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Javier Díez-Palomar

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Objectives

Adults Learning Mathematics (ALM) – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

· Research and theoretical perspectives in the area of adults learning mathematics/numeracy
· Debate on special issues in the area of adults learning mathematics/numeracy
· Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Adults Learning Mathematics – An International Journal

In this Volume 11(2)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editorial</td>
<td>4</td>
</tr>
<tr>
<td>Javier Diez-Palomar</td>
<td></td>
</tr>
<tr>
<td>Novice teachers reflect on their instructional practices while teaching adults math</td>
<td>7</td>
</tr>
<tr>
<td>Lynda Ginsburg</td>
<td></td>
</tr>
<tr>
<td>The intricacies of assessing numeracy: Investigating alternatives to word problems</td>
<td>14</td>
</tr>
<tr>
<td>Kees Hoogland and Birgit Pepin</td>
<td></td>
</tr>
<tr>
<td>Making the most of PIAAC: Preliminary investigation of adults’ numeracy practices through secondary analysis of the PIAAC dataset</td>
<td>27</td>
</tr>
<tr>
<td>Diana Coben, Barbara Miller-Reilly, Paula Satherley and David Earle</td>
<td></td>
</tr>
<tr>
<td>Survey of adult students with mathematical difficulties</td>
<td>41</td>
</tr>
<tr>
<td>Linda Jarlskog</td>
<td></td>
</tr>
<tr>
<td>Does adding mathematics to English language learners’ timetables improve their acquisition of English?</td>
<td>52</td>
</tr>
<tr>
<td>Jenny Stacey</td>
<td></td>
</tr>
</tbody>
</table>
**Editorial. “Opening Our Mathematical Eyes: Seeing Math in Everything We Do.”**

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We are glad to introduce a new issue of *Adults Learning Mathematics: An International Journal*. This issue includes some of the papers presented in the 2015 annual meeting of ALM, held in Washington D.C. The theme for that conference was mathematics as a component of every single human action. Actually, mathematics is closely embedded in our society in many different ways. A month ago, while facilitating a *Dialogic Mathematical Gathering* (DMG) via Skype with a group of adult learners in Barcelona, an interesting discussion came up: if everything is mathematics, then, where does mathematics come from? A man over sixty years old said: “we [human beings] invented mathematics.” This sentence provoked a lively discussion among participants within the DMG. Is mathematics a special language invented by humans, or mathematics is an attribute we observe in every *thing* in the World? Is mathematics a ιδεα in terms of Plato, or is just a physical property related to quantity, order, geometrical attributes that we can just *notice* via observation? Is mathematics outside our minds, or is it a product of our minds? The man who raised the discussion had a particular approach to it: according to him, mathematics is a human invention; it is just a tool in our hands to dominate the World. A woman reacted to him claiming that mathematics was discovered by ancient people. The discussion moved forward. Someone else said that actually, some people affirm that mathematics is in the origin of our *intelligence*, in the sense that ancient *homon* develop language skills (and cognition) as a consequence of learning to modulate songs and attributing them meaning to communicate important information to the other members of the group, like advertising danger (because coming a number of predators), food resources, location, etc. In a sense, we were able to develop a communicational system to share and build on a particular approach to perceive reality. That was a very interesting discussion. The man who raised the discussion still defended that mathematics was a tool “for men to dominate the group [and the World].”

Anyway, what seems clear from the discussion is that mathematics is present in our lives, and we use mathematics for almost everything. I was not present in Washington D.C. in 2015; however, looking through the articles included in this special issue I can try to imagine the important and significant discussions held on that occasion. First piece, Lynda’s work on *novice teachers*, invites us to have a look on how new teachers in mathematics tend to go back to their “own memories” on how to teach mathematics. That brings me back to my own memories, when I started to teach mathematics to adult learners. I did not have any experience in teaching at that time. Thus, what did I do? Teaching in the same way I saw my teachers of mathematics during my whole life up to that time. And that was my mistake, my huge mistake. I was not able to connect with the adult learners. It was like talking in a very different language to them. Some of them were even discouraged from attending my classes. Then, I learnt to listen to them. I learnt to give them a voice. I learnt to approach mathematics in other ways… probably ways that many of us would present as non-traditional or informal ways to do mathematics. But I learnt as well that those “ways to talk about mathematics” are very powerful, meaningful,
situated, connected... And, even more important, there were ways to connect “my mathematics” to “their mathematics,” doing something new, together. Lynda’s piece brought me back these memories, and I think that it is an important article to keep us on the need to include such thinking in our practices, as well as including that dimension in the professional training of in-service and pre-service teachers (especially those who want to work with adults).

Next article is about the use of word problems to assess numeracy. In this piece Kees Hoogland and Birgit Pepin explore the impact of using visual representations to introduce word problems to a set of adult learners, instead of using traditional verbal representations. Previous literature in our field suggests that students find difficulties in understanding problems when they are presented using text. In fact, at least in elementary education, students tend to feel mathematics harder when the teacher introduces problem solving. Algorithms (addition, subtraction, multiplication and division) are just “easy” for many students. However, knowing when they need to use each of them in the frame of a particular word problem is not that “easy.” In this article Kees and Birgit show that adult learners also find difficulties with word problems when using mainly text to introduce the problem. However, when they replace “words” by “visual representations,” then they observe a slightly, but relevant, improvement in terms of achievement in solving this type of problems. This result brings me back to Skemp’s notions about procedural learning and conceptual learning.

The third article is about the PIAAC dataset. Diana Coben, Barbara Miller-Reilly, Paul Satherley and David Earle introduce a preliminary investigation of adults’ numeracy practices through secondary analysis of the PIAAC. This is an international survey looking on adults’ numeracy worldwide, launched by the OECD. International datasets are important because they bring us data, which is comparable across countries. But, more importantly, international datasets become a sort of “points of reference” to re-think our work in order to improve it. Diana and her colleagues provide the analysis of the data belonging to the New Zealand set of data. They conducted a 4-steps procedure in order to analyse numeracy skills among New Zealanders, comparing them to the international dataset. Their approach was very open, even provocative, since they end their piece inviting us to conduct our own analysis of the data.

Next article is a very interesting piece on students with difficulties in mathematics. Linda Jarlskog brings us a study on ten adults who are experiencing some sort of cognitive difficulty (either dyscalculia, acalculia or other mental disorder). This piece is very innovative, since there are not that many studies in adult mathematics education dealing with adults who present some cognitive disorder. In addition, Jarlskog’ article includes a very detailed analysis of her work, which is also very well welcomed. We are just starting to understand how our brain works in order to process information and create cognitive answers to it. Kandel, for instance, have devoted his life to uncover the “secrets” of how our minds work, from the neuroscience. Today we know more about the physical and chemical processes involved in the higher mental functions. But still we need more input on this area. I have the feeling of being in the Ancient World, with the Greeks, looking to the sky, and wondering about the celestial spheres. Now we know that there is much more out there. Jarlskog’ work provide some insights coming out from a particular experience, with ten adults. Her research is an approach from the pedagogy to the learning dilemma, working with special adults. It opens the door to further research.

Last, but not least, the final article in this special issue is about combining mathematics and English to increase the chances for non-English speakers to learn mathematics. Jenny Stacey presents a study on ESOL in the UK. Previous studies have proved that teaching approaches involving the use of a foreign language as instructional medium are more successful than using language as “subject” of the learning, that is: learning English by itself (which many times is the traditional way to learn a language). Cummings, for instance, reports on a huge amount of studies that ESL gain from using the foreign language as a learning language for other subjects because the cognitive effort to understand the subject also “pushes” the learner into the use (and
understanding) of the foreign language (L2). Stacey’s study confirms these previous results with additional evidence from her set of data.

Hence, I conclude this editorial just encouraging the readers to enjoy these five articles, offering a great overview of how different fields and dimensions of our everyday lives connect in many ways to mathematics.
Novice Teachers Reflect on Their Instructional Practices While Teaching Adults Math

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Abstract
Over three years, eighty-two teachers in their first or second year of teaching participated in orientation programs for new adult educators. During the programs, they reflected on their own instructional practices when teaching mathematics to adults. The teachers identified the practice they were likely to overemphasize and explained why they were likely to do so, posting their responses to online course discussion boards. Almost half of the respondents reported they “primarily emphasize calculation skills” and shared various reasons for doing so. The remaining respondents reported emphasizing one of four other instructional practices. Teachers put forth a variety of justifications for the instructional practices they have been using. Professional development efforts will need to recognize and take account of the teachers’ beliefs, assumptions and current practices.

Key words: instructional practices, teaching, mathematics, adults

Introduction
Every adult education teacher was once a novice teacher. In the U.S., most states do not have a certification system requiring a formal educational program to prepare teachers for instructing adult learners who have returned to study to complete their high school education. Existing research on novice teachers has generally focused on those who are completing a university-based teacher education program for those planning to teach in elementary, middle or high schools (e.g., Horn & Campbell, 2015). However, two studies conducted outside of the U.S. have specifically focused on the perspectives of adult educators.

Beeli-Zimmermann (2015) examined the mathematical beliefs of five adult teachers in Switzerland who completed an intensive 8-day training and who are integrating numeracy instruction into their second language (German) instruction. She found that the teachers’ own school experiences influenced their beliefs about mathematics and their teaching preference.

The current study complements the work of Pratt and Beeli-Zimmermann by focusing on a relatively large sample of novice teachers who have had limited professional development but who are making their own decisions about how they are approaching adult numeracy instruction. In explaining the rationales for their decisions, they reveal their influences and intentions.
Methodology

Context of the Study

In one state in the United States, all new adult education teachers are required to complete a semester-long online orientation course during their first or second year of teaching. This course is delivered asynchronously and addresses multiple topics of relevance to adult educators including adult learning theory, career pathways, learning disabilities, reading and literacy, family literacy, assessment, English as a second language, as well as numeracy.

Each topic is addressed during two weeks of the course with relevant activities and discussion topics designed by a practitioner or researcher with expertise in that content area. Each topic segment includes an initial activity or assignment, a pre-taped webinar that participating teachers are expected to watch, followed by a second assignment related to the content of the webinar. During each implementation of the course, the same practitioners or researchers facilitated the topical online discussions.

Full-time teachers are expected to complete assignments for all topics while part-time teachers choose and complete assignments from a subset of the topics. At the end of the course, participating teachers develop and complete a culminating assignment for which they investigate some aspect of their practice. Some participants choose to focus on numeracy, though these investigations are not addressed in this paper.

The Task

This paper reports on the novice teachers’ responses to the initial activity for the Mathematics segment of the course. The assignment states:

1. Read the article: “Designing Instruction with the Components of Numeracy in Mind, Focus on Basics, v.9 (a), 14-20.
2. Reflect on your own teaching and consider whether you frequently find yourself in one of the "risk categories" as described in the article.
3. Explain why you find yourself there and what you may try to do differently.

The assigned article was written by the author of this article to complement a 2006 commissioned paper on the Components of Numeracy (Ginsburg, Manly, & Schmitt) and appeared in a journal produced for practitioners by NCSALL, a research centre funded from 1996 to 2007 to focus on adult basic education. In the article, five practices are described and identified as “risk categories” in that their overemphasis during instruction might limit the broader learning opportunities for adult learners. By the term “risks,” the author meant preferences, priorities, or maybe even ruts. In hindsight, the term “risks” was not the best term to use as it implies a level of danger that might be misconstrued. The list of practices was not meant to be comprehensive but rather a group of commonly seen instructional practices in U.S. classrooms and/or reflecting issues that have been explored in research relevant to adult learners, such as mathematics anxiety (e.g., Beilock& Willingham, 2014; Chinn, 2012; Evans, 2000) or embedding mathematics instruction within real-life contexts (Casey, Cara, Eldred et al, 2006; Stone, Alfield& Pearson, 2008). In the article, each practice is described and the relevant challenges from its overemphasis are explained.

The instructional practices described in the article are:

1. Primarily emphasizing calculation skills (procedures).
2. Focusing on the language aspects of word problems (key words).
3. Attempting to dissipate math anxiety.
4. Primarily dividing math content into distinct, non-overlapping topics.
5. Only embedding instruction within real-life contexts

Each participant selected one risk category as his/her primary instructional practice. As the participants read the entire article before responding to the questions, the position advocated in the article (that ideal instruction would balance competing priorities and demands in order to enhance and enrich learning opportunities) may have influenced their responses. We have no way to judge if this is so, but it would be interesting for future research to explore any impact reading about or discussing ideal instruction may have on teachers’ descriptions of their own practices.

The Subjects

Between October 2012 and January 2015, seven cohorts of novice adult education teachers completed the orientation course. All full time teachers completed the mathematics topic and some part-time teachers chose the mathematics topic as one of their optional topics. Across the seven cohorts, there were a total of 82 teachers who completed the mathematics assignments. A few additional teachers self identified as “I don’t teach math” so they were excluded from this study. Only a few of the teachers were primarily mathematics teachers; the majority of the teachers were teaching multiple subjects which may have included literacy, science and social studies as well as mathematics.

Findings

As shown in Table 1, each of the instructional practices was identified by some of the participating novice adult education mathematics teachers as their primary instructional practice or goal. However, almost half of the teachers saw themselves as “primarily emphasizing calculation skills” in their instruction.

<table>
<thead>
<tr>
<th>Practice</th>
<th>Number (n=82)</th>
<th>Percentage (rounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Primarily emphasizing calculation skills</td>
<td>37</td>
<td>45%</td>
</tr>
<tr>
<td>2. Focusing on language aspects of word problems</td>
<td>9</td>
<td>11%</td>
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<tr>
<td>3. Attempting to dissipate math anxiety</td>
<td>15</td>
<td>18%</td>
</tr>
<tr>
<td>4. Dividing math content into distinct, non-overlapping topics</td>
<td>10</td>
<td>12%</td>
</tr>
<tr>
<td>5. Only embedding instruction within real-life contexts</td>
<td>11</td>
<td>13%</td>
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Each participating teacher explained why he or she primarily uses a particular instructional practice. The explanations they provided for their choices are described for each practice. For the first practice (primarily emphasizing calculation skills), the large number of responses could be categorized into five categories. Since there were fewer participating novice teachers that identified each of the other four practices, their responses could not be as meaningfully categorized.
Practice #1. Primarily emphasizing calculation skills

Thirty-seven participating novice teachers (~45%) said that they “primarily emphasize calculation skills.” Their reasons for doing so fall into five categories of responses. The categories are listed here with one or more examples of teachers’ reasons.

