SYNERGY: WORKING TOGETHER TO ACHIEVE MORE THAN THE SUM OF THE PARTS

Te Piringa – Mā pango, mā whero, ka oti

Proceedings of the 19th International Conference of Adults Learning Mathematics – A Research Forum (ALM-19)

Auckland, New Zealand
26 – 29 June, 2012

Organised by the National Centre of Literacy & Numeracy for Adults, University of Waikato, New Zealand
Hosted by Auckland University of Technology (AUT), New Zealand
Editors: Anestine Hector-Mason and Diana Coben
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About ALM

Adults Learning Mathematics – A Research Forum (ALM) is an international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults.

ALM was formally established in July 1994 and is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number 3901346). The company address is 26, Tennyson Road, London NW6 7SA, UK.

ALM’s objectives are the advancement of education by the establishment of an international research forum in the lifelong learning of mathematics and numeracy by adults by:

- Encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;

- Promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels for the public benefit.

ALM’s vision is to be a catalyst for the development and dissemination of theory, research, and best practices in the learning of mathematics by adults, providing identity for the profession and internationally promoting and sharing knowledge of adults’ mathematics learning for the public benefit.

On the ALM website http://www.alm-online.net/ you will find pages of interest for teachers, experienced researchers, new researchers and PhD students and policy makers:

- **teachers**

  The work of members include many ideas for the development of practice and is documented in the Proceedings of ALM conferences, the *Adults Learning Mathematics - International Journal* and in other publications. We have started work on producing a searchable database.

  This page is also an opportunity for members to share resources and teaching ideas.

- **experienced researchers**

  The organisation brings together international academics, allowing the sharing of ideas, publication and dissemination via the conferences and the academic refereed *Adults Learning Mathematics - International Journal*.

  Here you will also find ideas, descriptions of projects, requests for help from researchers.
- new researchers and PhD students

The conferences and other events allow a friendly yet challenging environment to test out ideas and develop work.

This page will also enable exchange of ideas and work between times.

- policy makers

The work of the individuals in the organisation has a history of influencing policy in various countries including Australia, the UK and Denmark.

ALM holds an international conference each year at which members and delegates share their work, meet each other and network.

ALM Conference Proceedings and the online Adults Learning Mathematics - International Journal (ALMIJ) are available from the website.

ALM members live and work all over the world, the ALM members page will put you in touch with our regional activities and representatives. ALM welcomes new members; please contact the Membership Secretary. Contact details are on the ALM website.

ALM is managed by a Board of Trustees elected by the members at the Annual General Meeting (AGM) which is held at the annual international conference.
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Preface

The 19th international conference of Adults Learning Mathematics – A Research Forum (ALM19) was held in Auckland, New Zealand - a first for New Zealand and only the second time an ALM conference has been held in the Southern Hemisphere.

ALM19 was organized by the National Centre of Literacy and Numeracy for Adults and hosted in Auckland by Auckland University of Technology (AUT), 26th - 29th June, 2012.

The conference theme was ‘Synergy - working together to achieve more than the sum of the parts. Te piringa - mā pango, mā whero, ka oti’.

In keeping with the ‘synergy’ theme, papers discussing synergies with other competencies or contexts to enhance the skills needed for work and life in a constantly changing world were particularly welcome. The Proceedings reflect this theme, with contributors exploring, for example: the relationship between mathematics, learning and teaching and various workplaces; between numeracy and literacy; beliefs about mathematics and learning; and teaching and learning in diverse settings, including prisons, polytechnics and universities, as well as the workplace.

The idea of synergies also inspired the decision to run the international ALM19 conference alongside several national (New Zealand) events. These were the National Centre of Literacy and Numeracy for Adults’ 2012 Symposium, Māori Caucus and Workplace Hui and the first day of the 2012 conference of the New Zealand Adult Literacy Practitioners Association (ALPA).

The ALM19 conference was attended by 76 researchers, practitioners and policymakers from Australia, Ireland, New Zealand, the United Kingdom and the United States of America.

ALM19 sessions were open to delegates from these other events and vice versa, giving New Zealand-based practitioners and researchers the opportunity of international networking across the spectrum of research and practice on adults learning mathematics and their international counterparts the chance to find out more about research and practice – in both adult numeracy and adult literacy - in New Zealand.

Note
All conference presentations submitted for publication and meeting the editors’ requirements for style and presentation were peer reviewed and are published in the ALM19 Conference Proceedings.

Presentations for which no paper was submitted are represented by their programme abstract.
Acknowledgements

We are grateful to the many people who have contributed to the ALM19 conference and the production of these Proceedings, including:

• the participants, without whom there would have been no conference and no Proceedings;

• Janet Hogan, who made the complex task of organising the ALM19 international conference alongside a suite of national events look easy;

• Dr Pat Strauss, Kevin Roach and AUT colleagues for hosting the conference so splendidly;

• The University of Auckland Mathematics Education Unit (MEU), especially Professor Bill Barton, Dr Barbara Miller-Reilly and Dr Judy Paterson

• The University of Auckland Grafton Hall, especially Dr Greg Oates

• the ALM Officers and Trustees for ensuring continuity of the organisation between conferences;

• Dr Anestine Hector-Mason, Chief Editor of ALM’s conference proceedings - Anestine has been responsible for the peer review process and essential behind the scenes preparation of these Proceedings;

• Sarah Cowley for expert technical editing of the ALM19 Proceedings.

Their work is greatly appreciated and hereby acknowledged by the ALM19 Programme Committee: Niki McCartney, Diana Coben and Sally Davies.
Keynote Presentations
This paper presents some findings from the 2006 Adult Literacy and Life Skills Survey (ALL) on the numeracy demands of New Zealand workplaces. Though numeracy activities at work may now fit within a more developed IT setting, the six activities that the ALL survey asked about are very likely to be still relevant in a wide range of workplaces. The paper considers these activities across the whole working population, then analyses by occupation groups, and by measured numeracy skill to look at the match between activities and skill. A simple analysis of the combinations of reading, writing and maths practices provides an indicator of diversity and frequency of numeracy and literacy demands.

Introduction

The Adult Literacy and Life Skills Survey (ALL) was an international survey involving 11 OECD countries (including New Zealand, Australia, Canada and the USA). New Zealand took part in 2006. ALL collected information from a nationally representative household-based sample of adults aged 16-65 years old. It measured skill levels in literacy, numeracy and problem solving, and collected a wide range of socio-demographic, educational and work-related information. Results are published in an international comparative report (OECD & Statistics Canada, 2011). The Ministry of Education has published extensively on New Zealand findings. See for example Satherley, Lawes & Sok (2008).

ALL background information included how often people participated in numeracy activities at work. The ALL data therefore provides the scope to investigate workplace skill utilisation through analysing numeracy activity with literacy activity at work, and with actual numeracy skill.

The paper first looks separately at each of the six numeracy activities for occupation groups. We then derive an index of frequency and diversity of participation across the numeracy activities, and again consider patterns for occupation groups. The paper looks at literacy activities and overall patterns of combination with numeracy activities for occupation groups. The paper finally considers numeracy activity together with numeracy skill levels in order to obtain an understanding of activity/skill match and mismatch.

Numeracy in the workplace

Table 1 documents the question text and response options for the numeracy activities at work question in ALL which was asked of people who were currently working or who had worked in the last 12 months.
Table 1: The ALL numeracy activities at work question

<table>
<thead>
<tr>
<th>Question text and frequency options</th>
<th>Numeracy activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>How often do/did you do each of the following as part of your main job? Would you say at least once a week, less than once a week, rarely or never?</td>
<td>Measure or estimate the size or weight of objects</td>
</tr>
<tr>
<td></td>
<td>Calculate prices, costs or budgets</td>
</tr>
<tr>
<td></td>
<td>Count or read numbers to keep track of things</td>
</tr>
<tr>
<td></td>
<td>Manage time or prepare timetables</td>
</tr>
<tr>
<td></td>
<td>Give or follow directions or use maps or street directories</td>
</tr>
<tr>
<td></td>
<td>Use statistical data to reach conclusions</td>
</tr>
</tbody>
</table>

This aimed to be a set of activities that are common in many kinds of jobs, though they may be done at greatly different levels of complexity or intensity. The ALL survey was undertaken in New Zealand in 2006 and work-related numeracy activities are now likely to be undertaken within more technologically advanced settings. However, we consider that these activities are very likely to be still relevant in a wide range of workplaces.

Six graphs (Figures 1-6 below) show the differing patterns of each of the six activities across occupation groups.

Figure 1 shows that trades workers measured and weighed things much more often than other occupations. This is consistent with general knowledge of their work, so it provides some assurance of the validity of these measures. Weighing and measuring was also a common activity for machine workers and agriculture and fisheries workers. Clerical workers and professionals were least likely to weigh or measure things.

Managers were far the most likely to work on prices and budgets – see Figure 2. Nearly 80 percent of managers do pricing and budgeting at least once a week, and only 2 percent say they never do this. This is also an expected finding for whatever kind of small or large organisation managers work in.

Figure 3 shows that counting to keep track was a very common activity across all occupation groups. This demonstrates that this aspect of numeracy is a core work skill regardless of occupation. Timetabling was also very common – see Figure 4 – particularly for managers, professionals, technicians and trades workers.

Using directions or maps was less common but fairly consistent across occupation groups – see Figure 5. The frequency of using maps for work may depend on the size of the community a person lives in.

Using statistics at work was not so common for many occupation groups – see Figure 6. As expected, managers and professionals were the most likely to do this. That managers are more likely to use statistical data than professionals or technicians may reflect that management, even in small businesses, is likely to involve inferences based on summarised information, which may not be the case for some professional or technical occupations.
Possibly less expected is that almost one in 10 labourers reported that they used statistics to form conclusions. Inspecting the data showed that the largest specific occupation of these respondents was freight handlers. Perhaps they use statistical data in checking performance against averages or targets, or tally up progress.

Figures 1-6 show occupation groups along the horizontal axes, and proportions for how often people reported the activities were done are on the vertical axes.

**Figure 1: Frequency of measuring or estimating the size or weight of objects, by occupation**

**Figure 2: Frequency of calculating prices, costs or budgets, by occupation**
Figure 3: Frequency of counting or reading numbers of keep track of things, by occupation

Figure 4: Frequency of managing time or preparing timetables, by occupation

Figure 5: Frequency of giving or following directions or using maps or street directories, by occupation
Summarising across the six numeracy activities

A next step to try to understand these patterns more is to aim for a more summarised measure across the six numeracy activities. We prepared a simple index of frequency and diversity of numeracy activities using the following steps:

- Assign codes 1, 2, 3 and 4 for the four frequency options: never, rarely, less than once a week, at least once a week for each activity.
- Add the codes across the six activities to get a total score between 6 and 24.
- Divide the resulting distribution into thirds – low, medium and high. The cutpoints were: 6-15 low; 16-19 medium; 20-24 high. To be high, a respondent therefore had to have at least two 4s.

This is only one of many ways of doing this, but it gives a simple combination indicator of frequency and diversity of numeracy activities. Someone scoring high on this index often does several different numeracy activities.

The index, however, is only an approximate measure of the intensity of numeracy activity, since it doesn’t account for people who do, for example, only one activity very frequently or all the time. For example, someone whose job is solely to weigh up goods and pack them would score 4 (at least once a week) for that, but 1 (never) for the other numeracy activities, so they’d get a low total of 9.

Figure 7 shows a very wide variation across occupation groups in the proportions of people who are high or low on this index. Managers did more numeracy activities more often than any other occupation group. Next were trades, professionals, and technicians. Service and sales workers, and labourers were the least likely to do lots of numeracy activities frequently. However, it’s perhaps surprising that 14 percent of labourers are high on the index. A check of the data shows that mostly these people were messengers and deliverers, building labourers and freight handlers, and they engaged in weighing, pricing, counting and following maps frequently.
Also perhaps surprising is that 7 percent of managers were low on the index. The data shows that many of these managers were restaurant or hotel managers in small enterprises. Perhaps they’re front-of-house people and someone else does the GST (Goods and Services Tax) returns, the accounts, stocktaking and ordering.

![Figure 7: Diversity/frequency of numeracy activities at work, by occupation](image)

We see an overall picture showing that numeracy activities are important for most occupations. But how does numeracy combine with literacy in the workplace?

**Literacy activities at work**

The ALL survey asked about respondents’ participation at work in six reading activities and five writing activities. Table 2 lists these activities which are the same for reading and writing except that *Diagrams and schematics* is not included as a writing activity. The frequency options are the same as for numeracy activities.

**Table 2: Reading and writing activities at work**

<table>
<thead>
<tr>
<th>Reading activities</th>
<th>Writing activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letters, memos or emails</td>
<td>Letters, memos or emails</td>
</tr>
<tr>
<td>Reports, articles, magazines or journals</td>
<td>Reports, articles, magazines or journals</td>
</tr>
<tr>
<td>Manuals or reference books including catalogues</td>
<td>Manuals or reference books including catalogues</td>
</tr>
<tr>
<td>Diagrams or schematics</td>
<td>Directions or instructions</td>
</tr>
<tr>
<td>Directions or instructions</td>
<td>Bills, invoices, spreadsheets or budget tables</td>
</tr>
<tr>
<td>Bills invoices, spreadsheets or budget tables</td>
<td></td>
</tr>
</tbody>
</table>
We constructed similar frequency/diversity indexes for reading and writing activities at work as for maths.

Figure 8 shows a broadly similar pattern across occupation groups for reading activities compared to numeracy activities. A key difference is that, while managers, professionals and technicians were similarly likely to rate high for reading diversity/frequency, managers were noticeably more likely than professionals and technicians to rate high on numeracy.

Trades and agriculture and fisheries workers were less likely to undertake reading activities than numeracy activities.

![Figure 8: Diversity/frequency of reading activities at work, by occupation](image)

Figure 9 shows that managers did more different writing activities more often, compared to even professionals. To some extent, we see this pattern because our index includes a measure of diversity; the data shows that managers do more different kinds of writing than professionals and technicians.

![Figure 9: Diversity/frequency of writing activities at work, by occupation](image)
Numeracy and literacy activities at work

Finally we compiled an overall index that combines diversity/frequency of reading, writing and numeracy activities. This index takes the values 17-68. Again we divided the spread into thirds: low, medium and high. This gives an opportunity to look at how numeracy and literacy activities are combined in workplaces. It relates to the 2012 Adults Learning Mathematics conference theme of synergy by engaging with the joint numeracy and literacy demands in different occupation groups.

Figure 10 shows that managers combined literacy and numeracy activities to a very high degree, followed by professionals and technicians, followed in turn by clerical and trades workers. But these latter two occupations had different mixes of literacy and numeracy; trades workers did more numeracy, and clerical workers did more literacy particularly writing.

Numeracy activity and numeracy skill

In this section of the paper we turn to looking at the relationship between numeracy activities and actual numeracy skill.

In ALL, numeracy skill meant the knowledge and skills required to effectively manage the mathematical demands of diverse situations (OECD & Statistics Canada, 2011, p. 14). ALL measured respondents’ numeracy skill through assessments using booklets of test items. Respondents were assigned scale scores, and people working (or who had worked in the last 12 months) had an average of 276 score points. The 5th percentile was 182 and the 95th percentile 360 – which provides an indication of the spread of the numeracy skill distribution amongst the working population.

The following analysis helps answer the following questions. How well do numeracy skill and numeracy activity line up? Are people with strong numeracy skills doing lots of numeracy activities at work? Are people who do lots of numeracy activities strong numerically? How well do workplaces sort people into work that fits their skills from a numeracy point of view?

Figure 11 shows numeracy skill (as directly measured in the ALL survey) and how frequently and how many numeracy activities people actually do (as measured by the frequency/diversity index). We see a moderate relationship only for those with a low numeracy activity index (ie in
the bottom third of the distribution of the index and scoring less than 16). People with medium or high levels of numeracy activity (i.e., in the top two-thirds of the distribution and scoring 16 or more) had no significant difference in average numeracy skill.

In other words, within the group who do little numeracy activity at work, doing more numeracy activity was somewhat associated with higher numeracy skill. But for people who do a medium or high level of numeracy at work, we see no association with numeracy skill. The alignment of numeracy activity to numeracy skill occurs only at low levels of numeracy activity.

![Figure 11: Numeracy score (with 95% confidence intervals) by frequency/diversity index of numeracy activities](image)

**Numeracy skill levels**

The ALL survey grouped numeracy skill into 5 levels, where level 3 is the level considered as the minimum for full participation in a knowledge society and economy. Table 3 gives the descriptors for each level. (OECD & Statistics Canada, 2011, p. 15)

**Table 3: ALL numeracy levels**

<table>
<thead>
<tr>
<th>Level</th>
<th>Descriptor</th>
<th>Score point range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Tasks in this level require the respondent to show an understanding of basic numerical ideas by completing simple tasks in concrete, familiar contexts where the mathematical content is explicit with little text. Tasks consist of simple, one-step operations such as counting, sorting dates, performing simple arithmetic operations or understanding common and simple percents such as 50%.</td>
<td>0-225</td>
</tr>
<tr>
<td>Level 2</td>
<td>Tasks in this level are fairly simple and relate to identifying and understanding basic mathematical concepts embedded in a range of familiar contexts where the mathematical content is quite explicit and visual with few distractors. Tasks tend to include one-step or two-step processes and</td>
<td>226-275</td>
</tr>
<tr>
<td>Level</td>
<td>Descriptor</td>
<td>Score point range</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>estimations involving whole numbers, benchmark percents and fractions, interpreting simple graphical or spatial representations, and performing simple measurements.</td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td>Tasks in this level require the respondent to demonstrate understanding of mathematical information represented in a range of different forms, such as in numbers, symbols, maps, graphs, texts, and drawings. Skills required involve number and spatial sense, knowledge of mathematical patterns and relationships and the ability to interpret proportions, data and statistics embedded in relatively simple texts where there may be distractors. Tasks commonly involve undertaking a number of processes to solve problems.</td>
<td>276-325</td>
</tr>
<tr>
<td>Level 4</td>
<td>Tasks at this level require respondents to understand a broad range of mathematical information of a more abstract nature represented in diverse ways, including in texts of increasing complexity or in unfamiliar contexts. These tasks involve undertaking multiple steps to find solutions to problems and require more complex reasoning and interpretation skills, including comprehending and working with proportions and formulas or offering explanations for answers.</td>
<td>326-375</td>
</tr>
<tr>
<td>Level 5</td>
<td>Tasks in this level require respondents to understand complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information, draw inferences, or generate mathematical justification for answers.</td>
<td>376-500</td>
</tr>
</tbody>
</table>

**Numeracy skill activity mismatch**

Figure 12 shows the proportions of the working 16-65 year olds at the different levels of numeracy skill, for each of the low, high and medium ranges of the diversity/frequency of numeracy activity. This analysis provides a way of thinking about skill mismatch – where people have high numeracy skill, but low numeracy activity or vice versa. This relates to the synergy theme of the ALM conference. How well do numeracy skills and numeracy work line up in the workplace?

We acknowledge that the analysis of this paper is not the only way of tackling this question. Using the ALL survey data, we can measure the frequency and diversity of numeracy activity, but not its complexity. So perhaps some people with low numeracy skill may be often undertaking a lot of different numeracy activities that are relatively simple, and therefore within their capability. Other kinds of research would be needed to investigate this dimension. Any measure of skill match depends on the concepts, measures and categorisation of both numeracy activity and numeracy skill.

The paper proceeds with looking at mismatch in terms of numeracy skill levels compared to frequency/diversity of numeracy activity while recognising the limitations of this approach.

Figure 12 shows that nearly 40 percent of the low numeracy activity group were at numeracy level 3 or above – a ‘skill excess’ group. Also the Level 4/5 numeracy skill people with medium numeracy activity group could be considered as having skill excess. These two subgroups are highlighted on the graph with black ovals. A third of the high numeracy activity people have
level 2 or below numeracy skill – a skill shortfall subgroup. The medium numeracy activity group with level 1 skill could also be considered as skill shortfall. The latter two subgroups are highlighted with red ovals in Figure 12.

![Figure 12: Diversity/frequency index of numeracy activities, by numeracy skill level](image)

Figure 12: Diversity/frequency index of numeracy activities, by numeracy skill level

Figure 13 puts together the three skill match and mismatch groups together: skill shortfall, skill match and skill excess for the 16-65 year olds who are working or worked in the last 12 months. Across all numeracy skill levels, the respective proportions were: 17 percent, 64 percent and 19 percent.

![Figure 13: Distribution of numeracy skill/activity match and mismatch](image)

Figure 13: Distribution of numeracy skill/activity match and mismatch

We have an indication that, across the working population, nearly one in six did more numeracy activities than appear to be supported by their actual numeracy skills. And around one in five were in the opposite situation – they undertook less numeracy activity than they could easily manage.

The Ministry of Education has published an analytical report (Earle, 2011) that looks at skill match in ALL somewhat differently from this paper. It groups literacy and numeracy activities
by combining them into these three groups: financial literacy and numeracy; intensive literacy; practical literacy and numeracy. This contrasts with this paper’s focus on numeracy activities as such. In addition, Earle used ALL’s document literacy skill domain rather than numeracy. The match categories included partial shortfall and partial excess. Table 4 shows the proportions (Earle, 2011, p. 22). They are broadly consistent with this paper’s analysis, given its different approach. Either way, we have some evidence that New Zealand’s workplaces could have a better synergy between numeracy and literacy activities and the relevant skills.

Table 4: Match and mismatch of literacy and job practices from Earle (2011)

<table>
<thead>
<tr>
<th>Match/mismatch category</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skills excess</td>
<td>2</td>
</tr>
<tr>
<td>Partial excess</td>
<td>12</td>
</tr>
<tr>
<td>Matched</td>
<td>44</td>
</tr>
<tr>
<td>Partial shortfall</td>
<td>33</td>
</tr>
<tr>
<td>Skills shortfall</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

Our final step is to decompose Figure 13 by occupation groups. Figure 14 shows this for four key occupation groups. The skill shortfall group is smaller for professionals, who, on average, have strong numeracy skills and are highly educated. But trade workers and managers have more than 20 percent in the skill shortfall group.

Figure 14: Distribution of numeracy skill/activity match, for selected occupations

Earle (2011) derives a similar skill shortfall pattern for occupation groups. Technicians and trade workers, service and sales workers, machine operators had relatively high skill shortfall proportions (Earle, 2011, p. 23).
As mentioned previously, we should note that some of the managers and trade workers in the skill shortfall group may be doing lots of different numeracy activities often, but perhaps they entail quite low level, simple maths and are actually within their capability.

Programme for the International Assessment of Adult Competencies

New Zealand is participating in the OECD Programme for the International Assessment of Adult Competencies (PIAAC), which is a successor to ALL. With about a dozen countries, New Zealand will join a second round of PIAAC where the data collection is in 2014 and the results will be available from May 2016. About 25 countries have already participated in PIAAC’s first round data collection and their results will be released in October 2013. PIAAC measures literacy and numeracy consistently with ALL and adds a new technology-related problem solving skill. For more information on PIAAC, see OECD (2010). PIAAC has a strong focus on work skills linking to the importance of the OECD’s Skills Strategy (OECD, 2012).

PIAAC numeracy activities at work

The PIAAC study has substantially revised ALL’s numeracy activities, aiming to provide measures of both high and low level numeracy activities.

Table 5: PIAAC numeracy activities at work

<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate prices costs or budgets</td>
</tr>
<tr>
<td>Use or calculate fractions, decimals or percentages</td>
</tr>
<tr>
<td>Use a calculator – either hand-held or computer-based</td>
</tr>
<tr>
<td>Prepare charts, graphs or tables</td>
</tr>
<tr>
<td>Use simple algebra or formulas</td>
</tr>
<tr>
<td>Use more advanced maths or statistics such as calculus, complex algebra, trigonometry or use of regression techniques</td>
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Information derived from these PIAAC questions will enable further understanding of the relationships between numeracy skill, numeracy activities and work.

Summary

• Large proportions of most occupation groups undertook numeracy activities at work.

• On average, managers undertook more types of numeracy activity more often than other occupation groups. Professionals, technicians and trade workers rank next to managers on this aspect.

• Managers were most likely to undertake many literacy and numeracy activities often.

• Undertaking numeracy activities for work is moderately associated with measured numeracy skill only for people who do little numeracy activity.
• About one in six workers did more numeracy activity than appears supported by their numeracy skill. About one in five workers appeared to do less numeracy activity than they could manage.

• Trades workers and managers were occupations where numeracy skill shortfalls are over-represented.

• PIAAC will provide future measures of numeracy skill and numeracy activities.

**References**


Mathematical Tribes: Our Wars and Intermarriages

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The mathematics community is itself made up of many culturally distinct groups. How do we interact? How do we destroy each other? How could we improve our tribal behaviour?

The struggle for influence and resources involves us all, and, as in world affairs, often means we are fighting our friends, or at least undermining them.

Professor Bill Barton gave delegates a tour through our micro-politics, and an examination of our weapons of war, courting rituals, and funeral rites.
Paper Presentations
Doing Mathematics in the Workplace: A Brief Review of Selected Literature

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The aim of this review of selected literature on research into mathematics in the workplace is to offer researchers from the field of adult mathematics education an opportunity to become more familiar with this specialised area. In recent years much progress has been made, building on earlier research, but with increasingly nuanced understandings. In particular, sociocultural activity theory has played an important role, and several articles reviewed here have drawn on this theory. Their focus has been on gaining a comprehensive understanding of what workers actually do involving aspects of mathematics, set in the rich context of a functioning workplace. This encompasses taking account of workplace artefacts (technological and otherwise), various forms of communication (verbal and non-verbal), different forms of skills, and even the concept of boundary crossing; most importantly, how workers learn.

Introduction

Throughout history and across cultures, people have used mathematical and other tools and symbols in their daily activities. At any age, work (paid or unpaid) to produce goods and services can take many forms and have a variety of purposes. Apart from the work of professional mathematicians, doing mathematics at work is generally the means to an end external to the discipline. In relation to the theme of the 2012 Adults Learning Mathematics (ALM) conference, Synergy – working together to achieve more than the sum of the parts, it is clear that workplace mathematics is a case in point. Specifically, the combined interactions of workers at all levels, suppliers and consumers of goods and services, internal and external, all contribute to the end result.

Over a decade ago, I published¹ a comprehensive review of research into workplace mathematics (FitzSimons, 2002). I noted the evolution in research, from an identification of topics recognisable in school mathematics curricula of the day — itself an arbitrary selection from the discipline (Ernest, 1991) — towards more nuanced studies which attempted to understand activities involving mathematics within their respective socio-cultural contexts. This paper aims to provide a synthesis of selected workplace research over the last decade or so, with a view to answering the question of what knowledge has been gained since then. The focus is on the ‘world of work’ in relation to the mathematics that workers use mostly unconsciously, transform, or even create locally, and communicate through language, gestures, signs, and other non-verbal communication, in order to address the ever-evolving problems of their particular workplace activity.

¹ Bear in mind that there is a considerable lag between the production of a manuscript and its eventual publication in hard copy, although this is being reduced with online technologies.
Research Methodologies and Important Theoretical Concepts Used in the Workplace Literature Reviewed

The studies discussed in this literature review have adopted predominantly qualitative methods, with most adopting ethnographic approaches and many utilising socio-cultural activity theory, based on the work of Vygotsky and Leont’ev (see, e.g., Roth & Lee, 2007, for a comprehensive review). In these approaches, the unit of analysis is the activity itself which is generally undertaken by a group of people, (such as a work unit) in order to satisfy a motive (such as the production of a good or a service, in their broadest interpretation). Various actions are undertaken to achieve a range of goals supporting the activity, and, in turn, these depend on unconscious operations or skills. (For a further description of Activity Theory in relation to mathematics education, see FitzSimons, 2008.)

Any breakdown in this system, including communication, is an opportunity for learning to take place — for workers and for researchers. If the worker’s necessary skills are not in place (e.g., needing to learn a new IT technique), this becomes an immediate goal in the ongoing activity. As the context changes, different goals and different actions come into play for the worker. A feature of workplace activity is that problems arise on a daily basis that cannot always be predicted in advance, and locally new solutions must be generated. For researchers, a breakdown in a routine activity offers the possibility for gaining deeper insights as participants are probed for more detailed explanations of things that were previously taken-for-granted.

Communication — in its many forms — is, of course, a central issue in activities associated with work; and the concepts of boundary crossing and boundary objects are raised by many researchers (see Akkerman & Bakker, 2011, for a comprehensive review). Workers at all levels interact with others within and beyond their practice: between the work unit and customers or suppliers, internal or external; or between various organisational layers. Communication can encompass dialogue, written texts, diagrams, signs, symbols, and gestures. People often need to communicate across the boundaries of their particular work setting and, since mathematics is often an integral part of the work process, mathematical thinking and reasoning is likely to be embedded in this communication. Workplace artefacts such as graphs, tables, spread sheets, historic records of production, data collection and output (including quality control), are likely to be in use as well. As a means of communication between different groups, these boundary objects — as they have come to be known in the literature — are often wrongly assumed by their producers and users to be transparent, leading to communication breakdowns. For example, if a novice worker measures a length to be 32mm, but has neglected to allow for the space before zero on the ruler, and the person receiving the measurements does not think to query whether the technique of measurement was correct, because it is generally taken for granted, then there will be a communication breakdown which could ultimately prove to be costly in terms of time, money, and/or safety.

In the studies reviewed here, data collection has generally included participant observation, interviewing, and collection of artefacts. Not surprisingly, many articles have focused on the use and the influence of technology as an integral component of contemporary workplaces.

Any formulation of skills is time- and place-specific, and is often embedded in labour relationships and broader social structures encompassing social actors, institutions, as well as social values and norms. Wedge’s (2000) understanding of workplace technology views mathematics as being integrated in four dynamically inter-related dimensions: (a) technique/machinery; (b) work organisation; (c) vocational qualifications; and (d) workers’
competences. In addition, as part of this multidimensional view of workplace mathematics, Wedege (2011) provides a definition of technological competence, which she defines as “workers’ capacities (cognitive, affective, and social) for acting effectively, critically, and constructively” (p. 3) to challenges in the technological workplace.

**Research Findings**

The literature reviewed was drawn from over 50 refereed journal articles published over the last decade or so. Articles were selected on the basis of their capacity to illustrate: (a) differences in approaches to mathematics-related tasks between school students and workers, (b) graphic details of some mathematics-related activities actually carried out by particular workers, (c) differing structural resources available in the workplace, (d) workers’ attitudes to school mathematics or the vocational mathematics intended to be relevant to their future occupations, and (e) how technology-enhanced artefacts can help workers to uncover and find meaning in mathematics rendered invisible by automation.

**Problem Solving: Plumbers vs. Students**

One objective of many studies is to make comparisons between the workplace and the school setting. Jurdak and Shahin (2001) documented, compared, and analysed the nature of spatial reasoning by plumbers in the workplace and by school students while constructing the ‘same’ solids from plane surfaces. The authors drew on Activity Theory for its potential to explain the differences between the two settings in terms of motives, tools available and accessible, and constraints. Data were collected from a plumbing workshop with five experienced adult plumbers having little or no school experience, and five tenth-grade students (2 girls and 3 boys), while constructing a cylindrical container of capacity one-litre and height of 20 cm.

Based on the method of structural analysis, they identified differences in the types and sequence of each group’s actions as well as in the degree of complexity. Jurdak and Shahin (2001) contrasted the two groups:

In the course of constructing the task container, both the plumber and the environment changed. The plumber started with perceptual action, responded to what has been executed by continuously reviewing the model container, executed more actions, and so on. The goals for container construction changed as the container evolved and its actuality became possible. This interaction elicited critical skills such as recognizing opportunities or problem finding, knowing when and how to apply skills that have been learned in other contexts, and exploiting properties of the present situation. (p. 312)

On the other hand,

The students’ interaction with their physical environment was minimal and they approached the problem of constructing the task container by implementing the procedure that was derived directly from the classroom practice. ... they relied [almost] exclusively on mnemonic and cognitive tools by reading the problem, selecting the formula, calculating the unknown, and writing the answer. ... The students showed little control over the problem solving process, they strongly believed in the power of formulas and algorithms and hence did not feel the need for self-monitoring other than checking the correctness of their calculations. (p. 312)

Jurdak and Shahin (2001) observed that, whereas in school mathematics is a conceptual tool, detached from the situations which give it meaning, in the workplace mathematics is a concrete
A tool which takes its meaning from the situation at hand to solve problems that may arise within that context. They concluded that although using mathematics in the workplace is more meaningful, school (i.e., formal) mathematics has more power and is more generalisable beyond the specific context of application.


Magajna and Monaghan (2003) conducted a case study of six CAD/CAM technicians who design and produce moulds for glass factories; their research focused mostly on volume calculations. The technicians’ work was in many ways related to mathematics (e.g., constructing shapes, calculating or managing the cutting operations on machine tools). To help describe the structure of emergent goals, Magajna and Monaghan followed Saxe’s four parameter model:

1. **Activity structures**: In order to understand a particular aspect of observed practice it is necessary to “consider the whole production cycle from the client’s initial order to the manufacturing of the mould and the use of the mould in the production of bottles in the (distant) glass factories” (p. 105).

2. **Social relations/interactions** influence the emergent goals.

3. **Conventions and artefacts**: Although some methods used to determine the volume may be traditional, the introduction of computers and CNC machines has also led to new methods.

4. **Prior understandings** brought by individuals, both constrain and enable the goals that individuals construct in practices.

Magajna and Monahan (2003) found that “there was an evident discontinuity between the school mathematics used and the observed mathematical practices. This discontinuity was evident at both a subjective and an objective level” (p. 117). At the objective level, school-like concepts and practices, such as linear equations, trigonometry and Pythagoras’s theorem, were used in the context of practice, even though the technicians did not necessarily understand all the mathematical background. School mathematics knowledge was used ‘as is’, without being questioned discussed or modified. However, discontinuity was also evident at the subjective level when the technicians claimed that there was no school mathematics in their jobs.

Magajna and Monahan (2003) also found that technology in this CAD/CAM workplace was so all-pervasive that it virtually structured the technicians’ activity and played a crucial role mathematically. Not only were their mathematical procedures shaped by the technology they used, but their mathematical understandings were used as a means to achieve technological results. Mathematical correctness was negotiated amongst the technicians and also with customers and contacts in the glass factory, in contrast to the school situation.

Third, reflecting the findings of Jurdak and Shahin (2001), Magajna and Monaghan (2003) found that in mathematical breakdown situations the practitioners either simply chose another mathematical or construction method or, more commonly, overcame the problem by technological means rather than mathematising the cause of the breakdown in the sense of reasoning about mathematical procedures. They “did not solve problems involving mathematical abstractions in this workplace” (p. 120). In both cases, the workers’ actions should be interpreted within the goal-oriented behaviour of their occupation: Technicians’ work is embedded within the imperatives and constraints of the factory’s production activity cycle which included: (a)
time pressures demanding almost immediate solutions; (b) many mathematical procedures being frozen in the technology, often invisible, and poorly understood by practitioners who had limited control over them; (c) the final product and not the mathematics was what counted (i.e., mathematical correctness does not necessarily mean technological correctness); and (d) a lack of practitioner confidence in understanding the complex procedures involved.

Telecommunication Technicians: Tool Mediation

Triantafillou and Potari (2010) studied a group of technicians in a telecommunication organisation. Guided by an Activity Theory framework, they identified, classified, and correlated the tools that mediated the technicians’ activity, and studied the mathematical meanings that emerged.

The technicians’ typical daily activity was to fix a number of reported faults in the local underground copper-wiring network. Their major working tools were wire pairs that were bundled together into cables consisting of between 100 and 600 pairs. This wiring network started from the main organization building in the center of the city, went through specific boards, the telecommunication closets, and then distributed to a number of boxes around the area of the closet, and finally was directed to the local subscribers. The largest part of this network was underground. The technicians, in order to accomplish their daily main activity, had to perform a series of actions that were usually sequential. These actions were to: (a) trace the reported pair of wires on a set of physical tools (the telecommunication closet and the boxes around the area) by using the information given on an instruction sheet, (b) use a technical map to trace the underground wiring network from the closet to the subscribers’ boxes, and (c) use two measuring instruments ... to locate the exact point of fault. (p. 281)

In relation to these three actions, Triantafillou and Potari categorised the tools as (a) mathematical (communicative, processes, and concepts), and (b) non-mathematical (physical and written texts), which they presented in a comprehensive systemic network (see p. 290, Figure 5). They found that the technicians’ emerging mathematical meanings in relation to place value, spatial, and algebraic relations were expressed through personal algorithms and metaphorical and metonymic reasoning, indicating the situated character of their mathematical knowledge. Triantafillou and Potari identified differences from typical mathematical representations in the technicians’ explanations or their physical tools and written texts, in keeping with the findings of Magajna and Monaghan (2003). However, they also noted that a number of processes and strategies shared the same structure and characteristics with those developed in a school context.

From these relatively small-scale studies, I turn to a major UK research project where the concept of boundary crossing has been an important component. Kent, Noss, Guile, Hoyles, and Bakker (2007) construed the term *techno-mathematical literacies* (*TmL*) to emphasise both the mediation of mathematical knowledge by technology and the breadth of knowledge required in the context of contemporary technology-rich workplaces that are both highly automated and increasingly focused on flexible responses to customer needs. They were also interested in how boundary objects “may facilitate effective communication between and within work teams and between work teams and customers” (p. 67). In the first phase of their research they carried out ethnographic case studies in 10 companies; in the second they carried out design experiments: I discuss findings from each, in the manufacturing and financial services sectors.
Process Improvement in Manufacturing

Kent, Bakker, Hoyles, and Noss (2011) conducted several case studies of process improvement in manufacturing companies, focusing on measurement aspects. Here, I discuss two: a production line bakery and a pharmaceutical packaging company.

**A process improvement [PI] project in a large cake production line bakery.** Kent et al. (2011) observed the work of a PI team in a factory which involved cakes continuously moving on conveyor belts within linear ovens many metres long. The entire production line was several hundred metres long, with every stage of the baking process monitored using a combination of automated and manual measurements. In the PI process, the workers used measurements throughout the whole baking process to develop a capacity profile: a summary chart of speeds of the different machines and processes, intended to make visible bottlenecks in the process which could be targeted for improvement.

However, management had not been aware of the importance of creating a culture in which workers appreciated the need to measure and to take results seriously. Because the workers had not received any training in these requirements for PI, Kent et al. (2011) found that, initially, team members with less formal education were under-skilled in this kind of work. Measurements were taken inconsistently, from variable location points in relation to the cake and the machinery, and were not systematically recorded, hindering the optimisation techniques used in PI. After a 2-week project to improve performance on this production line, the workers’ views and actions were transformed through their participation in acts of measurement, reading the graphs, noticing things that were previously invisible to them, and thinking about solutions. Nevertheless, Kent et al. noted that even when PI recommendations are made on logical (mathematical) grounds, there are often good practical reasons (e.g., cost) for not accepting them.

**Overall equipment effectiveness [OEE] in a pharmaceutical packaging company.** In this company, tablets were brought to be packed into foil and plastic blister packs, with a number of these packaged together into a cardboard box. Just prior to the research taking place, manually operated packing lines, each run by six operators, had been replaced by fully automated packing lines, operated by one technician. The production manager described the activity as:

> Literally you pour tablets in one end and feed it cartons, film, foil, leaflets—and out of the other end comes a sealed pack, and groups of packs boxed and labelled—all automatically; all the operator does with that is to stack it on the pallet … the main thing is the ability to be able to tweak the machine to keep it going at the optimum speed we require. (Kent et al., 2011, p. 757)

Instead of using hand tools and having physical access to many parts of the old production process, the technician now had to operate the line through a computerised control panel—one level of abstraction away from the physical process, also from the measurement process. Kent et al. (2011) noted that there was more to tweaking and running machines well than recognised by management; especially managing product changeovers which varied significantly between operators. This required the technician: (a) to understand the physical mechanisms of the machine and to be able to identify and remediate problems, or (b) to be able to communicate what was wrong to the skilled maintenance engineers in the factory. It also required operators to be able to make sense of the measurements and (idiosyncratic) graphical data generated by the packing machine’s internal computers.
Overall equipment effectiveness [OEE] combines three generic variables for production, Performance, Quality and Availability, to construct a more abstract measure concerned with maintaining a balance between the three variables. Although OEE measures created a visibility about the system for senior managers, they were not discussed with the operational employees. Kent et al. (2011) concluded that technology brings an extra layer of complexity to measurement. Although it is commonly assumed that automation leads to a reduction in human error, many managers expressed the concern that it also leads to less engagement with the production process, and several preferred to have employees measure and report manually. Tweaking of machinery by operators highlights the importance of how their tacit knowledge interacts with their codified knowledge to get the job done effectively.

Financial Services Workers

One case study by Kent et al. (2007) involving design experiments was set in the financial services sector and focused on the annual pension statement, a boundary object designed to facilitate boundary crossing between the company and customers. However, this pension statement routinely failed in its communicative role, largely due to the invisible factors of the underlying mathematical-financial models not being made available either to customers or to the Enquiry Team. Straightforward customer enquiries were to be resolved immediately by telephone, if possible. However, automation of the IT system, intended to ensure the accuracy of information sent to customers, had actually disempowered employees who lacked any real understanding of the models and calculations, inhibiting their communications with colleagues and customers.

Although the mathematics involved in finance seems superficially similar to secondary school mathematics, Kent et al. (2007) observed that in the workplace context every mathematical procedure, no matter how simple, is part of a whole range of decisions and judgments about complex processes or products. Employees need to be able to mathematically appreciate computer outputs, interpreting them in their context, and recognizing which components are hidden by the IT system. They also need to be able to reason about the mathematical models embedded in the system in terms of the key relationships between product variables (e.g., percentage rates, management fees, sales commissions) and their effect on outputs presented in the form of graphs or tables.

Bakker, Kent, Hoyles, and Noss (2011) detailed an intervention in the same industry where they designed technology-enhanced boundary objects (TEBOs) in order to improve employees’ understandings of the mathematics behind the mortgages they sold (i.e., their technomathematical literacies). Their goal was not to teach employees the mathematics behind boundary objects such as the mortgage package, but to engage with the different aspects of their underlying mathematical models. They developed software to model or reconstruct actual practice, using data drawn from a current account mortgage (CAM) which integrated a regular current account with the property mortgage. The company had previously published a booklet containing a standard repayment graph which was actually mathematically unrealistic. Bakker et al. (2011) described it as pseudo-mathematical: mathematical in form but not in function. Interviews revealed that

the sales agents were largely unaware of any but the simplest relationships between interest rates and repayment schemes, even though they were able to explain mortgages in general financial terms (e.g., capital and interest)…. sales agents saw the different interest rates as labels for instruments: annual rates as labels attached to a mortgage arrangement or a savings account, monthly rates as labels attached to credit card or loan debts. (p. 29)
In other words, sales agents had not understood the mathematical meanings or relationships. One TEBO reconstructed the mortgage graph in a spreadsheet with all the input variables and calculations made explicit in order that the graphs could be produced according to the input variables that matched different customer scenarios. Another TEBO modelling credit-card debts revealed to participants the problems with paying back only the obligatory monthly charge. Anecdotally, this intervention seemed to have improved employees’ understanding and confidence, but they were forbidden by management to use the TEBOs in actual practice.

Apprentice Electricians Learning to Bend Pipes

In an empirical study framed by cultural-historical activity theory, specific differences between college and workplace were identified and theorised, not by boundary crossing but by the concept of personality, discussed below. Roth (2012) investigated (a) the geometrical practices of electrician apprentices learning to bend electrical conduits in college and on the job, and (b) how they handled the relation between differing practices. The requirements for doing well in the two activity systems were very different: exhibiting knowledge of trigonometry in one, and doing a good job that makes bending and subsequent pulling of wires practical in the other. Formal trigonometry was the reference in the classroom, whereas the codified rules of practice were the main reference on the job.

In college, students intending to become electricians are taught conduit bending theory, and are required to study basic trigonometry. The textbook provides “magic circles” to help calculate such functions as the sine, cosine, and tangent. Apprentices carry out extensive calculations and measurements to determine angles, their positions on the tubing, and the distances between the angles. Once a student has calculated the distances, s/he uses measuring tape and bender to produce the tubing such that it properly bypasses the obstacle provided. However, in their practical conduit bending class, apprentices encounter a specialised conduit bender on which much of the required information is inscribed, rendering the trigonometric calculations superfluous (see Fig. 4, p. 7 [online first version]).

This study illustrates the radical differences between the (mathematical) practices of bending electrical metal tubing, in college and in the workplace, calling into question the usefulness of vocational courses which emphasise formal mathematics that is treated as irrelevant in the workplace. Nevertheless, the electrical apprentices managed to move between the different activities with a sense of coherence, integrating these differing experiences as part of the electrician personality they develop. Having gone to college allows the electricians to work according to the formal and legal requirements of the electrical code, while meeting practical workplace constraints.

In addition to the experienced differences between college and workplace, the electricians’ discourse about those differences was both topic and resource in their workplace conversations. However, in the case of mathematics, there was little cross-reference between college and workplace, in story telling or in practice. According to Roth, stories encode not only practical knowledge but also the very process of subjectification, which describe the changes within an activity system (school, work); and personality, the changes the individual undergoes as s/he moves repeatedly between systems of activity. Being able to talk about the contradictions between the two systems is as much part of being a recognized practitioner as is competent practice.
Chemical Spraying and Handling

The processes of preparation, application, handling, storage and transport of chemicals are key elements of a range of economically significant industries, and place high demands on workers’ literacy, and especially numeracy skills. Many of these skills are acquired during employment on-the-job or in associated off-the-job training. However, a substantial body of research evidence demonstrates that such skill transfer is achieved only with difficulty, and that numeracy skills are highly context-dependent. FitzSimons, Mlcek, Hull, and Wright (2005) undertook 13 case studies of enterprises which used chemicals extensively in industries including rural production, amenity horticulture, local government, outdoor recreation, and warehousing.

The mathematical processes and strategies used by workers to undertake calculations included: estimation, written methods or basic calculator use; oral or written communication of mathematics to other workers, and the interpretation of their mathematics; consultation with prescriptive calculations sheets and with historical records or online data; and completion of up-to-date records of chemicals used and the corresponding amounts. Complex contextual factors included date/time of spraying; block area; specific crop to be sprayed; crop stage; weed/pest/disease targeted; chemical group, rate per hectare, litres of spray applied, method of application; temperature, wind speed, wind direction, rainfall, humidity; and variations in equipment used, from small-scale backpacks to broad-acre mechanised spraying.

The accuracy of calculations was highly dependent on these factors as well as the degree of accuracy available on the equipment used. Importantly, economic and legal contingencies are strongly implicated within these contextual factors. Safety standards are critical, both in terms of the risk of spillage or misapplication of chemicals harmful to people (workers and consumers) and/or the environment, even causing spoilage of the end-product itself.

For these workers, learning on the job is largely experiential, with opportunities for them to become enculturated into communities of practice through interrelationships with other employees. Supervisors are often involved in initial training and check regularly on work practices; in some cases novices are given trial areas to spray and then asked to account for any discrepancies in expected area coverage and/or spray consumption. Most workplaces place a strong emphasis on ensuring that workers are prepared to check before acting, of being unafraid to ask a ‘dumb’ question.

Learning in these kinds of workplace differs significantly from formal institutionalised education in that it is rich in context, supported by historical records, and mediating artefacts such as tools, equipment, manuals, charts, and so forth, as well as communication of a qualitatively different kind from the classroom. In this workplace environment the object is satisfactory task completion, with much more at stake than appropriately accurate calculations. (See FitzSimons & Wedege, 2007, for a more comprehensive review of related literature.)

Conclusion and Implications

It is pleasing to see a burgeoning of research in this subfield of mathematics and work, especially in ALM’s own journal and conference proceedings. Several conclusions may be drawn, each with implications for further research.

It is abundantly clear that school and workplaces in general are completely different activity systems, even though learning permeates both. In the language of Activity Theory, their motives
and goals rarely intersect, even though their operations may in fact do so. School and workplace have two different logics, as shown by the data with respect to technology, behaviours, and values placed on cognitive skills.

Automation has had a major impact on many industries, particularly in more developed countries, with a consequent reduction in the visibility of the mathematical processes it envelops. The term black box originated in the 20th century, but Maß and Schlöglmann (1988) argue that as a process it dates back to the Stone Age: Its success relies on its relatively context-free transfer of results, where knowledge without understanding can be passed from developers to end-users relatively easily. However, it is this hidden complexity—not only in technology but permeating many workers’ daily activities (see also paper by Keogh, this volume)—which presents a great challenge to workplace mathematics researchers and vocational mathematics educators. It is worth reiterating the importance of the interaction between the tacit knowledge of workers and the codified knowledge of the workplace.

A recurrent question concerns the relationship between formal vocational mathematics education and actual workplace practice. On the one hand, in traditional trade-related vocational mathematics programs there are contradictions between school and workplace practice—as shown by Roth (2012); see also FitzSimons (2002). On the other hand, competency-based training programs, which require only the atomised mathematical skills immediately visible, can disempower workers if they have no real mathematical understanding to build upon. Yet, the very nature of contemporary work and society requires people being able to cope with, and even contribute towards, inevitable unforeseen changes and ever increasing degrees of complexity—and this needs real meaning making in mathematics.

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**References**


A Workplace Contextualisation of Mathematics: Visible, Distinguishable and Meaningful Mathematics in Complex Contexts

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A recent doctoral research survey conducted by the authors seems to confirm the paradox that people deny their use of mathematics, while simultaneously recognising the role of mathematics in their work and the frequency of its use. A deep analysis of case studies carried out in different workplaces offered a degree of triangulation of this finding, albeit in what may be described as low-skilled jobs. The methods used to disambiguate and calibrate the underpinning mathematics, in companion documents, highlight that the mathematics that support work activity may be distinguishable and visible when viewed through a particular lens. Although such mathematics concepts may occur at low levels of complicatedness in terms of curriculum, they seem to serve the worker in multiple, complex contexts that hinder recognition of the depth of prior learning, for want of a suitable framework. It may be that mathematics in the workplace is more readily recognised when accompanied by its attendant properties that comprise the work setting. This paper reports on:

• how the synthesis of the findings of a workplace mathematics survey and the analysis of case studies, exposed the complexity of the workplace-contexts in which, often uncomplicated mathematics tools and concepts are used, and
• the evidence-based Context-Complexity Protocol developed to reflect the sophistication of the circumstances in which mathematics knowledge, skills and competence are deployed.

This paper argues that a Workplace Contextualisation of Mathematics, capable of reporting the level of complicatedness of mathematics in the workplace, and taking account of the range of sophistication of the situations in which it is used, enables mathematics visibility in the workplace and all that that implies.

Key words: Invisibility, commonsense, complicatedness, complexity, workplace-contextualisation.

“Looking at the Workplace through Mathematical Eyes” is a research project that seeks to align the mathematics that people actually “do” in work, with the formal provisions of the National Framework of Qualifications in Ireland (NQAI, 2003). It is located in jobs that do not seem to feature mathematics activity, prima facie, and that may be described as comprising ‘commonsense’ and skills that are ‘just part of the job’; the underpinning mathematics being
hidden somehow from the awareness of the worker. Many explanations have been offered for this paradox across a spectrum of political, educational, industrial, technological, economic, psychological, personal and anthropological themes (Strasser, 2003; Hoyles, 2008; Wedege, 2010; Williams & Wake, 2007; Coben, 2000; Coben, 2003; Coben & Thumpston, 1995; Benn, 1997; Benn & Burton, 1993; Fennema, 1979; Klinger, 2009; Bourdieu, 1977; Wake, 2005; Wedege & Evans, 2006; Wedege & Evans, 2006; Kent, Noss, Guile, Hoyles, & Bakker, 2006). This paper, in conjunction with slides originally presented at the ALM 19 Conference, Auckland 2012, reports on the latter stages of a novel approach to workplace mathematics research, begun in Ireland in 2009.

The obstacles encountered in the course of this work and the innovative solutions developed are described in a series of companion papers (Keogh, Maguire, & O’Donoghue, 2010b; Keogh, Maguire, & O’Donoghue, 2010a; Keogh, Maguire, & O’Donoghue, 2010c; Keogh, Maguire, & O’Donoghue, 2012). Of principal interest here is the mathematics Use/Denial paradox, i.e., the encapsulation of mathematics in a workplace context, the power of the workplace to render such mathematics invisible until required by the workplace circumstances and how they were explored by a combination of in-depth case studies and a National Survey of People at Work in Ireland.

