Objectives

Adults Learning Mathematics – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field, which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

• Research and theoretical perspectives in the area of adults learning mathematics/numeracy
• Debate on special issues in the area of adults learning mathematics/numeracy
• Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

The ALM International Journal will be published twice a year.
ISSN 1744-1803

Editorial Team:

Dr. Javier Díez-Palomar, Universitat de Barcelona, Spain [Chief Editor]
Dr. Anestine Hector-Mason, American Institutes for Research, Washington, DC, USA [Editor].
Graham Griffiths, Institute of Education, University of London [Guest Editor]
Dr. Katherine Safford-Ramus, Saint Peter’s College, Jersey City, New Jersey, USA
Dr. Chris Klinger, University of South Australia, Adelaide, Australia
Kees Hoogland, APS - National Center for School Improvement, Utrecht, the Netherlands

Editorial Board:

Prof. Alan Bishop, Monash University, Melbourne, Australia
Prof. Marta Civil, University of North Carolina, Chapel Hill, U.S.
Prof. Diana Cohen, Kings College London, UK
Dr. Jeff Evans, Middlesex University, London, UK
Dr. Gail FitzSimons, Monash University, Melbourne, Australia
Prof. Gelsa Knijnik, Universidade do Vale do Rio dos Sinos, Brazil
Prof. John O’Donoghue, University of Limerick, Ireland
Prof. Wolfgang Schloeglmann, University of Linz, Austria
Prof. Ole Skovsmose, Aalborg University, Denmark
Dr. Alison Tomlin, Kings College London, UK
Prof. Lieven Verschaffel, University of Leuven, Belgium
Prof. John Volmink, Natal University Development Foundation, Durban, South Africa
Prof. Tine Wedege, Malmö University, Malmö, Sweden
Adults Learning Mathematics – An International Journal

Special Issue

Critical Moments in Adult Mathematics Education

In this Volume 9(1)

Editorial
Anestine Hector-Mason

Construing Mathematics-Containing Activities in Adults’ Workplace Competences: Analysis of Institutional and Multimodal Aspects
Lisa Björklund Boistrup & Lars Gustafsson

From Standards-led to Market-driven: A Critical Moment for Adult Numeracy Teacher Trainers
Martyn Edwards

New PIAAC Results: Care is Needed in Reading Reports of International Surveys
Jeff Evans

Integrating Real-World Numeracy Applications and Modelling into Vocational Courses
Graham Hall

Counting or Caring: Examining a Nursing Aide’s Third Eye Using Bourdieu’s Concept of Habitus
Maria C. Johansson

A Workplace Contextualisation of Mathematics: Measuring Workplace Context Complexity
John Keogh, Theresa Maguire, & John O’Donoghue
Editorial

Anestine Hector-Mason
American Institutes for Research
Washington, DC, USA
ahector-mason@air.org

We are pleased to deliver the first issue of Volume 9 of *Adults Learning Mathematics: An International Journal*, and to present you with six compelling articles that add, tremendously, to the advancements of knowledge in the area of adult numeracy. The authors of these articles come from various walks of life and bring some unique perspectives to the intellectual discussion of adult mathematical thinking and mathematical capacity, and the implication of these for the realities of our daily experiences in life, school and the workplace. The content and discussions in all of these articles remind us, not only of the scholastic value of mathematics, but also of its sociocultural necessity, and especially its necessity in the workplace. By their work, all of the authors show remarkable dedication to inquiry of adults learning mathematics, while making some very important contributions to the field.

The articles in this special issue comprise the first volume of papers drawn from the submissions to the 20th International Conference of Adults Learning Mathematics – A Research Forum. The conference, held at the University of South Wales in Caerleon (UK), was concerned with ‘critical moments,’ and these articles, in a range of ways, have addressed this. Some take the idea of a significant moment in time from a global perspective (e.g., Edwards and Evans) while others take a more local, personal view (e.g., Hall and Johansson). Despite their varied topics, you will notice that most of the articles focus on a core theme: mathematics in everyday life, which includes mathematics in the workforce.

The first article, *Constructing Mathematics-Containing Activities in Adults’ Workplace Competences*, is an example of what Wedege (2010b) labels as sociomathematics: “a research field where problems concerning the relationships between people, mathematics and society are identified, formulated and studied” (p. 452). In this article, Lisa Björklund Boistrup and Lars Gustafsson utilize a multimodal, semiotic perspective to examine video and interview data on adult work competencies while construing mathematics-containing themes in two workplace settings (road haulage and nursing). In doing so, the authors reinforced the idea of mathematics as social activity that involves measuring, which is key to the mathematics adults in all cultures do or experience in their everyday lives and in the workplace.

In article two, *From Standards-led to Market-driven: A Critical Moment for Adult Numeracy Teacher Trainers*, Martyn Edwards examines the critical moments for adult numeracy teacher trainers, and discusses the issue of standards-led versus market-driven adult numeracy activities. Edwards reviews developments in adult numeracy teacher training in
England over the last 15 years, from a professional standards-led approach to the current deregulated “immersion in practice” model. He critically evaluates the DfES/Fento (2002) subject specifications for numeracy teachers, reviewing research into how these specifications were implemented in a range of different HE settings. Using Bernstein’s concept of vertical (expert subject) knowledge and horizontal (subject pedagogy and practice) knowledge, and Shulman’s seven types of teacher knowledge, he explores possibilities for designing more effective numeracy teacher education in the future. His recommendations include mathematics subject knowledge entry requirements, and opportunities to develop and critically reflect on subject pedagogy, including through practitioner-led research, throughout numeracy teachers’ careers.

Jeff Evans, in New PIAAC Results: Care is Needed in Reading Reports of International Surveys, presents a compelling discussion of the results from the Survey of Adult Skills, also known as PIAAC (Programme for the International Assessment of Adult Competencies), and opens a window of inquiry into questions concerning the relevance of the results and when and how to be sceptical when reading international survey reports. Evans’ discussion takes readers through a process of understanding these studies by considering conceptual issues, methodological validity of research design and execution, and presentation of the results. In doing so, the author identifies several sample items for numeracy published by the OECD (2012), and examines illustrative results identified by the Australian Bureau of Statistics. Evans’ work opens up questions concerning the relevance of the results, and their applicability to varied contexts – inquiry that can broaden discussions about the relevance and value of international mathematics assessments.

In Integrating Real-World Numeracy Applications and Modelling into Vocational Courses, Graham Hall describes practitioner research being conducted by vocational course tutors at a further education college in Wales, which aims to develop a framework of learning strategies that will interest and motivate students. Two key goals of the project are to develop students' numeracy skills within their vocational areas; and to help them to gain transferrable skills in critical thinking, creativity, teamwork and collaboration, and learning self-direction. A framework proposed by Tang, Sui, & Wang (2003) has been adapted for use in courses, and numeracy applications and modelling into vocational studies have been identified at five core levels (i.e., Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research) which are examined further through case studies that help to determine how these five levels are incorporated in engineering, construction, computing, and environmental science courses. Although the research is still underway, there appears to be some promise related to student motivation: teaching staff observed that improvements have occurred in student use of specialised mathematical vocabulary; their combined use of numerical and algebraic methods in problem solving; and abstract reasoning, and a deeper level of understanding of the mathematics used in problem solving.

The next article, Counting or Caring: Examining a Nursing Aide’s Third Eye Using Bourdieu’s Concept of Habitus is derived from an analysis of observations and an interview with a nursing aide who was followed and observed during her work in a semi-emergency unit in Sweden. In this article, Maria C. Johansson examines the mathematical complexities in the process of going from school to a workplace and the capacity of her research subject, Anita, to make the transition between different mathematical activities that both involve and require learning. Johansson posit that transition can occur in the workplace and be connected to critical moments in the execution of work tasks. The author adopts a social critical perspective in this article and initiates a discussion about the transitions between mathematical activities in the workplace and how the values given to these different activities can be understood.
In the final article, *A Workplace Contextualisation of Mathematics*, John Keogh, Theresa Maguire, & John O’Donoghue describes research they undertook, which identified the mathematics activity that underpinned low-skilled, low paid job, and aligned it with the National Framework of Qualifications in Ireland. The authors found that although the mathematics expertise deployed was modest in terms of complicatedness, it was used by workers in circumstances that were both sophisticated and volatile in varying degrees. The findings suggest that mastery of routine mathematics alone was a poor indicator of a person’s ability to ‘do the job’. The authors argue that there are varied factors exist and operate in ways that can ‘complexify’, otherwise routine, mathematics, which also pose consequences related to concealing the role and visibility of mathematics in the workplace.

Presented below are these six articles, which draw attention to the issues in workplace mathematics, the differences between school mathematics and workplace mathematics, and the mathematics in real life.
Construing Mathematics-Containing Activities in Adults’ Workplace Competences: Analysis of Institutional and Multimodal Aspects

Lisa Björklund Boistrup
Stockholm University
<lisa.bjorklund@mnd.su.se>

Lars Gustafsson
Malmö University
<lars.gustaf@telia.com>

Abstract
In this paper we propose and discuss a framework for analysing adults’ work competences while construing mathematics-containing “themes” in two workplace settings: road haulage and nursing. The data consist of videos and transcribed interviews from the work of two lorry-loaders, and a nurses’ aide at an orthopaedic department. In the analysis we adopt a multimodal approach where all forms of communicative resources (e.g., body, speech, tools, symbols) are taken into account. We also incorporate the institutional norms of workplace activities into the analysis. We coordinate a multimodal social-semiotic perspective with a learning design sequence model (Selander, 2008) which makes explicit the institutional framing. Adopting this framework enables us to understand learning as communication within a domain, with an emphasis on content matters, interpersonal aspects, and roles of communicative resources and artefacts. We describe a tentative theme (Measuring: precision through function and time), and illuminate how workplace specific resources for measuring provide efficiency and function.

Keywords: workplace, mathematics, competence, multimodality, learning, institutional norms, interpersonal

Introduction
An overarching aim of the project to which this paper is connected is to analyse and understand adults’ mathematics-containing work competences (Wedge, 2013). In doing this we want to investigate how we can learn from the workplace without taking assumptions of school mathematics for granted. In subsequent investigations and papers we will relate these findings to vocational education and general schooling. In this paper we propose and discuss an analytical framework for analysing mathematics-containing activities in adults’ work
competences where different functions of multimodal communication and institutional aspects are addressed. Two situations from our data, both video collected, will serve as a starting point for the article. We briefly describe them then address them later in the article.

In one situation, we followed two lorry-loaders when they loaded a trailer. As will be shown later on, one essential resource in this task was the loading pallets on which most of the goods were positioned. We will describe how we can identify measuring in the work performed by these lorry-loaders and how our analytical framework helps us to broaden our understanding of the mathematics-containing activity. In another situation we visited a nurses’ aide at an orthopaedic department. Her main responsibility was to put plaster cast on injured limbs. Measuring was also identified here and was elaborated using the framework.

Research on and approaches to workplace mathematics

In the literature concerning workplace mathematics we have distinguished a number of themes that are particularly relevant to us. They are described here and we pay extra attention to research in relation to measuring. In addition, different possible approaches for research in this field are described.

Mathematics in the workplace

Research on mathematical practices in the workplace has been carried out since at least the beginning of the 1980s: For example, the Cockcroft report (1982) which initiated several other studies.

Research on workplace mathematics has been described as a field which has passed through different phases (Bessot & Ridgway, 2000; FitzSimons, 2002, 2013; Hoyles, Noss, Kent, & Bakker, 2010; Wedege, 2010a). In the early years researchers presumed that mathematics was easily observable and visible in workplace activities, and frequently such studies resulted in (long) lists of mathematical contents described in “school mathematics” terms (Fitzgerald, 1976). Many of these studies have been criticized for having been conducted with, what has been described as a mathematical lens (Zevenbergen & Zevenbergen, 2009) or mathematical gaze (Dowling, 1996, 1998), or with a far too narrow conception of mathematics/numeracy (Harris, 1991; Noss, 1998).

Seminal works on the use of mathematics in informal workplace or everyday settings during the 1980s and 1990s draw attention to, for example, differences in strategies and cognitive structures between “school mathematics” and “out-of-school mathematics” and to the fact that schooled and un-schooled individuals perform and succeed differently in everyday and workplace practices as compared to school contexts (Lave, 1988; Nunes, Schliemann, & Carraher, 1993).

Research on workplace mathematics has, during recent years, been dominated by socio-cultural perspectives. Increasingly sensitive theoretical and methodological tools have been used to reveal the complexity of mathematical practices at work. One finding is the fact that mathematics in work is often hidden in activity, culture, social practice, and artefacts. This has been used to explain why it is so difficult to classify these mathematical practices in school-mathematical terms and, when so classified, how the complex use of mathematics in workplaces is reduced to simple computations, measurements, and arithmetic (Gustafsson & Mouwitz, 2008; Hoyles, Noss, & Pozzi, 2001; Keogh, Maguire, & O’Donoghue, 2010).
Mathematics as activity: The example of measuring

Bishop (1988) identified six pan-cultural activities which can be characterized as mathematical activities. These are: counting, locating, measuring, designing, playing, and explaining. In this paper, we focus mainly on measuring which, according to Bishop, is concerned with “comparing, with ordering, and with quantifying qualities which are of value and importance” (p. 34).

We are looking at practices which include measuring in a broad sense. Measuring is central in mathematical activities in people’s everyday lives and in workplace practices in all cultures. Several studies have shown the importance of measuring in different occupations (for an overview see, e.g., Baxter et al., 2006). Among these are studies on carpenters, carpet layers, nurses, process- and manufacturing industry workers, and so forth. Other more recent examples are Bakker, Wijers, Jonker, and Akkerman (2011) who write about the use, nature, and purposes of measurement in workplaces; a study of process improvement in manufacturing industry (Kent, Bakker, Hoyles, & Noss, 2011); a study of boat-building (Zevenbergen & Zevenbergen, 2009); and a study of telecommunication technicians (Triantafillou & Potari, 2010).