1. Meeting my learners’ wishes
   
   My students say, “Teach me how to do it. I don’t care why.”

2. Using the available workbooks
   
   I tend to focus heavily on workbooks that focus strictly on fractions, decimals, or percent. These seem like easy solutions to the larger problem, that students have either forgotten or never fully understood the fundamentals of math.

3. Efficient classroom management, especially given multi-level classes
   
   I have many students at low levels, yet in the same classroom I have those who score at 11th and 12th grade level. In order to offer individual attention where needed, students need something to work on when I am helping others.

4. Belief that mastery of computation must precede problem-solving
   
   The majority of my students lack the basic arithmetic calculation skills….I start off with this method and then move on to other methods such as word problems and real-life contexts. If a student does not have the basic skills they will not be able to move on to the higher level of mathematical reasoning.

5. Teacher’s personal comfort, ability and satisfaction
   
   It is very easy to get caught up in only focusing on the easily measurable, teachable, and observable, especially with learners who are lacking these skills.

   It’s easy to simply teach calculations – teach the steps, do examples, assign some worksheets, and voila! Now my students can do operations with fractions!

   That is the way I was taught. I find it just plain easier to talk about math in terms of calculations and operations. Admittedly, I don’t always understand the meaning of the procedures myself and thus find it difficult, if not impossible, to articulate meaning for my students.

   There is a comfort level there for me, and getting the right answer validates to me that I am doing a good job of conveying the material to my students.

Practice #2. Focusing on language aspects of word problems

This practice was identified by 9 of the novice teachers (~11%). It is often described by teachers as attending to the “key words” in word problems that ostensibly provide clues to which operation is required to solve the problem. Examples of such key words are: ‘in all’ (addition), ‘difference’ (subtraction), and ‘of’ (multiplication, particularly with fractions or percentages).

Among the reasons put forth by the teachers for emphasizing the key word strategy were that the strategy was perceived as efficient or that the strategy was presented in a textbook. A few of the teachers noted that their own strength was in literacy instruction, and thus they were less comfortable teaching mathematical problem solving by focusing on the mathematics. For example, two teachers commented:

   We try to use the language to help reason through the math. If the words present themselves, why not use them?

   I would like to explore the complex and messy nature of solving meaningful math problems; however, because of time restraints I often feel the need to simply teach language clues within word problems as an overall strategy.
**Practice #3. Attempting to dissipate math anxiety**

Many adult learners announce to their teachers that they are afraid of studying mathematics, have had prior negative experiences studying mathematics in school or have been told by parents or teachers that they are just not mathematically capable. All adult education teachers want to alleviate their students’ suffering and make it possible for them to learn the mathematics they need to learn. The fifteen teachers (~18%) who identified this practice as their primary instructional goal focused on creating a safe environment for their learners. However, upon reflection, the teachers note they may have been helping them too much for fear of adding to their struggles but may not have been actually helping them to learn the mathematics. Two teachers reported:

> I have a tendency to take a difficult math problem and break it into such small pieces that the bigger problem becomes lost. This may lead to frustration in some students because they still can’t grasp the bigger problem. I catch myself saying it’s only adding, subtracting, multiplying, and dividing, you just need to know when to do what. Well, if you don’t know when to do what, that’s probably not much help and could be frustrating.

> I think that sometimes they have learned through the years to become learned helpless. They feel that someone needs to help them. Sometimes when working with them I will take their pencil and show them the mistake, instead of letting them think out the problem for themselves and using a strategy to solve the problem.

**Practice #4. Dividing math content into distinct, non-overlapping topics**

Ideally, learners should come to see the connections across mathematical content, such as recognizing connections among fractions, decimals and percentages. But often the topics are taught in isolation. Ten teachers (~12%) identified with this practice, justifying their instructional practice by stating that their textbooks divide the topics and rarely mention any connections or that they divide the content into discrete topics because “it seems organized and simpler.” Among the teachers’ comments were:

> I’m short on time. I try to introduce concepts in small chunks because I feel there is so much to cover, and I don’t want to overwhelm students.

> I felt since I know what students need to know for the test, I could provide them with a map. We would cover one topic at a time and move forward as they understood and grasped the concepts.

**Practice #5. Only embedding instruction within real-life contexts**

Eleven novice teachers (~13%) chose this practice. Some of the teachers identified themselves as primarily teachers of English as a Second Language (ESL) and thus did not really address mathematics at all unless it came up in discussions of real-life situations. Others in this group suggested that they focused primarily on real-life contexts because the contexts provide authenticity, learner interest and engagement is high, or because they believe research has shown that this approach is desirable and recommended. Among their comments:

> When I was hired, I was told that my instruction should be relevant and rigorous.

> I find that students are more interested and eager to learn when they can relate the math to something they may encounter in a real life situation.

> Relating to real life math experiences is how I can best relate to the student because it’s what I know. One benefit of doing this is that the adults all take part in the teaching process [pooling knowledge].

What might you do differently?

As part of the assignment, the novice teachers had been asked to consider what they might do differently in the future. Almost all indicated they would try to incorporate a broader range of practices and priorities into their instruction. There were few comments that went beyond suggestions provided in the article. Since the participants were novice teachers only just
becoming accustomed to teaching mathematics to adults, they may not yet have been ready or prepared to modify their instructional practices without a support community. Further research on the process of teacher change, particularly among novice adult educators, could inform the development of useful supportive programs and environments.

**Conclusions**

Many novice adult mathematics teachers focus primarily on computation practice in their instruction even though many of them recognize the limitations of this practice. Their rationales indicate this is what they know and are familiar and comfortable with. Further, since so many adult education teachers have limited mathematics content backgrounds (Gal & Schuh, 1994), they may not have the deep content knowledge to go beyond computational instruction.

Some of the novice teachers’ beliefs and instructional approaches seem to be informed by their own experiences in school and they are choosing to teach as they remember themselves being taught. They may have difficulty picturing approaches that emphasize the development of conceptual understanding or the ability to apply mathematics in situations because they have never seen or experienced such practices in classroom instruction.

Further, some novice teachers’ instructional practices seem to be reinforced by the environments of the adult education programs in which they work. They mention relying on the resources that are available and the instructional practices implemented by colleagues. Programs are required to assess and report learners’ progress using standardized tests that require computational competence.

Some teachers believe they are actively aligning their instruction with their learners’ expectations and desires. Learners are hoping their time in adult basic education will be short so they are pleased to move quickly from one topic to another, in effect crossing off the items on the content checklist, encouraging the instructional practice of teaching content as distinct, non-overlapping topics. Most learners’ goals focus on achieving a High School Equivalency certificate. These assessments allow the use of calculators for almost all questions and require conceptual understanding since all questions are applications (word problem situations). Thus, some teachers emphasize key word strategies for dealing with these problems.

The development and implementation of effective professional development for novice adult mathematics educators requires recognition of the teachers’ various assumptions, current practices and rationales for those practices. Just as adult learners return to study with their own experiences in and out of school, their teachers bring their own sets of experiences, mathematical content knowledge and beliefs.

For the novice teachers in this study, the practices they identified were recognized during the second week of the mathematics component of the course, with the goal of considering how their preferred practices could be enriched by attending to other priorities. Surely, in an extended professional development course focused on teaching mathematics to adults, it would be valuable to engage in a deeper exploration of how teachers’ own experiences, beliefs and content knowledge guide their instructional practices and decisions. This study showed that different teachers rely on different instructional practices; during professional development, teachers can examine and discuss each others’ instructional strategies with a goal of developing a balanced approach that includes a variety of practices and priorities.

**References**


The Intricacies of Assessing Numeracy: Investigating Alternatives to Word Problems

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Abstract

Word problems are often used to assess numeracy, despite the growing number of reports on difficulties students encounter with this genre of mathematical problems. These reports contend that a large number of difficulties are influenced by the way the problems are presented, that is, with verbal representations of the problem situations. These difficulties are said to be associated with a form of suspension of sense-making. In this study, conducted in the Netherlands, we investigated the effect on adult participants’ performances of changing the representation of the problem situation, from verbal to image-rich. A controlled randomised trial was the main part of this investigation. Furthermore, we compared the results of adult participants with the results of a similar trial which was held with students from primary and secondary education. The study showed that adult participants’ performances improved slightly with the change in representation, particularly on tasks in the content domain of measurement & geometry. These results were comparable with the results found of students from primary and secondary education, indicating that the effect is not related to age. The results could be of interest, however, for all practitioners involved in the work of numeracy task design.

Key words: numeracy, assessment, word problems

Introduction

In most recent approaches in adult numeracy research, adult numeracy is defined as a complex, multifaceted, and sophisticated construct, incorporating the mathematics, communications, cultural, social, emotional and personal aspects of each individual in context (American Institutes for Research, 2006; Coben, 2003; Geiger, Goos, & Forgasz, 2015). As a consequence, learning and assessing numeracy in authentic situations is often advocated (Frankenstein, 2009).

A closer look at lesson or test materials used in numeracy education in many countries, however, reveals that most assessment materials consist of word problems or of items assessing procedural arithmetic skills. The same is the case in the Netherlands where, despite the country’s high rankings in international comparisons, there are persistent complaints about the literacy and numeracy levels of young adults in vocational education and in the workplace. As a result, in 2010 a Literacy and Numeracy Framework (LaNF) was introduced in the Netherlands, with a compulsory numeracy examination at the end of the vocational educational tracks.
(Hoogland & Stelwagen, 2012). After a lively debate on the assumed value of procedural skills for (young) adult learners, a compulsory numeracy examination has been implemented which consists of 45 mathematical problems of which 15 are strictly procedural problems and 30 arecontextual problems. Many teachers and mathematics educators have questioned the relevance of assessing vocational students this way, and they perceive a gap between the numeracy used by their young adult students in everyday life and (future) work, and numeracy as operationalised in the final examinations (Hoogland, 2006; Hoogland & Pepin, in press).

A study in 2011 and 2012 in the Netherlands focused on the idea that using image-rich numeracy problems contributes to bridging the gap between common classroom practice in numeracy and more sophisticated numeracy concepts (Hoogland, 2016). Part of this study was a controlled randomised trial with almost 32,000 primary, secondary, and vocational students, to investigate the effect on students’ performance of changing the representation of the problem situation from verbal (word problem) to image-rich (mixture of picture and words). The trial revealed that students performed better on image-rich numeracy problems than on otherwise equivalent word problems (Hoogland, 2016), indicating that students are less hampered by the many difficulties with word problems that are frequently reported (Verschaffel, Greer, & De Corte, 2000; Verschaffel, Greer, Van Dooren, & Mukhopadhyay, 2009). In an experiment in 2013 in the Netherlands the results of this trial were replicated with adult participants. The results are shown in this article and a comparison is made with the results of students from primary and secondary education. It revealed which types of tasks particularly, in both populations, benefitted most from the change from a verbal description of the problem situation, to a mainly depictive description of the problem situation.

**Theoretical perspectives**

This study is part of a larger research project to investigate alternatives to the persistent and problematic use of word problems to teach and assess students’ ability to deal with numerical problems originating in everyday life. This ability of students is often labeled as numeracy or mathematical literacy, although these concepts have been and are still evolving (Coben, 2003; Geiger et al., 2015; Ruthven, 2016). The sometimes superficial use of the concept is also criticised (Jablonka, 2015).

In current classroom practice, word problems are used predominantly to teach and assess these abilities. Many researchers, however, report serious difficulties in using word problems to assess these abilities (Verschaffel et al., 2000; Verschaffel et al., 2009). The reported difficulties can be related to the steps the problem solver is expected to take to solve the task at hand. Figure 1 shows the diagram used in PISA as a schema for the relevant steps in the problem-solving process. Similar diagrams are used in related research on problem solving and modelling in mathematics education (Blum, Galbraith, Henn, & Niss, 2007; Burkhardt, 2006; Lesh, Post, & Behr, 1987). The reported difficulties seem to appear mainly in the two horizontal steps in the diagram: “formulate the mathematical problem”, and “interpret the mathematical results”. In the first step (formulate) students are reported to look at these problems with a strong “answer-getting mindset” (Daro, 2013) and a calculational approach (Thompson, Philipp, Thompson, & Boyd, 1994), as if the problem was limited to the right-hand vertical step of the problem-solving process and that solving problems of any kind means getting the “right answer” by conducting a series of operations on the numbers in the problems. In the third step (interpret) students are reported not to take common-sense considerations about the problem into account (Greer, 1997).

We conjectured that the use of images from real life would strengthen the association with real-world situations (Palm, 2009) and therefore decrease the suspension of sense-making (Schoenfeld, 1992) and the strong calculational focus (Thompson et al., 1994). As a paraphrase of the most used definition of word problems (Verschaffel, Depaepe, & Van Dooren, 2014), we
suggested the following definition for such image-rich problems: “Image-rich numeracy problems can be defined as visual representations of a problem situation wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical reasoning to numerical data available in the problem representation”.

Cognitive psychology also offers theories and insights on the effect of depictive and descriptive representations on creativity and problem solving (Schnitz, 2002; Schnitz, Baadie, Müller, & Rasch, 2010; Schnitz & Bannert, 2003). Schnitz and Bannert (2003) concluded that task-appropriate graphics may support learning and task-inappropriate graphics may interfere with mental model construction. Schnitz et al. (2010) stated that, to solve a quantitative problem, a task-oriented construction of a mental mathematical representation is necessary, provided that it is task-appropriate. Their line of reasoning is that depictive representations can help students to make a relevant mathematical mental model of the situation, and that depictive representations have a high inferential power because the information can “be read off more directly from the representation” (p. 21). This perspective added to the plausibility of our conjecture, which we tested in our empirical studies and also gives some indications in which kind of problems the effect might be strongest, that is, problems whereof the representation of the problem situation is beneficial for constructing of a (mental) mathematical model needed to solve the problem.