**National Survey of People at Work in Ireland**

The National Survey of People at work in Ireland was underpinned by a construct that juxtaposed Numerate Behaviour as defined by PIAAC (PIAAC Numeracy Expert Group, 2009) and four mathematics domains as defined by The Further Education and Training Awards Council in Ireland which is responsible for the accreditation of adult learning up to degree level (FETAC, 2009). The survey invited respondents to nominate their three key workplace skills, followed by enquiries regarding their awareness, and frequency of use, of mathematics concepts in the course of their work, without using formal mathematics terminology.

The Mathematics Use/Denial paradox was sustained by 1200 statements of key workplace skills, where mathematics was hardly mentioned. In contrast, role of mathematics, and the extent of its use was acknowledged by approximately 80% of respondents when asked about specific numerate behaviours in encounter with specific mathematics domains.

Furthermore, while recognising the dependence on workplace artefacts e.g., procedures, almost 40% of respondents reported that their job was not straightforward. This insight led to the authors’ awareness that the level of complicatedness of workplace mathematics was important in itself, but that the visibility of the use of mathematics was conditioned by the complexity of the workplace context. In other words, two vastly different workplaces could be underpinned by the same level of mathematics knowledge skills and competence. A plausible corollary is that while the mathematics in the classroom may be taught in homogenous contexts, fairly routine mathematics may find expression in widely different workplace contextualisations, differentiated by complexity, and not complicatedness.

**Workplace Context-Complexity**

In the course of this present work, more than 30 factors were identified as having an influence on the workplace context. These may be summarised as Accountability, Clarity, Familiarity, Stressors and Volatility, each further characterised by a number of sub-domains. A more comprehensive account of workplace complexity is included in a doctoral thesis due for
publication in the summer of 2013. It may be readily recognised that Accountability, for example, extends beyond a simple alignment with the authority of ‘being in charge’. It was quite clear in each case study that the chain of command acted as a conduit for blame and disciplinary action. The depth of accountability seemed to be a function of materiality and immediacy, limited by the scope for exercising initiative and permission to make decisions. It is especially noticeable in collaborative work where the impact of an error in a preceding activity may be borne, in a tangible way, by a peer engaged in a dependant activity.

As a consequence, the authors developed a Workplace Contextualisation of Mathematics model and with it, an evidence-based protocol with which to capture the variance in context-complexity between jobs underpinned by similar levels of mathematics knowledge, skills and competence, Figure 1.

![Workplace Contextualisation of Mathematics](image)

**Figure 1. Contrast between the NFQ workplace profile extended by the Workplace Context-Complexity protocol.**

This illustrates that the context in which work takes place can vary across a spectrum of factors, each contributing to mathematics invisibility to some extent. It can be seen that the National Framework of Qualifications (NFQ) is silent on these workplace dimensions, giving testimony only to the degree of complicatedness of the mathematics. It is the authors’ view that the capacity to reflect the individual’s competence in his/her experience of the workplace may be a first step in enabling the recognition of inherent mathematics ability, and a platform for continuing development. Nevertheless, the ‘commonsense – anything but mathematics’ self-perception persists, even though the survey provides substantial evidence regarding the awareness of the role of mathematics and the frequency of its use in work. In this light, it would seem that the invisibility of mathematics is neither absolute nor permanent, but may be temporary and all-pervasive across a range of workplace activities.
Mathematics Invisibility

The evidence provided by the case studies underpinning this present research, and corroborated by the National Survey of People at Work in Ireland, seems to sustain the Mathematics Use/Denial paradox. Mathematics is at once invisible and recognised for its role and frequency of use. That this can persist may be a function of the way work is done and talked about. The basic tenets of Cultural Historical Activity Theory (CHAT) may offer the beginnings of a coherent explanation (Roth & Lee, 2007; Engeström, Miettinen, & Punamäki, 1999; Vassilieva, 2010), building on (Leont’ev, 1978). In essence, CHAT accounts for human activity in terms of motivation toward an object, experienced by an individual(s), drawn from a community, among whom labour is divided, who form, and are formed by, rules and whose activities are moderated by a range of artefacts. Participants in an activity are thought to be most conscious of their motives, less so of the actions they take in pursuit of their object, and least aware of the ‘automatic operations’ that supports their actions. Networks of activity systems typically depict the encounter of separate activities and the coincidence of their desired outcomes.

The authors offer an extension to this perspective that places the subject at the hub of a number of activities between which a worker may glide seamlessly in the course of their work, in Figure 2.

![Figure 2. Subject-centric Activity System](image)

Here, the Subject is observed to be engaged in a Principal Task, typically denoted by the job title, comprising a sequence of actions underpinned by mathematics (and other) knowledge skills and competence. In the event of a breakdown in routine or expectation, the activity is fundamentally changed in almost every respect as the subject begins a familiar or ‘first response’ problem-solving routine. Should this achieve the desired outcome, the subject returns to the Principal Task, otherwise, the Subject may engage with Peers on a formal (or informal) basis for support. The Peer to Peer interaction may involve a different community, artefacts, rules etc., and may resort to Problem Solving procedures that may or may not pre-exist, depending on the problem, and may escalate to more senior personnel or specialists until the task returns to normal. All the
while, the subject(s) is in informal and pervasive activity with work colleagues to maintain good working relations.

While this account of a controlled, structured workplace is useful for illustration, the evidence of the case studies is of a more subtle dynamic in operation which is unpredictable to some extent, yet stable enough to permit the development of a range of coping strategies to deal with multiple combinations of situations conditioned by multiple factors. The underpinning mathematics knowledge skills and competence resides at the unconscious level, being applied as automatic operations until promoted to ‘actions’ depending on the lens of scrutiny. That this is in contrast to the mathematics classroom, where mathematics learning is the principal motive, surrounded by mathematics actions, may have the effect of containing mathematics in the context of the classroom exclusively, and may not readily come to mind in the context of work. In this way, mathematics and related terms and ideas, may evoke a school experience at the Motive and Actions levels of consciousness, while at the same time, be relegated to the amorphous, automatic operations role in the service of multiple activities that comprise the context of work.

Discussion

The findings of the survey recounted here, taken together with the evidence of the case studies alluded to, suggest that references to mathematics invoke recall of a context other than the workplace, perhaps the mathematics classroom. This may account for the Mathematics Use/Denial paradox and contribute an understanding regarding mathematics invisibility. The workplace contextualisation of mathematics suggested herein may offer a protocol with which to differentiate between workplaces in a way that complements the identification of the underpinning mathematics. That this contextualisation of mathematics could extend the meaning of the NFQ may enable the recognition of workplace learning acquired in tacit, non-formal and informal means. In this way, mathematics in the workplace could be made more visible as distinguishable, discrete components, in the service of complex contexts and in so doing, construct a platform for people, employers and providers of learning opportunities to engage in continuing, workplace-relevant development, employability, and inter-sector mobility for the individual in the long term.

References


Behind the Headlines: Authentic Teaching, Learning and Assessment of Competence in Medication Dosage Calculation Problem Solving in and for Nursing

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This article addresses a key issue for mathematics educators preparing students for work: how should teaching, learning and assessment be designed to meet the mathematical demands of the workplace, especially when those demands are safety-critical? We explore this question through a discussion of our interdisciplinary research on numeracy for nursing, focusing in particular on the characterization and authentic assessment of competence in medication dosage calculation problem solving, where errors can and do cause death.

Introduction

Medication errors by nurses make headlines. An error can have disastrous consequences for the patient, for example: ‘Mother-of-four dies after blundering nurse administers TEN times drug overdose’ (Daily Mail Reporter, 2011), and for the nurse; in a recent extreme case this led to suicide: ‘Nurse’s suicide highlights twin tragedies of medical errors’ (Aleccia, 2011).

Concern in the nursing profession and the wider public, fuelled by headlines such as these, has led to a growing number of studies, both of qualified nurses, for example, Grandell-Niemi et al. (2003) in Finland and Hoyles et al. (2001) in the UK, and of nursing students, for example, in the USA (Rainboth & DeMasi, 2006), the UK (Coben et al., 2010; Jukes & Gilchrist, 2006) and Australia (Eastwood, Boyle, Williams, & Fairhall, 2011).

Medication errors, characterised as adverse drug events (ADEs), have been categorized by Bates et al. (1995) in terms of whether actual or potential harm from medicines is caused to the patient. Where harm is manifested in the patient ADEs are classified as:

- Preventable: errors in prescribing, dispensing, calculating, preparing or administering the drug.
- Non-preventable: for example, where the patient is correctly administered the medication for the first time but subsequently has an adverse drug reaction that could not have been predicted by current technologies.

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• Harm that can be minimised: for example, when, following correct administration, a drug rises to toxic levels in the blood; harm can be minimised by careful monitoring of blood levels of the drug.

Types of ADEs are shown in Figure 1, below.

![Figure 1: A model to describe the types of medication incident (NPSA, 2009, p. 6)](image)

Our focus in this article is on the development and assessment of the vocational skills, knowledge and understanding required to minimize preventable errors by nurses in prescribing, dispensing, calculating, preparing or administering drugs; we characterize this area as medication dosage calculation problem-solving (MDC-PS). The jury is still out on the part played by nurses’ numeracy skills in such errors. Indeed a recent review found insufficient evidence to suggest that medication errors are caused by nurses’ poor calculation skills, although the author concludes that more research is required into calculation errors in practice (Wright, 2010). We believe that calculation skills are only part of the picture, as shown in the definition of numeracy and the model of competence in MDC-PS set out below (Figures 3 and 4). We developed the model as members of the NHS Education for Scotland (NES) interdisciplinary Expert Numeracy Reference Group (hereinafter: the Reference Group), building on earlier work outlined below. The Reference Group undertook research on the ‘Benchmark Assessment of Numeracy for Nursing: Medication Dosage Calculation at Point of Registration’ project commissioned by NES (Coben, et al., 2010); this work is at the heart of this article.

The outcomes of the NES research programme (Coben, et al., 2010; Sabin et al., 2013) were reported to the UK regulatory body, the Nursing and Midwifery Council (NMC) in 2010. These outcomes subsequently part-informed the construct and content of the MDC-PS competencies within the NMC’s Essential Skills Cluster (ESC) for Medicines Management (NMC, 2010, p. 32) (see Table I).
In addition, when commenting on the MDC-PS competency model emerging from the NES research (see below), the NMC referred to the work of the Reference Group, stating in their ‘Advice and Supporting Information for Implementing NMC Standards for Pre-Registration Nursing Education’ that:

We have considered some new work in relation to calculations that has informed our approach to dealing with the numerical assessment in relation to the ESC on medicines management. The issues around standards of numeracy competence in the health workforce have been addressed by a team commissioned by NHS Education for Scotland. Programme providers may wish to take the following information into account when determining assessment criteria:

- An ESC assessment strategy for medication-related calculation that demonstrates competency across the full range of complexity, the different delivery modes and technical measurement issues.

- Assessment that takes place in a combination of the practice setting, computer lab and simulated practice that authentically reflects the context and field of practice.

- Diagnostic assessment that focuses on the full range of complexity, identified at each stage, and recognizing the different types of error (conceptual, calculation, technical measurement) which can then be linked to support strategies. \(\text{(NMC, 2011, pp. 60-61)}\)

The NMC statement falls short of directly advising programme providers to take the information into account when determining assessment criteria, saying only that they “may wish to take the [..] information into account”. Nevertheless, this is an important step towards the adoption in the UK of a more comprehensive, evidence-based approach to the development of professional competence in numeracy for nursing.

**The development of professional competence in nursing**

The *raison d’être* of professional nursing education programmes is to facilitate the development of practitioners who demonstrate professional competence in the following domains: cognitive competence (‘knowing that’ and ‘knowing why’); functional competence (‘knowing how’ and
skills); ethical competence (the embodiment of a professional value system); and personal competence (the ability to apply these competences in different practice situations) (European Communities, 2005; Weeks, Hutton, Coben, Clochesy, & Pontin, 2013). However, the traditional organisation of nursing (and most other) traditional vocational education systems has typically created an artificial distinction between the teaching and assessment of knowledge and the teaching and assessment of professional practice and values. These systems have in turn created an artificial theory-practice gap and an artificial knowledge-performance gap that has largely separated the teaching and learning of the professional body of knowledge from the teaching and learning of professional know-how and skilled performance (Lum, 2009; Weeks, Sabin, Pontin, & Woolley, 2013). This problem is complicated by the common practice of articulating competence and assessment problems in a word-based form and then requiring the interpretation and measurement of such abstract and descriptive competence statements in authentic practice environments.

When the professional know-how and skilled performance involves mathematics, as in MDC-PS, the problem is further exacerbated by the tendency for mathematical processes to become invisible in the workplace: “crystallised in ‘black boxes’ shaped by workplace cultures” as Williams and Wake (2007) put it. Other exacerbating factors include: problems in the transfer of learning between the classroom and the workplace (FitzSimons & Coben, 2009); incoherent nursing numeracy assessment regimes and invalid test items applied in high stakes testing (Coben, Hodgen, Hutton, & Ogston-Tuck, 2009); and problems with word problems in mathematics (Verschaffel, Greer, & De Corte, 2000). When the professional practice is safety-critical, as in MDC-PS, any disjuncture between theory and practice and between knowledge and performance may have serious consequences.

To address this central problem within the domain of MDC-PS in and for nursing in this paper we:

• Present the definition of numeracy and related criteria operationalized in our research;

• Review the problem of articulating MDC-PS competence in a word-based form;

• Propose a model that reflects the integration of the three competence sub-domains of MDC-PS (conceptual, calculation and technical measurement competence) and that articulates both the word-based definitions of ‘knowing that and knowing why’ and the authentic virtual representations of the ‘knowing how’ of MDC-PS competence that need to be manifested by the registered nurse in clinical practice; and

• Illustrate the essential design features of an authentic virtual learning and diagnostic assessment environment, based on the principle of authentic assessment, that facilitates the measurement of MDC-PS cognitive competence constructs (professional knowledge) and synthesizes these with the enculturation and sensitization of the nursing student to the MDC-PS functional competence (professional know-how and skilled performance) requirements of professional clinical nursing practice. This is aimed at bridging both the theory-practice and knowledge-performance gaps.

• Discuss the implications of our analysis for vocational mathematics education beyond nursing.

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Conceptualization of numeracy for nursing

The Reference Group was mindful of the terminological confusion and contestation around adults’ use and learning of mathematics, often characterized as numeracy (Coben & O’Donoghue). Nonetheless, we needed to define the domain within which we laboured. We decided to adopt the following generic definition of what it means to be numerate:

To be numerate means to be competent, confident, and comfortable with one’s judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (Coben, 2000, p. 35, emphasis in the original)

This definition was chosen because numeracy is characterized as the exercise of judgment with respect to specific issues in relation to the demands and affordances of a given context, in this case: nursing. It allowed us to explore issues of competence in MDC-PS and to delineate what we did and not consider to be numeracy issues in that context. Our formulation of the relationship between the domains involved in our analysis (numercy; healthcare numeracy; medicines management; and medication dosage calculation) is given below (Figure 2).

![Diagram showing the relationship between numeracy, healthcare numeracy, medicines management, and medication dosage calculation]

**Figure 2:** Medication dosage calculation in the context of numeracy and medicines management (Coben, et al., 2010, p. 13)

Evidence-based criteria for the assessment of numeracy for nursing

On the basis of the above conceptualization of numeracy for nursing, the Reference Group developed the following research-based criteria for the features of an effective nursing numeracy benchmark assessment tool, consistent with Gulikers et al.’s five-dimensional framework for authentic assessment (Gulikers, Bastiaens, & Kirschner, 2004). Gulikers identified five dimensions that inform the design and articulation of assessment environments:

1. Task: Students should be exposed to authentic tasks that involve integration of knowledge, skills and attitudes. The tasks should be meaningful and relevant to the
student and should reflect the full range of complexity, domains of practice and structure of the tasks as encountered in the real practice setting.

2. Physical context: The tasks should be learned and assessed in physical contexts that are as congruous as possible with the real physical practice setting; and should be undertaken and assessed using the typical tools available in the setting and along similar time-frames available to undertake the real tasks.

3. Social context: The social context of the practice setting should be as closely aligned as possible in respect of the individual or groups of professionals typically engaged in problem-solving and undertaking the tasks.

4. Criteria: Assessments should be centred on criterion-referenced outcomes, should be based on the criteria used in professional practice and should be realistic and transparent in respect of the processes and outcomes expected in the practice setting.

5. Form/result: Competence should be demonstrated and measured in respect of professionally relevant results that are observable and subject to multiple indicators of learning.

Work towards a benchmark assessment of numeracy for nursing undertaken by the Reference Group focused on the ‘task’, ‘physical context’ and ‘social context’ design features of authentic assessment environments, with student performance data measured against the ‘criteria’ and ‘form/result’ dimensions of Gulikers’ framework. The framework was employed for evaluating the authenticity and construct validity of a proposed benchmark assessment tool and associated assessment environment for MDC-PS in higher education institution (HEI) and practice settings. The framework was also used to evaluate nursing students’ perceptions of congruence between the authentic assessment environment and medication dosage problem-solving and computation requirements in practice settings (Coben, et al., 2010).

The Reference Group proposed the following evidence-based criteria for the assessment of numeracy for nursing supported by Gulikers’ (2004) framework. Such assessment should be:

- **Realistic:** Evidence-based literature in the field of nursing numeracy (Hutton, 1997; Weeks, 2001) strongly supports a realistic approach to the teaching and learning of calculation skills, which in turn deserve to be tested in an authentic environment. Questions should be derived from authentic settings. A computer-based programme of simulated practice in drug calculations, formative testing, with feedback on the nature of errors made, has been shown to develop competency in medication dosage calculation, which can also be demonstrated in the clinical areas (Weeks, Lyne, & Torrance, 2000). Exposure of students to real-world situations is recommended (Weeks, 2001).

- **Appropriate:** The assessment tool should determine competence in the key elements of the required competence (OECD, 2005; Sabin, 2001).

- **Differentiated:** There should be an element of differentiation between the requirements for each of the branches of nursing (Hutton, 1997).

- **Consistent with adult numeracy principles:** The assessment should be consistent with the principles of adult numeracy learning teaching and assessment, having an enablement focus (Coben, 2000).
Diagnostic: The assessment tool should provide a diagnostic element, identifying which area of competence has been achieved, and which requires further intervention (Black & Wiliam, 1998). Thus it should “provide information to be used by students and teachers that is used to modify the teaching and learning activities in which they are engaged in order better to meet student needs. In other words, assessment is used formatively to ‘keep learning on track’” (Wiliam, 2006).

Transparent: The assessment should be able to demonstrate a clear relationship between ‘test’ achievement and performance in the practice context (Weeks, Lyne, Mosely, & Torrance, 2001).

Well-structured: The assessment tool should provide:

- a unique set of questions with a consistent level of difficulty;
- a structured range of complexity; and
- the assessment should take place within a defined framework, at points by which students can be effectively prepared, while allowing time for supportive remediation (Hodgen & Wiliam, 2006).

Easy to administer: the assessment should provide the opportunity for rapid collation of results, error determination, diagnosis and feedback (Black & Wiliam, 1998).

(The Coben et al., 2008, pp. 96-97)

The problem of articulating MDC-PS competence in a word-based form

Word-based competence rubrics (such as that in Table 1, above), statements and regulatory body advice are common features of traditional competence-based education and training programmes. However, in questioning whether competence can be described and communicated through language in accurate and unequivocal terms, Lum (2009, p. 76) states that

the entirety of present policy-making and practice in education, not only in the UK but increasingly elsewhere, is underpinned by the assumption that such descriptions are possible. It is extraordinary that these strategies have gained such widespread acceptance and been afforded such unqualified official approval while the very assumption upon which they are based seems hardly to have received any attention. What makes this all the more remarkable is that there would appear to be profound and irrevocable difficulties with the idea that competence can be specified in clear and precise terms.

Ultimately, this difficulty in accurately defining and describing particularly functional competence (know-how and skilled performance) in a word-based form, results in the potential for variable interpretation of the required competence in vocational and practice-based professions like nursing. For example, returning to the NMC descriptors of competence in MDC-PS (Table 1), without a shared and unified understanding of ‘conceptual’, ‘calculation’ or ‘technical measurement’ competence, or ‘unit dose’ and ‘sub and multiple unit dose’ calculations, etc., misinterpretation or variable interpretation by educators and students can (and does) occur. To counter this problem within the MDC-PS competence domain (and other vocational mathematics domains) we have proposed that both ‘knowing that’ and authentic ‘knowing how’ representations of MDC-PS competence can be modeled to illustrate the
fundamental principles of how competent MDC-PS should be manifested in authentic practice environments.

**Authentic assessment and modeling and measurement of MDC-PS competence in virtual and practice-based environments**

Mueller (2005, p. 2) defined authentic assessment as “A form of assessment in which students are asked to perform real-world tasks that demonstrate meaningful application of essential knowledge and skills”. In the nurse education context it is not necessarily practicable, appropriate or ethical to ask students to perform real-world tasks in the real world with real patients. Nevertheless, students must “demonstrate meaningful application” of the “essential knowledge and skills” of MDC-PS if they are to practise safely and effectively as professional registered nurses. The solution to this conundrum may be found through authentic assessment undertaken in a safe, controlled environment, in which assessments are designed to be truly representative of performance in the field and assessment criteria seek to evaluate essentials of performance against well-articulated performance standards. The hallmark of authentic assessment environments is their capacity to measure the meaningful application of cognitive competence (knowledge) and functional competence (know-how and skills) in realistic contexts, together with the provision of a rubric (diagnostic framework) against which to measure performance (European Communities Education and Culture, 2008; Sabin, et al., 2013). Above all, “within reasonable and reachable limits, a real test replicates the authentic intellectual challenges facing a person in the field” (Wiggins, 1989, p. 706).

Authentic learning and competency assessment environments that meet these criteria support the bridging of the theory-practice and knowledge-performance gaps (Boud, 1990; Coben, et al., 2010; Gulikers, et al., 2004; Lowes & Weeks, 2006; Weeks, 2001; Weeks, et al., 2001; Weeks, et al., 2000; Weeks & Woolley, 2007). This is critical in the domain of MDC-PS and other vocational mathematics domains where assessment schedules must reliably and validly measure the construction, synthesis and meaningful application of the mathematical and measurement knowledge, problem-solving and professional skills that underpin safe and effective professional practice.

For the purposes of extrapolating and operationalizing a definition of dosage calculation competence based on the above definition of numeracy and consistent with the criteria set out above, the Reference Group adapted a competency model derived from an initial premise described by Weeks, Lyne and Torrance (2000) and elaborated by Authentic World Ltd³. Figure 3 illustrates a competence model that provides generic definitions for the three sub-elements of MDC-PS competence (conceptual, calculation and technical measurement); and Figure 4 illustrates an analogous competence model that reflects exemplar computer-generated iconic representations of the requirements for solving an injection-based dosage calculation problem. Comparison of these two models highlights the difficulty noted by Lum (2009) in accurately describing competence in precise detail in a word-based form; and shows how analogous computerized iconic modeling adds essential pictorial detail for the student in both constructing schemata (internal cognitive representations of an individual’s world) and learning competent MDC-PS practice, and for the educator and clinician who need to assess competence requirements against a defined rubric (e.g., see Table I).

³ [http://www.authenticworld.co.uk/](http://www.authenticworld.co.uk/)
Figures 3 and 4 articulate the interrelationship between the three sub-elements that combine to form competence in medication dosage calculation problem-solving. The confluence (central white section) of the model highlights how all three sub-elements must be practised concomitantly in order to achieve a correct dosage or rate solution. Conversely, an uncorrected error in one or more of the three sub-elements of the competency model WILL result in a medication dosage calculation error in clinical nursing practice. Consequently, we argue that it is imperative that nursing students are supported in the development of schemata and competence in MDC-PS and undertake systematic diagnosis of cognitive and functional competence in authentic assessment and clinical practice environments (Sabin, et al., 2013; K. W. Weeks, et al., 2013).

Figure 3: MDC-PS Competency Model (generic word-based definitions)
Knowing that, knowing why and knowing how

Weeks, Hutton, Coben, Clochesy and Pontin (2013) describe and illustrate the design features of an authentic virtual MDC-PS learning and diagnostic assessment environment. The central theoretical perspectives that informed the design of the Authentic World safeMedicate and eDose authentic virtual learning and diagnostic assessment environments are illustrated in Figure 5. In addition to our focus on competency modeling we provide a short summary of these perspectives here.

The design of the virtual environments is underpinned by a central constructivist and situated cognition perspective that actively engages learners in knowledge construction, and their active
engagement with the context-bound artefacts (functional objects) and tools of the social and clinical environments within which the professional knowledge is to be applied. These perspectives, together with the articulating cognitive apprenticeship, cognitive style in mathematics and non-threatening features of the learning and assessment environments, are fully explored by Hutton, Coben, Clochesy, and Pontin (2013).

This theory of learning lies in stark contrast to traditional didactic transmission methods of nurse education that not only fail to engage the learner actively in knowledge construction, but also largely rely on the use of abstract word problems to describe the clinical and social features of medication dosage problems and to assess cognitive competence. Figure 6 illustrates a typical word-based ‘complex’ essential skills medication dosage calculation problem in this genre.

![Prescribed dose, Aminophylline 200 mg, Dispensed dose, Aminophylline 250 mg / 10 ml. What volume should be drawn up for injection?](image)

**Figure 6: A typical word problem used to assess medication dosage calculation problem-solving ability**

Word problems of this type are highly structured and formalized, unlike “inherently ambiguous and open-ended” authentic challenges (Wiggins, 1989, p. 706). They inform the learner of the ‘prescribed dose’ and the ‘dispensed dose’, etc., a luxury not afforded to the registered nurse in clinical practice. In reality, as illustrated in our competency model (Figures 3 and 4) the competent registered nurse is required to understand conceptually and interpret this numerical information from prescription charts and medication ampoule labels, etc., to calculate an accurate numerical value for the dose and to perform an accurate technical measurement of the dose in an appropriately selected measurement vehicle (in this case a syringe). Hence, in contrast to the word-based problem, Figure 7 illustrates an example from the authentic diagnostic assessment environment that represents the same medication dosage problem as that described in words in Figure 6.

The example illustrates the presentation of the problem in an authentic form, together with diagnostic feedback of conceptual competence, calculation competence and technical measurement competence. Macdonald et al (2013) further inform this premise and illustrate examples of competence assessment for ‘unit dose’, ‘multiple-unit dose’, ‘sub-unit dose’ and ‘conversion of SI unit’ MDC-PS calculations. Note that the images in Figure 7 are screenshots and do not show the dynamic nature of the assessment, whereby students select appropriate vessels, drag and drop pill icons into containers and ‘pull up’ liquid medicine into a syringe. These dynamic iconic computerized models provide exemplar benchmarks for competent performance meeting the requirements of the NMC ESC hierarchical rubric illustrated in Table I.
Figure 7: Computerized iconic model illustrating conceptual, calculation and technical measurement cognitive competence diagnostic feedback for a typical essential skills ‘complex dose’ calculation.
Weeks, Higginson, Clochesy & Coben (2013) have explored a grounded theory of schema construction for medication dosage calculation problem-solving (MDC-PS). Within this theory, and as illustrated in this paper, we have stressed the importance of the student actively engaging in and with the medication dosage artefacts (functional objects) within which essential numerical and measurement information is embedded (e.g., prescription charts, medication ampoule labeling, syringes, etc.). Students engage in what Treffers (1987) calls “horizontal mathematization”, i.e., the mathematization of contextual problems, in which they come up with mathematical tools which can help to organize and solve a problem located in a real-life situation.

Of critical note, our findings suggest that ‘seeing’ and interacting with these context-bound physical artefacts, and their iconic representations, rather than the mere description of the dosage problem in a word-based form, is an essential factor for many nursing students in the construction of accurate schemata for MDC-PS and for subsequent development of competence in MDC-PS.

Similarly, Lum (2009, p. 102) noted that in terms of developing schemata and vocational capability there is an imperative for moving beyond descriptions and engaging in ‘learning to see’ the vocationally relevant features of the world:

The important point here is not so much the difficulty we have putting these things into words (important though that is) so much as the idea that we are surrounded by features of the world, be they physiognomies, facial expressions, functional objects, meaningful behaviours, all of which we must learn to see. In coming to recognize features of the world around us, many of the capacities we develop we will have in common with others. Importantly, however, a considerable part of what it is to be vocationally capable consists in being able to apply schemata which enable us to ‘see’ those features of the world which are relevant and perhaps even unique to a particular vocational role.

We argue that promoting the simultaneous ‘seeing’ of the context-bound physical artefacts (functional objects) of medication dosage calculation problems and supporting the construction of semantic connections with abstractions in the form of dosage calculation equations and word-based competency statements, both supports accurate schemata construction and improves our shared and unified understanding of competency requirements in vocational mathematics. In the absence of advancing this process we are unlikely to close the theory-practice and knowledge-performance gaps created by traditional vocational education systems.

Implications for vocational mathematics education beyond nursing

In this final section we review the implications of our work on MDC-PS for vocational mathematics education beyond nursing. We begin by summarising the implications for education and training of recent research on adult numeracy for work and life set out by FitzSimons and Coben in their entry in the UNESCO-UNEVOC International Handbook of Technical and Vocational Education and Training (FitzSimons & Coben, 2009) and then consider what our work adds to this picture.

FitzSimons and Coben state that:

1. Numeracy skills cannot be isolated and taught out of context.
2. The expectation that numeracy skills will be directly transferred from classroom to workplace or applied unproblematically is unsustainable. Rather, mathematical knowledge must be transformed into partly or potentially new context-specific knowledge, integrating experience-based judgment with the social and cultural norms of the workplace.

3. The training environment should employ simulation techniques.

4. Students should be aware of the variability between training and workplace environments and the inherent variability of workplace tasks.

5. There is no optimal site of learning for work (viz. the academy or the workplace).

6. Integrated curricula, problem-based pedagogies, and the development of generic skills of communication and reflection, as well as problem solving, modelling situations, planning, and teamworking skills are needed, rather than linear, hierarchical curricular models, unrelated to students’ lives (FitzSimons, 2002).

7. Normatively, mathematics education for the workplace should be intended to enhance the knowing of workers both subjectively and objectively, so that as individuals they may be empowered as ‘knowledge producers’ as well as ‘knowledge consumers’, i.e., to be technologically, socially, personally and/or democratically numerate.

8. Mathematics should be presented in contexts that make sense to the learner.

9. Qualitative and quantitative information should be valued together with an ethical approach to people and the environment.

10. Learning how to question critically and to listen, as part of enhanced communication skills, should be valued, particularly with regard to data handling and interpretation.

11. Following Wake and Williams (2001), there should be an emphasis on:

   a) using relatively ‘low level’ mathematics in quite complex situations and contexts;

   b) encouraging experiences of a diversity of conventions and methods,

   c) having students experience activities where the mathematics is embodied in context and to use artefacts with which they have become familiar;

   d) preparing students to transform their existing mathematical knowledge to make sense of activities in unfamiliar workplace situations;

   e) having students design spreadsheet programmes for modelling and for the recording, processing and analysis of data; and

   f) making students aware that that there are many and varied ways to solve any problem.

12. In addition, Hoyles et al. (2002) recommend:
a) an ability to perform paper and pencil calculations and mental calculations as well as calculating correctly with a calculator;

b) calculating and estimating (quickly and mentally), including understanding percentages;

c) multi-step problem solving;

d) use of extrapolation;

e) recognising anomalous effects and erroneous answers when monitoring systems;

f) communicating mathematics to other users and interpreting the mathematics of other users;

g) developing an ability to cope with the unexpected; and

h) a sense of complex modelling, including understanding thresholds and constraints.

(summarised from FitzSimons & Coben, 2009)

We endorse these points. Our work adds comprehensive breadth and depth specifically on numeracy for nursing, while drawing attention to the vital importance of authentic assessment, supported by authentic pedagogies, in all vocational mathematics education. We believe our work offers a model of vocational mathematics education which others may wish to develop in other work contexts.

Finally, as has been noted above, the Reference Group is interdisciplinary; it comprises expert researchers who are qualified nurses and nurse educators as well as non-nurses: experts in adult numeracy education, quantitative research methodologies and psychometrics. Such interdisciplinary research collaborations are unusual in the field of vocational mathematics education but in our experience there is much to be gained from them.

References


Estimation and spatial sense are important elements of workplace numeracy. Though the former is often promoted in schools during number sense topics, the latter may be accounted for by school geometry and art. This paper explores both elements by drawing on the perspective of numeracy as social practice (Lerman, 2006; Street, Baker, & Tomlin, 2008) within the work of urban recycling and refuse collectors. By observing the numeracy related events of the operators’ daily work, and by interviewing them about their numeracy practices, it describes how making estimations and having some spatial sense are well embedded in the roles of the collection operators and probably in other occupations. Many critical decisions are made during the work of the operators, and these often depend on either or both of these two elements. Among implications which may be drawn from this, it suggests that educators need to ensure that spatial awareness and estimation be included in future learning programmes, in numeracy training for learners of any age.

Introduction

This article describes part of the research work being carried out for an MPhil thesis. This work originated with a short study on how numeracy might be situated in urban refuse or rubbish, and recycling removals (Chana & Kane, 2010). Increasing weight (social, financial and political) is being placed on how to use quantities of recycling and refuse materials, and in particular where to store the latter. While visiting a landfill in East Auckland, Chana and Kane observed the landfill being covered with clean fill, namely soil, clay, concrete, rock, and sand. This cover needs to be several metres deep, as the site would eventually be turned into a park over the next few years. Though the remedial work in facilities like these could lead to other numeracy research opportunities, it was decided that the focus of the initial study should be on the numeracy practices of the operators who collect recycling materials or refuse in their purpose-built trucks.

These vehicles have two steering wheels (dual control); the normal steering position is used for travel to and from the day’s run, while the second left hand drive wheel (kerb-side) is for bin collection. Each operator uses a control stick for two claws or arms which hydraulically grasp, lift and empty each bin, before returning it to the verge. The arms protrude and retract from the left side of the truck, and may reach out to 2 metres. The truck cab has mirrors for either side of the vehicle, a digital scale reader for the load, a camera above the truck hopper to check on contents as each bin is emptied, and both a bin counter and an operational hour counter which register on the monitor screen.
Two aspects of numeracy identified from the earlier research were the operators’ use of estimation, and their sense of space. These are now the current research focus, and those experiences with operators are presented here. Estimation was not solely about number sense or crunching numbers; there were situations where timing, loading, and the slope of a road also had to be taken into account in order to make estimations. With a sense of space, several features resulted from this, including the use of mirrors and angles, positioning the hydraulic arms on the kerb side of a truck to grip each bin securely, reacting to road shape and width, and working with location generally. These will be discussed as this paper proceeds, though there is the need to first describe what spatial sense might be, as well as some preliminary implications for numeracy educators who work with adult and other learners.

Describing a sense of space

When we think about the common activities in our everyday lives, the use of space or having some spatial sense is familiar. Consider the following: returning a pot or a pan to a shelf in a cupboard after drying the dishes; deciding where to hang washing knowing that a sheet will need a longer line than a shirt; instructing a 16-year old how to carry out a three-point turn in a car driving lesson, or listening as they try to let the clutch out far enough for the selected gear to engage, but not to stall; and manoeuvring a trolley through a supermarket aisle, searching for items while avoiding other trolleys and shoppers. These are part of a large list of common activities where space is integrated in each activity - possibly known, definitely used, though probably overlooked, during the active work in each task.

When we draw out the spatial words or phrases from the situations above, (where, longer line, areas, three-point turn, manoeuvring, out far enough, avoiding) the evidence of our reliance on space and position in these activities is there. The words are indicators of how ingrained this spatial sense is, as both literacy and numeracy events (Barton, 2006); each activity relies on a space or positioning decision, and description, albeit brief. Though these tasks seem straightforward, there are other more complex work-based situations where spatial and positional decisions require a deeper sense of spatial reasoning. For instance, some organisations require job applicants to undergo spatial assessments. One resource for these kinds of occupations, by Wiesen (2003), prepares candidates for the spatial components of psychometric testing. These provide trainees with practice challenges to “think about how flat and solid objects can be rotated, put together, turned over, and folded” (p.1), and go beyond those everyday situations listed earlier.

Sorby (2003) acknowledges there is more to this sense of space, and that spatial skills are not simply “a unitary construct . . . spatial ability consists of mental rotation, spatial perception, and spatial visualization”. Brennan, Jackson, and Reeves (1972, as cited in Del Grande, 1990) agree that spatial perception does not consist of a single skill or ability, and they include attributes such as “hand-eye coordination, visual discrimination, figure-ground relationships, and language and perception” (p.14). Sutton and Williams (2007) concur with Sorby and Wiesen in that spatial ability does involve “mental rotation of objects” (p.3); they add two other aspects - that an understanding is needed to see how “objects appear at different angles . . . (and) how objects relate to each other in space” (p.3). Donohue (2010) includes a fourth aspect of spatial sense or ability, an understanding of three dimensions (or 3D).

Kozlowski & Bryant (1977) believed that “laypeople seem to view (spatial abilities) in terms of their sense of direction” (p.590). To further examine this, the authors invited a group of first semester college students to take part in three experiments around their college campus; each
was asked first to rate their own sense of direction. They found that students who rated themselves with good senses of direction seemed intrinsically motivated to improve their abilities, whereas those who rated themselves as having poor senses of direction seemed ultimately self-fulfilling.

In summary, it appears that there are many common activities which rely on our having a sense of space, or having some degree of spatial ability. The key consideration here is the degree of spatial ability - manoeuvring a trolley through a supermarket aisle does not have the same spatial demands as manoeuvring a collection vehicle through a narrow city street, while attempting to align the arms with bins on the verges. Though we are all imbued with some sense of space, and many have a sense of positioning, as Sorby (2003) said this is “not a unitary construct”, and there are clearly other aspects such as the mental visualisation and manipulation of objects, needing inclusion.

Methodology

This research seeks to build on these various concepts of spatial sense by conducting a case study that draws on the perspective of numeracy as social practice, and to explore the situated work practices of urban recycling and refuse collectors, as well as exploring how estimation and spatial sense could be useful in those roles. The research principles which support this work are derived in part from Brown, Collins, and Duguid (1989), who explain that any situated learning model contributes to an individual’s richer understanding of the world (workplace) with their artifacts and how they then use these. This situatedness is continued by Barton (2006) who places what people do in their everyday lives, or their natural settings (Denzin & Lincoln, 2005), with numeracy as an integral set of social practices.

In addition, Stake (2005) suggests case studies such as this are instrumental since although the work of the refuse and recycling collection operators is in itself interesting, their work exposes the wider issues of spatial ability and estimating numeracy, which they use in their everyday practices. As Flyvbjerg (2006) asserts, though case studies may sometimes be challenged with their breadth, they more than make up for that with their depth. In this case study that depth is hopefully being achieved by work shadowing and interviewing several collection operators during their daily work routines.

Preparing for the case study

As with any other research, access to the sources of information is critical to the conduct of the study. Access to the operators was possible after meeting with the onsite managers of two companies collecting waste and recycling materials around Auckland. One company collects only recycled materials and has thirteen to fourteen owner-operators, each with his own vehicle. These drivers contract to the company; they are responsible for the usual responsibilities such as taxation, and the maintenance of their vehicle. Other companies, including the second one in this study, also collect waste in their recycling zones.

This second company employs its operators so the ownership of the vehicles lies with the company. As with the first company, the vehicles are stored on site, and though each driver is reasonably knowledgeable about trucking, lifting, and loading operations, the maintenance of the truck is the responsibility of the company. This company has a fleet of recycling and waste vehicles, so it employs two sets of drivers, with similar tasks, though different load limits and destinations.
The managers of both companies were met on site. Posters inviting their operators to participate in this project were placed on noticeboards. I was also permitted to briefly speak at a monthly “tailgate” meeting to one of the groups of drivers in the second company, providing individual information forms and consent forms. The operators were initially bemused that someone might be interested in their work. They were open and co-operative and shared their experiences. Semi-structured questions had been prepared to elicit the operators’ told experiences of work and their thoughts on how they developed the necessary skills for this.

Diagrams and field notes helped to record the activities and skills more fully. As a consequence, during the semi-structured interview in some instances, several questions could be answered at the same time, alleviating the need to work through every question. Before interviewing, respondents were shown the questions and I emphasised that these would guide our conversation while the work continued.

Unexpectedly, one company decided to withdraw from its New Zealand operation so the collection work was subsequently taken over by the other. This meant the operators in the company being taken over needed more time to come forward if they wished to take part; they had larger issues beyond the scope of this research.

It was also essential that interviewing be paused on some locations and roads with limited room to manoeuvre the large vehicle, or when queuing traffic was attempting to pass. Driving and collecting required greater concentration by the operator; continuing would have been a distraction. The reward was that in some instances the operator would break the silence asking whether I had noticed some part of the action or the bin placement as we were departing the site. With the respondents occasionally broaching subjects like this it provided both interviewer and interviewee with a more flexible instrument to range over other unforeseen issues.

**Preliminary analysis**

Each interview with a collector was taped with the collector’s permission. Interviews were then transcribed, though sometimes this was a challenge with the background noise of the truck. The field notes taken during the observation assisted with the transcriptions, particularly with the sketches of certain addresses and the roads we travelled on. These would remind me of events or locations en route, where the collector made a comment or needed to respond to the conditions in a certain way.

After reading and rereading the transcriptions, five quite broad categories or markers stood out; these coincided with similar markers in the earlier short study (Chana & Kane, 2010). So the preliminary analysis consisted mainly of allocating the data from the interviews, observations, and field notes into these broad categories. Examples of these are given in the table below:

<table>
<thead>
<tr>
<th>Category or Marker</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Counting &amp; Number Identifying &amp; accuracy</em></td>
<td>Probably 9 times out of 10 I’ll do it the same, but today because I’m doing it slightly differently, I’ve got 220 bins on, if I keep going around the outskirts here where I want to go might be another 50 bins, so that will bring me up to 270 to 280-ish (J, int).</td>
</tr>
<tr>
<td><em>Estimation including measurement</em></td>
<td>the weight shows on here as being heavier when you’re facing downhill, and lighter when you’re facing uphill....so</td>
</tr>
</tbody>
</table>
Table 1

As stated, this research is concentrating on the aspects of spatial sense and estimation. These aspects did coincide at times during the runs, though for the purposes of this paper, and the findings which follow, these two major elements have been discussed separately. Some of the operators’ actions resonated with the previous spatial abilities such as aligning objects at different angles, and visualising in three dimensions. These need to be explored in more depth as this study unfolds.

Initial findings and discussion

I – The Estimation Element

Estimation has several variables in the everyday work of the refuse and recycling operators. Their actions as they drove were drawn from a need to anticipate the confluence of expected bin numbers, of the increasing weights in their loads, of their locations, and the time of the day, alongside what they knew about those particular routes. Drawing on these experiences they had also learned of the variations in bin weights from season to season, and also the variations in bin weights when collecting across different socio-economic areas. Van Groenestijn (2004) describes these actions as numerate behaviours since these operators solve authentic problems, always on the spot, needing decisions and responses. Some of these operators’ responses to research questions are presented in this section alongside observations being made during the run. Counting, timing, spatial sensing, measuring, and location are discussed next.

Bin Counting

Operators gradually gain experience on their allocated routes, and after some weeks anticipate how many bins will be out on a day. They notice for instance how many bins are on the side of a street, whether there are flats or heavily tenanted properties, and if a property is permitted to have more than one bin. They also see new residences appearing, and therefore more bins. They even recognise when a resident’s bin is late.

One recycling operator, J, keeps quite accurate records, and could show how the overall bin count varied in the same area, but in two different months.
In May (late autumn in New Zealand) this run was 965 bins – so a fourth load was needed for that. In July the same run was 841 bins so the difference of 120-130 bins means we could do it in three lots of 270-280. (J, interview notes)

Monitoring is critical for recycling operators since they have weight limits for each load. Due to a compaction factor (cubic space in the truck hopper is divided by the weight) which the recycling plant stipulates, and for commercial sensitivities is not described here, their loads cannot exceed four tonnes. This is because the recycled materials are more difficult to separate if they are compacted beyond this. Taking more on board can be expensive, as J explains.

If we look at another case, there’s a load which is close to 900 bins and I’ve tried to do three runs of 300; I’ve done it, come in underweight... for the day but overweight for an individual load. And that’s one I’ll be penalised for.

Operators are often challenged on runs with less bins but heavier weights. J showed a record of one load of 310 bins weighing 3.78 tonnes; however a smaller number of bins collected during a second load (under 300 bins) weighed 4.12 tonnes. They must make decisions about exit routes with a load, bearing in mind the remaining bins, and attempting to get as close as possible to the (ideal) count of 300 bins.

Knowing their areas assists with planning exit routes. Collecting the final 30 to 40 bins is dependent on knowing those roads. One day an operator was expecting a total of around 840 bins. However due to it being winter and an expectation of less weight, he expected correctly there were another 3 to 4 bins left in a nearby cul-de-sac to complete his second load of 275 bins; once collected he was confident that one more load would finish his day.

**Timing**

In the summer months bin numbers are higher and individual bin weights increase. Recyclers typically collect four loads on each daily run; more outdoor functions such as barbecues are held, accounting for the increase. In winter months however, bins go out less, so three loads are usually collected per run. With experience S stated that he could go to streets and predict ten minutes collecting here, half an hour there, and so on.

The early starts they make are usually to empty bins along the main roads before traffic becomes too dense. Before 7 am the traffic flow is generally good, but between 7am to 8am operators expect to add 15 to 20 minutes to their runs. During school holidays there is less traffic on the road and these result in shorter peak traffic hours. During term operators stay clear of schools near starting and finishing times. And long weekends with Monday holidays have collections on the following Saturday. Then every bin is out. J suggested that they were always estimating how far they can go with their current load before returning to the recycling plant or transfer station to unload.

The constant use of these heavy vehicles also means there are stresses on the equipment. The vehicles have a stop-start motion which is tough on brakes; consequently, these are constantly checked, while tyres last about three months before needing replacement. And as the refuse operators collect smaller bins with no load limits they can have the arms on their trucks go up and down about 1400 times a day, so the belts, rams, and hydraulics have scheduled maintenance.
Spatial Courtesies and Sensing the Load

Residents do not always follow recommended bin loads or placements on grass verges or sidewalks. There are many occasions when operators approach overfull bins, and as S describes, we need to “square them up” on the lift (to avoid spillage). There is also the need to be careful when returning lighter bins onto sloping verges, as these have become less stable when empty. Care is taken to avoid spillage since operators must then descend from their cabs, pick up the loose debris, and “spend the next two or three minutes writing” (a sticker for the bin); and “if we do that for every twentieth bin it’s a bit unrealistic time wise” (S’s interview).

The health and safety situation when alighting from a truck cab is well known in transport and associated industries. Quite serious injuries result if slipping and falling from the cab (almost 50 inches or 125cm); the impact when falling to the ground from the height of the cab is rated at 12 times a person’s body weight. Some useful safety software with graphical animation, Determine your Impact Force, has been created (Trucking Safety Council of BC, 2011, with mandatory safety posters in depot staffrooms.

There are instances where collection vehicles (sometimes from other companies) may be in the same road. Since recycling bins are collected every second week, there are more refuse bins, so some accommodations need to be made, with a spatial courtesy in use. Operators look for passing areas along the road. In South Auckland there are some areas which still use refuse bags; their collection vehicles employ a driver and two to three runners. This operation is generally quicker since the truck does not have to stop as much with runners often able to collect bags from several properties at once.

The hopper has some scales for monitoring the weight of a load, with a digital reading available inside the truck cab. The scales are near the rear door, so the operators wait until they are on level ground to obtain accurate readings of their scales. More experienced operators can sense a heavier bin load when activating the control stick for the grabbing arms. It “feels heavier if the arms are slow or strain so probably concrete or dirt is in there” (J’s interview).

Measuring and Localities

Each driver becomes more aware of their truck’s capacity, and as the weight of their load increases, recycling operators in particular factor in other variables. J explains in his interview:

We have about 4 tonnes (3.96 actually) worth of recycling capacity. My truck’s about 10,070kg when empty. Add my weight, and a full tank of diesel, so the full weight will be 10,260kg; if diesel is a quarter full, it is a bit under 10,200kg.

When S judged that his truck was about two-thirds full he would stop regularly to check the weight of his load. On some light days he was confident enough not to have to read the scales since he estimated a total 9½ tonnes altogether, from only 750 bins.

Operators see growth of housing and usually an increase in the number of bins along some roads. One found that an area he had been collecting in over two years had increased from 160 bins on one street to now being over 240 bins. “The area’s been subdivided and gradually over time as the houses have sold people have moved in so that . . . 240 bins is pushing the limits of my weight” (J’s interview). He requested the depot manager renegotiate the boundaries of his collection area, and another operator in an adjoining area assisted by taking 20 or 30 bins.
II – The Spatial Awareness Element

The equipment onboard

Every operator collects from what would normally be the passenger or kerb-side of their vehicle. It is critical then to become familiar with their cab mirrors since naturally they would drive their vehicles from the roadside of the cab. Right hand side small mirrors are used for checking roadside hazards since the driver has shifted to the left of the cab for collection. Left hand side mirrors are for lining up the arms with each bin; they often need to check over their left shoulder for bins that are not straight. These left hand mirrors are positioned so the midpoint of the near arm is visible, for a quicker stable pick up. Also, with each bin being tapered outwards from the bottom to the top, the arms grab about halfway up for effective lifting, preventing bins from sliding into the hopper of the truck.

Inner city collections

There are older (usually inner-city) suburbs, with streets of villas, often without their own driveways. This means there is less off-street parking, since these residences were constructed before automobiles were invented. This impacts on what can reasonably be collected since operators on those runs need to get out of their trucks and move the bins between parked residents’ vehicles onto the roadside. Hence they might collect only 80 to 90 bins an hour. One of the recycling contractors has even employed a runner to wheel the bins out on the Monday he works in the inner city.

As these residents depart for work, workers arrive and usually park their vehicles in the newly vacant kerbsides. In suburbs which are further out, there is usually more off-street parking, and those operators collect between 200 to 300 bins an hour. The runs have been allocated accordingly, given the constraints imposed by location. One other feature of the inner-city suburbs is that some of the lanes are quite narrow. So when cars park on both sides of these narrow roads, the collection trucks pass through with margins sometimes of just 2 to 4 cm on each side.

Other Hazards and Spatial Courtesies

With the nature of stop – start collecting, along with the size of the trucks, traffic queues form when collectors have to stop at narrower parts of roads. This situation is stretched when operators approach road works in their collection zones. Fortunately, an arrangement exists where roadworkers usually move the bins away from their working area to make it easier for the collection.

Where there are buildings being constructed, trades people tend to park anywhere there is a space along the road since their driveways are inaccessible; that impacts on the kerbside access for bin collectors. “We get a lot of builders, plasterers, pavers . . . they park down one side of the street and you’re trying to go down there - it becomes quite a challenge” (D, interview). Again operators have to get out of their trucks to move bins.

Cul-de-Sac positioning

No-exit streets usually have semi-circular ends and this presents challenges to operators. Driveways and tree planting were usually designed before collection vehicles existed, with curved sidewalks and grass verges requiring some manoeuvring for trucks to align with. Trucks often need to be driven onto the next pavement to line up with a bin from the previous residence,
which is hazardous if low-hanging tree branches are in the lifting zone. Though the arms may reach 2 metres, the little arcs in the cul-de-sac often mean that some bins are just out of reach of a truck in a chord-like position to that semi-circle.

Several operators invent their own approaches when collecting in cul-de-sacs. For instance, G used an ‘Exit not Approach’ strategy, driving as if exiting the cul-de-sac, then backing up to line up more closely to the bins, and picking these up as he drives out. Some drivers had knocked bins and in the process damaged their indicator lights.

In one of the newer subdivisions there are lower kerb heights which are just above the driveway entrances they are adjacent to. This allows the operator to drive his truck much closer to the bins, sometimes running along the kerb to collect bins, but there is less tyre damage.