Measuring is closely linked to estimating, and the boundaries between these activities are not obvious. Adams and Harrel (2010) have, as part of a more extensive study, presented observations and interviews from four occupations, and concluded that experienced workers often replace measuring with estimation. One important conclusion is that estimation is a complex activity that is learned by experience, and is based on a different rationality from conventional school-methods for measurement which may focus on units and calculations (at least in secondary school). In this article we will use the term measuring linking to the concept of activity (i.e. doing) rather than the generic label measurement, to address the human activity of measuring. We also include estimating in the concept of measuring.

Adopting a subjective approach when researching adults’ competence

In the literature on mathematics in the workplace, two approaches can be identified (Wedege, 2013). In the subjective approach, the interest lies in mathematics as part of personal needs and professional competences in working communities and in various situations. In the general approach, the interest lies in societal demands or demands made from the perspective of school mathematics. Drawing on Bernstein’s (2000) pedagogical models, performance and competence, Wedege (2013) also identifies professional competence as construed from the workplace rather than taking school mathematics as a starting point. In the research described here, we draw on the subjective approach when we strive towards capturing the mathematics-containing activities within workers’ competences. In this article we present a tentative finding of what could be called a theme in professional competence within the sectors of nursing/caring and vehicle/transport. Adopting our analytical framework from this article, we are able to construe wider themes between activities in two sectors of work. These themes will in subsequent research and papers be connected to a general approach when we compare our findings to the demands made within school.

In this article, we draw on the notion of competence. Ellström (1992) describes competence as an individual’s readiness for action with respect to a certain task, situation or context. Wedege (2001) concurs and opposes a view of competence as consisting of “objective” competencies defined as being independent of individuals and situations. According to Wedege (2001), competence is:

• always linked to a subject (person or institution)
• a readiness for action and thought and/or an authorisation for action based on knowledge, know-how and attitudes/feelings (dispositions)
• a result of learning or development processes both in everyday practice and education
• always linked to a specific situation context (p. 27).

The term competence can be further understood from two perspectives: (a) formal competence in terms of authorisation; for example, that a person has adequate education for a given position; and (b) real competence in terms of whether a person will really be able to demonstrate the abilities that are identified; for example in a particular certification (Wedege, 2001; 2003). In terms of our research interest here, the second meaning is more relevant.

**Addressing the socio-political through the notion of institutional framing**

We position this paper in a socio-political paradigm – paradigm is here understood according to Lerman (2006) – in mathematics education. This is connected to sociology and critical theories (Valero & Zevenbergen, 2004; see also Ernest, Greer, & Sriraman (Eds.), 2009). Mathematics incorporates means for understanding, building, or changing a society (Mellin-Olsen, 1987). Skovsmose (2005) acknowledges this (see also Jablonka, 2003; Gellert & Jablonka, 2009), whilst also stressing that mathematics does not hold any intrinsic good; instead mathematics can be used for different purposes in society and people’s lives. Thus, there is a need to address the role of the use of mathematics in society and in this article we incorporate institutional aspects of workers’ mathematics-containing activities.

We view the institutional context as always present. An early example of a theoretical discussion of this is given by Popkewitz (1988), who considers institutional framings as one way to address social and critical aspects in studies of school mathematics (see also Mellin-Olsen, 1987). Also, in work-places the institutional context and societal dimensions are always present (e.g., Salling Olesen, 2008). Here are included dominant discourses, the use of artefacts developed over time, the division of time, established routines, workplace structures, and authoritative rules (Selander, 2008, drawing on Douglas, 1986). A similar view is described by Bishop (1988, p. 36) when he writes about the development of units, and systems of units: “there is a clear progression, with the main idea being that of the stronger the environmental and social need the more detailed, systematic and accurate the measure”. As we will show in our analysis and findings, what constitutes an accurate measuring unit may be quite different in the workplace from what is usually emphasised in school.

Institutional aspects were addressed by Wedege (2010b) when she proposed the concept of sociomathematics. She described sociomathematics as both a subject field combining mathematics, people, and society, and a research field. We are also inspired by FitzSimons and Wedge (2007) who adopted Bernstein’s (2000) concept of horizontal and vertical discourses (see also FitzSimons, Mlcek, Hull, & Wright, 2005). Bernstein refers Vertical discourse to knowledge within a discipline, such as academic mathematics. This knowledge is coherent and systematic; the horizontal discourse refers to contextual knowledge and a relevant example for us is the context bound mathematics used and developed in the workplace. In the study by FitzSimons et. al (2005), activity theory (Engeström, 2001) was adopted as a theoretical framework, and the main findings were that mathematically straightforward skills become “transformed into workplace numeracy competence, when the complexities associated with successful task completion as well as the supportive role of mediating artefacts and the workplace community of practice are taken into account” (p. 49).
Analytical framework

In this section we present our analytical framework where a theory of communication – multimodal social semiotics (e.g., Van Leeuwen, 2005) – is coordinated with a model of a learning design sequence. Design is here understood in a broad sense, for example including both aesthetic and functional aspects. The term coordinate implies that the two theoretical approaches are compatible with respect to underlying assumptions (Prediger, Bikner-Ahsbahs, & Arzarello, 2008).

Learning as multimodal communication

In this article we attempt to problematise learning in order to avoid the term learning becoming a black box (Ellström, 2010). Ellström refers to the term, black box, to learning as it is in studies on innovations in workplaces. Learning is here described as a key concept, but it is not really spelled out how it is operationalised in the studies. We view learning as closely connected to human activity and understood as meaning-making towards an increased communication in the world through the communicative resources of a discipline (Selander & Kress, 2010; see also Björklund Boistrup, 2010). Learning in a work-place constitutes, at least in part the competence that the worker gains over time. This competence is not something fixed, but changes and may evolve over time. In operationalising learning, we discuss knowing that is part of workplace activities, and hence the worker’s competence, rather than discussing learning as such. By using the term knowing instead of knowledge we want make clear that we do not take into account an objective knowledge “out there” to be learnt. Instead, knowing is viewed as constructed and construed in communication among humans throughout history (Foucault, 2002; see also, e.g., Delandshere, 2002; Valero, 2004b, Volmink, 1994). What valid knowing is and how it is demonstrated in communication is not set in stone. At different times throughout history, the perception of what qualifies as important knowing has changed and will continue to do so.

In this article we draw on a multimodal approach when we adopt social semiotics as part of an analytical framework (Van Leeuwen, 2005). In a multimodal approach, described by Selander (2008; see also Björklund Boistrup & Selander, 2009), all modes of communication are recognised. Communication in a multimodal perspective is not understood in the same way as communication in a narrow linguistic perspective, focussing on verbal interaction only. Rather, all kinds of modes are taken into consideration, such as gestures, and gazes, pictorial elements and moving images, sound, and the like. Modes are socially and culturally designed in different processes of meaning-making, so that their meaning changes over time. It is also the case that “content” in one kind of configuration (e.g., as a measure on a dip stick), will not necessarily be exactly the same content in another configuration (e.g., as a number on a device for filling the oil):

Different representations of the world are not the “same” in terms of content. Rather, different aspects are foregrounded. In verbal texts we read linearly, within a time frame, whilst a drawing will be read within a space frame. And a graph does not represent a knowledge domain in the same way as numbers does [sic]. The modes that are “chosen” in a specific situation reflect the interest of the sign maker, and they are therefore not arbitrary. (Björklund Boistrup & Selander, 2009, p. 1566)

We argue for the importance of understanding multimodal communication to be able to fully understand a phenomenon such as mathematics knowing and learning in a workplace. In social semiotics, three meta-functions are often operationalized in analysis (Halliday, 2004). Halliday focused mainly on written and spoken language in his work but in this article, drawing on Van
Leeuwen (2005), we adopt the meta-functions in connection with a multimodal approach. These meta-functions are: the ideational, the interpersonal, and the textual. In Morgan (2006), these functions are used with a focus on the construction of the nature of school mathematics activity. In this article, we start out with the meta-functions as used by Kress et.al. (2001; see also Björklund Boistrup & Selander, 2009). The ideational meta-function is related to human experience and representations of the world (Halliday, 2004). Here there is a possibility to address the content, the “what-question” of a communication. In this article we look for measuring activities and resources in lorry-loaders’ and nurses aides’ practices and competences. The interpersonal meta-function is about how language (used in a broad sense in this article) enacts “our personal and social relationships with the other people around us” (Halliday, 2004, p 29). In this article we examine the roles of measuring activities for and in relations between the people involved. The textual meta-function is related to the construction of a “text” and this refers to the formation of whole entities (Halliday, 2004). With a multimodal approach, the term text refers to multimodal ensembles which are communicatively meaningful and part of the overall pattern of the actual communication. Here we are interested in what roles resources and communicative modes play in the measuring activity.

A model for understanding learning in other-than-school contexts

We draw on a model where a multimodal approach is connected to an institutional framing (Selander, 2008; Selander & Kress, 2010): a design theoretical perspective of learning.

This first model (Figure 1) gives the general principles for how communication, learning, and knowing can be addressed without starting from the perspective of a school setting, but considering meaning-making and learning as something always present. The starting point, the “situation”, is here to be taken as any other-than-school setting, for example a workplace. The worker and his/her work are embedded in a social practice with different kinds of social norms and with different semiotic resources at hand. The duration of the process that the model

Figure 1. A learning design sequence (Selander, 2008, p. 16).
captures can be rather short (seconds) but also longer (like hours or days). Selander (2008) writes:

In many instances we are put in situations where we try to figure out the challenge and what standpoint and action that is meaningful. It could be situations where we ask ourselves if the bus ticket still is of value or if we can swap a book, for example given as a present, for another one in the book store. In each such micro situation we also learn something about what is usual or “proper”, about restrictions and regulations etc. And there are also moments of creativity when we try out different solutions. (p. 14-15)

It could be possible to use this general learning design sequence to analyse what a person is doing at work. A person who is performing a well-known task is now and then met by an explicit learning purpose while working. It may be a situation where an innovation of some kind is needed in order to facilitate the work (Ellström, 2010). Even more explicit is the learning purpose when the person is new at her/his job. Even though the model by Selander (2008) is relevant for a study of learning and knowing mathematics at a workplace, we find the next model more suitable for our purpose. The reason for this is that we, as the research team, change the situation when we are present, and even more when we pose questions during the filming of the activity. The model that we use as our analytical frame is the Semi-Formal Learning Design Sequence.

![Semi-formal - LEARNING DESIGN SEQUENCE](image)

*Figure 2. A learning design sequence – semi-formal (Selander, 2008, p. 17)*

The idea behind the semi-formal learning sequence in Figure 2, is that the starting point is a setting (not a situation, as before) where the learner is confronted by an articulated learning purpose. In our case it is mainly the workplace that constitutes the setting where there are institutional norms affecting what is taking place and what is counted as relevant knowing. When we as researchers pose questions (see below for a description of our interviews), the worker is invited to meta-reflect on her/his work. There are then transformations taking place when the worker communicates through her/his actions, engaging with different artefacts, and then describes and explains the working process through speech and gesture, and so forth. In this sense both the primary transformation unit – the actual work – and the secondary
transformation unit – answering our questions and showing us the tools and processes of the work – are going on at the same time.

Methodology

The research design of the qualitative study for which this article is written is a case study. When using the term *case study*, we draw on Yin’s definition (1989):

A case study is an empirical inquiry that:

- investigates a contemporary phenomenon within its real-life context, when
- the boundaries between phenomenon and context are not clearly evident, and in which
- multiple sources of evidence are used (p. 23)

The phenomenon we are interested in here is to learn more about mathematics within the work and competences of lorry-loaders and nurses’ aides. More specifically we are interested in how we can analyse the complexity of mathematics interwoven in work. Our data gathering methods consist of:

- **Videos** which were filmed at one or two visits at each work-place. We followed one worker (or two), who was doing her/his regular work, with a hand held camera for about one hour. As a back-up, we also recorded sound with additional sound recorders. In total we visited six work-places, three in each sector.

- **Apprenticeship interviews** which were performed when possible during the filming. With apprenticeship interviews we mean that we took the role of a person trying to learn the work processes that the worker was engaged in. We then posed curious questions to the worker during her/his work.

- **Photographs** which were taken during our visits with a special focus on signs, notices, artefacts, etc.

- **Interviews** which were performed after the first visit at the workplace. These have so far been performed by Maria C Johansson as part of her PhD process (Johansson, 2013; in preparation). We used excerpts from these interviews to inform our understanding of the video data.

This article is based on data from two workplaces: one road-haulage company and a plaster unit at an orthopaedic department of a hospital.

Analysis and example of findings: measuring activities

The analytical framework that we present in this article is connected to the subjective approach mentioned earlier (Wedege, 2012). Here we pay attention to adults’ work competences and the activities we can construe as being possible to connect with mathematics. Our emphasis is on learning/communication in a workplace setting and we view workers’ actions as communication, as well as learning and knowing.

In the following, we utilise the analytical framework of the three meta-functions outlined above (ideational, interpersonal, textual) to describe the measuring activities as part of lorry-loaders’ work competence and of nurses’ aides’ work competence. The kind of measuring that we focus on in this analysis is what Bishop (1988, p. 34) labels “quantifying qualities which are of value and importance.” We also use the Learning Design Sequence model above, in how we
view the presence of the institutional framing. The three meta-functions are actually interwoven and it is an analytical construction to tease them apart. This may, in a systematic and structured way, bring forth findings that we otherwise would not capture. This also causes the same “events” to turn up more than once in the analysis, but with different emphasis.

Lorry-loaders

At the road-carrier company, we visited lorry-loaders, one of whose tasks was to load trailers according to specifications provided in written forms. The form was developed by administrative staff in the office. While we were there, one trailer was loaded using forklifts, and in discussions we were told about the written loading form (see Figure 3) which specified, for example, the number of pallets, the weights of the goods, the companies’ names for delivery (these names have been deleted in the photograph), and where different pallets were intended to be unloaded. In the first excerpt, the two lorry-loaders are talking with two persons from the research project before they start to load one trailer. One of them, Con (pseudonym), describes how they decide whether to load the trailer in one or two layers (Excerpt 1). The transcripts are made multimodally. In Excerpt 1, we identify Time, Speech (what people say and how they say it), Body (what people do including resources and artefacts), and Gaze (where people look).

