Mathematical Eyes: 
A Bridge between Adults, the World and Mathematics

Proceedings of the 18th International Conference of 
Adults Learning Mathematics - A Research Forum (ALM)

Hosted by:

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Edited by: Theresa Maguire, John J. Keogh and John O'Donoghue
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About ALM

Adults Learning Mathematics – A Research Forum (ALM) was formally established in July 1994 as an international research forum with the following aim:

To promote the learning of mathematics by adults through an international forum, which brings together those engaged and interested in research and development in the field of mathematics learning and teaching.

Charitable status

ALM is a Registered Charity (1079462) and a Company Limited by Guarantee (Company Number 3901346). The company address is: 26, Tennyson Road, London NW6 7SA.

Objectives of ALM

ALM's aims are the advancement of education by the establishment and development of an international research forum in the life-long learning of mathematics and numeracy by adults by:

- encouraging research into adults learning mathematics at all levels and disseminating the results of this research for the public benefit;
- promoting and sharing knowledge, awareness and understanding of adults learning mathematics at all levels, to encourage the development of the teaching of mathematics to adults at all levels, for the public benefit.

ALM Activities

ALM members work in a variety of educational settings both as practitioners and researchers, improving the learning of mathematics at all levels. The ALM annual conference provides an international network which reflects on practice and research, fosters links between teachers and encourages good practice in curriculum design and delivery using teaching and learning strategies from all over the world.

Board of Trustees

ALM is managed by a Board of Trustees elected by the members at the Annual General Meeting which is held at the annual international conference.
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How to become a member

Anyone who is interested in joining ALM should contact the membership secretary. Contact details are on the ALM website: http://www.alm-online.net

Membership fees for 2012

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Low waged – contribute between full and unwaged rate
Preface

Mathematical Eyes:
A Bridge between Adults, the World and Mathematics

18th International Conference Adult Learning Mathematics
The Institute of Technology Tallaght. Dublin, Ireland.

The Institute of Technology Tallaght is delighted to have hosted the 18th International Conference of Adults Learning Mathematics in June 2011, and to have welcomed the International Research Community to Ireland.

The conference came at a time of change in terms of the school mathematics in Ireland. The largest curriculum reform in second level mathematics in Ireland, ‘Project Maths’, was in the process of being implemented on a phased basis (see www.projectmaths.ie). The emphasis of this reformed curriculum is on forging strong links between the mathematics taught in the classroom and real world applications.

‘Project Maths’, aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience.

Project Maths Development Team (2012).

However, the concept of mathematics is strongly linked to each individual’s own school experience, is often considered abstract, and not relevant to everyday life. At community level, there was a sense that ‘Project Maths’ was new and unknown, and as a consequence, feared. In response, the Institute took the decision to build confidence in mathematics at community level, by opening the community’s ‘Maths Eyes’. The initiative ‘Looking at Tallaght with Maths Eyes’, took the innovative and novel approach of sowing the seeds of the importance of mathematics competence in real life situations in the local community. To provide synergy between the conference and the community initiative, the theme of the International conference was ‘Mathematical Eyes: A Bridge between Adults, the World and Mathematics’. The conference mirrored, to a large extent, the theme of the local initiative, ‘Looking at Tallaght with Maths Eyes’.

Central to the thinking and planning of the community initiative from the outset, was the notion that the target audience was the Community of Tallaght; not just the schools. As a result, a number of different approaches were incorporated into the initiative to introduce and reinforce the concept of having ‘Maths Eyes’, and making the link between mathematics and the real world. The key aims of the initiative were to:
• Develop the ‘Maths Eyes’ of the Tallaght Community
• Help the Tallaght Community to make the link between mathematics and the real world e.g. using ‘Maths Eyes’ to think about water usage and conservation
• Build people’s confidence in their use of maths in their life
• Empower people and build their confidence in own maths knowledge and skills because:
  a. empowered parents are more confident in supporting their children’s learning
  b. more confident citizens can make more informed evaluations of the information that bombards them every day, and
  c. empowered people have a better understanding of the impact of their actions and decisions in their life, work and leisure
• Build a positive image of mathematics
• As a community, celebrate the hosting of an International Conference at the Institute.

Engagement with community
In order to keep the community focus and reach as many stakeholders as possible, a range of different events were hosted. These included: a ‘curiosity campaign’, a range of family events, library activities, and exhibitions (See Figure 1).

![Figure 1. Overview of ‘Looking at Tallaght with ‘Maths Eyes’ community engagement](image)

A ‘curiosity campaign’ is a recognised marketing approach which aims to create interest by generating curiosity about a particular theme or brand. The curiosity campaign carried out as part of ‘Looking at Tallaght Through Maths Eyes’, took familiar places/pictures/activities and linked them, in novel ways, to real world mathematics through challenges/statements. In all, a series of 10 election sized lamppost-posters were developed. Six of the posters were general in theme and four were specifically targeted to prompt awareness of water use and conservation, so as to resonate with the current priority of SDCC; one of the funding partners.
Over 500 of the colourful posters were erected on lampposts around the Tallaght area in early June 2011. The campaign was supported by our media partner ‘Tallaght Echo’, who ran the posters as advertisements without comment in their newspaper over the weeks of the campaign. After a short period, and in a dedicated supplement in the local newspaper, the mathematics that underpinned each poster or challenge was revealed and explained. In addition, an information leaflet, similar to the newspaper supplement, was distributed and made available through several outlets in the local area. The uncovering of the posters launched a week of ‘Maths Eyes’ activities in the area surrounding the Institute.

Among the other strategies to engage the community, was the collaboration of the Mathematics and Computing lecturers at IT Tallaght to develop of a ‘Maths Eyes-Walk’ podcast, which was designed to guide people on a 40-minute walk through Tallaght. This was available to download via the South Dublin County Library website (2011). A ‘mathematics’ trail through a local park was also developed through collaboration between Mathematics lecturers in IT Tallaght, and the SDCC Parks Department. For this event, families were invited to exercise their minds and their bodies by following the trail of clues through a park located in the heart of Tallaght. Finally, a street workshop was designed and delivered in a busy square to engage passersby with the theme of ‘proportion’, set in the context of the amount of water used in various domestic tasks, and how to conserve it.

In addition, South Dublin Libraries were key partners in hosting a programme of events tailored for all age groups including; pre-school children and their parents, primary school children and their teachers, teenagers, adult learners and their educators, and families; indeed, everyone. As part of this, the Library assembled collections of picture books, featuring mathematics themes and vocabulary for young children, hosted a ‘mathematics’ trail within the Library, facilitated a workshop about developing mathematics skills with numeracy sacks for pre-schoolers, and hosted a family ‘mathematics’ day.

Using mathematics ‘problem-pictures’ and the posters (Figure 2) are excellent ways to help individuals of all ages to develop their ‘Maths Eyes’. The best ‘problem-pictures’ for developing ‘Maths Eyes’ are snapshots, taken of familiar things, that capture some aspect of real life mathematics. It was decided to use the concept of engaging the community with mathematics through these ‘problem-pictures’ and posters in two ways. Firstly, a set of photographs of everyday items and locations in the Tallaght area was assembled. The mathematics group at IT Tallaght added statements to these images to prompt the viewer of the photograph to explore the image for some mathematical aspect. In doing this, a range of images and questions were chosen so that not just shape, but other mathematical aspects e.g. number, pattern etc., would be suggested for exploration in the set of ‘problem-pictures’. This collection of ‘problem-pictures’, was assembled as an exhibition entitled ‘Solve-It’ and held in the foyer of the Institute. An interaction sheet for those looking at the exhibition was constructed. In keeping with the emphasis on whole community
engagement, a second exhibition of photographs was assembled in collaboration with the local Tallaght Photographic Society. The amateur photographers in the Society were asked to explore the Tallaght area with their ‘Maths Eyes’ open. From the images taken and exhibited, the most suitable photographs were selected, jointly, by an external photographer and Institute’s mathematics staff. An interaction sheet was constructed for this exhibition, entitled ‘Tallaght Cubed’. The public were invited to view and comment on the photographs which were exhibited in the County Library. This exhibition was a fine example of what can be achieved by maximizing the human capital in the locality.

Linking with those in Education
A number of activities were developed to encourage participation from those involved in all levels of education. In partnership with the Schools Liaison Officer from Dublin City Libraries, all primary schools in the local area were encouraged to ‘Show Their Maths Eyes’, and attend a showcase event in the Institute. To encourage participation, all schools were invited to send a representative to an information session on ‘Developing Maths Eyes’ that was facilitated by the Institute. Subsequently, one of the ‘Maths Eyes’ Project staff visited schools to explain the initiative to them and to encourage their participation. These visits were requested by the schools and facilitated through the South Dublin Library. In all, 14 of 28 schools in the area attended the showcase, comprising 600 children (senior infants – sixth class) and 100 adults came to show their ‘Maths Eyes’. In some cases, schools took a whole-school approach, while others limited their participation to a single class. The National Centre for Excellence in Mathematics and Science Teaching and Learning (http://www.nce-mstl.ie/), facilitated ‘Fun with Maths’ workshops for all children attending. The event was supported by national media figure John Murray (Light Entertainment broadcaster, Radio Telefís Éireann).

The timing of ‘Looking at Tallaght Through Maths Eyes’ was determined by the dates of the International Conference and was not ideal from the secondary school perspective. Hence, the decision was made to concentrate on those who would expect to enter secondary school in September 2011. The ‘Project Maths’ team presented ‘My Child and Project Maths’ to parents of children in the final year of primary school. The presentation gave parents the opportunity to find out more about ‘Project Maths’ and how it will impact on the mathematics education of their children.

To engage with those who are involved in numeracy teaching and learning in adult basic education, the Institute of Technology Tallaght, in partnership with the National Adult Literacy Agency (http://www.nala.ie/), hosted an event entitled ‘Sharing innovative numeracy teaching and learning: Adult, Youth and Community showcase’. Over 100 numeracy tutors from the Adult Education sector, and their students, showcased how they linked mathematics to real life. Exhibits included: the development and use of numeracy sacks, numeracy and art, photography, architecture, money and exchange, and fossils and botany. All those showcasing their work also attended a number of workshops on innovative teaching of mathematics in adult education.

A resource pack, ‘Developing Maths Eyes: An Innovative Approach to Building a Positive image of Mathematics, 2011’, was developed and made available to all schools/adult and community education centres in the area.
**Strength in Partnership**

A key to the success of ‘Looking at Tallaght through Mathematical Eyes’ as a community initiative, was the strong partnership approach among the key stakeholders in the Tallaght Community. Partners included: Institute of Technology Tallaght (many departments), South Dublin County Council, South Dublin County Libraries, Dublin West Education Centre, Tallaght Photographic Society and South Dublin Vocational Education Committee. The key media partner was the Tallaght Echo newspaper. There was support from the National Centre for Excellence in Mathematics and Science Teaching and Learning, National Adult Literacy Agency, Irish Mathematics Society and the Learning Innovation Network.

To strengthen the links between community and the International conference, a Community / Conference get-together was held in the Institute of Technology Tallaght. The event was a unique and successful opportunity for engagement between the local community and International conference participants. The Institute was delighted to be awarded ‘Centre of Expertise’ status for Adult Mathematics and Numeracy from the National Centre for Excellence in Mathematics and Science Teaching and Learning.

Building on the success of the resource pack for ‘Developing Maths Eyes’ and the other associated ideas, the organising committee decided to make the resources and learning approaches more widely available. To this end, the images and exhibitions associated with ‘Maths Eyes’ are now digitised and available online at [www.haveyougotmathseyes.com](http://www.haveyougotmathseyes.com).

Finally, I would like to thank the ALM Trustees, the local Organizing Committee, the local schools and Community, the Institute’s Staff, Management and Hospitality teams, contributors and attendees, all of whom helped to make ALM 18 a very informative, successful, memorable and enjoyable Conference.

Terry Maguire  
Chair of ALM 18 Organising Committee.
Acknowledgements

ALM18 Organising Committee

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Catherine Byrne                  John O’Donoghue
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Editors: Terry Maguire, John Keogh, John O’Donoghue
Section 1

Plenary Presentations
Adults mathematical literacy - starting a Grundtvig Multilateral Network

Lars Gustafsson

In March 2011 the European Commission organized the conference. It’s always a good time to learn. The final conference on the Adult Learning Action Plan was in Budapest. The background of the conference was to discuss what measures and priorities should be done in a new Adult Learning Action plan that should be formulated against the background of the EUROPE 2020 strategy following the now finished Lisbon strategy. During the conference adults mathematical proficiency was recognised as an urgent - and until now much neglected - issue in need of more attention. I was contacted by a representative of the Commission who encouraged me to consider starting a Grundtvig Multilateral Network (GMN) to address the issue of adult numeracy. If this will be realised, it will be organised through the Swedish National Center for Mathematics Education, University of Gothenburg who will take the full responsibility for the application and administration of the project. I know since a long time ago that within the ALM community there are researchers and practitioners with both extensive knowledge about and devotion for adults mathematical proficiency, that could make a significant contribution to the development of this field on an European level. In the workshop I will first give an introduction to what a GMN is and what is required by the organizations participating, followed by a dialogue about themes, directions and priorities for such a work and finally to an invitation for you to announce a preliminary wish to participate.

Biography

Lars Gustafsson has a background as a teacher of mathematics and science in liberal adult education. He is now working at the National Center for Mathematics Education (NCM), University of Gothenburg where he is responsible for issues concerning adults learning of mathematics in formal and informal settings. He has been carrying out developmental work in adult education and has been responsible for pre-service and in-service teacher training courses on adults’ learning mathematics. He has also been engaged in commissions at national and international level. He is co-author of the NCM-reports on Adults and Mathematics – a vital subject and the ‘Validation of adults proficiency’ - fairness in focus’.

Plenary Video is available at http://www.it-tallaght.ie/events/alm18conference/index.html
Images of Numeracy

Kees Hoogland
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Replacing word problems by image-rich numeracy problems in mathematics and numeracy tests, can have a significant positive affect on students’ results on these mathematics and numeracy tests. This is the outcome of a large scale experiment in the Netherlands.

Using image-rich numeracy problems, contributes to bridging the gap between common classroom practice in numeracy and more sophisticated numeracy concepts.

In 2010, a new Dutch Framework for Numeracy (Referentiekader Rekenen, Ministerie van OCW, 2009) was laid down in Law. Numerous stakeholders in education are now constructing or using numeracy tests. This research’s findings can have a major influence on the testing of numeracy and mathematics in primary, secondary and vocational education. The Ministry of Education and the testing industry have followed this research closely.

Key words: Numeracy concepts, numeracy problems, images, image rich contexts

Biography
Kees works as a researcher and an international consultant on mathematics and numeracy education at APS – the Dutch National Center for School Improvement.

Kees studied Mathematics at the University of Leiden and specialized in educational research and the theory of mathematics education. He worked as a mathematics teacher, teacher educator, schoolbook author, curriculum and test developer, editor and project leader of complex educational projects all over the world.

Plenary Video is available at: http://www.it-tallaght.ie/events/alm18conference/index.html
In this paper, I would like to share some thoughts about the sophistication of numeracy concepts and the representations of these concepts in lesson materials and tests. In my opinion, in lesson materials and test items, the sophistication of numeracy concepts is way ahead of the sophistication of numeracy problems. A lot of work must be done in bridging the gap between common classroom practice in numeracy and more sophisticated numeracy concepts.

In a recent research project, I was given the opportunity to research a part of that bridge: how can we design numeracy problems (for lesson materials or test items) which are closer to the sophisticated concepts?

Also, part of that research was to put into practice these newly designed numeracy problems and compare the results in a strict experimental setting with numeracy problems which are aimed at the same mathematical goals but in a less sophisticated way.

**Sophistication of numeracy concepts**

Much has been written on definitions of numeracy and numeracy concepts. In “Adult Numeracy: review of research and related literature” (Coben, 2003), there was an extensive analysis of the conceptual issues regarding the defining of numeracy. Over many years, David Kaye collected and put together those definitions which were presented at several ALM conferences (Kaye, 2010).

Maguire and O’Donoghue (2002) did some excellent work in creating a framework which gave an overview of the various concepts of adult numeracy education being used at present. In their view, these concepts could be arranged along a continuum of increasing levels of sophistication.

![Adult Numeracy Concept Continuum of Development](image)

*Figure 1. Sophistication of numeracy concepts*

This framework is discussed thoroughly in “A Review of the Literature in Adult Numeracy: Research and Conceptual Issues” (American Institutes for Research, 2006).
I summarize it here. The concept of numeracy in the formative phase, refers to basic mathematical or, sometimes, specifically numerical or quantitative skills which adults are deemed to need to function effectively in society. In this view, numeracy is a basic skill acquired normally in childhood; in many numeracy strategies, what adults are deemed to need is simple arithmetic. This view is reflected in the widespread use of lesson materials with a strict focus on performing arithmetical operations.

Numeracy, in the mathematical phase, is broader and puts the emphasis on the use of mathematics in daily life. In this phase, numeracy includes often numbers, money and percentages; aspects of algebraic, geometric, and statistical thinking; and problem-solving based on the mathematical demands of adult life. This view of numeracy has been quite influential in, for instance, the United Kingdom (Adult Numeracy Core Curriculum) and in the Netherlands (Realistic Mathematics Education (Gravemeijer, 1994)).

All of the most recent approaches to defining (adult) numeracy fall into Maguire and O'Donoghue’s integrative phase. In this phase, numeracy is viewed as a complex, multifaceted and sophisticated construct incorporating, in context, each individual’s mathematics, communication, cultural, social, emotional, and personal aspects. These more integrative approaches to numeracy have become influential over the last few years, as illustrated by projects to define numeracy instructional content standards such as the Programme for International Student Assessment (PISA), and the Adult Literacy and Lifeskills (ALL) Survey. The numeracy definitions in these projects specify the intended cognitive outcomes of adult numeracy education and/or emphasize the need for the individual to adjust to the increasing technological demands of the knowledge economy.

**Time line**

Of course, it is not only the sophistication of the definition which is represented in this figure 1. In another way, it is also a time line. Over the decades, mathematics education has had several goals. In most countries, at least one of the goals is to equip individuals to deal with the quantitative side of the world around us. In general, there is a broad consensus that it is important that individuals can cope with quantitative side of problems which they encounter. However, on the question of how to equip students to do that, the answers varied over time. In the 1950s and 1960s, the predominant answer to this question was: “Teach students the basic operations with numbers. A good mastering of these operations is the most important tool to deal with the quantitative issues you encounter in the world.” In that era it was the most sensible answer. All manufacturing activities, all technical designs and all building of machines were done by engineers who did the necessary calculations with pencil and paper. Sometimes, they used the help of a slide rule to make some rough guesses. It is fair to say that, in that period of our history, it was the most sensible answer.

In the 1970s and 1980s, things changed. More and more calculators were a normal phenomenon in everyday life and computer generated graphs and diagrams entered the worlds of newspapers, books and television more frequently. Also, the key phrase was integration of knowledge. At the same time in education, the outside world was brought in more and more.

In mathematics the way to bring in the world was through word problems: descriptions of situations from reality in such a way that a quantitative question could be asked, closely
followed by the actual question. All sorts of questions of this kind can be found in mathematics and numeracy textbooks all over the world.

For a comprehensive overview of all topics related to word problems, I recommend Verschaffelt et al. (2009)’s very readable publication: “Worlds and Worlds – Modelling Verbal Descriptions of Situations.”

In the last twenty years, the focus has changed even more to the most sophisticated definitions of numeracy, namely authentic education, real life problems, problem solving and creativity. Literacy and numeracy were integrated in tasks which equipped students for the future.

However, at the same time in several countries during the last ten years, some retrograde movements have been visible (Wilson, 2002; van den Heuvel-Panhuizen, 2010) and there has been a societal demand to focus on basic skills. Has the pendulum swung too much to the right hand side of the model? Has the further sophistication of the numeracy concepts created too wide a gap with the common sense ideas on teaching and learning arithmetic?

It is certain that we can see a gap between the sophistication of the concepts mentioned in policy documents and the representation of those policies in lesson materials and test materials.

**Gaps between policy and practice**

In the Netherlands, we use a representation of the sophistication of numeracy concepts which is derived from Maguire & O’Donoghue (2002)’s framework.

![Diagram of Sophistication of Numeracy Concepts](image)

Figure 2. Sophistication of numeracy concepts, adaptation of Maguire & O’Donoghue (2002)
In 2008, a new Numeracy Framework was introduced in the Netherlands (Ministerie van OCW, 2009; Hoogland & Stelwagen, 2012). If you look at the broadness of the included topics (numbers, proportions, measurement & geometry, relations) and the policy language surrounding it (aiming at functional use in practical (job) situations), the Dutch Numeracy Framework can be positioned at the right hand side of the diagram. It aims really at numeracy. In the first generation of textbooks published for the framework, you see a significant shift to the left side of the model. Some textbooks focus mainly on basis arithmetic skills. Other textbooks take a middle ground by focusing predominantly on word problems. There are only a few examples of learning materials which blend exercises, connections to the real world and multimedia to show dynamic simulations of concepts. As an example, we give you two samples from the lesson material called Gecijferd! (Hoogland & IJzerman, 2010).

![Figure 3. Sample of Gecijferd! (Hoogland & IJzerman, 2010)](image)

![Figure 4. Sample of Gecijferd! (Hoogland & IJzerman, 2010)](image)

Most of the testing materials, published around the new framework, can be placed at the middle or at the left side in the model. I show you some examples from a government
document (SLO, 2011) which exemplify the new numeracy framework and give the audience an idea about what the testing will look like. I think translation is unnecessary.

![Example from Rekentoetswijzer 2F (SLO, 2011)](image)

**Figure 5. Examples from Rekentoetswijzer 2F (SLO, 2011)**

I think this phenomenon is not unique to the Netherlands. A closer look at learning or test materials used in many different countries reveals that most materials consist of word problems or of exercises with formal arithmetic skills. Undoubtedly, the sophistication of the concepts runs way ahead of the sophistication of the learning and testing materials. In this era of technology and multimedia, a next step can and should be made to bring real quantitative problems – problems as individuals face them - into learning or test materials by using real life images.

**Designing image-rich numeracy problems**

It is increasingly possible to bring real life into learning materials by making use of multimedia techniques. If, for instance, the learning material is web based, you can insert images, video clips, animations, voice-over etc. In recent years, some such examples have been developed in the Netherlands. See, for instance, the web based numeracy lesson materials Gecijferd12 and Gecijferd34 on the website [www.gecijferd.nl](http://www.gecijferd.nl) (Hoogland & IJzerman, 2010).
However, even when the learning material is confined to paper, much can be done to introduce real life in ways other than by lengthy descriptions of contexts. Careful use of relevant images is the key here. The following are two examples.

**Example: Fridge thermometers**
At the top is a question which I found in a regular adult numeracy text book. I show the two steps in which I made some changes to it without losing the maths skills being tested. Below I explain the underlying thinking behind these changes:
The drawing is replaced with a picture. Drawings send only a message that the mathematics is hidden in a made-up context. Pictures immediately connect the mathematics with the real world. Do not underestimate the effect of such messages on the students’ views of the goals of mathematics lessons. The multiple choice alternatives were nonsensical and debilitating. Consequently, they were discarded. The language is brought down to the essence of the problem. Indeed, you now lose the background of the actual employee. However, there is hardly any benefit in describing that background only in words. Without it, we have still a recognisable question from real life. From the title ‘Fridge thermometers’, the students can imagine easily a situation where this might be relevant. If the skill is reading temperatures in fridges or in other situations where temperature measurement is important, then the students should be taught to do that. The last image reflects this wider context.

*Example: Percentage discounts*

The following example is a question taken from the first test according to the new Dutch Numeracy Framework (Ministerie van OCW, 2009; Hoogland & Stelwagen, 2012). You can guess probably what the Dutch text means from the drawing but to avoid any doubt, it reads: ‘What is the approximate percentage discount on this houseplant?’ (Cito, 2009).
I think this question is fairly representative of current practice in numeracy word problems i.e. a drawing rather than a picture because, probably, it is cheaper and faster to produce.

The question repeats, unnecessarily, in words, a part of the picture. The word, ‘approximate’, is added to avoid any confusion or debates in the classroom. The question is the ‘wrong way around’ – in real life we rarely want to know this. Therefore, the question is not ‘imaginable’ as a real life problem.
The alternative is quite simple. Life is all about making choices but, in numeracy education, we can do without the silly multiple choice model. The drawing is replaced with a picture of a real houseplant. It would be even better if you could have a picture of this houseplant in a garden centre with the real discounts printed or handwritten on a little chalk board. The question can be imagined. You can think of a person wanting to perform this task. You can expand the question easily by taking pictures of more and different plants.

From these examples, we can try to formulate a definition of a numeracy problem which reflects these considerations:

‘A numeracy problem is an imaginable question about the real world which can be solved by mathematical techniques. A numeracy problem in learning materials uses a minimum of language and a maximum of real life images. The images are essential in solving the problem.’ This definition became the starting point of a research project. In the research project, numeracy problems were designed according to the definition. However, even more interestingly, they were tested on a large number of students from all levels of education (10-18 years) in an experimental setting. In this setting, these newly designed numeracy problems were compared to problems which were equivalent to the mathematical goals they addressed, but were quite different in the way in which the reality was represented. The next section describes this research in more detail.

**Researching image-rich numeracy problems**

In 2010, a grant from the Dutch programme for evidence based educational research (Onderwijsbewijs) was awarded to research the use of real life images in numeracy test materials.

In the research design, the set up was described as follows:

“The concept of numeracy implies that the mathematics involved is connected to reality. Current practice of testing the connection to reality is predominantly made by language-rich word problems. The language used to connect the mathematics to the real life is, for many students, a serious barrier to showing their mathematical knowledge and skills in a way that gives credit to their level of mathematical knowledge and skills. The research presented investigates the effect of incorporating image-rich contexts in numeracy test items. The hypothesis from theories on multimodality learning on the one hand, and from theories and concepts on numeracy learning on the other, is that using image-rich test items will have a positive affect on the results of the students on such tests. This hypothesis is tested in a randomized and rigorously matched experiment.”

**Research setup**

In this research, the affect on students’ results of incorporating more image-rich test items in numeracy tests was measured in a rigorously randomized way. A digital test of 24 numeracy items was constructed. Each item had two equivalent versions; one being image-rich and one being language-rich. The items and the test as a whole were linked to the new Dutch Numeracy Framework (Referentiekader Rekenen) which was implemented in 2010. In May and June 2011, more than 7,000 students, who were between 10 and 18 years of age and from over 60 schools across the Netherlands, took this test. For each participant, a new test
was generated automatically and randomly by choosing randomly each item of the test from the two versions. This methodology fitted all the requirements of experimental research. I give a few examples of the A-variant and the B-variant. Again, I think translation is unnecessary and only the look and feel requires to be experienced.

**A-variant**

Harry ging in het voorjaar 2008 op vakantie naar IJsland. In IJsland gebruikt men de IJslandse Kroon (ISK). Tijdens de vakantie gold ongeveer € 100 = ISK 13 400 en ISK 100 = € 0,74627. Een IJslands tijdschrift kostte ISK 670.

Hoewel euro kost dit tijdschrift uit IJsland?
€ 

**B-variant**

![I&I Iceland] In IJsland gebruikt men de IJslandse Kroon (ISK).

€ 100 = ISK 13 400
100 ISK = € 0,74627

Hoewel euro kost dit tijdschrift uit IJsland?
€ 

**Figure 9. Two versions of a numeracy problem**

**A-variant**

Je gaat de wand van een keuken betegelen.
De afmetingen van de wand zijn 2,60 m bij 5,20 m. Per vierkante meter gebruik je 25 tegels.
Je kunt alleen hele dozen van 50 tegels kopen.

Hoewel dozen moet je kopen voor deze wand?
[ ] dozen
The hypothesis being tested was that replacing language-rich test items with image-rich items in mathematics and numeracy tests would have a significant positive affect on students’ results of these mathematics and numeracy tests. In this preliminary part of the research program, the first findings support this hypothesis and provide, also, indications of which characteristics of image-rich test items lead to a stronger or lesser effect. Eventually, the full research will result in design criteria for numeracy and mathematics test items which aim to connect numeracy and mathematics with real life and avoid test items with an excessive amount of language.

This research’s results are highly relevant to the Netherland’s current educational policies. In 2010, a new Dutch Framework for Numeracy (Ministerie van OCW, 2009) was laid down in Law. At present, numerous stakeholders in education construct or use numeracy tests. This research’s findings can have a major influence on the testing of numeracy and mathematics in primary, secondary and vocational education. The Ministry of Education and the testing industry have followed this research closely.
References

SLO. (2011). Voorbeeldtoets bij Rekentoetswijzer 2F. Enschede: SLO.
Eeny – Meeny – Miny – Mo: Count and be Counted!

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Selecting from the history of Number - from the tally stick, to the musical harmonies, to the quantum numbers – this seminar shows that it was intelligent insight, and not the existing facts, that led to the unfolding of new truths in mathematics. It links the disenchantment with mathematics and science in our schools and colleges to the tedious disenchanted world of common sense ‘reality’ and public opinion. Using, as examples, two new inventions - the UDDER balloon and the Intelligence Meter – it draws some surprising conclusions on how our world, and our educational processes, might be re-enchanted and re-invigorated.

Key words: facts, truths, education, numbers, metaphors.

Biography:

Conleth D. Hussey B.A., B.A.I., PhD, ScD
Head of Department, Civil Engineering & Materials Science.
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Professor Hussey is a well celebrated contributor and speaker on mathematics and science education in Ireland. He has a specific interest in the re-enchantment of mathematics and science education at all levels. He has a record of academic teaching, research supervision and innovation for which he became the first academic at the University of Limerick (UL) to achieve a Doctor in Science for work actually carried out at UL.

He is widely published in optical waveguide theory and design; optical fibre device theory, design and fabrication. He has invented some new devices and processes and successfully transferred device technology industry.

Other interests currently include: the relationships between post-Cantorian set-theory and the fundamentals of the quantum theory; the mathematical physics of the quantum properties of fields and materials, in particular, non-equilibrium phase transitions and optical interactions with materials.

Plenary Video is available at http:\www.it-tallaght.ie\events\alm18conference\index.html
One story about Eeny – meeny – miny – mo (a single, a pair, one more, the many) is that it relates to a prehistoric manner of counting that has been passed down through history in children’s play. Written records only go back to the 18th century. However, in the spirit of not letting the facts get in the way of the truth, I choose to believe the children’s lineage tale.

**Facts and Truths**

Inhabitants of Terry Pratchett’s Discworld can all agree that the directions up and the vertical are the same. On Earth, however, the inhabitants of Ireland can have reasonable agreement on the direction of up, but compared to Ireland, people who live near the equator have a horizontal up and people who live in New Zealand have a vertical up that is down! So, the direction up is a local fact and not a global truth.

The claim that there are no whole numbers greater than one million has one million facts to support it, but it is still not true. Yet we can declare that there are more whole numbers than there are atoms in the universe and know that it is true.

Newton’s First Law of Motion makes the ridiculous declaration that - A body remains at rest or in uniform motion in a straight line unless acted on by a force - which no experiment on earth can ever prove, yet we know that it is true.

Likewise, there is no meter stick or protractor accurate enough to experimentally prove the truth of Pythagoras’s Theorem, and again we know it to be true.

Truths are like Winnie-the-Pooh’s Heffalumps, difficult to describe, but we always know one when we see one.

**Truths must be Declared!**

A strange result from Kurt Gödel, one of the founders of modern mathematics, is the demonstration from 1931, that there are more truths than proofs, or in my language, there are more truths than facts. This means that, typically, a new truth has to be declared before it is proven! A proof, if one is available, is always undertaken retroactively.

For instance, in 1919, when Einstein received a telegram saying that astronomical observations had confirmed his theory of relativity, a student asked him what he would have done if his predictions had been refuted. ‘In that case,’ replied Einstein, ‘I’d have to feel sorry for God, because the theory is correct.’

The American Declaration of Independence holds that it is a self-evident truth that ‘all men are created equal’. Of course, this truth can never be supported by facts; there is no meter capable of measuring the equality of men. Indeed, this year the population of the world will reach 7 billion, so there are nearly 7 billion facts that contradict this truth of human equality. Like the mathematical truths, human equality can only be declared, and society subsequently organised to live in fidelity to this truth.

**Numbers and Sums**

The most primitive recorded method of counting is that of notching a tally stick, or equivalently, by the use of “counters” (e.g. pebbles, in Latin: calculus is a pebble). Shepherds
would notch a stick for each member of their flock going out to pasture in the morning. By checking the notches one-by-one in the evening as the sheep were being penned for the night, the shepherd could check if he had lost (or gained) any during the day, and take appropriate action.

With tally counting there is, as yet, no concept of number, there is simply a one-to-one correspondence between the sheep and the marks on the stick. This form of counting is present today in prayer beads which keep an automatic tally of the prayers without the distraction of the count. This is also how computers “count” today, so that talks of the human brain being replaced with an artificial intelligence in the near future are greatly exaggerated!

The great Stone Age settlements (~ 5000BC) in Sumer, situated in modern day Iraq between the Euphrates and Tigris rivers, record the first “Sums”, and with good reason. Stone Age agrarian life was full of risks: drought, flooding, infestation, and other natural disasters, capped throughout antiquity by wars. Farmers had often to borrow to get themselves through the lean months, while hoping that nothing would prevent them from bringing in crops that will allow them to repay their debts. In ancient times, failure to repay loans could cost farmers their land, possessions, enslavement of family members, or their own freedom. This was mankind’s first encounter with negative numbers and they have carried a ‘bad’ name ever since.

In order to prevent the destabilization that occurred when large portions of the population were forced off the land or into debtor’s prison for failure to repay loans, there developed throughout the ancient Near East a tradition of clean-slate edicts. These “proclaimed justice” or decreed “economic order” and “righteousness” by cancelling debts and restoring forfeited land to farmers. Clean-slate proclamations date from almost as early as the first interest-bearing debt, starting in Sumer around 2400 years BC, they continued for 2,000 years as a periodic and regular economic renewal based on freedom from debt-servitude and as compensation for the loss of access to a livelihood due to the privatisation of the land.

### Infinity – Discrete vs. Continuous

The Greeks deepened the concept of number to include the natural whole numbers, the negative numbers – formed by extending the rule of subtraction beyond the mere reverse of addition, and then on to the fractional numbers, formed by extending the rule of division beyond the mere reverse of multiplication, obtaining the ratios of two whole numbers - the so-called rational numbers.

The decimal representation of the rational numbers has either a finite number of decimal places or a repeating pattern of a finite number of decimal places.

When Hippasus (500BC) declared that $\sqrt{2}$ [the length of the diagonal of a square of side 1 through the application of Pythagoras’s theorem] was a number - a fractional number that did not consist of the ratio of two whole numbers - a so-called irrational number requiring an infinite number of decimal places to represent it, the outraged and scandalised Pythagoreans put him to death, in order to keep secret this new irrational truth.

Greek thought had no concept of infinity; it held that human thought was finite and that no human thought could, by definition, comprehend infinity and so had no business going there! A problem that freaked the Greeks was the relationship between numbers as discrete entities and the number line as a continuous geometrical entity.
For instance if we take decimal representation of another irrational number, Pi, viz: 3.141592653589793238462643383279502884197169399375105820974945923078164062 8620899862803482534211706798214809865132823064470939406095505822231725359408 128…
The question arises “On the number line, what is the next, or adjacent, discrete number to Pi? This question still freaks us today!

The traumatic encounter with irrational numbers, like Pi and $\sqrt{2}$, led Greek thought away from number and it concentrated instead on geometry - which Euclid recorded.

Common Sense or TRUTH

Euclid’s geometrical system was based on a set of “common sense” axioms, one of which is that the “the whole is greater than the part”. However, when we apply this axiom to the modern concept of the infinity of natural whole numbers we run into a difficulty.

There is an infinity of natural whole numbers $[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,…]$. The odd numbers $[1,3,5,7,9,11,13,…]$ are a definite part of the natural whole numbers. When Georg Cantor, the inventor of set theory, declared that the number of odd numbers is equal to the number of natural whole numbers, one of his fellow mathematicians claimed that ‘it was repugnant to common sense’ (irrational!), but at least they did not kill him. However by using the shepherd’s approach of tallying the odd numbers in a one-to-one correspondence with the natural whole numbers, it was found that they were, indeed, equal in number. In this case” the part is as big as the whole”. Euclid’s common sense axiom was found to be untenable when applied to ‘infinite multiplicities’ and it had to be discarded.

The Rubin vase provides an example where ‘the part is as big as the whole’, at different times a different part of the picture is perceived as the whole; the two parts cannot appear simultaneously as a whole.

As another example of a situation where the part is as big as the whole; take a whole human being; if she gets her hair cut she is still a whole human being, if she loses a finger she is still a whole human being, if she loses a leg, she is still whole, denying that the remaining part of the human is not equal to the whole human leads into ethical territory.

Truths and Metaphors

New truths, by their very nature, must contradict the existing body of knowledge and public opinion; otherwise they would not be new! Declaring a new truth shatters all existing categories of knowledge and as a consequence, always sounds irrational from the point of view of current “reality”.

The declaration of a new truth invariably requires the courage and audacity to declare that, within the current known world, something is what it is not! Truths, therefore, make their first appearance as metaphors.

Hippasus: $\sqrt{2}$ is a number.
Cantor: The number of odd numbers is equal to the number of whole numbers.
Archimedes: Eureka! The height of the water is the volume of the crown.
Newton: The moon is an apple!
(Heavenly bodies are the same as earthly ones.)
De Broglie: The particle is a wave!
Shakespeare: The world is a stage!
Me: The latex glove is a balloon. (e.g.: an UDDER balloon.)

**Mathematical Declarations**

Pythagoras’s theorem states that, for a right triangle; the square on the hypotenuse, \( z \), is equal to the sum of the square on the other two sides, \( x, y \), viz;

\[ x^2 + y^2 = z^2 \]

Tables of integer solutions of this equation, \( a^2 + b^2 = c^2 \), \((a, b, c)\), [e.g. (3:4:5), (5:12:13), (7:24:25), (8:15:17), (9:40:41)] were known to the Pyramid builders and predate the geometrical theorem by millennia.

Pythagoras’s leap of faith was to declare the validity of the formula for right triangles even when the values of \( x, y, z \) were non-integers, which his theorem subsequently proved.

Fermat, in declaring his ‘last theorem’ in 1637, took a reverse route to Pythagoras. Fermat’s hypothesis asserted that there were no positive integer solutions to the following equation;

\[ a^3 + b^3 = c^3 \text{ or generally } (a^n + b^n = c^n, \text{ for } n>2) \]

Even though Fermat claimed to have a proof, his proof could never be found. Over the years, every attempt to prove Fermat’s hypothesis failed, and every attempt to disprove it also failed! Yet every failure pushed the boundaries of mathematics forward, each failure was a better failure than the previous one! Andrew Wiles finally published a proof of Fermat’s theorem in 1995, 358 years after Fermat’s declaration. In the eight years leading up to his proof, in order to save himself from ridicule, and presumably, from claims of irrationality and childishness from his colleagues, Andrew Wiles isolated himself and worked in secret on the one problem that had attracted him to mathematics at the age of 10.

**To be a Militant Mathematician**

To be a mathematician is therefore to have the courage to defend and be faithful to a new truth even if it contradicts and defies older certainties. To put it in more philosophical language – the concept must always be vulnerable to the truth of the situation. Since the Greeks, this courage has had to be carried on pain of death, annihilation, marginalization, or ridicule. Indeed, with mathematical precision, we can assert that the ethical term EVIL amounts to the enforced defence of the concept over the situation.

Every detective, Miss Marple, Hercules Poirot, Sherlock Holmes, Fr Brown etc, who painstakingly follows the clues and not their prejudices, is a militant mathematician. The dysfunctional clinician House is a militant mathematician. When he treats the most likely symptoms first, and when that treatment fails, he moves on to the next symptom, etc…, he follows the – try again, fail again, fail better, - approach of scientific research.

Ever since Pythagoras pronounced that “Number is the essence of all things, and the organisation of the Universe as a whole… is a harmonious system of numbers and their
relations” scientists have operated in fidelity to this pronouncement. For instance, Mendeleev’s Periodic Table (1869) of the Chemical Elements shows the harmony of the chemical elements with respect to the number eight. The chemical behaviour of the smaller elements repeat in cycles of eight and the larger elements display a more complicated harmony superimposed on this cycle – a symphony almost, captured very eloquently on Soddy’s representation from 1911.

Pythagoras also proclaimed that “the musical harmonies [could be] reduced to mathematical relations”, and indeed, the behaviour of a simple guitar string bears this out - successive doubling of the length of a guitar string decreases the pitch by one octave. In turn, the modes of vibration of the guitar string, which correspond to whole number multiples of a half wavelength, find their modern echo in the quantum states of the elementary particles and their associated quantum numbers. In their turn Newton and Einstein operated in fidelity to Galileo’s pronouncement that “Mathematics is the language with which God has written the universe”, and all modern physicists follow suit.

Crisis of TRUTH

Mathematics is one of the key areas where Truths can be declared and defended with unassailable proof. However we must not confuse Mathematical Truths with the profusion of numbers, polls, and economic statistics that assail us in the modern world at every turn. When Brian Dobson reads out, to four significant figures, the ISEQ Index, the NASDAQ, the DOW, and the FTSE, every night on the Six One News, he is not relating any truths nor is he clarifying anything. In fact, he is adding to the general confusion by implying that our knowledge of these numbers is somehow significant to our daily lives, all under the respectable camouflage of numbers. The old truism that ‘There are three kinds of lies; lies, damned lies, and statistics.’ is now more relevant to our media than ever before.

The present economic, ecologic and ideological crises arise from the abandonment of truths in favour of ‘common sense’ public opinion, informed by the prevailing ideology and reinforced by newspaper “experts”. (There are three kinds of liar: liars, damned liars, and experts.) The crises can only be overcome with the mathematical approach of allowing the truth of the situation to predominate over the economic facts or ‘the reality’ of the situation. The false concept of today’s world is that the entire value of the history and culture of humanity can be expressed in monetary numbers alone and that these values can be exclusively acquired and controlled by small sections of the population to the detriment of the remainder. The truth of the situation is that, under the rule of this false concept, our bountiful earth is being destroyed and the viability of existing species, including homo-sapiens, is coming into question. Man is being deprived of his culture i.e. his humanity, and is being reduced to his basic animal substructure - to join the rat-race you have to become a rat! A Dark Age is descending as the monetisation of all values threatens to become a form of neo-feudalism with the economic bottom 90% increasingly driven into debt by the wealthiest 10%. All of the gains in human culture of the last 2000 years are being eroded daily. Perhaps it is time to insist on a new Clean-slate proclamation; for an economic renewal based on freedom from debt-servitude and to demand compensation for the loss of access to a livelihood due to the privatisation of the products of human culture and human ingenuity. Perhaps it is time to declare the following truth to ourselves and our children. ”You are a child of the Universe, you are entitled to be here, and you are entitled to a living which is your birthright as a member of the human race".
A Dark Age or a New Golden Age

Ireland's 'golden age' began in the sixth century and lasted well into the ninth century. With its rich culture of the Book of Kells, the Derrynaflan Chalice, the harpists, the bards and the clans, Ireland was known as an 'Island of Saints and Scholars'. Its missionaries and teachers were a beacon of scholarship and learning for the rest of Europe which was languishing in its 'Dark Age' following the fall of the Roman Empire. The flowering of literature and learning in early Ireland is firmly linked with the rise of Christian monastic schools which, operating at a remove from hierarchical church, were to establish religious centres of learning in many other parts of Europe.

In 563, Columcille (Columba) went to Iona, off the coast of Scotland, to convert the Picts. In 591, Columbanus went to France, and later into other parts of Europe, to establish monastic schools, the most famous of which was Bobbio in north Italy. In 633, Fursa went to England and France to establish Irish foundations of learning and in 635, Aidan became the first bishop of Lindisfarne (England).

What was it that drove these men to endure lives of hardship and exile? Could it be anything other than to live in fidelity to the truth that “All men are equal” rendered in their time by St. Paul’s universalist egalitarian doctrine that; “There is neither Greek nor Jew, neither slave nor free man, neither man nor woman, all are one before Jesus Christ”.

Over 1000 years later, after the repeal of the Penal Laws, came one of the most extraordinary developments within Irish Catholicism: the resurgence of this specifically Irish version of monasticism. Self-sacrificing, hard-working and again at a far remove from the hierarchical church and its power, they did their works where the needs were greatest, where the hardships were relentless, and the conditions were brutal.

In chronological order, came the Presentation Sisters, the Irish Christian Brothers, the Presentation Brothers, the Sisters of St. Brigid, the Brothers of St. Patrick, the Sisters of Charity, the Sisters of Mercy, the Loreto Sisters, and the Holy Faith Sisters.

In the age-old missionary tradition there emerged the Missionaries of the Blessed Sacrament, the Maynooth Mission to China, and the Society of St. Patrick for Foreign Missions and the Medical Missionaries of Mary.

Surely these men and women were driven by fidelity to the same universalist egalitarian Pauline doctrine as their forerunners had over a millennium previously.

With time, however, the words of the doctrine became empty and detached from the underlying truth, so that fidelity to the letter of the doctrine, rather than to its spirit, made room for the EVIL of which we have become all too well aware in recent times.

So today many of the religious monastic and missionary orders are in tatters. In spite of this, however, the Irish monastic and missionary tradition still carries on, but now, in more secular terms - but not exclusively so - with SVP, GOAL, TROCAIRE, Bothar, Gorta, etc. I would hazard that Bob Geldof’s Band Aid (St Bob!!) and even the UN peacekeeper role of the Irish Army are still rooted in this tradition.
Irish Nobel Prize Winners: Saints and Scholars

I have no reservation in claiming that our Nobel Laureates mark a fidelity to the tradition of Ireland’s Saints and Scholars. We have five Saints and five Scholars and it is interesting to see what overarching irrational and impossible truths were declared by them.

The Saints:

All declared: “We can live in Peace!”

The Scholars:

All Declared their Truth through Metaphor!

Ernest Walton is one of the unsung heroes of Irish Science, together with Cockcroft, he “split the atom” while at the Cavendish Laboratory in Cambridge. Their declaration was: “The indivisible ‘atom’ is divisible”.

What New Truths Can We Declare to Make Our Lives Count Today?

Can we today have the courage to even imagine a society that lives in fidelity to the Axiom of Equality that “All men are equal”, without the fear of appearing ridiculous in the face of common sense “reality” and public opinion?

Can we, in turn, dare to imagine an educational system that operates in fidelity to this Axiom? An educational system that is based on the following declarations:

“All human beings are equally intelligent.”

“All truths are transmissible to all human beings.”

“All human beings are equally creative”.

I hope that these sound ridiculous and irrational, as I can then be confident of their truth!

On the strength of the Axiom of Equality, I can even propose an Intelligence Meter. It is called a thermometer and, I would claim, since it gives the same number for all, is as valid an intelligence indicator as anything currently available. On the Fahrenheit scale at 98.6 we would all get A1’s. Suits me!
Summary

Mathematics is not about the economy, it is not about facts and figures, it is not about projects. Mathematics is about the joy and laughter that comes with the satisfaction of finding one’s own proof to a theorem. There is nothing as dreamy and poetic as mathematics, nor is there anything more radical or subversive because it stands and fights for truths. Mathematics is an art, but unlike poetry or painting, its patterns are more permanent and less ephemeral, because mathematical patterns are made with Ideas. It is by concentrating on the essence of mathematics as a creative activity that we can have any chance of re-enchanting mathematics education in our schools.

In order to make our lives count we need to learn to see the truth of each situation with mathematical eyes, and to courageously defend that truth, and we need to resist the evil which, ostrich-like shuts our eyes and denies any truth outside of the facts and public opinion.

If at first you don’t succeed, try again. More than this- against the contemporary common sense of not risking the new for fear of failure - the mathematician is driven to go for the wager and – in the words of Beckett – to try again, fail again, fail better!
Project Maths

Bill Lynch
NCCA – Project Maths Co-ordinator

Project Maths is a phased developmental initiative in post-primary mathematics under which revised syllabuses at both Junior Certificate and Leaving Certificate are being introduced over a three-year period. The project commenced in 2008, when 24 schools began the first phase of change in the way mathematics is taught and learned. National roll-out of the changed syllabuses began in September 2010. Greater emphasis is being placed on student understanding of mathematics, through active engagement in mathematical investigations set in real-life contexts and through the development of problem-solving skills.

The conference presentation will provide an overview of the project and its current stage of implementation. It will focus on the teaching and learning approaches being adopted under Project Maths, with examples of topics from the syllabuses which relate to everyday applications of mathematics. These examples will also illustrate the changed emphasis being given to understand and to problem solving.

Biography:

Before joining the NCCA in 1998, Bill had over 25 years of post-primary teaching experience. Initially as an Education Officer and, following his appointment in 2002, as a Director with the NCCA. He has served on the PISA National Advisory Committee and on the Task Force for the Physical Sciences, as well as on steering committees for national syllabus implementation programmes in science, technology and mathematics.

His current responsibilities include developments in the science and technology subjects at both junior cycle and senior cycle and he leads the developments for post-primary mathematics under Project Maths.

Plenary video is available at http://www.it-tallaght.ie/events/alm18conference/index.html
But what use is this 'maths stuff' in the real world? - One practitioners' quest to provide answers

Ciarán O’Sullivan

One of the questions that mathematics educators are frequently posed by learners of all ages engaged in mathematics is the question of how mathematics occurs and is useful in the ‘real’ world. Providing answers to such questions which are at a level appropriate for the learner posing the question, and which will engage the learner in further mathematical activity, is a challenging task for the educator. In this paper, the author seeks to outline some approaches that he has taken as a practitioner with a variety of learners of all ages to show how statistics and algebra occur and are useful. Firstly, the paper will discuss, briefly, a preparatory mathematics course designed for adults hoping to return to third level education. Secondly, examples providing some answers to the questions that are asked by students in relation to how useful algebra might be in the 'real world' are discussed in detail. In particular, examples of linear, quadratic and other equations in action in the areas of pizza cutting, motion, electrical circuits, kitchen shelf-bending, modelling car shock absorbers, fitness testing, and ‘green’ mathematics are explored. Feedback from a variety of learners using some of these examples is also discussed. Finally, other rich sources of tasks on websites and from the use of data-capture equipment are also discussed.

Biography:

The presenter, Ciarán O’Sullivan, has been a practitioner in the area of mathematics education for 27 years. He has been a lecturer in Mathematics in Dept. of Mechanical Engineering, ITT Dublin for the last 13 years and prior to this, he taught for 14 years at second level. He is interested in engineering mathematics education, adults learning mathematics, student retention (in particular the role of mathematics support in this), and numerical techniques.

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Section 2

Paper Presentations
Financial literacy of microcredit clients – Results of a qualitative exploratory study and its implications for educational schemes

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This paper presents a reanalysis of semi-standardised interviews with self-employed women in the informal sector in Nicaragua using Lindenskov and Wedege's (2001) working model for numeracy. The reanalysis illustrates that the women’s numeracy skills are not limited to counting and calculation skills when dealing with financial matters. All of the interviewees display a broad understanding of patterns and relations and there are indications of an awareness of data and chance. As money plays an important role in the description of these skills, they can be considered illustrations of financial literacy. If the livelihoods of these women are to be improved, it is suggested that these skills that the women already possess should be investigated more systematically in order to facilitate more targeted educational interventions in lieu of standardised approaches.

Key words: financial literacy, qualitative research, informal sector, developing countries.

This paper reanalyses data collected for a Master’s Thesis which studied the planning skills of women working in the informal sector in Nicaragua (Beeli-Zimmermann 2008). The informants received a microcredit of US$ 100 to start or enlarge a business. In line with the conference theme, “mathematical eyes” are used to reanalyse the interviews in view of statements illustrating the women’s numeracy skills. As most of these statements relate to financial phenomena in a broader sense, they can also be considered to be a description of the women’s financial literacy skills.

The first section of this paper describes the background of the study, followed by a presentation of the methods employed for data collection and analysis. Selected results are reported and conclusions are presented in the final section.
Background

The 19 women interviewed for this study live in the city of Matagalpa, Nicaragua, one of the poorest countries in Latin America. Matagalpa is the centre of coffee production which contributes to the fact that the city is characterised as an area of medium and low level poverty, while its surrounding areas are characterised by high levels of poverty\(^1\). All of the interviewed women are associated with a local branch of SOS Children’s Villages International, in Spanish Aldeas Infantiles SOS, from now on referred to as Aldea SOS. On the one hand, they benefit from child care provided at this institution, on the other hand they have access to further education and training, ranging from basic literacy to practical classes such as sewing or personal development\(^2\). In this context, they also have the possibility to receive a microcredit of US$100. This credit is provided by an international non-governmental organisation called Trickle-Up\(^3\) and administered by two local organisations.\(^4\) The women receive two tranches of US$50 each, within the timeframe of three months. They have to pay back the money one year later, again within three months.\(^5\) The conditions for this credit are that (a) the money has to be invested in setting-up or enlarging a business and (b) credit takers attend some workshops. The initial study focused on how the women planned for the spending and paying back of the credit (Beeli-Zimmermann 2008).

The interviews with the women were conducted in May/June 2008. On average they lasted 30 minutes (minimum 18 minutes, maximum 50 minutes). Some of them were held on the premises of Aldea SOS, others were held at the women’s work places or in their homes. In some instances third parties were present. All of the interviews were conducted in Spanish, they were recorded digitally, and then transcribed. Translations (by the author) were only undertaken as needed for quotations in presentations or publications. The informants can be described as having the following characteristics:

- Age: At the time of the interviews, the women were on average 32 years old with the ages ranging between 21 and 51 years. The majority (70%) was younger than 35 years.
- Educational background: A majority of the women (72%) completed six or more years of formal schooling. Three of the women even completed secondary schooling (eleven years) and thereby met the requirements for admission to university. Only one of the interviewees had never attended formal school.\(^6\)
- Type of business: All women worked in the informal sector, selling a wide variety of products including foodstuffs, drinks, second-hand clothing, cosmetics or accessories. Ten of the women worked in a fixed location in the city (market booth or specific corner in the

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\(^1\) The National government has categorised all Nicaraguan departments as well as all communities with respect to their level of poverty, namely low, medium, high and severe poverty. Extreme poverty, in this case, has been defined as households which have two or more unmet basic needs such as adequate living space, adequate construction materials used for housing, access to drinking water and sanitation, access to basic education for school-aged children and economic dependency (Instituto Nacional de Información de Desarrollo 2008, p. 27f., translation by the author).

\(^2\) As many of the women have a family history of violence, abuse or drugs, some also receive psychological counseling.

\(^3\) For more information on Trickle-Up see http://www.trickleup.org/.

\(^4\) Besides Aldea SOS a local communal development organisation called Organización para el Desarrollo Municipal, ODESAR, who also administers credits in the surroundings rural areas is involved.

\(^5\) In the meantime, this condition has been changed by Trickle-Up, the credit is now a grant which does not have to be paid back.

\(^6\) The informants’ educational background is slightly above the average for Nicaragua: 78% of the sample completed primary school, while in the general population only 73% completed primary school.
street), eight had no fixed location (they sold their products while walking through the streets) and two worked from home.

- Microcredit: Eight women had received their credit one year prior to the interviews meaning that they were in the process of paying it back. The other eleven had just received the first tranche of US$50.

In addition to the interviews further information was obtained through informal talks particularly with Aldea SOS staff, and from files which Aldea SOS set up and maintained while the women were their clients.

Methodology

As indicated in the previous chapter, the basic approach of this study is qualitative. Data were collected with semi-standardised interviews, they were then analysed with the method of qualitative content analysis (Mayring 2008) using the computer programme MAXQDA. The interview guideline as well as the framework for the initial analysis were developed on the basis of a specific planning model by Friedman and Scholnick (1997). The interview guideline encompassed the following thematic areas: context (family and business), task (financial aspects of the business) and planning (use of the credit and pay-back process). Particularly the latter two areas are relevant for this reanalysis.

The theoretical basis for the reanalysis presented in this paper is provided by Lindenskov’s and Wedege’s working model for numeracy (Lindenskov and Wedege 2001). This model is based on four interrelated dimensions: media, context, personal intentions as well as skills and understanding. All of these dimensions encompass a number of specifications. For this study the focus will be on the fourth dimension “skills and understanding”, which includes the following aspects: dealing with and sense of (1) quantity and numbers, (2) dimension and form, (3) patterns and relations, (4) data and chance, (5) change and (6) models (Lindenskov and Wedege 2001, p. 7). The working model’s terms were used as a basis for the codes employed for the qualitative content analysis and they constitute the deductive categories. In addition, inductive categories were added in order to further analyse and specify the deductive categories. For all categories, criteria for an allocation to the respective category were formulated and illustrated with an anchor example. This led to a code system with up to four levels, as the following illustration shows:

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7 Socio-demographic and educational information on the individual was obtained from the women’s files maintained by Aldea SOS.
Within the dimension of “skills and understanding”, the category “patterns and relations” has proven to be particularly fruitful. The criteria for statements to be allocated to this category were specified as follows: Statements have to describe a quantitative understanding of specific concepts (e.g. credit with interest; savings account), relating two (or more) linked concepts to each other (e.g. income and time). They therefore need to include identified elements, specify relationships between them and have an explanatory power. Anchor examples are “Business is best in December, or after the harvest.”, or “I don’t have a set income, it depends on the sales.”.

In the following section some selected results are presented. While time and many other concrete materials, such as specific foodstuffs, are also mentioned in the interviews, the focus will be on statements relating to finances. Similar to other discussions or definitions of specific terms, there is no generally agreed upon definition of the term financial literacy, let alone a standardised measurement (Huston 2010). For the purpose of this paper, financial literacy is understood as “as knowledge of financial concepts and the skills and attitudes to translate this knowledge into behaviors that result in good financial outcomes.” (Sebstad, Cohen and Stack, p. 2).

Results

According to Wedege, specific quantitative skills cannot be presented in an isolated manner: “whether or not an adult knows mathematics can only be answered after the questions such as who, where, when, what and related to what” (Wedege 1999, p. 206) have been addressed. With the description of the informants in the previous section, the first of these questions has already been answered at a general level. In this section, the results will be organised in a
manner that takes care of the other questions. The basic chronological sequence that the
women follow in their use of the microcredit, namely that of planning for and starting (or
enlarging) a business and running it afterwards, is used to illustrate the women’s
understanding of specific patterns and relations. In addition, a third section will present other
relevant statements, particularly relating to educational issues.

Starting a business

There are two important aspects when setting up a business, that of adequate (financial)
resources and that of estimating the income to be made. Many of the women were very aware
of the fact that the lack of financial resources was the main reason for them not to be working:
“I did not work, because I had nothing to work with.”, or

“I can do such things, but sometimes, one does not have a beginning, right? In order to
have a good business, which moves, one has to have money.”.

While some women simply expressed gratitude for being given the microcredit, others were
aware that this was a credit with special conditions:

“Because with this, ODESAR is not charging any interest. And it gives me a one year
grace period, do you understand? So [if I had not received your credit] I would have had
to go to another institution with loads of interest.”, or

“There [financial institutions], they offer us credits, but one has to pay daily quotas or
fortnightly. And sometimes, when sales are low, one does not get these quotas. And
with the credit you offer we have the option to collect the money over some time, over
the time given.”.

It is interesting to note that many of the women clearly see that this money is their working
capital and they talk of “investing it into products” or “buying a stock”. Many of them are
aware of the fact – and this is also an aspect that is clearly transmitted at one of the workshops –
that this money needs to be invested in productive matters and not be used to fulfil personal
wishes, such as buying a mobile phone or “personal luxuries”. A specific observation in this
context is the level of abstraction of the terminology used by the women. Not all of them talk
about their investments in abstract terms such as products or stock. When asked how they had
spent their credit money, some simply listed the specific things they had bought with it.8

Another consideration of starting a sales business is the frequency of sales to be made, as
income directly depends on that. The women consider two factors in this context, namely the
choice of products offered and the location of their business: “Well, in having things that
others don't have so that the customers come here.”, or “With the credit I bought more
merchandise and made more clients.”, or

8 As the questions relating to this aspect have not always been phrased in the same way, a reliable quantitative assessment of
the answers is not possible. However, a more systematic inquiry into the level of abstractness, as it is reflected in the language
used by the women, would be another interesting topic of research, as this could be an indication of the awareness of their
numerical and/or financial knowledge.
“In September there is more movement because other kinds of fruit come into the market. A kind of plum, oranges. And these fruits sell better, there is more demand.”, or

“But finding a little place there, in the centre of the city. One has to look for it and be on one’s toes. That you are where the money moves. Yes, because in a well off area, nothing moves, no one is out. One has to look for a place in the centre.”.

Generally, statements referring to this aspect of generating sales are vivid illustrations of the proportionality principle, however, in some instances there is also an aspect of dealing with, and sense of, data and chance, particularly for those women who keep books. However, as this aspect is more relevant in the phase of running a business, it will be discussed in the next section. It is interesting to note that, in general, the women mention the principle of direct proportionality – the more they sell, the more profit can be made, even if the profit per unit sold is not high:

“Investing in things which one sees are popular, which sell more [...]. One will earn little, but will sell quickly.”.

The statements also reflect the women’s experience of phases during the week or year when business is slow or going well. And they know that in some cases their products have to be adapted:

“When school starts again I offer backpacks, and school shoes. And in December I sell clothes, clothes for boys and women.”.

This last issue is an indication that the two phases of starting and running a business cannot always be clearly separated, particularly, sales volume is something to be considered in both phases. The same is true for other aspects discussed in the following section.

**Running a business**

In this section various aspects of financial management are discussed. Financial management includes basic accounting such as keeping records of income and expenditure, calculating profits and deciding their use (mainly reinvestments or savings) or debt management.

One key message imparted at the microcredit workshops is the fact that the women have to know what they spend their money on and that they are required to keep books. However, the interviews revealed that this is one of the areas where theory and practice do not match as only some women kept written records of their sales. Many of those who did, however, found it helpful and, from the following statements, it can be seen that the form in which information is available (in this case written) also has an impact on what is done with this information:

“For example in these things, at the end of the week we take stock to see how much we have spent, how much money we have and how much we will buy.”,
“Because this helps to, so that I control my expenses. This helps me that I don't spend too much. It helps me to plan and know on which days I earn.”.

A careful analysis of spending and income allows calculation of the achieved profits and provides a solid basis for any decisions regarding what to do with the profit. On a general level, all women know that profit is what remains after all expenses have been paid:

“But if I have an employee, I earn nothing. But if I don't have an employee, I get to keep the extra income. Yes, because in the maize business [when making tortillas] you have to buy wood, calcium hydroxide, pay for the milling […] And if you have an employee you have to pay her.”.

When calculating their profit margins, the women not only stress the importance of buying cheap merchandise:

“I buy in Managua, my merchandise. I have to look for the cheapest. If I buy here, it's very expensive. Everything. So I have to go down there in order to be able to sell anything. Because if I buy here, well, what will I earn? One does not earn anything.”.

They are also aware of the impact of intermediary trade:

“Yes, before I bought from her. And there I earned very little, because she was the one earning. And so I bought my own merchandise and it turned out a little cheaper.”.

Once the women have made profits, the key question is how to use them; Saving or Reinvesting? Most women do both, some of them according to specific proportions: “Half [of the profit] for merchandise, half I save.”. Saving money is particularly important regarding the pay back of debts such as the microcredit. In this context, another specific strategy is taught in the workshops, namely that of saving little by little, which is an approach employed by many women:

“We are collecting a share of what we are selling. And every now and then, not daily, but every fortnight, we see how much we have collected and go and buy dollars. Maybe five, one, two or three.”.

While not being asked systematically for it, thirteen out of the nineteen women mentioned this little by little approach spontaneously – an indication that this principle is transmitted more effectively than the bookkeeping requirement. In this context it is worth noting that some women are aware of the risks of fluctuating exchange rates between the Nicaraguan Córdoba and the US dollar: “It’s better little by little, because the dollar could rise.”. Apart from saving little by little to collect the money to pay back their credits, many women have other plans for the money they can save from their businesses, mainly securing their housing 10 or

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9 The women receive and have to pay back their credit in US dollars. While Nicaragua has its own currency, the Nicaraguan Córdoba, dollars are used very widely. At the time of the interviews (June 2008), 100 córdobas were worth 5.2 US$ with the per capita GDP for 2008 being at 1130 US$.

10 The housing situation for the women differs markedly, while some have their own brick house with a connection to the sewage system, running water and electricity, there are others who do not have any of these amenities and live in wooden shanties.
providing for their children. The basic need for security has been expressed by many women. The desire for their businesses to grow can be considered another expression of this need for security, as the following statement illustrates:

“My dreams are to enlarge the shop to have a source of life. Well, above all, for that. Not so much to enrich myself, but rather to have a way of living decently.”

While looking for instruments to help them achieve security, some women also mentioned the possibility of having a savings account at a bank. One woman already possesses an account, which she is using to save money to pay back the credit. And there are other women who would like to have an account in the future.11

The last aspects to be discussed are risk and debt management strategies employed by some of the women. One of the interviewed women sells three different kinds of cosmetics targeted at an upper class clientele. She uses two interesting strategies to secure her income when making a sale, but also when acquiring new customers:

“They have to give me half the money, when I go. When I bring the product, they have to pay me the rest of the money. And if they do not pay the rest, I do not give them the product.”.

She even has a specific strategy for the acquisition of new customers:

“If I do not know this person, then the one who has to vouch for her is you, because I do not know this person, so who will pay for that perfume or that Eau de Cologne? Nobody, so you are the one who has to vouch for it.”.

A less refined way of managing debts is used by some other women who keep lists of people to whom they have given credit: “Yes, in a booklet I keep taking notes: XY owes me that much.”. In doing so, they remarked on different benefits of such lists, both cognitive and social, namely that they ease the load on their working memory and that the people in question cannot challenge the fact that they owe money.

Overall, it can be said that when starting and running their businesses, the women show many of the aspects listed by Lindenskov and Wedege (2001) in the fourth dimension of their numeracy working model. When adding and subtracting sums to calculate their profits, they deal with quantity and numbers; when running their business throughout the year and planning for the products to be offered, they show a sense of patterns and relations; and when keeping and analysing books, they deal with data. As all of these activities related to managing money they can also be considered to be a display of financial literacy.

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11 Poor people rarely make use of formally provided financial services and are not encouraged to do so. The field of microfinance which includes, but is not limited to, microcredits, tries to address this gap between formal institutions and the need of poor people. Traditionally banks have not provided services for poor people for many reasons, two prominent ones being proportionally high transaction costs and the lack of securities of the poor.
Other issues

As has been seen in the previous two sections, there are indications that educational intentions of the workshops associated with the microcredit achieve differing levels of success. It has to be stressed that no systematic inquiry of this issue was made in the frame of the initial interviews, therefore, caution has to be exercised with the generalisation of the following statements. However, they are considered to be observations worthwhile of further inquiry.

In the overall context of poverty reduction efforts, specific training is often suggested to strengthen poor people’s assets. In this sense Lekoko and Garegae (2006), and many others, recommend that basic book-keeping skills are taught to street vendors. However, there are indications from the interviewed women that teaching particular skills, without taking into account the learners' previous experiences, is not always successful:

“But in the workshop we haven't learned it like this. […] You know, in the workshop we made a frame like, like this [draws while talking] with a line here […] And in this block here we put the quantity, mnh, the price. No, it seems it was here […] It's really nice what is in the booklet. What is happening now is that I don't remember well, how, how we put it.”.

Furthermore, there were a number of women who spoke in a somewhat distant manner about keeping books (“They have told us to do it.”), and a few of the group who just received the first tranche of their credit had not started keeping records for some reason, but often promised they would start soon. Such statements can be seen as indications, that the imparted skills might not be considered useful or do not integrate into the women's knowledge and are therefore not applied – by no means a new insight (see for example Archer and Jeng (2006) or Barton and Papen (2005))!

Conclusions

The previous statements have clearly shown that the interviewed women possess a variety of financial literacy skills. Qualitative research has an important role to play when it comes to a good understand and to specific descriptions of the characteristics of such skills and of the ways they are achieved. Studies like Nabi, Rogers and Street’s (2009), describing seven individuals with little or no formal education from Pakistan in their everyday literacy (and numeracy) practices, contribute to an in-depth understanding of why learning is not linear and methodologies based on such a linearity and ignoring previously acquired skills, cannot be successful. A one-size-fits-all approach adopted by many interventions – such as the workshops delivered to the interviewed microcredit clients – is hardly adequate to strengthen an individual’s (financial) literacy assets. These women possess not only very diverse types of knowledge, they also have different levels of abstraction when talking about their knowledge – a fact which needs to be taken into account when planning for their further education.

Providing targeted educational content to such a specific group of people, requires an in-depth knowledge of their needs in order to plan and deliver adequate educational measures. Considering context is a fundamental and frequently discussed term in the numeracy debate, it
is important for an increasing number of studies such as the one presented in this paper or Lekoko and Garegae (2006) or Gahamanyi (2010) which are not conducted “in the western hemisphere and developed nations of the world” (Naresh 2008, p. 9) to find their way into the fora of relevant organisations in exactly those nations in order to broaden various stakeholders’ understanding of contexts.
References


This paper discusses some challenges faced when teaching mathematics to adults in the U.K. and some research that is being undertaken in response to one of these challenges. In the paper, approaches used for teaching and learning mathematics to adults in contexts, often known as numeracy, are discussed. They include notions of transference of skills between contexts, some research background and analysis of the topic. This uses a model developed while teacher training, that aims to investigate the complexity of teaching mathematics in context, and to highlight the tensions between learning relevant mathematics skills in the workplace and those in education contexts. It also briefly explores assumptions made when teaching Functional Mathematics in relation to transferability and motivation.

The second half of the paper discusses research that is being undertaken with learners of mathematics or numeracy in the workplace, members of a trade union, in relation to the notions of motivation and skills transference.

The challenge is teaching and learning mathematics with individuals, who are not motivated by the ‘beauty of mathematical patterns’, but who are more motivated to understand the mathematical ideas underpinning work, vocational studies or life skills they were interested in. During my years of teacher training, I have tried to understand what makes this teaching of mathematics in a vocational context so demanding, especially for teachers of mathematics.

Socio-cultural researchers (Lave, 1997) (Wenger, 1988), contend that mathematical ideas, in all contexts including education, are deeply embedded in the context of a social situation. Hence, teaching and learning mathematics within a context requires knowledge of the culture, language and symbols used in the social context. This often requires talking to vocational teachers or people using mathematics in the workplace, learning what Vygotsky called the ‘semiotic mechanisms’ of language, tools and diagrams used for the job or vocational training (John-Steiner & Mahn, 1996). This ability, to identify key ‘semiotic mechanisms’ in social contexts and link them to mathematical concepts, requires abilities outside the experience of regular mathematics teachers, but something that teachers of numeracy, and teachers who work with adults to develop their mathematics knowledge, regularly work towards.

Part of this process of investigation has led me to develop a model of analysis that tries to illustrate this challenge visually. The model works with two contexts of learning mathematics i.e. education and the workplace on the horizontal axis, and the approaches used to teach or learn the mathematical concepts on the vertical axis. The different approaches and skills used to learn abstract mathematical ideas in a classroom is identified as separate to the development of mathematical concepts to solve problems in vocational contexts. Using a Vygotskian understanding of mental functioning, the knowledge co-constructed in the vocational context...
uses the language and tools of that context, and this will be distinctive from the knowledge constructed in the classroom.

Distinctive but still related. Currently, learning mathematics in the vocational context in the U.K. education system can be described as the identification of the key mathematical concepts to another context, or the identification in one vocational context and applying it to another. This notion of application or transference of concepts has been described as a ‘complex process of transmission, transformation, and synthesis in the co-construction of knowledge’ (John-Steiner & Mahn, 1996). Indeed, Vygotsky refers to the ability ‘to “decontextualise” and then “recontextualise” signs, as tools in action as a form of mastery that constitutes higher mental functioning’ (Penuel & Wertsch, 1995). Nevertheless, the Functional Mathematics approach to teaching being introduced across the U.K. requires teachers and learners to be able to develop these skills. The criteria state that:

‘Assessment must focus on functionality and the effective application of process skills in purposeful contexts and scenarios that reflect real-life situations’ (Ofqual, 2009).

