immediate personal values and goals. The effects of encounters with a contrasting learning situation in college on students’ images of mathematics were therefore significant and the findings support Volmink’s (1994) suggestion that the learning situation has the potential to change the views of the individual even within a society where there is an established public image.

Students’ images of mathematics in this study were largely built on memories of their previous experiences of learning mathematics. Through the superimposition of memories these images were continually changing, as new experiences either added strength to existing elements or served to obscure them. These images were dynamic and subject to change as students were exposed to new and different learning experiences. Contrasting experiences in college compared to school superimposed new elements on to students’ existing images that quickly seemed to obscure those from the past, resulting in positive changes over a short period of time. In the transition from school to workplace there appear to be significant benefits, therefore, in providing learning experiences for students with mathematics during vocational training that contrast with those in school.

Conclusions

In this study, the construct of a personal image of mathematics provides a useful way to represent the set of beliefs, attitudes and emotions that influenced student behaviour in different learning situations. Although images are founded on memories of prior experiences, often built up over some time, exposure to more recent contrasting learning situations can significantly shape students’ images of mathematics. This study shows the potential of contrasting experiences of learning mathematics during vocational training in college to obscure or replace negative school-based images of mathematics.

Mathematics teachers in FE colleges are in a powerful position to facilitate the development of more positive attitudes to mathematics and change negative images in students with prior experiences of failure and disaffection. The transformation of images is, however, dependent on the provision of new and contrasting learning situations through the use of appropriate pedagogy, curriculum and classroom culture. The gains
evidenced in this study, of better engagement and understanding, are personally valuable to students but also help them develop more positive dispositions towards using mathematics in the workplace. Changing students’ images of mathematics is therefore an important aspect of teaching mathematics during vocational training with a beneficial impact for both the individual and the economy.

The main intention of this paper is to raise some emerging issues for discussion from the study regarding the nature of students’ perceptions of mathematics and the interaction with their learning. The notion of a personal mental image of mathematics is proposed here as a construct with scope for further development but with the potential to describe and explain students’ changing perceptions and responses to mathematics. In the transition from school to workplace, understanding these changing images of mathematics may well provide a better understanding of how learning experiences in college can be designed to enable the development of more positive images than those associated with students’ negative encounters in the past.

References


A Ten Year Examination of the Mathematical Performance and Learning Styles of Mature Students and Traditional Students in Ireland

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Abstract: Students enrolled in particular service mathematics programmes in the University of Limerick (UL) in Ireland are given a diagnostic test. The diagnostic test is administered in the first mathematics lecture of the term and students are not forewarned about it. Preliminary analysis of the diagnostic test data revealed that many students were struggling with basic mathematical skills but in particular with algebra and arithmetic (Gill 2006). Further analysis highlighted the significant decline in diagnostic test performance and the changing profile of service mathematics students between 1998 and 2008 (Faulkner et al 2010). The profile of students in 2008 was starkly different from that in 1998 with one of the most outstanding profile changes being the increase in mature students entering service mathematics programmes. A comparison of the performance of traditional (students entering UL directly from second level education in Ireland) and mature students in the diagnostic test and in service mathematics was carried out. The findings of such comparisons and some of the possible reasons for such findings are outlined in this paper.

1. Introduction

Diagnostic testing has become a commonly used tool worldwide to help identify mathematical weaknesses which may be present when students enter a higher education institution (Tall and Razali, 1993; Edwards, 1995; Edwards, 1996; Hunt and Lawson, 1996; Lawson, 1997; Todd 2001; Engineering council, 2002; Malcolm and McCoy; 2007; Faulkner et al 2010; Gill et al 2010; Treacy and Faulkner 2015). It is used in the University of Limerick (UL) in Ireland since 1997 in order to identify such weaknesses in students basic mathematical competency levels on entry to third level education. Since the introduction of diagnostic testing in UL students’ performances have declined significantly over time (Gill, 2006; Faulkner et al 2010; Gill et al 2012; Treacy and Faulkner 2015). Research has also been carried out which compared students’ mathematical performance on entry to UL, i.e. diagnostic test performance, with end of term service mathematics examination performance. This research details how students’ performances in these 2 examinations relate to each other, outlines how performances have changes over time and focuses on how mature students get on in each examination in particular (Faulkner et al 2010).
2. Context for this Study

2.1 Gaining Access to Third Level Education in Ireland

Students in Ireland complete 5 years of second level education which can be broken up into the Junior Certificate program, which takes three years to complete, and the Leaving Certificate program, which takes two years to complete (Gill et al 2012; Faulkner et al 2013). Students entering second level education at age 12-14 approximately start by beginning the Junior Certificate programme which acts as a precursor to the Leaving Certificate programme which they tend to complete at the age of 17-19 approximately. Each certificate can be successfully completed by passing summative examinations in all subjects at the end of the programmes. Mathematics in second level education in Ireland is offered at three levels; Higher Level, Ordinary Level and Foundation Level.

Students’ performances in the Leaving Certificate programme can grant them direct entry to third level education in Ireland. Students’ performances in their final examination of 6 subjects are used to determine what higher education programmes they are eligible for. The system works by assigning a specific number of points to each grade received in Leaving Certificate state examinations. For example a student who got a Higher Level A1 grade is awarded 100 points whereas an Ordinary Level A1 grade is worth 60 points. Each third level programme has a minimum number of accumulated points which students must have to gain direct entry through the Central Applications Office (CAO) system. For example a student who wishes to gain direct entry through the CAO system to a third level programme requiring 560 points could do this by getting 5 Higher Level A1 grades and 1 Ordinary Level A1 grade (Faulkner et al 2013).

2.2 Background Information: The University of Limerick

The University of Limerick is a higher education institution based in the south of Ireland. It is the largest provider of second level teacher education in the country with a primary focus on Science and Technology subjects (Sahlberg 2012). The institution is currently organised into faculties, the largest of which is the Science and Engineering faculty consisting of 10 different departments. One of these departments is the Department of Mathematics and Statistics. One of the roles of the Department of Mathematics and Statistics is to support its client departments (Science, Technology, Business, Computers and Engineering) by delivering service mathematics modules to them. Service mathematics modules are generally delivered to degree programmes where mathematics is needed but is not the main focus of the degree.

Two of the service mathematics groups in UL form the focus of the research detailed in this paper: Science and Technology students. The number of students enrolling in programmes in which Science and Technology mathematics are required increased significantly over the course of this research (i.e. between 1998 and 2008). In 1998 there were 202 and 305 students enrolled in Science and Technology mathematics respectively which rose to 303 and 374 in 2008 (Faulkner et al 2013). The increased number of students within these service mathematics cohorts is due in part to the increased number of mature students entering UL; which went from 0.3% to 6.7% in Technology mathematics and from 1.5% to 4.0% in Science mathematics (See table 1) (Faulkner 2012).
2.3 Background Information: Diagnostic Testing in the University of Limerick

The catalyst for the initial implementation of diagnostic testing in UL in 1997 was mathematics lecturers’ concerns regarding students’ basics mathematical competency levels on entry to UL. The test which consists of 40 questions was deemed a suitable tool to identify if students had mathematical weaknesses on entry and to direct them to get mathematics support if this was found to be the case (Faulkner et al 2010; Gill et al 2012; Faulkner et al 2013; Treacy and Faulkner 2015).

During the life-span of this research (1998-2008)¹ almost 6,000 students took the diagnostic test in UL. This number increased to almost 10,000 by 2014 (Treacy and Faulkner 2015). Students are asked to complete the diagnostic test in their first Science or Technology mathematics lecture in first year without prior warning. No changes have been made to the diagnostic test, in content or structure, since it was first introduced in 1997 to the close of this research. This allows for direct comparisons to be made across year groups, student type etc. Students who get 19 out of 40 or below in the diagnostic test are deemed to be ‘at risk’ of failing service mathematics in UL². An ‘at risk’ student is advised to use the free mathematics support provided by the university as they are considered not to have the minimum basic mathematical skills necessary to pass their service mathematics modules (O'Donoghue 1999; Faulkner et al 2013).

2.4. Background Information: The Diagnostic Test Database

All of the diagnostic test data gathered on Science and Technology students since 1997 has been stored in a large database. The database between 1998 and 2008 contains information on almost 6,000 students. A sub-group of this database has been analysed for the purpose of the research outlined in this paper. Over 60 items of information are detailed on each student for example: how they performed on each individual question in the diagnostic test, each student’s performance in each topic on the test (e.g. Algebra, Arithmetic), their mathematical performance prior to entering UL, their gender, the degree programme within which they are enrolled etc. Information regarding whether a student is considered traditional or mature is also outlined; “A mature student is “anyone over the age of 23 who has gained access to UL on the basis of an interview” (Faulkner et al 2013; p.652).

The next section outlines students’ performances in the diagnostic test over time to give further context for the comparisons between mature and traditional students in terms of mathematical performance carried out in this research paper.

¹Data on students gathered in 1997 has not been used as diagnostic testing was only carried out on Technology students however from 1998 both Technology and Science mathematics students were examined.

²This figure was refined after the analysis in this paper was carried out to someone being considered as ‘at risk’ if they receive 18/40 or below in the diagnostic test (Faulkner 2012).

2.4.1 Performance in the Diagnostic Test over Time: Some Background

Previous research carried out on the UL database revealed the significant change in the profile of students entering Science and Technology mathematics between 1998 and 2008 (Faulkner et al 2010). There was a significant change in the percentage of mature students. There was an increase of 6.4 and 5.0 percentage points of mature students in Technology and Science mathematics respectively. There was also a decrease in the percentage of students entering
these programmes with Higher Level second level mathematics of 7.8 percentage points and 17.4 percentage points in Science and Technology respectively (see table 1) (Faulkner et al 2010). More recently published research demonstrates that this declining trend has continued from 2008 - 2014 (Treacy and Faulkner 2015). Faulkner et al (2010) found that the aforementioned changes to the student profile within these service mathematics groups was matched with an increase in the percentage of students considered to be ‘at risk’ of failing service mathematics as measured by the diagnostic test data. In Technology mathematics the percentage of students considered to be ‘at risk’ increased from 32.8% in 1998 to 46.4% in 2008 and it went from 21.3% to 46.0% for Science mathematics students.

Faulkner et al (2010) also concluded that the changing student profile within this service mathematics group was a major contributing factor to the large increases in the proportions of students considered to be ‘at risk’ of failing service mathematics in UL. A shift in student intake like this has been shown to affect overall mathematical performance in other institutions also such as Coventry University in the United Kingdom (Lawson 2003; Faulkner et al 2010). The changing profile of this cohort of students and its contribution to the declining diagnostic test performance was the initial catalyst for the research detailed in this paper. An extensive examination of the mathematical performance of traditional and mature students is detailed in section 5. The next sections outline the research question and hypothesis and the statistical methodology employed in this research.

Table 1 Breakdown of Student Demographics in Technology and Science mathematics (1998 & 2008).

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>2008</th>
<th>1998</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Technology mathematics</td>
<td>Science mathematics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traditional students</td>
<td>304 (99.7%)</td>
<td>339 (90.6%)</td>
<td>199 (98.5%)</td>
<td>282 (93.1%)</td>
</tr>
<tr>
<td>Mature Students</td>
<td>1 (0.3%)</td>
<td>25 (6.7%)</td>
<td>3 (1.5%)</td>
<td>15 (5.0%)</td>
</tr>
<tr>
<td>Higher Level</td>
<td>125 (41.0%)</td>
<td>124 (33.2%)</td>
<td>112 (55.4%)</td>
<td>115 (38.0%)</td>
</tr>
<tr>
<td>Ordinary Level</td>
<td>179 (58.7%)</td>
<td>215 (57.5%)</td>
<td>87 (43.1%)</td>
<td>167 (55.1%)</td>
</tr>
</tbody>
</table>

Note –10 students in Technology mathematics and 6 students in Science Mathematics are not included in this research as they are international students or have completed a previous degree, diploma or certificate in another institution prior to entering UL and so do not fit under any of the sub-headings in table 1.

3. Research Question and Hypothesis

Research Question:

What is the difference between the performance of mature and traditional (i.e. those coming directly from second level education) students in an initial mathematics diagnostic test and a service mathematics examination?

Research Hypotheses:
It is hypothesized that traditional students will outperform mature students in the diagnostic test and the service mathematics examination and mature students will improve in their mathematics test performance over time.

4. Statistical Methodology

All of the data analysis carried out with this research was done using the statistical software package SPSS for Windows. The database used consisted of data on 5,949 students in total.

Descriptive statistics such as means, percentages, standard deviations and co-efficient of variation were used to describe trends by year within the data. To examine if statistically significant differences existed between the means of two groups Independent samples t-tests were used. The equality of variances assumption was satisfied for all independent samples t-tests carried out. Statistically significant associations between the qualitative variables were determined using chi-squared tests and significant differences between the three or more means were determined using Analysis of variance (ANOVA). A 5% level of significance was used for all tests and no adjustment was made for multiple testing. Assumptions for all tests used were satisfied before the tests were carried out. The spread/ dispersion of results were summarised using the standard deviation and coefficient of variation (standard deviation/mean x 100). A significant difference between the variances of 2 groups was examined using Levene’s test for equality of variances (Faulkner 2012; Faulkner et al 2013).

5. Mature Students in UL and a Comparison of their Mathematical Performance with Traditional Students

One of the most significant changes to the student profile within the lifespan of this research was the increase in mature students entering the services mathematics courses in question. Section 5 provides some background on mature students and their academic experience prior to entering UL and outlines a comparison of the performance of mature students and traditional students in the two mathematics examinations in question i.e. the diagnostic test and service mathematics examinations.

5.1 Mature Students in UL in 2008

As previously mentioned mature students, in an Irish context, are students who are 23 years of age or older. All of these students have no previous qualifications in UL and have not completed the Leaving Certificate in the last 5 years. Some mature students gain access to UL degree programmes through a 1 year intensive access course which consists of a variety of subjects depending on the students’ choice of degree programme. Upon passing access course examinations and/or on the basis of an interview, mature students can gain places in degree programmes. The next section outlines the comparison of performance between traditional and mature students in the two examinations in question.

5.2 Comparison of the Mathematical Performance of Mature and Traditional Students in 2008

The majority of mature students have not studied mathematics for a number of years. This is evident in their diagnostic test results which can be seen in figures 1. Mature students have mean diagnostic test scores (expressed as a percentage of correct answers out of 40 questions) below that of the traditional students coming directly from Leaving Certificate. For both Science and Technological mature students the vast majority, with the exception of a few outliers, are classified as being ‘at risk’ of failing their end of semester examinations.
Initially 29 (85.3%) of the mature were considered to be ‘at risk’ of failing service mathematics due to their diagnostic test performance. In spite of this 25 (71.4%) of the mature students were successful in their service mathematics examination (see table 2 and figure 2). Upon carrying out an analysis of the standard students, it emerged that 194 (62.9%) of the Ordinary Level students were considered to be ‘at risk’ of failing service mathematics (see figure 1). In general, the Ordinary Level students’ performed poorly over the course of the semester with 142 (40.2%) of them failing service mathematics (see figure 2). The largest proportion of students who failed service mathematics was contained in the Ordinary Level students’ category. Their mean service mathematics result is also considerably lower than any other sub-category of student and was found to be statistically significantly different (p < 0.001) (see table 2). 19 (10.2%) of the Higher Level students were considered to be ‘at risk’ of failing service mathematics (see figure 1). Their mean performance in the diagnostic test is 65.7% and they maintain this high performance as they have a mean performance of 70.0% in service mathematics. The Higher Level students have a failure rate of 17 (7.4%) confirming that they are definitely not the reason for the traditional students having a lower mean performance than the mature students in service mathematics (see table 2 and figure 2).


The mature students are clearly less prepared mathematically than the traditional students on entry to UL however improve on average over the course of one semester. The fact that the mature students improve so much compared to the traditional students over time is unexpected as generally a students’ diagnostic test performance is predictive of their service mathematics performance (Faulkner et al 2013). Why then do mature students perform so poorly on entry to UL and improve so much over the course of the semester? These findings warrant further discussion which is outlined in section 6.

Table 2 Performance of different sub-categories of standard and non-standard students in the diagnostic test and end of semester examination for Science and Technology students in 2008.

<table>
<thead>
<tr>
<th></th>
<th>Mean Diagnostic Test Result (as a percentage)</th>
<th>Mean End of Semester Result</th>
<th>Percentage who Passed Service Mathematics</th>
<th>Total Number of Students in Each Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mature Students</td>
<td>28.3 (16.4) (n=34)</td>
<td>54.9 (27.9) (n=35)</td>
<td>71.4%</td>
<td>40</td>
</tr>
<tr>
<td>Ordinary Level Students</td>
<td>44.7 (11.8) (n=308)</td>
<td>43.5 (18.3) (n=353)</td>
<td>59.8%</td>
<td>382</td>
</tr>
<tr>
<td>Higher Level Students</td>
<td>65.7 (13.5) (n=187)</td>
<td>70.0 (18.3) (n=230)</td>
<td>92.6%</td>
<td>239</td>
</tr>
</tbody>
</table>
Education and Skills: July 2012.


Novice Teachers Reflect on Their Instructional Practices While Teaching Adults Math

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Abstract
Over three years, eighty-two teachers in their first or second year of teaching participated in orientation programs for new adult educators. During the programs, they reflected on their own instructional practices when teaching mathematics to adults. The teachers identified the practice they were likely to overemphasize and explained why they were likely to do so, posting their responses to online course discussion boards. Almost half of the respondents reported they “primarily emphasize calculation skills” and shared various reasons for doing so. The remaining respondents reported emphasizing one of four other instructional practices. Teachers put forth a variety of justifications for the instructional practices they have been using. Professional development efforts will need to recognize and take account of the teachers’ beliefs, assumptions and current practices.

Key words: instructional practices, teaching, mathematics, adults

Introduction
Every adult education teacher was once a novice teacher. In the U.S., most states do not have a certification system requiring a formal educational program to prepare teachers for instructing adult learners who have returned to study to complete their high school education. Existing research on novice teachers has generally focused on those who are completing a university-based teacher education program for those planning to teach in elementary, middle or high schools (e.g., Horn & Campbell, 2015). However, two studies conducted outside of the U.S. have specifically focused on the perspectives of adult educators.

For adult and higher education teachers, Pratt (1998) developed and tested (Collins & Pratt, 2011) a framework of five teaching perspectives that reflect the beliefs and intentions of teachers. These include Transmission (delivering content), Apprenticeship (modelling ways of being), Developmental (ways of thinking), Nurturing (personal agency), and Social Reform (bettering society). While this framework is applicable to adult education, it is not specifically geared to teaching mathematics to adults or to novice teachers who are just beginning their adult education careers.

Beeli-Zimmermann (2015) examined the mathematical beliefs of five adult teachers in Switzerland who completed an intensive 8-day training and who are integrating numeracy instruction into their second language (German) instruction. She found that the teachers’ own school experiences influenced their beliefs about mathematics and their teaching preference.

The current study complements the work of Pratt and Beeli-Zimmermann by focusing on a relatively large sample of novice teachers who have had limited professional development but who are making their own decisions about how they are approaching adult numeracy instruction. In explaining the rationales for their decisions, they reveal their influences and intentions.
Methodology

Context of the Study

In one state in the United States, all new adult education teachers are required to complete a semester-long online orientation course during their first or second year of teaching. This course is delivered asynchronously and addresses multiple topics of relevance to adult educators including adult learning theory, career pathways, learning disabilities, reading and literacy, family literacy, assessment, English as a second language, as well as numeracy.

Each topic is addressed during two weeks of the course with relevant activities and discussion topics designed by a practitioner or researcher with expertise in that content area. Each topic segment includes an initial activity or assignment, a pre-taped webinar that participating teachers are expected to watch, followed by a second assignment related to the content of the webinar. During each implementation of the course, the same practitioners or researchers facilitated the topical online discussions.

Full-time teachers are expected to complete assignments for all topics while part-time teachers choose and complete assignments from a subset of the topics. At the end of the course, participating teachers develop and complete a culminating assignment for which they investigate some aspect of their practice. Some participants choose to focus on numeracy, though these investigations are not addressed in this paper.

The Task

This paper reports on the novice teachers’ responses to the initial activity for the Mathematics segment of the course. The assignment states:

1. Read the article: “Designing Instruction with the Components of Numeracy in Mind, Focus on Basics, v.9 (a), 14-20.
2. Reflect on your own teaching and consider whether you frequently find yourself in one of the "risk categories" as described in the article.
3. Explain why you find yourself there and what you may try to do differently.

The assigned article was written by the author of this article to complement a 2006 commissioned paper on the Components of Numeracy (Ginsburg, Manly, & Schmitt) and appeared in a journal produced for practitioners by NCSALL, a research centre funded from 1996 to 2007 to focus on adult basic education. In the article, five practices are described and identified as “risk categories” in that their overemphasis during instruction might limit the broader learning opportunities for adult learners. By the term “risks,” the author meant preferences, priorities, or maybe even ruts. In hindsight, the term “risks” was not the best term to use as it implies a level of danger that might be misconstrued. The list of practices was not meant to be comprehensive but rather a group of commonly seen instructional practices in U.S. classrooms and/or reflecting issues that have been explored in research relevant to adult learners, such as mathematics anxiety (e.g., Beilock& Willingham, 2014; Chinn, 2012; Evans, 2000) or embedding mathematics instruction within real-life contexts (Casey, Cara, Eldred et al, 2006; Stone, Alfield& Pearson, 2008). In the article, each practice is described and the relevant challenges from its overemphasis are explained.

The instructional practices described in the article are:

1. Primarily emphasizing calculation skills (procedures).
2. Focusing on the language aspects of word problems (key words).
3. Attempting to dissipate math anxiety.
4. Primarily dividing math content into distinct, non-overlapping topics.
5. Only embedding instruction within real-life contexts

Each participant selected one risk category as his/her primary instructional practice. As the participants read the entire article before responding to the questions, the position advocated in the article (that ideal instruction would balance competing priorities and demands in order to enhance and enrich learning opportunities) may have influenced their responses. We have no way to judge if this is so, but it would be interesting for future research to explore any impact reading about or discussing ideal instruction may have on teachers’ descriptions of their own practices.

The Subjects

Between October 2012 and January 2015, seven cohorts of novice adult education teachers completed the orientation course. All full time teachers completed the mathematics topic and some part-time teachers chose the mathematics topic as one of their optional topics. Across the seven cohorts, there were a total of 82 teachers who completed the mathematics assignments. A few additional teachers self identified as “I don’t teach math” so they were excluded from this study. Only a few of the teachers were primarily mathematics teachers; the majority of the teachers were teaching multiple subjects which may have included literacy, science and social studies as well as mathematics.

Findings

As shown in Table 1, each of the instructional practices was identified by some of the participating novice adult education mathematics teachers as their primary instructional practice or goal. However, almost half of the teachers saw themselves as “primarily emphasizing calculation skills” in their instruction.

Table 1

*Primary instructional practices identified*

<table>
<thead>
<tr>
<th>Practice</th>
<th>Number (n=82)</th>
<th>Percentage (rounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Primarily emphasizing calculation skills</td>
<td>37</td>
<td>45%</td>
</tr>
<tr>
<td>2. Focusing on language aspects of word problems</td>
<td>9</td>
<td>11%</td>
</tr>
<tr>
<td>3. Attempting to dissipate math anxiety</td>
<td>15</td>
<td>18%</td>
</tr>
<tr>
<td>4. Dividing math content into distinct, non-overlapping topics</td>
<td>10</td>
<td>12%</td>
</tr>
<tr>
<td>5. Only embedding instruction within real-life contexts</td>
<td>11</td>
<td>13%</td>
</tr>
</tbody>
</table>

Each participating teacher explained why he or she primarily uses a particular instructional practice. The explanations they provided for their choices are described for each practice. For the first practice (primarily emphasizing calculation skills), the large number of responses could be categorized into five categories. Since there were fewer participating novice teachers that identified each of the other four practices, their responses could not be as meaningfully categorized.
**Practice #1. Primarily emphasizing calculation skills**

Thirty-seven participating novice teachers (~45%) said that they “primarily emphasize calculation skills.” Their reasons for doing so fall into five categories of responses. The categories are listed here with one or more examples of teachers’ reasons.

1. **Meeting my learners’ wishes**
   
   My students say, “Teach me how to do it. I don’t care why.”

2. **Using the available workbooks**
   
   I tend to focus heavily on workbooks that focus strictly on fractions, decimals, or percent. These seem like easy solutions to the larger problem, that students have either forgotten or never fully understood the fundamentals of math.

3. **Efficient classroom management, especially given multi-level classes**
   
   I have many students at low levels, yet in the same classroom I have those who score at 11th and 12th grade level. In order to offer individual attention where needed, students need something to work on when I am helping others.

4. **Belief that mastery of computation must precede problem-solving**
   
   The majority of my students lack the basic arithmetic calculation skills….I start off with this method and then move on to other methods such as word problems and real-life contexts. If a student does not have the basic skills they will not be able to move on to the higher level of mathematical reasoning.

5. **Teacher’s personal comfort, ability and satisfaction**
   
   It is very easy to get caught up in only focusing on the easily measurable, teachable, and observable, especially with learners who are lacking these skills.

   It’s easy to simply teach calculations – teach the steps, do examples, assign some worksheets, and voila! Now my students can do operations with fractions!

   That is the way I was taught. I find it just plain easier to talk about math in terms of calculations and operations. Admittedly, I don’t always understand the meaning of the procedures myself and thus find it difficult, if not impossible, to articulate meaning for my students.

   There is a comfort level there for me, and getting the right answer validates to me that I am doing a good job of conveying the material to my students.

**Practice #2. Focusing on language aspects of word problems**

This practice was identified by 9 of the novice teachers (~11%). It is often described by teachers as attending to the “key words” in word problems that ostensibly provide clues to which operation is required to solve the problem. Examples of such key words are: ‘in all’ (addition), ‘difference’ (subtraction), and ‘of’ (multiplication, particularly with fractions or percentages).

Among the reasons put forth by the teachers for emphasizing the key word strategy were that the strategy was perceived as efficient or that the strategy was presented in a textbook. A few of the teachers noted that their own strength was in literacy instruction, and thus they were less comfortable teaching mathematical problem solving by focusing on the mathematics. For example, two teachers commented:

   We try to use the language to help reason through the math. If the words present themselves, why not use them?