<table>
<thead>
<tr>
<th>Time</th>
<th>Speech</th>
<th>Body</th>
<th>Gaze</th>
</tr>
</thead>
<tbody>
<tr>
<td>03:11</td>
<td>The ideal is that you can get it [the trailer] fully loaded. But for this trailer we load now, well, we, I, have seen the loading form before.</td>
<td>Nods now and then. Points at trailer. Two hands 20 cm apart [marking the top and bottom of form].</td>
<td>Looks at research staff.</td>
</tr>
<tr>
<td>03:21</td>
<td>Then I know that we can load it without the dual goods. Dual goods means two pallets on top of one another.</td>
<td>Moves hands up and down. Right hand above left hand with a distance in between.</td>
<td>Looks at research team.</td>
</tr>
<tr>
<td>03:27</td>
<td>Then we utilize the whole space to make the safest solution possible.</td>
<td>Moves flat hands from center and out [indicating a surface]</td>
<td></td>
</tr>
</tbody>
</table>

Later on, when the workers started loading, the use of the pallets becomes clearer. Each pallet was positioned “horizontally” in the trailer. In this case there is room for two pallets beside each other along the trailer’s width. If instead they were positioned vertically there was room for three. This was also explained in a communication after the trailer was finished loading. Con explained how the size of the pallets, 800 mm x 1200 mm, makes this possible. In Figure 3 the pallets in the trailer are shown and it is also possible to get a glimpse of the loading form that Con describes and shows with his hands early in Excerpt 1.

In the following, we describe our analysis where we operationalise the social semiotic meta-functions and where we also coordinate with the learning design sequence by Selander (2008). The concepts from the Learning Design Sequence (Figure 2) are in italics and the analysis is mainly organised through the meta-functions.

- Ideational meta-function: We analysed the data from the lorry-loaders, looking for human experiences and representations of the world (the content, the what-question) in relation to the measuring we could construe. We then construed a measuring activity in the institutionally framed setting where the lorry-loaders used the loading pallets (i.e. resources) as measuring units for the actual goods to be carried. Here the workers did not use the measuring means and units normally used in school, such as using a measuring tape to find out the two lengths in centimetres, and then calculate the area.
- Interpersonal meta-function: When analysing the data from the lorry-loaders for personal and social relationships, we were able to capture how the informal measuring activity via the pallets entailed their involvement in the process on behalf of the
customer, and also gave a certain amount of control to the loaders. Our assumption is that the use of pallets as measuring resources saves time, which in the end lowers the cost for the customer. This may be seen as one purpose with the use of the pallets. The pallets were also communicative resources for the two lorry-loaders who, almost without any talking, communicated on how to position the pallets on the trailer when carrying them on the forklifts. When Con told the research team about his work we could identify engagement and an interest in making clear what he meant and generally in his work. This analysis is based on his speech and the many gestures. During this meta-reflection there were many transformations between speech and gestures.

- Textual meta-function: When looking at the multimodal text that was communicated to us as visitors through actions, speech, gestures, etc., we analysed the roles of, in this case, the informal measuring activity through the resources of the pallets. Our finding is that the pallets took the role of facilitating the measuring, as they provided a measuring function in themselves as well as a means for efficiency and effectiveness. Another resource, the written form, made the measuring activity visible for people involved. As shown in Excerpt 1, we could identify how there are transformations between different communicative modes which also forms the activity. One transformation goes from the written loading form to the loading process. This transformation concerns both media (from written form to physical activity) and modes (from writing in words and symbols to speech, body movements, and gaze). During the loading, Con ticks off the things that are loaded, an activity which constitutes a new transformation.

- Institutional norms: The loading form is normally used at this workplace and formed the situation. In this workplace, its use is a long-standing tradition. The pallets are standardised according to the transport sector regulations.

**Nurses’ aide (plastering)**

In the orthopaedic department of a hospital, we visited a nurses’ aid who specialised in plastering. During our visit, she put plaster on an arm and hand of a patient who had an injury to his thumb. In this situation we were mainly silent and the chat was between the nurses’ aide and the patient. For this example we have chosen only to present pictures. In Figure 4 some details from the room where it took place are shown. It is also possible to see how the nurses’ aide rolls out dry plaster wrap on the arm. The analysis mainly is focused on this action.
In the following we describe our analysis of measuring activity from the work performed by this nurses’ aide. Similar to the previous section, the concepts from the learning design sequence (Selander, 2008) are in italics and the analysis is mainly organised through the social semiotic meta-functions.

- **Ideational meta-function**: We were able to construe a measuring activity where the setting was a room that was designed for plastering. There were boxes with different kinds of plaster stored on shelves and there were appropriate tools present (resources). Prior to the actual plastering process, the nurses’ aide measured up with the dry plaster wrap directly on the patient’s arm. The aide then used the first measuring as a unit and made repeated folds based on this unit before finally adhering it to the patient’s arm. The resource for measuring here is the plaster itself.

- **Interpersonal meta-function**: This plastering activity is very important with respect to the patient (in the healing process). Measuring directly on the arm may then be the most accurate. It should also look neat and tidy (caring about the patient). The nurses’ aide described the procedure of plastering to the patient as she worked. This also seemed to act as a calming function and simultaneously gave her an opportunity for meta-reflection on her activity. Here we could identify interest and interaction.

- **Textual meta-function**: The plaster has several roles here. The main function was to stabilise the arm and hand during the healing process. Moreover, it fulfilled a measuring function, and its correlation to the length of the arm was part of the function. The transformations took place both during the primary and secondary transformation unit. In the primary transformation unit, one example is where a specific distance on the arm was transformed from the body to the plaster (resource) by the nurses’ aide when she measured up. This unit was then transformed to a longer piece of plaster during the repeated folds. In the secondary transformation unit, there were transformations from modes such as body and artefacts into speech when the nurses’ aide explained the process to the patient.

- **Institutional norms**: Methods for plastering are designed together at this workplace. Some may be general between hospitals, and some are specific to this workplace. Speed is important: Another patient is waiting, but the long-term function for this patient is the highest priority.

**A general theme of measuring activity: Precision through function and time**

Here we connect the two cases described above and we construe a general measuring activity between the two sectors of vehicle and transport, and of nursing and caring, which we expect to be found in many workplaces within these sectors.

*Ideational.* This measuring activity is an alternative to school-traditional precision measuring with tools. The worker uses “rough” measuring units. At a first glance it seems like function, result, and/or time is superior to precision. At a second glance we interpret that the accurate precision for the loading process or healing process is accomplished through this workplace specific measuring activity. “Rough” in this case does not contradict that the method is well adapted to the situation and that accuracy is judged by the situational needs and constraints/restrictions.”
Interpersonal. In this activity we captured relationships between the worker and the workplace and (in) directly the customer (company or patient). Ethical considerations are that it is important to do a good job so that the customer is satisfied. This could, for example, include economic considerations such as not to spend too much time which would increase the cost and decrease the profit. Interpersonal aspects also concern what the employer may impose on the workers (a good job, customer satisfaction, expediency). Also aesthetic aspects, such as “looking neat and tidy”, are part of what is regarded as a good job and what may make the customer satisfied.

Textual. The workplace specific resources for measuring provided efficiency and functionality. Resources can then have the role of facilitating the work; for example, the task is completed more quickly through the use of “rough” measuring units. When a measuring needs to be recorded, appropriate documents are included.

Institutional. Resources contribute to standardisation within the workplace, as well as between workplaces. Written forms can take this role as well as other resources for measuring. Notions concerning a “good job” also concern the institutional framing. The client is thus part of the institutional framing.

Concluding discussion

As stated previously we position this article within a social and critical paradigm. For our work, this quote by Valero and Zevenbergen (2004) is particularly relevant:

In mathematics education it is always possible to ask whose knowledge is being represented in society, schools and classrooms, and with what effects for the different participants in it. The recognition of the different and multiple positions that social actors can adopt in relation to and with the use of (school) mathematical knowledge is at the core of discussions of equity, social justice and democracy in mathematics education. (p. 2)

They continue by arguing that such social aspects are essential to an understanding of mathematics education practices in broader institutional contexts (see also Valero, 2004a). At the same time, such aspects form this broader understanding of the social. In terms of research on mathematics-containing activities in workplaces, our standpoint is that such an understanding incorporates an interest in whose and what kind of knowing is represented in school mathematics, and also how this is connected to the broader social context. We know from earlier research (Fahrmeier, 1984; Lave, 1988; Masingila, Davidenko, & Prus-Wisniowska, 1996; Nunes Carraher, Carraher, & Schliemann, 1985; Nunes, et al., 1993; Scribner, 1985) that the mathematics that can be construed from workplace activities has connections to, but is not the same as, school mathematics. One way to put it is that workers’ voices are missing in the school context, often also in prevocational studies.

The complexities of the workplace could be brought into school mathematics if we want to represent also the knowing of workers in different sectors. This is described by Steen (2003) in this way:

The contrast between these two perspectives—mathematics in school versus mathematics at work—is especially striking (Forman and Steen 1999). Mathematics in the workplace makes sophisticated use of elementary mathematics rather than, as in the classroom, elementary use of sophisticated mathematics. Work-related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features is found in typical classroom exercises (p. 55; our emphasis).
What we have accomplished through utilising our analytical framework, based on a multimodal approach and a design theoretical approach, is to connect the people, the workers and their competence, to the workplace, and to the institutional framing. The three meta-functions have served the purpose of connecting the content (ideographic) – the measuring, with relations between the people involved (interpersonal), with a special attention to the roles of resources (textual). The model by Selander (2008; see also Selander & Kress, 2010) helped us understand the institutional framing, and also the different kinds of communications that took place when we, on one hand, observed the work-processes, and, on the other hand, posed questions about it. What became clear to us in the analysis is what measuring accurately (Bishop, 1988) may mean in a workplace context, for example that precision for the loading process or healing process was accomplished through workplace-specific measuring units.

We would argue that our research is part of a development of research methods and analytical frameworks sensitive enough to do justice to the complexity and to the power of mathematical practices other than school-mathematics, for example, in workplaces. Included here is a view of the worker as self-governed and competent (Wedege, 2001) as well as an approach that there is much to learn from workplaces that can be brought into vocational education and training (VET) settings. This article is consequently an example of a study within what Wedege (2010b) labels as sociomathematics: “a research field where problems concerning the relationships between people, mathematics and society are identified, formulated and studied” (p. 452).

**Acknowledgements**

This paper is written as a part of to the research project “Adults’ mathematics: From work to school”, which is supported by the Swedish Research Council in 2011-2014. We thank Gail FitzSimons and Tine Wedege for constructive comments to earlier versions of the paper. We also thank Maria C Johansson and Marie Jacobson for gathering and discussing some of the data, from which we draw our analysis.

**References**


Björklund Boistrup, L., & Selander, S. (2009). Coordinating multimodal social semiotics and an institutional perspective in studying assessment actions in mathematics classrooms. In V. Durand-
Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), Proceedings of CERME 6, Sixth Conference of European Research in Mathematics Education (pp. 1565-1574). Lyon, France: Institut national de recherche pédagogique.


Wedge, T. (2003). Konstruktion af kompetence(begreber) [The construction of (concepts of) competence]. *Dansk Pædagogisk Tidsskrift* [Danish Pedagogical Magazine], 03, 64-75.


From Standards-Led to Market-Driven: A Critical Moment for Adult Numeracy Teacher Trainers

Martyn Edwards
Sheffield Hallam University
<m.edwards@shu.ac.uk>

Abstract
There has been a shift in the training of numeracy teachers in England away from a highly regulated 'standards-based' approach to teacher training towards one that seeks to engage employer groups and stakeholders in determining the training needs of teachers in further education. This shift has taken place within the context of rapid reform to numeracy and mathematics curricula for post-16 learners. The planned curriculum changes have again highlighted the shortage of qualified numeracy teachers needed to implement national policy initiatives, and has brought numeracy teacher training onto the policy agenda once again. This paper uses Bernstein's notions of vertical teacher knowledge and horizontal teacher knowledge to consider how trainee teachers may be supported to bridge the gap between their own mathematical knowledge and their classroom practice as numeracy teachers. It also draws on Shulman's seven types of teacher knowledge to make these connections. Recommendations made relate to the entry criteria for adult numeracy teachers, allowing 'time and space' to reflect with other trainees rather than 'immersion in practice', the benefits of practitioner-led enquiry to develop innovative pedagogies, and enhanced links between further education and school-based mathematics and between further education and higher education.

Key words: numeracy, teachers training, standards-based approach

Introduction and policy context for adult numeracy teacher training in England
The Moser report (DfEE, 1999) signalled the introduction of the ‘skills for life’ policy in England with a commitment to raise the literacy and numeracy skills of adults. This policy initiative was introduced in the context of a largely casualised teaching workforce where literacy and numeracy teachers often existed on the margins of further education and were sometimes perceived as lacking the subject or occupational expertise often associated with teachers of academic or vocational subjects (Lucas, 2007). The introduction of ‘subject specifications for teachers of adult literacy and numeracy’ (DfES/FENTO, 2002) sought to address this by ensuring “that all new teachers [of literacy and numeracy] are equipped with the appropriate knowledge, understanding and personal skills in their subject, in order to put them on a par with teachers in any other subject” (Lucas, 2007, p.127).
The drive to raise the subject knowledge of literacy and numeracy teachers in England through the introduction of the DfES/FENTO (2002) subject specifications was to some extent subsumed within the wider ‘equipping our teachers for the future’ initiative (DfES, 2004) that sought to raise the subject knowledge of all teachers in further education. This was partly driven by a critical Ofsted report (2003) into the initial training of further education teachers that found little systematic development of the specific skills and understanding needed for effective subject specialist teaching and that the lack of this specialist dimension to be “a major shortcoming in the present system of FE teacher training” (Ofsted, 2003, p.23).