Figure 1. A model of mathematical literacy in practice. From OECD (2013a) (p. 26)

**Design of alternatives to word problems**

In order to counteract these tendencies and the associated difficulties we designed tasks that were more “authentic” by changing the representation of the problem situation from descriptive to mainly depictive (Hoogland, 2016; Hoogland, Pepin, Bakker, de Koning, & Gravemeijer, 2016). Those tasks were incorporated in an instrument to measure students’ performance on both word problems and image-rich problems in a randomised controlled way. In a trial with students from primary and secondary education our conjecture was confirmed (Hoogland,
The intricacies of assessing numeracy

Although the conjecture was confirmed, the results were not straightforward. The students’ scores on image-rich problems were slightly higher (2%), which was significant, but with a small effect size \((d = .09)\) and the effect of better performance was most noticeable in tasks in the domain of measurement & geometry. The research question for the study reported here is: Does a replication of the original trial with adult participants show the same patterns and results as the original trial with primary and secondary students? In this paper we report on that replication of the original study with adults who participated in the “Groot Nederlands Rekenonderzoek (GNRO)” [Great National Numeracy Survey], a research initiative by the public broadcasting organisations VPRO and NTR, supported by the Netherlands Organisation for Scientific Research (NWO). Individuals of all ages and of all places in the Netherlands could register as participants on the GNRO website and could engage in a series of mathematical tests. We report the results from the trial with students from primary and secondary education in adapted format in the results section for easier comparison.

The Dutch context

For the international reader, we provide some information on the Dutch educational context. In the Netherlands in 2010 the “Referentiekader Taal en Rekenen” [Literacy and Numeracy Framework (LaNF)] was introduced as a guideline for Literacy and Numeracy education in the age range of 4–18 years (Hoogland & Stelwagen, 2011; Ministerie van OCW, 2009), followed by a very similar version for adult education.

<table>
<thead>
<tr>
<th>Framework</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIMSS 2015 – 8th grade</td>
<td>Number</td>
</tr>
<tr>
<td>PIAAC 2016</td>
<td>Quantity &amp; Number</td>
</tr>
<tr>
<td>PISA 2015</td>
<td>Quantity</td>
</tr>
<tr>
<td>Dutch LaNF 2010</td>
<td>Numbers</td>
</tr>
</tbody>
</table>

*Note.* Presented by similarity (horizontal).

The content domains in these frameworks resemble the categories used in the international frameworks on numeracy and mathematical literacy, such as TIMSS, PISA and PIAAC (Mullis & Martin, 2013; Organisation for Economic Co-operation and Development (OECD), 2013b; PIAAC Numeracy Expert Group, 2009). Table 1 gives an overview of the content domains used in the various frameworks. It is noteworthy that in the Dutch framework there is more emphasis on proportions, including fractions and percentages, and an absence of focus on uncertainty, chance and data (representation).

Method

The instrument

To measure the effect of the change in representation of the problem situation on the performance of participants we used an instrument that was used in both the trial with students...
in primary and secondary education and in this replication study with adult participants. The trials were held with Dutch language items (Hoogland, 2016); English translations of these items are available under open access (Hoogland & De Koning, 2013). The instrument consisted of 24 items of which 21 items were designed in two versions: word problem and image-rich problem. For every participant a test was composed randomly with 10 or 11 items in each version. The randomly selected items were presented in random order for each participant. In this case a randomised controlled trial was built into the test. Both versions of each item had an equal chance of being selected, independent of any other variables, measured or not.

In Figure 2 we show an example of two versions of an item. The items are translated to English for better readability. In the test, each item was presented as a screen-filling problem with an open numerical answer field. The tasks in the research instrument were validated and tested in earlier research activities (Hoogland et al., 2016). The complete set of tasks can be found under open access via the Dutch institute DANS/NWO (Hoogland & De Koning, 2013).

In Table 2 we give an overview of the items in the instrument, evenly distributed across three domains of the LaNF: numbers, proportions, and measurement & geometry. Three tasks in the instrument were in the domain of relations, and were only presented in one version, because of the already visual nature of the items.

Table 2.
Overview of tasks in the used instrument

<table>
<thead>
<tr>
<th>item</th>
<th>Domain: Numbers</th>
<th>item</th>
<th>Domain: Measurement &amp; Geometry</th>
<th>item</th>
<th>Domain: Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td>i04</td>
<td>TV + DVD</td>
<td>i01</td>
<td>Apples in bag</td>
<td>i03</td>
<td>Travel time</td>
</tr>
<tr>
<td>i05</td>
<td>Change</td>
<td>i02</td>
<td>Fuel usage</td>
<td>i06</td>
<td>Recipe</td>
</tr>
<tr>
<td>i09</td>
<td>Money pile</td>
<td>i11</td>
<td>Double glazing</td>
<td>i07</td>
<td>Price magazine</td>
</tr>
<tr>
<td>i12</td>
<td>Kitchen tiles</td>
<td>i13</td>
<td>Water bottles</td>
<td>i08</td>
<td>AEX index</td>
</tr>
<tr>
<td>i16</td>
<td>Hamburgers</td>
<td>i14</td>
<td>Bedroom tiles</td>
<td>i10</td>
<td>Scale model</td>
</tr>
<tr>
<td>i17</td>
<td>Cough syrup</td>
<td>i19</td>
<td>Cake tin</td>
<td>i15</td>
<td>Endive</td>
</tr>
<tr>
<td>i18</td>
<td>Public debt</td>
<td>i21</td>
<td>Chocolate boxes</td>
<td>i20</td>
<td>Winter tires</td>
</tr>
</tbody>
</table>
**Participants**

The research conducted for this paper was a trial with 420 participants from the GNRO research. Table 3 shows the distribution of gender and age categories of these participants. The GNRO was, after registration, an open access public test held in 2013. We cannot consider these participants as a representative sample of Dutch adults. However, we consider the distribution over age and gender diverse enough to draw some tentative conclusions on the results in comparison with the results of the original trial with students from primary and secondary education.

<table>
<thead>
<tr>
<th>Age</th>
<th>n (%)</th>
<th>Gender</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-19</td>
<td>15 (5.3%)</td>
<td>Male</td>
<td>115 (40.4%)</td>
</tr>
<tr>
<td>20-29</td>
<td>61 (21.4%)</td>
<td>Female</td>
<td>170 (59.6%)</td>
</tr>
<tr>
<td>30-39</td>
<td>66 (23.2%)</td>
<td>Not stated</td>
<td>135</td>
</tr>
<tr>
<td>40-49</td>
<td>64 (22.5%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>31 (10.9%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60-69</td>
<td>39 (13.7%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td>9 (3.2%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not stated</td>
<td>135</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Total sample is 410. *n* is number with percentages taken on stated age and gender in parentheses.

The original trial was conducted in October and November 2011. In that trial 31,842 students from 179 schools geographically spread across the Netherlands, participated. For convenience in comparing we show the results of the participants in the trial with students from primary and secondary education in this section. Table 4 shows the number of participants from the educational streams in the Dutch school system.

<table>
<thead>
<tr>
<th>Age</th>
<th>n</th>
<th>Gender</th>
<th>n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11-12</td>
<td>969 (3.1%)</td>
<td>male</td>
<td>15,310 (49.7%)</td>
</tr>
<tr>
<td>12-19</td>
<td>30,222 (96.9%)</td>
<td>female</td>
<td>15,465 (50.3%)</td>
</tr>
<tr>
<td>Not stated</td>
<td>680</td>
<td>not stated</td>
<td>1,067</td>
</tr>
</tbody>
</table>

*Note.* Total sample is 31,842 participants. Age group 11-12 is primary education, age group 12-19 is secondary education. *n* is number with percentages taken on stated age and gender in parentheses.

In this original trial we assumed the sample to be representative of Dutch students in the age group 11–19 years.
The statistical analysis focused on the difference in scores on the A-version and the B-version of the 21 paired problems. We conducted a classical analysis using mean, standard deviation, t-tests, and Cohen’s d as effect size to get a general idea of how the separate items contributed to the overall result we found (Cohen, 1988). As a caveat regarding the effect sizes note that we are not dealing with the most common cycle in educational research of measurement – intervention with the participants – measurement. The effect size category lists of Cohen (1988) or Hattie (2009) do not apply to this situation. Changing the representation of the problem situation is not an educational intervention. We are investigating what is the effect on participants’ behaviour of such a change and not measuring what they have learned from an intervention or a “treatment”.

Results

We present for both trials the results in the same table format for easier comparison. We compared the results of adults with the results of students from primary and secondary education. We focus in this comparison on the overall test and the results at item level. For the overall result on the test on the data collected in the GNRO we conducted a t-test on the mean scores on the A- and B-version items for each participant. We found that the difference in mean was .011 with standard error .001 and p = .184 (n.s.).

On item level we conducted a two-sided t-test with pooled variances to evaluate whether for each item the scores on the two versions differed significantly. We used a common effect size index, namely Cohen’s d, for a first general conclusion. The results are shown in Table 5. We found in four paired problems that the scores on the B-version were significantly higher than the scores on the A-versions with effect sizes ranging from .16 to .59. Furthermore, we found in one pair of problems that the scores on the A-version were significantly higher than the scores on the B-version with an effect size of .04. In the 16 other items the differences between the scores were not significant. The results in this replication trial were in most aspects in line with the results in the earlier large-scale student trial, which is discussed in more detail below.

Table 5.

Results from the GNRO trial, mean and t-test results

<table>
<thead>
<tr>
<th>Item</th>
<th>N</th>
<th>Mean (SE) A</th>
<th>Mean (SE) B</th>
<th>t-test</th>
<th>effect size d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>version A</td>
<td>version B</td>
<td>p (</td>
<td>T</td>
</tr>
<tr>
<td>i1</td>
<td>218</td>
<td>.899(.020)</td>
<td>.872(.024)</td>
<td>.374</td>
<td></td>
</tr>
<tr>
<td>i2</td>
<td>215</td>
<td>.823(.026)</td>
<td>.800(.028)</td>
<td>.544</td>
<td></td>
</tr>
<tr>
<td>i3</td>
<td>220</td>
<td>.836(.025)</td>
<td>.800(.028)</td>
<td>.337</td>
<td></td>
</tr>
<tr>
<td>i4</td>
<td>205</td>
<td>.951(.015)</td>
<td>.916(.019)</td>
<td>.150</td>
<td></td>
</tr>
<tr>
<td>i5</td>
<td>196</td>
<td>.898(.022)</td>
<td>.772(.028)</td>
<td>.000 ***</td>
<td></td>
</tr>
<tr>
<td>i6</td>
<td>209</td>
<td>.895(.021)</td>
<td>.929(.018)</td>
<td>.218</td>
<td></td>
</tr>
<tr>
<td>i7</td>
<td>223</td>
<td>.749(.029)</td>
<td>.751(.031)</td>
<td>.955</td>
<td></td>
</tr>
<tr>
<td>i8</td>
<td>207</td>
<td>.662(.033)</td>
<td>.690(.032)</td>
<td>.537</td>
<td></td>
</tr>
<tr>
<td>i9</td>
<td>209</td>
<td>.593(.034)</td>
<td>.540(.034)</td>
<td>.274</td>
<td></td>
</tr>
<tr>
<td>i10</td>
<td>212</td>
<td>.821(.026)</td>
<td>.870(.023)</td>
<td>.162</td>
<td></td>
</tr>
<tr>
<td>i11</td>
<td>203</td>
<td>.493(.035)</td>
<td>.774(.028)</td>
<td>.000 ***</td>
<td>.59</td>
</tr>
<tr>
<td>i12</td>
<td>212</td>
<td>.811(.027)</td>
<td>.789(.028)</td>
<td>.559</td>
<td></td>
</tr>
<tr>
<td>i13</td>
<td>202</td>
<td>.896(.021)</td>
<td>.858(.024)</td>
<td>.233</td>
<td></td>
</tr>
<tr>
<td>i14</td>
<td>212</td>
<td>.472(.034)</td>
<td>.433(.034)</td>
<td>.423</td>
<td></td>
</tr>
<tr>
<td>i15</td>
<td>226</td>
<td>.774(.028)</td>
<td>.825(.027)</td>
<td>.198</td>
<td></td>
</tr>
<tr>
<td>i16</td>
<td>219</td>
<td>.872(.022)</td>
<td>.866(.024)</td>
<td>.845</td>
<td></td>
</tr>
</tbody>
</table>
The results of the large scale trial have been published before (Hoogland, 2016). Table 6 highlights only those results that are necessary to make the comparison with the replication sample of this study. For this comparison only we incorporated \( p < .10 \) as a category – it is not used for further statistical inferences. For the overall results on the test on the data collected in the large-scale school trial, we conducted a \( t \)-test on the mean scores on the A- and B-version items for each participant. We found that the difference in mean was .019 with standard error .001 and \( p < .001 \) (**). On item level we conducted a two-sided \( t \)-test with pooled variances to evaluate whether for each item the scores on the two versions differed significantly. We again used the effect size index, Cohen’s \( d \), for similar conclusions. The results are shown in Table 6.

### Table 6

**Results for the large-scale school trial, mean and \( t \)-test results**

| Item | N version A | version B | Mean (SE) version A | Mean (SE) version B | \( t \)-test \( p (|T|>|t|) \) | effect size \( d \) |
|------|-------------|-----------|---------------------|---------------------|-------------------------|-----------------|
| i1   | 15,878      | 15,964    | .716 (.004)         | .720 (.004)         | .424                    |                 |
| i2   | 15,986      | 15,856    | .525 (.004)         | .483 (.004)         | .000 ***                | .08             |
| i3   | 15,785      | 16,057    | .314 (.004)         | .290 (.004)         | .000 ***                | .05             |
| i4   | 15,835      | 16,007    | .826 (.003)         | .833 (.003)         | .131                    |                 |
| i5   | 16,038      | 16,804    | .720 (.004)         | .828 (.003)         | .000 ***                | .26             |
| i6   | 15,775      | 16,067    | .631 (.004)         | .640 (.004)         | .102                    |                 |
| i7   | 16,065      | 15,777    | .404 (.004)         | .416 (.004)         | .042 **                 | .02             |
| i8   | 16,298      | 15,544    | .303 (.004)         | .299 (.004)         | .420                    |                 |
| i9   | 16,069      | 15,773    | .221 (.003)         | .213 (.003)         | .085 *                  | .02             |
| i10  | 15,882      | 15,960    | .495 (.004)         | .525 (.004)         | .000 ***                | .06             |
| i11  | 15,850      | 15,992    | .145 (.003)         | .310 (.004)         | .000 ***                | .39             |
| i12  | 15,871      | 15,971    | .466 (.004)         | .438 (.004)         | .000 ***                | .06             |
| i13  | 15,931      | 15,911    | .619 (.004)         | .641 (.004)         | .000 ***                | .05             |
| i14  | 15,889      | 15,953    | .040 (.002)         | .046 (.002)         | .080 *                  | .02             |
| i15  | 15,793      | 16,049    | .394 (.004)         | .388 (.004)         | .264                    |                 |
| i16  | 15,921      | 15,921    | .803 (.003)         | .815 (.003)         | .005 ***                | .03             |
| i17  | 15,986      | 15,856    | .803 (.003)         | .787 (.003)         | .000 ***                | .04             |
| i18  | 15,847      | 15,995    | .153 (.003)         | .168 (.003)         | .000 ***                | .04             |
| i19  | 15,932      | 15,910    | .247 (.003)         | .284 (.004)         | .000 ***                | .08             |
| i20  | 15,925      | 15,917    | .130 (.003)         | .164 (.003)         | .000 ***                | .10             |
| i21  | 16,044      | 15,798    | .188 (.003)         | .256 (.003)         | .000 ***                | .16             |

**Note.** \( N \) is number of items tested. Mean is mean score on items (with standard error in parentheses) \( P(|T|>|t|) \) is result of \( t \)-test, unpaired, unequal with hypothesis that difference in score is 0; \( * p < .10, ** p < .05, *** p < .01 \). Cohen’s \( d \) is effect size. Version A is the word problem; Version B is the image rich numeracy problem.
In the large-scale school trial we found with \( p < .10 \) in 11 paired problems that the scores on the B-version were significantly higher than the scores on the A-versions with effect sizes ranging from .02 to .39. Furthermore, we found in five paired problems that the scores on the A-versions were significantly higher than the scores on the B-versions with effect sizes ranging from .02 to .08.