   I would like to explore the complex and messy nature of solving meaningful math problems; however, because of time restraints I often feel the need to simply teach language clues within word problems as an overall strategy.
Practice #3. Attempting to dissipate math anxiety

Many adult learners announce to their teachers that they are afraid of studying mathematics, have had prior negative experiences studying mathematics in school or have been told by parents or teachers that they are just not mathematically capable. All adult education teachers want to alleviate their students’ suffering and make it possible for them to learn the mathematics they need to learn. The fifteen teachers (~18%) who identified this practice as their primary instructional goal focused on creating a safe environment for their learners. However, upon reflection, the teachers note they may have been helping them too much for fear of adding to their struggles but may not have been actually helping them to learn the mathematics. Two teachers reported:

I have a tendency to take a difficult math problem and break it into such small pieces that the bigger problem becomes lost. This may lead to frustration in some students because they still can’t grasp the bigger problem. I catch myself saying it’s only adding, subtracting, multiplying, and dividing, you just need to know when to do what. Well, if you don’t know when to do what, that’s probably not much help and could be frustrating.

I think that sometimes they have learned through the years to become learned helpless. They feel that someone needs to help them. Sometimes when working with them I will take their pencil and show them the mistake, instead of letting them think out the problem for themselves and using a strategy to solve the problem.

Practice #4. Dividing math content into distinct, non-overlapping topics

Ideally, learners should come to see the connections across mathematical content, such as recognizing connections among fractions, decimals and percentages. But often the topics are taught in isolation. Ten teachers (~12%) identified with this practice, justifying their instructional practice by stating that their textbooks divide the topics and rarely mention any connections or that they divide the content into discrete topics because “it seems organized and simpler.” Among the teachers’ comments were:

I’m short on time. I try to introduce concepts in small chunks because I feel there is so much to cover, and I don’t want to overwhelm students.

I felt since I know what students need to know for the test, I could provide them with a map. We would cover one topic at a time and move forward as they understood and grasped the concepts.

Practice #5. Only embedding instruction within real-life contexts

Eleven novice teachers (~13%) chose this practice. Some of the teachers identified themselves as primarily teachers of English as a Second Language (ESL) and thus did not really address mathematics at all unless it came up in discussions of real-life situations. Others in this group suggested that they focused primarily on real-life contexts because the contexts provide authenticity, learner interest and engagement is high, or because they believe research has shown that this approach is desirable and recommended. Among their comments:

When I was hired, I was told that my instruction should be relevant and rigorous.

I find that students are more interested and eager to learn when they can relate the math to something they may encounter in a real life situation.

Relating to real life math experiences is how I can best relate to the student because it’s what I know. One benefit of doing this is that the adults all take part in the teaching process [pooling knowledge].

What might you do differently?

As part of the assignment, the novice teachers had been asked to consider what they might do differently in the future. Almost all indicated they would try to incorporate a broader range of practices and priorities into their instruction. There were few comments that went beyond suggestions provided in the article. Since the participants were novice teachers only just
becoming accustomed to teaching mathematics to adults, they may not yet have been ready or prepared to modify their instructional practices without a support community. Further research on the process of teacher change, particularly among novice adult educators, could inform the development of useful supportive programs and environments.

**Conclusions**

Many novice adult mathematics teachers focus primarily on computation practice in their instruction even though many of them recognize the limitations of this practice. Their rationales indicate this is what they know and are familiar and comfortable with. Further, since so many adult education teachers have limited mathematics content backgrounds (Gal & Schuh, 1994), they may not have the deep content knowledge to go beyond computational instruction.

Some of the novice teachers’ beliefs and instructional approaches seem to be informed by their own experiences in school and they are choosing to teach as they remember themselves being taught. They may have difficulty picturing approaches that emphasize the development of conceptual understanding or the ability to apply mathematics in situations because they have never seen or experienced such practices in classroom instruction.

Further, some novice teachers’ instructional practices seem to be reinforced by the environments of the adult education programs in which they work. They mention relying on the resources that are available and the instructional practices implemented by colleagues. Programs are required to assess and report learners’ progress using standardized tests that require computational competence.

Some teachers believe they are actively aligning their instruction with their learners’ expectations and desires. Learners are hoping their time in adult basic education will be short so they are pleased to move quickly from one topic to another, in effect crossing off the items on the content checklist, encouraging the instructional practice of teaching content as distinct, non-overlapping topics. Most learners’ goals focus on achieving a High School Equivalency certificate. These assessments allow the use of calculators for almost all questions and require conceptual understanding since all questions are applications (word problem situations). Thus, some teachers emphasize key word strategies for dealing with these problems.

The development and implementation of effective professional development for novice adult mathematics educators requires recognition of the teachers’ various assumptions, current practices and rationales for those practices. Just as adult learners return to study with their own experiences in and out of school, their teachers bring their own sets of experiences, mathematical content knowledge and beliefs.

For the novice teachers in this study, the practices they identified were recognized during the second week of the mathematics component of the course, with the goal of considering how their preferred practices could be enriched by attending to other priorities. Surely, in an extended professional development course focused on teaching mathematics to adults, it would be valuable to engage in a deeper exploration of how teachers’ own experiences, beliefs and content knowledge guide their instructional practices and decisions. This study showed that different teachers rely on different instructional practices; during professional development, teachers can examine and discuss each others’ instructional strategies with a goal of developing a balanced approach that includes a variety of practices and priorities.

**References**


The Intricacies of Assessing Numeracy: Investigating Alternatives to Word Problems

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Abstract

Word problems are often used to assess numeracy, despite the growing number of reports on difficulties students encounter with this genre of mathematical problems. These reports contend that a large number of difficulties are influenced by the way the problems are presented, that is, with verbal representations of the problem situations. These difficulties are said to be associated with a form of suspension of sense-making. In this study, conducted in the Netherlands, we investigated the effect on adult participants’ performances of changing the representation of the problem situation, from verbal to image-rich. A controlled randomised trial was the main part of this investigation. Furthermore, we compared the results of adult participants with the results of a similar trial which was held with students from primary and secondary education. The study showed that adult participants’ performances improved slightly with the change in representation, particularly on tasks in the content domain of measurement & geometry. These results were comparable with the results found of students from primary and secondary education, indicating that the effect is not related to age. The results could be of interest, however, for all practitioners involved in the work of numeracy task design.

Key words: numeracy, assessment, word problems

Introduction

In most recent approaches in adult numeracy research, adult numeracy is defined as a complex, multifaceted, and sophisticated construct, incorporating the mathematics, communications, cultural, social, emotional and personal aspects of each individual in context (American Institutes for Research, 2006; Coben, 2003; Geiger, Goos, & Forgasz, 2015). As a consequence, learning and assessing numeracy in authentic situations is often advocated (Frankenstein, 2009).

A closer look at lesson or test materials used in numeracy education in many countries, however, reveals that most assessment materials consist of word problems or of items assessing procedural arithmetic skills. The same is the case in the Netherlands where, despite the country’s high rankings in international comparisons, there are persistent complaints about the literacy and numeracy levels of young adults in vocational education and in the workplace. As a result, in 2010 a Literacy and Numeracy Framework (LaNF) was introduced in the Netherlands, with a compulsory numeracy examination at the end of the vocational educational tracks.
After a lively debate on the assumed value of procedural skills for (young) adult learners, a compulsory numeracy examination has been implemented which consists of 45 mathematical problems of which 15 are strictly procedural problems and 30 arecontextual problems. Many teachers and mathematics educators have questioned the relevance of assessing vocational students this way, and they perceive a gap between the numeracy used by their young adult students in everyday life and (future) work, and numeracy as operationalised in the final examinations (Hoogland, 2006; Hoogland & Pepin, in press).

A study in 2011 and 2012 in the Netherlands focused on the idea that using image-rich numeracy problems contributes to bridging the gap between common classroom practice in numeracy and more sophisticated numeracy concepts (Hoogland, 2016). Part of this study was a controlled randomised trial with almost 32,000 primary, secondary, and vocational students, to investigate the effect on students’ performance of changing the representation of the problem situation from verbal (word problem) to image-rich (mixture of picture and words). The trial revealed that students performed better on image-rich numeracy problems than on otherwise equivalent word problems (Hoogland, 2016), indicating that students are less hampered by the many difficulties with word problems that are frequently reported (Verschaffel, Greer, & De Corte, 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009). In an experiment in 2013 in the Netherlands the results of this trial were replicated with adult participants. The results are shown in this article and a comparison is made with the results of students from primary and secondary education. It revealed which types of tasks particularly, in both populations, benefitted most from the change from a verbal description of the problem situation, to a mainly depictive description of the problem situation.

**Theoretical perspectives**

This study is part of a larger research project to investigate alternatives to the persistent and problematic use of word problems to teach and assess students’ ability to deal with numerical problems originating in everyday life. This ability of students is often labeled as numeracy or mathematical literacy, although these concepts have been and are still evolving (Coben, 2003; Geiger et al., 2015; Ruthven, 2016). The sometimes superficial use of the concept is also criticised (Jablonka, 2015).

In current classroom practice, word problems are used predominantly to teach and assess these abilities. Many researchers, however, report serious difficulties in using word problems to assess these abilities (Verschaffel et al., 2000; Verschaffel et al., 2009). The reported difficulties can be related to the steps the problem solver is expected to take to solve the task at hand. Figure 1 shows the diagram used in PISA as a schema for the relevant steps in the problem-solving process. Similar diagrams are used in related research on problem solving and modelling in mathematics education (Blum, Galbraith, Henn, & Niss, 2007; Burkhardt, 2006; Lesh, Post, & Behr, 1987). The reported difficulties seem to appear mainly in the two horizontal steps in the diagram: “formulate the mathematical problem”, and “interpret the mathematical results”. In the first step (formulate) students are reported to look at these problems with a strong “answer-getting mindset” (Daro, 2013) and a calculational approach (Thompson, Philipp, Thompson, & Boyd, 1994), as if the problem was limited to the right-hand vertical step of the problem-solving process and that solving problems of any kind means getting the “right answer” by conducting a series of operations on the numbers in the problems. In the third step (interpret) students are reported not to take common-sense considerations about the problem into account (Greer, 1997).

We conjectured that the use of images from real life would strengthen the association with real-world situations (Palm, 2009) and therefore decrease the suspension of sense-making (Schoenfeld, 1992) and the strong calculational focus (Thompson et al., 1994). As a paraphrase of the most used definition of word problems (Verschaffel, Depaepe, & Van Dooren, 2014), we...
suggested the following definition for such image-rich problems: “Image-rich numeracy problems can be defined as visual representations of a problem situation wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical reasoning to numerical data available in the problem representation”.

Cognitive psychology also offers theories and insights on the effect of depictive and descriptive representations on creativity and problem solving (Schnotz, 2002; Schnotz, Baadte, Müller, & Rasch, 2010; Schnotz & Bannert, 2003). Schnotz and Bannert (2003) concluded that task-appropriate graphics may support learning and task-inappropriate graphics may interfere with mental model construction. Schnotz et al. (2010) stated that, to solve a quantitative problem, a task-oriented construction of a mental mathematical representation is necessary, provided that it is task-appropriate. Their line of reasoning is that depictive representations can help students to make a relevant mathematical mental model of the situation, and that depictive representations have a high inferential power because the information can “be read off more directly from the representation” (p. 21). This perspective added to the plausibility of our conjecture, which we tested in our empirical studies and also gives some indications in which kind of problems the effect might be strongest, that is, problems whereof the representation of the problem situation is beneficial for constructing of a (mental) mathematical model needed to solve the problem.

![A model of mathematical literacy in practice](image)

*Figure 1. A model of mathematical literacy in practice. From OECD (2013a) (p. 26)*

**Design of alternatives to word problems**

In order to counteract these tendencies and the associated difficulties we designed tasks that were more “authentic” by changing the representation of the problem situation from descriptive to mainly depictive (Hoogland, 2016; Hoogland, Pepin, Bakker, de Koning, & Gravemeijer, 2016). Those tasks were incorporated in an instrument to measure students’ performance on both word problems and image-rich problems in a randomised controlled way. In a trial with students from primary and secondary education our conjecture was confirmed (Hoogland,
2016; Hoogland, Bakker, De Koning, Pepin, & Gravemeijer, submitted). Although the conjecture was confirmed, the results were not straightforward. The students’ scores on image-rich problems were slightly higher (2%), which was significant, but with a small effect size \( (d = .09) \) and the effect of better performance was most noticeable in tasks in the domain of measurement & geometry. The research question for the study reported here is: Does a replication of the original trial with adult participants show the same patterns and results as the original trial with primary and secondary students? In this paper we report on that replication of the original study with adults who participated in the “Groot Nederlands Rekenonderzoek (GNRO)” [Great National Numeracy Survey], a research initiative by the public broadcasting organisations VPRO and NTR, supported by the Netherlands Organisation for Scientific Research (NWO). Individuals of all ages and of all places in the Netherlands could register as participants on the GNRO website and could engage in a series of mathematical tests. We report the results from the trial with students from primary and secondary education in adapted format in the results section for easier comparison.

### The Dutch context

For the international reader, we provide some information on the Dutch educational context. In the Netherlands in 2010 the “Referentiekader Taal en Rekenen” [Literacy and Numeracy Framework (LaNF)] was introduced as a guideline for Literacy and Numeracy education in the age range of 4–18 years (Hoogland & Stelwagen, 2011; Ministerie van OCW, 2009), followed by a very similar version for adult education.

#### Table 1

<table>
<thead>
<tr>
<th>Framework</th>
<th>Categories</th>
<th>TIMSS 2015 – 8th grade</th>
<th>PIAAC 2016</th>
<th>PISA 2015</th>
<th>Dutch LaNF 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Number</td>
<td>Quantity &amp;</td>
<td>Quantity</td>
<td>Numbers</td>
</tr>
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<td>Algebra</td>
<td>Geometry</td>
<td>Number</td>
<td>Space &amp; Shape</td>
<td>Proportions</td>
</tr>
<tr>
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<td>Data &amp; Chance</td>
<td>Dimension &amp; Shape</td>
<td>Change &amp; Change</td>
<td>Measurement &amp; Geometry</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pattern, Relationship &amp; Change</td>
<td>Uncertainty &amp; Data</td>
<td>Relations</td>
</tr>
</tbody>
</table>

*Note: Presented by similarity (horizontal).*

The content domains in these frameworks resemble the categories used in the international frameworks on numeracy and mathematical literacy, such as TIMSS, PISA and PIAAC (Mullis & Martin, 2013; Organisation for Economic Co-operation and Development (OECD), 2013b; PIAAC Numeracy Expert Group, 2009). Table 1 gives an overview of the content domains used in the various frameworks. It is noteworthy that in the Dutch framework there is more emphasis on proportions, including fractions and percentages, and an absence of focus on uncertainty, chance and data (representation).

### Method

#### The instrument

To measure the effect of the change in representation of the problem situation on the performance of participants we used an instrument that was used in both the trial with students
in primary and secondary education and in this replication study with adult participants. The trials were held with Dutch language items (Hoogland, 2016); English translations of these items are available under open access (Hoogland & De Koning, 2013). The instrument consisted of 24 items of which 21 items were designed in two versions: word problem and image-rich problem. For every participant a test was composed randomly with 10 or 11 items in each version. The randomly selected items were presented in random order for each participant. In this case a randomised controlled trial was built into the test. Both versions of each item had an equal chance of being selected, independent of any other variables, measured or not.

![Figure 2. An example of an item in two versions: word problem and image-rich problem.](image)

In Figure 2 we show an example of two versions of an item. The items are translated to English for better readability. In the test, each item was presented as a screen-filling problem with an open numerical answer field. The tasks in the research instrument were validated and tested in earlier research activities (Hoogland et al., 2016). The complete set of tasks can be found under open access via the Dutch institute DANS/NWO (Hoogland & De Koning, 2013).

In Table 2 we give an overview of the items in the instrument, evenly distributed across three domains of the LaNF: numbers, proportions, and measurement & geometry. Three tasks in the instrument were in the domain of relations, and were only presented in one version, because of the already visual nature of the items.

Table 2.

<table>
<thead>
<tr>
<th>item</th>
<th>Domain: Numbers</th>
<th>item</th>
<th>Domain: Measurement Geometry</th>
<th>item</th>
<th>Domain: Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td>i04</td>
<td>TV + DVD</td>
<td>i01</td>
<td>Apples in bag</td>
<td>i03</td>
<td>Travel time</td>
</tr>
<tr>
<td>i05</td>
<td>Change</td>
<td>i02</td>
<td>Fuel usage</td>
<td>i06</td>
<td>Recipe</td>
</tr>
<tr>
<td>i09</td>
<td>Money pile</td>
<td>i11</td>
<td>Double glazing</td>
<td>i07</td>
<td>Price magazine</td>
</tr>
<tr>
<td>i12</td>
<td>Kitchen tiles</td>
<td>i13</td>
<td>Water bottles</td>
<td>i08</td>
<td>AEX index</td>
</tr>
<tr>
<td>i16</td>
<td>Hamburgers</td>
<td>i14</td>
<td>Bedroom tiles</td>
<td>i10</td>
<td>Scale model</td>
</tr>
<tr>
<td>i17</td>
<td>Cough syrup</td>
<td>i19</td>
<td>Cake tin</td>
<td>i15</td>
<td>Endive</td>
</tr>
<tr>
<td>i18</td>
<td>Public debt</td>
<td>i21</td>
<td>Chocolate boxes</td>
<td>i20</td>
<td>Winter tires</td>
</tr>
</tbody>
</table>
Participants

The research conducted for this paper was a trial with 420 participants from the GNRO research. Table 3 shows the distribution of gender and age categories of these participants. The GNRO was, after registration, an open access public test held in 2013. We cannot consider these participants as a representative sample of Dutch adults. However, we consider the distribution over age and gender diverse enough to draw some tentative conclusions on the results in comparison with the results of the original trial with students from primary and secondary education.

Table 3.
Number of Participants from GNRO: Age Groups and Gender

<table>
<thead>
<tr>
<th>Age</th>
<th>n (% )</th>
<th>Gender</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>15 (5.3%)</td>
<td>Male</td>
<td>115 (40.4%)</td>
</tr>
<tr>
<td>20-29</td>
<td>61 (21.4%)</td>
<td>Female</td>
<td>170 (59.6%)</td>
</tr>
<tr>
<td>30-39</td>
<td>66 (23.2%)</td>
<td>Not stated</td>
<td>135</td>
</tr>
<tr>
<td>40-49</td>
<td>64 (22.5%)</td>
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<tr>
<td>50-59</td>
<td>31 (10.9%)</td>
<td></td>
<td></td>
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<tr>
<td>60-69</td>
<td>39 (13.7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td>9 (3.2%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not stated</td>
<td>135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Total sample is 410. n is number with percentages taken on stated age and gender in parentheses.

The original trial was conducted in October and November 2011. In that trial 31,842 students from 179 schools geographically spread across the Netherlands, participated. For convenience in comparing we show the results of the participants in the trial with students from primary and secondary education in this section. Table 4 shows the number of participants from the educational streams in the Dutch school system.

Table 4.
Number of participants in original trial: Age groups and gender

<table>
<thead>
<tr>
<th>Age</th>
<th>n</th>
<th>Gender</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-12</td>
<td>969 (3.1%)</td>
<td>male</td>
<td>15,310 (49.7%)</td>
</tr>
<tr>
<td>12-19</td>
<td>30,222 (96.9%)</td>
<td>female</td>
<td>15,465 (50.3%)</td>
</tr>
<tr>
<td>Not stated</td>
<td>680</td>
<td>not stated</td>
<td>1,067</td>
</tr>
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</table>

Note. Total sample is 31,842 participants. Age group 11-12 is primary education, age group 12-19 is secondary education. n is number with percentages taken on stated age and gender in parentheses.

In this original trial we assumed the sample to be representative of Dutch students in the age group 11–19 years.
Statistical analysis

The statistical analysis focused on the difference in scores on the A-version and the B-version of the 21 paired problems. We conducted a classical analysis using mean, standard deviation, $t$-tests, and Cohen’s $d$ as effect size to get a general idea of how the separate items contributed to the overall result we found (Cohen, 1988). As a caveat regarding the effect sizes note that we are not dealing with the most common cycle in educational research of measurement – intervention with the participants – measurement. The effect size category lists of Cohen (1988) or Hattie (2009) do not apply to this situation. Changing the representation of the problem situation is not an educational intervention. We are investigating what is the effect on participants’ behaviour of such a change and not measuring what they have learned from an intervention or a “treatment”.

Results

We present for both trials the results in the same table format for easier comparison. We compared the results of adults with the results of students from primary and secondary education. We focus in this comparison on the overall test and the results at item level. For the overall result on the test on the data collected in the GNRO we conducted a $t$-test on the mean scores on the A- and B-version items for each participant. We found that the difference in mean was .011 with standard error .001 and $p = .184$ (n.s.).

On item level we conducted a two-sided $t$-test with pooled variances to evaluate whether for each item the scores on the two versions differed significantly. We used a common effect size index, namely Cohen’s $d$, for a first general conclusion. The results are shown in Table 5. We found in four paired problems that the scores on the B-version were significantly higher than the scores on the A-versions with effect sizes ranging from .16 to .59. Furthermore, we found in one pair of problems that the scores on the A-version were significantly higher than the scores on the B-version with an effect size of .04. In the 16 other items the differences between the scores were not significant. The results in this replication trial were in most aspects in line with the results in the earlier large-scale student trial, which is discussed in more detail below.

<table>
<thead>
<tr>
<th>Item</th>
<th>N</th>
<th>Mean (SE)</th>
<th>$t$-test</th>
<th>effect size $d$</th>
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<td></td>
<td>A</td>
<td>B</td>
<td>version A</td>
</tr>
<tr>
<td>i1</td>
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<td>.899(.020)</td>
<td>.872(.024)</td>
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<td>215</td>
<td>.823(.026)</td>
<td>.800(.028)</td>
<td>.544</td>
</tr>
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<td>220</td>
<td>.836(.025)</td>
<td>.800(.028)</td>
<td>.337</td>
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<td>.951(.015)</td>
<td>.916(.019)</td>
<td>.150</td>
</tr>
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<td>.000 ***</td>
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<td>.751(.031)</td>
<td>.955</td>
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<td>.662(.033)</td>
<td>.690(.032)</td>
<td>.537</td>
</tr>
<tr>
<td>i9</td>
<td>209</td>
<td>.593(.034)</td>
<td>.540(.034)</td>
<td>.274</td>
</tr>
<tr>
<td>i10</td>
<td>212</td>
<td>.821(.026)</td>
<td>.870(.023)</td>
<td>.162</td>
</tr>
<tr>
<td>i11</td>
<td>203</td>
<td>.493(.035)</td>
<td>.774(.028)</td>
<td>.000 ***</td>
</tr>
<tr>
<td>i12</td>
<td>212</td>
<td>.811(.027)</td>
<td>.789(.028)</td>
<td>.559</td>
</tr>
<tr>
<td>i13</td>
<td>202</td>
<td>.896(.021)</td>
<td>.858(.024)</td>
<td>.233</td>
</tr>
<tr>
<td>i14</td>
<td>212</td>
<td>.472(.034)</td>
<td>.433(.034)</td>
<td>.423</td>
</tr>
<tr>
<td>i15</td>
<td>226</td>
<td>.774(.028)</td>
<td>.825(.027)</td>
<td>.198</td>
</tr>
<tr>
<td>i16</td>
<td>219</td>
<td>.872(.022)</td>
<td>.866(.024)</td>
<td>.845</td>
</tr>
</tbody>
</table>
The results of the large-scale trial have been published before (Hoogland, 2016). Table 6 highlights only those results that are necessary to make the comparison with the replication sample of this study. For this comparison only we incorporated $p<.10$ as a category – it is not used for further statistical inferences. For the overall results on the test on the data collected in the large-scale school trial, we conducted a $t$-test on the mean scores on the A- and B-version items for each participant. We found that the difference in mean was .019 with standard error .001 and $p<.001 (***)$. On item level we conducted a two-sided $t$-test with pooled variances to evaluate whether for each item the scores on the two versions differed significantly. We again used the effect size index, Cohen’s $d$, for similar conclusions. The results are shown in Table 6.

**Table 6.**

Results for the large-scale school trial, mean and $t$-test results

| Item | N version A | version B | Mean (SE) version A | version B | $t$-test $p(|T|>|t|)$ | effect size $d$ |
|------|-------------|-----------|---------------------|-----------|-----------------------|----------------|
| i1   | 15,878      | 15,964    | .716 (.004)         | .720 (.004) | .424                  |                |
| i2   | 15,986      | 15,856    | .525 (.004)         | .483 (.004) | .000 ***              | .08             |
| i3   | 15,785      | 16,057    | .314 (.004)         | .290 (.004) | .000 ***              | .05             |
| i4   | 15,835      | 16,007    | .826 (.003)         | .833 (.003) | .131                  |                |
| i5   | 16,038      | 16,504    | .720 (.004)         | .828 (.003) | .000 ***              | .26             |
| i6   | 15,775      | 16,607    | .631 (.004)         | .640 (.004) | .102                  |                |
| i7   | 16,065      | 15,777    | .404 (.004)         | .416 (.004) | .042 **               | .02             |
| i8   | 16,298      | 15,544    | .303 (.004)         | .299 (.004) | .420                  |                |
| i9   | 16,069      | 15,773    | .221 (.003)         | .213 (.003) | .085 *                | .02             |
| i10  | 15,882      | 15,960    | .495 (.004)         | .525 (.004) | .000 ***              | .06             |
| i11  | 15,850      | 15,992    | .145 (.003)         | .310 (.004) | .000 ***              | .39             |
| i12  | 15,871      | 15,971    | .466 (.004)         | .438 (.004) | .000 ***              | .06             |
| i13  | 15,931      | 15,911    | .619 (.004)         | .641 (.004) | .000 ***              | .05             |
| i14  | 15,889      | 15,953    | .040 (.002)         | .046 (.002) | .080 *                | .02             |
| i15  | 15,793      | 16,049    | .394 (.004)         | .388 (.004) | .264                  |                |
| i16  | 15,921      | 15,921    | .803 (.003)         | .815 (.003) | .005 ***              | .03             |
| i17  | 15,986      | 15,856    | .803 (.003)         | .787 (.003) | .000 ***              | .04             |
| i18  | 15,847      | 15,995    | .153 (.003)         | .168 (.003) | .000 ***              | .04             |
| i19  | 15,932      | 15,910    | .247 (.003)         | .284 (.004) | .000 ***              | .08             |
| i20  | 15,925      | 15,917    | .130 (.003)         | .164 (.003) | .000 ***              | .10             |
| i21  | 16,044      | 15,798    | .188 (.003)         | .256 (.003) | .000 ***              | .16             |

Note. $N$ is number of items tested. Mean is mean score on items (with standard error in parentheses) $P(|T|>|t|)$ is result of $t$-test, unpaired, unequal with hypothesis that difference in score is 0; *$p<.10$, **$p<.05$, ***$p<.01$. Cohen’s $d$ is effect size. Version A is the word problem; Version B is the image rich numeracy problem.
In the large-scale school trial we found with $p < .10$ in 11 paired problems that the scores on the B-version were significantly higher than the scores on the A-versions with effect sizes ranging from .02 to .39. Furthermore, we found in five paired problems that the scores on the A-versions were significantly higher than the scores on the B-versions with effect sizes ranging from .02 to .08.