The DfES/FENTO (2002) ‘subject specifications for teachers of literacy and numeracy’ were replaced in 2007 by ‘new overarching professional standards for teachers, tutors and trainers in the lifelong learning sector’ (LLUK, 2007a) and an application of those standards for specialist teachers of adult numeracy (LLUK, 2007b). These new professional standards were followed by a qualification framework, workforce regulations and the imposition of highly prescriptive learning outcomes that sought to regulate the competencies trainee teachers were expected to demonstrate during initial teacher training. Nasta (in Lawy and Tedder, 2009, p.56) described this policy model as driven by a “linear notion that the standards must be specified first, then regulations and qualifications must be developed that incorporate the standards, and only at the final stage are a curriculum and assessment model to be developed that will form the basis of what trainees actually experience”.

Two research projects were carried out by the National Research and Development Centre (NRDC) into the DfES/FENTO (2002) ‘subject specifications for teachers of numeracy and literacy’. The earlier of these studies (Lucas et al., 2004) was based on nine universities that piloted the subject specifications alongside their initial teacher training courses whilst the later study (Lucas et al., 2006) drew upon a larger sample of mostly in-service courses delivered by both universities and colleges. The key foci of these research projects included an exploration of how the subject specifications were being translated and re-contextualised into teaching practice; different approaches taken to delivering the subject specifications; and the balance to be struck between subject specific knowledge, pedagogic knowledge and practical teaching skills (Lucas, 2007). The two NRDC projects led to a number of peer-reviewed publications by the researchers involved in the projects (Lucas, Loo and McDonald, 2006; Lucas, 2007; Loo, 2007a; Loo, 2007b). These discussed issues relating to the increased subject knowledge of numeracy (and other ‘skills for life’) teachers and the relationship of that increased subject knowledge to classroom teaching practice using Bernstein’s (2000) notions of vertical teacher knowledge and horizontal teacher knowledge.

Whilst a body of literature began to emerge specific to adult numeracy teacher training as a result of the two NRDC studies (Lucas et al., 2004; Lucas et al., 2006), this literature did not explicitly take account of the more developed debates on the nature of subject knowledge needed for teaching mathematics in schools (e.g. Ball and Bass, 2003; Davis and Simmt, 2006; Ball, Thames and Phelps, 2008; Hodgen, 2011). It is appropriate in considering subject knowledge for teaching adult numeracy to engage with the wider debate of subject knowledge for teaching mathematics in schools, particularly given the research that has taken place into the longer-established subject knowledge enhancement courses (formerly called mathematics enhancement courses) that are by universities to prospective trainee mathematics teachers for secondary schools (e.g. Adler and Davis, 2006; Askew, 2008; Stevenson, 2008; Adler et al., 2009).

The change of government in the UK in 2010 resulted in a shift of educational policy on teacher professionalism away from centralised government-control through a standards-based
and regulatory system towards one that afforded greater autonomy to employers to determine the professional qualifications their teaching workforce needed to respond to the needs of the learners and employers they seek to serve. The Lingfield review of teacher professionalism in further education (BIS, 2012, p.5) did confirm the need for specialist pre-service or early in-service teacher training for “lecturers in the foundation skills of literacy and numeracy”, albeit within the context of the revocation of the statutory regulations for teacher qualifications in further education. What Lingfield did not attempt to do was define what constitutes foundation skills in numeracy (whether it includes functional mathematics for 14 to 19 year-olds or GCSE mathematics, for example) or the specific outcomes trainee teachers should be expected to demonstrate during initial teacher training.

This article seeks to develop Bernstein's notions of vertical teacher knowledge and horizontal teacher knowledge found in the literature relating to adult numeracy teacher training in England by comparing it with Shulman's seven categories of teacher knowledge found in the literature from the more established subject knowledge enhancement courses offered by universities for intending mathematics teachers in secondary schools. Bernstein and Shulman's theoretical models will be used to analyse post-hoc three teacher training activities drawn from courses designed to meet the subject knowledge requirements of the DfES/Fento (2002) subject specifications for adult numeracy teachers.

Throughout this article the term ‘numeracy’ is used to distinguish the curriculum taught to post-16 learners in vocational contexts from ‘mathematics’ as the curriculum taught as a compulsory subject in schools. Similarly ‘numeracy teachers’ refers to those teachers qualified or training as specialist teachers of adult numeracy and ‘mathematics teachers’ to those qualified or training as specialist teachers of mathematics in secondary schools. The use of these terms to distinguish between curricula and job roles does not imply that such a simplistic division between numeracy and mathematics exists. Indeed, as will be seen in the later section critical moment in a changing policy context, the labels numeracy and mathematics can be used to signal the ideological perspectives of policy-makers and as such be subject to different interpretations. For a flavour of the debate on the use of the terms numeracy and mathematics see the papers presented by Kaye in earlier conference proceedings of this journal (Kaye, 2002; Kaye 2010).

**Subject specifications for adult numeracy teachers - Bernstein's vertical teacher knowledge and horizontal teacher knowledge**

The two NRDC studies (Lucas et al., 2004; Lucas et al., 2006) into pilot courses designed to meet the requirements of the FENTO ‘subject specifications for teachers of numeracy and literacy’ identified three different types of participant on the courses studied. These included very experienced practitioners who also held management posts and staff training roles in colleges; practicing teachers with some classroom teaching experience; and new entrants to teaching with little teaching experience. Each group had different expectations from the course with the most experienced wanting “a high level of theoretical content that would … provide them with a synoptic perspective on their specialism” (Lucas, Loo and McDonald, 2006, p.341) whilst the newer entrants to teaching wanted an emphasis on practical teaching to prepare them for teaching practice. Lucas, Loo and McDonald (2006) applied Bernstein’s notions of horizontal teacher knowledge and vertical teacher knowledge to understand the distinction between theoretical and practical knowledge for teachers and ways in which the courses attempted to bridge these two types of knowledge through what Bernstein called ‘re-contextualisation’.
An examination of the FENTO subject specification for adult numeracy (DfES/FENTO, 2002) shows that it consisted primarily of Bernstein’s ‘vertical knowledge’ separated into the sections of number and numeric operations, geometry and spatial awareness, statistics, and working with algebra. It was primarily ‘vertical knowledge’ in the sense that the specification required an academic or theoretical understanding of the content that was independent of context or experience. A closer inspection of the elements listed in the specification revealed that most of them approximated to topics that might be found on the first year of a course in GCE Advanced Level mathematics (level 3 on the English National Qualifications Framework) whilst other topics were identifiable from the content required for higher level tier of GCSE mathematics syllabi (level 2 on the English National Qualifications Framework). The specifications immediately raised the questions of (i) how the courses can be justified as being at level 4 on the national qualifications framework (equivalent to the first year of undergraduate study) when the content was clearly a repetition of level 3 study, and (ii) how all the elements listed in the specifications can be covered in a course of one-year part-time duration.

The first of these two questions relating to academic level was the simplest to answer. In the case of the experienced practitioners seeking a theoretical and synoptic perspective of mathematics this ‘level 4-ness’ could be justified as being demonstrated through the adoption of a connectionist approach to mathematics that emphasised relational understanding over procedural understanding (Skemp, 1976; Askew, 1997). For new entrants to teaching it was the requirement for 60 hours of practical experience in teaching adult numeracy that were seen to bring the ‘level 4-ness’. In both cases there were significant challenges for numeracy teacher trainers supporting trainees in the process of re-contextualising vertical teacher knowledge of mathematical content into horizontal teacher knowledge of classroom practice in teaching adult numeracy.

The second of the two questions posed more difficulties for course designers with different approaches taken by awarding bodies and universities to the problem of achieving coverage of the specifications within the learning hours available. Lucas (2007) identified that whilst national awarding bodies adopted a ‘standards-based approach’ that emphasised ‘coverage’ and ‘mapping’ in the competency tradition, universities were more innovative in a ‘knowledge-based approach’ where they chose which elements of the specifications to emphasise and in what depth to explore them.

Three examples, one from a course that I delivered at Thames Valley University, another from a course delivered by LLU+ at London South Bank University reported in the proceedings of the 13th annual international conference of Adults Learning Mathematics (Stone and Griffiths, 2006), and a third from one of the NRDC pilot studies (Lucas et al., 2004; Lucas et al., 2006) illustrate ways in which universities developed innovative ‘knowledge-based approaches’ towards the DfES/FENTO (2002) subject specifications:

**Example 1: Thames Valley University**

One element of the DfES/FENTO (2002) subject specification within the statistics section required knowledge of discrete probability distributions. The direct contact-time available to the trainer to teach this topic was a single session of four hours duration, albeit with the expectation that trainees would engage in self-directed study to further their knowledge outside of the taught session. There were several problems with this. Discrete probability distributions include rectangular, binomial and Poisson distributions. Each of these constitutes a topic in its own right worthy of more than four hours of direct contact-time. Furthermore, knowledge of discrete
probability distributions does not easily translate to strategies for teaching adult numeracy learners. Interestingly, coverage of the normal distribution was not required by the DfES/FENTO (2002) subject specifications since this is a continuous rather than discrete probability distribution, even though an understanding of the normal distribution is arguably more relevant to teachers than the discrete probability distributions due to its usefulness in interpreting assessment results for large populations, understanding IQ scores, and so on.

The trainer made the decision in planning the session to teach both the continuous probability distribution (normal) and the discrete probability distributions (rectangular, binomial and Poisson) within the four hour session. Being aware of the impossibility of teaching such a range of mathematical knowledge within four hours the trainer elected to see the content as a vehicle towards meeting an overarching course aim rather than specific content to be covered. The overarching aims of the trainer were (i) to provide trainees with the opportunity to carry out self-study in pairs on an area of mathematics unfamiliar to them and then teach that concept to the rest of the group, (ii) appreciate the uses of mathematical modelling (e.g. the normal distribution to interpret IQ scores and the Poisson distribution to predict volcanic activity), and (iii) to make links with own practice as teachers of adult numeracy.

**Example 2: LLU+ at London South Bank University**

Stone and Griffiths (2006, p.148-149), in reflecting upon their experiences as numeracy teacher trainers at LLU+, argued that:

making teachers ‘do some hard sums’ and giving them some background information on personal and social factors affecting learning was not really equipping them to teach their subject. … Clearly, something was missing. At LLU+ the feedback from our own teacher training programmes was that while the course sessions were fun and participants were exposed to [an] imaginative variety of teaching methods, they did not feel they were learning as much as they would have liked that would be useful to them in the numeracy classroom. To this end, we began enriching our programmes on offer with opportunities to explore mathematics and numeracy at a basic level and to discuss and evaluate ways to teach it.

This extract appears to indicate a similar orientation to the trainer in example 1 where a commitment to overarching course aims allowed the subject specifications to be interpreted creatively. In the case of the two trainers at LLU+ the overarching course aims appeared to include learning as fun, modelling variety in teaching methods, valuing the ‘student voice’, and ensuring relevance of activities to participants’ professional practice.

**Example 3: Broken keys activity**

Loo (2007) describes an activity used by one of the institutions in the NRDC studies called ‘broken keys’. This involved trainees creating problems for others in the group to solve using mathematical functions. These were then linked to word cards and picture cards to illustrate the links between algebraic symbolism and real life. Finally the trainees were encouraged to reflect on how the approaches could be applied to the teaching of topics from the Adult Numeracy Core Curriculum (DfES, 2001).

Whilst the starting point to the ‘broken keys’ activity was drawn from the ‘working with algebra’ section of the subject specifications a commitment on behalf of the trainers to overarching course aims such as modelling the Standards Unit approaches of learners creating problems, multiple representations and encouraging discussion (Swan, 2005) can arguably be inferred from the teaching approach described.
Subject knowledge enhancement courses for schoolteachers in secondary mathematics - Shulman’s seven major categories of teacher knowledge

Subject knowledge enhancement courses (previously known as mathematics enhancement courses) are well-established in many English universities offering Post-Graduate Certificate in Education (PGCE) courses for intending mathematics teachers in secondary schools (Sheffield Hallam University, 2013). These courses are usually offered as short part-time courses to graduates who have already been offered a place on secondary mathematics PGCE courses. They are designed to meet the needs of new entrants to teaching whose undergraduate degree is not in mathematics but in a related subject such as engineering or finance. Since such courses are more established and theorised than those developed to the DfES/FENTO (2002) subject specifications that are the subject of this article it is worth considering what lessons can be learnt from them, and whether those lessons are transferable to adult numeracy teacher training.

Shulman (1986), in developing a theoretical model for teacher knowledge that can be applied to mathematics (and adult numeracy) teacher training, defined the seven major categories of teacher knowledge shown in figure 1. The first four of these categories related to generic teaching skills and these were the mainstay of teacher education programmes at the time. These four categories were seen as relevant to all teachers irrespective of the subject-specific context of their teaching. Shulman acknowledged the crucial importance of these four categories for teaching but went on to propose three further categories that he termed content knowledge, curriculum knowledge and pedagogical content knowledge.

‘Content knowledge’ includes knowledge of the subject to be taught and how it is organised, including an understanding of which concepts are central to the discipline and which are peripheral (Ball, Thames and Phelps, 2008). This type of knowledge can be related to the expectations of the most experienced practitioners in Lucas, Loo and McDonald’s (2006) study of pilot DfES/FENTO courses who wanted a high level of theoretical content to provide them with a synoptic view of their specialism.

‘Curriculum knowledge’ relates to knowledge of the full range of courses available to teach particular subjects and topics at a particular level, including the range of instructional materials available (Ball, Thames and Phelps, 2008). It also includes ‘lateral curriculum knowledge’ (what is being taught to learners in other subject areas) and ‘vertical curriculum knowledge’ (what has been taught in the subject in previous years, and what will be taught in subsequent years).