**Comparing results**

The overall result on performance in this study with adult participants was 1.1 percentage point higher scores on image-rich problems. This was in line with the overall results we found in the large school trial, that is, 1.9 percentage point higher scores on image-rich problems. In almost all items the effect of higher scores on the B-version occurred with a very small effect size, in other items it did not occur. In one item the effect was even opposite. We synthesised the results in Table 7.

Table 7.

*Comparing results of adult and students from primary and secondary education*

<table>
<thead>
<tr>
<th>Domain</th>
<th>Population</th>
<th>A &gt; B</th>
<th>A = B</th>
<th>B &gt; A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers</td>
<td>Adults</td>
<td>43%</td>
<td>14%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>71%</td>
<td>14%</td>
</tr>
<tr>
<td>Meas. &amp; Geom.</td>
<td>Adults</td>
<td>0%</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>14%</td>
<td>71%</td>
</tr>
<tr>
<td>Proportions</td>
<td>Adults</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>Students</td>
<td>14%</td>
<td>43%</td>
<td>43%</td>
</tr>
</tbody>
</table>

*Note.* A > B means the results on the word problem version are significantly larger \( (p < .10) \). A = B means the results are not significantly different \( (p < .10) \). B > A means the results on the image-rich problem version are significantly larger \( (p < .10) \).

Solving problems from the domain of measurement & geometry seems to benefit the most from a depictive representation in both populations. For problems in the domain of numbers we see no beneficial effect for either representation, although the deviation is much larger for the adult population. In problems in the domain of proportions only the student population seems to benefit to some extent from depictive representations. We found three tasks in the domain of measurement & geometry that in both trials showed a significant better performance for the image-rich versions. They are shown in Figure 3. This finding corroborates our earlier findings that the change in representation of the problem situation has the greatest positive influence on the performance of the participants in tasks from the domain measurement & geometry. Indeed, in these cases the depictive representation of the problem situation could arguably be beneficial to form a (mental) mathematical model necessary to solve the problem, such as estimating the area in item 11, calculating the content in item 16, and estimating the content in item 21.
Discussion

Assuming the diagram of problem solving in Figure 1 contains essential steps for the solving process (going from the problem situation to the situation model and on to the mathematical model), we argue that the mental activity needed for the necessary steps in the process is interdependent on the mathematics domain of the task. So following the reasoning of Schnotz et al. (2010), in the domain of numbers the mathematical model is primarily computational and thus one dimensional. In that case a mainly depictive representation was presumed not to contribute considerably to the ease with which problem solvers make sense of the situational or mathematical model. In this domain most items gave no significant difference, even one opposite effect. In the domain of proportions the mathematical model is in general more complex than in the domain of numbers, because there is always some activity of (relatively) comparing quantities or comparing a quantity with a whole. A mainly depictive representation was assumed to be beneficial here. At the same time a counter-effect is possible if the mathematical model and the depictive representations are not mutually beneficial, which might lead to an increased complexity experienced by the participant. For tasks from the domain of proportions one could not make a plausible straightforward prediction, whether a mainly depictive representation could help the solvers to construct the appropriate mathematical model and hence help them in solving the problem in a successful way.

In the domain of measurement & geometry the underlying problem situation in itself is two- or three-dimensional. So, a mainly depictive representation of the problem was assumed to help the problem solver to create the appropriate (mental) mathematical model. We saw in both trials...
that of the four items that significantly favour the image-rich numeracy problem, three are in the domain of measurement & geometry, so this assumption is supported by the data.

In this replication, we found fewer tasks with a significant difference between the A- and the B-versions. With smaller samples, small increases in performance cannot be labelled as statistically significant. Nevertheless the findings give enough incentive for further research in the design of numeracy tasks and the way reality is (re)presented in those tasks.

Conclusions

Word problems are a dominant feature of both classroom teaching and assessment of numeracy worldwide, and also of large-scale international assessments, like TIMSS, PISA, and PIAAC (Mullis & Martin, 2013; OECD, 2013b; PIAAC Numeracy Expert Group, 2009). Lessons learned from these assessments have been brought together recently, see for instance Tout and Gal (2015). Despite these efforts and despite the dominant use of word problems to teach and assess people’s ability to solve practical numerical problems, not much research has been conducted that systematically focuses on the effect on students’ performance of changing the verbal representation of the problem situation to a mainly depictive representation or a more authentic representation of the problem situation.

The original trial and this replication have limitations. The participants in the adult sample were not representative of all adults in the Netherlands. And although the replication strengthened some of the conclusions from the earlier large-scale school trial, the conclusions were still based on a limited number of items. More research is necessary to establish whether the results hold for other sets of problems that are paired in the same way as in these trials. The overall difference in results is small and effect sizes related to those results are in most cases very small. Slavin (2016) has recently stated, in a Huffington Post blog “What is a Large Effect Size?”, that in educational studies using a randomised controlled trial, effect sizes are seldom found over 0.2, however. The conclusions on the effects of a change in representation of the problem situation can thus be labelled as tentative. At the same time, the results of the change were significant and consistent, and not influenced by other variables, so there is, arguably, enough justification to speak of a small but robust effect.

Our suggestion is that in the task design of future assessments the representation of the problem situation should be taken into account as a factor when interpreting the results.

Acknowledgements

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Making the most of PIAAC: Preliminary investigation of adults’ numeracy practices through secondary analysis of the PIAAC dataset

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Abstract
The Programme for the International Assessment of Adult Competencies (PIAAC) assesses key information processing skills and collects information on how often people undertake a range of activities at work and in everyday life. We are exploring what secondary analysis of online anonymised PIAAC data can tell us about adults’ numeracy practices. In the process we review the accessibility and user-friendliness of the data for novice researchers and practitioners in the hope of encouraging them to explore this rich resource and give a brief account of our experience of the process of accessing publicly-available PIAAC data for secondary analysis.

Key words: assessment, numeracy, adults, mathematics

In this paper we explore what secondary analysis of data from the latest international Survey of Adult Skills in the Programme for the International Assessment of Adult Skills (PIAAC1), can tell us about adults’ numeracy practices. In the process we are also reviewing the accessibility and user-friendliness of the dataset. We hope to encourage the exploration of this rich resource by practitioners and researchers, including those with little previous experience of working with large datasets. This is important because, as Hansen and Vignoles (2007, p. 1) point out “In the last few decades, there has been an unprecedented increase in the availability and quality of large-scale data sets that are suitable for use in education research. Analyses of these data have the potential to radically improve the robustness and generalisability of educational research”. In a still young but growing field such as adults learning mathematics, this is especially important.

We focus on secondary analysis of anonymised publicly-available PIAAC data. We draw our understanding of secondary analysis from Dale, Watham and Higgins (2008, p. 520): “Secondary analysis is generally understood as the analysis of data originally collected and analysed for another purpose”. In addition, Heaton (1998) says “Secondary analysis involves the use of existing data, collected for the purposes of a prior study, in order to pursue a research

interest which is distinct from that of the original work; this may be a new research question or an alternative perspective on the original question”. She adds that secondary analysis may be undertaken either by the original researchers or others.

The OECD has published two analytical reports on PIAAC (OECD, 2013a, 2016a). They relate to the first and second rounds of the PIAAC survey in participating countries. The New Zealand Ministries of Education, and Business, Innovation and Employment have published three initial reports: Ministry of Education & Ministry of Business, Innovation and Employment (2016a, 2016b, 2016c). Secondary PIAAC analysis will tackle research questions and topics beyond what is covered in the initial reporting, or take different or more in-depth approaches. Examples include:

- integrating different parts of the PIAAC dataset so as to generate new knowledge and understanding of associations and relationships
- building or refining conceptual or statistical models.
- exploring specific themes rather than taking a broad focus.

This paper explores an example of secondary analysis of numeracy practices in everyday life and work that takes a different perspective from the analysis of OECD (OECD, 2016a, pp. 97-113) which constructs indices that group together tasks involving similar activities.

**What is PIAAC?**

PIAAC is an international survey that assesses key information processing skills of adults of working age in Literacy (reading) and Numeracy and collects information on how often they undertake a range of related activities in work and everyday life (OECD, 2013c). Two additional assessment components are optional for participating countries: Reading Components (Sabatini & Bruce, 2009); and Problem Solving in Technology-rich Environments (OECD, nd-b). PIAAC builds on previous international surveys: the 1994-1998 International Adult Literacy Survey (IALS) (OECD & Statistics Canada, 2000); and the 2003-2006 Adult Literacy and Life Skills Survey (ALL) (Satherley, Lawes, & Sok, 2008; Statistics Canada & OECD, 2005).

PIAAC is planned as a repeating survey with a ten-year cycle. The first cycle was undertaken in two rounds, with a third round scheduled for 2016-2019. Round One included 24 countries’ with data collected in 2011/12 and findings published in 2013 (OECD, 2013a). New Zealand, together with eight other countries, participated in Round Two, with data collection April 2014 – February 2015. Results were released in June 2016 (OECD, 2016a).

Some key New Zealand findings are:

- New Zealand adults’ literacy and problem solving skills are on average among the highest in the OECD
- New Zealand adults’ numeracy skills are on average higher than the OECD average
- Although there are significant differences in skills between ethnic groups, average literacy and numeracy skills have been rising faster among Māori and Pasifika than in the total New Zealand population

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2 Normally 16-65 years but this can vary, e.g., Australia extended the range to 15-74 years.
3 Round 1 PIAAC countries were: Australia, Austria, Belgium, Canada, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Japan, Korea, The Netherlands, Norway, Poland, Russian Federation, Slovak Republic, Spain, Sweden, United Kingdom, the United States.
4 Round 2 PIAAC countries were: Chile, Greece, Indonesia, Israel, Lithuania, New Zealand, Singapore, Slovenia, and Turkey. Round 3 PIAAC countries will be: Argentina, Colombia, Hungary, Kazakhstan, Mexico, Peru and the United States of America (the latter repeating PIAAC in subpopulations).
• Overseas-born New Zealanders have on average higher literacy and numeracy scores than overseas-born people in any other country
• While there are no differences in average literacy and problem solving skills between men and women, men have higher numeracy skills on average than women.

The PIAAC survey is carried out by: interviewing a sample of at least 5000 adults in each participating country in their homes; collecting a broad range of information through a Background Questionnaire; and assessing skills in the PIAAC domains. The language of assessment is normally the official language or languages of each participating country. Depending on their computer skills, participants either enter their responses to the assessment items of the main skill domains on the assessor’s laptop computer or complete a paper version using printed test booklets. The Background Questionnaire is administered face-to-face in the respondent’s home by an interviewer who enters the answers into a laptop computer. All aspects of countries’ implementation of PIAAC is strictly monitored and quality-assured by the PIAAC Consortium. PIAAC governance is provided by a Board of Participating Countries.

The Background Questionnaire collects data on participants’ educational background, skills used at work (for those currently or recently in employment) and in other contexts such as the home and the community. For example, people are asked about their voting habits, volunteering, languages spoken, political efficacy and health. The following variables are covered: demographic characteristics; other personal characteristics (including learning disposition and self-assessed health status); education and training characteristics; work characteristics; self-assessed mathematics skills for work; self-assessed reading and writing skills for work; and skill use in everyday life.

Respondents with very low literacy skills (as assessed by some initial questions) are not assessed in the main skill domains, but instead go directly to the Reading Components assessment. This covers “the basic set of decoding skills that enable individuals to extract meaning from written texts: knowledge of vocabulary, ability to process meaning at the level of the sentence, and fluency in reading passages of text” (OECD, nd-a, p. 1).

In PIAAC, proficiency is considered as a continuum of ability involving information-processing tasks of increasing complexity defined as ‘proficiency levels’ (OECD, 2013b, p. 64). Six proficiency levels are described for Literacy and Numeracy (Levels 1 to 5, and below Level 1) and four for Problem Solving in Technology-rich Environments (Levels 1 to 3, and below Level 1). These summarise what adults with proficiency scores in each skill domain can do. The ‘average’ individual with a proficiency score in the range defining a level will successfully complete items located at that level approximately two-thirds of the time. PIAAC measures cognitive skills through test items that have a range of contexts: work-related, personal, society and community, and education and training. The OECD has released a number of sample test items (OECD, 2016b, pp. 21-23; 26-28; 31-32). The work context of PIAAC’s cognitive skill assessment is complemented by self-reports on generic skills required in work, including interpersonal skills and physical skills.

As for earlier international skill surveys, PIAAC published results include: average scores on each skill domain for countries and population subgroups; comparisons over time and with other countries; and proportions of different sub-populations reaching different benchmarks for each skill domain.