**Comparing results**

The overall result on performance in this study with adult participants was 1.1 percentage point higher scores on image-rich problems. This was in line with the overall results we found in the large school trial, that is, 1.9 percentage point higher scores on image-rich problems. In almost all items the effect of higher scores on the B-version occurred with a very small effect size, in other items it did not occur. In one item the effect was even opposite. We synthesised the results in Table 7.

Table 7.

*Comparing results of adult and students from primary and secondary education*

<table>
<thead>
<tr>
<th>Domain</th>
<th>Population</th>
<th>A &gt; B</th>
<th>A = B</th>
<th>B &gt; A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
<td>Adults</td>
<td>43%</td>
<td>14%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>71%</td>
<td>14%</td>
</tr>
<tr>
<td>Meas. &amp; Geom.</td>
<td>Adults</td>
<td>0%</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>14%</td>
<td>71%</td>
</tr>
<tr>
<td>Proportions</td>
<td>Adults</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>43%</td>
<td>43%</td>
</tr>
</tbody>
</table>

*Note.* A > B means the results on the word problem version are significantly larger ($p < .10$). A = B means the results are not significantly different ($p < .10$). B > A means the results on the image-rich problem version are significantly larger ($p < .10$).

Solving problems from the domain of measurement & geometry seems to benefit the most from a depictive representation in both populations. For problems in the domain of numbers we see no beneficial effect for either representation, although the deviation is much larger for the adult population. In problems in the domain of proportions only the student population seems to benefit to some extent from depictive representations. We found three tasks in the domain of measurement & geometry that in both trials showed a significant better performance for the image-rich versions. They are shown in Figure 3. This finding corroborates our earlier findings that the change in representation of the problem situation has the greatest positive influence on the performance of the participants in tasks from the domain measurement & geometry. Indeed, in these cases the depictive representation of the problem situation could arguably be beneficial to form a (mental) mathematical model necessary to solve the problem, such as estimating the area in item 11, calculating the content in item 16, and estimating the content in item 21.
Figure 3. Three examples from the domain of measurement and geometry with a significantly higher score on the image-rich version.

Discussion

Assuming the diagram of problem solving in Figure 1 contains essential steps for the solving process (going from the problem situation to the situation model and on to the mathematical model), we argue that the mental activity needed for the necessary steps in the process is interdependent on the mathematics domain of the task. So following the reasoning of Schnotz et al. (2010), in the domain of numbers the mathematical model is primarily computational and thus one dimensional. In that case a mainly depictive representation was presumed not to contribute considerably to the ease with which problem solvers make sense of the situational or mathematical model. In this domain most items gave no significant difference, even one opposite effect. In the domain of proportions the mathematical model is in general more complex than in the domain of numbers, because there is always some activity of (relatively) comparing quantities or comparing a quantity with a whole. A mainly depictive representation was assumed to be beneficial here. At the same time a counter-effect is possible if the mathematical model and the depictive representations are not mutually beneficial, which might lead to an increased complexity experienced by the participant. For tasks from the domain of proportions one could not make a plausible straightforward prediction, whether a mainly depictive representation could help the solvers to construct the appropriate mathematical model and hence help them in solving the problem in a successful way.

In the domain of measurement & geometry the underlying problem situation in itself is two- or three-dimensional. So, a mainly depictive representation of the problem was assumed to help the problem solver to create the appropriate (mental) mathematical model. We saw in both trials
that of the four items that significantly favour the image-rich numeracy problem, three are in the domain of measurement & geometry, so this assumption is supported by the data.

In this replication, we found fewer tasks with a significant difference between the A- and the B-versions. With smaller samples, small increases in performance cannot be labelled as statistically significant. Nevertheless, the findings give enough incentive for further research in the design of numeracy tasks and the way reality is (re)presented in those tasks.

Conclusions

Word problems are a dominant feature of both classroom teaching and assessment of numeracy worldwide, and also of large-scale international assessments, like TIMSS, PISA, and PIAAC (Mullis & Martin, 2013; OECD, 2013b; PIAAC Numeracy Expert Group, 2009). Lessons learned from these assessments have been brought together recently, see for instance Tout and Gal (2015). Despite these efforts and despite the dominant use of word problems to teach and assess people’s ability to solve practical numerical problems, not much research has been conducted that systematically focuses on the effect on students’ performance of changing the verbal representation of the problem situation to a mainly depictive representation or a more authentic representation of the problem situation.

The original trial and this replication have limitations. The participants in the adult sample were not representative of all adults in the Netherlands. And although the replication strengthened some of the conclusions from the earlier large-scale school trial, the conclusions were still based on a limited number of items. More research is necessary to establish whether the results hold for other sets of problems that are paired in the same way as in these trials. The overall difference in results is small and effect sizes related to those results are in most cases very small. Slavin (2016) has recently stated, in a Huffington Post blog “What is a Large Effect Size?”, that in educational studies using a randomised controlled trial, effect sizes are seldom found over 0.2, however. The conclusions on the effects of a change in representation of the problem situation can thus be labelled as tentative. At the same time, the results of the change were significant and consistent, and not influenced by other variables, so there is, arguably, enough justification to speak of a small but robust effect.

Our suggestion is that in the task design of future assessments the representation of the problem situation should be taken into account as a factor when interpreting the results.

Acknowledgements

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The Liar Paradox: Looking Through Bertrand Russell's Mathematical Eyes

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The insistence that numbers created at the stroke of a pen can spontaneously acquire value without requiring some human sweat equity has the same inconsistency as Gotlob Frege's foundations of mathematics that was exposed by Bertrand Russell over 100 years ago. The system that denies the human source of all values leads to its own perverse nightmarish reality in which the frictionless motion of this economy is lubricated by the blood and sweat of those very human beings whose role it denies.

Introduction: The Resolution of Frege’s and Russell’s Dilemma

At the turn of the 20th century, the British mathematician and philosopher Bertrand Russell wrote a letter to the German mathematician Gotlob Frege in which he outlined what has subsequently become known as the Liar Paradox (Badiou, 2008, Article 2.11, p 20). Frege had been reworking the foundations of mathematics following the invention of set theory by Georg Cantor but his system was based on the premise that once a concept was expressed in a formalised language the set of terms described by that concept actually existed. He was also still restricted by the Greek concept of the whole, which implied the complete coincidence between a concept and the extension of that concept; the unity of the name with what was being named.

The Liar Paradox can be understood as a sort of unravelling of the two of these suppositions: the conjuring of existence from language and the notion of the whole, which in order to be whole must contain its own name. Take the self-referential statement, S: This statement is false. If S is true, then S is false, on the other hand, if S is false, then it is true that S is false; but, because S is saying precisely that S is false, S is true. Either way leads to a contradiction or paradox in that no thing can be described by the self-referential statement S even though it is a well-constructed lingual expression. Following Russell’s letter any defence of the whole could only revert to dogma and had no place in mathematics.

In response to the undoing of Frege’s foundations, mathematicians had to make two new decisions of thought, or axioms, to preserve the consistency of set theory. The German mathematician Ernst Zermelo proposed the axiom of separation in which a language statement could only extract a subset of terms from an already existing set. The Hungarian mathematician John von Neumann proposed a further axiom of foundation prohibiting the formation of a totalising whole, as the set of all sets, for since such a set would have to belong to itself, it is strictly forbidden on pain of the Liar Paradox.
Similarly the perpetual motion instability of the frictionless capitalism of Bill Gates’ dreams, that manifests as the economic, ecologic and humanitarian crises currently plaguing our world and threatening not only human culture but human existence itself, can be located in the assumption that money value can be created *ex nihilo* in the modern economy where new money is only ever created as a debt on society. However, we cannot come to terms with the inconsistencies of the money economy without examining how money itself is created, so this is our first task.

**Money Creation: Doing the Numbers!**

In economics textbooks, money is defined with three different functions: as a *store of wealth*, as a *medium of exchange*, and as a *unit of account*. These functions have stimulated much argument as to whether money gets its value from the commodity form of the material, such as gold or silver, from which the money token is made, or whether money is a social construct getting its value from the sponsorship of the sovereign power. Nowadays this argument has been resolved by the actual electronic practices of the banks themselves, so that it is only today that we can understand the essence of what money is and, indeed, what it always was. Money *is* a social construct, it is *created* by the bank as a *book-entry*, and it finds its physical embodiment only in electrons stored as digits in the computer of the issuing bank; it cannot be converted into gold.

All new money is created today in the form of a *book-entry* in the bank; specifically, it is created in the form of a *double-entry* of two equal numbers on the debit and credit sides of the bank’s account (Cencini, 1988, p 58-59). Prompted by a request from one of the parties to a real economic transaction, the *ex-nihilo double-entry* opens the line-of-credit that can be drawn down by the requesting party, and it also keeps account of that transaction. Created at the stroke of a pen or keyboard, the *ex nihilo double-entry* cannot represent real value and must therefore consist of *units of account*, pure numbers that rest on both sides of the bank’s account. So, the *double-entry* amounts to nothing in itself; it is what Adam Smith referred to as an empty vehicle. This *unit of a-count* is the same unit that we use for counting on our fingers *it is a pure number*.

When the line-of-credit is drawn down into an individual’s credit account, having paid her wages for example, then this number enters a new phase of its existence and converts to *true* money representing a store of wealth, or, more subtly when it is drawn down by the state to create the *fiduciary* or nominal money issue then again the number enters a new phase and becomes acceptable as the medium of exchange for real goods and services. The completion of a real economic transaction, as recorded by the subsequent entry in the individual’s account, thereby converts the pure number representing a unit of account in the bank’s book into a pure number representing true money in the individual’s credit account. The completion of a transaction in the real economy provides the alchemy required for Adam Smith’s empty vehicle, the *unit of account*, to acquire its load to become *true* money.

Here is our first possible source of confusion; *true* money having no physical dimension is *expressed* as a pure number and the *unit of account* representing the line-of-credit, *is* a pure number. All of the wishful thinking in the world will not convert a pure number into *true* money: the type of money that represents a store of real wealth or that is acceptable in exchange for real goods and services in the economy.
The Origin of Money and Number: The Talisman

Our enquiry into the intimate relationship between money and numbers starts with an understanding of the talisman, the indentured fetish of the primitive savage. From the Greek word *telesma* meaning complete, a talisman is the object that completes another object. The simplest talisman consists of a split stone, or stick, whose interlocking halves, when accepted by two parties to a transaction, acquired the potential of being a pre-literate form of an indentured contract, or a *bond*, for once the halves are matched or *trued* the contract is incontestable. In isolation, each half of the split object, each bearing silent witness to the contract, gains a supplementary spirit in an enchantment process that probably marks the point of origin of the symbolic human domain, the point of origin of human culture itself.

The most primitive method of taking a-count is that of tally-counting. Shepherds would notch a stick for each member of their flock going out to pasture in the morning. In the evening, by checking the notches one-to-one with the sheep as they were being penned for the night, the shepherd could check if he had lost, or gained, any during the day, and take appropriate action. With tally counting there is, as yet, no concept of number, there is simply a one-to-one correspondence between the sheep and the marks on the stick. Once the flock of sheep was to be put up for sale the tally stick was split in half lengthwise, with each half reflecting either a debt or purchase obligation. One half of the stick, the *stock*, was left at home to keep a-count of the real stock, the flock of sheep, where it played the role of the conventional tally stick. The second half of the stick, the *foil*, was brought to market to relieve the burden of bringing the whole of the flock, and being found to be a socially acceptable representation of the real stock in the purchase obligation, became charged with a non-material, or ideal, supplement and so became a fetish, a *thing* in which the social contract or *bond* between the two parties was represented: it became money.

Just as today’s electronic bank money sheds light on the essence of what money is and indeed what it always was, irrespective of its various disguises throughout the ages, so the understanding of number throughout history must start with the most modern theory of what a number is. Here we must take a detour into Cantor’s creation of the ordinals which are the raw material from which numbers are extracted.

Unlike their Greek forebears who commenced with unity, von Neumann’s adaptation of Cantor’s process started with the empty or *void* set:

\[
\{ \}.
\]

This strange set having no elements represents, or names, the indeterminate nothingness, and is deemed to exist in the mathematical *decision of thought* called the *axiom of the void*. The *void* is therefore the primordial existent of set theory. Von Neumann then proceeded to create a second set containing this one existent element. This second set is written as:

\[
\{\{\}\}.
\]

Gathering the two initial sets into a third set yields another new set;
The process continues with each successive new set constructed by the accumulating all of the previous sets as its elements, so that the fourth set of the sequence is

\[\{\{\}, \{\{\}\}\}, \{\{\}, \{\{\}\}\}, \{\{\}, \{\{\} \}, \{\{\}\}\}\}\].

And this sequence of sets, all conjured from the void, proceeds to infinity, and beyond!

Finding the above construction cumbersome, mathematicians give names to each of the sets in the sequence as presented in Table 1 and proceed to operate only with the names when undertaking their calculations.

### Table 1: Constructing the Ordinals

<table>
<thead>
<tr>
<th>name</th>
<th>longhand</th>
<th>shorthand</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Uncounted)</td>
<td>Nothingness</td>
<td></td>
<td>pure indeterminacy</td>
</tr>
<tr>
<td>0</td>
<td>{}</td>
<td>{0}</td>
<td>the “name of the void.”</td>
</tr>
<tr>
<td>1</td>
<td>{{}} = {0}</td>
<td>{0}</td>
<td>there is only one void.</td>
</tr>
<tr>
<td>2</td>
<td>{{}, {{}}} = {0, 1}</td>
<td>{0, 1}</td>
<td>etc.</td>
</tr>
<tr>
<td>3</td>
<td>... = {0, 1, 2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>... = {0, 1, 2… n-1}</td>
<td></td>
<td>as the general term.</td>
</tr>
</tbody>
</table>

Examining the shorthand column of Table 1, it is clear that the process can be summarised in an algorithm in which the successor of any set consists of the elements of that set together with its name, i.e.

\[S(p) = \{\text{elements of } p, p\}\].

So, for example:

\[S(2) = \{\text{elements of } 2, 2\} = \{0,1,2\} = 3\].

So the exercise of putting the name of something inside of that thing creates something different from the original thing, it creates its successor. For example putting, 0, the name of the void, inside the brackets of the empty set \{\} creates the singleton of the name of the void, \{0\}, or 1. The difference, or gap, between 0 and 1, is the minimum difference, a sort of quantum of difference, and is the same difference as that between each successive ordinal, including the uncounted gap between the indeterminate nothingness and its name. The members of this sequence of sets, known as the ordinals, proceed from the void in such a manner that each successive set contains all of the prior sets. Every ordinal, therefore, has the void as one of its elements.
We can now proceed to the definition of a number: *A number (N) is the conjoint givenness of its ordinal matter M (N), and a part of that matter called its form F (N)* (Badiou, 2008, Article 12.2, p.102). A number can be represented as follows:

\[ N = [M (N), F (N)] \quad \text{i.e.} \quad [\text{matter, form}] \]

The remaining part of the *matter* constitutes the *residue* and is denoted by R (N). Reassembling the *form* with the *residue* restores the whole of the *matter*. The designation of *form* and *residue* is arbitrary in that each is a complementary part of the same *matter*; however, for a particular number N, substituting the *residue* for the *form* leads to the negative of that number:

\[ -N = [M (N), R (N)] \]

Whether the original number N is positive or negative in the first place is not arbitrary, as it depends on where the void is located, with respect to the *form*, once the ordinal matter is split in two; if the void is located in the *form* then the number is positive, and if it is located in the *residue* then the number is negative.

From the vantage of this description of numbers, it is only now that we can retrospectively assert that, beyond mere counting, the birth of numbers took place with the simple gesture of cleaving the notched stick along its length, where each half of the stick became the unambiguous counterfoil of the other half. I think it is safe to assert that the birth of money took place with the same gesture and at much the same time! However the intimate relationship between money and number goes well beyond analogy. In the money economy the void is human existence and social human practice. So when money is issued into the economy on the basis of some new human creation it is issued as a positive number, a credit, into somebody’s account, and when human sweat equity is uncounted then the resulting money is issued as a negative number, a debt, into somebody’s account.

The line-of-credit created by the *ex-nihilo double–entry* is neutral and worthless in itself, yet it provides an opening to possibilities; in this regard it is but the modern embodiment of the split stick of the primitive savage, worthless in itself, but full of possibilities.

**Fiduciary Money and its Betrayal**

When the *ex-nihilo double entry* is drawn down by the sovereign power to issue new money into the economy this is referred to as the *fiduciary*, or nominal, money issue. James Tobin, who won the Nobel Prize in Economics in 1981, described the fiduciary issue as follows:

The community's wealth now has two components: the real goods accumulated through past real investment and fiduciary or paper "goods" manufactured by the government from thin air. Of course, the nonhuman wealth of such a nation "really" consists only of its tangible capital. But, as viewed by the inhabitants of the nation individually, wealth exceeds the tangible capital stock by the size of what we might term the fiduciary issue. This is an illusion… (Tobin, 1965, p 675)
Tobin’s fiduciary issue *illusion* is nothing of the sort. It is the implication that the value of the fiduciary issue is created from thin air, *ex nihilo*, that is the illusion. The fiduciary issue is the monetary embodiment, the objectified image, of the symbolic human domain; its value arises from human existence and social human practice in its historical entirety: neither the banknotes nor the coin-of-the-realm have any relevance to a polar bear. While fiduciary wealth does not exist as physical wealth on top of the real assets of the community, individuals can behave as if fiduciary wealth was real because, at an individual level, money is readily exchangeable for physical assets.

Currently throughout the global economy, when a sovereign state requests an issue of new money from the central bank; the bank gives the state a line-of-credit, and, in turn, it records this as an interest bearing debt in the form of Government bonds. The banking sector has an effective monopoly on the supply of money to the sovereign state, with the result that in order to have a growing money supply, the public must carry an overall burden of a *faster growing* interest bearing debt, which can therefore *never* be repaid.

So the issuing of the fiduciary money as interest bearing debt resembles the religious concept of Original Sin, as the indelible stain on the soul of every child who comes into the world.

Privatising the fiduciary issue and issuing it as interest bearing debt denies the social human source of its value and betrays the fiduciary trust of the community as it gracelessly denies the spiritual substance of humanity and unleashes a savage destructive nihilism.

**True Money and the Immortal Stock Paradox**

Consider the payment of a wages bill by a firm which requests a line-of-credit from the bank for the payment. The bank opens the line-of-credit by entering its two equal numbers on the debit and the credit sides of its balance sheet. At the instant the wages are paid, the number on the bank’s debit side gets drawn down into the credit side of the worker’s accounts and the number on the bank’s credit side gets drawn down into the debit side of the firm’s account, and the firm takes possession of the new goods. At this new phase of its existence, the number in the worker’s credit account: the wage, is *true* money whose face value is honoured by society at large when it is proffered for purchases in the economy. *True* money remains faithful to the social contract that money obtained for today’s goods will purchase equivalent goods tomorrow.

However, it is worth taking a closer look at the firm’s debit account which contains a number equal to the sum of the prices of the physical goods or *stock* that have yet to be sold from the warehouse. As there can be no unity between the name of a thing and the thing itself, there can be no unity between the price of the good and the good itself. It would be a big stretch of the imagination to declare that the firm’s numerical debt is *true* money, albeit in a negative embodiment. When physical output is expressed in the form of a negative number, or debt, the realisable money value of the very mortal and perishable goods against which the debt is incurred cannot always match the numerical value of the debt. A positive physical quantity, two loaves of bread, represents positive wealth and can be seen and touched, but *minus* two loaves of bread is a negative magnitude with no physical dimension, it is a pure negative number and, as such, it can
only play the role of a *unit of account* to keep track of the repayment of the debt, in true money, as the sale of the goods progresses.

The overall impossibility of treating a debt as true money, and not as a unit of account that is regularly *true* to its underlying asset, does not imply that it cannot be *true* in some individual cases. For instance, the successful sale of the daily bread at the anticipated price would mean that the baker could clear his debts and, indeed, there could also be individual cases where the physical stock actually increases above the amount reflected in the debt through a purchased sheep giving birth to a lamb, for instance. However once the unit of account is issued into the economy as a true money debt at *interest*, the ability to *true* the unphysical debt to physical reality becomes universally impossible. Although negative numbers can follow the law of compound interest indefinitely, the real future output against which the immortal debt has a claim cannot grow at compound interest for long (Soddy, 1933, p28)

**The Mortgage Grip-of-Death and the Limitless Money Paradox**

When a bank issues a mortgage, it opens the line-of-credit with its *double-entry* of numbers, equal to the price of the house, in its accounts book. In this case as no *new* economic output prompts the mortgage transaction, neither of the two entries that are drawn down can represent true money in this case where both sides of the double-entry can only remain *units of account* whose role is the keep account of the transaction. Yet, the new number in the seller’s credit account *acts* as if it is true money as it has purchasing power which is honoured by society at large. When this new number is added to the existing money supply it leads to a general increase in prices as the increased money supply is spread over the same real economy as before. *Inflation of the money supply, a sort of stealing from the past, is the first burden that the wider society must bear as a consequence of treating this ex nihilo number as true money.*

As the interest bearing debt of the mortgagee is repaid in instalments over the term of the loan, the number in his debit account is gradually reduced, and, in tandem, the money amount of each instalment is gradually withdrawn from the economy so that the original integrity of the money supply is restored at the final repayment of the capital. Any money that is used to pay interest on this mortgage must therefore come from somewhere else in the economy. *The payment of interest, a sort of stealing from the future, is therefore the second burden on the rest of the economy that arises from treating the ex nihilo number as true money.*

However if the numbers on both sides of the *ex-nihilo double-entry* are treated as if they are true money, they can, without question, cause real activity in the economy, which reinforces the illusion that the numbers are actual *securities* or *assets*. This illusion gives rise to the stock and bond *markets*, where these *assets* trigger more transactions and the creation of even more ex-nihilo numbers and, as there is no physical stopping point, the process can continue *ad infinitum*. This apparent limitless supply of money exposes the inconsistency of mortgage money, which, as well as causing the double jeopardy of inflation and interest, leads to the perpetual motion of the stock and bond market number machines that brought about the global economic meltdown in 2008. It is worth examining, in detail, the numbers involved in that meltdown.
In spite of the 10% fractional reserve rule for the high street banks, in 2004, large investment banks, such as Goldman Sachs and Merrill Lynch, had an asset to equity leveraging ratio of 23, equivalent to a 4% reserve. By 2007 this ratio had risen to 30 or above, equivalent to a 3% reserve. By 2008 the Royal Bank of Scotland were operating at a 2% reserve and Lehman Brothers were down to 1%.

However the financial crisis was finally triggered by a run on the shadow banking system which had allowed private financial institutions to generate as much credit as they wanted without any reserve requirement. So that in August 2008, the entire banking system held actual cash reserves of a total of $50 billion while it was clearing daily trades of $3,000 trillion, with every one of these trades carrying an unconditional promise to pay hard cash whenever it was asked for, in spite of the astronomical annual leveraging ratio of over 10 million involved. Incredibly, the stock market is back in action today with the same mountain of fictitious cash assets seeking out the next bubble!

In turn, interest bearing debt is also listed as an asset by the bank and as such it too is treated as if it has liquidity. The banks had been creating risky loans that they knew would never be repaid because the mortgage applicants weren't truly qualified to borrow as much money as they did. (Many of the applicants were cynically called Ninjas, “No income, no job, and no assets”). These risky debts were chopped up into securitized bonds and sold to insurance companies, retirement funds and foreign investors. But the regulators and ratings agencies turned a blind eye because there was a lot of money about and everyone involved seemed to be getting very rich and indeed, a lot of economic activity was stimulated.

In 2007, nearly $14 trillion worth of complex-securitized products were created through this process. Financial firms that had been providing the full face-value for securitized bonds got worried that those bonds might contain toxic subprime debts, so they reduced the amount of money they would lend on the bonds. When Lehman Brothers defaulted in September, 2008, the downward spiral accelerated and the entire financial system crashed. It was at this point that Federal Reserve chairman Ben Bernanke stepped in and provided blanket guarantees on the financial assets of banks and shadow banks alike. This was the beginning of the impossible attempt by the governments of the world to fill the bottomless hole of bank debt by imposing the austerity on their human societies that continues today, while the banks are back in business!

**Stand Up and be Counted!**

Over one hundred years ago the American journalist and political economist, Henry George, proposed that a tax on land values to the full extent of the economic rent and that the distribution of the proceeds as a universal dividend would be the way to restore socially generated values to society and that all other taxes could and should be abolished, thereby removing disincentives to creativity and entrepreneurship in the economy (George, 2006). The 21st century brings the demand for a full tax on economic rent back on the agenda as, perhaps, the only way to regulate, not only land, but the acute problems of scarce resources and environmental destruction. But more importantly, at the start of the 21st century, it is the fiduciary issue, in its entirety, the socially generated supplement to the community’s wealth, which must be reclaimed from private banking institutions and distributed as true money for the benefit, indeed
for the survival, of humanity and social human culture. The mountains of fictitious stock market assets must be dumped into the bottomless hole of fictitious bond market debt in order to clean the slate of human existence, in order to begin again with a money system that is thoroughly understood so that we don’t need to rely on an invisible hand to sort it all out.