**Restarting once they return from unloading**

There are times where an operator needs to leave an unfinished street. If they are close to the weight limit, especially recyclers, they cannot fit more bins, so they need to mark where they finish to restart on their return (or advise another operator if there is a breakdown). Various landmarks are used, as in choosing a house next to a clearing or a sign; also the angles of the last empty bins compared with the remaining full ones might also indicate this.

**Planning the route . . . and a challenge**

Once they have collected along a road, many operators avoid travelling the same road again. This is difficult if the number of routes is few. Others take more of a critical path approach. During one load, J came to a set of crossroads. He had already collected the bins on four sides of one block (Figure 1) so there were three blocks remaining. However, he wished to complete the other three blocks without retracing any of his route. Less than a minute after studying his map, he resumed the run having visualised how he would complete the run.
Putting yourself in his position, which way should J drive to collect the bins on all the other sides, still keeping left . . . but without driving along any completed side? Is there more than one way of doing this? (i.e., without ‘retracing’ any tracks . . .)?

**Figure 1**

### Discussion and concluding thoughts

Considering spatial abilities and considerations for adult learning

A few of the operators had worked in other fields using large vehicles, and this experience (often non-schooled) assisted them with their current work. Barton (2008) suggests that many people had ‘brought highly-developed skills and competencies, some unrelated to the curriculum’ to their everyday demands. One of those operators (J, interview) who had that experience suggested that there were others in their group who had initially struggled without that experience, but with persistence managed to become skilled enough to operate the collection trucks. That begs questions about the spatial awareness that people might already be applying, and what is needed in the work places. How many of our adult learners will need to engage in similar forms of spatial awareness (and estimation) in future careers?

As this research has been unfolding, one becomes more aware of other events where spatial awareness is involved. Earlier this year (2012), there was a change in some key road rules in New Zealand. For over thirty years we had driven using a rule where drivers gave way to all traffic on their right. The country’s roads have now reverted to what used to happen, or the “if you’re turning right give way” rule (New Zealand Transport Authority, 2012). A second change is what now happens in an uncontrolled T-intersection. Figure 2a shows what happened pre-25 March, if three vehicles simultaneously approached an uncontrolled intersection. The change states that “traffic from a terminating road (bottom of the T) should now give way to all traffic on a continuing road (top of the T)” (New Zealand Transport Authority, 2012) – Figure 2b.
Earlier this year, the New Zealand Transport Authority began ‘re-educating’ drivers on New Zealand roads after a lengthy public advertising campaign; drivers had to reorient themselves at these intersections. Large advertisements were placed in all forms of the media, and a website (http://www.nzta.govt.nz/traffic/around-nz/road-user-rule.html) was established; people could assess their own driving skills on interactive diagrams.

This retraining of motorists through a mass campaign of key messages alongside spatial images and activities is not a new approach, though perhaps the scale is unusual. Sorby (2009) writes of engineering training and the need not to assume what spatial awareness people have, and to be more proactive with encouraging those skill sets:

Most engineering faculty have highly developed 3-D spatial skills and may not understand that others may be struggling with a topic they find easy . . . They may not believe that spatial skills can be improved through practice falsely believing that this particular skill is one that a person is either born with or not. (p.478)

Gee (2005) advocates the use of video games for learning, and suggests that one of their many useful features is the feeling of power as the controls are manipulated some distance from the virtual world of the game. He adds that “humans feel expanded and empowered when they can manipulate powerful tools in intricate ways that extend their areas of effectiveness” (p.8). Another training opportunity I observed was when a tutor explored some spatial skills in a class of English for Speakers of Other Languages (or ESOL). Their migrant students learn directional prepositions and phrases such as ‘under, below, over, next to, to the left, on the right, clockwise’, and so on. They meet these in situations as diverse as supermarket aisles and shelves, public transport routes, and map reading. So this intersection of numeracy (which is well embedded in many) with literacy helps them to become more familiar with both aspects.

Spatial skills for learning

This paper considered what I believe is an overlooked or an as yet incomplete aspect of Numeracy – functioning with spatial skills. Many mathematical learning tasks traditionally require learners to answer questions on number calculations alongside some Measurement themes such as metric conversions, and the reading of scales to measure lengths. As described above, there is ample evidence in one workplace (at least) where spatial awareness and
estimation are critical elements, and essential for on-the-spot decision making. Might these not also be worthy then for learners to explore?

Coben (2003, as cited by Kaye, 2007) suggests that ‘numeracy is a notoriously slippery concept’. After observing what these recycling and refuse collectors do, I believe their practices might not be defined as ‘slippery’ so much, rather these are less well known or appreciated elements of numeracy. Their work has civic, environmental and commercial aspects. Their need for some sense of space is predictable since one only need observe their work from afar. What is perhaps not as anticipated is the scale of this element in tandem with estimation which the operators practise on several fronts. How these are accounted for in formal prior learning will be examined in the next part of the research.

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Some Correspondences and Disjunctions between School Mathematics and the Mathematical Needs of Apprentice Toolmakers

A New Zealand Perspective

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This paper examines the school mathematics curriculum in New Zealand (with a focus on numeracy) and the mathematical requirements of toolmaking (a branch of mechanical engineering requiring sophisticated mathematical skills applied to fine measurements). Correspondences and disjunctions are found between toolmaking mathematical requirements and the school mathematics curriculum, as evidenced in the New Zealand Curriculum, the Numeracy Development Project and the National Certificate of Educational Achievement (NCEA). These are discussed in conjunction with interviews with toolmakers and toolmaking educators on what mathematics they use most in practice and how they use it. While the school and toolmaking mathematics curricula were found to correspond well, some important disjunctions were found between the approach to mathematics at school and in toolmaking, and between school assessment and the thorough knowledge required in the workplace. Since toolmaking and other branches of mechanical engineering have internationally common skills, and the New Zealand school mathematics curriculum is similar in many respects to other countries, the conclusions presented here should have applicability to other vocations and countries.

Introduction

This paper reviews some relationships between school mathematics and the mathematics of toolmaking, an important branch of mechanical engineering characterised by fine measurement. The paper is divided into six sections: an introduction; a literature review; methodology; key findings; discussion; and implications and conclusions for the wider vocational context.

Mathematics is recognised as an important part of our society. It impinges on most aspects of our personal lives and particularly on our ability to perform efficiently in the workplace. Most western governments regard numeracy as a crucial aspect of mathematics and in recent decades have made efforts to enhance both school and post-school numeracy education (Carraher, Carraher, & Schliemann, 1985; Coben, D., Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V., 2003; Evans, 2000; FitzSimons, 2002; FitzSimons, Mlcek, Hull, & Wright, 2005; Kane, Patel, & Rawiri, 2006; Satherley & Lawes, 2009; Wedge & Evans, 2006)
Definitions of numeracy

Numeracy, however, is a comparatively modern term that includes more than skill in performing calculations, or getting the ‘right’ answer. It is a contested political and social issue that, today, contains social and communicative components, as well as involves the ability to relate numbers to context. Official statements of numeracy reflect the broad consensus that numeracy is important. This is evident in school mathematics curricula statements (e.g., the Number and Algebra strands) in the New Zealand Curriculum (NZC), which call for contexts for numeracy where students are to:

Learn to estimate with reasonableness, calculate with precision, and understand when results are precise and when they must be interpreted with uncertainty. Mathematics and statistics have a broad range of practical applications in everyday life, in other learning areas, and in workplaces. (Government NZ, 2010, p. 26)

The NZC (for years 1 – 13) sets out the framework for school mathematics education in New Zealand; the Numeracy Development Project ([NDP) (numeracy objectives, curriculum and assessment for years 1 to 8) and the National Certificate of Education (NCEA) (curriculum and assessment regimes for years 11 to 13 - ages 16 to 18). These are intended to work in conjunction with each other to provide smooth, progressive links of both content in mathematics and numeracy between primary school, secondary school, and the workplace. The NCEA assessment is standards based with grades Non Achieved, Achieved, Achieved with Merit, and Achieved with Excellence.

Another definition of numeracy not only embeds numeracy firmly within mathematics but includes the important component of decision making (Coben et al., 2003; FitzSimons, G., Mlcek, S., Hull, O., & Wright, C., 2005; Marr & Hagston, 2007). To be numerate means to

Be competent, confident, and comfortable with one’s judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (Coben 2000, p. 35, quoted in Coben, D. et al, 2007, p. 95)

Coben’s definition is highly applicable to toolmaking, where appreciation of the appropriate degree of accuracy, or ‘tolerance’ (maximum allowable difference between the specifications and the finished product) is required for each task. This is important, because the processes of measuring and machining in toolmaking are never perfect, as is also the case in all other science and engineering disciplines. The tolerances define the precision required, and vary greatly from situation to situation. Unnecessary precision wastes time and money whereas insufficient precision leads to an ineffective or faulty product.

The result is that the toolmakers have to develop strategies for performing the machining, and are thus constantly confronted with decisions about choice of mathematical method and the appropriate degree of accuracy required. The strategies a toolmaker chooses will depend on the tools available, the acceptable tolerance, and the skill of the individual toolmaker (Marr & Hagston, 2007). Thus, the NZC’s and Coben’s definitions of numeracy both give useful insights into toolmaking numeracy: the first because it describes aspects of apprentices’ school learning experiences, and the second because it emphasizes the highly contextualised nature of numeracy in toolmaking and the need for mathematical strategies to be developed for each situation.
**Background to toolmaking**

Toolmaking is a technically demanding branch of mechanical engineering that requires sound calculation skills which are applied in mathematics, physics, and engineering contexts. While measurements must often be as precise as 20 microns (0.02 mm or roughly one fifth the thickness of a piece of paper), there are some toolmaking contexts (e.g., the aeronautics and space industries) that require precision of just 3 microns. In other situations where precision is not so important, toolmakers make quick ‘near enough’ estimates. Toolmakers make calculations frequently, and while computers are now regularly employed there is still a need to develop strong mental arithmetic skills.

These arithmetic skills are linked to mathematical contexts that are practical and concrete, rather than theoretical and abstract. Although the mathematics involved rarely extends beyond NCEA Level 1 (year 11), the lateral and multi-step thinking they employ in problem solving is sophisticated and frequently involves lengthy discussion with other toolmakers. This gives a strongly socio-constructivist element to their work.

Toolmakers are an essential group in the manufacturing sector of the economy where they make and service a wide range of machine components. Globalisation and economic restructuring have significantly reduced the number of toolmakers in New Zealand and other Western countries. The toolmakers interviewed for the dissertation all expressed their fears of toolmaking skills being lost to New Zealand as the older generation of toolmakers retires, and bemoaned the fact that there were just 1164 toolmakers recorded in the New Zealand 2006 Census (Government NZ 2006) and only 12 apprentice toolmakers in 2011. This, they agreed, was insufficient to meet future needs.

Toolmakers acknowledge the importance of the numeracy in their work, although they do not usually use the term ‘numeracy’. They prefer to use ‘maths’, by which they mean understanding number, and confidence and skill in performing calculations. Toolmakers see themselves as ‘practical’ people for whom finding the ‘right answer’ means reputation and money, and while they acknowledge that understanding is an important precursor to obtaining the right answer, mistakes and delays in calculation are not tolerated.

While toolmaking apprentices need to learn to “calculate with precision” (Government NZ 2010, p. 26), they also learn short cuts to find estimates that are ‘near enough’ (e.g., so-called ‘friendly fractions’ based on approximations ‘that work’ and are quickly and easily applied). Toolmakers pass these short cut techniques on to one another, thus providing a rich culture of shared experiences that combines technical skill with mental agility. The result is that while on one level toolmaking may appear to be a series of procedures applied in mechanical fashion, on the other it is full of creative ways of doing things that demand the individual think beyond the square and apply “elementary tools in sophisticated settings” (Steen, 2001, p. 108).

Computer Aided Design (CAD), Computer Aided Machinery (CAM) and Computer Numeric Control (CNC) have revolutionized the engineering industry in recent decades. It appears that their main effect on toolmaking numeracy has been to increase the number and sophistication of tasks toolmakers are able to do. Because tasks can be planned and performed more quickly and with greater precision than previously, it might seem that using computers would reduce the need for numeracy. However, this is not the case - modern toolmakers still do hundreds of estimates and calculations each day, often in their heads. Learning basic numeracy skills (and engineering techniques) remain as important as ever.
Literature review

This section reviews the literature pertaining to contrasts between school and workplace mathematics: abstract versus concrete thinking; current understandings of the relationship between numeracy and mathematics; the effectiveness of studying advanced mathematics to increase numeracy competence, and social constructivism in the workplace.

Firstly, the role of common sense is important in workplace numeracy where the concrete nature of the problem guarantees a ‘real’ solution. The same is not true of academic mathematics where emphasis is often placed on demonstrating that a solution is findable rather than actually finding it (FitzSimons & Mlcek, 2004; FitzSimons, G., Mlcek, S., Hull, O., & Wright, C., 2005; FitzSimons & Wedege, 2004).

This means that numeracy in the workplace is not so much about understanding abstract concepts but about “applying quantitative skills in subtle and sophisticated contexts” (Steen, 2001, p. 108). Thus, since the contexts of toolmaking mathematics are very practical, complex and sophisticated, they frequently require creativity to find solutions to particular problems. In contrast with academic mathematics (either at school, or in the university) no thought is given to proving formulae and theorems, generalizing results or extending them to more complex, hypothetical situations.

However, although workplace mathematics relies on concrete rather than abstract understanding, this belies the complexity of the way in which it is used. Thus:

The contrast between mathematics in school and mathematics at work is striking. Mathematics in the workplace makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics. Work-related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. (van der Kooij & Strässer, 2004, p. 319)

Secondly, new views of what constitutes numeracy have altered the traditional view of numeracy’s subordinate role in bridging school mathematics and the workplace. Numeracy is no longer just a small, albeit special, subset of school mathematics but can be seen as a separate subject in its own right. That is, numeracy and mathematics are “complementary aspects of the school curriculum. Both are necessary for life and work, and each strengthens the other. But they are not the same subject” (Steen, 2001, p. 108).

This has important ramifications for the classroom, since none of the features of how numeracy is applied in the workplace outlined above “are found in typical classroom exercises” (van der Kooij & Strässer, 2004, p. 319). While the classroom may still be regarded as necessary, it is nevertheless an artificial training ground for preparing students to apply mathematics in a vocational context.

Thirdly, one response of employers to the perceived deficiency in numeracy skills has been to require students to complete more advanced mathematics courses. Commenting on this, Steen writes that “more mathematics does not necessarily lead to increased numeracy” and, “seldom do students gain parallel experience in applying quantitative skills in subtle and sophisticated ways” (p. 108). Since formal study and assessment of numeracy in New Zealand schools currently stops
at Level 1 NCEA (typically 16 years of age), then requiring students to study more advanced mathematics may not necessarily serve the development of workplace numeracy skills well.

Finally, traditional school teaching approaches in mathematics do not emphasize the social components of mathematics. Working effectively together to create new approaches and ways of thinking is now part of the NZC (Government NZ, 2010). This is in line with the actualities of the toolmaking workplace environment that requires authentic problem solving and cooperating in small groups with “shared responsibilities … [and] the development of metacognitive skills, such as critical thinking, learning to learn, planning and problem-solving” (FitzSimons, G., Mlcek, S., Hull, O., & Wright, C., 2005, p.4). Toolmakers have long known this.

Methodology

While there is a substantial literature on mathematics in the workplace, very little is known in the wider education community about the specific mathematical skills required for mechanical engineering and toolmaking or how they are used there. The dissertation on which this paper is based is thus primarily a qualitative investigation of what mathematics toolmakers need to know, how they use it, and how toolmaking mathematics differs from the school mathematics. The methodology for the dissertation comprises two parts: collection and analysis of document data on toolmaking including, NDP and NCEA curricula and assessment, and collection and analysis of interview data from toolmakers on their perceptions of what numeracy they use in the workplace and how they use it.

Comparisons were made of both document and interview data to identify common themes, including issues raised by the toolmakers themselves. Specific topics relating to those themes were placed in hierarchical categories which were then analysed to reveal correspondences and disjunctions in curricula and practice between toolmaking mathematics and school mathematics, with a focus on numeracy. A small, “purposive sample” (Punch, 2009, p. 162) was selected from so-called “key informants” in the industry assumed to represent various points of view and perspectives (Sarantakos, 1993, p. 183). Interviews of experienced current practitioners, toolmaking educators and secondary school technology teachers with a toolmaking background provided a comparison of perceived mathematics and numeracy needs and skills within the industry, and perspectives of students’ capabilities and needs. Table 1 illustrates some of these skills, capabilities, and needs.
Table 1: Toolmakers and their backgrounds

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Secondary school perspective</th>
<th>Toolmaking training perspective</th>
<th>Toolmaking curricula perspective</th>
<th>Current toolmaking involvement</th>
<th>Business perspective</th>
<th>New Zealand trained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laurence</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Pablo</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>David</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Fraser</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Michael</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Lyle</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Significance of the research

An exploration of toolmaking curricula and assessment resources, as well as discussions with toolmakers may increase our knowledge of what mathematics is used in toolmaking and how it is used. Since toolmaking mathematics curricula are the same as for other branches of mechanical engineering, a focus on toolmaking mathematics may also give us important insights into how numeracy is used in other branches of mechanical engineering. Knowledge of the correspondences and disjunctions between school mathematics and workplace mathematics curricula, assessment and practice could point to possible future development of mathematics curricula and numeracy programs in both workplace and formal education sites.

Much of school mathematics and numeracy curricula in New Zealand possess close similarities with those in other countries. The same applies to mathematics curricula for toolmaking and other mechanical engineering trades. Since toolmakers readily transfer their skills between countries, then many of the conclusions presented here may well have applicability outside of the New Zealand context (e.g., Holder, 2009).

Key Findings

This section provides results from a comparison of workplace and school mathematics through: interviews with toolmakers; the curricula for toolmaking mathematics; the Numeracy Development Project, and the curricula and assessment regime of NCEA Level 1 Mathematics and Number (year 11, age 16 years).

Workplace and school mathematics compared – interviews with toolmakers

A key finding from an analysis of the data suggests that the toolmakers all tended to regard workplace mathematics as real and school mathematics as unreal. One commented that Pythagoras “became a reality” only after he had entered the workplace. A second said that “toolmaking maths has a purpose” and led to greater motivation to learn because you can actually “see a purpose for everything [you do]”. A third felt that in toolmaking you can take “the initiative to use applied calculations … to step out of the square and come up with something that you set yourself”.

Another key finding is that the toolmakers regarded much of school mathematics as irrelevant. Only a ‘little bit’ of algebra was used, and one toolmaker claimed to have learned transposing
formulae “in chemistry and physics classes”. Two toolmakers felt that studying more advanced mathematics was at the expense of certain other skills, like calculating volumes of compound objects from measured dimensions.

In addition, the toolmakers all emphasised the importance of comprehensive number skills to perform basic calculations and to be “confident in using” them, e.g., converting metric quantities like metres to millimetres, and extending this to conversions for areas and volumes. Finding sensible estimates of calculations to immediately identify nonsensical answers was also mentioned (c.f., Government NZ, 2010). Simple calculations were used ‘all the time’ e.g., in setting up a machine. Confidence using and converting between fractions, decimals and percentages were regarded as crucial, and were “lacking in [students coming] out of high school … a real problem”.

The toolmakers also emphasized the need to develop lateral thinking skills and multi-step problem solving, because you do not know ‘what’s coming in the door next’ and there is ‘no formula that pops into your brain straightaway’ so you have to ‘sit down and think about a way of doing it’.

The curricula for toolmaking mathematics

The official New Zealand Government qualification for mathematics and mechanical engineering apprentices is contained in Unit Standard (US) 21905 (NZQA, 2010e). Numeracy skills are usually employed in contextual situations such as: area and volume calculations; lengths and angles calculated in right-angled triangles; tables and graphs; physics calculations, and conversions of unit names and symbols from mechanics, electricity and heat.

Some of these ideas and calculations form part of the Levels 2 and 3 NCEA Physics curricula. However, changes in physics curricula in recent years have reduced the amount of mechanics related to toolmaking, as well as heat, pressure, sliding and static friction (NZQA, 2010e, 2011c). Similarly, problems involving vector addition of velocities with their important contextual applications of trigonometry and Pythagoras have been removed from Level 1 and 2 mathematics curricula. Volumes of compound solids are also not part of school mathematics curricula. The result is that mechanical engineering apprentices begin their training with little or no contextual understanding of many physics and mechanics concepts and their associated mathematical applications that were once taught in secondary education.

The Competenz Student Workbook contains the assessment task for US 21905 (Competenz, 2006). The tasks are all oriented to mechanical engineering applications, including the definitions of units and their conversions. The important basic concepts surrounding decimal place value, ratios, the equivalence of fractions, decimals and percentages, are not assessed but receive extensive coverage in both the Study Notes and the Open Polytechnic’s student resources (Glaeser, Curry, & Mortlock, 2010). This suggests that toolmaking educators recognise that contextual application of number must be preceded by conceptual understanding of number.

While substitution in formulae is not specifically mentioned as a necessary skill in either US 21905 or the Competenz Student Workbook, it is frequently assumed and required in the assessment tasks. Solving equations has no section of its own but is included and demonstrated in the section on transposing formulae (Competenz, 2006). It does not, however, form part of the assessment. When questioned on the relevance of the toolmaking curricula to practice, the
toolmakers conceded that those sections of US 21905 they did not use were relevant to other branches of mechanical engineering.

In summary, toolmaking curricula, teaching resources and assessments all lay emphasis on being able to accurately and confidently perform computation in engineering contexts. The teaching resources begin by reviewing basic concepts on numeracy before applying them contextually. Thus, both conceptual and contextual understanding is seen as important to toolmaking numeracy.

The Numeracy Development Project

The 1990s saw governments in the United Kingdom, Australia, and New Zealand set up programmes in schools to increase numeracy levels. The latest stage of this process in New Zealand is the NDP. Originally based on the Australian model Count Me In Too⁴, the NDP focuses on communication with reasoning rather than passive reception of rules (Nicholas, 2006).

The NDP aims to develop students’ understanding of and competence with number, to demonstrate numeracy and thereby increase overall mathematics achievement. It also emphasises and encourages flexible thinking to enhance students’ problem solving ability and thus improve their achievement “in Number and Algebra and in the other strands of the mathematics and statistics learning area” (Government NZ, 2008b, p. 2). Students’ conceptual thinking is developed by teaching a range of numeracy strategies to solve problems drawn from ‘real’ contexts rather than concentrating on the rote-learned algorithms of the past. Students are encouraged to develop strategies (rather than learn procedures) to solve problems. Exercises in manipulating materials are used as bridges to understanding until students are able to develop their own abstract understanding. Solving problems mentally and developing abstract thinking are especially important to the NDP.

The NDP identifies a hierarchy of eight stages of understanding. Toolmaking and nursing require the highest levels: Advanced Multiplicative–Early Proportional Part-Whole (AM) and Advanced Proportional Part-Whole (AP) (Government NZ, 2008a). AP thinking includes being able to solve ratio problems e.g., dividing 135 cans of baked beans in the ratio 2:3, a problem which appeared in a year 11 Level 1 NCEA Number examination paper (NZQA, 2010c). While there appear to be no published statistics for the percentage of students who were able to do this problem, a review of a year 12 Level 2 Algebra examination paper stated that “many of the candidates who did not achieve showed poor basic numeracy skills and algebra skills” (NZQA, 2010d, p. 2). The proportion of students who eventually reach Level 8 AP, a satisfactory level needed for vocations like nursing where safety is crucial (Gillham & Chu, 1995; Gillies, 2004), is unknown.

⁴ Count Me In Too is a numeracy project operating across New South Wales Department of Education and Training primary schools: http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/
Development of Part-Whole thinking is important to toolmakers (e.g., finding fractions, ratios, decimals and percentages from their equivalents. Table 2 highlights some of these.

**Table 2: Some toolmaking numeracy skills and their corresponding NDP stages**

<table>
<thead>
<tr>
<th>Toolmaking numeracy skill</th>
<th>NDP level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read decimals to three places</td>
<td>Advanced Additive, AA</td>
</tr>
<tr>
<td>Equivalent fractions; rounding decimals; estimates of percentages and decimals; squares and square roots</td>
<td>Advanced Multiplicative, AM</td>
</tr>
<tr>
<td>Multiplication of decimals; recurring decimals; standard form; ratios; rates; inverse ratios</td>
<td>Advanced Proportional, AP</td>
</tr>
</tbody>
</table>

(Government NZ, 2008c, 2008d)

The curricula and assessment regime of NCEA Level 1 Mathematics and Number

Until 2011 the NCEA assessment on Level 1 Number was contained in Achievement Standard Solve straightforward number problems in context AS 90151. In 2011 it was replaced by Achievement Standard AS 91026 Apply numeric reasoning in solving problems with little change in the curricula, but significant changes in the assessment.

An examination of the 2011 AS’s pertaining to Pythagoras and Trigonometry showed that they continue to be closely aligned in curricula and assessment with the requirements of toolmaking, as they were before 2011 (NZQA, 2010a, 2010b). An analysis was made of the skills examined in the 2006 examination paper AS90151 (NZQA, 2007) and those in an internal resource exemplar “Carbon Credits” for AS 91026 in 2011 (NZQA, 2011a) with those in the curricula (see Table 3).
Table 3: NCEA, toolmaking and NDP curriculum skills and inclusion in NCEA assessment tasks

<table>
<thead>
<tr>
<th>Skill or Aspect in NCEA</th>
<th>Toolmaking Curriculum</th>
<th>NDP</th>
<th>Assessed in AS 90151 in 2006</th>
<th>Assessed in AS 91026 “Carbon Credits” Exemplar 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reason with linear proportions</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Use powers (including square roots)</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Understand operations on fractions, decimals, percentages, and integers</td>
<td>YES</td>
<td>YES</td>
<td>Not integers</td>
<td>Not integers</td>
</tr>
<tr>
<td>Use rates and ratios</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Exchange rates only</td>
</tr>
<tr>
<td>Know commonly used fraction, decimal, and percentage conversions</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Know and apply standard form, significant figures, rounding, and decimal place value</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Rounding only</td>
</tr>
<tr>
<td>Apply direct and inverse relationships with linear proportion</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Extend powers to include integers and fractions</td>
<td>Not fractional powers</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Apply everyday compounding rates</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
</tbody>
</table>

The analysis of the questions in the 2006 examination paper, AS90151, showed the following assessed aspects: find a straightforward percentage; find a straightforward percentage decrease; find a straightforward fraction remainder; read a recipe for information and calculate a weight; calculate the original amount before GST (General Sales Tax) was added; calculate the original amount before depreciation; find a percentage increase using scientific notation, and explain using appropriate calculations which of two investment schemes is better, one of which involved compound interest. A student needed to get two of ten possible opportunities correct to gain an Achieved level (NZQA, 2007). Several of the questions were of AP level in the NDP, and the last question for Excellence was challenging.

A similar analysis of the internal resource exemplar ‘Carbon Credits’ for AS 91026 in 2011 provided no evidence that Advanced Proportional thinking was required at all. A student could gain Achieved by: calculating a simple percentage; calculating a fraction of a whole number, and performing an exchange rate problem involving money. Moreover, the student merely needed to provide evidence of “using three different appropriate methods”, but “does not have to fully solve the problem …” (NZQA, 2011b, p. 6). The pre-2011 requirement for obtaining a correct
answer appears to have been relaxed. An Excellence student was required to devise and explain a complex strategy involving several steps to solve the problem, and to follow through a chain of calculations to obtain the correct answer. This was less challenging than the 2006 assessment and none of the numeric skills involved AP thinking.

**Discussion**

This section focuses on correspondences and disjunctions within and between the literature and findings in the following order: workplace and school mathematics compared; curricula; assessment, and implications.

**Workplace and school mathematics compared**

The interviews with the toolmakers were closely aligned with the conclusions of the literature. For example, the toolmakers acknowledged the usefulness of geometry, trigonometry, physics and certain aspects of algebra. However, some of them would have agreed with Marr and Hagston’s (2007) comment that classroom mathematics is regarded as “useless, abstract, and taught without relevance” because they did not see the relevance of school mathematics to the workplace, and felt that the school mathematics lacked application to context.

In addition, the toolmakers frequently commented on the open-ended, technology-dependent and multi-step nature of their mathematics which reflected the comments of Steen (2001) and van der Kooij and Strässer (2004). Thirdly, a strongly constructivist approach came through the interviews, which differs from traditional classroom teaching.

**Curricula**

The NZC is intended to set up smooth, progressive links of content and numeracy between the NDP, the NCEA and workplace mathematics. While the toolmaking contexts were more complex than those required at school, the mathematics skills toolmakers used in practice were closely aligned with those of NCEA Level 1 mathematics in Number as well as Geometry, Pythagoras and Trigonometry.

Investigation of the content of toolmaking curricula contained in US 21905 and the Competenz Student Workbook (Competenz, 2006) demonstrated that basic numeracy skills are important to toolmaking, especially knowledge and understanding of decimals, fractions, percentages (and their equivalents) and conversions of units. While the toolmakers used calculators often this did not replace the need for sound mental arithmetic skills.

There is a clear connection between the skills in the NDP and toolmaking requirements. For example, ratios and percentages in NDP Book 8 (Government NZ, 2008d) also appear in the Open Polytechnic Learning Guide for toolmaking (Glaeser et al., 2010). Toolmaking requires the ability to operate in all of the three highest NDP levels viz., AA, AM and AP. The NDP and toolmaking numeracy also correlated with the stated numeracy aspects to be assessed in AS 91026 (see Table 3).

**Assessment**

The NDP follows a socio-constructivist approach that corresponds well with the toolmaking workplace practice which requires comprehensive knowledge and mastery applied in multi-step
contexts with lateral thinking. Qualified toolmakers were also expected to have the higher thinking skills of Merit and Excellence, especially problem solving strategy skills.

However, there were several major disjunctions with regard to the NCEA.

One disjunction is that since only one third of numeracy skills are actually assessed there is no emphasis on comprehensive knowledge. Another disjunction has arisen following the changes to the Number Achievement Standards in 2011 (replacement of AS 90151 by AS 91026) and their accompanying “reduction in rigour”, the correspondence of objectives between the NDP and NCEA may no longer apply. This applies also to another original NZC objective to “calculate with precision” (Government NZ, 2010, p. 26). A further cause for concern is that since there are many opportunities to gain Achieved, we cannot ascertain the specific skills a person can do from the NCEA grade alone.

The 2011 changes in Level 1 NCEA mathematics also appear to have created a disjunction with the NDP over attaining the objectives of AP (Advanced Proportional) thinking. Even an Excellence grade in Level 1 NCEA does not require demonstration of NDP Advanced Proportional [AP] thinking. In one of several regular and extensive reviews of the NDP, Young-Loveridge (2010) states that there is a clear indication from the New Zealand documents (e.g., The New Zealand Curriculum (Government NZ, 2007) and the more recently published (and controversial) Mathematics Standards for Years 1 to 8 (Ministry of Education 2009)), that students should be AP thinkers by the end of year 8. However, only “12% of year 8 students” reach this level (Young-Loveridge, 2010, p. 29). It is not known what proportion of students eventually reach AP level by the end of year 11. Finally, comprehensive mastery of numeracy skills is crucial for developing the sophisticated, lateral and multi-step thinking necessary for proficiency in the toolmaking workplace. This is incompatible with the current NCEA policy of “heading towards” an answer. Hence the emphasis placed on basic number skills in toolmaking teaching resources (Competenz, 2006). By the time apprentices qualify as toolmakers, they need mastery of all the higher order thinking skills of Level 1 Excellence in mathematics plus detailed application to the toolmaking context.

Implications and Conclusions

From time to time the media focusses on insufficient numeracy skills, but seldom with an appreciation of the importance indirect lateral thinking skills have in the workplace. Quick-fix strategies which provide the learner with no more knowledge, skill or understanding beyond what is required to perform the immediate needs of the job, can no longer be regarded as an effective strategy in remedying numeracy issues (Marr & Hagston, 2007). Thus, while employer frustration over lack of numeracy skills among beginning apprentices may be understandable, strict adherence to narrow definitions of numeracy (Marr & Hagston, 2007) becomes self-defeating in the very area employers consider to be the most important - the real world.

Two aspects for curricula development emerge from this paper. Firstly, toolmaking mathematics is used in an integrated approach with physics and technology in practical situations which provide contexts for numeracy. There is currently a lack of engineering and other vocation-specific contexts in mathematics (and also physics) curricula in the secondary school that leaves students with little or no quantitative exposure to important toolmaking situations requiring such background. There is thus a case for extending the Number and Measurement strands into Level 2 NCEA and incorporating contextual applications, especially of fine and small measurements. Secondly, it would appear that all essential toolmaking numeracy skills are contained in Level 1
mathematics achievement standards, but complete mastery is required to meet the needs of the workplace. A numeracy programme based on NDP principles and Advanced Proportional thinking, focusing on mastery and context, with assessment standards defined by the actual needs of the workplace would be much more suitable for students considering toolmaking as a career. This cannot be delivered by the more advanced mathematics Achievement Standards currently available in NCEA Levels 2 and 3.

Finally, the reduction in rigour demonstrated in AS 91026 *Apply numeric reasoning in solving problems* will have significant ramifications for numeracy levels among senior secondary school students and professional vocations. Since Levels 2 and 3 mathematics courses have no specific numeracy assessment requirements, then less emphasis is being given in the secondary school curriculum (driven by assessment regimes) to numeracy at a time when it is being emphasised by the NZC.

**References**


Connecting the dots for a more numerate New Zealand

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The Literacy and Numeracy for Adults Learning Progressions, the Literacy and Numeracy for Adults Assessment Tool, and Pathways Awarua form part of the educational infrastructure developed by the Tertiary Education Commission to support New Zealand adult learners strengthen their literacy and numeracy skills. This paper outlines the development, implementation and impact of these national resources on the literacy and numeracy teaching and learning of New Zealand adults.

The numeracy challenge in New Zealand

The challenge for a more numerate New Zealand is the close to one million adults who are currently below the level considered necessary to meet the demands of life and work in the 21st century. The labour-force demands of a modern economy are complex but it is clear that if New Zealand is to improve or maintain its position in the world economy it must develop a workforce with high levels of generic and technical skills (Satherley, Lawes & Sok, 2008). The impact of having poor literacy and numeracy skills has become greater as the number of unskilled manual and manufacturing jobs has declined and more jobs, such as those in the service sector, require higher levels of numeracy and computer skills. Low literacy and numeracy levels have been identified as contributors to New Zealand’s relatively low productivity (Tertiary Education Commission, 2008).

Numeracy poses particular problems for employability, because if numeracy skills are not used in employment they are likely to decline (Parsons & Bynner, 2005). This suggests a vicious cycle in which a numeracy skills deficit that impedes access to employment gets worse with lack of use, restricting access even further. This is less the case with literacy, where most adults have frequent exposure to written information and communications. Prior to 1996 there was very little systematic information on the literacy and numeracy skills of New Zealand adults. The research that had been conducted on these skills was small in scale and limited in quality and coverage (Benseman, 2008). Most studies had relied on self-report and used small samples of specific populations such as the unemployed, prisoner, and industry groups.

New Zealand’s participation in the second round of the International Adult Literacy Survey (IALS) in 1996 was the first comprehensive study of adult literacy ever undertaken in New Zealand. It was followed by participation in 2006 in the Adult Literacy and Life Skills (ALL) survey. In both surveys proficiency was graded into five skills levels with level one representing the lowest ability range. Level three was regarded by experts as being the minimum required for adults to meet the “complex demands of everyday life and work in the emerging knowledge society” (Walker, Udy & Pole, 1997, p.1). The results of the 2006 survey were virtually unchanged from 1996, and indicated that approximately 51 percent of the workforce were at
levels one and two of the ALL survey and did not have the numeracy skills to participate effectively in a modern society (Satherley, Lawes & Sok, 2008).

While many people with low literacy and numeracy skills are already participating and making valuable contributions across New Zealand society, they would be better positioned to take advantage of the opportunities created by economic development if they were able to strengthen their literacy and numeracy skills.

**Responding to the challenge**

Work on improving the literacy and numeracy skills of adult New Zealanders began in 2001 with the publication of the New Zealand Adult Literacy Strategy (Ministry of Education, 2001). The strategy described three key ways to raise the levels of literacy and numeracy: increasing opportunities for literacy and numeracy learning; developing capability of providers; improving the quality of the provision.

In 2008 the Tertiary Education Commission (TEC) published an action plan for literacy, language and numeracy outlining a tertiary educator sector programme of work (TEC, 2008). The action plan aimed to improve the workforce literacy and numeracy skills of adults by increasing learning opportunities, improving sector capability and building a national infrastructure. The action plan took a systemic approach to building tertiary education sector capability that would ensure a progressive increase in the number of learners receiving support to improve their literacy and numeracy. A total of over 165 million of additional funding was allocated in Budget 2008, for the period 2008 to 2011, to support building the demand, supply and capability of literacy and numeracy learning opportunities (TEC, 2008).

A key component of the action plan was building a national educational infrastructure to ensure that literacy and numeracy provision became part of core tertiary education practice. Developing nationally available tools has been a specific priority of the infrastructure work programme and includes the theoretical framework for embedding, the Learning Progressions for Adult Literacy and Numeracy (TEC, 2008a and b), the Literacy and Numeracy for Adults Assessment Tool, Pathways Awarua, and resources on the national website (literacyandnumeracyforadults.com).

A key mechanism for strengthening the literacy and numeracy of adults has been to embed the provision into all level 1 to 3 courses funded by the TEC. The primary advantages of embedding are that it allows literacy and numeracy support to be contextualised and relevant to learners, and that it reaches the broadest range of learners. The number of learners receiving literacy and numeracy through embedded support has increased from 12000 in 2010 to 65000 in 2012.

The National Centre of Literacy and Numeracy for Adults was established in 2009 to be the focal point of professional development in the sector. The work of the Centre is profiled on the national website (www.literacyandnumeracyforadults.com). Alongside this work is the delivery of qualifications in adult literacy and numeracy and at the start of 2012 a total of 2200 educators had enrolled across the range of qualifications.

**The national resources that form the educational infrastructure for literacy and numeracy**

The TEC has developed a suite of national resources to support educators and organisations working with adult learners to strengthen their numeracy and reading skills. These are the
Learning Progressions for Adult Literacy and Numeracy (Learning Progressions), The Literacy and Numeracy for Adults Assessment Tool (assessment tool) and Pathways Awarua. Together the resources help answer the three key questions for effective learning illustrated in Figure 1.

![Figure 1 Using national resources to address key questions for effective learning](image)

The Learning Progressions, developed in 2006, provide a framework and language for describing what adult learners know and can do at successive points as they develop their skills in literacy and numeracy. The Learning Progressions were developed as a set of continuums. Each continuum describes how adult learners build their expertise, with each step along the continuum representing a significant learning development. The framework is a useful guide to identifying the next learning steps for adult learners.

While the Learning Progressions and the ALL survey both address literacy and numeracy they do so in different ways and for different purposes. Although the two measures are not directly comparable it is possible to find some general points of alignment between the two measures. The descriptors in the Progressions can be compared with the descriptors of the levels in ALL. Also when the assessment tool was developed, there was an opportunity to collect some empirical data using a limited number of items from the ALL survey to test where they sat with respect to the Learning Progressions. From these two approaches, it can be concluded that a learner who has successfully achieved at step 3 or higher on the reading progressions and at step 5 or higher on the numeracy progressions would be very likely to be assessed at level 3 or higher on the ALL survey.

The assessment tool is an online adaptive tool developed to provide robust and reliable information on the reading, writing, vocabulary and numeracy skills of adults. The assessment tool aligns a learner’s literacy or numeracy competencies with steps on the Learning Progressions. The primary purpose for developing the assessment tool was to provide diagnostic information for educators to inform the development of learning interventions that match learners’ needs and strengthen their literacy and numeracy skills. The assessment tool allows learners to track their progress over time and enables educators and organisations to evaluate the impact of their teaching interventions. Data from the assessment tool also provides a framework for informing government, at a high level, on the literacy and numeracy skills of learners. This data forms part of the information used to inform future policy and investment decisions in the tertiary sector.

Pathways Awarua forms a third component of the educational infrastructure. It is an online learning system designed to be interactive and engaging to adult learners. Pathways Awarua focuses on developing the learner’s competence in reading, writing and numeracy. The learner study path is separated into individual learning components called modules, and the modules are ordered into a sequence called a learning pathway. In each module, the learner is given...
background information and examples, and is encouraged to participate and practise their new knowledge by completing self-assessing questions and activities. Correct answers to these questions allow learners to accumulate points, leading to achievement certificates as recognition of their progress. The learner experience has a sophisticated set of user interactions and content features including text to speech, images, videos, animations, and both dragable and touchable elements.

Pathways Awarua has proven to be a successful system, with approximately 800 educators using it with 5000 learners in its first year of operation. An evaluation of learners and educators who used the system from July to December 2011 showed that Pathways Awarua had a significant and medium-sized impact on the numeracy achievement of the learners who had completed at least 300 points on the system. Almost all (95%) of the learners were enthusiastic about their experiences of Pathways Awarua with 87 percent believing it had strengthened their numeracy skills. Ninety-four percent of the educators identified benefits in using Pathways Awarua, both for themselves and for their learners.

**Future direction**

The TEC signalled its ongoing commitment to developing its educational infrastructure in the Adult Literacy and Numeracy Implementation Strategy stating that “the literacy and numeracy infrastructure is a valuable and internationally renowned asset which supports the government’s vision for accessible opportunities to raise the literacy and numeracy skills of adult New Zealanders” (TEC, 2012, p 11). The strategy confirms the ongoing and key roles of the Learning Progressions, the assessment tool and Pathways Awarua in the national educational infrastructure.

**References**


During the last decade, in New Zealand, resources and systems have been implemented to enhance tertiary students’ literacy and numeracy development. In New Zealand, the Literacy and Numeracy for Adults Assessment Tool (AT) was developed to provide teachers with diagnostic information about tertiary students’ current knowledge and skills in reading, writing and numeracy: with the Tertiary Education Commission requiring use of the AT in some qualifications. However, little research is available on the synergy of adult numeracy and literacy or the pedagogical consequences of this interaction. Does neglecting numeracy also put students’ literacy at risk? This research study examines connections between students’ numeracy and literacy (skills and knowledge), using the AT data collected for Foundation Studies: Whitinga (FS) students for 2012. This paper reports the preliminary findings, and discuss consequences for the teaching and learning of adult numeracy.

Introduction

Over the last decade embedding literacy within the mathematics classroom has become a priority. The assumption for many researchers and policymakers is that improving students’ literacy will improve students’ mathematics and in particular raise numeracy levels. But is this true? Is it necessary to establish a high level of proficiency in literacy to succeed in mathematics? Perhaps the opposite is true and improving a student’s numeracy improves their literacy. By neglecting students’ numeracy are we putting students’ literacy at risk? Are numeracy and literacy competencies developed in isolation or do they develop together?

Both authors of this article, working at Unitec, a New Zealand (NZ) tertiary institute, have been grappling with many of these questions. Since the year 2010, pre-degree and diploma programmes in the NZ tertiary education sector have been required to administer a reading and numeracy assessment to all students: the Literacy and Numeracy Assessment Tool for adults (AT). The authors, a lecturer in the Foundation Studies: Whitinga (FS) pre-degree and diploma programme and a teaching and learning advisor, wondered if data generated by the AT might yield information on the connections between numeracy and literacy. We report here on our analysis of the reading and numeracy AT data collected for Unitec FS students. Our findings show that literacy and numeracy are, indeed, quite strongly linked and we discuss the consequences of this strong connection for teaching and learning, for tertiary institutions, and for national policies.
Background

Definitions of Numeracy and Literacy

Although the concepts of literacy and numeracy now appear extensively in educational discourses these two words have a fairly recent origin. The 1959 Crowther Report, in the United Kingdom (UK), used numeracy and literacy to indicate skills required in managerial positions (Walls, 2004). By the 1982 UK Cockcroft Report the concept of numeracy had changed to become the understanding of mathematics needed for everyday life (Walls, 2004). Since then the use of the concepts of numeracy and literacy has grown to the point where they are now considered core competencies in the educational curricula of the UK, Australia and NZ (Walls, 2004).

In the development of a numeracy concept a range of meanings and definitions have appeared. Definitions range from knowledge of basic number operations and strategies to ability to confidently use mathematics in real life contexts (Diez-Palomar, 2011). Surprisingly this range of meanings occurs within the NZ education sector with the primary and secondary sector understanding numeracy as number operations and strategies and the tertiary sector focusing on use of mathematics in real life contexts.

Part of the confusion lies with the implementation, in 2001, of the NZ Numeracy Project into primary schools then intermediate schools, and later into some secondary schools. The Numeracy Project primarily aimed to improve students’ number skills and knowledge labelling this as numeracy. Consequently numeracy, in pre-tertiary level education in NZ, is now thought of as the knowledge, skills and strategies for number operations (Walls, 2004).

The NZ Ministry of Education, however, defines numeracy as “the ability to use mathematics effectively in our lives, at home and in the community” (Ministry of Education, 2001b). The Tertiary Education Commission (TEC) has based its work around enhancement of numeracy and literacy skills using the Ministry of Education’s broader definitions:

"Literacy is the written and oral language people use in their everyday life and work; it includes reading, writing, speaking and listening. [...] Numeracy is the bridge between mathematics and real life. It includes the knowledge and skills needed to apply mathematics to everyday family and financial matters, work and community tasks (Tertiary Education Commission, 2008, p.6)."

In this paper the authors will use these Ministry of Education definitions of numeracy and literacy.

Embedding Literacy and Numeracy

Within the tertiary education sector in NZ there has been growing concern over students’ numeracy and literacy competencies. The 1996 International Adult Literacy Survey (IALS) and the 2006 Adult Literacy and Life Skills (ALL) survey results show a significant part of the NZ population unable to perform literacy and numeracy tasks at a level required by everyday life demands. Across NZ, 51% of adults’ numeracy skills fall below this level, whilst approximately 1.1 million adults or 43% of adults aged 16 to 65 have insufficient literacy skills (Ministry of Education, 2007).
The Adult Literacy Strategy, “More Than Words” (Ministry of Education, 2001a), has been an important step in recognising the gap between tertiary learners’ numeracy and literacy competency levels and the demands they face, as well as in outlining responses beyond just the increase of provision. Government funded resources and systems, related to enhancing numeracy and literacy skills, over the last ten years include: the Learning Progressions; a range of professional development options for tertiary teaching; capability building within tertiary institutions; and the AT. Ensuring numeracy and literacy are embedded in tertiary education includes assessing the numeracy and literacy competencies of learners, understanding the programme’s literacy and numeracy requirements, and instituting teaching based on the gap between these. Different models of the embedding process exist, however central to this embedding is the understanding that numeracy and literacy development are combined with vocational teaching and learning (Casey, H., Cara, O., Eldred, J., Grief, S., Hodge, R. & Ivanič, R., 2006).

The Learning Progressions and the Assessment Tool

The Learning Progressions were developed to provide a framework for adult learning in literacy and numeracy. The framework splits literacy and numeracy skills and understanding into a variety of learning areas, for example Proportional Reasoning, Measurement, Comprehension and Vocabulary. Within each of these learning areas the framework describes a series of six general levels, Steps, through which a learner progresses as they increase their knowledge. In this way the Learning Progressions provide a tool for gauging a student’s current knowledge in a learning area (defined as their Step Level) and provide information on how to advance the student’s understanding (Literacy and Numeracy for Adults Te Arapiki Ako, 2011).

The AT provides diagnostic information about learners’ strengths and needs based on the Learning Progressions. Assessments are available for Reading, Writing, and General Numeracy (which includes measurement), using a database of more than 2000 questions written in a New Zealand adult context. As an online task, a student’s assessment can be set as adaptive (the question difficulty is changed in response to the learner’s previous answers) or non-adaptive. The AT aggregates individual student’s correct and incorrect answers into a scale score with a standard deviation (SD). From this score the AT determines the student’s Step Level within a Learning Progression (Literacy and Numeracy for Adults Te Arapiki Ako, 2011).

The Data

Unitec Foundation Studies

In the FS programme Unitec supports students who need further qualification for entry into diploma and degree programmes: Levels 1-3 of the New Zealand Qualifications Framework (NZQF). Since 2011 the TEC has required students studying at these levels to complete the AT. With approximately 500 students enrolled each semester in the FS programme, Unitec has gathered considerable data on FS students’ numeracy and literacy.

Both online reading and writing AT assessments are available, however FS lecturers most commonly use the reading assessment as an indicator of literacy level: marking of the reading assessment occurs immediately online and is less time consuming than the writing assessment, which lecturers need to hand mark and moderate. Similarly two different assessments are offered for numeracy. FS lecturers prefer using the General Numeracy assessment at the beginning of the
semester: the standard deviation for this 30 question assessment is less than that of the 15 question Numeracy Snapshot assessment (more often used at the end of a semester).

Analysis

By compiling the AT Reading and General Numeracy score data the authors compared FS students’ literacy and numeracy levels. With 18 months of FS student data we hoped to have a large database for the comparison. Unfortunately accessing the 2011 data has proved more difficult than anticipated, however, the authors’ retrieved information for 359 students with both Reading and General Numeracy assessment data records from the beginning of Semester 1 2012: we hope to increase the data set with Semester 2 2012 records later this year. The data provided the following information: student name; gender; ethnic group; assessment name (chosen by lecturer); assessment strand (reading or general numeracy); scale score with SD; progression step; and date sat. Of the 359 students, 114 were male and 245 female. The ethnic groupings were self-selected by students and yielded a range of responses. The authors categorised these into 5 groups: Māori (44); NZ European/Pākehā (152); Pacific Island (87); Asian (7); and other (69).

The data included two measurements for each of reading and numeracy: a scale score (the assessment mean and SD); and a Step level (a more general band of means). Figure 1 below shows an Excel graph of the linked scale scores for each student.

![Figure 1. Excel graph of reading and numeracy scores](image)

Figure 1 shows a possible relationship exists between students’ Reading and General Numeracy assessment scores. Entering the scale score data into the SPPS programme, produced a Pearson correlation coefficient of 0.531, showing a strong relationship between students’ reading and general numeracy scores. Using Step Levels data for an SPSS Chi-squared test, between students’ Step Levels in reading and numeracy (Appendix 1), produced a Chi-squared value of 95.8 with 25 degrees of freedom. The test gives a probability of 0.000 for the null hypothesis, that reading and numeracy scores are not related, and thus confirms the strong relationship between students reading and numeracy.

The following table shows the SPPS Chi-squared test results for relationships between reading and numeracy Step Levels and gender or ethnic group.
Table 1. SPSS Chi-squared test results.

<table>
<thead>
<tr>
<th>Test</th>
<th>Pearson Chi-squared value</th>
<th>Degrees of freedom</th>
<th>Probability</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender and Reading Step</td>
<td>1.30</td>
<td>5</td>
<td>0.935</td>
<td>Accept null hypothesis of no relationship.</td>
</tr>
<tr>
<td>Gender and Numeracy Step</td>
<td>7.74</td>
<td>5</td>
<td>0.171</td>
<td>Not significant. Accept null hypothesis of no relationship.</td>
</tr>
<tr>
<td>Ethnic group and Reading Step</td>
<td>85.90</td>
<td>20</td>
<td>.000</td>
<td>Reject null hypothesis. Relationship found.</td>
</tr>
<tr>
<td>Ethnic group and Numeracy Step</td>
<td>42.88</td>
<td>20</td>
<td>.002</td>
<td>Reject null hypothesis. Relationship found.</td>
</tr>
</tbody>
</table>

Within the cross tabulation for ethnic group and both reading and numeracy (Appendix 2) the Māori, Asian, and other ethnic groups students numbers were as expected. Two groups, however, showed student numbers different from those expected. In numeracy, at Steps 3 and 4, there were fewer NZ European/Pākehā students and more Pacific Island students than expected. This reversed at Step 5 where there were more NZ European/Pākehā students and fewer Pacific Island students. Step 6 yielded a similar result to Step 5, however; only 44 of the total students were represented at this Step level.

The reading/ethnic group cross tabulation was similar to the numeracy/ethnic group cross tabulation with fewer NZ European/Pākehā students and more Pacific Island students than expected at Step 3 and more NZ European/Pākehā students and fewer Pacific Island students at Step 5. Reading Step 2 was similar to Step 3; however Step 2 contained only 29 of the total students. Step 6 Reading, with a total of only 11 students, showed similar results to Step 5.