Even though the concept of transference remains contentious, the assumption is that learners will be motivated to learn in a context relevant to their studies and then develop the ability to transfer the mathematical skills into different contexts; this actually underpins the Functional Skills teaching approach.

Indeed, during my pilot research on mathematics in the workplace, the first interviewee described a notion which sounded like transferability of mathematics between contexts when he spoke of ‘…. maths at work being built on ideas recalled from those learned at school. … area = length x breadth, is used to solve problems at work.’ He spoke about learning ‘Π at school’ which now helps ‘to go onto to make cogs in engines’. Another example of using Π was ‘producing a Pitch Circle Diameter (PCD) for drilling holes on a wheel to fit onto studs’.

So there is an awareness of learning at school that is ‘transferred’ or ‘applied’ to another context e.g. the workplace.

**Functional Mathematics and Transferability**

This notion of transferability or ‘extrapolation to another context’ (Dept of Education, 2012), is included in the Functional Mathematics now being proposed in the U.K. curriculum. The definition of Functional Skills is “those core elements of English, Mathematics and ICT that provide individuals with the skills and abilities they need to operate confidently, effectively and independently in life, their communities and work”.

The underpinning approach to assessment of Functional Mathematics is that mathematical skills can to be tested in any context, so someone learning ratios in a catering context may be tested on these skills in a health context, thus testing the notion of ‘functionality’ or ‘transferability’. This approach is partly to answer the criticism of employers and the CBI that young people coming straight from school and college do not have good mathematical skills (CBI, 2011)(DIUS, 2007) (Leitch, S., 2006) (Wolf, 2011).
Exploring the ideas of transferability, extrapolation or functionality, the Venn diagram in Figure 1, illustrates mathematics with functionality as being situated somewhere between the ‘esoteric’ mathematics, that is taught as a distinct subject in the classroom (Stech, 2008), and the particular mathematical ideas that are used to solve problems in the workplace. The ‘intersection’ represents the notion of numeracy or the common mathematical skills that are needed in vocational subjects but also taught as a topic in mathematics. For example, time is used in hairdressing when booking appointments, millimetres are used in motor vehicle maintenance when using a micrometer to measure disc brakes, the concept of positive and negative are used in tailoring when adjusting a pattern to a larger or smaller size.

The approach to teaching being promoted is that identifying and developing the mathematical skills in a vocational context the learner is interested in, will then help them ‘transfer’ the identified skill to another context, making them more ‘functional’. Another challenge for teachers is that not every vocational area covers the curriculum which is tested in Functional Mathematics, so this approach can only be used with certain topics in certain vocational contexts.

Further analysis of the teaching and learning skills needed to teach numeracy, or mathematics in context, The spreadsheet in Figure 2, was used to adjust the ingredients in the Cake Bake Company in 2009.
This spreadsheet is used in the workplace to solve problems in production and could also be used in an education context to develop understanding of formulae and ratios within the spreadsheet. The model of analysis shown in Figure 3 (Kelly, 2010), distinguishes the learning approaches on the vertical axes into two categories viz., developing discrete numeracy (mathematical) concepts and problem solving while identifying contexts of learning on the horizontal axis i.e. education and the workplace.

Figure 3. A model of analysis for learning numeracy in Context

Utilising the model to analyse the skills needed in the Cake Bake spreadsheet, the formulae in the spreadsheet used in the workplace is situated in the top right hand quadrant of Figure 4, using mathematics skills solving production problems in the workplace.

Figure 4. Model of analysis for teaching and learning numeracy in different contexts- employers’ perspective.

The ‘back of the envelope’ calculations for a single cake at work uses mathematical concepts located in the bottom right hand quadrant. Developing the spreadsheet at work helps to make sense of the larger problem of cake production, but requires an understanding of the use of mathematical formulae underpinned by basic numeracy concepts. The multiplication, addition and division calculations become part of the process for problem solving will be developed in the bottom left hand quadrant, in an education context developing discrete mathematical concepts.

The decision-making process in the workplace is useful to understand in its complexity. Often workplace scenarios are deemed as simplistic or are stripped of their complexity to concentrate on the numeracy skills, thus loosing the context. One of the challenges of
supporting learning in different contexts is recognising when to develop the numeracy concepts and when to move into the more complex contexts to recognise when calculations can inform a broader decision-making process. The teaching and learning approach in Functional Maths also relies on this problem solving approach. Thus, ‘Functional Skills’ appears situated above the horizontal axis, using ‘Problem Solving’, whether within the workplace, vocational or mathematical subjects.

Figure 5. Model of analysis for teaching and learning numeracy in different contexts – Learning Approaches.

One of the challenges of relying on this approach is that learners, who do not have the understanding of the discrete mathematical topics that underpin the problem, still need to be given time to understand the concepts. For example, the spreadsheet uses formulae and some learners may need time needs to recap formulae before applying that concept to the problem. The particular skills development needs of the learner can put a lot of strain on vocational teacher or assessors, who are not given the time to develop their learners’ skills, and may not have a good understanding of these concepts themselves.

Another example of a vocational context and the analysis of mathematical skills and teaching and learning approaches used, was developed with a group of trainee numeracy teachers in 2010. The question investigated was, ‘How many bricks does it take to build a wall 1m$^2$ in the U.K.? Bricks measure 215mm x 102.5mm x 65mm the space between the bricks is 10 mm of mortar.

Figure 6. Model of analysis for learning numeracy in different contexts – Construction.

Initially, teachers and students work in the top right hand quadrant exploring the problem in an education environment. However, in order to solve the problem, learners must understand
metric measurement systems and concepts of volume, so some time may be spent recapping those ideas, essentially working in the lower right hand quadrant, developing discrete mathematical concepts.

In construction work in the U.K. (in the lower left hand side other the model), calculations are all carried out in mm, so there is no need to know about metres. Also, in the U.K. it is known that it takes 60 bricks to build a metre square wall, so the calculation is unnecessary. Interestingly in India it takes 54 bricks to build a metre square wall, so the question is culturally specific in terms of workplace and country environment.

The analysis points to the need for teachers and learners to be able move in and out of contexts to utilize mathematical ideas fully, again, perhaps supporting the notion of transferability. However, it also points to the complexity of teaching and learning within that context and the skills needed by teachers to understand different contexts such as the workplace, as well as the mathematical skills and knowledge.

The model helps illustrate the tension in developing ‘real’ mathematics problems with an education context. The reality of this question is that, although it tests maths skills in an education context, these calculations are not relevant and not needed in the workplace as the answer is already known. This creates a tension for assessors and trainers who work in construction, as the learners on an apprenticeship programme, who may have to achieve functional mathematics for their main qualification, may not see the relevance of learning the underpinning mathematics skills of a notion that is an ‘industry standard’. This is also often the attitude of the employer.

This section explored some of the main issues facing teachers, trainers and assessors in developing a functional approach to mathematics. The next section discusses research that has developed from working with these tensions over a number of years.

**Researching motivation and transferability applied to learning mathematics at work.**

So a Functional Skills approach to teaching mathematics is underpinned by the assumptions that: a) learning mathematics that is relevant to work, or life, will enable people to be more confident and effective in their lives,  b) transferability of mathematical concepts between contexts is possible and c) teaching this way will be of value to the individual, community of practice and society.

These assumptions are linked to notions of motivation; that teaching mathematics will be more motivating if linked to real life contexts. As a teacher of mathematics in further education for over 20 years, many of these years spent developing mathematics in context, and as a teacher trainer for over 10 years, I have been interested in these notions of context, motivation and transferability. As a consequence I am undertaking research into the development of knowledge and skills when using or learning mathematics at work in the U.K.

by focusing on the individual learners’ notions of value and their own understanding of the development of mathematical concepts, and by exploring the learners’ own mathematical journey, their motivations for learning considering the links to identity and empowerment, while also discovering any connections they make between the transferability of mathematical concepts between education and work. The individuals in the research will be union members, who will be learning maths while at work.

This research will have an epistemological aim, in that it will seek to contribute to the body of knowledge on how teachers may better understand the development of adults mathematical skills connected to notions of motivation and transferability of maths skills in different contexts.

The notion of transferability remains contentious. Some researchers consider the notion of transferability as limited by the uniqueness of each context of learning and the particular
history, social and cultural contexts changing the sense of the mathematics in those different contexts. (Lave, 1997) (Wenger, 1988). Other researchers question the notion of transferability due to the ‘invisibility’ of mathematics in real contexts (Coben, 2003) (Wedege, 2000).

Another group of researchers suggest the possibility of transferability describing how the psychological impact of learning mathematics in schools develops a particular way of thinking that is transferable (Stech, 2008), while others believe mathematics to be ‘the most universal mode of thought’ (D’Ambrosio, 2007) that underpins whole cultures and their perspectives. Reflecting on the pilot research, I have found that interviewees make links, either positive or negative, to their school or college mathematics learning experiences and so identify some notion of transference.

The research questions will be focused on What motivates adults to learn maths at work? This reflection will seek to identify the value to the individuals of mathematics used or learned at work. Exploring the individuals’ learning journey will entail a discussion on the motivations to learn connected to notions of identity and empowerment.

Does union membership in any way influence learning maths at work? As the target group is union members, this question seeks to identify any relationship between union membership and learning, again linked to notions of empowerment and identity. Union activism can be seen as a group activity, learning can be seen as an individual activity. Do the two viewpoints create a tension in this situation, or is there complementarity?

How do adults prefer to learn or use maths at work? This question seeks to use learners’ own notions of success by identifying connections to, or transference of, mathematics skills used at work and concepts learned in an educational context.

**Methodology**

As the researcher, I take a critical realist perspective, recognising that value is a socially constructed concept linked to power, knowledge and individual experience. This perspective is informed by my career as a lecturer, who has been developing numeracy skills and knowledge in Further and Higher Education with students and teachers since 1981.

This work will be underpinned by my belief that education, at whatever level, is valuable if it is linked to the empowerment of the learner and enables them to live a fuller life that contributes to the betterment of themselves, their environment and society. The work is therefore underpinned by an ethical and a political dimension of mathematics education that reflects the ideas of D’Ambrosio when he states that ‘Mathematicians and math educators must accept, as priority, the pursuit of a civilisation that recognises notions of equity, value in diversity, solidarity with, and co-operation with the other (the different) in maintaining quality of life and dignity for all’ (D’Ambrosio, 2007).

The research will be based on the Vygotskian proposition that ‘all human mental functioning is socioculturally, historically and institutionally situated’ (Penuel and Wertsch). As such, the research will take into account the context of the ‘use or learning’ of mathematics, union membership, the workplace, the learners’ own sense of self and place in the community, the learners’ personal understanding of mathematics, their own skills and knowledge, motivations for learning, empowerment and impact on their lives. As the researcher, I will seek to identify motivations linked to notions of empowerment as described by the interviewees, while trying not to influence them by my own understanding of identity and empowerment.
Research methods

The empirical field of research will be members of a trade union within the workplace. This group is chosen as they are a relatively new group of learners since learning mathematics through trade union systems developed relatively recently through the development of the Skills Pledge and Skills for Life strategy (Union Learn, 2007).

The samples of people to be interviewed will, in some respects, be representative in that they will be members of a union, however, the choice of the workplaces will be initially referred by the Equalities and Diversity officer in the Education Department for Unite the Union, and then through local union members, hence the sample can be defined as a ‘snowball sample’ (Dowling, P. & Brown, A., 2010).

The Equality and Diversity officer has offered to facilitate the identification of potential interviewees. I will also utilise her offer to ensure a balance in the number of males and females who are interviewed. In the initial phase I will seek a balance of males and females and will utilise the support of the union officer, and other regional education officers who have offered to help, to identify sites where I can find workers who fit into the required criteria. I may also need to identify key union education officers or ULRs within the organisations, who can support the distribution of brief information sheets on the research, and, possibly, questionnaires in phase two, to help me identify volunteers who would be prepared to be interviewed. I will follow the ethical guidelines established for this project through the Ethics Committee at LSBU.

In-depth interviews will be undertaken. In phase one, I am building on the learning from the pilot interviews that I have already undertaken to inform the questionnaires and the interview processes. These interviews will also contain a discussion of the learners’ mathematical journey, using this as a vehicle to identify the key motivations and trigger points for learning mathematics at work and for identifying the effect on the learners’ lives. I will seek to use this method to highlight the historical, cultural, social and personal aspects of learning mathematics experienced by the interviewees, by analysing the responses in relation to research literature with respect to identity and motivation.

The table below shows some predicted variables to be discussed in the interviews, links to the relevant questions and identifies the range of possible connections identified through the pilot research:
Table 1. Predicted variables discussed in Interviews

<table>
<thead>
<tr>
<th>Links to questions: How or why learn /use maths?</th>
<th>Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why? (1, 2)</td>
<td>Motivations/ triggers to learning /using</td>
<td>Work, personal, family</td>
</tr>
<tr>
<td>Why? (1, 2)</td>
<td>Impact of learning</td>
<td>Description of change in learner’s behaviour</td>
</tr>
<tr>
<td>How? (3)</td>
<td>Learners’ reflections on relationship between learning in educational establishment and at work.</td>
<td>School, college experiences</td>
</tr>
<tr>
<td>How? (3)</td>
<td>Use or learning of mathematics Transferability</td>
<td>Problem or project; mathematics skills or knowledge identified; approaches used</td>
</tr>
<tr>
<td>How? (2, 3)</td>
<td>Context of learning</td>
<td>Workplace or community</td>
</tr>
<tr>
<td>Why? (1, 2)</td>
<td>Gender</td>
<td>Male, female</td>
</tr>
<tr>
<td>How and why? (2)</td>
<td>Union membership</td>
<td>Importance of membership in relation to learning</td>
</tr>
</tbody>
</table>

The process will be repeated in the second phase to ensure a range of interviewees and saturation of data.

Conclusion

This research is motivated by a life of teaching mathematical skills to adults through vocational courses in different contexts. The development of Functional Skills as the latest approach advocated in the U.K. to develop some learners’ mathematics skills, provides a useful backdrop to the research since it relies upon the notions of motivation through relevant contexts and the ability to transfer mathematical concepts between different life contexts. The research will further investigate the motivations of adults to learn mathematics while at work and explore notions of transferability by understanding what learners themselves see as skills they are using in a work context that they might have developed elsewhere. It will also develop these ideas in a Union context as all of the learners are in work but engaged in learning through Union support. Early research shows this context will increase the range of motivations for learning.
References


This article outlines a research project into maths teaching and learning in a prison education centre. It describes the research plan, tools, methodology and setting and then describes the progress in the first stage of the PhD, entitled ‘Facilitating Mathematics Teaching and Learning within the Irish Prison Education Service: A Model for Future Development’. The aim of the research is to identify the factors that motivate individuals to engage in mathematics learning in Irish prison education and the factors that enable them to continue to participate. The next stage of the research will continue to investigate the mathematics learning and teaching taking place and will reflect on the effects of the programme on both teacher and prisoner. It will reflect on how mathematics learning affects the learners’ view of themselves.

Key words: mathematics; prison; learning, crime, education.

Research into education in prison is rare as most research in the area covers other aspects of crime and prison (Costello 2003) and insider research (Rooney 2008) is less common. In 2010 the author started a PhD, having taught for thirty years in prisons and adult education, and is currently part of an education service to Dublin prisons, that offers education at basic, examination and degree levels. This last thirty years was a time of great change in both penal and adult education systems. In 1981, adult education advocacy was developing in Ireland (Aontas 2011) and a national literacy agency (NALA) was established in 1976 to deal with literacy and numeracy for adults. The Whittaker Report, initiated in the early 1980’s and published in 1985, advocated prison reform and its recommendations are still debated (2011). Prisoner numbers were less than half of what they are now (Irish Prison Service 2011) and most of the new prisons in Ireland had not been built. Formal training for prison education was not offered to new teachers; instead teachers learnt informally from others, both prison and teaching staff. In recent years more opportunities have developed at third level to train in adult education, but not specifically in prison education (WIT, NUI Maynooth).

This research will consist of four distinct stages. In the first stage, i.e. the exploratory stage, the students and their environment will be described, including their learning histories, motivation, psychology and habitus of prison education. This will lead to insights which will inform the next stage, that of the action research cycle. Here the implementation of good practice will promote student engagement in learning mathematics. The cycle of students and peer and teacher reflection of planning, acting, observing and reflecting will lead to improved approaches. The next phase will consist of repeated action research cycles where reflection is followed by planning and action, then followed by observation and further reflection which continues the repeated action research cycle. The aim is ultimately to develop a model of good
practice to engage and support students in mathematics teaching and learning in prison education. The research questions are:

- How can mathematics ability and mathematics learning be monitored effectively in Irish prison education?
- What pedagogical resources enable mathematics learning in this setting?
- Can mathematics learning promote prisoner self efficacy?

As the author had trained initially for mainstream education, adapting to a different setting became part of teaching practice. After many years teaching different subjects (English, Mathematics, Literacy, Numeracy, Citizenship, Personal Development and Effectiveness) in such an area of innovation, reflection is inevitable. Critical thinking (Seabright 2006) involves questioning of assumptions and moving between areas that are emotional and rational. In prisons there is often a sense of changing of roles, relationships and identities in order to facilitate the smooth flow of institutional life and this has been outlined by many prison ethnographers (Bhatti 20, Irwin 2008 and Wilson 2010 and Twiss 2008). Observers of prison education have used terms such as “miraculous metamorphoses” (Nielsen 2011) and “borderland” (Wright 2005) to describe the changing dynamics and uncertainty. This research aims to describe the habitus of the classroom in such a setting and reflect on how learning takes place in an out of field (mainstream school) setting by fitting the culture of the prison. The reflection process, in which the author engages, is similar to cognitive dissonance (Festinger 1956), or cognitive disequilibrium (Piaget 1952), where a discrepancy is felt between the everyday reality and that which is already known or believed. The individual then has to reflect on how best to implement changes to ease the dissonance.

**Teachers**

Teachers in prisons in Ireland are recruited from the secondary school sector and their qualification must be in the subject they are teaching (Teaching Council 2011). Generally, they have a pedagogical qualification also which is a generic training course designed for secondary schools. While there is debate on the necessary qualification for teaching adults in prison, there is broad agreement that one needs to know one’s subject and have the skills to transfer this (Delaney 2010). In adult and prison education, a further skill needed is the ability to respect and understand the learners (Twiss 2008). Training in basic skills e.g. literacy and numeracy, varies and is not mandatory for teaching in prisons as the teachers are appointed to the scheme by the local Vocational Education Committee, and the criteria used is the same generally as for mainstream teachers. Mathematics teachers’ qualifications are under scrutiny currently in Ireland, as due to a shortage of teachers of mathematics in Ireland, many are teaching “out of field” (Ni Riordáin 2010, Flynn 2011).

There is no single training course for the extra skills that have been noted to be of benefit as a teacher of basic skills or in prison. These include as the ability to stay present, to have empathy and stay professional. Other personal characteristics that have been listed as useful include authenticity, hardness, enthusiasm, assertiveness, and an appropriate communication style (Twiss 2008, Marquard 2011). The Aristotelian concept of “phronesis” or the ability “to do the right thing, in the right manner, at the right moment” is fitting here (Marquard 2011). Prison teachers can have a degree of discomfort in discussing with mainstream colleagues the nature of their work (Irwin 2008, Twiss 2008) and can at times feel an interdependence with their students, leading to feelings of being equally marginalised. Concerns expressed by prison teachers for their students’ education, learning difficulties and dignity are voiced at
times against a culture of obedience and acquiescence demanded by the prison regime (Bhatti 2010).

The Setting

Cloverhill is a relatively new remand prison in the western suburbs of Dublin. Numbers on remand i.e. awaiting trial, have grown in recent years in Ireland due to changes in legislation. Remand prisoners in the past had been kept in other prisons but, due to the rise in numbers, a special remand prison was deemed necessary. Prisoners are remanded and denied bail due to the nature of the offences, the danger of reoffending, of not appearing in court, or breaking bail conditions. (Freeman 2008). Findings in research conducted into young people on remand in Ireland (Freeman 2008) shows similarities to remand prisoners of all ages. The stresses associated with remand imprisonment include the loss of freedom, uncertainty about court dates and outcomes, and loss of the presumption of innocence while imprisoned. Statistics in Ireland (IPS 2004, 2005) state that the number of people on remand is almost equal to those sentenced, which matches statistics abroad (Freeman 2007, Hodgkin 2002).

Education in a remand prison can be more complex than in sentenced prisons due to the possibility of transfer (between cells and other prisons), numbers of inmates per cell and the mental instability of being in a “remand state of mind” (Wilson 2010). However, informal feedback from the agencies within the prison suggests that it has benefits. Overall, while education in a remand prison may be more difficult for those who wish to direct their own learning, the option of face to face education offers an opportunity to make decisions, be something other than a prisoner and exercise control over books and teaching materials for example (Wilson 2010).

There was a delayed opening of the Cloverhill school and several interested bodies involved have since maintained strong connections, giving progress reports and updates in their annual reports (Prison Chaplains 2006-2009, Cloverhill Prison Visiting Committee 2005-2009). Prison education in Ireland has been available to all prisoners for almost forty years in its present format, yet was offered to prisoners and those awaiting transportation as long ago as the 18th century. Activities available to prisoners are more limited in remand prisons than to those in many sentenced prisons and include school, part time work at cleaning, laundry, kitchen or the prison yard. Integrated sentence management is being implemented in other Irish prisons for sentenced prisoners but not yet in Cloverhill for the sentenced prisoners there (IPS 2011). Teachers offer a range of subjects from basic education level upwards.

Learner Profiles

Traditionally the profile of prisoners in Ireland (O’Mahoney 1997, Seymour et al 2005) and internationally (http://bjs.ojp.usdoj.gov/index.cfm?ty=pbdetail&iid=1118 ) has included some of the following traits: early school leaving, violence, abuse, trauma, family and/or community history of alcohol and/or drug abuse, (Muth 2005), mental illness (O’Neill 2011), family changes or bereavement, special learning needs, (Murphy et al 2000) and membership of the Travelling Community (Linehan 2003). Those represented in prisons are mostly from poor backgrounds but even though crime is committed by all classes, white collar crime is treated differently (O’Donnell 1997). Currently 20-25% of prisoners in Cloverhill are foreign nationals (Visiting committee 2009) many of whom are ESOL students. Some who are imprisoned do not fit the traditional model of a prisoner, and have committed a crime once and may never reoffend but most are the early school dropouts, the neglected children (Finlay
2011). All share the reality of being in remanded in custody and deprived of their freedom. Most are resilient individuals with immense skills learnt through everyday life e.g. working, homelessness, relationships, sport etc., which remain invisible and often, with scaffolding, (Vygotsky 1986) quickly these skills can become visible.

**Prison Education Strategy Statement**

The strategy statement for Irish prison education (Warner 2001) states that the aim of the prison education service is to provide a high quality, broad and flexible programme of education that meets the needs of those in custody through helping them: to cope with their sentence, to achieve personal development, to prepare for life after release, and establish the appetite and capacity for lifelong learning. Within that strategy statement, there is scope to develop mathematics education to meet the needs of those in custody, by helping them to cope with their sentence by alleviating boredom, offering mental challenges to keep sharp, to achieve personal development by learning skills and accrediting those already learnt, and to prepare for life outside by gaining qualifications and by developing maths eyes to critique economic and citizenship issues in a more adult way, and by offering information on further education and training in related fields.

National Accreditation is available for those attending mathematics classes in prison education centres in Ireland. The choice is to follow either uncredited courses or those accredited by FETAC and the Department of Education (Holland 2011). Students can choose between State Examinations, Junior Certificate and Leaving Certificate, (school) and FETAC accreditation options. Both have equal status for further education and for training. In Ireland the annual examinations create a great deal of interest in the media, and many prisoners see this as a higher status than FETAC which is completed locally and quietly. A major motivation to sit examinations is the wish of their parents, children or partners for them to do so, and it is a form of rehabilitation, along the lines of the story telling project e.g. “Story Book Dads” (IPRT 2011).

Education in prisons in Ireland is available to those serving sentences and on remand for any length of sentence (Chaplains Reports 2009) Probation and Welfare, unlike many other services which only engage with those who serve a minimum sentence (Guardian, P&W, IPRT). Thus prisoners that present for education can be just recently admitted, or can be in prison for a long time and may have been waiting for an opportune time to start or may have required assistance such as advocacy (DIT). In the experience of the author, the primary concern of the teacher for those starting school is to engage them at the point of entry and help them to find a goal to encourage them to return for another day. The reasons that people in prison join education classes are many. Research from Costello (2003) looked at factors that led prisoners to engage in third level study in a Dublin prison and the author used this as a benchmark to assess the motivation for those attending in this prison. Reasons included for family, personal development, for court, for social reasons (DIT), it can be humour, (EPEA and metamorphoses) and the term “smuggle” (HMP) and “stealth” (McElligot 2007).

The basic education standard of most prisoners is low in most European countries, including Bulgaria, UK, Slovenia and Ireland. (http://ec.europa.eu/education/grundtvig/doc/conf11/ghk_en.pdf ). The author in this study is reflecting on the teaching of mathematics at all levels from basic through to intermediate and advanced. Research has been limited in this subject area although Irish research (Kett 2003) shows levels of prisoners with literacy and numeracy of approximately 50%.
Methodology

The proposal for this research had first to be submitted to the Research and Ethics Committee of the Irish Prison Service for approval. This required a description of the research, the rationale and the inclusion of all worksheets and evaluation sheets and possible questions to be included in interviews and consent sheets for all participants. All interviews that were taped had to be destroyed and a copy of the research was to be made available. These procedures are entirely appropriate as the participants are vulnerable people and the procedure is designed to protect them. After approval was granted, the research started in autumn 2010 when the integration of the reflective sheets into mathematics class was introduced. The rationale for the sheets was based on research nationally and internationally into adults returning to basic skills in community settings (Safford 2008, Lindberg 2005) and prison education research (Costello 2003, O’Higgins 2002). The author wrote a reflective journal also to monitor the effectiveness of the classes and the thoughts, observations and feelings during and after.

The research plan was outlined by the author to the prisoners as part of the process of making a personal learning plan: (http://www.probation.ie/pws/websitepublishing.nsf/attachmentsbytitle/NESF+Re-integration+of+Prisoners/$FILE/NESF+Re-integration+of+Prisoners.pdf), and a reflective journal, both of which are components of a course on “Personal Effectiveness and Mathematics Learning”, which could eventually be submitted for accreditation. This would build up to a greater understanding of their own learning styles and intelligences and would help them and the teacher to teach in the style that best suited them at first (Felder 1988). The assessment sheets were drafted after research by the author from sources such as Adults Learning Mathematics archives and researchers who have compiled data the benefits of developing a learner’s mathematics history from research on adults and mathematics education (Safford 2008). A maths learning history graph (Lindberg 2005) was adapted to introduce the idea of trend graphs and co-ordinate geometry and positive and negative numbers while reflecting prisoners’ good and bad memories of learning mathematics. An example was shown and the sample answers were discussed, and afterwards, each person in the class did their own graph. The turning points were provocative in discussions as they showed key events that changed a trend (Lindberg 2005). The original was drawn in Geogebra and the prisoner was encouraged to draw it by hand first and then to draw it on Geogebra also as a project for his Mathematics folder. A reflective diary was designed for use by prisoners to fill in at regular intervals at the end of class, with the option of a group or individual feedback or both. This was to enable reflection on their learning styles of doing, feeling, watching and thinking (Kolb 2006) in order to encourage thinking about how they are doing things and how they are thinking about what they do in this setting.

A key objective was to capture in this research was what it was that motivated prisoners to return to school in prison in the first place and then to continue to engage with it, as motivation is key in the rehabilitation and re-integration of prisoners (NESC). The method used was a radial chart with “motivation” at the centre and possible factors on the extreme points. The factors were based on research in Irish prisoner engaging in third level study in a Dublin prison (Costello 2003). The sample is shown and then they fill in the blank version. Discussions can follow on the actual diagram, the name of the shapes, the radials, the size of the angles and the size of the angles if more or less radials were there, the unit of measurement to measure angles and the total degrees in a rotation. Some other motivating
factors can be added which are specific to personalities or cultures in the class. The key learning is that the mathematical graph is being used to demonstrate real life data, based on a topic of interest. The learning of the concepts is embedded at the same time as the reflection process is beginning, all of which fits in well with Project Maths as mathematics learning occurs best when it is connected to real life experiences and activities (Project Maths 2011).

The second phase of the research i.e. the interviews, was scheduled to commence later in the year to allow time to reflect on the implementation of the first stage of the plan. This would provide a distance between the author’s work as a teacher and as a researcher. They were to be conducted in a semi-structured format outside of the author’s normal teaching schedule. Qualitative open ended interviews suit a prison environment as conversation is a critical in prison, and such interviews yield rich data for prisoners on their histories (Muth 2005). This format would allow them to express their strengths and their needs and thus facilitate the teacher to develop an holistic view of the learner. Adult basic skills learners may have little experience of learning in a formal school setting but have proved adept at learning in other situations (Smilkstein 2003 in Beebe 2006), and the interview process aimed to capture that. The questions listed (Figure 4) cover all possible topics and are based on question used in other prison research (Costello 2003). They offer a starting point although it is unlikely that such prompts would be needed as prisoners generally enjoy talking and so qualitative interviews (Muth 2005) offer an opportunity for expression. The lack of discussion opportunities in prisons for the inmates to engage in “intellectual talking” (Wilson 2010) and to use new words that they are unsure of pronouncing, means that the chance to engage in open-ended interviews is welcomed. One result that emerged, and is echoed by Wilson’s research, is the number of prisoners who are passionate about higher level maths and who wish to work on it alone in their cells. The option of single cells in Irish prisons is dependent on an inmate having a part time job; so many dedicated learners opt for part time school so they can work part time and have a cell to study alone in at night. Higher level learning needs to be offered as a matter of course, but not at the expense of literacy and numeracy provision, which is the priority in prison education (IPS 2004) and in mainstream schools, in order to achieve social justice and equity in Ireland (Quinn 2011).
Figure 1: Maths Learning History Graph Sample

Figure 2. Mathematics Learning History Graph.
Sample Motivators: Family, Personal Development, Court, Qualification, Work, Use time constructively, Achievement, Boredom, New Interest

Figure 3. Mathematics Motivation
1. How long have you been studying Maths?
2. What Maths course are you studying this year?
3. How is it going?
4. Are you glad you started?
5. Could you summarise your motivations for doing Maths while in prison?
6. Did you do study any Maths before coming to prison?
7. Why you are studying this particular Maths course?
8. Are you working towards any qualifications in Maths?
9. Do you intend to continue with your Maths studies on your release?
10. Do you think your Maths study will stand to you on your release?
11. At the moment, how important are your Maths studies to you?
12. What do you see as the benefits of following a Maths course while serving a prison sentence?
13. Is there anything, other than the qualifications, to be gained from studying Maths while in prison?
14. Do you think you would have undertaken any Maths study if you had never come to prison?
15. Do you think there’s a particular type of prisoner that undertakes Maths study or would everyone do it if they could?
16. Do you think they might differ from those of students on the outside?
17. Is there anything else you want to tell me or think I should know?

**Figure 4. Questions and question order generally used in interviews / discussions during class.**

**The Findings**

The primary finding was that the research was welcomed by both prisoners and prison staff, easing the research at the start. Many prisoners had been interviewed at various stages in their lives, particularly those who were not first time offenders (Seymour 2008, Costello 2003, O’Higgins 2002, O‘Mahoney 1997, Hassi 2008, Holland 2011, Chaplains 2005-09, Visiting Committees 2004-2010). Research into prisons is not unusual and visiting committees and
other concerned bodies have accessed prison regularly to ensure standards are maintained and in the process interview inmates.

Initial results recovered from the learning journals were descriptive of the activity completed that day, such as fractions, measuring and offered little reflection. Some said that they now realised that they were good at mathematics or that they now liked it, but this insight was generally expressed verbally on the way out of the classroom door. As a result, the author started to do a short “vox pop” each class in the last five minutes to see what people had learnt that day, how they saw themselves as mathematics learners and what they would like to start with the next day. The author also recorded personal reflections and integrated this learning into subsequent classes. What emerged through this activity was that the self image of the prisoners was changing and becoming more positive, expressing pride in their skills and insights. Part of this process was that they could state now that they did not like something that had occurred in class, such as the way the teacher explained something or the way some other learners behaved. In the past this was rarely expressed, instead people just left the class. This led to a rise in self esteem as the dominant narrative (Fleming 2003) was being rewritten by a counter narrative (Terry 2010).

Neural science offers an explanation for this (NRC 2000) as learning changes the nature of the brain it organises and reorganises it. As different parts of the brain are developed at different stages of development, brain development happens over time. The initial data from this research shows that the self-image of the prisoner as a learner on the point of re-entry to education may not have changed much since he was a school boy, while in fact his life experiences in the meantime equated to learning and this causes new synaptic connections in his brain. While this growth is not in awareness in the initial period after entry to the class, it appears to emerge after a time in a learning environment that is not competitive but intellectually stimulating, and is expressed by reflection.

The descriptions of how a prisoner feels while in class and doing activities such as examinations appear to mirror the “Flow” (Csikszentmihalyi 1996), also described as “How to live life as a work of art, rather than as a chaotic response to external events”. Comments have been expressed such as “Time passed so fast”, “I never felt as happy as when I was in here”, “I never believed I could do it before”, show that individuals are carried along by the engagement in education. Features of the “Flow Experience” include a clear understanding of the goals and rules and concentration, as happens when people are in an education setting and focus. Time appears to be distorted and is not felt to pass and feedback from the teacher is direct and immediate. There is an element of control as people are there voluntarily and the activity is balanced between the ability and a challenge. The activity is rewarding as mathematics gives a sense of satisfaction, and prison ethnography in UK (Wilson 2010) notes a great interest in higher level mathematics. There is a loss of feeling of self consciousness which means that all awareness is limited to the activity (Csikszentmihalyi 1996).
The effect on the learner of the teacher putting energy into understanding the learner and allowing them to build on their own experiences, allows them to trust when they are offered challenges and support. Emotions and interest are part of change and teachers’ acceptance of the learner and interest in their learning enables change (Zull 2002).

Conversations convey care, and a state of preparedness and a willingness to act. Caring relationships are formed through connections between people (Gilligan 1992), and of being present to another (Noddings 1999). Care has been defined as an expression of hope for the future (Wright 2004) and inspires a love of learning. To be shown respect and consideration enables a person to respond in kind. In prisons, the task is to find the correct degree of caring, a midpoint between absolute distance and abolition of all distance (Wright 2004).

Care transforms learners in prisons (Mageehon in Wright 2004) and further research on its effects in this setting may be worthwhile. Reflection and mindfulness practice through using a visual of a learning plan can bring a learner into the present and out of other stories that repeat the former stories of inadequacy or grandiosity (Moss 2007). The mapping of this data on their learning plans and mind, can lead to discussions on the role of mathematics in mapping data from the physical world as well as the inner world. Future research into the effect of learning in this environment will expand with reflections on affect and cognition (McLeod 1992 and Schlöglmann 2001). Future research will also focus on areas such as gender, class and local community impact in learning mathematics in prison (Lerman, 2000) as well and the role of humour and metamorphosis, or swapping identities, as in acting for the duration of the class as a real student or as a teacher in a peer tutoring situation and reflecting on the experience.
References


Wedge, T. (1999). To Know or not to Know - That is a Question of Context. Educational Studies in Mathematics

Wilson, Anita. (2010). Goodwill and Good Fortune Obstacles and Opportunities for Level 2 learners in local jails. Prisoners Education Trust. p. 28

An International Comparative Study of Adult Numeracy in New Zealand and the UK – what can we learn from surveys?

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We report on our work in progress on a comparative study of adult numeracy/mathematics education in New Zealand and the UK. This paper follows on from the paper presented by Barbara Miller-Reilly at the ALM17 conference in Oslo, Norway, in 2010, where she discussed our survey of key UK research projects in the field of adults learning mathematics. In this paper we attempt to compare our two countries in the wider international context, as shown in international surveys of, or relevant to, adult numeracy. Our review of the survey evidence has convinced us of the need to be careful in interpreting survey results and cautious in using them to compare populations.

Key words: adult, quantitative literacy, numeracy, mathematical literacy, survey, international, national, comparative

We approach the work outlined in this paper as academics with backgrounds in the United Kingdom (UK) (Diana Coben) and New Zealand (Barbara Miller-Reilly). The idea for a comparative study came to us as a result of our conversations about what was happening in our respective countries with respect to adult numeracy education. Over the last ten years, both countries have established policies that are intended to raise the level of adult numeracy and literacy of their populations of working age. England’s Skills for Life strategy (DfEE, 2001) pre-dated New Zealand’s Literacy, Language and Numeracy Action Plan 2008-2012 (TEC, 2008) by seven years. It seems to us that there might be lessons to be learned from a systematic comparison of our two countries’ attempts to solve essentially the same problem. Since embarking on our project with a review of the current situation in England (Miller-Reilly, 2010), Diana Coben has been appointed Director, Research and Policy, of the New Zealand National Centre of Literacy and Numeracy for Adults, thereby giving us a valuable opportunity to work together on our project from within New Zealand.
Why compare?

When contemplating any international comparative study it behoves researchers to ask ‘why compare?’ What insights might be gained from such an approach that would not be available in a national study?

The benefits of cross-national comparisons have been summarised as follows by Linda Hantrais:

When researchers from different backgrounds are brought together on collaborative or cross-national projects, valuable personal contacts can be established, enabling them to capitalise on their experience and knowledge of different intellectual traditions and to compare and evaluate a variety of conceptual approaches.

Comparisons can lead to fresh, exciting insights and a deeper understanding of issues that are of central concern in different countries. They can lead to the identification of gaps in knowledge and may point to possible directions that could be followed and about which the researcher may not previously have been aware. They may also help to sharpen the focus of analysis of the subject under study by suggesting new perspectives.

Cross-national projects give researchers a means of confronting findings in an attempt to identify and illuminate similarities and differences, not only in the observed characteristics of particular institutions, systems or practices, but also in the search for possible explanations in terms of national likeness and unlikeness. Cross-national comparativists are forced to attempt to adopt a different cultural perspective, to learn to understand the thought processes of another culture and to see it from the native's viewpoint, while also reconsidering their own country from the perspective of a skilled, external observer.

(Hantrais, 1995).

We endorse these points. We hope that our comparative study will lead to fresh, exciting insights and a deeper understanding of adult numeracy in our respective countries. We were fortunate to have already established personal and professional contact through ALM, strengthened through Diana Coben’s working visit to New Zealand in 2008, during which the idea for the study arose. We feel that the advantages of an international comparative study outweigh the risks, so long as we bear in mind Hantrais’ advice. For example, she warns of problems in cross-national comparative research, particularly if linguistic and cultural factors, together with differences in research traditions and administrative structures, are ignored. She also highlights difficulties in managing and funding cross-national projects and accessing comparable data and stresses the need to clarify concepts and research parameters, for example, in defining the unit of observation.

To offset these problems Hantrais advocates a process of trial and error in the management of projects and urges researchers not to disregard major demographic variables when re-analysing existing large-scale datasets since these may indicate greater intranational than international differences. She advises that researchers should attempt “to establish comparable groupings from the most detailed information available - the raw data - and to focus on the broader characteristics of the sample”. Accordingly, “The solution to the problem of defining the unit of observation may be to carry out research into specific organisational, structural fields or sectors and to look at subsocietal units rather than whole societies”. In new studies she advocates replicating the research design and using the same concepts and parameters simultaneously in two or more countries on matched groups, while remaining alert to the dangers of cultural interference, ensuring that discrepancies are not forgotten or ignored and
being wary of using what may be a sampling bias as an explanatory factor. Finally, in interpreting the results, “findings should be examined in relation to their wider societal context and with regard to the limitations of the original research parameters” (Hantrais, *passim*, 1995).

### Survey evidence of levels of adult numeracy in the UK and New Zealand

We decided we needed to clarify the nature and extent of the issue that our study addresses so we decided to review survey evidence of adult numeracy in the UK and New Zealand. This review is the focus of this paper. In undertaking it we are acutely aware of the difficulty of defining and assessing adult numeracy (Coben et al., 2003).

There are three surveys of adults of working age relevant to our study: the International Adult Literacy Survey (IALS), undertaken in three phases in the 1990s; the Adult Literacy and Lifeskills Survey (ALL), the successor survey to IALS, undertaken in two phases in 2003 and 2006-08; and the Programme for the International Assessment of Adult Competencies (PIAAC), which is currently underway.

In addition to these ‘adult’ surveys, we are also reviewing data from the Programme for International Student Assessment (PISA) (OECD, 2000-ongoing). PISA is relevant to our study because it sought to assess students’ abilities to apply mathematical concepts, skills and understanding to authentic problems that arise in real world settings. The primary focus of PISA is on the ability to apply mathematical knowledge and thinking to a whole variety of situations, including what PISA calls inner mathematical settings, not only ‘everyday functional mathematics’ (Adams, 2003). PISA also gives us a recent direct comparison of New Zealand and the UK, with large samples of participants assessed on the same measure.

The domains and measures used in these surveys vary: IALS assessed Quantitative Literacy, Prose Literacy and Document Literacy; ALL assessed Numeracy, Prose Literacy, Document Literacy, Problem Solving and Health Literacy; and PISA assesses Reading, Mathematical and Scientific Literacy.

The most relevant of these domains for our study are defined in the relevant survey documentation as follows:

Quantitative Literacy in IALS is:

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12 We did not have access to the raw data from these surveys so in this respect we are not following Hantrais’ lead.

13 PIAAC is assessing Literacy, Numeracy and Problem Solving in Technology-Rich Environments as well as collecting information from respondents concerning their use of key work skills in their jobs and including language components (see [http://www.oecd.org/document/35/0,3746,en_2649_201185_40277475_1_1_1_1,00.html](http://www.oecd.org/document/35/0,3746,en_2649_201185_40277475_1_1_1_1,00.html)). The UK is participating in PIAAC; the NZ Ministry of Education is currently assessing options for New Zealand’s potential participation.

14 PISA surveys have been conducted every three years since 2000 with one domain the main subject each time. Reading was the main subject in 2000, mathematics in 2003, science in 2006 and reading in 2009; Mathematical Literacy will be the focus of the 2012 PISA survey.
the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a checkbook, calculating a tip, completing an order form, or determining the amount of interest on a loan from an advertisement.


In the ALL survey:

Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations.

(Desjardins, Murray, Clermont, & Werquin, 2005, p. 292).

The Numeracy scale in ALL was designed to be broader than the IALS Quantitative Literacy scale, “going beyond applying arithmetic skills to a wider range of mathematical skills (e.g., use of number sense, estimation, statistics)” (Lemke et al., 2005, p. 1).

Another ALL domain, Problem Solving, is also relevant to our study. In ALL:

Problem solving involves goal-directed thinking and action in situations for which no routine solution procedure is available. The problem solver has a more or less well defined goal, but does not immediately know how to reach it. The incongruence of goals and admissible operators constitutes a problem. The understanding of the problem situation and its step-by-step transformation based on planning and reasoning, constitute the process of problem solving.


Mathematical Literacy in PISA is broader than Numeracy in ALL in that it assesses:

…an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen.

(OECD, 2003, p. 24)

Comparing New Zealand and the UK in international surveys

We begin here with a review of data from the two extant international surveys of adults of working age: the International Adult Literacy Survey (IALS) and the Adult Literacy and Lifeskills Survey (ALL). However, two problems arise in seeking to use these surveys to compare the UK and New Zealand. The first is a problem of participation: both the UK and New Zealand participated in the IALS survey (OECD & Statistics Canada, 1997, 2000) but only New Zealand participated in the ALL survey (OECD & Statistics Canada, 2011). Countries that participated in IALS and ALL are shown in Figure 1 below.
The second problem concerns the measures: Quantitative Literacy scores from the earlier IALS survey cannot be compared directly with Numeracy scores in ALL because of the different conceptual basis of the measures, as expressed in the definitions above. The evolution of the IALS, ALL and PIAAC domains is shown in the following chart.
In order to mitigate these problems as far as possible we have started by comparing key findings for the UK and New Zealand in IALS, the only ‘adult’ survey in which both countries participated. Then, since although New Zealand took part in the more recent ALL survey the UK did not, we have turned to the Skills for Life (national) surveys for data on England, the largest jurisdiction within the UK; the Skills for Life surveys were undertaken in 2002-03 and again in 2010.

We have also broadened our international focus to include eight other OECD countries which participated in both IALS and ALL: Australia (AUS), Canada (CAN), Hungary (HUN), Italy\(^\text{15}\) (ITA), The Netherlands (NLD), Norway (NOR), Switzerland (CHE) and the United States of America (USA).

Hence, in this paper, we present key findings for New Zealand and the UK in each of the following surveys: Quantitative Literacy in IALS; Numeracy in the Skills for Life survey; Numeracy and Problem Solving in ALL; and Mathematical Literacy in PISA (mainly in PISA 2006 and 2009). Within this overall framework, we have looked more closely at some aspects of the survey results, for example, we have compared the Mathematical Literacy scores of New Zealand students in PISA with Numeracy scores of young adult New Zealanders in ALL. For each survey we have looked at the mean scores and proportions of the populations at various levels, of all the participating countries.

\(^{15}\) Data for Italy in IALS were delayed and are published separately Gallina, V. (Ed.). (2012). *La Competenza Alfabetica in Italia: Una ricerca sulla cultura della popolazione*. Milan: F. Angeli.
Key findings from the IALS and ALL international surveys

**Key findings for New Zealand in IALS**

Some of the key preliminary findings for New Zealand from IALS are listed as:

- The distribution of literacy skills within the New Zealand population is similar to that of Australia, the United States and the United Kingdom.
- Approximately one in five New Zealanders is operating at a highly effective level of literacy.
- New Zealanders do less well at document and quantitative literacy than at prose literacy.
- Māori with tertiary qualifications have literacy profiles similar to those of tertiary educated European/Pākehā.\(^{16}\)

(Walker et al., 1996, p. 1).

A later report (Culligan, Arnold, Noble, & Sligo, 2004, p. 5) extended the analysis of the New Zealand IALS data with the “primary aim of determining which demographic characteristics could predict low literacy proficiency levels (as indicated by IALS)”\(^{1}\). The report found that:

educational attainment level was the strongest predictor of subsequent adult literacy proficiency. However, it was also noted that the effect of educational attainment could be mediated by a variety of other factors, not least of which were early childhood family and school experiences.

(Culligan, et al., 2004, p. 53).

This predictor variable was the “strongest overwhelmingly” for all three categories of literacy (prose, document and quantitative) (Culligan, et al., 2004, p. 5). The authors also found that the “predictor of secondary strength was Ethnicity”, also for all three categories of literacy.

In this analysis it was found that those people who identify as Asian Peoples, Pacific Peoples or Māori would appear to be more at risk of low literacy proficiency (as measured by the IALS) than those who identify as European. However, it was also noted that the IALS measured proficiency in English literacy only and that within the Asian and Pacific Peoples category especially, caution has to be taken to ensure that literacy proficiency is not confounded with language and perceptual proficiency.

(Culligan, et al., 2004, p. 53).

After these two factors it was found that the predictive effects of the other variables were similar in their predictive strength. “Variables of interest within the NZ sample included four labour force factors: labour force participation, occupation type, industry type, and income level” (Culligan, et al., 2004, p. 6).

Culligan et al also noted that “participation in adult training increased as literacy proficiency increased”, and that those who were “employed within the higher occupational categories, the

\(^{16}\) Pākehā is a Māori word for New Zealanders of European descent.
professional and business occupations, were also the more likely to participate in adult training”.

Ethnicity was another interesting demographic characteristic influencing participation in adult training programmes. Those with a level 1 literacy proficiency are similar in their participation rates across ethnicities, whereas those within level 2 show an extremely high participation rate of Pacific Peoples over and above the other ethnic groups.

(Culligan, et al., 2004, p. 5).

Key findings for the UK in IALS

The UK took part in the second round of IALS in 1996, with England, Scotland and Wales participating together (Carey, Low, & Hansbro, 1997) while Northern Ireland took part separately (Sweeney, Morgan, & Donnelly, 1998). The results showed that 51% of the adult population was estimated to be performing below the minimum required for coping with the demands of life and work in the knowledge society (Houtkoop & Jones, 1999, p. 36), with 23% at the lowest level (Level 1) and 28% at Level 2 (OECD & Statistics Canada, 1997, p. 151). The UK’s poor results in IALS led, amongst other things, to the inception of the Skills for Life strategy to improve adult literacy and numeracy in England (DfEE, 2001), bringing unprecedented attention and funding to what had been a neglected area of educational provision.

IALS was heavily criticised in the UK by Mary Hamilton and David Barton, who questioned the validity of the test, arguing from the perspective of the New Literacy Studies that it provides only a partial picture of literacy; that culture is treated as bias; and that the test items do not represent the real-life items as claimed (Hamilton & Barton, 2000). Alain Blum, Harvey Goldstein and France Guérin-Pace also cast doubt on the validity of IALS. They offered the following recommendations for any future surveys:

1. The psychometric criteria used by IALS do not provide a satisfactory basis for country comparisons. The one-dimensional models used fail properly to explore the complexity of the data with the result that the conclusions of IALS may well be over-simplifications about the state of literacy in the member countries. These criteria need modification.

2. There is a need to carry out sensitivity analyses of the assumptions made in any item response modelling. In particular, multi-dimensional models should be explored and rankings of item difficulties compared between countries.

3. Attention should be directed at providing greater validity and recognising that absolute comparability may not be achievable. The survey data should be viewed as potentially casting light on factors that are locally specific and not amenable to simple scale comparisons between countries.

4. Country comparisons should be carried out at task or ‘small task set’ level with particular attention paid to translation issues and cultural differences.

5. Multilevel modelling needs to be considered in all analyses of the data in order fully to explore within-country variability.

6. A variety of alternative procedures need to be explored for combining and reporting items with clearly set out assumptions that are used.
The IALS survey, as it stands, should be treated with caution at national level and more so at an international level. The instability of the item success hierarchies due to a combination of linguistic and cultural differences shows that the survey cannot be used on a comparative basis. The operation of translation leads to important biases in the estimated levels of the tasks. The scoring and the processing of omissions in the IALS survey also resulted in a biased assessment of the ability levels due to unequal motivation on the part of interviewees which was not taken into account.

On the basis of our analyses, it is not possible to assume that IALS measures only literacy. It seems to measure a combination of different factors: motivation (reflected in the different ways of filling in the questionnaire), understandings of what items mean, and differences in test taking behaviour more generally. We are not arguing against any kind of international comparative study. Indeed, we think they can be useful. However, we do want to make both the constructors and the users of such surveys more aware of the complexities of design and interpretation, and the caveats that need to be entered about their use.

(Blum, Goldstein, & Guérin-Pace, 2001, pp. 243-244).

The Skills for Life Surveys in England

The UK government decided against participating in the next international survey, ALL, despite advice to the contrary (Carey & Morris, 1999) and instead established the Skills for Life survey of 2002-03 in England to assess adults’ proficiency in Literacy, Numeracy and Information and Communication Technology (ICT) (Williams, Clemens, Oleinikova, & Tarvin, 2003).

Outcomes from the Skills for Life survey cannot be aligned with IALS, as Greg Brooks notes in a recent paper, worth quoting at length for the light it shines on the real politik surrounding these surveys, and as an example of the difficulty – in this case impossibility - of comparing like with like in different surveys:

the Skills for Life survey used no IALS items, and used computer administration rather than paper-and-pencil. The results turned out rather differently too: 16% (rather than IALS’s 22%) were deemed to have literacy skills at or below Entry level (= at or below IALS Level 1), but 47% (rather than IALS’s 23% for “quantitative literacy”) were deemed to have numeracy skills at or below Entry level. The British government was not too displeased with the literacy figure, but the alarmingly high numeracy figure led it to set a new preferred criterion for “less than functional numeracy”, namely “at or below Entry level 2”. This took the figure for poor numeracy down to 21%, much more acceptable, and much closer to the IALS figure. But this too seems to have been a purely pragmatic decision, without theoretical justification.

There is no principled way of estimating where the boundary between UK Entry levels 2 and 3 falls within IALS Level 1 (which does not have subdivisions), so it is impossible to know how the new UK criterion for poor numeracy maps to IALS. It is also impossible to know whether the differences in estimates for less than functional literacy and numeracy between IALS and the Skills for Life survey are genuine, or artefacts of the different items and modes of administration.

(Brooks, 2011, p. 5).
The National Research and Development Centre for Adult Literacy (NRDC\textsuperscript{17}) was highly critical of the Skills for Life survey, mainly on the grounds of inadequate piloting and reliability (Howard et al., 2004). Nevertheless, a follow-up Skills for Life survey was undertaken in England in 2010 in which “The literacy and numeracy assessment tools utilised were the same as those used in the 2003 survey to ensure absolute comparability with the 2003 survey” (BIS, 2011, p. 2). With respect to numeracy, the 2010 Skills for Life survey found that standards had declined:

Three quarters (76 per cent) of respondents achieved Entry Level 3 or above in numeracy, with one quarter (24 per cent) scoring below this level. This represents a small decline in numeracy levels as 79 per cent achieved Entry Level 3 or above in 2003.

(BIS, 2011, p. 4).

**Key findings for New Zealand in ALL**

The Second International ALL Report includes a snapshot of New Zealand’s position in the survey:

Australia, Canada and New Zealand consistently score about average on all four scales. Hungary performs about as well as the United States on the prose and document literacy scales. Hungary also performs as well as Canada, Australia, New Zealand and Bermuda on the numeracy scale, and scores just above Italy on the problem solving scale.


Analysis of New Zealand ALL data by Lane (2010) finds three key factors can account for a large part of the variation between people aged 25-65 in their literacy and numeracy skills: completed education, language background and computer use.

Upper secondary education and tertiary education is strongly associated with higher literacy and numeracy whereas lower secondary education is associated with lower literacy and numeracy.

People with English as first language have a considerable advantage in literacy and numeracy skills (tested in English). People whose first language is not English are at less of a disadvantage if their home language is English.

Computer use is strongly associated with higher literacy and numeracy, especially combination of work and home computer use. In addition, computer use is associated with intensive and extensive reading, writing and numeracy practices. Work computer use or non-use divided jobs broadly into those that required higher literacy and numeracy and those that did not. There is also a large overlap between groups of people with low literacy and low numeracy, and the group of people who did not use a computer at work (Lane, 2010).

The ALL survey included an oversample of Māori adults. This allowed meaningful analyses of the distribution of literacy and numeracy skills among the adult Māori population of New Zealand

\textsuperscript{17} Greg Brooks and Diana Coben both contributed to NRDC’s critique of the 2002-03 Skills for Life survey.
For all four ALL skill domains more than half of Māori adults in 2006 had low skills – level 1 or 2 skills. Employed Māori adults in 2006 had substantially higher skills than Māori adults not working. Skill levels are also strongly related to education levels.

(Satherley & Lawes, 2009, p. 6).

**Key findings for New Zealand in PISA**

In New Zealand 4,824 students from 170 schools took part in PISA 2006. The overall key results follow:

The mean mathematical literacy performance of New Zealand’s 15-year-old students in PISA 2006 was above the OECD mean.

Only 5 out of the other 56 participating countries had significantly higher\(^{18}\) mean performances than New Zealand.

There was no significant change in mathematical literacy performance of New Zealand’s 15-year-old students between 2003 and 2006.

Compared to (the other 29) OECD countries, a relatively larger proportion of New Zealand students were highly proficient in mathematical literacy and a relatively smaller proportion had low proficiency in mathematical literacy.

The five top-performing countries had larger proportions of students achieving the highest level of proficiency and smaller proportions of students with low proficiency compared with New Zealand.

(Caygill, Marshall, & May, 2008, p. 6).

**Key Mathematical Literacy findings for the most recent PISA survey, PISA 2009, are outlined on the New Zealand Ministry of Education website as follows:**

New Zealand students’ overall mathematical literacy performance (519) was significantly higher than the average for the OECD countries (496).

Five OECD countries and six non-OECD partner countries or economies performed better than New Zealand, four OECD countries were similar, and the other 49 countries had a significantly lower performance.

New Zealand girls and boys achieved a similar mean mathematical literacy performance.

New Zealand’s 15-year-olds’ mean mathematical literacy performance did not change between 2003 and 2009.

Performance in mathematical literacy is also measured in terms of proficiency levels\(^{19}\) that links student achievement to specific levels of competency and describes the types of mathematical tasks that students proficient at each level would typically be expected to perform.

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\(^{18}\) Throughout this report, the term ‘significantly’ refers to statistical significance at the 0.05 level. See the ‘Definitions and technical notes’ at the end of this report for further details.

Nineteen per cent of New Zealand students were top performers in mathematical literacy (achieved proficiency Level 5 or higher).

Fifteen per cent of New Zealand students did not reach the baseline proficiency level\(^{20}\). This was a similar proportion to Australia. In the United Kingdom and the United States the proportions were larger.

(Ministry of Education).

**Mathematical Literacy scores in PISA 2006 – international comparisons with the UK**

Mathematical Literacy was a minor subject in both the PISA 2006 and 2009 surveys. A sub-sample of students was assessed in Mathematical Literacy and there were fewer questions than in Science (in 2006) and Reading (in 2009). The PISA 2006 results reported are estimates for the whole population (in the case of England only, not the whole of the UK), based on the performance of students who were presented with Mathematical Literacy test items.

Eighteen countries had mean scores for mathematics\(^{21}\) which were significantly higher than that of England. In twelve countries the difference in mean scores to that in England was not statistically significant. Twenty-six countries had mean scores which were significantly lower than England.

The mean score for mathematics in England was not significantly different from the OECD average.

Of the eighteen countries with higher mean scores (where the difference was statistically significant), twelve were members of OECD. Seven OECD countries had mean scores significantly lower than England (Spain, United States, Portugal, Italy, Greece, Turkey and Mexico).

Seven of the countries with mean scores significantly higher than England are in the European Union (Finland, the Netherlands, Belgium, Estonia, Denmark, the Czech Republic and Slovenia). Six EU countries were significantly lower than England.

In contrast to science, the spread of attainment in mathematics was not large compared with other countries. While the proportion at the lowest levels was similar to the OECD average, the proportion at the highest levels was slightly below the OECD average.

Males scored significantly higher than females in mathematics. This was the case in 35 of the 57 participating countries.

(Bradshaw, Sturman, Vappula, Ager, & Wheater, 2007, p. vii)

The national report on PISA 2009 in England found that England’s performance in mathematics in PISA 2009 did not differ greatly from that in the 2006 PISA survey outlined above (Bradshaw, Ager, Burge, & Wheater, 2010).

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\(^{20}\) The baseline proficiency level in PISA is Level 2.

\(^{21}\) ‘Mathematics’ here refers to Mathematical Literacy on the PISA scale.
Comparison of Mathematical Literacy scores of New Zealand students in PISA with Numeracy scores of young adult New Zealanders in ALL

There are a number of sampling differences between PISA and ALL:

PISA 2006 is based on data from 4,824 students, compared with 1,082 respondents aged 16-24 in ALL. PISA samples 15-year-old students and excludes students who have had less than a year of English instruction as well as students in Māori immersion programmes. The 16-24 age group in ALL does not have these restrictions; it also includes a considerably higher percentage than PISA of people who mainly speak a language other than English at home, because of the large number of migrants who have arrived in New Zealand after the age of 15.

In spite of these sampling differences, the results of the two studies are similar where they can be compared, which is in relation to gender, socioeconomic status, main home language and ethnicity.

(Lane, 2011, p. 91).

No significant gender differences were found in the 16-19 age group in ALL for Numeracy (nor were there significant differences in the 16-24 age group) (Lane, 2011). These results are in line with PISA 2009, which showed no significant gender difference in New Zealand in Mathematical Literacy (OECD, 2010), although PISA 2006 had shown an advantage for boys (Caygill, et al., 2008; Lane, 2011).

Analysis of PISA 2006 using an index of economic, social and cultural status (ESCS), which included parental education, showed increasing Mathematical Literacy with increasing ESCS, in line with the results for parental education and deprivation in ALL

(Lane, 2011, p. 94).

The relationship of Numeracy to main language spoken in the home among people aged 16-24 in ALL was in line with the results of PISA 2006 for Mathematical Literacy. There is no difference between students whose main home language was English those with a different main home language. This contrasts with the significantly higher mean Reading Literacy scores and Document Literacy scores for the former group in PISA and ALL respectively (Lane, 2011; Marshall, Caygill, & May, 2008).

PISA 2006 showed the same pattern of mean Numeracy scores for ethnic groups among 15-year-olds as among those aged 16-24 in ALL, namely comparable means for Europeans and Asians, which were significantly higher than the comparable means for Māori and Pasifika (Caygill, et al., 2008; Lane, 2011).

**International comparisons of scores: Quantitative Literacy scores in IALS and Numeracy scores in ALL**

A comparison of New Zealand and UK Quantitative Literacy scores in IALS with other countries

In the final OECD report of the International Adult Literacy Survey (OECD & Statistics Canada, 2000) a comparison was made of 22 countries’ mean scores on Quantitative Literacy in the survey of the adult population (aged 16-65). Figure 3 below (OECD & Statistics
Canada, 2000, p. 15) shows box plots of the mean scores of the Quantitative Literacy scale, with .95 confidence interval and scores at 5th, 25th, 75th, and 95th percentiles of these scores, where countries are ranked by mean scores (population aged 16-65, 1994-1998).

Figure 3: Mean scores of the Quantitative Literacy scale, with .95 confidence interval and scores at 5th, 25th, 75th, and 95th percentiles of these scores, where countries are ranked by mean scores (population aged 16-65, 1994-1998) (OECD & Statistics Canada, 2000, p. 15).
Before comparisons are made between countries it is necessary to determine groups of countries whose mean scores are not significantly different to each other. Table 1 compares countries’ performances on the IALS Quantitative Literacy scale, by listing countries whose mean score is not statistically different from that of the comparison country.

**Table 1: Comparing countries’ performance on the IALS Quantitative Literacy scale**

<table>
<thead>
<tr>
<th>Comparison country</th>
<th>Countries whose mean score is NOT statistically different from that of the comparison country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>Czech Republic, Norway</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>Denmark, Norway</td>
</tr>
<tr>
<td>Norway</td>
<td>Czech Republic, Denmark</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>Finland, Belgium (Flanders), Canada</td>
</tr>
<tr>
<td>Finland</td>
<td>Netherlands, Belgium (Flanders), Canada</td>
</tr>
<tr>
<td>Belgium (Flanders)</td>
<td>Finland, Netherlands, Canada, Switzerland (French), Switzerland (German), Australia, United States, Switzerland (Italian)</td>
</tr>
<tr>
<td>Canada</td>
<td>Finland, Netherlands, Belgium (Flanders), Switzerland (French), Switzerland (German), Australia, United States, Switzerland (Italian)</td>
</tr>
<tr>
<td>Switzerland (French)</td>
<td>Belgium (Flanders), Canada, Switzerland (German), Switzerland (Italian)</td>
</tr>
<tr>
<td>Switzerland (German)</td>
<td>Belgium (Flanders), Canada, Switzerland (French), Australia, United States, Switzerland (Italian)</td>
</tr>
<tr>
<td>Australia</td>
<td>Belgium (Flanders), Canada, Switzerland (German), United States, Switzerland (Italian)</td>
</tr>
<tr>
<td>United States</td>
<td>Belgium (Flanders), Canada, Switzerland (German), Australia, Switzerland (Italian)</td>
</tr>
<tr>
<td>Switzerland (Italian)</td>
<td>Belgium (Flanders), Canada, Switzerland (French), Switzerland (German), Australia, United States, New Zealand, Hungary</td>
</tr>
<tr>
<td>New Zealand</td>
<td>Switzerland (Italian), Hungary, United Kingdom</td>
</tr>
<tr>
<td>Hungary</td>
<td>Switzerland (Italian), New Zealand, United Kingdom</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>New Zealand, Hungary, Ireland</td>
</tr>
<tr>
<td>Ireland</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>Slovenia</td>
<td></td>
</tr>
</tbody>
</table>

This type of table was used to present the PISA 2009 results OECD. (2010). *PISA 2009 Results: What Students Know and Can Do: Student performance in Reading, Mathematics and Science (Volume I)*. Paris: OECD. p. 134); data for it are extracted from Figure 2.3C in the IALS final report OECD, & Statistics Canada. (2000). *Literacy in the Information Age: Final report of the International Adult Literacy Survey*. Paris and Ottawa: Organisation for Economic Cooperation and Development and Statistics Canada..
When countries are ranked by their mean scores on the Quantitative Literacy scale, New Zealand and the United Kingdom are ranked within a group of three countries, including Hungary, since there was no (statistically) significant difference in the mean scores between these three countries (OECD & Statistics Canada, 2000, p. 21). There are also no significant differences between the next highest mean scores of seven countries. Included in this group are Canada, Australia and USA. Seven more countries have significantly higher mean scores than this group. Four countries have significantly lower mean scores than the New Zealand/Hungary/UK group.

Some international comparisons of New Zealand’s scores in the Numeracy and Problem Solving scales with those of the nine other countries who participated in the ALL survey are reviewed in the next section.

**Numeracy and Problem Solving Scores in ALL – international comparisons with New Zealand**

In the Second International ALL Report (OECD & Statistics Canada, 2011) a comparison was made of ten countries’ mean scores on Numeracy, and nine23 countries’ mean scores on Problem Solving in the survey of the adult population (aged 16-65). Figure 4 below shows box plots of the mean scores of the Numeracy scale and mean scores on the Problem Solving scale, with .95 confidence interval and scores at 5th, 25th, 75th, and 95th percentiles of these scores, ranging from 0 to 500 points, where countries are ranked by mean scores (population aged 16-65, 2003 and 2008).

---

23 The United States and Switzerland (Italian) did not field the Problem Solving skills domain.
Figure 4. Comparative distributions of Numeracy and Problem Solving skills scores - countries are ranked by mean scores (OECD & Statistics Canada, 2011, p. 41).

Table 2 compares ten countries’ performances on the ALL Numeracy scale, by listing countries whose mean score is not statistically different from that of the comparison country.
Table 2: Comparing ten countries’ performance on the ALL Numeracy scale

<table>
<thead>
<tr>
<th>Comparison country</th>
<th>Countries whose mean score is NOT statistically different from that of the comparison country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>The Netherlands</td>
</tr>
<tr>
<td>The Netherlands</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Norway</td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>Canada, Australia, New Zealand, Bermuda</td>
</tr>
<tr>
<td>Canada</td>
<td>Hungary, Australia, New Zealand, Bermuda</td>
</tr>
<tr>
<td>Australia</td>
<td>Hungary, Canada, New Zealand, Bermuda</td>
</tr>
<tr>
<td>New Zealand</td>
<td>Hungary, Canada, Australia, Bermuda</td>
</tr>
<tr>
<td>Bermuda</td>
<td>Hungary, Canada, Australia, New Zealand</td>
</tr>
<tr>
<td>United States of America</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td></td>
</tr>
</tbody>
</table>

When these ten countries are ranked by their mean scores on the Numeracy scale, New Zealand lies within a group of five countries including Hungary, Canada and Australia, since there was no statistically significant difference in the mean scores of these five countries (OECD & Statistics Canada, 2011). Three countries have significantly higher mean scores than this group (Switzerland, The Netherlands and Norway). Two countries have significantly lower mean scores than the group (USA and Italy).

Table 3 compares nine countries’ performances on the ALL Problem Solving scale, by listing countries whose mean score is not statistically different from that of the comparison country.

Table 3: Comparing nine countries’ performance on the ALL Problem Solving scale

<table>
<thead>
<tr>
<th>Comparison country</th>
<th>Countries whose mean score is NOT statistically different from that of the comparison country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Netherlands</td>
<td>Norway</td>
</tr>
<tr>
<td>Norway</td>
<td>Netherlands, Switzerland</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Norway</td>
</tr>
<tr>
<td>New Zealand</td>
<td>Canada, Bermuda</td>
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<tr>
<td>Canada</td>
<td>Australia, New Zealand, Bermuda</td>
</tr>
<tr>
<td>Bermuda</td>
<td>Canada, New Zealand, Bermuda</td>
</tr>
<tr>
<td>Australia</td>
<td>Canada, Bermuda</td>
</tr>
<tr>
<td>Hungary</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td></td>
</tr>
</tbody>
</table>

There was no statistically significant difference in the mean scores of New Zealand, Canada and Bermuda (OECD & Statistics Canada, 2011). Three countries have significantly lower mean scores than this group (Australia, Hungary and Italy). Three countries have significantly higher mean scores than New Zealand (Switzerland, The Netherlands and Norway).
Mathematical Literacy scores in PISA 2006 – international comparisons with New Zealand

Figure 5 shows the box plots of the mean scores on the PISA Mathematical Literacy scale, with .95 confidence interval and scores at 5th, 25th, 75th, and 95th percentiles of these scores, where countries are ranked by mean scores. The box plot for the OECD average (i.e., mean) is listed as an entry on this Figure. The unshaded section shows countries whose means are not significantly different from that of New Zealand. The darkest shading indicates countries whose means are significantly higher than New Zealand. Those countries with lighter shading have means significantly lower than New Zealand.
Figure 5: Means and distributions of mathematical achievement in PISA 2006 (Caygill, et al., 2008, p. 13). Note: * denotes non-OECD (partner) countries. These countries are not included in the OECD average.
As shown in Figure 5, the mean Mathematical Literacy performance of New Zealand 15-year-olds was significantly above the mean for the 30 OECD countries who participated in PISA 2006. Altogether 58 countries participated in PISA 2006. The seven countries whose mean scores were not significantly different from New Zealand’s mean score included Canada, Australia and Switzerland. The UK scored significantly lower than New Zealand, as did Norway, Hungary, the USA and Italy. The UK mean score did not significantly differ from the OECD mean score and was significantly higher than the mean scores for the USA and Italy.

Percentages of the adult population in different countries at each level of Quantitative Literacy in IALS and numeracy in ALL

Comparative distribution of Quantitative Literacy levels in the adult population in IALS

Figure 6 gives a comparison of the distributions of Quantitative Literacy levels in the adult population (aged 16-65) in 22 countries in the IALS survey. These histograms were ranked by the proportions in Levels 3-5, making it easier to compare how countries differ in the proportions of people with higher and lower levels of literacy skills.

New Zealand ranks 15th and the UK ranks 16th by proportions at Levels 3-5, with very similar distributions of percentages of the population at each level. A number of the 22 IALS countries had similar proportions in Levels 3-5 as for New Zealand or the UK. For example, Switzerland (German) ranked 10th, Canada (11th), Australia (12th), Switzerland (Italian – 13th), the USA (14th) and Hungary (17th) and all these countries show somewhat similar
distributions to New Zealand and UK. Norway, ranking 3rd, The Netherlands (6th) and Switzerland (French – 7th) had higher proportions in Levels 3-5.

**Comparative distribution of Numeracy and Problem Solving levels in the adult population in ALL**

The first graph in Figure 7 shows the percentages of the adult population at each Numeracy level. As in Figure 6 above, these are anchored at the boundary of Levels 2 and 3, marking the boundary between lower Numeracy skills (Levels 1 and 2) and higher Numeracy skills (Levels 3-5). The second graph in Figure 7 shows the percentages at each Problem Solving level, but these are anchored at the boundary of Levels 1 and 2.

![C. Numeracy scale](image)

![D. Problem solving scale](image)

**Figure 7: Distribution of Levels of Numeracy Skills and Problem Solving Skills (ALL 2003 & 2008) for the adult population (16-65). (OECD & Statistics Canada, 2011, p. 43).**

Comparative distributions of Numeracy scores by levels show that New Zealand (49%) and four other countries – including Australia (50%), Canada (50%) and Hungary (49%) – all show similar proportions of respondents scoring at Levels 3-5. The corresponding proportion in the USA (41%) is lower, and that for Italy (20%) much lower. The Netherlands (63%), Norway (60%) and Switzerland (61%) have the highest proportions at Levels 3-5.