When confronted with the impossibility of perpetual monetary self-expansion, and the economic, ecologic, and humanitarian crises caused by the assumption that money can spontaneously acquire value, humanity must courageously make new decisions of thought, and declare its own axioms of the void, of separation and of foundation by asserting that the proper material basis for economic reality is in its own existence and social practice, and that this must be reflected in a true money system and not in self-referencing debt money.

There are many objective crises in our world today, but our biggest crisis is a subjective one, our biggest challenge is to find a new subjectivity. Once we know that money only acquires its value from human society and human existence then the onus is upon us, as human beings, to declare this truth and operate in fidelity to it, rather than blindly following the status quo. We must open our mathematical eyes and see mathematics in everything we do, and have the courage to blow away the mists of inconsistency and paradox, so that we may live sane and healthy lives and not be led in blind acquiescence to madness and extinction.

**References**


Survey of Adult Students with Mathematical Difficulties

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Abstract
This paper relates to one of the test procedures being used in Sweden, used to establish if students need a more thorough investigation of their mathematical difficulties. This paper mainly describes the test process and the results from 10 test subjects. The paper also refers to parts of the research forming the basis for the test process. The paper shows how teachers in their everyday work can use the understanding researchers have of mathematical difficulties and the methods they have developed. The purpose is not to drive the research on mathematical difficulties.

Key words: mathematical difficulties, survey, levels of knowledge

About Mathematical Difficulties
As early as 1919 mathematical difficulties were observed by the Swedish doctor Salomon Eberhard Henschen, who later, in 1925, made connections to damages in the brain. Henschen, practicing in Uppsala, Sweden, characterized the difficulties he observed as acalculia (http://en.wikipedia.org/wiki/Salomon_Eberhard_Henschen, 2014-11-01). It was not until the mid 1970’s before further comprehensive studies of mathematical learning disabilities were conducted (Kling et al, 2011). After this period developments have been rapid including the designation of dyscalculia and the causes of mathematical learning disabilities scrutinized, formulated and discussed (Östergren, 2013). Today it is self-evident that specific development disorders and dyscalculia are defined in the most established systems for diagnosis, DSM and ICD.

Mathematical difficulties are usually divided in four categories (Adler, 2010); acalculia, dyscalculia, pervasive mathematical disorders and pseudo dyscalculia.

According to the Swedish professor Arne Engström (Engström, 2016) researchers do not agree on how dyscalculia shall be defined or the criteria to use. Also, they are not in agreement on how common dyscalculia is. Additionally, the researchers do not know enough about the reasons. This should be kept in mind whilst reading this paper.

Acalculia
A student exhibiting acalculia lacks the ability to calculate and to learn to calculate which mostly depends on measurable brain damage. A student having acalculia lacks the ability to count to 10 or add low numbers. Acalculia is unusual and is found only in a few per thousand of the population (Adler, 2007).
Dyscalculia

Dyscalculia together with dyslexia is today regarded as a subgroup of specific learning disorders. The diagnosis for dyscalculia, within the two most established international systems for diagnosis DSM and ICD, has been modified from proven experiences over the years. Today DSM has reached the fifth version (American psychiatric association, 2013) of its definition, DSM-5:

The Diagnostic and Statistical Manual of Mental Disorders
Dyscalculia is an alternative term used to refer to a pattern of difficulties characterized by problems:
- processing numerical information
- learning arithmetic facts and
- performing accurate or fluent calculations.

If dyscalculia is used to specify this particular pattern of difficulties, it is important also to specify any additional difficulties that are present, such as difficulties with math reasoning or word reading accuracy.

Difficulties in processing numerical information means an inability or reduced ability to deal with numbers. This is due to a dysfunctional number sense and in many cases lacking the ability to connect our Arabic symbols to numbers. Inability to recognize numbers often leads to the inability to deduce which of two numbers is the largest (the mental number axis, schema for numbers) and to perceive low numbers that are larger than four (Butterworth, 2010) as anything else than a series of “ones” (Halberda et al, 2008). Students having these difficulties are often seen counting on their fingers. Processing numerical information also means being able to understand and realize how numbers are influenced by simple arithmetic operations for adding and multiplication.

Difficulties to learn arithmetic facts also include remembering arithmetic facts, like simple additions. It stems from dysfunctional processes for absorbing, storing and recalling information from the long term or working memory (Kulcian et al, 2014). Difficulties to quickly fetch arithmetic facts from the memory are called automation difficulties. Number sense and working memory are important for the ability to calculate accurately or fluently (Klingberg, 2011).

The International Statistical Classification of Diseases and Related Health Problems
Specific disorder of arithmetical skills

Involves a specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus.

Arithmetical difficulties:
- associated with a reading or spelling disorder (F81.3)
- due to inadequate teaching (Z55.8)

When dyscalculia is being diagnosed according to DSM-5, difficulties in mathematical reasoning and in the working memory should also be included. The mathematical reasoning that DSM-5 includes is the abilities for:
• logical reasoning when tasks are solved
• apply strategies for solutions
• apply mathematical concepts, facts, procedures and methods when tasks are solved.
(Students’ methods are especially noted when tasks are solved, by checking if students use the correct rule of arithmetic or when the correct rule of arithmetic is used but the result is wrong.) (Adler, written tutorial, 2014-11-04)


Students having dyscalculia do have a normal level of talent as clarified by the WHO diagnosis. Later studies point out the number of students having dyscalculia can be compared to the number of students having dyslexia and this accounts for 3.6 to 6.5 per cent of the population (Butterworth, Yeo, 2010). A dyscalculia diagnosis is valid for a period of one year for children and two years for adults.

**Pervasive Mathematical Disorders**

There are also students with pervasive mathematical disorders. As these disorders are pervasive they are not specific for learning and understanding mathematics. They influence all learning. These students need adapted teaching where learning is allowed to consume more time and contents may be simplified. These students are more consistent in their achievements compared to students with dyscalculia.

**Pseudo Dyscalculia**

This fourth category (Adler, 2010) is about students with emotional blockings experiencing specific mathematical disorders. These students are discovered as they get surprised that the survey was so easy. Pseudo dyscalculia, which is most common among women, is treated with support from a psychologist, welfare officer or similar. It is also called math anxiety.

**Cognitive Disabilities**

Cognitive disabilities with a broad impact on many abilities are called pervasive. The cognitive disabilities specifically affecting students’ abilities to learn and understand mathematics are called specific mathematical disabilities or disorders. Two examples of pervasive proficiencies are intelligence and processing speed (Adler, 2007).

Students only having difficulties specific for mathematics usually have dyscalculia. Dyscalculia assumes the specific mathematical disabilities depart from the other more functioning abilities.

Mathematical disabilities are often caused by a combination of specifically mathematical and otherwise pervasive difficulties. Rickard Östergren (2013) proposes weak working memory in combination with weak number sense is a risk for students to develop MLD. Others have reached different conclusions. For example Deary, et al, (2007) suggest reduced intelligence combined with reduced number sense is a common reason for development of MLD. However, it has also been suggested intelligence is the same as working memory capacity (Ackerman, et al, 2005). In addition it is noted that general abilities like phonological consciousness, processing speed and executive attentional processing have great importance for MLD, (Östergren, 2013).

According to DSM-5, a weak number sense and its limitation in understanding digits and numbers is the main difficulty contributing to dyscalculia.
The Pedagogical Survey

Execution

The pedagogical survey that scores for pervasive and for mathematics specific development disorders was done in four phases; three screenings and one skill test in mathematics using the following material from Kognitivt Centrum in Malmö.

- Reading screening III (from 16 years), Adler, 2012
- Writing screening III (from 16 years), Adler, 2012
- Mathematics screening III (from 16 years), Adler, 2010
- Skill test for mathematics (secondary school and adults), Adler, 2008

Every screening lasts approximately one hour. The skill test takes exactly 5 minutes.

The test material has been bought from Kognitivt Centrum in Malmö. The authorized distributor prohibits further distribution. Therefore, it cannot be included as an annex. Due to this the article focuses on the students’ results rather than the questions asked. Choosing this test material is a consequence of Sweden being a small country and this is the test material that is readily available in Swedish.

All tests were done individually even if the skill test may be done in a group. Doing them individually enables observing aspects such as how easy students understand instructions and the time taken to complete different tasks. The most time consuming duty for the teacher is to compile the results and write recommendations for more thorough investigations and the bases for these.

Testing students’ reading and writing abilities is done since these are important to know about. For example, if students don’t have the fine motor skills to write and understand what they have written themselves, of course, the situation is made more complicated. The same applies if they cannot sketch or understand geometrical shapes. Students having difficulties to read may have problems to understand what the question is in a problem solving task. Also, texts may become incomprehensible when words are pronounced wrongly. These are a few examples.

Professor Arne Engström has the opinion that calling three of these tests screening is wrong because the word screening is used when a complete population is tested. (Engström, 2016)

About the ten students

I came in contact with eight of the ten students that were surveyed at my place of work (Komvux, Lund). The remaining two students, Josefin and Elise, heard about my investigations and wanted to take advantage of the opportunity to be in the survey and get more help for their difficulties. These two additional students were younger. Four of the students at Komvux were my own. The other students at Komvux were recommended for the survey either by a teacher or by the welfare officer. The designation Ma Gr in the table means a pre-secondary school course in mathematics. The other designations are courses at secondary school.

Table 1 shows background information about the 10 tested students. It displays age, if there are previous diagnoses, which course they are taking and how they describe their own difficulties.
Table 1.
About the ten students.

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
<th>Previous diagnosis</th>
<th>Studying</th>
<th>Student’s own description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marie</td>
<td>24</td>
<td>Dyslexia</td>
<td>Ma 2b</td>
<td>Bad memory. Good at per cent because of much shopping. Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Jan</td>
<td>24</td>
<td>No</td>
<td>Ma 2b</td>
<td>Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Mårten</td>
<td>31</td>
<td>No</td>
<td>Ma 2c</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Carina</td>
<td>21</td>
<td>No</td>
<td>Ma Gr</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Amanda</td>
<td>24</td>
<td>No</td>
<td>Ma 3b</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Emma</td>
<td>27</td>
<td>Dyslexia</td>
<td>Ma 1a</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Jonny</td>
<td>21</td>
<td>Post traumatic stress, probably dyscalculia</td>
<td>Ma 1a</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Josefin</td>
<td>15</td>
<td>No</td>
<td>Ma Gr</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Elise</td>
<td>16</td>
<td>Dyscalculia</td>
<td>Ma 1b</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Johan</td>
<td>20</td>
<td>Dyslexia</td>
<td>Physics 1a</td>
<td>Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
</tbody>
</table>

Here is a short description about the mathematical brain before we discuss the results for the 10 students.

The mathematical brain

Mathematics encompass various cognitive processes where different help systems collaborate. Therefore, the whole brain is used for mathematics. Both difficulties in reading and doing mathematics have the origin in deficits in both halves of the brain, even though reading depends mostly on the left half and number sense and calculus ability stems from the right half. (Adler, 2014. Network meeting dyscalculia, Stockholm).
The frontal lobes are an important part when doing new calculations. The brain also has an area called IPS where number facts are stored. This area may be blocked in any person. Typically students having a dysfunction in the IPS may say “I know but I cannot get it out”. Problems with IPS can be compensated by training. To succeed in mathematical studies it takes more from the brain than to succeed in reading or writing. The calculus itself encompasses more cognitive processes (Campell, 2004). At least, when looking at all the components needed for problem solving you can understand the whole brain is used for mathematics; good understanding of reading, linguistic understanding, phonological consciousness, persistence, attentiveness, a good capacity to automate, a well-functioning working memory that is not too slow, good speech recognition, mathematical reasoning etc.

Results for Ten Surveys

Tables 2 and 3 show how to interpret skill test results. The skill test consists of 50 simple calculations like e.g. 10 – 3 + 2. The skill test is normalized due to a standard population. It measures according to a profile and a normal distributed Stanine score in the range from 1 to 9. Students achieving a Stanine score of 1 or 2 are among the 11 % with the lowest result according to the standard population the test has been normalized against. Students achieving Stanine score 3 may also have great difficulties with speech recognition. These students probably have to some extent compensated their deficit number sense with a strong working memory. Stanine scores for skill tests are determined according to table 2.

Table 2.
The stanine scores.

<table>
<thead>
<tr>
<th>Stanine</th>
<th>Score (No. of right – wrong)</th>
<th>% of students in the normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>≤1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2-6</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>7-10</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>11-14</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>15-19</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>20-24</td>
<td>24</td>
</tr>
<tr>
<td>7</td>
<td>25-31</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>32-40</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>41-50</td>
<td>7</td>
</tr>
</tbody>
</table>

Profiles for skill tests are determined according to table 3.

Table 3.
Profiles for the skill tests

<table>
<thead>
<tr>
<th>Profile</th>
<th>The student works…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>slowly and does maximum 10 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>R</td>
<td>slowly and does maximum 10 tasks but makes few mistakes – three or less</td>
</tr>
<tr>
<td>S</td>
<td>at normal pace and does at least 11 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>T</td>
<td>at normal pace and does at least 11 tasks but makes few mistakes – three or less</td>
</tr>
<tr>
<td>U</td>
<td>fast and does at least 25 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>V</td>
<td>fast and does at least 25 tasks but makes few mistakes – three or less</td>
</tr>
</tbody>
</table>
Table 4 shows the screening and skill test results for the 10 students. In the table well-functioning abilities are shown as “X” while a “?” means this should be investigated further. Boxes with question marks are also shaded for clarity. A “?” in any of the blue fields indicates possible dyscalculia.

Table 5 shows a summary of the results for the pedagogic surveys. The students’ shortened names are given in the lilac headline. The table gives an insight into what is investigated. The yellow rectangle above the table shows the test designers’ instructions for how to interpret the number of difficulties resulting from the mathematics screening.

Table 4.

Collocation of the survey

<table>
<thead>
<tr>
<th>Ability</th>
<th>Ma</th>
<th>Ja</th>
<th>Må</th>
<th>Ca</th>
<th>Am</th>
<th>Em</th>
<th>Jon</th>
<th>Jos</th>
<th>Eli</th>
<th>Joh</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number structure</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>Read and write numbers</td>
</tr>
<tr>
<td>Number concept/perception</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>Largest, ordinal number</td>
</tr>
<tr>
<td>Reading digital clock</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Numbers on clock</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Hand on clock</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Recognizing letters</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>Read different fonts</td>
</tr>
<tr>
<td>Phonological consciousness</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>Rhyme simple words</td>
</tr>
<tr>
<td>Read single letter words</td>
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<td>Read complex words</td>
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<td>Mobilize words</td>
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<td>State words that...</td>
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<td>Name things</td>
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<tr>
<td>Order word sequences</td>
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<td>Order texts</td>
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<td>Read text</td>
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<td>Retell</td>
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<tr>
<td>Writing ability</td>
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<td>Copy text and figure</td>
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<td>Free writing</td>
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<td>Motional quickness</td>
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<td>Copy model</td>
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<td>Motional rhythm</td>
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<td>Perceptual completeness</td>
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Table 5.
Summary of the ten pedagogical surveys

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<td>8</td>
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<td>Q</td>
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<td>6</td>
<td>S</td>
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<tr>
<td>1</td>
<td>V</td>
</tr>
<tr>
<td>8</td>
<td>V</td>
</tr>
</tbody>
</table>

Marie shows signs of mathematically specific development disorders and problems with the working memory. Jan needs support to train his basic abilities for his own benefit and to definitely exclude his deficiencies depend on mathematically specific development disorders. Jan also needs encouragement to read more. If this training does not give the desired effect a more thorough investigation is required.

Mårten has automation difficulties and problems recognizing numbers. He almost certainly has dyscalculia or mathematically specific development disorders.

The survey indicates Carina is not a weak student but she has a complicated learning as she is concerned about several pedagogical building blocks. A deepened psychological and medical judgment shall then be conducted as a complement.

The survey shows there are problems with the mental number axis, and there may be difficulties with the working memory and concentration. Perhaps could some of these contribute to the resistance against math studies. The difficulties with the mental number axis, which in her case are apparent when the numbers are not written, should be possible to remove by number axis training. Therefore, currently no deeper pedagogical investigation on mathematical difficulties is recommended. Instead a working memory test is recommended to find out how Amanda could train mainly the visual working memory.

The survey clearly points out the already known dyslexia and mathematically specific development disorders that should be dyscalculia. A deeper dyscalculia and dyslexia investigation along with psychological and medical judgment is needed. It is also necessary to further define Emma’s dyslexia to prevent her being hindered in future studies at teachers college.

The survey shows Jonny has and mathematically specific development disorders and possibly also dyscalculia. A complementary deeper psychological and medical judgment shall then be conducted. Not the least, the reasons for Jonny’s concentration problems need to be established. The survey also show Jonny has a lack of concentration, reduced visual working memory, shape comprehension and ability to visually scan. A working memory test
Josefin has mathematically specific development disorders (since her mathematical difficulties deviate that much from her ability to read and write) and problems with the working memory, especially the visual, which in this context indicates complicated learning. Emotional blockings magnifies her learning problems. A psychological and medical judgment is needed as a complement.

Elise has mathematically specific development disorders (since her mathematical difficulties deviate that much from her ability to read and write) and problems with the working memory, especially the visual, which in this context indicates complicated learning. Strong feelings she has difficulties to control may also aggravate her learning problems. A psychological and medical judgment is needed as a complement.

Johan has read and write difficulties hindering him to succeed in math and physics. To alleviate these difficulties Johan needs to meet a specialist that can support him in using tools to ease taking notes, structure texts and understand concepts and explanations he has written himself. To get concepts and texts read out loud to him may also help since words he reads often are pronounced erroneously and become completely different words. Johan also needs a new, fresh dyslexia investigation prior to his studies at college.

Discussion

Each of these students exhibits issues with the mental number line, even when the ability test shows a high Stanine score. For some of the students the difficulties with the mental number line are a clear sign of difficulties associated with their number sense. The two students that achieved well in the ability test show problems with attention. One of them also showed problems with persistence/ concentration which may explain the difficulties with the mental number axis.

Six of the seven students having Stanine score 1-3 showed apparent signs for mathematics specific development disorders. One might have practiced too little. Out of these students Mårten exhibits signs of dyscalculia since there was a limitation in specific difficulties, except those linking to the visual. For Mårten the mathematics specific development disorders deviated a lot from his better functioning pervasive abilities. The survey showed Mårten had difficulties with persistence/ concentration and learning ability. Despite this, I see as a teacher of this student, these two general abilities function well. After the survey the student was allowed to use multiplication tables when doing written tests, which probably relieved some stress! After this he has been doing well.

For another student, Emma, having a low Stanine score (2) using multiplication tables when doing written tests was imperative for her to succeed with the course. It is obvious corrective actions do not have to be far away!

For the two students that produced a Stanine score of 8, one clearly exhibits dyslexia and the other shows a lack of attention and not being used to confront obstacles with problem solving.

The student with a Stanine score of 6 showed signs of mathematics specific development disorders and troubles with the working memory and concentration. These students’ difficulties are similar to those Östergren gives as an important source for mathematical difficulties, MLD.

Several students showed pervasive development disorders in combination with mathematics specific development disorders. One of these students, Carina who I have in my class, does not work at all in the large group where she is currently placed, since the size of the group hinders her from speaking up and communicating her thoughts. Her pedagogical survey made my headmaster act by offering her individually adapted support.

The ability for mathematical reasoning has not been investigated. This ability will be put forward in later investigations when instructive material for this has been developed. More investigations are needed to be able to exhaustively link results to contemporary research.
About more thorough pedagogical investigations

The pedagogical screenings investigate cognitive building blocks needed for students reading, writing and spelling. Depending on what is pointed out by the screenings deeper pedagogical investigations might be needed. These should be conducted by a team consisting of a medical doctor, a psychiatrist and possibly also a speech therapist and a social worker. For adults, in case it is not possible to engage all these staff, a medical doctor shall conduct a complementary investigation. The doctor’s role is to establish if there is anything else that can explain the difficulties. Today the requirement for a psychologist has been relaxed.

Memory tests, Rey-complex figure test and Rawens matrix is often used by more exhaustive investigations. Rawens matrix is a logical non-linguistic intelligence test. Rey-complex figure test shows if the spatial ability is defective.

When doing the survey it is important to remember that a poor educational experience can influence the results. In these cases the lack of previous education should be made up. If this does not help a deeper investigation shall be called upon as fast as possible so the required support can be provided as soon as possible.

Experiences from the surveys

Some of the students benefited directly from the survey as they were allowed to compensate difficulties in written tests and by doing so were able to attend higher mathematics classes. Other students have finally been offered support, although unfortunately not by remedial teachers. Another experience was the tests and the collected results made clear to the school management what students mathematical difficulties may look like and which help they might need. This has contributed to the decision that Komvux in Lund will hire a remedial teacher.

References


**Abbreviations**

- DSM: The Diagnostic and Statistical Manual of Mental Disorder
- ICD: The International Statistical Classification of Diseases and Related Health Problems (WHO)
- IPS: Intraparietal Sulcus
- ANS: Approximate Number System
- MLD: Mathematical Learning Disability
Increasing Accessibility of Mathematics Education through Open Sources

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Abstract
This article examines MyOpenMath, a free open-source software that is used in combination with open source textbooks, as an example of the broader issues related to open educational resources (OER). MyOpenMath is currently funded by Lumen Learning as part of their OER initiatives. A detailed look at MyOpenMath in the teaching of developmental mathematics shows strengths and weaknesses of the program, using the author’s experience in a face-to-face algebra class and as a complete online course.

Key words: open sources, mathematics, learning

Introduction
For several years I have been dissatisfied with modern developmental mathematics textbooks, in terms of content, size, and costs (Jorgensen, 2010). Also, the software that publishers provide for their textbooks, in my opinion, often have an unnecessary number of bells and whistles. The software is expensive as well. Yes, publishers will bundle a textbook with its respective software, or include a digital book within the software, but it is still expensive. Costs for higher education in the United States can be quite expensive and thus discourage the most disadvantaged from enrolling. In looking at what I might do to help, at least in a small way, I decided to look at open source options. For two semesters I have used MyOpenMath: two sections of face-to-face intermediate algebra, and one online course that combined beginning and intermediate algebra.

Background
MyOpenMath is funded by Lumen Learning which promotes open educational resources (OER). They claim reduced costs for students and higher success rates. Lumen Learning’s website states:

By replacing expensive commercial textbooks with free, high quality open content, open educational resources (OER) represent a largely untapped opportunity to make education more affordable, while at the same time improving student success.

Lumen Learning was founded to 1) expand a collective vision of what is now possible with OER, and 2) help K-12 and higher education institutions, faculty, and learners take full advantage of its benefits to cut costs and strengthen student learning. (Lumen Learning, n.d.)

The MyOpenMath program “was born out of free, open source software developed by David Lippman, a community college math professor in Washington State in the United States, starting in 2005” (MyOpenMath, n.d.). It runs on the open source, IMathAS platform. David Lippman and Lumen state:

Both feel very strongly that a free and open version of MyOpenMath should always exist, without advertisements. They are also committed that all the content remain open as well, so it
could be moved to a local install of the IMathAS software if desired or needed. To accomplish this, they are adopting a "freemium" model, where the basic service will remain free with community-based support for faculty, but additional services are available to institutions for a fee. Those services include instructor support, content support, and administrative services. Students will never be directly charged, as ensuring student access to high quality materials is our number one priority.

After hearing that a local community college had adopted a campus OER initiative, I decided to explore the mathematics options they had been using. I should note that my department adopts a textbook for all faculty to use and we have common final exams. Fortunately, my department is flexible and allows some individual faculty to experiment from time to time with other options. I received permission to try it out and spent some time over a two-week period to set up my first course. What follows is a practical look at what I experienced in using the program.

MyOpenMath

Course Set-Up

I found the program to be fairly intuitive. There is a training course with videos to help you get started. A feature I especially appreciated was a Quickstart section to get the basic information. Figure 1 is a screenshot of the Course Settings page. You set up a course name and an enrolment key which is sent to students, along with the course ID. Among the options is a Late Pass feature. This allows student who miss a deadline to still complete the work. I found this feature helpful because things come up in students’ lives that may hinder their ability to complete an assignment on time. However, having a limit on the number of late passes still encourages students to keep up with their work.

Content

Figure 2 shows how a course looks in MyOpenMath. There are several course templates with textbooks and questions that an instructor can copy. I chose the one that is closest to what my department covers and then I customized it. I can renumber, retitle, change the order, etc. I can even add additional material in various forms. One of my preferences is to add links to Khan Academy videos when they add helpful additional clarification.

It is possible to blend material from different open courses. For example, none of the intermediate algebra open courses cover equations of circles. Yet, this is one of the expected learning outcomes established by my department. So, I went into an open-source college algebra textbook’s conic sections material and copied the particular section on circles and included it in my course. This took only a few minutes.

The textbook within the software is formatted in a simple and readable format. As figure 3 shows, the course that I used has black and white pages. There are good explanations of concepts followed by examples. There were also numerous problems, with answers, for extra practice by students. Much of the content has explanatory videos built in.
Assignments

When you copy an open-source course from the MyOpenMath template, it has prearranged questions that can be used for assignments. In setting up the course you can preview the questions and decide whether or not to keep them. There are also other questions that you can add. I have noticed various degrees of rigor so you can choose the level of difficulty you would like to have.

There are numerous options including: point values, number of tries at a question (and whether or you want new numbers generated for each try), whether you present one question at a time or the entire assignment at once, when you want to provide a score or feedback with correct answers, whether a late pass can be used, etc.
A feature I like is a “Message instructor about this question” link that can be added to each question. If a student sends a message to me with a question on one of the problems, I can see the question and their response and reply back. Figure 4 shows a view of a question and you can see the message link (this is an instructor view, the “Show Answer” button would not be shown in student view).

Figure 2. Screenshot of Course Outline.

If I notice a glitch with one of the problems that has already been assigned (this has happened a few times), I have an option to withdraw it. Also, I can manually give a student credit for a problem (for example, if they had the correct answer but could not input it correctly) or reset it so they can try again with different numbers. I can also set it up so that each student gets different numbers for the same problem. Once an assignment has been completed, students can continue to practice the problems and new numbers are generated. Of course, I can assign various end and start dates to each assignment.