What does this mean?

The analysis shows a strong link between students’ numeracy and reading. However correlation does not infer causation: finding a correlation is not saying that improving students’ reading improves their numeracy, nor is it implying that improving students’ numeracy improves their reading. However, such a strong link shows that a student’s reading won’t improve without their numeracy improving and that a student’s numeracy won’t improve without their reading improving.

This strong link could also occur if the General Numeracy assessments generated by the AT relied on a high literacy content or, alternately, if the Reading assessments generated by the AT contained a high numeracy content. Investigation of the AT reading questions reveals no numeracy content. The numeracy AT, however, does contain a literacy component. On investigation the authors noted that the question design uses predominantly visual image and short simple sentences with the reading level for most questions at the lower reading Step Levels. If facilities are available the assessment questions can be read aloud, however no lecturers used this feature in semester 1 2012. Unfortunately an online numeracy assessment cannot completely remove a literacy component and this might affect the strength of the observed relationship. The authors, however, consider the low level of reading demand in the numeracy questions would have a very small impact on the strength of the relationship.
Having established a relationship between reading and numeracy, the authors decided that valuable insight might be gained by investigating students that did not fit the relationship (the extreme values). Students’ scale scores included the value for 1 SD (Lane, 2012): the probability that a student’s score falls within 1 SD of the mean is 68%. All SDs lay between 26 and 87, however only 27 of the total 718 scores had SD’s above 40 and more than half the scores had SD’s between 27 and 30 with a mean SD of 29.

To find students with significant differences in reading and numeracy scale scores, the authors of this paper decided to look for students whose difference in scale scores fell well beyond the mean 2 SDs (48): the probability that a students’ score falls within 2 SDs of the mean is 95%. Of the 359 students 16 showed more than 150 points difference between their numeracy and reading scores, with 7 having a higher numeracy score than their reading score and 9 a higher reading score than their numeracy score.

These 2 groups of students exhibited distinct differences in gender, age and possibly first language status. The 7 high numeracy/low literacy students were all male, younger students, 17 - 28 years of age, and one noted as having a learning disability. Data on first language status was not given, although by examining permanent residency status (versus NZ citizenship) and stated ethnicity the authors deduced that 5 may not have English as a first language. Unitec offers language course to non-first language English speakers and this may explain the low numbers of high numeracy/low reading students in the FS programme.

The high reading/low numeracy students comprised 6 males and 4 females, with 3 of these students over 30 years of age and 2 permanent residents, possibly non-English first language speakers. None had identified learning disabilities. One possible explanation for these older students is that life and work experiences and practices have maintained or enhanced reading without similarly improving numeracy.

Māori students often feature in low success rates for schools and tertiary institutions (Statistics New Zealand: Tatauranga Aotearoa). Our analysis shows FS Māori students were not overrepresented in lower Step Levels of numeracy or literacy and the data appeared to follow the general FS distribution of Step Levels. With only 44 students identified as Māori, a larger data base might yield different results; however FS offers Māori class cluster groups which include a strong Māori kaupapa (principles of Māori education). So perhaps the Māori AT data reflects the Māori teaching and learning practices of the FS programme.

In contrast the AT data reveals that Pacific Island students are overrepresented in the lower Steps Levels and underrepresented in the higher Steps Levels in both literacy and numeracy, especially in comparison to NZ European/Pākehā data. The 2006 ALL Survey showed low levels of literacy and numeracy of Pacific peoples (Ministry of Education, 2007). In light of this, further research on Pacific Island students’ literacy and numeracy is suggested, particularly in the NZ context.

**Implications for Teaching and Learning**

Few studies have looked at the links between students’ numeracy and literacy. A personal conversation with Chris Lane (2012), who has been investigating the total TEC AT data base for the Ministry of Education, illuminated an even stronger relationship between reading and numeracy. Roe and Taube (2004) found a similar correlation of 0.57 between the overall reading
and mathematics scores of Norwegian and Swedish students in the PISA 2003 data (4595 students).

The PISA data offers a breakdown into mathematics topics and question types and this study found stronger correlations with some mathematics topics than others. In particular, change and relationship, and open questions highly correlated with reading, whereas space and shape and multi-choice questions showed a weaker correlation (Roe & Taube, 2004). Roe and Taube interpreted this correlation as causation and concluded that mathematics teachers should teach more reading comprehension. That mathematics itself is a language is, at times, forgotten: perhaps greater proficiency in the language of mathematics in fact improves students’ literacy and reading. Should we actually be exhorting literacy teachers to teach more numeracy?

In the drive to improve students’ numeracy and literacy it is assumed that: improving literacy is more important than improving numeracy; and improving literacy will lead to an improvement of numeracy. Often the numeracy component of a students’ education is ignored. In the authors’ experience professional development for teaching numeracy and literacy often ignores the numeracy component and focuses only on literacy activities and discussion. Alternatively, numeracy professional development may be poorly attended by many teachers who feel that they teach literacy only.

From personal experience, some literacy and numeracy advisors admit to having little understanding of numeracy or to being mathematics phobic: institutions are disadvantaging their students by employing advisors in literacy and numeracy that have this poor numeracy focus. An examination of the links between numeracy and literacy show that students’ cannot improve one without the other and teachers; teaching organisations and policy makers need to ensure that opportunities are provided to improve both. We jeopardise students’ literacy if we don’t also improve numeracy and vice versa.

The authors would like to suggest the following ideas to ensure literacy and numeracy are much more strongly interwoven into tertiary teaching and learning:

- Teaching and learning strategies that include building awareness in students that literacy and numeracy skills and knowledge are connected: improvement in one requires improvement in both.
- Professional development in embedding literacy and numeracy that includes both literacy and numeracy development.
- Tertiary learning providers ensuring that professional development programmes link both numeracy and literacy to classroom and workplace practice and that staff can deliver both.
- An equal focus on numeracy development with a centre or central resource that is available, in institutions or at state/national level, to teachers and programmes.

**Conclusion**

Using the 2012 AT data for FS students, our study shows a strong correlation between the students’ numeracy and literacy results. In tertiary education we need to recognise students’ literacy won’t improve without improving their numeracy and students’ numeracy won’t
improve without literacy improvement. Tertiary policies and practices must reflect the link and ensure students’ develop their numeracy and literacy in tandem.

Finding a link between students’ numeracy and literacy is just a beginning, we now require greater detail on the nature of this relationship. Is this link present in all aspects of literacy or just for reading? Is this link also present in other aspects of mathematics? Why is there an overrepresentation of Pacific Island students at lower Step Levels? Further investigation of these issues is needed, along with development of teaching and learning strategies, policies and practices that will improve both literacy and numeracy for our students. In neglecting students’ numeracy we may place their literacy at risk.

References


## Appendix 1

Table 1. Chi-Squared Test of Reading and Numeracy Step Levels

<table>
<thead>
<tr>
<th>Numeracy Step Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>Expected count</td>
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<td>0.3</td>
<td>0.4</td>
<td>0.1</td>
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<td>1</td>
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<td>Count</td>
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<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Expected count</td>
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<td>1.3</td>
<td>1.8</td>
<td>0.5</td>
<td>0.1</td>
<td>4</td>
</tr>
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<td>64</td>
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<tr>
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<td>8</td>
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</tr>
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<td>Total</td>
<td>2</td>
<td>29</td>
<td>113</td>
<td>159</td>
<td>45</td>
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<table>
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<tr>
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<th>3</th>
<th>4</th>
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### Appendix 2

#### Table 2. Chi-Squared Test of Numeracy Step Levels and Ethnic Group

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#### Table 3. Chi-Squared Test of Reading Steps and Ethnic Group

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The skill of reading with understanding is essential in any adult mathematics classroom where context is used to promote learner engagement. A wide range of contexts may be used which can increase the literacy demands for the learners. The development of mathematical concepts and skills, for example, are promoted through investigations and problem solving. The New Zealand Literacy and Numeracy for Adults Assessment Tool also uses context to evaluate mathematical competency. Here learners may need to engage with up to thirty different contexts in the process of displaying their mathematical knowledge. The relationship between literacy and mathematical development is evident in working with mathematical texts and worksheets. Reading with understanding is a challenge for many learners and may inhibit their grasp of the subject and their ability to answer word problems. Literacy is more than understanding the vocabulary of mathematics and the context in which it is presented. In this paper, ways of assisting the learner to meet the heightened literacy demands are explored going beyond the use of vocabulary lists and word bombs.

Introduction

Most teachers in foundation and bridging education courses for adults in New Zealand are aware of the need to develop both the literacy and numeracy skills of their students. The Unitec Faculty of Social and Health Sciences\(^5\) has this statement in their Innovation in Teaching and Learning Strategic Plan (2012-2016): “Academic literacies are to be embedded in all programmes to improve student success” (p. 1). The academic literacies include numeracy and literacy. As a teacher of mathematics, meeting the literacy needs of mathematics students is a new and different challenge. The literacy needs include reading mathematical text, understanding it in order to do some maths and communicating the results of the calculations, relating them back in the text.

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\(^5\) Unitec Institute of Technology is a large New Zealand technical institute which aims to provide students with work-relevant qualifications. Foundation or developmental mathematics is largely taught within the Department of Foundation Studies in the Faculty of Social and Health Sciences.
The focus of this paper is to explore the reading aspect of literacy in mathematics. This paper will also explore the needs of adult students studying the maths which focuses on numbers, fractions, decimals and percentages, and the level of numeracy needed in their everyday lives as members of families, communities and in the workplace. The mathematics content is covered in the Learning Progressions for Adult Numeracy (Tertiary Education Commission, 2008). The reading demands of the courses are considered and some interesting approaches to meeting those demands are offered.

Background

Many maths teachers share the opinion that they are not qualified to assist learners in learning how to read mathematical text (Chandler-Olcott, Masingila, Hinchman & Doerr, 2011). A mathematics teacher is often under time pressure so there is a worry that spending class time on reading will reduce the time spent on the mathematics itself. Most students, including those who have English as a second language (ESOL) experience difficulties accessing maths problems. The problem is how to help all students with their reading needs so that they can make sense of word problems, become independent problem solvers and, in the process, broaden their mathematical understanding. The reading needs of learners are partly determined by the style of the mathematics lesson.

The classroom environment under consideration is one which promotes the development of understanding, reasoning, communication and solving problems alongside the development of mathematical skills. Students often work in collaborative groups in which they share ideas and problem solving strategies as well as discussing the problems, their understandings and the mathematics involved. This constructivist model, together with the diversity of approaches to learning mathematics, increases the literacy demands for the student. In contrast, in the transmission model, or the ‘tell-show-do’ approach to learning mathematics, the teacher asks the questions, the learner replies and the teacher evaluates the answers (Borasi, Siegel, Fonzi & Smith, 1998). Here the learning revolves around the teacher rather than the student, and the literacy demands are interpreted and mediated by the teacher. A feature of the constructivist approach to learning mathematics is that learners build on their existing knowledge and understanding to construct meaning for themselves. This is promoted through the use of contextual problems.

The encouragement to use context in mathematics education is the product of the 1980s mathematics reform movement which supported the use of contextualised maths problems and also open-ended problems which encourage students to explore different methods of solution (Boaler, 2002). When these reform ideas were implemented in New Zealand it was suggested that the teacher should introduce the topic with word problems, thus giving context to the mathematical ideas, instead of using them at the end as applications which is the traditional order. The use of context however has given rise to equity issues across the wide range of student abilities and it may be argued that traditional maths teaching which used procedural methods to teach concepts and skills directly was more equitable (Boaler, 2002). Efforts must be made so that equity for all is achieved or maintained. Some methods which can be used include group work, scaffolding and class discussions to introduce problems and any challenging ideas, unfamiliar contexts or vocabulary.
Problematic Areas

What is the role of context in mathematics? The underlying assumption that numeracy (and mathematics) should be relevant to real-life, where possible, gives rise to the use of many and varied contexts. For the adult learner, who is often reluctant to learn mathematics, the use of context shows them how useful mathematics is (Shield, 1998). Word problems are used to engage the student and stimulate their thinking. Many contexts are not authentic and not related to the life experiences of the students, which contrasts with those used when students study numeracy in a workplace environment. A theme that emerged in a study of the use of context in an adult numeracy classroom was “that students rarely seemed to relate context-based mathematical word problems to life outside the classroom” (Oughton 2009, p. 20). The contexts used are usually realistic but not authentic. Many contexts can be considered as camouflage contexts or “painted on” contexts. A common strategy in working with such word problems is to ignore the context, extract numerical information and solve the problem.

Context is also a feature of assessment. In New Zealand all programs at Levels 1, 2 and 3 are required to use the national Literacy and Numeracy for Adults Assessment Tool to assess learners’ numeracy and literacy levels. The learners are “tested” before and after doing a course. There are a variety of forms of these tests. At Unitec, we use an on-line adaptive version for general numeracy for all our students. The questions appear well presented with both text and diagrams or pictures. The pictures engage the student with the context of the question but the diagrams require interpretation and linking with the text. The learner needs to make sense of both the individual context and the maths content of the questions of which there are about 30. Some concern has been expressed as to whether the reading level of the text matches the reading level of the students. A comparison between literacy and numeracy is currently being carried out by my colleagues.

To some degree, literacy skills are transferred from other areas into the maths classroom. However, a review of literature on the subject of literacy transfer concluded that “most studies reveal a relatively low degree of correlation between reading performance with different sorts of material requiring differing background knowledge and reading strategies” (Mikulecky, Albers, Peers, 1994, p. 21). Many studies have investigated the transfer of skills between the classroom and the workplace and between content areas. Berryman and Bailey (as cited in Mikulecky et al. 1994, p. 5) claim that “learning transfers best when it is done in real situations in which both knowledge and strategies are learned at the same time”. Not only does the maths classroom lack the realism of the workplace but the skills needed are different.

The reading demands of mathematics are different from those of other subject areas. For example, the reader does not have to sort out the facts from theory or argument as in sciences or history. Generic approaches to learning to read may not be the most helpful in mathematics as they focus on common skills and strategies and not those specific to the subject. Numerous studies have shown that learners can improve their ability to read with understanding when they are given instruction into reading in their specific content area (Hall, 2005).

The difficulties of reading mathematics are well known to both students and teachers of mathematics. Braselton & Decker (1994, p. 276) declare that “Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than in any other sentence”. Mathematics is a very compact subject and it is important that student performance in mathematics is not held back by readability issues. Word length and sentence length are often considered as the key constraints when determining the readability of
texts but in mathematics there are other major additional factors. These include the use of numbers, symbols, diagrams, tables and special vocabulary. Mathematical text needs to be unambiguous. Every word is important particularly the small ones, e.g., reduced by 40% and reduced to 40% have different meanings. The order of the words may be vital (e.g., “take 8 away from 17” is quite different from “take 17 from 8”). The text often contains hidden or assumed concepts and relationships. For example: is the answer to the question “how many 5’s are there in 25?” one or five? The mathematical text is often embedded in a real world context to make a word problem for the learner to solve. There are multiple ways to address the difficulties of reading in mathematics.

A common approach in addressing mathematical literacy is to focus on vocabulary. There are several different sets of vocabulary in play in the maths classroom. Among these are the vocabulary of mathematics, the vocabulary of the context and the vocabulary of mathematical symbols. Learners without the required vocabulary cannot engage with the task. Many words used in mathematics look familiar to the student as they are used in different contexts but the words have specialised maths meanings e.g., volume may not mean loudness. Confusion arises when the teacher and the learner use the same word but with different understandings. For example the word “difference” has something to do with subtraction in mathematics but is a word often used by the learner without that meaning.

Learning materials used in the classroom can be many and varied but most courses use a text book. It is difficult to find an appropriate one to use with adult classes at this level. The level of the mathematical content is usually that covered by primary-aged children who need different contexts for their mathematics. On the other hand there is often a mismatch between the text level and the maths content where the reading level of the text is up to 3 years above that of the students (Braselton & Decker, 1994). Mal Shield (1998) reported from a study of seven mathematics teachers that it is the teachers rather than students who read mathematical textbooks. In the analysis of one textbook, Shield commented on the density of the material and the number of ideas that had to be remembered by the reader. He also commented that there are hidden messages in text books. They are often written using the “tell-show-do” or transmission model which may conflict with the teachers’ beliefs although they may match students’ perception of how to learn maths. There is a need for students to develop their skills by doing exercises which are usually well provided for in mathematical textbooks.

The style used by most writers of mathematics problems is different from that of writers in other core subjects. The intention of the writer is to provide unambiguous text. This is often intended for another mathematician to read and not necessarily to be easy for a student of mathematics to make sense of. Other difficulties with the style include strange uses of tense and indirect communication of information as commented on by Helen Oughton (Oughton, 2009). The reader’s prior content knowledge and knowledge of context often need to be drawn on (Barton, Heidema & Jordan, 2002). The structure of the text is also different. Instead of the main idea being given in the first sentence of the paragraph, the question needing to be solved is usually written as the last sentence. This gives rise to “template” approaches to reading mathematical word problems. They help the readers to decode the written text and to reorganise it, getting the main ideas into a more useful form for solving the problem. One useful approach is suggested by Barton (Barton et al, 2002). They suggest to

- Read through the problem quickly to get general picture
• Ask questions – find the question, work out what you need to know to answer it and the sequence of working required
• Reread to find details, data and facts
• Write the equation or calculation and compute answer
• Reflect on the answer, communicate the answer, then check whether the answer is reasonable.

Another template approach is advocated by Braselton & Decker (1994) and involves the use of a graphic organise to guide the problem solver. This is a diamond shaped template setting out a five step approach. These steps include restating the problem, finding the necessary information, planning and carrying out the calculations then reviewing solution for reasonableness.

Less formulaic approaches to problem solving use transactional reading strategies involving the learner and text and sense making (Borasi, Siegel, Fonzi, & Smith, 1998). Transactional reading strategies have been advocated to encourage meaning-making and engagement of the reader with the text (Borasi, Siegel, Fonzi, & Smith, 1998). These strategies include talking through the text with a partner, writing important ideas on cards which can then be shuffled into a different order, making drawings to represent the text, and acting out the procedure. Mikulecky (1994) on the other hand describes similar approaches called metacognitive reading strategies which include making predictions, asking questions, summarising and clarifying.

“Scaffolding a task” is another strategy often used to assist the learner. The task itself may be scaffolded to lead the learner through the task by breaking the task up into a sequence of smaller steps. Alternatively the teacher may scaffold material by reading the text aloud to the students and discussing any difficulties in interpretation (Chandler-Olcott, Masingila, Hinchman, & Doerr, 2011). To scaffold mathematical tasks the teacher must predict the difficulties, prepare vocabulary, introduce or explain the context, identify and prepare the mathematical processes needed, ask the learner questions and encourage the learners to create their own questions and to make sense of the material for themselves. Scaffolding can be regarded as the middle step of three required to produce independent learners (Chandler-Olcott, Masingila, Hinchman, & Doerr, 2011). The final stage is for the learner to apply mathematics independently and in new, different contexts. The adult learner often needs to learn how to learn and their independence as a learner is scaffolded by scaffolding the task.

Solutions – Numeracy and Literacy Experts Working Together

To achieve some practical solutions that we could use in our classrooms, I drew on the experience gained at a workshop, presented by Helen Doerr of Syracuse University, USA, at the New Zealand Association of Mathematics Teachers 10th Biennial Conference in Auckland, in September 2007. The workshop was entitled “A Conversation on Math and Literacy”. We arranged a meeting between some maths tutors and some literacy specialists to consider the challenges of reading and making sense of mathematical text. We pooled our separate knowledge to set up some ground rules for helping students to read mathematical text.

From these discussions and after reviewing some of the literature on reading we felt that maths teachers need to be explicit about meeting the literacy demands in the mathematics classroom. We have observed some unsuccessful reading strategies used by students in the maths classroom.
These include skim reading the text for the numbers, ignoring the context, and when the correct answer is obtained the student goes on to the next problem. For us, it is important that the student learns from doing the “problem”, makes sense of the answer and relates it to the context. In a similar way we have observed students with a “have calculator can get answer” approach where the numbers are fed into the calculator and buttons are pressed until the correct answer is achieved. Other students apply the strategy of highlighting the important words and make up the “story” and are unaware of the true “story”. We were keen to provide an alternative reading strategy which is generic and non-formulaic and would encourage independent learners.

Before using a mathematics text, vocabulary needs to be considered along with any prior knowledge or mathematical concepts that the learners should be able to draw on. This includes both the mathematical vocabulary and the contextual vocabulary. There are many ways of addressing vocabulary issues, from glossaries, concept maps, word bombs, cloze activities, crosswords to mention a few. Word bombs (or lists of vocabulary) give learners an opportunity to create glossaries with the meanings of the words; alternatively they can be used to create mind maps in which the learners can link words and mathematical ideas. There are many ways to make the language of mathematics meaningful. The next step is to intervene while students are reading some text.

It was felt that explicit guidance should be given to assist learners to make sense of the mathematical text. They need to understand the three-part structure of many mathematical word problems. The problems start by setting the scene with a story or picture, followed by some information in words, numbers, tables or other mathematical language. The question or questions are usually at the end. It is important that the learner does not expect to make sense of the text by reading it through once from the beginning to the end. Mathematics teachers observed how they, themselves, read mathematical text. They observed that they read and reread the text both the whole and in parts, making connections between text, diagrams, tables of data and questions. They frequently paused to make sense of their reading and asked their own questions of the text and checked for consistency.

We found that the questions “what did I notice?” and “what did I wonder?” to be the most useful questions to use in a generic approach to reading maths word problems. They encourage the learner to consider the big picture of the text as well as the detail and have the positive effect of helping the learner to engage with the text. Once engaged with the maths the learner finds it more natural to think about the suitability of the solution and whether there are other solutions. Mathematics teachers need to be aware of the complexities of reading mathematical texts. They should reassure learners that reading maths text is non-linear and that it is OK to read, reread, check and recheck as often as necessary to understand the text. Teachers need to explicitly teach reading strategies and model them in the classroom.

**Conclusion**

It cannot be assumed that adult learners will be able to read and understand the text used in mathematical work sheets, activities and word problems. The richness of context in realistic problem solving adds another layer of complexity on top of the already demanding mathematical text. Teachers who are aware of the difficulties that a learner can encounter should teach how to read mathematical text. The questions “what did I notice?” and “what did I wonder?” are very useful in engaging students with the text, leading the him/her into understanding the word problems and thus, learning.
References


http://www.academia.edu/1602564/How_Secondary_Mathematics_Teachers_Address_the_Literacy_Demands_of_Standards-based_Materials


The Impact of Teaching on the Mathematics Learning of Two Prisoners

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This paper presents the findings from a small case study conducted with a purposive sample of two adult prisoners who agreed to participate in a teaching program designed to strengthen their mathematical understanding. The study was informed by research conducted with school children into the use of manipulative materials with problem-solving activities embedded in real-world contexts to develop learners’ mathematical understanding and reasoning within a positive and constructive learning environment. The researchers wanted to explore whether the adoption of such a teaching program would enable adult learners, who had struggled with mathematics, to improve their understanding and use of mathematics. One of the researchers works within the New Zealand prison system, where low levels of literacy and numeracy are common amongst prisoners. Findings from this study, while not generalizable due to the study’s small sample size, highlight opportunities for large-scale research into how teaching mathematics for understanding can improve adult learners’ attitudes towards and utilisation of mathematics in everyday life.

Introduction

Many adults who struggle with mathematics have reported the negative influence of their school classrooms, teachers, and instructional methods on their ability to learn and use mathematics (Bibby, 2002; Biddulph, 1999; Carroll, 1994). Some have experiences so negative that they develop anxiety, phobia, or even hatred towards mathematics. Such attitudes become entrenched, and can affect students’ self-confidence, ability, and inclination to use mathematics in daily life, limiting career choices and income (Boaler, 2008). These effects often flow through to their children, either through negative attitudes, or because the adults felt unable to teach their children mathematics or help with homework.

Research indicates common issues amongst struggling mathematics students can be remediated through using a reform-based approach of teaching for mathematical understanding. A small case study was conducted with two adult prisoners, for the purpose of investigating and testing these new approaches and methods for teaching mathematics with adult learners. A discussion of the literature on numeracy issues amongst adults, mathematics learning difficulties and reform-based approaches to improving mathematics instruction is followed by an outline of the rationale and methodology used for the study. Results of the study are outlined in detail, followed by a discussion of the implications for educators and learners.
Literature on Teaching Mathematics for Understanding

Technological advancements in recent years have increased the demand for strong numeracy skills, both in the workplace, in home, and community contexts (Gilbert, 2005). However, there is a considerable gap between numeracy demands and adult numeracy levels. The 2006 Adult Literacy and Life Skills (ALL) Survey showed that half of New Zealand adults have numeracy skills below the level needed to function successfully in the modern world (Satherley, Lawes & Sok, 2008). In particular, adults of Māori and Pasifika ethnicity, the unemployed and younger adults (16-24 year olds), performed particularly badly. Likewise, mathematical learning difficulties are more prevalent amongst children from minority and lower socio-economic homes (Ginsburg & Pappas, 2007).

Recent surveys of New Zealand’s prison population indicate that about 80% of prisoners lack functional levels of numeracy (Department of Corrections, 2009). Preliminary assessment results from the Tertiary Education Commission’s (TEC) Literacy and Numeracy for Adults Assessment Tool (LNAAT) of 240 prisoners in programs for embedded literacy and numeracy show that they lack knowledge of higher-level multiplication and division facts, common factors, place value over 10,000, decimal place-value, fractions, percentages, decimals and exponents. These topics are at the upper steps of TEC’s Learning Progressions (TEC, 2008) and are critical for becoming numerate and functioning successfully in the workforce.

New Zealand’s future workforce will come from a growing population of predominantly young, minority, and lower socio-economic communities who currently have lower numeracy levels that limit their employment opportunities (Te Puni Kōkiri, 2011). Hence it is vital for all learners to become numerate through effective teaching.

Research on people with mathematics learning difficulties (MLD) (over one-third of learners) and mathematics disabilities (MD) (6% of people) points to common issues amongst struggling learners, including deficits in working memory, processing, retrieval, cognitive/metacognitive thinking, and long-term memory (Allsop, Kyger, & Lovin, 2007). Other problems include learned helplessness, passive learning, attention difficulties, low academic achievement, delayed mathematics understanding (Mazzocco, 2007), and limited awareness of mathematical pattern and structure (Mulligan, 2011).

Traditional mathematics instruction, using memorization of rules, facts, and algorithms, has been identified as both difficult and limiting for many learners, especially those with MLD (Baroody, Bajewa & Eiland, 2009; Skemp, 2006). A reform-based approach is based on teaching for mathematical understanding. This involves meaningful mathematics instruction, emphasizing relational understanding, problem-solving strategies, communication, and reasoning, using a problem-based approach where learning is embedded within authentic contexts (Allsop et al., 2007; Anthony & Walshaw, 2008; Boaler, 2008; Pesek & Kirshner, 2000; Skemp, 2006). Establishing a classroom community in which all learners have a sense of belonging and engage with mathematics as a collaborative process is important (Anthony & Walshaw, 2008). This involves building relationships, encouraging participation, using small group processes, giving quality feedback, and structuring and modelling appropriate mathematical discourse that leads to higher levels of thinking.

Relational understanding of mathematics is defined as “knowing both what to do and why,” in contrast to instrumental understanding, which is seen as “rules without reason” (Skemp, 2006, p. 89). Skemp argues that relational understanding helps learners to appreciate the connections
between different aspects of mathematics, enabling them to adapt their understanding to new
tasks. Pesek and Kirshner (2000) found that students taught for relational understanding
outperformed a group who received instrumental instruction followed by relational. The
relational instruction encouraged students to construct their own connections using
communication, questioning, and concrete materials. These two researchers argue that reforming
mathematics education requires a “reorientation of classroom norms and practices” (p. 539).

Research on grouping practices shows that students in mixed-ability classes not only perform
better academically than peers in ability-grouped classes, but also fare better in terms of career
achievement and economic wellbeing (Boaler, 2008). Mixed-ability classes used complex,
multilevel, multidimensional problems, providing opportunities for students to do challenging,
complex work. Higher-achieving students explained ideas to others, in the process deepening
their own understanding. This created an environment of communication, support, open-
mindedness, thoughtfulness, and respect for others.

Problem-based tasks have advantages over teaching the mathematics then applying it (Davis,
1992). Boaler (2008) describes teachers who want their students to be challenged by problems
that stimulate them to think and reason their way to solutions. Through this process they build
genuine understanding of mathematical concepts. It is important to link tasks to learners’ life
experiences, knowledge and interests, together with creating task challenge based on learners’
current mathematical competencies (Anthony & Walshaw, 2008).

The literature reviewed informed the teaching approaches used in this study; that is, teaching for
mathematical understanding through the use of relational understanding, problem-solving
strategies, communication, questioning and reasoning, and the use of concrete materials to
construct connections. The following two sections discuss the rationale for the study and the
methodology used.

**Rationale**

The researchers identified both a gap in the research around improving prisoners’ mathematical
skills and issues with how numeracy was being taught in New Zealand prisons. They developed
this study to investigate the impact on two prisoners of a mathematics program using an
approach that emphasized relational understanding.

**Methodology**

**Participants**

Two prisoners volunteered to participate in the study, on the basis of a previous survey of a
wider group of adults, in which they identified that they had difficulties with mathematics.
Pseudonyms are used to preserve participants’ privacy (see Table 1).

<table>
<thead>
<tr>
<th>Learner</th>
<th>Bob</th>
<th>Jim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Range</td>
<td>30s</td>
<td>50s</td>
</tr>
<tr>
<td>Gender</td>
<td>Male</td>
<td>Male</td>
</tr>
<tr>
<td>Ethnicity</td>
<td>NZ European</td>
<td>NZ Māori/Pākehā</td>
</tr>
</tbody>
</table>
Procedures

A mixed-methods approach was taken to conducting the study. Individual diagnostic interviews as well as paper-and-pencil versions of TEC’s LNAAT were used to assess numeracy before and after the teaching program. Learners’ responses to assessment tasks were analysed against the Learning Progressions for Adult Numeracy Framework and key areas for teaching identified. Five lessons were planned utilising TEC’s online numeracy resources for educators (TEC, 2011), as well as numeracy fact sheets, worksheets, and marking guides from the BBC’s (2011) Skillswise website for adult learners.

Prison security constraints denied access for prisoners to online educational resources, and resources brought into the prison were limited to paper and plastic items, or items already available within the prison workshop. Activities were created that enabled learners to utilize manipulative materials for conceptualising the mathematical learning. These included card sorts, card games for multiplication and place value, grouped paperclips, number lines, fraction sheets to cut and re-assemble, coloured fraction segments and fraction bars, deci-mats (rectangular paper divided into thousandths), and an adaptation of the deci-wire (Young-Loveridge & Mills, 2011) called a “deci-string”.

The teaching approach used by the first author focused on creating a positive learning environment and building learners’ trust and confidence in her, in mathematics, and in their own abilities. Strategies used included:

- Actively avoiding language such as “That’s wrong”, “No” etc., asking instead: “Is there another way to do that?” or “What about using another strategy,” always trying to acknowledge a learner’s solution as valid, then asking if there is a different way of solving the problem.

- Holding discussions about topics, rather than lectures (e.g., “How would you solve this problem?” or “What have we learned lately that could help here?”)

- Sitting with the learners as they worked out solutions with manipulatives and participating in games with them.

- Reflection by learners at the end of each lesson on their learning, and at the start of the next session on their use of previous learning over the preceding week.

Lessons were weekly and supplemented by the BBC Skillswise resources and homework activities to embed and extend the learning. Vocational instructors were encouraged to monitor how the learners were coping with their learning and homework between sessions. Written reflections on each teaching session enabled the researcher to adjust her teaching plans according to progress at each session.
Results

In this section, findings from an initial survey are presented, followed by a discussion of the impact of the program.

Initial attitudes towards mathematics

Table 2 presents learners’ responses to a survey about their attitudes towards mathematics conducted about a month prior to the assessments and teaching program.

Table 2: Responses to statements and questions on views about mathematics

<table>
<thead>
<tr>
<th>Item</th>
<th>Statement</th>
<th>Bob</th>
<th>Jim</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Some people have a maths mind and some don’t.</td>
<td>Agree</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>2</td>
<td>You can be good at maths without understanding it.</td>
<td>Agree</td>
<td>Disagree</td>
</tr>
<tr>
<td>3</td>
<td>Adding, subtracting, multiplying and dividing are only a small part of maths.</td>
<td>Agree</td>
<td>Agree</td>
</tr>
<tr>
<td>4</td>
<td>It is OK for learners to make mistakes in maths.</td>
<td>Agree</td>
<td>Agree</td>
</tr>
<tr>
<td>5</td>
<td>There is always a best way to do a maths problem.</td>
<td>Agree</td>
<td>Agree</td>
</tr>
<tr>
<td>6</td>
<td>Men are better at maths than women.</td>
<td>Strongly disagree</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>7</td>
<td>Maths is not creative.</td>
<td>Strongly disagree</td>
<td>Strongly agree</td>
</tr>
<tr>
<td>8</td>
<td>Knowing why an answer is correct in maths is just as important as getting the right answer.</td>
<td>Strongly agree</td>
<td>Agree</td>
</tr>
<tr>
<td>9</td>
<td>Working hard at maths leads to success in maths.</td>
<td>Strongly agree</td>
<td>Agree</td>
</tr>
<tr>
<td>10</td>
<td>Which face matches how you feel about maths now?</td>
<td>Neutral</td>
<td>Neutral</td>
</tr>
<tr>
<td>12a</td>
<td>How much did you like maths when you were in Primary school?</td>
<td>Very negative</td>
<td>Very negative</td>
</tr>
<tr>
<td>12b</td>
<td>How much did you like maths when you were in Secondary school?</td>
<td>Very negative</td>
<td>Very negative</td>
</tr>
</tbody>
</table>
Learners were asked why they had given a particular response in Question 10, as well as several other open-ended questions (see Table 3).

Table 3: Responses to open-ended questions

<table>
<thead>
<tr>
<th>Item</th>
<th>Statement</th>
<th>Bob</th>
<th>Jim</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Have your attitudes changed since leaving school? How and why?</td>
<td>I have avoided any maths as much as possible.</td>
<td>Not really. I just can’t seem to understand the subject or see its importance.</td>
</tr>
<tr>
<td>15</td>
<td>Have you ever felt anxious about maths? Why?</td>
<td>Yes, because I’ve never fully understood it enough.</td>
<td>Yes, because I have always found it a very difficult subject to get a grip on.</td>
</tr>
<tr>
<td>16</td>
<td>What impact has maths had on your life?</td>
<td>If I was better at maths from primary school … I’d say it’s had a big impact overall.</td>
<td>Quite a bit at school, I felt inadequate and it caused problems between me and my maths teacher.</td>
</tr>
</tbody>
</table>

Of interest was the long-term impact of their negative experiences with mathematics learning on their lives, particularly their avoidance of mathematics, the anxiety they experienced, and the ongoing feelings of inadequacy resulting from their experiences of mathematics learning at school.

Impact of Teaching Program on Learners’ Attitudes

Table 4 summarizes the participants’ responses to questions about their attitude towards mathematics as part of the assessment.

Table 4: Responses to questions about attitudes towards mathematics

<table>
<thead>
<tr>
<th>#</th>
<th>Question</th>
<th>Bob Pre</th>
<th>Bob Post</th>
<th>Jim Pre</th>
<th>Jim Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>How much do you like maths? (1=Hate to 5=Love)</td>
<td>3.5</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>How good do you think you are at maths? (1=Awful to 5=Great)</td>
<td>2.5</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>How confident do you feel in doing maths during your daily life? (1=Not at all to 5=Very)</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The shift in attitude from the initial assessment (Pre) to the final assessment (Post) almost seven weeks later was notable, with Bob showing moderate improvement in confidence about his abilities, and Jim showing considerable improvement in enjoyment of the subject and confidence in attempting problems he previously would have avoided. This shift in learners’ confidence is an important outcome, with learners no longer constrained by past negative experiences, and free to continue learning and using mathematics in their lives. As Bob so succinctly stated after just one session, “Maths is no longer the ogre I thought it was.” Vocational instructors reported that
the learners seemed to enjoy the lessons. They looked forward to future lessons and tested each other’s mathematics knowledge during the week at work.

Table 5 presents the steps on the TEC Learning Progressions in numeracy framework for both the initial (Pre) and final assessment (Post) interviews. Both learners made progress of between half and two Steps on at least four out of the six Learning Progressions assessed, notably in the areas of calculating fractions, whole number and decimal place value, the use of partitioning strategies, and improved number fact knowledge. They made genuine attempts to solve more problems, with Bob making only five guesses in his summative assessment versus nine in his initial assessment. In particular, there was greater use of appropriate strategies to solve problems.

Table 5: Steps on the Learning Progressions framework for strategies and knowledge progressions before (Pre) and after (Post) the Intervention programme

<table>
<thead>
<tr>
<th>Learner</th>
<th>Bob</th>
<th>Jim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Progression Step</td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Strategy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additive</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Multiplicative</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Proportional</td>
<td>4</td>
<td>2-3</td>
</tr>
<tr>
<td>Knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Sequence</td>
<td>3-4</td>
<td>4</td>
</tr>
<tr>
<td>Place Value</td>
<td>3</td>
<td>4-5</td>
</tr>
<tr>
<td>Number Fact</td>
<td>2</td>
<td>2-3</td>
</tr>
</tbody>
</table>

Table 6 presents the results of paper-and-pencil versions of the TEC LNAAT in General Numeracy, Steps 2-5 before (Pre) and after (Post) the Intervention programme. Paper-based, non-adaptive versions of the LNAAT were used because prisoners were not permitted access to the online adaptive version of the tool. Questions in the pre and post assessments were randomly selected by the LNAAT to be of comparable difficulty.

Table 6: Results of General Numeracy assessments from TEC Assessment Tool

<table>
<thead>
<tr>
<th>Learner</th>
<th>Bob</th>
<th>Jim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Progression Step</td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Step for Number Strategies &amp; Measurement</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Scale Score</td>
<td>608 +/- 34</td>
<td>531 +/- 28</td>
</tr>
</tbody>
</table>

It was interesting to note that both learners scored lower on their post-assessments than on their pre-assessments. Jim’s drop in score was within the margin of error (i.e., not statistically significant for the norms for the LNAAT, which have been established over a large national cohort), whereas Bob showed a statistically significant drop in score. This phenomenon has been
observed on occasion with the LNAAT assessments, where learners’ assessment scores drop in post-assessments after their initial assessments. Some possible explanations include ‘assessment fatigue’ amongst learners who are too frequently assessed, over-confidence (“I know this already, I don’t have to try hard”) and in some cases, where there is a gap of some months between learning and subsequent assessment and the knowledge or skills gained had not been practised, learning decay may have occurred (Sayre & Heckler, 2009). Both learners admitted later to rushing their way through the summative paper-based LNAAT assessments, making silly mistakes, and not checking their work.

Of greater interest was that although the score was lower on the paper-based post-LNAAT assessments, both learners showed improvements in the interview post-assessments in at least four of the six domains. The individual interview assessment, with its focus on identifying which strategies a learner knows and can use, probably provides a more valid and reliable picture of a learner’s understanding and use of mathematical strategies and knowledge than the paper-based LNAAT. The initial diagnostic interview showed that both learners had knowledge gaps with higher-order multiplication/division facts and place value, but in particular, they really struggled with fractions, percentages and decimals.

In the first lesson, learners were told that the focus of the teaching program was on strategies they could use to solve mathematical problems quickly and easily in their heads, rather than using a calculator or pencil and paper. This could help them check whether they had been charged the correct price, or determine if they were getting the best value (i.e., not being “ripped off”). That idea appealed to them!

**Multiplicative/division facts and strategies**

The researcher used a short written multiplication/division facts test and a card-sort activity, separating out the facts they knew from those they didn’t. Bob in particular had no immediate recall of his multiplication facts for most of the times-six, seven, eight and nine tables. This highlighted that both forms of assessment, with unlimited time, failed to pick up Bob’s lack of immediate recall of multiplication facts, indicating that he probably used additive strategies to solve higher-order multiplicative problems. The learners were assigned homework with cards to check their knowledge throughout the week.

The researcher taught a variety of partitioning strategies such as rounding up or down to the nearest 10 or 5, and doubling and halving, to enable them to more effectively manage multi-digit multiplication/division problems. The use of coloured sticky notes to model partitioning strategies was highly effective. At the next lesson a week later, learners were able to provide several solutions to the problem “39 x 21” based on partitioning strategies. As learners progressed to division strategies, they were able to apply their increased multiplication/division knowledge to solving “72 divided by 4” by using partitioning strategies.

**Place Value**

In teaching Place Value, the key was to emphasize that the decimal place must never move and that each place to the left of the decimal point was a tenfold increase, whereas each place to the right of the decimal point was one tenth as large (i.e., tenths, hundredths and thousandths, etc.). Drawing a place-value chart helped the learners to visualize the increases and understand where numbers were to be placed. An activity using a calculator to challenge one another to add or subtract numbers of varying place value proved very popular and challenging for the learners. The learners seemed to relate well to measurement examples, such as there being 1,000 grams in
a kilogram. Homework provided practice converting money and metric measurements, interpreting place value charts and calculating using 10s, 100s and 1000s. The learners seemed to find these exercises relatively straightforward.

Fractions
The researcher commenced this lesson by having each learner cut up coloured copies of a round pizza into enough even slices for four people. She then said “Another four people have suddenly arrived for dinner, so how will you cut the pieces up to have enough slices for everyone?” – They cut each of the quarters into two eighths. She then got the learners to cut different coloured pieces of paper into halves, thirds, quarters, fifths and sixths and label each of the pieces accordingly. She got them to compare the different sizes and note any relationships, e.g., between thirds and sixths and then borrow similar fraction pieces from one another to make improper fractions. Together they discussed how an improper fraction could be represented as a mixed number, e.g., 8/6 could be written as 1 2/6 or 1 1/3. They discussed the terms denominator (the total number of pieces the whole is cut into) and numerator (the number of pieces shown in the fraction), and added the terms to the mathematics word bank on the board.

The learners then created a number line between 0 and 1 using fractions of various sizes, to help emphasize that unlike whole numbers which got bigger as the number got larger, fractions got smaller as the denominator got larger. Jim commented that: “you could almost go to infinity with the size of fractions”. Bob was initially withdrawn, not engaging in this activity, until he was invited to explain why 1/20 was smaller than 1/5. He explained that it was “One divided into 20 pieces, so each piece would be very small, compared to one divided into five pieces” and he started to engage with the activity again. The researcher particularly wanted Bob to engage with this activity, as he had said in his diagnostic interview that his previous tutoring on fractions had gone “too fast, too hard” and he had “tuned out”.

The use of coloured magnetic fraction segments was revolutionary! The learners were able to stack up the various segments, compare sizes to work out the difference say between 2/5 and 3/6, and demonstrate how to make a whole circle out of different sized fractions. They then progressed onto working with fractions represented by coloured magnetic fraction bars, creating ‘families’ of fractions which divided evenly into one another, measuring how many pieces of each type of fraction made a whole. As the lessons on fractions progressed, if they experienced difficulties, they were referred back to their fraction bars to work out solutions. The learners enjoyed the Count-3 fractions game, especially when it got strategic towards the end.

Percentages
The lesson on percentages built nicely on the work on fractions, having the learners work with deci-mats to calculate percentages such as 10%, 30%, 60%. A discussion of sale discounting ensued and the learners understood the value of comparing sale prices for the best value. Combining a number line and place value chart was helpful in converting fractions to percentages, and then to decimals. Bob was particularly effective at converting percentages to decimals.

Decimals
The use of decimal strips, deci-mats and a ‘deci-string’ as manipulatives, were useful in helping the learners conceptualise decimal place values. The learners’ understanding of fractions became very important when understanding the relationship between decimals, fractions and percentages
and being able to convert between them. Bob in particular had several attempts at solving problems on the whiteboard. There was a poignant moment when he realised he’d missed a step in one problem, stood back and then had another attempt at solving the problem. He would not have done that four weeks prior! Understanding of addition and subtraction of decimals was good, but another lesson could have helped to consolidate understanding of multiplication and division of decimals.

**Improved Confidence**

Jim, who had shown a very negative attitude towards mathematics in the initial survey and diagnostic interview, yet had performed at Step 4 in the majority of the diagnostic categories, revealed considerable general mathematical knowledge and competence during classes. Initially shy to put forward his answers, as the classes progressed, he grew in confidence and was usually the first to offer well-considered solutions to a problem. He was diligent in completing his homework and showing his workings or strategies used. The researcher emphasized to the learners at the outset that she was more interested in seeing how they worked out their answers, rather than whether they got them all correct. As the classes progressed, Jim became more relaxed and willing to ask questions about other mathematical topics and even have a joke about a homework question that he got wrong through inattention.

Bob, who had initially shown a somewhat negative attitude towards mathematics, but seemed more confident in his own abilities, actually only performed at Step 3-4 across the diagnostic categories and only at Step 2 in Number Knowledge. This lack of knowledge of number facts, particularly higher-order multiplication facts, was a significant impediment to his mathematical learning. He preferred using a calculator, but often could not explain why his answer was correct. As the classes progressed, Bob became more engaged in the subject and was willing to offer solutions. During the last two classes, he often wrote out his solutions to problems posed on the whiteboard.

**Discussion and Implications**

Assessment data indicated that both participants were ‘struggling learners’, constrained by negative experiences and memories of school mathematics learning and a ‘maths avoidance’ mentality. Comments such as “I hated maths” showed that they had neither enjoyed, nor succeeded under traditional mathematics instruction. The teaching program deliberately created a learning environment that was positive and constructive, using methods that fostered understanding of mathematics (Allsop et al., 2007; Boaler, 2008; Pesek & Kirshner, 2000; Skemp, 2006). Encouraging open discussion of principles and methods in a positive context (Anthony & Walshaw, 2008), resulted in learners becoming confident to ask wide-ranging questions and offer a variety of solutions to problems.

Validation by the researcher of learners thinking their way through to solutions was crucial in building their confidence in the class, themselves, and their abilities. Their increased enjoyment of mathematics was another important outcome. Several months after the program, when the researcher was establishing a subsequent group of new learners, one prisoner commented: “You taught Jim maths last year, didn’t you? OK, well, we’ll do your class then.” Six prisoners immediately signed up. Evidently other prisoners had noticed Jim’s increased self-confidence, attributing it to his participation in the program. Jim also joined the new class to continue developing his numeracy skills, and now participates in work activities requiring higher-order
numeracy. He recently wrote a letter commenting on his newfound confidence with multiplying decimals, at the age of 57!

Focusing initially on solving multiplication and division problems using partitioning strategies with whole numbers helped learners later solve more complex problems involving decimals. This is a form of the “mathematical learning residue” referred to by Davis (1992). The researcher spent some time in class deconstructing word problems, to help the learners identify what the question was about and how to solve it mathematically. It would be beneficial for tutors to have learners spend more time deconstructing word-based numeracy problems, to build their skills in effectively solving problems.

The use of manipulative materials such as paperclips, number-lines, fraction segments and bars, deci-mats and deci-strings was critical for learning. The learners’ engagement with tactile activities, creating arrays of ‘fraction families’, or dividing up strips of paper into equal portions – as one learner exclaimed “they must look and be fair!” was remarkable. However, more could have been done to transform the kinesthetic learning into mental imaging, by having the learners draw representations of the manipulatives, before moving to abstractions (Ministry of Education, 2008).

Also effective was the use of games with cards, calculators, and charts to challenge the learners to practise what they had just learned. Consistent with Anthony and Walshaw’s (2008) findings, the learners greatly enjoyed communicating, solving problems in novel ways, and the challenge of a competitive environment.

The biggest challenge faced was the ‘tyranny of time’. Pressure was created by the researcher’s decision to teach too many topics in the time allocated. It might have been better to focus only two sessions on multiplicative strategies and place value knowledge followed by three solid sessions on fractions. It was disappointing that learners’ understanding of percentages and decimals remained weak. Ideally the lessons should have been more frequent (every two to three days) to reinforce learning and build upon the learners’ knowledge. Subsequent discussions with vocational instructors at the Printshop have identified a Level 2 Trade Maths Unit Standard (US64) for the learners to work towards. Instructors have been asked to give learners work-related numeracy problems to maintain their new knowledge.

**Conclusion**

On reflection, the researcher had only begun to scratch the surface with creating connections via relational understanding (Skemp, 2006). She often commented to the learners “I am not as concerned about whether you get the answer correct, but that you can show me how you solved it”, and would underscore this in marking their homework. Notable moments included Bob noticing the relationships between fractions, percentages and decimals and beginning to convert back and forth between these. However, it became evident to the researcher that she needed to develop greater relational understanding of mathematics herself, in order to use this effectively with learners.

The findings of this small case study indicate the importance of teaching mathematics for understanding in the school sector, so fewer adults emerge with the difficulties shown by our participants. Given the small sample size, the findings of this study are not generalizable. However, educators and researchers are urged to expand the efforts made in this research to examine the value of this kind of intervention for larger populations.
References


Lucky Colours of Sunshine: Explaining the Mathematics of Chance Gambling

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The supportive environment of ALM has seen me develop these ideas over some years to a level of quality I am now happy to share more widely. Potential gamblers ought to know why they are going to lose their money long term. This interactive game and accompanying analyses, reveal the key mathematics - of randomness, game structure and distribution of results, which account for the outcomes of commercial chance gambling. Graphs are constructed from participants’ data showing the changing distribution of results over increasing play. Discussion of gambling illusions takes place while the game is played. Experiential and theoretical knowledge of why chance gambling is unprofitable in the long run is made accessible.

Introduction

Mathematics teachers are uniquely placed to complement the common sense understanding of many people that: the gambling industry makes profits from its players. A brief adolescent or adult curriculum is proposed to teach the mathematics which explains players’ losses. From the collection of participants’ game data, outcomes from pure chance games are graphed, then compared with predicted outcomes, to counter the intuitive cognitive distortions readily associated with gambling. Consideration of game structure, and calculation of how the odds of game outcomes differ from payment odds, followed by basic practical introduction to probability distributions, provides material for several mathematics lessons, which may be used from middle high school onwards with varying sophistication. This paper includes material for higher levels, including use of combinatorics to calculate probabilities, and dynamic software programs which enable display of longer run outcomes.

Elsewhere, I (Smith, 2003) have presented quantitative findings linking poor probability knowledge to illusions of control of chance outcomes, well known as a correlate of problem gambling. That work brought awareness, and disappointment, that the lack of understanding of probability concepts was, and is, widespread, among gamblers and non-gamblers; welfare advisors, psychologists, gambling researchers, conference participants, even mathematics teachers! That impression has become stronger over the years.

Social context of gambling education

In addition to poor understanding of applied probability concepts, the prevalence of problem gambling among youth is around 5%. Electronic gambling machines (EGMs) remain the most damaging form of gambling in Australia, warranting our educational efforts. Commercial gambling has become a major growth industry in many parts of the world. Over the past few
decades there has been unprecedented expansion in gambling availability, participation and expenditure. Growth has been particularly strong in jurisdictions where EGMs and large urban casinos have been widely introduced, for example Canada, the United States, Australia, New Zealand and South Africa. Other countries, including the United Kingdom, are currently undergoing this expansionary phase. Increasing legitimacy and social acceptance of legal gambling, the intersection of gambling and financial technologies, impacts of the internet and mobile telephony, economic globalisation, and the spread of gambling to formerly non-gambling settings (Abbott & Volberg, 1999), have facilitated the evolution of commercial gambling.

Myths and illusions about gambling can have a number of sources, and be of a variety of types. They include mistaken expectations and recall of winning, illusions of control, and erroneous perceptions of independence and chance events. Articulating common myths and illusions may help in their avoidance, but another approach is to understand the actual truths of the situation.

Carefully designed teaching at a middle secondary or adult level, can demonstrate emphatically why it is practically impossible to win over the long run on commercial chance gambling. Mathematics fully describes the outcomes from betting on pure chance games. Short-term results are variable within a range of possibilities. Long-term results are less variable, with an ever-increasing guarantee of long-term net loss. The results from chance gambling are a randomly patterned reflection of the structure of the game being played. We need carefully designed teaching regarding the long-term unlikelihood of winning.