Shulman’s final category of ‘pedagogical content knowledge’ sought to define that specific knowledge about a subject that is unique to teachers of the subject. It includes an awareness of what makes particular topics conceptually easy or difficult for learners to understand; the most useful analogies, illustrations, examples, explanations and demonstrations that can be used to support learning whilst remaining consistent to the integrity of the subject matter; and common conceptions and misconceptions of particular topics typically held by learners at different ages or ability levels (Ball, Thames and Phelps, 2008). Interestingly, Shulman’s approach was quite different to that of subject specifications and prescribed learning outcomes adopted by FENTO and its successor bodies in that he “did not seek to build a list or catalogue of what teachers need to know in any particular subject area” but instead “sought to provide a conceptual orientation and a set of analytic distinctions that would focus the attention of the research and policy communities on the nature and types of knowledge needed for teaching a subject” (Ball, Thames and Phelps, 2008, p.392).
By analysing Shulman's categorisation of different types of teacher knowledge it becomes apparent that his content knowledge related most closely to Bernstein's vertical teacher knowledge whilst Shulman's curriculum knowledge and pedagogical content knowledge are more akin to Bernstein's horizontal teacher knowledge.

There are currently two dominant views on the subject knowledge that mathematics teachers in secondary schools need to know to effectively teach their subject (Bell, Thames and Phelps, 2008). The first view is that they need to know whatever mathematics is in the curriculum at the level they are intending to teach plus some additional years of further study at a higher level of mathematics. The second view is that they need to know the mathematics in the curriculum at the level they are intending to teach, but that this should be a ‘deep understanding’ incorporating aspects of Shulman’s ‘pedagogical content knowledge’ (Shulman, 1986). The notion of deep understanding in mathematics is evident in the literature in a number of guises. Ma (1999), for example, refers to ‘profound understanding of fundamental mathematics’ whilst Adler and Davis (2006) use ‘understanding mathematics in depth’ to describe their conceptualisations of subject pedagogical knowledge.

**Bringing together the theories of Bernstein and Shulman**

Bernstein’s notion of the re-contextualisation of vertical teacher knowledge into horizontal teacher knowledge applied by Loo (2007a; 2007b) to adult numeracy teacher training and Shulman’s seven categories of teacher knowledge applied to secondary mathematics teacher training (Ball and Bass, 2003; Davis and Simmt, 2006; Ball, Thames and Phelps, 2008; Hodgen, 2011) can be brought together by considering the three examples of teacher training activities discussed earlier.

In example 1 the teaching of discrete probability distributions was discussed. Knowledge of discrete probability distributions (rectangular, binomial and Poisson) fits comfortably within Bernstein's vertical teacher knowledge in that it provides teachers with a synoptic view of their specialism. The re-contextualising of that vertical teacher knowledge into horizontal teacher
knowledge is more problematic since the pedagogical techniques adopted of peer-led teaching and mathematical modelling could have been achieved more effectively through studying a numeracy concept drawn from the curriculum that trainees were being trained to teach, rather than through an unfamiliar mathematical topic that trainees themselves experienced as conceptually difficult. It could be argued, for example, that it would be more beneficial for teacher trainers to model the use of a 'washing line' strung across the classroom to order the probability of events occurring on a scale of 0 to 1 rather than being required to teach discrete probability distributions in the tradition of Bernstein's vertical teacher knowledge as a proxy for Shulman's pedagogical content knowledge.

In example 2, discussed earlier, the difficulties teacher trainers experienced in supporting trainees to re-contextualise Bernstein's vertical teacher knowledge into horizontal teacher knowledge was even starker. In this case the phrase 'do some hard sums' was contrasted negatively with what teacher trainers saw as necessary to equip trainees to teach adult numeracy effectively. Their response was to enrich the programmes (presumably by adding what they considered to be more relevant pedagogical content knowledge) to the content prescribed by the subject specification. In this case, it could be argued that the trainers' pedagogical content knowledge replaced, or at least marginalised, the vertical teacher knowledge found in the subject specification in such a way as to obviate the need for the re-contextualisation by trainees of different types of teacher knowledge.

The broken keys activity described earlier in activity 3 resonates with the first example in that mathematical functions do not feature in the adult numeracy core curriculum (DfES, 2001). Nevertheless they appear to have been used with some success to introduce Shulman's pedagogical content knowledge by proxy through the use of Standards Unit (Swan, 2005) approaches to teaching mathematical functions. In spite of the apparent success of this approach it could again be argued that using the algebraic notation of functions unfamiliar to trainees adds an unhelpful layer of conceptual difficulty that clouds the more pressing concern of how to effectively teach the basic algebraic concepts found in the adult numeracy core curriculum (DfES, 2001).

**Critical moment in a changing policy context**

In recent times teaching has been practiced within a rapidly changing policy context (Ecclestone, 2008; Earley et al., 2012). This has led to changes in the way that the teaching role and teacher professionalism has been conceptualised, along with related changes within teacher training itself. It is within this context that a 'critical moment' for adult numeracy teacher training may emerge.

Current government policy in England raises the expectation that all school-leavers without the GCSE mathematics pass expected of sixteen year-olds should be required to retake the full GCSE in mathematics if they progress to full-time further education (DfE, 2013). Additionally, those school leavers progressing to full-time further education who have already achieved the GCSE mathematics pass expected of school-leavers should be required to continue to study mathematics to a higher level rather than being allowed to discontinue mathematics at age 16 as previously (ACME, 2012). Such an approach is seen by policy-makers as promoting the more rigorous and academic study of mathematics rather than the development of numeracy skills for vocational learners through qualifications such as adult numeracy and functional mathematics. Such curriculum reforms are seen by policy-makers as ensuring the UK can compete with leading industrialised nations (Vorderman, 2011).
Recent policy initiatives in teacher training for schools have included encouraging high-achieving graduates to enter teaching through targeted bursaries and to encourage school-centred initial teacher training (SCITT) consortia to provide teacher training as an alternative to more traditional university-led provision (DfE, 2010; DfE, 2011). Such an approach to teacher training assumes that the acquisition of subject content knowledge at a high level should be attained prior to entering teacher training, and that the practical skills of teaching itself are acquired as a 'craft' by working alongside practicing teachers. The speech by the Secretary of State for Education to the National College (Gove, 2010) expressed the view that "Teachers grow as professionals by allowing their work to be observed by other professionals, and by observing the very best in their field …" and that "teachers … improve their craft by learning from others while also deepening their academic knowledge" (my emphasis). The dichotomy between teaching as a craft and teaching as a profession was challenged by Kirk (2011) who argued that whilst teaching generates substantial personal craft knowledge, often in the form of tacit knowledge, it also required engagement with a broader type of knowledge that "… implies a professional duty to keep in touch with the literature of teaching and learning, and indeed to contribute to it as a way of raising the level of public and professional debate on teaching and learning" (Kirk, 2011).

Similar tensions have been experienced in the training of further education teachers to those found for schoolteachers. The Lingfield Report (2012) recommended the revoking of the regulatory framework for teachers in further education and called for new qualifications for teacher training to be developed by an employer-led ‘guild’. However, Lingfield (2012, p.33) also called for a strong professional identity for further education teachers underpinned by increased autonomy to develop innovative pedagogies specific to the vocational focus that is unique to further education. Such practitioner-led enquiry hinted at by Lingfield (2012) is not new to further education. Previous initiatives have included the practitioner-led research initiative (NRDC) and the teacher enquiry funded projects (NCETM). Such initiatives were consistent with Hoyles’ (1975) notion of extended professionalism and sit comfortably with emerging measures of professional esteem such as chartered mathematics teacher status and chartered status for further education teachers. In reflecting upon such initiatives, however, it is necessary to sound a cautionary note concerning the culture within the further education sector that can mitigate against such initiatives. The using research to enhance professionalism in further education project (Economic and Social Research Project) identified that whilst practitioner research had a significant role to play in shaping the professional identities of those teachers that engaged in it, the benefits were often undermined by managerialist cultures within colleges where short-term gains, such as compliance with national policy agendas, hindered practitioners from asking more fundamental and critical questions about their practice (Goodrham, 2008).

The reforms to the post-16 mathematics curriculum described earlier in this section are a case in point where the shortage of qualified mathematics teachers to deliver the policy initiative has led to the launch of a government-subsidised six-day training programme intended to "further develop the skills of those currently teaching functional skills, preparing them to teach GCSE maths" (Education and Training Foundation, 2013). Such a quick-fix approach to numeracy training appears unlikely to provide teachers with the space or time to gain Bernstein's vertical teacher knowledge and re-contextualise it into horizontal teacher knowledge, nor to acquire those aspects of Shulman's subject pedagogical knowledge critical for effective teaching of numeracy to 'second-chance' learners in further education. Regional training programmes promoted as up-skilling teachers of numeracy by "enhance[ing] their knowledge so that they can teach GCSE effectively" (EMCETT, 2013) is likely to lower the status of numeracy teachers and undermine the gains made through the introduction of specialist
teacher training for adult numeracy teaching rather than raise the quality of numeracy teaching. The ambitious targets set to engage post-16 learners in the study of mathematics up to the age of eighteen is laudable, as is the intention to enhance the subject knowledge of teachers so that they can effectively meet the challenges of the new curriculum. These targets and intentions need to be matched by a strategy for recruiting high quality graduates into teaching mathematics and then providing specialist teacher training courses to support them to re-contextualise their own knowledge of mathematics into effective numeracy pedagogies for further education. Similarly, experienced teachers of vocational subjects cannot be expected to retrain to teach GCSE mathematics without first being provided with the opportunities to increase their own mathematical knowledge to the standards that would be required for teaching in any other curriculum area.

Conclusions and recommendations

Mathematics subject knowledge should be a prerequisite for new entrants to numeracy teaching, whether for new entrants to teaching or for experienced teachers retraining to teach numeracy from other curriculum areas, in the same way that the best graduates and those with substantial vocational experience are sought as teachers for other academic and vocational subjects. Whilst it is unlikely that a consensus can be reached amongst the mathematics community on the detail of the content and level necessary, it is nevertheless important for the status of numeracy that minimum entry criteria be developed. These criteria should be credible when compared with entry requirements for teaching in other academic and vocational areas of further education.

Numeracy teachers should be given opportunities to build upon and extend their own mathematical knowledge and subject pedagogical knowledge throughout their careers, including at Masters level. They should be given support, time and space to develop innovative numeracy pedagogies related to the particular vocational contexts and specialist settings they encounter within further education. Supporting practitioner-led enquiry holds much promise as an effective form of continuous professional development for numeracy teachers.

Whilst acknowledging the benefits of observing the best teachers to learn the 'craft of teaching', it is also necessary to allow teachers the time and space to reflect on their professional learning with other trainee teachers. Such an approach is more likely to develop the critical skills to adapt to the fast-changing and policy-driven culture of further education than immersion in practice. The benefits gained from the subject specialist teacher training in adult numeracy from 2002 need to be maintained and strengthened if the challenges of post-16 curriculum reform are to be met.

Opportunities for developing links between further education and school-based mathematics and between further education and higher education should be grasped. These links can be beneficial both to share effective practice in teaching mathematics and to identify the nature of numeracy pedagogies specific to the contexts and learners in further education.

References


New PIAAC Results: Care Is Needed in Reading Reports of International Surveys

Jeff Evans
Middlesex University, London NW4 4BT, UK
<j.evans@mdx.ac.uk>

Abstract
Results from the Survey of Adult Skills, also known as PIAAC (Programme for the International Assessment of Adult Competencies), were recently made available for 24 participating countries. PIAAC involves several developments in relation to the earlier international “adult skills” surveys (IALS in the 1990s and ALL in the 2000s), notably the use of computer administration of the survey. In this paper, I focus on understanding these studies, by considering conceptual issues, methodological validity of research design and execution, and presentation of results. I consider several of the sample items for numeracy published by OECD (2012). And I discuss illustrative results from Australia made available in February 2013, by the Australian Bureau of Statistics. The paper shows when and how to be sceptical when reading international survey reports. It also opens up questions concerning the relevance of the results, and the other types of research that may be needed, in different national and local contexts.

Key words: adult skills, international assessment, and mathematics

Introduction

From October 2013, results from PIAAC (Programme for the International Assessment of Adult Competencies) for 24 participating countries have been available (OECD, 2013a, 2013b). PIAAC aims to provide information as an international comparative survey, successor to IALS (during the 1990s) and ALL (2000s), and it has many similarities with national studies, such as Skills for Life in the UK. Unlike the school level surveys (TIMSS, PISA), which gain access to “captive populations” in schools, PIAAC needs to use a combination of household survey and educational testing methodologies. It involves developments from the adult earlier studies, in several ways.

The first round covers a greater range of countries (24, two thirds of which are EU members, with the rest from North America, East Asia and Australia) – though all are advanced industrial economies. It focuses on three domains or “competencies” – Literacy, Numeracy, and now Problem Solving in Technology Rich Environments (PSTRE). It uses computer administration, which has a number of consequences, in particular allowing adaptive routing of respondents (see Section 3), and making the survey results available more quickly and more accessibly. In addition, PIAAC has implemented a number of methodological and fieldwork improvements, for example, tighter specification and regulation of sampling and fieldwork standards than in previous international surveys (OECD, 2013b, pp. 47-61).
PIAAC is designed to be repeated, in order to build up time series data for participating countries. This longitudinal aspect would aim to facilitate the study over time of the correlations of the performance outcomes with relevant social or attitudinal variables.

In Section 2 I sketch international the policy context, including the conception of Lifelong Learning (LLL) promoted by the survey’s sponsor, the OECD (Organisation for Economic Cooperation and Development). In Section 3, I describe the survey aims, and the underlying conception of numeracy. In Section 4, I consider how this conception is deployed in the measurement process, and other aspects of methodological validity that need to be considered for international performance surveys. I also focus on the need to consider the way that the survey results are reported, since this crucially affects the way “the findings” are perceived by various categories of readers. In Section 5, I discuss some illustrative results from Australia, and in Section 6, I return to focus on the effects of international surveys like PIAAC on the developing educational policy context worldwide.

The international policy context

Educational policy is currently being developed on a world-wide scale, with supranational organisations acting as key agencies for change. Increasing globalisation and competitive economic environments are leading national governments to seek competitive advantage – which is “frequently defined in terms of the quality of national education and training systems judged according to international standards” (Brown, Halsey, Lauder & Wells, 1997, pp. 7-8). Results from surveys like PIAAC (and PISA) seek to provide measures of a country’s progress according to international standards.