Distributions of Problem Solving scores ranked by proportions of respondents scoring at Levels 2 and above (9 countries) show New Zealand (71%) occupying middle ground, similar to Canada (70%), Switzerland (71%) and Australia (68%). Hungary (59%) has the second smallest proportion scoring Level 2 and above, The Netherlands (78%) and Norway (77%) the highest. “The proportions of the adult population with high Problem Solving proficiencies (Levels 2, 3 and 4) are on average about 20 percentage points higher than for the other three skill domains” (OECD & Statistics Canada, 2011, p. 44).

Discussion

Ranking by scores on the different mathematical measures in three international surveys: IALS, ALL and PISA

Table 4 ranks the mean scores for ten countries on the different mathematical measures in three international surveys: Quantitative Literacy in IALS, Numeracy and Problem Solving in ALL, Mathematical Literacy in PISA 2009. This ranking should be treated with caution because, as we have seen, Quantitative Literacy in IALS does not align with Numeracy in ALL and Mathematical Literacy in PISA is a broader measure than Numeracy in ALL and very broader than Quantitative Literacy in IALS. Also, the populations sampled differ in that PISA samples 15 year olds, not adults, as in IALS and ALL. Countries with the highest mean scores are listed in the first row and so on. Countries whose mean scores are not statistically different from each other are listed in the same row. The ten countries are Australia (AUS), Canada (CAN), Hungary (HUN), Italy (ITA), The Netherlands (NLD), Norway (NOR), New Zealand (NZ), Switzerland (CHE), the United Kingdom (UK) and the USA, where scores are available.

Table 4: Ranking of Mean Scores for Australia (AUS), Canada (CAN), Hungary (HUN), Italy (ITA), Netherlands (NLD), Norway (NOR), New Zealand (NZ), Switzerland (CHE) and USA. Note: *, #; §; ∞; No significant difference between these groups of means

<table>
<thead>
<tr>
<th>IALS</th>
<th>PISA 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative Literacy</td>
<td>Mathematical Literacy</td>
</tr>
<tr>
<td>ALL Numeracy</td>
<td>ALL Problem Solving</td>
</tr>
</tbody>
</table>

NOR NLD USA; AUS; CAN; CHE; 25
UK; NZ; HUN

NLD; CHE NOR NZ; CAN; AUS; HUN USA ITA

NLD; NOR CHE
NZ; CAN AUS HUN ITA

CHE; CAN
NLD; NZ; AUS NOR; UK; HUN; USA ITA

The highest-scoring countries that we are considering, The Netherlands, Norway and Switzerland, seem largely to hold that position in all these scales, apart from Norway’s score in PISA 2009. Italy is consistently low in ALL (in Numeracy and Problem Solving) and in PISA 2009 (in Mathematical Literacy).

However, there seem to be inconsistencies in New Zealand’s ranking on these scales. It appears that New Zealand’s score in Quantitative Literacy in IALS is lower than one might expect, judging from the scores in the other three scales. For example, there is no significant difference between New Zealand and the UK in IALS in Quantitative Literacy but New

25 The overall mean for Switzerland has been estimated by taking the mean of the scores for the three main Swiss language groups (French, German and Italian).
Zealand scores significantly higher than the UK in PISA 2009 in Mathematical Literacy. Also New Zealand scores significantly lower than the USA in IALS (Quantitative Literacy), significantly higher than the USA in ALL (Numeracy) and also in PISA 2009 (Mathematical Literacy). Another inconsistency is that New Zealand scores significantly lower than Australia in IALS (in Quantitative Literacy) but there is no significant difference between these two countries in ALL (Numeracy) and PISA (Mathematical Literacy) and a small, but significant, difference in favour of New Zealand in ALL (Problem Solving).

The ranking and significant differences between the other seven countries seem to be similar in ALL (Numeracy and Problem Solving) and in PISA (Mathematical Literacy). The USA scores are significantly below New Zealand, Australia and Canada in ALL (Numeracy) and in PISA 2009 (Mathematical Literacy) but, in IALS, the USA score is not significantly different from those of Australia and Canada and is significantly greater than New Zealand’s score. New Zealand has changed its relative position with respect to Australia, Canada and the USA in the surveys reviewed here. The mean for Australia and Canada in the IALS Quantitative Literacy scale was significantly higher than New Zealand’s mean on this scale but, for the Numeracy scale in ALL, there was no significant difference between the means for New Zealand, Australia and Canada. In addition, the USA’s mean for the Quantitative Literacy scale in IALS was also significantly higher than New Zealand’s mean on this scale but, in the ALL survey, New Zealand’s mean for the Numeracy scale is significantly higher than that for the USA.

As we have stated before, it is not possible to know if the UK would have changed its position in these international rankings since it did not take part in the ALL survey. In any case, given the trenchant criticisms of IALS noted above, it behoves us to treat the IALS results, in particular, with caution. It seems that these results may support the views of Alain Blum, Harvey Goldstein and France Guérin-Pace who cast doubt on the validity of IALS and indicate that the IALS survey “cannot be used on a comparative basis”. The results above seem to illustrate that survey data in IALS is “not amenable to simple scale comparisons between countries” (Blum, et al., 2001, pp. 243-244). Even more caution must be applied to attempts to compare between surveys.

Comparative distribution of levels in the adult population for Quantitative Literacy in IALS and Numeracy in ALL in eight countries

The fact that the Prose Literacy scale and the Document Literacy scale used in the IALS and ALL surveys were very similar has allowed researchers to investigate changes in these measures from 1996 to 2006, in particular, and to compare these changes in New Zealand with those in Australia, the English-speaking parts of Canada and the USA (Satherley, Lawes, & Sok, 2008). One result was, for example, that “within the New Zealand adult population, the subpopulation with very high (Levels 4 or 5) Prose Literacy skills has shrunk, and the subpopulation with very low (Level 1) Prose Literacy skills has shrunk substantially” from 1996 to 2006 (Satherley, et al., 2008, p. 7). In addition,

for all four countries [New Zealand, Australia, Anglophone Canada and USA] the subpopulations with very low (level 1) Prose Literacy and very high (levels 4 or 5) Prose Literacy has shrunk. This pattern exists across most of the countries that took part in both the IALS and ALL survey.

(Satherley, et al., 2008, p. 16).
Because the Quantitative Literacy scale in IALS and the Numeracy scale in ALL are very different measures, we cannot compare the mean scores for these scales, so it is not possible to investigate the changes in mean scores over ten years, as Satherley et al. have done for Prose and Document Literacy in IALS and ALL (see above).

We could instead ask the question: did the distribution of Quantitative Literacy/Numeracy skills in New Zealand change between 1996 (IALS) and 2006 (ALL)? Figure 8 compares the histograms for eight countries for the Quantitative Literacy scale in IALS (OECD & Statistics Canada, 2000) and for the Numeracy scale in ALL (OECD & Statistics Canada, 2011) using data supplied in these reports.

![Figure 8: Comparison of proportions at each level for Quantitative Literacy in IALS and Numeracy in ALL: Proportions (%) in Levels 1-2 and 3-5 for The Netherlands (NLD), Switzerland (CHE), Norway (NOR), Canada (CAN), Australia (AUS), New Zealand (NZ), Hungary (HUN) and the USA – ranked by proportions in Levels 3-5 in ALL.](image)

It appears that for Australia, Canada, Norway and the USA the Numeracy measure in ALL resulted in a higher proportion of the adult population (aged 16-65) achieving at Levels 1-2 than occurred for Quantitative Literacy in IALS. Conversely, the Numeracy measure in ALL resulted in a smaller proportion of the adult population (aged 16-65) achieving at Levels 3-5 than occurred for Quantitative Literacy in IALS for these four countries. For New Zealand, The Netherlands, Switzerland and Hungary little change occurred in these percentages.

It seems that the change of scale (Quantitative Literacy in IALS to Numeracy in ALL) may have affected the results for some countries. The largest change is 13% less in the proportion of the adult population at Levels 3-5 in the USA from IALS to ALL. This seems to indicate that the differences in Figure 6 and 7 between seven of the eight countries we are comparing may be negligible. Any differences seen in Figures 6, 7 and 8 could be the effect of the
change of the scale used in each of these surveys, lending more support to the concerns of Alain Blum, Harvey Goldstein and France Guérin-Pace, that survey data in IALS is “not amenable to simple scale comparisons between countries” (Blum, et al., 2001, pp. 243-244).

Conclusion

Our review of evidence from the surveys outlined above has convinced us of the difficulty of comparing the results of different surveys, largely because of their different target populations, sets of countries and scales used. As Hantrais (1995) warns, in interpreting results, “findings should be examined in relation to their wider societal context and with regard to the limitations of the original research parameters”.

Nevertheless, it is clear from our analysis that New Zealand and the UK both have a problem of low levels of what might be termed adult numeracy, as measured in the surveys we have reviewed. An international comparison of initiatives to address this problem seems to be worthwhile.

Accordingly, in further work, we plan to look at upskilling interventions in the UK and New Zealand, including standards, curricula, assessment regimes, initial tutor training and continuing professional development, as well as the policy context and organizational and other relevant factors.

In doing so we shall be mindful of Hantrais’ advice to replicate the research design and use the same concepts and parameters simultaneously in both countries on matched groups, while remaining alert to the dangers of cultural interference, ensuring that discrepancies are not forgotten or ignored and being wary of using what may be a sampling bias as an explanatory factor (Hantrais, 1995, passim).

We envisage that our bi-national study might form part of a larger international comparative study of adult numeracy, capitalizing on the international connections we have made through ALM and our other contacts.
References


Houtkoop, W., & Jones, S. (1999). Adult numeracy: An international comparison. In M. van Groenestijn & D. Coben (Eds.), Mathematics as Part of Lifelong Learning:


Lane, C. (2011). *Factors Linked to Young Adult Literacy*. Wellington, NZ: Ministry of Education.


Being competent in mathematics: adult numeracy and common sense

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There is a plethora of literature regarding the concept of numeracy. For some scholars, numeracy concerns to a set of basic skills involving the ability to carry out easy computations or arithmetical operations. For other scholars, numeracy points out to a more broad perspective that involves a range of knowledge, skills and supporting processes that enable adult to manage mathematical demands in a range of different real life situations. Van Groenestijn and Schmitt (2000) define numeracy as the link between the knowledge gained in a mathematics classroom and students’ ability to handle real-life situations requiring mathematical knowledge and skills. According to Colleran and O’Donoghue (2000), numeracy refers directly to real world situations. They clarify that these situations may be meaningful for learners. This is the same concept underpinning Gal’s definition (2005). Coben (2002) clarifies that we need to distinguish between adult numeracy learning, adult numeracy as skill, adult numeracy education and adult numeracy practice, because we risk to adding to the confusion surrounding the term. Benn (2001) connects numeracy with the idea of active citizenship.

In this paper we draw on data coming from a European research project titled Inbalance. The main aim of this project is to design and disseminate a European Numeracy Framework to validate adults’ competences in mathematics. We have elaborated a five level scale drawing on three different variables: type of context (real life situation and problem transparency), type of information (all tasks are open ended, as is in real life situations), and type of operation (van Groenestijn, 2002, Gal, van Groenestijn, Manly, Schmitt and Tout, 2003). We used a survey and personal interviews with adult learners in order to validate such a scale. During the process of validation, some interesting issues regarding numeracy and common sense arose. In this paper we further explore how school mathematics may (or may not) interfere with mathematical competence in order to understand real situations.
adding to the confusion surrounding the term. Benn (2001) connects numeracy with the idea of active citizenship.

In this paper we draw on data coming from a European research project titled *Inbalance*. The main aim of this project is to design and disseminate a *European Numeracy Framework* to validate adults’ competences in mathematics. We have elaborated a five level scale drawing on three different variables: type of context (real life situation and problem transparency), type of information (all tasks are open ended, as is in real life situations), and type of operation (van Groenestijn, 2002, Gal, van Groenestijn, Manly, Schmitt and Tout, 2003). We used a survey and personal interviews with adult learners in order to validate such a scale. During the process of validation, some interesting issues regarding numeracy and common sense arose. In this paper we further explore how school mathematics may (or may not) interfere with mathematical competence in order to understand real situations.

**Inbalance: an attempt to clarify numeracy for adult learners**

*Inbalance* is a research project lead by Roc Midden Nederland, supported by the European Union. The main objective of this study is to create a European framework in terms of numeracy, to measure the different levels adult learners may have regarding their mathematical competence. This project establishes the standards in numeracy in line with the priorities of the Common Quality Assurance Framework (CQAF) (European Commission, 1995) by improving the quality of teachers in mathematics for adult learners.

*Inbalance* comes out from a deep discussion regarding the meaning of numeracy. One of the scholars involved in this study, Mieke van Groenestijn, wrote her Ph.D. dissertation on numeracy26. After going through a number of previous works, she came with a broad definition of numeracy, grounded on a realistic approach close to the one lead by the Freudenthal Institute (2011). Drawing from this approach, *Inbalance* defines numeracy as a particular behaviour involving mathematical skills to manage a situation or solving a problem in a real life context. In this regard, numeracy involves actions such as identifying and locating, acting upon (order / sort, count, estimate, compute, measure, model), interpreting and communicating. Of course, numeracy is more than just “do basic arithmetic”. Numeracy involves quantities, numbers, dimension, shapes, patterns, relationships, data, chance, change, and an extended list of mathematical topics. From a teaching approach, it is important to pay attention to the type of information used for teaching mathematics. Teachers may use objects, pictures, numbers, symbols, formulas, diagrams, maps, graphs, tables, texts, etc. to introduce a new task.

Behind this picture, *Inbalance* assumes that numeracy concerns four different domains: everyday life, work-related, societal or community and further learning. Numeracy occurs everyday. People face numerate situations all the time. We need to bring into operation our competences on numeracy to take decisions, to manage tasks, about work, hobbies, personal development and interests. The list of everyday tasks involving numerate behaviours could be endless: handling money and budgets, shopping, time management, travelling, planning holidays, assuming risks, dealing with uncertainties and chance, playing games, sport, reading statistics on the newspapers, using measurements at home, cooking, paying the bills, and more. As Niss (1995) wrote in his work, mathematics is always present, even if it appears to be “invisible”.

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Numeracy also has a work-related dimension, which is crucial. According to van Groenestijn et al. (2011), work-related situations “often are more specialized than those seen in everyday life.” (p. 5) Van Groenestijn and her colleagues mention different situations such as completing purchase orders, totalling receipts, calculating change, managing schedules, budgets and project resources, using spread sheets, organizing and packing different shaped goods, interpreting control charts, reading blueprints, tracking expenditures, predicting costs, etc. In order to be able to manage this type of situations, people need to use their mathematical skills.

But this is not the only space or domain where mathematics (numeracy) is needed. Numeracy is also a competence to develop a critic citizenship (Skovsmose, & Valero, 2022). Our current societies are full of examples of people taking agency in their societal and community environments. Cases such as the revolutions in the Arab countries, the Indignados movement in Spain, the protests such as the 15-M movement in London, Paris, New York, Madrid, Barcelona, Tokyo, and over 300 other main cities around the World, are just a few examples that illustrate how people takes lead in terms of agency. Numeracy becomes also an essential component in order to organize and manage these activities.

In fact, numeracy also open the door for further learning, since most of the times learning opportunities demands a solid numerate background.

Recent research (van Groenestijn, 2002; Coben, O’Donoghue, & FitzSimons, 2000) and results from projects based on numeracy such as Mathematics in Action (van Groenestijn & Lindenskov, 2007) suggest that numeracy is considerably improved by using a realistic and systematic methodology based on the needs of individual learners and their everyday practices. Drawing on this prior knowledge, Inbalance introduces a concept of numeracy based on realistic situations. Instead of seeing Mathematics as an abstract set of concepts and rules, the concept of numeracy is presented as a human activity. Learning should provide adults with “guided” opportunities to use their own knowledge of mathematics to solve “real” situations. Inbalance provides more than a hundred exercises based on a number of different real-life situations, such as shopping, banking, currency change, travelling, cooking, measuring distances, reading the statistics in the news, etc. In all exercises numeracy comes as a resourceful set of skills to “mathematize” the situation. The challenge presented to adult learners is to successfully go from the “horizontal mathematization” to the “vertical” one (Treffers, 1978, 1987).27

Method

This paper comes out from the fieldwork conducted in Spain under the frame of the research project Inbalance. The fieldwork is part of the process of validation of the European Numeracy Framework (ENF) developed in that project, according to the criteria presented in the last section of this paper. The target group were adult learners in the European countries involved in this study: The Netherlands, Austria, Finland, Spain, and United Kingdom. In

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27 Treffers (1978, 1987) defines horizontal mathematization as a type of situation in which students use their mathematical competences to organize and solve the problem contextualized in a real-life situation. In our field (adult mathematics education), this points out directly to the concept of “previous learning” or “mathematical background” that all adult learners already have. Vertical mathematization is the process of reorganization within the mathematical system itself, that is: making meaningful connections between mathematics concepts and ideas. This is the goal so far, in terms of adults learning mathematics: try to guide them / help them to make these meaningful connections. In terms of Freudenthal: “horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols.” (Freudenthal, 1991).
order to set up a reliable basis for the quantitative analysis of the data, the members of the team conducted a stratification of the sample based on gender, age, number of years of formal schooling and country of residence at the time of the survey. Figure 1 summarizes the process of stratification.

<table>
<thead>
<tr>
<th></th>
<th>NL</th>
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<th>FINLAND</th>
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</thead>
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<td>Y1</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Y2</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Y3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<tr>
<td>Y4</td>
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<td>20</td>
<td>20</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>275</td>
</tr>
</tbody>
</table>

Figure 1. Stratification in country validation samples (absolute values).

In this figure we use the terminology $Y_n$ to indicate the number of years of formal schooling of the individual. Next figure clarifies the scale that we constructed in order to stratify the sample according to this criteria.

<table>
<thead>
<tr>
<th>Code</th>
<th>Criteria</th>
<th>ISCED skill level (UNESCO, 1997)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>Not yet at ISCED 1</td>
<td>ISCED LEVEL 1: Equivalent of the level of skill obtained at the successful end of “primary education which generally begins at ages 5-7 years and lasts about 5 years.”</td>
</tr>
<tr>
<td>Y2</td>
<td>ISCED 1 but not more ISCED 2</td>
<td>ISCED LEVEL 2: Equivalent of the level of skill at end of first stage of secondary school up to the age of 14 or 15 years.</td>
</tr>
<tr>
<td>Y3</td>
<td>More that ISCED 1 but not yet ISCED 2</td>
<td></td>
</tr>
<tr>
<td>Y4</td>
<td>ISCED 2 but not more ISCED 2</td>
<td></td>
</tr>
<tr>
<td>Y5</td>
<td>More than ISCED 2</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. The International Standard Classification of Education (ISCED).

Definitions of ENF skill scale and codes aligned with ISCED skill levels. Source: van Groenestijn, Kanes & Diez-Palomar, 2011.

According to these criteria, ENF discriminates numerical behaviours on a scale of 5 different levels, in which levels 2 and 4 are aligned with ISCED level 1 and 2 respectively. For this reason, $Y_2$ and $Y_4$ are the two main groups in terms of sample stratification.

To validate the ENF the members of the team constructed a validation instruments consisting of 20 pencil-based items. Items were chosen from a bank of items prepared by writers using reasoned methods. Writers used a 5-level scale to create the items included on the bank. This 5-level scale was constructed based on van Groenestijn theoretical approach to numeracy presented in the last section of this paper. According to the three different types of complexity variables associated with numerate behaviour, items should range from a level-1 degree of complexity, up to a level-5 degree of complexity. The levels of complexity result from the sum of difficulty in terms of type of content, type of information and type of operation.
involved in the situation presented to the learner. Figure 3 clarifies the algorithm created by van Groenestijn and colleagues (2011) to measure the numerate behaviour of individuals.

<table>
<thead>
<tr>
<th>level</th>
<th>Total # points</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC1</td>
<td>1-2-3</td>
</tr>
<tr>
<td>NC2</td>
<td>4-5-6</td>
</tr>
<tr>
<td>NC3</td>
<td>7-8-9</td>
</tr>
<tr>
<td>NC4</td>
<td>10-11-12</td>
</tr>
<tr>
<td>NC5</td>
<td>13-14-15</td>
</tr>
</tbody>
</table>

Figure 3. Notional Complexity scales associated with complexity variables.

The 20 pencil-based items were presented in two different tests (A and B). Adult learners were requested to complete the two questionnaires, with a little break in between of each other. Results obtained in both questionnaires were used to validate the consistency of the answers and the reliability of the instrument itself, according to the criteria used to build it.

In Spain we conducted the survey in two different groups (classrooms) of adult learners, in one school placed in Barcelona. Adult learning in Catalonia ranges from *neolectors* (new readers, people who are just getting into reading and writing), to *certificate* (people who are already confident enough in reading and writing, and then start to convert what is part of the curriculum in elementary education), and finally to *graduate* (aligned to the last strand in terms of compulsory school – middle school). These two groups were part of the *certificate* level. A total of 67 individuals participated in the survey. Learners ranged from 24 to 87 years old, mostly women. Half of them reported 6 years or less of formal schooling, although the mode was 10 years of formal schooling. Drawing on this information, the group of individuals involved in the Spanish sample surveyed were people with a “mid-level” of schooling proficiency, that is: with a certain degree of formal schooling experience.

During the survey conducted in Spain, an interesting and unexpected trend arose from adult learners’ responses to one of the 20 items presented in the questionnaires. The item requested the learner to figure out the exact time displayed on a watch at a later period. The item presented the image of a watch, showing 7h 20min o’clock. There were three more pictures on the item: Vienna, Salzburg and a train in between these two cities (see the figure 4). The question was: *At what time is the traveller in Salzburg?* Another additional sentence completed the whole item, providing the crucial information for the adult learner to be able to answer correctly the question: *Travel time 2 hours 40 minutes.* We expected an addition as solution of this problem.
The answers of the participants in the survey to this question were unexpected. The correct answer involves a “unit conversion” from minutes to hours. However this did not happen. For this reason, I organized an additional session (a debriefing) with the same groups of adult learners to provide them feedback on their answers, and discuss about this particular item. During the session we went back to the questions of the questionnaires A and B. We conducted a session with each group. The structure of the session was in both cases the same. First, I requested adult learners to solve the problems proposed in each item. They had a 2-5 minutes time slot to work together before giving the answer. Then, time for clarification followed. I asked learners to explain and justify their answers. Finally, I came up with the correct answers. We did the same with all the items in both questionnaires. Sessions were videotaped. Special attention was paid to the controversial item about the watch and the unit conversion from minutes to hours.

Results and discussion

During the fieldwork to validate the ENF 5-levels scale created within Inbalance to measure individuals’ numerate behaviours we collected dozens of questionnaires filled by adult learners. Our goal was to compare the results obtained in test A and B, to analyse the consistence among them. Any difference between both questionnaires may lead us to either doubt about the right answer on the items used, or the correctness of our scale. For this reason, we were really focused on “discovering” these types of inconsistencies as indicators of such “errors”.

After a few questionnaires were analysed, nothing was unusual, and data seemed to confirm that our model was working as was supposed to do. However, looking more carefully to the learners’ answers, something started to claim my attention. Questionnaire after questionnaire, I started to see the same answer to the same item: 9:60 hours as the answer to the following question: *At what time is the traveller in Salzburg*, when this traveller starts to travel at 7:20 hours and the trip longs for 2:40 hours? What brought my attention up was the second part of
the answer: sixty minutes. I was surprised because I was expecting 10 hours, not 9:60 hours. The same answer came up from many different questionnaires. Almost everybody did the same “mistake”. Next two figures illustrate one of the most common responses from adult learners involved in this survey.

Figure 5. Antonia’s survey. Item *At what time is the traveller in Salzburg?*

Figure 6. Maria’s survey. Item *At what time is the traveller in Salzburg?*
These two figures suggest that Antonia and Maria28 though in the same way, to solve the problem. They came up with the idea to do addition. In order to apply this operation, they wrote down just the numbers, without considering the meaning of these numbers. The fact that neither Antonia nor Maria wrote down the word “hours” (or “minutes”), insinuate that they thought on the problem as a “typical” problem of addition, thus they used the classic algorithm mechanically, without making any type of meaningful connection with the context of the problem. It seems that Antonia and Maria focused their thinking on the techniques, not on the meaning.

Maria and Antonia illustrate the most common type of answers of these two groups of people. Less people tried to use in some way the information about hours and minutes. This is the case of Joaquina (see figure 7). Joaquina did the same kind of calculation as Antonia and Maria did. However, she wrote down the word “hours” (horas), which suggest that in certain way she was aware that she was operating with units of time, even though the use of addition did not produced the right answer. In fact, the second part of the answer (the 60) was the problematical one. During the debriefing all participants (adult learners) were happy with the first part of their answer: the 2 (hours). This was not problematic. Second part was trickier. What does it mean to have 60 minutes? As soon as I asked that question, several people on the audience came up with the same answer: “Sixty minutes means one hour”. Thus, when the question is direct, without context, the answer is so clear, and no doubts appear. However, the mystery of the first answer (9:60) was still in the air.

28 All names used in this paper are pseudonyms, to protect the real identity of the persons involved in the fieldwork conducted.
My first reaction to this unusual answer was “did they copy the answer one from each other?”, “did they cheat?” It was hard to think such a thing, because this type of behaviour is not common among adult learners who come to the school to learn because they want, not because they are forced to, and in addition they do not have any special requirement or pressure to pass any kind of test or examination. It was even harder to think such a thing, because people in this school are encouraged to share their thoughts, and discuss about them. Hence, “to copy” was not on the possibilities. Thus, what was the reason for that “mistake”?

Drawing on the prior literature in mathematics education, I knew that unit conversion is a tricky topic, which provokes hard times to learners (Voigt, 2002). Students tend to build on routines to cope with the complexity of mathematics activities. Teachers drawing on realistic or reform-based mathematics, use to spend some time negotiating the meanings of the different procedures to solve the activities of unit conversion. This is not new and it is well know in our field. We also know that learners (children education) use to provide the answers that they think the teacher is expecting, hence sometimes children does not really think about the mathematical concept, but they do what the teacher “says” they have to. According to the scientific literature in education, this is one of the main facts that distinguish children from adults in terms of education: adults use to draw on their previous background, to solve the problems. So, for them, learning involves also sense making. It is really difficult for an adult learner to learn something without establishing meaningful connections to previous knowledge or experiences. For this reason, adults’ answers use to be more “different” from what is a “formal” answer (the type of answer that someone could find within a textbook, for example). One of the main arguments from authors working in adult mathematics education is that school should provide to adult learners this “other type of knowledge” more academic, because they already have the non-formal or informal one.

However the answer to this item was amazing because it suggested that sometimes adults also follow the same patterns as children: they also learn the techniques in a mechanical way, without making sense to them. They were doing the sum as a decontextualized one. 7:20 plus 2:40 was 9:60. That’s all. Nevertheless, 9:60 in terms of time does not make any sense. What does it mean 9:60? What kind of hour is that one? Why not one, but so many adult learners wrote down their hand-outs the same answer? It was maybe chance? Was it on purpose? If so, why? All these questions came to my mind, and I did not have any data to be able to suggest an answer. For this reason I decided to conduct additional work on that, and I went back to the same people, to ask them “why”. Next quote illustrates adult learners’ answers to my questions, looking for figure out the reason of such mistake:
1. Javier: This is a trip that last two hours and forty minutes... So he says at what time he

2. Arrives at Salzburg.

3. Antonio: At ten, at...

4. Josefina: Nine... nine... and

5. Antonio: At ten o’clock.

6. Josefina: It is forty, fifty and sixty. Or at nine seventy?... Nine sixty!

7. Antonio: At ten o’clock...

8. Javi: Nine sixty...

9. Josefina: Nine sixty, isn’t it?

10. Antonio: Nine sixty means ten!


13. Javi: Aha, this is what I wanted to tell you, because almost everybody gave this answer.

14. It is OK, it is Ok... but you said 9:60.


16. Josefina: Yes, because we were in class with Vicente the other day...

17. Antonio: I was not here that day.

18. Josefina: ... about seconds, degrees, and all that...

Josefina provides a hint that explains why almost all of them did the same answer. In line 16 she says “Yes, because we were in class with Vicente the other day”. I was asking what does it meant, that sentence. Josefina told us that during the last few weeks, Vicente (the teacher) was teaching them the units to measure the circle, in terms of angles. Vicente explained that they were doing a big number of activities involving transformations from minutes to degrees, working with the number system in base sixty. Somehow, when they saw the question related to time units, some of the people went back to the procedures they were using fro a number of weeks now, and used them right without thinking (mechanically). That’s why their answer was mathematically correct, but with no sense in real terms.

However, this possible answer to this behaviour was not entirely satisfactory to my understanding of the phenomenon. What was behind this “easy” way to apply the mathematical procedures? Why almost all learners produced the same answer? Even more important, why almost nobody questioned that answer and asked what does it mean? Why all of them seemed to be satisfied with the answer? No much information came out from the debriefing.

Conclusion: Are adult learners “institutionalized” by the school?

In this paper I presented an example of a session with adult learners working with “real” situations. Drawing on previous literature, real contexts for mathematical activities should provide a meaningful learning, especially with adult learners which already have plenty of experiences in their daily lives. According to van Groenestijn (2002) real situations are the best fit for adult learners to promote them to learn mathematics and develop their numerical skills. Research provides evidence that suggest this connection between learning and real life (Colleran, & O’Donoghue, 2000; Gal, Groenestijn,
Theoretical approaches to learning also suggest that connection (Flecha, 2000). However, here is a counterexample of that statement: a group of adult learners having troubles with an activity framed in a real context. Most of the adult learners were not able to go from the horizontal matematization to the vertical one (Treffers, 1978, 1987). It did not happen until the entire group went back to the question, during the debriefing, and though all together about the most common answer. The dialog among all participants makes explicit the connection between mathematical concepts and procedures, and real meaning of the answer. After a while, all learners understood the situation, and some of them were also able to suspect the meaning of to use a numerical system in base sixty, while making calculations and converting quantities from one unit to another one.

The “mistake” reported in this paper is really common also when working with children. There is a big amount of literature that reports how children use to apply the mathematical rules and procedures without thinking on the real meaning of what they are doing, but just using them mechanically. Doing to the special nature of adults’ learning (it is supposed to be more base on meaningful connections, rather than “memorization”), we expected this not happen in this field. Nevertheless, it does. Some authors claims that there is a process of “institutionalization” that may explain such a behaviour. After a while, students (children) tend to provide what they think are mathematically correct answers (answers that try to follow either teachers’ or books’ examples). However, we know that sometimes even if the calculations are right, the answer does not makes any sense (what does it mean to have a negative surface after using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve $x^2$ in an $x^2 + ax + c$ equation?). The example reported in this paper suggests that adult learners may also experience “institutionalization”, as children do. However, not all types of questions have the same potential to provoke “mechanical” answers. Adults involved in the Inbalance workshop answered 20 items, and only one was provoking what I call “institutionalized” answer. According to Plaza and his colleagues, (2004) adults may better solve problems when they are able to “see” (make visible) the question, rather if the problem is so abstract. Real situations help learners to make questions more concrete (and related to their daily life experience). However, the example provided in this paper suggests that there this is not always true. Many questions come out of that assert: what kind of questions / items provoke “institutionalized” answers? Why adult learners provide a scholar answer rather than build on their own previous knowledge about real life? Why they feel happy with an answer that does not makes sense in real life terms? Why the item on unit conversion provoked this mathematical behaviour, and not the other 19 items? Those are some questions that arose from this paper, which needs further research. This paper opens a range of new studies with the potential to extend our knowledge about how adults learner mathematics, to better improve our practices in this field.
References


Ottawa: Statistics Canada. (pp. 137-191).


Family Math for Adult Learners

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Margarida César, University of Lisbon, Portugal
Maria Lo Cicero, GRIM, University of Palermo, Italy
Benedetto Di Paola, GRIM, University of Palermo, Italy

This paper will address families’ involvement in their children’s learning of mathematics. We present evidence from parents and other relatives regarding learning mathematics in Portugal, Spain, France, United Kingdom, and Italy, drawing on FAMA’s findings. FAMA, Family Math for Adult Learners, is a project funded by the EU Commission, which seeks to explore what the best practices are for promoting the involvement of parents and other relatives in learning-support activities with their children, in order to help them to improve their performance in mathematics. FAMA aims to create a European network for discussing these kinds of issues. Drawing on prior research, we know that parents (as adult learners) face different challenges when supporting their children with mathematics. Some parents deal with the changes in teachers’ methodologies for teaching mathematics (Jackson & Epstein, 2006; Remillard & Jackson, 2006). They draw on their prior experience to help their children, but they discover that nowadays, teachers use “new” strategies to solve problems or to do calculations without using algorithms (Díez-Palomar, Menéndez, & Civil, 2011). Many parents have a “clear” social representation about what “mathematics” is for them, and “what” are they expecting from the teachers. Finally, there is also an important number of parents who struggle with mathematics, because a long time has elapsed since they were learning mathematics, and now they are not able to mobilise resources and knowledge about how to solve equations, deal with fractions, and do pre-calculus, among other topics (Allexshat-Snider, 2006).

What is FAMA?

FAMA – Family Math for Adult Learners, is a research project funded by the European Union under the GRUNTVIG programme. FAMA came into existence to promote successful actions in the field of family mathematics education. This field of knowledge and practice is receiving increasing attention, because of the changes which have occurred in the last decades regarding the importance of knowledge to our society, and the importance of having a high quality education to be able to cope with the current challenges in our society.

Recent theoretical contributions in the field of educational sciences argue that we live in a time in which learning is not purely individual anymore. People learn both inside and outside the classroom, by reading books, but also by interacting with peers. Learning has escaped from the classroom as a privileged educational transmission site (Aubert, et al. 2008). We are surrounded by spaces saturated with information. In order to learn what Newton’s binomial means, we probably look for information in a textbook, in our
notes, we ask a friend for help, but also we have a look on Wikipedia, we search for some video clips on Youtube, or we visit educative websites looking for further explanations. In other words, now knowledge and access to it, has become diversified. In this context, families are experiencing big challenges.

Children still look for help with their math homework at home. A few decades ago, when a child had a problem with his math homework, he used to ask his father or his mother, for answers. The parent either pick up the book and review the examples with the child until s/he understood them or they avoided that responsibility. Now things are different. Parents from different regions in the World are said to complain that mathematics now is so different from the mathematics they remember learning in the school. Books are also not helpful or perhaps excessively difficult and unclear. When they open the Internet, the breadth of information is such that they need to ask for a guideline to help them decide what is valuable and what is not.29

The teaching of mathematics has undergone major transformations in recent decades. In fact, this field is relatively new, as evidenced by the work of Cantoral and Farfán (2003). Mathematics education has experienced some unprecedented changes as suggested by the amount of different current theoretical approaches, methodologies, etc. existing today. From mid-20th Century up to now, mathematics teaching and learning has experienced continuous reform. These changes explain the differences in teaching approaches used by teachers today, compared to those used only a generation ago. When families complain that they do not recognize the methods and strategies used to teach mathematics to their children, what they are referring to is the curriculum change on the one hand, and perhaps, the negative effects of time causing us to forget some of what we learnt in school on the other. In any case, families around the World seem to complain that they cannot, or lack the know-how, to help their children with mathematics.

Why FAMA?

According to data provided by the Michigan Department of Education, children spend 70% of their waking hours (including weekends and holidays) outside of school. That the participation of families in the learning process of the children is a predictor of success or failure is something that has been known for decades. In 1984 Walberg published a study concluding that family participation in education is twice as predictive of students’ academic success as family socioeconomic status. Some of the more intensive programs in that field had effects that were 10 times greater than other factors. In fact, the earlier in a child’s educational process family involvement begins, the more powerful the effects. When parents and other relatives are involved, their children have higher grades, test scores, and graduation rates, better school attendance, increased motivation, and fewer episodes of violent behaviour. The impact of family involvement in the education of their children justifies our interest in this area of work. As we have seen in the previous section, the constant changes in the field of mathematics teaching and learning, have added to many families feeling that they are not able to help their children at home (Diez-Palomar, & Kanes, 2012). For this reason, FAMA – Family Math for Adult Learners, draws together the efforts of many authors who have worked to create resources that will enable families to provide this help.

29 FAMA provides a database with learning resources (in mathematics) for families. Further information: http://www.familymath.eu
However, not all forms of family involvement help children to improve their academic achievement (INCLUD-ED, 2009). Sometimes families attend schools just to complain, or to generate an atmosphere or to confront the teachers. This type of engagement has become one of the most popular of the Family involvement strategies as illustrated even by the non-academic media. The success of this form of involvement in the academic performance of children is controversial. The latest studies show that families who focus their involvement on promoting children’s efforts and working together with them, obtain better results in terms of children’ performance, than families who avoid working together with their children. It is more likely for children whose families are engaged to end up with good grades, rather than those who do not have this support.

There are several ways in which families can support their children’s learning of mathematics. During the last twenty years, several researchers have dedicated their lives to studying how parents help their children, and what the most effective ways to do that are, from the point of view of school success. One of the most prominent authors to have studied this relationship is Joyce Epstein (2002). For many years, Epstein defined five types of family involvement viz., parenting, communicating, volunteering, learning at home and decision-making. Table 1 summarizes these five types of involvement.

Table 1. Types of family involvement according to Joyce Epstein

<table>
<thead>
<tr>
<th>Type 1 - Parenting</th>
<th>Assist families with parenting skills and setting home conditions to support children as students. Also, assist schools to better understand families.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 2 – Communicating</td>
<td>Conduct effective communications from school-to-home and from home-to-school about school programs and student progress.</td>
</tr>
<tr>
<td>Type 3 – Volunteering</td>
<td>Organize volunteers and audiences to support the school and students. Provide volunteer opportunities in various locations and at various times.</td>
</tr>
<tr>
<td>Type 4 – Learning at home</td>
<td>Involve families with their children on homework and other curriculum-related activities and decisions.</td>
</tr>
<tr>
<td>Type 5 – Decision Making</td>
<td>Include families as participants in school decisions, and develop parent leaders and representatives.</td>
</tr>
</tbody>
</table>


Research in this area has continued, and the need to review previous classifications relating to family involvement in education is becoming increasingly clear. One of the main results obtained in INCLUD-ED, the current research with greater resources and higher scientific quality in Europe, is the classification of the different forms of family involvement. Researchers from INCLUD-ED point out that the type of involvement that produces more success, from the standpoint of academic achievement and future academic opportunities, is what they call educative participation. They define this type of involvement as family and community members participat[ing] in students’ learning activities, both, during regular school hours and after school. Family and community members participate in educational programmes which respond to their needs (INCLUD-ED, 2009). The success stories presented as evidence show cases in which families and schools worked together to achieve better learning opportunities and outcomes.  

30 It is well-known Emmanuel Chaunu’s caricature about two parents and a teacher, looking critically at a child, because his bad grades. Fifty years later, in 2009, the same picture: two parents, a teacher, and a child, with bad grades. But now, parents are looking critically at the teacher, because the bad grades of their child.

31 Further information: http://creaub.info/included/
Table 2. Types of family involvement according to CREA

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<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Informative</td>
<td>Parents are informed about the school activities, school functioning, and the decisions which have already been made. Parents do not take part in those school decisions. Parents’ meetings consist of informing families about these decisions.</td>
</tr>
<tr>
<td>2. Consultative</td>
<td>Parents have a limited impact on decision-making. Participation is based on consultation with families. They participate through the school’s statutory bodies.</td>
</tr>
<tr>
<td>3. Decisive</td>
<td>Community members participate in decision-making processes by becoming representatives in decision-making bodies. Family and community members monitor the school’s accountability in relation to its educational results.</td>
</tr>
<tr>
<td>4. Evaluative</td>
<td>Family and community members participate in students’ learning processes through helping evaluate children’s school progress. Family and community members participate in the general school evaluation.</td>
</tr>
<tr>
<td>5. Educative</td>
<td>Family and community members participate in students’ learning activities, both during regular school hours and after school. Family and community members participate in educational programmes which respond to their needs.</td>
</tr>
</tbody>
</table>


This combination of ways of including families, community and school is crucial to increase, in an effective way, the results of the larger part of students, not just the ones who already have good scores. Epstein and colleagues have added a new type of family involvement to their previous scheme (see table 1). This is what they call Collaborating with the community, defined as: [to]coordinate resources and services from the community for families, students, and the school, and provide services to the community.32

Family involvement in the mathematics education of their children, in Europe

The discussion about family involvement is the more and more common. As we have mentioned in the last section, there is a plethora of evidence showing the correlation between family involvement and academic performance. In one of the most popular forums for discussion regarding mathematics education, a reference to an article in Time Magazine was published. In this reference, Annie Murphy Paul (2011) presented an article entitled Tiger Moms: is Tough Parenting Really the Answer? In this article, Murphy Paul compared the involvement of families in USA and China, two countries with different results in the last round of PISA. According to Murphy Paul, Chinese children obtain better scores because they are educated within the idea of hard work and responsibility. In contrast, children from Western countries are freer, their parents are more permissive, even lazy sometimes, so children’ scores are lower in international assessments such as PISA. The international discussion worldwide is focusing on the type of parent (family) involvement, rather than on what really makes children successful. FAMA analyses what these practices are, that really produce success.

<table>
<thead>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>1970</td>
<td></td>
<td>1990 – 1993</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Many laws between 1918 to 1945</td>
<td>1970</td>
<td>1980s</td>
<td>1990s</td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
<td>1985</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>1974</td>
<td>1993</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td></td>
<td></td>
<td></td>
<td>1994 – 1996</td>
</tr>
<tr>
<td>Liechtenstein</td>
<td>1971 - 1972</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td></td>
<td></td>
<td>1988</td>
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</tbody>
</table>


**France**

Over the years, there has been a modification of the definition and structure of families and social codes in France. A family with a mother and a father plus the kids, is not the only recognized definition. Actually, many different arrangements can be considered e.g. single parents, remarried parents, and gay families. This societal change (Beck…) increases the lack of mutual understanding that already exists between families and the school.

Considering those facts, it is important to establish a better cooperation between schools and families to facilitate a mutual understanding of their needs, of both the school and parents. Schools need parents to be more involved with their children’s education and this can occur only if parents better understand school’s expectations and teaching methods.

We think that Families have the feeling that schools exclude them when they want to get involved. Concrete solutions should be found to facilitate a dialogue and
opportunities to exchange views, besides scheduled meetings. Actually, meetings are not favourable to easing dialogue.

Parents should be a “teaching” interface between schools and children. But due to a lack of skills and knowledge, and due to problems they face understanding school methods, their involvement with children homework is weak. Nowadays, methods of teaching are far from what parents experienced when they were studying. The lack of a good relationship between parents and schools could be explained by a lack of communication between schools and families. Improving this relationship could help parents understand better, what is expected and how teachers educate at schools (Thin, 2009). Actually, in France, teachers expect a greater involvement of parents with their children’s education, not only regarding homework but in everyday life. They consider that parents should integrate educational activities when spending time with their children.

When looking closely at family mathematics involvement with their children in France, both literature and resources show that French parents often ask for external help such as tutoring, after-school classes, private teachers, students, etc. The kind of involvement and external help sought also depends on different factors e.g parents’ economical and social class, education level, and the children’s school level.

Considering studies, it seems obvious that there is a link between parental involvement in children’s mathematics and their social level, as well as their involvement and their children’s school level. According to a study by Marie Gouyon on behalf of INSEE (French National Institute of Statistics and Economic Studies) in 2004 – “L’aide aux devoirs apportées par les parents” (“Parents’ homework support”) - children from primary and secondary schools benefit more frequently from parents’ help than children in higher grades.

Whatever their social backgrounds, the study shows that mothers spend more time helping their children [with mathematics], than fathers. When they are involved, it is mostly as a substitute for mothers. On average, parents support their children around 15 hours per week.

Half of the mothers with no or low qualifications, feel overwhelmed regarding their children’s homework (starting from elementary school). With qualified mothers the percentage is only 14%. But even if they are overwhelmed, 82% help their children. Mothers with low qualifications tend to spend more time helping their children that qualified mothers and this may be due to the fact that low qualified mothers mostly have part time jobs.

Whatever the amount of time spent helping their children with homework, around three-quarters of parents feel satisfied regarding the support given to their children. But even if they feel satisfied, 1 parent out of 5 often feels not having had enough skills to help their children. This feeling is linked to parents’ school level. More than half of mothers without any qualifications declare having a lack of competencies when helping their children, when only 5% have a higher school diploma. This gap remains clear when the child is attending high school, where half of qualified mothers and all of non-qualified mothers feel overwhelmed. It is the same among fathers.

The level of Parents’ support falls during the period of scholarship while the support of other relatives rises. Thus, 31% of students in elementary school receive help from siblings, friends or grandparents, increasing to 43% in secondary school and 47% in high school.
Another important homework support in France is tutoring. There are different kinds of tutoring e.g. free classes, after school tutoring with a teacher, paid for courses, and parents who pay a teacher or student to come to the home to help their children with their homework. On average, 9% of students receive tutor support.

In elementary schools, tutoring is mostly oriented to French classes while in secondary schools it is mainly mathematics’ support.

Demand for tutoring support comes from 25% of parents and 15% students. This demand can also come from teachers in 6 cases out of 10 cases.

The kind of tutoring chosen by families depends on the parents’ social level. Families attending school tutoring sessions are mostly from low social classes (working class families, unemployed parents with low or no qualifications and single parents). Those attending paid for courses are mostly from a higher social class (teachers, executives, merchants or heads of companies).

In 2001, the French government set up a Local Agreement for Scholarship Accompaniment (CLAS). This program is developed along 2 axes i.e. on the one hand, students (from elementary school to high school), and on the other, families.

After school, students have access to a tutor to help them with their homework, as well as cultural activities. Families receive help regarding their children’s scholarship and can also get involved with different activities. This program aims to give families a better understanding of the school’s expectations and methods:

This plan contributes to after school support for 170,000 children and young people with their personal homework, through homework support actions, methodological explanations, cultural activities and, more generally, provide an extra pedagogy aiming to give them confidence. Families find hospitality, advice, and accompaniment through the different steps of scholarship and, if they want to, they can get involved in coaching actions.

They (employees) intend to facilitate the relationship between families and the School, to help parents following up and understanding their children’s scholarship

(CATONI – circulaire interministérielle).

Another successful experience in France is CRAVIE - a study and action plan on parenting support.

The regional working group comprised socio-educational workers initiated by CRAVIE and who worked for 2 years on “the active participation of parents to their children’s scholarship success within the school field – support to parenting”. They based their research and actions on the fact that families feel devalued and powerless regarding their children’s scholarship and this is especially true among low social class families where children challenge their parents’ knowledge and skills. Parents want their children to succeed but cannot help them. This has implications for the educational legitimacy parents have when facing their children.

Children, specifically from this social field, challenge their parents’ knowledge and know-how. This fact is accentuated through the fact that society does not value their competences, this brings us to consider the legitimacy of the educational role of parents facing their children.
When developing experimental actions, with an active participation of the School to inform and motivate parents in order to set up an active partnership among them, the result is positive. Parents have access to the school codes and therefore can transmit it to their children. Parents / school relationship is better and contributes to student scholarship success.

**Italy**

Family involvement in education, in Italy, has been produced in many ways, very different from each other. There are various experiences and actions which have resulted even in educational movements, such as Reggio Emilia, which is known internationally.

Family involvement or participation has been conceptualized in Italy as *partecipazione* (New, & Cochran, 2007). This concept illustrates the active participation of families, teachers, children and other members of the educative community in the activities and decisions related to learning and teaching in Italian schools. New and Cochran (2007) place its origin in the late 1960s and early 1970s. According to them, *partecipazione* was connected with another term *gestione sociale*, meaning “community-based management.” The latter concept refers to the sharing of responsibilities in the management of educational institutions and the services offered to all members of the educative community, not only teachers, but also including families, caregivers, and other members of the educative community. This form of participation emerged as a way of democratizing public services in Italy.

*Partecipazione* is one of the main features that define Reggio Emilia (Hall, Horgan, Ridgway, Murphy, Cunneen & Cunningham, 2010). According to one of the scholars that has studied this successful action, Carlina Rinaldi (2007), Hall, Horgan, Ridgway, Murphy, Cunneen and Cunningham (2010) define *partecipazione* as “the sharing and co-responsibility of families in the “construction” and “management” of the *nido.*” (p. 71). As these authors write, *partecipazione* is much more than just “participation” in the English sense of the word. It is much more closely related to the idea of partnership with families, in the sense defined by Epstein (1995, 2002), since it includes all the elements listed by her (see table 1). More explicitly, *partecipazione* is:

… an educational Project that has as its base and as its principal objectives, relationships, communication and solidarity, characterized by dialogue and Exchange, where the presence of the families is as essential as the children’s and staff’s role as protagonists, where the necessary co-responsibility of the educational process is created. School must be a place of participation and Exchange, and education an on-going process of dialogue and listening among children, teachers and parents.

(Hall, Horgan, Ridgway, Murphy, Cunneen & Cunnigham, p. 72, quoted from Ghiardi, 2002, p. 33).

Malaguzzi, in the conference of the Gruppo Nazionale Nidi held in 1984, stated:

… family participation was not a choice but part of the identity of the *nido*, the children’s right beside being the parent’s right. Deference of and
expansion of services could only come about with family understanding, solidarity and support, achieved by parents coming to the nido not to be instructed and educated on parenthood but to bring their parental knowledge. They would then see the nido as a place where value was attributed to them and they could attribute value to childhood as a social and cultural heritage.

(Hall, Horgan, Ridgway, Murphy, Cunneen & Cunningham, p. 73, quoted from Rinaldi, 2007, p. 26)

As time passed, the partecipazione became institutionalized and began losing its open and participatory sense. In 1974 a regulation was established by law for this type of family involvement. As explained by New and Cochran (2007), this regulation came to formalize a situation that was running for many years i.e. the loss of spontaneity in the organizational forms and ways of involving families. With this law, the Committee created a council where families are represented through an election process in the school. Only in towns were the sense of participation has remained strong, the idea of partecipazione still survives.

Portugal

In Portugal, the involvement of families is managed by the Confederação Nacional da Associaçao de Pais (CONFAP). Participation is often reduced to the representation on school boards. This lack of real opportunities for participation has a long history in Portugal. A study in the early 1990s, focused on the relationship between schools and families, created a typology of countries in Europe according to the degrees / opportunities for families to participate in the education of their children (Bogdanowicz, 1994). Portugal appeared in the group of countries with fewer opportunities for families to participate. This is a trend that still remains nowadays (Silva, Monteiro, & Moreira, ).

While in most EU countries the participation of families in education has gradually been incorporated to the current educational laws, in Portugal this process has taken a long time, and is relatively recent (see figure 2).

The relationship between school and families is a multifaceted relationship. The implications from parent-teacher meetings at school include, for example, the set of activities that students do in their homes (school assignments, homework, extra work, etc.). This is where controversy appears, because the activities that families conduct at home are more difficult to keep under the control of the school. In addition, families often complain about the lack of support from teachers to enable them to better support their children. However, the relationship using mail, written notes, phone calls or informal conversations at the front door of the school are relatively frequent in Portuguese schools.

The involvement of families in the teaching of mathematics began to change during the 1990s, especially because of the influence of the methods applied in English-speaking countries. Drawing on the reform of mathematics, family involvement became more regular in those countries. Teachers adopted the use of worksheets, and families were supposed to answer the activities presented within the worksheets, together with their children. Such methods are still used even today, as in the curriculum ‘Everyday Math ‘for example. In Portugal, during the 1980s, a program called Parceria Escola – Familia – Communidade was launched. One of the main activities of this program was to promote families’ collaboration with teachers in teaching mathematics. Families were
requested to solve the activities presented in worksheets, with their children, at home (Moreira, & Sampaio, 2000). This project has continued to operate since then. At the beginning of the 21st Century, teachers started to promote practices such as the use of school libraries to engage families in learning the type of mathematics that their children are learning in the school.

This project has continued working since then. At the beginning of the 21st Century successful actions were implemented, such as the use of school libraries to engage families in learning Mathematics using the same educative approaches as teachers use with their children. Such practices have demonstrated improving students’ grades.

Additional training for families is another successful action that has been underway in Portugal since the late eighties. Examples are projects such as *Mathematics, Parents, children and Teachers*. The main aim is to engage families in the process of mathematics teaching both at home and at school. Teachers provide workshops for families in order to teach (to share) their teaching methods with families. A number of different projects have been conducted following that objective, such as *Parental Involvement in the Core Curriculum*, for instance. The type of activities that are usually conducted at home focus on a range of mathematical topics, as summarized in figure 3.

**Figure 3. Mathematical activities conducted by Portuguese families at home.**

<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation of measures related to household items</td>
</tr>
<tr>
<td>Study of figures using design tools</td>
</tr>
<tr>
<td>Reading math textbooks</td>
</tr>
<tr>
<td>Problem solving</td>
</tr>
<tr>
<td>Recollection of data to bring it to the classroom as part of a school project / task</td>
</tr>
<tr>
<td>Mathematical games to play within family</td>
</tr>
<tr>
<td>Preparation of schemes regarding issues addressed in the classroom</td>
</tr>
<tr>
<td>Homework</td>
</tr>
</tbody>
</table>


**Spain**

In Spain, family involvement has a long tradition, although systematic involvement regulated by law is relatively new.

At the beginning of the 20th Century there is already an example of family involvement as a common practice in one of the more successful educative centres at that time: the *Institución Libre de Enseñanza* (Free Institution of Education). This school was a landmark in Spanish education between 1876 and 1936, when the fascist regime in Spain almost killed ILE. One of the educational principles of this centre was the cooperation between families and teachers in the educational process (Núñez, & Servat, 1988).

Civil War and subsequent dictatorship was a step backwards for Spain in many ways, and education was one of them. After this historical period of time, during the
democratic transition in the mid-seventies, people in Spain met in enthusiastically
democratic organizations, such as neighbourhood associations, and other grassroots
groups. The Ley General de Educación [General Law of Education] passed in 1970,
established the roots to define family involvement within the school. Many families felt
the opportunity to engage in a more active way within the learning process, taking
decisions regarding the curriculum and other components of the teaching methods.
However, the first enthusiasm quickly led to a power confrontation between teachers
and families, resulting in the creation of the Asociaciones de Padres de Alumnos –
APAs – [Parents’ Students Associations]. Family involvement became to be managed by
these organizations. In a sense, the creation of the APAs was a way to intercept
families’ involvement in the schools, because since that moment, all families and other
relatives or caregivers’ demands, started to be managed by a board of elected
representatives. Hence, it became the representatives the ones who assumed the
relationship with the school board. This process of bureaucratization was the starting
point for the loss of motivation that many families felt regarding participation, because
the APAs were relegated to manage extracurricular topics, such as the type of food
provided in the school for lunch, organization of the school parties, recreational
fieldtrips, etc. The academic dimension of the educative process remained under
teachers’ control. Teachers requested families to attend individual appointments to talk
about their children’ behaviour. Family involvement is reduced to this individual
relationship dominated by the teacher. This type of participation has been called
informative participation (INCLUD-ED, 2009).

As a consequence, the process of bureaucratization produced an important decrease of
family participation (Álvarez, 1987). After the political reform of the educational law
(LOE, 2006), families consolidated their role as participants within the educative
process through APA’s (now AMPAs – Mothers’ and Fathers’ Students Associations).
Their competences are out-of-school activities, holidays, and a number of collateral
issues such as “wearing uniform or not”, meals for the lunchtime, etc. However, parents
do not have the possibility to participate within the teaching and learning process.
Nothing related to curriculum is checked with them. This type of decisions belongs to
teachers.

This has been the major type of family involvement in Spain, for the last decades.
However, for the last 10 years, some critical movement has begun to problematize
family involvement in Spain. A number of different actions have emerged all over the
Spanish territory. Some regional governments started to include family-school
relationships within their agenda. There is much evidence of this change.

However, the case that better illustrates this transformation is Learning Communities.
The first one was created in 1978, in an adult school, and it has taken nearly 10 years for
this action to be implemented in elementary education. In the mid 1990s, three schools
in the Basque Country decided to become Learning Communities. Since then, the
transformation has been unstoppable. Families' participation in school practices totally
changed. The sense of solidarity and radical democracy from earlier times came back,
but with a new approach viz., dialogic participation. Families, as well as other people
(educative actors), worked together with teachers. In this way, they became involved,
not just within the school activities, they transformed the school itself, which became
part of their personal lives too. Among Learning Communities there is a plethora of
examples of decisive, evaluative and educative participation (INCLUD-ED, 2009).
Bureaucratic barriers are broken, and new ways to manage participation are created,
which are more efficient and transparent. The international scientific community has
confirmed the impact of this transformation. The students' performance in these Learning Communities improved dramatically (Díez-Palomar, Gatt, Racionero, 2011; Elboj, & Niemelä, 2010; García, Mircea, & Duque, 2010; Díez-Palomar, & Flecha, 2010). This is also the case of the Mathematical Workshops for Families. In the last years many examples have been conducted in some regions in Spain, such as Catalonia, with very positive results (Díez-Palomar, Menéndez, & Civil, 2011).

The current increasing participation of families within educative actions starts a new target for ALM: the teaching for family members (adult learners) who attend adult schools looking for help in order to better support their children at home.

United Kingdom

Family learning in U.K. draws on the concept of family literacy originated in the United States. In England, the initial focus for family learning appeared as a strategy to improve children’s learning, in the earlier 1990s. This process culminated with the National Literacy Strategy. According to Mallows (2008), “Family literacy also grew throughout the 1990s, after the Adult Literacy and Basic Skills Unit (ALBSU) imported the Kenan model\(^{33}\) of family literacy from the United States.” (Mallows, 2008, p. 6). During 1994, ALBSU implemented four demonstration programs in different deprived areas in England and Wales. According to Brooks et al. (1996), the programs produced significant gains in the literacy achievement of both children and families. From then, family literacy practices started to recognize the importance of family literacy practices involving both children and parents.

As stated by Mallows (2008), “in the UK, in the years 1994-96 a co-funding scheme of grants to support some 400 smaller programmes was instituted by ALBSU, and in 1995 its remit was extended to include supporting the development of effective programmes in literacy, language and numeracy for children and young people and changed its name to the Basic Skills Agency (BSA).” (Mallows, 2008, p. 7). These programmes were conducted in a number of different settings, such as family centres, baby clinics, day nurseries, libraries, after-school clubs, travellers’ sites, playgrounds, churches and homes. Many different actors were involved in the provision of family training e.g. schools, adult community colleges, further education colleges, voluntary organisations, educational business organisations, community associations, social services and healthcare organisations.

Drawing on these experiences, family training in UK included a set of components, namely: a) opportunities (for children’s literacy development), b) recognition (in their literacy practices), c) interaction (with children to develop their literacy), and d) modelling (of their own literacy practices).\(^{34}\)

Findings obtained in United Kingdom indicated that the educational background of adults with a care responsibility for students are positively correlated with the learning support school students receive in their studies of mathematics. Nevertheless, among

\(^{33}\) The “primary goal of the model programs is to break the intergenerational cycle of illiteracy, under education, and poverty by improving parent’s skills and attitudes toward education, by improving the children’s learning skills, by improving parent’s childcare skills, and by uniting the parents and children in a positive educational experience.” (Darling, & Hayes, 1989, p. 9) The components of this model are a) parent literacy training, b) parenting and parent education, c) early childhood education, d) human-resources development, and e) evaluation methods.

\(^{34}\) This model was developed by Hannon & Nutbrown (1997).
adults surveyed in the frame of the FAMA research project, there is a strong motivation within families of lower educational background, to support their children’s learning. Interestingly however, it was not the need for a stronger mathematical background itself, but the kinds of pedagogical approaches that should be adopted in the home, that were most frequently identified as barriers in providing the support needed.
References


Moreira, A.G., & Sampaio, M. (2000). A parceria entre a escolar, a família e a comunidade: a descoberta da matemática e a dinamização da biblioteca como formas de envolvimento


Adult Students' Response to Images of Mathematics in Advertising

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Learners' affective responses to mathematics have long been a central theme in research in adults' mathematics education. One important set of influences on these responses has been the images of mathematics circulating in society. In an ongoing study we have extracted advertisements containing mathematical images from the UK daily press (Evans, Tsatsaroni & Staub, 2007); the first phase of this work involved both a quantitative analysis of the advertisements found, and semiotic readings of selected advertisements. This paper reports on the next phase, where adult students in a university, divided into those who were considered to be 'acculturated' to mathematics and those who were 'not', were interviewed for their responses to a sample of the advertisements found in the first phase.

Key words: images of mathematics; advertisements; acculturation to mathematics; positioning in practices; affect; emotion.

Adults’ Mathematical Thinking and Emotions

It is inappropriate to believe that there is such a thing as the cognitive power of a particular person. We operate well or badly in learning, and more especially in problem-solving, according to the drive provided by our emotions. Reason is powered by emotion ...


People’s beliefs and feelings about mathematics are influenced by ‘the experience of formal mathematics education … [and] stereotypes reinforced by popular media, or personal expectations conveyed … by significant others such as peers and close relatives.’

(FitzSimons, 2002, p.45; emphasis added).

A number of researchers have studied how media images condition the identities of learners and potential users of mathematics (Mendick, Moreau & Epstein, 2007). Our earlier work focussed on images of mathematics in film and advertising (Evans et al., 2007). In this paper I focus on advertisements, since they are clearly a powerful cultural tool in ‘mature capitalism’, with many resources dedicated to producing them and an acknowledged ability to attract attention (Leiss, Kline & Jhally, 1990), – and they are also reasonably easily accessible for analysis.
The overall project has been planned to comprise three phases:
1. Content analysis of advertisements (n=76) extracted from our sample of nine daily UK newspapers monitored daily during Sept. – Nov. 2006 and Jan. – March 2008.
2. Analyses of interviews with students (n=24) representing members of ‘target audience’ for selected advertisements.
3. Interviews with advertising industry practitioners (account planners, ‘creatives’), currently ongoing.
This paper reports initial findings from Phase (2).

Theoretical Considerations and Research Questions
In this paper, I present illustrative results for Phase (1), and a fuller range of results for Phase (2).

Research Questions for Phase (2)
RQ1 To what extent are ‘images of mathematics’ in selected advertisements perceived as such by audiences?
RQ2 What kind of reactions, including emotional ones, are evoked by these advertisements?
RQ3 How are readers/consumers ‘positioned’ vis-à-vis mathematics by these advertisements?
RQ4 How may these various responses vary according to ‘acculturation to’ mathematics, and by social position (e.g. social class, gender, age group, etc.)?

Rather than using the imprecise term ‘affected by’ in RQ3, we have used the term *positioned*, to specify the way that a particular adult is ‘located’ in a given situation. Here we are interested in understanding an adult who is reading an advertisement, and, in the case of the interviews, being asked about his/her reactions. Within a discursive approach (Evans, 2000), *positioning* includes both cognitive and affective aspects (thinking, attitudes, ideas on a situation) and access to material resources (social support, money, time, and power).

Similarly, the term *acculturation* (Bishop, 1988) is now used in preference to the less well-defined term ‘familiarity’ (discussed in Evans, 2000, pp.219-20), to indicate an appreciation of, and an alignment with, the ideas and values of a discourse.

Methodology
At the stage of extracting advertisements, we had to specify our criterion for selecting an advertisement as having a ‘mathematical image’ (Evans et al., under review). Our criterion was the presence of (one or more of) the following:
Figures 1 and 2 give examples of advertisements selected from the newspapers in the sampling periods.

Figure 1. Doughboy advertisement for Intel and Hewlett-Packard (Times, Oct. 2006)

For Phase (2), we conducted semi-structured interviews, which included the presentation of four or five of the advertisements found in Phase (1), and some life-history questions; the approach was broadly modelled after that used by Evans (2000, pp.135-149). The advertisement in Figure 3 (exceptionally) had not been located as part of the main sampling procedure but was substituted in most of the interviews for Figure 1, as being simpler and more likely to evoke a range of reactions from the audience – and hence, as more appropriate. The set of questions posed to interviewees for each of the selected advertisements is given in the Appendix.
As representatives of adults who might form the ‘audience’ for the advertisements selected for the interview, we chose university students. Students are normally interested in, and responsive to, advertising, and they are easier for university-based researchers to recruit than are members of the general public. The university chosen was a ‘post-1992’ (former polytechnic) in London, known to have a diverse student body, which therefore would be closer than many HE student bodies to representing the population of adults in the U.K.

The non-acculturated students (n=12) were volunteers from students attending the university’s Numeracy Support section, approached by the tutor, who was also doing the interviews. The acculturated students (n=12) were students of final year degree programmes in statistics and economics, who were approached by a post-graduate student, who did most of these interviews.

It was expected that this rather ‘purposive’ sampling procedure would produce differences between the two groups of students if the concept of acculturation had any predictive validity.

![Figure 2. Lift Off Rocket advertisement for Early Learning Centre (Daily Mail and Times, Oct. 2006)](image-url)
Results

Here we present a selection of results.

Quantitative Analysis of the Advertisements

The first advertisement to be shown in the majority of interviews (18 of 24) was the BIC Infinity advertisement; of course, when addressing interviewees, we did not call it that, since the point of Question T: ‘What do you think this advertisement is about?’ (see Appendix), was to see what the interviewee recognised in the advertisement; see Figure 3. (Note that the first six interviews conducted presented a different advertisement as the first to be considered by the interviewee.).

Figure 3. BIC Pen advertisement [http://adsoftheworld.com/media/print/bic_infinity]

The responses to Question T, for those that were shown the BIC advertisement, are given in Table 1. The numbers for this and all subsequent ‘quantitative’ analyses are of course small (n = 24 or fewer). Thus the results presented here can be considered as suggestive only.

Table 1. Crosstabulation of respondent’s recognition of mathematical content in BIC advertisement by mathematics background (Acculturated vs. Non-acculturated) (n=18)

<table>
<thead>
<tr>
<th>Maths background * BIC advert Recognition Crosstabulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Maths background Non-acculturated Acculturated</td>
</tr>
<tr>
<td>Total</td>
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<td></td>
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</tbody>
</table>
Here we see that one third (4 of 12) of the ‘acculturated’ students (mostly statistics majors) recognised the advertisement as possibility representing the symbol for infinity – compared with none of the 6 ‘non-acculturated’ students. In addition (and relatedly), one third of the non-acculturated students (2 of 6) recognised nothing mathematical, as compared with only one twelfth of the acculturated. Despite the small numbers, Table 1 suggests that there may be differences between the two groups in terms of recognising mathematical content in advertising images.

We now consider the feelings expressed by interviewees in response to the Rocket advertisement.

**Table 2. Crosstabulation of respondent’s feelings evoked by the BIC advertisement by mathematics background (Acculturated vs. Non-acculturated) (n=18)**

<table>
<thead>
<tr>
<th>Maths background * BIC advert Feelings Evoked Crosstabulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>Maths background</td>
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<tr>
<td>Acculturated</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

In Table 2, in contrast with Table 1, we find little difference between the two groups in terms of expressing feelings about the advertisement: 4 of 6 (67%) of the non-acculturated, 9 of 12 (75%) of the acculturated group.

**Researchers’ Reading of the Advertisement**

Different members of the ‘target audience’ for an advertisement may ‘read’ the advertisement in different ways. In this subsection, we give an illustration of one adult’s response to the Lift Off Rocket advertisement (Figure 2). But first we summarise our ‘semiotic’ reading of the advertisement as researchers (Evans et al., under review).

The advertisement from the *Daily Mail* and the *Times*, October 2006 clearly aims to promote the Early Learning Centre (ELC) brand, in particular the Lift Off Rocket toy. The two visuals (photo of the boy and ‘speech bubble’) link together the Rocket, which represents stimulation of the child, and the equation, which represents the child’s bright future. The strapline “Give him the Lift Off Rocket and who knows what he’ll grow up to be”, cleverly reinforces the link between the object and the parent’s hope – linked with anxiety – for the child’s future.
The role of the equation or formula is worth scrutinising. It seems rather ‘complex’, so can be taken to represent highly developed knowledge, thus supporting the brand’s claim to nurture children’s intellectual development. The formula seems plausible, though it is difficult to say if it is ‘true’ or ‘correct’. In any case, the formula represents achievement in mathematics / science, and doing well in mathematics / science is understood by the readers of these newspapers to contribute to doing well in life.

The advertisement addresses middle class readers (Times, Daily Mail). The readers of the advertisement are positioned as ‘knowing’ socially, if not scientifically. They know that it is possible to preserve or improve one’s social position through children’s education – and they aspire to that. In the background is the parents’ knowledge that many children do not do well in mathematics / science – so that can be a source of anxiety. However,... Early Learning Centre toys can help!

**An Interviewee’s Reading of the Advertisement**

Here we consider Interview 11 in more detail. ‘Dani’ (not her real name) was female, aged 35-44, and of ‘White British’ background. Her highest mathematics qualification was at GCSE (16+ exam), with grade C (a good grade). Her most recent employment was as a non-qualified teacher of art, and she also practices as a sculptor. Her course at the University was Postgraduate Certificate of Education in Art & Design, and to become a Qualified Teacher, she had to pass a numeracy test.

The interview begins with the first advertisement (see Figure 3).

(Interviewer) J: I am going to start off by showing you some advertisements, take your time to have a read of them and then I am going to ask some questions about them. She shows the student the BIC advertisement.

J [leaves 7 seconds for reading the advertisement]: What do you think this advert is about?

S: Well the first thought that I had is an 8 on its side. I think of 8 as a figure of change ...

(Interview transcript, p.2).

We see that the student “recognises” the BIC advertisement as being about the figure 8 – and attributes particular meanings to that – but does not mention the symbol for infinity (cf. Table 1).

Later, after showing the student two other advertisements, the interviewer shows the ELC advertisement.

S [takes 3 seconds to read]: Oh, look at the rocket – and I think he is going to grow up ...

[4 lines] ...
I look at this and I say that wouldn’t have been me – because not only, to have that going on in his head he must have come from a set of parents who’ve also had those combinations.…

[2 lines] …

Now when I look at this as a mum, I just think I am sick to death of people saying everyone has the fair…same opportunity to this – they don’t. I don’t believe that. Now what I am aware of also that that image is saying … [stumbles over words]… you can still try, that’s what that makes me feel. There’s an element of - you can still try! Who’s to say you can’t try!?

Now [...] I bet you [addressing interviewer / numeracy tutor] could work that out – you know there would be some formula where you could actually break that down and to me if that’s what mathematics is about – all those years – no one actually spelled that out, I am furious, I am furious to think no-one actually said look these are things we can learn because they’re methods, they’re formulas, they’re little languages – well I have missed out, I have missed out.... [2 lines] …

but I...I...I would like more of this [points to advertisement] in my life…

[1 line] ...

I would like to work out what the equations are…There is this child still in me, you know, that wants to build that rocket as a sculptor ...

[2 lines] …

I look at this [...] link from education to opportunities of your working life, you know I think this child could grow up and be rich. If they have got this in their head [points to equation], and this rocket and that environment to learn those kind of equations, he’s going to be up there with the league of scientists [...]

(Interview transcript, pp.55-6).

Discussion

The interview suggests that this student also sees the link, earlier visualised in the researchers’ reading, between the Rocket toy, representing an opportunity to stimulate the child, and the equation, representing a potentially bright future for the child. This interviewee clearly feels a parent’s anxiety, and hope, for the child’s future.

‘Dani’ moves through a series of positionings during the interview, and in particular, in the discussion of the Rocket toy. She was positioned by her family / social class background, as a child lacking support from her parents: “… my dad, I didn’t really see much of him with regards to my learning – my mum, always on the brink of anxiety …” (p.4, transcript). As a pupil, she has pleasant memories of primary school. But at secondary level, she is in difficulty and (seen as) ‘difficult’, when her teacher did not appear to like her; she now considers him to have ‘bullied’ her: “I get so upset, I have had to overcome some real development hurdles with getting to grips with mathematics” (transcript, p.7). Later in the interview, she experiences a fantasy of ‘stamping out teachers like that’ (p.8).
Thus, as an adult, she is able to re-evaluate her earlier relatively weak positioning as a pupil unsupported by parents or teacher, despite still regretting what she lacked then. She sees herself as having done her best at school, and can re-experience anger on behalf of herself as a child. She now speaks and acts from more powerful positionings (and related identities) in the several adult practices in which she participates: (i) as a confident artist / sculptor; (ii) as a reflective (though not yet ‘qualified’) teacher; and (iii) as a mother, determined to help her daughter in the ways she knows of. The resilience she shows is inspiring, and shows what many adults are capable of, given a chance.