Such mathematical content can be developed with interaction from participants in a hands-on session. Remarkably, such relevant simple explanation of the key concepts affecting gambling outcomes seems new to many. This paper challenges the notion that being vague about the likelihoods of gambling outcomes is satisfactory for the playing public, gambling counsellors, or educators. People should have specific understanding of the principles, as applied to gambling, by which those games work for the owners, and denude the players of funds over time.

**Key teaching points**

Key misunderstandings in pure chance gambling, such as on electronic gaming machines, include a lack of:

- understanding of independence and randomness;
- understanding of the basic loss making structure of the game; and
- appreciation of the tendency of variable short-run chance results to congregate around the average mathematically expected result in the longer run. That is, that the variability of net outcomes reduces in larger samples.

So these are items we could try to teach. Note that here the lack of understandings has been set up differently from how they would be described as myths or illusions. We are dealing with the intuitive mistakes we tend to make, in terms of that mathematics knowledge which would explain the correct understandings, rather than focussing directly on the myths and illusions. This is a key difference in this approach compared with some counselling approaches. Instead of concentrating on what misunderstandings people have, we are asking what understandings do they need? And then, how do we teach them?
Mathematics explains the outcomes of chance gambling

Not only does luck move around the room, as gamblers like to say, it also makes a regular distribution, (a Binomial one, but Normal enough over time). The results generated from chance gambles over time mirror the structure of the game being played. The structures of commercial gambling games set up the players to lose, in that the total of prizes offered is less than the average amount of betting needed to generate those prizes. That gives a house margin, how the operator is ensured a profitable business. In a rather strange way, this means that each time you have a win that is actually when you ‘lose’, because you have not been paid a prize commensurate with the unlikeliness of the win you’ve just had. You are typically paid a smaller prize than the frequency or rarity of the winning event would require, were it a fair game in the mathematical sense. Over time, if you keep playing, these insufficient winning payouts leave you further and further behind, as you also experience the normal range of losing bets. For example, if you were paid at 4 to 1, (equivalent to a one in five chance), on what is really a one in six chance, (e.g., on a dice throw), then after 120 one dollar bets you would expect to be an average of $20 behind.

Over a short period of play the random distribution of results may be more or less favourable toward you, when compared with the average expected loss. It may even give you a short term win, but longer term the chance of gaining results differing markedly from the expected rate of loss diminishes practically to nothing. This patterning of results over time, increasingly reflecting the inherent game structure, has been called the law of large numbers. It is quite possible to understand this without talk of normal curves, distribution theory, and the central limit theorem, which typically are only addressed by advanced mathematics students.

Students can be shown how the difference in pricing between the chance of a winning result and a lower corresponding payout generates guaranteed loss for players over time. Once grasped, extension of the concepts to more sophisticated chance gambling games, like electronic gambling machines is conceptually straightforward, although the detail is almost prohibitively complicated, and sometimes unavailable. We need practical direct education about this. Gambling examples should not be left to individual teachers to offer, nor be buried in mathematical curricula about statistics, which have purposes other than warning about commercial gambling, so that important lessons are not learnt.

Problems in teaching about gambling probabilities

Gambling not covered in textbooks

Mathematics teachers do not always cover gambling in their probability teaching. In their celebrated textbook Goos, Stillman, & Vale (2007), dividing the teaching of mathematical content into six areas, including a large chapter on “teaching and learning chance and data”, have no gambling contexts. Indeed, when mathematics teachers have given gambling examples in teaching probability, it has often been to teach the principles of probability generally, not directed to why one will lose on commercial chance games, not directed to, what is for us here, the relevant application of probability knowledge.

Probability concepts are difficult

Mathematics teaching is not always effective, and probability concepts may be particularly difficult to grasp. There is research which questions the efficacy of knowledge about probability in gambling situations. It has long been held that general mathematical understanding does not
inoculate against common intuitive errors about gambling (Peard, 1991; Ayres & Way, 2001). In recent years, Canadian researchers have addressed these issues (Benhsain & Ladoucer, 2004). Williams and Connolly (2006:62) questioned whether “learning about the mathematics of gambling change[s] gambling behavior?” Australians Lambos & Delfabbro (2005:93) have suggested “a basic understanding of mathematics, statistics or gambling odds is unlikely to be a protective factor in problem gambling because gamblers can pick and choose which information they chose to apply when the information is applied to activities in which they have a personal interest.” As the early investigator of illusions of control held: Pure chance is especially difficult for people to understand and accept, as there is no contingency, no meaning and most importantly, no control (Langer, 1977).

Applying concepts to new contexts can be difficult. If we do not engage students, sometimes quite concretely, they do not always get it. There has been too much assumption that because university students undertake courses about statistical tests, for example, they should understand how those principles would apply in a gambling context. Teaching needs to be explicit and concrete to maximize effective communication.

Teaching about gambling is controversial

Even the type of gambling examples we may use in schools are fraught with controversy, as we overstep the realm of social education, and impinge on parental prerogatives about children’s education (Smith, 2012). However, we do not need to introduce actual commercial gambling games in order to teach about them. Dangers with teaching the mathematics of gambling include introducing gambling to students: just the fun, not the maths. Introducing commercial games; teaching rules, initiates/grooms students for gambling.

Past educative attempts

A wide variety of people have published school level gambling curricula with some mathematics content. Mostly either curricula have been too restricted, like that mandated by Shaffer, the Harvard addictions guru (Shaffer, Hall, & Vander Bilt, 2000), which dealt only with randomness and independence, which is important, but will only explain why you did or did not win on the previous go, not why you are going to lose overall, OR, curricula have been overly large, like the mammoth Queensland Government document (2006), which deals with some of the relevant mathematics, but has weeks or months of other marginally relevant material to wade through. Such curricula do not get to the point effectively, and are too big to be widely adopted by teachers who already have busy teaching programs and crowded curricula. The main points about mathematics knowledge as applied to gambling have not often been effectively taught. In Australia, the Productivity Commission (2010: 9.2), recommended “Given the risk of adverse outcomes, governments should not extend or renew school-based gambling education programs without first assessing the impacts of existing programs.”

Amongst those mathematically inclined, much of my commentary falls into the category of the ‘bleeding’ obvious. The point is that it is not as obvious as some may think, and to be understood, requires teaching. People will not intuit mathematical understandings which took mathematicians hundreds of years to discover. High school education seems to be the most appropriate place.
The classroom activities

We assume that students will receive other theoretical and practical examples about probability much as they do now, but a few sessions may also be usefully devoted directly to understanding why chance gambling produces losses for the players. Interest in the structure of a gambling game may be stimulated by playing one: This is a game of selecting one colour from four, called: Lucky Colours of Sunshine (Smith, 2006, 2011).

If we pay a net prize of twice the single unit bet for a correct colour selection, we get an interesting and viable game. It is more heavily biased against the player than most commercial chance games, excepting large lotteries, Kino and scratch cards, but it is more effective as a demonstration for its simplicity. Played in multiples of twelve its analysis has marvellously simple arithmetic. Participants need to record their colour guesses, the colour that comes out, and the amounts won or lost:

Table 1. Recording Individual Results from playing Lucky Colours of Sunshine

<table>
<thead>
<tr>
<th>Game</th>
<th>Your choice of Colour</th>
<th>Colour picked out</th>
<th>Win or Loss</th>
<th>Running total – number of wins</th>
<th>Running Total – money won or lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>White</td>
<td>Red</td>
<td>Loss</td>
<td></td>
<td>-1 = -1</td>
</tr>
<tr>
<td>2</td>
<td>Blue</td>
<td>Blue</td>
<td>Win</td>
<td>1</td>
<td>+2 → +1</td>
</tr>
<tr>
<td>3</td>
<td>B.</td>
<td>R.</td>
<td>L.</td>
<td>-1 → 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Yellow</td>
<td>W.</td>
<td>L.</td>
<td>-1 → -1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B.</td>
<td>B.</td>
<td>Win</td>
<td>2</td>
<td>+2 → +1</td>
</tr>
</tbody>
</table>

As we play successive rounds, we may enquire as to who has won and who is still ahead. After two goes, we will very likely still have some people who are in front.

If we play twelve times we will get a sufficient spread of results for instructive purposes. After playing 12 times the average expected number of wins is three, giving gross returns of 9 units, thus a net loss of three units. Those who have won less than three times have been unlucky and have lost more than average, those who have won more than three times have been lucky, but still most of the lucky will find that they have not won overall. Doing better than average will not generate an overall win. Only the very lucky who have won more than 4 times will be ahead, and they will be few, and likely not far ahead. Of course, it is theoretically possible that someone has won big, but such freak results are just that.

The collective results of the group may be tabulated, then graphed, as has been done in Figure 1, from a group of 17 people playing 12 games.
This group did slightly worse than average losing just over 3 units on average. While there were 2 winners out of 17, over 12 games the net group result closely resembled expectation. If we begin again and play another set of 12, we will get similar results, except that it is very likely that different people will be amongst the very lucky, thereby demonstrating the inability of the lucky to maintain their status in a random situation.

**Dispelling myths and illusions: Questions to ask during play**

Who is competitive when they play a game? How do you feel about winning/losing? Do you think you can improve if you try hard or practice? Do you think you can influence the colour drawn? Do you think you can do anything to become more successful in guessing the colour? If you are ahead after this set, will you be ahead on the next set?

In playing this game, people may be asked to reflect upon the intuitive illusions about gambling. They may be aware of their competitive spirit in a game over which they can have no influence on the result. They may be aware that they are concentrating, and trying to feel the forthcoming colour. Repeated play will demonstrate that this does not work.

One becomes aware of how powerful the illusions of control are. We desire control of the situation. We are successful as humans, individually and as a species, when we have been able to recognise patterns. However, that is in situations where patterns have causal significance, and hence meaning. But, here we attribute meaning in circumstances without significance; circumstances that are random, and not just casually random, but causally random, i.e., in Australia and many other jurisdictions, by law, EGMs are designed to be random.

At a more advanced level of mathematics teaching, we may predict the distribution of results from the group, and then compare the actual results. If we take our sets of results and average
them all, we shall see that indeed the expected results are achieved. This is incredibly powerful, showing that the mathematical explanation of the randomness of what is happening, concurs strongly with reality, providing a predictive model much more successful than any other stratagem you may favour, such as rituals, talking, petting the machine, holding your body in a particular way, or any contribution that may be made by your deceased relatives or significant others.

**Analysing the game**

Consider the odds. Your chance of winning is one out of four, which is called 3 (against you) to 1 (for you). Your average expectation is one win out of four, three losses out of four. But when you win you are only paid 2 to 1. You would need to be paid at 3 to 1 on your wins to match the outlay on your expected losses. That is, you need a winning prize to equal your expected losses, in a (mathematically) fair game. This mismatch between the payment odds and the real odds is fundamental to commercial chance gambling.

We can calculate the expected average rate of loss, which I much prefer to “return to player” (RTP). When a bank pays you interest, they do not call their return to customer 105%. We should stop talking in this confusing way about loss making gambling games.

\[
\text{RTP} = \frac{\text{Favourable outcomes} \times (1 + \text{Prize})}{\text{Total equally likely outcomes}}
\]

\[
= \frac{1(1+2)}{4} = 75%
\]

Rate of loss \(= 1 - \text{RTP}\)
\(= 25\% \) of turnover

In summary, for the basic teaching:

- **Play the game in sets of 12** over several lessons.
- Record individual results.
- Consider odds.
- Consider average expectation.
- Graph group results, note spread, wins and losses.
- Do it again, a few times.
- Compare individual results; variation and losses.
- Total individual results and average, graph (and compare to Binomial/Normal curve).
- Be amazed that maths all along could tell us what distribution of results we would get from playing this random chance game.

A broader gambling curriculum might contain numerical literacy about social facts of gambling, e.g., national loss rates per capita, its socio-economic distribution, effects on local economies,
etc., and non-mathematical material around risk and quality of life issues, from which there is now plenty to choose.

More background mathematics for higher and lower levels

A decision tree lets us see the range of possibilities from multiple goes. Although each of the 64 sequences of outcomes from 3 goes is unique, some of them share characteristics when viewed as combinations instead of permutations. Thus there are 27 ways to get 1 win from 3 goes, 27 ways to get no wins, and so on, as begun in Figure 2:

![Figure 2. Mapping the Permutations of Wins and Losses](image)

Examining the probability distribution

We may also consider how the probability distribution varies as the sample size, i.e., number of games, increases. The probability of a particular number of wins from a set number of goes is given by the formula:

\[
Pr(w) = \frac{n!}{w!(n-w)!} (p)^w (1-p)^{n-w}
\]

where:
- \( Pr \) = probability
- \( w \) = no. of wins
- \( n \) = total no. of goes
- \( p \) = pr. of a win on each go
- \( 1 - p \) = pr. of a loss

For example:

\[
Pr(0) = \frac{12!}{9!(12-0)!} \left( \frac{1}{4} \right)^0 \left( \frac{3}{4} \right)^{12-0} = 0.0317
\]

The most common outcome for 12 goes is 3 wins (24% of the time), giving a loss of 3 betting units. Mathematics teachers understand standard deviations, so I offer this information to impress you: The formula to calculate a standard deviation in a binomial distribution is given by:

\[
\sigma = \sqrt{np(1-p)}
\]

After 12 goes, there is an average loss of 3 units, with \( \sigma = 4.5 \). Quite a few winners. After 100 goes, average loss of 25, with \( \sigma = 13 \). So, about 3% are still winners. There is an average loss of 100 units for 400 goes, with \( \sigma = 26 \), i.e., a winner is almost 4 standard deviations away. Not a popular place! If we played this game 400 times, a set of 400, we would need a stadium holding 10,000 players to have the likelihood that one or two players, up to a handful, might still be ahead. The likelihood of loss over increasing play is dramatic.

To compare the results from 12 goes and 36 goes, they may be graphed with matching average expectancy and breakeven lines, by varying the scale on the two graphs’ axes. In figure 3, to make the visual comparisons easier, the results are grouped in the 36 games, generating graphs
with equal area under the graphs, i.e., total pr =1. The probability of getting results to the right of
the breakeven line reduces as games increase, and there is increasing crowding around the
average expectancy.

![Probability distribution of game outcomes](image)

**Figure 3. Probability distribution of game outcomes.**

Player results from 36 games could be graphed, with the individual movement of players
between the 12 and 36 game graphs noted. Most players will regress toward the mean. Once the
negative expectation on chance gambling is understood, it is the increasing certainty of that
losing outcome over time which seems crucial to understand. Other resources can round out the
teaching on distributions. Computer simulation of regression to the mean is a useful activity.
Extension to gambling which is not purely chance based, such as sports betting, introduces new
complications, but the difficulty in overcoming a house margin long term remains an important
point.

We can go on to show the expected distribution of wins (and corresponding net outcomes) over
increasingly large periods of play. Here this is shown in a freely available dynamic display using
the maths software program Mathematica (Wolfram, 2010).^6^

All the parameters are variable. Figure 4 below is an example of the output of the software
program to illustrate what it does. This shows the expectations of numbers of wins from 24
games, with the average expectancy and the breakeven line. The number of wins may be counted
along the abscissa. As you increase the number of games played, a decreasing proportion of the
graph is on the win side of the breakeven line.

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^6^ Footnote: This is initially available at http://ftp.physics.uwa.edu.au/pub/Mathematica/VU/Smith/, provided you
first download a free CDF player in order to view it.
Secondly, computers enable us to play the game a vast number of times at the flick of a button. Actual long run outcomes may thus be found in an instant. The long run no longer takes a long time! This second remarkable dynamic display in Mathematica allows you to play the game in sets of 12, from 100 to 100,000 times, up to a total of 1.2 million games, showing the actual distribution of results of sets of 12, compared with expectation. I don’t know the underlying software, but when you play 1200 games you can see variation from average expectation, but when you play 1.2 million games no deviation from expectation is visible. I find it flabbergasting. It is the very point I am trying to make; how reversion to the expected loss will guarantee taking your money over time on all chance games of this sort, such as the pokies (EGMs).

Figure 4. Expected average distribution of results from 24 goes.

Figure 5. Expectation and actual distribution of wins from 100 repeated sets of 12.
In my view, gambling provision is mostly a fraudulent activity, perpetrated on the insufficiently informed. A few other examples of teaching about the basic mathematics of gambling have impressed me. There is a Canadian website on “How Gambling Really Works”, (Addictions Foundation of Manitoba, 2012) which explains the key teaching points about the mathematics of gambling, but is aimed at gamblers, so should be used with discretion. Tim Falkiner (2001) also used visual examples, such as simulation of roulette results on “even-money” (18:19) bets, in his teaching software.

I hope this example of how the mathematics of gambling may be taught by demonstration will stimulate you to have suchlike included in your local curriculum.

**Another ALM song**

The conviviality of ALM conferences has included a lot of singing. Here, with apologies to Terry Britten, Graham Lyle and Tina Turner I offer a chorus very much on the topic. Everybody sing:

> What’s maths got to do with it, got to do with it, when it comes second to emotion?
> Maths has a lot to do with it, lot to do with it, even if second to emotion!

Stand by for further developments...

**References**


Building Capacity in the Adult Numeracy Workforce after IALS:
Developments in Professional Development for Adult Numeracy Education in New Zealand and England

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In this paper we shall outline the present situation with regard to adult numeracy in New Zealand – policy, infrastructure and practice since the International Adult Literacy Survey (IALS) in the mid-1990s - as part of our ongoing international comparative study of adult numeracy in New Zealand and the United Kingdom. We focus in particular on the effort in both countries after IALS to build capacity in the adult numeracy workforce through professional development.

Key words: adult, numeracy, policy, infrastructure, practice, international, comparative education research

An international comparative study of adult numeracy in New Zealand and the UK – the story so far

This paper reports on the latest stage of an ongoing comparative study of adult numeracy education in New Zealand and the UK, with particular reference to England. The study was conceived in 2009 when Diana Coben (from the UK) and Barbara Miller-Reilly (from New Zealand) compared notes about our countries’ attempts to raise the levels of numeracy of the working-age population. We felt that there was much we could learn from each other’s experience and we decided to embark on a systematic comparative study.

We are bearing in mind Hantrais’ (1995) comments on the benefits of international comparative research as our project progresses; these are summarised here as follows:

- Researchers can capitalise on their experience and knowledge of different intellectual traditions to compare and evaluate a variety of conceptual approaches.

- Comparisons can lead to:

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7 Policy with respect to education, including adult numeracy and mathematics education, is different in each of the four UK home nations (England, Wales, Scotland and Northern Ireland); we decided to focus in the first instance on England as it is the largest UK jurisdiction.
• fresh, exciting insights and a deeper understanding of issues that are of central concern in different countries

• identification of gaps in knowledge

• identification of possible directions about which the researcher may not previously have been aware

• sharper focus of analysis of the subject under study by suggesting new perspectives.

• Cross-national projects give researchers a means of confronting findings in an attempt to identify and illuminate similarities and differences, not only in the observed characteristics of particular institutions, systems or practices, but also in the search for possible explanations in terms of national likeness and unlikeness.

• Cross-national comparativists are forced to attempt to adopt a different cultural perspective, to learn to understand the thought processes of another culture and to see it from the native’s viewpoint, while also reconsidering their own country from the perspective of a skilled, external observer.

Our project began formally in 2010 when the Royal Society of New Zealand’s Bilateral Research Activities Programme funded Barbara to visit the UK to develop an international comparative research project between New Zealand and the UK. Diana facilitated this project, enabling Barbara to meet with leading UK practitioners, researchers and other stakeholders. In her paper for ALM17 (Miller-Reilly, 2010) Barbara outlined the state of play in the UK and reviewed relevant UK-based research projects.

In the following year we compared our two countries in the wider international context as shown in international surveys (Coben & Miller-Reilly, 2011). We focussed in particular on the International Adult Literacy Survey (IALS) (OECD & Statistics Canada, 2000), the Adult Literacy and Lifeskills Survey (ALLS) (Desjardins, Murray, Clermont, & Werquin, 2005; Satherley, Lawes, & Sok, 2008) and the Programme for International Student Assessment (PISA) (OECD, 2010) as well as the 2003 and 2011 Skills for Life surveys in England (Harding et al., 2011; Williams, Clemens, Oleinikova, & Tarvin, 2003). Our review confirmed that these surveys show New Zealand and the UK to have similar proportions of the population of working age scoring below the level required for full participation in a modern knowledge economy, while also convincing us of the need for caution in the interpretation of survey evidence, especially where this is based on different definitions of numeracy and associated concepts.

In 2011 Diana moved to New Zealand to direct the National Centre of Literacy and Numeracy for Adults (the National Centre), thereby enabling us to work together from within New Zealand. In 2012 we have focused on a comparison of policy, infrastructure and professional development in adult numeracy education in New Zealand and England. Niki McCartney, Associate Director of the National Centre, with her expertise in professional development in adult numeracy in New Zealand, has joined our team to develop this paper8.

8 Diana Coben and Niki McCartney are writing here in a personal capacity; this paper should not be considered to represent the view of the National Centre of Literacy and Numeracy for Adults.
In the following sections we sketch the policy context in New Zealand and England, with a particular focus on professional development in adult numeracy education.

New Zealand

In New Zealand “Numeracy is the bridge between mathematics and real life. It includes the knowledge and skills needed to apply mathematics to everyday family and financial matters, work and community tasks” (TEC, 2008c, p. 6).

Before IALS was undertaken in the mid-1990s, supporting adults to improve their literacy and numeracy was a somewhat marginal activity in New Zealand. Numeracy tended to be subsumed under the umbrella term of literacy, as shown by the Adult Literacy Strategy launched in 2001 (Walker et al., 2001). That Strategy had three key elements:

- Developing capability to ensure adult literacy providers deliver quality learning through a highly skilled workforce with high quality teaching resources;
- Improving quality systems to ensure that New Zealand programmes are world class;
- Increasing opportunities for adult literacy learning by significantly increasing provision in workplaces, communities, and tertiary institutions.

It was underpinned by four principles:

- gains for learners will be achieved as quickly as possible;
- programmes will match learners’ needs in content and pace;
- best practice, evaluation, and research will guide programme development;
- programmes will be suitable for the wide range of learners.

(Walker, et al., 2001, p. 3)

Despite the apparently exclusive focus on literacy, by the time ALL data were gathered in New Zealand in 2006 (Satherley & Lawes, 2009) a numeracy professional development initiative was underway. The initiative began in 2004 under the auspices of the Foundation Learning Strategy, part of the government’s first Tertiary Education Strategy which ran from 2002 to 2007 (New Zealand Ministry of Education, 2002). In a good example of inter-departmental coordination within government, the Foundation Learning Strategy built on the Adult Literacy Strategy (Walker, et al., 2001) and the Adult English as a Second or Other Language Strategy (New Zealand Ministry of Education, 2002) and echoed the Employment Strategy (Department of Labour, 2000). The goal of the Foundation Learning Strategy was to ensure that foundation learning resulted in gains for learners and, over time, in significantly improved literacy, numeracy and language in the population as a whole. The government funded initiatives as part of the Foundation Learning Strategy under the heading of Learning for Living, one of which focussed on numeracy. The numeracy initiative began as a research and evaluation project covering a number of programmes and developed into a professional development project aimed at growing expertise in the sector.
From the findings of the first round of Learning for Living exploratory projects, three regional clusters with a focus on numeracy were formed in which five to six providers, tutors and managers came together with an external facilitator to introduce new content and teaching ideas. In the first phase (implementation) cluster meetings were held over two days every three weeks during a six-month period. The second phase (consolidation) supported tutors’ new professional knowledge and embedded numeracy teaching practices within classrooms. The third phase (extension) extended the cluster groups to tutors of other organisations with the aim of establishing and strengthening communities of practice that would sustain new numeracy practices. Professional development in these phases was not linked to qualifications. Indeed, before 2006 there were no specialist literacy, language and numeracy (LLN) teaching qualifications in New Zealand but progress has been rapid since then. As noted in *A Literature Review of International Adult Literacy Policies* undertaken by the National Research and Development Centre for Adult Literacy and Numeracy (NRDC):

New Zealand’s adult LLN workforce has traditionally been largely untrained (TEC, 2008c). However, over the last several years, New Zealand has invested heavily in improving its LLN teaching workforce, and has established workforce development as one of its primary policy goals. According to Benseman and Sutton (2008, p. 5), while implementing workplace learning has been New Zealand’s primary focus over the last several years, significant emphasis has also been devoted to building “the infrastructure necessary to support tutor training, develop teaching resources and offer a flexible range of professional development opportunities”.

(NRDC, 2011, p. 37)

The tertiary literacy and numeracy sector began to professionalise in 2006 with the development of two National Certificate tutor qualifications. These qualifications are both at National Qualification Framework (NQF) Level 5 (equivalent to the first year of an undergraduate degree course) and described as “entry level qualifications at the beginning of a qualifications pathway that aims to lift the literacy and numeracy skills of adult learners in New Zealand, across a wide variety of learning settings” (TEC, 2009a, p. 4). The National Certificate in Adult Literacy Education (NCALE) Educator was designed for specialist literacy, language and numeracy tutors and the NCALE Vocational was designed for vocational or workplace trainers to support them in embedding literacy, language and numeracy into their programmes. In 2011 the names of the NCALE qualifications were changed to include numeracy for the first time. They are now known as the National Certificate in Adult Literacy and Numeracy Education (NCALNE) with the suffixes ‘Educator’ and ‘Vocational’, as before. Beginning in 2008, a range of university qualifications was also developed, offering progression for suitably qualified practitioners. Auckland University of Technology (AUT) developed a Masters in Adult Literacy and Numeracy Education (Level 9), the University of Waikato developed a Diploma (Level 6) and a Postgraduate Diploma (Level 8) in Adult Literacy and Numeracy Education and in 2010 Victoria University launched a Diploma (Level 7) in Adult Literacy and Numeracy. The Ministry of Education provided 500 scholarships as an incentive for practitioners to enrol in the National Certificate programmes and further scholarships for university qualifications.

A draft quality mark was completed by the New Zealand Qualification Authority (NZQA) in 2004 and in the following year the Ministry of Education developed key competencies and draft descriptive standards. From 2007 the NZQA Foundation Learning Quality Assurance (FLQA) framework required “all tutors working in foundation learning to be working towards a New Zealand recognised qualification in adult literacy, numeracy or language teaching; or have
relevant experience, that is appropriate to their role” (NZQA, 2007, p. 2) and to undertake continuing professional development. In the event, FLQA was not implemented and in 2010 the TEC and NZQA advised the sector that new literacy and numeracy requirements would replace the FLQA requirements.

Meanwhile, in 2008 the TEC published the Literacy, Language and Numeracy Action Plan 2008–2012, Ako Tūāpapa (TEC, 2008c) which aimed to improve the literacy and numeracy skills of adults by increasing learning opportunities, improving sector capability and building a national infrastructure. In the same year the Skills Strategy Action Plan (Department of Labour, 2008) was launched by the Department of Labour on the basis of work by the Skill New Zealand Tripartite Forum. The Tripartite Forum brought together key stakeholders (government Ministers and officials, Business New Zealand, the New Zealand Council of Trade Unions and the Industry Training Federation) to work in partnership to implement the Skills Strategy “to ensure New Zealand individuals and organisations are able to develop and use the skills needed in the workplaces of the future” (Department of Labour, 2008, p. np). This partnership approach marked a distinct change from previous skills development work undertaken in New Zealand. The Skills Strategy also strengthened the role of Industry Training Organisations (ITOs) “as the interface between industries and tertiary education, ensuring that industries’ skill requirements drive the provision of education and training for industries” (Department of Labour, 2008, p. np).

Meanwhile, the embedded literacy and numeracy resources trialled during the Learning for Living project developed into the Adult Literacy and Numeracy Learning Progressions, published in 2008 by the Tertiary Education Commission (TEC) (TEC, 2008b). The Learning Progressions are designed to indicate what learners know and can do at successive points as they develop their expertise in literacy and numeracy learning and can be used as a guide to identify the next steps for adult learners. They are also used to identify the level of demands of a course and/or programme. The Numeracy Learning Progressions are organised in three strands: ‘Make sense of number to solve problems’; ‘Measure and interpret space and shape’; and ‘Reason statistically’. The Learning Progressions have become a key component of the New Zealand adult literacy and numeracy infrastructure organised around the framework of the ‘Three Knowings’ (‘Know the demands’, ‘Know the learner’ and ‘Know what to do’). The Learning Progressions underpin the Literacy and Numeracy for Adults Assessment Tool (the Assessment Tool) launched in 2010 and now required to be used pre- and post-tuition for all TEC-funded literacy and numeracy programmes at National Qualifications Framework (NQF) Levels 1–3. In addition, all tutors working with learners whose literacy and/or numeracy is assessed at NQF Levels 1 to 3 are required to embed literacy and/or numeracy in their teaching, whatever their specialism. This is a challenging requirement in what has become New Zealand’s embedded literacy and numeracy approach. This means that, for example, vocational tutors are required to teach the literacy and numeracy required for their trades, with professional development support from the National Centre and their organisation. Guidance on embedding, geared to a range of settings, has been published by the TEC (TEC, 2008a, 2008d, 2009b, 2009c, 2009d).

The roll-out of the Learning Progressions required a national professional development programme of work which began in 2008 with regionally-based clusters of Institutes of
Technology and Polytechnics\textsuperscript{12} (ITPs), followed in 2009 by clusters of Private Training Establishments\textsuperscript{13} (PTEs). Each cluster received three days of free professional development in each of numeracy and literacy. There were generous financial incentives to organisations for staff to attend the professional development workshops (NZD 250,000 for each ITP and NZD 75,000 for each PTE). The work programme was ambitious, innovative and progressive - and it was met with resistance from some tutors who were not specialists in literacy and numeracy and who did not see improving learners’ literacy and numeracy as their responsibility.

In 2009 the National Centre was established, hosted by the University of Waikato in partnership with Te Whare Wānanga o Awanuiārangi, funded primarily by the TEC and tasked with building the capability and capacity of the sector through research-informed professional development, research and critical engagement with policy and practice at a national and international level. The National Centre’s numeracy professional development offerings evolved in response to sector need, aligned to a three-phase development strategy as follows: (1) Introduction to embedded literacy and numeracy; (2) Towards a climate of embedded literacy and numeracy; and (3) Sustainable whole organisation embedded literacy and numeracy practices.

The varied experience, capability and capacity of both individuals and organisations called for wide-ranging professional development offerings with a focus on building the capability of educators in developing embedded literacy and numeracy policy and practices across the whole of their organisations. In the period since 2009, these have included:

- Introduction to numeracy
- Developing number knowledge and strategy
- Understanding statistics
- Understanding fractions, decimals, percentages and ratios
- Numeracy Learning Progressions; Intensive numeracy (Entry level)
- Building educators’ confidence in numeracy
- Embedded numeracy in carpentry, horticulture, agriculture, automotive, hairdressing, hospitality, retail and business
- Moving from Step 4 to Steps 5-6 of the Learning Progressions
- Engaging the priority learner groups (Māori, Pasifika and Youth)
- Interpreting Assessment Tool results to inform teaching
- Strategic planning for whole organisation embedded literacy and numeracy
- Curriculum development for embedded literacy and numeracy

\textsuperscript{12} There are currently 18 ITPs in New Zealand http://www.nzqa.govt.nz/providers-partners/about-education-organisations/itps-in-new-zealand/.
\textsuperscript{13} There are currently 629 PTEs in New Zealand http://www.newzealandeducation.com/private-training-establishments.html.
• Literacy and numeracy resource development

In 2010 *Getting Results in Literacy and Numeracy* was published by the TEC, providing an update of what had been achieved since the 2008 Action Plan and outlining the next steps for the tertiary education sector. It reported that the components of a national, professional literacy and numeracy infrastructure are in place, comprising: quality assurance; teaching and learning resources\(^\text{14}\); qualifications and professional development opportunities for educators; and assessment tools and funding systems with a strengthened focus on results. It set out the agenda for the next period, from 2011, when most entry-level tertiary provision would include literacy and numeracy: “By 2013 100,000 tertiary learners will participate in programmes that include literacy and numeracy. This will be achieved within current funding levels using the Investment Planning process.” (TEC, 2010, p. 3).

Accordingly, in 2012 specified funding for literacy and numeracy in tertiary education organisations (TEOs) was withdrawn and literacy and numeracy is now treated as ‘business as usual’ for funding purposes. Also in 2012, the TEC published the *Adult Literacy and Numeracy Implementation Strategy*, which includes this statement on professional development of the sector:

> This workstream remains a key ongoing priority to ensure organisations and educators are well equipped to understand and apply best practice for the optimal benefit of their learners, especially Tertiary Education Strategy priority learner groups. The TEC has supported developing qualifications pathways for literacy and numeracy educators, and contracts the National Centre of Adult Literacy and Numeracy to provide professional development and act as a focal point for the sector. While the literacy and numeracy infrastructure and funding opportunities are in place, continuous improvement through professional development is needed to drive further success.

(TEC, 2012a, p. 5)

The TEC also published its proposed *Indicators for Literacy and Numeracy Provision and Gain* (TEC, 2012b), including a measure of the difference between initial and progress scores from the Literacy and Numeracy for Adults Assessment Tool (LNAAT or the Assessment Tool), where that is statistically significant, to be known as the ‘Literacy and Numeracy (LN) Gain Indicator’.

In the same period, the National Centre’s professional development has moved to a ‘whole organisation’ approach:

> Where literacy, language and numeracy provision is central to the whole organisation at all levels, ranging from strategic leadership and management to delivery of practice. This includes embedding literacy, language and numeracy in teaching and learning programmes across a range of learning aims and goals and providing all learners with opportunities to progress and achieve qualifications.

(Skills for Life Support Programme, 2010, p. 2)

This approach provides a good practice framework for the development of policy, strategy and processes within organisations to ensure that literacy and numeracy concerns permeate the

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\(^\text{14}\) The “teaching and learning resources” referred to in *Getting Results* were augmented in 2011 by Pathways Awarua, an online literacy and numeracy learning tool accessible through the National Centre’s literacyandnumeracyforadults.com website.
organisation and are not regarded only as the tutor’s responsibility. Whole organisation initiatives include: explicit literacy and numeracy targets and goals, embedded literacy and numeracy induction modules for all new staff; reworked organisational processes and documentation, including curriculum development and registration-related processes; conversations with learners about their assessments and results; improved student management systems; reviewed human resource and stakeholder engagement processes, and self-assessed reviews of organisations’ practices with respect to literacy and numeracy, aligned with the New Zealand Qualifications Authority’s (NZQA) External Evaluation and Review (EER) processes.

Although progress in adult numeracy (and literacy) across the tertiary education sector as a whole has not been formally evaluated, it is clear that New Zealand has come a long way since Learning for Living in 2004. A ‘New Zealand approach’ to embedding literacy and numeracy is emerging and becoming mainstream across the sector. Tutors have worked hard to improve their literacy and numeracy teaching practice and deepen their understanding of embedded literacy and numeracy pedagogy. The sector is professionalising, tutor capability has improved and understanding has increased and more people are becoming qualified in adult numeracy and literacy. However, as noted above, there is still no requirement for tutors to be qualified. Ako Aotearoa, the National Centre for Tertiary Teaching Excellence, is currently (2012) undertaking a consultation about a possible voluntary accreditation scheme for tertiary educators of all subjects.

Numeracy is no longer marginalised in New Zealand and its profile is undoubtedly much higher than in 2004. The Learning Progressions framework, with its common language to describe numeracy and literacy, has raised awareness across the tertiary education sector of numeracy as a domain in its own right. Numeracy-focused programmes in workplaces are improving individuals’ numeracy skills and practices and in some cases seem to be contributing to improved company productivity, something the National Centre will be investigating in the coming months. Doctoral and other postgraduate study and research in adult numeracy is underway and successful practitioner Action Enquiry projects in numeracy have been undertaken, with results shared with the sector at a Workplace Literacy and Numeracy Hui (meeting) in Auckland in 2012. This meeting was part of the National Centre’s 2012 Symposium, which was aligned with the ALM19 conference, thereby bringing the opportunity for the first time to New Zealand practitioners and researchers to attend, on their home turf, the only international conference geared to adults learning mathematics.

England

In England numeracy covers the ability to:

- understand and use mathematical information
- calculate and manipulate mathematical information
- interpret results and communicate mathematical information.

(DfES, 2000)

In response to the UK’s poor performance in IALS the government commissioned a Working Group chaired by Sir Claus Moser to advise on ways in which the government’s plans for basic

15 see http://www.nzqa.govt.nz/providers-partners/external-evaluation-and-review/
skills provision for adults can be supported and developed to achieve the target to help 500,000 adults a year by 2002. The Moser Report was published in 1999 (DfEE, 1999) and led to the launch of the *Skills for Life* strategy to improve adult literacy and numeracy in England (DfEE, 2001). As in New Zealand, an infrastructure for literacy and numeracy education was developed. In England this took the form of national standards, curricula and qualifications for learners in adult literacy, numeracy and ESOL at Entry level, Level 1 and Level 2, with targets for the numbers of learners to achieve specified levels by specified dates; national tests and diagnostic assessment materials; quality assurance and funding regimes; professional qualifications for those teaching and supporting adult literacy and numeracy learning; and the establishment of the National Research and Development Centre for Adult Literacy and Numeracy (NRDC). Free literacy, language and numeracy tuition was made available to all adults in England without a Level 2 qualification.

Ambitious Public Service Agreement (PSA) targets were set from the outset with the literacy and numeracy of 2.25 million adults in England to be improved between the launch of *Skills for Life* in 2001 and 2010, and a milestone of 1.5 million in 2007. This was part of a wider government effort to tackle adult skills gaps by increasing the number of adults with the skills required for employability and progression to higher levels of training. In 2006 the targets for 2020 set out in the Leitch *Review of Skills: Prosperity for All in the Global Economy - World Class Skills*, which covered the UK as a whole, included an ambitious target for “95 percent of adults to achieve the basic skills of functional literacy and numeracy, an increase from levels of 85 percent literacy and 79 percent numeracy in 2005” (Leitch, 2006, p. 3). The process for measuring progress towards the 2011 *Skills for Life* PSA targets is set out in a government document published in 2010 (The Data Service, 2010).

National standards for adult literacy and numeracy were established in 2000 (QCA, 2000), with ICT (Information and Communication Technology) added in 2005 (QCA, 2005). The standards follow a common format and relate to the pre-existing Key Skills of Communication, Application of Number and ICT (Glover, 2003). They also match the National Curriculum requirements for English, mathematics and ICT for state schools (see [http://www.education.gov.uk/schools/teachingandlearning/curriculum](http://www.education.gov.uk/schools/teachingandlearning/curriculum)) and (for ICT) the National Occupational Standards for Information Technology ([http://nos.ukces.org.uk/Pages/index.aspx](http://nos.ukces.org.uk/Pages/index.aspx)).

The national adult literacy and numeracy curricula are aligned with the national standards and organised in five steps: Entry levels 1, 2, and 3, progressing to Levels 1 and 2, with Level 2 broadly equivalent to the level of basic skills expected to have been achieved at age 16 (DfES, 2001a, 2001b). An *Adult Pre-entry Curriculum Framework for Literacy and Numeracy* (DfES & BSA, 2002) caters for adults with learning difficulties and disabilities. The Adult Numeracy Core Curriculum covers Number, Measures, Shape and Space, and Handling Data (DfES, 2001b). The *Numeracy Progression Overview* shows how learners’ numeracy skills can progress across the levels (Excellence Gateway, nd).

Adult literacy and numeracy qualifications for learners geared to the national standards and the curricula were also established as a key part of the *Skills for Life* infrastructure. These are being phased out from August 2012 in favour of new Functional Skills qualifications in English, mathematics and ICT, rolled out from September 2010. The Functional Skills Support Programme (FSSP) for post-16 providers closed at the end of August 2010. FSSP was the

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16 [see http://www.skillsforlifenetwork.com/functional-skills](http://www.skillsforlifenetwork.com/functional-skills)
successor programme to (in turn) the *Skills for Life* Quality Initiative (SfLQI), the *Skills for Life* Improvement Programme (SfLIP), the *Skills for Life* Support Programme (SfLSP) and the Learning and Skills Improvement Service (LSIS). LSIS is being phased out but it will continue to provide functional skills advice, materials and resources for post-16 providers until August 2013.

Diagnostic assessment materials for literacy, language and numeracy at Entry level 1 to Level 2 were published in 2002, with additional materials for assessing learners on the Pre-Entry curriculum framework and materials for assessing the need for support for dyslexia. The numeracy diagnostic assessment materials include a tutor guide together with learner materials from Pre-Entry Milestones 5-8 to Level 2, a calculator checklist, analysis of numeracy errors and guidance for structured interviews (DfES, 2002a).

Unlike New Zealand, the UK did not participate in ALLS. Instead, a national survey of adult basic skills was undertaken, with results published in 2003 (Williams, et al., 2003). The survey showed that of the adult population aged 16–65 in England, 15 million (47 percent) have numeracy skills at or below Entry level, with 6.8 million of these classified at Entry level 2 or below. We pointed out the difficulties in aligning the outcomes of this survey with ALL in our paper for ALM18 (Coben & Miller-Reilly, 2011), nevertheless, it appears that New Zealand has a similarly serious problem with low levels of adult numeracy.

*Skills for Life* was ‘refreshed’ in 2009 with the publication of *Skills for Life: Changing lives* (DIUS, 2009). This updated the original strategy from 2001 and aligned *Skills for Life* with the reforms associated with the Leitch Review of Skills (Leitch, 2006). The document set out a new focus on improving employability, a strategy for raising demand for literacy and particularly numeracy skills, and new proposals to increase the flexibility and responsiveness of *Skills for Life* provision, including the intention that by 2011 all Skills Funding Agency-funded providers would offer embedded literacy, language and numeracy through adopting a whole organisation approach (Skills for Life Support Programme, 2010, p. 2). The priority learner groups identified within the refreshed strategy are: people who are unemployed and on benefits; low-skilled adults in employment; offenders in custody and those supervised in the community; and other groups at risk of social exclusion (DIUS, 2009, pp. 7-8).

Meanwhile, the NRDC was established in 2002 as an independent university-based research and development consortium dedicated to conducting research and development to improve literacy, numeracy, language and related skills and knowledge. For the first time funding was available for research in adult numeracy and NRDC researchers took up the challenge, with the first major review of research and related literature in the field (Coben et al., 2003). As well as many studies covering numeracy alongside other domains, including major studies of learners (Rhys Warner & Vorhaus, 2008) and teachers (Cara, Casey, & Mallows, 2009), numeracy-specific studies followed. These included: effective practice in numeracy teaching and learning (Coben et al., 2007), an investigation of adult learners’ expectations of numeracy classes and their motivations for returning to study numeracy (Swain, Baker, Holder, Newmarch, & Coben, 2005); measurement (Baxter et al., 2006); formative assessment (Hodgen, Coben, & Rhodes, 2010); a review of initial assessment tools for adult numeracy (Brooks, Coben, Davey, & Rhodes, 2006) and Maths4Life (see below).

These studies shed light on various aspects of adult numeracy teaching and learning, including this snapshot of adult numeracy teaching and learning in England in the mid-2000s from the Effective Practice in Numeracy Teaching study. The study found that:
The most common form of organisation was whole-class and learners working individually. There was less group or collaborative work, and it was less typical to find learners working with, and learning from, each other.

Most teachers followed a set scheme of work and it was rare for them to incorporate learners’ personal interests. The main pedagogical approach employed was for teachers to show learners procedures, breaking concepts down into smaller parts and demonstrating examples. Resources were generally used to enhance and support learning. The use of worksheets was widespread, and there was little use of practical apparatus, games or ICT.

Teachers generally had adequate subject knowledge, gave clear explanations and provided a variety of learning activities. It was less usual for teachers to differentiate work, make connections to other areas of mathematics, or ask higher-order questions to encourage higher-level thinking or to probe learners’ blocks and misconceptions.

Mutual respect between teachers and learners was high, and learners felt free to express themselves. Teachers were invariably enthusiastic and gave learners lots of praise and encouragement. They also usually monitored learning and gave learners feedback.

On the whole, learners were generally highly engaged; they were often challenged and stretched; they were given time to gain understanding, and the majority had their individual needs met.

(Coben, et al., 2007, pp. 44-45)

On the basis of these studies teaching guides and teaching and learning materials were developed, for example, on measurement (Tomlin, nd) and using calculators (Newmarch, Rhodes, & Coben, 2007). From 2004 to 2007 the NRDC Maths4Life project aimed to stimulate a positive approach to teaching and learning in numeracy and mathematics. Maths4Life combined research and practice to produce resources that provide guidance for all those involved in the delivery of numeracy and non-specialist mathematics (Maths4Life NRDC, nd). The research report, Thinking Through Mathematics (Swain & Swan, 2007) studied the impact on teachers and learners of the introduction of teaching approaches and resources which emphasised connected and challenging ways of teaching mathematics to post-16 learners and gave the research background to the resources. In 2007 Maths4Life transferred to the National Centre for Excellence in Teaching Mathematics (NCETM, https://www.ncetm.org.uk/).

In 2011 a second Skills for Life Survey was undertaken (Harding, et al., 2011). This showed a decrease in numeracy levels, with the proportion of respondents classified at Entry level 3 or above having declined from 78.6 percent in 2003 to 76.3 percent in 2011 and the proportion of respondents classified at Entry level 2 or below having increased from 21.4 percent to 23.7 percent.

Also in 2011, Professor Alison Wolf’s Review of Vocational Education was published (Wolf, 2011). This introduced principles to guide study programmes for young people on vocational routes post-16 to ensure that those who had not secured a good pass in English and mathematics GCSE (the examination normally taken at age 16 in England) continue to study those subjects and gain skills leading to progression into employment or further learning.

Professional development in adult numeracy in England should be seen against this background. Before 2007, staff were not required to undertake professional formation but it was encouraged by managers and employers as a demonstration of currency of teaching practice. Numeracy
professional development from the 1970s to 1990 was extremely patchy. With some notable exceptions, what little there was tended to reflect “the practice-based, atheoretical focus of numeracy teaching – questions of ‘how’ took precedence over questions of ‘why’” (Coben & Chanda, 1997, p. 380). In 1990 a national accreditation framework was launched, offering professional training opportunities for numeracy tutors in England and Wales funded by local education authorities (LEAs) and the Adult Literacy and Basic Skills Unit (ALBSU). Less training was available (or demanded) for numeracy than for literacy and numeracy remained the ‘poor relation’ of literacy in the period up to and following IALS (Coben & Chanda, 1997).

Following the inception of Skills for Life, the government’s Adult Basic Skills Strategy Unit, in conjunction with its partners, introduced a new framework of specifications and qualifications for teachers and those who support the teaching and learning of adult literacy, numeracy, and ESOL. The ‘Key Messages’ of The Skills for Life Teaching Qualifications Framework User’s Guide are that:

For the first time, the expertise of these specialist teachers is being formally recognised through qualifications required of new specialists joining the profession.

The new specialist qualifications:

- recognise discrete roles, responsibilities and skills needs required by those who teach and support learning of adult literacy, numeracy and ESOL;
- promote clear pathways of progression and personal development;
- complement, and are required in addition to the generic initial teaching qualifications.

In November 2002 Success for All (DfES, 2002b) reinforced the principle that all teachers should be qualified to teach. The Department is committed to securing an appropriately qualified workforce in all contexts.

(DfES, 2003, p. np)

Despite this initiative, an inspectorate report in 2003 found that expertise in teaching numeracy was lacking and it was too often taught by rote rather than by understanding numerical concepts (ALI/OfSTED, 2003). In the following year, the Smith Report, the first major report on mathematics teaching since the Cockcroft Report of 1982 (DES/WO, 1982), acknowledged that teaching and learning adult numeracy is challenging and demanding for both teachers and learners (Smith, 2004). In the same year, at the ALM11 conference, Terry Maguire and John O’Donoghue noted that few countries had a comprehensive national system of provision for the training and development of the adult numeracy teaching workforce (Maguire & O’Donoghue, 2005).

In 2007 generic teaching qualifications for those teaching in the lifelong learning sector were introduced, comprising: an initial award, Preparing to Teach in the Lifelong Learning Sector (PTTLS); the Certificate in Teaching in the Lifelong Learning Sector (CTTLS); and the Diploma in Teaching in the Lifelong Learning Sector (DTTLS); alongside subject-specific qualifications for teachers of literacy, numeracy and English for Speakers of Other Languages (ESOL); numeracy teachers are required to have Level 3 Mathematics (BIS, 2007). The 2007 Regulations introduced professional status to the non-university tertiary education sector (known as the
lifelong learning sector or Further Education (FE) and skills sector): Qualified Teacher Learning and Skills (QTLS) and Associate Teacher Learning and Skills (ATLS). QTLS status (for which DTLLS is a pre-requisite) is required for all new teachers in a full teaching role. ATLS status is required for all new teachers in an associate teaching role; CTLLS is required for ATLS (Skills for Business, 2008). The 2007 regulations for teachers and trainers in Further Education (FE) set out requirements for 30 hours continuing professional development (CPD) a year for full-time teachers, pro rata for part-time teachers, with a minimum of six hours a year. From 1 April 2012, holders of QTLS status are also qualified to teach in maintained schools. The Evaluation of the Further Education Teachers’ Qualifications (England) Regulations 2007 found that providers considered that having an equivalent to Qualified Teacher Status (QTS) in the school sector gives a clear message that the professionalism of FE providers is on a par with that of schools (GHK Consulting, 2012).

Alongside these developments the Institute for Learning was formed by teachers, trade unions and employer bodies as an independent and practitioner-led professional membership body, with voluntary membership for teachers, trainers and assessors in the FE and Skills sector. Under the 2007 Regulations, newly qualified teachers had to register with the Institute for Learning in order to become a Registered Teacher. They were then required to become a Qualified Teacher by first completing the PTLLS award and then completing either the Certificate (CTTLS) or Diploma (DTTLS) (depending on their teaching role). They would then need to undergo professional formation in order to become a Licensed Practitioner.

As noted in Jackie Ashton and Graham Griffiths’ paper for the ALM16 conference, the academic year 2008/09 saw a growing focus on numeracy in the Skills for Life Improvement Programme (SfLIP), with the development of a range of programmes for numeracy, including targeted sessions for adult numeracy teachers (Ashton & Griffiths, 2010). The authors noted that although large numbers of documents, teaching, learning, assessment and professional development resources and materials have been produced, such as the Maths4Life pack of strategies for teaching and learning (Maths4Life NRDC, nd),

Yet there is concern that these resources have not yet had the impact that was intended.

The teachers in our study have shown that they value such resources but that the existence of these resources is not enough. We note that the field as a whole contains a wide range of practice and experience and that practitioners can learn from each other. What they value is the opportunity to meet and discuss the use and development of resources and ideas. It would therefore seem important to give as much opportunity for this to happen as possible. The requirement for teachers to undergo 30 hours of professional development each year would seem to help such a position although the cost of organising such meetings beyond one institution may be a threat.

(Ashton & Griffiths, 2010, p. 46)

In 2009 NRDC published its report Teachers of Adult Literacy, Numeracy, and ESOL: Progress towards a qualified workforce (Cara, et al., 2009). The report found that in both numeracy and literacy there has been an increase in the number of teachers working in the sector and in the amount of provision. There had also been a steep rise in the percentage of teachers across all three subject specialisms who are deemed ‘fully qualified’, from 35 percent to 48 percent. Nevertheless, the report found that there was still much to be done, with almost one-fifth (18 percent) of numeracy teachers having no teaching qualifications. Work-based learning providers, in particular, were found to employ large numbers of part-qualified or unqualified teachers. Key
Skills and Functional Skills teachers had a lower proportion of both ‘fully qualified’ and ‘unqualified’ teachers. Over 90 percent of teacher training providers reported that they had no extra capacity to increase their provision due to staffing, funding and recruitment issues. Across England there were big differences between the availability of teacher training courses across the regions, but a consistently large disparity between the supply of subject-specific teaching qualifications and integrated pathways to achieve those qualifications. Overall, the report found that the supply of courses was sufficient to qualify literacy and numeracy teachers in the workforce who still needed a subject-specific teaching qualification within a period of two years and those who are unqualified and need an integrated pathway within five years. However, looking ahead to 2020, the report calculates that:

the number of literacy teachers needed to meet the World Class Skills targets will be at least 7,400 for literacy and 12,000 for numeracy. From this initial analysis it is clear that, while for literacy a shift in the balance of provision may be sufficient to meet current and future demand, for numeracy there needs to be a major increase in the availability of all types of teacher training.