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5 Participating countries can increase the sample size to provide better information on specific sub-populations; e.g., New Zealand oversampled 16-25 year olds and Māori.
6 The PIAAC Background Questionnaire is at http://www.oecd.org/site/piaac/BQ_MASTER.HTM.
7 In some countries, PIAAC assessment has also been conducted in widely-spoken minority or regional languages.
8 Problem Solving in Technology-rich Environments is assessed only on a computer, so only participants who have sufficient computer skills and choose to use a computer are assessed on this domain.
PIAAC breaks new ground by: expanding the range of skill domains measured; including information on the skills of adults with levels of proficiency below Level 1; expanding the self-reported measures of the use of skills at work; introducing self-reported measures of qualifications matched to work; using computers to administer this kind of international assessment; making data publicly available for review and secondary analysis on an unprecedented scale via the PIAAC website; having an online version of the assessment publicly available, and not specifying Level 3 as a benchmark (OECD, 2010, p. 4). In IALS, Level 3 was considered to be the minimum skill level required to cope with the demands of modern society (OECD & Statistics Canada, 2000, p. xi). However, while many higher-level jobs require Level 3 or above, there is no evidence that everyone needs to be at Level 3. This change is significant because, as Black and Yasukawa (2014) point out, the Level 3 criterion has been used by powerful institutions to promote a crisis discourse in adult literacy and numeracy.

For more detail about PIAAC constructs, methods and how the survey was undertaken in participating countries, see the PIAAC Reader’s Companion (OECD, 2016b).

**Numeracy in PIAAC**

Numeracy is defined in PIAAC as:

> the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life … Numerate behaviour involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways. (OECD, 2012, p. 34)

PIAAC directly measures numeracy proficiency through the Numeracy assessment. This provides average scores on the PIAAC Numeracy scale for the whole population, or for subgroups.

PIAAC also collects numeracy-related information via the Background Questionnaire. The interviewer asks respondents how often they undertake seven numeracy practices in work (if appropriate) and everyday life. For example:

The following questions are about activities that you undertake as part of your job and that involve numbers, quantities, numerical information, statistics or mathematics.

- In your job, how often do you usually calculate prices, costs or budgets?
- Use or calculate fractions, decimals or percentages?
- Use a calculator – either hand-held or computer based?
- Prepare charts, graphs or tables?
- Use simple algebra or formulas?
- Use more advanced mathematics or statistics such as calculus, complex algebra, trigonometry, or use of regression techniques?

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9[http://www.oecd.org/site/piaac/publicdataandanalysis.htm](http://www.oecd.org/site/piaac/publicdataandanalysis.htm)

10A public online version of the PIAAC survey is at [http://www.oecd.org/skills/ESonline-assessment/](http://www.oecd.org/skills/ESonline-assessment/).

11The Numeracy domain in the ALL survey is comparable with the Numeracy domain in PIAAC but not with the Quantitative Literacy domain in IALS.

12The international version of the PIAAC Background Questionnaire includes the first six of these numeracy activities. New Zealand asked for the seventh activity, ‘measure or estimate the size or weight of objects’, to be added as a national extension. This activity was in the 2006 ALL survey and it was found to be analytically useful as it helps identify groups of people whose numeracy activities have this simple practical purpose.

13The same questions are asked in the past tense for people who are not currently working, but who worked in the last 12 months.
• Measure or estimate the size or weight of objects?

The frequency options are: Never; Less than once a month; Less than once a week but at least once a month; At least once a week but not every day; Every day. These data provide measures of the frequency and diversity of participants’ numeracy activities. The activities can be analysed separately, or a range of options can be developed for deriving an index of numeracy activity by combining the data across the activities.

The Adult Literacy and Life Skills Survey (ALL) asked similar questions, and since analysis of these questions, together with occupational characteristics, provided a coherent picture, we have some good assurance of the validity of these measures (Satherley, Lawes, & Sok, 2009).

Analysing adults’ numeracy practices: issues and types of analysis

A range of types of analysis may be used to explore questions about adults’ numeracy practices. Simple univariate tabulations can provide a ‘big picture’ view while multivariate analysis such as multiple regression can provide measures of the contributions of different factors to associations with numeracy practice. For example, frequent practice of numeracy activities at work may be associated with: higher levels of education; numeracy-related fields of study; specific groups of occupations; and higher measured numeracy skill. Such analysis can show which factors are most strongly associated with frequent numeracy practice and how much increase in frequency of numeracy practice is associated with one unit of measured numeracy skill, whilst holding other factors constant.

PIAAC data cannot tell us to what extent frequent numeracy practice causes high numeracy skill, so researchers should not use ambiguous language such as ‘leads to’, ‘brings about’, ‘influences’, or ‘is linked with’. In any case, it may be that numeracy practice, opportunity or requirement to undertake numeracy practice, and numeracy skill are all mutually reinforcing. Even where data show a strong association between two factors (for example, numeracy practice and numeracy skill) whilst controlling for other factors, we cannot infer that a change in one factor will result in a change in the other, either on an individual or a group level. Another limitation is that PIAAC does not provide a measure of either the intensity or complexity of numeracy activity. For example, finance analysts doing nothing but calculating costs and budgets would report the same way as someone who worked on costs and budgets for 10 minutes every day. Researchers also need to be aware of issues relating to continuous and categorical variables. For example, PIAAC frequency options are five separate categories, although for some analytical purposes it may be legitimate and useful to derive a continuous frequency variable from the discrete categories.

Some other methodological or technical issues users should be aware of include:

• The 2016 PIAAC report (OECD, 2016a) presents OECD averages based on the 28 OECD countries participating in Round 1 or Round 2. This differs from the averages presented in (OECD, 2013a) based only on Round 1 countries.

• Numeracy skill is directly measured, whereas participation in numeracy activities is self-reported.

• The PIAAC data are rich enough to provide scope for different indicators of a construct, such as job-skill mismatch. What indicator is the most useful will depend on the purpose of the analysis, or the need for comparability with published findings across countries.

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\(^{14}\text{We recommend novice researchers should consult a text book covering quantitative research methods in the social sciences (e.g., De Vaus, 2014; Frankfort-Nachmias, Nachmias, & DeWaard, 2015).}\)
Where a user analyses PIAAC data across all or many participating countries, the large sample may mean that nearly all differences are statistically significant. This will not apply for an analysis for one or a few countries, and very fine analysis with many variables for a single country may entail large sampling errors and therefore few significant differences.

Some questions about numeracy activities that PIAAC secondary analysis can help answer

In this section we pose some questions to demonstrate the process we went through in our exploration of the PIAAC dataset and comment on how these questions could be answered through secondary analysis of the PIAAC data. The OECD has published an initial analysis of skill use at work and in everyday life (OECD, 2016a, pp. 98-100). This is a country comparison that aggregates activities and summarises frequencies into an index. Our paper aims to suggest more in-depth and more detailed analysis focused on more specific research questions beyond that of the OECD report.

**How often do people perform numeracy activities at work and in everyday life?**

PIAAC numeracy activities at work and in everyday life and frequency categories can be shown in tables as simple frequency tables, i.e., activity categories can denote rows with frequency categories, denoting columns.¹⁵

**What are the patterns for different groups in numeracy activities at work?**

The PIAAC Background Questionnaire sheds light on the part numeracy activities play in different kinds of work for different groups of people. PIAAC classifies Occupations as: Managers; Professionals; Technicians; Clerical; Service & sales; Agriculture & fisheries workers; Trades; Machine workers; and Labourers. Even at this broad level, analysis of the 2006 New Zealand ALL data show distinct profiles of numeracy activity for different occupations, for example, managers, trades workers, technicians, professionals, and agriculture and fisheries workers engaged in relatively frequent numeracy practice compared to clerical, labourers, machine and service and sales workers (Satherley et al., 2009). Broad PIAAC Industry sector groups include: Agriculture & fisheries; Manufacturing; Construction; Trade; Transport & communications; Finance & real estate; and Health & education. Analysis of PIAAC data will enable researchers to identify sectors with unexpected pockets of high or low numeracy activity. We can also look at the association between numeracy practice and measured numeracy skill to answer questions such as:

- How likely are people with strong numeracy skill to have jobs that entail frequent numeracy activity?
- Where in the economy do we see areas of job-skill mismatch? One way of tackling this question is developing an indicator of job-skill match that compares actual numeracy skill with the frequency of undertaking numeracy activities. A possibility is grouping respondents into (a) high, medium and low numeracy skill and (b) high, medium and low frequency of undertaking numeracy activities at work; then derive a third grouping: over-skilled, under-skilled and matched; then we could investigate industry or occupation patterns for this indicator (see OECD, 2016a, pp. 129-143).

Analysis by socio-demographic characteristics can also show different numeracy activity patterns for different groups by: gender; age group; education level; and field of study. New Zealand’s oversampling of 16-25 year olds and Māori will allow these groups’ numeracy activities to be examined in detail.


¹⁶ PIAAC dataset categories and sub-categories are italicised for ease of reference.
What are the patterns for different groups for numeracy activities in everyday life?

We can analyse numeracy activities in everyday life similarly to work contexts. In this case analysis is likely to focus on socio-demographic characteristics and measured numeracy skill.

What similarities or differences exist for numeracy activity at work and in everyday life?

By looking at participants’ numeracy activities at work and in everyday life, we may discern patterns of global differences between these two contexts for individuals. For example, if we were interested in calculator use in work compared to in everyday life, we could cross-tabulate these variables.

What similarities or differences exist for numeracy and literacy activity at work?

Literacy activities at work included in the Background Questionnaire covered eight reading and four writing activities: Read directions or instructions; Read letters, memos or emails; Read articles in newspapers, magazines or newsletters; Read articles in professional journals or scholarly publications; Read books; Read manuals or reference materials; Read bills, invoices, bank statements or other financial statements; Read diagrams, maps or schematics; Write letters, memos or emails; Write articles for newspapers, magazines or newsletters; Write reports; and Fill in forms. One approach to making sense of this level of detail is to summarise the numeracy activities by creating a frequency index. For example, we could assign a numerical code for the frequency options and add the codes to obtain a total score, which would be a measure of the frequency and diversity of numeracy activities undertaken. Someone with a high score often engages in several different numeracy activities. We could generate similar indices for reading and/or writing. This would allow us to study the characteristics of, for example, people who score highly on both numeracy and literacy activities at work.

What similarities or differences exist for numeracy and literacy activity in everyday life?

We could take a similar approach to explore numeracy and literacy activity in everyday life. For example, we could look at questions about whether numeracy and literacy activities in everyday life go together for many people or not. What are the characteristics associated with undertaking a lot of different numeracy and literacy activities? Can we identify characteristics that are associated with seldom undertaking few numeracy and literacy activities?

What can PIAAC tell us about adults’ financial capability?

PIAAC can shed light on financial capability, for example, through a study of the relationships between directly measured numeracy skill and the self-reported numeracy activity of calculating prices, costs or budgets. Similarly we could study literacy skill and the literacy activity of reading bills, invoices, bank statements or other financial statements. Acknowledging that numeracy activity is self-reported, inferences about financial literacy skill may still be possible.

What are the patterns of work-related numeracy activity together with ICT activities?

The PIAAC Background Questionnaire asks how often people participate in the following ICT activities: use email; use the internet in order to better understand issues related to your work; conduct transactions on the internet; use spreadsheet software; use a word processor; use a programming language to program or write computer code; participate in real-time discussions on the internet. These could be analysed using a similar approach to the comparisons between numeracy and literacy activities.

What are the characteristics of people who seem to be matched or mismatched on numeracy skill at work and numeracy activity? Are Field of study of highest qualification, or Occupation related to the match or mismatch?

To what extent people’s numeracy skill aligns with their numeracy activity is an important issue. A working-age population might contain significant proportions who could easily manage more
(or a higher level of) numeracy activity than they actually engage in, or significant proportions who are attempting numeracy activity that their skills do not support. In these cases, a work skills policy issue seems to emerge about whether policy levers should be applied to support a more efficient allocation of numeracy skills and numeracy activity.

One approach to better understanding numeracy skill and activity mismatch would be a multivariate analysis of Numeracy activity, Measured numeracy skill, and Main field of study of highest qualification, and Occupation. This could build on an OECD report on the first round of PIAAC entitled The System-level Causes and Consequences of Field-of-study Mismatch (Montt, 2015).

What changing patterns for young people’s numeracy activities can we see on their pathways from school to tertiary education to work?

Here we could identify subgroups of 16-25 year olds who are: at school; in formal tertiary education; in full-time work; or combining work with study. We could identify what characteristics are strongly associated with participating in numeracy activities in everyday life (including study). These could include respondents’ age; gender; level of highest qualification; the level of qualification they are studying for; the main subject of qualification they are studying for; the type of educational institution they are studying at; whether they are also undertaking non-formal or informal study; and their occupation and numeracy activities at work (for those currently or recently working).

How have patterns of numeracy activity at work changed over time?

The ALL survey asked how often people participated in six numeracy activities as part of their job. The ALL activities were: Measure or estimate the size or weight of objects; Calculate prices, costs or budgets; Count or read numbers to keep track of things; Manage time or prepare timetables; Give or follow directions or use maps or street directories; and Use statistical data to reach conclusions. The first two on the above list of activities were re-asked, the first in New Zealand’s PIAAC survey only, and the second internationally. Using these data, we could look at how often workers undertook the numeracy activities in common between ALL and PIAAC, and investigate whether changes over time are associated, for example, with changing work, with changing occupational composition, or changing levels of education.

We turn now to a brief account of our experience of the process of accessing publicly-available PIAAC data for secondary analysis.

Using PIAAC Data Explorer to access PIAAC data

Information on PIAAC Public Data & Analysis is available online17. The OECD PIAAC Gateway website Data Tools › Datasets and Tools18 is designed to give users the tools needed to analyse the PIAAC dataset. The Data Tools section provides information on how to analyse a chosen dataset and Data Files are available, categorised as National, International, and Trend. Users are advised to read the guide What You Need to Consider(AIR PIAAC Team, nd) and watch online Distance Learning Data Training (DLDT) modules19 to learn about appropriate statistical procedures and methods of analysis before accessing the PIAAC datasets. Users may also wish to consult a survey statistician or a psychometric expert for technical advice.

The PIAAC International Data Explorer (IDE)20 is a web-based application for accessing PIAAC data that does not require any advanced statistical knowledge or specialist software. It is a point-and-click interface for creating statistical tables and charts and exploring levels of adult skills and demographics.

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17 http://www.oecd.org/site/piaac/publicdataandanalysis.htm
18 http://piaacgateway.com/datasets/
19 http://nces.ed.gov/training/datauser/#PIAAC
There are four steps to progress through when using the PIAAC Data Explorer, supported by the DLDT modules. We outline our journey through the process as follows.