In terms of our (researchers’) distinction between persons ‘acculturated’ and those ‘non-acculturated’ to mathematics, we see that she is preparing to cross the boundary on the basis of opportunities afforded by the university she is attending. This is another inspiring aspect of the interview with this particular adult.

Conclusions

1. In the quantitative analysis of the set of interviews (n = 24), we have been able to show some differences between students who are ‘Acculturated’ (to Mathematics) and ‘Non-acculturated’ students; however, given the small sample size, these results must be considered as suggestive only. In the qualitative analysis, we have displayed some affinities between the researchers’ reading of one of the advertisements presented in the interview and the experiences and perceptions of one of the interviewees.

2. The interview (no. 11) was chosen on the basis of the clarity of the views and feelings expressed, and the interest of what ‘Dani’ said. It shows how such interviews can help to describe and to interpret adults’ reactions to a given set of advertisements, in this case including images of mathematics. However, among the limitations of our approach, we must acknowledge that the process is somewhat reactive - that is, the choice of advertisements will arguably condition the sort of talk elicited from interviewees. The five advertisements shown to the majority of the interviewees here (of which Figures 1 and 2 are examples) are such as to appeal to, and to be emotionally moving for mature students who have already become parents. However, this is not inappropriate for a sampling strategy that aims to recruit interviewees that will be broadly representative of the adult population at large.

3. Further research in this approach will include:
   - interviews with advertising practitioners, on the processes leading to the creation of such advertisements (Stage 3 of the present project, currently underway)
   - investigation of other media fields for their images of mathematics (cf. Mendick et al., 2007)
   - similar studies of the prevalence of mathematical images in other settings, which can be carried out using the kinds of analyses described here.

4. Uses for teaching and learning:
We can use such interviews (or similar) in at least two ways:

- to understand the contexts, especially the rich variety of these, that adult students bring to Adults’ Mathematics Education
- to illustrate for teaching the way that mathematics is used in one particular form of popular culture (Evans, Tsatsaroni & Staub, 2007; Williamson, 1978).

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References


Evans, Tsatsaroni & Czarnecka (under review). Mathematical images in advertising: Constructing difference, shaping identity, in global consumer culture.


Appendix. Interview Questions in Phase (2)

Here are the questions posed for each of the selected advertisements:

(Question T) What do you think this advert is about? [Interviewer shows advertisement]

(Contexting question C) Does this remind you of any of your current activities or experiences – in general?

(M) Would you say there are any images of mathematics or mathematicians or people using mathematics in the advert?

(F) What feelings do these images bring up for you?

(Contexting Question R) Now can you tell me about any sorts of earlier activities or experiences it may remind you of or any feelings it brings up?
This paper regards Ethnomathematics as a bridge to cultural dialogue, bringing implicit or explicit mathematical practices to the learning process, valuing personal qualities and life experience. In this matter, it will be presented as a graduating experience between Brazilian natives, that reflects the ways that prevail in their own culture, in a dialogue between their cultural mathematics concepts and the national curriculum. So this group of native teachers, gathering three different ethnic groups, learns and creates educational strategies, which remind their young students that they are active citizens that may use mathematics as other learning artefacts socially, politically and ethically to strengthen their culture.

In South Bahia, Brazil, a small city called Cumuruxatiba, Pataxó, Pataxó Hâ-hâ-hâe and Tupinambá natives, from different locations and with very different life histories, got together for the first stage of their intensive course. Instead of students going to the University, I found one University that went to the students regarding their needs, wishes, interests and difficulties.

As an outside participant, in a stage of a graduation course for Native School Educators, founded in Intercultural Education, I observed an ethnomathematics reality. From this experience, I understood, by the words and actions I witnessed, that the main purpose of this event is to value the culture of local and native people, helping them to reinforce and retain their values, behaviour and knowledge alongside Western society, by respect, solidarity and cooperation (D’Ambrosio, 2007).

The first step in this big journey is held with the children being taught by those native educators. In this sense, these teachers intend to know the multiple realities they live in and with, to create and apply strategies that promote in their students, the consciousness that “reality is conceived to be a dynamic, relational, dialectical system (…)” (Restivo, 1980, p.11), in which we are together, living with each other and not against each other. So it’s important to be open to dialogue in the way David Bohm (2003) presented it. In a way, all parts win. I observed in Cumuruxatiba that this dialogue in reality, was not always achieved but was permanently remembered and intended.

35 These are three of the main ethnic groups struggling to reinforce their culture in South Bahia – Brazil, since they inhabit in the area of the first and more intensive contacts with the colonizers, just in the heart of discovery coast.
Ethnomathematics and intercultural dialogues

This graduation programme emerged from a movement that began in the seventies with the substantiation of the perception of natives leading their own formal schooling, in a change of attitude. But only in the nineties did the State of Bahia have the first native teacher course.

Since then, many significant changes have occurred, and in Bahia, the year 2002 marked the strengthening of the dialogical movement between the natives and the government, passing through conflict to harmony. This has been a hard intercultural dialogue, nourished by the constant demands of the native people.

Native School Education

It is important to notice that school has now a very different connotation for those natives. It is not an imposed school, with subjects they don’t relate to. Now it is a way to achieve universal knowledge, and to organize their own knowledge, reinforcing their languages and cultures, before native and non-native people in an universal way.

Guiding lines

In this sense, there were established three main guiding principles for Native School Education:

1. A specific, differentiated and intercultural character, that promotes multiple languages studies

2. A continuous and contextualized process of teaching-learning, that reflects the local, regional and global realities in social historical levels, by reference to research-action methods of knowledge

3. Respect for cultural life experiences and specificities in the organization of school time and space.

The practices

Supported by the previous guidelines, the responsibility for this particular event (local university coordinators and native committee) chose to establish themselves in a small place near most of the student’s villages, and where they could develop the activities asked, in their own terms. Therefore, the ‘when and where’ are decided as a result of dialogue.

In this dialogical environment, we watched mothers with their sons in their lap analysing Ubiratan D’Ambrosio’s (1999) proposal of a new concept of curriculum, focused on literacy, materacy [numeracy] and technoracy [technology-literacy]. Understanding how these three strands of the Program Ethnomathematics reach for the communicative, analytical and technological instruments of each culture, reinforces critical thoughts with a dialogical purpose (Freire, 2008).
C., a Tupinambá girl says:

“This new vision [Ethnomathematics] has been shaping and constructing a new perspective of the learning process, and understanding allows us to write a new paragraph to this story, a more dynamic and objective one. (Free translation)”.

This dynamic and objective paragraph is translated in all actions e.g. the children’s presence, the work places, the permanent interaction and creativity, and the connections made between their traditional knowledge and ways, and the goals defended by Ethnomathematics. They said frequently how important it was to them to find a subject that supports their practices.

These indigenous educators showed extreme creativity in the concretization of all tasks, firmly presenting their critical analysis of the questions brought up in classes, revealing a strong acceptance of the D’Ambrosio proposal.

**Onça’s Game**

The Onça’s Game has been used to explain the numeracy concept. It was explained to me how important this game is for teaching the young to act as team. So the grandfathers teach their grandsons that if dogs (cachorros) hunt the onça in a team effort, they should achieve their purpose, otherwise they will be destroyed by the enemy.

For the natives, this is a very important tool for an essential lesson, in the same way, they think they should take this tool with different purposes. In other words, they think it is easier to teach the children geometry figures, angles and lines with a game they all know so well.

![Jogo da Onça](image)

**Figure 1, Onça’s Game**
When asked to reflect and construct an abacus, they all found very interesting and diverse ways to do it. They tried to find the best way to understand, and to explain to their students, mathematical operations with the resources they had at that time. The figures 2 and 3 show us a few examples of the work.

![Abacus Image 1](image1.png) ![Abacus Image 2](image2.png)

**Figures 2 and 3. Abacus.**

They all recognise as an important remark for their daily routines as teachers, the learn/teach experiences lived in Ethnomathematics classes. So F., a Pataxó Hã-hã-hãe, explains:

“My purpose for now on (…) is to identify, interpret, collect and systematize my people’s knowledge, and to work them with a different regard, acting like mediator and data organizer between my people, the school and the surrounding society”.

(Free translation)

At this level, CD., a Tupinambá, claims that this Ethnomathematics classes “(…) will provide a new attitude in the classroom.” And, she adds, “I learned how to construct new concepts, this course, although it lasted a short time, it made me realised several values, in relation to mathematics I learned in formal school.”. (Free translation)

Reading their testimonies after witnessing the reality they shared with me, I understood how important is to implement Ubiratan D’Ambrosio’s Program Ethnomathematic as a bridge between cultures, knowledge and men.

The education should be a bridge built by men, sustained by those teaching and learning in the field. As Marco Polo described to Kublai Kahn, in his adventures,” a bridge needs the stones, different sized and shaped stones. Even the best outlined arch needs stones to support itself”. 
References


The role of mathematics in engineering practice and in the formation of engineers

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It is commonly assumed that increasing the levels of mathematics competence, in general schooling and in engineering education, is a necessary precursor to solving the shortage of engineers. There is, however, little empirical research on mathematics usage in modern engineering practice. This paper presents the findings of a study that explores the use of mathematics in engineering practice in Ireland. Findings show that mathematical thinking has greater relevance than curriculum mathematics to engineering work practice. Problem solving skills and logical thinking are cited as key influencers of engineering work performance. Engineers attribute their own interest in mathematics to good teachers. The respondents confirm that their feelings about mathematics impacted their choice of engineering as a career.

Key words: mathematics usage; engineering practice; affect; mathematical thinking.

It is claimed that “engineering has never mattered more” (Sheppard, Macatangay, Colby, & Sullivan, 2009). However while engineering expertise is seen as key to sustaining a modern technology economy and to the advancement of civilisation, the interest of young people to pursue careers as engineers has diminished, in western Europe and the USA in particular. This trend has been evident since more than 30 years and continues today.

Traditional engineering careers do not interest modern young people to the same extent as a half century ago. Technology usage and associated practices have changed significantly in the developed world. Also, young people’s ranges of interests, skills and activities have altered dramatically in the same period. Hence, there are good grounds to consider that the traditional understanding and even practice of engineering might be adjusted to be, at once, more valuable to modern industry and more interesting to prospective young engineering students.

Mathematics is perceived to be the “the key academic hurdle” in the supply of engineering graduates (Croft & Grove, 2006; King, 2008). In Ireland, too many students with demonstrated higher mathematical ability choose not to stick with numerate studies (Higher Education Authority, 2010) and there is a disaffection in students toward mathematics and, by extension, other numerate studies.

For decades, mathematics has been regarded as the fundamental knowledge underpinning engineering practice (James & High, 2008). However, there is a belief among some practising engineers that the mathematics they learned is not applicable to their work (Cardella, 2007). Also, with advancements in knowledge and technology, it
is asserted that teaching “engineers to think analytically will be more important than helping them memorise algebra theorems” (Katehi, 2005). This paper presents the results of a study that explores:

- the role of mathematics in engineering practice and
- the role of mathematics affinity (during school years) in the formation of engineers.

The paper is structured under the following headings:

- Background
- Methodology
- Data collection and analysis
- Findings
- Discussion
- Conclusions
- Future work

**Background**

This research was inspired by the observation that students’ difficulty with higher-level school mathematics is often blamed for the declining number of entrants to engineering degree courses (Prieto et al., 2009).

It is commonly asserted that mathematics is foundational for careers in technology and innovation (National Academy of Sciences, National Academy of Engineering, & Institute of Medicine, 2007). Technological competence is by far the most important source of economic growth and economic studies show that as much as 85% of measured growth in US income per capita is due to technological change (Borrus & Stowsky, 1997; Boskin & Lau, 1992; Boskin & Lau, 1996; Grübler, 1998; Solow, 1957).

In Ireland, mathematics achievement is a strong predictor of third level persistence (Mooney, Patterson, O’Connor, & Chantler, 2010). However, Ireland’s PISA performance in mathematics is below the OECD average score and declining (Perkins, Moran, Cosgrove, & Shiel, 2010). In Ireland, there are two state administered exams: the junior certificate at mid secondary (age 15) and the leaving certificate at completion of secondary (age 18). Students sitting these exams can choose either the ordinary level curriculum or the more advanced higher level curriculum. Participation in higher level mathematics in Ireland is low, with only 45% of junior certificate mathematics students and 16% of leaving certificate mathematics students taking the higher level paper (State Examinations Commission, 2011). Also in Ireland, less than 8% of entrants to level 8 honours degree programmes in university choose engineering and technology programmes compared to 34% choosing Humanities & Arts (Higher Education Authority, 2010).

Mathematics is generally perceived as a “special” subject. A survey of junior certificate students found that almost 60% found mathematics difficult and less than 50% found the subject interesting (National Council for Curriculum and Assessment, 2007). Compared to most other subjects, mathematics concepts are more abstract and students...
mostly consider mathematics study to be dissociated from society or human experience. For many, it appears mathematics may be seen as an uninteresting chore, where the more gifted can marshal some competence for the special purpose of passing an exam, but which inspires little desire to continue its practice in general life. Most students see mathematics as being about procedures and techniques rather than concepts and principles (Ernest, 2004; Greer & Mukhopadhyay, 2003; Sam & Ernest, 1998; Schoenfeld, 1992; Smith, 2004; Strawderman, date unknown).

The declining interest in engineering careers is evident in the dramatic reduction of CAO points (competitive points system for entry to third level education in Ireland based on leaving certificate grades) required for entry into level 8 engineering programmes in Ireland over the past 30 years. There is also a reported shortfall of engineers in Ireland, it is projected that the domestic supply of high-skilled computing and electronic graduates alone is “not sufficient” to meet future demand to 2013 (Forfás, 2008). In the UK, a requirement for an extra 587,000 engineers in the period up to 2017 is reported (Elliott, 2009). The United States faces a shortfall of almost two million technical and analytical workers over the next ten years (McKinsey, 2011).

“Mathematics is part of the life-blood of engineering” (James & High, 2008). Students wishing to pursue an engineering degree course are required to demonstrate proficiency in mathematics, in Ireland the requirement for level 8 accredited engineering courses is a minimum C3 grade (55-59.9%) in higher level Leaving Certificate mathematics. It is asserted that the minimum mathematics requirement contributes to students’ hesitancy to pursue an engineering degree course and there is a link between students’ experience of second level subjects and their perception of future careers. (Lynch & Walsh, 2010).

While many students have “no idea” what role mathematics will play in their future careers (Wood et al., 2011), most view engineering education as further engagement in school science and mathematics (Brickhouse, Lowery, & Schultz, 2000). “Some see mathematics as the gateway to engineering, paving the way for sound design; others see mathematics as a gatekeeper, denying entry to otherwise talented would-be engineers” (Winkelman, 2009). Many third level engineering students struggle with the mathematics in their courses (James & High, 2008), “it is now generally accepted that students entering the tertiary level suffer a lack of mathematical skills and no longer find mathematics to be an enjoyable subject … this decline in mathematical skills leads students to avoid overly analytical subjects in later years of degree programmes” (Irish Academy of Engineering, 2004).

Research concerning the type of mathematics used in engineering practice is sparse (Alpers, 2010b; Cardella, 2007; Trevelyan, 2009). According to Alpers (2010) there are only a few studies of engineers’ usage of mathematics because “they are not easy to conduct”. Of the few studies of mathematics in engineering, most use qualitative methodologies and many involve observing engineering students rather than studying practicing engineers. Difficulties with investigating mathematics usage of “real engineers” is that access to engineers is difficult and with many different branches and job profiles within engineering, there is “nothing like ‘the’ engineer” (Alpers, 2010a).
Methodology

This research explores the role of mathematics in the formation of professional engineers and in engineering practice in Ireland. A mixed methods study is employed, whereby an initial quantitative survey is followed by an explanatory qualitative stage. The results of the initial quantitative survey are presented in this paper. An initial quantitative approach was considered necessary as the professional engineering population in Ireland comprises a diversity of engineering disciplines, roles and functions in many different types of organisations (Engineers Ireland, 2011).

An online interactive survey instrument was used to measure (i) mathematics used in engineering practice; and (ii) mathematics affinity (during school years) in the formation of professional engineers. In Ireland professional engineers are awarded the professional title of Chartered Engineer. To achieve this title engineers must attain defined educational standards followed by the development of a broad set of competences which are assessed by Engineers Ireland.

Measuring mathematics usage

The role of mathematics in engineering practice is about engineers’ use of mathematics and the extent they engage with mathematics in their ordinary work. In this study mathematics usage is measured with respect to the following:

- Curriculum usage (de Lange, 1999; de Lange & Romberg, 2004).
- Thinking usage (Schoenfeld, 1992).
- Engaging usage (Csikszentmihályi, 1992; McLeod, 1992; Schunk, Pintrich, & Meece, 2010).

The methodology used to measure the use of curriculum mathematics, derived from de Lange’s mathematics assessment pyramid (de Lange, 1999; de Lange & Romberg, 2004), is shown in figure 1. Mathematics usage is measured with reference to three dimensions:

- Domain, this refers to the five mathematics domains specified in the new “Project Maths” syllabi in Ireland (National Council for Curriculum and Assessment, 2010).
- Usage type, the three main usage types are reproducing, connecting and mathematising (de Lange, 1999; de Lange & Romberg, 2004).
- Level refers to academic levels.
Type 1 usage (reproducing) is the usage of mathematics through knowledge of facts and concepts. Type 2 usage (connecting) is usage of mathematics by making connections within and between different mathematics topics and integrating information in order to solve problems where there is a choice of strategies and mathematical tools. Type 3 usage (mathematising) is usage of mathematics by extracting the mathematics embedded in a situation and using mathematics to develop models and strategies and translating mathematical models into real world solutions.

Type 4 (thinking usage) relates to mathematical modes of thinking used in the workplace (Schoenfeld, 1992) and type 5 (engaging usage) concerns the affective domain whereby it is believed that attitudes, beliefs, values and emotions play a central role in mathematics learning (McLeod, 1992). Self-efficacy is an important predictor of motivation (Bandura, 1986) and previous emotional experiences with an activity are important predictors of achievement (Schunk, et al., 2010). Failure at mathematical tasks reinforces fear and dislike of the subject, while success leads to pleasure and confidence and a sense of self-efficacy, the resultant improved motivation leads to more effort and persistence (Ernest, 2011). Csíkszentmihályi developed a theory of intrinsic motivation called ‘flow’ to account for feelings of a deep sense of accomplishment or enjoyment when a person succeeds in an activity that is perceived as difficult or worthwhile. The availability of too many choices today has increased uncertainty and reduced flow (Csikszentmihályi, 1992).

In this survey, engineers rate their mathematics usage on a Likert-type scale (1 = not at all; 2 = very little; 3 = a little; 4 = quite a lot; 5 = a very great deal).

Measuring mathematics affinity (during school) in the formation of engineers

In this part, engineers are asked to rate their enjoyment of school mathematics and how, in their view, young people’s affective engagement with mathematics could be improved. They identify the events, experiences, aptitudes or other factors within and outside of school that contributed to their interest in and learning of mathematics and
they rate the degree to which their feelings about mathematics impact their choice of engineering as a career.

**Data collection and analysis**

In conjunction with Engineers Ireland, the representative body for professional engineers in Ireland, the online survey was distributed by email to 5,755 (424 women) chartered engineers who have a minimum level 8 qualification or equivalent and at least four years professional experience. While every chartered engineer registered with Engineers Ireland was given the same opportunity to participate in the survey, it cannot be verified that engineers who volunteered to participate in the study comprised of a random sample. 365 valid responses, with 39 (10.7%) from women, were received from a variety of engineering disciplines, roles and positions as shown in figure 2. Despite being only a small fraction of the chartered engineering population in Ireland the response rate is within the sample size required for precision to within 0.15 units (on a Likert scale with five outcomes) and 95% probability that the findings from the survey questionnaire represents the population of chartered engineers in Ireland. The data was analysed using Minitab statistical software.

![Engineering Disciplines and Roles](image)

**Figure 2. Survey participants.**

**Findings**

There are three main findings:
- Mathematical thinking has a significantly greater relevance to engineers’ work compared to “curriculum” mathematics
- Engineers say that the way they felt about mathematics at the time of choosing a career path had a significant impact on their choice of engineering
- Affective variables are major contributors to interest in and learning of school mathematics.
Specific analysis of the engineers’ responses showed that:

- Almost a third (32.3%) of the responding engineers say that they could perform satisfactorily in their current job without higher level leaving certificate mathematics, figure 3.

Figure 3. Engineering performance without higher level leaving certificate mathematics.

- Engineers rate their “curriculum” mathematics usage in work in the range “very little” to “a little”. The “functions” domain is least used by the engineers while the “number” domain has the greatest usage. Domains include: statistics and probability (D1); geometry and trigonometry (D2); number (D3); algebra (D4) and functions (D5), figure 4.
Figure 4. Engineers’ mean curriculum mathematics usage by domain.

- Overall, engineers rate their mathematics “thinking usage” in the previous 6 months as “quite a lot”. Over the lifetime of their engineering careers, engineers’ thinking usage is highest when they were within 2 years of graduating. Thinking usage reduced when the engineers were within 3 to 5 years since graduating and there were further reductions in thinking usage when the engineers were within 6 to 10 years since graduating and greater than 10 years since graduating, respectively, figure 5. It is noted that thinking usage rates significantly higher than “curriculum usage”.

Figure 5. Mathematical thinking usage over the lifetime of engineering career.
• The strongest “modes of thinking” resulting from mathematics education that influence engineers’ work performance are problem solving strategies, logical thinking, critical analysis, modelling and decision, figure 6.

**Figure 6. Modes of thinking used in engineering practice.**

• The engineers rate the necessity for a specifically mathematical approach in work, the degree that they actively seek a mathematical approach and their enjoyment of mathematics in the range “a little” to “quite a lot”. Engineers rate their confidence using mathematics as “quite a lot”. The degree of engineers’ negative experiences when using mathematics is rated in the range “not at all” to “very little”, figure 7.

**Figure 7. Engaging mathematics usage.**

• 80% of the engineers surveyed enjoyed mathematics at school at the levels of “quite a lot” and “a very great deal”, figure 8.
• The three most popular views on how young people’s affective engagement with mathematics could be improved are: usefulness/practical applications & examples (24.86%), relevance to modern living (20.11%), and teacher/teaching (17.08%), figure 9.

• The main events, experience, aptitudes or other factors that contributed to the engineers’ interest in and learning of mathematics are shown in figures 10, 11, 12, 13, 14, 15, 16 and 17 and include:
Within school:

- Teacher (Primary and Secondary school)
- Success (Primary and Secondary school)
- Enjoyment (Primary and Secondary school)
- Easy subject (Primary and Secondary school)
- Recognition (Primary school and Secondary years 1 and 2)
- Practical applications (Secondary school years 1 and 2)
- Relevance to Science (Junior certificate and upwards)
- Competitions (Secondary school)
- Relevance to engineering / careers (Leaving certificate).

Figure 10. Factors within primary school contributing to interest in mathematics.
Figure 11. Factors within secondary school (years 1 & 2) contributing to interest in mathematics.

Figure 12. Factors within secondary school (Junior Certificate) contributing to interest in mathematics.
Figure 13. Factors within secondary school (Leaving Certificate) contributing to interest in mathematics.

- **Outside of school:**
  - Family/Parent’s interest/encouragement (Primary and Secondary school years)
  - Engineers in family (Primary and Secondary school years)
  - Helping in family business (Primary and Secondary school years)
  - Toys, Games, Puzzles and Competitions (Primary and Secondary school years)
  - Activities/ sports that required numeracy (Primary and Secondary School years)
  - Interest in engineering/ mechanical things (Secondary school years)
  - Peers (Secondary school years 1 and 2 and Junior Certificate years)
  - Careers/ College/Points (Junior Certificate and Leaving Certificate years)
  - Computers (Secondary School years)
  - Usefulness / practical applications (Leaving Certificate years)
  - Ambition (Leaving Certificate years).
**Figure 14. Factors outside Primary School contributing to interest in mathematics.**

**Figure 15. Factors outside Secondary School (years 1 & 2) contributing to interest in mathematics.**
What events, experiences, aptitudes or other factors
within and outside of school contributed to your interest in and learning of mathematics?

7. OUTSIDE SCHOOL - SECONDARY - Junior Cert

Figure 16. Factors outside Secondary School (Junior Certificate) contributing to interest in mathematics.

8. OUTSIDE SCHOOL - SECONDARY - Leaving Cert

Figure 17. Factors outside Secondary School (Leaving Certificate) contributing to interest in mathematics.

- 75.9% of engineers state that their feelings about mathematics impacted their choice of engineering as a career in the range “quite a lot” to “a very great deal”, figure18.
To what degree did your feelings about mathematics impact your choice of engineering as a career?

![Pie chart](chart.png)

- **Category**: 1 = Not at all, 2 = Very little, 3 = A little, 4 = Quite a lot, 5 = A very great deal

**Figure 18. Degree that feelings about mathematics impacted engineers’ career choice.**

**Discussion**

It is worth noting at the outset that 84% of the engineers surveyed took higher level Leaving Certificate mathematics and that 80% of the sample enjoyed mathematics at school at the levels of “quite a lot” and “a great deal”. This is in contrast to the majority, indicated by the fact that only 16% of Leaving Certificate mathematics students opts for the exam at higher level.

Despite the widespread view that higher level mathematics competence is critical to a technology economy and necessary for engineering, almost a third of engineers surveyed agreed that they could perform satisfactorily in their current job without higher level Leaving Certificate mathematics. Consistent with this, these engineers rate the extent of their usage of curriculum mathematics at work at under “a little”.

Despite the low usage of curriculum mathematics, the engineers surveyed show a significantly higher mathematical thinking usage compared to curriculum mathematics usage. This indicates that engineers, when carrying out their work, regard thinking usage e.g. problem solving strategies as more important than curriculum mathematics. This finding agrees with Ernest’s view that mathematics comprises of explicit knowledge and “know how” that comes from the experience of working with mathematics which he describes as personal knowledge of mathematics (Ernest, 2011). Similarly Schoenfeld’s framework for mathematical thinking includes the “knowledge base” as one component and four other components: problem solving strategies, monitoring and control, beliefs and affects and engagement in mathematical practices which concerns the ways experts engage in mathematics (Schoenfeld, 1992).

The strongest modes of thinking, resulting from mathematics education that influence work performance found in this study are: problem solving strategies, logical thinking, critical analysis, modelling, decision making, accuracy/confirmation of solution, precision/use of rigour, organisational skills and reasoning. These are important
findings as “the use of mathematics within the job of an engineer is not necessarily self-evident to an undergraduate student, and hence it is not easy for students to make a connection between what they are learning at university and what they will be doing after graduation” (Wood, et al., 2011). The importance of problem solving strategies is consistent with much of the literature on engineering education that calls for a problem solving approach to engineering education. For example, Jane Grimson (2002) is of the view that industry would prefer graduates who have “current know how to solve immediate problems” in addition to a sufficiently theoretical background to cope with future engineering challenges. Grimson calls for engineering education to encourage students “to exploit the power of engineering tools in order to tackle real-world problems” (Grimson, 2002).

The engineers who participated in this study enjoyed school mathematics and achieved high results in their Leaving Certificate mathematics exam, and they are also very confident in using mathematics. Teachers and teaching and affective factors are found to be the main contributors to engineers’ interest in and learning of mathematics. In the case of the engineers who responded to the survey, they attribute their success to “good” teachers and teaching. Engineers noted that “encouragement” was key to their good mathematics teaching in primary school, while in secondary school the survey participants describe good teachers as “enthusiastic”, “engaged”, “interesting”, “strict”, “good at maths” and “willing to answer questions”. In contrast it is “bad” teachers and teaching who are frequently blamed for poor performances in mathematics and for negative attitudes in mathematics education (Boaler, 1997; Fryer & Levitt, 2010; Goodchild & Grevholm, 2009).

Affective factors noted by the engineers within their school mathematics experience include: success (motivation), enjoyment (feelings), easy subject (views), recognition (motivation), practical applications (value), relevance to science (value), competitions (motivation) and relevance to engineering/ careers (value). Outside of school, sociocultural experiences are the main influences on the engineers’ interest in, and learning of, mathematics. This is consistent with affective theory (Schunk, et al., 2010), with the view that societal beliefs influence children’s learning of mathematics (Schoenfeld, 1992) and with the view that “knowledge is usually learned in social contexts” (Ernest, 2011).

Three quarters of engineers rate the degree to which their feelings about mathematics impact on their choice of engineering as a career at either “quite a lot” or “a very great deal”. On average, the engineers surveyed rate the degree their feelings about mathematics impact on their choice of engineering as a career at “quite a lot”. This is a significant finding.

Conclusions

A key finding in this study is the high impact engineers’ feelings about mathematics have on their choice of engineering as a career. This fits well with the assertion that students view engineering education as further engagement in school science and mathematics (Brickhouse, et al., 2000). There is thus a need to further explore how a revised second-level mathematics education could generate greater affective engagement.
Another key finding is the lesser relevance of curriculum mathematics to the engineers’ work compared to mathematical thinking. This outcome is consistent with the views that “engineers should no longer be taught mathematics as if they were mathematicians” (Janowski, Lalor, & Moore, 2008; Manseur, Ieta, & Manseur, 2010). The findings of this study suggest that integrating mathematics education within engineering courses or adopting a problem solving learning approach would improve students’ mathematical “know how” and would also improve students’ perception of the usefulness and relevance of mathematics in engineering education and increase students’ motivation to engage in mathematics.

The modes of thinking usage identified by the engineers in this study could be broadly described as problem solving strategies. The ability to solve problems is at the heart of mathematics and it is recognised that problem solving strategies in addition to mathematical knowledge are a “central feature of successful mathematics learning” (Cockcroft, 1982). Problem based learning methodologies are not new in engineering education, for example the Conceive-Design-Implement-Operate (CDIO) approach to improving engineering education aims to produce graduates who are “ready to engineer”. The CDIO model of engineering education is based on active learning environments and a major part of the CDIO content is about engineering reasoning and problem solving, experimentation and knowledge discovery, system thinking, personal skills and attitudes and professional skills and attitudes. (Crawley, Malmqvist, Östlund, & Broduer, 2007) In this study, engineers are of the view that by demonstrating the usefulness of school mathematics, the practical applications and examples, young people’s affective engagement with mathematics would improve.

**Future work**

With a view to developing a deeper understanding of the three main quantitative findings, secondary qualitative data will be studied in the next phase of this research.

**Acknowledgements**

We wish to thank all survey participants for their time. We are grateful for the assistance given by Damien Owens, Registrar Engineers Ireland, James Reilly, Statistician Institute of Technology Tallaght and Carton House, sponsor of prize for survey participants.
References


Understanding mathematics in the workplace is a key but difficult issue facing the lifelong learning sector. There is a view that the lack of mathematical understanding is a key component in the production of errors that leads to problems for the economy and is in need of remedy (e.g. see Bynner 2002). It has been noted that the “transfer” of mathematical knowledge from the classroom to the workplace is at best a problematic area (Evans 2000). Some researchers have looked at the skills required in particular employment areas (for example, see Hoyles et al 2002, Coben et al 2010). These studies do not provide simple answers. One of the difficulties is the localised nature of mathematical usage in the workplace. The fact that one hairdressing salon uses one set of skills does mean that all hairdressers are required to display the same skills. In this paper, it is proposed that a model that looks at changing practice offers a way of understanding the skills required for the workplace. The conference considered the notion of the development of “mathematical eyes” (Maguire 2003) in support of the workplace; this model may form a component lens.

Key words: mathematics, numeracy, workplace, model, change.

We are all aware of the type of problems that have informed the skills agendas up until now: the nurse who gives the wrong dose for an injection; the builder who orders the wrong amount of materials; the administrator who puts in time as a decimal. But there are other problems:

- The Millennium Bridge. In designing the bridge, the engineers and designers had not taken into account the lateral forces involved when large numbers of individuals behave in certain ways (in this case beginning to walk in step). The solution was not particularly difficult and did not require new knowledge per se, but rather the engineers and designers were not expecting this problem.
- The use of probability in the courts. The mistaken application of an assumption of independence and the collection of ‘hard data’ led to an erroneous calculation of likelihood in the case of twin cot deaths in the UK. This has led to the false imprisonment of a number of individuals.

This paper will consider a variety of situations and propose a series of conjectures and models that future work could consider.

*‘Turn and face the strain’ is a line from the David Bowie song ‘Changes’. The idea of turning around to look at issues appeared to be consistent with the idea of looking at change that the paper discusses.
Background

It is perhaps obvious that mistakes, such as those presented in the introduction, can be ‘one-off’ errors due to pressure of time or some other similar reason, but it has been felt that there is likely to be a more systematic basis for at least some of these issues. Following a series of studies, the ‘numeration problem’ has been identified. The report of the Moser committee (1999) set out the ‘Skills for Life’ agenda and has informed policy up to the present day. The committee used data from the ‘British Cohort Study’ of the Centre for Longitudinal Studies (see http://www.cls.ioe.ac.uk/studies.asp?section=000100020002) to identify that something like a quarter of the population had low skills in relation to numeracy. This was followed up by the Skills for Life Survey (2003) which confirmed the problem with further details of skills at entry level. The Leitch review (2006) has emphasised the link between the skills of the population and the impact on the economy and argued a causal relationship that would require a certain level of achievement in order for the UK to maintain a high economic profile.

In contrast, studies such as those involved in the Adult Learners Lives project (see http://www.nrdc.org.uk/content.asp?CategoryID=424&ArticleID=378) show that individuals do not feel that they lack skills. Additionally, critics of the large scale surveys note that the type of questions asked are not really related to use of skills in the real world (see, for example, Colwell 2002). Indeed, studies of mathematics in the ‘real world’ (e.g. Carraher 1991, Lave et al 1984) have suggested that the type of activities that we engage in the world about us do not map easily to (or from) the classroom. Carraher and team investigated the ‘everyday’ calculations that street sellers undertook when selling their wares compared to performance with pencil and paper tests. The young people were almost always correct (98% of the time) when dealing with their own produce but this fell to 34% when dealing with classroom ‘sums’. The grocery shopping investigation identified a range of complex thinking that involves consideration of non-mathematical context as well as a range of ways of calculating costs when looking at decision making with supermarket goods. This showed how simplistic the notion of ‘best value’ is - a notion that is often used in numeracy tests. See the example question in Figure 1 below.

![Figure 1: Exemplar ‘best value’ questions](patmore and Woodhouse 2009 p14)

What is interesting here is the different interpretations that can be put on the data that has been collected and presented. One might say a numeracy issue at a high level. The past few years have seen an emphasis of decision making formed on the basis of data collection. While this is understandable, there are dangers in using such data uncritically and assuming that ‘performance indicators’ are actually ‘real’, rather than the indicators that they were initially intended to be. It is proposed that taking such data as ‘real’ is another type of numeracy issue that also needs to be tackled (along with the higher level issues noted above).
Some scenarios for consideration.

In this paper I wish to consider a number of scenarios which, it is hoped, will offer some evidence for the conjectures and agenda that is proposed in this paper.

Scenario 1
In the early ‘90s, a college manager calculates the average pass rate of examinations by finding the (unweighted) mean of individual pass rates. This practice changes when the new data management systems are introduced to the college. There is a decrease in the average pass rate for the year following the change.

Unpublished anecdote.

Scenario 2
A group of senior nurses discuss the drug dosage requirements of a patient when the frequency requirements are reduced from every 6 hours to every 12 hours. The nurses come up with a number of different answers as to when the first of the 12 hours doses start.

Hoyles, Noss and Pozzi 1999.

Scenario 3
A report identifies that, while there is some resistance to financial literacy programmes, there are times in people's lives and contexts, when they are likely to be more receptive to financial education, information or guidance: these are sometimes known as ‘teachable moments’. Examples include couples who are planning to set up home together or to get married – or who are separating or divorcing; parents before, or soon after, the birth of a child; students at college or university who are facing, often for the first time, the challenges of managing their own personal finances; people who are starting work; and people who are approaching retirement.

Munday 2011.

Scenario 4
A team of designers are employed to build a pedestrian bridge. The project design produces a new style of bridge with a sleek look longer than others existing at the time. The bridge is opened and a large mass of people begin to cross. The bridge begins to move quite dramatically causing closure and a solution to the swaying problem is constructed.


Scenario 5
A stylist of a Toni and Guy hairdressing salon explains the type of mathematics used in their salon. She explains that in order to recreate the same design across the whole Toni and Guy chain, it is important for trainees to understand the angle of various cuts.

http://www.youtube.com/watch?hl=en&v=GQGW6FJWfDM&gl=US
Some conjectures and a proposed model

The following three conditions are considered. It is conjectured that for workplace situations that each of the conditions is held.

Condition 1 ‘Problem solving’ in a situation involves numeracy.

This is perhaps not likely to be contentious in the adult mathematics world but could be an issue for some. In a sense it can be seen as a trivial observation, that any situation can be mathematised if we wish (perhaps looked at through ‘mathematical eyes’). The trouble is that this could be seen as a type of cultural imperialism (for example, see Dowling 1995) and a trivial mapping. The intention here is to imply that ‘real world’ workplace problems can be seen as having an appropriate mathematical interpretation and that such an interpretation is likely to offer insights to proposed solutions. It is not suggested that such knowledge is privileged in some way but a helpful part of the problem solving toolbox. Indeed, ‘mathematical solutions’ may be rejected but the mathematics needs to be considered when coming to choices. It is perhaps fairly clear that most problems in the workplace involve resource allocation in some form. Such resources would involve some combination of time, staffing and physical resources, and this would lead to a possible mathematisation.

Condition 2: The numeracy skills that people need for their everyday activities are by-and-large undertaken appropriately and errors are not systematic.

This is a more difficult and perhaps more contentious issue. The nurses in Hoyles et al, noted above, suggest that many problems can have multiple, ‘good enough’ solutions. This is not to suggest that there are times where problems can be more systematic but that they are rare rather than commonplace. In one sense, the conjecture that this condition holds in the workplace could be seen as having a natural basis, in that if an individual is undertaking a task day after day, then there is a good chance that they will find solutions to the issues that are raised. It is clear that the solution may not sit comfortably with everyone. For example, there are cases of a small set of staff that refer required ‘sums’ to a particular individual. The difficulty comes when something unusual happens, which takes them outside their normal practice, like the individual for whom the unexpected sum turns up. Or indeed, like the college manager for whom the external environment and expectations have changed. Or perhaps, like the Millennium Bridge designers for whom the engineering mathematics was not particularly complex but the design was relatively unique. Or perhaps the individuals for whom the ‘teachable moment’ happens when the usual has changed.

Condition 3: Localised numeracy features far more than universal features.

This condition may also be a cause for thought. In the UK, a government department (DfES 2004) had tried to argue that there exists a ‘top ten’ numeracy skills that most employment situations required (see Appendix). While some of these sound reasonable (‘plan the use of time effectively’ perhaps) others sound stretched as a universal feature to say the least (‘calculate area and volume accurately’). If we take the case of the Toni and Guy hairdressers, and the angles used across all their salons, then there may be a need to train for certain angles. But what about the independent salon? It is less likely that an angle needs to be measured accurately. This does not mean that there is no
numeracy skill being employed but rather that is a somewhat different skill needed for
the chain salon than the independent.

These conditions and the conjecture that workplaces satisfy them need some testing.
Nevertheless, they have been written with some deliberate contingencies built in. It
would be interesting to find problems that cannot be interpreted non-trivially from a
numeracy perspective, it would be interesting to find examples of systematic numeracy
errors in a workplace and it would be interesting to find a universal numeracy concept
in a particular workplace environment. But individual cases may just ‘prove the rule’
rather than act as serious counter examples. In any case, for the purpose here these are
taken as conditions (i.e. assume that we have a workplace where conditions 1-3 hold).

If condition 2 is true then would that suggest that there is no problem? The issue is
condition 1 combined with the ever-changing world in which we live. This is perhaps
made more complex by condition 3, in which specific working practices can mean
responding to change may not be the same for all.

Model: The following is proposed as a model for understanding workplace
mathematics (WM) / numeracy.

**Workplace Mathematics 1 (WM1): A changed environment is the driver for a
changed skillset.**

For example, in an employment situation, this might mean changes in technology that
require a different set of interpretation skills, and that in order to respond to this,
workplace mathematics 2 (WM2) individuals need to be able to self identify their
current skillset, and be assisted in recognition of what change is required, as well as the
development of those skills.

The proposed model assumes that the changing environment, in which the workplace
exists, is key to investigating needs. A separate issue is responding to this, and the
model suggests that there is a need to assist workplaces in identifying appropriate
professional development.

**Diagrammatically:**

![Diagram](image_url)

**Figure 2. Model for development of skills in the workplace.**
Research

The above conjecture and model suggest research work. There is work in investigating the three conditions, in applying the model to existing data as well as exploring other situations and gathering more data. For example, an investigation into what happens in workplace situations under changed circumstances would identify some further numeracy ‘problems’ in particular cases. It should be noted that these investigations could work at a range of levels and not just include ‘basic’ mathematics.

Professional development

This model suggests a number of professional development issues for teachers.

WM2.1. ‘Identification of existing skills’. The model itself requires a change of view. Not seeing mathematics as a set of ‘basic skills’ that can be applied to situations, but understanding that real life is a complex situation, which requires a wider understanding, and that individuals bring their own ‘funds of knowledge’ (see Street et al 2005). Getting trainee and existing teachers (as opposed to researchers) to analyse the way people really employ number in problem solving in the real world would be a major step forward.

WM2.2. ‘Identification of development needs’. The ability to consider how a changed situation may require a different set of skills would be of enormous benefit to an education professional. This would require a teacher to work with someone from a real world or employment context to understand the range of issues concerned. This would involve a numeracy specialist to appreciate that there are a range of ways of approaching a problem and that numerical answers are not the only issue.

WM2.3. ‘Development interventions’. There have been a range of interventions in teaching and learning numeracy and mathematics that could be employed once the above two aspects have been considered.

Discussion

The world of work is as much about responding to a changed environment as providing goods or services. Occupational standards provide a snapshot of the workplace that is very useful for analysis, but does not reflect what happens over time. This paper suggests looking at the changing needs of the workplace and how organisations and individuals respond. Reframing the ‘numeracy problem’, as a much wider issue than dealing with those considered to have ‘basic skills’ needs, may also be a way forward. Seeing the development of numeracy as an issue for all of us, rather than for someone else (i.e. someone with a basic skills need) may help. It is likely to be the case that many individuals do not feel that they do have a need – and that will be true at all levels of society – so we need ways to:

a) show that a changed world requires a changed skillset, that will involve some form of numeracy (in most cases)

b) compare and contrast the new model to traditional ways understanding numeracy

c) that this need not be a painful process

d) identify what next steps are needed.

Some research and development work will be needed to identify examples of resources for training.

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References


Appendix

DfES Employer Toolkit Skills for Life: Make it your business

The Top Ten number … skills needed in most places of work

1. Arrive at work on time and plan the use of time effectively
2. Write down sequences of numbers accurately
3. Understand the importance of accuracy in number calculations
4. Make calculations using addition, subtraction, multiplication and division
5. Make necessary calculations of fractions, decimals and percentages
6. Weigh and measure to required tolerances
7. Use calculators accurately
8. Use estimating skills
9. Make money calculations, including checking pay slips, accurately
10. Calculate area and volume accurately
Family mathematics/numeracy: identifying the impact of supporting parents in developing their children’s mathematical skills

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For a number of years, parents have been encouraged to become involved in their children’s learning. This has led to ‘family learning’ provision of various types being developed and funded. Specific funding of Family Language, Literacy and Numeracy (FLLN) has promoted this type of provision, including the Family Numeracy classes run by LLU+ at London South Bank University. There have been a number of studies looking at parental involvement in their children’s learning, though less so with a focus on the perspective of the parents (although see Abreu and Cline 2005 and Civil et al 2008). The numeracy team at LLU+ have started a small scale, pilot investigation into the impact of the provision on their ability to support their children. Previous authors (McMullen and Abreu 2010) have noted that such parental support means that parents are engaging in some aspect of teaching and that this requires some form of training. This study involves interviewing parents about their motivations for learning, their views on their ability to support their child’s learning, and the extent to which the courses involved have assisted this process. The data collected so far indicates heterogeneity in motivations although some possible categories are emerging which may assist planning for such programmes.

Keywords: family, adults, mathematics, numeracy, parents, impact

Parental involvement in school education has been the object of various studies, for example the Impact project (Mertens and Vass 1990), in which it was argued that such involvement assisted the development of children’s learning. Family learning programmes have been provided to help support such parental involvement by developing the skills of the adults. A study of more general adult numeracy provision (Swain et al 2005) has demonstrated that helping their children is one of the motivations for adults to attend classes. LLU+ was asked to run a number of these programmes by two London boroughs and the authors decided to use the opportunity to undertake a small scale, pilot study into the impact of such programmes.

The data collected suggests that family mathematics programmes attract participants with a range of skills and confidence. We have noted some different categories of parents and some positive aspects of the programmes. We have also suggested further research studies.
The study
This study is an investigation into the impact that various Family Mathematics programmes have had on parents in two types of provision. The researchers have supported programmes in two London boroughs. In one borough two 60 hour programmes have run in early years centres and in the second borough seven 30 hour programmes have run in primary schools.

The learners were asked to complete short questionnaires on joining the classes and volunteers took part in semi structured interviews. The questionnaires were intended to find out why the parents joined the course and the type of support that they already provided for their children. The purpose of the interviews was to find out from the parents examples of how they support their children and the elements of the course that may have assisted this, or aspects that may need further development.

It was decided that the class teachers (the authors) would collect the data, including conducting the interviews, as there was a wish to minimise disruption and make the learners feel comfortable during data collection. The authors are aware that the participants are likely to be positive about the programmes (and that some of the questions chosen may be seen as ‘leading’, see Appendix) and so it was important to us that participants discussed specific examples of interventions. In other words, the authors were more concerned with how the provision may have helped rather than whether it did.

It is perhaps unsurprising, given the multicultural nature of the London boroughs, that these groups contain a variety of backgrounds although we note that the groups (so far) are made up almost exclusively of women and contain very few participants identifying themselves as ‘white British’. This may say something about the wider relevance of the research and suggests an investigation into those that attend, and those that do not, would be worthwhile.

Other literature
There is a small but growing body of (international) literature on Family Mathematics provision. McMullen and Abreu (2009) found that many parents were unclear about current teaching methods and were reliant on their children’s explanations, but the children often had difficulty explaining. Their research concludes that participating in different mathematical approaches allows parents to be more positive and understand their value.

Abreu and Cline (2005) looked at the impact of children’s home culture on their maths learning in school. They looked at school maths in school as a different social practice to school maths in the home. They found few differences between groups of parents apart from some language issues. However they report that their research shows that parents do not find it easy to teach their children at home and argue that parents need support with both how maths is taught in school and strategies for bridging the home-school gap.
Ginsburg and Farina (2008) explored the roles women take when attempting to solve mathematical problems with their children. They conclude by advising that ‘adult educators should be sensitive to helping parents consciously prepare for this work.’

Russell (2002) identified different categories of why and how parents help their children with maths. She also concluded that ‘parents who do not have an up-to-date understanding of pedagogy and school/curriculum structures have difficulties in supporting their children’s maths, irrespective of their social class or mathematical ability’.

However, Cai (2003) found that parental involvement is a statistically significant predictor of their children’s mathematics achievement. Brooks et al (2008) also reported that FLLN has ‘contributed at home to parents’, especially mothers’, empowerment through learning, and improved children’s educational prospects’. Therefore, the potential for parents to support their children with mathematics is something that should be examined and cultivated.

In order to discuss the data that is collected the team has found it useful to describe two types of provision. One type (Type I) of provision focuses on children’s learning and the content of the school curriculum; adult skills are discussed and developed as a secondary feature. Another type of provision (Type II) focuses on the development of adult skills with an awareness of children and the school curriculum. We claim that Family Mathematics programmes lie on a continuum between these two types with the longer programmes at the children centres closer to Type II and the shorter programmes closer to Type I. It is possible to see that Type I provision focuses on subject pedagogic knowledge while Type II is more of a mix between subject and subject pedagogic knowledge in the sense of Shulman (1985).

The data

The research is ongoing and at the time of the report, 14 learners had been interviewed: 3 from each of the longer courses and 8 from the shorter. The following quotes are selections from the interviews which illustrate the type of responses obtained in the research.

On changes (question 1)

“There’s a lot of maths English I did not know. … I never heard of it and I learn a lot now. Which changes my feelings, I become a lot more confident. Because I lost a job before because of maths…. I come for my kids, to help in the future as well.” (Parent B)

“Things like fractions and long division. Just having different ways of working them out – like you have got the grid and for percentages the bubbles. Just having those tools and being able to use those when working out. Knowing how to break things down was handy. I wasn’t particularly good at times tables so having those sorts of things has helped quite a lot.” (Parent D)

“It’s changed a lot actually. Since I left school twenty years ago, I didn’t do any maths.” (Parent E)
“Some of the stuff you’re teaching, I’m familiar with it but it has opened my mind up a bit more. I’ve known it but it’s woken me up. Like with the hexagon. I’ve known it from school but I’d forgotten it.” (Parent H)

“Maths is OK – not like a bunch of numbers – like confusing . . . not scared of doing maths now or numeracy – just give me that confidence to say yes, I can carry on” (Parent N)

“The folder you gave me I took it home and showed the girls. They’ve showed me, I’ve showed. I’ve helped them, they have helped me, I have got a lot out of it to tell you the truth...we have shared.” (Parent K)

On how the course impacts (question 2)

“For example yesterday. He is doing minus and plus. I always use my hand, I always say look there is 4 (shows fingers) minus..., ... because he is in reception, I always use my hand to help him or maybe marbles. I always use something like that for him to gain understanding. Because you helped me a lot, give me ideas how to teach him.” (Parent A)

“Anything we are doing is helping. When I look at what we are learning back home, and what we are learning here, its different ways... When I come to class and learn myself how to do the maths, I understand my children, the way they are going.” (Parent E)

“I cook with my children. So we did an activity where I took a bottle and asked could you get 1 litre in this small bottle? This was with the eight year old. He could see that you couldn’t have 2 litres in 500 ml” (Parent G)

“The things we did with the games and that with the rabbits, it makes it more exciting. (My children) really liked that one. My kids are 8 and 6. They wanted to win – there were arguments over the game. Can we play it again!!” (Parent H)

“She’s more willing to let me help her now. Mummy’s not shouting. I have patience now – realise there’s more than one way to reach the same answer. She likes the dice game (Mr McGregor’s Garden) and the number bond matching cards.” (Parent I)

Table 1 below displays some key characteristics of the parents and their children as described in the interviews. These characteristics relate to the situation at the start of the course.
<table>
<thead>
<tr>
<th>Main Motivation</th>
<th>Own Mathematics</th>
<th>Child’s Mathematics</th>
<th>First Language</th>
<th>Home Activities</th>
<th>Prog. Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - Improve own maths and help children</td>
<td>Always loved maths. Assessed at L1</td>
<td>Confident at the moment but child is very young</td>
<td>Spanish</td>
<td>Counting</td>
<td>II</td>
</tr>
<tr>
<td>B - Improve own maths</td>
<td>Poor – caused job loss. Assessed at E3</td>
<td>Very young – too young to say</td>
<td>Tigrinya / Amharic. Has done ESOL classes</td>
<td>None noted although discusses counting</td>
<td></td>
</tr>
<tr>
<td>C - To help child</td>
<td>Grade C at GCSE but feels she is not good at maths. Low confidence.</td>
<td>Worried that child will not enjoy maths.</td>
<td>English</td>
<td>Counting</td>
<td></td>
</tr>
<tr>
<td>E – To help children and improve own maths</td>
<td>Likes maths but not confident. Used to be confident in own country. L1</td>
<td>Not very confident. Thinks maths is ok.</td>
<td>Somali</td>
<td>Had tried to help children but didn’t always understand the questions.</td>
<td></td>
</tr>
<tr>
<td>F - Help children</td>
<td>Lacked confidence. No qualifications in maths. E3 assessed.</td>
<td>Lacks some confidence but enjoys maths.</td>
<td>Chinese</td>
<td>Tried to help but didn’t understand school methods.</td>
<td></td>
</tr>
<tr>
<td>I - Improve own maths</td>
<td>Lacks confidence and didn’t like maths at school.</td>
<td>Confident – loves maths</td>
<td>English</td>
<td>Already doing some activities at home with her child</td>
<td></td>
</tr>
<tr>
<td>J - Joined mainly to be able to help child with maths</td>
<td>Not confident in own maths – no maths qualifications but did well in initial assessment</td>
<td>Quite confident and enjoys maths</td>
<td>English</td>
<td>Not doing any activities at home – unable to help due to lack of patience (caused by not knowing different methods)</td>
<td></td>
</tr>
<tr>
<td>K - Help children</td>
<td>No maths qualifications. Lacked confidence</td>
<td>Enjoys it / fairly confident</td>
<td>English</td>
<td>Had problems helping due to gaps in own skills and knowledge</td>
<td></td>
</tr>
<tr>
<td>L - Help children</td>
<td>Fairly confident with own maths but struggles to help due to language and differences in layout / methods</td>
<td>Confident but not very interested</td>
<td>Danish and Somalian</td>
<td>Had problems helping due to not knowing current methods and English words.</td>
<td></td>
</tr>
<tr>
<td>M - Help children</td>
<td>Struggled with maths / lacks confidence but good at mental arithmetic</td>
<td>Not confident</td>
<td>English</td>
<td>Had been leaving the child to do it themselves. Also lacked knowledge of current methods and vocabulary.</td>
<td></td>
</tr>
<tr>
<td>N - Came to be able to help children</td>
<td>Hated maths – found it confusing. Scared of it.</td>
<td>Enjoy it / fairly confident</td>
<td>English</td>
<td>Hadn’t really done anything. Hadn’t been aware of the maths around her – had ignored it.</td>
<td></td>
</tr>
</tbody>
</table>
Some categories of parents
The following have emerged as categories of the parent participants.*

1. Confident in own maths / joined to help child:

   A – “has gained ideas about helping her child and knowledge about school methods (but has also been able to make sense of what she learned at school).”

   G – “confident in own maths but not in helping child due to only knowing one method and not knowing how to explain or teach her child; gained knowledge of different methods, ideas for helping and confidence; worried about confusing her children.”

   L – “fairly confident in own maths but struggles to help child due to unfamiliarity with current methods used in this country and mathematical vocabulary in English.”

These individuals expressed very positive attitudes towards mathematics, confidence in their own abilities and joined mainly to help their children (although A mentioned improving her mathematics as well). They were already helping their children through activities but were not completely confident about how to help them – either now or for the future. It is interesting to note that their children are also confident in mathematics. One of them had joined a type I course and two had joined a type II course (this may not be important to the participant and may just be a case of opportunity and location). It is not surprising therefore that they reported gaining knowledge about school methods, finding out what their children are learning and getting ideas for how to help them at home i.e. the pedagogy. One of them was worried about confusing her child but has now gained confidence in their ability to help. This learner mentioned particular strategies and methods such as using compensation for subtraction and the lattice method for multiplication, as being particularly helpful. There is a sense that the learners in this group feel strongly that they should be using the current school methods rather than attempting to get their children to follow the methods they themselves learned at school and this appears to have motivated them to join the classes.

2. Low confidence in own maths but enjoys it / joined mainly to help child:

   E – “Low confidence is linked to not being able to help due to language issues and not having studied mathematics for a long time. Used different methods in her own country; child not confident with mathematics. Improved understanding of mathematics language through doing written questions; focus on vocabulary of the course. Being able to help her child has improved her own confidence and reports her child has improved a lot.”

   F – “Her low confidence is also linked to language; children previously didn’t understand her methods; through the course she gained knowledge of UK school methods and improved her language (spoken and written). This learner connected with her child through a course resource using a school method that he understood. Now she thinks maths is ‘magic’. Her son’s confidence, independence and ability in mathematics have also improved.”
H – “Enjoyed mathematics but had forgotten some of it and is not very confident. She was already helping with homework but has gained knowledge of methods, strategies and resources for helping her child. She has found using games fun and has reminded her of some forgotten mathematics. Her child has shown some improvement in mathematics.”

J – “Her low confidence is linked to not having passed any mathematics qualifications and thinking she was not capable of doing it. She was unable to work with her child because she only knew one method and had tried to coerce her child into using it. This learner gained an understanding of current methods and that there is more than one way of working things out. She is now able to help her child and is more aware of what her child is doing in school. She gained confidence in her own mathematics but still has gaps. She mainly enjoyed doing test papers while on the course.”

This is an interesting group in that they have expressed an enjoyment of mathematics but are not confident in their abilities. For two of these people, this lack of confidence may be related to the fact that English is not their first language. ‘Before the course sometimes I didn’t understand the questions’ (E). Their children vary in their level of confidence, but while three of them enjoy mathematics the other thinks it is just ‘ok’. All of them had tried to help their children at home but with limited success, consistent with McMullen and Abreu (2010) noted above. It is therefore understandable that these people are very keen to find a way to be able to help their children with mathematics. The two ESOL learners report having benefitted from an improved understanding of the mathematical language as well as knowledge of the school methods. They hint at an improved connection with their children through their knowledge of the school methods in this country which are different to those learned in their own countries, (c.f. Abreu and Cline (2005)) although the parents in this study did not express resistance to the alternative methods. They also mention methods and resources that have been particularly helpful, such as the box method for multiplying and the 100 square for counting. The learner who also wanted to improve her own mathematics reports that her improved ability to help her own child has, in turn, improved her own confidence. Indeed, all of them have gained some confidence, want to continue learning and take qualifications in mathematics.

It is interesting to compare this group with the first group. Despite the obvious difference in confidence in their personal skills there are similarities i.e. they both express a lack of knowledge of current school methods. However, the learners in this group seem to have had problems trying to help their children that appear to have been connected with their different approaches from their children: “before I just used my formula and he can’t understand (F) and When you try to sit down and tell her where she’s gone wrong she don’t want to hear. She’s more willing to let her brothers help ‘cause they work things out different. I’m from the old school and they know the

* Note that the descriptions should be understood as reported rather than statements of fact. It would make the English more difficult to read if the text consisted entirely of ‘X reported that…’ type statements.
modern ways.” (J). These problems were alleviated when they discovered it was permissible to use different approaches and they were able to show their children that they were learning the same methods that they knew. Another difference is that this group seem to rely on sitting down and helping with homework whereas the first group talked about doing activities with their children: “When it comes to homework that has to be done before they can play. I’ve always sat down and worked with the kids’ work” (H).

C. Low confidence / did not enjoy mathematics/ joined for different reasons

B – Poor maths caused her to lose her job; has language issues; gained new knowledge (UK methods) and improved understanding of mathematics language. Has gained confidence. Not helping young child at the moment.

C – Did not enjoy mathematics at school (due to the teaching style). Learner has gained enjoyment of mathematics due to teaching styles on the course matching her learning. Has gained confidence and ideas for teaching her child; developed a passion for passing on her knowledge using teaching strategies from the course e.g. talking to her child about toy shapes.

D - This learner did not like mathematics; bad experiences at school. But her attitude towards mathematics has improved due to the practical teaching styles on course and links to relevance in everyday life. Has enjoyed learning current school methods and found them helpful for improving her own skills. Has gained confidence in her own ability and ability to help children with mathematics, e.g. Dominoes for number recognition. Her child has improved.

I – Did not like maths at school and was not confident in it. Has problem areas such as fractions and percentages. She decided to take up the opportunity to improve her own maths (although initially recorded that she was attending for herself and her child). Already doing some activities with her child and speaking to her teacher, so was aware of what her child was doing in school but a bit worried about the future. Enjoyed being part of a group that were there for similar reasons and had similar issues with maths and enjoyed doing activities and making things to take home. Found the games useful e.g. Mr McGregor’s garden; playing the game helped her work on her child’s counting skills – (she was saying one before she had moved rather than after moving one space). Really enjoyed doing the maths assessments and would have liked to focus even more on her personal skills gaps. She liked the teaching methods used on the course and the way things were broken down to make them easier to understand. Also liked having the sheet that gave information about what the children will be doing in school up until year 6 in different topics.

K - Lacked confidence and did not have any maths qualifications. Found it hard to help children due to gaps in personal skills, low confidence and lack of knowledge of current methods/difference in methods taught. Took approach of sharing what she had learned and taking turns to take on role of teacher at home – teaching each other.
Found it fun and learned together. Makes her happy to be able to share what she has learned. Found resources helpful e.g. fraction wall helpful in understanding and explaining concept of fractions. Finds children’s methods easier to understand. Able to teach child about loans. Children’s maths has improved.

M - Struggled with maths / low confidence. ‘I always struggled with maths; I would take much longer to work things out than other people. I have to keep relearning rules, it doesn’t stick.’ Very good at mental arithmetic. Had tended to let child do it by himself but now able to help – used resource (fraction wall). Feels that a text book would have helped her to help him before as she lacked knowledge of methods. Has been helped by the course – better understanding of methods, especially fractions.

N - Hated maths. Low confidence. Lack of awareness. ‘To me maths is just a bunch of numbers – confusing’. Joined to avoid passing on negative feelings. Gained awareness of maths around her. Has learned some simpler methods for multiplication such as lattice / Egyptian that has given her confidence: ‘How you do maths in a different way’ and has gained awareness ‘Opening my eyes on maths’ which has helped her to help her children through everyday activities. ‘Before I just carry on walking but now by counting things and recognising shapes they are aware of certain stuff.’ Does lots of activities now with her children – has confidence to support them. Understands the need for being able to use maths in the everyday, but also not scared of doing maths any more.

This is the largest group in this study. They have joined for different reasons although five out of the seven recorded their main reason as wanting to help their children. They may be the most difficult to persuade to join such courses due to their negative experiences and feelings about mathematics. For the learners who joined to help their children, it appears that a strong desire to do so has overcome their own feelings and lack of confidence with mathematics. Three of them were already helping their children at home to an extent but have gained greater confidence in their ability to do so and are very keen to use the new ideas and activities they have been exposed to with their children. On the other hand, some of the group were not helping their children with mathematics at home at all. One learner has not mentioned helping her child at all as a reason for joining the course and does not claim to help her child at home. Nevertheless she does help with counting although does not consider this to be mathematics. She has gained confidence and learned new things in mathematics and mathematics language. These learners point out teaching styles and strategies on the course as being particularly attractive to them; an important issue as they did not enjoy mathematics at school. Enjoyment, having fun and seeing the relevance appear to be important factors in making these learners feel more positive. Reference is made to finding the children’s methods simpler and appreciating having things broken down to make them easier to understand. The difference in teaching methods seems to have helped change their attitudes towards mathematics and they are keener to engage with maths as a result (c.f. McMullen and Abreu (2009) where parents reported an improvement in their feelings about mathematics). They also mention resources as being helpful to their understanding and supporting their explanations to their children.
Discussion

Overall it is noted that participants on both programme types have identified a range of positive aspects of the courses and a variety of reasons for their attendance. There are parents who have their own skills higher in their sights although we also note that they feel this will help them assist their children later. There are parents who feel that the UK education system is something of a mystery to them some of whom also need significant language support. There are parents who want to learn methods that help their children learn.

From the position of a provider it seems to be important that such courses encompass a variety of aims in order to meet the range of needs of the participants. We argue that such courses should be taught in an interesting and engaging way that highlights that mathematics can be fun. It appears however, that a more detailed initial assessment and diagnosis of learner motivations may be helpful in responding to parents' needs and in the planning and delivery of provision.

It seems that knowledge of current school methods is a priority for most and there is certainly evidence that tallies with findings of Russell (2002), namely that parents’ lack of knowledge about current school methods makes it difficult for them to support their children’s developing mathematics skills, regardless of their own skills in mathematics. However, this seems to be more strongly connected with giving support with homework as quite a few of the learners, some of whom lacked confidence in their own mathematical ability and even had quite negative feelings about mathematics, were already doing activities with their children that involved numeracy. A lack of these activities does appear to be linked to mathematical confidence and awareness. Yet, there are many other things that different people gain from such Family Mathematics courses. Our findings are consistent with Abreu and Cline (2005) although we have looked at it from the point of view of the parents’ feelings about mathematics rather than the perspective of the child. Here the differences seem to be that the ‘confident parents’ gained more confidence from knowledge of school methods, while the less confident parents needed to develop their own mathematics, as well as knowledge of school mathematics, before feeling equipped to help. However, all parents were able to express some impact on their ability to work with their children. One strong finding is the importance of teaching a variety of methods, making the learning fun and engaging through a multisensory approach and breaking things down when trying to support learners who had negative experiences of mathematics earlier in their lives. This is where family learning can have such a strong role to play in reaching out and changing attitudes towards mathematics and benefitting the children of these families in turn.

Also, following Abreu and Cline (2005), the authors note that many parents are challenged by the notion of taking on a teaching role with their children. Indeed parents are likely to take on a range of roles with respect to the programmes. They are learners themselves, they are taking on a partial teaching role (perhaps best described as teaching assistant) and to some extent they are becoming learning champions to encourage their children. It is noted that the parents in this study (as in others) have a range of skills and confidence and it would be of interest to study how parents identify with these various roles. Following Gee (2001), many researchers are now interested in using identity as a
tool of analysis. Noting the heterogeneity of the learners in our study such an approach may uncover some important messages for those planning family learning programmes.

**Conclusion**

This pilot research suggests a number of possible follow up studies. There is a possible quantitative study with a relatively large set of learners identifying the mix of motivations for attending. There is a possible smaller scale, in depth longitudinal study tracking the changes in motivation i.e. whether the motivations for joining a course are the same motivating factors that keep learners engaged with the learning or whether they change as the courses progress. The majority of learners who joined these courses stated that their main reason for doing so was to help their child, yet a significant number of them became very interested in improving their personal mathematics skills. Is this purely so that they can help their children or is something else happening? There could also be a similar in depth longitudinal study of the impact of specific activities on the parents and their children. For example, do some activities have a greater effect on the ability of parents to support their children’s developing numeracy or does it depend on the individual learners?

In addition, as we noted above, there is a possible study into the composition of such family learning groups and how their own perspectives and identities prepare them for the various roles they may undertake. A final question we might ask is to what extent we should be teaching parents how to teach their children and how much we should be concentrating on improving their personal mathematics skills and sharing ideas for activities they could do at home.
References


Appendix: Questions and prompts for semi structured interviews

1. In what ways has the course changed your own mathematics / numeracy knowledge? Confidence? Feelings about mathematics? If not given ask for an example.

2. Can you describe a recent time when you have successfully helped your child with mathematics? Did you use any of the resources from the course? Did you use any suggestions from the course? What age is the child you are talking about?

3. Can you describe a recent time when you found it difficult to help your child with mathematics? What do you think would have helped? Do you think it was due to a gap in your own mathematics skills, lack of knowledge of what methods to use with your child, the right words to use or something else? What age is the child you are talking about?

4. (If not already answered) Do you feel you have a better understanding of what your children are learning in mathematics / how they are taught? Examples

5. Have you noticed any changes in your child’s mathematical skills and knowledge / confidence / feelings about mathematics / attitude to learning mathematics?
Financial Literacy Competencies - the story so far

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This is a report on a current Financial Literacy Project organised under the Grundtvig programme with 8 European partner countries. The project started in October 2010 and is due to be completed in September 2012. The UK partner was LLU+ in Year One and Learning Unlimited in Year Two.

The project aims to improve the financial literacy competencies of adult learners in order to prepare them for the challenges and temptations of the consumer society and to prevent situations of financial indebtedness. In order to accomplish this, a toolbox for adult learners, a handbook for trainers and a curriculum will be developed that will be shared across the nine countries.

The project aims to base the development of these resources on local research in each of the partner countries. At ALM 18 we will be in a position to disseminate research from year one and to discuss the proposed development of resources for year two of the project.

The research from the UK identifies investment in a large number of resources produced by the Financial Sector but limited investment and awareness in the Education sector.

This paper has three main aims. Firstly to introduce the background to the FinLiCo project, secondly to describe the process of involvement in year one and thirdly to report on the UK situation in relation to financial literacy and its relevance to adult numeracy and adults learning mathematics.

Background

The project is called Financial Literacy Competencies for Adult Learners (FinLiCo). The project aims to ‘improve the financial literacy competencies of adult learners in order to prepare them for the challenges of a consumer society’. The project is funded through the European Grundtvig programme for adult education and involves 8 other countries (partners): Austria, Cyprus, the Czech Republic, Italy, Portugal, Slovakia, Slovenia, and Switzerland (as a silent partner). Portugal is the lead partner. The proposal submitted by the partners to the EU argued that such work was important “in order to prepare [adults] for the challenges and temptations of the consumer society” and to “prevent situations of financial indebtedness”.

The project aims to impact not only on adult learners' competencies, but also use new training methodologies and resources that reflect current educational thinking.

This project is based on findings from an earlier ‘Financial Literacy’ project (http://www.financial-literacy.eu/cms/). This original project found that some European countries had developed examples of good practice aimed at minimising and/or
overcoming the lack of financial literacy in adults. However despite excellent efforts more and more families had to deal with debt and efforts to reduce indebtedness had not been successful. The project argued that this lack of progress was in part due to the short-term nature of such proposed solutions.

FinLiCo - the first year

The first part of the project involved each of the members of the project carrying out research into the policies and the provision of resources currently available in each country.

The research has included a review of the policies and resources available in each country as well as carrying out a survey of people involved with training and developing financial resources. The survey of views and knowledge of relevant professionals and experts about financial literacy was collected in the other partner countries as well. This target group included organisations that had produced resources and individuals in the education sector. The organisations were contacted by telephone and e-mail, the individuals were contacted through e-mail with a request to complete an online questionnaire.

The questionnaire was standardised across all of the partner countries. In the United Kingdom, the questionnaire was distributed to the Adult Numeracy Teacher Network (facilitated by LLU+), an article was written in the Basic Skills Bulletin and specialists in the field of financial capability were contacted to include their views. The questionnaire was provided on-line for self-completion.

The information collected has been shared through the course of two face to face meetings of all of the partners involved where a set of priorities have been agreed to produce the final products of the project. These will comprise a set of resources, referred to as a ‘toolbox’, a handbook for trainers and a set of curriculum for particular groups.

The initial research identified such a wide range of existing materials and target groups that the original scope of the project had to be re-defined. The current focus (October 2011) of the project is summarised as:

The target audience of this project is citizens with little or no competencies in financial literacy.

Our main aim is to produce some materials that will help these citizens to overcome this lack of competencies. We will produce materials specially designed for trainees, as well as materials for trainers. Furthermore, we will develop a curriculum related to financial literacy and, whenever possible, to adapt this curricula to different target groups such as entrepreneurship, young adults, families, etc. The development of these materials also aims to improve pedagogical approaches and the management of adult education organisations. Moreover, all this process will contribute to the promotion of a dialogue between educators and the community including those with social responsibility in finances.

(FinLiCo, 2011).
The UK partner has undergone considerable changes during the first year. The UK partners have changed as a result of the economic situation. LLU+ took over from another organisation that had been involved in planning but had gone into liquidation before the project started. During year one LLU+ was closed down by its hosting university, however the numeracy specialists involved in the FinLiCo project established a social enterprise called Learning Unlimited and the work continues.

Financial Literacy Competencies in the EU/ Financial Capabilities in the UK

In the early stages of the research it became apparent that there are many different terms being used to describe this field of work. For the purposes of the project the consortium defined “Financial Literacy” as the ability to understand finance. More specifically, it refers to the set of skills and knowledge that allow an individual to make informed and effective decisions through their understanding of finances. The Organisation of Economic Co-operation and Development (OECD) has been involved for the past decade in issues of Financial Literacy and Education has provided the following definition for “financial education”:

Financial education is the process by which individuals improve their understanding of financial products and concepts; and through information, instruction and/or objective advice develop the skills and confidence to become more aware of financial risks and opportunities, to make informed choices, to know where to go for help, and to take other effective actions to improve their financial well-being and protection.

(OECD 2005).

In the UK the term ‘financial capability’ rather than ‘financial literacy’ or ‘financial education’ has been used for some time. Shaun Mundy in his review of financial capability in the UK gives a very wide definition of this term.