(Cara, et al., 2009, p. 6)

Meanwhile, an earlier report by some of the same NRDC team focused on the teaching qualifications of numeracy teachers, their personal skill levels in mathematics and English, and how these qualifications relate to the progress of their learners (Cara & de Coulon, 2008). The summary report’s conclusions are worth quoting in full:

Making use of comprehensive and unusually rich data sets we are able to make a number of distinctive and new claims with regards to how teacher qualifications affect learners’ progress in adult numeracy.

• Experience matters. Number of years’ experience teaching numeracy was found to positively affect learners’ progress in and attitude to numeracy.

• Subject knowledge is also of prime importance. There was dissent in the field at the introduction of the entry requirement of personal maths skills at Level 3 for courses leading to numeracy teaching qualifications. It was argued that Level 3 was too high and that it would discourage people from applying. However, this research strongly endorses the requirement. Learners’ improvements in numeracy were mostly due to teachers who held qualifications in maths at Level 3 and above. No effects on improvements were detected for numeracy teachers holding qualifications at Level 2 compared to those teachers who did not hold this qualification.

• There was also a positive effect where teachers held numeracy qualifications at Level 6 or above. What’s more these teachers also impacted positively on the attitude of their learners to maths use in their everyday life. However, they also appeared to impact negatively on the confidence of their learners in their numeracy skills after their course had finished.

(Cara & de Coulon, 2008, p. 7)

In 2011 Lifelong Learning UK (LLUK) undertook a review of qualifications for learning professionals in England in two phases (LLUK, 2011a, 2011b). This found that the professionalization of the sector has continued apace, with 57.2 percent of teaching staff in
Further Education (FE) colleges holding a Level 5 or above teaching qualification in 2009-10, compared with around 52 percent in 2006-07.

From November 2012 LLUK’s successor organization, the Learning and Skills Improvement Service (LSIS) is undertaking a further review with the overall aim of producing qualifications that: have been simplified and renamed; are widely valued and taken up by employers and practitioners; are used by the Further Education (FE) sector to evidence their commitment to quality; and are widely recognised as supporting career progression (LSIS, 2012). The new qualifications will be introduced from September 2013 onwards. This independent review is linked to the government’s reform of the FE and skills system for adults aged 19 and over in England, New Challenges, New Chances (BIS, 2011).

Meanwhile, the final report of the independent review panel established by the government’s Department of Business, Innovation and Skills (BIS) and led by Lord Lingfield, on Professionalism in Further Education, was published in October 2012. The Panel’s findings include the following statement:

We find that lecturers teaching remedial literacy and numeracy, and those working with students with learning difficulties or disabilities, cannot be regarded as relying heavily on past qualifications or experiences as the basis of their practice, but that they should achieve pre-service or early in-service specialist qualifications to at least the level of our recommended new Certificate in Further Education; we trust that the Learning and Skills Improvement Service (LSIS) will take this observation into account in its review of qualifications.

(BIS, 2012b, p. 2)

The report goes on to state:

Remedial provision we hope to see gradually cease as a major function of FE, as soon as the government’s current reforms make this practicable, leaving schools to deal more effectively with foundation skills.

(BIS, 2012b, p. 2)

Whether this turns out to be unduly optimistic remains to be seen.

Also in 2012 a prospectus for a new body, the Further Education Guild, has been published (BIS, 2012a). This is envisaged by the government as an employer-led partnership providing a focal point for all FE and Skills sector interests in taking forward the professionalism flowing from the Lingfield Review (BIS, 2012b). The prospectus states that key functions and features of the Guild are likely to include:

- acting as an overarching body with end to end responsibility for professionalism and vocational education across the sector, including to:
  - own professional standards and codes of behaviour for members;
  - develop appropriate qualifications for people working in the sector through which people can progress; to support individual, subject specific and corporate CPD;
working at a strategic level to help bring in expertise across the sector; and

support employer recognition of professionalism;

• offering institutional and individual membership, both of which would be on a voluntary basis:

  o corporate membership of the Guild, entailing commitment to standards of workforce professional development and qualifications.

  o for the individual there would a strong emphasis on support development and progression through high impact CPD fully recognised and linked to additional higher level qualifications at Level 5 and 7;

• seeking to enhance the reputation and status of the sector as a whole through providing a single, collective focus for raising standards of professionalism and being a custodian of excellence. Again, this would be closely linked to individual colleges and providers being able to obtain “Chartered” status as a public stamp of recognition;

• an employer-led partnership drawing in employee representative organisations and sector bodies concerned with workforce development. Development proposals will need to consider the form of organisation needed for a future Guild and how this would be best constituted in FE, given the current landscape.

(BIS, 2012a, p. 7)

IfL is a partner to the Guild, with other independent membership bodies including the two lead partners: the Association of Colleges (AoC) and the Association of Employment and Learning Providers (AELP).

Some points of comparison

Similar IALS results have led to government responses in both New Zealand and England aiming to raise the literacy and numeracy of working age adults from similar base levels. Adult numeracy and literacy are moving from the margin to the mainstream in both countries, with some alignment with the school sector. The importance of numeracy is now more widely recognized in both countries and the standardisation of policy and infrastructure across numeracy and literacy has raised the profile of numeracy.

Policy stability helps. The New Zealand policy framework and infrastructure are still emergent but coherent against the background of a relatively stable policy environment. England’s equivalent policy framework and infrastructure are longer-established but there has been a churn of agencies and qualifications pathways since 2001. The situation is still changing fast, with the shift to Functional Skills underway and the outcome of the LSIS FE Teacher and Trainer Qualifications Review to come in 2013, the year in which LSIS will close.

New Zealand also has a single post-compulsory (tertiary) education sector, while tertiary education in England is divided between the FE and Skills sector and the Higher Education (HE) sector, with adult numeracy and literacy located in FE, traditionally regarded as the Cinderella sector (Norton, 2012). A unified tertiary sector does not in itself produce parity of esteem between the sub-sectors, but it may help.
There are some close links and an interchange of ideas between New Zealand and UK-based professionals and academics active in the field (including our project). However, there are significant differences in our national contexts, most notably in the size of population. New Zealand has an estimated resident population of 4.44 million in 2012, of whom approximately two thirds are of working age (Statistics New Zealand, 2012), while the population of England was 53,000,000 in 2011, with an estimated working-age population of just over 38 million in 2010 (ONS, 2012). In both countries, priority is given to those for whom there is evidence of serious literacy and numeracy problems, although the mechanisms for prioritization and the priority categories are somewhat different.

Thanks largely to the research funding made available from government and other sources through NRDC, much more research and development has been undertaken in England than in New Zealand and indeed many New Zealand research reports cite NRDC research extensively. Hence much more is known about learners and educators and about effective practice and professional development in England than in New Zealand.

The New Zealand approach to embedding literacy and numeracy is distinctive and ambitious, with implications for whole organisation support and professional development. It is based on NRDC research (Casey et al., 2006) but it differs from the integrated model found to be most effective in that study. In fact the NRDC study found that

> Where a single teacher was asked to take dual responsibility for teaching vocational skills and LLN, the probability of learners succeeding with literacy and numeracy qualifications was lower.

(Casey, et al., 2006, p. 6)

This is exactly what is expected of tutors in New Zealand. As Benseman and Sutton note in their *Synthesis of Foundation Learning Evaluation and Research in New Zealand Since 2003*:

> Research into integrated provision is important if we are to be confident that our models of integrated practice are in fact achieving the anticipated learner LLN skills gain that UK research has demonstrated to be possible. Specialist LLN teaching expertise is required to enable the integration of LLN into vocational programmes. We need to understand what can be realistically achieved by upskilling vocational tutors, investigate the extent to which integrated programmes explicitly teach literacy, language and numeracy and understand the nature of the specialist support required.

(Benseman & Sutton, 2007, p. 8)

A small practitioner action enquiry study of embedded literacy and numeracy is underway in the National Centre but further research is needed in order to answer Benseman’s and Sutton’s call.

Both New Zealand and the UK are participating in the successor to the IALS and ALL surveys, the Programme for the International Assessment of Adult Competencies (PIAAC) (OECD, 2004) with results for New Zealand expected in 2016 and the UK in 2013. This may give a welcome boost to efforts to improve adult numeracy in both countries.
What have we learned about building capacity in the adult numeracy workforce?

From the NRDC evidence it is clear that professionalisation is an issue of central concern and one that is key to building capacity in the adult numeracy workforce. In both countries, issues remain around building teachers’ knowledge, skills and understanding of mathematical content and pedagogy, especially in embedding numeracy in vocational and other areas.

Professional qualifications pathways have been established but are not yet a requirement for teaching adult numeracy and literacy in New Zealand while in England there has been a major shift since 2007 towards professionalising the adult literacy and numeracy workforce as part of wider FE and Skills sector reform involving credentialisation, and the introduction of qualified teacher status linked to qualifications. The UK government has made clear its intention to simplify teaching qualifications in the FE and Skills sector; much will depend on what form that simplification takes.

As Sylvia Johnston noted in her paper on professional development for the adult numeracy workforce presented at the ALM14 conference, adult numeracy teaching is at best still only a semi-profession (Johnston, 2008). She identified a number of factors that need to be taken into account in considering the future planning of and research into professional development of the workforce:

a) We do not really know who comprises the workforce. Encouraging governments to collect data on this would appear to be an important step.

b) We do know that workforce is diverse, and we do know that professional development crucially must meet their individual needs. This has to imply flexibility of provision.

c) An approach being developed by some countries is the formal professionalisation of the workforce. For some this may provide advantages, for others disadvantages. A starting point could be greater universality of initial training provision.

d) Such formalisation leads to the creation of formal qualifications. In a context where much development happens in informal contexts through informal mechanisms, we need to find effective ways to formalise the informal without destroying its nature.

e) Professional development can meet individual personal needs of the teacher but it also meets both social and economic needs. This can lead to tensions.

f) Some approaches to development are seen as more effective than others by providers and researchers, but these may not be the same as those preferred by teachers themselves. Approaches which focus on short workshops remain the most common.

g) The content of development may focus on mathematics or on pedagogy or both. Rarely does content appear to address issues of belief, values and self efficacy despite these issues being acknowledged as crucial for personal and team development.

h) There is an implicit assumption in most of the discussion about teacher development that teacher change is sought. We need to be clear about what we are seeking to do in promoting professional development and to acknowledge that at times, enhancing current practice may be more effective an outcome than changing practice.
We know almost nothing about the impact of professional development on student learning. Evaluating effectiveness must surely take account of this dimension.

On the basis of our research we echo Johnston’s call. In saying that, we are mindful that with respect to her point (a), for example, while NRDC research has produced data on adult numeracy teachers in England, equivalent data are not available in New Zealand; effective planning for professional development surely requires that such data are routinely available to all who need it. We strongly endorse Johnston’s final point about the importance of evaluating the impact of professional development on student learning.

We believe that professionalisation in adult numeracy should: be based on evidence-informed professional development with clear pathways to qualifications; be critical, internationally aware and geared to the cultural and historical context; build on practitioners’ experience, enthusiasm and commitment; and develop practitioners’ subject knowledge. All this, we believe, should be seen as part of a whole organisation approach to support for adult numeracy and literacy, geared to improving outcomes for learners.

Conclusion

Our review of the development of adult numeracy in New Zealand and England since IALS has given us the opportunity to look back at a period of recent history that we have all lived through and to see it to a certain extent through each other’s eyes. In our study we are constantly balancing attention to detail with the need to discern broad trends, such as the movement of adult numeracy education from the margin to the mainstream and the professionalization of the adult numeracy workforce. Some things are easily recognisable in each other’s histories, systems and approaches; others we have had to explain to each other in some detail. The use of common terms can be misleading when they signify something different in each country, for example, ‘embedding’ numeracy and literacy is developing in a radically different way in New Zealand and England. Overall, we concur with Hantrais’ view that comparisons can lead to “fresh, exciting insights and a deeper understanding of issues that are of central concern in different countries” (Hantrais, 1995).

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Academic Numeracy as a Framework for Course Development

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This paper defines academic numeracy around three features: competence, confidence and critical awareness of both the context of mathematics and students’ own relationship with mathematics. It then uses this definition to frame a course and assessment for 1st year nursing students to develop their mathematics skills needed for their degree.

Introduction

What does it mean for nurses to be numerate? How can universities assist in increasing nursing numeracy so students become more competent, confident and aware of the mathematics needed in their future career? This paper will first define numeracy in the university setting and then describe one course developed, at the University of Southern Queensland, around this concept of numeracy in terms of competence, confidence and critical awareness. It will show the results of some data from an evaluation of the effectiveness of the course in terms of these three factors and the consequent improvements to the next iteration of the course.

Academic Numeracy

There have been numerous discussions on the definition of numeracy (Coben et al., 2008; Ellerton, Clarkson, & Clements, 2000; FitzSimons & Coben, 2009; Galligan & Taylor, 2008; Kemp, 1995, 2005; Maguire & O’Donoghue, 2002; Miller-Reilly, 2006). From these discussions five main elements emerge that are important in the context of numeracy for nursing students:

1. numeracy is more than just basic skills,
2. it is different from school mathematics,
3. it is embedded in context and may have multiple layers,
4. it may change over time, and
5. it involves both competence and confidence.

Maguire and O’Donoghue (2002) provided a useful framework to conceptualise the numeracy maze (Figure 1). This framework exemplifies the first and third element of numeracy.
Phase 1 reflects the basic (simple) numeracy model. In this phase, for example, fractions, decimals and percentages are revised. This approach is rejected by most in the numeracy field as much of what is needed may be basic mathematical skills, but the approach may be very different in context. On the surface it may appear that a nurse would use proportional reasoning, for example, to calculate a dose, but in reality a variety of approaches could be taken. Phase 2 corresponds to functional literacy and quantitative literacy. So, for example, fractions, decimals and percentages are couched in contextual examples of drug calculations. However, numeracy is more complex than that. Maguire and O’Donoghue (2002) are joined by others in the area to agree that numeracy is both sophisticated and multifaceted (Phase 3). As well as mathematics, it includes cultural, social, personal and emotional aspects. Here, for example, what are nurses’ past emotions about and confidence in studying mathematics? Do they understand the implications of their numeracy knowledge? What are their experiences and knowledge in nursing already that can be shared by the student nursing community?

The second of the five main elements of the definition of numeracy, that it is different from school mathematics, can be summarised in FitzSimons’ (2006) work. She brought a new dimension to the numeracy debate by focusing on Bernstein’s concepts of vertical and horizontal discourse. While vertical discourse centres on mainly school mathematics, horizontal discourse is closely linked to numeracy as it is related to on-going practices; is affective; has specific immediate goals and is highly relevant. She emphasised that these discourses are different with different practices and that vertical discourses will not guarantee numerate activity.

The third and fourth element of numeracy relates to numeracy being “dynamic and contextually bound to time and place” (van Groenestijn, 2002 in FitzSimons, 2006). We must acknowledge the time dependency of numeracy. The course described in this paper was developed in the first decade of the 21st century and changes are already noticeable in this current decade. While there has been debate over multi-literacies in these ‘New Times’, no similar debate has emerged in mathematics and numeracy. Zevenbergen (2004) introduced the notion of multiple numeracies. She poses the question of whether there are different forms of numeracies in these changing times.

![Adult Numeracy Concept Continuum of Development](image)

**Figure 1. A continuum of the concept of numeracy showing increased level of sophistication from left to right (from Maguire & O’Donoghue, 2002).**
times and concluded that we may need to re-theorize knowledge, skills and dispositions to re-conceptualise definitions of numeracy relevant to workforce needs.

Finally there is an element of numeracy that appears critical in nursing: “numeracy under pressure” (Coben et al., 2008). Nurses need a sense of confidence and competence and as Coben defines numeracy (2000):

To be numerate means to be competent, confident, and comfortable with one’s own judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (p. 48)

Nursing is a very specific vocational career and numeracy in this context is reflected in a definition of health numeracy:

The degree to which individuals have the capacity of access, process, interpret, communicate, and act in numerical, quantitative, graphical, biostatistical and probabilistic health information needed to make effective health decisions.

(Golbeck, Ahlers-Schmidt, Paschal, & Dismuke, 2005, para. 2)

To the five elements of numeracy outlined above, must be added the context of the university setting, to create a definition of academic numeracy. In 2005 my colleague Janet Taylor and I (Galligan & Taylor, 2005) developed a definition of academic numeracy:

a critical awareness which allows the student to situate, interpret, critique, use and perhaps even create mathematics in context, in this case the academic context. It is more than being able to manipulate numbers or being able to succeed at mathematics. (p. 87)

For the nursing context, the definition of academic nursing numeracy needs to include ideas of integration with the cultural, social, personal and emotional. It needs to incorporate ideas of confidence and competence. It also needs context, location and time as central ideas to the definition. Within this research, I defined academic numeracy as:

- mathematical competence in the particular context of the profession and the academic reflection of the profession at the time;
- critical awareness of the mathematics in the context and in students’ own mathematical knowledge and involves both cognitive and metacognitive skills; and
- confidence highlighting its deeply affective nature (Galligan, 2011a)

The paper first outlines the course and assessment in terms of this definition of academic numeracy and then highlights some of the assessment results.

**The Course**

Prior to 2006, a number of approaches had been taken to develop nursing students’ numeracy levels at the USQ (Galligan & Pigozzo, 2002). In 2006 USQ’s nursing program was reaccredited with the Australian Nursing and Midwifery Council (ANMC). This meant courses, especially those offered in first year, were revised. When planning the reaccreditation, it was decided by the Department of Nursing, that nursing students needed to develop some key academic skills in first
semester of first year, as a separate course. Two new integrated first year nursing courses were developed (Lawrence, Loch, & Galligan, 2010) that included Information Technology and mathematics (one course) and literacies skills (second course). The aims of the first course were to develop students’ numeracy and Information Technology skills directly linked to their degree. These skills were addressed by embedding aspects of the other courses taken in the students’ first semester and course content encountered later in the program.

The course described in this paper, consisted of 10 × 2 hour tutorial style sessions, six of which were numeracy related. Table 1 highlights some of the mathematics needed and the context in which it is seen, and each tutorial session concentrated on one of these modules. While the concepts of decimals, fractions, percentage, proportion, measurement, and scale are all studied at the primary level at school, many adults have not mastered these concepts and this has implications in nursing (Hilton, 1999; Pirie, 1987).

Table 4. Mathematical content of course

<table>
<thead>
<tr>
<th>Module</th>
<th>Mathematics content</th>
<th>Nursing examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic and Formulas</td>
<td>basic operations, fractions, decimals, squares, and order of calculations and use formulas</td>
<td>Body Mass Index, Lean Body Weight and Ideal Body Weight</td>
</tr>
<tr>
<td>Graphs and Charts</td>
<td>read single scale graphs; read and construct patient charts; draw graphs with appropriate scale, title and labels and units; and interpret graphs</td>
<td>patient charts; drug profiles; graphs found in nursing articles</td>
</tr>
<tr>
<td>Rates and Percentage</td>
<td>calculate percentages of given values; convert to and from decimal fractions to percentages; express two quantities as a rate; determine quantities from given rates;</td>
<td>determine drip rates; pay rates; use of % burns calculations; % concentrations of drugs</td>
</tr>
<tr>
<td>Ratio and Proportion</td>
<td>manipulate equivalent fractions and ratios;</td>
<td>read a percentile chart; drug calculations</td>
</tr>
<tr>
<td>Measurement</td>
<td>identify the units used in the metric system; convert between units of measurement; convert from ordinary to scientific notation; multiply and divide by powers of 10 and multiply and divide by decimals</td>
<td>read syringes</td>
</tr>
<tr>
<td>Drug Calculations</td>
<td>problem solving</td>
<td>read drug calculation problems correctly; recognise the different types of drug calculations; recognise the solutions and units needed in drug calculation problems</td>
</tr>
</tbody>
</table>
In addition features of the course included: Social presence in the online format; videos of a nursing practitioner; video clips of adults the context of everyday life; photos of drugs and their use in some assessment tasks; cartoons; and extra optional material.

The course took the definition of academic numeracy, outlined in the introduction, and incorporated these features into teaching and assessment sections of the course (Galligan, 2011b) as summarised in Table 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>Course components</th>
<th>Numeracy component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maths Relationship scale</td>
<td>Critical Awareness</td>
</tr>
<tr>
<td>2</td>
<td>Discussion Forum</td>
<td>Critical Awareness</td>
</tr>
<tr>
<td>3</td>
<td>Self-Test</td>
<td>Competence &amp; Confidence</td>
</tr>
<tr>
<td>4</td>
<td>6 maths tests</td>
<td>Competence</td>
</tr>
<tr>
<td>5</td>
<td>Post-test</td>
<td>Confidence, Competence &amp; Critical Awareness</td>
</tr>
</tbody>
</table>

In class, students were asked to discuss and rate their past relationship with mathematics (item 1). They were also directed to read two articles on the relationship between mathematics and nursing. Using these two exercises as a basis they were asked to reply to the questions “Describe your previous experiences with mathematics in a couple of sentences” and “How do you think mathematics…will be important for you as a nursing student, and later as a professional” in an online forum (item 2).

In 2008, this data was coded into a 5 point scale, overall, about one-quarter of the students (206 students) claimed that they “disliked” or “hated” mathematics. An example of each category is highlighted below:

- I ended up changing from Maths B to A but not before developing a loathing for it!! (Hate)
- Maths I do not like it. For me it’s like great mystery (Dislike)
- Sometimes numbers just don’t make sense to me…I do blame the teachers and their inability to find why maths perplexed me so much, but in high school I do blame myself…now I find maths much better, I am no longer afraid of numbers and can grasp the theory and at least try to put it in practice – even if the answer is wrong. (Neutral)
- Maths has always been an enjoyable experience for me. (Like)
- I learned mathematics at school and really love mathematics…(Love)

As part of a first assignment ask, students were asked to complete a 32-item maths test (Galligan, 2011b) where the mark they received was on the completion of the test, not whether they got it right or wrong (item 3). For each question students were asked to rate (on a five-point scale) their confidence, and reflect on their answers in relation to their competence, confidence, and critical awareness. They also completed six short competence tests through the semester (item 4) and
one Post-test (item 5) where they were asked to reflect on their current numeracy status. For item 5, marks were also allocated to their competence.

Generally, students performed well on the pre-test, with a median mark of 78%. An error analysis identified decimals, fractions, multiplicative thinking, careful reading, as well as an overall understanding of the connectedness of mathematics, as potential areas of improvement for students (Galligan, 2011b). However, there were significant issues with both under-confidence (right but not confident) and over-confidence (wrong and confident). In addition, an analysis of one class’s reflective comments (25 students) on confidence against their pre-test confidence scales suggested students’ reluctance to comment on confidence if they were wrong and highlighted lack of confidence even when right.

These comments and analysis, in combination with the data analysed on error from the whole cohort, provided more detailed error profiles of initial concern for six questions: estimation, average, conversion, formula, proportion and scale. While there were some comments from students on the importance of these mathematics skills in nursing, they only appeared in questions that were specifically related to nursing, for example, reading a syringe. At this stage most students did not specifically relate uncontextualised mathematics problems to a skill needed in nursing. Some students had a vague awareness of the connections between the mathematics they had learned at school, the mathematical skills needed in their profession and the mathematics needed in the next three years of their academic life. It was rare for a student to write a comment such as the one below:

It is imperative for me to get this mathematical side of things such as, measurements, percentages, multiplications, divisions, fractions, down pat as it is a huge responsibility to have as a future Registered Nurse, to administer the right dosages to other people in need, so that they can feel better, they rely on that, and my aim in life is to help others in need, and I cannot let anyone down or hurt anyone in the process.

After one semester, there was significant improvement in students’ mathematical confidence and competence related to nursing numeracy. There was an overall decrease in students’ over-confidence (wrong and confident) and a decrease in students’ confidence in the six ‘over-confident’ questions identified in the pre-test. From the post-semester survey and students’ reflective comments, the majority of students found the course useful, relevant and helpful in building confidence and competence. There was evidence that students were using checking mechanisms to ensure their answer was correct. Students also had fewer comments about their lack of knowledge in some areas, which was reflected in the increase in results and confidence in almost all of the questions and student comments about learning some specific skills in the course. For example, one student wrote at the end:

This semester has been very helpful for me. …, it was good to revise it and be able to practise it and put it into the nursing context for the future. I have only improved slightly from the readiness test to the [end] test but relating to the feedback – confidence levels – I felt much more confident the second time …, most other areas I just need to revise and practice the questions in a nursing context. The main thing I am still having a little trouble with is drug calculations but with practice and revision plus more work on it next semester, I will hopefully become confident with it. Overall the course was very beneficial to me.
Discussion

While there were improvements in all but one of the questions, there were still four areas of concern, i.e., in reading, rounding, decimals, and algebraic manipulation in the post-test. This deeper understanding of students “stuck points” had consequences for course improvement. In the next iteration of the course there was:

- An increased emphasis placed on reading a question carefully and correctly, de-construct and re-construct into a new form, as well as on looking and checking the answer. Students sometimes mentioned making ‘silly’ errors. Any attempt to engage students in addressing these ‘silly’ mistakes may be of crucial importance in nursing.

- Explicit teaching of rounding decimals and the consequences of incorrect rounding in the context of nursing.

- Revisiting the basics of the number system, particularly decimals. This could be via highly contextual scenarios. For example, the time question of turning decimal hour time (i.e., 1.2 hrs) into minutes could be tested in the CMA’s during the semester and more scaffolded learning objects inserted into the material.

- Provision of more examples of reading syringes, particularly the 1 mL size. Links to other areas of nursing and everyday life may also increase the effectiveness of highlighting the importance of reading carefully. Highlighting this error from past students’ tests may provide triggers to students’ thinking about the units.

- More linkages of rearranging formula with context. For example, in a rate problem the drug may be administered at 8mL/minute and the volume is 200 ml to go through, how much time is required? While many can do this intuitively, students have two formulas which are used regularly in their degree program although perhaps less so in their careers (Hoyles, Noss, & Pozzi, 2001):

  \[
  r = \frac{\text{total volume}}{\text{total time}} \quad \text{and} \quad \text{time} = \frac{\text{total volume}}{\text{rate}}
  \]

  infusions of the volume to be infused = \(\frac{\text{volume in drops}}{\text{time in minutes}}\) and a related formula for time to finish infusion: \(\frac{\text{time in minutes}}{\text{volume in drops}} = \frac{\text{volume in drops}}{\text{volume to be infused}}\) Being able to manipulate these equations, aids in understanding the formulas so they are not simply black box approaches to nursing numeracy. Whether understanding this level of mathematics is crucial to nursing or just useful is unknown.

These are just the results from the pre- and post-test data. There were other CMAs which gave a finer grained look at the numeracy in nursing. Some of the results in these CMAs reinforced the issues highlighted above, but also highlighted areas that may need more attention. These included:

1. Thinking fractionally:

   - If students see the first part of a drug calculation, formula \(\frac{\text{dose required}}{\text{dose in stock}} \times \text{stock volume}\) as a fraction of the stock volume it may prevent some errors in the checking stage of calculation.
• If students think of problems such as \( \frac{6}{7} = \frac{15}{?} \) as equivalent fractions, it may help in the reflection/checking stage.

2. Thinking decimally:

• Students need to take notice of scale in between units. Not only can this be emphasized in syringe problems, but also in graph reading. The error needs to be highlighted to alert students.

• The concept of division by a decimal creating a larger number is poorly understood so some specific scaffolding may help.

• The concept of cancelling zeros and moving decimal points is poorly understood. Students come from school with these approaches, but have an incomplete understanding of why these ‘tricks’ work. Students can use a calculator to do the problems, but again a checking/estimation mechanism is important.

3. Thinking proportionally:

• In the CMAs there were a number of questions which revealed some students’ inability to think proportionally at different levels. This is linked again to the question like \( \frac{6}{7} = \frac{15}{?} \).

• There appears to be some misunderstanding of percentages, especially in relation to the decimal equivalent.

• Thinking proportionally, decimally, fractionally could then be combined into thinking multiplicatively (but highlighting the connections).

4. Thinking statistically:

• Students were able to do an average problem, with just some revision.

• Students had more trouble reading a frequency histogram, without explicit instruction.

• There were some more statistics in the course, where students were asked to interpret journal articles that were used in other courses. This was performed very badly, with most students ignoring any statistical terminology or conclusions. If nurses are required to read and understand these articles, then statistical literacy is of importance and this will need to be addressed.

5. Thinking mathematically:

• In many quizzes students demonstrated a lack of a logical step by step approach to problems.
• Students appeared not to see the correct notation and the importance of it.

• Rounding appropriately was a major issue and there is a need to highlight why and when it is needed.

• Using estimation is a skill underutilised. Students need to think of the implications of the answer or result.

6. Thinking like a nurse:

• Safety of the patient is crucial, so it is essential to link the mathematics done with the consequences.

• Thinking logically and systematically is crucial in nursing (I recently interviewed a nutritionist in a hospital who said these are skills needed in hospitals).

• Day to day reporting of patients’ results, etc., needs to be communicated clearly to other people, such as doctors or other nurses

• Making an error can have consequences for both patient and nurse.

• When the pressure is on, the nurse must be confident that any numeracy undertaken is correct.

Conclusion

This paper outlined a definition of academic numeracy that was mapped on a course for first year nursing. The definition was able to drive the assessment and test whether the assessment was successful in improving students’ competence, confidence and critical awareness. It was able to highlight areas of improvement for the next iteration of the course. Future challenges include delivering this course in the online and blended environment and providing the support for other lecturers in the course to ensure they have the same understanding of the nature of teaching mathematics in the context of a nursing environment.

References


Students enter university with a variety of mathematical backgrounds. Some are not adequately prepared for the mathematics involved in their preferred, non-specialist mathematics courses. Many bring emotional conflicts about mathematics which affect their ability to learn. The University of Wollongong recently appointed a Mathematics Support Lecturer to its Learning Development team whose role is to provide assistance to students who find the mathematics involved in their courses “challenging”. This paper looks at the development of this role with emphasis on the requirement of mathematics to “synergise” with other courses, contexts and competencies within university studies and examines the broad cultural contexts of students seeking support and the issues – often affective and motivational – faced by students who seek support at this level.

Introduction

Students enter university with a wide variety of backgrounds in mathematics. Often these backgrounds are not strong, with some students having avoided studying mathematics as part of their final secondary school subject choices. Other students, although succeeding in mathematics at school, may have been out of formal education for a while and may perhaps have forgotten “fundamentals”. Students may encounter mathematics in many different courses in their university studies and many are surprised – even alarmed – by the need to pursue it again.

Moreover, students at any level of mathematics or statistics may occasionally – or more often – need help to overcome difficulties in the subject. Thus, there is a great need for assistance in mathematics or statistics topics across a range of faculties at university level (MacGillivray & Wilson 2008; Croft, Harrison & Robinson, 2009). MacGillivray and Wilson (2008) comment that mathematics and statistics support “is an area in which a small quantity of resources in the overall university scene can produce enormous dividends in student learning, confidence and fulfilment of potential” (p. E2).

At the University of Wollongong (UOW), a large regional university in New South Wales, Australia, support in mathematics and statistics was available from approximately 1993, through Learning Development (a section of the Academic Services Division) which provides assistance to students in academic language and literacies. In 1996, however, various institutional changes saw the lecturer transferring to the School of Mathematics and Applied Statistics and the position removed. The Learning Development section stayed without any mathematics or statistics support provision, apart from a little part-time assistance at one of the remote campuses, until research led to a fixed-term, contract position at the main campus being created and filled at the
beginning of 2011. Consequently, the position of Mathematics Support Lecturer in 2011 was regarded as “new”.

The focus of this paper is to look at the synergies created within the first twelve months of the position while attempting to develop a worthwhile, viable support system for students who need assistance with mathematics involved in their non-specialist mathematics courses. The challenges have been varied; the rewards worthwhile. These are discussed below, beginning with the challenges.

The Challenges

How to attract students?

A major challenge was to work out who the students were that needed and/or would benefit from this new support and where they might be found. What courses were they doing? Were there any courses where students needed help more than in other courses? What were their “educational biographies” (Schuetze & Slowey, 2002, p. 315), for example, what were their previous levels and experiences of mathematics education? And for what reasons would they seek assistance?

To address this challenge it was necessary to visit each faculty in turn, as well as certain schools within each faculty and to meet with appropriate representatives who would pass on information to relevant lecturers and their students. As no advertising had been set up, it was also necessary to develop promotional materials that would reach students. Further, there was the matter of determining the appropriate procedures and protocols to be followed. Although this challenge was time-consuming, it did, however, effectively begin the process of establishing synergy with faculty members and other university staff.

Developing this synergy with faculties and schools depended on the faculty or school involved. The School of Mathematics and Applied Statistics willingly encouraged students in first year non-specialist, or foundation specialist mathematics courses to use the service. By contrast, it appeared that personnel from some schools were reluctant to collaborate, or else the students themselves did not take up the support. Of course, this is unfortunate as several of these students may have benefitted from accessing support. The most attracted were lecturers in Primary Education and their students, many of whom had gained entry to that course without an “ideal” mathematics background; several School of Engineering and general science students also willingly participated as the service was readily promoted by their lecturers.

Thus it seemed that the development of synergies with faculties could be determined by the personnel involved. University school and faculty representatives need to see the work of Learning Development in a way that it is acceptable – in fact vital – that their students identify problems and seek to overcome them. One of UOW’s own aims, according to its website, is to produce graduates with certain “graduate qualities … so that they can become informed and responsible and to be an independent learner, problem solver and effective communicator” (University of Wollongong, 2011). The website directs lecturers to the Learning Development section for help in incorporating strategies into their course procedures that will assist students to achieve these attributes. Until all academics realise that not all students have the same abilities and that support is absolutely essential for some of them to succeed in all subjects, however, Learning Development will need to continue to effectively promote its aims and the benefits of collaboration.
How would the support work?

Offers were made to lecturers in various disciplines for “team-teaching” procedures to take place with the idea that the Learning Development lecturer could teach the necessary mathematics in their courses as an embedded procedure; however, there was no uptake of this idea. This is disappointing, as with collaborative teaching, each teacher brings different expertise to the situation, such as in-depth pedagogical and discipline-content knowledge (Quinlan, 1998), and the opportunity for learning from each other by sharing experience and teaching styles can be beneficial as a “reciprocal learning process” (Zhou, Kim & Kerekes, 2011, p. 134). Perhaps staff felt that trust had not yet been established or perhaps they were not willing to dedicate time – or did not have sufficient time available in their heavy schedules – to work out a collaborative strategy (Zhou, Kim & Kerekes, 2011).

Some mid-semester workshops were developed that focussed on generic skills with the potential for them to be varied according to the specific needs of participating students; however, attendance at these workshops was poor. Thus the main focus of the support provision has been with one-to-one or small group consultations. Croft (2000), writing about the development of the successful mathematics learning support centre at Loughborough University (UK), concurs that this form of help is the most popular: “feedback from students shows that they appreciate this service most of all, and we have little doubt that if it were not available students would be less likely to use the Centre” (p. 437).

The graph below shows the percentages of students by subject areas who accessed the service in 2011; (subject areas have been combined within their schools or faculties)17. The graph especially illustrates the large proportion of Education students who used the support provision.

17 Both General Science courses are run by the School of Mathematics and Applied Statistics; “low” refers to a compulsory course for students such as those studying Environmental Science or Geology who do not have a background in calculus-based Mathematics; “high” refers to a mathematically more advanced course aimed at those possibly wishing to specialise in Mathematics or who may study Physics, for example. “Other” refers to students who accessed support once only, for example (but not exclusively) Medical Science students requiring assistance in interpreting the statistical results/analysis in a medical report.
In addition to the one-to-one support, the Faculty of Education, through the faculty’s Student Support Advisor\textsuperscript{18}, were keen to adopt the idea of a weekly lunchtime “drop-in” session. The mathematics support lecturer and the lecturer in Primary Education (Mathematics) had already established a firm synergy built on a shared vision of supporting students, together with a mutual trust and understanding. The first drop-in session proved so popular that it was decided to schedule two per week, with one a more formal session run by the course lecturer and the other a “Q & A”-type session, where students could bring their own questions and the group contribute ideas and discussion. All sessions of both types were well attended and this practice has continued, with the course lecturer adding a drop-in session to her course schedule. Most students who attended these drop-in sessions were studying the Graduate Diploma of Primary Education; and most did not seek further support through 1:1 or small-group consultations. It must be said that these drop-in sessions were also suggested to other schools and faculties, but the idea was not adopted.

**What range of mathematics would be covered?**

Faculty of Education students, in particular, have presented with a great array of problems, ranging from the primary school levels of mathematics that they will be required to teach, through to some pre-tertiary level concepts, which are given as course material mainly so that students become capable in, for example, reading statistical results, or other examples which will arise as part of their teaching duties. Students in other courses have requested review of lower secondary concepts through to advanced calculus topics. In fact, at times there has been demand for mathematics or statistics support by specialist Mathematics course students. Unfortunately, the current Learning Development lecturer does not have expertise in all requested topics and thus this has been seen as an equity issue for those students.

A further equity issue was that mathematics support was not initially available at all campuses; however, part-time assistance has now been organised and is available at the relevant remote campuses. Work is also in progress to develop resources to enable student access to support through internet and email procedures.

**Student diversity**

It is one thing to have attracted students, to have resources in place, to have determined how to deliver support and to have become familiar with the courses from which students are likely to seek help, yet it is certainly another thing to provide each individual student with the most appropriate support possible in order to ensure they are learning effectively and achieve their aims. Students present with a variety of reasons for seeking support: they will be of diverse backgrounds, ages, abilities, personalities; they will have followed any of several pathways into university; their cultural origins will vary, as will their learning styles and strategies, attitudes towards mathematics and their need to pursue support. For each of these students, “the focus of learning support tends to be on building mathematical fitness, confidence and transferability, all with reference to the specific course being taken” (MacGillivray & Wilson 2008, p. E13).

A student’s level of confidence can be low or affected for a variety of reasons: it could be that they feel overwhelmed with the amount of work needed in their study; they may be unfamiliar with the particular topic being covered; perhaps they do not understand a concept and have never previously experienced this sense of not understanding; perhaps they did not enjoy mathematics at school and have set up a barrier towards learning it again; their background may not be strong

\textsuperscript{18} UOW Student Support Advisers offer liaison and information services to students.
because of various gaps in their schooling; they may themselves have experienced poor quality teaching in their own mathematics career, which has turned them against it; or as one student put it simply: “some people are good at things, some are not”. Further, some students may feel shame in needing to approach Learning Development and perhaps may even not come forward because of embarrassment. For many, “seeking help is associated with failure or loss of face” (Clegg, Bradley & Smith, 2006, p. 111). Thus synergy between the Learning Development lecturer and student is essential — only by developing a rapport with each student can one build their confidence which, in turn, helps the building of that “mathematical fitness” (MacGillivray & Wilson 2008, p. E13).

During the first twelve months of support availability, females outnumbered males (>3:1); there were only a few whose first language was not English and only one Indigenous Australian. Many of the students were employed as well as studying — often on a full-time basis. Most students were not in their first year of study, although some stated that they wished the Mathematics support had been available in previous years. Ages varied but students were, in the main, “mature-aged returners to study” rather than those who proceeded to university directly from school. This bears out the findings of Burton, Taylor, Dowling and Lawrence (2009) in their evaluation of mature-aged students’ learning strategies as compared to those of school-leavers. They comment that “mature-age students are typically conscientious and responsible, efficient, self-disciplined and organised, and have high aspirations for academic success. ... Mature-age intend to do well” (pp. 75-76). In fact, one particular mature-aged student’s decision was, in her words, “to be proactive” and to initiate help before she had problems.

Mathematics issues with which students have so far presented have ranged from higher-level mathematics and statistics to problems with basic computation — the latter students are possibly “dyscalculic” (Trott 2011). There have also been several students with varying degrees of mathematics anxiety (Taylor & Galligan, 2006), who regard their ability with mathematics in a very negative light and have little confidence of succeeding in their course because of their perceived lack of skills. Some of the Primary Education students are especially anxious about teaching the subject. Hodgen and Askew (2007), citing Buxton (1981) state: “for many primary teachers, their relationship with mathematics is fraught with anxiety and emotion, much of it relating to their negative experiences of school maths” (p. 469). Hodgen and Askew (2007) also express apprehension that many of this type of potential teacher may tend to oversimplify maths for their own students in order to “protect” them from it, with the possible outcome that their students will find mathematics tedious and “irrelevant” (p. 482). This is a real issue, as many of these students are hopeful of being allocated classes “with the little ones” as they believe that level will be “easier” to teach. But, as Bibby (2002) states: “if ... primary teachers’ subject knowledge is mediated by powerful feelings rooted in their autobiographies ... then this will impact on the ways in which that knowledge is used professionally in the classroom” (p. 706). Bibby also links the concept of “shame” to some teachers, where “feelings of shame include threat to ‘professional identity’ as well as ‘personal and social aspects of identity’” (p. 708). Thus synergy is absolutely involved in supporting students such as this: a great deal of working with them to instil confidence in their ability, of guiding them to discover that they indeed can do maths, and that mathematics is definitely relevant, not only to their teaching profession, but, of course, in their own “everyday lives”.

Other students bring with them a variety of problems. In some cases this has prompted the need for negotiated collaborative support through different agencies such as the Counselling and Disabilities units; in turn, these units also often refer students to Learning Development.
Students may take very different amounts of time for support to be effective. Several students have used their consultations as “one-offs” to iron out a lingering problem they have not been able to solve through “normal channels” (lecturer, tutor, fellow students, for example); others attended for the full year. It is absolutely necessary to distinguish between those who use the service more as a crutch and those who are in genuine need of support, however, and to try to encourage all to become efficient, successful, independent learners.

Of course there will be some students who do not attend Learning Development. This may be because they have many commitments outside university and possibly consider Learning Development support as an extra obligation in their already-too-busy schedule. Others may find support given by their lecturers, tutors, peers, or other means, sufficient for them. Possibly some students feel all right about their mathematics even though their results might demonstrate otherwise. Others, however, may feel embarrassment or shame and a feeling of failure at needing support, or have perhaps organised their own means of coping (Clegg, Bradley & Smith, 2006, p. 112). As the availability of support becomes more widely known and is advertised by word-of-mouth, it is hoped that these students may feel more comfortable about attending.

Developing resources

Learning Development soon accumulated a professional Mathematics library with the purchase of several relevant Mathematics books. Many lecturers in various schools and faculties, especially the school of Mathematics and Applied Statistics and the Faculty of Education, were exceptionally generous both with their time and also in providing the support lecturer with access to printed and electronic material appropriate for their students.

Consideration was given to purchasing a tablet PC for use in Learning Development and a project was undertaken exploring its viability. A 2007 model was given on loan from the School of Mathematics and Applied Statistics, which was used mainly for developing resources rather than during 1:1 consultations because, unfortunately, it was found that there was inadequate provision of storage or software capabilities whereby the student could take any part of the session away with them for review, which is an extremely important part of the learning process. This is disappointing because, as Loch, Galligan, Hobohm and McDonald (2011) point out, the portability of tablets is a distinct advantage, especially for revision, practice and reflection: if the screen- and voice-capture technology of a tablet PC is used together with annotations using the stylus, the student can take away a complete recording of their support session to which they may refer in their own time later, if necessary19.

Early on in the tenure of the position, it was seen that students lacking the more basic mathematical concepts – for some reason or other – were not necessarily catered for by resources previously developed by the University. Therefore it was decided to embark on a project aimed at developing interactive web-based resources for the most common topics that students accessing the support required. These resources need to be user-friendly, and be aimed very much at the more “maths-anxious” students. The agreement governing the project was not reached easily, however. There is a definite need for university personnel to understand the reason that basic mathematics resources are needed at university level: that, for example, the range of students now is greatly diverse and, especially, that the requirements for entry to

19 By the time of the ALM Conference, a laptop tablet PC had been purchased by Learning Development. Investigations are underway looking at the problems outlined.
courses are no longer bound by success in particular school-based mathematics. MacGillivray and Wilson (2008) argue that:

Universities need to guard against the dogma of denial of the importance of mathematics, and ensure that the totality of help for their students in numeracy, mathematics and statistics is espoused and sustained. ... Learning Support in mathematics and statistics has become an increasingly important component in this totality, building individual confidence and repairing weaknesses (p. 33).

Put quite simply, if background knowledge is missing, students do not have adequate means to pass their course.

To this end, as well, several print-based resources have been developed on each topic necessary for students requiring support at a “basic” level. They are aimed specifically at adult, maths-anxious students and incorporate a great deal of explanation about the relevance of each topic. The resources, soon to be accessible through the Learning Development website, are now situated within the Learning Development unit, which is situated in a non-prominent position within the university, thus reducing embarrassment a student may feel about being seen taking such resources, although this placement does have the disadvantage that the resources may be difficult for students to locate.

To assess or not to assess?

In the relatively short life of mathematics support availability at UOW, lecturers from some schools and faculties have been keen to adopt diagnostic testing. They argue that, in order to recommend to students that they will need extra support, they must be able to back such a recommendation with evidence gained from assessment. Such diagnostic procedures can be demeaning to many students (Clegg, Bradley & Smith, 2006, p. 112) by emphasising yet again the inadequacies of their mathematical skills; by the time they reach university, students know their weaknesses. Further, “the act of being assessed is one that has considerable emotional resonance” (Boud & Falchikov 2006, p. 406).

Therefore it was decided that the support lecturer write a diagnostic test that catered particularly to such students. Following the methods of Galligan (2011); Price, Stacey, Steinle, Chick, and Gvozdenko (2009); and Egea (2004), a model was written which would not only give the student a hint of a means of working out answers to problems posed (through supplying method answers for each set of choices) but also required them to rate how confident they felt for each topic. Importantly, the test had no time limit. For incorrect answers, separate explanations were supplied which would indicate to the assessor the reasons that those answers were incorrect. It was very disappointing to discover that this model was rejected by the particular school for which it had been written. It is hoped that the school – and others – will, however, use the model as a self-diagnosis for their students.

The challenge of whether or not to use diagnostic testing may perhaps be seen as evidence that some lecturers consider these students as fitting a “deficit” model. Cole, reviewing Hartley, Hilsdon, Keenan, Sinfield and Verity’s (2011) book Learning Development in Higher Education, suggests that educational institutions may still see Learning Development as “institutional first aid” and that the use of such terms as “diagnose”, [and] ‘intervene’ ... imply that the agency is still not with the student, which undermines the notion of the independent learner” (p. 6). Further, Black and Yasukawa (2011) suggest that “deficit thinking” ...amounts essentially to the process of ‘blaming the victim’” and add that “one of the significant outcomes of deficit
approaches ... is the negative and sometimes debilitating effects they can have on people’s self-image, their identities” (p. 3).

The equity issue

One of the concerns for Learning Development is that of providing equitable support – that is, of course, support for any student who needs it. For this to be possible, the present lecturer could be joined by specialists such as those who can confidently teach further topics of each of mathematics and statistics. Funding administrators need to be convinced of this, with the present Mathematics Support position seen as a starting point only. As MacGillivray (2009) reports: “students who choose to specialize [sic] in mathematics and/or statistics are as likely to access this learning support as much as non-specialists” (p. 458).

Croft, Harrison and Robinson (2009) use the term “holistic” to describe their “vision of mathematics support” and recommend that “help needs to be available when students need it and from staff who are well-suited to provide it” (p. 113). Currently at UOW, support is provided by course lecturers in their consultation times and, for some courses, by Peer Assisted Study Sessions (PASS). Not all students are comfortable in either setting, however, as “seeking help is somehow associated with failure or loss of face” (Clegg, Bradley & Smith, 2006, p. 111). In fact, one mathematics lecturer reported to the support lecturer that she had been “underwhelmed” by student attendance during her consultation hours. Perhaps these students may benefit from the more anonymous support provided by Learning Development.

By contrast, in 2011, the first tier of a program entitled Successful Transitions was implemented by Learning Development. This series of seminars runs at remote campuses between Orientation Day and the commencement of the academic year. It is aimed at new enrolling students, introducing them to certain strategies that they will need at university but with which they may not yet be familiar. The program was very well-received by both staff and students. In 2012, a component called Mathematics: Marvellous or mystifying was run as part of the Successful Transitions program, in which students were asked to analyse their feelings towards Mathematics with the aim of challenging them to realise that they do indeed possess – and regularly use – mathematical skills and, especially, problem-solving skills. As well, it aimed to confirm to them that if they felt inadequate or anxious about mathematics, then they certainly were not alone. By introducing the mathematics support lecturers at each campus, the program sought to reassure students that support and assistance is available should they need it and that it is definitely acceptable to seek such support. This program is seen as part of an equity program and is funded by a Social Inclusion grant. Its aim is to target, reach and support an increasingly diverse cohort of students such as rural, remote, Indigenous, or mature-aged; however, as Burton, Taylor, Dowling and Lawrence (2009) put it, “acknowledging diversity is one thing – achieving inclusiveness is another” (p. 78).

Retention of students

It is hoped that the work of Mathematics support through Learning Development will be seen as more than simply a means of enabling greater retention of students. However, this seems to be an aim for universities in general, and implies that economic aims, not educational aims or the aims of individual students drive the financing of such support. Students themselves, with the assistance of professional guidance, must make their own decision whether or not to stay on at university. All circumstances should be taken into account, including, but not exclusively, potential failure.
MacGillivray and Croft (2011) suggest that “a university’s approach to retention should be a positive one and ... it should provide students with opportunities to improve their grades rather than just addressing gaps within their knowledge” (p. 201). As suggested previously, this is in line with UOW’s mission of Graduate Qualities. Mathematics support through Learning Development must be for the benefit of each student even if, in the long term, that student decides not to continue their university course – the outcome obtained may be as simple as that student developing a more positive attitude about mathematics that they may carry into the community.

The Rewards

“Success”?

Although this topic has been mentioned under the heading Rewards, it actually brings with it a further challenge! Yes, there has been “success” – students have passed their courses who may perhaps otherwise not have done so. The challenge is, of course, to determine just how much influence the support had in students obtaining these results. MacGillivray and Croft (2011) suggest that it is worthwhile collecting qualitative data from students; for this purpose, a short evaluation form was emailed to students at the end of the year. Unfortunately not many replies were received, however all responses acknowledged satisfaction with the support received and students felt it had helped in their studies. This could be supported by quantitative data (obtained from faculties) which might show, for example, improvements in course outcomes. Even with substantial data, however, causality cannot be implied! Cole (2011), in fact, suggests that for Learning Development, this is the “biggest challenge: ... determining what kinds of data will best illustrate impact and identifying ways in which it can fund such evaluation and analysis” (p. 6).

Further, “success” may mean different things to different people – for the student, it may be as simple as being able to apply a mathematical concept with understanding, although, of course, in general most would probably acknowledge that their aim is to pass their course! For the university administration, “success” may mean that a student will eventually graduate – and become a “success statistic”. For the mathematics support lecturer, it can be the emotion felt on seeing that a student understands, retains that understanding of, and can apply a concept about which they had asked for help. The words: “now I see!” said by one individual student have more meaning than any set of crunched numbers! The problem is, of course, that for the support lecturer’s position to be maintained, statistics which prove – or at least imply “success” must be produced.

Conclusion

This synergy of mathematics support within a university setting can be seen to have involved, and to continue to involve, a great many collaborations between the mathematics support lecturer and students, faculties and colleagues. Further synergy has grown through communication with fellow external university lecturers who have given expert advice and assistance – this has been particularly valuable in the setting up of the position and resources. As well, a valuable synergy has grown through the opportunity to participate in some research and other projects. However, by far the main synergy has been found in working with a diverse range of students, the majority of whom are hard-working, dedicated, and possess “I-won’t-let-this-subject-beat-me!” attitudes.
UOW’s Learning Development mission statement includes the following: “To develop synergies between teaching, governance, professional development and research as they relate to improving and facilitating student learning, the acquisition of academic literacies, the acquisition of discipline appropriate English language, and numeric thinking”, (Learning Development Policy and Procedures Manual (2011)). Developing and maintaining these synergies is vital for the success of all students in their venture to become professional graduates who exhibit UOW’s Graduate Qualities. It is through these synergies that more than the sum of the parts can be achieved.