There is always a learning curve for students in how to input answers into any software system and that is the case with MyOpenMath. However, there is a preview button that allows a student to see how the program interprets what a student has typed in, before it is submitted. There are also helpful tips as to the format of the response. Additionally, there is access to mathematical symbols. Graphing and shading of graphs is fairly intuitive to most students.
There is a grade book that records scores. This can be exported. It is also possible to incorporate MyOpenMath into some learning management systems (e.g., Canvas) although I have not tried that yet.

**Example 53.**

\[ 4x - 20 = -8 \]

We have two numbers on the same side as the \( x \). We need to move the 4 and the 20 to the other side. We know to move the four we need to divide, and to move the twenty we will add twenty to both sides. If order of operations is done backwards, we will add or subtract first. Therefore we will add 20 to both sides first. Once we are done with that, we will divide both sides by 4. The steps are shown below.

\[
\begin{align*}
4x - 20 &= -8 & \text{Start by focusing on the subtract 20} \\
+ 20 + 20 &= \text{Add 20 to both sides} \\
4x &= -12 & \text{Now we focus on the 4 multiplied by } x \\
\frac{4}{4} &= \frac{-12}{4} & \text{Divide both sides by 4} \\
x &= 3 & \text{Our Solution!}
\end{align*}
\]

Notice in our next example when we replace the \( x \) with 3 we get a true statement.

\[
\begin{align*}
4(3) - 20 &= -8 & \text{Multiply 4(3)} \\
12 - 20 &= -8 & \text{Subtract 12 - 20}
\end{align*}
\]

**Figure 3. Screenshot of Textbook.**

**Other Features**

There are other features such as the ability to set up student groups and forums. It is also possible to email the entire class. There are calendar and learning outcome options as well.

**Advantages and Disadvantages**

From my experience there are a number of advantages that make it worthwhile to consider adopting MyOpenMath. First, it is free to students and the institution (unless you want to pay for additional support). Second, being relatively simple it is easily used on a smart phone. This allowed me to reply to students’ questions on the go. Students are also able to have their work with them all the time and work on it when they have a chance. Third, it is highly customizable so I can design it the way I want and not have to rely so much on the content and rigor of a single textbook and its accompanying software. Fourth, I like the way it makes it easy to communicate with students through its email system and through the message-my-instructor feature.
With the embedded textbook and videos, I have found that MyOpenMath works well for an online course as well as a supplement to a traditional face-to-face class.

There are some drawbacks. First of all, I have found some glitches in homework problems where correct answers are sometimes rejected. It is not frequent but there have been a few. As noted above, it is easy to exclude or withdraw questions. At the time of writing, there is a current problem where the notification system is not automatically notifying me when a student has a question.

Another drawback is that there are only a few open source textbooks in mathematics to choose from. So far, I have found everything I need but more options would be nice.

The biggest potential drawback is future support. Will the program continue to be maintained and improved over time? Right now the prospect looks good but commercial sources keep improving what their systems can do and faculty may gravitate towards those. I still maintain that the simplicity and accessibility offered through open educational resources makes it worthwhile over commercial products. Feedback from my students has been very positive.
References


http://lumenlearning.com/our-story/

Mathematics in the Gabonese Education System

Komlan Mensah
Gabon

In the Gabonese education system, mathematics is compulsory and is taught intensively from primary to secondary school levels and some university faculties of sciences, technics medical and mathematics school “math sup.”

Since 2009, UNESCO has been cooperating with foundation for peace and the ministry of Education, in promoting science Education through especially the organization of a National Mathematical contest “The Pythagoras” for grade 11 and 12 science Students. The access of youth to knowledge and particularly to science, technology and mathematics stand as a national priority, in accordance with the overall goal which is to build an Emerging Society. As a matter of fact, promoting science Education will contribute to the development of quality human capital needed for sustainable development and the advent of a knowledge society, composed of at least forty ethnical groups

At the most basic level, mathematics is very important and essential in the conduct of everyday life. In engineering, commerce, natural and social sciences advanced mathematical concept and techniques are indispensable tools for future career.

Mathematics teaching and learning strategies

The structure of lessons plan in the classroom is:
- Activities (Brainstorming questions and answers)
- Lesson (Teacher speaks, explains, rules, definitions…)
- Applications (teacher and students)
- Exercises (students’ assignment, homework)
- Tests (exams …, quiz)

The lessons are designed so that by their competition, students can write, solve, and explain their own mathematics problems, using notions, terms, concepts, vocabularies, illustrations, definitions, rules, theorems, axioms, formula. The National mathematical contests helps students to work hardly and best their knowledge by studying in small groups, lecture, discussion, games, debate
- For mathematic Olympiads tests

Mathematics in Adult education

In Gabon, there are two categories of adults who learn mathematics:
- Adult workers studying mathematics to earn university certificate for self improvement, promoting or job security young
- Learners of mathematics for access to higher education (mathematics superiors) math sup

Besides Gabonese researchers in mathematics (Research Laboratory of mathematics) ENS is in charges of three public universities
- University Omar Bongo
- University of sciences and Techniques of Masuku
• University of Medical sciences
• Elite and other high institutions as
• ENS (for research)
• IAI (Laboratory of Applied Mathematics and Computer Sciences.
• NB: Mathematics Society of Gabon (Société mathématiques du Gabon) is promoting sciences education in Gabon.

The Gabonese winner prize 1500
Institute TATA (INDIA)
RAMANUJAN 2011 president of mathematics Society of Gabon is Dr Philibert NANG professor of Mathematics.

Conclusion

In Gabon, the mathematics contest represents a real motivation teachers, students and parents the impact is felt throughout the country. Science Technology and mathematics are the national priority the Gabonese government, in accordance with the motto: Promoting school of Excellency for an emerging Gabon by science education at all level. The creation and promotion of mathematics clubs is high schools helps students to overcome “the black beast”, to understand explain and solve their “daily mathematical problems” in classroom and in the community!
Is it possible to integrate in the mathematical syllabi, our local languages or activities take from ethno mathematics practices?

Recommendation further action

The suggestion is to cover the gap between the past and the future by collecting examples from the traditional culture on one hand, and examples from information technology and software on the other hand. In doing so, a student may enrich his experience and enlarge his better understanding and learning of mathematics in Africa, particularly in Gabon.

References

UNESCO (2010-2011) 7e edition (World Data on Education)
Africa Institute for mathematical sciences South Africa. Babacar fall (2007) survey of ICT and Education in Africa Gabon country report
THE GABONESE EDUCATIONAL SYSTEM

The Gabonese educational system—French speaking country— is quite different from the English speaking countries system most especially American educational system, some of the differences are mentioned below:

- Chronology of studies in the Gabonese educational system
- Structure of the Gabonese educational system
- Presentation of administrative personnel of high school or “collège”.

I - Chronology of studies in the Gabonese system

- **Crèche et Garderie:** (Nursery School) (1 to 2 years)
  Safe child, child care. Day care is typically an ongoing service during the specific periods such as the parents’ time at work; but you can generally only find them in Libreville.

- **Ecole Marternelle** “Kindergarten” (3 to 5 years)
  A real good school, good introduction to learning lots of stories and songs, helps kids to learn French. It is composed of tree sections:
  - Petite Section (3 years)
  - Moyenne Section (4 years)
  - Grande Section (5 years).

- **Primary School:** It is composed of three sections:
  - CP (Cours Préparatoire 1 et 2)
  - CE (Cours Elémentaire)
  - CM (cours moyen 1 et 2)

  These classes are all taught in French language; French will be one of the subjects taught, at least for the first couple of years. The school year is divided into three trimesters of three months duration (from September to July).

  Pupils or students are graded on a scale of 0 to 10; 5/10 is a passing grade, but more often than pupils don’t achieve it. There is much higher rate of failure here than in the United States, the reasons for this are: the poorer quality of instruction, the greater severity of grading, and the lack of emphasis on education as opposed to the other duties and distractions in a pupil’s or student’s life.

  **NB:** Special problems at this level:

  There’s shortage of trained competent teachers (as well as a shortage of schools); many good teachers don’t want to leave Libreville, the administrative capital; when they are assigned for a small village; they aren’t paid, nor housed very well; as a result, they show up late - in the school year - and leave early. The quality of instruction varies but is generally low - teachers are often sent out without proper preparation, personal problems lead to excessive drinking, poor planning and lack of professionalism and strike.

  - Concours d’entrée en 6e : at the end of the primary school (CM2), students take a standard national exam which allows them to continue their studies in secondary schools.

- **Secondary school:**
A- Premier cycle: (6e ; 5e ; 4e ; 3e)

In this section, we have four levels: 6e, 5e, 4e and 3e. These classes are thought in both lycées and “collèges”, the difference being that lycée continue through the second cycle and are found in larger (generally provincial capital) cities, have more teachers, students and subjects taught. “Collèges” are found in most towns or cities of any size; they have usually between 600 and 2000 students and lycées, between 1000 and 3500 students. Since the students come from villages they’re sometimes living away from home either with relatives, in dorms, or totally unsupervised.

Again, there’s a very high rate of failure at these levels; kids have other interests, not all the teachers are good; sometimes students are passed to the next class anyway, especially if they’re relate to someone important.

In general, students are supposed to stay in school until they’re 16 years of age. However, many don’t. If a student fails the same grade twice or the first time with badly grade (less than 7/20) he’s allowed to repeat the class or changes school because in “collège” or lycée the passing grade is 10/20”. Also, many students just voluntary drop out of school – many girls get pregnant, and either quit or fail; they may also be working on a plantation or running a household. Boys are generally unsupervised and uninterested (or greatly interested in other things like getting girls pregnant, playing soccer, smoking and drinking) and just quit going to school. However, even if a student is kicked out of school for absenteeism, conduct or poor grades, he sometimes simply shows up in another school in village with fake papers and resumes his studies. Students who pass each trimester receive a certain sum of money (24.000 FCFA = 40 USD) from the government, for encouragement and to defray the expenses of their education – to make up to the family for the loss of wages or help on the plantation. It’s not a big sum of money USD40 but it’s sometimes very important to the family. Still, students don’t seem to think about it until the end of the trimester, when it’s too late… then they or their parents will show up at your house and ask you to change their grades!

In some schools, mostly private or religious, report cards are distributed every month and the third month of a trimester students take a comprehensive exam. The quality of education can vary greatly from school to school because of the materials you have to work with the support of administration (creating a working atmosphere, providing materials, overseeing discipline…)

The quality of the teachers … some people’s idea varies according to the native intelligence of the predominant tribe in the region.

Different kinds of schools which exist: public, private (catholic, protestant…) technical, vocational school and ENS “Ecole Normale Supérieure” for secondary school teachers training and ENI “Ecole Normale des Instituteurs” for primary school teachers training and for technical training teachers ENSET “Ecole Normale Supérieure de l’Enseignement Technique” and universities.

At the end of 3e academic year, the last year in the “Premier Cycle” students take another national exam, the B.E.P.C, which permits them to continue on the “Second Cycle”. By the time they get to 3e, students should be able to carry on a decent conversation in English, compulsory in their final oral exam. The percentage of students who passed the B.E.P.C exam can vary from 0 to nearly 100.

B- Second cycle : (Seconde, Première, Terminale)
These classes are taught in lycées. Only a very small percentage of students gets this far, usually more boys than girls. At the end of Terminale, students take another series of exams called the "BAC", successful completion of second cycle which confers both a diploma and the right to continue on to the university.

At this point, a student should be able to read and write just about fluently in English, and to carry on a normal conversation.

There are fewer students than ever at this level; classes are therefore smaller, and students are generally more serious, so the pass ratio is somewhat better – still, the BAC is a difficult exam, and even a pretty good student could have a very hard time passing it.

At the beginning of the “second cycle” students are placed in one of four or five different programs (generally divided between arts and sciences), according to what they’ve been good at before, with little or no importance being attached to what they want to study. This type of channeling continues through the university.

II – Structure of the Gabonese educational system

The Gabonese system of education is, for historical reason, closely modeled after its French counterpart. There are twelve years of pre-university education, divided as follow:

A/ Primary school “Ecole primaire”

Five years of instruction (2003 new reform) sometimes preceded by la “Crèche ou Garderie” Nursery school or Kindergarten and les “classes maternelles”.

Classes at Primary school:

- Cours préparatoire (two years) : CP1, CP2
- Cours élémentaire (one year) : CE
- Cours moyen (two years) : CM1, CM2

B/ Secondary School (Enseignement Secondaire)

1- Premier cycle: four (4) years of education.
Classes called in ascending order:

Sixième (6e), Cinquième (5e), Quatrième (4e) and Troisième (3e)

Students go to “Collèges or lycées”

2- Second cycle: three (3) years of education
Classes called in ascending order:

Seconde (2nde) Première (1ère) Terminale (Tle)

Students go to “lycée, Mission Schools, vocational or technical schools”

NB: Different kinds of secondary schools.

- Lycée public (seven years, premier and second cycles) usually located in provincial capitals.
- Collège d’Enseignement Secondaire (four years, premier cycle only)
- CES public, usually located in smaller town.
Figure 1 Science and Technology students’ diagnostic test performance in 2008 by sub-category.

Figure 2 Science and Technology students’ service mathematics performance in 2008 by sub-category.

**Note:** 75/382 Ordinary Level students and 52/239 Higher Level students did not sit the diagnostic test. 29/382 Ordinary Level students and 9/239 Higher Level students did not sit their service mathematics examination.
6. Discussion

6.1 Discussion on Traditional and Mature Students’ Performance in the Diagnostic Test and Service Mathematics

Mature students are sometimes said to be deficient in the basic skills needed for effective study in higher education (Richardson 1995). They have also, in general, studied significantly less mathematics than their younger counterparts (Relich et al 1994). They tend to perform poorly in mathematics diagnostic tests on entry to higher education when compared to the standard students due to numeracy problems (O’Donoghue 1995; O’Donoghue 1996; Kaye 2002; Maguire et al 2002). Several reasons which have been suggested for this poor performance include:

- Being out of practice with formal mathematics (O’Donoghue 1999).
- A lack of confidence in a formal education setting (Bowl 2001).
- Weak mathematical backgrounds on entry to higher education (Golding and O’Donoghue 2005).

The initial challenges which mature students face however are likely to have been counteracted by their motivation to succeed. Mature students have been found to be more motivated by intrinsic goals than younger students in higher education (Pierce 1995; Forgasz 1996; Murphy and Roopchand 2003). In addition to this mature students tend to exhibit more desirable approaches to academic learning. They are less likely to adopt a surface learning approach in higher education than younger students. Younger students tend to acquire a surface learning approach to education due to their reliance on it in their final years of secondary school. The prior life experiences of mature students however promote their tendency to adopt a deep learning approach over a surface learning approach (Richardson 1995; Jacobs and Newstead 2000; Smith 2002).

As well as their tendency to adopt more desirable approaches to academic learning, mature students have been found to frequently avail of mathematics support services (Gill and O’Donoghue 2007). During the academic year 2008, UL provided support tutorials specifically for mature students in addition to one-to-one consultations with members of staff in the mathematics learning centre. The MLC data in UL highlights the frequent use of the centre by mature students in all service mathematics courses. Data collected over the time period 2009-2011 found that typically over 50% of attendances to the MLC are made by mature students (O’Keeffe 2011). Engagement with support services such as these have been found to have a positive impact on service mathematics grades for those students (Mac an Bhaird et al 2009; Gill et al 2010). Their research gives an insight into mature students’ progress with mathematics over the course of the semester.

7. Conclusions

There was an increase of 6.4 and 3.5 percentage points of mature students in Technology and Science mathematics combined between 1998 and 2008. The increase in mature students was matched with a decrease in the percentage of students entering service mathematics in UL with Higher Level Leaving Certificate mathematics. The changing profile of Science and Technological mathematics students between 1998 and 2008 was found to be a major contributing factor to the declining standards in mathematical competency as measured by the UL diagnostic test of students entering UL (Faulkner et al 2010; Faulkner et al 2013).
The analysis of traditional students’ performance against mature students’ performance revealed that mature students’ diagnostic test score was below that of the traditional students however their mean service mathematics performance was higher than that of Ordinary Level traditional students and only slightly below that of Higher Level Leaving Certificate Students. This finding was surprising as students’ performance in the diagnostic test has been found to be predictive of their service mathematics examination performance (Faulkner et al 2013). This unexpected finding was found to be partially due to mature students improving in their mean mathematics performance over the course of the semester and Ordinary Level students not making the same improvements. Potential reasons examined for such improvements in mature students’ performance can be summarised as follows:

- Their motivation to succeed (Murphy and Roopchand 2003).
- Their tendency to adapt desirable approaches to learning (Smith 2002).
- Their frequent use of available mathematical support services (Gill and O’Donoghue 2007).

Mature students improve greatly in their mathematical performance over time for many reasons. We should learn more from this than just the fact that mature students are likely to be resilient and motivated if we want to improve retention rates along with the quality of the output of our undergraduate population.

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- CES privé, protestant or catholic secondary school sometimes, continues through second cycle.

- Collège / lycée d’Enseignement Technique et Professionnel (three years)
  Students take the same general courses as well as mechanics, woodworking, Electricity, Business, Management, commerce…

**NB1-Teachers schools**

Ecole Normale des Instituteurs (ENI) for primary schools teachers training;
Ecole Normale Supérieure (ENS) for secondary schools teachers training;
Ecole Normale Supérieure de l’Enseignement Technique (ENSET) for technical schools teachers training.

**NB2-National Examinations**

At the end of Primary School, the Premier Cycle and the Second Cycle, all students must pass a national exam in order to receive a diploma. The exams are written and/or oral on each subject studied during the year. The student receives one final grade which is a weighted average of grade in each subject. A passing grade is 50%.

1. **C.E.P.** (Certificat d’Etudes Primaires)
   Examination at the end of Primary school. Those failing must repeat CM2 or else drop out. They may also take the Concours d’Entrée en Sixième (6e) in order to enter Secondary School in a Collège or a Lycée.

2. **B.E.P.C.** (Brevet d’Etudes du Premier Cycle)
   Examination after your year of Secondary School (classe de Troisième)
   The percentage of students who pass varies widely from school to school, from 35% to almost 100%. If a student has had passing grades during the year but fails the B.E.P.C he passes nonetheless into seconde (first year of second cycle). On the other hand, if he passes the B.E.P.C but he has had insufficient grade during the year, he must repeat the classe de Troisième.
   English subjects at B.E.P.C are oral exam. The mathematics, sciences and other subjects exams are written.

3. **BAC.** (Baccalauréat)
   Examination at the end of the second cycle (classe de Terminale).
   Begins with written exams, which are followed by oral exams. Students who score from 50% and over pass.
   Those who score between 40% and 50% must take additional oral exams (Epreuves orales de contrôle), in order to receive the Baccalauréat diploma.
   Note that going to school and successfully passing courses do not guarantee a diploma. The BAC is the exam which means everything without it; it is nearly impossible to go on to the University, unless you pass Special Entrance University exams in Economics or laws only.

**NB3- Secondary School Education“‘L’Enseignement Secondaire”**

A/ Premier cycle (in a CES or a Lycée)
All students in a given class take the same subjects: Français, Mathématiques, Histoire-Géographie, Anglais, Science de la Vie et de la Terre, Instruction Civique, Education Physique et Sportive, Physique et Chimie. In the second two years (4ème and 3ème), Espagnol or another language might be added: Arabe.

**B/ Second Cycle (in a Lycée or certain catholic and protestant collèges)**

In addition to being divided into three classes, second cycle students are also separated by “Series”. The courses that a student takes are determined by the serie he is in. The student is placed in a serie at the beginning of his first year (classe de Seconde). Students are in either:
- 2nde LE (Literature and Economics) or
- 2ndeS (Scientifics)

Further orientation occurs in Première. At most lycées publics, series A, B, C, D are common:
- Series (A1-A2): emphasis on literature and languages
- Serie B: emphasis on general education and economics
- Serie C: heavy emphasis on mathematics and sciences
- Serie D: heavy emphasis on natural Science, Physics, Chemistry and mathematics.

There are other series, most of them technical in nature, which are programmed in economics, engineering electronics (F2) mechanics (F1)/(E), Business accounting (CG), Office management (ACA)/commerce(ACC)

These are taught in technical high school “Lycées techniques”. There are at present three Lycées techniques in Gabon.

The Ministry of Education places great importance now, on training Gabonese technicians and engineers.

**III- Secondary school administrative**

**A/ School officers**

1. Proviseur: Head of a lycée  
   Principal: Head of a CES public  
   Directeur: Head of a CES privé or lycée Technique

2. Censeurs : Assistants to the proviseur (vice-principal)  
   Censeur1: Assistant1 in change of Premier Cycle  
   Censeur2: Assistant2 in change of Second Cycle

They are in charge of academic matters, scheduling and of overseeing national examinations held in his school.

   Censeur vie scolaire: in charge of students’ school problems.

3. Surveillant Général: in charge of student discipline. He may have some assistants, called surveillants adjoints if the school is large.

4. : Intendant: Financial officer in charge of budgeting, expenditures and of dormitories (internats)

5. Sécrétaire: school secretary in charge of registration, school files.

6. “Planton: office boy or “gopher”
It is a good idea to ask about the exact organization of your school when you are introduced to Proviseur, Principal or Director.
**B/ Class officers**

1- Professeur titulaire/principal:
Teacher in charge of any class problems and of calculating averages at the end of each trimester.

2- Chef de classe:
A student chosen because of high grades or elected by his classmates to serve as a liaison between the class and the administration, to maintain order in the absence of a teacher, to keep attendance records… also called the “responsable de classe”.
Investigating the Impact of Literacy Skills in the Adult Mathematics Classroom

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Abstract
There has been a focus in recent years on the high levels of literacy that are being demanded in the mathematics classroom. Increasingly, learners are required to interpret word-based mathematical problems and provide written explanations for solutions. There is concern that, as a result of the language-intensive components of these tasks, learners with weak literacy skills are not able to answer questions of which they are mathematically capable. This study aims to investigate this issue from the perspective of adult learners. Participants completed two examinations: the first consisted of mathematical word problems while the second abstracted the mathematical procedure from the word problems by removing extraneous language and context. The results show that participants performed significantly better in the second case.

Key words: Literacy skills, adults, mathematics

Language, Literacy and Mathematics
Mathematics can be recognised as a language in its own right, a language which has its own vocabulary, grammar, symbols and punctuation (Ellerton and Clarkson, 1996). The teaching of mathematics, however, takes place within a spoken language, such as English (Zevenbergen, 2001). This spoken language is an essential element of the teaching and learning of the subject (Gorgorió & Planas, 2001). It is the vehicle for communication within a mathematics classroom and provides the tool for teacher-student interactions (Smith and Ennis, 1961). Language permits mathematics learners to ask and answer questions, to convey their understanding and to discuss their answers with others. It also plays a significant role in the processing of mathematical text and the interpretation of questions (Hoosain, 1991).
Changes to Second Level Mathematics in Ireland

In September 2010, in light of a number of concerns regarding students' performance in mathematics at all levels, the Irish Government introduced a national initiative called Project Maths. This initiative is a major reform of second level mathematics education. The overall aim is to teach mathematics in a way which leads to real understanding (Department of Education and Skills (DES), 2010). Project Maths involves changes to what students learn in mathematics, how they learn it and how they will be assessed. The initiative is designed to ensure an appropriate balance between understanding mathematical theory and concepts, and developing practical application skills. There is a much greater emphasis placed on students' understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to their everyday experiences.

Project Maths was introduced nationally in second level Irish schools in September 2010. The new curriculum, which identifies five strands of mathematics (Statistics and Probability, Geometry and Trigonometry, Number, Algebra and Functions), was implemented using a phased approach over a number of years, and the assessment in the examinations was adapted as each strand was rolled out. The assessment reflects the increased prominence of problem-solving and applications in the teaching and learning of mathematics and there is a greater emphasis on reading and understanding the problems. Despite emerging evidence of the positive impacts on students' experiences of learning mathematics, many challenges remain, chiefly with the implementation process and the availability of teaching and learning resources. However, further concern has also been expressed regarding the perceived literacy demands of the revised mathematics syllabus (Cosgrove et al., 2012; Jeffes et al., 2013).

Concerns Regarding the Perceived Literacy Demands

The greater stress on word problems and applications in Project Maths has led many teachers to express concern. They feel that students with low literacy levels and students for whom English is not a first language are struggling with comprehension of the material and the wordy nature of some of the questions: ‘the language used when phrasing a question poses a major problem for students whose literacy skills would be weak, they can therefore not answer a question they are mathematically capable of doing! This is a major issue!’ (Cosgrove et al., 2012:72). Many students (including those studying at Higher Level) have also expressed difficulties with interpreting word-based problems and with providing written explanations for their solutions to mathematical problems (Jeffes et al., 2013). Students also appear to lack confidence when asked to draw conclusions from a considerable amount of written information (Jeffes et al., 2013).

Context for the Study

For the past fifteen years funding towards effecting a significant increase in the number of students from lower socio-economic groups participating in higher education has been provided by both public and private sources in Ireland. (National Plan for Equity of Access to Higher Education 2008-2013). As a result, 15% of all first-time entrants to higher education in Ireland are now mature students, with numbers continuing to rise (Higher Education Authority (HEA), 2015). For example, the Dublin Institute of Technology (DIT)’s ‘Access Student Strategy’, which aims to ensure wider participation and equality of outcome in higher education, has as its target for 2020 a mature student quota of 20% of total student numbers, in addition to a young adult Access student quota of 7% of total student numbers (DIT, 2010). The growing number of Access students in higher education has been linked with increased literacy problems amongst students (DES 2005). It has also coincided with the introduction of Project Maths in second level
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schools with its afore-mentioned emphasis on literacy and language. This research aims to investigate the effect (if any) which Project Maths can have on Access students' mathematical performance and to view the initiative from their perspective.

The Study

Access Foundation students in the DIT are mainly mature students (23 years of age or older) and young adult students (below 23 years of age) from socioeconomically disadvantaged and educationally disadvantaged backgrounds. In essence, Access students are mainly “non-traditional” students. Schuetze and Slowey (2002) state that with regard to the framework of equality of opportunity, the term “non-traditional” tends to refer to socially or educationally disadvantaged sections of the population, which includes those from working class backgrounds, ethnic minority groups, immigrants and in certain cases, women.