There is no generally accepted definition of the term ‘financial capability’. However, at its core it means: having the knowledge, understanding, skills, motivation and confidence to make financial decisions which are appropriate to one’s personal circumstances . . .

A person’s financial capability is best judged by their actual behaviour. Someone can be financially literate (in the sense that they have the knowledge, understanding and skills which would enable them to manage their personal finances well) without necessarily being financially capable, as demonstrated by their actual behaviour. For example, people who understand the importance of shopping around before buying financial products or services, and know how to do so, can be regarded as financially literate (as regards that issue) but cannot be regarded as financially capable if, in practice, they never shop around before choosing financial products or services.

(Mundy, 2011).

The use of this different terminology signalled for us that there had been developments for some time in this field in the UK, led by the financial services industry that were very different from our experiences in teaching adult numeracy.
UK policy and resource research

In 2006 a report called *Establishing a Baseline*, was commissioned by the FSA (Financial Services Authority) after the *Financial Risk Outlook 2006* identified a priority risk being ‘people having to take increasing individual responsibility for their financial affairs.’ (FSA, 2006a) The initial survey was undertaken with researchers at Bristol University who surveyed over 5,300 people. This survey distinguished five components to financial capability:

- Making ends meet
- Keeping track of your finances
- Planning ahead
- Choosing financial products
- Staying informed about financial matters

Based on the findings of this survey and as part of the National Strategy for Financial Capability, the FSA published *Financial Capability in the UK – Delivering Change* (FSA 2006b). This included a seven-point action programme, including projects targeted at specific groups the survey had highlighted as having the most to gain from improved financial capability.

In April 2010, the Consumer Financial Education Body (CFEB) took over from the FSA trying to improve financial education in the UK and help people avoid debts. During 2010 it published the results of a number of surveys. One report, *Transforming Financial Behaviour*, took a psychological approach (Eliot et al 2010). This report identified effective drivers of the behaviour of individuals when dealing with the financial world. The report sought to enhance the effectiveness of delivering the financial capability messages.

The other major intervention pursued the recommendations of the Thoresen Report (Thoresen, 2008) to establish a major, free public money advice service. CFEB launched the Money Guidance Pathfinder project in Northern England from April 2009 to March 2010 with a budget of £12 million. This had the task to determine a range of models for achieving greater access to generic financial advice on a national scale, taking account of future developments in financial services markets and, in particular, personal accounts. The pathfinder project was then itself the subject of a major evaluation, with the main objective of assessing whether such a money advice service should be provided at a national level. This was a large survey involving over half a million visits to the Money Guidance pages of the Moneymadeclear website, the data collected showed approximately 25,000 people had sought face-to-face guidance and nearly 4,000 by telephone (Kempson. et al 2010).

The key findings identified a clear need and demand for a money guidance service. This has in fact been put into practice and since April 2011 the CFEB itself has been transformed into the Money Advice Service.

In 2011 The Centre for British Teachers (CfBT), commissioned a ‘perspective report’ from Shaun Mundy to review the current situation. This was published as Financial Capability: Why is it important and how can it be improved? This report sought to answer the questions: what is financial capability? What are the benefits? And what are the most effective ways of helping people?
Mundy identifies that financial capability means different things to different people, for ‘a person who has little money to have an accurate picture of how much they have available at any given time, because the consequences could be severe if they ran out of money.’ ‘A wealthy person may, on the other hand, only need a broad idea of how much money they have on them, but would benefit from understanding how to invest their money (Mundy, 2011 p4). He highlights that some countries confuse financial capability or financial literacy with entrepreneurship. He also recognised that claims that the financial problems of a country can be solved or a person empowered through improving financial capability may be a ‘seductive’ idea. However, to promise ‘both a free market and increased consumer welfare’ may be built on a ‘belief in the effectiveness of financial literacy education . . . lacks empirical support.’. He sees financial capability as an ability to be able to recognise when you need help. This can be thought of as being similar to being aware when you need to go to the doctor for a health check. During 2011 a new online survey (the Big Money Test) is also being undertaken by Lab-UK and the BBC; to explore the many different aspects of people’s relationship with money, looking at areas like knowledge, emotions, attitudes and habits. At the point of writing this report the research is on-going.

One interesting aspect of the research is that the person engaged in completing the questions online gets instant feedback on their attitudes and financial behaviours via short videos pre recorded to respond to certain answers. This interesting development provides a more interactive way of collecting data and performs an instant pedagogic role for the participant.

**UK online Survey**

The aims of the survey were to identify initiatives in the financial literacy field and identify any national concerns the project should take into consideration. There was considerable time pressure to complete the survey and so only a total of 39 respondents answered in the time available.

**The respondents**

The respondents were nearly three-quarters female and over 80% were graduates. The chart below shows that nearly three-quarters of the respondents were teachers or teacher trainers and that only a very small fraction considered themselves financial specialists.
Financial literacy Initiatives in the UK

This chart indicates where respondents considered initiatives on financial literacy take place. Most thought that state educational (69%) or private institutions (28%) were responsible for the initiatives. While others (28%) thought the banks provided / made available financial literacy to the public. Given the background of most respondents it is not surprising educational institutions are considered so often. Given what is reported above it is interesting to note that awareness of the initiatives by national government is so low.

Biggest obstacles to financial literacy in the UK

The biggest obstacles to developing the financial literacy skills in the UK were identified as no funding, low mathematical knowledge, no national policy and the growing complexity of financial products. The response about the lack of funding for financial education is interesting given the amount of money spent on national strategies through the FSA. The lack of funding suggests that training in financial literacy in the UK has to be embedded into other programmes in the education sector.
Initiatives related with financial literacy in the UK

There was an awareness of a range of initiatives, materials and products related to financial literacy. The responses included the identification of national initiatives. The highest recognition was of the work carried out by NIACE, such as their development of a financial capability framework. However, the development of resources funded by the Office of Fair Trading (OFT) which have focused on the literacy and numeracy skills for the consumer in a variety of contexts was also quoted.

Other organisations involved with financial capability such as the Citizens Advice Bureaux (CABx) Service and the Financial Service Agency (FSA) were also identified. Some particular websites such as Money Made Clear, Money matters and the BBC Raw were also identified. Some responses highlighted more institution-based initiatives and identified the Personal, Social Health and Economic education (PSHE) programme in UK schools that has a financial literacy aspect. Many cited locally run programmes in particular colleges or schools such as “Sound as a pound” or ‘Managing your money’ that use resources from one of the national initiatives or are linked to one of the national curriculum initiatives such as ASDAN and Free Standing Maths Qualifications (‘Free Standing Maths Qualifications’ have been developed by the mathematics education community to support the development of mathematical thinking in innovative ways).

How is the information about financial literacy provided or made available to the public?

This diagram shows that educational leaflets plus Internet and social networks are considered the main sources for information about financial literacy.

Teaching and training strategies
Strategies used to introduce content in current training courses were mainly simulations, discussion, online resources and case studies. Videos seemed to play a minor role in current teaching approaches or strategies.

**Topics important to financial literacy**

The respondents were also asked what were the most important topics in the training courses that they knew about. The main ones identified were:

- Budgeting, borrowing (dealing with debt) and understanding the benefits system
- Comparing prices of different products and services
- Percentage discounts and interest rates
- Cheques, opening bank accounts
- Financial language/jargon busting
- Basic maths skills

It is interesting to note the overlap with the main ‘chapter headings’ identified during the FinLico project, which are discussed below.

**Organisations surveyed**

In addition to the self-completion questionnaire there was also a survey carried out of key organisations that were identified as being important to the development of financial literacy in the UK. The organisations contacted were:

- Bank of England
- Confederation of British Industry (CBI)
- Citizens Advice Bureau (CAB)
- Consumer Financial Education Body (CFEB)
- Prudential
- CfBT Education Trust
- Office of Fair Trading (OFT)
- NIACE

The questions asked of these organisations are given in Appendix 1a and a summary of the results are given in Appendix 1b. The response that was received from CFEB was most useful, as it is (in its revised version) so important to the development of the UK national strategy. The response included a number of links to various advice bodies including its own successor body the ‘Money Advice Service’. In response to the question, “Do you feel there is enough financial literacy education in the UK?”, the response was:

> “Perhaps too much emphasis is placed on day-to-day budgeting skills and accessing benefits or entitlements. More ought to be placed on promoting attitudes and behaviours behind key financial decisions. This is both for day-to-day financial management as well as for longer-term financial decisions”.

This same theme, to move away from the more basic and everyday aspects of personal finance is referred to again in response to question 5. “What topics do you feel should be covered in financial literacy programmes?”
“More than simply related to budgeting and benefits – understanding the importance of taking key financial decisions as early as possible to reap maximum wellbeing – understanding how to benefit from advances in technology for financial planning.”

These responses raise an important issue in the focus of future work in financial literacy. There appears to be a distinction between services that provide advice and information on using financial products and Financial Literacy education services to enable individuals to have skills to investigate financial matters themselves.

It is also significant that so few of the other organisations felt able to participate in this research. The impression given by those whose response was a refusal was that they did not feel able to comment on a policy position. Even though the introduction we gave them made it clear we were more interested in provision than policy information. It may also be that since the preferred term in the UK is now Financial Capability that our request under the name of Financial Literacy was not given any priority for a response.

**Developing the ‘Toolbox’ for year two**

One of the key outputs from the FinLiCo project is to produce a wide range of resources, which can include various activities and games, physical examples to manipulate, paper-based questions, websites for information and interactive activities such as personal budgeting. These collectively are called the ‘toolbox’. To co-ordinate the development of these resources across the eight partners a set of ‘chapter headings’ was agreed. The partners each contributed their interpretation of the headings to form a framework for the development of the resources.

The final list of headings agreed was as follows:

- Personal
- Critical Thinking
- Risks
- Planning and Investment
- Budgeting
- Glossary
- Savings
- Income (and taxes)
- Basic Mathematics and Adult Numeracy
- Financial Products
- Indebtedness
- Credit
- Monetary System
- Shopping and consumer rights
- Business Strategy
To give an indication of the development of the ideas for this framework we can look at an example of one of them – Basic Mathematics and Adult Numeracy. The UK contribution was:

The underpinning skills for making all financial decisions; this can include interest rates, balances, earning and spending, calculating cost of a loan. Skills particularly focus on calculating and estimating to two decimal places, percentages (increase and decrease), fractions (1/3 off) probability (risk), extracting information from complex tables, interpreting charts and graphs.

The composite definition now agreed is:
- The daily use of arithmetic
- The presentation, and contribution, of information using graphs
- The elaboration, and interpretation, of family budgets in common use every day.

These are the essential and elementary aspects of math for this project:
- Mathematics to help in financial counting
- The underpinning skills for making all financial decisions which can include interest rates, balances, earning and spending, calculating cost of a loan.

Skills particularly focus on,
- Calculating and estimating to two decimal places.
- Percentages (increase and decrease)
- Fractions (1/3 off)
- Probability (risk)
- Extracting information from complex tables
- Interpreting charts and graphs.

This implies that a person is able to reason, analyze, formulate and solve problems in a real world setting. He/she is able to use Basic Operations, needs to know how to add, subtract, find a percent, depending on how he/she approaches the problem, and/or the context, needing to know decimals and how to work with fractions, understanding charts, graphs and tables.

Basic mathematics is useful for everyday transactions and financial actions. In addition basic mathematics oriented towards financial issues would be extremely helpful.

In the UK especially, there are many excellent basic numeracy exercises, about percentages, estimates, dimensions, such as, to:

- be aware of the presence of the mathematical presence and concepts in our everyday life
- be able to calculate distances, cooking, gardening, reading a plan: all these activities need basic understanding of mathematical concepts and problem solving strategies
- be able to estimate, make use of basic operations
- be able to read charts, plans, timetables.
At the discussion in the ALM 18 workshop it was noted that a focus on personal finance can be used as an attraction for people to attend an adult numeracy class, who might not attend a more general course.

Discussion

Research by the UK financial sector has identified the need for more financial capability skills development and support amongst the population. To this end many organisations have developed informative and interactive websites in response to the identified need. There seems to be general agreement that financial capability includes budgeting, keeping track of finances, planning ahead, choosing appropriate financial products and staying informed about financial matters. However, it would appear that people are more interested in developing these skills at particular times in their lives, usually at a time of change. This could be when starting a job, starting a new course at college or university, dealing with a purchase or a debt, becoming a parent, becoming unemployed or retiring. Mundy (2011) identifies these as “teachable moments”. The research also shows that people do not usually want to plan too far ahead and are often over confident with the evaluation of their own financial skills.

Most of the investment for research and the development of websites have been developed through the financial sector, including sections of the national government. They have targeted certain social groups and provide useful, up to date sources of information. A comprehensive list of sites is provided in Appendix 2.

However the survey of educationalists identified a gap in both awareness and knowledge of teachers and trainers in the provision of financial capability skills development for adults. It also points to a lack of provision of courses to develop financial capability skills for particular groups of learners.

While there has been some development in the UK schools provision linked to the PSHE programme there appears to be little on offer in either Further or Higher Education. This despite the fact that young adults may be confronted with extra travel and fees costs for the first time in their lives. In Further Education there is some evidence of financial capability skills development within other larger courses or programmes of study but this is not consistent and appears to be on an ad-hoc basis. The research conducted in the UK for the FinLiCo project has identified a ‘disconnect’ between the surveys, products and behavioural messages developed through the financial sector and those financial capability training, courses and resources available and targeted through the education sector.

Conclusion

This report summarises the work done by the adult numeracy team of teacher trainers now at Learning Unlimited (previously LLU+ at London South Bank University) as the UK partner in FinLiCo. This is a European project that aims to develop a set of resources and materials to address the financial literacy competencies (FinLiCo) of the public in general and specific groups as identified. The background research initiated at the beginning of the project identified the work that has been funded in the UK by the financial sector, which is considerable and well developed. A small-scale survey, of
mainly educationalists, identified a completely different set of concerns and not much awareness of the developments in and by the financial sector.

The FinLiCo team has developed a framework of competencies summarised under 16 headings. These are now being used to guide the development of a wide-range of resources to be shared amongst the eight partner countries in the second year of the project.
References


Big Money Test https://www.bbc.co.uk/labuk/articles/money/faq.html (accessed Oct 28th)


Appendix 1a

The questions used were as follows:

1. What do you think are the main issues associated with Financial Literacy in the UK? (debt, knowledge of financial products, budget management, investment, savings, general financial knowledge, getting value for money, other?)

2. Are you aware of any Financial Literacy education or related advice and guidance initiatives or programmes? (please list any you are aware of and the organisations involved)

3. Do you feel there is enough Financial Literacy education in the UK? (If not, what are the barriers and how could this be improved?) (educational leaflets, print and broadcast media, adverts, books, internet, social networks, other?)

4. How do people find out about financial literacy education programmes in this country?

5. What topics do you feel should be covered in financial literacy programmes?

6. Are you associated with any financial literacy programmes? If yes, do you have any contact details of the organisers?
<table>
<thead>
<tr>
<th>Name of organisation</th>
<th>Type of organization</th>
<th>Latest stage in process</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of England</td>
<td>The UK national bank</td>
<td>Email request</td>
<td>No response</td>
</tr>
<tr>
<td>Confederation of British Industry (CBI)</td>
<td><strong>Main UK business association</strong></td>
<td>Email request</td>
<td>Interview not given but gave link to Financial Skills Partnership</td>
</tr>
<tr>
<td>Citizens Advice Bureau (CAB)</td>
<td>UK wide network of advice centres</td>
<td>Email correspondence</td>
<td>Some information provided and interview pending</td>
</tr>
<tr>
<td>Consumer Financial Education Body (CFEB)</td>
<td>This is a UK government funded public advice service now called “The Money Advice Service”</td>
<td>Interview questions completed</td>
<td>The response is summarised.</td>
</tr>
<tr>
<td>Prudential</td>
<td>International finance company based in UK (established originally as a finance company)</td>
<td>Email request</td>
<td>Felt that they did not have the resources to respond</td>
</tr>
<tr>
<td>CfBT Education Trust</td>
<td>Education consultancy and service organisation</td>
<td>Email request</td>
<td>Interview not given but referred to relevant recent report</td>
</tr>
<tr>
<td>Office of Fair Trading (OFT)</td>
<td>The UK’s consumer and competition authority.</td>
<td>Email request</td>
<td>Unable to give policy views in an interview</td>
</tr>
<tr>
<td>NIACE</td>
<td>Major national adult education organisation</td>
<td>Email request and telephone response</td>
<td>Staff in financial literacy unavailable for comment</td>
</tr>
</tbody>
</table>
ASDAN Budgeting activities produced for awarding body with the cooperative college
http://teacher.beecoop.co.uk/?q=node/120

The Citizens Advice Bureau has produced a range of resources and reports on Financial Capability in the UK. The main advice site is http://www.financialskillsforlife.org.uk/ and the reports can be found at http://www.financialskillsforlife.org.uk/index/partnerships/financialskillsforlife/fsfl_resources_publications/fsfl_rp_publications.htm. In addition there is advice for young people on the Money Talks site including an interactive game on running a car http://www.citizensadvice.co.uk/en/moneytalks/MT-Toolkit/

CfBT Education Trust has recently (2011) produced a perspective report Financial capability: Why is it important and how can it be improved? This report is aimed at “those with an interest in the development of national financial capability strategies and those who are developing or implementing financial capability programme” http://www.cfbt.com/evidenceforeducation/pdf/FinancialCapability.pdf

Direct Gov Lots of links to useful websites about money, taxes and benefits http://www.direct.gov.uk/en/MoneyTaxAndBenefits/ManagingMoney/index.htm

Dius -Read Write Plus Reading and writing and calculating with money- Entry 3 – focus on decimal points http://rwp.qia.oxi.net/learning_material/portal/reading-writing-and-calculating-with-money_num_e3/m04/t19/index.htm

Financial capability framework The UK developed a framework for financial capability in 2000 which has been used to structure training and qualifications http://shop.niace.org.uk/adult-financial-capability.html.

Financial Services Authority (FSA) – advice on wide range of consumer advice http://www.fsa.gov.uk/Pages/consumerinformation/scamsandswindles/index.shtml The FSA has also produced a number of reports including Establishing a Baseline which reports on a survey of financial capability of 5300 adults, Consumer Research http://www.fsa.gov.uk/pubs/other/fincap_baseline.pdf

Martins Money Tips Advice website http://www.moneysavingexpert.com/ and weekly tips on how to save money by email MartinsMoneyT@moneysavingexpert.com

There is also the separate group, Consumer Financial Education Body (CFEB), set up by the FSA http://www.cfebuk.org.uk/index.shtml and since April 2011 it has become the Money Advice Service http://www.moneyadvice service.org.uk/

Money made Clear -Loads of information on home, everyday money, cards and loans, mortgages, insurance, pensions & retirement, savings and investments, tax and benefits http://www.moneymadeclear.org.uk/tools/budget_planner.html –

Money Matters a practical guide to family finance Many interactive pages such as: using an ATM, checking change, budgeting, Bank and Building society charges video, Money word search http://www.moneymatterstone.com/default.htm

Mums net Shopping and discount tips as well as loads of other information on parenting http://www.mumsnet.com/

NIACE has also produced a range of resources from projects linked to financial capability and learning. These can be found at http://www.niace.org.uk/current-work/area/financial-learning

Office of Fair Training (OFT) – consumer advice and activities www.ofit.gov.uk/skilledlogo

Personal Finance Education Group (PFEG). Online resources on financial awareness developed for schools, primary, secondary and 14-19 year olds, funded by banks and other financial institutions http://www.pfeg.org/
Younger people Financial advice aimed at students - A quick quiz to see your financial style and then 10 areas with a quiz for each e.g. banking, borrowing, bills, debts, etc

http://www.moneymakesense.co.uk/ -.
New Dutch Numeracy Framework

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Very recently, a new framework for literacy and numeracy was implemented in the Netherlands (Ministerie van OCW, 2009). In this paper we describe the background of the new framework and some of its characteristics. We give, also, some insights to the problems facing the implementation of such a framework in schools.

Key words: Numeracy framework, Implementation, the Netherlands

Why a new framework?

In the 1980’s and 1990’s, Dutch education focused on meta-cognition; high order skills; cooperation; using ICT; integrating knowledge; connecting knowledge with the real world; bringing the real world into the classrooms; and on self-regulated learning. Innovations in education were aimed at preparing students for the future with broad and higher order skills. Many countries pursued a similar aim.

In the early years of this century, some concern arose when some students at the lower strands of the educational system had severe problems in reading the school textbooks. Another concern was the observation that many students of all ages used a calculator for calculating problems like 3 x 5 or 6 + 18. In Dutch elementary schools, the common practice is realistic mathematics education (Gravemeijer, 1994). This is an instruction theory which focuses on context-based learning through understanding mathematical principles. Students are encouraged to see the world through mathematical eyes and to reflect on their individual problem solving techniques. Although the mathematical skills of the Dutch students are ranked among the top ten in the world, a broad societal concern arose about the apparently decreasing “technical mathematical skills and plain arithmetic skills”. Concurrently in the Netherlands, there was an upcoming populist political climate with an undercurrent of retro-romantic rhetoric about many issues including basic skills in reading, writing and arithmetic. Key to this rhetoric was a kind of inverse utopia: once there was a time when all the children were able to write and spell correctly; to read literature; and to do all calculations by mental arithmetic or by orderly pen-and-paper algorithms. Indeed it was an utopia.

An expert panel “Doorlopende Leerlijnen Taal en Rekenen” [Continuous Learning Strands Literacy and Numeracy] was established with the task of developing a framework for literacy and numeracy for children aged 8 to 18. In Dutch, it is called “Referentiekader Taal en Rekenen” (Expertgroep Doorlopende Leerlijnen Taal en
There is not a good translation in Dutch for literacy; language (Taal) is used instead. Likewise there is not a good translation for numeracy; arithmetic (Rekenen) is used.

With a framework aimed at a broad range of school levels, the goal is to provide more continuous learning strands through the different school levels. The transition from primary school to secondary school (age 12), and the transition from secondary to vocational education (age 16) or University (age 17 and age 18), are seen as especially problematic. Those transitions generate everlasting complaints that the lower level keeps delivering students with decreasing levels of language and arithmetic. However, the most important goal of installing a new framework was the improvement of the students’ results in numeracy and literacy tests. Those tests are designed by the government and made obligatory for all students at some point in their school career. The assumption is that this will significantly increase the overall level of literacy and numeracy (or arithmetic and language) in Dutch society.

**The Numeracy Framework**

The framework consists of eight levels which are: 1F, 1S, 2F, 2S, 3F, 3S, 4F, and 4S. For the numeracy framework, this is limited to six levels: 1F, 1S, 2F, 2S, 3F, and 3S. The levels 4F and 4S are considered to be mathematics.

From this point forward, this paper describes the mathematical side of the framework.

![Figure 1. Levels in Dutch Numeracy Framework (Ministerie van OCW, 2009)](image_url)

The levels 1F and 1S are for 12-year-olds. The level, 1F, stands for “fundamental” level and the level, 1S, stands for “strive” level. Level 1F is fixed on the benchmark that 75% of the current primary school population reaches this level. The policy aim is that this must grow to 85%. Level 1S is fixed on the benchmark that 50% of the current primary school population reaches this level. The policy aim is that this must grow to 65%.

The levels, 2F and 2S, are for 16-year-olds. The level, 2F, is considered to be the level that every citizen in the Netherlands should reach because it is the required level to participate in society and to cope with the quantitative aspects of the surrounding world. When reached, it is the level which must be consolidated by the students over the whole school period. From the framework itself, or from the underlying report, it is not clear directly what this level should be exactly. It is mentioned that it should be, at least, the median of the students’ results in the lower vocational strands. In a series of pilot-examinations over the years, the levels will be established.
The levels, 3F and 3S, are for 18-year-olds. The level, 3F, is the target for vocational students at the end of their vocational education and for students at the end of their pre-higher or university education.

**The content of the framework**

All leading frameworks on numeracy and mathematical literacy consist of four or more categories.

In Pisa, there are the following four big ideas (De Lange, 1999) (Dekker e.a., 2006):

- Quantity and Number sense
- Shape and space
- Change and relationships and
- Data analysis.

In the IALS/ALL, comparative studies the following, comparable, distinction is made (van Groenestijn, 2002):

- Quantity and number
- Dimension and shape
- Patterns and relationships
- Data and chance and
- Change.

These are all examples where the goals of mathematics education (in primary and secondary school) are broad and encompass connections between several subdomains of mathematical knowledge.

In this tradition, the expert panel “Doorlopende Leerlijnen Taal en Rekenen” [Continuous Learning Strands Literacy and Numeracy] arrived at the following four domains in the numeracy framework:

- Numbers (Getallen)
- Proportions (Verhoudingen)
- Measurement & Geometry (Meten & Meetkunde) and
- Relations (Verbanden).

Within those domains, a further specification is made about the way in which the skills have to be mastered. In each domain the expert panel distinguishes between:

- Functional use (functioneel gebruiken)
- Ready knowledge (paraat hebben) and
- Knowing why (weten waarom).

This breakdown is visualized in the picture below. The four quadrants are the domains. In the inner circle, you see the functional use. In a circle around that, you find the ready knowledge required for functional use. In the corners, you find the more reflective approach (How is it working? What are the connections?).
This is also an indication and an acknowledgement that the use of mathematics for a large part of the population is relevant because of its *functional* use. It is simply not sufficient to only focus on ready knowledge and the mastery of basic skills.
If we zoom in, we can see the way the goals are formulated. Here are some translations, just to get an idea.

**Upper left corner: Knowing why**
- Relating numbers to situations
- Check a calculation
- Choosing an appropriate way of calculating: by pen and paper or by using the calculator and
- Verify calculations and reasoning.

**White circle: Ready knowledge**
- Correct use of parenthesis
- Writing fractions, percentages, powers and roots as a (rounded) decimal number with a calculator and
- Using negative numbers.

**Inner circle: Functional use**
- Translating a situation into an operation and
- Rounding numbers in a meaningful way.

These are all quite general formulations of the goals. What exactly is meant by them is laid out in some examples in the report. However, the interpretation is influenced heavily, also, by publishers and test producing companies which bring “translations” of the framework into materials for lessons and tests for the market.

**Calculator issues**

Debates on the framework and on the state of mathematics education in the Netherlands intensified, also, the discussion on the role of a (graphic) calculator in education. There is an ongoing debate about its role. The opinions vary from, “One should forbid calculators in education all together. They are to blame for the terrible state of our students’ mathematical skills.”, to “Calculators are part of our life. In the 21st century, one should teach students how to use them properly and effectively. Education without calculators should not be in schools but in a museum.”.

From the more retro-romantic rhetoric, it is clear that students should do all mathematics without a calculator. Pen-and-paper are considered to be innovative enough (which, ironically, could be a truth since, nowadays, most students seem to be attached permanently to their highly advanced technological gadgets). From the innovators, who are aiming at preparing students with 21st century skills, a general plea is made for using technology.

From an international perspective, this discussion is settled already. In PISA and TIMSS, calculators may be used by students who are familiar with them in their school practice. In countries where the calculator is not yet an implemented education tool, students can do the test without a calculator.

In our experience it is the first time, in a highly technology advanced country such as the Netherlands, that advocacy to abolish the calculator in education seems to have gained ground. Is it a reaction against using untimely or inappropriate technology in education or is it an retro-romantic Luddite kind of resistance? The future will tell.
In the Netherlands, a compromise is found for almost every problem, in the end. The present situation is that 20% of the problems in the final examinations should be solved without calculator and for the remaining 80% a calculator is allowed. However, the debate continues.

**Implementing the framework**

For the implementing the framework, four different groups of students can be distinguished as follows:

- Tertiary students in the vocational strands
- Secondary school students in the pre-vocational strands
- Secondary school students in the pre-university strands
- Primary school students.

**Tertiary students in the vocational strands.**

In the Netherlands, there is quite a large system of tertiary vocational education (age 16-20). It is called mbo. It is populated by over 60% of all the people attending some sort of school. In the mbo during the last 10 years, a lot of development was put into competency based education; a constructivist idea of teaching vocational skills in a real situation and with tasks resembling closely future jobs. The skills are evaluated by portfolio, masterpieces, and oral presentations. Although bound by centrally made agreements on the skills which need to be mastered, the quality of which is controlled by Inspectorate of Education, schools are free in the way in which they build their lessons and test the students’ skills.

As a result of the introduction of the Numeracy Framework, a generic system of testing numeracy was superimposed on top of this system. The same National examinations, for all students and rated by grades were introduced (thus implying a rigorous fail/pass system). The contrast between these two types of examinations created quite a stir.

As from 2015, all students in vocational education will do a final examination in either numeracy 2F or numeracy 3F. This examination will consist of a digital test of some 60 items with a testing time of 90 minutes. For most questions, the student must provide a numerical answer. Those answers are to be scored automatically by computer software. For some other questions, when numerical answers are not possible, there will be multiple choice answering.

![Figure 4 Example of a numeracy problem level 2F (College voor Examens, 2011)](image-url)
Even without translation you can get an idea of the sort of problems.

In the forthcoming years, through a system of pilot examinations, the right level and a suitable pass mark must be established.

One of the serious problems with such a generic pass mark is that some sectors of vocational education focus on technology and technical jobs. Students from these sectors do not have a lot of difficulty with numbers and mathematical problems. However, a large part of vocational education is preparing students for a job in Health Care and Social Care. From tradition, these students do not have a lot of affinity with mathematics. The concern of the teachers of these students is that a generic test with a general pass mark will lead to massive failures amongst those students. No one is willing to have failure rates of, say, 80% amongst Health Care and Social Care vocational students, who are essential for the future of an ageing population. A problem which has not yet been solved.

Another effect of introducing generic demands is the massive introduction of numeracy lessons for all students. Independent of the way in which a school chooses to implement these lessons, there is suddenly a huge need for numeracy teachers. Until now, there is no suitable education for teachers of numeracy or arithmetic skills. Also, a numeracy teacher is not an official title. In vocational education every teacher is allowed and encouraged (and, sadly sometimes forced) to teach every subject.
Now, the reality is that, even if sufficient teachers are found, often they are not competent in the subject itself, let alone in the specialized content knowledge. This imposes an additional problem in respect of the quality of numeracy education.

**Secondary school students in pre-vocational strands**

The stream for prevocational education (vmbo, 12-16 year old) is quite large; some 40% of all students are in this stream. Since 1992, mathematics education in this stream has been influenced by international developments and tests like PISA. It transpires that the mathematics curriculum has about a 70% overlap with the new numeracy framework. Nevertheless, the Ministry of Education and many schools see the two subjects as separate. Separate lessons and separate testing are planned in many schools. Both “subjects” are in the pass-fail mode. There is a commonly held view that there is a clear distinction between mathematics (formulae, graphs) and arithmetic (operations). In their respective curricula however, they are almost completely about the same issues (numbers, geometry, relations, proportions). Educational consultants, like ourselves, advise schools to aim at integrating mathematics and arithmetic and even recommend to connect it with other subjects which have quantitative components. Steps towards organizing it in such a more connected and intertwined way are both slow and small.

**Secondary school students in the pre-university strands**

In the pre-university strands (havo, vwo, 12-18 year old) there are four different kinds of mathematics: ‘A’, ‘B’, ‘C’ and ‘D’, very much specified to the different student groups. Mathematics ‘A’ and ‘C’ are oriented very much on functional mathematics and mathematical literacy. Mathematics ‘B’ and ‘D’ are more technical and abstract and very useful for students who pursue technical or natural science studies. More than 90% of all students do a final examination in one of these types of mathematics. Nevertheless, there was a broad discussion in the media about the poor basic skills of these students. Many university professors complain about the very low basic skills of the students entering college.

In passing a mathematics ‘A’ examination, students use a (graphic) calculator to solve mathematical literacy type problems. However, some entrance tests for universities consist of abstract and complex arithmetic problems to be solved with pen and paper. There seems to be a mismatch between the curriculum attained by the students and the curriculum expected to have been attained at the next educational level. These students pass, relatively easily, the currently developed numeracy tests for the framework. This is not very surprising given the fact that the framework overlaps for a considerable part with the mathematics curriculum. Nevertheless, the Ministry of Education insists on keeping the final testing in numeracy in place, in addition to the mathematics examinations. One wonders how long it will last.

**Primary school students**

In primary schools, there is still very little knowledge about the framework to be implemented. In general, primary schools trust their textbook and the tests produced by Cito, the ubiquitous testing company in the Netherlands. For primary school teachers, the framework does not seem to have a major influence on their daily routine. Consequently, even the goal of getting the primary school teachers acquainted with the framework is proving to be a difficult job.
The future

The developing of a Dutch numeracy framework fits with the general trend of establishing overall frameworks for complex domains of cognition. In Europe, there is already a European Language Framework and several projects have worked on a European Numeracy Framework such as the “In Balance” project in which several European countries participate.

In the Netherlands, the focus on basic skills has led to a new numeracy framework which has a considerable overlap with the mathematics curriculum. For now, numeracy and mathematics are both organized and tested separately. In the long run, we predict that the numeracy parts will be integrated into the mathematics curriculum and that, more than now, the mathematics curriculum will focus on mastering basic skills, not only arithmetic skills but, also, the basic algebraic skills. In the worst case scenario, the mathematics curriculum will be reduced to only basic arithmetic and algebraic skills.
References

SLO. (2011). Voorbeeldtoets bij Rekentoetswijzer 2F [Test example in Numeracy Test Indicator]. Enschede: SLO.
This preliminary qualitative study will focus on the mathematical experience of commencing tertiary adult learners undertaking a degree in either science or engineering who take at least one mathematics module as part of their course. Previous research has been carried out to examine the effect that pre-semester bridging courses (Headstart Maths at the University of Limerick) have on commencing adult learners’ mathematics self-concept. This study will concentrate on the mathematical experience of the adult learners, only some of whom attended Headstart Maths, once they have commenced their mainstream college course. The study was conducted as part of a voluntary support tutorial run for both the science and engineering adult students throughout their first semester with the analysis being based on reflective journals maintained by the adult learners.

The University of Limerick (UL) defines an adult learner as any individual aged 23 years or older who is currently a member of the student body. In the academic year 2010/11, almost 12% (11.99) of the commencing first year cohort who studied mathematics as a service module (primary fields of study include Engineering, Science, Technology or Business) were adult learners (O’Keeffe, 2011). Adult learners face many difficulties when returning to study mathematics including a lack of confidence in their mathematical abilities in addition to harbouring negative attitudes, beliefs and feelings towards mathematics (Coben, 2003). All of these factors result in students having low self-efficacy beliefs and, additionally, displaying symptoms of mathematics anxiety.

Ashcraft & Moore (2009) define mathematics anxiety as “a person’s negative affective reaction to situations involving numbers, math, and mathematics calculations”. Mathematics anxiety has previously been highlighted as a factor in attrition rates (Jones, 1996), under achievement (Reyes, 1984), and under performance (Cates & Rhymer, 2003), particularly in timed, high-stakes settings (Ashcraft & Moore, 2009). In a study carried out by Sewell thirty years ago (Sewell, 1981), it was shown that at least half of the adult population surveyed displayed negative feelings towards mathematics. All of these studies, and others, emphasize the importance of the potential effects that mathematics anxiety has on the overall mathematics performance of adult learners, irrespective of true ability.

Self-efficacy is defined as “people’s judgement of their capabilities to organize and execute courses of action required to attain designated types of performance” (Bandura, 1986:391). Pajares and Graham (1999) showed that there exists a relationship between self-efficacy and performance and that a higher level of self-efficacy may in fact moderate the influence of mathematics anxiety. A higher level of self-efficacy in an
individual means that they are more effortful, attempt more cognitively challenging problems, and persist longer (Pajares & Graham, 1999). Klinger (2006) investigated attitudes, self-efficacy beliefs, and math-anxiety of pre-tertiary adult learners using a self-designed pre- and post-survey. He found that although students initially shared the negative views reported in the broader population, these views changed significantly by the conclusion of the programme. He concluded that adults’ perceptions of, and capacity to engage with, mathematical content was strongly influenced by early learning experiences, but, like other authors had found in previous studies (Burton, 1987), confirmed that these perceptions could be changed.

In this paper, the authors investigate adult learners’ ideas and experiences of mathematics based on written reflections maintained by the commencing adult learners. These reflections are gathered as part of a weekly support tutorial run by the Mathematics Learning Centre (MLC) at UL during the first semester of the academic year 2010/11. The concepts of mathematics anxiety and self-efficacy are recurring themes when dealing with adult learners of mathematics and so attention was paid to see if these themes would be evident within the journals collected as part of this study. Previous research on reflective journals (Farrell, 2007; Lanigan, 2007) have found that students who engaged with the reflective journal activity not only experienced a deeper understanding of the content, but experienced a greater awareness of their own style of learning, and, additionally, acquired more self confidence as well as a positive change in attitude. For these reasons, and also so that we could have a weekly report of students progress, the students were asked to maintain the reflective journals during their commencing semester.

Two different cohorts of adult learners, engineering students and science students, participated in the study with the aim of identifying differences between the experiences of both groups, if they exist. Initially it was found that the science students displayed higher levels of self-efficacy than their engineering colleagues and lower levels of mathematics anxiety. Over the course of the semester the gap between the two groups, in terms of both self-efficacy and mathematics anxiety, decreased with both groups displaying more belief in their mathematical capabilities and abilities by the end of the semester.

**Background to the study**

The students (N=22) partaking in this study are commencing first year Science or Engineering students. They are all adult learners, aged 23 years and older, and had varied knowledge and prior learning experiences due to the number of years since they last studied mathematics (anywhere from less than 3 to more than 20 as can be seen in Figure 1.), and the fact that some of them are from countries other than Ireland. In addition to this, some of these adults will have been enrolled in the mature student access certificate course which is a one-year pre-degree course designed for individuals who feel that they lack the skills to undertake, successfully, a degree course or do not have the necessary qualifications to enter a course directly. Mathematics is one of the core modules on the access course and so adults who participated in this course will have received a refresher course on mathematical fundamentals, from numbers through to basic calculus, in the year prior to commencing their degrees.
In addition to the access course, adult students have the opportunity to attend a pre-semester bridging course called *Headstart Maths* run by the MLC in the weeks leading up to the start of the college term. Gill (2010) showed that courses like this can have a positive effect on students’ mathematical self-concept. In past years, a one week course (Headstart1) was run but based on feedback from students it was expanded for a second week (Headstart2) and more advanced topics relevant to engineering mathematics (e.g. vectors & complex numbers) were included (Gill, 2010).

Headstart Maths is a two week bridging course that covers foundational mathematical knowledge that every commencing student is expected to know. Topics covered in the first week include Number Systems, Equations, Factorising, Graphing Functions, Problem Solving and Logs and Indices. The second week progresses to topics that would normally be introduced in late post-primary education e.g. Differentiation and Integration, Vectors, Complex Numbers and Advanced Algebra.

It can be seen from Figure 2 that very few engineering students (EM) attend the bridging courses or participate in the access course. This is due to the fact that most commencing adult engineering students already meet the requisite mathematical requirements for entry to degree level engineering programmes. On the other hand the majority of the science students gain access to their degree either through the access course or by interview through the Mature Student Office. Once they are known to the
Mature Student Office they are informed of any additional bridging course (including Headstart Maths) that runs prior to the commencement of the term and so more science than engineering students typically attend the Headstart program.

**Methodology**

Once the adult learners have gained entry onto their chosen degree, there are a number of mathematics support provisions in place for them organised by the MLC. A weekly support tutorial is offered to both the science and engineering students in addition to their regular timetabled classes each year. These support tutorials are run in the evening (normally 6pm) and commence in week 3 of the term and run until the end of term (week 12). The adult students in both science and engineering mathematics are allotted a separate tutorial to the traditional students as it is felt that the adults will require a slower pace and more supplementary detail within the tutorial which may not be necessary for the traditional students.

As part of the requirement of the support tutorials this year, the adults were asked to maintain a reflective journal on their mathematical experiences during each week. The reflective journals provided the adult learners with a means of reflecting on their learning while providing us, the teachers, with valuable insight into the learning experience from the perspective of the adults. In week 2, the tutors outlined what exactly a reflective journal is and also provided the adults with a sample for them to see. Additionally, a number of sample questions, taken from literature and websites relating to mathematics journal writing, that the students could ask themselves to assist in the writing of their reflections, were provided. Reflective writing is something that the majority of these students would have been unfamiliar with, especially in a mathematics class, and so providing them with as much support and provisions from the start was essential to encourage them to maintain the journals. The journals were collected from the students at the end of the semester or at the start of the following semester (some students held onto the journals for study purposes) and the findings from the journals are presented here.

**Students’ Experience of Mathematics**

In this section, we examine the students’ reflections on their mathematics classes in order to investigate their overall experience of mathematics itself. We make comparisons between engineering and science students to highlight any differences that may exist between the two cohorts. The following results are presented in three different subsections based on a midterm examination that typically takes place in week 7 of the semester in both science and engineering mathematics. The first subsection focuses on the pre-midterm weeks (weeks 2-5); the second subsection focuses on the weeks around the midterm (weeks 6-8); and the final subsection concentrates on the end of term weeks (weeks 9-12). The midterm examination is worth a substantial amount of the final grade, viz., 20% in science mathematics and 30% in engineering mathematics. Additionally, it is the first formal examination that most of the adult learners will have taken in quite a while and therefore is a significant and milestone event in their commencing college studies.

**Pre-Midterm Weeks**

Science Mathematics comments:
“After 6 weeks of maths (including the HSM) I find myself to be reasonably confident in the class. My ability to work out problems seems to be quite good”.

“Just finished week 3 and I have to say that it went pretty well, certainly a lot better than this time last year (access course), when I couldn’t even add fractions”.

Engineering Mathematics comments:

“I feel totally lost in the lectures and feel well behind everyone else in the room. I’m starting to think I picked the wrong course and at this stage I cannot see myself lasting to Christmas”.

“Everything seems to happen so quickly and I did not have too much time to review lectures and do the homework. Time is flying…..All is very fast and more complicated; I’m starting to think I’m too slow”.

In the first weeks of term it is clear that the science mathematics students are more comfortable and appear more confident in their capabilities and their ability to deal with the workload and learning involved in their course. The engineering mathematics students, on the other hand, experience high levels of anxiety early on in the term. This could be due in part to the fact that they are expected to cover more material than their science counterparts and that there is an expectation that all engineering students have studied “Higher” level mathematics for their Leaving Certificate. Unfortunately it is outside the scope of this study to determine if these factors are, in fact, responsible for the high levels of mathematics anxiety displayed by the students.

Midterm Weeks

The midterm weeks are analysed in three sub categories i.e. pre-exam, post exam (after the exam but without the result been known), and post result. This allows us to gain a deeper insight into the experience of the adults and also to be able to place the comments in context depending on when they occurred around the midterm examination.

Science Mathematics comments (pre-exam):

“Overall I’m still enjoying maths but am still very nervous about the exam”.

“I think I may have overdosed and find myself confused and dazed once more. Simple processes like adding and multiplying functions is catching me out”.

“I’m not panicking; I’ve kept up with the course in the preceding weeks”.

Engineering Mathematics comments (pre-exam):

“I’m getting a bit nervous, which is good I suppose because it means that this exam is important to me. It will be my first exam in college and I want to do well. I’m doing nothing else but maths at the moment and I’m starting to enjoy it. I’m getting more confident and things are starting to make some sense, at last”.

“I don’t feel too bad about next week’s midterm, not saying I’m going to do brilliantly but I don’t think that I will fail, which I couldn’t have said at the beginning of the semester or if the support tutorial wasn’t there”.

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“… I feel more confident now and I still have a few days to prepare the last questions. I know I can do it”.

It is clear that both groups of students feel more confident at this stage of the semester although the thought of an examination does appear to increase anxiety levels in both groups. The engineering students display more belief in their abilities and comment that they are finally starting to make sense of the mathematics being presented to them.

Science Mathematics comments (post exam):

“I’m pretty confident I did ok in the exam. I had all questions answered in 30 min which gave me 15 minutes to go back over each one to ensure I had not made silly mistakes”.

“Just had the midterm, was really nervous beforehand. The biggest thing was to start writing something. I’m pretty sure I haven’t failed, in fact I’m pleased already with how I have done”.

“I think I did ok, overall I’m pleased with my effort. Could never have seen myself here 2 years ago. I can now do maths as well as my 17 yr old nephew. Great experience!”.

Engineering Mathematics comments (post exam):

“I wasn’t too happy coming out the midterm as I did a real mess out of question 2. But looking back at it I’m not too upset because for some strange reason I feel more confident going forward because I feel I have come a long way and I’m starting to understand the maths more”.

“Had the exam yesterday, felt pretty good about it while doing it and after but seeing the answers and talking to some people I don’t think I did as well as I imagined, but probably I didn’t do too bad either. I hope to get 15-20% of the 30%. Wouldn’t have gotten 5% if I didn’t have the support tutorials to help me and give me more confidence about the exam”.

Both groups of students appear confident after the examination is completed and are pleased with how they believe they did in the examination. Science students seem slightly more confident in their capabilities and less anxious than the engineering students.

Science Mathematics comments (post result):

“I’m pretty chuffed, got an A for the first time since primary school. A lot of my fears have dissipated with this result. I’m much more confident with my abilities and can now visualise myself finishing this degree”.

“Result 16.5 out of 20 - excellent, I’m delighted. It is a huge improvement on my 13 out of 40 on the first day of lectures (diagnostic test) … I’m well capable of learning maths as long as I put in the effort and avail of the help and resources provided. For me maths has been de-mystified, it is still difficult but this is third level”.

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Engineering Mathematics comments (post result):

“Absolutely delighted with my result 26%! I would never have dreamt that I could get much after the first week or two. This is going to give me a real boost going towards the final exam, takes a lot of pressure off my shoulder now about even just passing the module”.

“As I expected I got 14.5%, I still feel disappointed but it is time to move forward and concentrate on the next exam”.

Overall, it is clear that closer to the midterm the difference in mathematics anxiety and self-efficacy levels between the science and engineering students, which was evident in the early weeks, has decreased. Both groups of students appear confident in the week leading up to the midterm examination. The midterm examination appears to “release” the pressure that has been experienced in the previous weeks among the students. Even if they do not score as high as other students in their class they seem to feel more confident about their abilities and their capability of doing mathematics and of moving forward.

Towards the end of the term, both groups of students display a higher level of self-efficacy. Having done well in the midterm examination both groups of students feel confident and believe that they can succeed in mathematics.

Post Midterm Weeks

Science Mathematics comments:

“I’m revising for the final exam and I’m pleased with how I’m tackling the exam papers from previous years, particularly when I can complete a question without looking back at similar examples and solutions”.

“One week left to end of the term exams. I should do well in Maths. I’m not sure if I should try to work on particularly weak areas or really reinforce my medium to strong skills. Should I settle for a good B or push for an A?”.

Engineering Mathematics comments:

“I have the maths exam on the first day of exams. Will be good to get it out of the way. I must say I’m not as worried about it as I thought I would be or as I was when I thought about it in week 1! ... In fact it has probably gone from being the exam I was most fearful of after a few weeks to being 2nd from the bottom of my worrying list”.

“I still get stuck on small things but the more I do the more I understand it. With a week and a half to go to the final exam, even though I don’t want to jinx myself, I think I’ll be ok”.

It is interesting to note that science students appear to place higher expectations on themselves in terms of achieving a better grade than their engineering colleagues. This could be due to the fact that the science students displayed higher levels of self-efficacy and lower levels of mathematics anxiety from the early weeks and so are in a better overall position than the engineering students at the end of the semester.
Additional findings from the Journals

During the term, the students offered other insights into their mathematical experience that were not directly related to the module or the examination. Some of these insights are now presented and discussed.

The students found the support tutorials very helpful during the semester. The key pedagogical aim of the support tutorial was to convey the importance of conceptual understanding to the students. In addition to this, some of these individuals would have been out of education for a long period of time and so it was felt that stressing the importance of understanding a concept, rather than just being able to apply a procedure, was essential. The reasoning behind this is, due to the fact that both science and engineering students are expected to apply their mathematical knowledge regularly to problems set in unfamiliar contexts, having a deeper understanding of mathematical concepts is essential to assist in the solving of these problems. The support tutorials also offered the adults an opportunity to get to know each other in a smaller setting than the lecture or traditional tutorials and helped to build their confidence and belief in their capabilities.

“Just had a really good support tutorial. Focused on vectors. Everything was explained very well and I understood all the reasoning” (SM).

“The tutorials are really helping with the matrices. I’m starting to get the big picture and make the links: Cross product and the determinant of a matrix” (SM).

“After the second support tutorial I feel I might have a chance to get through the course” (EM).

“The extra group sessions are really good, especially the more I’m getting to know the lads the more confident I feel about asking questions” (EM).

The students also discussed which components of their courses they found particularly difficult, and why. This offers us, as teachers, an insight into what students think about particular topics and also gives us valuable feedback on sections of the course which might need more explaining or additional time.

“Word problems” presented the science mathematics students with problem formats and presentations they were not familiar with and thus were conceived by them as a major obstacle. In particular the concept of optimisation where the adults were asked to apply the mathematical techniques that they have covered to a real life problem proved to be challenging.

“I don’t like these types of questions, I think about maths in terms of numbers and rules, now to solve a question I have to come up with my own functions”.

“Differentiation is not the problem, finding what to do is!”.

For the engineering students, complex numbers appeared to be a challenging topic. The fact complex numbers relies on other foundational knowledge, e.g. trigonometry, made this topic more challenging for the adults.

“We have started complex numbers, it was ok until we got to the polar form. I don’t like and I don’t get these trigonometric functions”.

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“I’m finding complex numbers hard, I’m not use to working in radians”.

**Conclusion**

Over the course of the semester, the initial difference highlighted between the science and engineering students’ mathematical anxiety and self-efficacy appears to diminish. The science students, perhaps due to their participation in the access and Headstart courses, are in a stronger position from the start compared to their engineering counterparts. Additionally, the demands of the engineering course, e.g. the expectation of higher level mathematics, places more pressure on the engineering students especially adult learners returning after a period of time out of formal education. By the time the midterm examination happens, both groups of students appear to have settled into college life and score well in the examination. This builds confidence and has the overall effect of lessening mathematics anxiety within the students.

A lot of valuable information was gathered from the reflections of the students. Comments regarding particular topics or study techniques provided us with an insight into how the students were learning and studying mathematics. Traditionally this type of information is difficult to come by as students do not want to say that they find something difficult in front of their friends and most study by themselves, which means that they do not feel the need to discuss how they study with anyone. In light of this, we would recommend that other teachers involved in similar programs should encourage their students to maintain a reflective journal, as reading and considering their students’ reflections will offer valuable insights into the teaching and learning that is happening with their class.
References


Relevance as a Bridge between College-Preparation Adults and the World

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Relevance is important for learners and this may be especially true for adults in remedial Algebra college-preparatory courses. For many topics in Algebra, however, relevance can be somewhat elusive. Students are often unsatisfied with explanations from instructors on why they have to take a math course. Most remedial Algebra textbooks in the United States claim to have real-world application word problems but those problems are oftentimes contrived and unrealistic. Is Algebra relevant? What do we mean by relevance? In this study, relevance is situated within a conceptual framework based on Skovsmose’s notions of dispositions, intentions, and actions (Skovsmose, O. 1994, 2005). This paper shares the results of a qualitative study in which relevance is examined from the student perspective. Through interviews of remedial Algebra students at a university [in the United States of America], five types of relevance were found: practical, process, professional, intrinsic, and empathetic. Personal meaning and relevance of Algebra to students does not always match what is intended by faculty and textbook authors. Five recommendations are offered to better deal with relevance including potential future research questions. This area of research is important given the role that college mathematics plays in access to college degrees. Perhaps changing how we address relevance can help build a bridge to the world; a bridge students are willing to cross.

Key words: Algebra, relevance, higher education, motivation, developmental mathematics.

Context and Purpose

While teaching a remedial Intermediate Algebra class at a university recently, I was asked the perennial question, “When is this ever used in real life?” Another student interjected that he knew the answer that all math teachers should give for that question: “It is used at the National Aeronautics and Space Administration.”. He said that statement will satisfy 95% of students who are wondering about the relevance of mathematics. For the rest of the semester the running joke was “this is used at NASA”. While everyone would laugh, the underlying issue of personal relevance was serious for me. This and other interactions with students have caused me to think more deeply about this topic. As I work with underperforming or failing students, I have wondered if their performance is a lack of ability on their part or, rather, a lack of motivation because they see no reason to become engaged with learning mathematics. What makes
Algebra personally relevant? What is relevance? Is there a relevance bridge to the World?

Each semester, I feel more and more guilty. Am I lying to my students, particularly those not planning to study in math or science-related fields? Is what they are learning worth their time and effort? My time and effort? Students who pass my class can continue on to pursue a college degree. If they do not pass, the students are unable to proceed on to get a degree, unless they repeat the course until they can pass it. For most students at my institution, they must also go on to pass College Algebra. I have become more and more disillusioned with Algebra requirements for the masses. Most people will never use Algebra in their lives outside of maybe some basic equation solving. Regardless, I tell my students that it is still valuable, even if they never use Algebra again. It helps to develop critical thinking and problem-solving skills. At least, I hope it does.

Sometimes, I cringe when writing a rational expression equation on the board like this one:

\[
\frac{x + 1}{2x + 3} - \frac{3}{x - 4} = \frac{-3}{2x^2 - 5x - 12}
\]

Why is the student majoring in history or film required to know how to solve this? I believe in the importance of general education and I further believe that relevant mathematics should be a part of that general education. Much of Algebra, however, is not relevant, at least from a practical use perspective. Textbooks try to make Algebra sound relevant with problems like:

The first generation of the iPhone \(^\text{TM}\) has an approximate volume of 4.968 cubic inches. Its width is 0.46 inch and its face has the dimensions \(x\) inches by \(x + 2.1\) inches. Find the dimensions of the face in inches.

(Larson, p.xii).

The problem above is actually touted in the introduction of the book as an example of “a wide variety of real-life applications” (p. xii). Would anyone want the dimensions of an iPhone while happening to know the approximate volume to the thousandths place and also happening to know that one dimension is 2.1 inches more than another?

Apparently, some of the reasons for requiring Algebra have remained unchanged for over 150 years in the United States. The following quote is from the introduction to an Algebra textbook written in 1848 [Note: Excerpts from antiquarian textbooks show gender bias through the use of male pronouns. That reflects the state of education and the gender bias of the times. I have retained the original language in the quotation with that understanding.]:

A parent often inquires, “Why should my son study mathematics? I do not expect him to be a surveyor, an engineer, or an astronomer.”. Yet, the parent
is very desirous that his son should be able to reason correctly, and to exercise, in all his relations in life, the energies of a cultivated and disciplined mind. That is, indeed, of more value that the mere attainment of any branch of knowledge.”

(Ray, 1848, p. v).

From the same time period, here is another statement from an Algebra textbook arguing in favor of studying math as part of a liberal education:

If the design of studying the mathematics were merely to obtain such a knowledge of the practical parts, as is required for business; it might be sufficient to commit to memory some of the principle rules, and to make the operations familiar, by attending to the examples… But a higher object is proposed, in the case of those who are acquiring a liberal education. The main design should be to call into exercise, to discipline, and to invigorate the powers of the mind. It is the logic of the mathematics which constitutes their principal value, as a part of a course of collegiate instruction. The time and attention devoted to them, is for the purpose of forming sound reasoners, rather than expert mathematicians

(Day, 1841, pp. iii-iv).

This mental discipline argument continues to persist. Dudley (2010), a retired mathematics professor who taught for 30 years, recently made a strong point that the vast majority of Americans will never use higher math skills, like Algebra. So why did he spend a career teaching math? “It is to teach the race to reason” (p. 613). However, some would disagree that reasoning ability is learned through math (Lemire, 2002) and much of the past research on transfer of skills to other domains has been inconclusive (Atherton, 2007). My search of the literature shows few experimental studies since 2000 have been done on the transfer of reasoning skills although there is some recent evidence that students can be taught critical thinking skills through a math course if it is designed to do so (Aizikovitsh and Amit, 2010; Sanz de Acedo Lizarraga, Sanz de Acedo Baquedano, Goicoa Mangado, and Cardelle-Elawar, 2009; Benander and Lightner, 2005).

Regardless of whether Algebra is taught for utilitarian value or for critical reasoning skills, many students struggle while learning it. The Carnegie Foundation for the Advancement of Teaching reports that up to 60% of community college students have to take at least one remedial math course (Carnegie Foundation, n.d.). Only about two-thirds of college students taking developmental math courses successfully complete those courses (Gerlaugh, Thompson, Boylan, and Davis, 2007). Developmental math courses are often geared towards preparation for College Algebra which is generally considered to be a preparation for calculus. However, the vast majority of college students in the United States that take College Algebra do not go on to take a calculus course (Herriott and Dunbar, 2009). Presumably, College Algebra serves to satisfy a general education requirement for most students who take it.
Some mathematics educators are beginning to question the relevance of Algebra for the masses (Shaughnessy, 2011). I am among those questioning relevance. When I look at my students who are not successful, I believe that most of them are capable of learning Algebra. I wonder if those students do not see a personal connection to learning math. Even many of my successful students are just trying to check off a general education requirement. I am intrigued by the relevance, if any, that students attach to Algebra. Knowing more about that may be a key to help more students succeed (or reconsider the college mathematics curriculum). The purpose of this paper is to examine what is meant by relevance from a student’s perspective. I will first discuss my conceptual framework and positionality and then describe a qualitative pilot study conducted with Intermediate Algebra students.

**Conceptual Framework**

The primary theoretical framework is Ole Skovsmose’s (1994, 2005) theory of disposition, intention, and action; or more specifically related to education: disposition, intention of learning, and learning as action. Skovsmose is a critical math educator and connects his theory to the development of critical thinking skills. While I do not primarily use a critical theory lens, I find Skovsmose’s description of the learning process and the importance of meaning from the students’ perspective to provide a particularly suitable theoretical framework.

Skovsmose describes the common practice in mathematical education of focusing on mathematical concepts. He does not disagree with a curriculum and pedagogy that includes the teaching of concepts; however, he does disagree with making it the first priority. For him, “concepts are not delivered, they are constructed” (2005, p. 85). He prefers a focus on the student’s educational task which is related to a student’s intentions to do the task which, in turn, is related to the student’s dispositions. “The essential thing is to bring meaning to the educational actions of the students – and that meaning is not the sum of the meanings of different concepts” (2005, p. 92). I find this to be a helpful way to look at the issue of relevance and concur with Skovsmose’s (2005) point:

> As a researcher it is important for me to suggest a framework for research that does not obstruct the possibilities for investigating meaning from this broader point of view. The investigation of the meaning of concepts is not sufficient for an investigation of meanings that students might assign to their tasks.

( Skovsmose, 2005, p. 88).

A paradigmatic switch is suggested that is not so narrowly focused on mathematical concepts. Meaning, from a student’s perspective, becomes key in his theory of disposition-intentions-action. Action, or rather *learning as action*, refers to intentional action where the student has a choice. Indeterminism and awareness are necessary ingredients. For Skovsmose, and for me as well, learning has to be performed by the learner; receiving is not learning. The learning process is not just about a student having
an experience with a concept. As social beings, it is more complex than that as students are simultaneously having experiences with a teacher, classmates, and other aspects of life – a network of experience (2005, p. 91).

**Intentions** are described as being directed toward action or the personal meaning of the action. Intentions are the goals and reasons that make activity an action. Skovsmose contends that goals cannot be implanted in students; they must be identified and accepted by the students (2005, 91). Therefore, students need opportunities to investigate goals and reasons. In the broader view, students can have intentions that are different from what the school expects. In fact, Skovsmose states that it is quite common for students to develop what he refers to as *underground intentions* (2005, p. 93). Intentions emerge from students’ dispositions.

**Dispositions** are the source of intentions but not the cause. They are mediated by the individual and contain two essential elements. The first is *background* which is how the term is commonly used but is specifically defined by Skovsmose as the “socially constructed network of relationships belonging to the history of the social group to which the person belongs” (2005, p. 89). The second element is *foreground* and is defined as “the set of opportunities that the student’s interpretation of his or her socially determined opportunities as ‘real’ opportunities” (2005, p.97). This is an important aspect of the theory because it places the student in a larger life situation recognizing that there are many potential influences on students that will impact intentions.

A conceptual model of Skovsmose’s theory is cyclical. Dispositions are the source of intentions and intentions lead to actions. As students reflect on their actions and the consequences of those actions, their dispositions are modified and the cycle continues. Relevance for me is the personal meaning of action, the intentions which emerge from the dispositions of students. The term *relevance* is often assumed to mean practical value. However, Sealey and Noyes (2010) problematized the term by finding three different categories of relevance from students’ perspectives: practical, process, and professional. *Practical* refers to the value or usefulness of the concepts, *process* is related to transferable skills (e.g., problem-solving), and *professional* relevance means the exchange value (e.g., being able to get a degree and go on to a career).

My broad definition of relevance, as addressed in the pilot study, is the purpose for which students engage in the learning process; not the designed reasons by curriculum planners, faculty, or textbook authors. Relevance is connected with the reasons that students have for making efforts to learn.

**Pilot Study**

Skovsmose states that “the issues to be considered, when meaning in mathematics education is discussed, become very complex” (2005, p. 93). To start to get a feel for that complexity I conducted a small pilot study in which I qualitatively examined
students’ dispositions and intentions towards learning Algebra. Before discussing the methodology and findings, I will address my positionality.

Positionality

It is important for me to reflect on my positionality for two reasons. The first is so that I am aware of my own position relative to the participants in my study. This will help allow me to better see through their eyes and also understand my own weaknesses in ever being able to do so completely. The second reason is be clear to the reader as to my perspective and potential biases.

The Soviet Union launched their first Sputnik satellite one month after I started kindergarten. National attention was focused on competing with the Soviet Union and as a result I seemed destined to be an engineer because I enjoyed and excelled in math. That idea was planted somehow in me and I pursued that even though I did not truly understand what an engineer was.

As an engineering undergraduate in the freshman honors calculus program I almost switched to becoming a mathematics major. However, in a sophomore Linear Algebra class I started to feel like I was just playing around with numbers. The intrinsic value of studying math was not enough. To have more meaning for me, I needed the math to be grounded in practical application. After graduating in engineering, I never actually worked as an engineer but used my undergraduate study as a basis for later graduate work in science and education. I am now several years into a second career and have chosen to teach something I have always enjoyed. I see my role as a developmental mathematics teacher as helping underprepared students to be successful in math and also in college (and also in life).

I believe that by not being a pure mathematician, but understanding mathematics, allows me to stand in a rather unique position from which I can be critical of current math education. As I challenge colleagues on the relevance of Algebra, I can see defensiveness on their part. “Of course what we teach is relevant.” Yet, they struggle with specific practical relevance examples in significant areas of the curriculum. They also accept the transference of problem-solving skills learned in math to other domains as axiomatic. “Of course math increases critical thinking skills.” I think I can stand apart from mathematics and use my vantage point to question current practices.

It is important for me to also mention my privileged status. I am a white male who grew up in white, suburban, middle-class Seattle. One could say I was blessed with good aptitude for math and education. This aptitude was nurtured by my parents, teachers, and others. I realize that many of the students I work with have backgrounds quite different from mine. I cannot put myself entirely in their shoes but being aware of the differences helps me to listen better and be open to their perspectives, values, and issues.
Method

Because this research is about how students make sense of their math experience, a qualitative approach was chosen. In fact, the type of methodology is what Merriam (2009) refers to as basic qualitative research in which researchers are interested in “(1) how people interpret their experiences, (2) how they construct their worlds, and (3) what meaning they attribute to their experiences” (p.23).

Four successful students in an Intermediate Algebra course volunteered to be interviewed using semi-structured, open-ended questions. They were asked about their math background including their earliest remembrances of math as well as their feelings towards math over the years. They were also asked about their purpose for taking this course and the personal value of what they were learning.

To provide a context for the students’ comments, a participant-observer approach was also used. This included observation of one week of the class (four academic hours) as well as an interview with the instructor and an examination of the textbook. As a participant in the class, I assisted the instructor when the students were doing assigned problems in small groups by going around to each group and listening and answering questions. This made the students more comfortable around me and more willing to volunteer to spend time outside of class in an interview.

Each participant was interviewed for 20 to 30 minutes. Transcripts were made of the class sessions as well as interviews of the students and the instructor. The transcripts were first coded for any type of reference to relevance, purpose, or value in what the students were learning. Then, similarities between the comments were examined after which several themes related to relevance became apparent.

Research Site and Participants

The research was conducted within the Developmental Mathematics Department of the university where I teach in the western United States. Sources of data, as noted above, include a one-week observation of a remedial Intermediate Algebra class, examination of the textbook, an interview with the instructor, and interviews with four students.

The instructor for the class was willing and even eager to be involved. Prior to conducting this research, the two of us had several discussions about pedagogy and the relevance of Algebra. While we have not always agreed, I have been impressed with his desire to learn more about teaching and to collaborate with others. He has been teaching developmental math for three years and is continually developing and modifying his teaching philosophy and approach. He chose to be referred to as Professor X in this report of the research.

The four students who were interviewed had all been successful in the class, receiving A’s and B’s in exams. They all volunteered to be interviewed. Each chose their own alias for this report. Jared is a Pacific Islander in his mid-twenties who has always considered himself to be a “math person.” He is in remedial math because it has been a
few years since his last math class. Giovanna is from Bolivia and moved to the United States after high school. She is in her early forties. She enjoyed math in the early grades but later found Algebra to be hard. The farthest she went in high school was the equivalent of Intermediate Algebra, the current course she is taking. Jan is a middle-aged, white woman returning to college after her children have grown. She took the bare minimum math requirements in high school and struggled with Algebra. Finally, there is McKenzie, a young woman of 22 years old. She has never felt like she has hated math, but has nevertheless put off taking her math requirements until the last possible moment.

Results

The students made reference to each of the three types of relevance noted in the research of Sealey and Noyes (2010): practical, process, and professional relevance. In this study, two other types of relevance were also found that may be more prevalent in older students: intrinsic and empathetic relevance.

Practical relevance

All of the students were asked why they were taking this particular course. I was particularly interested in hearing if they noted that the content of the class would have some practical usefulness in their lives, now or in the future. Jared, who is potentially interested in being an engineer, could describe a general usefulness for math, at least compared with other subjects:

“In my mind I’m still geared as that I’m going to use math a hell of a lot more than what I’m going to learn about [other subjects] like certain paintings. Stuff like that. You know what I mean? I think math is so much more needed now. Or like when I’m older because then I can... everything I’ve done I’ve just realized, ‘Oh, there’s math in this’, but there’s not always ‘Oh, I learned about this painting’, and this relates to this.”.

McKenzie stated that she actually uses some of the math that she learned earlier on in the semester. In her job as a catering consultant she uses some proportions and even some of the modeling techniques. She could not think of any specific examples at that time during the interview. However, she saw little practical value in the current topic of quadratic equations or in the College Algebra class that she would have to take. Referring to quadratics, she said:

“Well, it’s just a lot of numbers and scribbles and stuff. I love seeing things that I learn and then applying it, but now doing the quadratic forms and stuff like that it doesn’t … I wish I could see a cool outcome. Like what it actually is for; for me”.

Interestingly, Giovanna enjoyed the fun of her kindergarten class in Bolivia where there was a little shopping place in the room and where the students could use play coins to buy things. She expressed no value in the practical utility of what she was currently learning.
Process relevance

If the content of Algebra has little practical value, is there some other value to taking the course? I asked Jared, who had the most enthusiastic attitude towards math of the interviewees, if learning math improves critical thinking or problem solving in general. He responded immediately, “Oh yeah.” Jared believes that math, in particular over an English class, helps one to critically think because it is so hard that “you just have to sit down and take it one step at a time and think about it.”. He made an interesting analogy that is from the mental discipline argument for learning math, it exercises your brain:

“I used to play football. If I trained my body to just be an athlete I may not be good at basketball, but I’m still able to play. I’m still able to run up and down the court with basketball players, and I’m able to jump with them, and I’m able to do my best. I may not be the best with them, but because I’ve trained at football I may well translate it over to trying to be the best at basketball. Or any other sport. So yeah, I do believe that critical thinking in math can, and probably does translate over to English and any other subjects”.

While Jared is challenged by the hard work of math, McKenzie said, “I might just get more frustrated, and like don’t care.”.

Giovanna, in answer to those who do not want to learn math because they are never going to use it in real life, stated that, “The more that you can put in your head, the more that you can handle things, the smarter that you get, that you can make better decisions. You’re making your brain work.”. When asked specifically if math would make her a better critical thinker or problem solver in other areas other than math she immediately replied, “Of course.”. She followed that by referring to learning patience and improving concentration and discipline.

Professional Relevance

As one might expect, professional relevance was noted by these adults as they are concerned with the ability to use their education to get a job. Jan, the oldest of the interviewees, was “toying around with the idea of going back to being a teacher.”. One reason she was taking math was “to leave the doors open” for herself. She also needed to do well and get at least a ‘C’ to move on. Her test scores had been declining as the material was getting harder.

Like Jan, Giovanna was a little tentative about her future and was also trying to leave doors open. “I always dreamed to be a genetic engineer when I was little, and they always told me that there was a lot of math involved with it. So if one day I decide to go to my dream, I will have the foundation device.”. Jared expressed his desire to be a civil engineer eventually so the math “wouldn’t hurt me.”. He is currently a Deaf Studies major so I asked him about that. He wants to be an interpreter because that pays well and he can then work and pay to go to school to study engineering.
McKenzie does not need Intermediate or College Algebra to get her degree in Culinary Arts. However, she wants to get her associate degree in case she changes her mind about the culinary profession. Taking Intermediate Algebra is just a means to prepare for the next course so she can get her degree.

**Intrinsic Relevance**

Two students mentioned an intrinsic value to learning math. Regardless of any other type of relevance they saw, they believed in learning for learning’s sake. Jan did not necessarily need to be enrolled in school. When asked about the value of this math course she said:

> “The value is just because I want to. I didn’t catch it the first time, and the value to me is that I really want to understand it this time. And I spend a lot of time studying and I’m not getting it super well, but I’m just grateful for every little bit I am understanding.”.

She repeated this sentiment in the interview saying “You know, this will sound really dumb but it’s fun to get it. It just feels good to get it.”. And later, “When I said a while ago that it was fun for me to know it, it really does. It brings me joy just to know a little bit more about this.” She also added that she has come to understand more about herself and why she did not get Algebra the first time.

Jared is motivated for the excitement of doing math. “Just to be able to look at something and be like, ‘Man, that looks really hard.’ But then if you did this... it’s just exciting. To me it’s exciting.”. He expressed the value of learning but was a little torn between learning for learning sake and the more practical relevance:

> “To me, too much knowledge... you can’t have enough. So yeah, it kind of sucks because it takes up time that technically I’m paying for, and I would rather learn it some other time. Because if I absolutely knew I wasn’t ever going to use it again, I would be like, “Why don’t we just move on to something else, and if we can get time I’d love to learn it.” It’s one of those things that it’s kind of... yeah if it’s not going to help me I’d rather skip over it, but if we have time I would love to learn it because that way I could go back to someone else and be like, Hey, I know how to do it.”.

Both Jared and Jan explained that they had something to prove to themselves. For Jan, it is almost as if there is some therapeutic effect: “I don’t feel like I succeeded in math the first time, and I just want to feel like I’ve grown and matured and I can succeed this time.”. Jared, who seemingly likes personal challenges said:

> “When I do my homework and I get it wrong, I’m like “Crap. Why did I get it wrong?” Then I’ve got to think about it and go over my things, and then I’ll be like, “Freak. I forgot the negative.” You know? Then I’ll be alright, and I’ve proved to myself that I know what I’m doing and I know how to do it.”.
It is interesting to note that Professor X said in his interview “it’s the goal that anybody should have in teaching a class: that you have emotional changes in your students” … “either they believe that they can learn now, or they believe that they can learn better, or something touches them in there that says, This is great. This is interesting stuff I had to do.”.

**Empathetic Relevance**

During the interviews, the students expressed value in this math course for the purpose of preparing them to help others. Math is difficult and as they are “getting it” they see a need to help others get through it as well. For lack of a better term, I am calling this empathetic relevance. They are empathetic to seeing the struggles of others, as they have perhaps struggled themselves.