**Step 1**

In the first step we chose a **Subject (Numeracy)** from three options: **Literacy; Numeracy**; and **Problem Solving in Technology-rich Environments**. Note that **Adults (16-65)** is the only option for **Age** at this point. After the choice of **Numeracy**, a fuller screen appears. Initially there is a choice of which Background Questionnaire dataset to use: **PIAAC 2012, ALL 2003 or All years/studies**; we chose **PIAAC 2012**.

Next we selected a dependent variable from the 72 Categories and Groups available. Then we selected a jurisdiction or group from the 20 national and 17 sub-national entities available, as well as the OECD average. Under the Category **Scale Scores**, and Sub-Category **Skills**, the Measure **PIAAC Numeric: Numeracy is already selected**. Under the Group **International**, we chose the jurisdiction **OECD average**. We clicked on **Select Variables** to move to Step 2.

**Step 2**

At Step 2, we selected independent variables (from a choice of 334) in order to examine the strength of associations between these and the dependent variable chosen above. Under the Category (and Sub-Category) **Major reporting group** we chose the variable **All Adults**. In addition, under the Category **International Background Questionnaire**, and Sub-Category **Skill use – literacy & numeracy**, we chose the following six variables:

- **Skill use work – Numeracy - How often - Calculating costs or budgets**
- **Skill use work – Numeracy - How often - Use or calculate fractions or percentages**
- **Skill use work – Numeracy - How often - Use a calculator**
- **Skill use work – Numeracy - How often - Prepare charts graphs or tables**
- **Skill use work – Numeracy - How often - Use simple algebra or formulas**
- **Skill use work – Numeracy - How often - Use advanced math or statistics**

We then clicked on **Edit Reports** to move to Step 3.

**Step 3**

Step 3 allowed us to choose the types of statistics we wanted to report, collapse any variable response categories, adjust table layouts and refine the formatting of the reports to be generated, using the variables chosen above. To summarize, we trialled the use of the PIAAC International Data Explorer (**PIAAC IDE**) by exploring information about the percentage of adults currently or recently in work who self-report the frequency of their engagement in given numeracy practices in their work contexts, and their average Numeracy scale scores for each category. We focused on the PIAAC jurisdiction **OECD average**, then chose as variables **Skill use work** for each of the six specified numeracy skills in order to generate the average PIAAC Numeracy scale scores and the percentages of adults aged 16-65 in each usage response (frequency) category.

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21http://piaacdataexplorer.oecd.org
The PIAAC International Data Explorer generated seven draft reports. These seven reports can be reviewed, edited, deleted or copied. New reports can be created, new formats and statistics chosen. We selected Averages and Percentages from the drop-down menu Statistics Options.

We clicked on Build Reports to move to Step 4 to view the completed reports.

Step 4

At the fourth step PIAAC IDE generated a report for each of the six variables of Numeracy skill use at work. Each report lists Averages of the Numeracy scale scores and Percentages of the population (each with Standard Errors) for the response categories: Never; Less than once a month; Less than once a week but at least once a month; At least once a week but not every day; Every day. Each report table can be viewed by selecting the report name from the drop-down menu. At this stage, we could preview and select information displayed in these tables or charts.

In addition, PIAAC IDE will generate (on request) comparisons between frequencies of skill use with significance tests for each of the six variables (Numeracy skill use at work) for the statistic specified. This allowed us to compare, for example, the means of two frequency groups to see if the difference is statistically significant. For this trial run we requested these significance tests be generated for each numeracy skill for both percentages and averages. Our use of large samples will mean that almost all differences will be statistically significant. (This issue is discussed in the section above entitled “Analysing adult numeracy practices: issues and types of analyses”). Our purpose in this instance was to examine the tables that would be generated by PIAAC IDE.

We then chose Export reports and downloaded our selected reports as Excel worksheets, allowing us to manipulate the data generated. Tables 1, 2 and 3, below, were produced through such a reorganisation of these Excel original reports/tables. At this stage we could have chosen instead to download the reports as HTML or Word documents.

The first six reports, one for each of the six variables of Numeracy skill use at work, are summarised in Table 1 (below).

Table 1

Average Numeracy scale scores and percentage of adults for frequency options for Numeracy skill use at work variables

<table>
<thead>
<tr>
<th>Jurisdiction</th>
<th>Never</th>
<th>Less than once a month</th>
<th>Less than once a week but at least once a month</th>
<th>At least once a week but not every day</th>
<th>Every day</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD Average</td>
<td>264</td>
<td>290</td>
<td>285</td>
<td>286</td>
<td>290</td>
</tr>
<tr>
<td>OECD Average</td>
<td>257</td>
<td>291</td>
<td>291</td>
<td>291</td>
<td>291</td>
</tr>
<tr>
<td>OECD Average</td>
<td>254</td>
<td>284</td>
<td>284</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>OECD Average</td>
<td>261</td>
<td>298</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>OECD Average</td>
<td>260</td>
<td>296</td>
<td>296</td>
<td>296</td>
<td>296</td>
</tr>
<tr>
<td>OECD Average</td>
<td>270</td>
<td>305</td>
<td>305</td>
<td>305</td>
<td>305</td>
</tr>
</tbody>
</table>

NOTE: The Numeracy scale ranges from 0 to 550. Some apparent differences between estimates may not be statistically significant.

Average (Avge); Percentage (Percent); Standard Error (std error).

SOURCE: Organization for Economic Cooperation and Development (OECD), Program for the International Assessment of Adult Competencies (PIAAC), 2012.

Reports used for this table were generated using the PIAAC International Data Explorer. http://piaacdataexplorer.oecd.org

22The data used were the OECD Average for PIAAC 2012.
Looking at percentages in Table 1 across the six Numeracy skill use at work variables (rather than within each skill), we found that the percentages of adults responding Never with regard to the category, Numeracy skill use at work - Use a calculator was the lowest (30%) (of all the six numeracy skills) and the percentages of adults in the response category Every day was the highest (37%) (of all six skills). The use of calculators in the workplace therefore emerges as very common, since less than one third of adults in, or recently in, work indicate that they never use them at work.

The next similar frequency response pattern is Use or calculate fractions or percentages with 46% indicating Never and 23% indicating Every day, although Calculating costs or budgets is similar, with 49% and 21% respectively in these response categories. In other words, almost one half of adults in, or recently in, work reported that they never used these two numeracy skills at work. Focusing on averages across the six Numeracy skill use at work variables (rather than within each skill), we found that the averages of Numeracy scale scores within the frequency option Never are the lowest (of all frequency categories) for all six skills. These percentages and averages for the less complex Numeracy skills may indicate that adults with the lowest numeracy skill are choosing to work in jobs that entail infrequent numeracy activity. Equally it may indicate the converse – employers may recruit staff in a way that closely matches skills to the job, or infrequent numeracy practice may lead to skill loss. If the latter, this would support the findings of research on the British Cohort Studies that showed adults’ skills diminishing with lack of use (Bynner & Parsons, 1998, 2000).

For the more complex numeracy skills, Numeracy skill use at work - Use advanced math or statistics shows percentages of adults in the response category Never as the highest (86%) (of all six skills) and percentages of adults in the response category Every day as the lowest (2%) (of all six skills). The next similar (but less striking) skill response pattern is Prepare charts graphs or tables, with 60% indicating Never and 6% indicating Every day, although Use simple algebra or formulas is similar, with 55% and 17% in these response categories respectively. Averages of numeracy scale scores for Numeracy skill use at work - Use advanced math or statistics are somewhat higher within most response categories. These percentages and averages may suggest that people with strong numeracy skills have jobs that entail frequent numeracy activity.

Table 2
Frequency of use of a calculator in a work context. Significance tests of the differences between either percentages or averages, calculated for each of the five frequency options

<table>
<thead>
<tr>
<th>Difference in percentages between variables</th>
<th>Difference in averages between variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>Less than once a month</td>
</tr>
<tr>
<td>Never</td>
<td>Diff = 22</td>
</tr>
<tr>
<td>Less than once a month</td>
<td>Diff = 22</td>
</tr>
<tr>
<td>At least once a week but not every day</td>
<td>Diff = 7</td>
</tr>
</tbody>
</table>
| Every day | Diff = 9 | P-value = 0 | Diff = 9 | P-value = 0 | Diff = 9 | P-value = 0 | Diff = 9 | P-value = 0 | Diff = 9 | P-value = 0 |<table>
| LEGEND: |
| < | Significantly lower. |
| > | Significantly higher. |
| x | No significant difference. |

NOTE: Within country comparisons on any given year are dependent with an alpha level of 0.05.

OECD Average, PIAAC 2012
Organization for Economic Cooperation and Development (OECD)
Program for the International Assessment of Adult Competencies (PIAAC)
Generated using the PIAAC International Data Explorer.
Examples of the types of reports generated by PIAAC IDE in the form of tables of differences between percentages and averages, with associated p-values\(^2\) are shown in Tables 2 and 3, below, for two numeracy skills (percentages and averages are combined).

To see how one percentage (or average) compares with those for other frequencies, users should read across the row for that value in Tables 2 and 3. The displayed symbols indicate whether that value is significantly higher, significantly lower, or not significantly different from the value associated with that column. The p-value indicates the probability with which a difference in percentages (or averages) between frequency groups as large as observed here could occur by chance, if there were actually no difference. The customary significance level is 5% (0.05). The p-value must fall under this significance level for the results to be deemed statistically significant. As expected, most differences were statistically significant with very low p-values.

**Table 3**

Frequency of preparation of charts, graphs or tables in a work context. Significance tests of the differences between either percentages or averages, calculated for each of the five frequency options.

<table>
<thead>
<tr>
<th>Difference in percentages between variables</th>
<th>Difference in averages between variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never</td>
<td>Never</td>
</tr>
<tr>
<td>Less than once a month</td>
<td>Less than once a month</td>
</tr>
<tr>
<td>Less than once a week but at least once a month</td>
<td>Less than once a week but at least once a month</td>
</tr>
<tr>
<td>At least once a week but not every day</td>
<td>At least once a week but not every day</td>
</tr>
<tr>
<td>Every day</td>
<td>Every day</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Never</th>
<th>Never</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than once a month</td>
<td>Less than once a month</td>
</tr>
<tr>
<td>Less than once a week but at least once a month</td>
<td>Less than once a week but at least once a month</td>
</tr>
<tr>
<td>At least once a week but not every day</td>
<td>At least once a week but not every day</td>
</tr>
<tr>
<td>Every day</td>
<td>Every day</td>
</tr>
</tbody>
</table>

**LEGEND:**

<  | Significantly lower.

>  | Significantly higher.

\(\times\)  | No significant difference.

**NOTE:** Within country comparisons on any given year are dependent with an alpha level of 0.05. Generated using the PIAAC International Data Explorer.

**Conclusion**

Overall, we found our exploration of the publicly-available PIAAC dataset stimulating and challenging in equal measure. While the scale of the dataset may appear daunting to novice researchers and practitioners, we would encourage readers to undertake their own exploration, using the range of support and tools for analysis available online. Our focus here has been on adults’ numeracy practices: just one of the many areas of interest on which data are available in this rich resource. Ultimately, the choice of focus lies with the reader. Our advice is: start with a simple question – something that intrigues you - and take it from there.

\(^2\) See below for an explanation of p-value.
References


Survey of Adult Students with Mathematical Difficulties

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Abstract
This paper relates to one of the test procedures being used in Sweden, used to establish if students need a more thorough investigation of their mathematical difficulties. This paper mainly describes the test process and the results from 10 test subjects. The paper also refers to parts of the research forming the basis for the test process. The paper shows how teachers in their everyday work can use the understanding researchers have of mathematical difficulties and the methods they have developed. The purpose is not to drive the research on mathematical difficulties.

Key words: mathematical difficulties, survey, levels of knowledge

About Mathematical Difficulties
As early as 1919 mathematical difficulties were observed by the Swedish doctor Salomon Eberhard Henschen, who later, in 1925, made connections to damages in the brain. Henschen, practicing in Uppsala, Sweden, characterized the difficulties he observed as acalculia (http://en.wikipedia.org/wiki/Salomon_Eberhard_Henschen, 2014-11-01). It was not until the mid 1970’s before further comprehensive studies of mathematical learning disabilities were conducted (Kling et al, 2011). After this period developments have been rapid including the designation of dyscalculia and the causes of mathematical learning disabilities scrutinized, formulated and discussed (Östergren, 2013). Today it is self-evident that specific development disorders and dyscalculia are defined in the most established systems for diagnosis, DSM and ICD.

Mathematical difficulties are usually divided in four categories (Adler, 2010); acalculia, dyscalculia, pervasive mathematical disorders and pseudo dyscalculia.

According to the Swedish professor Arne Engström (Engström, 2016) researchers do not agree on how dyscalculia shall be defined or the criteria to use. Also, they are not in agreement on how common dyscalculia is. Additionally, the researchers do not know enough about the reasons. This should be kept in mind whilst reading this paper.

Acalculia
A student exhibiting acalculia lacks the ability to calculate and to learn to calculate which mostly depends on measurable brain damage. A student having acalculia lacks the ability to count to 10 or add low numbers. Acalculia is unusual and is found only in a few per thousand of the population (Adler, 2007).
Dyscalculia

Dyscalculia together with dyslexia is today regarded as a subgroup of specific learning disorders. The diagnosis for dyscalculia, within the two most established international systems for diagnosis DSM and ICD, has been modified from proven experiences over the years. Today DSM has reached the fifth version (American psychiatric association, 2013) of its definition, DSM-5:

The Diagnostic and Statistical Manual of Mental Disorders
Dyscalculia is an alternative term used to refer to a pattern of difficulties characterized by problems:
- processing numerical information
- learning arithmetic facts and
- performing accurate or fluent calculations.

If dyscalculia is used to specify this particular pattern of difficulties, it is important also to specify any additional difficulties that are present, such as difficulties with math reasoning or word reading accuracy.

Difficulties in processing numerical information means an inability or reduced ability to deal with numbers. This is due to a dysfunctional number sense and in many cases lacking the ability to connect our Arabic symbols to numbers. Inability to recognize numbers often leads to the inability to deduce which of two numbers is the largest (the mental number axis, schema for numbers) and to perceive low numbers that are larger than four (Butterworth, 2010) as anything else than a series of “ones” (Halberda et al, 2008). Students having these difficulties are often seen counting on their fingers. Processing numerical information also means being able to understand and realize how numbers are influenced by simple arithmetic operations for adding and multiplication.