Methodology

The authors decided to use a mixed method approach by combining both qualitative and quantitative methods of research. The use of multiple methods was decided upon in order to get an in-depth understanding of the research. The study evaluates Access students’ opinions of Project Maths and compares their scores in a traditional style mathematics examination (which reflects mathematics education in Irish second level schools prior to the implementation of Project Maths) with their scores in a Project Maths style examination.

Methodological Consideration: Comparability of Test Questions?

When conducting this research the authors were conscious of the fact that the questions in the Project Maths style examination may be deemed to have a higher level of mathematical sophistication than the traditional style examination questions. The ‘Adult Numeracy Concept Continuum of Development’, which was developed by O’Donoghue and Maguire (2002), demonstrates that conceptual understanding of adult numeracy is a three-phase continuum in which the level of sophistication increases from Phase 1 to Phase 3 (see Figure 1).

\[ \text{Figure 1. A continuum of development of the concept of numeracy showing increased level of sophistication from left to right (Maguire and O’Donoghue 2002)} \]
In the context of this continuum, the traditional style examination questions align with Phase 1 and the Project Maths examination questions align with Phase 2. The authors acknowledge the value of using examination questions that can be directly compared within the framework of this model, and indeed would recommend that future research incorporate such considerations. Clearly there is an opportunity here for further investigation with a focus on evaluating any confounding effect of using different phase questions on differences in performance on the two examinations. However the authors designed their research to mimic the current shift in state mathematics assessment in Ireland. This shift has increased the literacy demands on second level students in an education system which is effectively labelling each style of question as the same, thus a strong case can be made for direct comparability.

Participants

The participants in this study were fifty Access students who were enrolled in a year-long Foundation Programme in an Irish Higher Education Institute (HEI). The study took place in the 2014/15 academic year. Mathematics is one of six core subjects that all students are required to pass, along with two elective choices, in order to complete the programme. Upon successful completion of the programme, students are granted direct entry onto an undergraduate programme of their choice in the HEI. The aim of the programme is to equip them with the skills to meet the minimum entry requirements of such undergraduate programmes.

Of the participants, 75% were male and 25% female. The majority (78%) were Irish nationals with 71% speaking English as their first language. Other nationalities (such as German, Russian, Congolese and Somali) accounted for 22% of students. Ages ranged from 17 to 54 years with a median age of 31 years. All of the data was collected by the authors in 2014 in the participants’ first semester of the programme.

Quantitative data

In order to get a quantitative measure of the effect of Project Maths on Access students the authors decided to compare the scores of students in a Project Maths style examination with their scores in a traditional style mathematics examination. Each examination consisted of ten questions from the Junior Cycle Number strand and each question was taken from Irish second level textbooks and previous State examination papers. Students had fifty minutes to complete each examination. The questions based on the Project Maths method of assessment reflected the emphasis on understanding, problem solving and applications. The questions in the traditional style examination were technically the same questions but had numbers changed and were mathematical procedure and skill-based only with the removal of any context or language. For example:

Project Maths Style Examination Question:

Usain Bolt, the fastest man on earth, has a stride length of \( m \) when he is at full stride. In a 100m sprint, how many strides would Usain take to cover the final 30m when he is at full stride?

Traditional Style Examination Question:

Evaluate \( 46 \div \frac{2}{3} \)

Students completed the traditional style examination first and then the Project Maths style examination directly afterwards. Ten marks were awarded per question. Each student received a mark out of 100 for each assessment.

At the end of the Project Maths style examination, there were also three closed-ended questions. The questions explored which examination the participants preferred, which examination they
found more difficult and whether their English language skills had had an impact on their performance in the Project Maths examination.

Qualitative data

In addition to the three closed-ended questions at the end of the Project Maths style examination, there were also a number of open-ended questions which all participants were invited to answer. The questions enquired about the main differences between both examinations, the students' opinions of Project Maths and what could be done to help Access students become accustomed to the changes brought about by Project Maths. The responses to these questions were transcribed, analysed and arranged into themes by the authors.

Results and Findings

Quantitative data

A paired-samples t-test was performed on the pairs of examination scores. The mean score on the traditional examination (M:47.44; SD:19.44) was found to be statistically significantly different (t(df=49)=2.717, p=0.009) to that on the Project Maths examination (M:41.94; SD:19.54). See Figure 2 below for a comparison of the mean scores. A 95% confidence interval for the mean difference on the tests for students on this Access programme was calculated as (1.918, 10.199). The effect size given by Cohen's d is 0.38.

Figure 2. The mean scores on the tests were statistically significantly different. The plot shows the mean scores with 95% confidence intervals for the mean performance of Access Programme students in the Higher Education Institute on each test.

The response rate on the three closed-ended questions was between 70% and 86%. Of those who responded 54% preferred the traditional style examination, 62% found the Project Maths examination more difficult and 89% believed that English language skills were an important factor in their performance in the Project Maths examination.

Qualitative Findings

Qualitative data analysis was carried out on the Access students’ responses to the questionnaire data and several strong themes emerged under each question which provides further insight into students’ quantitative performances in the Traditional and Project Maths examination papers.
Upon analysis of the question “In your opinion, what were the main differences between the Project Maths and the Traditional Style questions?”, three themes emerged from the 41 responses. The dominant distinction which students made between the two examinations, which appeared in 63% of student responses, was that the Project Maths examination used words and involved analysis, thought and real life context while the Traditional examination was seen as being much easier and “just numbers”. One student summarised this viewpoint by stating: “Project Maths is full of reading and more thinking while traditional is very straightforward maths”. A smaller proportion of students (22%) made the distinction between the examinations in a similar way while stating that the Project Maths was better despite the fact that it was considered more difficult as it was “useful – it allows you to think about a real situation – traditional is the opposite.”. The final theme which emerged in terms of the differences between the two examination papers was mentioned in 7% of student responses which stated that the Traditional mathematics was familiar to them and the Project Maths was not: “Project Maths is confusing- I’m familiar with traditional maths”.

Students were also asked “What is your opinion of Project Maths?”. Upon analysis of this data (for which there were 43 respondents), three major themes emerged. Interestingly, the dominant opinion on Project Maths mentioned by 58% of the Access students was their belief that Project Maths is better than traditional maths as it encourages real understanding of real life contexts. One student stated that “it allows for a better understanding as you could be familiar with the scenario, it’s not just symbols”. Of the students, 16% reported that they found Project Maths to be difficult because of the language used in it: “I think it’s good but they should use visual aids too to help people who struggle with text” with another students stating that “it’s very unhelpful if a student is dyslexic or has attention difficulties”. The same proportion (17%) of students noted that they found Project Maths difficult for reasons relating to basic arithmetic that they struggle with leaving it difficult to tackle the word problems “…I struggle with fractions so that was an issue for me”. One student detailed their general frustration with Project Maths being done to a “difficulty understanding what needs to be done” and finding it very “time consuming”.

The final question that students were asked that resulted in the emergence of one significant theme was “What can be done to help Access students become accustomed to the changes brought about by Project Maths?”. Of respondents to this question, 71% mentioned the facilitation of more practice for Access students with this type of mathematics in the form of homework, assignments or practice in class “we need more time to learn and lots of interaction – bring back the fun!”. Two students requested that maths vocabulary could to be taught to help them decode the Project Maths problems a bit more strategically. One student suggested that basic arithmetic and algebra needed to be strong before students could tackle Project Maths problems with another student backing this up by stating that “a mixture of both the traditional way and the Project Maths way” would be best.

The qualitative analysis of the students’ questionnaire data supports the quantitative findings that students find the Project Maths examination more difficult while also providing some further insights into why this might be. Although difficulties with the Project Maths paper are expressed by several students it should be noted that almost 60% of students supported Project Maths as a better way of teaching and learning mathematics allowing for real understanding of the material to take place in spite of the difficulties expressed. The qualitative findings allow us to see the complexity of the issues that Access students are likely to have with the Project Maths style, teaching and learning in an Irish context.
Discussion: Challenges faced with Language and the Learning of Mathematics: Supporting our Students

Along with the findings of the ‘Research into the impact of Project Maths on student achievement, learning and motivation’ (Jeffers et al., 2013) and ‘Teaching and Learning in Project Maths: Insights from Teachers who Participated in PISA 2012’ (Cosgrove et al., 2012) reports, this study highlights concern for learners in how they manage the literacy demands of Project Maths. Statistically significant differences were found in the results of student scores in a traditional style mathematics exam with their scores in a Project Maths style exam. To support this, 89% believed that their English language skills were an important factor in their performance in the Project Maths examination. In effect, language and literacy skills had acted as a barrier to the learning of mathematics. These findings highlight the importance that literacy skills have on the teaching and learning process. A learner can have excellent mathematical ability but this is futile unless they can competently communicate and understand the language in which they are being taught and examined.

However this is not just a problem for adult learners. Primary school children’s difficulties with mathematics have been summarised under four main headings: memory difficulties, language and communications difficulties, literacy difficulties and difficulties with low self-esteem (Krick-Morales, 2006). Language and literacy skills therefore play a key role in the biggest challenges for students trying to learn mathematics at any age. Much of the research in the area of mathematics education emphasises the importance of enabling students to use mathematical language effectively and accurately. The development of such a skill involves an ability to listen, question, discuss as well as read and report (Into Learning, 2015). All of these skills are now at the core of the reformed mathematics curriculum in second level education in Ireland so it is more important than ever that an importance is placed on the expression of mathematical ideas in order to develop mathematical concepts (Jeffers et al., 2013). One of the reported causes of failure in mathematics is poor comprehension of the words and phrases being used. Some of the language used within the mathematics classroom has dual meanings in everyday life and some of the vocabulary will only be found in a mathematical context (Halliday, cited in Pimm, 1987). Both of these vocabulary types can cause confusion to the learner in their own right so as mathematics educators we must familiarise ourselves with the mathematics register and how imperative it is to use precise language when teaching mathematics (Khisty & Chevl, 2002).

Discussion plays a significant role in the acquisition of mathematical language and in the development of mathematical concepts. Our students may be helped to clarify ideas and reduce dependence on the teacher or lecturer by discussing concepts and processes with other students. Discussion with the teacher or lecturer has also been found to be extremely useful. Advice has been given from a research perspective that the teacher or lecturer should assist students, as the need arises, by supplying the students with the mathematical language necessary to help them to express or clarify their ideas more accurately (Khisty & Chevl, 2002). This enables students to clarify mathematical ideas particularly where context could be causing difficulty in the formulation of ideas (Gibbs & Orton, 1994). Again, from the perspective of a young child learning mathematics, recommendations are made that the use of symbols and mathematical expressions should follow engagement with oral reporting and discussion of the mathematical topic in questions. Such an approach could be adopting in an adult mathematics education context also (Into learning, 2015).

Conclusions

Mathematical ideas are understood by making connections between language, symbols, pictures and real life situations (Haylock & Cockburn, 2003). Research into young childrens’ mathematical development found that without sufficient language to communicate the ideas
being developed, to interact with peers and their teachers, mathematical development can be seriously curtailed (Perry and Dockett, 2005). The same developmental issues in mathematics must be considered in light of the findings within this research in which students, some of whose first language is not English, with others having poor literacy skills, are attempting to engage with word-heavy mathematical questions.

Language in mathematics has been shown to result in difficulties for students whose first language is English due for example to unfamiliarity with contextualised mathematics problems or issues relating to ‘miscues’ in word problems (e.g. 25 would be a common answer to a question such as ‘John has now collected 18 tokens. That is 7 more than he has last week. How many did he have last week?’) (Haylock and Thangata, 2007). Such challenges are evident from the research detailed in this paper and they appear to be amplified for Access students studying mathematics where English is not their first language or they have been out of formal education for many years and basic literacy is an issue.

**Future Research**

A natural generalisation of this research would be to expand it to include students on Access programmes in Higher Education Institutes across Ireland. In addition it would be of interest to collect and analyse data on contributory factors to the participants' language ability such as their English language proficiency and whether they have learning disabilities such as dyslexia. Finally it would be very worthwhile to reconcile the research with the ‘Adult Numeracy Concept Continuum of Development’, as discussed in the Methodology section.

**References**


ALM As Chameleon:
Tracing the Evolution of the Organization through Its Literature

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Abstract:
Adults Learning Mathematics – A Research Forum published its first conference proceedings in 1995. Since then the organization has published proceedings of the annual conference and, since 2005, a peer-reviewed journal. The articles contained in both publications reflect the changing landscape of adult mathematics education over two decades and the adaptation of the organization to shifts in clientele, global economics, and national policy fluctuations.

This paper revisits the content of the proceedings from 1994 to 2013 in order to identify the themes that have ebbed and flowed over the years as well as those which remain central to the conversations of the researchers and practitioners who comprise ALM. Paraphrasing the theme of the conference, this paper hopes to open the eyes of the reader to ALM past in order to propose a vision for its future.

Introduction
I have been a member of Adults Learning Mathematics – A Research Forum (ALM) since its inception and have attended every annual conference. While 22 years is just a drop in the historical bucket, this score of years has seen dramatic swings in the politics and economies of the countries represented by the membership. This has resulted in a shift in viewpoint about the purpose of education, at times narrowing the vision to employability rather than education of the “whole person.” In fact, one proposed measure of tertiary institutions in the United States is the salaries of the institute’s graduates.

Reflecting on the focus of ALM over the years I perceived that it too had shifted. Being a mathematician at heart, I felt the need to confirm or disprove that perception with numerical evidence. At the same time, a numerical analysis served the purpose of identifying under-researched areas within our field, concrete numbers that might inform and direct future research, mine and others. Taking an opposite viewpoint, it would be equally valuable to determine well-researched topics that could support research proposals. This paper aspires to provide the reader with a qualitative analysis of the quantitative results.

Methodology
Earlier research on doctoral dissertations and journal articles identified “themes” for adult mathematics education research (Safford-Ramus and Rotondo, 2012). These earlier themes were used to create the rows of a spreadsheet whose columns were the ALM conference years. The doctoral themes proved to be an imperfect fit for the articles from the ALM proceedings and a few themes had to be added to the list. Each article from the proceedings was read to determine its primary theme. A pen and pencil tally was kept and the total entered into the spreadsheet. Some themes from the original template were not used and were discarded. In the end there were 22 themes identified as primary or secondary. These were then grouped into six thematic areas: classroom, affect, teacher, student, citizen, and theoretical framework. The totals can be found in Appendix A.
Findings
The Conferences
While creating the spreadsheet, I found that the conferences fell naturally into four clusters. During the first five years (1994-1998), the organization was finding its footing and conference sessions explored definitions of numeracy and the research field “adult mathematics education.” The first four conferences had no specific theme. The papers conveyed a strong flavor of learning for empowerment. There was a robust practitioner involvement and participation. Memories flooded back of the excitement we all felt at finding kindred spirits interested in the striking and the nuanced differences between pedagogical and andragogical mathematics instruction.

The conferences from 1999 to 2004 advanced the groundwork that was laid in those early years. This was a period of growing government support for adult mathematics education. The economies were strong in most member countries and adult mathematics education benefitted. ALM as an organization was gaining a foothold in the research community. Members were being invited to serve on government panels and this was reflected in presentations at the conferences. It was during this period that the organization registered as a charitable company in the United Kingdom.

The conferences over the next 6 year period had two central foci. The 2003 and 2004 conferences focused on the learners as householders, workers, and parents. The conferences in the years 2005 through 2007 circled back to the heart of the organization – connecting practitioners and researchers. This is highlighted in the themes: “Connecting Voices” in 2005 and “Crossing Borders” in 2006. In 2007 the organizers invited participants to take a deep breath and look back at how far we had come with an invitation to map out a future path for adult mathematics education. The 2008 conference returned to the learner, particularly his role as a citizen in a democracy and the empowering value of numeracy for that role. Then the global recession hit.

The role of the learner as a worker came to the forefront in the titles of the conferences in 2009 and 2010, *Numeracy Works for Life* (2009) and *Maths at Work – Mathematics for a Changing World* (2010). This reflected the economic climate and the need to upgrade or alter the numeracy skills of the workforce. However, the final 5 conferences moved towards a view of the learner from a holistic standpoint. They situated her in the world and invited the learner to see the mathematics in their daily life and in her surroundings. This is not a new idea – threaded throughout all the proceedings is the task of making the invisible mathematics in student lives visible. But these latest conferences made the goal specific in their titles: *Mathematical Eyes: A Bridge Between Adults and the World and Mathematics* (2011), *Adults Learning Mathematics – Inside and Outside the Classroom* (2014) and *Opening our Mathematical Eyes to See Maths in Everything* (2015). That “world” is all enveloping and includes the trinity of work, household, and citizenship.

The Proceedings
The Adult Mathematics Classroom
32% of the articles in the proceedings were directly connected to instruction and assessment. While there were peaks and valleys in the area of curriculum, the assessment and mathematical thinking themes persisted at a steady rate through the years.
Citizenship
Presentations focused on education for citizenship accounted for 24% of the articles but their intensity varied wildly over the 22 conference proceedings. It is interesting to note that critical numeracy, numeracy for empowerment, continues as a topic. European Union members are currently grappling with a flood of refugees. Numeracy education will play a role in the massive enculturation effort required to absorb these new arrivals into political societies that are different from those left behind.

Theoretical Framework
Both research and practice should be rooted in recognized theories of the ways in which students, in particular adult students, learn. 20% of the articles addressed topics that defined the field or were concerned with research methods or adult learning theories.

Teachers
10 percent of the articles focused on teachers – their perspectives on teaching or professional development.

Affect
Only 9% of the articles dealt with affect, a major inhibitor of learning for adult mathematics student. The smallest percentage of all, 5% of the articles, explored the student perspective. Grouped together, these articles account for only 14% of the articles despite their critical role in adult student retention and success.

Totals by Theme within Year

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Reflections on the Findings
We have a strong body of work that focuses on the classroom. It might be time to review and amalgamate these results in order to provide classroom teachers with practical advice and good quality teaching materials. Building a theoretical framework had strong representation in the early years but has received less attention as the organization matured. That framework, however, reflects a twentieth century white, middle-class research base. Only 9 of the proceedings articles addressed gender. Eleven were concerned with language. A generation gap even exists in the definition of numeracy skills. In the cashless world of the Millennials, the ability to give correct change is mote. Perhaps the theoretical framework would benefit from a “tune-up” to better reflect the multicultural, technological society of 2015. Professional development of teachers, a key area in dissertation and journal literature is under-represented at our conferences. I regard high quality teaching as the heart of the ALM mission yet only 7.5% of the presenters spoke directly about PD.

As ALM approaches its “silver anniversary”, I suggest that the time has come to consolidate and update the research in our field. I believe that there are several areas that could be the subject of ALM monographs: classroom methods, adults as citizens, adults as mathematics students, and professional development for mathematics teachers of adults. The time is ripe for us to learn from the research sitting on shelves in our organizational vault and to set a course to explore uncharted territory so as to continue to serve as a beacon for adult mathematics education.

References
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Abstract: A personal journey with mathematics is presented with the ways in which the subject was taught (including algorithms and rote learning) in the recent past, to what is taught presently, which has an emphasis on strategic thinking. There are different approaches and strategies students can use to solve problems and comparisons between the new methods and the old are made. Holding the interest of students and strategic methods are important in mathematics education, but basic mathematical knowledge is a crucial foundation.

Introduction

Many things have changed in New Zealand over the last 50 or so years. In the early 1960s, Heather Martin and Dale Simkin (the author’s parents) were sitting their School Certificate examinations which were for students aged 15–16 years old. New Zealand’s measurements were in imperial units (pounds, yards, feet, inches, etc.) and its currency was in New Zealand pounds, shillings, and pence. School Certificate examinations were very difficult and quite a lot of students did not succeed in passing. Heather Martin was able to pass the examinations on her first attempt and she was then able to work in medical laboratories in New Zealand and England, become a mother, and then apply her knowledge to teach and mentor students (including the author) for over 35 years.

Dale Simkin was not able to pass the School Certificate examinations on his first attempt, but he was able to on his second attempt and, from doing so, he entered the workforce as a quantity surveyor. Simkin could see how the mathematics he learnt at school could now be applied in his work and the mathematics now had meaning (D. Simkin, personal communication, April 30, 2015). His work involved calculating all the items and labour required to erect large commercial or industrial buildings so that developers could fairly quote the cost of construction. Simkin’s employer did not allow him to use any of the then “modern” equipment for his calculations which included slide rules, Ready Reckoner tables, or the latest Facit mechanical calculating machine. This machine had a handle which was cranked at speed in different directions to achieve the answer (D. Simkin, personal communication, April 30, 2015). For the first year of employment he calculated feet and inches by feet and inches using duodecimals with the factor being 12. This he got very good at by breaking the inches into percentages of the 12 for the calculation. In his second year, an Italian electronic, but mechanical, calculating machine called an Olivetti was purchased. While he was allowed to use it, he could get the answer manually ahead of the time it took for the noisy mechanical machine to calculate the answer. One project Simkin worked on as a quantity surveyor was for a new building for the Napier Public Hospital in 1968; Napier is a city on the East Coast of New Zealand’s North Island. He calculated the amount of reinforcing steel needed for the construction of the new building (D. Simkin, personal communication, April 30, 2015).

In 2015, 50 years on from Heather Martin and Dale Simkin sitting their School Certificate
behind the methods are fascinating. Jakow Trachtenberg developed a speed system of basic mathematics. Trachtenberg was in a concentration camp when he created his system and, without any paper or pens, he was able to occupy his mind with numbers (Cutler & McShane, 1960). On late night television, New Zealand has infomercials and one infomercial was for Brainetics developed by Mike Byster. Mathematics did appear to be commercialised, but the children showed a lot of enthusiasm when answering the questions (see https://www.brainetics.com/m2/). The methods taught in the infomercial focussed on clever tricks and the children would have required basic mathematical abilities and a good memory to recall all the different methods. Therefore, strategies do require students to also have basic mathematical abilities and a good memory.

Rote learning and using algorithms also require a good memory, however there are not as many varieties in the way students use these methods. There was an American television programme called Murder, She Wrote. In New Zealand, and in other countries, there does appear to be so much stigma around the word “rote” that it can be thought of as murder if someone mentions the word (Murder, She Rote could be the name of a mathematical television series). A Canadian newspaper article reported on a study conducted by a team of neuroscientists regarding the important role of rote memorisation (Brean, 2014). The researches show that rote learning is essential for further mathematical learning.

To rote, or not to rote; that is the question. A balanced approach is needed in New Zealand to go forward and multiply. Both rote learning and strategic methods have their place in New Zealand schools and tertiary institutions. If only one method is being taught, some learners are missing out. There appears to be a push towards strategic methods and this can be pulled back a little to re-introduce the knowledge behind the strategies. New Zealand does not need to go right back to the basics, but it needs a combination of strategy and knowledge which was the initial aim of the Number Framework and the NDPs. The pendulum may have swung in favour of strategy, but it is not too late to find middle ground by bringing back the basics.

New Zealand certainly has been on a big journey for a small country. Its measurements have predominately changed from imperial units to metric units, its currency has changed from New Zealand pounds to dollars, and basic mathematical learning has been overhauled with an emphasis on strategic methods. Just like the Napier Public Hospital building is being demolished, things come and things go. Basic mathematics is the foundation for strategic thinking and new methods will come and go, but the focus on assisting students overcome any anxieties and helping them discover the possibilities of mathematics needs to stay.

References

Our ALM chairperson has promoted “seeing with your maths eyes”. How far might we take this concept into areas of advocating public policy? My first ever paper for ALM was on the social content of numeracy teaching, and pointed to the relationship between gun availability, and death from guns. Generally I have argued that we should provide our students with access to mathematically encoded social knowledge. As numeracy educators we want our students to have the capacity to engage in public discourse. But what are our expectations of ourselves? Are there areas where mathematical people should be making a difference in public debate? What understandings do we have, that many of the general public do not? To cut to the chase with the most prominent example in current times: - Regarding climate variability, what does our capacity for data interpretation require us to say?

To begin, as is now an Australian tradition, I would like to acknowledge the people who were/are custodians of the land around here. Was it Sitting Bull’s people, forced away to the Black Hills by the European invaders from the land of my ancestors? Or the Nacotchtank and Piscataway, Algonquian peoples scattered far and wide, but still retaining a presence? The latter apparently. Of course, the same colonising nation also invaded New Holland, establishing Australia, very much to the detriment of the peoples who were already there. OK, it’s history, and we are not going to remedy all historical crime, but there are parts of the world today where, in effect, supported by US, UK and Australian policy, the story reads just like another chapter of Bury My Heart at Wounded Knee.

ALM has been a wonderfully supportive forum. Not always demanding intellectual rigor, it’s a place where ideas can be tossed around. With this paper I want us to think about not only what broader course content we might offer, but whether we should be looking for new roles and new markets.

- Which knowledge could you highlight which is socially important, but mathematical in description?

- What do you address in your classes now? How might we expand the range of what we think is important to talk to our students about?

- What should people who are reasonably mathematically competent, be saying to the wider world? And how? A mathematical outreach to the general public?
At ALM, we also like to have a bit of fun, so my talk is subtitled: **Or, how I was upstaged by Pope Francis, the Dalai Lama, President Obama, and even Neil Young!** I’m pleased to say all these chaps have been stealing my thunder betwixt my initial idea and the presentation of this paper. I am also outperformed by Marilyn Waite, in a keynote speech at the ALM conference, who provided detailed examples of the sort of data I’m suggesting we use in classes.

Yes, upstaged by the Pope. Joke: Did Francis hear that I was giving this talk, and decide to get in first with his recent encyclical, *Laudato Si’ On Care for Our Common Home*? I’m with David Hume on miracles and much else, but what a breath of fresh air this man Francis is. Like the invention of optic fibre, and Google Earth, his election is a modern miracle. He has sidestepped the age old Catholic/Judaic anthropocentrism (God created everything for human use), and restored Mother Nature to a central place in canonical thinking.

The Dalai Lama, who I once had the good fortune to meet briefly, remarked on his 80th birthday the week before the ALM conference, (July 5, 2015) “the damage being done to the environment doesn’t have… immediate impact on the mind. We don’t notice it until it is often too late. That’s why it will be more effective if taking care of the planet becomes part of our day to day life.” President Obama has met with the Chinese leadership and presented new plans to get action on carbon emissions underway. And the great North American songster Neil Young? ... Later...