Giovanna was very clear on this aspect of her motivation:

“It makes me excited so I can teach other people. Like, for example, I know that a lot of Hispanics are struggling with math, so I want to learn everything the right way so I can teach as well. I don’t want to just keep it for myself, I want to be able to help.”.

Professor X’s pedagogy is conducive to facilitating this relevance. He has a strong focus on working in groups which was evident in my observations. This has been helpful to Giovanna:

“A lot of help comes from when the professor makes us work in teams or groups, because he is right when he says that’s the only way you can learn. You can teach to your classmates and they can teach you, and that’s the only way to learn. I haven’t found any other easy way.”.

Jared is appreciative of the opportunity to help others: “I love being able to be like, ‘Oh, I know how to do that.’ I love being able to be like, I can help you with that. I know how to do that.”.

Giovanna also wants the ability to help her children. McKenzie stated that, “If I’m never going to use them [quadratic equations] then I don’t see another benefit unless I’m helping my kids try to do quadratics.”. For McKenzie, if no other relevance is that important at least she can fall back on this. For Jan it was maybe not so much to help her kids to learn math as to show her abilities and be an example of the merits of working hard: “I’m proving it to myself more than anything, but I want them [children] to see that I can work hard and do this.”.

**Findings**

Only vague references were made to practical relevance by the students. Jared talks about it in general. It is almost as if he has some blind faith in what he has been told
regarding math in the real world. McKenzie thought there had been at least a little real world application earlier in the semester but saw none later, including the current week’s topics on quadratics and imaginary numbers. The textbook (Sullivan and Struve, 2007) tries to sound relevant but uses the following to create interest at the beginning of the chapter:

One of the more unusual sports is found in Millsboro, Delaware – Punkin Chunkin. Participants catapult or fire pumpkins to see who can toss them the farthest (and most accurately). Interestingly, this bizarre ritual is an application of a quadratic function at work.

(Sullivan and Struve, 2007, p. 547).

I doubt that any of the pumpkin launchers calculated the trajectories of their pumpkins.

Another example of contrived relevance can be found in an “application” exercise at the end of the chapter. It sets the stage by saying that your boss is the president of a corporation and his approval ratings have been fluctuating each month. The boss discovers that “the following function described his approval rating for that year where $x$ is the month: $R(x) = 3x^2 - 10x + 48$” (Sullivan and Struve, 2007, p. 618). I doubt that any boss would discover that. It is understandable with examples like that that adult students would not see much real world value, except perhaps for those going into math, engineering, or the sciences.

In class, Professor X made general references to applications of math through modeling the real world. Some fields that use math were noted but not explained in any level of detail. It is almost like he was saying “trust me, it is used in the real world”. In one class he stated, “So just because you have not used it in your life, and just because you won’t use it in the next week or two weeks other than in this class does not mean it’s not a useful concept”. How can a student relate to that?

When practical relevance fails to excite then the other forms of relevance are used by students to provide the motivation they need to succeed. Beyond doing what they have to do to get their degree, there is a belief in the mental discipline aspect of learning math. To some, a large importance was placed by these students on the intrinsic value of doing something that is hard and meeting the challenge. To my surprise, most referred to a desire to help others get through math in the future. It is as if math is inevitable and natural and to be somebody you have to do it. Powerless to fight the math system, or unaware of the issues, they at least vow to help others.

**Conclusion**

There is much we can learn from students to inform curriculum planning and pedagogy. There are at least five types of relevance practical, process, professional, intrinsic, and empathetic. Do the ways that they find meaning in what they do match the designs of
the math curricula and materials? If not, why not? Are we building a bridge to the world that some do not want to cross? Is there a bridge to nowhere?

My concern is that students see the Algebra-based general education math curriculum as imposed, disingenuous, rigid, and designed with a gatekeeping function. This could in turn result in student resistance, unquestioning compliance, cynicism, or distrust. If critical thinking is an important aspect to the math curriculum, then these potential results are inconsistent with that premise. There are five recommendations that I would suggest:

- Be honest with students regarding the purpose of learning Algebra. It is easy to find clear practical applications of some mathematical concepts but not others, like factoring polynomials by hand or solving rational expression equations. We should be able to point to other applicable types of relevance when practical relevance fails. Also, if some word problems are contrived (and most seem to be) then we should call them contrived from a practical relevance viewpoint and tell them what type of relevance does come into play.
- Acknowledge that there are different types of relevance from students’ perspectives and use that in designing appropriate curricula. Perhaps one-size Algebra does not fit all. Perhaps Algebra curricula tend to focus on the traditions of mathematics education that are no longer appropriate.
- More consideration should be given to access to higher education. Should general education College Algebra have a gate-keeping function? I would suggest not. A student who has trouble factoring a third degree polynomial may be perfectly able to learn other math concepts that have more personal meaning and relevance.
- Reconsider the Algebra-based general education curriculum. Trying to reform the traditional Algebra curriculum will undoubtedly result in cries of a loss of rigor. I reject that argument. Rigor can be redefined to include depth of learning rather than the breadth of the topics covered. I believe a liberal arts general education math course can be designed to be just as rigorous as a college Algebra course and, at the same time, provide stronger practical and personal relevance for students.
- Conduct additional research. Is mental discipline (process relevance) a valid argument? Mathematics educators put a lot of stock in the fact that it is valid. In what specific ways does it help mental discipline? How do we know that? What can we do to foster it? In addition, this study looked at successful students but what about relevance, or lack thereof, is seen by failing students and those on the margins? Perhaps math educators need to problematize their own perceptions of relevance. Certainly a more specific vocabulary regarding relevance is necessary. Other future research questions might include: What are students’ perceptions of possibilities and “real” opportunities for learning Algebra? What impact does the context of their learning (e.g., instructor, textbooks, classmates, etc.) have on those perceptions? As intentions emerge from dispositions, how do students act? As students reflect on their experiences
during the course, how are dispositions affected? How do dispositions, intentions, and actions differ between successful and unsuccessful students?

**Final Thought**

A recent large study of 57 colleges in seven states [in the US], found that only 20% of college students [having been] referred to math remediation [classes], are successful in starting and persisting through a developmental math sequence, and then, passing their freshman-level, gate-keeper mathematics course (Bailey, Jeong, & Cho; 2010). Considering the demographics of the students, the study also found that success rates were lower for males, African Americans, part-time students, and those in vocational programs.

José (not his real name) was a student of mine. He was born in Mexico and was brought to Los Angeles, California by his parents when he was a toddler. He is part of a large, somewhat dysfunctional, family. His father was an abusive alcoholic. José dropped out of high school and never even tried to earn an equivalency diploma. At 31 years old, unmarried and jobless, he came to college for the first time. He was able to enroll in my open access institution after taking what is called an ability-to-benefit test. In the Fall, he was in my Beginning Algebra class. He told me: “I want to be successful at something for the first time in my life.”. He was eventually successful in passing Beginning Algebra and is now in Intermediate Algebra. José wants to major in behavioral science and eventually be a counselor and mentor to youth. When I am helping him learn how to divide polynomials using long division, I say to myself, “Why does he have to learn this? What’s the point for him? He is capable, but am I wasting his time?” I want a curriculum that provides access for students like José and helps them develop a deep personal meaning for learning mathematics. While José is capable, he struggles to see the point. Will he be able to persist through the string of Algebra courses still remaining for him? José needs a bridge to the opportunities he sees for himself to help others in the world.
References


Section 3

Paper Presentations
Individuals may engage in actions in the workplace that are underpinned by mathematics knowledge, skills and competence while denying their use of mathematics or dismissing it as commonsense. That people may know more about mathematical ideas and techniques than they think they do, and use them more often than they realize, may imply that mathematics can somehow become invisible to them to some extent.

This paper explores the potential of Cultural Historical Activity Theory (CHAT) to explain how a worker’s experience of ‘mathematics invisibility’ may be influenced by the context in which it occurs. Bernstein’s observations regarding Discourse are examined for insights into how this invisibility may be intensified by the ways in which people talk about and recognize mathematics.

That mathematics-supported activity may abound in the workplace, albeit concealed by multiple surrounding elements, suggests that the mathematics may not be described, adequately, in terms of level of difficulty alone. Furthermore, the role of mathematics in work may not be identified easily nor measured in any absolute way, but may be observable as expressions of numerate behaviour in complex, dynamic contexts. In this light, a tentative framework for a contextualisation of mathematics in the workplace is offered here, in order to coordinate the complementary dimensions of complicatedness and complexity, to capture mathematics in the workplace more fully, to enhance mathematics visibility for the individual and to create a platform for innovation and change.

Key words: Activity Theory, Numerate Behaviour, Mathematics Invisibility,
Individuals may engage in actions in the workplace that are underpinned by mathematics knowledge, skills and competence while denying their use of mathematics or dismissing it as commonsense. That people may know more about mathematical ideas and techniques than they think they do, and use them more often than they realize, may imply that mathematics can somehow become invisible to them to some extent.

As part of a multi-faceted approach to mathematics in the workplace, this paper first explores Cultural Historical Activity Theory (CHAT) for its potential to explain how a worker’s experience of Mathematics Invisibility may be influenced by the context in which it occurs.

In addition, Bernstein’s observations regarding Discourse are examined for insights into how this invisibility may be intensified by the ways in which people talk about and recognize mathematics.

A tentative framework for a contextualisation of mathematics in the workplace is offered in order to coordinate the complementary dimensions of complicatedness and complexity. Such a framework may help in the reconciliation of the formal, academically biased account of mathematics complicatedness with the informal, socially biased account of workplace complexity to enhance visibility and transcend inter-discourse boundaries. This may provide the opportunity for the individual to revise the self-perception of “not being a maths person”, and enable the creation of a platform for change.

**Cultural Historical Activity Theory**

Cultural Historical Activity Theory (CHAT) is widely used to describe the interdependent constituents of human activity. CHAT, first graphically illustrated in the 1980s (Engeström, Miettinin, & Punamäki, 1999; Engeström, 2001; Engeström, 1987; Engeström & Cole, 1997; Leont'ev, 1978; Roth & Lee, 2007), posits that human activity comprises subject(s), gathered in a community, who form and are formed by rules, among whom labour is divided, and who are joined in common cause in pursuit of an objective, even if they may be differently motivated. Although latterly criticized for its limitations (Vassilieva, 2010), CHAT may still retain the power to explore many of the factors that influence human activity, including the use of mathematics in work, and offer an explanation for the paradox of denying the use of mathematics in work while, at the same time, being observed to use mathematics to solve problems and manage mathematics containing situations.

In the modern workplace, individuals report their job as comprising their motivation, or as summarized by their job title. The performance of their work features actions that are specific to the task, and these are underpinned by automatic operations, considered to be commonsense, or “just part of the job”(Cohen, 2000). In the event of a process breakdown, such automatic operations are brought to the fore, demanding a higher level of consciousness, returning to invisibility when the problem has been resolved (Wake & Williams, 2007). The recognition that few workplaces comprise a single, self-contained, stable process, inspired the continuing evolution of CHAT to account for more complex activity systems.

**Activity System**

The concept of Activity System is that of an activity, expanded to acknowledge the reality that human activity is not something that occurs in isolation. Rather it takes place in a social
context and interacts with a range of surrounding activities, the stability of which may depend on the extent to which their common expected outcome is understood and shared, figure 1.

This representation draws attention to the role of shared outcome, as being crucial to the success of collaborating Activity Systems, each with their own subject, rules and so on. The focus of this research is somewhat different in that it seeks an explanation for invisible mathematics as experienced by the individual as s/he engages in work, guided by procedures, constrained by time and resources, and conditioned by specifications. Individual human activity in work may have a stake in the outcomes of a stream of preceding, parallel and dependent activities each with their own characteristics, invoking different responses, underpinned by appropriate knowledge, skills and competence.

**Fractal Activity System**

In the earlier stages of this research, the process of developing the tools to reveal the mathematics knowledge, skills and competence at the kernel of work practice, (Keogh, Maguire, & O'Donoghue, 2010), suggested that organizations may be fractal i.e. comprising activity systems that seemed to be supported by other, similar, activity systems of ever finer detail, each with their own properties, Figure 2.
The central triangle (ABC), echoes the interdependence of the six nodes of the employee’s core activity. The yellow triangle (DEF), depicts Peer / Peer activity, involving negotiation and workload sharing. The green triangle (GEF), represents Worker /Team Leader interaction concerning load volume, timetables, special conditions and extra requirements. The red triangle (GDE), suggests preceding activities, and sub-activities, on which the core activity depends for accuracy and completeness. Other triangles represent Employee/Customer, and multiple possible engagements with other activities both inside and outside the organization. Each activity may be characterized by different communities, rules, artifacts and division of labour but may be underpinned by the same set of mathematics and other resources. Activities may comprise a range of adjacent and sub-activities, depending on the degree of magnification required. From the point of view of this research, the “subject” remains the same. In this way, it seems that the worker /subject, glides seamlessly through networks of integrated Activity Systems in the course of his/her daily work and life, occasional breakdowns excepted. Each transition may necessitate the performance of actions directed towards a particular goal, moderated by artifacts, guided by rules and the community or communities of practice, among whom the work load is divided. The actions performed in each activity system may comprise automatic operations that draw on the subject’s repertoire of mathematics knowledge skills and competence. A possible consequence is that, while activities may be easily differentiated by the changing composition of the nodes and may evoke different behaviours from the subject, the underpinning mathematics knowledge skills and competence may become ever more amorphous from the subject’s point of view. The point is, that the same mathematics knowledge skills and competence are subsumed in unconscious operations in support of multiple, interleaved activity systems, such that the anonymity of the mathematics is continuously reinforced.

While each Activity Theory node may exert its own influence on mathematics invisibility, this work now turns to the mediating power of discourse, i.e. language that has been adapted, developed and maintained as the vernacular of the activity’s internal communications, serving to include some and exclude others.

**Discourse**

Discourse may feature words, expressions, abbreviations and acronyms, drawn from a common language, but bearing meanings particular to the participating community. The presence of mathematics concepts and ideas may not be acknowledged for activity theoretical reasons, but may be referred to in ways that are coherent to the subject(s) of the activity system alone.

Bernstein provides a useful framework to support the analysis of discourse and how knowledge is developed, accumulated and circulated, under conditions determined by the segment in which it occurs and is acquired (Bernstein, 1999; Bernstein, 2000; Fitzsimons, 2005). Developing an understanding of the nature and substance of local discourse may contribute an explanation for the denial/use of mathematics in workplace settings.

The prevailing discourse within communities may be characterised as local and specific in order to optimise interactions between persons and their surrounding contexts. The extent to which knowledge is circulated is biased by the social structure of the community and the structure of that knowledge. This chimes well with the notion of the activity system being object-oriented and artifact mediated, occurring within a community that shapes and are
shaped by rules and among whom labour is divided. However, admission to the community, especially in the workplace, may also be constrained by formal and informal rules concerning status and access (Lave & Wenger, 1991), which serve to confine the circulation of knowledge to the “insider” and exclude the “outsider”, however s/he may be defined (Wake & Williams, 2007).

In the course of this research, it became apparent that workplaces are replete with words and phrases that take on particular and local meanings. It seems possible that mathematics activity, at once invisible, unconsciously performed, denied through self perception, referred to in terms understood by the activity system’s subjects alone, compounds the difficulty for the “outsider”, and may only be detected by methods of enquiry capable of transcending these obscuring barriers.

Many workplace activities are underpinned by relatively simple mathematics, but they can occur in diverse situations and may be applied in multiple, conditioned, unpredictable and possibly complex combinations. Complicated mathematics may be more evident and more readily detected, perhaps due to the surrounding discourse being more formal. The terms complicated and complex are often used interchangeably and may have the effect of misrepresenting mathematical properties depending on their usage context, rather than as complementary descriptors of mathematics knowledge skills and competence in workplace settings.

Complicatedness and Complexity

Complicatedness, for the purposes of this research, indexes the degree of difficulty to understand (Summers, Brookes, Holmes, Gilmour, & O’Neill, 2000). The National Framework of Qualifications in Ireland, (NFQ) (NQAI, 2003) is graduated in 10 levels from the foundational and concrete at the lower end, rising to the abstract, innovative, theoretical and ground breaking at the other. The framework is oriented towards academic accomplishment to assure the quality and consistency of awards across the spectrum of disciplines. The pertinent knowledge structure and accompanying discourse are necessarily standardized in the interests of transparency and progression within Ireland and across Europe, and to assure the parity of the qualifications awarded by accredited providers of learning opportunities. The degree of complicatedness associated with a subject, is precisely described by the associated curricula and assessed at clearly defined intervals. The social activity of work is less amenable to descriptive precision and may be more easily characterized in varying degrees of complexity.

Complexity

Complexity is a contested notion, at once meaning intertwined and interdependent such that components are meaningless when separated and impossible to re-assemble (Grootenboer, 2010) or as comprising multiple decision-making agents, each exerting influence on the behaviours of any fraction of the whole (Hurford, 2010) or as an inherently subjective sharing properties with, and being understood from, the perspective of other systems. More appropriately to the workplace, complexity may be a property that a system shares with other systems with which it interacts including the observer or controller (Casti, 1994). These observations are made in reference to naturally occurring ‘systems’ and reflect the
view of this present work insofar as it indexes the unpredictable, novel, and diverse aspects of humans engaged in work.

This current research features a number of case studies where the mathematics activity observed tends towards the lower end of the NFQ scale (Keogh, Maguire, & O'Donoghue, 2010). Although simple concepts in themselves, they are deployed in various combinations, subject to a range of conditions that are unpredictable to varying extents. The implication is that a worker’s academic awards may not record their observable competence developed in the workplace, albeit based on lower accredited levels of knowledge and skills.

Experience in the workplace is highly sought after by employers, often cited as a key prerequisite of prospective job applicants. In this regard, experience refers to the job-relevant knowledge, skills and competence having been aggregated through informal and non-formal learning “on-the-job”. It may reflect the explicit and tacit exposure to variation, innovation and creativity leading to enhanced levels of judgment and performance when compared to the novice. Such experience may not raise the level of complicatedness of the job knowledge and skills content according to the NFQ, but may independently extend the required competence. The corollary is that recent graduates may possess the necessary knowledge and skills to be viable candidates for a job, but may not have sufficient, experience-induced competence-complexity to actually “do” the job.

The apparent failure of the NFQ to provide for levels of competence-complexity acquired in non-formal and informal ways, may have the effect of restricting the progress of the worker, and by concealing their mathematical abilities, confirm their self-misperception of “not being a maths person”. This paper suggests that the mathematics knowledge, skills and competence in work contexts are used in dynamic combinations in response to, and in support of, a range of situations that vary in complexity. That this property tends to be overlooked argues for a more complete way of documenting the application of mathematics in the workplace that is capable of accounting for both complicatedness and complexity.

**Contextualisation of Mathematics for the Workplace**

The NFQ adequately provides for the vertical progression of knowledge and skills through 10 levels. These levels could be accompanied by an indication of the associated complexity in the context of work. However, any attempt to align complexity with the existing framework, would suggest that there are sharply defined stages of transition between gradations of complexity. Instead, there appear to be pivotal points at which the depth of complexity is transformed viz., range of variables, precision of task definition, selection conditions, availability of known solutions, predictability, novelty, autonomy and responsibility.

At the lower end of the spectrum, competence is demonstrated in contexts that are tightly defined, occupied by a supervised, unvarying role, performing tasks that are unproblematic and one-dimensional, with little or no scope for insight or learning. Complexity at the upper end, reflects contexts that are highly diverse and unpredictable, defining a role that is autonomous and responsible, presenting novel challenges in new situations, requiring the development of insights and solutions, while identifying learning limitations and overcoming them. Transition across the complexity spectrum is marked by the accumulated competence, in the fuller sense, over time and as opportunities allow. Figure 4, illustrates that complicatedness may be determined in terms of the NFQ levels, complemented by an assessment of complexity varying between two poles viz., Simple and Complex. The introduction of a third dimension recognizes that competence may be developed at different rates in different mathematics, as well as other, domains, figure 4.
The development of a protocol to determine workplace context-complexity is continuing in parallel with an across-case and within-case analysis. It is clear that relatively simple mathematics knowledge and skill may be deployed in a wide variety of circumstances ranging from the simple and straightforward at one end of the spectrum, to the sophisticated, high risk (with embedded material consequences) at the other. Guided by this protocol, it will be possible to differentiate between the circumstances in which mathematics knowledge skill and competence may be deployed and, in this way, capture the depth of mathematics understanding required, rather than the level of complicatedness alone.

Summary

Mathematics activity may be present in workplaces but may be denied as commonsense or anything but mathematics (Coben, 2000). This may be explained by a combination of Cultural Historical Activity Theory, the absence of shared Discourse and the impact of other, background factors. That it is invisible to some extent, exerts a constraining effect on long term employability and well-being of the worker (Bynner & Parsons, 1997; Bynner & Parsons, 2005), and it may also be symptomatic of a tendency to equate what s/he “does” with what s/he “knows”, rendering hidden knowledge skills and competence unavailable for development.

The identification of such cognitive abilities requires the application of tools specifically designed to strip away the occluding layers (Keogh et al., 2010) so that they may be calibrated for complicatedness. But this metric alone is insufficient to reflect that routine knowledge and skills may be deployed in diverse contexts, in the performance of unstructured roles, in response to unpredictable demands, requiring the development of new insights and resulting in new learning. Each work element in isolation may be simple and easy to understand, yet applied in situations that are constrained and conditioned by multiple
factors. This paper suggests that complexity is a feature of the modern work activity and is a necessary dimension to the contextualisation of mathematics for the workplace.

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References


Incorporating Study Skills Training into an Elementary Algebra Course

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Pass rates in developmental mathematics courses are very low across the United States and the attrition rates are high. The purpose of the intervention undertaken at an urban community college was to address the above problems. Five instructors incorporated study skills training into an elementary algebra course in sections randomly selected for treatment. Other sections taught by the same instructors were designated as control. All participating instructors attended a workshop on study skills training conducted by an expert in that area. The workshop included outlines of seven mini-lessons as well as suggestions on how to incorporate them into the course. A detailed description of the mini-lessons is presented in this paper. The results show that the attrition rate was significantly lower in the treatment section and that the pass rates were slightly higher, although not significantly so.

Key words: developmental mathematics, study skills, mathematics learning, student retention.

Background

This paper was inspired by a grant funded study, conducted during Fall 2010 semester at the Borough of Manhattan Community College (BMCC) of the City University of New York, USA. The purpose of the study was to test a specific approach for increasing pass rates and reducing attrition rates in a developmental mathematics course offered at BMCC. In the past, the mathematics department at BMCC has undertaken a number of initiatives aimed at improving success in developmental courses. These included Supplemental Instruction in which tutors attended classroom sessions in order to enhance student learning. In addition, the tutors also helped students outside of the formal classroom setting on a weekly basis. Other initiatives included faculty workshops designed to improve teaching methods as well as student workshops which reviewed subject matter in preparation for exit examinations.

Notably, none of the aforementioned initiatives addressed such important student variables as study skills, time management, attitudes, and persistence. Research, however, has documented that only about 25% of the variation in students’ performance is attributable to the quality of teaching of the subject matter. Another 25% is explained by students’ affective variables such as attitudes, study habits and skills, dispositions, and math and test anxiety (Bloom, 1976; Nolting, 2008), with the remaining 50% attributable to cognitive entry skills.
Aptitude, prerequisite knowledge of the subject. According to Nolting, students’ affective characteristics are the most neglected area in colleges today. A number of researchers have explored the affects of mathematics study skills training on student learning and grades. Paul Nolting (1991) did research on students who repeated an algebra course one to four times. The experimental group in his study received math study skills training in the form of a one-hour math study skills course which met for two hours a week for the first eight weeks of the semester. The results indicated that the experimental group had a significantly higher passing rate than the control group.

Valencia Community College in Orlando, Florida, received a Title III grant to examine the effects of math study skills courses on their Elementary Algebra passing rates. Their model involved providing a one-hour math study skills course that ended at the midterm. Their findings indicated a significant improvement in students’ grades.

West Virginia Wesleyan College in Buckhannon, examined the effects of teaching study skills to students with disabilities. The instructor taught mathematics study skills in her Elementary Algebra course during the first half of the semester. The pass rate in her class was significantly higher than the average course pass rate for the school (Houghton Mifflin, n. d.). Based on the above evidence, it was apparent that training in mathematics study skills may have a positive affect on student performance in Elementary Algebra. Consequently, the authors decided to incorporate study skills training into developmental mathematics instruction. Ideally, we would have liked to offer a separate mathematics study skills course, but because this was not a feasible alternative at our institution, we decided to include formal study skills training in selected sections of Elementary Algebra.

**Intervention**

Five instructors teaching two or more sections of an Elementary Algebra course participated. Each instructor incorporated study skills training into one section of the course. The instructor’s other section was taught without any specific intervention.

The instructors were given a workshop conducted by a consultant (Dr Peskoff) who shared his expertise on incorporating study skills into the developmental mathematics classroom. The consultant developed a series of lesson plans based on a study skills workbook written by Alan Bass (2008). The lessons covered a variety of topics such as “navigating” the textbook, effective note taking, completion of homework assignments, preparing for exams, and coping with mathematics anxiety. The workshop was offered immediately preceding the beginning of the semester and was three hours in duration. Throughout the workshop, instructor participation was encouraged. The following handout, prepared by Dr Peskoff, was distributed at the workshop and served as a reference for instructors during the semester.

**Integrating Study Skills and Time Management Strategies into the Classroom**

The goal of this workshop is to enable instructors to motivate mathematics students to study effectively and to help them learn the appropriate study skills. Each student will receive a copy of the Bass workbook entitled “Math Study Skills”. However, it is the instructor’s job to see that his or her students are willing and able to use the information. This workshop will begin by asking the audience to participate in a “hands on” exercise to enable them to appreciate the important role that study skills play in effectively learning mathematics.

Subsequently, we will then discuss the most important points contained in each chapter of the Bass workbook, emphasizing how you can implement them in your classroom. The material will be organized into seven different topics. The first topic discusses four chapters from Bass and the remaining six topics discuss one chapter each. It is recommended that you...
require students to read the appropriate chapters in Bass before you review them. You can assign the exercises as homework, or perhaps do one or two of them from each chapter in class as part of your presentation.

**Suggested Timetable:**

In order to enable students to benefit from the material as soon as possible, it is suggested that all of the information be initially presented (and then later reviewed) during the first twenty to thirty minutes of the first seven to eight class meetings (assuming a course meets twice per week for one hour and forty minutes). Accordingly, each of the seven topics can be presented in approximately one-half hour. Nonetheless, you may choose to spend proportionately more time on certain chapters (such as math anxiety) and less on others (such as your class notebook).

It should also be noted that throughout the course of the entire semester (even after the first eight meetings), students should always be reminded of appropriate study skills (such as completing homework) whenever necessary.

In other words, after the initial presentation over the first few weeks, the material should still be constantly reinforced, especially when students appear frustrated or apathetic.

**Topic One: Introduction, What Makes Math Different, Learning Styles, and Retention**

Note: Because this topic covers four chapters, it may take more than one-half hour to cover in class.

Chapter One: Introduction

You should ask students about their past experiences studying math. You will probably get a lot of feedback! Many students will say they felt frustrated or even angry in former math classes. Ask them why. Use their responses to focus on ways to improve their understanding of math and achieve success in your class. Motivate the students to read Bass precisely because by helping them study more effectively, it will decrease their anxiety and increase their success. Remind students that the skills learned here can be used in future math courses also.

Chapter Two: What Makes Math Different

Ask students how learning math is different from learning English, Psychology, or History. See what they have to say. Bass refers to math as a “skill-based subject.” Similarly, I emphasize that there is a focus on understanding and problem solving rather than mere memorization. Remind students of the importance of doing homework.

You may wish to give students a simple in-class exercise such as “list three important reasons to do homework after every class.” Actually there are more than three reasons, and we will discuss them together during this workshop.

This may be an opportune time to remind students of the benefits of using the mathematics laboratory for help with homework.

Chapter Three: Learning Styles

You may briefly want to mention the three learning styles (visual, auditory, and kinesthetic) presented in Bass. The goal here should be to let students know that not everyone learns the
same way but that you will try your best to accommodate each student’s best style of learning (which is not an easy task).

If time permits, you can have students complete the “yes/no” exercises for each learning style in Bass.

Whatever learning style works best for a student, he or she should be encouraged to practice solving problems as much as possible.

Chapter Nine: Retention and General Study Strategies

This chapter contains a lot of information which may be difficult for you to present within a limited time frame. You might want to discuss the importance of note cards as an important retention technique. Present one of the examples from Bass in class and perhaps ask students to create one or two note cards of their own for the next class. Some students may like note cards and others not, but as Bass states to his audience “You owe it to yourself to try note cards.” We will discuss these in more detail during the workshop.

Some of the other important strategies that you should discuss (also listed on p. 72 Bass, 2008) include doing homework effectively (and regularly), reading your textbook, creating a vocabulary list, reworking and reviewing notes. Keep in mind that students often have limited time and get easily frustrated, so although these approaches can be presented, they may not be the best alternative for everyone.

Nonetheless, I would strongly recommend that you constantly emphasize the importance of reviewing class notes and doing homework after every class meeting. Of course, you can monitor this either electronically or by simply collecting or “spot checking” homework in class.

Topic Two: Math Anxiety

Many students feel very anxious and even “dread” studying math. Although there are many reasons why math anxiety develops, many students have had negative experiences in the past (perhaps stemming back to elementary school) when they attempted to study math.

Your focus should be on helping students feel more comfortable studying math. The best way to achieve this is to practice math (even if it’s the last thing a student feels like doing) as much as possible.

You may wish to first discuss the “avoidance” behaviors Bass presents (2008, p. 27) which accompany math anxiety and then review the seven strategies Bass lists (2008, p. 28) as a “cure” for math anxiety. We will review these during the workshop.

For homework (or if you prefer, in class), you may wish to implement a simplified form of the Self-Talk exercise in Bass. Ask students to write down a negative thought they have about studying math and then a positive thought that can replace it. Subsequently, ask them to write a negative behavior and then a positive behavior that can replace it. We will also review these during the workshop.

Topic Three: Managing Your Time

Most of this material is not unique to the study of mathematics. However, it is harder to be motivated to manage one’s time when (a) you are extremely overwhelmed by job and family responsibilities and (b) you are trying to find time to study a subject you find difficult, if not
frustrating. You should emphasize these constraints to your students so they know you understand the challenges they face.

In addition, you may wish to briefly review the list of strategies presented in Bass. In particular, I would emphasize maintaining a regular study schedule which includes homework after every class. Lack of study leads to lack of understanding which leads to “cramming” which often leads to failure and the perpetuation of a vicious cycle of math anxiety. Disciplined study, however, can lead to success!

**Topic Four: Your Class Notebook**

Bass recommends using a three ringed notebook but I’m not sure how many students will comply with this idea. A good approach to emphasize is that a student’s notebook should be used to organize his or her work and should contain different sections. A notebook should be thought of as a portfolio which contains handouts, class notes, homework, exams/quizzes, and perhaps a glossary of vocabulary terms (although some students may prefer to have the glossary as part of their class notes).

The key here is **organization** so no material gets misplaced.

**Topic Five: Your Textbook and Homework**

One of the most important challenges for students is how to effectively use their textbook (and ancillary materials).

Bass recommends surveying the material, surveying the assigned homework, and then reading the section. You may wish to go through this process using one of your homework assignments as a model. Keep in mind though, that many students do not read the textbook. They simply use it (or photocopied pages from it) to do homework. Perhaps some students can achieve success in this manner, but you should try to emphasize the importance of using the textbook to accompany and enhance the material presented in class rather than merely for homework assignments.

Doing homework is perhaps one of the most important activities of all. Bass (2008, p. 51) discusses “How to do Homework.”

The key point to emphasize to your students is that they should review their class notes and handouts before attempting a homework assignment to ensure that they remember the material they learned. Most assignments go from easier to harder problems so students may get stuck near the end of an assignment. They should be motivated not to give up. If they are stuck, they should mark the question, and ask their instructor (or tutor) to review it as soon as possible.

Another strategy that Bass presents to avoid frustration and anxiety is to always end on a “positive note” with an easier problem that you know how to do, perhaps from an earlier lesson.

You may wish to model the process of completing a short homework assignment (perhaps five problems) for your students, including what to do when you “get stuck.” This will be discussed in more detail during the workshop.

**Topic Six: Class Time and Note taking**

The key points here are arriving on time to class, being an active listener during class, and taking organized notes during the lesson. Each lesson should begin with a clean sheet of
paper (and the date and topic written on top). Bass recommends that students should “space your notes out” so that details can be filled in later. Although it is important to take comprehensive notes, it is equally important to listen. The emphasis should be on “understanding, not dictation.”

Usually, during the course of a lesson, students will be given an opportunity to solve problems to practice what they have learned. You should try to implement this strategy as much as possible. Walk around the room looking at students’ work and encourage them to go to the blackboard.

Both Bass and I emphasize the importance of asking questions in class. In sum, students should be encouraged to ask questions, but too many questions from an underprepared student may impede the learning of others.

**Topic Seven: Test Taking**

It can be argued that if a student has been studying regularly, then preparing for an exam should require minimal effort. This may be slightly exaggerated, but nonetheless, if students have implemented the correct study skills, then they should essentially need to review their class notes, homework, and practice exams (or sample questions) to prepare for an examination. Bass emphasizes the importance of a practice test as a dress rehearsal and I agree. Accordingly, students should be encouraged to stay up to date and avoid cramming so that they can prepare confidently for an exam primarily by reviewing and practicing what they have already learned.

As an instructor, it is useful to provide students with frequent short quizzes and practice problems so that they not only have plentiful material to practice but can also feel confident.

You should emphasize that an exam does not have to be stressful if one feel adequately prepared.

Bass recommends that while taking an exam, students should first write down any necessary formulas before looking at the questions. Subsequently, they should “survey the entire test,” do the easiest problems first followed by the hard problems, always be “mindful of your time,” and finally review the entire exam with any time that remains.

(Peskoff, 2010, complete).

**Discussion and Conclusion**

The workshop was successful. Subsequent interviews with the participating instructors confirmed that all were completely satisfied with the workshop and found the information presented to be both useful and relevant. Moreover, they reported that the workshop thoroughly prepared them to teach study skills to their students. At the end of the semester, the instructors reported that it would be advantageous to have more time allotted to teach mathematics study skills (without sacrificing the time needed to teach mathematics).

The sections in which students received study skills training had a significantly lower attrition rate than the control sections. This difference may be attributed to the fact that participation in a formal study skills program (as part of a mathematics course) contributes to better time management skills, increased confidence, and a lower level of anxiety. All of these factors tend to lead to an increase in persistence and a decrease in learned helplessness. The pass rates were slightly higher in the “study skills” sections than in the control sections, but the difference was not statistically significant. One reason for this result might be that both the “study skills” and control sections had the same amount of total class time. In the
“study skills” sections, some of this time had to be utilized to teach study skills in lieu of mathematics content while in the control sections, all of the class time was dedicated to teaching mathematics. This discrepancy can be regarded as a “double edged sword.” The authors hope to correct this imbalance in future studies by designating additional class time for teaching study skills so that the time used to teach mathematics does not need to be sacrificed. This extra time may be included in the course as a supplementary “laboratory” hour dedicated to the learning of study skills.

In conclusion, although our study clearly demonstrated the benefits of incorporating study skills into a developmental mathematics course, we believe that the amount of time spent teaching mathematics content should not be reduced. In other words, the time allotted for study skills training should supplement rather than replace the time allotted for mathematics instruction. In addition, the authors plan to investigate the effect of study skills training in college level mathematics courses.
References


A Mature Student Maths Programme

Padraig Kirwan

This programme was developed by the Centre for the Advancement of Learning of Maths, Science and Technology (CALMAST) in Waterford Institute of Technology, arising out of an initiative by ESB-Electric Ireland to retrain some apprentice electricians as electrical engineers. The purpose of this programme was to prepare the students for the mathematics modules on the B.Eng. in Electrical Engineering in CIT and DIT.

These workbooks combine:
- interactive exercises from www.mymathlab.com/global
- video tutorials and animations from www.mathcentre.ac.uk
- "quick-reference" leaflets from www.mathcentre.ac.uk

as a blended learning module that is ideal for the self-motivated learner who is keen to improve their maths skills and prepare for mathematics courses at third-level. Support for this project was received from the National Digital Learning Repository.
On the role of language and interpretative fluency in addressing mathematics anxiety in adult learners

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Practitioners in further and higher education face considerable challenges in their efforts to achieve effective learning outcomes for ‘at risk’ students who experience varying degrees of distress arising from their inability to cope with mathematical or numeracy content encountered in what they understood to be non-mathematical topics, or topics that supposedly demand no pre-requisite mathematics skills. Case notes from one-to-one consultations with such students demonstrate the existence of a range of common characteristics that are largely independent of age, gender, area of study, or the means by which students come to their study pathways. These characteristics include confusion, lack of confidence, negative perceptions, narrow focus and, arguably most significant of all, a lack of appreciation of the concept of mathematics as language. Drawing on ideas that follow a connectivist approach to learning allied with lessons from English language proficiency, this paper discusses the role of language in mathematics learning. In addition, it outlines a pedagogical framework whereby math-averse behaviours may be mitigated and mathematics anxiety reduced, through the development of a mathematics variant of ‘metalinguistic awareness’. This refers to the ability to reflect on and analyze mathematical language which promotes the interpretative fluency that students need to attend to, and facilitate, the acquisition of propositional and procedural knowledge relevant to their mathematics learning needs.

Key words: mathematics anxiety; interpretative fluency; language; adult learners.

Adult mathematics learning practitioners in further and higher education frequently encounter students who experience varying degrees of distress arising from their inability to cope with mathematical or numeracy content encountered in what they understood to be non-mathematical topics and courses, or those that supposedly demand no pre-requisite mathematics skills. Such content and skills are often considered implicitly to be ‘assumed knowledge’ or capable of being addressed through a brief introduction or review during one or two initial lectures and tutorials before moving on to the ‘real’ topic content, where students will be expected to immediately apply the reviewed mathematics. Many of these students will be at risk of failing or withdrawing from their study as a direct consequence of their induced distress, prospects that serve to further fuel their anxieties. Often, they manifest math-averse behaviours, typically reporting that they ‘hate maths’ and were ‘never any good at it’ at school, so that their situation is backgrounded by lack of confidence and expectations of failure. Moreover, they frequently self-identify as victims of poor early mathematics learning experiences so that, typically, their mathematics aversion and anxiety are learned
responses arising from episodes that, in many instances, involved humiliation and even bullying (Klinger, 2004). Thus, deep-seated psychological barriers may exist to impede students’ progress in learning mathematics and it is not uncommon to find that their focus is on attaining a ‘quick fix’ – that is, superficial solutions to overcome immediate difficulties in response to extrinsic factors, rather than being open to mathematics learning at a deeper level that would require them to be fully engaged through intrinsic motivation.

To be meaningfully successful with such students – that is, to provide a transformative learning experience by which they may become increasingly self-sufficient in their mathematics’ learning – calls for practitioners to have a significant appreciation of a range of issues that involve the students themselves, the learning environment and a variety of pedagogical approaches. In discussing these various aspects, I draw upon experience gained empirically over a number of years from work with math-averse and math-anxious students. This is extended through consideration of ideas that follow a connectivist approach to learning (Klinger, 2011a) allied with lessons from English language proficiency and the role of language in mathematics learning (Klinger, 2011b) and concludes with an outline of a particular pedagogical framework whereby math-averse behaviours may be mitigated and mathematics anxiety reduced both through the development of metacognitive awareness and a variant of ‘metalinguistic awareness’ – specifically, the ability to reflect on and analyze mathematical language.

**Expectations and characteristics**

In higher education settings, there is often a mismatch between institutional expectations in terms of students’ preparedness for the content demands of their courses and students’ expectations in terms of the skills and specific knowledge that will be required of them in order for that content to be accessible (Lawson, Croft & Halpin, 2003). The divide can be significant, with Cox (2001, p. 848) for instance, reporting a decade ago that ‘…in many areas staff expectations are twice that which the students can actually achieve.’. Such observations are not confined to courses that are overtly mathematical but extend to discipline areas in which mathematics is a service subject (for example, psychology, social administration, health sciences, commerce, and economics), where students may be admitted without having to meet mathematics prerequisites despite the fact that many of the topics have considerable quantitative content and material that is at least conceptually mathematical. This is a rising trend in universities seeking to boost their enrolments by relaxing or eliminating pre-requisite requirements, with Wilson and Macgillivray (2007, p. 30) noting that ‘in quantitative areas this has resulted in the removal of higher level mathematics as a prerequisite for any field of study and of the standard algebra and calculus based course for many areas’. Whereas teaching staff might assume that students entering such courses should nonetheless have been adequately prepared during pre-tertiary education and thus be ready to respond positively to lectures and course materials that review the ‘basics’, commencing students might reasonably respond to the lack of prerequisite mathematics with the expectation that they will not encounter overt mathematical content.

The disparity in these respective views is aggravated by the inadequate preparation afforded by school mathematics curricula and the extent to which educators within universities fail to
appreciate how little mathematics is known by commencing students, particularly those who have recently completed secondary education (Sutherland & Dewhurst, 1999). This is compounded by the ‘the rapid shift in the widening participation agenda has resulted in many universities being required to engage directly with an increasing proportion of relatively ill-prepared students’ (Pokorny & Pokorny, 2005 p. 465). Lack of alignment, generally, between the assumptions and expectations of students and educators has been identified as an important factor in student attrition rates (Lake, 1998), and where the misalignment occurs in conjunction with affective, cognitive, and behavioural factors associated with math-aversion and mathematics anxiety, and which are typically accompanied by intense emotions, students must be considered to be particularly at risk of failure and withdrawal.

There is a range of learning and behavioural characteristics that tend to be common to such students and these are largely independent of age, gender, area and level of study, and the pathways by which they come to their studies. These include confusion, lack of confidence, negative perceptions, narrow focus, and assessment-driven motivation – all of which are customarily associated, in teaching and learning literature, with shallow or surface learning styles and typically accompanied by avoidance behaviour. Although math-averse and mathematically anxious students typically exhibit these characteristics in relation to their mathematics learning activities, one should not assume generally that they will display them similarly in learning contexts and situations that do not involve mathematics. Indeed, in other discipline areas, their learning style may well be considerably deeper, and so the point here is that surface learning behaviours and attitudes associated with mathematics are often reactive, being driven by anxiety, habits and expectations, all of which likely stem from unsatisfactory prior learning situations and negative early encounters with mathematics education.

**Language and interpretative fluency**

Whereas the foregoing attributes are not exclusive to mathematics learning, a further characteristic is distinctively so, and that is a lack of appreciation of the concept of mathematics as language. What I have previously termed the ‘confluence of mathematics and language’ (Klinger, 2011b) involves both conceptions of the linguistic context of mathematics – variously: rhetorical, metaphorical, and literal (respectively, mathematics is like a language; mathematics as language; and mathematics is language) – as well as the language of mathematics, i.e. the superset of natural language comprising the metalanguage referred to by linguists as the ‘mathematics register’.

Rincke (2011) has explicitly associated language learning with the teaching of mechanics in science education, acknowledging Vygotsky’s 1962 seminal work in recognising a relationship between learning in science and learning a language, and stating that this ‘relation to language learning offers one possible way to improve our understanding of learning processes experienced by the students’ (Rincke, 2011 p. 230). This viewpoint stems particularly from the Vygotskian perspective that learning arises first from dialogue, so that individuals make sense of new ideas by working through them in concert with associated discourse with instructors and peers to construct meaning. Thus language is not an ‘add-on’ to the teaching of science but is central to such meaning-making activity ‘when students are introduced into new ways of thinking and talking about the world’ so that ‘the process of
internalising new ideas… originates in the social plane [and] individuals construct their meaning with respect to the social language which they experience in the given [learning] situation’ (Rincke, 2011 p. 232).

If this is so for the teaching of science, the confluence of mathematics and language must make it even more so for the teaching of mathematics, where students are exposed both to mathematics as a new language in its own right, and to the metalanguage of the mathematics register with its specialised application of natural language. Thus, from the students’ perspective, the classroom utterances of the instructor might as well be made in a foreign language (which can fuel or reinforce anxiety) while, from the instructor’s perspective, the utterances of the students may appear imprecise or even nonsensical, so that it can be difficult to gauge the extent to which the meaning-making of individual students aligns with the intended learning goals. Moreover, the language divide within mathematics classes extends beyond the direct and specialised applications of terminology. Morgan (1998), advocates devoting greater attention to the development of students’ critical language awareness, while Chapman (2003) observes that students’ discourse becomes increasingly mathematical in form as their mathematical understanding develops. As Wagner (2004, pp. 12,13) expresses it, ‘language counts in mathematics education discourse because we have no direct access to the objects of mathematics; we can only access the language we use to point at these abstract objects … we need to draw out the voice of students as we investigate mathematics learning discourses’. Wagner continues, observing in particular that:

…mathematics students who see themselves manipulating symbols, unaware that these symbols are representations of something else, miss significant aspects of mathematics. Mathematical symbols, whether they are algebraic, geometric or words spoken to refer to mathematical objects, should be looked through, not looked at. Wagner (2004, p. 186).

That is, mathematical symbols in and of themselves, while demanding our attention for the purpose of manipulation and syntactic precision, serve merely to provide a window by which we may look upon and consider the actual objects of interest.

Multi-dimensional lack of correspondence between student and the mathematics learning situation, encompassing bilateral expectations, learning styles, and discourse, is a ready source of multiple tensions that, if left unchecked, can confound the efforts of instructors to teach effectively and impede students’ opportunities to learn. Individual tensions might be addressed in a variety of ways, according to specific circumstances and details, but the objective here is to consider a more holistic approach which aims to both acknowledge the spectrum of concerns that can arise, and position practitioners’ responses within an overarching framework aimed at reducing or eliminating these tensions through pedagogical practices that reflect distinctions between the conceptual and linguistic aspects of mathematics teaching and learning, while integrating content with students’ everyday experience and ‘commonsense’.
A framework for effective practice

Some students who struggle with mathematics learning, including those who experience overt mathematics anxiety, will openly identify themselves as math-averse, stating that they ‘hate maths’ and labelling themselves as non-mathematical individuals (frequently associated with common stereotypes); they then project this self-image onto their learning situation so that they approach their study with self-defeating attitudes and corresponding behaviours. In contrast, other students will have no particular explicit view of their ‘mathematical self’ beyond recognizing their present struggle and may well identify themselves as effective learners in non-mathematical settings so that their focus is likely to be on specific content difficulties as an externally-imposed condition not of their making and beyond their control to influence. In both cases, and particularly when combined or compounded by the circumstance of having to engage with mathematics learning contrary to their expectations, the students’ perceptions of their role as learners, the practitioner’s perceptions of the students and the learning situation, and the interaction between the two, are critical factors. While there will be tensions arising from the personal and social contexts of the mathematics learning circumstance, there will likely be a further tension from these differing role perceptions – student motivation being primarily (even exclusively) assessment-driven in the pursuit of extrinsic goals, while a dedicated and passionate practitioner is likely to be motivated by the desire to ignite students’ interest in learning the subject matter for its own sake, as something intrinsically of value.

The first element of the framework, then, must be to bridge these divides. For instance, by:

1. challenging the math-averse student’s preconceptions of ‘ordained ineptitude’ through demonstrations, for example, that as adults capable of functioning effectively in a complex world they already possess at least the potential to engage successfully with overtly mathematical concepts and techniques. This, then, involves exposing and debunking the attitudes that underpin negative self-efficacy beliefs, recognizing that these stem predominantly from prior unsatisfactory mathematics learning situations (and hence highlighting the essential need to not repeat or reinforce those experiences. Indeed, it is likely that all students have a number of actual capabilities that they do not recognise as being intrinsically mathematical (e.g. ‘commonsense’ abilities to assess, estimate, recognise patterns, evaluate probability) and a good starting point is to engage in awareness-raising activities;

2. beginning the development of metacognitive awareness that both promotes an informed re-assessment of mathematical identity and contextualises mathematics and mathematics learning activities as cognitive and communicative processes rather than procedural methodologies to be employed as a computational ‘add-on’ to other endeavours; and

3. negotiating a better alignment of students’ and practitioner’s motivation through continuing discourse about the meaning and purpose of the mathematics being taught and learned. Practitioners need to adopt an appropriate degree of pragmatism about students’ motivations while nonetheless being constantly alert for new possibilities and opportunities to stimulate an intrinsic interest in the subject – not by proselytising, but by highlighting connections with students’ life and study inclinations and pursuits.
The second element of the framework involves a departure from traditional behavioural and constructivist practices towards aspects of connectivism (Siemens, 2008), encapsulated in the view that ‘…knowledge is distributed across networks and the act of learning is largely one of forming a diverse network of connections and recognizing attendant patterns’ (p. 10). That is, by adopting teaching approaches that explain and demonstrate how the context and methods of mathematics are revealed through its application, mapping these onto concepts (and language) with which the student is otherwise familiar and confident in ways that align with their various skills and existing understandings of the world. Via commonsense and intuition, metaphor, and analogy, learners are thus encouraged to first understand mathematics in terms of things that are familiar and which ‘make sense’ to them, before moving on to work more directly with mathematical concepts and procedures in terms of the more traditional discourse of the discipline. The goal is to develop students’ ‘at-homeness’ with mathematical concepts, processes, and language gradually and in context, much as a second-language learning development takes place (Klinger, 2011a; 2011b). In this way, new mathematical material is embedded by forming connections with students’ existing knowledge networks so as to create familiarity by association and thus add leverage to the learning task. As students advance and gain confidence, they will naturally have access to an increasing number of opportunities to overcome negative perceptions and attitudes and may begin to develop an appreciation of the intrinsic value of the subject, thus bringing about a motivational shift in their engagement.

The third and final element of the framework draws heavily on the linguistic connections put forward previously to promote the development of students’ ‘interpretative fluency’ (Klinger, 2011b), whereby they gain confidence in their ability to read and write mathematics so that it ‘makes sense’, and acquire the skills to understand, interpret, generalise, and communicate their mathematical knowledge. Thus, the imperative in this part of the framework is to approach the teaching of mathematics as language, both directly in terms of the linguistic and grammatical aspects of the symbolical formalism of written mathematics and increasingly frequent use of, and reliance upon, the mathematics register, and in the nature of the discourse that takes place within the mathematics classroom. The value of adopting a strong language focus for the teaching of mathematics is encapsulated in the following quote from Wagner (2004, abstract):

> Because language is the primary medium through which mathematical understandings are shared, the form of the discourse in mathematics classrooms is significant. For fluid communication, it is necessary for teachers and students to use language as though it accurately represents their mathematical ideas. However, when attention is directed toward language, new possibilities can be opened up for seeing more clearly the nature of the classroom discourse and the nature of mathematics itself.


This focus on language and discourse includes, particularly, the ‘meta-discourse’ (Lemke, 1990) by which the practitioner engages students in discussion about the mathematical language and concepts to which they are being introduced. This encompasses not merely the technical ‘jargon’ of the mathematics register but also the syntactic and semantic features
associated with everyday, informal, language used in a descriptive sense within class discussion. Here, the practitioner must always exercise caution to not imbue such descriptions with their potentially misleading common language associations without, at the very least, drawing students’ attention to nuances of meaning where technical usage differs from everyday understandings. Similarly, students are to be encouraged to use everyday language along with formal mathematical terminology to discuss their understanding of new concepts while also themselves drawing attention to differences where these arise.

Such emphasis on discourse accords with Chomsky’s (1957; 1965) ideas of surface form and deep structure in modelling the relationship between language and thought. Meta-discourse relates to Chomsky’s notion of ‘deep structure’ and it is through such discursive interactions between instructor and learner (and indeed between learners) that meaning is negotiated and by which the practitioner may gauge the extent of a student’s understanding or misunderstanding and thereby gain the opportunity to move on conceptually, on the one hand, or review the present material on the other, which may, from time to time, identify a need to revisit yet earlier material since meta-discourse can reveal weak, conditional, or flawed understanding of precursor concepts. Of course, students must be able to ‘do’ the mathematics, not just talk about it. However, while a student may well carry out a mathematical procedure without error, the result alone need not imply an understanding of what has been undertaken whereas discourse alone – or, ideally, alongside the application of procedure – offers the practitioner with a definitive assessment of the student’s interpretative fluency and comprehension of both concept and application.

This approach, involving as it does linguistic appreciation and application, corresponds with, and extends, what Lemke (1990, p. 217) refers to as ‘triadic dialogue’; that is, the three-part *initiation, response, follow-up* (IRF) cycle that is a common pattern in classroom discourses. According to Atweh, Forgasz and Nebres (2001, p. 209), triadic dialogue ‘acts as a conduit for the relay of mathematical language and concepts … creating an environment that is potentially rich in mathematical language.’. For this to be an effective technique however, students must be compliant in the dialogic exchanges and initiating questions need to be authentic rather than formulaic and restrictive. The matter of student compliance can be problematic in a class situation where mathematics anxiety is a strong referent since afflicted students may well feel inhibited from engaging in discourse due to lack of confidence and recalled trauma from the consequences of ‘getting in wrong’ when speaking out in early-experience mathematics classes. Thus it is vital that the first element of this proposed framework be fully addressed both at the outset and as an ongoing feature of the mathematics learning environment.

## Conclusion

It is most unfortunate that many adult learners have histories of poor, even appalling, early mathematics learning experiences, leaving them with adverse perceptions of mathematics and a diminished capacity to engage effectively with the subject. It can be just as unfortunate to view such students as being in deficit and thus in need of remediation, a generally unhelpful and inappropriate perspective that presents as a problem with the student at its epicentre.
The discussion and framework put forward here have been proposed as an alternative that calls for a re-evaluation of traditional mathematics teaching activity so as to provide a positive learning experience in every respect, both in terms of responding sensitively and creatively to their apprehensions and anxieties and by adopting pedagogical approaches that are informed by an appreciation of the value and role of language in mathematics instruction. The most effective teaching practice occurs when the teaching style and quality of the learner/practitioner interaction promote a high degree of mutual respect and trust that moderates the disparities between their respective perceptions of their roles. In this, a focus on the nature and quality of discourse and meta-discourse within the mathematics teaching and learning environment is argued to be paramount in engaging students more effectively than can be achieved otherwise, promoting the development of their interpretative fluency, and offering them opportunities to experience a level of success in mathematics that has hitherto eluded them.
References


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The challenges faced when integrating numeracy and literacy in the development of a ‘Real World Maths’ training pack

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The National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), in conjunction with the National Adult Literacy Agency (NALA), received a tender to develop two numeracy training packs (Level 3 and Level 4) for use by Ireland’s national training and employment authority ‘Foras Áiseanna Saothair’ (FÁS). Each of these packs consisted of three elements; a learner pack, a tutor pack and a tutor resource pack. The team developed the mathematical content, pedagogical approaches, tutor guidelines and resources for the training packs. The content is aimed at adolescents engaged in vocational education programmes but it could also be useful to mature adult students returning to education. When developing the numeracy training packs, many challenges were encountered by the authors. Such issues and valuable lessons learned on the integration of numeracy and literacy are highlighted in this paper.

As the National Training and Employment Authority, FÁS, operates training and employment programmes throughout Ireland and provides a recruitment service to jobseekers and employers, an advisory service for industry, and supports community-based enterprises. Teaching mathematics in context and in a way that relates to learners’ lives was a primary aim of FÁS when issuing a tender for the development of ‘Real World Maths’ packs. FÁS also required the packs to be designed in a way that would assist learners to achieve certification on Levels 3 and 4 of Ireland’s National Framework of Qualifications (NFQ). As mathematics teachers, teacher educators and researchers who are aware of the many benefits of relating mathematics to ones’ interests and lives, the NCE-MSTL (National Centre for Excellence in Mathematics and Science Teaching and Learning) team, in conjunction with NALA (National Adult Literacy Agency), began developing the pack. There were two packs to be developed – one at Level 3 called ‘Application of Number’ and one at Level 4 called ‘Mathematics’. These titles refer to those of the particular FETAC Awards to which the packs were tailored. FETAC - the Further Education and Training Awards Council – is the awarding body responsible for Levels 1 – 6 of the NFQ.

As outlined above, each of these packs consisted of three elements; a learner pack, a tutor pack and a tutor resource pack. The FETAC Award Specifications for Level 3 Application of Number and Level 4 Mathematics, specifying the required learning outcomes, were provided by FÁS prior to commencing the development of the packs. The packs were developed for use by FÁS Community Training Centres (CTCs) nationally. These CTCs cater for early school leavers and between the ages of 16 and 21.

The development team encountered many numeracy and literacy challenges. The content was similar to secondary school mathematics but the focus was predominantly on how the content
relates to the real world and to the learners’ interests. This was not always an easy task and was coupled with concerns for low literacy levels. Research has highlighted that numeracy and literacy issues are rife at all levels of education in Ireland, and indeed internationally, and so these challenges are not unique to early school leavers.

Numeracy and literacy concerns in Ireland

In November 2010, the Department of Education and Skills (DES) published a draft national plan to improve literacy and numeracy in schools. The DES (2010: 9) defined literacy as reading, writing, speaking, viewing, and listening effectively in a range of contexts as well as to refer to as “a flexible, sustainable mastery of a set of capabilities in the use and production of traditional texts and new communications technologies using spoken language, print and multimedia”. The relevance of technology for our young people’s education and future is of utmost importance and will be discussed in more detail later in this paper. The DES (2010: 9) goes on to define numeracy as “the capacity, confidence and disposition to use mathematics to meet the demands of learning, school, home, work, community and civic life”. Importantly to this project, the DES further explain the key role of applying mathematics, understanding its uses in the broader context as well as in the study of other disciplines. Again, this had a major influence on how the packs for FÁS were researched and developed. The need for, and publication of, the DES draft national plan highlights the concern that exists for the literacy and numeracy of our young people. Such concerns are now outlined.

Numeracy concerns in the development of the pack

According to the DES (2010: 11), “the teaching and learning of mathematics in Ireland requires even greater attention than literacy”. While the report is predominantly focused on mainstream primary and secondary schools in Ireland, it also highlights the important role of the DES in supporting numeracy and literacy in the Youthreach programme. The Youthreach programme caters for early school leavers and is delivered through a number of providers, including Vocational Education Committees (VECs) and FÁS CTCs. The DES report refers to a recent evaluation of VEC Youthreach Centres which indicated that one of the greatest challenges facing Youthreach is the development of the learners’ literacy and numeracy skills.

In mainstream second level mathematics, there are many concerns relating to the teaching and learning of mathematics. Each year, the national media reports on the large failure rate at Leaving Certificate (final State examination) mathematics. There is a low level of uptake at Leaving Certificate for the highest level of mathematics, referred to as Higher Level, with only 16 per cent doing so in 2010. Internationally, The Programme for International Student Assessment (PISA) reports show that Ireland has significantly fewer students at higher proficiency levels than the OECD average (6.7% compared to 12.7%) (Perkins, Moran, Cosgrove and Shiels, 2010). These high proficiency levels refer to students’ level of problem-solving, reasoning, communication and reflection of findings, for example. Such concerns are heightened for early school leavers many of whom have significant numeracy difficulties.

In developing the ‘Real World Maths’ pack, such concerns were to the fore in the minds of the project team. The curriculum required learning activities which should be relevant, challenging and imaginative, building on interests and themes that attract the learner. In order to achieve this, the project team needed to be aware of the topics and activities of interest to the young learners for whom these packs were intended, and to apply numeracy in that
context. In conjunction with this, in developing the pack, the project team needed to take effective account of literacy issues.

**Literacy concerns in the development of the pack**

NALA is an independent, membership-based organisation, committed to making sure people with literacy and numeracy difficulties can fully take part in society and have access to learning opportunities that meet their needs. NALA defines ‘literacy’ as involving

“listening and speaking, reading, writing, numeracy and using everyday technology to communicate and handle information. But it includes more than the technical skills of communication: it also has personal, social and economic dimensions. Literacy increases the opportunity for individuals and communities to reflect on their situation, explore new possibilities and initiate change.”

(NALA 2011, p. 8)

NALA has also specifically defined its understanding of ‘numeracy’ as

“a lifeskill that demands the competent use of mathematical language, knowledge and skills. Numerate adults have the confidence to manage the mathematical demands of real-life situations such as everyday living, work-related settings and in further education, so that effective choices are made in our evolving technological and knowledge-based society.”

(NALA, 2004, p. 5)

Literacy and numeracy are therefore seen as integrated practices that are always bound up in broader social practices and that vary according to context, purposes and domains of life. NALA’s Evolving Model of Curriculum Development for adult literacy and numeracy education (NALA: 2009) (see Figure 1), puts learners at the centre of curriculum development and takes their real world contexts as the starting point for the process. Methods build on learners’ everyday uses of literacy and numeracy and focus on developing literacy and numeracy mobility across different contexts and purposes. This has much in common with the NCE-MSTL’s ‘real world maths’ approach and enabled effective cooperation between staff from both organisations in carrying out this project.

![Figure 1. An evolving model of curriculum development for adult literacy and numeracy education (NALA).](image-url)
In taking account of literacy issues, the team paid close attention to issues of readability, clarity and user-friendliness in the finished packs (NALA 2006; NALA 2009), and included resources and guidelines for non-text-based teaching and learning activities to engage a range of learning styles.

A major consideration in relation to the particular setting in which the packs will be used was the FÁS strategy to integrate literacy across the curriculum in CTCs. Research and practice in Ireland and abroad indicate that integrating literacy and numeracy across the curriculum and into all phases of a further education and vocational training programme, is highly effective both for literacy and numeracy development and for learning and achievement in the main vocational programme (Hegarty & Feeley 2009; McSkeane 2009; Casey et al, 2007). In addition to providing specific tuition in literacy and numeracy, CTCs have been encouraged to build literacy and numeracy development into the other subjects and activities in the Centre and at every phase of the programme. FÁS and NALA have collaborated to develop a range of supports for the centres, including a third-level professional development programme accredited by National University of Ireland (NUI) Maynooth (the NALA-NUI Certificate Course in Integrating Literacy), and NALA has developed guidelines and resources to support a whole-centre approach to integrating literacy (Ni Chinneide 2002; NALA 2003; NALA 2006b). The team was aware that further supports for CTCs would be provided in 2011 to advance their strategy to integrate literacy and numeracy across the programmes, including further training for CTC staff in integrated approaches, and the maths packs were developed in that context.

As the focus in the CTC integrated materials development to date had been on the real world context of the learners’ lives inside the Centre – in particular their vocational programmes – it was decided to develop the real world maths materials based on other contexts of the young learners’ lives ‘outside’ their formal education and training programme. The content draws on themes and topics that the young people, consulted by the team, said would be of interest to them and to their circle of friends. The materials were also designed to assist the learners to gain certification for their mathematics learning, using topics and themes that would be meaningful and interesting to them.

**Development of the ‘Real World Maths’ Pack**

**Levels, Learning Outcomes and Assessment**

As mentioned earlier, the pack was based on two levels – Level 3 and Level 4. These levels are determined by the Further Education and Training Awards Council (FETAC) which is the statutory awarding body for further education and training in Ireland. A module descriptor for both levels was provided to the project team. This module descriptor listed the content to be covered under each unit. There were two such units in Level 3 comprising of ‘Number’ and ‘Measurement and Capacity’ and four units in Level 4 comprising of ‘Number’, ‘Geometry’, ‘Algebra’ and ‘Data Handling’. The learning outcomes associated with each one were similar to that at Junior Certificate level (State examination after three years of schooling) and can be viewed in Appendix 1. The assessment for Level 3 was in the form of a portfolio/collection of work (100%). The assessment for Level 4 included portfolio/collection work (80%) as well as an examination (20%).

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Content and Activities

There was a clear discrepancy between the content at Levels 3 and 4 with Level 4 progressing on from Level 3. Unit 1 in both levels was ‘Number’ and consisted of basic operations and concepts. This was perhaps the most challenging unit. It consists of a large number of learning outcomes, many of which pose great difficulty for all learners of mathematics e.g. addition, subtraction, multiplication and division of fractions. The pack aimed to make such concepts clear at a conceptual level while also maintaining the learners’ interest and enthusiasm. There has been much research on conceptual understanding and the need to move away from a reliance on a procedural, rote-learning approach to learning mathematics which focuses on memorisation and reproduction of the material rather than on a deeper understanding of the concept (e.g. Case and Marshall, 2004; Ramsden, 1992 and Skemp, 1978). This focus on conceptual understanding presented the project team with one of their toughest challenges.

Activity Sheets

For all learning outcomes, an activity or activities were developed for the learner pack. Each activity was named around the topic or task of interest to the learner. For example, natural numbers were introduced in an activity called ‘Playing Darts’. The project team felt that the naming of activities was important since it was more likely to attract the learners’ attention rather than deter the learner by using mathematical terms such as ‘Natural Numbers’. All members of the project team followed the same format when developing the activities. Once the activity was named, the learning outcome(s) were listed in clear, simple English e.g. the learner will be able to recognise natural numbers. The activity sheet then progressed to a brief description of the mathematics involved in the particular task and also listed what the learner may need to know prior to completing this activity sheet. For example, in order to complete ‘Playing Darts’, the learner may need to familiarise themselves with the game of darts. For each activity sheet there were a number of tasks to be completed. In all cases, the first task was used as an example with answers and explanations provided. Another task or two was then set out for the learner to complete. This was called ‘Now you try this’. All answers to these sections were provided to the tutors in their tutor pack. Many activities involved the learner taking part in an activity, collecting data or doing some research related to the mathematical concept. Learners were then guided to practice sheets which allowed them straightforward practice of routine skills. The activity sheet concluded with the assessment briefs which learners could complete once they felt they has mastered the previous tasks and practice sheets. These assessment briefs were then handed to the tutor and used as part of the overall assessment grade.

In addition to the learner pack, a tutor pack was developed which consisted of answers to all tasks, practice sheets and assessment briefs. Guidelines were also provided to tutors on how they might approach the teaching of each of the activity or adjust the activity to suit their learners’ needs specifically.

Finally, a resource pack was provided for the CTCs which included all templates for hands-on activities advised in the learner packs. For example, fraction circles were used in both Level 3 and Level 4 unit ‘Number’, and so a template for tutors to produce their own was necessary. A list of useful websites and other recommendations was also included in the resource pack.
Feedback from CTC learners and tutors

During the early stages of development, feedback on the activities developed was sought through a meeting with two learners and two tutors in a CTC in Co. Kildare. This feedback proved invaluable providing the development team with a further insight into the learners’ interests as well as tutors’ opinions on how the activities would, or wouldn’t, work in practice.

All members of the NCE-MSTL and NALA project team were alerted to the findings from this meeting so that the activities could be developed accordingly.

Pedagogical Approach and Blum’s Modelling-Process

The way in which the mathematical concepts were advised to be taught was as important as the content itself. Blum’s modelling process of real world problems was the framework used for the development of the activities (see Figure 2 overleaf). This model begins with a real world context or problem which is then transformed into a mathematical problem. Mathematics is used to solve the problem and the final phase involves relating the solution back to the real world context.

![Figure 2. Adapted from Blum & Leiß’s Modelling Cycle (Cited in Blum & Ferri, 2009: 46)](image)

This model is often used in the teaching and learning of mathematics in schools and research has shown that students can see the utility of mathematics by using such an approach. A study by Osawa (2002) used a relay problem to demonstrate how mathematics can be applied in the real world. Students collected data by taking part in a relay race and Osawa (2002: 89) explained how “students had never shown such an interest in regular classes”. Blum and Ferri (2009:47) highlight the advantages of this mathematical approach:

- Help students to better understand the world,
- Support mathematics learning (motivation, concept formation, comprehension, retaining),
- Contribute to develop various mathematical competencies and appropriate attitudes,
- Contribute to an adequate picture of mathematics.
Gainsburg (2008) reports on the importance and urgency of connecting classroom mathematics to the real world. She concludes that teachers at second level make connections to the real world regularly, but are mostly brief and do not actively engage students. This project aimed to not only make connections, but to engage the learners in the process and stimulate their interest in the subject. In order to engage the students, it was essential they were actively involved in the activities. Active learning was therefore central to the development of the pack. Prince (2004:225) summarises some advantages of active learning:

- Better student attitudes and improvements in students’ thinking skills
- Much higher levels of energy and participation
- More and better questions and answers from students
- Surpasses traditional modes of instruction for retention of material.

Another important feature of the pack in terms of pedagogy is the use of technology in the classroom. Technology plays an important role in our daily lives and in the workplace. Zevenbergen (2011:88) highlights the increased role of technology in the lives of younger people and proposes that “young people have developed novel ways of working in relation to numeracy, which are often different from those of past generations”. It was therefore imperative that in the development of our pack we included technology, be it by referring to recent technological devices of interest to the students e.g. iPad, iPod, Smart Phones etc., or by incorporating technology into the activities e.g. using Excel/SPSS in statistics and highlighting concepts using GeoGebra. It is also essential that tutors are aware of such technologies given that their students will need to meet these high-tech demands in the working world.

**Training Tutors in the ‘Real World Maths’ Pack**

The learner, tutor and resource packs for both Level 3 and Level 4 were introduced to tutors from CTCs around the country. Training days took place in Limerick and Dublin and were carried out by NCE-MSTL and NALA staff. The aim of these training days was to familiarise the tutors with the three packs. It was important to explain Blum’s modelling cycle so as tutors could understand how the activities were developed and how best to implement them in their own classrooms. It was also necessary to put some of the activities from each level into practice with the tutors and to highlight to them the importance of adapting the activities to fit the needs of their own students.

The training days also allowed the development team to receive written feedback on the packs. It should be noted that the packs were not yet at the final stage when the training days took place but tutors were given access to the draft copies of both Level 3 and Level 4 packs. Feedback was very positive in relation the content of the packs. Some quotes from the tutors in relation to what they learned at the training day or what he/she may try in their own mathematics teaching and learning include:

- “How to apply maths into everyday life” (Tutor 1)
- “Yes – [I will] make the maths practical and relevant” (Tutor 2)
“How to relate maths to more real life situations–It was very interesting” (Tutor 3)

“It reminded me to incorporate more real life examples into lessons” (Tutor 4).

There were concerns highlighted by a number of tutors in relation to assessment of the pack at Level 4. At the training day, the types of questions that may be asked in the 20% exam at Level 4 were provided to tutors. These exam questions were not developed by the project team and did not reflect the real world approach incorporated into the activities in the pack. Following the training day, such concerns were brought to the attention of FÁS and the examination questions were altered to complement the pack. This was a very important issue as the tutors expressed strong concerns for the success of the pack if the examination did not reflect this real world maths approach.

Challenges and Lessons Learned

One of the main challenges the project team faced was the short time scale within which the packs were required to be developed. The team received notice of the tender in early October with the view to having the packs developed by early December.

In terms of numeracy and literacy, there were many challenges and indeed lessons learned. The NCE-MSTL team comprised maths and numeracy experts while the NALA team provided literacy expertise and its experience of working with practitioners in the intended context to integrate numeracy and literacy with other learning. Pitching the mathematics at a suitable level was extremely important as well as promoting teaching for understanding and a shift from the didactical, routine approach. In addition, it was essential to ensure that each of the concepts was related to the real world using topics of interest to the learners. Much research, thought and discussion between the team members was critical. Once the activities were developed they were sent to the team in NALA to ‘plain English’ the text and proof read. In many cases, this involved a lot of editing and re-drafts. The NCE-MSTL team became more aware of the literacy levels of the learners and the language required as the packs developed further.

Conclusion

Teachers and researchers of mathematics education alike, need to be aware of the numeracy and literacy levels of early school leavers and mature adult learners. All content and resources produced should reflect this and be tailored to suit the needs of the learner. The numeracy experts in the development of this pack needed to be careful of their use of language making sure it was appropriate to the literacy levels of the intended learners. Putting the mathematics into real life context throughout the packs was welcomed by the tutors. In addition, it is hoped that this approach will stimulate the learners’ interest in mathematics thus improving their numeracy levels and importantly, enabling them to put it into practice in their daily and working lives.