The idea of Lifelong Learning (LLL) is central to the conceptualisation of adult numeracy (and literacy). In international policy debates, LLL has been much contested, e.g. between “humanistic” (learning for the whole person) and “economistic” (human capital) approaches (Evans, Wedege, & Yasukawa, 2013). In this connection, it is important to consider work done both within the UNESCO programmes (e.g. Guadalupe, 2013), and by the OECD.

Here I focus on the OECD, PIAAC’s sponsor. OECD’s view of LLL aims to promote the development of knowledge and competencies enabling each citizen to actively participate in various spheres of globalised social and economic life. It also promotes a broad view of the context of learning, and a weakening of the distinction between formal and informal education. At the same time, it emphasises the citizen’s need to acquire and update a range of abilities, attitudes, knowledge and qualifications over the life-course, and hence the individual learner’s responsibility for their own education (e.g. Walker, 2009). Some of the consequences of these commitments will be discussed below; see also Tsatsaroni & Evans (2013).

The European Union (EU) is working closely with OECD on PIAAC. For supra-national institutions like the EU, the area of Lifelong Learning provides a domain where they can make a legitimate policy intervention, since, in a globalised world, a focus on labour mobility makes LLL a supra-national concern. This provides a basis for OECD’s and EU’s actions, leading to the promotion of the “skills and competencies agenda”, in all sectors of education and training (Grek, 2010). More generally, the OECD and the EU are disseminating ideas and practices that strongly influence national policy making around the world. These include the promotion of expertise in creating comparable datasets, and new forms of “soft governance” of national educational systems, encompassing the production and dissemination of knowledge, and of comparative data such as educational and social indicators, and peer reviews involving country and thematic reviews. These practices allow countries to measure the relative success of their education systems and to shift policy orientations accordingly, while increasingly facilitating the role of these supra-national organisations themselves to be “governing by data” (Ozga,
2009). Overall, one of the effects of international studies like PISA and PIAAC is to contribute to a “comparative turn” in educational policy-making and to a “scientific approach” to political decision-making (Grek, 2010).

The PIAAC Survey

PIAAC’s main objectives were presented by Andreas Schleicher (2008) of the Education Directorate at OECD – as helping the participating countries to:

- Identify and measure differences between individuals and across countries in key competencies
- Relate measures of skills based on these competencies to a range of economic and social outcomes relevant to participating countries, including individual outcomes such as labour market participation and earnings, or participation in further learning and education, and aggregate outcomes such as economic growth, or increasing social equity in the labour market
- Assess the performance of education and training systems, and clarify which policy measures might lead to enhancing competencies through the formal educational system – or in the work-place, through incentives addressed at the general population, etc. (pp. 2-3).

The PIAAC objectives thus appear to comprise a “human capital” approach, coupled with social concerns (Evans, Wedege & Yasukawa, 2013).

In the framework used by OECD, literacy, numeracy and problem-solving in technology rich environments are the three competencies which PIAAC aims to measure. In the OECD’s approach, competencies are internal mental structures, i.e. abilities, capacities or dispositions embedded in the individual […] Although cognitive skills and the knowledge base are critical elements, it is important not to restrict attention to these components of a competence, but to include other aspects such as motivation and value orientation. (PIAAC Numeracy Expert Group, 2009, p. 10)

Numeracy is defined for the purposes of designing the items for PIAAC as:

the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.

(PIAAC Numeracy Expert Group, 2009, pp. 20ff.)

This is put forward as a basis for conceptualising mathematical thinking in context. However, in order to produce measures of numeracy, the idea of numerate behaviour is put forward, that is:

the way a person’s numeracy is manifested in the face of situations or contexts which have mathematical elements or carry information of a quantitative nature. […] inferences about a person’s numeracy are possible through analysis of performance on assessment tasks designed to elicit numerate behaviour. (PIAAC Numeracy Expert Group, 2009, p.10)

This led to specifying the following dimensions of “numerate behaviour” that can be used to guide the construction of assessment tasks:

- **context** (four types): everyday (or personal), work, society and community, further learning
- **response** (to mathematical task - three main types): identify / locate / access (information); act on / use; interpret / evaluate.
- **mathematical content** (four main types): quantity and number, dimension and shape, pattern and relationships, data and chance
• *representations* (of mathematical / statistical information): e.g. in text, tables, and/or graphs.

Each item can be categorised on these four dimensions, along with its estimated difficulty.

PIAAC also aims to produce affective and other contextual data that can be related to the respondent’s performance. This includes demographic and attitudinal information in a Background Questionnaire (BQ), and self-report indicators on the respondent’s use of, and need for, job-related skills at work; see OECD (2013b) for the BQ’s conceptual framework, and CSO, Ireland (2013) for a copy of the BQ.

Each country interviewed at least 5000 adults, normally 16-65 years of age. PIAAC’s default method of survey administration is by laptop computer, although paper-based testing was used in IALS / ALL (and PISA up to now). This facilitates the use of *adaptive routing*, which estimates the “skill level” of the respondent from a few initial responses, and then administers more appropriate items (in terms of difficulty) throughout the rest of the interview.

**Understanding PIAAC’s conceptual framework and methodology**

In seeking to understand PIAAC and other adult skills surveys and their results, I consider how the interpretation of such studies needs to be related to their conceptual bases and methodological decisions, as well as choices about presentation and reporting and arguments about the range of applicability of the findings (Tsatsaroni & Evans, 2013; Hamilton & Barton, 2000; Radical Statistics Education Group, 1982 / 2012).

Generally, surveys rely on aspects of the research design, responding to reasonably well-understood criteria of *validity*, to enhance and to monitor the measurement and sampling procedures. It is important for literacy and numeracy researchers, teachers and policy makers to be able to consider these, when the results of a survey are presented and discussed. Here I consider the likely effects of certain design features of the survey, and their realisation in the field, in terms of the following aspects of validity:

1. the *content validity* of the definitions of numeracy and numerate behaviour (“types” or categories of items, as described above)
2. the *measurement validity* of the items presented, including the administration and scoring procedures (“qualities” of items)
3. the *reliability* of the measurement procedures
4. the *external validity*, or representativeness, for the national population of interest, of the results produced from the sample (See Evans, 1983, for a fuller discussion.)

**Content validity**

I am using the term *content validity* in this paper to refer to the extent to which a measure represents all aspects of a given concept, as it is defined. The definition of numeracy used by PIAAC (and, earlier, ALL) is based on the four dimensions of numerate behaviour stipulated above. Each item can be categorised on these four dimensions, and the proportion of items falling into each category can be controlled over the whole scale, so as to make the operational definition of numerate behaviour more explicit, and the content validity of the overall set of items more open to scrutiny. In PIAAC numeracy, the proportion of items falling into each category of *mathematical content, context,* and *response* is controlled (OECD, 2013b, p.28). This allows test designers to stipulate the proportions of the items that are from each type of

---

1 Literacy and PS-TRE items are characterised by a similar, though not identical, set of dimensions (OECD, 2013b, pp. 21-34).

2 Respondents are presented with initial computer-based tasks; anyone uncomfortable with these takes an alternative pencil-and-paper version of the main tests.
each key dimension, and from different levels of difficulty — for example, the proportion of “data and chance” items of medium difficulty.

However, in an international survey, this can provide only a general, transnational definition, and one needs to question how well it “fits” adults’ lives in any particular country. For example, the four types of context (everyday / personal, work, society and community, further learning) can be specified only in a rather general way – they may or may not represent the repertoire of actual specific social practices or social contexts in which any particular respondent might engage, in his/her life. Thus we need to examine a set of items that a particular sample member might be asked to respond to.

Measurement validity
What I call here “measurement validity” refers to the extent to which the responses to the set of items administered to a respondent actually capture what the conceptualisation of numeracy specifies; this will depend on the actual range of items used. As with most large-scale educational assessments, the full set of the items used is not made public, while the survey is on-going. Nevertheless, four illustrative items are presented on several websites (e.g. CSO, Ireland, 2013), and in the Appendix.

This sample of four PIAAC or “PIAAC-like” numeracy items were published to represent the more than 50 that might potentially be presented to any PIAAC respondent (OECD, 2012). Like any sample, of course, these four items cannot represent the full range of combinations of content, context, responses required, and difficulty levels. Nevertheless, it may be useful to consider them here, since they give some specificity to the more general characterisation of numeracy in the survey discussed in the previous subsection. For two of the items, the mathematical contents are framed by Everyday / Personal or Work contexts; for the other two, Society and community contexts. They appear to combine realistic images of the problem at hand with school-like test rubrics, providing the questions that need to be answered, presumably by applying the correct mathematical procedures. Thus these items represent a hybrid type of task.

In any particular country, we need to ask how well these sorts of tasks – such as making precise calculations (as in sample item 3), making precise readings from the appropriate scale (as in item 2), or detecting changes in a time series graph (as item 1) – might represent adults’ social practices and everyday lives in that country. We should also ask whether tasks such as these would tap or encourage what we would consider as mathematical thinking about potentially challenging tasks. Sample item 4 certainly appears to represent a more challenging task for most adults in many of the countries surveyed by PIAAC in the current round.

Measurement validity also requires procedures designed for the administration of the survey to be standardised in advance across all countries, e.g. design specifications of the laptops and software to be used, and rules for access to calculators and other aids. As with any survey, full appreciation of the validity of procedures requires assurance of how these procedures are followed in the field; see OECD (2013b, pp. 47-61). This is even more crucial when results are compared across countries using different fieldwork teams.

---

3 These levels of difficulty are estimated by Item Response Modelling procedures; see subsection 4.5 below.
4 Round 2, including a further 9 countries, is now underway.
5 The overall distribution of numeracy items included by contexts was: Everyday / Personal – 45%, Work – 23%, Society – 25% and Further learning – 7% (OECD, 2013b, p.28).
6 Respondents in the first round of PIAAC, completed in 2011-12, were supplied with hand held calculators and rulers with metric and imperial scales, for use during the interview.
**External validity**

External validity includes the representativeness of the sample for the population of interest; thus, the 5000 or more adults (usually aged 16-65) selected for the sample in each country need to represent the population of that country. We can scrutinise, for any participating country, the sample design and other key aspects, such as the incentives offered to those selected for the sample, in order to encourage agreement to be a respondent. Again, judgments about the effectiveness of these procedures depend partly on knowledge of actual field practices.

However, it is important to realise that any result from such a sample, whether the mean score for a country, or a difference (e.g. by gender) in percentages of items correct, is only an estimate for the corresponding population value (namely, the mean, or the size of the difference in percentages), for the whole country. Of course, we would like to know about the population value – but this is not possible with certainty, since we only “know” about a subsample. So virtually every numerical result that we produce with a sample survey cannot be considered exact, but should have a “tolerance”, a margin of error, on either side of the sample-based estimate.

Thus, if we consider the PIAAC Numeracy results from OECD (2013a), we would find that the first four countries are:

Japan (288) … Finland (282) … Netherlands and Belgium (280)

This appears as a clear ranking – before we realise that a 95% confidence interval for the country score for Finland would be approximately 280 to 284, and for Netherlands and Belgium, approximately 278 to 282: thus these countries have overlapping confidence intervals, and so their performances are not really able to be differentiated.

Similarly, the differences between the Netherlands and Belgium and the next three ranked countries (the Scandinavians) are not “statistically significant”, again because of the variation that we must always expect in results based only on samples. So what appeared to be a neat ranking of the top 7 dissolves into Japan at the top, followed by a group of six countries, within which one cannot really differentiate performance on the PIACC Numeracy survey (OECD, 2013a, pp. 79-80).

**Reliability**

The comparability of test administration across countries and across interviewers, and especially assuring the use of the same standards and practices in marking, has been a problem with past international surveys. Computer presentation and marking of test items can be expected to help greatly with reliability (assurance that the survey will produce the same or very close results, if it were to be repeated, using the same procedures). But it may tend to undermine content validity, if it reduces the range of types of question that can be asked; for example, it is difficult to construct an item which can validly assess a respondent’s reasons for his/her answer, when the item is computer-marked. This trade-off between content / measurement validity and reliability is a well-known dilemma in research design.

Further, the strengthening of reliability may lead to concerns about loss of another aspect of external validity, namely ecological validity, i.e. whether the setting of the research is representative of those to which one wishes to generalise the results. For example, the on-screen presentation of tasks may not be representative of the settings in which respondents normally

---

7 The margin of error depends on the degree of “confidence” desired in the estimate, but is normally 2 standard errors for a 95% confidence interval.

8 The confidence intervals produced here are only approximate for the sake of illustration: I have estimated the margin of error for country scores based on an inspection of Figure 2.6a (OECD, 2013a, p.80), and have used the idea of countries “with overlapping confidence intervals”, instead of the broadly equivalent idea of countries “differing by an amount which is not statistically significant”.

---

Evans, New PIAAC Results.

Adults Learning Mathematics – An International Journal
carry out tasks involving numeracy, and so may not facilitate their “typical” thinking and
behaviour responses. Again, similar dilemmas arise for much educational assessment.

**Beyond methodology: variations in interpretation and reporting**

This discussion of several aspects of the validity of the survey shows the importance of sound
research design – and also of the way field work is accomplished. However, a number of key
issues in interpreting the uses and effects of the survey go beyond the technical issues around
methodological validity (Radical Statistics Education Group, 1982 / 2012). They include the
way that the survey’s measured scores are interpreted / reconceptualised in presentations and
reports of various interested parties. This aspect is of course not under the complete control of
the survey’s sponsors: for example, the media and certain national interests have often offered
conflicting interpretations (“spin”) of results of international surveys. These processes require
an understanding of the policy context and the ideological debates that surround the reception of
results in a particular country, as well as the global education policy discourse.

Several examples can be given of the need for care and scepticism about the reporting and
interpretation of these results; see e.g. EERJ (2012), on the way that PISA results have been
reported and used, and in particular, Carvalho on the “plasticity of knowledge” (2012, pp. 180-83).
One problem is that an adult’s performance on one of the subtests such as numeracy cannot
simply be expressed as the proportion correct – since adaptive routing means that respondents
were presented with different sets of items, some “harder”, and some “easier”. So Item
Response Modelling is used to (“psychometrically”) estimate a standardised score (e.g. for
PIAAC: scores 0-500, mean 250, standard deviation 50). Then, the numerical score is usually
related to one of five general “levels” of literacy or numeracy to make it meaningful; see OECD
(2013b, pp. 69-70).