Adult education has long been about improving the skills and knowledge of students so that they can further their personal aims. Our somewhat participatory, somewhat democratic societies have an ethos of moral equality and rights to have a say. In the linguistics of a past era: we’ve also wanted to empower our students to participate in public discourse. And how do they do that? Amongst the many other knowledges they would need, they need to be able to access important mathematically expressed information. I’ve characterised that as accessing mathematically embedded social knowledge. By that I mean useful information that most of us should know which is numerical in description; bringing greater public awareness of the mathematical components of important public issues. Mathematics teachers have always given *ad hoc* advice about money handling; which bank accounts to use, to pay off your credit card each month, and so on. Here are some other areas of numerical discussion:
Critique of public policy discourse.

**Interpreting the business case for public infrastructure proposals.** In Melbourne we had a bad run with: the largest water desalination plant in the southern hemisphere, totally unused in the four years after its construction; “myki” - a malfunctioning, and billion dollar cost-overrun, new public transport ticketing system; a 10+ billion dollar road tunnel with a poor business case fortunately stopped by removal of the government, but not before committing toward a $billion to their business mates. Mathematically capable people need to engage in public debate around such proposals.

**Efficiencies.** By this I mean, for example, information about which form of heating or cooling costs less, or has a smaller environmental footprint. Which type of device has lower lifetime costs? Is it worth getting solar power on the house you live in? What is the saving if a Texas couple put in a washing line, instead of using the tumbler drier all the time?

**Guns.** My first ALM talk, in Boston in 2000, preceded by the Adult Numeracy Network in Chicago, was titled “The Social Context of Australian Numeracy Teaching” wherein my colleagues in Australia, were arguably world leading in having meaningful adult content in their numeracy teaching (Schliemann, 1998). Giving an example of mathematically embedded social knowledge I focused on the relationship between gun availability and death from guns. This graphic in Figure 1 was made in Washington more recently. I can’t vouch for its details, but you get the point:

![Figure 1. Stop handguns before they stop you](image)

We have a saying in Australia. “Only in America.” We say it when we read about a young child in a supermarket, sitting in a shopping trolley, pulling a gun out of mother’s
handbag, and shooting her. Frankly, “US people, from America*, thank you, I personally believe”, you are hung up on this. In Australia we can go through our whole lives without ever handling a gun. A program of reduced gun types, tighter new licenses, buy backs, systematic cache reduction, culture change, etc, starting slowly, could take 30 years to really have an impact in the U.S., but if you’d started fifteen years ago when I told you to, you’d be halfway there by now. (That was a joke Joyce.) OK, I know it’s not easy:

A well regulated Militia, being necessary to the security of a free State, the right of the people to keep and bear Arms, shall not be infringed. U.S.A. Constitution (1791)

People’s militia vs. a modern state? Anachronistic, ludicrous, almost meaningless. Either the right to bear arms includes bazookas, rocket launchers, drones and nuclear weapons, OR you can be restricted in which arms you may bear. But it will take new laws and a refreshed Supreme Court to come to this conclusion.

Gambling.

My own efforts in mathematics education have focused on gambling (Smith, 2012), discussed elsewhere in these proceedings. But consider figure 2 below. This rich information gives us a good talking point for adults. What does it mean? Should there be a response to information like this? Pokies/slots losses go with poverty and with less education, and with availability of slots. A strong correlation is visible, r=-0.67. [On the SEIFA scale a higher number is higher socio-economic status; wealthier suburbs to the right.]

Figure 2. Gambling losses by SEIFA Index of Relative Socio-economic disadvantage: Melbourne Metropolitan Municipalities, 2011/12.
The in-session contribution by participants at the ALM conference raised a number of issues. Pre-empting my later comments one teacher indicated that she liked to teach about climate change. Others expressed difficulty addressing social issues as they might be regarded as political, not allowed in their employment contexts.

Catastrophic Consequences of Climate Change

I think we should be talking about the **catastrophic** future impacts of climate change. Visit London while you still can, one of humanity’s greatest achievements. We face massive unintended consequences that you haven’t thought of. The topic I want to address most is really the one of the moment. I’ll let that great public communicator Neil Young explain:

> ... the jet stream has been disturbed by warming ocean water temperatures, resulting in erratic and extreme weather patterns. The disturbance is caused by global warming, which is caused by humans creating more CO2 [and other gasses, viz. CH4] than the planet can consume with plant life. The CO2 rises in the atmosphere and creates a layer around the earth keeping heat in. (Young, 2014 p.318)

Why did people not get it? Why did they not understand the world’s situation. Why did the media, especially TV networks, downplay and ignore the obvious so consistently, making it virtually impossible for the masses to grasp reality as far as the importance of climate change was concerned. (Young, 2014 p.310)

The normalcy bias refers to our natural reactions when facing a crisis. It causes smart people to underestimate the possibility of a disaster and its effects. People believe that because something has never happened before... it never will. We are all guilty of it... it's just human nature. [People] almost always are too complacent, because they cherish the illusion that when things start to go bad, they will have time to extricate themselves... It never works that way. Events move much faster than anyone expects... History usually doesn't evolve in a slow and orderly way; often it leaps forward in disorderly, chaotic jumps. (Biggs, 2008)

Basic public understanding of mathematics runs to simple arithmetic, simple probability, maybe linear change. Do we think of change as gradual; in terms of a mathematical function, a straight line, perhaps exponential? But change can be incredibly dynamic, and an apparent equilibrium a long way from a previous one. Here is some numeracy we could perhaps talk about: phenomena that have parameters. Humans can live within a wide temperature range, but there are other species that have it a bit tougher. Take the world’s largest and incredibly magnificent coral reef, a bigger mass of life than the U.S. eastern coast conurbation where our conference took place. It’s 1000s of miles along the north-east coast of Australia. Figure 3 shows coral full of life. Figure 4 is a picture of once colourful coral, no longer so colourful.
Coral reefs can only survive within a strict temperature range. The slightest temperature change can disrupt the delicate balance of the coral reef ecosystem. When temperatures deviate too much, the algae become disconnected from the host cells because they lose their cell adhesive function (Tchernov, 2004). A deviation of 1-2 degrees for 5-10 weeks is enough to cause bleaching (Buchheim, 1998). The result is that the corals are left colourless and without ability to produce energy. If the corals lose their zooxanthellae for too long, the coral host will die. Without the zooxanthellae to photosynthesize, the corals have no way to produce energy.

**Methane levels.** In 2010, methane levels in the Arctic were measured at 1850 nmol/mol, a level over twice as high as at any time in the 400,000 years prior to the industrial revolution. Historically, methane concentrations in the world's atmosphere have ranged between 300 and 400 nmol/mol during glacial periods commonly known as ice ages, and between 600 to 700 nmol/mol during the warm interglacial periods.

The Earth's atmospheric methane concentration has increased by about 150% since 1750. Methane has a large effect for a brief period (a net lifetime of 8.4 years in the atmosphere), whereas carbon dioxide has a smaller effect for a long period (over 100

**Very inconvenient.** Did Al Gore’s (2006) graphs in Figure 5 frighten you? They shocked me. They showed very long term comparisons between gases in the atmosphere, studied from ice cores, and global temperatures. They go up together, one after the other, very forebodingly.

![Figure 5 Temperature and CO2 Records (McInnes, 2015)](image)

**Species Extinction** So how can we, who are relatively mathematically literate, contribute more to public education / discussion about such matters? It’s not new, that human activity is causing species extinction. You’ve been doing this for a while. Are you seeing the new NorthWest passage, the polar bears having to drown or become landlubbers? In Figure 5 Australia’s iconic koala hangs on.
Figure 5 Marsupial Koalas are listed as ‘vulnerable’.

If you don’t start paying serious attention, it’s going to get worse. Is that really how you want to live?

And finally, here’s a quote from an interesting book I was rereading on my US tour (Chatwin, 1987).

“The idea of returning to an ‘original simplicity’ was not naïve or unscientific or out of touch with reality. [I disagree. ‘Sophisticated efficiency’ is the new mantra.]

‘Renunciation,’ Bruce said, ‘even at this late date, can work.’

“I’d agree with that’ said Arkady. ‘The world, if it has a future, has an ascetic future.’ ”

Asceticism Now! We need to do something about our over-consumption, or eventually we are going to be prosecuted for the long term damage our conduct causes. On that cheery note: Who was the world’s most famous ascetic? There was a contender just before my lifetime: Mahatma Gandhi. Also a contender for Man of the Century, although Sarojini Naidu remarked “it costs a lot of money to keep Gandhiji poor.” (Mehta, 1993) But also very famous as an ascetic was Francis of Assisi, who recently gained a namesake. We see from the recent encyclical Francis did not choose the name lightly. You could do worse than listen to him on this topic:
Reducing greenhouse gases requires honesty, courage and responsibility, above all on the part of those countries that are more powerful and pollute the most. International negotiations cannot make significant progress due to positions taken by countries that place their national interests above the global common good. Those who will have to suffer the consequences of what we are trying to hide will not forget this failure of conscience and responsibility, (Francis 1, 2015, p.169)

References


Chatwin, B. (1986) The Songlines, Jonathan Cape


*this comic meme, a parody of an answer by an unfortunate beauty pageant contestant, was popular with my hosts at the conference.*
Our intuitions can mislead us: Contrasting intuitive and mathematical understandings about chance gambling.

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Gamblers often have superstitious and irrational beliefs. Important intuitive biases related to gambling include the representation bias, over-weighting of small probabilities, misunderstanding randomness, memory distortion, biased attributions, beliefs about the role of luck, the illusion of control, chasing losses, and just world beliefs. These will be explained and contrasted with true understandings, often mathematically expressed. Mathematical explanations for the surety of long term loss on commercial chance games are fourfold. The outcomes from machine based casino games and Australian EGMs are random and independent, the structure of the sequence of possible outcomes is calculable, each game has a calculable house margin which gives the expected rate of loss on that game, and distribution theory proves that the variability of expected outcomes reduces over larger periods of play effectively guaranteeing the long term rate of loss. An innovative basic teaching exercise, and XL simulations are shown.

In Australia, there is some enthusiasm for providing well developed mathematics teaching in schools; to effectively explain why you’ll lose your money on long term commercial chance gambling (Lowe, Smith & Money, 2015). Both the gambling treatment community and school educators have tended to overlook that mathematics explains the outcomes from chance gambling (Smith, 2012a). Very few people have a good understanding of that mathematics (Smith, 2003).

In Melbourne, we have conducted a study over 4 weeks of cross-curricula teaching of 14–15 year-olds, to assess the efficacy of an innovative curriculum. After the teaching, year 9 students performed significantly better on our assessment than they did beforehand, and better on most measures than year 10 students who were not taught (Lowe, Smith & Money, 2015). The present paper selects the most important of the mistaken psychological illusions which apply to gambling, then in response identifies the key mathematics, and showselements of mathematics teaching, which provide truthful understanding of the gambling phenomena.

Misconceptions

A fairly comprehensive list of mistaken intuitions or psychological biases was provided by Wagenaar (1988). Perhaps the most important of the intuitive biases that people have, and the ones most strongly supported by empirical research into gambling, are the representation bias, over-weighting of small probabilities, misunderstanding randomness, availability, biased attributions, beliefs about the role of luck, the illusion of control, chasing losses and just world beliefs.

The representation bias (Tversky & Kahneman, 1971) refers to the belief that short-term sequences of evidence should reflect long-run probabilities, which is not true, and often leads
to the well-known gamblers’ fallacy in which events, such as heads in coin-tosses, or wins in general, are seen as more probable if they have not occurred for some time.

**Over-weighting of small probabilities.** People often treat unlikely outcomes as more likely than they are, engaging in “massive risk seeking for small probabilities” (Kahneman, 2011). They will prefer to have a bet for a large prize with a very small probability, regardless of the mathematically implied value of that bet. “Risk seeking for low-probability gains may contribute to the popularity of gambling.”

**Misunderstanding randomness**

The tendency to see patterns in randomness is overwhelming. We are pattern seekers, believers in a coherent world. Random processes produce many sequences that convince people that the process is not random after all. (Kahneman, 2011)

**Availability** refers to a tendency to base judgements (e.g., the profitability of gambling) on salient cues which come easily to mind, such as previous large wins, rather than on objective assessment of all wins and losses that have occurred (Tversky & Kahneman, 1973). People typically remember their wins more than their losses. This also covers observational selection bias where we notice more frequently examples of what we are looking for.

Belief in **personal luck** (Griffiths, 1994; Teed, 2012) often leads to assuming one’s personal odds of winning are better than the objective odds, or that outcomes are influenced by conjunctions of events or circumstances, such as the right person or atmosphere being present at the gambling venue. The **illusion of control** (Langer, 1975) leads to results similar to beliefs about luck, but refers more specifically to over-estimations of personal capacity to influence outcomes. This perception typically arises in situations where people have a strong intention to achieve outcomes, and where people perceive (incorrectly) a link between their actions and the outcomes, such as when they are personally involved, or make choices while participating in chance-based activities. In 1975 Ellen Langer, now a Professor at Harvard and a pioneer of Mindfulness, defined the illusion of control, as “an expectancy of personal success inappropriately higher than the objective probability would warrant.” Measures of this illusion have been found to correlate with poor probability knowledge, for example in a University cohort (Smith, 2003).

**Biased attributions** (Gilovich, 1983) work in tandem with the illusion of control in the sense that gamblers will tend to attribute failures to external factors such as bad luck, and successes to their personal skill, thereby maintaining their perception of control despite evidence to the contrary.

**Chasing losses** may be partially interpreted as an example of the sunk capital fallacy. Until you write off a lost asset it is still valued at what it was once worth. If you stop gambling when behind, you are accepting the reality that you have lost. It is only possible to win the money back (although incredibly unlikely) by continuing to gamble.

Finally, although perhaps more influential as motivation than in the perception of outcomes, a strong **belief in a just world** or that a person deserves to be rewarded, can lead gamblers to be convinced that outcomes should eventually turn out in their favour, or that effort should eventually be rewarded (Lerner & Simmons, 1966). Such a belief can lead to odd beliefs or behaviours, including a belief that croupiers or gaming machines are “unfair” or personally
against the gambler, or a tendency to personalise the task by talking or shouting at gaming machines as if they were genuine competitors or rivals. Generally we may call it an example of a positive expectation bias.

Mental shortcuts, sometimes called heuristics, are commonly used in everyday life to facilitate quicker decision making (Gigerenzer et al, 1999) and, in the case of biased attributions and illusions of control can be psychologically beneficial at times. Heuristics are very important to real time effective decision making under conditions of uncertainty. However, when applied in a gambling context, any or all of these heuristics can lead to over-confidence or over-estimations of success and may lead to excessive gambling. Evidence in support of this view emerged from a number of studies employing a speaking aloud method, in which subjects were asked to verbalise their thoughts and rationalisations aloud while gambling. Studies using this method found that over 70% of verbalisations recorded during gambling sessions were irrational (Ladouceur et al, 2001), and that many of the biases described above were being used. Many studies have also confirmed a strong relationship between scores on measures of problem gambling, and people’s susceptibility to cognitive biases related to gambling (Cunningham, Hodgins & Toneatto, 2014). Mistaken intuitions about probability often survive considerable contradictory evidence (Tversky & Kahneman, 1982).

The goal of all forms of gambling is to win more than is lost. However, the reality is that all popular forms of gambling provided by the various commercial operators are based on the absolute certainty that, ultimately, the gambler will lose more than they will win over time. Most gamblers know [agree?] that the ‘odds are in favour of the house’ but they also believe that they can ‘beat the odds’ (Grivas et al, 2010, 735).

This is not a comprehensive list of biases which may affect gamblers. Gamblers are also subject to the biases that we are all prone to: confirmation bias, in-group and bandwagon tendencies, anchoring and others, a list which may be extended rather extravagantly (Wikipedia, 2015), without the strong evidentiary basis of the above, and without specific relevance to gambling.

Although the teaching undertaken in Melbourne includes reference to a cut down list of misconceptions, below, the underlying idea is that correct mathematical understanding utterly undercuts these misconceptions, so that what is needed is not so much discussion of the illusions, as successful teaching of the mathematical explanations of gambling outcomes.

However during teaching about the mathematics of gambling it is important to ask questions about students’ feelings and attitudes. They should become aware of mistaken feelings about luck, and how they feel when winning and losing, and reflect on whether or not their performance can improve through practice.

**Simplified Misconceptions, as used in Lowe, Smith & Money, (2015)**

- A run of losses won’t continue for long - it will even out soon.
- Better to go for a large prize than a smaller one, since the gain is greater if you win.
- There is a pattern in the results - if only I can find it.
• Many people are winning – it’s my turn soon.
• I am a lucky person. I just haven’t had a lucky day for a while. I feel lucky now.
• I have the power to ‘will’ my numbers to come up in gambling games.
• When I lose it is just bad luck, or the machines are rigged.
• I deserve to win at gambling – maybe today.
• Using skill I can get better results at gambling.

Mathematical Explanations

The overarching mathematical explanations for the surety of long term loss on commercial chance games are fourfold. The outcomes from machine based casino games and Australian electronic gambling machines (EGMs) are random and independent, the structures of the sequence of possible outcomes is calculable, each game has a calculable house margin which gives the expected rate of loss on that game, and mathematical distribution theory proves that the variability of expected outcomes reduces over larger periods of play effectively guaranteeing the long term rate of loss. Or, saying the same thing differently:

• Chance gambling outcomes are random events, with the result for each outcome being independent of previous outcomes.
• The ‘risk versus reward’ understanding has its mathematical interpretation as ‘gamble expectation = probability times payout.’ Commercial gambling is constructed so that this favours the house. We need to distinguish between the odds which are implied by the prospective payout and the real odds, which are worse (Smith, 2012a).
• A short-term experience at gambling can have a reasonable chance of producing a profit, although losses are more likely.
• Longer-term experience will give outcomes that are highly predictable: average losses per game come closer to the theoretical value built into the design of the game, and the actual spread of actual losses per game, each side of the theoretical value, become relatively smaller, and so long-term gambling is highly likely to lead to substantial loss (Smith, 2012b).

Teaching in Summary

There is not space here to fully outline the recommended teaching, but that is available elsewhere (Smith, 2012b). This graphic is from an Excel spreadsheet (Money, Smith & Lowe, 2014) which runs my game Lucky Colours of Sunshine, many times, so that the effect of a house margin may be observed. In this example, the game was set to run 100 trials, selecting one colour from four, and set to pay a 2:1 prize for a successful (automatic randomly selected) colour choice match, which has a chance of 3:1. We see in this particular example, that matches were less than the long term expectation of 25%, resulting in a bigger loss than usual.
Figure 1 Lucky Colours spreadsheet set for 2:1 payout on a 3:1 game.

Illustrated in Figure 2 below, from Money, Smith & Lowe (2014), is an exercise to plot the actual results from a series of 23 repeated series of bets of different sample length, i.e. 10, 100, or 1000 goes. It shows how the range of results narrows in a larger sample, and how in the long run the overall actual outcome congregates around the rate of loss expected from the structure of the game. This was an Excel simulation game set to a house margin similar to electronic gaming machines in most jurisdictions.

Figure 2. Box-and-whisker plots showing actual spread of percentage gain or loss for 10, 100 and 1000 games, with expected return 85% of outlay.

This is a middle school level, (Years 9 or 10), exercise which does not require teaching the notion of the Normal curve, nor standard deviation, to make the telling point that over a long period of play you are increasingly unlikely to be winning, and increasingly likely to achieve a rate of loss set by the game’s structure, delivered by the ‘predictable’ interaction of randomness and the likelihoods of individual outcomes.
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Does Adding Mathematics to English Language Learners’ Timetables Improve their Acquisition of English?

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Abstract
This enquiry based project set out to find out if adult English language learners, known as ESOL (English for Speakers of Other Languages) learners in the UK, might benefit, in terms of their acquisition of English, from studying maths. This research has been conducted at a medium sized FE college in the East Midlands where I teach. I evaluate this in two ways, firstly by analysing learners’ results, and secondly by asking experienced ESOL teachers to observe and reflect on an ESOL Maths session. This project found a correlation between attending a maths class and improved English language exam results over 5 cohorts of students. In addition, ESOL teachers noted many and varied opportunities for English language learning in an ESOL Maths class, with higher levels of learner participation and confidence than seen in language classes. I recommend that we offer ESOL maths to ESOL learners, and that we reassess maths teaching for all learners, ESOL and English speakers, as a triad: conceptual understanding, procedural competence and language acquisition.

This article was first published after proceedings at the BSRLM (British Society for Research into Learning Mathematics)/BERA (British Education Research Association) conference, Durham, UK, 2015. The research was carried out for the dissertation phase of a Master’s degree in Education.

Key words: addition, English language learners, mathematics

Introduction
The learners involved in this project are all people who have voluntarily signed up for ESOL Maths, and may have just arrived in the UK, or been here for many years. They may have opted to come to the UK for work or family reasons, or been subjected to political or social persecution in their country of origin. The learners are all over 16 years of age; most are 19 or over. The primary motivation for many of the learners attending ESOL Maths classes is to improve their English, and this can be for a number of reasons, including improving their job prospects or helping their school aged children.

ESOL Maths learners form part of a number of wider educationally-based communities, namely mathematics learners in the UK, ESOL learners in the UK, and, globally, those whose first language (L1) is not English who are learning maths in English. They may come from many countries and cultures which can be very different, both from the UK and each other, in terms of the content of maths lessons and assessments, mathematical symbols and language, and the value placed on mathematics learning.
Methodology and methods

In order to ascertain whether adding ESOL maths to ESOL learners’ timetables has an impact on their acquisition of English I compared the results of ESOL learners who have studied for maths qualifications with those who have not. This was small-scale quantitative analysis based on an ESOL intake of approximately 130 learners each year, 11% to 18% of whom enrol for ESOL Maths.

It should be born in mind that this is a small-scale investigation based in one college which has one ESOL Maths teacher, namely myself, although some higher-level learners do attend an English speakers maths class if it falls on a more convenient day for the learner. Entry-level English learners are not encouraged to attend English speakers Entry-level classes as they are deemed to need specialist help. As such I examined the results of the whole population of ESOL learners at this college, of whom the ESOL maths students can be seen as a subset of the total population (Cohen, Manion & Morrison, 2000).

I chose to analyse the data to see if these observations can be evidenced in some way by improved ESOL results, but in order to triangulate this information ESOL colleagues were asked to observe an ESOL Maths session and report back on their findings. This mix of quantitative and qualitative analysis can be seen as an opportunity for triangulation of data (Coben, 2003).

The analysis does not tell us why any correlation occurs, highlighting the limitations of data analysis (Bell, 1993) and a positivist approach (Stacey, 2013). It may be that the amount of teacher contact time has an effect, or that adult ESOL learners who opt for ESOL maths are more highly motivated than those who do not. A correlation might also be for reasons unrelated to ESOL Maths; that it may be that any subject taught, from sport (Hately-Broad, 2006) to flower arranging, could have the same effect, particularly if the learner has some prior knowledge of the subject.

Observers were given three questions for consideration: Firstly, can they see any advantages of ESOL Maths, where the language is less overtly taught than in an ESOL class? Secondly, do the students exhibit skills that they were unaware of, or that surprise them for the level those students are at in their English? Finally, does it make the teachers reassess the learners’ language skills levels? The questions are open-ended to draw full responses (Ribbins, 2006), but can be seen to be connected to allow for corroboration (Richards, 2009). A Likert scale was included to discover strength of feeling (Bell, 1993). I asked observers to rank the usefulness of ESOL Maths in improving English acquisition on a scale of 0 to 10, where 0 is ‘of no use’, and 10 is ‘extremely useful’.

There is potential for any observer or interviewer to have an impact on a situation and peoples’ responses, as observed in other investigations, where observers become aware that they are an acknowledged presence in the room, and that this is disturbing the normal flow in some way (Brown 2001). This is known as the Hawthorne effect, and it might affect both myself as the teacher or the students in the ESOL Maths class.

One of the negative issues with conducting this study myself is that the outcome might be affected by my involvement. For instance there could be an issue with learners and colleagues giving less than honest answers to the questions. This is known as the halo effect (Cohen et al., 2000), where the previous knowledge of the participants affects their judgements.

Quantitative Findings

The analysis of the data seems to clearly show a correlation between attending a maths class and English language acquisition at my college, as the percentage of ESOL Maths learners
with ESOL passes varies from 87.5% to 100%. This compares with the performance of the group without maths classes of between 62% and 84.5%.

There is a consistent positive correlation between opting for ESOL Maths and passing ESOL exams, as there is a minimum of 10.5% and a maximum of 32% improvement in ESOL Maths students’ performance compared to the non-maths cohort (Figure 1).

![Percentage Comparison of learners ESOL results with and without ESOL Maths classes](image)

*Figure 1*. Percentage comparison of learning ESOL results with and without ESOL maths classes

In Figure 2 it can be seen that the impact on the cohort as a whole is low:

![Percentage achievements of ESOL learners over 5 years](image)

*Figure 2*. Percentage achievements of ESOL learners over 5 years
Qualitative Findings

Three of the four teachers who were able to observe classes were ESOL teachers, but the fourth was a Maths teacher. Interestingly the comments from the Maths teacher were very similar to those of the ESOL teachers, and I have felt it unnecessary to differentiate between the two which will also help to preserve the anonymity of the teachers involved.

All of the observers noted language teaching taking place in the observed sessions. One teacher expressed surprise at “the amount of language that was used in Maths and therefore it was a language lesson based on maths” and also said that the session was “very interesting and definitely beneficial to the learners as they were exposed to a different type of language use.”

Another was surprised by the level of English and fluidity of the language use: “I was surprised by the good level of English used by the students, the vocabulary was very fluid and the student’s understanding of maths on the whole was of a very good standard”. The teachers also commented on the opportunities available to learners to practice pronunciation, such as in a place value recap session all learners practiced using the ‘th’ sound with tenths, hundredths and thousandths. One commented that “time was given to reinforcing pronunciation, spelling of numbers and vocabulary.”

Three out of the four teachers when asked to rate the usefulness of maths sessions in improving English acquisition on a simple Likert scale, where zero was ‘of no use’ and ten was ‘extremely useful’, rated the usefulness at 10, extremely useful. One teacher did not use the scale, and commented that “It would be more useful if the group was not of such differing levels, so language could be more easily structured”, that “some of the language used was more advanced than might be expected for some of the learners”, but did note that the learners “were engaged and attentive”.