Difficulties to learn arithmetic facts also include remembering arithmetic facts, like simple additions. It stems from dysfunctional processes for absorbing, storing and recalling information from the long term or working memory (Kulcian et al, 2014). Difficulties to quickly fetch arithmetic facts from the memory are called automation difficulties. Number sense and working memory are important for the ability to calculate accurately or fluently (Klingberg, 2011).

The International Statistical Classification of Diseases and Related Health Problems
Specific disorder of arithmetical skills
Involves a specific impairment in arithmetical skillsthat is not solely explicable on the basis of general mental retardation or of inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus.

Arithmetical difficulties:
- associated with a reading or spelling disorder (F81.3)
- due to inadequate teaching (Z55.8)

When dyscalculia is being diagnosed according to DSM-5, difficulties in mathematical reasoning and in the working memory should also be included. The mathematical reasoning that DSM-5 includes is the abilities for:
• logical reasoning when tasks are solved
• apply strategies for solutions
• apply mathematical concepts, facts, procedures and methods when tasks are solved.
(Students’ methods are especially noted when tasks are solved, by checking if students use the correct rule of arithmetic or when the correct rule of arithmetic is used but the result is wrong.)(Adler, written tutorial, 2014-11-04)


Students having dyscalculia do have a normal level of talent as clarified by the WHO diagnosis. Later studies point out the number of students having dyscalculia can be compared to the number of students having dyslexia and this accounts for 3.6 to 6.5 per cent of the population (Butterworth, Yeo, 2010). A dyscalculia diagnosis is valid for a period of one year for children and two years for adults.

**Pervasive Mathematical Disorders**

There are also students with pervasive mathematical disorders. As these disorders are pervasive they are not specific for learning and understanding mathematics. They influence all learning. These students need adapted teaching where learning is allowed to consume more time and contents may be simplified. These students are more consistent in their achievements compared to students with dyscalculia.

**Pseudo Dyscalculia**

This fourth category (Adler, 2010) is about students with emotional blockings experiencing specific mathematical disorders. These students are discovered as they get surprised that the survey was so easy. Pseudo dyscalculia, which is most common among women, is treated with support from a psychologist, welfare officer or similar. It is also called math anxiety.

**Cognitive Disabilities**

Cognitive disabilities with a broad impact on many abilities are called pervasive. The cognitive disabilities specifically affecting students’ abilities to learn and understand mathematics are called specific mathematical disabilities or disorders. Two examples of pervasive proficiencies are intelligence and processing speed (Adler, 2007).

Students only having difficulties specific for mathematics usually have dyscalculia. Dyscalculia assumes the specific mathematical disabilities depart from the other more functioning abilities.

Mathematical disabilities are often caused by a combination of specifically mathematical and otherwise pervasive difficulties. Rickard Östergren (2013) proposes weak working memory in combination with weak number sense is a risk for students to develop MLD. Others have reached different conclusions. For example Deary, et al, (2007) suggest reduced intelligence combined with reduced number sense is a common reason for development of MLD. However, it has also been suggested intelligence is the same as working memory capacity (Ackerman, et al, 2005). In addition it is noted that general abilities like phonological consciousness, processing speed and executive attentional processing have great importance for MLD, (Östergren, 2013).

According to DSM-5, a weak number sense and its limitation in understanding digits and numbers is the main difficulty contributing to dyscalculia.
The Pedagogical Survey

Execution

The pedagogical survey that scores for pervasive and for mathematics specific development disorders was done in four phases; three screenings and one skill test in mathematics using the following material from Kognitivt Centrum in Malmö.

- Reading screening III (from 16 years), Adler, 2012
- Writing screening III (from 16 years), Adler, 2012
- Mathematics screening III (from 16 years), Adler, 2010
- Skill test for mathematics (secondary school and adults), Adler, 2008

Every screening lasts approximately one hour. The skill test takes exactly 5 minutes.

The test material has been bought from Kognitivt Centrum in Malmö. The authorized distributor prohibits further distribution. Therefore, it cannot be included as an annex. Due to this the article focuses on the students’ results rather than the questions asked. Choosing this test material is a consequence of Sweden being a small country and this is the test material that is readily available in Swedish.

All tests were done individually even if the skill test may be done in a group. Doing them individually enables observing aspects such as how easy students understand instructions and the time taken to complete different tasks. The most time consuming duty for the teacher is to compile the results and write recommendations for more thorough investigations and the bases for these.

Testing students’ reading and writing abilities is done since these are important to know about. For example, if students don’t have the fine motor skills to write and understand what they have written themselves, of course, the situation is made more complicated. The same applies if they cannot sketch or understand geometrical shapes. Students having difficulties to read may have problems to understand what the question is in a problem solving task. Also, texts may become incomprehensible when words are pronounced wrongly. These are a few examples.

Professor Arne Engström has the opinion that calling three of these tests screening is wrong because the word screening is used when a complete population is tested. (Engström, 2016)

About the ten students

I came in contact with eight of the ten students that were surveyed at my place of work (Komvux, Lund). The remaining two students, Josefin and Elise, heard about my investigations and wanted to take advantage of the opportunity to be in the survey and get more help for their difficulties. These two additional students were younger. Four of the students at Komvux were my own. The other students at Komvux were recommended for the survey either by a teacher or by the welfare officer. The designation Ma Gr in the table means a pre-secondary school course in mathematics. The other designations are courses at secondary school.

Table 1 shows background information about the 10 tested students. It displays age, if there are previous diagnoses, which course they are taking and how they describe their own difficulties.
Table 1.
*About the ten students.*

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
<th>Previous diagnosis</th>
<th>Studying</th>
<th>Student’s own description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marie</td>
<td>24</td>
<td>Dyslexia</td>
<td>Ma 2b</td>
<td>Bad memory. Good at per cent because of much shopping. Developed math disliking called “ouch-math”. Difficult to multiply simple numbers. Difficult to concentrate. Likes calculus. Dislikes writing. Reads a little, excellent self confidence. Likes calculus despite difficulties. The four basic operations are difficult and he forgets what has been heard /seen. Good image memory. Good problem solver, curious, patient and positive.</td>
</tr>
<tr>
<td>Jan</td>
<td>24</td>
<td>No</td>
<td>Ma 2b</td>
<td></td>
</tr>
<tr>
<td>Mårten</td>
<td>31</td>
<td>No</td>
<td>Ma 2c</td>
<td></td>
</tr>
<tr>
<td>Carina</td>
<td>21</td>
<td>No</td>
<td>Ma Gr</td>
<td></td>
</tr>
<tr>
<td>Amanda</td>
<td>24</td>
<td>No</td>
<td>Ma 3b</td>
<td></td>
</tr>
<tr>
<td>Emma</td>
<td>27</td>
<td>Dyslexia</td>
<td>Ma 1a</td>
<td></td>
</tr>
<tr>
<td>Jonny</td>
<td>21</td>
<td>Post traumatic stress, probably dyscalculia</td>
<td>Ma 1a</td>
<td>Tried to study Ma 3b three times. Easily stressed and quits when feeling pressured. Counting fingers. Daily math works well.</td>
</tr>
<tr>
<td>Josefin</td>
<td>15</td>
<td>No</td>
<td>Ma Gr</td>
<td></td>
</tr>
<tr>
<td>Elise</td>
<td>16</td>
<td>Dyscalculia</td>
<td>Ma 1b</td>
<td></td>
</tr>
<tr>
<td>Johan</td>
<td>20</td>
<td>Dyslexia</td>
<td>Physics 1a</td>
<td></td>
</tr>
</tbody>
</table>

Here is a short description about the mathematical brain before we discuss the results for the 10 students.

**The mathematical brain**

Mathematics encompass various cognitive processes where different help systems collaborate. Therefore, the whole brain is used for mathematics. Both difficulties in reading and doing mathematics have the origin in deficits in both halves of the brain, even though reading depends mostly on the left half and number sense and calculus ability stems from the right half. (Adler, 2014. Network meeting dyscalculia, Stockholm).
The frontal lobes are an important part when doing new calculations. The brain also has an area called IPS where number facts are stored. This area may be blocked in any person. Typically students having a dysfunction in the IPS may say “I know but I cannot get it out”. Problems with IPS can be compensated by training. To succeed in mathematical studies it takes more from the brain than to succeed in reading or writing. The calculus itself encompasses more cognitive processes (Campell, 2004). At least, when looking at all the components needed for problem solving you can understand the whole brain is used for mathematics; good understanding of reading, linguistic understanding, phonological consciousness, persistence, attentiveness, a good capacity to automate, a well-functioning working memory that is not too slow, good speech recognition, mathematical reasoning etc.

Results for Ten Surveys

Tables 2 and 3 show how to interpret skill test results. The skill test consists of 50 simple calculations like e.g. $10 - 3 + 2$. The skill test is normalized due to a standard population. It measures according to a profile and a normal distributed Stanine score in the range from 1 to 9. Students achieving a Stanine score of 1 or 2 are among the 11% with the lowest result according to the standard population the test has been normalized against. Students achieving Stanine score 3 may also have great difficulties with speech recognition. These students probably have to some extent compensated their deficit number sense with a strong working memory. Stanine scores for skill tests are determined according to table 2.

Table 2. The stanine scores.

<table>
<thead>
<tr>
<th>Stanine</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score (No. of right – wrong)</td>
<td>≤1</td>
<td>2-6</td>
<td>7-10</td>
<td>11-14</td>
<td>15-19</td>
<td>20-24</td>
<td>25-31</td>
<td>32-40</td>
<td>41-50</td>
</tr>
<tr>
<td>% of students in the normalization</td>
<td>4</td>
<td>7</td>
<td>12</td>
<td>17</td>
<td>20</td>
<td>17</td>
<td>2</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Profiles for skill tests are determined according to table 3.

Table 3. Profiles for the skill tests

<table>
<thead>
<tr>
<th>Profile</th>
<th>The student works…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>slowly and does maximum 10 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>R</td>
<td>slowly and does maximum 10 tasks but makes few mistakes – three or less</td>
</tr>
<tr>
<td>S</td>
<td>at normal pace and does at least 11 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>T</td>
<td>at normal pace and does at least 11 tasks but makes few mistakes – three or less</td>
</tr>
<tr>
<td>U</td>
<td>fast and does at least 25 tasks and makes many mistakes – four or more</td>
</tr>
<tr>
<td>V</td>
<td>fast and does at least 25 tasks but makes few mistakes – three or less</td>
</tr>
</tbody>
</table>
Jarlskog, Survey of adult students with mathematical difficulties

Table 4 shows the screening and skill test results for the 10 students. In the table well-functioning abilities are shown as “X” while a “?” means this should be investigated further. Boxes with question marks are also shaded for clarity. A “?” in any of the blue fields indicates possible dyscalculia.

Table 5 shows a summary of the results for the pedagogic surveys. The students’ shortened names are given in the lilac headline. The table gives an insight into what is investigated. The yellow rectangle above the table shows the test designers’ instructions for how to interpret the number of difficulties resulting from the mathematics screening.

Table 4.
Collocation of the survey

A ? in any of the blue fields indicates possible dyscalculia. Investigate further!

<table>
<thead>
<tr>
<th>Ability</th>
<th>Ma</th>
<th>Ja</th>
<th>Må</th>
<th>Ca</th>
<th>Am</th>
<th>Em</th>
<th>Jon</th>
<th>Jos</th>
<th>Eli</th>
<th>Joh</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>Read and write numbers</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>?</td>
<td>X</td>
<td>Largest, ordinal number</td>
</tr>
<tr>
<td>7</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>Read different fonts</td>
</tr>
<tr>
<td>9</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>State words that...</td>
</tr>
<tr>
<td>10</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>Read single letter words</td>
</tr>
<tr>
<td>12</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>X</td>
<td>X</td>
<td>?</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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Instructions for red fields

All correct: emotional blockings/knowledge gaps

2-8 errors: might be mathematically specific development disorders

≥ 9 errors: pervasive mathematic specific development disorders or complicated learning
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<th>Item</th>
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<td>30</td>
<td>Room perception</td>
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<td>Perceptual speed</td>
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<td>Eye-hand coordination</td>
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<td>General knowledge</td>
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<tr>
<td>43</td>
<td>Planning ability</td>
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<td>44</td>
<td>Time planning/time perception</td>
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**Table 5. Summary of the ten pedagogical surveys**

- **Marie**: Shows signs of mathematically specific development disorders and problems with the working memory. Jan needs support to train his basic abilities for his own benefit and to definitely exclude his deficiencies depend on mathematically specific development disorders. Jan also needs encouragement to read more. Jan also needs support to train his basic abilities for his own benefit and to definitely exclude his deficiencies depend on mathematically specific development disorders. Jan also needs encouragement to read more. If this training does not give the desired effect a more thorough investigation is required.

- **Mårten**: Has automation difficulties and problems recognizing numbers. He almost certainly has dyscalculia or mathematically specific development disorders. The survey indicates Carina is not a weak student but she has a complicated learning as she is concerned about several pedagogical building blocks. A deepened psychological and medical judgment shall then be conducted as a complement.

- **Carina**: Shows problems with the mental number axis, and there may be difficulties with the working memory and concentration. Perhaps could some of these contribute to the resistance against math studies. The difficulties with the mental number axis, which in her case are apparent when the numbers are not written, should be possible to remove by number axis training. Therefore, currently no deeper pedagogical investigation on mathematical difficulties is recommended. Instead a working memory test is recommended to find out how Amanda could train mainly the visual working memory. The survey clearly points out the already known dyslexia and mathematically specific development disorders that should be dyscalculia. A deeper dyscalculia and dyslexia investigation along with psychological and medical judgment is needed. It is also necessary to further define Emma’s dyslexia to prevent her being hindered in future studies at teachers college.

- **Amanda**: Shows Jonny has and mathematically specific development disorders and possibly also dyscalculia. A complementary deeper psychological and medical judgment shall then be conducted. Not the least, the reasons for Jonny’s concentration problems need to be established. The survey also show Jonny has a lack of concentration, reduced visual working memory, shape comprehension and ability to visually scan. A working memory test
is required.

Josefin has mathematically specific development disorders (since her mathematical difficulties deviate that much from her ability to read and write) and problems with the working memory, especially the visual, which in this context indicates complicated learning. Emotional blockings magnifies her learning problems. A psychological and medical judgment is needed as a complement.