Now, this may well be more informative than simply reporting the percentage of adults in a
country that are categorised as “literate” or not, as was formerly done. But as in other national
and international surveys, there is debate about use of a simple and one-dimensional
characterisation of an adult’s numeracy. For example, Gillespie (2004) referring to the first UK
Skills for Life survey (done using a similar methodology to PIAAC) notes: “The findings
confirm that for many, being ‘at a given level’ is not meaningful for the individual, as levels
embody predetermined assumptions about progression and relative difficulty” (p. 1). Part of this
scepticism flows from the finding that many adults have different spiky profiles, due to
distinctive life experiences (Gillespie, 2004, pp. 4-6). Thus, some adults may find items of type
A (say, “data and chance”) more difficult than type B items (e.g. “dimension and shape”) – and
others find the opposite.

Similarly, some policy-makers attempt to stipulate “the minimum level of numeracy needed
to cope with the demands of adult life” in their particular country. But this notion too is
questionable, since such generalising claims group together adults with different work, family
and social situations, and different literacy / numeracy demands on them; see Black &
Yasukawa’s (2013) discussion of current debates in Australia.

These sorts of concerns about validity and interpretation are shared by users of all surveys
which include assessments, especially those that aim to make comparisons across countries, or
over time. Nevertheless, such concerns must be assessed for any survey, where results aim to
inform policy or practice.

---

9 And this may disadvantage some groups of respondents more than others, e.g. older ones more than younger. (I am
indebted to one of the anonymous referees for this suggestion.)
Some illustrative results for PIAAC from Australia

A summary of the methodology and results from Australia was made available in February 2013, by the survey contractor, the Australian Bureau of Statistics (ABS, 2013). This illustrated the sorts of results that were made available in each of the participating countries in October 2013. Here I give three examples.

Figure 1. Overall results from PIAAC for Literacy and Numeracy: Australia, 2013

**Source:** ABS (2013)

Figure 1 allows us to read off the proportions of Australian adults at different skills levels. Approximately 44% (7.3 million) of Australians aged 15 to 74 years had literacy skills at Levels 1 and 2, a further 39% (6.4 million) at Level 3 and 17% (2.7 million) at Levels 4/5. For the numeracy scale, approximately 55% (8.9 million) Australians were assessed at Levels 1 and 2, 32% (5.3 million) at Level 3 and 13% (2.1 million) at Level 4/5. One could also compare literacy and numeracy levels for subgroups, e.g. residents of different Australian states (using other data). For example, the Australian Capital Territory recorded the highest proportion of adults at Level 4/5 (23%) numeracy. We can also ask about gender differences, of interest in much earlier research; see Figure 2.

Figure 2. Proportion at each PIAAC numeracy level, by sex: Australia 2013. **Source:** ABS (2013)
In Figure 2, there appears to be little difference in the proportion of males and females at each level of the numeracy scale. However, a higher proportion of males (17%) attained scores at Levels 4/5, compared with females (9%), as seen from the graph.

We can look at age differences too, over the age group surveyed in Australia: 15-74 (a wider age range than required by PIAAC protocols); see Figure 3.

![Figure 3. Proportion at each numeracy level, by age: Australia, 2013. Source: ABS (2013)](image)

The data suggest that proportions of people at Level 1 are highest among the oldest age groups (people aged 60 years and older), and lowest in the younger age and middle-aged groups (people aged 20 to 49 years) for numeracy skills.

**Discussion: Possible effects of international surveys and “countervailing forces”**

In previous sections we have described the developing role of a globally promoted type of pedagogic discourse promoted by transnational organisations, which asserts adults’ need for certain rather generic skills, and individual countries’ needs to assess these in a comparative way. Basil Bernstein’s analysis (2000) of the structuring of pedagogic institutions and discourses and his focus on changing forms of educational knowledge and practices, along with related work (e.g. Moore with Jones, 2007), can illuminate and critique shifts in the mode of governance of educational policy, in which international surveys like PIAAC are used (by a number of policy actors) to play a role (Tsatsaroni & Evans, 2013).

The international studies of adults, like IALS and PIAAC, have no systematically thought out curriculum associated with them (unlike TIMSS and PISA). Yet the existence of such a “curriculum” is arguably implied in the definition of numeracy (see Section 3 above) and the use of existing classifications of mathematical content. Tsatsaroni and Evans (2013) originally predicted that there was “a strong possibility that PIAAC could reinforce this type of pedagogic discourse, and the surveys could tend to work as an exemplary curriculum type which indirectly prescribes what knowledge the adult populations in all societies should value, strive to acquire, and demonstrate” (emphasis added). In the event, Christine Pinsent-Johnson’s more recent paper (2013) on adult literacy shows that this “possibility” has already materialised in the Essential Skills in Canada, “a competency-based compendium of employment related ‘learning outcomes’ that integrates [international testing] constructs”. Ontario, Canada’s largest province, has recently begun to use a new curriculum that was put together using these constructs: “A hypothetical and abstracted literacy devised for large-scale testing has been transposed into a...
pedagogy that is distinct from schooling and academic literacy practices, and disconnected from personal, community and work literacy practices” (Pinsent-Johnson, 2013, p.2).

There are a number of possible effects of such performance surveys, which may represent high stakes for adults and the countries involved. An obvious negative effect is the pathologising of countries which do not “perform” to standards – not necessarily by the survey’s sponsors, but by sections of the media, political parties, and new educational agencies, such as national assessment bodies. (cf. “PISA shock”, discussed in EERJ, 2012).

The emerging discourse supported by international surveys may also have effects on teachers’, learners’, researchers’ and citizens’ ways of understanding adult literacy and numeracy. Knowledge can come to be seen as generic skills, flowing from a decontextualised imagining of the adult’s everyday practices. To the extent that different social groupings and different countries embrace such ideas, they may have restricted access to the countervailing principles of thinking that disciplinary or professional forms of knowledge can provide.

Now, “disciplinary knowledge” can also be understood as “powerful (mathematical) knowledge” (Young, 2010), or as “big ideas” in mathematics education (Lerman, Murphy & Winbourne, 2013) – that is, as ideas that have rich applicability in a range of fields. One example of a big idea in mathematics / statistics that was illustrated several times at the ALM-20 conference is the idea of conditional probability. This idea occurs under many guises: as having the right denominator for your proportions, in arithmetic; or in reporting research results (e.g. percentage of items correct) for the appropriate population; or in appreciating the difference between the probability of testing positive for x, given that you have disease x – and the probability of having disease x, given that you test positive for x, which is vital in understanding medical test results (Gigerenzer, 2003; O’Hagan, 2012.) However, for big ideas to be fully appreciated by learners, a coherent curriculum for adults’ mathematics is necessary.

As for positive effects, we must investigate whether international surveys afford opportunities for further research. One can relate performance scores to demographic and attitudinal data from the Background Questionnaire, and/or further information available on numeracy related practices and “use of skills” at work; see OECD (2013a, pp. 101-140) for such analysis, at the international level. These studies may also provide a context for certain types of national studies, or local qualitative studies, to supplement or to probe Background Questionnaire results; for example to investigate why residents of the Australian Capital Territory might have recorded the highest proportion of adults at Level 4/5 for numeracy (23%) (See above). There are also some examples of use of results from earlier international surveys, e.g. PISA and TIMSS, to study wider educational and social questions (see Tsatsaroni & Evans, 2013).

Resources for researching additional interesting questions suggested by the preliminary results are now more accessible than before. OECD makes available, on their website, datasets from PIAAC – and software for data analysis – for research purposes (see http://www.oecd.org/site/piaac/#d.en.221854).

In the international adult numeracy community, we can look to alternative research programmes to assert the value of alternative conceptions of educational knowledge, and to critique developments in adult educational policy issues, including literacy and numeracy. From within adult numeracy, or what can be called adults’ mathematics education (Evans et al., 2013) – we can illustrate ways to challenge the currently dominant ideas of numeracy and adult skills. For example, Diana Coben and colleagues have challenged the conventional “deficit” characterisation of practising adults’ (nurses’) numeracy, and argued that the high-stakes testing programmes used have often deployed instruments which lacked reliability, validity, and authenticity (Coben, 2000). Hoyles, Noss, Kent & Bakker (2010) go beyond a narrow definition

10 And lifelong learning more generally (Evans, Wedege & Yasukawa, 2013).
of numeracy to develop a richer conception of “Techno-mathematical Literacies” (TMJs), informed by the affordances, flexibilities and demands of information technologies, and document its use by middle ranking UK professionals, in decision-making in specific workplaces. Mullen & Evans (2010) describe demands on citizens’ numerate thinking and learning, emphasising the social supports made available (by government and other institutions), in coping with the 2009 currency conversion to the euro in the Slovak Republic. Gelsa Knijnik and her colleagues (e.g. Knijnik, 2007) describe work with the Landless Movement in Brazil, facilitating their learning to recognise, to compare, and to choose appropriately from academic and/or “local” knowledges, in carrying out their everyday practices. The proposals of Knijnik and colleagues and Hoyles et al. are clearly moving towards the formulation of alternative, coherent curricula based on the big ideas that their researches are pointing towards, and helping to develop. Coben and her colleagues are working to develop alternative methods of assessment for professional practitioners.

Powerful knowledges of these kinds can empower on a broader social basis, through knowledge located in the disciplines, professional practice, or other established practices of adults’ “lived experience”. The aim of educational researchers must be to support the development of potentially powerful knowledge (Young, 2010), like numeracy and literacy, and to prevent their being reduced to narrow competencies.

To summarise, it seems clear that PIAAC and other international surveys will be key background features in educational policy discussions and educational research for the foreseeable future. These surveys will have a range of effects, some of which will be a focus of struggle involving their transnational sponsors, countries and their citizens. PIAAC itself includes a complex set of measures, and offers the opportunity to relate them in a range of ways. Like all studies, because of its conception and its methodology, it tends to highlight and to emphasise particular aspects of the world it surveys – such as a generic conception of numeracy and literacy, and the use of measures understood as comparable across a globalised world – and to play down others. It is therefore essential for all those interested in adult numeracy and literacy to read its results carefully and sceptically.

**Acknowledgements**

I thank Anna Tsatsaroni, Tine Wedege and Keiko Yasukawa for useful discussions concerning the arguments in this paper. I also thank other colleagues in ALM, and in particular the audiences at the ALM-20 conference, for stimulating exchanges, and the anonymous referees for careful and helpful comments. Appreciation is due also to colleagues in the PIAAC Numeracy Expert Group for valuable discussions in the course of our work.

**References**


Numeracy – Sample Item 1
This sample item (of difficulty level 3) focuses on the following aspects of the numeracy construct:

<table>
<thead>
<tr>
<th>Content</th>
<th>Data and chance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Interpret, evaluate</td>
</tr>
<tr>
<td>Context</td>
<td>Community and society</td>
</tr>
</tbody>
</table>

Respondents are asked to respond by clicking on one or more of the time periods provided in the left pane on the screen.

**Numeracy – Sample Item 2**

This sample item (of difficulty level 3) focuses on the following aspects of the numeracy construct:

<table>
<thead>
<tr>
<th>Content</th>
<th>Dimension and shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Act upon, use (estimate)</td>
</tr>
<tr>
<td>Context</td>
<td>Every day or work</td>
</tr>
</tbody>
</table>

Respondents are asked to type in a numerical response based on the graphic provided.

**Correct Response:** Any value between 77.7 and 78.3

**Numeracy – Sample Item 3**

This third item (of difficulty level 1) in the set focuses on the following aspects of the numeracy construct:

<table>
<thead>
<tr>
<th>Content</th>
<th>Dimension and shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Act upon, use (measure)</td>
</tr>
<tr>
<td>Context</td>
<td>Every day or work</td>
</tr>
</tbody>
</table>

Respondents are asked to type in a numerical response based on the graphic provided.

**Correct Response:** Any value between -4 and -5
Evans, New PIAAC Results.

**Numeracy – Sample Item 4**
This sample item (of difficulty level 4) focuses on the following aspects of the numeracy construct:

<table>
<thead>
<tr>
<th>Content</th>
<th>Quantity and Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Act upon, use (compute)</td>
</tr>
<tr>
<td>Context</td>
<td>Community and society</td>
</tr>
</tbody>
</table>

**Wind Power Stations**

In 2005, the Swedish government closed the last nuclear reactor at the Barseback power plant. The reactor had been generating an average energy output of 3,372 GWh of electrical energy per year.

Work continues in Sweden on installing large offshore wind farms using wind power stations. Each wind power station produces about 6,000 MWh of electrical energy per year.

**For your information:**

Electrical energy is measured in Watt-hours (Wh).

1 kW = 1,000 Wh
1 MW = 1,000,000 Wh
1 GW = 1,000,000,000 Wh

**Correct Response:** One of the three values (no values between): 595, 596 or 600.

*Source: CSO, Ireland (2013)*
Integrating Real-World Numeracy Applications and Modelling into Vocational Courses

Graham Hall
Coleg Meirion-Dwyfor, Dolgellau, United Kingdom, LL40 2SW
<graham.hall@gllm.ac.uk>

Abstract
Practitioner research is in progress at a Further Education college to improve the motivation of vocational students for numeracy and problem solving. A framework proposed by Tang, Sui, & Wang (2003) has been adapted for use in courses. Five levels are identified for embedding numeracy applications and modelling into vocational studies: Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research. These levels represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work. Case studies are presented of the incorporation of the five levels of application in engineering, construction, computing, and environmental science courses. In addition to student motivation, teaching staff observed that improvements have occurred in: use of specialised mathematical vocabulary; the combined use of numerical and algebraic methods in problem solving; and abstract reasoning, and a deeper level of understanding of the mathematics used in problem solving. A difficulty which has not yet been fully resolved is the reconciliation of a problem solving and project based approach to numeracy, and the requirement by some Examination Board numeracy syllabuses to assess specific mathematical methods.