Paired work involving verbal problem solving was taking place between learners who would not normally speak to each other during a session, one of whom had previously refused to participate in paired work during ESOL sessions. One teacher said “Student X does not speak in English, but spoke here with other students she does not normally interact with”. Another commented that the session gave “non-speakers” “a chance to participate”. A learner with extremely low verbal language skills was clearly prepared to attempt questions and to make mistakes which had not been seen before. “Student Y really tries and has a go, not seen that in an ESOL class” and “I could see some students were very timid but these still participated”.

The positive response to set tasks was noted by all observers and surprise was expressed at the level of maths attempted and achieved during the sessions: One commented that “maths skills…can build confidence” and “The confidence the ESOL learners gain in tackling mathematical problems will allow them to gain confidence in learning other subjects”. It seems that in this class we do not have an issue with maths anxiety, as identified by many researchers, but that we may have an issue with English anxiety for ESOL learners. It may be that those learners who opt for ESOL Maths classes are those who are maths confident.

Observers commented on the increased level of participation compared to ESOL classes, and how beneficial this was for learners, as paired work “fostered greater communication in English”. Even those with confidence issues “still participated” in the paired work: “The shyest learner in the group from the lowest level language class clearly…felt able to answer the teacher because the focus was on maths not English”. Learners “responded well and found the experience useful and relevant”, even those “with strong educational backgrounds in maths”. Observers noted that “learners clearly felt more confident and more able to answer questions” and that the learners both responded well to the tasks set and performed well in their completion. There was “good interaction with resources/activities…working individually or in pairs”.

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Observers were generally surprised to note the level of language performance shown by the learners, and felt this was improved compared to ESOL classes. One observer noticed that “the focus is on maths where some learners who may be weaker in language are able to do better” than in an English class, as they are using other skills, not just English. Two students performed consistently better, according to one observer, than they would have done in an ESOL class in terms of speaking and listening skills. All of the students seemed to be performing at a “good level of English” according to one observer.

Observers did not feel that any change to ESOL exam levels set was needed, but some did feel more confident that learners might achieve.

**Conclusion and recommendations**

Whilst caution is required due to the statistical insignificance of the sample size (Cohen et al., 2000) the implication here is that ESOL learners’ English acquisition might be further enhanced by placing them in ESOL maths classes, based on their English language level. This adds to the current knowledge in my college and perhaps elsewhere in the UK, and might be useful when considering maths provision for ESOL learners. It contrasts with a recent change of practice in the USA (Kersaint, Thompson & Petkova, 2013), where current thinking is that English language learners be placed in maths classes according to their level of mathematics knowledge.

Observation of ESOL Maths classes did seem to cause teachers to refine and extend their thinking about ESOL Maths and its usefulness in developing language skills, which can be seen as evidence of increased levels of language activity in mathematics in many countries, including the UK (Brown, 2001; FitzSimons, 2002).

The observations seem to support the idea that although mathematical language should be the focus of maths classes (Barwell, 2002; Fletcher and Barr, 2009; Monaghan, 2009), there is enough other language occurring for learners to benefit in terms of English acquisition (Adler, 2001; Clarkson, 2009).

Observers noticed the importance of the teacher interface with learners, and this supports the need for specialist maths teachers to enable learners to make progress with their maths (Brown, 2001; FitzSimons, 2002).

The benefit of resources developed for use with English language learners benefitting all maths learners has been previously examined (Adler, 2001), and I too have found these resources useful with all maths learners, hence the recommendation that we consider the importance of maths language teaching in maths classes and move to a triad approach: conceptual understanding, procedural competence and language acquisition.

**References**


PRESENTER ABSTRACTS
Blended and Active Learning Math in a Mixed-Setting Classroom

Mrs. Gisella Aitken-Shadle and Mrs. Claire Finn

Abstract
This workshop will explore the use of blended learning math, which combines traditional instruction with computer-assisted learning, in a mixed setting classroom, with low level learners, special education students, and English language learners, all in one class of two hours every day. This presentation will examine key ideas drawn from class experience of active learning incorporated with blended learning, looking at how it leads to a more beneficial student learning environment and greater constructive engagement that challenges students while benefitting the different groups within the diverse class population. It will also provide tips on incorporating these techniques into a math classroom at all levels. Participants will also have the opportunity to get a taste of the everyday experiences of students in such a class to provide concrete illustrations of the ideas discussed in the presentation.

CMAST and QEP: Developing 20/20 Mathematical Vision in a STEM Environment

Ms. Corisma Akins and Mrs. Shakira Hardison

Abstract
This multimedia interactive session targets individuals who are directly responsible for identifying and implementing strategies that support student success in mathematics and its application across curriculums. This session will provide participants with an overview of effective collaboration and broad-based involvement originating from an institution’s Quality Enhancement Plan (QEP) and a STEM Student Success Program, the Center for Mathematical Achievement in Science and Technology (CMAST). The presentation will target three specific initiatives (classroom instruction, online tutoring, and QEP Mathematics Clinic), demonstrating an impact via effective implementation of learning strategies in a co-curricular environment, with sustained student success outcomes. Participants will be provided with: (1) a blueprint for the inclusion of co-curricular collaborations and strategies to energize the classroom; (2) proven, structured methods within a STEM learning community setting, which equips students for success in what has historically been viewed as a high-failure gateway course; (3) shared resources designed to improve basic the skill set necessary for student success and matriculation; and (4) practical application strategies that utilize effective collaborations and pooled resources to impact retention. In addition, the facilitators will also discuss their experiences from a “lessons learned” perspective from the ongoing QEP and CMAST implementations along with subsequent strategies.
The Effect of Language and Culture on the Learning of Mathematics

Mr. Darren Allen

Abstract
Long thought by most math teachers to be a subject insulated from the vagaries of language, research has shown that the learning of math (from numeracy to more abstract and advanced concepts) is significantly affected by religion, culture, language, and socioeconomic status. The presenter will discuss research in the area and site-specific instances where these influences affect the learning of math and will help to explain some of the well-publicized “failings” in American mathematics education. We will finish the discussion with some basic linguistic and cultural recommendations to increase the success of our diverse student bodies, while being sensitive to cultural and religious differences.

Logbook – From Research Tool Towards Learning Tool in Adult Mathematics Education

Ms. Charlotte Arkenback-Sundström

Abstract
I developed the structured logbook as a research tool for getting to know a group of apprentices’ work-based learning in various stores. The aim was to find out how they use and communicate numbers and mathematical relationships in work activities. Logbook notes were then used as the basis for dialogue conversations about activities where numbers are used in the workplace. The conversations were structured as a combination of group supervision and study circle. The structured logbook has proven to be a useful learning tool in mathematics education of adults, for both the students and me as a teacher. In Sweden, there are no courses in numeracy in adult education; however, there are introductory courses before the mathematics courses at the secondary level. Since January 2015, I have taught a new course: Preparatory Course in Mathematics—From the professional and everyday math to school mathematics. The students’ logbook notes are the starting point for dialogue-based mathematics learning. The teacher’s role is to be the moderator and the “translator” to the formal school mathematics. Applied mathematics and problem solving is a shared responsibility of the group; the goal is that everyone should have the opportunity to discover and develop their mathematics containing vocational and living skills.
Examining the Mathematical Literacy of 15-Year Olds and the Numeracy Skills of the Age Cohort as Adults

Dr. Alka Arora and Ms. Emily Pawlowski

Abstract
To what extent does mathematics proficiency of 15-year olds predict the proficiency of their age cohort as adults, looking at variation between countries and between demographic groups within countries? This paper will examine the mathematical literacy performance of 15-year olds in the Program for International Student Assessment (PISA), focusing on the 2003 administration, and it will look at the performance of the relevant age cohorts that participated in the 2012 Program for the International Assessment of Adult Competencies (PIAAC), which assessed adults aged 16 to 65. The results will focus on cohorts from the United States and will look at how the performance of various demographic groups (such as gender, race/ethnicity, and U.S. region) compares over time. PISA and PIAAC are both international assessments of skills and competencies conducted by the Organization for Economic Cooperation and Development (OECD). The target population for PIAAC includes the cohorts that participated in PISA 2000, 2003, 2006, and 2009, and there is considerable commonality in the way in which the skills of numeracy and mathematical literacy are conceptualized and defined in the two studies, so comparing PISA and PIAAC performance offers evidence concerning the relative growth and development in proficiency for these age cohorts.

Shifting From Mathematical Worksheets to Meaningful Tasks

Mrs. Cynthia Bell

Abstract
With the new assessments and standards come shifts in instruction. Perhaps one of the most exciting and challenging is moving toward providing meaningful and engaging tasks that allow students to make connections and expand their understanding of concepts and procedures. Participants will learn to make this shift, discovering how to turn an ordinary worksheet into a meaningful, rigorous task.
Teaching Functions and Proportional Relationships

Mrs. Cynthia Bell

Abstract
Ever wondered how to introduce functions in a meaningful way to learners or how to connect the concept of functions to their applications in everyday life? Functions can be a daunting content area to teach, but the instructional shifts of coherence, deep understanding, and application can help. This session will provide ideas and methods for deepening student understanding of the concept of linear functions and their connections with proportional relationships and rates by engaging participants in easily duplicated tasks and activities.

Teaching Matters! Turning High-Quality Standards Into Successful Mathematics Learning

Dr. Diane Briars

Abstract
High-quality standards and curricula are necessary for students to successfully learn mathematics, but are insufficient. Teaching matters! What teaching practices most effectively support all students’ attainment of the conceptual understanding, procedural fluency, and mathematical problem-solving and reasoning capabilities required for high-level mathematics learning? This session presents eight research-based Mathematical Teaching Practices along with the conditions, structures, and policies needed for successful implementation as described in a recent publication of the National Council of Teachers of Mathematics (NCTM): Principles to Actions: Ensuring Mathematical Success for All.

Mathematics Professional Development With MOOCs

Mrs. Delphinia Brown

Abstract
The growth of Massive Open Online Courses (MOOCs) has exploded in the last few years. This global phenomenon has enabled access to free web-based instruction for lifelong learners and professionals. Anyone and everyone can take advantage of these expanded learning opportunities, but researchers and practitioners are still settling on answers to some questions such as: What exactly is a MOOC? Are they all alike? Can MOOCs be designed for learning and not just scale? How can open courses be used to develop an awareness of math in everyday life? As their promise is explored and debated, this session create a space for learning and talking about professional development with MOOCs.
Making the Transition From Teacher-Centered to Student-Centered Instruction in Mathematics

Dr. Ronnie L. Brown

Abstract
In the effort to better serve today’s adult learners in mathematics education, there is the need to fully understand differences in the populations of students. The two unique populations this presentation focuses on are the traditional student versus the adult learner. We define traditional students as those immediately or soon after out of high school. Adult learners are categorized as more of a mature population with possible large breaks in their schooling and even carrying the added responsibilities of family and/or full-time work. The two populations are very different and cause for the need to teach to them differently. Instruction for the traditional group is usually teacher-centered. While this may be generally adequate for this group, adult learners are different in characteristics which will need for a more student-centered instructional style. The presentation show the differences and the hopeful movement from behaviorist to constructivist methods.

Visual Thinking in Adult Learning Using Spatial Intelligence

Mr. Arthur Conroy

Abstract
Mathematics teachers are rediscovering the power of visual thinking and its relationship to mathematical thinking. This presentation describes research that illustrates how adults use hidden patterns of geometry to make everyday decisions in their personal and professional lives.
Putting an Old Head on Young Shoulders (Math Teaching and Learning)

Dr. Anthony Cronin

Abstract

A recent report titled, “Student Evaluation of Mathematics Learning Support: Insights From a Large Multi-Institutional Survey” was launched by the Irish Mathematics Learning Support Network. In this study, 1,633 first-year students from across nine Higher Education Institutions (HEIs) in the Republic of Ireland took the survey that asked students to evaluate their experience with Maths Learning Support (MLS). Of the respondents, 13.5% of respondents were classified as mature students (also called adult learners, are classified in the Republic of Ireland as a student that is 23 years of age or older on January 1 of the year of registration). A statistically significant higher proportion of mature students (62%) than traditional students (32%) availed of MLS. Mature students reported different needs and motivations for seeking MLS. Mature students were more likely to use MLS simply because it was there for them and they wanted to access extra help. In contrast, the traditional students were more motivated by assessment demands. Qualitative feedback illustrated that for mature students MLS is a mathematical lifeline. Mature students were more positive in their praise of MLS than their traditional counterparts, and their experiences with MLS played a more significant role in their retention than in that of other students. Low self-efficacy in mathematics seemed to inspire mature students to avail of MLS rather than shy away from it.

Inquiry Learning and Mathematics in the Workplace

Dr. Diane Dalby

Abstract

Mathematics qualifications for young people and adults in England are increasingly including references to problem solving, and there is agreement that these skills are necessary for the workplace. Despite the importance attached to this aspect of mathematical development, nurturing the relevant skills in mathematics classroom remains challenging. The EU project Mathematics and Science for Life (Mascil) aims to tackle this by supporting teachers with approaches that combine inquiry learning with connections to the world of work. These approaches support students to develop skills that equip them for solving mathematical problems in different contexts. By examining a range of materials, based on problems from the workplace, we will explore how dimensions of inquiry learning can be addressed, what students gain from adopting different roles in these situated tasks, and how students can develop valuable skills from using workplace scenarios in mathematics lessons.
Geometric Tesselations: Travel With M.C. Escher to Italy, Spain, Morocco, and Turkey

Ms. Carol Desoe

Abstract
Learn how the Dutch artist, M.C. Escher, magically changed geometric tessellations into his unique prints. Photos of architectural tilings used to create visual stimulation in otherwise barren landscapes in Europe, Asia, and Africa will be used to explain the transformational geometry underlying Escher’s art. Discover that Middle Eastern art and architecture also reflect many Islamic beliefs, as the artist and mathematician were one and the same. Participants will learn how to construct triangular and square grids, 12- and 16-pointed stars, and imaginative Escher-like designs using the “nibbling technique.”

Promoting Productive Struggle

Mr. Fred Dillon

Abstract
What does productive struggle look like? How can we encourage students to be engaged when success does not happen instantly? We will work a geometry task, look at a video case based on it, and then examine key steps that were used to keep students on task, discussing and moving forward with their learning. (Video and task are available through NCTM.)
Mathematics Online: A Success Story

Dr. Margie Dunn

Abstract
Online mathematics courses can offer accessibility and flexibility to the adult student. This presentation describes the challenges and successes of an online general education mathematics course offered at a private, not-for-profit online college with a primarily adult student body (average student age is 38 years). This course was developed bearing in mind best practices both within mathematics and online education. A standard Liberal Arts mathematics textbook is used, with topics chosen to appeal to the adult student. Students are provided with just-in-time instruction via Khan Academy to refresh prerequisite knowledge. Homework and quizzes (through Pearson’s MyMathLab) are traditional, while discussion board activity provides opportunity for group work on extended problems. Students individually submit complete solutions to similar extended problems. The presentation discusses the role of the instructor and rubrics in the online environment. The course runs 15–20 sections every 8 weeks; thus, the issue of scale will also be addressed. Different styles can be accommodated, but instructors must facilitate actively and effectively within the discussion boards while refraining to assess student work. It is up to each group to decide if their solution is correct. Examples will be provided, and student and instructor experience and satisfaction will be presented.

“When Will I Ever Use This Stuff?” Making Statistics Relevant

Mrs. Mary Gore

Abstract
This presentation describes two projects that were designed to prove to students that they would use statistics and mathematics outside the classroom. In the first project, students applied statistical concepts learned in class to read and analyze peer-reviewed articles in their fields of study. In the second project, students interviewed professionals in their field about how they use math and statistics in their job. The aim of these projects was to help students discover that quantitative literacy is valuable for their professional, civic, and personal lives. Data from 300 community college students will be presented. Additionally, the presentation will provide a forum for session participants to share and discuss best practices, such as project-based learning, in mathematics and statistics courses.
Representing and Using Ratios in a Common Core Context

Mr. Will Johnston

Abstract
Proportional reasoning is central to the middle school curriculum in the United States. Many educators are surprised that the Common Core State Standards do not refer to ratios represented as fractions. The differentiation between ratios and rates has also changed for many teachers. In this interactive workshop participants will explore reasons not to consider ratios as fractions and will go beyond “cross-multiply and divide” using tape diagrams, double number lines, tables, and graphs as well as equations to represent and solve problems involving ratios, proportions, and percentages.

Mathematics Ancient and Modern—Using the History of Mathematics With Adult Learners and in Teacher Education

Mr. David Kaye

Abstract
My focus on teaching mathematics to adults has always been motivated and informed by two conceptual approaches in parallel: the significance of recognising adult numeracy and the use of the history of mathematics as a teaching strategy. I have spoken of both at previous ALM conferences, and at ALM 22. I will review and develop the use of the history of mathematics. I will explore how and why using the history of mathematics can be a powerful resource, particularly for learners who have rejected other approaches. I will also show how building on my own experience with learners I have included using the history of mathematics in teacher education programmes. There will be an opportunity to participate in some activities that introduce some themes from the history of mathematics and how these can be used to make connections between mathematics and society. The discussion will develop these ideas further by considering teaching mathematics as a part of the history of ideas.
It Is Instruction Silly: Research-Affirmed Practices That Make All the Difference

Mr. Steve Leinwand

Abstract
This fast-paced, example-laden presentation will engage participants in a set of short activities and classroom-tested strategies to make mathematics far more accessible to all students of all ages. More specifically, we will explore a set of research-affirmed and easily adaptable instructional techniques that all teachers of mathematics should be expected to consistently employ.

U.S. Adult Numeracy Skills in the 21st Century Workforce

Ms. Saida Mamedova

Abstract
During the last decade, average numeracy scores of U.S. adults have dropped significantly from 262 to 253 (on a scale from 0 to 500), both of which are consistently below the international average. Compared to the international average (20%), a higher proportion of U.S. adults (30%) are at the lowest numeracy proficiency levels (level 1 and below level 1), and 24% of U.S. working adults scored at these low levels. Our special interest group session will use the results of the 2012 Program for the International Assessment of Adult Competencies (PIAAC) to explore U.S. adults’ performances in numeracy in relation to their international peers, and by demographic variables including gender, age, race/ethnicity, and nativity, as well as by employment status and educational attainment. In addition, we will provide information on the type and extent of numeracy skills adults use in the workplace and the relationship between numeracy skills and earnings. PIAAC is a large-scale international assessment conducted in 24 countries in 2011–12 with a nationally representative sample of 5,000 adults aged 16 to 65 per country. PIAAC assesses basic skills and a broad range of adult competencies, especially cognitive and workplace skills needed for successful participation in the global economy.
Mathulous Mathionistas: The Mathematics in Fashion

Mrs. Jacqueline Mogey

Abstract
Fashion relies heavily on mathematical concepts. This will be a discussion group to explore some of these concepts, perhaps some of the lesser acknowledged roles of mathematics in fashion, and we will look to potentially collaborate on some units of work or projects that could be used to meet some of the requirements of adult numeracy education.

Math: The Deciding Factor

Mrs. Jacqueline Mogey

Abstract
How does math play a role when our decisions are not obviously related to numbers? Are ANY decisions not related to numbers? I plan this to be a discussion group around ways in which practitioners of adult numeracy education can confidently bring the mathematical concepts of everyday decision making into their lessons.

Black Male Achievement in STEM: Impacting Lifelong Trajectory of Success With an Integrated Mathematics Education Approach

Mr. Fredric Navarre and Dr. Yvette Pearson-Weatherston

Abstract
Data shows that black males have the most acute racial and gender combination deficiency in math achievement and performance. Early motivation in STEM fields is key to increasing black males’ achievement in as well as their matriculation through STEM degree programs. Interest and understanding can be improved by integrating mathematical concepts into non-STEM subjects to demonstrate practical applications to everyday life in areas such as business and finance, team sports, and the arts. Using historical data, this study will explore reasons why black males struggle with mathematics and how their engagement in STEM may potentially resolve more issues than those related to academic achievement. Research shows students who develop interest in STEM-related subjects are more likely to attain lucrative careers after high school regardless of their post-secondary paths. Further, increasing the number of black males in STEM-related fields will strengthen the nation’s economy by addressing this group’s distinct challenges in high unemployment and incarceration rates.
Coping With Math Anxiety in College: From Research to Relief

Dr. Fred Peskoff and Dr. Leonid Khazanov

Abstract

Many adult learners feel overwhelmed by mathematics anxiety. Likewise, instructors often feel frustrated when encountering these students in their classrooms. This workshop will examine a number of coping strategies used by adult learners as they try to move from “distress to success” in their study of mathematics. After a brief discussion of the causes and “symptoms“ of mathematics anxiety, the participants will be asked to rank a list of 10 coping strategies in terms of their usefulness. Their results will be compared with the results found by the workshop leaders in their own research. Gender differences and differences between basic and more advanced mathematics courses will be discussed. The goal of the session will be to increase awareness of mathematics anxiety and the most effective ways to help adult learners cope with it so that they can experience success. Case studies of “real-life“ students will be presented for discussion and “diagnosis.” Most participants will have encountered students just like them at their own schools. A resource booklet will be distributed and active audience participation will take place throughout the session.

The Symbiotic Relationship of Research and Practice (Topic Group A and B)

Dr. Katherine Safford-Ramus and Mr. David Kaye

Abstract

Topic Group A/B was formed at ALM-4 to serve as a discussion group where researchers and practitioners could discuss the interplay of their roles and co-dependency of their work. The theme of those discussions has ranged widely over the intervening years but the underlying questions are always: What defines our field of “adults”, “mathematics”, and “education”? What research is being conducted in the field? How can ALM support the enhancement of practice through research results? The topic group meets for two sessions and is intended to be a discussion that serves the needs of the participants. Anticipating that there will be many new attendees this year, the organizer will present a brief summary of themes from previous years and then invite the group to determine the discussion path they would like to follow at ALM-22, keeping the theme of the conference , Opening Our Mathematical Eyes: Seeing Math in Everything We Do”, in mind. Prior to attending the first session, participants might like to read the following article by Sriraman and Törner: “The 2010 Banff workshop on teachers as stakeholders in mathematics education research from the Mathematics Enthusiast, February, 2014, Vol. 11 Issue 1, pp. 1-6.
An Exploration and Evaluation of “Maths Eyes” Projects and Events

Ms. Aoife Smith, Dr. Terry Maguire, Ms. Niamh O'Meara, and Dr. John O'Donoghue

Abstract

Maths Eyes is an initiative that emerged at the start of the 21st century in Ireland. The initiative focuses on developing an approach for making real-life maths more visible. The first Maths Eyes initiative “Looking at Tallaght with Maths Eyes” took place in June 2011 to coincide with the 18th International Conference Adult Learning Mathematics, which was hosted by the Institute of Technology Tallaght, Dublin, Ireland. Since then, many other community initiatives, school activities, and collaborative projects based around Maths Eyes have been undertaken. In addition to this, the concept of Maths Eyes as a resource to enhance mathematical numeracy and literacy in schools is currently being disseminated through education centres across the country of Ireland. Hence, it is evident that the initiative has gathered significant momentum in recent years. This research paper seeks to document the various Maths Eyes projects that have taken place to date. This will give rise to a categorised directory of Maths Eyes projects and events, which may help highlight the impact it has had in schools, workplaces, and communities. The paper will focus on a number of these projects, which will be discussed in greater detail, and an evaluation of each project will be presented.

Middle School Math Teaching and Learning

Dr. Michael Steele

Abstract

High academic standards for school mathematics are an important component to improving outcomes for secondary students. However, high standards alone are not enough to transform teaching and learning in the classroom. The National Council of Teachers of Mathematics’ Principles to Actions document identifies eight research-based effective mathematics teaching practices that teachers can use in their classrooms to support stronger student learning outcomes. In this session, we will use the new (and free) NCTM materials that support teacher learning about the effective mathematics teaching practices using rich tasks and narrative and video case resources. Specifically, our work in this session focuses on supporting teachers in using and connecting multiple mathematical representations and facilitating meaningful mathematics discourse.
The Role of Historically Black Colleges and Universities as Pathway Providers: Institutional Pathways to the STEM PhD Among Black Students

Dr. Rachel Upton

Abstract
The participation of diverse groups of individuals in the science, technology, engineering, and mathematics (STEM) academic and workforce communities is severely lacking, particularly in the context of the nation’s shifting demographic landscape. The need to broaden participation in STEM is particularly salient for those who identify as African American or Black. Historically Black colleges and universities (HBCUs) might play an especially critical role in increasing the number of Black STEM PhD holders. An examination of Black STEM PhD recipients’ institutional pathways to the doctorate can provide insight into who among Black students are earning STEM doctoral degrees. Specifically, this study seeks to examine the following questions: (1) How many STEM doctoral degrees were awarded to Black students overall and by discipline of study? (2) What proportion of Black STEM PhD recipients earned their doctoral degrees from HBCUs, and which HBCUs were the top producers of Black STEM PhDs? (3) What are the institutional pathways of Black STEM PhDs? (4) How do the characteristics of Black STEM PhD recipients (e.g., discipline of study, citizenship status, gender, first-generation college status, and level of graduate student funding and debt) differ by institutional pathways taken to the STEM doctorate?

Diagnosing Areas of Weakness Among Mature STEM Students at Third-Level, and Current Intervention Methods Employed to Combat these Weaknesses at The University of Limerick

Mr. Richard Walsh

Abstract
This paper presentation will report on the findings from diagnostic test data relating to the mathematical abilities of mature first-year undergraduate STEM students (n = 324) at the University of Limerick (U.L.) from 2010 to 2014. A focus will be placed on science and technology students. The diagnostic test instrument that is administered to all first-year science and technology students in their first mathematics lecture of the year will be described. The presentation will explore the mathematical areas that mature students in these disciplines find difficult, as well as the mathematics that they have a sound understanding of, upon entering a third-level undergraduate programme of study. The current intervention methods employed at U.L. for those diagnosed as “at risk” from the diagnostic test will be discussed.
Engaging Learners and Realizing the Development of Mathematical Practices

Dr. Trena Wilkerson

Abstract
Examining the roles of problem solving, perseverance, and reasoning in the context of learning mathematics as adults and in mathematics teacher education is essential to support the development of key mathematical practices of learners and mathematical teaching practices. We will engage in activities and discuss approaches that support adult learners and teachers of mathematics as they engage in problem solving.
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