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References


“It’s your money they’re after!”: Using advertising flyers to teach percentages

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Every day we get advertising material into our mail boxes. How do students evaluate this ‘junkmail’? When they read it, do they understand how the percentage discounts and the percentage of the actual price that they pay, are related? Can they calculate what the normal price would be, given the sale price of an item and the percentage discount? If money is put into savings, how are the interest rate and the dollar amount of the interest affected by their tax rate and the risk of losing their investment? This paper shows that students can use their ‘junkmail’ to learn how to calculate percentage increase and decrease, to calculate interest on hire purchase as opposed to interest on savings, and to calculate the original price of an item from the sale price, as well as its discount percent.

What is the importance of “Financial Numeracy”?

This paper is based on a presentation that focuses on the importance of financial literacy, and the importance of financial literacy in our world. Some historical reports are highlighted in this document to apprise the reader of the history of some terminologies and the import they have on today’s numeracy contexts. A discussion of the importance of financial literacy is presented to set the stage for a more specific discussion about the mathematics in advertising, which shows some implications for consumer numeracy.

Literature about what Numeracy is and why it is Important

The term “numeracy” was first used in the 1959 Crowther Report (Ministry of Education, 1959), which related the term to both quantitative reasoning and the ability to reason scientifically. Since then, literacy and numeracy have often been linked in such a way where one is subsumed under the other. This means that when writing material for adult numeracy, it is important to remember that the material needs to be accessible to students at all levels of literacy as well as embedding numeracy concepts into the material.

In 1982, the Cockcroft Report discussed an

ability to make use of mathematical skills which enable an individual to cope with the practical mathematical demands of his everyday life. The second is ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease.

(Department of Education and Science/Welsh Office, 1982, p. 39)
Maguire and O’Donoghue (2003) call this the mathematical phase, and assert that this phase is based on the problem solving requirements of everyday life, and often includes money and percentages as well as other, topics such as number, algebra geometry and statistics.

More recently, the 2006 Adult Literacy and Lifeskills (ALL) Survey (Lane, 2010), conducted through the Organisation for Economic Co-operation and Development (OECD), emphasised using the mathematics of real situations, including everyday life problems. From the ALL Survey, numeracy is considered to be “the knowledge and skills required to manage and respond effectively to the mathematical demands of diverse situations”. Also,

Numerate behavior is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors and processes.”

(Gal, van Groenestijn, Manly, Schmitt, & Tout, 2005, p. 142)

In 2006, New Zealand’s National Strategy for Financial Literacy defined financial literacy as

“the ability to make informed judgements and to take effective decisions regarding the use and management of money.”


In 2005, The Australia and New Zealand Bank and the Retirement Commission, instructed Colmar Brunton to do a Financial Knowledge Survey, to find out the level of financial literacy in the adult New Zealand population (ANZ, 2006). The findings from this showed that there was a strong correlation between financial knowledge and socio-economic status and that some basic financial terms were not well understood. As a result of this information, The Retirement Commission20 put some strategies in place to improve the financial literacy of New Zealanders (http://www.cflri.org.nz/financial-literacy/national-strategy).

Background and Context

Foundation students at Whitireia do not have sufficient secondary credits for immediate entry to tertiary education. Most of them want to further their education by training as nurses, paramedics, teachers or social workers, and many are likely to be from lower socio-economic groups. From all this literature, it seemed important to me that in writing teaching material for Foundation students, it would be very important to give the students examples that they could relate to from their everyday lives that would also give them some knowledge as to their spending and savings options. The module for teaching percentage calculations seemed to be a suitable place for me to combine both numeracy and some of the financial literacy skills. Many of our students are in the ‘low knowledge’ group, so their financial knowledge is limited. Advertisers are very persuasive and I tell them: “It’s your money they’re after!

Numeracy and Financial Numeracy in the Foundation Classroom

At the beginning of the course, the students do the National Diagnostic Test to ascertain their level on the Learning Progressions. About 5 percent are likely to be below Level 3 of the

national Qualifications Framework, so are considered to be disadvantaged quantitatively in their ability to function in society. These students are registered on Pathways Awarua, a self-paced course to help them achieve a better understanding of numeracy, while at the same time, attending their mathematics classes. For the 95 percent, numeracy techniques are embedded in the course material to help them to move from Step 3 to Step 5 of the Learning Progressions, the numeracy level that they need to be at for success in tertiary education.

Before we actually begin working with percentages, the students have some time to learn/revise basic numeracy techniques for the four rules of addition, subtraction, multiplication and division, as well as doing some work on the understanding of place value. The students then learn how to find 10%, 5% and 1% without a calculator so that they can work out discounts mentally, and check their numerical solutions to problems. After doing the mental calculations, the students can then use their scientific calculators to do the percentage calculations for the remainder of the work in the module.

The first topic we discuss is the percentage discount. The advertising flyers that are delivered at least twice a week contain information about all the items that are on sale at a discount that week. To begin with, we analyse one advertisement as a class, finding the discount and the percentage discount and setting out a logical set of statements and calculations to find the answer to the question:

“What percentage discount am I getting? Is the discount from one store, better than that from another, for the same item?”

The students then choose five different advertisements to find both the dollar discount of an item and its percentage discount. They also find the sale price percentage of the original price and discover that this, plus the discount percentage add to 100, leading to the equation:

\[
\text{Percentage of price paid} + \text{percentage discount} = 100\%
\]

Which is equivalent to the statement:- Sale price + discount = original price?

The students enjoyed doing these examples and there was considerable discussion about the various ‘bargains’ that were advertised. Very few of the students had known before, how to do this analysis, or how to evaluate what a ‘bargain’ really was. Some of the advertising material only showed the discount percentage and the sale price, so the next part of the topic looked at the question:-

“How do you find out what price you would have paid if advertiser only gives you the discount percentage and the sale price?”

Working from the percentage equation above, as a class, we discussed the fact that the sale price\% = 100\% - discount\%. We then set out the calculation for this, using the figures for the advertisement:

\[
\frac{100\%}{(100\% \text{ - discount\%})} \times \text{sale price} \text{ to do the numerical calculation.}
\]

The students then choose some advertisements from the ones supplied, to find out the original prices of the items they select. However, I did not complicate the issue at this point by taking the goods and services tax into account, because the differences that it made to the prices were negligible in terms of the answers to the calculation.
Another kind of advertisement that is aimed at low-income people aims to suggest that they buy an expensive good on hire purchase, at a very low weekly payment rate. As an example, we analysed an advertisement for a flat screen television with a cash price of $999 and a weekly payment rate of $8.88. One of the students was quite excited about this, saying that she could probably afford that, and it would be a fantastic thing to buy for her family. What was not very obvious was the length of the hire purchase contract. This was in the fine print at the bottom of one of the pages, and only in there did it state that the hire-purchase agreement was for 48 months.

Again, we discussed how we could calculate how much the TV would cost. Mixing up weekly payments with a term in months was an issue, because, in general, a month is longer than 4 weeks, so we decided to use 52 weeks in the year, and agreed that 48 months is 4 years. The calculation that we used was:

Weekly payment × 52 × time in years = total amount to pay.

The student was horrified that over the 4-year term, she would have paid almost twice the advertised price for the TV. The students then happily worked on several advertisements for hire-purchase from various retailers’ advertising flyers, and compared final prices, deciding that bargains for hire purchase did not exist.

Next we considered interest rates on savings and we got a list of the most well-known New Zealand financial institutions from the internet. These included banks, finance companies and credit unions. The column headings gave a variety of terms for deposits, a range of interest rates and the Standard and Poors credit rating of each one. There were questions around the relationship between the interest rate and the letters shown in the column for credit rating. Once the students realised that the credit rating was a measure of the risk that they would lose their money, they were able to make comparisons between institutions with the returns on investments – although this was not a definition used at this time.

From the example of the flat screen TV, whose cash price was $999, we did a class exercise using the highest interest rate deposit account we could find, to compare the interest amount on saving the money with the interest charged on the hire purchase over the same period of time. We worked out that the interest charged on the hire purchase was slightly less than four times the interest earned in a deposit account for the same amount of money, again, using numeracy techniques and estimation.

This time, we also had to consider that interest on savings was classed as ‘income’ for tax purposes, and that even on the lowest tax rate, 19 percent of the interest would be taken in tax, so even the $230 in interest would be reduced to just over $187 from a 19 percent tax rate. From there, the students looked again at the interest rates from the financial institutions, and factored in the tax that would be levied from this income.

Has this approach helped students to understand percentage calculations?

Four weeks after we had finished working on this module, the students did an evaluation which included problem-type questions using all the percentage and financial information we had covered. The students worked in self-selected pairs, and were allocated half an hour to come up with solutions to the problems. All but one of the pairs were able to solve the problems within the allocated time span. The pair that was unable to complete had been on Step 2 of the
Progressions, and were working through the Pathways Awarua program. These students were able to find the percentage discount, the total cost of a 3-year hire purchase contract and the interest on a savings account, but they ignored the tax calculation on the interest and were not able to calculate the original price from the sale price and discount percentage.

Has this helped the students to think more critically about advertising?

When the students evaluated the module, they were “very satisfied” with this approach to learning about percentages, and felt that it had helped them to make more considered decisions about their spending and saving. Most said that they would, in future, think more carefully about taking up hire purchase contracts to buy luxury goods, when saving the money would result in paying for the goods in half the time.

References


Other Reading


How Might Schools Contribute to the Poor Mathematics Skills of Adult New Zealanders?

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The 2006 Adult Literacy and Life Skills (ALL) survey has shown that more than half of New Zealand adults aged 16 to 65 do not have the required numeracy skills to meet the needs of everyday life or the so-called “knowledge economy.” This paper examines evidence from the school and tertiary sectors (Initial Teacher Education [ITE]) on students’ mathematical understanding and attitudes towards mathematics. Overall, the evidence highlights a systemic problem, with many ITE students entering teacher training with negative attitudes towards mathematics coupled with weak understanding of key mathematics concepts. Assessment at the end of the program shows that some ITE students graduate without their misconceptions being corrected, and then go on to become the mathematics teachers of our children. Developing the mathematical competence of teachers as adult learners is vital for breaking the negative cycle perpetuating poor mathematics learning for many citizens.

Introduction

New Zealand Ministry of Education policy places a high priority on all students achieving literacy and numeracy levels “that enable their success”. Improving outcomes in numeracy and mathematics is recognised as being of strategic importance to education in New Zealand. This focus is borne out by the survey of Adult Literacy and Life Skills (ALL) showing that more than half of New Zealand adults aged 16 to 65 years have numeracy skills below those deemed necessary to participate fully in a modern, high-skilled economy and for everyday life (Lane, 2010; Satherley, Lawes & Sok, 2008). Māori and Pasifika populations were skewed towards the lower end of the distribution for numeracy, with 75% of Māori and 86% of Pasifika at either level 1 or 2 (i.e., below the required level for meeting the needs of everyday life; Ministry of Education, 2007a, 2007b; see Table 1).

Table 1: Description of numeracy skills at each level on the ALL survey

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Perform simple one-step calculations</td>
</tr>
<tr>
<td>2</td>
<td>Execute one- or two-step calculations and estimations</td>
</tr>
<tr>
<td>3</td>
<td>Manipulate mathematical symbols, perhaps even in several stages</td>
</tr>
<tr>
<td>4</td>
<td>Complete multi-step calculations requiring some reasoning</td>
</tr>
<tr>
<td>5</td>
<td>Understand and use abstract mathematical ideas with justification</td>
</tr>
</tbody>
</table>

Over the past three or more decades, mathematics has moved from its position as an elitist discipline taught only to the most intellectually able students and used as a gatekeeper to sort
students, to now being recognised as important for all citizens to master; the catchcry now is “Mathematics for All” (Gates & Vistro-Yu, 2003). As well, there has been a shift away from traditional teaching methods emphasizing the memorization of rules, facts, and procedures (the so-called instrumental approach) to a reform-based approach that stresses the importance of conceptual understanding and the inter-connections between concepts and processes (the so-called relational approach; see Skemp, 2006). However, despite the fact that academics have been advocating a shift to the reform-based approach for at least three decades if not longer, educators and systems have been slow to adopt this way of teaching mathematics.

Research involving international comparisons of mathematics achievement has shown that many western countries (e.g., New Zealand, Australia, Canada, Great Britain, Ireland, and the United States) perform more poorly than Confucian heritage countries such as Singapore, Hong Kong, China, Japan, and Korea (Garden, 1996; Gates & Vistro-Yu, 2003; Leung, 2012). These results have provided a catalyst for western education systems to revise their curricula and accompanying support materials for teachers. A subtle shift has also occurred in the messages conveyed by successive curriculum documents.

For example, New Zealand’s mathematics curriculum (Ministry of Education, 1992), instead of framing its expectations in terms of content coverage at particular year levels, as did previous documents, presented its expectations in terms of outcomes – what students at particular levels in the school system needed to know and be able to do. This shift placed more pressure on teachers to ensure that their students achieved in mathematics at the levels expected. With this new focus on outcomes, it became clearly evident that students’ mathematics achievement varied systematically as a function of such variables as ethnicity and socio-economic status (as reflected in school decile). This contributed to increasing concern about the need for teachers to change their teaching approaches in order to cater better for diversity by using more effective pedagogy (Alton-Lee, 2003, 2011, 2012; Anthony & Walshaw, 2007, 2008).

The most recent version of the school curriculum (Ministry of Education, 2007c) represents a move away from a somewhat arbitrary collection of concepts and skills judged appropriate for students at particular year levels, towards identifying progressions in learning particular aspects of mathematics across the years of schooling. This focus on continuity came from research showing how children’s understanding and skills develops through a series of identifiable stages, as their thinking becoming increasingly sophisticated (e.g., Steffe, 1992). Another hallmark of the most recent curriculum is the intense focus on students’ thinking and ways of reasoning through problems towards solutions. The goal is no longer accumulating correct answers, but the nature (in terms of sophistication and efficiency) of the strategies used by students to solve problems. With this shift has come permission (and encouragement) for students to work out their answers in ways that make sense to them, rather than adopting the teacher’s “one right way” to get the answer.

This paper examines evidence from the primary, secondary, and tertiary (ITE) sectors, to investigate at how schools might contribute to the difficulties that adult learners eventually experience.

**Primary Schools**

New Zealand’s numeracy initiative, the Numeracy Development Projects (NDP), began a little over a decade ago (Ministry of Education, 2001). The NDP emerged as a result of the government’s Literacy and Numeracy Strategy, which was aimed at raising achievement in
mathematics as well as building the capacity of teachers to teach mathematics. Although the focus appeared to be on the mathematics learning of the children, one of the underlying goals was adult education through the up-skilling of the teachers. The project was informed by a trial of Count Me In Too, a program developed for students in the early primary years attending schools in New South Wales, then extended upwards from there (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould, 2005). Key features of the program included a framework outlining progressions in the mental strategies used for solving problems in Number, an individual diagnostic interview designed to identify each student’s stage on the framework, and a comprehensive program of professional learning and development (PLD) for teachers to build their own mathematical understanding and support their classroom teaching. Unlike previous PLD, schools had to commit all their teachers to participate in PLD, and the program involved a combination of workshops and in-class support by mathematics advisers over at least a year (more for those teaching at the upper primary and intermediate levels).

Virtually all teachers working at the primary and intermediate level were given the opportunity to participate in the program. Results from assessments of students at the end of the year’s PLD indicate that students progressed further than they would have without the program, but that numbers of students reaching particular levels of mathematics achievement fell short of expectations (see Appendix A from Young-Loveridge, 2010), according to the latest curriculum and the Mathematics Standards introduced several years ago (Ministry of Education, 2007c, 2009). Only about one third (36%) of students met the curriculum expectations at the end of Year 8 (i.e., to be advanced multiplicative thinkers, able to choose flexibly from a range of partitioning strategies to solve multiplication and division problems, including adding and subtracting fractions and decimals).

Secondary Schools

A secondary version of the NDP numeracy initiative was introduced into secondary schools in 2005. Approximately one third of secondary schools had participated before support was discontinued when Mathematics Standards were introduced into primary schools (Ministry of Education, 2009). Evidence from the assessment of students after a year of PLD showed that students in the first year at secondary school (Years 9) seemed to make little progress beyond that they had attained by the end of Year 8, the final year of primary schooling (see Appendix A). It was discouraging to see that a small but persistent group (about 7%) of students continued to use counting strategies (albeit counting on and skip counting) to solve addition/subtraction and multiplication/division problems. Unfortunately the Secondary NDP was not in place long enough, or implemented in a sufficient number of schools to have any substantial impact on the mathematics learning of the young adults who subsequently graduated from secondary schools.

Initial Teacher Education

Initial teacher education provides an opportunity to break the cycle of low mathematics achievement by ensuring that primary teachers (as adults) are well prepared with strong subject matter knowledge (SMK) of mathematics as well as good pedagogical content knowledge (PCK) (Ball, Hill, & Bass, 2005; Ball, Lubienski & Mewborn, 2001; Ball, Thames & Phelps, 2008; Ma, 2010; Rowland & Ruthven, 2011; Shulman, 1986). There has been some debate about whether more mathematics content knowledge is necessarily helpful to teachers. Researchers who have investigated the connections between SMK and PCK have found no clear linear relationship between these two categories (e.g., Askew, 2008; Ward & Thomas, 2008). Having tertiary level
mathematics is not necessarily an advantage for teachers of primary mathematics, although having limited understanding of mathematics may be a major problem for them.

Ward and Thomas’s (2008) study found that teachers with low levels of SMK also had low levels of PCK, but that those with high levels of SMK had a range of scores on the measure of PCK. That is, some teachers with high levels of SMK had low levels of PCK. This evidence supports the claims of several writers (e.g., Askew 2008; Moch, 2004) that there is a certain threshold level of SMK that is necessary for good teaching, but being able to meet this requirement is not sufficient on its own. This is consistent with Moch’s (2004) argument that a certain level of mathematics content knowledge is necessary but not sufficient on its own to be an effective teacher of mathematics. However, the bottom line is that you can’t teach what you don’t know!

In New Zealand, programs of initial teacher education are now expected to meet certain numeracy requirements (New Zealand Teachers Council, 2010). The document clearly states that:

Careful attention needs to be paid to the knowledge, skills and dispositions expected of student teachers in their preparation if there is an expectation of improving the quality of education for all children and young people (p. 2).

Later in the document, there is a statement about all students entering teacher education needing to be assessed for numeracy competency prior to entry, and those not meeting the requirements needing to meet these prior to graduation. Under the requirements for Academic Entry, the document refers to the University Entrance (UE) qualification (42 credits from the National Certificate of Educational Achievement [NCEA] Level 3 gained in the final year of secondary school, including 14 Numeracy in the two years prior). There seems to be an assumption that UE provides a satisfactory assessment of numeracy competency. However, recent research has shown that university admission status is not related to students’ numeracy (Young-Loveridge, Bicknell, & Mills, 2012).

Although overall, students with UE performed slightly better than those without UE, some students with UE were unable to subtract decimal quantities, multiply two 2-digit numbers, add related fractions together, convert a fractional quantity to a percentage, or work out the original price for an item discounted by one-third (see Appendix B). These results are consistent with those of other researchers who have revealed concerning gaps in prospective teachers’ mathematical understandings (Livy & Vale, 2011; Leikin & Jolfaee, 2011).

The New Zealand Teachers Council has articulated its expectations for what beginning teachers ought to know once they have graduated from the teacher education program (New Zealand Teachers Council, 2007). Notable among these is Standard One: Graduating Teachers know what to teach (under Professional Knowledge):

a) have content knowledge appropriate to the learners and learning areas of their programme.

b) have pedagogical content knowledge appropriate to the learners and learning areas of their programme.

A survey of 129 ITE (primary) students in the final semester of the last year of a three-year Bachelor of Teaching degree at a New Zealand university shows that many ITE graduates (as
adults) do not have the content knowledge in mathematics expected of Year 7 and 8 students (potentially their pupils), let alone the PCK that would enable them to anticipate the likely misconceptions of these children and help to overcome them (see Appendix C). In New Zealand, primary teachers are qualified to teach up to Year 8 (12 to 13-year-olds) whereas in many countries, students of this age are taught by specialist mathematics teachers in the secondary school system. As McArdle (2010) has pointed out, “a good teacher makes a difference” (p. 60), and teacher quality is a lynchpin in reforming education internationally. Hence it is vital for ITE programs to ensure that their students have the necessary understanding of mathematics that will enable them to teach for relational understanding, thus reflecting the shift from traditional to reform-based teaching approaches (Skemp, 2006).

**Attitudes towards mathematics**

An important issue that needs to be considered alongside mathematics achievement is attitude towards mathematics. Research shows that teachers’ attitudes towards mathematics can have a profound impact on their students’ mathematics learning (McGinnis, Kramer, Shama, Graeber, Parker, & Watanabe, 2002; Southwell, White, Way, & Perry, 2005). Teachers’ attitudes seem to influence their classroom practices, and this can impact on the attitudes and learning of their students.

Findings from the National Education Monitoring Project show that primary-aged students feel relatively positive about mathematics. Approximately 84 per cent of Year 4 students and 71 per cent of Year 8 students (averaged across three assessments) had positive attitudes towards mathematics (Crooks, Smith & Flockton, 2010). These findings are consistent with those from a survey of students entering an ITE program who were asked how they felt about mathematics at primary and secondary school, then currently as adults (see Table 2; Young-Loveridge, et al., 2012). There was a marked drop in positivity towards mathematics from primary to secondary school. Although the pattern had improved somewhat by the time these students began their ITE program as adults, it is important to note that more than one third of them (36%) either disliked or really disliked mathematics at that time.

**Table 2: Percentages of students entering an ITE program who liked (or really liked) mathematics**

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>UE</th>
<th>no UE</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=248</td>
<td>n=150</td>
<td>n=98</td>
<td></td>
</tr>
<tr>
<td>At Primary school</td>
<td>83</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>At Secondary school</td>
<td>56</td>
<td>59</td>
<td>50</td>
</tr>
<tr>
<td>Currently</td>
<td>64</td>
<td>62</td>
<td>66</td>
</tr>
</tbody>
</table>

**Conclusion**

The findings presented in this paper show that learners’ levels of achievement in mathematics are below expectations at primary and secondary school, as well as the first year of university at the beginning of an ITE program. At the end of a three-year program of teacher education (primary), some adult learners continued to have major misconceptions about aspects of mathematics, particularly in the areas of fractions, ratios, and proportions. This is consistent with literature showing that fractions are notoriously difficult to teach and learn (Anthony & Walshaw, 2007;
The errors made by the participants were very similar to those evident for upper primary-aged students (Drews, 2005; Swan, 2001; Young-Loveridge, Taylor, Hawera, Sharma, 2006).

There is considerable rhetoric about the idea that teacher education programs can only do a certain amount of the preparation for teaching and that beginning teachers must continue to learn for themselves once they are out teaching. However, the concern is whether or not the teachers with weak knowledge of mathematics actually recognise that this is a problem that needs to be addressed. There is limited opportunity to engage with mathematics (both content and methods) during the ITE program (72 hours of compulsory courses during the first half of the program but none in the second half). There is concern among staff about the use of written tests to assess their understanding on the grounds that this practice does not sit easily with a reform-based approach to teaching mathematics. Although individual task-based interviews might be seen as a better assessment tool, this is simply not practical for the numbers involved.

It is important to recognise that our ITE students, as adult learners, are almost certainly the product of traditional methods of teaching mathematics and have been taught using the instrumental approach (Skemp, 2006). As Pesek and Kirshner’s (2000) study shows so well, trying to teach relationally after learners have been taught instrumentally can be very difficult. Learners are not always receptive to the idea that having a strong conceptual understanding of what they have been taught (instrumentally) could be far more advantageous for them in the longer term. It is often not until the final year of the program that ITE students seem to become aware of the enormity of the challenge they will face when they are a fully-fledged teacher out in a school with their own classroom of learners. An optional course in the final year of the program attracts some students who recognise their weaknesses, as well as others who are strong mathematically and aspire to take on leadership positions in mathematics in their schools. Some students who opt to do no further mathematics beyond the two compulsory courses may never realise that they have gaps and misconceptions in their understanding that need to be addressed. As McArdle (2010) has pointed out, teacher quality is a lynchpin in reforming education internationally.

Educational reform is a difficult process that may take many years (Anthony & Hunter, 2009; Lamon, 2007). However, if we are raise the levels of mathematics understanding across the population, then educators in all sectors will need to play their part in the process. It is vital that primary teachers, as adult learners, are given adequate preparation in mathematics as part of their ITE program so they can teach the children in their own classrooms more effectively than perhaps they themselves were taught. It is time the cycle of poor mathematics achievement was broken.

References


A Review and Summary of Research on Adult Mathematics Education in North America (2000-2012)

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Research in adult mathematics education is spread across the publications of several disciplines – adult learning, mathematics education, and educational theory – or lies hidden in doctoral dissertations. This paper presents findings from a two-year study that identified journal articles and dissertations indexed by the major United States scholarly databases. Primary and secondary themes that emerged are used to categorize the findings. Interesting findings or trends are discussed although that analysis phase of the project is still a work in progress.

Introduction

At ALM-7 in 2000 I presented a paper that analyzed and categorized the research conducted in adult mathematics education during the period 1980-2000 (Safford-Ramus, 2001). That study examined both doctoral dissertations and journal articles, summarizing them separately. The paper was organized around the traditional journalists’ questions: Who, what, when, where, how, and why?

During the past two academic years the same approach was taken to examine published research over the intervening twelve years. Aided by a research assistant, Amelia Rotondo, dissertations and articles were identified and copies obtained for analysis. The work of reviewing each document was split between us–the dissertations fell to me while Amelia assumed responsibility for the journal articles. This report, therefore, presents each effort separately.

We attempted to follow the protocol used in the 2000 study but found that the categories had changed. The earlier study was organized by the themes of that ALM conference which were not a perfect fit for this work. New categories had emerged while others had lapsed. Changes were also needed for locating journal articles. The indexing agencies had changed radically since the earlier study. Articles were now identified using academic search engines and, for the most part, readily available in full-text format electronically.

The Journalists’ Questions

The first question posed was “Who is conducting research?” Was it an individual, possibly a doctoral student, a pair of researchers, or a team working as part of a large research project? Linked to that response was the issue of the funding source. Was the researcher independent or supported by the institution, a private grant, or government funding?

The second question asked was “What?” We looked at the type of research, qualitative or quantitative. Themes emerged as the abstracts were analyzed. Many studies had multiple themes
so we ended up classifying each document by primary and secondary theme. As stated earlier, we began our efforts with the themes from the 2000 project as a framework but quickly determined that the new study would be compromised by forcing the data to conform to the 2000 framework so we allowed new themes to be added and unused themes to be dropped.

The third, and rather important, question in the context of our study was “When?” Adult mathematics education takes place at several levels: adult basic education, adult secondary education, tertiary instruction—both developmental and collegiate, and graduate education. The same question could be answered by the year the research was conducted and the year it was reported.

To answer the next journalist’s question, “Where?” we focused on the institution where the study was conducted. In the United States there is variation in the location where adult basic and secondary instruction takes place. It could be a community-based organization site, a public property such as a library or school, a workplace, or at a community/junior college. We typified the institutions as academic (adult basic education site, community/junior college, or college/university), industrial, or government (military or prison).

The query, “How?” registered the source of the research data. Studies used student records, questionnaires or surveys, pre- and post-tests, classroom observations, and interviews. A few were intensive case studies. Finally, we asked “Why?” the study was undertaken. For example, was it a degree requirement or a report to a funding agency?

**Dissertation Research**

The majority of accredited universities in North America submit their doctoral dissertations to *Dissertations Abstracts International* (DAI) for listing or publication. The publisher, ProQuest, estimates that between 95 to 98% of all United States doctoral dissertations are included in DAI ([http://www.umi.com/en-US/catalogs/databases/detail/dai.shtml](http://www.umi.com/en-US/catalogs/databases/detail/dai.shtml), accessed October 8, 2012) An advanced search was conducted on the DAI index subject headings (SU) using the search argument “Adult” AND “Mathematics” AND “Education” for the years 2000 through 2012. Seventy seven dissertations, four of which are Canadian masters theses, were returned. Two of the dissertations had been included in the 2000 ALM-7 study. After a review of the abstracts, three others were discarded because they were not truly adult subjects, leaving a total of 72 dissertations for review. Appendix A lists the dissertations in chronological order.

Who and Why?

These are rather moot questions as the research was conducted by doctoral candidates to fulfill a degree requirement. They appear to have chosen a question sparked by a problem or situation in their work environment. While many acknowledged emotional or occupational support from colleagues, financial support was not apparent. No one seemed to be reporting a segment of a large, grant-funded project.

What?

Fifty-seven percent of the dissertations were quantitative studies while 36 percent were qualitative. Only seven percent were hybrids containing both quantitative and qualitative methodologies. Nineteen themes emerged as either primary or secondary foci. Figure 1 contains a graph by theme.
When?

The research was overwhelmingly tertiary in nature. Fifty-three dissertations focused on either developmental (22) or collegiate mathematics (31) while three addressed issues on the graduate level. Only ten examined issues in adult education: adult basic education (6) and adult secondary education (4). This is probably a reflection of the earlier questions about “Who and why?” The shift towards a doctoral credential at the community college and administrative levels of tertiary careers may have been the impetus for many of the candidates to seek an advanced degree. After reading a dissertation I often attempted to contact the author with a brief note of congratulations on their research. It was not uncommon to find that they had taken a different post after achieving their degree. From a reverse perspective, there is little impetus for an instructor at the ABE or ASE level to seek a doctorate as it would have little impact on their career path.

![Figure 1. Dissertations by Primary and Secondary Themes](image)

Where?

Not surprisingly, the majority of the doctoral research was conducted at tertiary institutions: 25 studies at community colleges and 30 in university settings. Geographically they were all over North America. Six of the more recent degrees were from Capella University, a for-profit institution.

How?

There were no trends evident in the instruments used to gather data. A variety of survey documents were employed, often designed by the researcher. The least used tool was classroom observation by a peer.
Journal Articles

A search of the Education Resources Information Center (ERIC) database was conducted using the search criteria: “Adult” AND “Mathematics” AND “Education”. The search was limited to full text and peer reviewed journals. 225 results were found. After each abstract was read, the citations were saved in folders labeled “Useful,” “Might Be Useful,” and “Not Likely.” Sixteen abstracts were saved as Useful. Forty-eight abstracts were saved as Might Be Useful. One hundred sixty-one abstracts were saved as Not Likely. The abstracts that were categorized as “Not Likely” were not appropriate for the study because they either discussed middle school mathematics, high school mathematics, or overall education. After the abstracts were categorized, the “Useful” and “Might Be Useful” articles were found.

All of the “Useful” articles were read, and out of the 16 articles, 13 articles were determined to be appropriate for the study. The other three articles were not useful because one discussed self-esteem in all classes, not just mathematics; one discussed how people’s adolescent education affected their level of education; and one discussed “young people’s” attitudes towards math and technology and how high schools need to teach the importance of math thinking. Then all of the “Might Be Useful” articles were read, and out of the 48 articles, 24 articles were determined to be appropriate for the study. There were 19 articles that were not useful because they mostly discussed elementary and high school mathematics or overall education, not solely mathematics. The other five articles could not be found.

A search of the EBSCO database was conducted using the same search criterion that was used on the ERIC database, but all 169 results were the same abstracts that were found in ERIC. No new articles were found. The search continued on the ProQuest Central database. In the ProQuest Central database, the search criteria used was: “Adult AND Mathematics AND Education” in Subject Heading from the date range 1999-2011. The search was narrowed by source type: Books, Conference Papers and Proceedings, Scholarly Journals, and Trade Journals. 96 results were found. Each abstract was read. 17 abstracts were saved as Useful. 14 abstracts were saved as Might Be Useful. Sixty-five abstracts were saved as Not Likely. After the abstracts were categorized, the “Useful” and “Might Be Useful” articles were found and read. Out of the 17 “Useful” articles, 8 were appropriate for the study. Three articles were not useful. Three articles had already been found in the ERIC database, and the other three articles could not be found. The “Might Be Useful” articles could not be found and therefore could not be classified as appropriate to the study.

Who and Why?

The researchers who conducted these studies picked topics in areas that they found interesting or areas that they noticed have problems. In most of the articles, the researcher used his/her classroom or institutional department as the setting. But in some of the articles, it is apparent that the researcher sought out participants for their study. For example, some researchers sent out requests to homes in a particular neighborhood or city.

What?

This question looks at whether the research method used was qualitative, quantitative, or both, and what were the primary and secondary themes. There was an even amount of qualitative and quantitative studies. Out of the 46 articles found, 16 were quantitative and 17 were qualitative. The rest of the articles used both qualitative and quantitative methods. Out of the 17 articles that
used qualitative studies, half of them used information gathered from other studies to discuss their research question. The rest of the articles used their own information gathered from observations and case studies conducted at adult basic education sites, community colleges, universities, homes, and workplaces. Out of the 16 articles that used quantitative studies, 13 used tests and questionnaires to gather information. One article used Raven’s standard progressive matrices, test number series, and self-report questionnaires. Another article used a ten-question survey, analyzed with a Simple Analysis of Variance and Two-Way Univariate Analysis of Variance. One article used statistical samples of entire populations. The other 12 articles that used both quantitative and qualitative research methods used questionnaires, tests, observations, and case studies to gather their information. One article was unclear whether or not a quantitative or qualitative study was conducted.

The primary themes varied greatly among the 46 articles. While reading some of the articles, it was hard to determine one primary theme because two or more themes were very important to the question studied by the researcher.

When?
There are five levels of adult mathematics learning that the articles focused on: adult basic education, adult secondary education, post-secondary developmental, undergraduate, and graduate. One study took place at both the adult basic education and adult secondary education levels. One study focused only on adult secondary education. One study took place at both the adult basic education and post-secondary levels. One was at the graduate level. Two studies were at the post-secondary development level, focusing on teachers. One study focused on both undergraduate and post graduate levels. One study focused on undergraduate and post-secondary development. It was unclear in one study what level of mathematics learning the researcher studied. Of the remaining 37 articles, 18 focused on undergraduate mathematics learning in community colleges, colleges, universities, and one researcher conducted their study in an undergraduate nursing school. Nine studies were conducted at the adult basic education level, and the remaining nine studies were conducted at the post-secondary development level.

Where?
Most of the studies were conducted outside of the United States. Out of the 46 articles, only 15 of the studies were conducted in the U.S. Eight of the studies were conducted in the United Kingdom, and four studies took place in Canada. Two did not state where the study took place. The rest took place in Europe, Asia, and Australia. These countries include Finland, France, Ghana, Greece, Ireland, Japan, New Zealand, Pakistan, Scotland, Spain, Sweden, and Turkey.

How?
Out of the 17 qualitative studies, half used information gathered from other studies. The rest used information gathered from observations and case studies. Out of the 16 quantitative studies, 13 used tests and questionnaires to gather information. The other three used methods that pertain to the specific study. Out of the 12 articles that used both, the researchers used tests, questionnaires, observations, and case studies.

Observations
We know a great deal about students who are at-risk and they have a similar profile to students at risk in elementary and secondary school: women and minorities. Math anxiety is perhaps greater
than that of the younger population because earlier negative experiences have had time to ferment in memory. On the other hand, self-efficacy proves to be the clearest indicator of success.

We know a little about interventions that can help students be successful. A few studies altered classroom methods and achieved some degree of success. A disturbing fact that emerged from the studies that varied the use of technology was that online courses carry a high risk of dropout or failure even for adult students whom theory alleges are self-motivated and desirous of online work. As a personal observation, there has been a rush in the United States to offer online courses to all students, particularly adults, without sufficient research to determine whether this is an appropriate delivery system for mathematics instruction. The studies included in this research suggest caution.

Adult students need assistance when moving to tertiary studies. Secondary completion and tertiary placement assessments are not aligned which is a detriment to a smooth transition for students who often find themselves in developmental classes after successfully passing the high school equivalency examination. Non-traditional and adult students need support services that help them successfully adjust to the college classroom — services that address their emotional fragility and academic skills.

**Concerns**

By the conclusion of this study, I have read about 100 doctoral dissertations from the past 32 years. It was my hope that I would see a steady progression of work that built upon earlier research and experiments designed to incorporate andragogical theory in practical classroom methodology. Sadly, this is not what I found. In some ways we have regressed. Few of the literature reviews referenced the body of earlier doctoral research and many of the theoretical frameworks were similar, based on decades-old theory. There is no coherence to the body of work and I found it to be illumination by firefly rather than spotlight. The geographic dispersal of the scholars and the lack of funding for research are just two possible reasons for this state of affairs.

Quantitative studies that predict student success continue to be pursued, probably because the data is readily available and analysis with statistical software is easy. Such studies are not intrinsically deficient but they revisit the diagnosis of the patient without exploring a possible cure for the ailments. They are terminal rather than seminal. The population of the United States continues to diversify yet I found scant attention paid to classroom methods or interventions that addressed the needs of minority students or women. Study after study determined that self-efficacy is the best predictor of success when all other factors are considered. How then, do we promote that quality and assist adult students to succeed in the mathematics classroom?

The lack of journal articles that our project found is very disturbing. Both the ALM proceedings and journal are missing. If we are writing about research and practice and no one indexes our work, how do we get the message out? In the topic group last year in Dublin I began the session with the old question “If a tree falls in the woods and no one hears it, does it make a sound?” We need to be proactive in advancing the indexing of work published by our members both in our publications and the wider circle of journals.
Moving Forward

At the conference this slide referenced an old Beatles song that offered the challenge “You say you want a revolution…” Based on this research study I suggest the following agenda for action:

- We need to design and assess interventions that help:
  - Decrease Math Anxiety
  - Increase Self-Efficacy
  - Initiate a Shift in Student Perspective.

- We have to align the secondary school proficiency assessments with the college placement tests.

- We need to design and teach adult and developmental mathematics courses that reflect the mathematics adults need to succeed in our democratic societies.

- We need to imagine and execute innovative tactics to jumpstart non-traditional students entering tertiary study
  - Intense, pre-entry classes
  - Short, repeatable modules
  - Effective, appropriate use of technology.

Adults Learning Mathematics—A Research Forum is the only organization dedicated to adult numeracy that spans the delivery system from adult basic education through graduate school and professional development. As we approach our twentieth conference I would respectfully suggest that we focus our work to illuminate the efforts of the thousands of instructors who teach millions of adult mathematics students across the globe.

References


Education Resources Information Center (ERIC), http://www.eric.ed.gov.


### Appendix A: North American Dissertations Published 2000-2012

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An Emerging Framework for Ethnography of Adult Mathematical and Numeracy Practices

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This paper presents a potential ethnographical framework for examining in-situ adult mathematical practices. It results from a meta-analysis of over 400 articles and reports published on adult mathematics and numeracy practices in the workplace, everyday life, and assorted other situations where numeracy is present (for example, sports events). The framework is also informed by a combination of my own academic sociolinguistic and mathematical backgrounds. Consequently, it draws on a synergy of insights from sociomathematics, ethnomathematics, social practices theory, and the history of mathematics. It is envisioned that this ethnographic framework may assist in excavating mathematical practices at multiple levels (semiotic, material, discursive and diverse others), and thus provide a way forward to offering, among other things some pedagogical insights on the teaching and learning of mathematics for adults.

Introduction

This paper sets out an incipient framework for an ethnography of adult mathematical practices. To some extent, it was “inspired” by the work of Dell Hymes and his well-known “Ethnography of Communication” (Hymes, 1964 & 1974). Saville-Troike took up the Hymesian framework and expounded its potential in more detail (Saville-Troike, 1982). As a sociolinguist, what struck me about the framework was the lack of any reference to mathematical practices. In fact, it is fair to say that mathematics seldom comes within the orbit of ethnographical considerations.

The current discourse on adult numeracy and literacy “skills” in New Zealand and many other OECD countries, which was brought to the radar screen by the recent All (Adult Literacy and Lifeskills) survey (for an overview in New Zealand, see Satherley & Lawes, 2007), has made numeracy practices an object of more intense investigation. The extant discussions of the results of the survey invariably equate numeracy ‘scores’ with numeracy ‘skills’. If ‘skills’ are seen to be things performed in real settings, then this ‘scores-equals-skills’ equation is a considerable pedagogical and indeed epistemological leap. An ethnography of maths practices will put this ‘scores-equals-skills’ equation under the spotlight.

Two related concepts have already arisen in the course of this discussion: mathematics and numeracy. In the literature definitions of adult numeracy are most usually defined with respect to mathematical practices as they are carried out in real every day, and employment contexts. However, it must be acknowledged that the majority who define numeracy in this way seldom
venture any definition of mathematics itself. There is then a certain sense that mathematics is taken for granted (Leng, 2002). It is seen as occupying an impenetrable domain of pure thought. There is thus a need “To haul this lofty domain (mathematics) from the Olympian heights of pure mind to the common pastures where humans toil and sweat” (Struick, 1986, p. 280); Struick cites Engels in Anti-Dühring.

Yet if an understanding of adult mathematical practices is to be entertained, some discussion of the contested area around a definition of mathematics is necessary. However, for the purposes of this paper, a potential definition will be left implicit within the emergent framework. An ethnography of mathematical practices opens up a space in which hegemonic, normalized, eurocentric mathematical norms can be interrogated (see Reinhart, 2012). Bakhtin (1982, 1986) refers to centripetal and centrifugal force at work in language formations – a concept I believe can be fruitfully applied in the mathematical sphere to enhance such an interrogation.

Rationale

An ambitious project then it is. My motivations, aside from those intimated above, are a love of mathematics, a desire to enhance mathematical pedagogy and andragogy, a concern for those who dislike maths, and most of all, an intense curiosity about what people really do when they do mathematics (practices that are often designated by agents as ‘common sense’ rather than mathematics).

Methodology

The method employed in this study falls under the general rubric of a meta-analysis. It is not strictly so. It has involved searching through hundreds of articles and viewing videos that explicitly or implicitly deal with actual mathematical practices. In other words, this project makes use of the labours of others and endeavours to seek a framework in the light of already explicated practices.

Ethnography of mathematics

What follows is an overview of what an ethnography of mathematics might consist of from a number of perspectives.

What is an ethnography of mathematics?

A broad definition of ethnography would be “people’s actions and accounts are studied in everyday contexts” (Hammersley & Atkinson, 2007, p. 3). However, Hammersley and Atkinson acknowledge the complex and contested nature of what ethnography is.

I have broadly divided an ethnography into three types and utilized the terms etic and emic first proposed by Kenneth Pike (1967). Firstly, there is an etic approach. I call this exo-ethnography. Here the focus is on bringing an outsider’s ‘grid’ or framework to the mathematical practices of people in their assorted cultural settings. Secondly, there is an emic approach. I call this endo-ethnography. The focus here is on an insider’s perspective (or a participant’s perspective) on mathematical practices. Finally, there is the concept of an auto-ethnography where a person examines their own mathematical practices. There are of course important philosophical concerns with each of these approaches that have been well discussed in the literature (for example Hammersley & Atkinson, 2007). Each can be seen to offer advantages and disadvantages. In what follows, the emphasis is on an etic orientation (an outsider’s view point).
Who does ethnography?
A range of researchers/practitioners from various disciplines have an investment in an ethnography of mathematical practices: ethnographers; anthropologists; historians; archaeologists; teachers and trainers; health and safety experts; policy makers; and of course mathematicians themselves.

Apart then from the obvious answer of researchers, an understanding of mathematical practices, both overt and covert, could well benefit all the disciplines. Mathematicians themselves may gain a deeper understanding of their practices which may in turn lead to alternative strategies and more elegant and significant mathematical outcomes. Across the “divide” in the humanities and arts, practitioners could benefit from the explicit and implicit practices uncovered in a range of disciplines including art, dance (Gadanidis & Borba, 2008), design and music (Beer, 2005).

Where might an ethnography be done?
The ubiquitous presence of mathematics in daily lives answers this question. It also opens up spaces and domains where practices maybe compared and contrasted, for example classroom maths “versus” street maths (Saxe, 1988).

Why might an ethnography be done?
Our intuitions about mathematical practices are often wrong and practices are often so embedded, they become invisible. Also, formal maths practices in classroom are often cut off from practical mathematical uses in more informal domains. As a result, many adults carry with them often stark memories of seemingly irrelevant high school mathematical experiences. The endless, seemingly pointless, array of decontextualized algebra problems often ‘bewitched, bothered and bewildered’ many students as to the purpose of mathematics.

Yet mathematics in many ways has been and still is a high-stakes game. In hospitals, for example, this is certainly the case. And the repercussions of $E=MC^2$ have reverberated into this century. Whether it is the wave/particle paradox that supports quantum physics or the mathematical considerations that makes all sports winnable, or losable (in my experience), we live in a world saturated with numbers.

An Emergent Framework

Hyme’s original ethnography of speaking framework used the pneumonic: S-P-E-A-K-I-N-G. It can be elaborated thus: 1. Setting and Scene; 2. Participants; 3. Ends; 4. Act Sequence; 5. Key; 6. Instrumentalities; 7. Norms and 8. Genre (Hymes, 1973). I bear this in mind as I ask the most basic of questions. It was Rudyard Kipling who coined the idea of “six faithful servants” (he called them serving men at the time). And so, for simplicity sake, this would be a good place to start.

Where is the mathematics being practised?
Here the primary consideration is given to the general environment where people are practising maths overtly or covertly. Words like event, situation, domain and episode come to mind. The ‘setting and scene’ of the Hyme’s mnemonic is in view here. Any environment from a lecture theatre to a cross-country event can be included (Figure 1).
What does the mathematics involve?

Needless to say, some professional mathematicians would delimit mathematics to what professional mathematicians do. However, this paper sees mathematics in its broadest context. If any emphasis is to be placed, it would be that mathematics has much to do with the ‘science of patterns’. We tend to presuppose that every maths event involves solving a problem. However, there is also an element of play, performance and patterning. Below we look at what mathematics might involve (from an etic point of view).

1. The material environment.

Here the material environment is under consideration in terms of ‘objects’ (see Riss, 2011). Some artefacts are obvious or explicit – of course textbooks, calculators, computers, white/black boards spring to mind in more formal mathematics settings. In informal settings, there are tools, instruments, blueprints, maps and plans. There are all manner of artefacts that mediate mathematical processes including the human body. Some artefacts are less obvious or implicit – particularly in the process of estimation. Hymes’ notion of ‘instrumentalities’ is in view here. Some artefacts (and processes) can be thought of in terms of “black boxes” (Williams & Wake, 2007). Here, by means of processes, instruments, routines, agreements, division of labour and so forth, mathematical processes become solidified and extremely opaque to outsiders. Indeed it would be the role of an ethnography of maths to ‘unpack’ such black boxes. Of note, the maths practices of the renaissance were very much reliant on artefacts. Oosteroff (2011) discusses the practices of Mutio Oddi as a case in point.

2. The semiotic environment

The material environment intersects with the semiotic environment (a key example is formulae). Semiotics refers to the sign systems humans “invent” and use to construct meanings. This area would include language, discourse and genres in general as well as multiple other sign systems. These might include, symbolic orders, multiple representations, inscriptions, traces, and hierarchies of abstractions.

Proxemics and in particular gesturing (see Rasmussen, Stephan, & Allen, 2004) can also be seen under the purview of semiotics. The notion of ‘genre’ from Hymes comes closest to this idea of semiotics.
3. The rules of the game.

Here, I allude to Wittgenstein and his notion of “language games” (Wittgenstein, 1953). In Hymes’ initial framework, ‘norms’ would fit with this realization. Formal mathematics is the subject par excellence which insists on norms, rules, conventions, theory, prescriptions, canons (e. g. Euclid), classifications, syllabi, assessments and policy. However, in all contexts of explicit or implicit maths use, there are norms, social norms, conventions, and “recognition and realization rules” (Bernstein, 1996).

Who does mathematics?

Here a lot of trajectories merge. Is the mathematical practice solo or social? What networks and communities are involved? There are issues around identity and subjectivities which inevitably lead to questions of power structures on one hand and psychological dispositions towards mathematics on the other. There are issues around whether maths understandings are purely cognitive or whether there is an embodied, situated dimension. Supposed binaries also come into play: expert/novice; teacher/student; foreperson/labourer; and official/athlete.

Implicated in the ‘who’ then are the dispositions that the ‘players’ may carry towards mathematics. Certainly maths anxiety is a phenomenon that can haunt many mathematical efforts.

How is the mathematics being done?

Firstly, I refer to something said earlier. Mathematics practices might be thought of in terms of problem, play and performance, and the search for patterns. Related to this is the supposed binary of concrete versus abstract. See Roth and Wang (2006) for an analysis of this binary. For practical purposes this binary will be invoked here, but with the understanding that it needs to be critiqued.

The concrete might involve such familiar dimensions of counting, measuring, shaping, forming, estimating, moving, calculating, proving, puzzling, grouping and so forth. The abstract could include such processes as: strategizing (local and/or global); abstraction; deduction; induction; pattern searching; analysis; exploration; modelling; encoding and decoding; tabulation; graphing; hypothesising and so forth. Of course some of these clearly overlap with the concrete processes previously mentioned.

At another level are the organizational processes of routines, division of labour, transfer, task decomposition, idiosyncratic procedures and assorted mathematical repertoires. Practices may be particularly implicit as for example in dance.

Why is the mathematics being done?

Again we return to the now familiar theme of problem, play, performance and pattern making.

Leading the fray among these is no doubt the problem solving dimension. Sometimes play turns into problem solving. Some of Riemann’s work/play with non-Euclidean geometries seemed to have had little use at the time. However, years later, it was to Riemann’s work that Einstein turned in order to develop both the special and general theories of relativity. Einstein was able to solve the problem of how gravity works with a new configuration of space-time that was a direct consequence of Riemann’s “play”.
The question of what are the purposes, goals, objectives is important, especially so for students learning maths and pondering its relevance to their lives.

**Conclusion**

Finally, I want to finish with a somewhat superficial and artificial table which compares ‘Academic mathematics’ with ‘real world mathematics’ (I use the terms loosely). However, the table is only indicative of deeper insights that might be achieved through an ethnography of mathematical practices.

**Table 1. A comparison of academic versus real world practices.**

<table>
<thead>
<tr>
<th><strong>Accuracy</strong></th>
<th><strong>Estimation</strong></th>
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<tbody>
<tr>
<td>One right answer</td>
<td>A number of possible answers</td>
</tr>
<tr>
<td>Individualist orientation</td>
<td>Social/collaborative orientation</td>
</tr>
<tr>
<td>Axiomatic</td>
<td>Pragmatic</td>
</tr>
<tr>
<td>Generalisable principles</td>
<td>Local problem solving strategies</td>
</tr>
<tr>
<td>Formal system of signification</td>
<td>Functional system of signification</td>
</tr>
<tr>
<td>Subject positioned in terms of maths ability</td>
<td>Subject positioned not in terms of maths ability</td>
</tr>
<tr>
<td>Context is secondary</td>
<td>Context is primary</td>
</tr>
<tr>
<td>Absolute knowledge</td>
<td>Provisional knowledge</td>
</tr>
<tr>
<td>Maths as a stand-alone entity</td>
<td>Maths as a practical social consciousness</td>
</tr>
<tr>
<td>Closed self-referential discourses and genres</td>
<td>Open routine discourses and genres</td>
</tr>
<tr>
<td>Specialised regulations on language – on definitions and strict manipulations of formal objects</td>
<td>Everyday uses of language – fluid definitions, multiple manipulations of informal objects</td>
</tr>
<tr>
<td>One is positioned as incompetent.</td>
<td>One is automatically positioned confident by self and others</td>
</tr>
</tbody>
</table>

The above table is simply a basic look and what a potential framework may achieve. Many areas of the incipient framework outlined above are not considered in the table. The table tends to focus on rules, semiotics and identity. Next on my agenda is to seek out a possible mnemonic for the framework and bring more order to it. There will also be the need to justify an etic approach to maths practices. Clearly the applications are many in uncovering what people actually do when they engage in mathematics practices. Hopefully, such an understanding will lead to enhancing pedagogical, assessment, syllabus and policy decisions. It may also help with an understanding of how transfer of mathematics across domains may be accomplished.