Acknowledgements

The authors would like to thank the following people for their contribution to the development of the ‘Real World Maths’ pack: Terry Maguire, Tim Brophy, Mark Prendergast, Paraic Treacy, Lisa O’Keeffe and Fergus Dolan.
References


NALA (2003). Skillwords Dublin: NALA

NALA (2004) Meeting the numeracy challenge – A development strategy for numeracy in adult basic education Dublin:NALA

NALA (2006a) Preparing learning materials: A guide for literacy and numeracy tutors Dublin: NALA

NALA (2006b) Missling the Tóbar: A resource pack Dublin: NALA

NALA (2009) Simply Put: Writing and design tips Dublin: NALA

NALA (2011) Providing leadership in adult literacy: Strategic Plan, 2011-2013 Dublin: NALA


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**Appendix 1 – Learning Outcomes for Level 3 and Level 4**

<table>
<thead>
<tr>
<th>Level 3, Unit 1 (Number)</th>
<th>LO1: Explain the concepts of natural numbers (N), integers (Z), and real numbers (R).</th>
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<tbody>
<tr>
<td></td>
<td>LO2: Demonstrate equivalence between common fractions, simple ratios, decimals and percentages by conversion e.g. ( \frac{1}{2} = 1:2 = 0.5 = 50% ).</td>
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<tr>
<td></td>
<td>LO3: Use a calculator to perform operations requiring functions such as +, -, ×, ÷, %, memory keys and the clear key.</td>
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<td></td>
<td>LO4: Give approximations by using strategies including significant figures and rounding-off large natural numbers.</td>
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<tr>
<td></td>
<td>LO5: Demonstrate accuracy of calculation by applying the principal mathematical functions i.e. +, -, ×, ÷, to natural numbers (N) and integers (Z), common simple fractions, and decimal numbers to two places of decimal.</td>
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<tr>
<th>Level 3, Unit 2 (Measurement and Capacity)</th>
<th>LO6: Describe shape and space constructs using language appropriate to shape and space e.g. square, rectangle, circle, cylinder, angles, bisect, radius, parallel, perpendicular.</th>
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<tbody>
<tr>
<td></td>
<td>LO7: Draw everyday objects to scale using a range of mathematical instruments.</td>
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<td></td>
<td>LO8: Calculate the area of a square, rectangle, triangle, and circle by applying the correct formula and giving the answer in the correct form.</td>
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<td></td>
<td>LO9: Calculate the volume of a cylinder by applying the correct formula and giving the answer in the correct form.</td>
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<td></td>
<td>LO10: Understand simple scaled drawings by working out real distance, location, and direction.</td>
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<td></td>
<td>LO11: Demonstrate metric measurement skills using the correct measurement instrument, and vocabulary appropriate to the measurement, to accurately measure length/distance, capacity, weight, time.</td>
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<tr>
<td></td>
<td>LO12: Calculate solutions to real life quantitative problems by applying the appropriate mathematical techniques to a variety of everyday situations and discussing the results to</td>
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include budgets, costings, time, quantity etc.

<table>
<thead>
<tr>
<th>Level &amp; Unit</th>
<th>Learning Outcome (LO)</th>
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<tbody>
<tr>
<td><strong>Level 4, Unit 1 (Number)</strong></td>
<td>LO1: Discuss the application of number to familiar real life situations</td>
</tr>
<tr>
<td></td>
<td>LO2: Calculate conversions to include: between currencies, between fractions, decimals and percentages, and from fractions to ratios and ratios to fractions, and from standard form to scientific notation and scientific notation to standard form</td>
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<td></td>
<td>LO3: Use appropriate strategies such as rounding off, places of decimal, significant figures, estimation, % error, to give approximations, where numbers are from the set of natural numbers (N) and from the set of integers (Z)</td>
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<td></td>
<td>LO4: Use a calculator with confidence to perform extended calculations, requiring functions such as +, -, ×, ÷, %, √, π, ¹/ₓ, scientific notation keys, memory keys and the clear key, while following the conventions of precedence of operations</td>
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<td></td>
<td>LO5: Demonstrate an understanding of the laws of indices and the rules of logarithms by using the laws and rules to simplify expressions, solve equations, and transpose formulae</td>
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<td></td>
<td>LO6: Differentiate between simple interest and compound interest by applying the appropriate given formula to a range of savings and credit options</td>
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<tr>
<td></td>
<td>LO7: Apply the percentage (%) function accurately to a range of everyday situations including gross income and net income, pay slips using appropriate statutory deductions, gross profit, net profit and loss on goods sold, VAT inclusive and VAT exclusive prices</td>
</tr>
<tr>
<td><strong>Level 4, Unit 2 (Geometry)</strong></td>
<td>LO8: Describe simple geometric shapes associated with the home and workplace</td>
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<td></td>
<td>LO9: Recognise folding symmetry and rotational symmetry in common shapes</td>
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<tr>
<td></td>
<td>LO10: Plot graphs of ordered pairs in the coordinate plane showing the relationship between two variables, using real life situations and the correct terminology</td>
</tr>
<tr>
<td></td>
<td>LO11: Use formulae for calculations in the coordinate plane correctly, including distance between two points, mid-point of a line segment, slope of a line, parallel lines, perpendicular lines, equation of a line, equation of a circle with centre (0,0) and radius r, and tangent to a circle</td>
</tr>
<tr>
<td></td>
<td>LO12: Construct, using drawing instruments, a variety of angles and simple geometric shapes to given criteria to include naming of angle types and sides associated with the shapes and angles</td>
</tr>
</tbody>
</table>
| | LO13: Solve practical problems by using the correct formula(e) to calculate the area and perimeter of a square, rectangle, triangle, and circle, giving the answer in the correct
<table>
<thead>
<tr>
<th>Level 4, Unit 3 (Algebra)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LO14:</strong> Apply standard axioms and theorems of geometry, including Pythagoras Theorem, to solve real life or simulated problems involving straight lines, parallel lines, angles, and triangles</td>
</tr>
<tr>
<td></td>
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<tr>
<td>Level 4, Unit 4 (Data)</td>
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<tr>
<td>--------------------------</td>
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<tr>
<td><strong>LO15:</strong> Discuss the presence of variables in a range of real life situations</td>
</tr>
<tr>
<td><strong>LO16:</strong> Solve algebraic equations including linear equations of one variable, simultaneous linear equations of two unknowns, and linear inequalities of one variable</td>
</tr>
<tr>
<td><strong>LO17:</strong> Construct algebraic expressions and formulae for real life situations using the correct terminology and including rearrangement of formulae</td>
</tr>
<tr>
<td><strong>LO18:</strong> Explain basic statistical concepts to include population, sample, dependent, independent and discrete variables</td>
</tr>
<tr>
<td><strong>LO19:</strong> Present information from data collected from the world wide web or other methods, in graphical and tabular form, including bar charts, pie charts, trend graphs, cumulative frequency curves, histograms and frequency tables.</td>
</tr>
<tr>
<td><strong>LO20:</strong> Calculate the statistics for measuring averages and dispersion of an array of data, to include calculating the mean, mode, and median</td>
</tr>
<tr>
<td><strong>LO21:</strong> Discuss findings, to include interpretation of results, and suggesting reasons for findings</td>
</tr>
</tbody>
</table>
In a 2011 case study which I conducted, adult learners of mathematics used a socially interactive board game (BAR) for the review of beginning and intermediate algebra. A board game was devised (with help of an advisory committee and based on the same materials from the texts in use) in which two or more teams compete to answer algebra questions, using hints provided if necessary. The case study looked for possible changes in levels of anxiety, confidence and competence after several sessions playing the board game.

Researchers and educators in the field of mathematics refer to literature from adult education to inform their research and practice (Evans, Tsatasaroni & Staub, 2007; Milller-Reilly, 2008; Hassl, Hannula & Nevado, 2011; et al). Much of the research addresses the andragogy of adults learning math. The traditional content-driven, teacher centred mathematics classroom is not always best suited to adult learners; alternatives that incorporate adult learner principles are explored (Lasry, Mazur & Watkins, 2008; Mesa, 2010; Schwarz, 2011; et al). Using the strengths of adult learners to enhance mathematics learning can mean incorporating social fun, with a community of learners playing a board game to overcome negative affect and to enhance both learner confidence and competence. Also, the work of researchers focused on SEL (Social and Emotional Learning) contribute to this idea and to the socially interactive game (Lantieri, 2009; Hunter, 2009).

A short introductory talk will place the study and the game in the literature of adult mathematics education. For this workshop, several boards will be available and workshop participants will play in teams to experience the challenges and rewards of this socially interactive board game. A game takes between 45 minutes to one and a half hours. A closing discussion will focus on the experience and insights of workshop participants and questions about the game and the case study.
Our previous discussion focused on the challenges of designing a college competency course that should provide a diverse population of young adults, and college students, with opportunities to improve their numeracy, and quantitative reasoning skills. One of the challenges involved negotiations of the meaning of “real-life” and “everyday mathematics” among committee members with various mathematical backgrounds. This contribution focuses on related questions of relevance and authenticity of problems: What (quantitative) contexts are (or potentially will be) perceived as authentic and relevant in the lives of our students? What kind of evidence may help us answer the question? We discuss these questions in the context of our first pilot course that was offered in the spring of 2010 with an emphasis on content analysis of students’ journals they kept during the semester as one of the course assignments.

Key words: numeracy, quantitative reasoning, college requirement, everyday mathematics.

The role of mathematics in “everyday life”, and debates about infusing school mathematics with important, authentic, “real-life” problems that prepare students for life, has been present in the literature for some time (Boaler, 1993; Lave, 1988; Forman & Steen, 2000; Steen L. A., 2001, etc.). In our previous discussion (Marcinek, Dias, & Piatek-Jimenez, 2010), we showed that the negotiations of the meaning of “everyday” or “real-life” mathematics is problematic even among colleagues at the same department (mathematicians, statisticians, mathematics educators). We also saw that disagreements about “everyday” or “real-life” problems often relate to two major aspects: ‘What is an authentic problem?’; and ‘What problems are (or will be) relevant in the lives of our students?’. The question, what is relevant and authentic, is, however, very sensitive to the educational context and the search for evidence has been very often focused on specific situations such as vocational mathematics, education, or mathematics at the workplace (see for example Hoogland, 2005; Keogh, Maguire, & O'Donoghue, 2009). Considerably less is known about what constitutes a relevant and authentic problem in the context of general education of college students with a wide range of interests and career orientations.

General Education is considered an important component of students’ education at many universities in the United States. Its overarching goal is to educate students to be successful in a rapidly changing global world regardless of their major or specialization.
Requirements are often part of General Education Programs, and our program at Central Michigan University consists of English and Mathematics competency requirements, and a newly approved competency requirement in quantitative reasoning. Students can fulfil competency requirements typically by taking certain coursework (“competency courses”) or demonstrating their competency (for example by taking a test).

In the previous paper (Marcinek, Dias, & Piatek-Jimenez, 2010), we discussed the challenges of designing a college competency course that should provide a diverse population of young adults, and college students, with opportunities to improve their numeracy and quantitative reasoning skills. A different set of issues arises when a newly designed numeracy course, for the general population of college students, needs to be implemented. In spring 2010, we went through the first phase of our implementation plan that included pilot testing the course and course materials. A subsequent phase will take place when a university-wide competency requirement in quantitative reasoning will come into effect. In this paper, we focus only on local issues of our pilot course implementation. We describe how we addressed the search for evidence of authentic and relevant mathematics in the lives of our students.

**Pilot course**

32 students enrolled in our pilot course. Professional orientation, (majors or specializations) of students enrolled in the class, included art (music, design, 21%), social sciences (psychology, sociology, political science, etc. 33%), natural sciences (8%), sport (8%), marketing and management (13%) and accounting and business (17%). The majority of the students were in their junior and senior years (3rd and 4th year) and for many of these students it was the last course with mathematical or quantitative background in their careers.

Along with the implementation of our pilot course, we tried to look for evidence that would shed light on the mathematical/quantitative reasoning students engage in, and are aware of in their everyday lives. As we were designing the course, we used various assessment tools that were considered as a possible data source.

**Table 1: Options for course assessment components**

<table>
<thead>
<tr>
<th>Assessment component</th>
<th>Type of information it provides</th>
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<tbody>
<tr>
<td>Test and Quizzes</td>
<td>Knowledge, skills and ability of their application</td>
</tr>
<tr>
<td>Team Projects</td>
<td>Strategies of solving rather complex problems set in authentic contexts</td>
</tr>
<tr>
<td>Team Presentations</td>
<td>Ways of formulating and presenting relevant quantitative arguments. Student’s perceptions of what constitutes a relevant quantitative argument.</td>
</tr>
<tr>
<td>Journal</td>
<td>Students’ perception and awareness of their “everyday” activities that involve quantitative reasoning or other numeracy skills</td>
</tr>
<tr>
<td>Portfolio</td>
<td>Collection of artefacts evidencing semester-long work</td>
</tr>
</tbody>
</table>
For our first semester pilot, we selected several assessment options from the list, but chose to focus closely on Journals, hoping that the data they provided could be used not only in student’s self-reflection and assessment, but also as a rich source of valuable information about how students perceive the role of quantitative reasoning in their everyday lives. Students were asked to follow guidelines for Journal keeping (Appendix A) and their journals were evaluated two times during the semester. We carefully documented and classified each journal entry with the goal of creating a:

- **Still Image**: How the students perceive the role of quantitative reasoning in their everyday lives? What “quantitative encounters” or *numeracy incidents* (Hoogland, 2008, p. 168) are they sensitive to? What does a snapshot of the world through our students’ eyes look like?

- **Moving Image**: Are the students becoming more sensitive to quantitative aspects in their everyday lives as a result of attending our course? Are the quantitative occurrences (journal entries) increasing in number, or covering a broader spectrum of “everyday” activities as the semester progresses?

### Collected Data

32 students attended the class and kept a journal of numeracy incidents in students’ lives. As the Appendix A indicates, data available for an analysis consisted of:

- More than 500 raw journal entries. Raw entries are brief notes, the purpose of which is to help students recall particular situations. Submission of these entries was not mandatory and only some were included in our analysis.

- About 330 student-selected and discussed journal entries, with enough detail for a disinterested reader to understand the context of the situation and strategies, mathematical ideas or tools employed in solving it. Student journals were evaluated based on the amount of detail provided in the discussion of these entries. They formed the most important data set for our analysis.

- 16 group-selected and presented journal entries. The items of individual students from the previous bullet were discussed by each group. Each group then chose 2 entries and presented them for the discussion in the class.

### Data analysis

In the first phase of the analysis, we tried to reduce the number of items by identifying multiple entries that shared common features (context, mathematics ideas, strategies/procedures etc.) For example, three entries by three students related to figuring out the final price of products (i.e. including a sales tax) were included in a single data item “shopping: sales tax”, ignoring irrelevant details, such as a type or nature of the products. We identified 164 unique items, weighted them according to the number of individual entries that fall under each item, and used these items in our content analysis. We use the word “item” when we
refer to these unique weighted items; we will use the word “entry” or “journal entry” whenever we need to refer to the original journal entries.

We selected several general categories (we will refer to them as domains) to simplify the data structure. Some domains were chosen before we started analysing the data, some arose in the course of data analysis.

**Mathematical Substance**

Our first domain was chosen to help us see recurrent mathematical content areas that students draw on in their everyday activities. All data items can be structured using the following categories:

- Elementary counting and additive/subtractive reasoning. (Do I have enough money to get a soda if I have $7.00 and want coffee and a bagel? Figuring out total hours worked; Keeping track of fluid intake during the day; Counting out change; etc.)
- Rates, Ratios and Proportional Reasoning. (Currency conversions; cost sharing - rent, utility bills and food among friends; estimating miles to travel on a ¼ of a tank before refuelling; etc.)
- Averages. (Averaging utility bills for budget planning; finding lowest possible grades in classes to meet a given GPA; Estimating car’s fuel efficiency; etc.)
- Geometry: Reasoning about shapes. (Building a snake cage; re-carpeting a room; etc.)
- Measurement: Length, Area, Volume and Time. (Building a snake cage; painting a wall; Scheduling activities to meet a given time limit; etc.)
- Units and unit conversions. (SI prefixes in units of computer storage – MB, GB, TB; imperial vs. SI units in IKEA store; etc.)
- Elementary probability (Chances of winning a football game)
- Entries with little involvement of quantitative reasoning (after reading Dan Brown’s book, I created a text based on my own Masonic cipher…).

**Strategy**

We tentatively identified several categories with the purpose of further exploration of the interplay between: (1) estimation and exact calculations; (2) mental and other strategies; and (3) various problem-solving strategies. However, only a limited number of entries provided explicit information about important aspects of students’ strategy. More sophisticated examples (that the students selected for a journal discussion) were presented in their final form without sufficient detail on the solving strategies. We therefore later excluded this domain from the analysis.

**Context**

We chose this domain to reveal types and kinds of everyday activities in which the use of quantitative reasoning and other numeracy skills appears. The data show considerable variability and overlap with the following general categories:

- Finance-related activities include balancing accounts, cost sharing, dealing with spending habits (shopping, credit card management), etc.
• Work-related activities are mostly related to various part-time positions students hold and include salary calculations, filling out time sheets, closing cash registers, fundraising planning, etc.
• School-related activities include calculations of/with GPA, credit hours, time planning and scheduling classes, tuition calculations
• Home and household-related activities include decorating projects, caring for a pet, cooking, etc.
• Commute-related activities mostly involve managing operating cost of a car
• Hobby involves a range of students’ interests. Travel and sport make up a significant portion of the category
• Health-related activities include exercising schedule, calorie count, fluid intake, etc.
• General curiosity. Quantitative reasoning entailed by this category does not refer to any purposeful activity or context and the students’ curiosity seems to be the only motivator. This category includes such questions as ‘I wondered how much are parking meters making in a month?’, or figuring out riddles and brain teasers.

**Context Specificity**

The context specificity domain was chosen to reveal possible relationships between the contexts in which numeracy skills are utilized and a student’s professional orientation. Do all data items refer mostly to general contexts shared by all students, or are the students’ future professional orientations (major or specialization) significantly shaping the contexts in which they utilize numeracy skills? In other words, is it possible to guess a student’s specialization by inspecting his or her journal entries, or are these entries too general to make that distinction?

**Data items as a function of time**

Students’ journals were evaluated two times: earlier during the semester (7 weeks after the beginning of semester), and before the end of the semester (after 15 weeks). It allowed exploring possible changes in the number or nature of journal entries, or in the level of quantitative reasoning sophistication in the course of the semester, and relating these changes to the course design; is there evidence that our course is helping the students open their mathematical eyes?

**Selected results**

The purpose of our study was to provide course developers with feedback on what situations or activities (that entail quantitative reasoning) students perceive as relevant or authentic. A thorough analysis of the data we collected is therefore mainly of local importance. As some results might be of interest to other researchers or curriculum developers, we will briefly discuss selected results in each of the above described domains.
Mathematical Substance

As we expected, there was clear dominance of very elementary mathematical concepts and unsophisticated quantitative reasoning processes. Items that dealt with sharing costs (monthly expenses at a dormitory, trip expenses) and various calculations involving grade point average (GPA) are examples of the most sophisticated quantitative reasoning that students reported in their journals.

Context

About 70% of all items referred to the context of dealing with finances. More than a half of these items (52%) were related to spending money (shopping, dining, finding a better deal) and less than 4% were related to savings (a plan to save for an engagement ring, making extra savings for car and insurance).

Another interesting observation relates to mathematical methods used in this category. Although finance-related items constitute a considerable portion of all items, all quantitative methods used there fall either into a category of elementary additive reasoning (balancing budgets) or multiplicative/proportional reasoning (sale tax, splitting utility costs). We did not see any use of typical concepts of financial mathematics such as interest, amortization or savings plan even though their application would be appropriate on several occasions.

Other significant contexts include commuting (18%; this includes calculations of time, mileage, car operating costs, etc.), school-related (11%) and work-related (6%) activities.

Context Specificity

As our competency course was designed to serve a general population of college students, we were interested to see whether students’ career orientations entail specific contexts or numeracy areas. Interestingly, the only specialization identifiable through journal entries was a Music Major. This specialization was clearly indicated by the use of specific quantitative measures (tempo, beats per minute). No other majors or specializations could have been identified through journal entries suggesting that most college students may benefit from a general-purpose numeracy course that comprises various contexts and aspects of numeracy and quantitative reasoning.

Items as a function of time

The above domains provide different perspectives to look at a still image of the world seen through the students’ mathematical eyes. With respect to our course design, we were also interested in seeing a moving image; if and how that still image changes in the course of the semester, and if any of the changes can be attributed to our course design. The data allowed us to identify differences at two points in time: after 7 and 15 weeks since the beginning of the semester.
Except for natural fluctuations, we did not observe any significant patterns in differences. The category with the largest relative increase in the number of entries (from 1 to 5) was a general curiosity category, and the increase can be explained by students’ exposure to similar problems in the class. Significant changes in other categories or in sophistication of quantitative reasoning were not detected.

**Summary**

The paper reported on a content analysis of journal entries of students who attended our pilot competency course in quantitative reasoning. We described domains we used to look at the data, and identified different categories that helped us structure the entries. Although the main purpose of the study was to inform our local curricular decisions with respect to our competency course, we presented several results that might be of interest to a broader community: (1) nearly all journal entries related to very elementary mathematical concepts and unsophisticated quantitative reasoning; (2) although quantitative reasoning about financial matters formed by far the greatest portion of all journal entries, all relied on elementary additive or multiplicative reasoning and none included typical mathematical concepts of financial mathematics (such as interest); (3) with a minor exception of music students, journal entries did not reflect students’ professional orientation, which indicates that their use of “everyday mathematics” can potentially be enhanced by a general-purpose numeracy course, and; (4) attributing changes in students’ numeracy habits (as evidenced by journal entries) to our competency course turned out to be problematic. We only saw a positive increase in the category of “general curiosity”; significant changes in other categories or in sophistication of quantitative reasoning have not been detected.
Appendix A

Journal Suggestions

You will be required to start a journal to keep track of mathematics you encountered and reflect on your mathematical experience. You will need a 3-ring binder in which you will be collecting entries (short narratives, newspaper clippings, photographs, etc.) related to your everyday use of mathematics and/or quantitative reasoning.

In many cases our use of mathematics or quantitative reasoning skills is subconscious and difficult to trace. Therefore try to make it a habit to reflect on quantitative or mathematical situations on a regular basis - every day or two.

Although there is no uniform template for the journal, it is suggested to keep it as a diary, especially at the beginning of the semester. These diary entries can be very brief, such as:

- 02/11 – checking change in the grocery store; estimating time of my arrival to Midland (by car)
- 02/12 – figuring out a better deal on AA rechargeable batteries on eBay – the same brand, one priced less but w/ shipping charges, another more pricey w/ free shipping
- 02/13 – estimating mileage before running out of gas etc…

The purpose of these short “daily” entries is to increase your sensitivity to mathematical situations around you. After collecting several of such entries, you will choose a few and provide a more detailed description and discussion – what strategies you used or tried to use to solve the problem, what kind of mathematics helped you and how exactly (diagrams, equations, ratios…) etc.

There is a considerable variety of mathematical or quantitative situations and you can encounter them in a store, as you talk to your friends or family or read a newspaper or book, when you cook, drive a car or work on a household or garden project or even maybe when you buy a lottery ticket... Try to make your journal entries varied to get a broad sample of quantitative or math-related life situations pertinent to your lives.

The instructor will collect and evaluate your journal 2x in the semester. Each evaluation is worth up to 5 points. It is expected that you will have at least 5 authentic entries with detailed discussion.

Other remarks:

- Authentic problems that you were not able to solve have special value – please highlight these items for a quick reference.
- If you add copies of bills or receipts, protect sensitive information.
- Entries related to college courses with mathematical or predominantly quantitative background should be excluded from the journal.
• If your entries contain mathematical symbols, sketches or diagrams, they do not have to be typed as long as they are legible and can be understood by others. Keep in mind that they will serve your group to initiate discussions and prepare for regular reports (See Course Information and Syllabus).
An exploration of the impact of functional skills: on young people and adults life chances and employability.

Ann McDonnell

The introduction of functional skills followed the Tomlinson report on 14–19 reform (2004) and the government’s response, the 14–19 Education and Skills White Paper (February 2005). Tomlinson stated that it was not difficult for young people to achieve grade C in the General Certificate in Secondary Education (GCSE) in English and Mathematics without achieving an acceptable level of Literacy or Numeracy. The government initiated a ‘sharper focus on the basics’ and promised to ensure that learners have a sound grounding in ‘functional skills’. (2005:4) Described by QCDA as:

…core elements of English, mathematics and ICT that provide individuals with the skills and abilities they need to operate confidently, effectively and independently in life, their communities and work (2011).

Whilst it is clear that Functional skills will replace Key skills by 2012 it is far less clear about the future of Adult Literacy And Numeracy Tests (ALAN tests) and their alignment to Functional skills principles content and context.

The link between unemployment/underemployment and insecure employment and basic or functional skills has been investigated by many researchers, notably Parsons and Bynner, who, using the statistical evidence from the 1970 British Cohort Survey, established a negative link between levels of basic skills and employment opportunities and its continued impact on adults throughout their lives (2005). This is also a factor for Young people who are Not in Education, Employment or Training and who account for 10% of young people between 16 and 19 in England (DFE Statistical bulletin Q3 2010). The Youth Cohort Study and the longitudinal study of young people (aged 16) in England:2007 surveys indicate that 81% of this number at 16 do not have a full level 2 qualification-5 General Certificate of Secondary Education exams (GCSE) at grade A* to C or an equivalent.(DFE Statistical bulletin YCS& LSYPE 2007)

This paper:

• Interrogates the Data available on current NEETs and their educational achievement

• Explores if the introduction of functional skills will have a positive effect on educational achievement at 16 and beyond

• Will discuss the introduction of functional skills and whether it will help to reduce the number of NEETs

• Asks if Adults may be disadvantaged if they continue to take ALAN tests in preference to Functional skills tests

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The Collaborative Group of GEMP as context-building knowledge and the relationship with the knowledge

Mesquita, M.; Franco, S.; Matias, M; Matos, I.; Madeira, M.; Marques, J.

This workshop aims to work with concepts developed by Paulo Freire, who is considered one of the most influential theorists in the field of adult education, by developing the foundations of libertarian education and, among others, the concepts of conscientization and situacionalidade. Our proposal is to develop an open discussion of these two concepts embedded in an Ethnomathematics praxis (as a process of education), arguing, under the sociology of mathematics, the love as a special case of cooperation.

The proposed discussion will be supported by looking at mathematics as power and social structure, cultural and value, by looking at education as a social institution controlled by the political and social processes, by looking at lifelong learning from a phenomenological point of view, and by looking at the adult's knowledge as a libertarian tool for the exercise of praxis (the dialectical relationship between action and reflection).

In this workshop, we propose to share the learning landscape of our meetings, a group of adults learning mathematics in Lisbon, and bring our urban and native ethnomathematics experiences.
In the 1990's, it was found that the numeracy skills set of the UK population had fallen below effective working practice which led to research being carried out by Sir Claus Moser. In 1999, the Moser Report outlined recommendations for the introduction of basic skills numeracy to be delivered to all 16-19 year olds and adults as preparation for working life. For 16-19 year olds, this was delivered in the form of 'Key Skills' until now. In 2009, it was announced that Key Skills was outdated and would be phased out with the introduction of Functional Skills in Pilot format until 2011, with Functional Skills being fully operational in 2011-2012. Functional Skills are designed to prepare learners for working life and involve a much more problem solving approach with more multi-step mathematical operations being needed to answer questions. This workshop will evaluate whether this redesign has supported learners in achievement and improved attitudes towards number. This will be demonstrated through initial assessment results and student questionnaires. There will also be an opportunity to answer some Functional Skills mock questions.
Education Projects in Honduras & Nicaragua

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During travels to Central America in 2010, I visited educational projects in Honduras and Nicaragua. Both countries have things in common such as poverty in urban and rural communities and poor educational infrastructure, but they differ politically. Nicaragua has had universal education as a high priority since its revolution in 1979, and in 2010, celebrated 30 years of its literacy campaign, recognised by UNESCO. In Honduras, state provided education is not effectively universally available. The educational communities I visited all displayed a hunger and enthusiasm for learning. Some projects are rooted in the community. In the Honduras SAT project, curricula and project work had been developed which would be directly relevant to the community itself. Other projects are concerned to give adults the opportunity to achieve their school education, denied in previous political climates. This paper describes some of the projects I visited, illustrated with some photos I took on the trip.

Key words: Central America, Community Education, SAT, 'Yo Si Puedo'

Note: A soft copy of the images contained in this paper is available from the ALM website at http://www.it-tallaght.ie/events/alm18conference/index.html

Background

In August 2010 I visited two countries in Central America.

Honduras

Military rule, corruption, a huge wealth gap, crime and natural disasters have rendered Honduras one of the least developed and least secure countries in Central America (BBC 2011).

Honduras had recently had a change of government as a result of a military coup. The main economic activity is banana and coffee growing, and more recently, as part of free trade agreements with the US, textiles. Honduras was devastated by Hurricane Mitch in 1998 which destroyed vast amounts of crops. Unemployment is high, and there is much violence. The country has high levels of poverty, state provision of education, especially in rural areas, is patchy and of poor quality, often interrupted by striking teachers who are currently fighting for six months back pay, and to oppose school autonomy (Kirkpatrick 2011, Merril 1995).

As the guest of Sarah Richards, I visited two rural communities in northern Honduras implementing SAT educational projects run by the BAYAN organisation, in an attempt to
counteract the poor state provision. Sarah was writing an evaluation of the project (Richards, S 2010).

I also visited Mayan ruins at Copan de Ruinas, seeing at first hand some of the best preserved Mayan carvings in existence. Each monument had the dates of the event it commemorated carved in several date forms. I thought they might be a good maths teaching resource, and on my return, I used my photos and a work sheet from BEAM (downloaded from www.beam.co.uk), in some training.

Nicaragua

In Nicaragua, I was on an educational study tour organised by the Nicaragua Solidarity Campaign (NSC) during which we visited many different projects from pre-school to adult education (see muswell.eu5.org).

Nicaragua had a successful revolution some 30 years ago in 1979, after which the revolutionary party, the FSLN (aka Sandinista), were elected to government and began many social reforms including in education. There followed ten years of a concerted campaign against the new socialist regime, economic and physical war was waged with much violence by the Contra movement, funded and supported by the USA. To put an end to war, a right wing neo-liberal government was elected, which reversed reforms such as free state education and signed up to agreements including the Central American Free Trade Agreement & the IMF. Implementing a policy of school autonomy, 87% of schools were privatised excluding about a million children. In 2006 the FSLN were again voted into power. Against this background of huge losses during the war and as a result of Hurricane Mitch (1998) and a reversal of socialist reforms during the neo-liberal regime, the current government are trying to redress inequalities in education, health, welfare, employment and other areas of life. When most schools’ fees were abolished again, the increase in enrolments was noticeable. There is ongoing work to ensure all children attend school, as an example, in some rural areas, parents are reluctant to send their children to school, which they have managed without themselves, and to improve retention at all levels. (NSCAG 2010, FEDH web site).

Maths education is particularly problematic, in 2009 less than 1% of people taking the tests for university entrance passed the maths papers. This might be due to an over emphasis in
vocational education over technical education. In addition, the infrastructure is badly in need of improvement and repair, and people on the north Atlantic coast have particular problems in accessing education due to poor roads, weather conditions and lack of provision in their own language of English.

Nicaragua is the second poorest country in Latin America; half the population live on less than $1 a day. It has a widespread state supported education and recently the government has been able to pay teachers bonuses to improve their poor pay through participation in ALBA (CAR 2010 b) the Latin American fair trade organisation. Despite a commitment of Latin American countries to invest 7% of GDP in education, Nicaragua spends only 3.7%, partly because of restrictions placed on them by the IMF, on the other hand it has a fantastic record of community and voluntary participation in providing education.

![Nicaraguan Street Seller, Leon](image)

Figure 2. Nicaraguan Street Seller, Leon

Before the revolution only about 20% of the population had access to secondary education, 75% of the population was illiterate. Since 1979, as well as trying to provide free education in general and strengthen teacher training, there has been a concerted effort to wipe out illiteracy. The current government inherited a literacy rate of 40% but, by the 30th anniversary of the start of the literacy campaign in 2010, over 95% literacy had been achieved, a feat recognised by UNESCO. The Government's next priority is to offer all adults the opportunity to achieve education to grade 6 primary level.

The FEDH (see www.fedh-ipn.org) is an association of educationists and members of civil society who campaign for education rights. They would like to push the government to achieve greater reforms in education, as an example to extend free education to baccalaureate level to all age groups. Education is considered a basic human right by most Nicaraguans.

**Community Secondary Education in Rural Honduras - The BAYAN SAT Project**

Although state education is, in theory, available to all communities in Honduras, in practice, many rural communities do not have local schools. Where schools do exist, provision is still patchy: teachers are too frequently involved in industrial action to try to improve their poor wages; the standard of teaching and teacher training are poor (Richards 2010).
The BAYAN organisation (see www.bayan-hn.org), in a collaboration which includes the Ministry of Education, have implemented the SAT system of secondary education in several rural areas in Honduras, and in the space of 10 years has expanded student numbers from several hundred to about 8,000. The organisation also provides continuing and initial training for teachers involved in the project and a system of coordinators, including maths specialists, who support individual communities' schools.

BAYAN is part of the Baha’i Community in Honduras, whose vision is ‘committed with the individual and collective well-being, excellence, innovation and transparency; with a solid ethical, moral and spiritual background, coherent in its actions; with a learning attitude and practice’ (BAYAN 2011). It is responsible for bringing ethical and social aspects to the SAT project.

SAT is a tutorial education system, developed in Columbia, which involves the whole community. Learning is based around projects which are of benefit to the community and classroom lessons. Teachers become members of the community, often living amongst their students, with whom they work in a peer relationship as co-workers. It promotes maths teaching which includes class discussion, use of learners’ errors, investigative questioning and discussion, a peer relationship between teacher (seen as facilitators) and student. Teachers stay with the learners for the full 6 years, so get to know them and their families, enabling flexibility to accommodate individual circumstances. The class should move forward together, no member should be left behind. The system is recognised by the ministry of education and its baccalaureate awards are accepted for entrance to further and higher education, teachers are paid by the state, other funding comes from abroad, in particular the USA. All members of the community are welcome on the courses, in practice, most students are young people of secondary school age.

The cooperative nature of learning, and learning through implementation of real projects, useful in the local community, have been shown to be very effective, especially in teaching and learning mathematics.

Figure 3. Maths class solving bread baking problem & casava growing project

However, the particular philosophies and assumptions about teaching that the SAT project calls upon are very new to Honduran teachers and trainers, and such a rapid expansion in provision has proved problematic for providing teachers who will be able to carry out the project in all its aspects. In addition, many teachers do not have sufficient mathematics knowledge and skills at the start of their teaching for the curriculum, however, they learn effectively together with their students, enhancing their own skills and clearly implementing
the co-worker nature of the relationship between teacher and student. The textbooks are carefully written to encourage a questioning and discovery approach to learning, bringing in social issues and values. Sarah Richards has written an evaluative report of maths teaching, learning and teacher training experience in the project (Richards 2010). Her conclusions are very supportive of the SAT maths pedagogy for teaching and training and find that the level of mathematics is higher in the SAT programme than in state programmes, partly due to the emphasis on development of mathematical concepts. However the pedagogy is not always implemented, largely because teachers and trainers themselves are coming from a low quality education system, exacerbated by the very rapid expansion in the number of teachers joining the project. She notes that maths is the subject most often taken in re-sit exams. Nonetheless, the maths component has achieved many notable successes.

Nicaragua

Yo, Si Puedo

Meaning 'Yes I can', this is a country-wide literacy campaign, giving adults the opportunity to learn to read and write at the first level. Course materials and pedagogy were developed in Cuba and adapted for use in Nicaragua (Torres 2009). Courses last for 13 weeks and are held in the community – in living rooms, community buildings, primary schools etc. Teachers are local people supported by a network of technicians, all volunteers. Cuban technicians also do continuous feedback with teachers to monitor the programmes. The teaching system, developed in Cuba, includes allocating letters to numbers, using discussion, usually of social issues, to find words which contain particular syllables, to broaden the vocabulary of words that can be read and written. Printed materials and video tape sessions are developed in collaboration with Cuban experts and then distributed to local centres together with video equipment. Graduation is by writing a short letter.

![Figure 4. Yo Si Puedo graduation letter & printed materials](image)

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Even these short courses are enabling members of the community to change their lives through being able to read and write, improve work prospects and realise their rights. (CAR 2010a).

There are now two more courses, also developed with Cuba and designed to be taught locally using video and texts: a 10 week interim course that enables participants to consolidate their new skills (Ya Puedo Leer); a 3 year course, Yo Sí Puedo Sequir, that will give every adult the opportunity to reach grade 6 of primary education; the government's current objective (CAR 2011).

Nicaragua is now able to claim nearly 96% literacy rates, amongst those adults fluent in Spanish, and who have no learning difficulties; an achievement recognised by UNESCO. However, there are sections of the population, such as the English speaking communities on the north coast, who are not using this scheme. They are lobbying to have the scheme translated and modified so that they can learn to read and write English.

2010 celebrated 30 years of the literacy campaign, commemorating the original *brigadistas* who went out from the towns to teach in the countryside, as well as recognising both the successes and the continuing work of the literacy campaign (CAR 2010a).
Vocational learning in Estelli

In another model for adult education, the Superemos foundation started, in 1999, to provide education and training for low-income families in northern Nicaragua. It is a non-profit but private community development organisation and aims for the local community to be able to take up its basic human rights to education and health care. It uses abandoned buildings in Esteli, a town to the north west of Managua, and the nearest town to the Miraflor National Park. Funded by a number of organisations in Europe and the USA, it is staffed by volunteers and paid workers.

Superemos is also home to the Christine Kid cooperative (CKC) which, in particular, provides education for women, offering them a better alternative than seeking work as employees in the low-wage economy. Superemos provides 80 to 100 scholarships to women on low incomes studying at the Estelli women's school, a Sunday school run by CKC, under the supervision of the Nicaraguan Ministry of Education. Here there is a rare opportunity for women to complete the national secondary baccalaureate qualification.

The cooperative also offers services and workshops such as counselling and personal development; computer skills; social programmes in the local penitentiary and surrounding neighbourhoods (barrios); teacher training and other vocational opportunities.

The Christine Kid Cooperative implements most of the skills training work promoted by Superemos in Estelli and nearby communities. Workshops include:

- metal work developing community projects such as a 'bicycle' pump to draw water from wells in the rural communities, building a children's playground, installing ecological lavatories as well as ornamental metal work
- a bakery providing produce
- a ceramics training program for women and young people, employing half a dozen people, they take on commissions from all over the country
- a garden and nursery producing fruit and vegetables for local sale
- a sewing workshop
- a pharmacy and health education for local people and in the surrounding villages.

These workshops not only provide vocational training but also a valuable income for the Speremos Estelli community centre (www.superemos.com).

Learning English in Miraflor National Park

We drove through the pouring rain of a dark rainy season evening, over a heavily rutted unmade road, a bone shaking experience, to reach Fuente de Vida coffee finca (farm), turned tourist hostel higher up in the stunningly beautiful Miraflor National Park. Here young people are taking part in the NEST project to learn English, so that they can become tour guides in the burgeoning tourist industry. Some of the women we met are members of a women’s cooperative who work in agriculture, education and tourism (and campaign for women to be able to play baseball!).

They work daily with the NEST project, which has been running for about 6 years, where people from England are helping teach English. There is a small computer laboratory powered by solar energy (but no internet access) and CD players are available (www.thenesttrust.org.uk). The learners themselves also become teachers of English in different local contexts.

Members of the group explained that English is difficult to learn for native Spanish speakers, and this is exacerbated by the wide variety of accents used by speakers of English. Of the tourists who visit the area some 20% are English speakers.
Although they have access to books written in English (fiction and factual books), they would like audio books to hear English, as there is no access to English radio or the Internet. The nearest Internet access is several hour’s drive away in Estelli, mostly a trip undertaken once or twice a week, as this is the place to pick up and post mail, do banking, use the internet and dance (at the disco).

**Union Workers' Sunday School**

The FNT (Frente National de los Trabajadores) represents 9 trade union confederations including education and health workers and CTCP, the self employed and informal sector workers’ union whose members work in low paid employment, such as street-sellers. Through IFOCATT, its Institute for training, the FNT provides free education to its members in various areas such as trade union issues, but in particular has a Sunday school, set up in 2006, offering formal education for workers from the informal sector, many of whom were not able to complete their education for economic reasons. The school offers classes covering the whole of the school curriculum from grade 1 basic literacy and numeracy through to grade 11 baccalaureate, the national secondary qualification after which people can study in higher education (NSCAG 2010). CTCP, which is recognised by the International Labor Organisation, is breaking new ground in trade union activity; most of its members are considered unemployed, and most are women.

The classes are free to participants and teachers are paid by a Danish organisation. In other similar projects, students must pay, at least for books and other resources, but here even these are provided without charge as the very low level of income of CTCP members is recognised.

Figure 9. Informal sector workers' Sunday school classes are popular and full

Students attend every Sunday for half a day and are expected to do a lot of study at home; not easy to fit round work and family, students must be dedicated and are highly motivated by the opportunity to get the education so long denied them. This was evident in the packed classes, the amount of self study, the enthusiasm and, as an example, the class where the teacher was away and one of the students was preparing to facilitate the class instead. This also illustrates the concepts of the parity between teachers and students, all members of the same community uniting in education. About 60 primary students and nearly 340 secondary students were enrolled.

Many of the workers who are starting with the primary years classes cannot read or write and have poor numeracy skills. This gives them an opportunity to develop these skills and prepare
for secondary classes. Primary grades are taught in an accelerated program that covers two primary years for each year of study. Each secondary grade is covered in one year of study. The Sunday school takes place at the Instituto Manuel Olivas, Managua, in a building used as a regular school during the week, and is staffed by school teachers. Although the government, which supports the Sunday school, agrees to provide printed resources for the primary classes, these are slow in coming so teachers are using their own school resources. Teachers are aware of the different needs of adult learners, and can feel frustrated by this: the need for support, to be patient, deal with learning differences etc. Teachers are trained. Primary level teachers have annual professional development summer schools, secondary level teachers meet monthly for evaluation and planning workshops.

Figure 10. Primary level literacy class at FNT Sunday school

The building itself, considered as an historic building, has an interesting history. It was built by Samosa, before the revolution, for the children of collaborators, the rich and elite. Since then, it had fallen into disrepair and had no resources. In the 1980s it had been reclaimed as a school for the people, but is still in a very poor condition, and there are few resources. Furniture is obtained when it can be, for example the tables and chairs are throw-outs from the university. The government has approved a programme to renovate the building, which has started, being funded by the Japanese Government. Resources, such as paper, are often donated by support groups, such as the Nicaragua Solidarity Campaign, based in London (see www.nicaraguasc.org.uk).

We talked to three early grade students who were very enthusiastic about their studies, the dedication and support of the teachers and class representatives. They described how learning to read and write and practice maths had started to transform their working lives. One of the women ran her own hairdressing business and previously had had to call on the help of her daughter and husband to help with the administration. The second student was a man who worked in the bus station helping passengers. He was a freelance giver of information who gained an income from tips from the bus companies when he sent passengers their way, being able to read time tables and other information was clearly helpful, but his ambition was to be a radio broadcaster. The third student did voluntary work through her church, but was finding that the ability to read also greatly enhanced her abilities to work effectively.

Involving the Whole family

At the school in the Las Cortesas rural community near Masaya, children are taking part in the Association for Integral Community Development (ADIC) project which aims to involve the whole family in projects, as many parents have missed out on their own education. The
work is supported financially in the U.K. by the Leicester Masay Link Group (leicestermasayalink.org.uk) and Cool Earth in Cornwall. One class described their current project which is about developing tree nurseries. Each student had chosen a seedling which they would nurture and take home to plant in the family home. The project looks at the requirements of running a tree nursery, fertilisation and conditions which are conducive to successfully growing trees.

![Figure 11. Mixed ages in classes enables students to catch-up with their education](image)

The project, called 'Trees for Life', is designed to build an understanding of the importance of the role of trees for the community and help to change attitudes towards them. In the last two years, classes of around thirty primary school children in the small village of El Pochote have been taking part, also learning of role trees play climate change, and how they can take action themselves. The message is passed on to parents at an annual tree-planting day (www.coolearth.org).

**Conclusion**

Honduras and Nicaragua are two of the poorest countries in Latin America. As part of the same region, they share many cultural legacies, but today they differ significantly in their political environment. Nicaragua, 30 years after a successful people's revolution, is now a representative democracy, Honduras has had long periods of military rule, with a newly elected Nationalist Party President in 2010. Since the revolution, before which only about a fifth of the population had access to education, and these the most well off and privileged, through difficult times such as the Contra wars and hurricane Mitch, universal education has been a priority, providing facilities for all ages to achieve literacy and secondary level qualifications. Thanks to the literacy campaign started by the 'brigadistas', brigades of young educated people travelling out from the cities into the countryside, the country has achieved just over 95% literacy; an achievement recognised by UNESCO. Adult classes, provided through union funding and other community projects, are popular and enthusiastically attended (CAR 2010a).
In Honduras, education is provided by the state, but the poor employment conditions mean that teachers are often on strike, and through lack of resources, many communities, especially rural communities, do not have access to primary and secondary education. The BAYAN SAT project is a new and exciting approach to secondary education. Based in rural communities it has developed a curriculum which involves whole communities and centres around practical projects that communities will be able to take forward, such as chicken farming, baking and agriculture. Although there are many problems, such as a lack of trained teachers, the project has achieved recognition by the Government, who accept the curriculum and assessments as part of the national baccalaureate qualifications, and in fact, recognise that the standard of the SAT education is often higher than that provided by the state.

![Image](image.png)

**Figure 12. Selling ribbon by the yard, Masaya, Nicargua.**

I gained a sense of communities and the Nicaraguan people developing educational opportunities through a number of different initiatives and approaches, and keen to redress past education inequalities, recognising that education forms one path out of unemployment and poverty and towards the power and understanding that knowledge and skills, such as literacy and numeracy, can bring. But these projects depend on organisations such as the FEDH and many overseas organisations donating resources, expertise and money, one of which is the Nicaragua Solidarity Campaign in the UK, always keen for more people to join them in supporting the Nicaraguan people.

The projects we learned about in Nicaragua were concerned to give people of all ages access to the government approved curricula and assessments, where as in Honduras the SAT project has its own curriculum and assessments which are recognised by the state, (although I saw nothing to suggest a SAT-like project could not be implemented in Nicaragua).

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References

news.bbc.co.uk/1/hi/world/americas/country_profiles/1225416.stm. Accessed 20.6.11
CAR (2010 a) Nicaragua turns the page on illiteracy once again Central America Report pp 10-11
Winter 2010
CAR (2010 b) What Latin America's alternative alliance means for Nicaragua Central America
Report p 3 Winter 2010
www.nscag.org/solidarity/ accessed 20.6.11
Kirkpatrick, Olivia (2011). Political party or social movement ? Central America Report Summer
2011
Richards, Sarah (2010) Improving Maths Education in Rural Honduras, Ford Foundation & Bayan
September 2010. Available on line at
accessed 20/6/11
Torres, M (2009) From literacy to lifelong learning: Trends, issues and challenges in youth and adult
education in Latin America and the Caribbean Revised report presented at the Regional
Conference on Literacy and Regional Preparatory Conference of the Sixth International
Conference on Adult Education (CONFINTEA VI),
“From Literacy to Lifelong Learning: Towards the Challenges of the XXI Century”,UNESCO Mexico
2008

Links

ALBA conference, London Metropolitan University, January 2011.
www.londonmet.ac.uk/depts/hal/research/clarc/events/alba.cfm
BAYAN www.bayan-hn.org/about.php
BEAM Mayan numeral worksheet www.beam.co.uk/uploads/mompdf/Maya numeral system.pdf
FEDH www.fedh-ipn.org/
Leicester Masaya Link leicestermasayalink.org.uk
Nicaraguan Literacy Campaign www.nicaraguasc.org.uk/campaigns/Education/educ_home.htm
Nicaragua Solidarity Campaign www.nicaraguasc.org.uk
NEST trust www.thenesttrust.org.uk
NSC Education Study tour 2010 muswell.eu5.org
SAT project www.bayan-hn.org/programs-sat.php
Superemos, Christine Kid Cooperative, Estelli community centre www.superemos.com
The Positive Impact of Storytelling in Mathematics for Adults

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I will discuss how the telling of short stories in mathematics can play a major part in enhancing the understanding, awareness and appreciation of mathematics among adults in the general public, including those with very little mathematical background. Such stories may include practical power, beauty, famous characters, Irish connections, motivation, drama, humour and much more. I have lots of experience telling stories in mathematics for the general public and I have conducted a wide variety of public events including Maths Week events, public talks, radio and TV shows and the annual Hamilton Walk and I have contributed to many newspaper articles. I will also discuss the positive feedback and some surprising consequences of the short story approach.

Key words: mathematics; stories; impact; adults.

A story in mathematics for adults may include some or all of the following eighteen topics: (a) history of mathematics, (b) humanity and famous characters, (c) beauty, (d) practical power/applications, (e) motivation, (f) Irish connections (even local ones), (g) drama, (h) word origins, (i) humour, (j) outdoor activities, (k) cultural connections, (l) tricks/magic, (m) puzzles, (n) freedom, (o) creativity/imagination, (p) research/unsolved problems, (q) deductive reasoning, (r) abstraction.

The above eighteen items came after reflection of many years of storytelling and are not part of a 'ticking the boxes' process in advance. Stories can be told with a natural flow without the need for checking which of the above items are included. From many years of experience, I find that short stories in mathematics can play a significant role in enhancing the understanding, awareness and appreciation of mathematics among adults of all ages. For every area in mathematics a relevant short story can be told.

A short story in mathematics can be as short as 30 seconds (and still have a major impact) or much longer if you like. Evidence for the beneficial impact of storytelling in mathematics comes from the positive feedback from:

- Conducting a wide variety of public events including the annual Hamilton walk, public talks, radio shows, TV shows, newspaper articles etc. Many adults, including those with very little mathematical background, have told me how my stories in mathematics have played a major role in enhancing their understanding, appreciation and awareness of mathematics.
• Conducting Maths Week events for the general public. See www.mathsweek.ie for the events taking place in Maths Week Ireland.

• Many years of third level teaching, including a course on the history of mathematics where there is an abundance of great stories. One mature student had a nice quote when she said that my stories in mathematics acted like a hook in helping her understand the mathematics.

• Giving talks for second level teachers. Also, I co-authored a book (O'Cairbre, McKeon, & Watson 2006) for teachers of Transition Year mathematics. Every second level school in Ireland received two copies of this book from the Department of Education and Science in 2006.

• Giving second level school talks to students.

• A module of twenty lesson plans (forty minutes each), related to the history of mathematics and stories, for Transition Year students that I co-designed with Stacy Carter, who is a second level teacher (Carter & O’Cairbre, 2011). The module was implemented by Stacy in a class of 21 second level students over a three month period in Spring 2010 and we then analysed the anonymous student feedback. The main results from our research were that all students changed their perception of mathematics for the better after the first lesson plan and also there was a sustained change of perception of mathematics for the better for all students after completion of the module.

Some Examples

I will now illustrate the power of storytelling in mathematics by discussing five examples below. Also, remember that every topic in mathematics has a relevant story that can be told.

Example 1: Origin of the equality symbol ‘=’

This example discusses the origin of the equality symbol =, which is probably the most common symbol in mathematics. This story is accessible to the general public.

The story goes as follows: In 1557 the mathematician, Robert Recorde, justified his adoption of a pair of parallel line segments for the symbol of equality “Bicause noe 2 thynges can be moare equalle”. (*)

One can still see the page where the equality symbol = first appeared with the above justification in 1557 (Eves 2000). Recorde was Welsh but he was living in Ireland at the time.
This 30 second story above has completely changed adults' perception of mathematics for the better. In one such case, a former student of my history of mathematics course here in NUI, Maynooth, was teaching in second level. The teacher told the students the above story about the origin of the equality symbol. The students found the story very interesting and enlightening. One particular student took it a step further. She told her parents the story over dinner that evening. Her mother wouldn't believe the story and so she raised it with the school Principal at the parent-teacher meeting the next day. The Principal didn't believe the story either and raised the matter with the teacher. The teacher showed the Principal the page from the 1557 book with the above justification (*) and so the Principal (and later the mother) finally believed the story.

Why did the mother and the Principal not believe the story? The main reason was that they thought mathematical symbols were just there and had no human connection. Then they realised that the equality symbol (and also all other mathematical symbols) were actually created by human beings and had their own individual stories from history. This completely changed the adults' attitude to mathematics for the better and the main reason for the change was that the above story humanised mathematics for them. This story also has a similar beneficial impact on many other adults that have heard the story.

This story also has an Irish connection because Recorde was living in Ireland at the time. Irish adults typically find Irish connections in stories about mathematics very appealing.

A variety of interesting questions and topics in mathematics can follow from the above seemingly innocent 30 second story. For example, an adult once asked me what was used before the equality symbol =. It was a revelation to the adult that the equality symbol was not always just there, and now they wanted to know how mathematics could have been done before the equality symbol. I told the person that often people used words like aequatur (meaning equal in Latin) instead of the equality symbol. The adult was now, for the first time, witnessing a piece of the evolution of symbolic mathematical notation (and also realising that mathematical notation is arbitrary in a certain sense), and it had a profound effect on that person. This 30 second story has at least seven of the eighteen topics mentioned in the introduction.

Example 2: The story of the sheet of music versus the music.

Mathematics essentially consists of an abundance of ideas. Number, line and rectangle are just some of the many ideas in mathematics. I find from experience in teaching/promoting mathematics with adults that it's a big shock for many people when they hear that number is an idea that cannot be sensed with the five physical senses. Numbers are indispensable in today's society and arise almost everywhere from football scores to phone numbers to the time of day.

The reason number appears practically everywhere is because number is an idea and not something physical. Many people think they can see the number two when it's written on the blackboard but this is not the case. The symbol for two on the blackboard is merely a symbol to represent the idea we call two. The number two cannot be physically sensed because it's an
idea. So, what is this idea we call two? If one looks at the history of number one sees that the powerful idea of number did not come about overnight. As with most powerful mathematical ideas its creation involved much imagination and it took a long time for the idea to evolve into something close to its current state around 2500BC. Here is one way to think of what the number two is:

“Think of all pairs of objects that exist; they all have something in common and this common thing is the idea we call two.”

One can think of any positive whole number in a similar way. One can only really see mathematical ideas like number with the eyes of the mind because that is how one sees ideas. Think of a sheet of music which is important and useful but it's nowhere near as interesting, powerful or beautiful as the music it represents. Similarly, mathematical notation and symbols on a blackboard are just like the sheet of music; they are important and useful but they are nowhere near as interesting, powerful or beautiful as the actual mathematics (ideas) they represent.

Many adults initially claim that they don't see mathematics in the physical world and the reason for this is because they are looking with the wrong eyes. These people are not looking with the eyes of their mind. For example, if you look at an aeroplane with your physical eyes you don't really see mathematics, but if you look with the eyes of your mind you may see many mathematical ideas that are fundamental for the design and operation of the aeroplane.

I find from experience that this 'sheet of music versus the music' metaphor has the power to completely change in a positive way what mathematics means to many adults.

**Example 3: Much ado about nothing - the story of zero**

One of the best book titles I have seen is Zero: The Biography of a Dangerous Idea (Seife, 2000). It gives an interesting account of the history of zero and has a very dramatic beginning with zero hitting a warship like a torpedo! I will return to this drama on the high seas later. Depending on how much time is available, one can give a short version or a longer version of the story below.

The first people to use a symbol for what we now call zero were the Ancient Babylonians. Around 300 BC zero was born as a position indicator in the sense that the Babylonians created it as a way to indicate a missing power of 60 in their number system (just like the way it's now used to distinguish 4076 from 476 in our current number system, by indicating a missing power of 10). The Babylonian symbol for zero consisted of two small slanted wedges and strangely was only used within a number and never at the end of a number. Before 300 BC a space had been left to indicate a missing a power of 60 but this was often ambiguous and didn't work well. The birth of a symbol for zero around 300 BC is a major event in the history of mathematics.
Now, the story doesn't end there. In fact the story of zero is just beginning. It's important to note that zero was not regarded as a number in its own right by the Babylonians, in the sense that zero did not interact with the other numbers via addition, multiplication etc. It was as if zero was a comma and the other numbers were letters. Zero was purely a position indicator and was on a much lower level than the other numbers. It's also interesting to note that the Egyptian culture at the time had no need for a zero in their hieroglyphic number system because their number system was not positional (Eves, 1990). Similarly, many other civilisations had no need for a zero in their number systems. Many different number systems have been used throughout history. One could say that number systems are like hairstyles - they go in and out of fashion.

The next major event in the story of zero happened around 500 AD in India. Now, if somebody said to you “I wish to make a comma into a letter”, you would probably think they were crazy. However, something very similar occurred in 500 AD in India, because at that time the people of southern India were the first to try and make zero into a number in its own right. Remember up until then zero was like a comma and the other numbers were like letters. It's fascinating to read the attempts of Brahmagupta and others to make zero into a number in its own right because it certainly was neither obvious nor easy (Kaplan, 2000). For example, they had great difficulty trying to make sense of division by zero, and we now know why, because it is impossible. Remember, nobody had ever even considered trying to make zero interact with the other numbers before. This Indian attempt was something that seemed completely bizarre and yet they succeeded. Our mathematics would be completely different today, and nowhere near as powerful, if the Indians had not tried to make zero into a number in its own right.

Our word zero is derived from the Arabic word cifr, which means empty, and cifr is a translation of the Hindu word sunya. The Arabs borrowed the idea of zero from the Indians and later introduced it into Europe.

I will now finish this story with a bang by returning to the drama on the high seas. In 1997, a billion dollar warship was dead in the water because of zero. Why? Well, it was because the new computer software tried to divide by zero and immediately the 80,000 horsepower became useless. It took two days for the engineers to get rid of the zero. They had protected the warship from all sorts of modern weapons but nobody thought of protecting it from zero.

The above story of zero can make a big impression on many adults who never realised that zero had such a long history.

**Example 4: A big sum for a little boy**

The beauty in mathematics typically lies in the beauty of ideas because, as already discussed, mathematics comprises an abundance of ideas. I believe that beauty is the most important feature of mathematics (O'Cairbre, 2009a).

From my experience in teaching/promoting mathematics with adults, I find that beauty is probably the most surprising feature for them. They initially react to my discussion about
beauty with shock, but soon they are actually accepting that beauty is a feature of mathematics. This completely changes their perception of mathematics for the better.

There are many examples of beauty in mathematics (O'Cairbre, 2009a). The following story is one of my favourites and makes a big impression on adults, even those with very little mathematical background.

A German boy, Karl Friedrich Gauss, was in his first arithmetic class in the late 18th century and the teacher had to leave the class for about fifteen minutes. The teacher asked the pupils to add up all the numbers from 1 to 100 thinking that would keep them busy while he was gone. Before the teacher left the room, Gauss put up his hand. Gauss had the answer and his solution exhibits both beauty and practical power. Gauss observed that:

\[
\begin{align*}
1+100 &= 101 \\
2+99 &= 101 \\
3+98 &= 101 \\
&\ldots \\
50+51 &= 101
\end{align*}
\]

So, \(1+2+3\ldots+100 = 50 \times 101 = 5050\)

Notice how Gauss' solution exploits the symmetry in the problem and flows very smoothly. Compare Gauss' approach to the brute force direct method of \(1+2+3\ldots\) which is very tedious and cumbersome and would take a long time. Both approaches will give the same answer but Gauss' solution has beauty and the other is tedious. Gauss' idea is also much more powerful than the \(1+2+3\ldots\) method because his idea can be generalised to solve more complicated problems, whereas you cannot really do much more with the \(1+2+3\ldots\) method. The power of the beauty in mathematics happens very often.

For those adults who are initially shocked by the notion of beauty in mathematics, this example from Gauss typically changes their attitude to mathematics, very quickly, for the better. They then agree that beauty can be a feature of mathematics and this was something that was beyond their wildest dreams beforehand.

**Example 5: Hamilton and the annual Walk**

Among all my ways of promoting mathematics among adults and the general public, this one has probably had the biggest positive impact on changing adults' perception of mathematics for the better. The story of Hamilton and the annual Walk contains all eighteen topics mentioned in the introduction of this paper. I will not give the full story of Hamilton and the Walk here on account of space limitations, but I will give some references below where one can get the full story.
William Rowan Hamilton (1805-1865) is Ireland's greatest mathematician and one of the world's most outstanding mathematicians ever. He was born in Dominick St. in Dublin and then spent his early youth on the banks of the Boyne in Trim, Co. Meath. Like most great mathematicians, his motivation for doing mathematics was the quest for beauty. He was successful in finding much beauty in mathematics. Also, his mathematics has turned out to be incredibly powerful when applied to science, engineering, computer games and animation, special effects in movies, space navigation and many other areas. See O'Cairbre, 2000 and O'Donnell, 1983 for more on Hamilton's life and works.

I organise the annual Hamilton Walk on October 16 which commemorates his famous creation of a strange new system of four dimensional numbers called Quaternions, in a flash of inspiration, on the banks of the Royal Canal in Broombridge, Cabra, in Dublin on October 16 in 1843. The Walk retraces Hamilton's steps from Dunsink Observatory to Broombridge. There is a plaque at Broombridge with the Quaternion formulas which Hamilton scratched on the bridge in a 19th century act of graffiti!

The Walk celebrated its 20th anniversary in 2010. See O'Cairbre, 2010 for a history of the Walk. The Walk typically attracts around 200 people from diverse backgrounds including staff and students from third level, staff and students from second level and many adults from the general public. Anybody interested in joining us on the Walk should contact me.

Quaternions freed algebra from the shackles of arithmetic and consequently Hamilton has been called the Liberator of Algebra. The mathematical community was stunned by Hamilton's audacity in creating a useful system of numbers that did not satisfy the usual commutative rule of multiplication (ab=ba) that exists in arithmetic. In Quaternions, the order in which the numbers appear is important in multiplication and this did not bother the creative Hamilton because this is what usually happens in nature. For example, consider an empty swimming pool and the two operations of diving into the pool he had first and turning the water on. The order in which the operations take place is quite important!

The word Liberator above captures the important feature of freedom in mathematics. Freedom arises in mathematics because as I mentioned before, mathematics comprises an abundance of ideas and one is free to think of any idea they want. Whether or not these ideas lead to anything useful or interesting is another matter. Many of the great breakthroughs in mathematics were achieved by great mathematicians being free to pursue any (seemingly strange) ideas they wanted, like Hamilton above. As Cantor once said: “The essence of mathematics lies in its freedom.”.

Quaternions now play a crucial role in computer games. One example of this, which always appeals to journalists, radio hosts, students and adults, is the fact that Lara Croft in Tomb Raider was created using Quaternions! Continuing with the theme of entertainment, Quaternions now play a major part in special effects in movies. For example, an Irish company called Havok used Quaternions in the creation of the acclaimed new special effects in the movie, The Matrix Reloaded, and also in the movie, Poseidon, which was nominated for an Oscar for its visual effects in 2007. Havok won an Emmy award in the US in 2008 for pioneering new levels of realism and interactivity in movies and games. Also, some of the
dramatic visual effects in the recent James Bond movie, Quantum of Solace, were created by Havok. It's worth mentioning here the interesting (and surprising for many) fact that the largest employer of mathematicians in the world is now the animation company, Pixar!

I will now let some adults speak for themselves in relation to Hamilton and the walk. Local Cabra resident, Jack Gannon, once said.

“On account of the Walk, Hamilton is in the folk consciousness of the local people”.

One of the surprising consequences of the Walk has been that Jack was inspired by the walk to write a ballad about Hamilton in 2003. After participating in the Walk, Jack returned to school as a mature student to study mathematics.

Aodhán Perry, of Cabra Community Council, captures the profound impact of the walk in a 2009 quote:

“The Walk has had a huge impact on the local community. In fact it has gone way beyond just being a walk because all the local school children and the community are extremely proud of Hamilton and their local connection with him. The Walk really has touched the local community in a big way. The fact that famous mathematicians and Nobel Prize winners mingle with school children and the local community on the Walk and at the bridge is a great experience. Also, not one but two local artists have been commissioned in recent times to do portraits of Hamilton which are then publicly displayed at the bridge during the walk.”.

Mick Kelly, from Swords, wrote the following:

“The Hamilton Walk was my licence to explore and express myself around the subject of mathematics. By the age of nine, I had decided I couldn't do mathematics, but I had also developed a strong interest in things technical and scientific and this created a conflict that simmered in the background of my educational and professional career for forty years. That is until I took part on the Hamilton Walk in 2005. That Walk had a profound effect on me. Hearing not only a Nobel laureate and a professor of mathematics sing Hamilton's praises, but also local poets, school children, balladeers and the Cabra community council, spurred me to turn my desire to celebrate Ireland's Science Heritage into action. That action turned out to be a family run business called Science Heritage Ireland.”.

The setting up of Mick’s business was another surprising consequence of the Walk.
Mick also wrote the following:

“By the 2007 Walk, I could sense flaws developing in the glass wall I had built around learning mathematics and found it strange but very uplifting to be answering queries from people about quaternion algebra. There was a special sense of magic at Broombridge on that fine Tuesday, October 16, 2007, when the canal bank was alive with children playing all kinds of mathematics games. I couldn't help but wonder how many bridges to the future the organisers of this Walk and maths week had created for
our children, one year into the Government's Strategy for Science, Technology and Innovation 2006-2013”.

There are more stories in O'Cairbre, McKeon & Watson 2006 and O'Cairbre, 2009b.

**Conclusions**

From my many years of teaching/promoting mathematics among adults, I find that telling stories changes their perception of mathematics for the better. I also find that storytelling can enhance their understanding, awareness and appreciation of mathematics. One of the main reasons for this is that stories bring out the humanity in mathematics. Other appealing features that can appear in stories in mathematics are beauty, freedom, creativity, practical power, humour, Irish connections and the history of mathematics.

**Recommendations**

I strongly believe that storytelling in mathematics is very beneficial for adults. This paper gives some of the reasons for that belief. I encourage anybody teaching/promoting mathematics among adults to tell some stories.

**Biography of Author**

The author is a senior lecturer in mathematics at the National University of Ireland, Maynooth. He obtained his Ph.D from the University of California at Berkeley.
References


(When) can we trust ourselves to think straight? – and (when) does it really matter?

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Challenging ideas have emerged from outwith the adult numeracy world about how adults use formal mathematical, logical (syllogistical), intuitive and heuristical (rule of thumb) thinking when they are learning or using mathematics and when they are making complex choices and decisions; and it appears likely both that adult numeracy practitioners and researchers have had few opportunities to discuss those ideas, and that those ongoing debates are largely uninformed by thinking from inside the adult numeracy world. This article sketches some of these ideas and suggests some implications for adult numeracy education.

Key words: heuristics; argument; intuition; probability

Section 1 – The scope of this article.

In my ALM-18 workshop I sketched ideas and research findings (from psychologists and philosophers based outside adult numeracy) about how and when humans reason effectively, and suggested some connections with research and debates in adult numeracy. Participants then worked on exercises based, in the main, on classic psychological experiments. The length and format of the workshop made it impossible and inappropriate to discuss the ideas fully or to record the discussions systematically, and the space available in this article makes it similarly inappropriate to try to discuss the research findings or the exercises in depth, or to draw out in any detail the implications for adult numeracy teaching, learning and research. Here I am offering a preliminary exploration of some interesting and inter-connected research which is hotly debated elsewhere, but – despite its likely significance for learners - may not yet have been much considered by adult numeracy practitioners. My hope is that this will stimulate thinking among adult numeracy practitioners about the implications for their work.

In Section 2, I sketch several overlapping ideas and evidence about probabilistic reasoning, heuristical thinking, syllogistical reasoning, and reasoning in abstract and “real life” tasks, looking at some psychological and philosophical perspectives.

In Section 3, I suggest some possible implications for adult numeracy practitioners and for adults who are making decisions and choices in complex situations.
Section 2 – some ideas from outwith adult numeracy about how humans reason

Systematic errors in human reasoning: the ideas of Daniel Kahneman and Amos Tversky

Daniel Kahneman and Amos Tversky provided evidence that when they are assessing probabilities in real life situations, people often fail to get the “right” answers, i.e. those that derive from, or are in agreement with, formal probabilistic thinking. For example in Kahneman and Tversky (1974), Kahneman, Slovic and Tversky (1982) and Kahneman (2002) studies are described which suggest that flying instructors, failing to understand regression to the mean, wrongly conclude that praising students causes them to underperform; that people wrongly think a long run of red on a roulette wheel is much more likely to be followed by a black than by another red; that people see patterns in a series of natural disasters where in fact the distribution is random; and that physicians, failing to use Bayes’ Theorem correctly, make major errors when interpreting information about mammogram results. Kahneman and Tversky also suggest that in making these probabilistic and other errors, people are relying inappropriately on a set of flawed heuristics. In my workshop we used the exercise below to explore one aspect - “representativeness”. The exercise, adapted from many similar ones cited by Kahneman and Tversky, invited participants to consider the following problem:

A panel of psychologists has interviewed and administered personality tests to 30 engineers and 70 lawyers, all successful in their respective fields. On the basis of this information, thumbnail descriptions of the 30 engineers and 70 lawyers have been written. I’m going to read you one of these descriptions, chosen at random from the 100 available descriptions. Please indicate – on a scale of 1 to 100 - your probability that the person described is an engineer.

“Jack is a 45-year-old man.....he is generally conservative, careful and ambitious. He shows no interest in political and social issues and spends most of his free time on this many hobbies, which include home carpentry, sailing and mathematical puzzles.”

Kahneman and Tversky suggest that most people give too much weight to their beliefs about whether the description of Jack is representative of their beliefs about engineers and lawyers, and thus fail to draw conclusions consistent with formal probabilistic thinking.

Fast and frugal thinking: the ideas of Gerd Gigerenzer and his colleagues

Gerd Gigerenzer and a group of colleagues take a different view of adults’ heuristical thinking. Their central idea is that humans have evolved a set of heuristics which are effective in a range of probabilistic and decision-making situations, and they offer evidence that in some situations heuristical thinking is more effective than formal mathematical (including probabilistic) methods. They suggest (Gigerenzer, Todd, & Research Group, 1999, p. 6) that “much of human reasoning and decision making can be modelled by fast and frugal heuristics that make inferences with limited time and knowledge” and that these heuristics “do not involve much computation, and do not compute probabilities and utilities.” They specifically reject as inappropriate a standard of human reasoning which relies on the kinds of
“unbounded rationality” on which, they say, traditional probability theory depends. Citing evidence about the behaviour of people in a range of situations including financial investment decisions, consumer predictions about the nutritional content of foods and opportunity cost choices, they critique much of what Kahneman and Tversky say, arguing that unbounded rationality is not and need not be the gold standard for human decision-making.

For example Gigerenzer (2007, pp. 28 - 30) describes a competition in which, using a strategy which he calls “the recognition heuristic”, he and a colleague backed the intuitive strategies of people who knew very little about stocks against the more informed strategies of a large group of “non acculturated to stock trading” entrants. The competition had offered all entrants a list of fifty equities and invited them to deal in these for a fixed period. Gigerenzer asked the “non acculturated to stock trading” people which of these equities (companies) they recognized, and used the top ten as his competition portfolio. His “recognition” portfolio did better than 88% of all the portfolios submitted, many of whom had used much more complex strategies. Gigerenzer argues that in many situations a simple, single-factor analysis compares favourably with more complex (and more mathematical) strategies. He suggests in effect that less (mathematical logic) is sometimes more (effective). He further argues that humans have evolved a capacity to choose (heuristically and effectively) the factors that are likely to be most significant in this kind of decision-making situation.

He also illustrates this with the following story (1999, p. 9), which provoked some knowing nods from my workshop participants:

“One philosopher was struggling to decide whether to stay at Columbia University or to accept a job offer from a rival university. The other advised him: “Just maximize your expected utility – you always write about doing this.” Exasperated, the first philosopher responded: “Come on, this is serious”.