Elise has mathematically specific development disorders (since her mathematical difficulties deviate that much from her ability to read and write) and problems with the working memory, especially the visual, which in this context indicates complicated learning. Strong feelings she has difficulties to control may also aggravate her learning problems. A psychological and medical judgment is needed as a complement.

Johan has read and write difficulties hindering him to succeed in math and physics. To alleviate these difficulties Johan needs to meet a specialist that can support him in using tools to ease taking notes, structure texts and understand concepts and explanations he has written himself. To get concepts and texts read out loud to him may also help since words he reads often are pronounced erroneously and become completely different words. Johan also needs a new, fresh dyslexia investigation prior to his studies at college.

### Discussion

Each of these students exhibits issues with the mental number line, even when the ability test shows a high Stanine score. For some of the students the difficulties with the mental number line are a clear sign of difficulties associated with their number sense. The two students that achieved well in the ability test show problems with attention. One of them also showed problems with persistence/concentration which may explain the difficulties with the mental number axis.

Six of the seven students having Stanine score 1-3 showed apparent signs for mathematics specific development disorders. One might have practiced too little. Out of these students Mårten exhibits signs of dyscalculia since there was a limitation in specific difficulties, except those linking to the visual. For Mårten the mathematics specific development disorders deviated a lot from his better functioning pervasive abilities. The survey showed Mårten had difficulties with persistence/concentration and learning ability. Despite this, I see as a teacher of this student, these two general abilities function well. After the survey the student was allowed to use multiplication tables when doing written tests, which probably relieved some stress! After this he has been doing well.

For another student, Emma, having a low Stanine score (2) using multiplication tables when doing written tests was imperative for her to succeed with the course. It is obvious corrective actions do not have to be far away!

For the two students that produced a Stanine score of 8, one clearly exhibits dyslexia and the other shows a lack of attention and not being used to confront obstacles with problem solving.

The student with a Stanine score of 6 showed signs of mathematics specific development disorders and troubles with the working memory and concentration. These students’ difficulties are similar to those Östergren gives as an important source for mathematical difficulties, MLD.

Several students showed pervasive development disorders in combination with mathematics specific development disorders. One of these students, Carina who I have in my class, does not work at all in the large group where she is currently placed, since the size of the group hinders her from speaking up and communicating her thoughts. Her pedagogical survey made my headmaster act by offering her individually adapted support.

The ability for mathematical reasoning has not been investigated. This ability will be put forward in later investigations when instructive material for this has been developed. More investigations are needed to be able to exhaustively link results to contemporary research.
About more thorough pedagogical investigations

The pedagogical screenings investigate cognitive building blocks needed for students reading, writing and spelling. Depending on what is pointed out by the screenings deeper pedagogical investigations might be needed. These should be conducted by a team consisting of a medical doctor, a psychiatrist and possibly also a speech therapist and a social worker. For adults, in case it is not possible to engage all these staff, a medical doctor shall conduct a complementary investigation. The doctor’s role is to establish if there is anything else that can explain the difficulties. Today the requirement for a psychologist has been relaxed.

Memory tests, Rey-complex figure test and Rawens matrix is often used by more exhaustive investigations. Rawens matrix is a logical non-linguistic intelligence test. Rey-complex figure test shows if the spatial ability is defective.

When doing the survey it is important to remember that a poor educational experience can influence the results. In these cases the lack of previous education should be made up. If this does not help a deeper investigation shall be called upon as fast as possible so the required support can be provided as soon as possible.

Experiences from the surveys

Some of the students benefited directly from the survey as they were allowed to compensate difficulties in written tests and by doing so were able to attend higher mathematics classes. Other students have finally been offered support, although unfortunately not by remedial teachers. Another experience was the tests and the collected results made clear to the school management what students mathematical difficulties may look like and which help they might need. This has contributed to the decision that Komvux in Lund will hire a remedial teacher.

References


**Abbreviations**

DSM The Diagnostic and Statistical Manual of Mental Disorder

ICD The International Statistical Classification of Diseases and Related Health Problems (WHO)

IPS Intraparietal Sulcus

ANS Approximate Number System

MLD Mathematical Learning Disability
Does Adding Mathematics to English Language Learners’ Timetables Improve their Acquisition of English?

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Abstract

This enquiry based project set out to find out if adult English language learners, known as ESOL (English for Speakers of Other Languages) learners in the UK, might benefit, in terms of their acquisition of English, from studying maths. This research has been conducted at a medium sized FE college in the East Midlands where I teach. I evaluate this in two ways, firstly by analysing learners’ results, and secondly by asking experienced ESOL teachers to observe and reflect on an ESOL Maths session. This project found a correlation between attending a maths class and improved English language exam results over 5 cohorts of students. In addition, ESOL teachers noted many and varied opportunities for English language learning in an ESOL Maths class, with higher levels of learner participation and confidence than seen in language classes. I recommend that we offer ESOL maths to ESOL learners, and that we reassess maths teaching for all learners, ESOL and English speakers, as a triad: conceptual understanding, procedural competence and language acquisition.

This article was first published after proceedings at the BSRLM (British Society for Research into Learning Mathematics)/BERA (British Education Research Association) conference, Durham, UK, 2015. The research was carried out for the dissertation phase of a Master’s degree in Education.

Key words: addition, English language learners, mathematics

Introduction

The learners involved in this project are all people who have voluntarily signed up for ESOL Maths, and may have just arrived in the UK, or been here for many years. They may have opted to come to the UK for work or family reasons, or been subjected to political or social persecution in their country of origin. The learners are all over 16 years of age; most are 19 or over. The primary motivation for many of the learners attending ESOL Maths classes is to improve their English, and this can be for a number of reasons, including improving their job prospects or helping their school aged children.

ESOL Maths learners form part of a number of wider educationally-based communities, namely mathematics learners in the UK, ESOL learners in the UK, and, globally, those whose first language (L1) is not English who are learning maths in English. They may come from many countries and cultures which can be very different, both from the UK and each other, in terms of the content of maths lessons and assessments, mathematical symbols and language, and the value placed on mathematics learning.
Methodology and methods

In order to ascertain whether adding ESOL maths to ESOL learners’ timetables has an impact on their acquisition of English I compared the results of ESOL learners who have studied for maths qualifications with those who have not. This was small-scale quantitative analysis based on an ESOL intake of approximately 130 learners each year, 11% to 18% of whom enrol for ESOL Maths.

It should be born in mind that this is a small-scale investigation based in one college which has one ESOL Maths teacher, namely myself, although some higher-level learners do attend an English speakers maths class if it falls on a more convenient day for the learner. Entry-level English learners are not encouraged to attend English speakers Entry-level classes as they are deemed to need specialist help. As such I examined the results of the whole population of ESOL learners at this college, of whom the ESOL maths students can be seen as a subset of the total population (Cohen, Manion & Morrison, 2000).

I chose to analyse the data to see if these observations can be evidenced in some way by improved ESOL results, but in order to triangulate this information ESOL colleagues were asked to observe an ESOL Maths session and report back on their findings. This mix of quantitative and qualitative analysis can be seen as an opportunity for triangulation of data (Coben, 2003).

The analysis does not tell us why any correlation occurs, highlighting the limitations of data analysis (Bell, 1993) and a positivist approach (Stacey, 2013). It may be that the amount of teacher contact time has an effect, or that adult ESOL learners who opt for ESOL maths are more highly motivated than those who do not. A correlation might also be for reasons unrelated to ESOL Maths; that it may be that any subject taught, from sport (Hately-Broad, 2006) to flower arranging, could have the same effect, particularly if the learner has some prior knowledge of the subject.

Observers were given three questions for consideration: Firstly, can they see any advantages of ESOL Maths, where the language is less overtly taught than in an ESOL class? Secondly, do the students exhibit skills that they were unaware of, or that surprise them for the level those students are at in their English? Finally, does it make the teachers reassess the learners’ language skills levels? The questions are open-ended to draw full responses (Ribbins, 2006), but can be seen to be connected to allow for corroboration (Richards, 2009). A Likert scale was included to discover strength of feeling (Bell, 1993). I asked observers to rank the usefulness of ESOL Maths in improving English acquisition on a scale of 0 to 10, where 0 is ‘of no use’, and 10 is ‘extremely useful’.

There is potential for any observer or interviewer to have an impact on a situation and peoples’ responses, as observed in other investigations, where observers become aware that they are an acknowledged presence in the room, and that this is disturbing the normal flow in some way (Brown 2001). This is known as the Hawthorne effect, and it might affect both myself as the teacher or the students in the ESOL Maths class.

One of the negative issues with conducting this study myself is that the outcome might be affected by my involvement. For instance there could be an issue with learners and colleagues giving less than honest answers to the questions. This is known as the halo effect (Cohen et al., 2000), where the previous knowledge of the participants affects their judgements.

Quantitative Findings

The analysis of the data seems to clearly show a correlation between attending a maths class and English language acquisition at my college, as the percentage of ESOL Maths learners
with ESOL passes varies from 87.5% to 100%. This compares with the performance of the group without maths classes of between 62% and 84.5%.

There is a consistent positive correlation between opting for ESOL Maths and passing ESOL exams, as there is a minimum of 10.5% and a maximum of 32% improvement in ESOL Maths students’ performance compared to the non-maths cohort (Figure 1).

![Percentage Comparison of learners ESOL results with and without ESOL Maths classes](image)

*Figure 1. Percentage comparison of learning ESOL results with and without ESOL maths classes*

In Figure 2 it can be seen that the impact on the cohort as a whole is low:

![Percentage achievements of ESOL learners over 5 years](image)

*Figure 2. Percentage achievements of ESOL learners over 5 years*
Qualitative Findings

Three of the four teachers who were able to observe classes were ESOL teachers, but the fourth was a Maths teacher. Interestingly the comments from the Maths teacher were very similar to those of the ESOL teachers, and I have felt it unnecessary to differentiate between the two which will also help to preserve the anonymity of the teachers involved.

All of the observers noted language teaching taking place in the observed sessions. One teacher expressed surprise at “the amount of language that was used in Maths and therefore it was a language lesson based on maths” and also said that the session was “very interesting and definitely beneficial to the learners as they were exposed to a different type of language use.”

Another was surprised by the level of English and fluidity of the language use: “I was surprised by the good level of English used by the students, the vocabulary was very fluid and the student’s understanding of maths on the whole was of a very good standard”. The teachers also commented on the opportunities available to learners to practice pronunciation, such as in a place value recap session all learners practiced using the ‘th’ sound with tenths, hundredths and thousandths. One commented that “time was given to reinforcing pronunciation, spelling of numbers and vocabulary.”

Three out of the four teachers when asked to rate the usefulness of maths sessions in improving English acquisition on a simple Likert scale, where zero was ‘of no use’ and ten was ‘extremely useful’, rated the usefulness at 10, extremely useful. One teacher did not use the scale, and commented that “It would be more useful if the group was not of such differing levels, so language could be more easily structured”, that “some of the language used was more advanced than might be expected for some of the learners”, but did note that the learners “were engaged and attentive”.

Paired work involving verbal problem solving was taking place between learners who would not normally speak to each other during a session, one of whom had previously refused to participate in paired work during ESOL sessions. One teacher said “Student X does not speak in English, but spoke here with other students she does not normally interact with”. Another commented that the session gave “non-speakers” “a chance to participate”. A learner with extremely low verbal language skills was clearly prepared to attempt questions and to make mistakes which had not been seen before. “Student Y really tries and has a go, not seen that in an ESOL class” and “I could see some students were very timid but these still participated”.

The positive response to set tasks was noted by all observers and surprise was expressed at the level of maths attempted and achieved during the sessions: One commented that “maths skills…can build confidence” and “The confidence the ESOL learners gain in tackling mathematical problems will allow them to gain confidence in learning other subjects”. It seems that in this class we do not have an issue with maths anxiety, as identified by many researchers, but that we may have an issue with English anxiety for ESOL learners. It may be that those learners who opt for ESOL Maths classes are those who are maths confident.

Observers commented on the increased level of participation compared to ESOL classes, and how beneficial this was for learners, as paired work “fostered greater communication in English”. Even those with confidence issues “still participated” in the paired work: “The shyest learner in the group from the lowest level language class clearly…felt able to answer the teacher because the focus was on maths not English”. Learners “responded well and found the experience useful and relevant”, even those “with strong educational backgrounds in maths”. Observers noted that “learners clearly felt more confident and more able to answer questions” and that the learners both responded well to the tasks set and performed well in their completion. There was “good interaction with resources/activities…working individually or in pairs.”
Observers were generally surprised to note the level of language performance shown by the learners, and felt this was improved compared to ESOL classes. One observer noticed that “the focus is on maths where some learners who may be weaker in language are able to do better” than in an English class, as they are using other skills, not just English. Two students performed consistently better, according to one observer, than they would have done in an ESOL class in terms of speaking and listening skills. All of the students seemed to be performing at a “good level of English” according to one observer.

Observers did not feel that any change to ESOL exam levels set was needed, but some did feel more confident that learners might achieve.

**Conclusion and recommendations**

Whilst caution is required due to the statistical insignificance of the sample size (Cohen et al., 2000) the implication here is that ESOL learners’ English acquisition might be further enhanced by placing them in ESOL maths classes, based on their English language level. This adds to the current knowledge in my college and perhaps elsewhere in the UK, and might be useful when considering maths provision for ESOL learners. It contrasts with a recent change of practice in the USA (Kersaint, Thompson & Petkova, 2013), where current thinking is that English language learners be placed in maths classes according to their level of mathematics knowledge.

Observation of ESOL Maths classes did seem to cause teachers to refine and extend their thinking about ESOL Maths and its usefulness in developing language skills, which can be seen as evidence of increased levels of language activity in mathematics in many countries, including the UK (Brown, 2001; FitzSimons, 2002).

The observations seem to support the idea that although mathematical language should be the focus of maths classes (Barwell, 2002; Fletcher and Barr, 2009; Monaghan, 2009), there is enough other language occurring for learners to benefit in terms of English acquisition (Adler, 2001; Clarkson, 2009).

Observers noticed the importance of the teacher interface with learners, and this supports the need for specialist maths teachers to enable learners to make progress with their maths (Brown, 2001; FitzSimons, 2002).

The benefit of resources developed for use with English language learners benefitting all maths learners has been previously examined (Adler, 2001), and I too have found these resources useful with all maths learners, hence the recommendation that we consider the importance of maths language teaching in maths classes and move to a triad approach: conceptual understanding, procedural competence and language acquisition.

**References**