By ethnographically comparing practices across domains it will assist in making classroom mathematics, whether in child or adult contexts, more relevant, meaningful and exciting.

**References**


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Roth, W., & Hwang, S. (2006). Does mathematical learning occur in going from concrete to abstract or in going from abstract to concrete? Journal of Mathematical Behavior, 25, 334-344
Divergent Learner Pathways: Exploring the Mathematical Beliefs of Young Adult Learners

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Maladaptive beliefs about mathematics have been described as those in which mathematics is characterised as facts, rules and procedures, thought to be learned primarily by rehearsal strategies, and perceived to reflect one’s intellectual abilities. These beliefs often develop during school years and may contribute to poor learner engagement in mathematics and subsequent poor mathematical achievement. Maladaptive beliefs may also be held by adults studying numeracy in a vocational context and contribute to negative attitudes and approaches to learning. The aim of this study was to explore the mathematical beliefs of young adult learners who had struggled with mathematics during their school years. Six young adults took part in individual interviews that explored their beliefs, attitudes and learning histories. The results indicate that the learners did hold maladaptive mathematical beliefs and that these beliefs influenced how they approached learning numeracy in the tertiary sector. Recommendations are made for an increased focus on improving learners’ beliefs as part of numeracy provision.

Introduction

Many adult learners who experienced difficulties with mathematics in the compulsory sector continue to struggle with numeracy in the tertiary sector. Despite the contextualised nature of the provision, comprehensive embedded numeracy processes, and the efforts of tutors, many young adult learners simply fail to master the numeracy content necessary for their chosen vocation. Research suggests many of these learners developed maladaptive beliefs about mathematics during their school years, and that these beliefs continue to negatively impact their cognitive and affective engagement in the topic (Briley, Thompson & Iran-Nejad, 2009). Little is known about the mathematical beliefs of young adults in New Zealand who are working toward their first qualification in the tertiary sector and who struggled with mathematics in school. This article presents one component of a larger study that explored the mathematical beliefs of six adults who struggled with mathematics, attended alternative education, and were enrolled in their first year of a Level Two qualification.

Background

Many learners develop maladaptive beliefs that negatively impact their behaviours, motivation, attitudes and dispositions toward mathematics (Muis, 2004). These learners believe and act as though mathematics is an unconnected series of rules and procedures that need to be memorised; that mathematical success is the product of innate ability, not effort; that answers to problems can be solved quickly or not at all; that mathematics knowledge is passed down only by sources of authority; and that mathematics is not useful in everyday life (Briley, et al., 2009; Mason, 2003; Schommer-Aikins, Duell, & Hutter, 2005). This narrow concept of mathematics is in contrast to that held by many mathematics researchers who see mathematics as deeply interconnected, inter-
dependant, aesthetic, intuitive and able to be learned through research, rather than directly transmitted (Reid, Wood, Smith & Petocz, 2005).

The narrow beliefs described above have relationships with behaviours contrary to those recommended by research-based reforms that emphasise the need for conceptual understanding over memorisation of rote procedures (Young-Loveridge, 2011). Additionally, a growing body of research suggests that beliefs of this nature act as constraints on learners’ construction of mathematical knowledge (Mason, 2003; Schoenfeld, 1989). For example, students who hold these beliefs are more likely to employ rehearsal strategies to memorise single-strategy solutions, in contrast to seeking conceptual understanding by exploring multiple solutions, making connections, sharing solution strategies and explaining to others (Hofer, 1999; Meyer & Parsons, 1996). Additionally, when the rote recall of a procedure is unable to be adequately applied to a problem-solving task, the learner may reduce effort early or stop trying completely as they are operating under the belief that mathematics problems can be solved quickly or not at all. This in turn may reinforce the erroneous belief that mathematics ability is ‘innate’, further eroding their self-efficacy and effort (Blackwell, Trzesniewski, & Dweck, 2007). In essence, maladaptive mathematical beliefs create a dissonance between how learners believe mathematics is learned and how it is actually learned.

Maladaptive beliefs are also theorised to lie at the root of poor affective responses to mathematics (Goldin, Rosken & Törner, 2009). Poor affective responses include, mathematics anxiety, fear and shame, and in some cases may lead to learned helplessness (Bibby, 2002; Yates, 2009). Evidence suggests that these factors are as damaging to achievement as low numeracy skills themselves, because self-efficacy, attitudes and emotions all affect how learners approach learning opportunities and engage with educational material (Cretchley, 2008; Grootenboer, Lomas & Ingram, 2008). Adults with poor affective responses tend to avoid environments where their lack of skills will be exposed. Where this is impossible, they often disengage and may resort to self-protection strategies such as absconding or shutting off, disguising their lack of knowledge through deception, verbal self-denigration or creating diversions (Bibby, 2002). The essential need to address adult learners’ affective reactions to numeracy instruction is emphasised in the literature (Coben, 2003, 2005), as much research has described the mental anguish and difficulties that many adult learners experience when required to engage with mathematics (Bibby, 2002; Carroll, 1994). Maladaptive mathematical beliefs contribute to poor affective responses that continue to influence learners throughout their adult lives.

Given the seriousness of mathematical beliefs and their possible consequences on learners’ attitudes and approaches to learning, it is important to determine whether maladaptive beliefs are held by adult learners. The aim of this study, therefore, was to explore the mathematical beliefs and attitudes of young adult learners who had experienced difficulty learning mathematics in school and were now studying numeracy in a vocational context. The method, findings, and discussion are described below.

**Methodology**

**Participants**

Six students attending two level one to three NZQA training programmes volunteered to take part in the study. The programmes were run through a single private training establishment. The students were aged between 16 and 17. Four were enrolled in an employment skills programme
and two were enrolled in a retail programme (see Table 1). Both the retail programme and the employment skills programme had a strong numeracy component.

All but one participant had been granted an exemption from school at the age of 14 and had attended an alternative education programme until the age of 16. For all six students, the current course was the first they had undertaken since attending alternative education. None of the participants had school qualifications and all were working toward the Level One and Two New Zealand National Qualification in Employment Skills. Three of the six students identified themselves as New Zealand Māori, one as Columbian and two as New Zealand European (see Table 1).

**Table 5. Participant profiles**

<table>
<thead>
<tr>
<th>Participant</th>
<th>Gender</th>
<th>Ethnicity</th>
<th>Age at time of exemption</th>
<th>Type of programme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aranga</td>
<td>Female</td>
<td>NZ Māori</td>
<td>14</td>
<td>Employment</td>
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<tr>
<td>Hone</td>
<td>Male</td>
<td>NZ Māori</td>
<td>15</td>
<td>Retail</td>
</tr>
<tr>
<td>Caleb</td>
<td>Male</td>
<td>NZ European</td>
<td>14</td>
<td>Employment</td>
</tr>
<tr>
<td>Shelley</td>
<td>Female</td>
<td>NZ Māori</td>
<td>14</td>
<td>Employment</td>
</tr>
<tr>
<td>Tori</td>
<td>Female</td>
<td>NZ European</td>
<td>14</td>
<td>Employment</td>
</tr>
<tr>
<td>Rod</td>
<td>Male</td>
<td>Columbian</td>
<td>14</td>
<td>Retail</td>
</tr>
</tbody>
</table>

**Procedure**

Individual semi-structured interviews were used to explore the participants’ beliefs about and attitudes toward mathematics. Interview questions included prompts that explored issues such as school history, anxiety, attributions of success and failure, definitions of mathematics and specific experiences that might have influenced their perspectives. The questions were designed to generate ideas, opinions and memories of personal experiences with mathematics. The interviews took place in a private room with only the researcher and participant present. Each took approximately one hour to complete and all six were recorded on a video recorder. Recordings were transcribed. Open coding was used to organise the data into meaningful units of analysis. These were organised into broader categories (e.g., study methods, relationship with teacher, emotional reactions) from which larger themes emerged. These themes are described in the findings below.

**Limitations**

The reliance on self-reporting inherent in using semi-structured interviews is problematic when attempting to identify underlying beliefs. Participant responses may be inconsistent, memories of specific events may be inaccurate and responses may be situated in a particular context. Moreover, these responses must be interpreted by the researcher as indicative of underlying beliefs. Future research design requires a range of data collection methods with a focus on triangulation that may include interviews, observations and surveys. With these limitations in mind the following chapter presents the findings of the research.
Findings

Interview responses

The content of the interviews was analysed and organised into five prominent themes that emerged from the data. These themes are explored under the following headings: concepts of mathematics; the purpose of learning mathematics; the importance of understanding mathematics; mathematical authority; and learning strategies. Each theme is treated individually, but each informs and is informed by the others.

Concepts of mathematics

Participants were asked to describe mathematics in their own words in order to provide an insight into their beliefs regarding their conception of mathematics. Five of the six participants described mathematics as a discrete subject that involved numbers and procedural rules. These participants’ descriptions were uniformly related to classroom contexts and made no reference to the utility of mathematics as a problem solving tool. For example, in response to the question, ‘Describe what you think maths is all about’ the participants responded:

Shelley: Numbers.
Aranga: Numbers.
Tori: Calculation.
Hone: Numbers.

There appeared little appreciation of what lay behind those numbers. Further themes emerged when participants were asked prompt questions. Mathematics was perceived to be a subject one was compelled to take part in, difficult, and simply a collection of discrete pieces of knowledge. For example, when asked how mathematics would be described to a friend:

Aranga: It’s just a hard subject that you have to do. You have to learn all those ways of doing things.
Tori: You have to solve equations, concentrate, practise.
Shelley: Counting and times tables and divided by and stuff.

Tori, Aranga and Shelley never spoke of mathematics outside a school context, despite the three of them having been out of school for at least two years and being in full-time study in an employment skills programme at the time of the research project. Rod and Hone shared similar beliefs, describing mathematics as a school subject with some relevance to application but that required one to apply pre-determined procedures to specific types of tasks.

In summary, the participants’ concept of mathematics reflected the belief that mathematics is a ‘subject’, discrete from other bodies of knowledge and real-world applications. Additionally, mathematics was conceptualised as a subject that required learning a multitude of isolated ‘methods’ to solve teacher-presented mathematical tasks. Mathematics was viewed as inherently difficult, and had notions of compulsion associated with it.
The purpose of mathematics

Participants were invited to speak about the purpose of learning mathematics. The responses of three of the participants’ indicated that the purpose of learning mathematics was to complete teacher-presented problems correctly, and showed no appreciation of the usefulness of mathematics to other contexts.

Rod and Hone, in contrast, linked mathematics to a context outside of the learning environment, that of dealing with money:

Rod: Money. If you want to give money out to people.

Hone: You need to know maths for money. So you don’t get ripped off.

Caleb, however, clearly thought of mathematics as a necessary skill vital to working in real-world contexts.

Caleb: Sort of like if you are like a builder you need the maths, the measurement part so you don’t stuff up.

In summary, three of the participants held that mathematics was purposeful for its utility in real world contexts. Two of these participants made links between mathematics and the context of using money, suggesting a future-focused view of learning mathematics. The other, Caleb, made links between learning mathematics and a specific vocational example that suggested a broader appreciation of the usefulness of mathematics in adult life. Three participants expressed views consistent with a view that the purpose of mathematics is to correctly answer mathematical questions while in a learning environment.

The importance of understanding mathematics

A third theme emerging from the data was that of the participants’ beliefs regarding the need to understand mathematics. The predominant view of the participants was that getting the answer correct was the ultimate goal of mathematics. Understanding mathematical concepts, while valued, was peripheral. While each of the participants stated that they thought it was important that mathematics made sense to them personally, further questioning revealed that for them the primary reason for understanding was to meet teacher expectations. For example, when asked ‘How important is it that maths problems/answers make sense to you?’ responses included:

Tori: Very, so I can explain to the teacher how I got the answer.

Shelley: Sort of important – I get bothered if I, um, don’t know what it is.

To explore further the role of understanding, the participants were asked if getting a mathematics answer correct was more important than understanding it. Four of the six maintained that it was. Further prompting explored views on how important it was to be able to describe how a problem was solved to someone else. While understanding was valued, participants held that getting the correct answer was the priority:

Aranga: Not that important, it’s better to get it right.
Caleb diverged from the others and expressed the idea that the purpose of understanding mathematical concepts was in order to learn more in the future. This was consistent with his view that mathematics was applicable to a wider context than the classroom:

Caleb: If you don’t understand it [mathematics] then you can’t use it to learn anything else.

In summary, the predominant view of the participants is that understanding a mathematical solution strategy or procedure was secondary to answering questions correctly. The participants’ perspectives linked with their views regarding the purpose of learning mathematics. Those who held that the purpose of mathematics was to answer teacher questions valued correct answers over understanding. In contrast, those who held mathematics to have a purpose outside the classroom context valued conceptual understanding.

Mathematical authority

The fourth theme was that of mathematical authority. Questions were asked to investigate whether participants held the view that mathematics knowledge must be verified by an external authority or, in contrast, is able to be self-verified through logic or reason. Five of the participants held views consistent with the belief that mathematical knowledge is held and verified by an external authority. The participants’ answers indicated that they relied on an external authority to determine mathematical success, rather than seeking to verify solutions using reason or collaboration. For example, they were asked ‘If two students disagree on the answer to a mathematics problem, what should they do?’, and replied as follows:

Hone: Teacher.

Aranga: Tell them to get a calculator.

Shelley: Use a calculator.

Tori: I’d get the teacher’s answer and also the calculator’s answer and maybe someone else’s.

Again Caleb’s views can be contrasted with that of the other participants in that he expressed views consistent with the belief that answers could be found and verified through reasoning. For example, in answer to the question above:

Caleb: They should work it out together and see if one is right, then the other could help him.

However, appeals to the teacher as the arbiters of mathematical success or failure were frequent and the preference for an external authority to provide a confirmation of their success was strongly held. The external authority was generally the teacher or a calculator, suggesting the view that all mathematics answers have a single correct answer.

In summary, understanding mathematics was viewed as a means to achieving the correct answers to pre-determined questions and being able to demonstrate this to the teacher. The participants relied on external sources to determine the accuracy of their answers rather than relying on their own conceptual understanding. Except for Caleb, ‘understanding’ mathematics did not appear to be valued for any future learning benefits.
Learning strategies

The fifth theme related to learners’ perceptions about how mathematics is learned. None of the participants gave an indication of knowledge of, or use of, learning strategies. They attributed their lack of mathematical achievement to not concentrating or listening to teacher explanations. Learning was expected to occur simply from passive exposure to mathematical information, indicating a belief in quick learning, that mathematical knowledge is learned quickly or not at all. Comments regarding teachers and tutors were consistent with this belief. The participants stressed that a good teacher or tutor was one able to explain rather than one who engaged learners in learning tasks:

Tori: Mrs __________. She was pretty good. She explained everything down to the last little bit. She would give us work from the board or from the book.

Caleb: She was real good, because if we didn’t get it, she would just keep explaining it until we did.

Caleb noted that a teacher who kept him in class after break to complete work was not as good because he failed to explain the mathematics:

Caleb: I didn’t enjoy my last teacher. He sort of picked on me a bit. Like um, he didn’t explain it more, he um, I don’t know. He made me stay behind and do it, but didn’t explain it.

All the participants described their mathematical difficulties in terms of passive responses to mathematical information, classes and teachers. When asked specifically why they struggled to learn mathematics, indications of disengagement and passive responses to teachers emerged frequently:

Tori: He [the teacher] could ramble on about so many things but I would never get it. [Prompt: And what were you thinking to yourself?] I don’t even understand. But I would usually just stay quiet.

Shelley: ‘Cause all my friends are in the class and we’d just talk all day instead of listening and when I didn’t listen she’d carry on talking [the teacher] and I didn’t listen.

Aranga: He would just start and I tried to keep up. I hated maths.

In summary, the majority of the participants held that mathematics is a discrete subject in which one is compelled to learn a collection of procedures to use in response to specific mathematical tasks. The purpose of learning mathematics was largely confined to the classroom context, although its utility was connected to societal uses by three participants. Understanding mathematics was considered important but less so than solving problems correctly; and solutions to problems required verification by an external authority, not through logic or reason. Finally, mathematics learning was approached passively, and believed to be the result of listening and concentrating.

Discussion

The findings of this study demonstrate that some learners in the tertiary sector hold maladaptive beliefs about mathematics and learning. The beliefs identified in this study are consistent with
those that have been associated with poor mathematics engagement and achievement in previous studies (Allsop, Kyger & Lovin, 2007). These include the belief that mathematics is largely a collection of procedures and rules (Mtetwa & Garofalo, 1989), that mathematics is only applicable to the classroom context (Boaler, 2002), that the role of a teacher is simply to transmit information (Taylor, Hawera, & Young-Loveridge, 2005) and the belief that answering mathematics problems correctly is more important than conceptual understanding (Briley et al., 2009; Reid et al., 2005).

This raises two immediate concerns. The first is the impact of these perceptions on how learners approach learning mathematics, and the second is the affective response these beliefs often promote. Firstly, how learners conceive of mathematics influences how they attempt to learn mathematics (Hofer, 1999). Learners who hold maladaptive mathematics beliefs have narrower strategic learning repertoires and less inclination to apply effective study techniques (Briley et al., 2009; Hekimoglu & Kittrell, 2010). Additionally, maladaptive beliefs appear to result in the setting of learning goals that focus on memorisation as a key learning strategy (Muis, 2007). Meyer and Parsons (1996) distinguished between learning strategies as ‘desirable’ and ‘undesirable’. Desirable learning strategies include strategic problem solving, taking a deep approach to learning, incorporation of group work, and explaining ideas and solutions to others. Undesirable strategies include memorising and single-strategy problem solving. The emergent beliefs and strategies mentioned by participants in this study were generally consistent with the ‘undesirable’ memorisation and single-step solution approach.

To compound the issue, the perfunctory learning strategies of attending and listening confine learning opportunities to class periods. Unfortunately, during class periods the participants are operating under the belief that passive listening should result in learning, rather than actively making connections or applications to real-world tasks. The reliance on passive reception rather than active exploration is an approach associated with poor performance and negative attitudes (Allsop, Kyger & Lovin, 2007; Reusser, 2000). In this study the interactions with teachers that the participants described were always passive, with the teacher explaining and repeating information. In the absence of adequate learning strategies, the application of effort in these environments was effectively reduced to attending.

Secondly, the beliefs that emerged from the interviews are consistent with those related to poor affective and motivational responses (Bibby, 2002). The participants in this study felt they had failed to learn mathematics during their school years. Unless effective initiatives are introduced it is probable, given such beliefs and approaches to learning mathematics, that other individuals like them will continue to experience difficulties, and will attribute their repeated failures to a lack of innate ability. This may lead to further entrenchment of negative emotions, attitudes and dispositions toward and about mathematics including, shame, mathematics anxiety and possibly learned helplessness.

The findings also highlighted the importance of positive beliefs about the purpose of mathematics. The inability of learners to see the utility of mathematics in their own lives may have an immediate impact on its perceived value and therefore on motivation (Weiner, 2010). The purpose of mathematics was held to be situated wholly within the classroom despite their having been in vocational training for some time. This demonstrates the persistence of beliefs, considering numeracy provision is designed to make quite explicit the links between mathematics and the vocational application. Moreover, participants viewed mathematics as an externally imposed requirement, something that ‘had’ to be done. These beliefs have been carried into the tertiary sector and are likely to erode motivation unless they are addressed.
Recommendations

The results of this study raise many challenges for private training establishments and for the tertiary sector in general. The tertiary sector must become more aware of how learner beliefs can undermine engagement and achievement in mathematics and, by association, numeracy. This will require two initial strategies. Research suggests that teachers in the compulsory sector who themselves hold maladaptive beliefs about mathematics tend to transmit them to their learners through their pedagogy, their resources and their attitude toward the subject (Philipp, 2007). Therefore, the first step will require tutors to examine their own implicit beliefs, an approach that has had success in the compulsory sector (Swarz, Hart, Smith, Smith, & Tolar, 2007). Secondly, tutors need to explicitly question and change learners’ negative beliefs while actively developing positive beliefs about mathematics. This goes beyond ‘making maths fun’; rather it requires a concentrated strategy to develop positive beliefs and self-regulated learning skills. Given the low numeracy skills of many learners studying in PTEs, and the correlation between low numeracy skills and negative beliefs, this objective must be an integral part of all programme design at levels one to three.

Future research

Further research is required to understand the impact of mathematical beliefs on learners with poor mathematics learning histories studying in the tertiary sector. Additionally, we need to understand the most efficient and effective ways of replacing negative beliefs with others that promote positive engagement, positive attitudes and positive dispositions to mathematics.

Conclusion

International research suggests that many lower achieving learners who struggled with mathematics during their school years leave the school system with maladjusted beliefs about the domain. These have an ongoing influence on their engagement in mathematics in subsequent study. This study suggests that maladaptive beliefs are also held by young adults in New Zealand studying toward vocational qualifications and indicates some of the ways these beliefs may undermine success. The tertiary sector has an opportunity to develop positive beliefs about mathematics in the context of vocational numeracy. Changing a learner’s beliefs may be a catalyst to a lifetime of learning.

References


Selective Ignorance and the Adult Numeracy Conundrum

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Mathematics is widely held in high regard within the community and there is a considerable body of research emphasising the value of mathematics (particularly functional numeracy in adults) as a social good that can build bridges and empower adults in ways that enhance independence and promote critical citizenship. Nonetheless, math-aversive attitudes and behaviours are prevalent in the adult populations of most western nations, with poor adult numeracy skills and overt adult innumeracy continuing as a perpetual and intractable problem despite decades of investment in training programs and initiatives to improve functional numeracy skills for life and work – this is the ‘adult numeracy conundrum’. Whereas the customary response by educators is to explore and articulate the costs and dysfunctions of innumeracy, ask who or what can be blamed, and devise new means by which it might be overcome, this paper approaches the matter from a very different perspective to consider the psychological and social functions of innumeracy as selective ignorance. It is argued that, notwithstanding the obvious opportunities for gain by exploitation of innumerate individuals, the persistence of this form of ignorance signals the presence of underlying factors that offer sufficient pay-off to induce self-perpetuating innumerate behaviours.

Introduction

There are … known knowns; there are things we know that we know. There are known unknowns; that is to say there are things that, we now know we don’t know. But there are also unknown unknowns – there are things we do not know we don’t know. (Rumsfeld, 2002)

The notion of the value of mathematics (particularly functional numeracy in adults) as a social good that can build bridges and empower adults in ways that enhance independence and promote critical citizenship finds considerable support in research literature, which also reports the high regard in which mathematics is broadly held within the wider community. Nonetheless, math-aversive attitudes and behaviours are prevalent in the adult populations of most western nations, with poor adult numeracy skills and overt adult innumeracy continuing as a perpetual and intractable problem despite decades of investment in training programs and initiatives to improve functional numeracy skills for life and work – this is the ‘adult numeracy conundrum’ (Klinger, 2011).

The customary response by educators is to explore and articulate the costs and dysfunctions of innumeracy, ask who or what can be blamed, and devise new means by which it might be overcome. The purpose of the present work is to explore another possibility. Innumeracy covers a class of ignorance, from not knowing various computational skills to a lack of appreciation of fundamental scientific concepts; it defies the rationale that promotes improved adult numeracy levels as a social good in terms of critical citizenship and its persistence has led me to consider
the psychological and social functions that it might perform and thus to arrive at a very different perspective – that of persistent adult innumeracy as ‘selective ignorance’.

In what follows, ‘ignorance’ will be used to mean ‘lack of knowledge’ and ‘ignoramus’ to mean an individual who is lacking in some specific knowledge. It is important to understand that this is not to say that an ignoramus is a ‘moron, imbecile, idiot or dolt’, all of which are offensive attacks on an individual’s level of intelligence. Being ignorant is not synonymous with being stupid. On the contrary, acknowledging one’s ignorance can be an indicator of great intelligence as it admits one’s limitations of knowledge.

Smithson (2008) makes the point that there are many ways in which ignoramuses are better off than those who are more knowledgeable. That is, there are pay-offs in being ignorant. So, I want to look first at the social value of ignorance in a broad sense to see if this sheds any light on the subject.

Social, psychological, and political dimensions of ignorance

The sociology of ignorance is a relatively new field and social theories of ignorance have largely emerged only in the last 10-15 years. Smithson (2008, p. 212) points out that ‘ignorance, like knowledge, is largely socially constructed’. Proctor (1995) coined the neologism ‘agnotology’ for the study of (culturally-induced) ignorance, according to which ignorance can be a product – a commodity even – to be ‘harnessed as a resource’ (McGoey, 2012, p. 1) and used, exchanged, and traded. Proctor (2008) further makes the point that it is remarkable that so little is known about ignorance, given its prevalence, variety, and consequence in our lives.

Either we know something or we do not – this is a true, binary statement. But it is also true that we may be either consciously aware of our state of knowledge or lacking in such awareness. Just as there are things that we ‘know that we know’ – conscious awareness of knowledge – there are other things that we ‘know we don’t know’. That is, we are consciously aware of a lack of knowledge – Rumsfeld’s ‘known unknowns’. We know that we don’t know when we are conscious of a gap in our knowledge. And of course it is often the case that we don’t know that we lack knowledge, so it follows that neither can we know the extent to which our knowledge is lacking. This is ‘meta-ignorance’ – Rumsfeld’s unknown unknowns. There is also a distinction to be made between being ignorant of something and ignoring it, which is a denial of knowledge, usually because the particular facts are unpalatable, inconvenient, or irrelevant.

Taxonomy of ignorance

Proctor (2008, p. 3) describes a taxonomy of ignorance: ignorance as a native state, or resource; ignorance as a ‘lost realm’ or ‘selective choice’; and ignorance as a deliberate and strategic ploy, each of which may be described thus:

Ignorance as a ‘native state’

Ignorance as a ‘native state’ or resource is much as it sounds – a lack of knowledge with no direct social, cultural, or political subtext in play. It is generally impersonal and non-social, such as scientific knowledge awaiting discovery, but can have a personal dimension, as in the educative sense where the aim is to reduce ignorance of particular material. In that sense, native ignorance can be benign – even beneficent – or it can be hurtful and malignant, as in ignorance of impending peril or the existence of an unknown tumour.
Ignorance as a deliberate and strategic ploy

Ignorance as a deliberate and strategic ploy is quite different but, again, can have positive or negative connotations. The key difference is that it is an act of agency imposed to fulfill specific social, cultural, or political functions whose roots run deep in all social structures and such ignorance ‘appears to be an inevitable part of any society’ (Simpson, 2000, p. 245). It is ‘both inescapable and an intrinsic element in social organisation generally’ (Moore & Tumin, 1949, p. 788). Benignly, many social interactions can depend on incomplete disclosure, privacy, secrecy – ranging from polite ‘white lies’ (“Do you think I’m putting on weight?”) to intentional deception, such as when throwing a surprise party.

As Prataknis and Aronson (1992) observe, ‘the functions of ignorance can be traced to the fundamentals of societal structures, and maintenance of ignorance of members of society can be seen to contribute to order and stability’ (p. 245). Organisations could not function effectively, if at all, in an all-seeing world where everything is known (and knowable) by all. Businesses have legitimate trade secrets and confidential practices. By definition, experts must know things that others do not and have a vested interest in protecting their specialised knowledge. Government departments and agencies could not carry out their legitimate functions without appropriate safeguards to protect private information and due process (see, for example, Ruskin, 1906; Moore & Tumin, 1949; Karsh & Siegman, 1964; and – for an informative summary – Simpson, 2000.)

On the other hand, there are consequences of deliberate, strategic ignorance that are decidedly non-benign to those on the receiving end, although of course they advantage the perpetrator. In the mid-1960s, Karsh and Siegman observed that:

> The division of labor [sic] implies not only knowledge but also notions about what people need not know and are not supposed to know. Thus, formal organizations can be seen as models of differential amounts of ignorance, or better yet, as structures of both knowledge and ignorance. …ways by which people deliberately attempt to keep others ignorant and how ignorance functions to protect existing statuses. (Karsh & Siegman, 1964, p. 141)

Many modern business corporations operate secretively not only to protect their competitive advantage but to assert power and influence by imposing ignorance on others. A notorious example was the conduct of tobacco companies in perpetrating and perpetuating public ignorance of their products’ harmful effects. Neither is the scientific community immune from exploiting popular ignorance – as actions on both sides of the climate change debate demonstrate – and in politics, deliberate, strategic ignorance is a common feature to undermine political opponents and create uncertainty in voters minds, making it easier to persuade the electorate by emotive, subjective rhetoric rather than by informed, objective and reasoned debate.

This has particular impact for the innumerate, as Smithson (2011, n.p.) observes: ‘Politicians don’t want a numerate electorate any more than they want a politically sophisticated one, so elected office-holders are unlikely to lead the charge to combat innumeracy.’ More than twenty years earlier, Paulos (1988) highlighted inaccurate media reporting of quantitative information and the public’s inability to detect the flaws, leaving people open to easy manipulation. He also lists other areas where innumerate adults are vulnerable to exploitation through strategic ignorance:

- financial mismanagement (e.g., of debts), especially regarding the misunderstanding of compound interest;
• loss of money on gambling, in particular caused by the gambler’s fallacy;
• belief in pseudoscience;
• distorted assessments of risks;
• limited job prospects.

Of course, these are all windfall opportunities for others, such as unscrupulous banks and retailers, fraudsters, gambling agencies, and what Smithson (2011, n.p.) calls “peddlers of various religions, magical and pseudo-scientific beliefs”. This is not to suggest that they can be blamed for innumeracy, or that there is a conspiracy to maintain it, but it would be naïve to expect opportunists to campaign for a less ignorant public.

Ignorance as a ‘lost realm’ or ‘selective choice’

Particularly interesting in the context of adult innumeracy is the notion of ‘selective ignorance’, which can arise in several ways:

• choosing to not inform ourselves when we know we’re ignorant. This is a ‘head in the sand’ type of selective ignorance – or ‘ignore-ance’;
• choosing to not question our state of ignorance by not asking ourselves, “Is there anything going on here that I don’t know about?”; and
• as a preference for reliance on others (‘experts’) to inform us rather than taking the trouble to equip ourselves with the knowledge and means to think for ourselves.

There are many situations where we believe it is actually better to not know; that is, to choose ignorance. We have all sorts of euphemisms for this – ‘turning a blind eye’, ‘ignorance is bliss’, and so on. We might even feign selective ignorance in our interactions with others – “I didn’t get your email. What was it you wanted me to do?”; “Oh, honey, I didn’t realise you were seeing your friends today – I can’t look after the kids, I’ve already arranged a game of golf with Tom, Dick and Harry!”

Many people approach quantitative information from a stance of selective ignorance. For all sorts of personally or socially valid reasons, intelligent, numerate individuals might choose to not ‘do the maths’. Other, no less intelligent but innumerate individuals, may choose to use their innumeracy as an excuse and, again, they do so for many personally or socially valid reasons. So here is a proposition, which goes to the heart of the title of this paper, ‘selective ignorance and the adult numeracy conundrum’:

**Proposition:** the value of numeracy is over-rated by its proponents and under-rated by the wider population.

I am not suggesting that specific, functional maths skills required of those in certain occupations or professions, such as nursing, for instance, are not important. Of course they are, and vitally so. That is not the point. I am talking about a far less specific aspect of numeracy in much broader contexts than ‘tools of the trade’. Consider statistics, as an example, since this is broadly accepted as the most widely used every-day mathematics apart from simple arithmetic. Descriptive statistics, such as measures of central tendency, have little value when presented
without information about the sampling method, size, distribution shape, and measures of deviation or variance. Likewise, inferential statistics (polling) without information about sample size, the method and politics of sample selection, confidence level or interval is meaningless. How many people consider questioning the lack of such information when presented with statistical evidence to support some claim? Clearly, this is a rhetorical question; given what we know of low functional numeracy rates in the adult population, one must suppose that very few people even have the capacity to ask about such things, let alone the ability to properly interpret the information if it is supplied.

So why do people not care (or care enough) to educate themselves appropriately, or to use the knowledge they have – to take personal responsibility for their lives? Much of the problem is that people ‘don’t know what they don’t know’. This unconscious ignorance, or meta-ignorance, means there is nothing to drive them to fill in the blanks. Of course, knowledge is culturally situated so some people are simply not interested in certain forms of knowledge (one might categorize this as conscious ignorance), or those forms are not culturally relevant. Nonetheless, even when they become aware of ‘known unknowns’, few may find in this sufficient incentive to correct their knowledge deficit because they see the perceived effort as too great to justify what they regard as an inadequate, if not futile, payoff.

Inadequate and futile, firstly, because decision-making over life’s big issues is far more likely dominated by qualitative (simply, as distinct from quantitative) arguments, subjective persuasion, and appeal to self-interest, much like Pascal’s Wager (which argues that a rational person should bet on the existence of God, and live virtuously, since a loss is at no cost – a finite stake – whereas the cost of losing the alternate bet is eternal damnation – an infinite stake). ‘Big issues’ are too important to be clouded by distracting quantitative and mathematical detail. And secondly, because most people are socially conditioned to defer to experts and authority in matters for which they, themselves, have neither the expertise nor the authority (Simpson, 2000). Even when second or third expert opinions are sought, the one that prevails is more likely to do so because it came from an adviser who had greater personal appeal than the others, or who instilled a greater subjective sense of confidence.

**Big issues**

The following examples are illustrative of ‘big issues’:

1. **The global financial crisis** may, in hindsight, have been avoidable. The clues lie in the mathematical modelling of financial risk, so specialised that it is way beyond the reach of the average shareholder. The public, in other words, relied implicitly on experts. However, the global nature of the risk meant that it fell outside the scope of the models because these depended on very precise local calculations mixed with quite crude estimates of global factors – which were falsely assumed to be of negligible consequence. The modelling both concealed ignorance of what was going on and encouraged analysts to ignore their ignorance until it was too late; what happened next was the result of political forces (Bouleau, 2011) over which the wider public had no control. Few individuals, regardless of their level of numeracy, could influence the outcomes or navigate their way to greater personal safety. The general public just had to ride out the storm.

2. **Sustainable growth** is an often heard buzz-phrase, particularly among entrepreneurs and politicians. Modern capitalist economies depend on continuing economic growth.
Yet ‘sustainable growth’ is an oxymoron when applied to any setting with finite resources because, over long time scales, any continuous growth follows the exponential function. Most people simply do not understand the exponential function – which fact ‘may be one of the greatest shortcomings of the human race’ (Bartlett, 1976, p. 395) – despite the vernacular usage of the term ‘growing exponentially’. They have no appreciation of just how rapidly the consumption of resources accelerates. They have no sound grasp of doubling time, what it means, or how to estimate it. They fail to appreciate the significance that for each doubling period, more resources must be consumed than have ever been consumed in the entire history of the growing quantity.

The great majority of the few who do understand this presumably choose to ignore it because its truth is inconvenient for their ideology or the consequences are so unpalatable that they refuse to accept them. The response is always the same: “We’ll find new resources, we always have…”; or “We’ll develop new technologies to solve that problem long before we run out of X”. Such is the force of this ideological stance that the simple mathematical truth is ignored – and worse, it is undervalued or mistrusted in its applicability to the ‘real world’.

3. **Climate change** has more proponents than it has sceptics. That does not make them right. Climate models are incredibly complex and rely on many assumptions. The data are rich, noisy and open to differing interpretations, even by those who claim to understand the science. That last term is itself problematic – ‘the science’, used in such contexts, is often treated as a proper noun: “The Science”. It is made to sound authoritative but in fact there is no single, definitive, and generally accepted science of climate or climate change. There are theories and there are models. There are experts who are proponents of human-induced global warming; there are other experts who argue against it. There are many facts but few certainties.

If experts fail to agree, how are the rest of us to decide? This is not a matter of how numerate we are. And it probably matters little, or less, if we are innumerate. There are lots of data but the calculations, the mathematics, do not help us. The problem is too complex. Yet the public is supposed to have a voice on the matter – as it should in democratic societies – so how are people to find that voice? How does the political will arise to act or to not act?

When it comes to the big issues, how many people say, “I don’t know enough to do the calculations myself but this is really important issue so I’ll take the time to learn how to do them. Then I’ll be able to be better informed.” Again, this is a rhetorical question. Most people do not respond that way. Public ignorance enables the media to exercise its considerable influence over public opinion (Anderson, 1998) and the greater the public concern, the more imminent the threat, and the less knowledgeable people feel, the greater their avoidance of information and endeavours to learn about the issue, which serves to further strengthen the power of the media. Furthermore, people become increasingly dependent on those in authority (i.e., government) to respond appropriately as the degree of complexity and seriousness of the issue rises, giving rise to a ‘psychological chain’ of ignorance, dependence, trust, and avoidance (Shepherd & Kay, 2011, p. 265).

That is, selective ignorance takes hold and the majority, without the capacity or interest to examine the science for themselves, adopt the position that it is less risky to assume, for instance,
that human-induced climate change is actual and that societies and individuals should act accordingly to mitigate the consequences – this response of risk-aversion again echoes Pascal’s Wager: if they turn out to be right, the actions will (may) steer the planet away from disaster; otherwise, what is there to lose? Actually, this is an entirely different matter to evaluate and determine; to act rashly, even with the best of intentions, could be extremely costly in many unforeseen ways – economically, politically, and socially. Conversely, climate-change sceptics have no tangible gains if they are correct (other than maintaining the status quo) but they potentially risk the future of the planet if they turn out to be wrong. It is a position that makes climate change sceptics easy targets for derisive criticism and accusations of being reckless and irresponsible.

Public policy, then, is more likely to be determined by subjective desires of risk minimisation and collective self-interest – but only to the point where measures introduced to combat climate-change impinge to an unacceptable level on individual self-interest. These things factor in public decision-making behaviour to a vastly greater extent than any amount of quantitative analysis, to the point where I submit that there is almost no case to answer in seeking to persuade the innumerate that numeracy is vital to critical citizenship.

**Lesser issues**

The foregoing behaviour is not confined to these really big issues. It is representative of the way people tend to behave in their daily lives. Paulos (1988, p. 80) says:

> Some people personalize events excessively, resisting an external perspective… People too firmly rooted to the center of their lives find such questions [as: How many? How long ago? How far away? How fast? What links this to that? Which is more likely?] uncongenial at best, quite distasteful at worst.

One might ask how many highly numerate people actually do the mathematics when it comes to deciding investment strategies on which their retirement pension (say) depends. And of those who do, how many of them trust themselves sufficiently to rely solely on those calculations and how many seek additional advice from financial and investment experts? In the end, what is the real basis for their strategic decisions? Consider the following example:

**Mow your own lawn or contract out?**

When it comes to deciding whether to buy a lawnmower and cut your own lawn or pay to have someone come in and cut it for you, do you sit down and work out the present value of the series of payments you will make to have the job done over the serviceable life of the mower you would have bought? Do you calculate how much the cost of purchasing the mower and the ongoing maintenance costs could earn you if deposited in an interest-bearing account? Do you then find the difference in these two values to see which strategy has the best financial return?

Of course, this omits the matter of factoring in the depreciation of the mower. And, in addition, there are many assumptions needed in order to do the calculations, including:

- an estimate of the expected service life of the mower that may or may not be purchased (which, in itself, might require some mathematics to decide the best value machine for the job);
• the cost of each lawn cutting job based on the best rate currently available and how much one estimates that will change over time;

• the best rate of interest that could be earned now and how that could change over time – and similarly the bank fees, tax liability of interest earned;

• what one’s time is worth – literally or in terms of the value of leisure time if cutting the lawn is not something to be regarded as a leisurely pursuit.

Even then, not all of the factors in the decision are directly quantitative, let alone mathematical. I am capable of doing those calculations but I never have. Or hundreds, even thousands, of others like them. Usually, I will base my decisions on estimates but I am just as likely to be swayed by subjective factors (for instance, the appeal of a shiny new piece of machinery, or the attraction of more time to play golf) or I will choose to obtain opinions from ‘experts’ – and even then, I will more likely go with the advice of the person that I best connect with. But maybe that is just me.

Even the most numerate get it ‘wrong’(?)

Paulos presents a scenario concerning mandatory health screening to illustrate how probability and risk are misunderstood by the innumerate. He supposes some form of cancer that has a 0.5% incidence rate and a test for the disease that is 98% accurate. From this, he finds the conditional probability of actually having cancer given a positive test result is only about a 20% chance and says this ‘unexpected figure for a test that we assumed to be 98 percent accurate should give legislators pause when they contemplate instituting mandatory or widespread testing’ (Paulos, 1988, p. 66).

The calculations are correct but the conclusion is problematic because Paulos chooses to not report the other side of the story, which is the chance of the screening giving a false negative result and this turns out to about one hundredth of 1%. In other words, screening is highly likely to be dependable if it gives the ‘all clear’. Moreover, even though 80% of the positive test results will turn out to be false, 20% will not be and this can be determined by more specific/explicit testing – importantly, the vast majority of actual cancer victims will have a chance of identifying the disease at a stage early enough, one hopes, for effective treatment. Trying to find 50 in 10,000 is needle-in-a-haystack territory, so actually finding 49 of them would appear to be a worthwhile outcome.

For whatever reason, Paulos neglects the context, which is screening for early detection of conditions that have substantially greater chance of successful treatment if diagnosed sufficiently early. Irresponsibly, in my view, he also inflates the impact of this illustration by choosing to use cancer as the subject disease, which itself is highly emotive, and then supposes a relatively low incidence rate, which substantially inflates the number of false positives. Now, was this selective ignorance or a strategy of ‘deliberate ignorance as a ploy’ by Paulos to make his case? His choice to reinforce the example by using emotive subject matter rather suggests the latter (no doubt to help emphasize the illustration for the benefit of the reader, rather than merely to sensationalize the point).

Conclusion

I have discussed ignorance and its various forms – some of its social, psychological, and political dimensions – coming, in particular, to ‘selective ignorance’. I put forward the proposition that
the value of numeracy is over-rated by its proponents and under-rated by the wider population and explicitly directed my argument away from occupational or professional contexts. To support that proposition, I have presented a range of real-world instances to illustrate that the innumerate and the numerate, alike, probably fare neither worse nor better than the other and are no more, or less, equipped, overall, to manage in a complex world.

It is evident that there are vastly many situations in which numeracy fails to help people to inform themselves better, so they may as well ignore it, and there are many contexts where the mathematics can be done, yet it is not a dominant feature of people’s decision-making behaviour. Certainly, there are many issues in which mathematics is a dominant factor – at least for the experts – but it is used just as correctly in wrong theories as in right ones. Moreover, its use to support contending theories that claim to be unequivocal creates confusion. Confusion means uncertainty. Uncertainty engenders mistrust (paradoxically, of experts, but not of governments). Mistrust motivates avoidance. And avoidance reinforces selective ignorance, which serves to perpetuate a general lack of adult numeracy and a disinterest in its correction.

Thus we are returned to the proposition and to the adult numeracy conundrum. As educators we can and do make a difference in terms of vocational mathematics but, for reasons that are now more evident, programs to more broadly promote numeracy via traditional approaches (including even the most innovative variants) – that is, via curricula that differ little from school mathematics – seem bound to make little or no difference in terms of overall adult numeracy rates. If we want to more effectively endorse the ‘product’ (adult numeracy), we need to re-think what is on offer and how we set about promoting it. If we want to overcome widespread innumeracy, we need first to find strategies to address effectively people’s natural propensity for selective ignorance.

References


Appendix A

Percentages of school students at each stage on the Number framework in 2003, 2005, and 2007 (boxes show the expected stage at particular year levels)

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<th>Y4</th>
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<th>Y7</th>
<th>Y8</th>
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# Questions and Responses

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<td>Sue used 8.3 metres of red material and 2.57 metres of blue material to make costumes for the play. How much material did she use altogether? (Correct Ans: 10.87)</td>
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<td>Ana bought 4.3 metres of rope to make skipping ropes, but only used 2.89 metres. How much rope was left over? (Correct Ans: 1.41 m)</td>
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<td>Tama and Karen buy two pizzas. Tama eats 3/4 of one pizza while Karen eats 7/8 of the other one. How much pizza do they eat altogether? (Correct: 13/8 or 1 and 5/8)</td>
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<td>If Ben got 72 out of a possible total of 90 marks, what percentage was that? (Correct Ans: 80%)</td>
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<td>Ans: 72/90</td>
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<td>Ans: 72/90 x 100</td>
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<td>Jo spent $60 on stationery. She got one-third off the original price, because she was a teacher. What was the original price? (Correct Ans: $90)</td>
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<td>Ans: 80</td>
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Appendix C

Percentages of ITE students at the end of their program who were successful on selected tasks (n = 129)

• 45% were able to add $\frac{5}{14}$ and $\frac{1}{7}$. (Note: 29% added across the numerators and denominators to get an answer of $\frac{6}{21}$).

• 73% were able to mark where the number 1 should be on a number line showing just 0 and $\frac{1}{3}$.

• 67% recognised that 0.5 x 840 is the same as 840 ÷ 2.

• 15% found the percentage corresponding to 72 out of 90

• 39% were able to order these fractions correctly: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{12}{10}$, $\frac{3}{5}$, $\frac{10}{8}$, $\frac{4}{10}$, $\frac{3}{4}$

• 52% were able to estimate that 45 x 105 is closest to 4600 (not 4000, 5200, 47250) (Note: 23% chose 47250 as their best estimate)

• 78% were able to estimate that 29 x 0.98 is less than 29 (Note: 15% thought that the answer is more than 29).

• 52% knew that there are many different decimals between 1.52 and 1.53 (Note: 28% thought there were no decimals in between these two numbers).

• 18% worked out that 29 ÷ 0.8 is the largest number compared to 29 + 0.8, 29 – 0.8, or 29 x 0.8 (Note: 42% thought that 29 + 0.8 was largest, and 31% thought that 29 x 0.8 was largest).
This presentation and hands-on workshop looked at the connections between literacy, language and mathematics in the construct of numeracy or mathematical literacy. The presentation considered a range of literacy and language based factors and issues related to the teaching and learning of (adult) numeracy. This relates to fundamental questions such as:

- What is numeracy (or quantitative or mathematical literacy)?
- What is the relationship between numeracy and mathematics?
- How do numeracy and mathematics connect to language and literacy?
- How do language and literacy impact on numeracy and mathematics teaching and learning?

The workshop considered these questions based on experience in the teaching of adults in adult numeracy, alongside the perspective of a number of different research, assessment and curriculum frameworks, both international and Australian. The implications for the teaching of numeracy and mathematics were discussed and a number of teaching strategies and activities were demonstrated that support and encourage students to incorporate aspects of literacy and language in their learning of numeracy and mathematics.
A Community of Practice (CoP) is a collection of people that share a common passion and meet regularly to talk about it and try to learn how to improve their practice. In Australian universities, Communities of Practice are becoming popular. At the University of Southern Queensland a new Community of Practice was formed in 2011. It is a gathering of lecturers who have a keen interest in the practice of teaching and learning in mathematics and statistics. This poster outlined the objectives of CoPs in general, and the aims of this CoP, called Mathematics and Statistics Teaching and Learning (MaST). The poster also outlined the progress so far and future directions.
Helping University Students Appreciate “The Inherent, Innate and Pervasive” Nature of Mathematics

Keith McNaught
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As a response to staff concerns about entrants’ literacy and numeracy, the faculty of Health Sciences at the University of Notre Dame Australia (UNDA) Fremantle campus, worked with academic support staff from the University’s Academic Enabling and Support Centre (AESC) to develop a Post Entrance Numeracy Assessment (PENA). The PENA was designed to parallel and complement the University’s PELA (Post Entrance Literacy Assessment). The PENA led to the articulation and discussion with regard to concerns about the range of mathematics courses offered in Western Australia. This led to staff discussion on the need for curriculum mapping between secondary schools and the tertiary sector, and ensuring that consultative mechanisms already in place were effective. The PENA testing and subsequent student counselling from staff indicated that significant numbers of Health Science entrants lacked an appreciation that mathematics would be inherent, innate and pervasive in many of their units. Whilst the level of mathematics necessary varies between Health Science courses (eg, it is higher for Biomedical Science than Outdoor Recreation programmes), mathematical knowledge, skills and understandings are pivotal to study success. The removal of prerequisites from university courses, and the issues around students seeking to maximise their ATAR (Australian Tertiary Admission Rank) score, warrant further discussion.
Developing Maths Eyes – A Successful Model for Building Confidence in Mathematics in a Community

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This presentation described an innovative and novel approach to sowing the seeds of the recognition of the importance of mathematics competence in real life situations. Traditional methods of mathematics teaching and learning have resulted in a maturing population who do not appreciate the mathematics they use in their everyday lives. These ‘everyday’ mathematics skills often involve the use of complicated mathematical ideas and techniques. However, many people often consider the mathematics they can do as ‘common sense’ and the tasks they can’t do as ‘mathematics’ (Coben 2000).

The presentation described a successful community initiative ‘Looking at Tallaght with Maths Eyes’. The initiative aimed to:

• Develop the maths eyes of the Tallaght community: (Every member of the community has maths eyes – they just need to be opened).
• Help the Tallaght community to make the link between mathematics and the real world. (A key focus was to encourage the community to use Maths Eyes when they think about their water usage and water conservation).
• Build people’s confidence in their use of maths in their life.
• Empower people and build their confidence in their own maths knowledge and skills. (Empowered parents are more confident in supporting their children’s learning; more confident citizens can make more informed evaluations of the information that bombards them every day and have a better understanding of the impact of their actions and decisions in their life, work and leisure).
• Build a positive image of maths.

The presentation outlined the different approaches that were used to encourage participation from a range of stakeholders. These included a community wide ‘curiosity’ campaign; the development and piloting of a resource pack for educators called ‘Developing Maths Eyes; An Innovative Approach to Building a Positive image of Mathematics’ (2011); primary schools showcase; adult learners showcase; exhibitions; the development of maths trails for the local parks and an audio maths ‘I-walk’ for Tallaght. In addition it described how the initiative has since been developed and is being rolled out nationally.
Part of my role at Otago Polytechnic, Dunedin, New Zealand, is to teach students in our Certificate in Health course the mathematics they will need in order to be able to dispense drugs to patients – without sending them into toxic shock. The mathematics involved is surprisingly difficult as it requires a lot of fractional and proportional thinking as well as rock solid place value understanding. Perfect for foundation level learners! In addition, some of the students are enrolled as distance learning students. Certificate in Health students typically hope to progress to a degree in Nursing, Midwifery or Veterinary Nursing.

My workshop showcased the resources I use to deliver this course to students – Moodle, Adobe Connect, Visualizer, animated Powerpoints and activities. I am a committed visual learner and my approach reflects this. I brought all my materials with me and was happy to share.
Numeracy and Informal Banking: What Does the Sou-Sou Tell Us About Numerate Behaviours?

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Sou-sou (or su-su) is an important way of saving money in many African and Caribbean countries through the revolving exchange of a fixed amount of money between close friends or family members for a specified length of time. As a social activity, the conduct of sou-sou requires strong communal ties built on trust. Over the years, while the attempt to showcase sou-sou as an important micro-financial activity in many developing nations has resulted in some research and literature on the topic, very little connection has been made between sou-sou (which involves numerate behaviours) and numeracy. Dr Hector-Mason opened a window of inquiry in the relationship between sou-sou and numeracy and presented information and materials that supported a rich discussion about sou-sou as a form of ethnomathematics.
Some Reflections on Adults’ Numeracy Competence from International and National Numeracy and Mathematical Literacy Assessments

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Based on the frameworks, item development and the results of adults’ and young people’s performance in a number of international and national numeracy and mathematical literacy assessments, this presentation highlighted a number of issues related to the teaching and learning of numeracy (or mathematical literacy). The assessments and frameworks used as the basis for the presentation included the Adult Literacy and Lifeskills Survey (ALL); the Program for International Student Assessment (PISA), and the New Zealand Adult Literacy and Numeracy Assessment Tool. The presenter has worked on all three assessments.

Issues addressed included the similarities and differences between the assessment frameworks and their associated test items; and some reflections on what some of the results to date tell us about how performance compares between the school-based assessment of 15 year olds in PISA and the two adult assessments – the household assessment of adults’ numeracy skills in ALL and the assessment of adult learners in the New Zealand Adult Literacy and Numeracy Assessment Tool. Note that in the trial of the New Zealand Adult Literacy and Numeracy Assessment Tool a number of ALL numeracy items were used to map the test against the NZ curriculum framework.