Gerd Gigerenzer and Adrian Edwards also offer (2003) some suggestions for moving “from innumeracy to insight”; they suggest that the apparent failure to think in accordance with Bayesian statistics is largely an effect of the choice of language used to describe the decision- or choice-making situation. Aware that doctors and patients often find risk assessment difficult, they suggest strategies for dealing with a range of situations. One of Gigerenzer’s examples, in the article cited above which was published in the British Medical Journal, was about how to assess the chances, given a positive medical test result, that the patient actually has a particular disease.

Gigerenzer and Edwards suggest that when the problem is posed using natural frequencies (“Three out of every ten patients experience….”) rather than conditional probabilities (“Patients have a 30% chance of experiencing….”), humans intuitively arrive at answers that are very likely to be in keeping with formal Bayesian reasoning. This simple change of language is, they suggest, very effective in helping medical practitioners and their patients to correctly interpret medical test results.

To illustrate this “from innumeracy to insight” idea, I offered in the workshop two versions of “Does Betty Like Mathematics?”. Aware of the ideas of Cosmides and Tooby (2000) - see discussion later in this paper - I hoped that a “mathematics and women” context would have enough emotional resonance to engage participants’ interest (and thus a particular kind of reasoning). I also deliberately created the Betty exercise to be a less
evocative context than the medical ones described by Gigerenzer and Edwards, since medical issues might have elicited emotional reactions which could not be dealt with in the workshop context. The Betty “data” in my exercise – below - were entirely fictional.

Readers might like to make initial guesses in response to each version of the exercise before trying any calculations, and might like, choosing to use Bayesian ideas or illustrative tree diagrams, to observe how close their initial guesses are to their calculated outcomes.

Does Betty like mathematics?

In this task you are given some information about women’s attitudes to mathematics, and about their chances of passing a particular mathematics test. Then you’re given some information about Betty, who you know has passed the test.

You are invited to try to infer “backwards” the likelihood that Betty liked mathematics before the test. (Adult numeracy / mathematics tutors will be aware that Betty may well change her attitude to mathematics depending on whether or not she passed the test, but we’re asking here about her prior attitude).

<table>
<thead>
<tr>
<th>Version A</th>
<th>Version B</th>
</tr>
</thead>
<tbody>
<tr>
<td>using the language of conditional probabilities</td>
<td>using the language of natural frequencies</td>
</tr>
<tr>
<td>The probability that a woman likes mathematics is 60%.</td>
<td>60 out of every 100 women like mathematics.</td>
</tr>
<tr>
<td>If she likes mathematics, the probability that she will pass a particular mathematics test is 90%.</td>
<td>Of these 60 who like mathematics, 54 will pass a particular mathematics test.</td>
</tr>
<tr>
<td>If she does not like mathematics, the probability that she will pass that test is 25%.</td>
<td>Of the 40 women who don’t like mathematics, 10 will pass the same test.</td>
</tr>
</tbody>
</table>

Let’s think about Betty, who has passed the mathematics test.

What’s the probability that Betty liked mathematics?

Philosophical views on probabilistic reasoning: James Logue and Jonathan Cohen

James Logue (1995) and Jonathan Cohen (in Logue, 2002) using philosophical positions about probability which are different from those which underpin the Kahneman and Tversky
work, and are also different from each other, have suggested that the Kahneman and Tversky “errors” are not in fact errors at all; that the insights they describe are in fact rationally based. Logue for example (1995, pp. 50 - 57) discusses a set of six key fallacies described by Kahneman and Tversky. Logue’s central argument is that probabilities are degrees of subjective belief rather than relative frequencies, and he suggests that Kahneman and Tversky, committed as they are to a relative frequency model, conclude that humans are “irrationally” discounting relative frequencies. People who “wrongly” – in “Kahneman and Tversky” terms – overestimate the likelihood that Jack is an engineer can be seen as rational under a subjectivist (or “personalist”) model of probability. Logue remarked

…any defensible personalist account of probability rejects such a simplistic reliance on frequency; if I am asked to judge the probability that a particular man described to me is a lawyer I can take into account not only anything I know about relative frequencies but also anything I believe as to which frequencies might be relevant, the necessity for coherence with my other beliefs, and the extent of my knowledge – and I may do all this on a quite different basis from the way in which I would act if asked to judge a long run of events

(Logue, 1995, p. 51).

In an article entitled “Are People Programmed to Commit Fallacies?” ( in Logue, 2002, pp. 195-220), Cohen discusses how the positioning of the adults involved (real life decision-maker or experimental subject) may affect their responses. In particular he suggests that where people are asked to reason in a framework in which they can call up construals of the task which are different from those in the mind of the experimenter, they reason in ways which surprise the experimenters. He suggests that most of the psychological experimenters, using a “Preconceived Norm Method” tested their subjects against a gold standard of formal probability calculus (often involving Bayes’ Theorem), and found them wanting; and he argues that unless subjects are primed at the start of the experiment to invoke this kind of thinking, they may well choose to invoke a different model of probability, one perhaps more related to their “real world” experiences. One of the exercises (similar to those reported by Kahneman and Tversky) I offered in my workshop was as follows:

There are two hospitals in the town, called Little Hospital (with on average 15 births per day) and Big Hospital (with roughly 45 per day).

In the town overall, the girl:boy births ratio is 50:50. But on some days of course, the ratio will be different.

Which hospital would have more days on which the girl:boy ratio is 40:60?

Kahneman and Tversky (1974, pp. 38-46) found that most subjects judged the probability of obtaining more than 60% boys to be the same in the small and large hospital, and they interpreted this result as showing that the subjects had ignored the effect of sample size (small samples are more likely than large ones to show uncommon distributions). But Cohen cites an experiment (Jones & Harris, 1982) in which, after subjects had had some prior experience which provoked them to consider the significance of sample size, they then
“successfully” invoked that model in the babies exercise. In the Jones and Harris research, people watched as first 5 and then 50 marbles were dropped haphazardly into a divided box, they were then asked to estimate how the marbles would end up distributed between the two parts of the box, were shown the actual results and were invited to comment. Cohen suggests that without this prior “teaching”, the subjects were perhaps inclined to call on their real world experience (in which hospital size seemed perhaps unlikely to influence the gender ratio). Perhaps this is another case of Gigerenzer’s “Come on this is serious”?

Cohen and Logue also discuss some classic “urn problem” experiments in which subjects were asked to revise some of their original probability judgements in the light of new evidence. The classic experiments suggest that people revise these estimates by less than would have been predicted by Bayesian reasoning, i.e. that they fail to meet the standards of Bayesian reasoning. Both Cohen and Logue (who themselves hold two different models of probability) argue that the experiment assumed that subjects were calling up a (frequentist) model in which probability is seen as the ratio of the number of a particular outcome to the total number of possible chances, whereas in fact people may have been using any one of several alternative views of probability. Viewed in the light of the frequentist model, people were getting the wrong answers, but in the light of the other models, their answers were not unreasonable.

In ALM-18, I introduced some urns problems in which I invited participants to first make intuitive probability assessments and then – if they were familiar with the mathematics involved – to use Bayesian ideas to calculate probabilities. Discussion of the urns problems evoked very energetic responses, so I have here included my original problem as well as a description of Logue’s argument about this kind of problem.

My workshop problem – drawn directly from the example used by Logue (1995, p. 52) – was as follows:

Two urns are filled with a large number of poker chips.

The first urn contains 70% red chips and 30% blue. *(I call this the “predominantly red urn” below)*

The second contains 70% blue chips and 30% red. *(the “predominantly blue urn”)*

The experimenter flips a fair coin to select one of the two urns, so the prior probability for each urn is 0.50.

He then draws a succession of chips from the selected urn.

Suppose the sample contains eight red and four blue chips.

What is your revised probability that the selected urn is the predominantly red one?
The classic experiments suggest that people usually conclude, intuitively, that the chances that the urn from which the sample was chosen was red were a lot less (ranging from about 0.5 to 0.75) than the Bayesian (97%) answer. In effect, they revised their probability estimates by less than the Bayesian answer which the experimenters took to be correct.

Logue argues (1995, pp. 52-53) that the experimenters were making two inappropriate assumptions; that one can assign prior probabilities in terms of given ratios (the distribution of red and blue in each urn) and that probabilities should be revised on the basis that the experimental set-up is genuinely random. Logue argues that the experimental subject is entitled to be at least slightly sceptical both that the experimenter is telling the truth about the original distribution, and also that the arrangement of chips in each bag is pure chance. So the subject in effect consistently applies Bayes’ Theorem (intuitively) but with reasonable assumptions about the figures that should be plugged into the formula; these cautious assumptions would lead the subject to revise his prior probabilities less than the experimenters had expected. Logue – who was an adult numeracy teacher in London in the eighties before resuming his interest in philosophy - also makes the telling point (1995, p. 53) that this caution “becomes more reasonable as the task becomes more important”, and this seems to me to be very significant for adult numeracy practitioners and learners who are sometimes engaged in real world – or simulated real-world – tasks as distinct from abstract ones.

In an interesting comment on his own “Norm Extraction” approach and that of the experimenters, Cohen also observes, perhaps rather acerbically, (in Logue, 2002, p. 214) that research that focussed on searching for circumstances in which people are likely to bring to bear some implicit understanding might be more useful than research which established that people err when they are not supported to invoke that understanding. I think many adult numeracy teachers, committed to helping students explore their understanding of mathematical concepts, might agree.

Abstract Reasoning and important decisions: Selection Tasks

Another series of classic results originating with experiments in which adults attempted Wason Selection Tasks (Wason, 1966) have been interpreted as indicating that they make systematic and significant errors in deductive (syllogistical) reasoning. But more recently Cosmides and Tooby (2000) have suggested that, compared with situations in which the experimental subjects are asked to work on abstract problems, people make many fewer errors when they are invited to work on problems which, whilst syllogistically identical, place them in situations where their conclusions have significant social or personal consequences. Cosmides and Tooby argue that this is because humans have evolved reasoning strategies which would have been effective in dangerous hunter-gatherer worlds. In my workshop, participants were invited to attempt and reflect on three Wason tasks below: the first is the classic Wason task, whilst I offered the others, adapted from Cosmides and Tooby, in an attempt to invoke varying degrees of social engagement from participants.

In each task, you are given four cards with statements on front and back; you are asked to investigate whether the set of four cards follows a rule, and you are asked which two cards
(you’re limited to two) you’d like to turn over to test whether the rule is being followed. Participants were invited to attempt the tasks in the order below and to reflect on whether any seemed easier than any other.

Each of these cards has a number on one side and a letter on the other.  
**“Any card with a consonant on one side has an even number on the other.”**
Which two cards do you want to turn over to check if the rule is being broken?

<table>
<thead>
<tr>
<th>D</th>
<th>3</th>
<th>8</th>
<th>E</th>
</tr>
</thead>
</table>

Donal is aged 18  
John is drinking alcohol  
Joan is drinking orange juice  
Anne is aged 29

**“Nobody under 21 is allowed to drink alcohol”**
Which two people do you want to question?

<table>
<thead>
<tr>
<th>John is doing something dangerous</th>
<th>Paul is wearing safety gear</th>
<th>George is not wearing safety gear</th>
<th>Ringo is not doing anything dangerous</th>
</tr>
</thead>
</table>

**“If somebody’s doing something dangerous they should be wearing safety gear”**
Which two people do you want to talk to?

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**Section 3 – some issues for adult numeracy practitioners**

Why does this cluster of ideas matter? There are several, often overlapping, constituencies who might be interested: adult numeracy / mathematics teachers and researchers; adult numeracy learners; adults who are trying to make complex choices and decisions inside and outside adult numeracy classrooms; educational policy makers / funders.
For example, teachers working with adults on statistical or probability at secondary and tertiary level have been supported to take account of the ideas of Kahneman and Tversky, and to some extent also the critiques raised by Gigerenzer, through for example a series of publications by Joan Garfield and Dani Ben-Zvi (2005) and (2007). And more recently, a project based at the Institute of Education in London, “Promoting Teachers’ Understanding of Risk in Socio-Scientific Issues” (http://www.riskatoe.org/, accessed 10 October 2011) has begun to support science teachers in secondary schools. But these authors comment almost entirely on learners and teachers at secondary level and in tertiary education; it seems likely that most teachers and learners in adult numeracy have not had opportunities to discuss the ideas.

There may also be implications for teachers, learners and researchers working in classrooms where the emphasis is on using mathematics to analyse everyday problems. In the UK and elsewhere, there is increasing emphasis on supporting adults (including those in adult numeracy provision) to use mathematical reasoning to analyse everyday problems. A UK government website spells this out clearly: “Functional Skills are not just about knowledge in English, maths and ICT. They are also about knowing when and how to use the knowledge in real life situations.” (http://www.direct.gov.uk/en/EducationAndLearning/QualificationsExplained/DG_173874 accessed 3 October 2011.)

Whilst the functional mathematics initiative does not insist that all real world problems are amenable to mathematical analysis, the default assumption is that they are, or at least that students in functional mathematics classrooms should be encouraged to use mathematical skills to deal with real life situations. In particular, the development of “functional mathematics” requires students to undertake activities described (Ofqual, 2009) as:

- “Representing – selecting the mathematics and information to model a situation”
- “Analysing – processing and using mathematics” and
- “Interpreting and communicating the results of the analysis”.

In the light of the Kahneman and Tversky findings, the challenges from Logue and Cohen, the ideas of Gigerenzer about the superiority of intuitive over “rational” thinking and the Wason results about the significance of meaningful tasks, how should learners and teachers take account of adults’ prior knowledge and previously successful (in life) heuristical methods for making decisions? Does the functional mathematics initiative require adults to eschew these previously successful methods and frameworks and substitute for them less reliable, more formal, often more mathematically demanding and perhaps more mathematically “respectable” strategies? Or, if leaving the heuristical methods temporarily at home, adults enter the functional maths classroom with the intention of adding mathematics to their problem-solving repertoire, where are they supported to review, synthesise, integrate or make evaluative choices among this richer range of strategies?

**Summary**

This article has looked at some ideas from outwith adult numeracy about how adults use syllogistical (logical), heuristical (rule of thumb) and intuitive thinking when they are learning, doing or using mathematics, and when they are engaging with complex choices and
decisions. When they are learning or doing mathematics, the default position for many learners and teachers is that formal logical thought is the ideal; that they should aim to proceed – explicitly or otherwise - in keeping with formal mathematical rules and norms. When adults are being supported in classroom situations to analyse complex issues the default position is two-fold; that the quality and robustness of the classroom-based analysis will be improved by formal mathematical analysis, and that this kind of thinking will bring similarly positive outcomes outside the classroom. Thus adults and their teachers are being encouraged to believe that mathematical / logical thinking styles will support the development of mathematical skills and knowledge and will improve the ability to analyse complex real-life situations; and that the knowledge, skills and practices thus developed will support choice- and decision-making in the adults’ lives as consumers, citizens and workers. Yet there is ongoing argument and counter-argument (outwith adult numeracy) about the idea that humans make systematic and stubborn errors in these situations; and these are ideas with which adult numeracy researchers and practitioners have only just begun to engage.

1

1, 2 Daniel Kahneman and Amos Tversky wrote very widely on adults’ statistical, probabilistic and heuristical reasoning, conducted experiments themselves and commented on many others. Whilst I have usually cited particular works, I have also very occasionally rather loosely used the term “Kahneman and Tversky ideas” to indicate the large body of ideas with which they are so strongly associated. If this usage is acceptable, it is only so if it helps readers navigate a complex web of ideas in this introductory paper.

1 Bayes’ Theorem suggests a way of recalcualting one’s probability estimates in the light of evidence obtained after the first estimates were made. Gerd Gigerenzer (2003, p. 46) offers a useful note on this, using as an example the problem of working out the significance of medical test results. My note below draws heavily on Gigerenzer’s text, which offers (unusually in explanations of Bayes’ Theorem) two ways of expressing the ideas, the first using probabilities and the second using natural frequencies.

You’ve had a positive test for a particular disease; what are the chances that you actually have the disease? The probability that you have the disease, given that you’ve had a positive result, depends on the incidence of the disease in the relevant population and the sensitivity and specificity of the test (how often people who have the disease actually test positive or negative, and how often people who don’t have the disease test positive or negative). If you know these figures, you can plug them into Bayes’ formula.

Using the terminology P(disease|positive) to mean the probability that you have the disease given that you’ve had a positive result, Bayes’ Theorem says

\[
P(disease|positive) = \frac{P(disease) P(positive|disease)}{P(disease) P(positive|disease) + P(no\ disease) P(positive|no\ disease)}
\]

Gigerenzer also offers a different representation of Bayes Theorem, using natural frequencies. If A if the number of people who test positive and have the disease, and B is the number of people who test positive and do not have the disease, then P(disease|positive) = A / A+B.
References


What is Mathematical Wellbeing? What are the Implications for Policy and Practice?

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This project will attempt to investigate the ‘usefulness’ of the capabilities framework as a means to empower adult learners to identify and, reflexively, consider the mathematical learning that they value. The methodological approach will then be outlined and justified as fit for purpose. The final part of the paper will then utilise the discourse terrain offered by the capabilities approach to sketch a theoretical landscape to show how a learner’s approach to learning mathematics can both impinge and provide substantive opportunities for improved mathematical wellbeing.

Key words: capabilities; wellbeing; mathematics; learning; reflexive; education.

Background to the capabilities approach

Imagine if someone asked you to describe the sort of mathematician that you aspire to be. Before continuing with the rest of this article, please just take a few minutes to visualise yourself in an appropriate setting and then consider the ways in which such a challenge would appeal to you, or not, as the case may be. Did you find yourself revisiting your schoolroom experiences of learning maths? Did you consider the mathematics that you use to resolve everyday problems? Did you think about how your values and beliefs affect the ways in which you approach and resolve mathematical problems? Did you think this isn’t for me? I bring these up as points for consideration, because teaching mathematics to adults is different to teaching children.

Adults do not just bring mathematical skills with them into the classroom; they bring lived experiences and beliefs about the purpose of mathematical knowledge and constructs (albeit messy ones) of teaching and learning (FitzSimons, 2002). Research into how adults learn mathematics in the United Kingdom (Coben et al, 2005, Smith, 2002, Swan & Swain, 2010), all converge on the findings that learning tends to be more intuitively meaningful, when adult learners hold a greater degree of agency over the direction and purpose of their learning.

However, many adult learners remain distrustful of their mathematical knowledge. Mathematics is often regarded as something that is unobtainable, outside of their control; something that other people do (Mendick, 2005), and despite poor experiences of learning, tend to want to learn mathematics ‘properly’ (Wedge, 2004). Adult learners often enter the classroom expecting to be taught a set of procedures, which they then assume must be correctly (re)produced, in order to ascertain ‘the’ correct answer (Skovsmose, 1994). Coben (2000), reveals insights into how and why such paradoxes can emerge. She posits that adults who can competently use mathematics in their everyday and working lives, tend to situate these mathematical strategies as ‘common sense’, rarely acknowledging them as a site of mathematical expertise to be imported into the classroom:
Mathematical knowledge is socially powerful; it enjoys high prestige and being knowledgeable is often treated as an indicator of general intelligence. Common sense by contrast, is regarded – or disregarded – as low level, practical everyday phenomenon, hardly noticed except when absent. Adults constantly undervalue the mathematics they can do, dismissing it as common sense.

(Coben, 1998, p1).

Tine Wedege (1999) provides a Bourdieuan theoretical framework to further explain this common response, positing that because mathematical strategies are often rooted in habitual behaviour, adults learning mathematics often instinctively revert to their habitual modes of learning, which, more often than not, revolve around the reproduction of fixed and authorised mathematical ‘truths.’ Thus, it is possible to begin to see how an adult, who is able to navigate a range of sophisticated problem solving strategies in everyday life, might become blinded by fear in a classroom situation.

With this in mind, asking learners to trust their ‘common sense’ yet simultaneously recognise that if left untrained, can restrict mathematical thinking, is both conceptually conflicting, and demanding. It requires learners to engage with the notion of a multiplicity of evaluative, learning, and mathematical spaces, which can only be navigated on developing a communicative agility that can cut across socially constructed barriers (Atweh and Brady, 2009).

Consequently, reflexive learning cannot be simplified into a notion of ‘empowering’ language of ‘you can do it’; there are systemic philosophical differences (particularly regarding the purpose of education and of mathematics) that require considerable conceptual unravelling (Coben, 2000). So, whilst a constructivist approach to obtaining knowledge tends to hold exciting opportunities for learners and teachers, to critically engage with the processes of learning (Swan & Swain, 2010); this author posits that to do so in a meaningful way, requires an authorised (Bernstein, 1977) curriculum space.

A curriculum space that is structured for the learning community to be able to collectively look below the surface of previous learning experiences, to disaggregate and disentangle the freedom dimensions of learning2 from the more instrumental achievement of a qualification gained (Walker, 2008). As such, this author is not constructing mathematical wellbeing as a theory of learning; neither can the approach offer epistemic insight into the validity of what constitutes mathematical knowledge. Nor can ‘Capabilities’ offer pedagogic insight into how an adult learner can learn mathematics, but instead is envisaged as a discourse terrain to create a reflexive learning space owned by the learner, to unpack and make sense of past and present experiences, as they unfold during the learning of mathematics.

Introduction to the Capabilities Approach

Coined by economic laureate and political thinker, Amartya Sen and diversified as a moral framework for justice by Martha Nussbaum, capabilities offers a multi-dimensional “freedom focussed approach to analysis that concentrates on the achievement (and lack of achievement) of human capabilities rather than other focal variables such as income, growth (and) … production” (Vizard & Burchardt, 2007, 21). The freedom focus centres on the twin notions of functionings and capabilities, which are summarised by Alkire & Deneulin (2009) as the following:

2 For example, the pedagogic exercise of developing mathematical reasoning through engagement in a mathematical discussion could provide the space for an adult learner to identify the capability of voice, confidence, and/or language to share their mathematical ideas, or even the capabilities to develop a sense of intellectual curiosity.
Functionings are surmised by Sen (1999) as ‘the various things a person may value doing or being’ (p75). Valuable ‘doings’ are described in terms of activities which may include being able to visit loved ones, having a good job, being literate and numerate, or being able to take part in a discussion. Valuable beings are described in terms of states of mind such as achieving happiness or feeling safe, valued, and respected. In terms of learning mathematics, valued functionings could include (but are by no means limited to) being able to ‘do’ fractions, check change in a shop, and take part in a mathematical discussion or feel safe and/or valued within the learning environment. A valued functioning can also combine the two domains; for example being able to write a cheque in the bank without a feeling of shame. Or may even include aspects of negotiating identity such as ‘feeling like a mathematician’.

Capabilities refer to the real and actual freedoms and opportunities that an individual needs to experience in order to achieve their valued outcome (the functionings). Learning capabilities, according to Walker (2008), typically include the capability of critical thinking, of imagination and of voice and often hold the key to expending and developing further valuable capabilities and/or functionings.

Research Questions

- To what extent do values and beliefs influence the ways in which these learners approached classroom mathematics?
- How do learner perceptions of formal mathematical structures affect their learning progress?
- What are the implications, for policy and practice, of using the concepts of capabilities and wellbeing for improving the experience of learning mathematics?

Sample

The learner participants have been drawn from a non-probability, purposively constructed (both in terms of participant demographic and learning context) sample base to gain a nuanced understanding (Bryman, 2008) from experiences of learning mathematics. The learner participants have been drawn from a small pool of experienced and specialist mathematical teachers who, to a varying degree, interweave mathematical discourse (as a pedagogic approach) into the processes of learning mathematics. In total, there is a sample of 11 adult learners (19+) from a variety of educational settings including:

- Discrete numeracy settings, where the learning of mathematics is the sole learning aim, including an adult education college, a residential women’s college, family learning provision within a primary school and work based learning (classroom assistance)
- Embedded numeracy provision, where learning mathematics tends to be part of a wider, often vocationally based, learning program including foundation tier (business), ESOL learners (Information Technology) and an access to Higher Education programs (nurses and teachers).

This research attempts to gain insight from the ways in which a small sample of individual learners ‘may have encountered learning in relation to historical conditions and factors that may have shaped the ways in which they have been able to live their lives’ (Snape & Spencer, 2003). Thus, the

3 English as a Second and Other Language
methodological approach stems from the assumption that the recollection of an experience is not an attempt to seek a “truth” about the past, but instead intends to gain a relational picture of how the individual makes sense / has made sense of their learning experiences; that is given the particular time, space, and context of the interview.

This, on the one hand, situates the inquiry within a subjectivist interpretation of knowledge but on the other, necessitates a theoretical agility to draw attention to the different ways in which the layers of structure, the social arrangements, may have served to inhibit or grow mathematical learning. In attempting to overcome what by many may be regarded as a binary division between the objective and subjective nature of knowledge (Bourdieu, 1993), I have decided on employing a variety of data gathering tools, starting with an open narrative approach to collecting stories about learning. I have then supplemented this data with a non-participatory classroom observation of a learner ‘being’ a mathematician and ‘doing’ mathematical problems, which I then followed with a semi-structured interview.

**Choosing (and supplementing) a narrative approach.**

In this research, it is the discursive practices, how participants makes sense of learning mathematics, that provide the insight into the capabilities and functionings that the individual participants might value and may want to foreground during their experiences of learning. However, rarely are adult learners free agents with neatly tied aspirations that are simply awaiting conversion into valued functionings (Walker, 2008). Rarely are their anecdotes of learning neatly tied packages of cohesive accounts of the past and as such, Rapley (2004) posits that it is simply not feasible to start an interview on the premise that a few well-prepared questions can elicit a coherent picture of the past. Bryman (2008) concurs and offers a narrative interview as an holistic and person-centred approach that allows the participant to frame their stories of how they feel, perceive, and do mathematics (Valero, 2008). Accounts, which can then provide some insights into how the narrators voice may have become authorised and/or silenced, contested and/or accepted (Squire, 2009) on inter and intra personal levels.

In recognising that an active role for the researcher remains a controversial area of scholarship, West (1990) argues that subjectivity, in itself, may not, by necessity be regarded as an implicit threat to the robustness of the data. Instead, he welcomes story telling as a narrative approach, as a fluid and open framework that enables different, and often, contradictory layers of meaning to emerge. Narratives which can thus provide a window for the researcher and audience to investigate how stories are structured, and consumed (Squire, 2009).

Ontologically, I also wanted to be able to discuss, in a meaningful way, what mathematics means to the individual participant and for them to describe the sort of a mathematician they aspire to be. However, such abstract questions are almost impossible to visualise let alone to answer, and so I planned to use the notes and transcriptions from the observation of a mathematical discussion, as a conduit to gain insight into the shifting nature of mathematical identities.

**Ethics**

It would be naive to view the narrative interview as a conversation free from the hierarchies of power; the researcher has, after all, set and then controls the agenda. However, in a Foucauldian tradition, Tamboukou (2009) suggests that telling stories is often a political act whereby, in addition to the
researcher, the narrator also holds multiple layers of motivations which may have led them to narrate a story in a particular way. The responsibility then of the researcher, according to Rapley (2004), is to be an active listener, and to interpret the stories in a sensitive and reflexive manner, which remains both in context and with regard for the circulating power relations that govern the processes of the research.

Findings

Given the size of the sample and the methodological approach, I feel it would not be useful to attempt to construct, at this stage, a multi dimensional measuring device for evaluating the impact of learning mathematics on the freedom to live a life that they value, or to identify the freedom dimension of learning for each of the participants. Instead, drawing from strands of critical mathematics pedagogy and post-structural literature on voice, I intend to use the interview space to explore difference by encouraging the participant to problematise their approaches to mathematics, with particular reference to the structuring factors of class, race and gender that have impacted on their opportunities to learn (Quinn, 2003). I then propose to use the richness of the participants’ descriptions to pool the emergent themes, to investigate the potential usefulness of the twin pillars of capabilities and functionings as a powerful tool to reflexively consider the freedom dimensions of learning mathematics. Thus far, I have completed the research cycle and, although I have by no means analysed the data, my initial attempts to organise and categorise has uncovered some promising emergent themes.

... on relationship with mathematics

According to Heather Mendick (2005), learners often hold a fluid, fragmented and, often contradictory, identification with mathematics, where mathematicians tend to be characterised as independent thinkers who are separated from, rather than connected to, the rest of the world. ‘Real’ mathematicians tend to be different to other people and the preliminary result of this research shows similar patterns of identification. Within this small sample, many of the learners have constructed their relationship through the lens of an idealised, often masculine, vision of a ‘mathematician’ and this tendency to view mathematics through a lens of ‘otherness’ is nicely captured by J’s remarks during her initial interview:

“... So it’s almost like hands off, I don’t know. So I think I have been brought up in that sort of environment although my father worked as an engineer so it’s almost like, well he’s the one that knows it all so ... yeah.” (J 2011).

... on the narrative of the ideal learner

‘M’ is studying mathematics on a foundation learning tier course in Business and like J, she voices her (in)ability to ‘do’ mathematics in relation to an ‘otherness’ that has been generated by her vision of an ‘ideal’ mathematician. M is not alone in her thoughts. According to Boaler (2009), speed and memory are often cited by learners as the key mathematical capabilities that are needed, in order to succeed:

“Yeah, it takes me so long and everyone will do it like that (clicks her fingers) and do it in their heads and stuff. I mean, there is this guy who kind of knows everything and so he will teach me it, but it will be like derrr (shows signs that she can do the maths), and then it’s gone.” (M, 2011).

‘J’ also refers to the importance of memory for ‘learning’ mathematics, but in doing so, her comments also provide some interesting insights into the shifting dynamics of the power structures that have moulded her experiences and approaches to learning mathematics:
“Well funny enough, the lady sitting next to me which is F (name of one of her peers). Err, she did it all in her head. She was taught back in Africa and blah blah blah blah blah and so that’s all in there (she taps her head) but she doesn’t know what to do with it. So, she’s working like a computer and I said no you’ve got it! You’ve got it, and she said I’ve got it, I’ve got it! So I said all you’ve got to be able to do, is to take it out of here (points to her head) and put it down on here (points to the tree of knowledge) and then you’ll realise what you know. She says I don’t understand, I don’t understand, and I said you’ve got it. Don’t worry, you’ve got it (pats my arm).” (J, 2010).

... on capabilities and well-being

Learning mathematics through empowering learners to construct their own mathematical schemata is by no means an ideologically neutral pedagogic approach. Learning mathematics discursively is premised from a constructivist approach to learning, which, according to Skovosmose (1994), opens the door to human agency thereby inviting power dynamics, and other social factors that affect learning, into the classroom.

‘H’ studying mathematics, through a full time IT course, is very frustrated by his performance in mathematics. ‘H’ has lived in the England for 2 years and has achieved a very high standard of English. ‘H’ has a very successful academic history and despite an impoverished childhood, was the first in his family to gain an undergraduate qualification. He too told a story of his struggle with mathematics. Throughout his interview, he demonstrated very effective and critical learning capabilities but was frustrated and angry with the pedagogic approach that is characterised by agentic choice:

“This kind of exercise it give you the possibility to go around ... this curriculum ... example if it doesn’t make me learn these things that I don’t know ... it doesn’t help me, he (it) destroy me little bit.” (H, 2011).

But he also shows frustration with himself for not being able to transfer his generic learning capabilities into the mathematical space:

“All this numbers you know, it was like some kind of magic ... I trying to (remember), these rules, all these things I learned before, but now they slipped and I couldn’t answer the questions. I was thinking, this is ridiculous to forget all these things but I don’t. With this course all the things that I learned they are all gone.” (H, 2011).

... on agency

Mathematical wellbeing is being posited as a shared language that can initiate collaborative discussions that encourage learners to reflexively forefront the mathematical technical skills and experiences, which they consider most valuable to their wellbeing. In this respect, the learner cannot be situated as the subject of their learning; they must own their mathematical processes and act as agent of their own critical wellbeing (Skovosmose, 1994). ‘D’ is another ESOL learner, but this time with a poor experience of learning in school, and who, in comparison to ‘H’, demonstrated an emerging sense of mathematical wellbeing:

“Yes as long as the problem is give you this liberty, yes I like it because ... on your own you can say, you know, this is my result because ... but I have been taught (in
Romania) and I got used with problems that give you something strictly asking (questions) so you look for the answer on thinking of those askings (questions).” (D, 2011).

But in recognising his emerging sense of the self throughout the interview, ‘D’ also made statements that lean towards the layers of complexities that lie behind the notion of agency. According to Swan and Swain (2010), there has been some progress towards supporting mathematics teachers to adopt pedagogic principles that require learners to reason, rather than to recall, answers. However, this approach could frustrate ‘D’, unless he is given the curricular space to consider reflexively his changing approach to learning mathematics:

“... but you tell me exactly what you want me. Tell me exactly how you want it to be done ... If you want something, you have to say what the askings (questions).” (D, 2011).

**Conclusion**

In summary, it is in the space where a discursive approach to learning mathematics is implemented in the classroom, that this author posits the need for a reflexive capability approach. This author posits mathematical wellbeing as a discursive space that can authorise curriculum designers, teachers and, most importantly, learners, to assess the impact of social arrangements (Walker, 2008). A multidimensional space that can accommodate questions from multidisciplinary approaches, from the philosophy of mathematics, to the relationship between structure and agency, of gendered, classed and racialised ways of mathematical behaviour and of wider discussions about notions of social justice.
References


Assessing for learning and quantitative literacy

Damien Raftery & Sharon McDonald

The learning outcomes of an introductory business quantitative techniques module are traditionally calculations focused, with students focusing on learning the mathematical steps in producing a numerical answer. We undertook an action research project (as part of the SIF 2 Repositioning Assessment for Learning project), which aimed to evolve a learning environment conducive for our students to engage more deeply. We focused on assessment, as "from our students' point of view, assessment always defines the actual curriculum" (Ramsden 1992). The issues we attempted to address included: broadening the assessment approach to include elements to promote quantitative literacy (the ability to know when and how to work with numbers in particular contexts, as well as to critically evaluate and communicate the results); encouraging meaningful engagement in a large lecture setting; and, the use of online resources and quizzes as a means of providing immediate feedback. The poster will summarise what we did, feedback from students and our reflections.
BBC Skillswise: developing resources around maths in workplaces

Michael Rumbelow

The BBC Skillswise maths website is re-launching in 2011 with new free-to-access online video clips, games, quizzes, mobile phone tools and printable resources, developed with the NRDC, NIACE and LLU+. The site will include a new 'Job skills' section designed to support tutors in making connections between maths and 14 types of workplace such as retail, hairdressing, security, catering, IT, nursing etc. This presentation will preview the new job skills resources and outline plans for the next phase of development on Skillswise.
At ALM-7 in 2000, a paper was presented that summarized published research in adult mathematics education. The journalists questions of “Who, What, When, Where, How, and Why” were investigated in light of the themes about which that conference was organized. This past academic year, work since 2000 was identified and the abstracts of the dissertations were reviewed using the ALM-7 protocol. This paper reports the findings thus far. The project will continue through the academic year 2011-2012.

At ALM-7 in Boston, I reported the findings from a research project that examined dissertations from the period 1980 to 1999 (Safford-Ramus, 2001a). The report was framed by the themes that the conference organizers had identified and followed a protocol that sought to identify clusters of work by asking the traditional journalists questions about each abstract. During the academic year 2010-2011, I began a follow-up project in order to determine whether earlier trends had continued, changed, or been replaced by new foci. Only the work on dissertation abstracts could be completed by the end of the year. During AY 2011-2012, the dissertations will be read in their entirety. A meta-analysis of their findings will be conducted. Journal articles will also be included in that analysis.

Methodology

A search of the Dissertation Abstracts International (DAI) database for the years 1999 to 2011 was conducted using the Boolean argument “Adult” AND “Mathematics” AND “Education” in the Subject Heading (SU). Sixty-eight dissertations abstracts were returned, five proved to not be appropriate for the study. Two of them had been reported in the 2000 article and three were not truly adult education. One major difference between the 2000 study and the current project is the ready availability of full text dissertations. These were downloaded for in depth study.

Abstracts were then coded using the template from the 2000 study. While most of the template remained valid, attempts to pigeonhole the new dissertations into the 2000 ALM themes failed. Primary and secondary themes that emerged were noted for resolution and possible consolidation at the end of the full dissertation analysis phase.

Findings

Who and Why?
This question examined the funding sources and investigators of the research. There was no evidence that the doctoral candidates were grant-funded by private or public monies. Each appeared to have recognized a problem or situation that interested them and developed research questions that explored some aspect of it. While generally not overtly stated, a
reader of the abstracts senses that the setting is the candidate’s classroom or institutional department.

What?
This question looked at the research methods used in the study, i.e. qualitative or quantitative, as well as the primary and secondary themes. In the ALM 2000 study, most of the research had been quantitative although qualitative studies had seemed to gain a foothold in the late 1990s. One supposition of this study was that the earlier quantitative work had identified the demographic characteristics of students who were “at risk” and that later work would report successful interventions or rich descriptions of student experiences in their own words. This did not, however, turn out to be the case. More than half of the studies were still quantitative. A few even appeared to be “quick and dirty” statistical analyses of old ground asking “Who is math anxious?” or “What demographic groups are more/less likely to succeed?” Figure 1 shows the breakdown by publication year. Figure 2 shows the breakdown by study type.

![Dissertations By Type](image)

**Figure 1. Dissertation Type by Year.**

While there was a spike in qualitative studies in 2000 there was an even greater spike in quantitative studies ten years later. During the presentation at ALM-18, I was asked to explain why I seemed opposed to quantitative studies and that question lingered in my mind later in the summer as I began to read the dissertations themselves rather than just the abstracts. While all of the candidates compiled thorough and interesting literature reviews, the quantitative studies continued to reinforce my bias. They either told me facts that I already knew—adult women returning to further education are math anxious—or produced new statistics with little or no discussion of possible interventions to overcome the problem. After three decades of studying the patient it would be reasonable to expect multiple studies about treatments and their success or failure.
The original 2000 ALM themes were assessment/frameworks/standards, contexts, instructional approaches, parents, teacher’s knowledge, theory, understandings, and workplace/vocational. These proved to be impractical for this analysis and had, in fact, been re-worked in 2000 when a version of the project was presented at the International Congress on Mathematics Education (ICME) in Japan (Safford-Ramus, 2001b). Figure 3 contains a breakdown by themes identified in the current study. It is possible that some themes will be merged and others may appear once the full dissertations have been read.
Many of the dissertations had strong secondary themes. The 2000 study did not look at secondary themes but they were striking enough in this study to merit attention. For instance, a study with a primary theme of math anxiety might contain significant discussion of a link to self-efficacy. Figure 4 shows the breakdown by secondary theme.

![Figure 4. Dissertations by Secondary Theme](image)

When?
Adult mathematics instruction is offered at five different levels in the United States: adult basic education (elementary school mathematics), adult secondary education, developmental (remediation at tertiary institution), baccalaureate, and graduate. Three parent-child studies did not fit into any of these categories. One study was multi-level. Six did not indicate the level in the abstract. Of the remaining 53, 81% of the instruction took place at the developmental (32%) or college (49%) level at tertiary institutions. Only 13% were at the basic (7.5%) or secondary (5.5%) level. This is a marked decrease from the 2000 study when 31% of the dissertations reported research at these levels. I would conjecture that this is the result of two factors. First, the lack of grant funding for adult education during this period provided little opportunity for doctoral research. Secondly, and probably a stronger reason, the individuals seeking these degrees were probably doing so to obtain or secure a tertiary post and would have access to tertiary subjects for their studies.

Where?
The sites where American adult mathematics education is implemented varies, sometimes influenced by the geography of the area. In many parts of the country, the community college houses instruction in elementary, secondary, and tertiary mathematics. Densely populated urban centers may have centers specifically dedicated to adult learning. Workplace skills or professional development are often provided at the work site. Six of the abstracts did not indicate where the research was conducted. The parent programs took place in a pre-school and two middle schools. Of the remaining 54 projects, 80% were conducted on either a community college (31%) or university (48%) campus. Five (9%) were at either an adult basic education center (4) or an adult high school (1). Six (11%) were offered onsite at either a workplace (9%) or prison (2%). These numbers are subject to rounding affect.
How?
This preliminary report does not include the answers to this question. The 2011-2012 academic year phase of the research will consist of a reading and analysis of the full dissertations. It will examine the particular instruments used in the studies as well as the theories that framed the work.

Discussion

This summary draws information from the abstracts of the doctoral dissertations published in the years 2000 to 2011. By its nature, an abstract gives a limited picture of the actual research conducted by the candidate and a richer picture emerges once the full dissertation is read. By ALM-19, that full-reading phase will be completed and I will be able to give a clearer picture of the doctoral research conducted since ALM-7. The 2011-2012 phase will also include data on journal articles over the same time period which form a smaller but still relevant, portion of the literature in adult mathematics education research in the United States.

While it is encouraging to see that mathematics educators are willing to conduct research on topics that interest them, it is discouraging to see the lack of focus that this independent work sustains. I am reminded of the sky on a clear night, each star provides a small, bright light that is pretty but would not help find the way down a dark road. We need a centralized effort by teams of researchers in order to develop meaningful research that helps adult mathematics students to be successful, to be numerate. At best, this calls for large-scale government funding for projects like the NRDC in the UK. Lacking that, I see a critical role for ALM to play in providing opportunities for collaboration within and between the countries represented by its members. Bringing the “starlight” of the individual member research into focus would allow us to shine a strong beam on critical research areas and make a real difference in the field of adults learning mathematics.
References


Professional Development for Middle School Teachers: A Summary of Recent Research in the United States Final Report

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This presentation is the culmination of the work, including ALM presentations, of several years. The report begins with a recapitulation of findings reported at earlier ALM conferences and then summarizes the materials that were located and reviewed during the academic year 2010–2011. While the quantity is disappointing, the trove may still contain information that is of use to educators interested in developing or pursuing professional development (PD) at the middle school level.

Earlier Conferences

ALM-16, London, England, Professional Development for Middle School Teachers: A Growing Adult Student Audience

The movement to strengthen the requirements for teaching at the middle school level has grown over the past twenty years, although individual states may have initiated higher standards even earlier than that. In its Professional Standards for Teaching Mathematics, the National Council of Teachers of Mathematics (NCTM) maintained that teachers of grades 5 through 8 should have at least fifteen semester hours of coursework in content mathematics, including the topics of number systems and algebraic structures, geometry and measurement, statistics and probability, and concepts of calculus (NCTM, 1991, pp. 137–139). The NCTM recommendations for greater mathematical knowledge were reinforced with the federal mandates of the No Child Left Behind (NCLB) Act of 2001. This legislation stated that all students should be taught by a highly qualified teacher (HQT) who holds at least a bachelor's degree, has obtained full state certification, and has demonstrated knowledge in the core academic subjects he or she teaches. Precise definitions of HQT requirements and enforcement of them were left to individual states, resulting in wide variation in interpretation.

To meet the NCLB requirements, the state of New Jersey instituted a middle school certification requirement for fifteen semester hours of college-level mathematics coursework,
excluding pedagogy. Candidates for the certificate must take the PRAXIS™ Middle School Mathematics Test. In 2010, the passing score was set at 152. Saint Peter’s College is located in New Jersey, and the middle school mathematics certificate program that the college adopted reflects the state requirement for fifteen semester hours of college-level mathematics, excluding courses in pedagogy. Since the state requires six credits for the initial elementary school certificate, the college initiated a three-course sequence: Elementary Mathematical Functions and Models for Middle School; Statistics, Probability, and Discrete Mathematics for Middle School; and Geometry for Middle School. Courses were cross-listed as graduate credit for practicing teachers and undergraduate credit for pre-service teachers and were offered during the 2009–2010 and 2010–2011 academic years. A dilemma faced by the program designers was the lack of a commercial textbook or materials for the functions course. It was this difficulty that inspired the research being reported in this paper.

ALM17, Oslo, Norway: Professional Development for Middle-School Teachers: A Summary of Recent Research in the United States

The textbooks used in the geometry and statistics courses in the Saint Peter’s sequence emerged from federally funded PD projects. Since it was likely that material useful for the functions course might have been produced from other projects, a study to locate potential sources was begun during the academic year 2009–2010 and continued in 2010–2011. Three databases were explored: the database of the U.S. Department of Education (USED); the database of the National Science Foundation (NSF); and the database of Dissertation Abstracts International. The U.S. Department of Education is the government agency that funds elementary and secondary education research. The monies are generally allocated by state with annual grants that are renewable. The NSF is the major funding agency for research in mathematics education. These grants are awarded by divisions within the NSF, are often multi-year awards, and permit collaboration across states. Dissertation Abstracts International is the repository for most dissertations published in the United States. Individuals seeking doctorates conduct research projects that can focus on PD and/or produce classroom materials.

The USED database search did not yield any useful leads. Results from the NSF search were much more successful. Their website (www.nsf.gov) was searched for expired (completed) grants, using the search argument “Middle School Mathematics.” Seven hundred sixty-five awards from the previous twenty years were found. The abstracts were downloaded, and those that indicated publications had been produced were noted. Contact information for the principal investigators (PIs) was saved. For the search in Dissertation Abstracts International, since the lead time for dissertations can be as much as seven years, it was decided that a search of dissertations from 2000 through 2011 was most likely to reflect the reforms advocated by NCTM. A Boolean search was conducted on the index term/code field with the arguments “Professional Development” AND “Mathematics” AND “Education” AND “Middle School.” Four abstracts were retrieved. Two additional abstracts surfaced as “Adult” AND “Mathematics” AND “Education.”

ALM-18 Dublin, Ireland

During the academic year 2010–2011, the analysis of the abstracts took place. Each NSF abstract was reviewed to determine whether it truly described a project in PD for middle school teachers (pre-service or in-service). Of the original 765, only 265 were deemed useful for the study. Attempts to contact PIs were made. E-mail addresses for 126 PIs were found;
118 were contacted via e-mail; 8 addresses proved invalid. A total of 29 PIs responded. Of the respondents, 18 provided us with websites and publications that might be applicable. The publications were reviewed but did not contain materials, although a few provided clues to websites that might be useful. All located websites were visited, reviewed, and classified as either courses, guidance for individuals preparing PD, or materials that might support PD activities. Alas, the Holy Grail of the project, a text suitable for the functions course, never surfaced.

**Courses**

Three projects produced materials that could be described as “courses.” The first, Building Algebraic Thinking through Pattern Function and Number-Professional Development Course for Middle Grade Math Teachers is an online course developed by the Education Development Center (EDC). The course is forty hours long and is intended to be taken over an eight-week period. It resides on a web portal titled Curriki that is dedicated to the sharing of curriculum materials by the world-wide education community.

The second, Seeing Math, is an online course developed by the Concord Consortium that allows teachers to create their own video case studies and then engages them in collegial discussions about teaching practice. The elementary school packet consists of twelve units, two of which are specific to algebra and three additional ones that are about mathematics education in general. There is a fee charged for the course, and it is available through a company called Teachscape (http://www.teachscape.com/). Their web page indicates that interested parties can obtain a free trial of the materials, but it was not clear whether one could request the mathematics series or just access to a sample module chosen by Teachscape.

A third project, Developing Mathematical Ideas (DMI), resulted in a series of books/seminars that can be used for PD. The series is published by Pearson. The material is organized in modules and the packet offers a facilitator’s manual, as well as a DVD and a casebook. Each module is designed to be used as eight three-hour sessions. Judging from the description, the course is more suitable for in-service instruction than for a university mathematics course. The Mathematics Leadership Program website describes it as:

> Developing Mathematical Ideas (DMI) is a professional development curriculum designed to help teachers think through the major ideas of elementary and middle-school mathematics and examine how students develop those ideas. At the heart of the materials are sets of classroom episodes (cases) illustrating student thinking as described by their teachers. In addition to case discussions, the curriculum offers teachers opportunities: to explore mathematics in lessons led by facilitators; to share and discuss the work of their own students; to view and discuss DVD clips of mathematics classrooms; to write their own classroom cases; to analyze lessons taken from innovative elementary mathematics curricula; and to read overviews of related research (Developing Mathematical Ideas [2009]; retrieved January 5, 2012, from http://www.mathleadership.org/page.php?id=33).

The material might be useful for adult tutor training, but the emphasis is on children, so a facilitator would need to determine its suitability for his/her population.
Professional Development Guidance

Four of the projects produced material that could be helpful to individuals planning or conducting PD. One could begin with a challenging report out of Michigan State University, *The Preparation Gap: Teacher Education for Middle School Mathematics in Six Countries*. The project, Teacher Education and Development Study in Mathematics (TEDS), compared the preparation of mathematics teachers across six countries. The document contains a thorough presentation of elementary (K–8) teacher preparation in the United States. One chapter is devoted specifically to the preparation of middle school teachers, with recommendations for future national direction (retrieved January 5, 2012, from http://www.educ.msu.edu/content/sites/usteds/documents/Breaking-the-Cycle.pdf).

A second project, the *Critical Mathematics and Science Synergy Project* report shared findings from a five-year PD collaboration of the University of Massachusetts–Lowell and the Hudson and Fitchburg public schools. The teachers took part in 160 hours of PD over the five years. Three full-day content knowledge workshops were held during each school year and summer institute, with an emphasis on the synergy between mathematics and science. The report speaks frankly about the difficulties in effecting change in teacher content knowledge and instructional methodology. While that report is not readily available, a national report on similar projects that were part of the systemic program can be found at http://www.pcgpr.com/graphics/NSFmathscience.pdf (retrieved January 5, 2012).

The last of the three reports, *Optimizing the Impact of Online Professional Development*, describes a study that offered online PD about algebraic reasoning to teachers across North America and then examined the effect of the course on teachers’ beliefs, content knowledge, and classroom behavior. “The overarching goal of the study was to identify the OPD design factors that maximize the conditions for effective professional development” (INTASC; retrieved February 5, 2012, from http://www.bc.edu/research/intasc/researchprojects/optimizingOPD/OPD.shtml). Findings from the project were shared in journal articles, one of which is contained in the bibliography (Carey et al., 2008).

A fourth project produced a multimedia kit. The kit, *Learning to Lead Mathematics Professional Development*, by Carroll and Mumme (2007), consists of a user’s guide, DVDs, and seven modules that contain the output of forty-four seminars. According to the publisher’s description, the modules incorporate research on adult learning into the mix of mathematics and instructional skills. The promotional material states that:

Unlike a typical guide for a specific professional development program, these materials focus on facilitation issues that are likely to be encountered in a variety of mathematics professional development situations. The multimedia kit demonstrates how leaders can deal with key issues, including:

- Managing productive mathematical discussions
- Working with teacher explanations of mathematical ideas
- Being mindful of equity issues in mathematics teaching
- Selecting appropriate mathematical tasks
- Deepening teachers’ knowledge of mathematics.
This set of mathematics seminars is the ideal resource for people charged with supporting teacher leaders and others who lead mathematics professional development. Potential users include curriculum leaders, math–science partnerships, university–district partnerships, and mathematics teacher educators (retrieved January 5, 2012, from http://www.corwin.com/books/Book227032#tabview=title).

Supporting Materials

This final category includes resource materials that could be incorporated into a middle school PD program. During the period 2000–2010, several NSF projects resulted in commercial middle school textbook series. Professional development efforts ran concurrently with and followed on from the crafting of the student texts. Since the projects are all described on their websites, I have in most cases quoted their description rather than paraphrasing, and perhaps distorting, the nature of the sites.

- Freudenthal Institute—USA (FI-US): http://www.fius.org/
  Information on projects Mathematics in Context and Mathematics in the City, and Classroom Assessment as a Basis of Teacher Change (CATCH). Also, material about mathematics education in the Netherlands. The site maintains a bookstore at which publications resulting from the various projects, as well as other material, are available for purchase. One item of interest to middle school PD is Romberg’s A Standards-Based Mathematics Assessment in Middle School—Rethinking Classroom Practice.
    This is an article about the CATCH assessment endeavor.
    This site is a bibliography of publications that emerged from the Mathematics in Context project referenced above.

- Center for the Study of Mathematics Curriculum: http://mathcurriculumcenter.org/
  Maintained by University of Missouri, this site supports three major databases: Mathematics Curriculum Literature Database, K–12 Mathematics Textbook Database, and a Curriculum Research Instrument Database.
  - Modeling Middle School Mathematics: http://www.mmmproject.org/index.html  
    Also housed at the University of Missouri, Modeling Middle School Mathematics is a PD program using video lessons and web-based Internet materials to examine each of the five NSF-funded middle school math initiatives Pathways to Algebra and Geometry, Mathematics in Context, MathScape, Connected Math Project, and MathThematics.

- Mathematics and Sciences Partnership: http://hub.mspnet.org/index.cfm/resources/  
  “MSPnet is an electronic learning community for the Math and Science Partnership Program. With the MSP program, the National Science Foundation implemented an important facet of the President’s No Child Left Behind (NCLB) vision for K–12 education. A major research and development effort, the MSP program responds to concern over the performance of the nation's children in mathematics and science. Institutions of higher education partner with K-12 districts and others to effect deep, lasting improvement in K–12 mathematics and science education” (retrieved January 11, 2012, from http://hub.mspnet.org/index.cfm/home).
• Problems with a Point: http://www2.edc.org/mathproblems/
  “‘Problems with a Point’ help students in grades 6–12 learn new mathematical ideas by building on old ones. Each problem or sequence focuses on one mathematical idea and also connects that idea with others. Varying in difficulty and approaches, these problems are useful for teachers, students, parents, math clubs, and home-schoolers. Problems are classified by topic, time required, suggested technology, required mathematical background, and habits of mind that students develop or use as they work. Synopses of the problems are keyword searchable. Answers and solutions are provided, and many problems include hints” (retrieved February 5, 2012).

• MERLOT Mathematics: http://mathematics.merlot.org/
  “MERLOT is a free and open online community of resources designed primarily for faculty, staff and students of higher education from around the world to share their learning materials and pedagogy. MERLOT is a leading edge, user-centered, collection of peer reviewed higher education, online learning materials, catalogued by registered members and a set of faculty development support services” (retrieved February 5, 2012).

• Annenberg: http://www.learner.org/resources/browse.html?discipline=5&grade=3
  “Annenberg Learner uses media and telecommunications to advance excellent teaching in American schools. This mandate is carried out chiefly by the funding and broad distribution of educational video programs with coordinated Web and print materials for the PD of K–12 teachers. It is part of The Annenberg Foundation and advances the Foundation’s goal of encouraging the development of more effective ways to share ideas and knowledge”.

  “Annenberg Learner’s multimedia resources help teachers increase their expertise in their fields and assist them in improving their teaching methods. Many programs are also intended for students in the classroom and viewers at home. All Annenberg Learner videos exemplify excellent teaching”.

  “Annenberg Learner resources can be accessed for free at Learner.org, or can be purchased through the Web site or by calling 1-800-LEARNER” (Retrieved January 11, 2012).

• Teacher Education Materials http://te-mat.org/
  “This site was developed to support professional development providers as they design and implement programs for pre-service and in-service K–12 mathematics and science teachers. In this database you will find:

  A Conceptual Framework that ‘highlights key elements critical to the design and implementation of effective professional development programs, with numerous links to relevant reviews of materials and practitioner essays’ and Reviews of Materials that is a ‘searchable collection of reviews is the heart of the database, intended to help K–12 mathematics and science professional development providers more readily select materials appropriate for their program goals. Reviews may be browsed by purpose, subject/grade level, topic area, features, or author and title, using the navigation bar to the left’ ” (Retrieved January 11, 2012).

“This project investigated the scale-up of an innovative integration of technology, curriculum, and teacher professional development aimed at improving mathematics instruction in grades 7 and 8” (Retrieved February 5, 2012). The website provides links to publications about the project as well as lessons that can be used with middle school students.

Conclusion

While the response rate was disappointing, the project did uncover resources that could be useful to faculty planning a course or professional development seminar for middle school teacher candidates. Furthermore, the availability of accredited online courses provides an opportunity for practicing teachers to deepen their conceptual knowledge of mathematics when a face-to-face class is impractical or impossible. This project had personal roots, my search for an appropriate text for a course in functions. While no one finding meets that need, I will draw on material from several of the websites during the 2012-2013 academic year iteration of the functions course at Saint Peter’s College. I hope that this compendium of resources proves useful to adult mathematics educators everywhere.
References


“This workshop will reflect on the world around us using ‘mathematical eyes’, using the 5th lens of understanding—the context from which we teach” (Maguire 2006). At the last conference, the presenter revealed the perceptions and attitudes of a small group of people from diverse vocational settings to their use of mathematics in the workplace and everyday life. The emergent results reinforced research on the invisibility of mathematics. The participants in this workshop will have opportunities to investigate the mathematics which is always visible but not always seen – using mathematical eyes to develop our own and our learners’ appreciation of mathematics.

This workshop was attended by a diverse group of individuals which included a numeracy tutor delivering numeracy to post 16 students in a further education college, a PhD student, and at least two university professors (Mathematics and Engineering). Therefore, an interesting reflection of different levels and perceptions of mathematics ensued. The presenter reflected on the previous year’s conference where perceptions of individuals’ understanding and use of mathematics in the workplace had been explored. This workshop endeavoured to identify mathematics in the real world following the theme of the conference and work undertaken by the organisers prior to the conference with teachers and young people.

The framework used was developed by Maguire some years previously and the presenter had worked with students in Belfast using the framework and further developing the approach in teacher training situations in both London and Birmingham in 2009/10. In her introduction to the workshop, the presenter reviewed the framework of Brookfield’s 4 lenses: theoretical literature, colleague experiences, autobiographies as teachers and learners, and students’ eyes. Using mathematical eyes gives a new way of using the world around us as a resource, providing new ways of looking at familiar objects and of considering familiar objects leading to new insights into teaching and learning. This approach requires the ability to develop penetrating questions, to pose problems and develop insights into how the problems can be solved. Consideration of what is the concept of learning was made; an increase in knowledge, abstraction of meaning and interpretation aimed at understanding. The use of mathematical eyes to develop this learning encourages individuals to look for the mathematics which is visible but not consciously seen by the individual.

The presenter had provided photographs reflecting her journey from Birmingham to Dublin and Tallaght, using real context she had experienced. The intention was to provide an opportunity of real life context using collaborative group-work and discussion and mathematical eyes. In England, for over 7 years, the collaborative approach has been welcomed by the national standards authority, Ofsted, which assesses mathematics provision (Swain and Swan 2007). This approach moves from teacher-led activity enabling individual approaches. Ofsted revealed in their report that high achievement was reflected where “teaching that focuses on developing students’ understanding of mathematical concepts and
enhances their critical thinking, reasoning, together with a spirit of collaborative enquiry that promotes mathematical discussion and debate” (Ofsted 2006).

The participants worked in pairs (of their own choice) and each pair was given one of the photographs of a specific leg of the journey. They were given a short time to develop a view of what mathematics they could see in their photograph and to then develop some mathematical questions they could share with the rest of the group.

As is often the case in the real classroom with a diverse group of learners, both in experience and knowledge, some competition developed which raised the level of questioning shared; not the usual “how many people are there in the photograph?” but more incisive advanced thinking in some cases. The discussion revealed a total involvement and enthusiasm for the task and a keenness to produce acceptable results.

This reflected the value of using the approach with individuals/groups at whatever level. There was insufficient time to solve some of the complex ideas, however, there was agreement on this being an approach worth considering in the classroom.

The presenter concluded the workshop which had focussed on the mathematics which is visible as one moves within the real world but which is not always seen unless one focuses using the mathematical eyes.
References

The Swiss National project, «Literalität in Alltag und Beruf» (LAB - http://www.literalitaet.ch) develops and tests specific learning environments of further education. They are designed for adults learning at a basic skills level and combine numeracy with literacy education by using computers and Internet based tools. A general aim of the program is to achieve, and where possible, to obtain and enhance the employability of the educationally disadvantaged. Other objectives are: (1) Basic standards and competence descriptions for literacy and mathematics, (2) development of learning resources for the use of written language and for numeracy education, and (3) low-threshold access to ICT tools for learners. Development and course work are accompanied by research, which aims to describe specific tracks and difficulties of adult learning as well as aspects of persistence.

The teaching and learning management system, ILIAS, is the technical base of LAB-course work, on one hand supporting learning processes on-site and in the virtual space; on the other, giving opportunities to develop basic skills in computer literacy.

This article describes a range of courses from the LAB sub-project Bern, which works with participating companies and public sector programs for reintegration into the labour market. Four learning environments illustrate the use of information technology and stress the role of computer literacy in the acquisition of basic skills.

Key words: computer literacy, further education, basic skills, information technologies.
Project «Literacy for everyday life» – LAB

The primary aim of the Swiss National program, «Literalität in Alltag und Beruf» (LAB - http://www.literalitaet.ch), is to achieve, and where possible, to obtain and enhance, the employability of the educationally disadvantaged. The LAB course work is designed as a so-called stepping stone program, enabling adults, skills wise and psychologically, to enter the broad landscape of further education in Switzerland. An accompanying research program looks for measurable gains in competences but also for positive changes in self-perception and self-efficacy (cf. Hilbe & Hollenstein, 2010, p. 192ff). More specific objectives of LAB are: (1) development of basic standards and competence descriptions for literacy and numeracy, (2) design and test of learning resources for the use of written language and for numeracy education, and (3) low-threshold access to ICT tools for learners.

Under the lead of the Research Centre for Reading (University of Applied Sciences, Northwestern Switzerland), and the Department of Educational Science (University of Bern), field partners are assigned to the development work. Like its predecessor, «Illettrisme and New Technologies» (INT, cf. Sturm et al., 2009), LAB works with companies and institutions in the public sector.

The open source teaching and learning management system, ILIAS, is used as technical base of the LAB course work (www.leap.ch), and its user interface is customised to the specific needs of LAB students (cf. Sommer & Studer, 2010, p. 46ff and Hollenstein, 2010, p. 265ff.). All LAB-courses focus on an enhancement of basic skills. An implicit use of information technology and therefore the acquisition of basic computer skills, offers considerable advantages – as we like to illustrate.

Experiences in the field

In the following we will depict exemplary learning environments, developed and applied by the Adult Education Centre Bern (Volkshochschule Bern, www.vhsbe.ch), which is a major LAB field partner. But first some context information. A cardboard factory in Deisswil near Bern was closed down in April 2010. In June 2010 a group of investors bought the bankrupt’s assets with the aim to build an industrial park for small and medium-sized enterprises, called «Bernapark» (www.bernapark.ch). Consequently, many former employees of the cardboard factory were laid off.

The Adult Education Centre Bern, was commissioned by the investors to run a specific educational program for the affected people. It marked the restart of professional development and it had to increase the chances of reemployment. Fostering motivation and self-esteem despite the participants’ difficult situation were primary goals. The course group was highly volatile. Some participants got new jobs during the course period or were working temporarily – dropping out and coming back. In the course of winter/spring 2010/2011 there were 15 men (6 men with immigration background and German as a foreign language) and 1 woman. These were placed in three learning groups. The course duration was 20 weeks. In each week the students spent two hours in class on-site.
The learning management system ILIAS supported the course administration and kept track of the fluctuating group environment. In addition to learning activities related to literacy and numeracy, the students got acquainted with the online environment. In their own home, each participant had a computer with an Internet connection at her/his disposal. ILIAS was “the only” way to get the learning material and to submit the results of individual work. Therefore, the participants could go over the course content at home at their own pace.

Also, in the on-site lessons, the learning management system was used to develop basic ICT skills like navigating the World Wide Web, communicating through email or chat, saving, organizing, finding and retrieving files in a local or ILIAS-based file system etc.

This learning activity is illustrated by the following course description written by a participant and directed to new students:

“ILIAS is a Learning Management System. In about 20 days you’ll learn the basic computer skills. Each course day has its own programme with different tasks: writing letters, calculating, to exercise grammar with verbs, nouns and adjectives. Brain jogging, cleverness and everything that little John didn’t learn, big John will learn it here. Via www.ilias.leap.ch you’ll come to this site. Most of the things are self-explanatory but you’ll have to use your little grey cells, … and you have to read up. (But I realised over time: asking our competent mentors gets you quicker there.)”

Four learning environments illustrate the use of ICT and the role of computer literacy for basic skills in literacy and numeracy.

1) Writing? – Please copy! (using MS Word)

The exercise, “3 Boroughs”, develops writing skills using a keyboard. Firstly, three short texts about communes in the neighbourhood of Bern have to be read. Secondly they have to be types accurately into an empty MS Word file. The focus is neither orthography nor grammar, rather it is just the perception of sentence composition, the use of a keyboard as a writing tool and the use of ICT-based spelling correction. One reassuring aspect of copying text is that you cannot fail. Using the keyboard was new for (almost) every participant. The word processor supports the writing process by underlining misspelled words with a red line and they may be correct easily. But the allocated texts contain quite a few words not contained in the software dictionary (e.g. names of Boroughs). So, a student has to decide; does an underlined word show a spelling error or is it correctly spelled and marked because the “computer doesn’t know it”.

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Reflection by the course mentors: “Writing with computer is a new challenge for participants and isn’t connected with orthography and grammar and the risk to be responsible for spelling mistakes. Most copy mistakes, e.g. distortion of letters, are problems of being able to use the keyboard. So, negative connotations with writing don’t persist. Effects like this can’t be attained by handwritten copies.

If there are words underlined and marked as faults, we explain the participants that “computers” don’t know everything, that they are rather “dumb”. So participants develop a feeling for monitoring and control. Everything that is marked is: (a) not necessarily their fault, and (b) not a scolding from an error free, judgmental authority. All the marked text passages do is to attract attention and to desire a decision. So participants reflect their writing and may diminish their fear of writing.”.

2) “Writing on the wall”, but nice looking (using MS PowerPoint)

“Presentation of a hobby” is a learning environment where students present their hobby on six slides, using the presentation software PowerPoint, to include: description, equipment, location, audience and aspects of practice. Examples are “Railways”, “Football (soccer)” or “Working in the woods”, i.e. felling trees – all composed of text and pictures. The aim is to characterise a hobby by self-authored texts and graphical elements, e.g. from the World Wide Web.

Some participants drafted on paper, others started with blank PowerPoint slides. Most of the students used a template containing six sheets. One participant created a flyer and described the requested content in text. Notable is the work of one participant, who said at the beginning of the course: “But I recognize just the ‘On’-button of the computer, I haven’t done anything before with such a machine”. In a later course session, he didn’t find the time to start “Presentation of a hobby”. One week later he handed in his slides in print. He had worked autonomously from home and loaded the file correctly into ILIAS. As a precaution, he also printed his work at home. During the session he retrieved his file on the server with a little help of his mentor. Proud and deeply satisfied he said: “I’ve done it myself!” (… and the mentor shared his feelings).
Reflection by the course mentors: “Presentation software has didactical benefits: (a) Already the default layout lets text look attractive. In contrast, handwriting on a white board may look rather shabby. Using presentation software, students produce easily and have alternative designs to choose from, without further graphic know how; (b) The software urges the students to concentrate on important points, texts are meant to support the verbal presentation; and (c) Text and images can be arranged and corrected. Colours help to make the information process via text more effective.

The focus lies in choosing central attributes of a hobby and in the composition of text and images – it’s not about ‘creative writing’.”

3) ”What do I get?” (using MS Excel)

Exercises using MS Excel allow the participants in developing numeracy skills, of course teamed with literacy skills. They learn to understand some mathematical concepts of calculation. As in other learning environments, templates were heavily used. Some formulas were already embedded in specific cells of the worksheet used by the students. The main benefit of this approach is that participants may “play” with the template asking and answering the classic question “What if …?” Some resulting models may be highly informative, others just nonsense – but the underlying calculation mechanism is trustworthy. Finally, it is a qualitative process to choose a numerical scenario as a result. By individually probing the template and by discussing them in the group, the understanding of calculation contexts and mechanisms improves. But there are benefits on the emotional level as well. Students use numbers and formulas in a more playful way than they had learned in school.
4) Writing daily reports

Our course participants liked repeatedly and steadily using the learning environments. One of these iterating exercises is called “Daily Report” at the end of each on-site session. Students summarize in a log: (a) learning activities in a session, (b) what they have learned doing them, and (c) an evaluation of these learning activities, to feedback to the mentors. In terms of subject matter, they ought to differentiate writing, reading, numeracy and computer use. The participants dedicate their text to one of these learning fields. Reacting to these reports, the mentors can connect them to personal learning aims. Often, reports are entitled “computer use”, but deal with writing, reading or numeracy. These reports are often finished at home where the filenames have to be edited, locally saved and uploaded to the learning management system.

Conclusions

In the beginning of each course, all of the participants are interviewed as to their interests and individual course goals, e.g. “In which domain would you like to improve?” and “Which field is less important for you?”. Almost without exception, the participants tell us initially that they would like to learn about computers. During the course work the focus shifts
remarkably to some other subject. One participant said “Now we are able to use the computer; but we would like to improve on our writing skills”.

We conclude that a didactical use of information technology and therefore the acquisition of basic computer skills, offers considerable advantages in terms of basic skills education:

- “Computer” and “Internet” are no longer obstacles to further education
- A didactical and well designed use of ICT helps to decompose individual blockages and supports adult learning processes on a motivational and voluntary level:
  - (a) It brakes with burdened “school”-biographies by providing new approaches to old problems
  - (b) «Computer classes» are seen as less deficit-oriented then traditional basic skills classes
- The mastery of classical formal aspects like orthography or calculating procedures is less important
- Server based tools like these implemented in ILIAS are used for learning activities, both on and off site; this practice fosters ICT literacy in a non obstructive, implicit and goal driven way
- A learning management system is a very useful way of staying on top of fluctuating course settings.

Last but not least, the continuous use of ICT is a positive predictor for measurable gains in literacy skills (cf. Sturm, 2009. p. 242f.).

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References

Hilbe, R. und A. Hollenstein: Förderliche und hinderliche Bedingungen für das Schriftlernen Erwachsener. Münster [etc.] 2010
Notter, P. et. al.: Lesen und Rechnen im Alltag. Neuchâtel 2006
Sommer, T. und M. Studer: Eine Online-Plattform als Lehr- und Lernumgebung. Münster [etc.] 2010
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ALM Bibliography Anthem (tongue in cheek)

To the tune of ‘We didn’t start the fire’ and apologies to Billy Joel

Crowther and Cockroft, Mathematics going soft
One and one not making two, Had employers in a stew
Making mathematics count, Was what they all talked about
Coben and O’Donoghue, formed ALM to sort it out

Chorus

We didn’t start the fire
‘Coz there are Adults learnin’ as the world is turnin’
ALM takes it higher, so everyone can make it
In their work and lives and on and on and on and ...

Kathy Safford, David Kaye, Anistine, USA,
Graham Griffiths, Catherine Byrne, Rinske from the Netherlands
Armin Hollenstein, Mieke van Groejnstein,
Brynhild, Hanne C, Liston M & Hussey C

Chorus

Beth Kelly, Jürgen Maaß, Chris Klinger, Tracy Part
Rasch G, Gramsci, Gates, Glaser, Hammersly
Geoff Wake, Cuffé, Keogh, Ubiratan D’Ambrosio
Carraher, Nunes T and Pinto de Santos

Chorus

Maguire T, L A Steen, Hoogland and Hembree
Niss, Noss, and Hoyles C, Harris, and Fitzsimons G
Wartofsky, Wenger E, Colleran and Leedy
Schoenfeld, Silverman, O’Cairibre and Jorgensen

Chorus

Lisa, Niamh from Limerick U, Tine, Double U
Evans, Williams, Iddo Gal and Zaeed too,
Eileen Goold, Johansen, Fennema and Johnston
Tout, Barr, Bishop, Cole
Use your name to fill this hole

Chorus
Seabright, Lynch B Barbara, Miller-Reilly.
Rumbelow, O’Sullivan, and a few that couldn’t come
Many hadn’t got the time, others didn’t have the dime
Tallaght isn’t glamorous so some have chosen Lichenstein

Chorus

Discourse, Activity, Tacit Rationality
Bernstein & Engeström, Mouwitz & Gustafsson
Real maths & Functional, Techno-mathematical
Easier to criticise, let’s open mathematics eyes

Chorus

This silly song is nearly done, hope you had a little fun
If your not mentioned this time, it’s ‘coz I couldn’t make it rhyme
Research, Lit review, case studies, more to do
Have to stop this foolishness and go and write the thesis!

Last chorus
We didn’t start the fire
‘Coz there are Adults learnin’ as the world is turnin’
ALM takes it higher, so everyone can make it
In their work and lives and on and on and on and ... \[...\]

John Keogh & Terry Maguire

ALM 18, Tallaght, Dublin, June 2011.