Key words: numeracy, vocational education, modelling, applications, assessment

Introduction
This paper describes practitioner research which is being carried out by tutors of vocational courses at a Further Education College in Wales. Students often begin vocational courses with a poor experience of school mathematics and lack enthusiasm to improve their mathematical skills. However, they will need to develop numeracy and problem solving as an essential aspect of their vocational training, for example: in subjects such as engineering or construction. The aim of the current project is to develop a framework of learning strategies which will interest and motivate students. It is hoped to develop students' numeracy skills within their vocational areas; and to help them to gain transferrable skills in critical thinking, creativity, teamwork and collaboration, and learning self-direction.
School mathematics in Britain, as in many other countries, is designed around a bottom-up academic model. Pupils learn mathematical methods within distinct topic areas such as: number, algebra and geometry, then work on example applications still within these same topic areas. The intention of the developers of mathematics syllabuses seems that pupils will progress to study subjects at an advanced level, such as sciences, where they will be able to make good use of the mathematical techniques they have learned. Figure 1 highlights the components of this bottom up academic model in Britain.

This model can present problems for students who leave school at the age of 16 to study a practical vocational course. They may view mathematics as a series of unrelated topics, some of which seem to have no relevance to their chosen profession. Algebra, in particular, is seen by many school leavers as having very little practical everyday.

**Methodology**

Students entering further education courses in engineering, construction, computing, and environmental science at post-16 age took part in questionnaire surveys. This allowed the researchers to better appreciate and understand the attitudes and abilities in numeracy developed by the students during their school education.

Clinical interviews (Ginsburg, 1981) were then carried out with a total of 12 students chosen from the range of courses. The students were asked to give a commentary on their reasoning whilst attempting to solve various mathematical problems. From an analysis of the interview transcripts, four particular difficulties were identified:

- Lack of specialised mathematical vocabulary. Students had difficulty describing features of graphs, equations and other mathematical entities.
- No strong connection between number and algebra in problem solving (Lee & Wheeler, 1989). Students made no attempt to understand relationships in formulae by substituting numerical values, and made no attempt to devise formulae to simplify the repetitive handling of numerical data.
- A preference for justification by concrete example. Students generally preferred to use manipulation and measurement of solid shapes to solve problems, rather than abstract mathematical reasoning.
- Misuse of standard algorithms which had been learned in a superficial manner without full understanding. Examples causing difficulty included formulae for areas and volumes, sides of triangles, and trigonometry.

It became evident that there was little to be gained by continuing to teach in a way which had already been unsuccessful for some students. A new approach was therefore attempted by teaching staff participating in the project, and forms the basis of this paper. Thought was given to the development of open ended and ill-defined numeracy problems to be presented to students which would realistically simulate work related tasks within the within their vocational areas.

A useful framework for introducing real world problems into mathematics teaching has been proposed by Tang, Sui, & Wang (2003) from work in China. Practitioner research during the current project has focussed on ways in which this framework could be successfully adapted for introduction into vocational courses at further education level. Five approaches are identified by Tang et al. for embedding numeracy applications and modelling: Extension; Special Subject; Investigation Report; Paper Discussion; and Mini Scientific Research. These approaches represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work designed by students themselves.

At the end of an academic year, students were re-interviewed to investigate changes in their perceptions of numeracy, and to investigate the extent to which their abilities in problem solving had developed.

**Applying Numeracy**

A distinction is made by a number of researchers (Hoyles et al., 2000; Dingwall, 2000; Coben, 2000) between mathematics, which is taken to be a set of quantitative methods, and numeracy which has wider links with the real world. Numeracy need not be at an elementary level but might include, for example, the advanced mathematics used by engineers or scientists. Numeracy requires knowledge of the real world context in which the problem occurs. It is essentially a practical problem solving activity drawing upon appropriate mathematical techniques, and the results obtained often need to be communicated to others in a way which is useful for decision making. The relationship between mathematics and numeracy developed by this model is illustrated in Figure 2.

![Figure 2. Relationship between mathematics and numeracy](image-url)
The approach taken by teaching staff during the current project is to work downwards within vocational area to identify numeracy tasks undertaken by practitioners in everyday work. Tasks are analysed in collaboration with students, and solved using mathematical methods which might be familiar or which might need to be learned at this stage. Additionally, the work provides opportunities for consolidating mathematical knowledge in broader topic areas. This approach is illustrated in Figure 3.

![Figure 3. Top-down vocational numeracy model](image)

Central to the numeracy approach which we are developing with our students is the MeE motivation model of Martin (2002) and Munns & Martin (2005), summarised in Figure 4:

![Figure 4. MeE motivation model of Munns & Martin (2005)](image)

The model focuses on the need to motivate students by presenting interesting learning activities, and the self-satisfaction that students can gain from engaging successfully with these activities. This can lead students to develop a personal engagement with the subject as a whole, through the enjoyment and sense of achievement which it provides, so that they become intrinsically motivated to develop skills and knowledge to a higher level.

Munns and Martin advocate the introduction of the most interesting work from the very start of a course, as a means of generating enthusiasm. Teachers may need to simplify tasks to ensure that students achieve a successful outcome and gain a sense of achievement. It is important that the students consider the tasks to be realistic, relevant and worthwhile.

During the current research activities, teaching staff realised that a number of interesting and motivating tasks might need to be presented, but they hoped that individual students would reach the point of engaging with the subject as a whole. From this stage on, the work of the
teacher would become much easier. The value of the subject would be clear to the students and they would be motivated to extend their knowledge and skills through independent learning.

**Naturally Occurring Numeracy**

In a number of vocational areas, numeracy tasks occur quite naturally in everyday work. Two examples produced by colleagues are (a) curved work in carpentry, and (b) expedition planning, which are presented here:

**Curved work in carpentry (Slaney, 2013)**

Amongst the more advanced practical skills taught to carpentry students are methods for constructing curved door and window frames of various designs. Designs have to be produced as a bench template for cutting the timber components.

A challenge presented to students was to construct a gate in the form of a Tudor Arch, which traditionally has the geometrical design illustrated in Figure 5:

![Figure 5. Geometrical construction method for a Tudor arch](image)

Students investigated the construction method:

- Two circles of equal radius are drawn to set the width AB of the arch. Two arcs AE and BF of these circles form parts of the completed arch.
- A square is constructed, with the distance between the circle centres 1 and 2 as the length of each side,
- The mid-point of the bottom edge of the square, CD, is used as a centre for constructing the upper arcs between E and F to complete the arch.

The group were interested to investigate the geometry of other traditional arch designs developed by masons and carpenters during different historical periods.

**Expedition planning**

Students who are training to become outdoor pursuits instructors are required to make reasonably accurate estimates of the time which expeditions will take over mountainous terrain as part of the procedure for safety planning. A mathematical formula known as Naismith’s Rule can be used for estimating journey time. This determines a time based on walking speed over flat ground, then adds extra time for the amount of ascent and descent necessary during the journey.
Students using Naismith’s Rule have found that the time calculations for expeditions in the mountainous area of North Wales are very inaccurate. This is due to wide variations in the time taken to cross different types of terrain. Walking speed is much slower across moorland, overgrown forest or wetland than along well constructed footpaths. Scrambling over rocks on mountainsides is particularly slow.

As a project, students have documented the actual times taken for the different stages of a number of expedition routes, and have related these to the nature of the terrain. They are attempting to develop a more accurate journey time formula to improve on Naismith’s Rule. This project is a good example of the application of the modelling cycle of Blum and Leiß (Keune & Henning, 2003).

The modelling cycle of Blum and Leiß considers the manner in which a mathematical modelling problem can be conceptually divided into two domains – the real world domain, shown on the left in Figure 6, and the mathematical domain shown on the right.

The modelling cycle begins in the real world domain, where it is necessary to identify the factors which are important to the outcome of the model. The relationships between these factors are then assessed in descriptive terms. Modelling then moves to the mathematical domain, where the factors are expressed in terms of a formula and numerical examples are run to generate modelling predictions. These modelling predictions are then related back to the real world domain and checked against actual observations. If necessary, the modelling assumptions can be revised and the model re-run, until an acceptable solution is found.

A particular value of this project has been to help students make a connection between algebra and number, with these mathematical methods employed together effectively in problem solving. Students often have difficulty in constructing algebraic formulae from theoretical relationships between variables, or from the identification of patterns in observed data. We have found that the plotting of graphs can provide a helpful conceptual link between number and algebra.

**Framework for Numeracy and Applications based on the work of Tang, Sui & Wang (2003)**

It is valuable for numeracy tutors to make use of naturally occurring numeracy tasks, such as the carpentry work mentioned above which forms an essential component of the course syllabus. However, it is sometimes necessary for tutors to develop additional applications to broaden the
mathematical and problem solving skills of vocational students. Many topics studied on vocational courses can, with imagination on the part of the tutor, provide opportunities for interesting and realistic numeracy problem solving.

Tang, Sui & Wang (2003) proposed a framework which we have adapted for use in vocational courses during this project. The original framework was intended to provide a practical structure in which mathematics students could apply their mathematical skills in realistic real world situations. Our approach is somewhat different, in that we have used the framework as a structure by which vocational students might investigate real world problems through the application of numeracy. Whilst the students of Tang et al. would be experienced in mathematical techniques but perhaps unfamiliar with their applications in the real world, our students would be familiar with the types of problems arising in vocational situations but might need to develop further mathematical skills to solve these. The overall aim in both cases is to develop practical numeracy problem solving skills, though from different starting points.

Five levels were identified by Tang et al. for incorporating applications and modelling into mathematics courses: (a) Extension, (b) Special Subject, (c) Investigation Report, (d) Paper Discussion, and (e) Mini Scientific Research. Examples of tasks illustrating each of these levels are presented below. The five levels represent a progression from applications set by the teacher, through increasing student involvement in the solution of real world problems, to totally independent project work designed by students themselves.

**Extension**

In this approach, students who have been studying a mathematical topic are presented with an ill-defined real world problem where they need to seek out additional data for its solution. As an example, consider the problem in Figure 7 which might be given at the end of a study of trigonometry. The problem cannot be solved without obtaining measurements.

A photographer is intending to travel on the London Eye, a large Ferris wheel in London. She wishes to take panoramic views across the city, but needs to be at least higher than the roof of the nearby County Hall building to do this. She would like to know how many minutes will be available for the photographic session.

**Figure 7.** Example of an ill defined problem

In this case, the student should obtain actual data, or at least reasonable estimates, for the speed of rotation of the wheel, its diameter, and the height of the adjacent building. This might be found by use of the Internet. The student is then free to devise his/her own method for numerical, graphical or analytic solution of the problem.

An example solution produced by a student is shown in Figure 8. Trigonometry has been used to obtain a formula linking the height of a car above the ground, H, to the diameter of the wheel, the height of the central axel above the ground, and the angle of rotation θ. A graph was
then plotted using a spreadsheet, and an estimate made of the time during which the car was above the height of the nearby building.

Figure 8. A possible graphical solution of the London Eye problem. The wheel has a diameter of 120m, and rotates in 30 minutes.

Special Subject

Students who have studied a vocational topic were given the opportunity to investigate the topic further through a quantitative project. This approach was used successfully with construction students who had been studying heat losses from buildings. After discussion of the insulating properties of different building components, students developed their own spreadsheets to determine the heat losses from a house. This allowed investigation of the effects of double glazing of windows, cavity insulation of walls, and insulation of the roof space, and gave a deeper understanding of the mathematics involved.

The model makes use of the dimensions of each wall of a room, and its construction material, to estimate heat losses. This heat loss is also dependent on the average temperature difference between the two sides of the wall. Heat losses are added for the floor and ceiling, to obtain total heat losses for the room. Allowance must also be made for heat loss through air ventilation in the room.

Figure 9. House heat loss
Investigation Report

For this approach, students gather their own primary data through surveys, laboratory or fieldwork measurements, then process the data using appropriate mathematical methods. In this way, it is hoped to gain a clearer interpretation of the data and to obtain insights which were not initially obvious.

As an example, geography students investigating coastal processes measured pebbles which were being transported along a shingle spit by wave action. Results and analysis are presented in Figure 10. The chart on the left plots the average dimension of pebbles against position along the beach. The shingle spit originates at a cliff line at Friog on the left of the chart, and extends out into the estuary for a distance of approximately 1.5km, past Fairbourne to Barmouth Ferry.

![Figure 10. A comparison of field data and a theoretical model for pebble erosion](image)

It was seen that although a mixture of pebble sizes was present at each location visited, there was a reduction in mean size during transport along the shingle spit. Discussion between students and their tutor lead to a hypothesis that the rate of size reduction would be proportional to the actual pebble size — large pebbles would be eroded more easily by wave action than small pebbles.

A spreadsheet numerical model was developed for a constant percentage reduction in size for each distance unit, leading to the familiar negative exponential curve. The theoretical curve produced in the spreadsheet model to the right was seen to closely reflect the best fit curve through the actual field data, supporting the initial hypothesis connecting rate of erosion directly to pebble size.

Paper Discussion

The approach used here is to present students with an interesting and challenging vocational mathematics task, then provide resources from books, journal articles or the Internet which will allow the students to teach themselves the necessary quantitative techniques for solving the
problem. This contrasts with the normal teaching approach in which the tutor provides instruction, and is intended to encourage students to develop as independent learners.

An example presented to computing students was to model an epidemic of a non-fatal illness such as influenza. Published articles were provided which explained the recurrence relations which form the Simple Epidemic Model (Keeling, 2001). The population is modelled as three groups:

- Susceptible: those who can catch the illness,
- Infected: those who have the illness, and could infect others, and
- Recovered: those who cannot catch the illness again and are no longer infectious to others.

In each time period, a number of people will catch the disease and move from the Susceptible to the Infected group. The number of persons infected will depend on the proportion of the population who are susceptible and infected, and come into contact. It will also depend on the infectiousness of the disease – a variable known as the Epidemiological Parameter.

In the time period, others will recover and move from the Infected to the Recovered group. This number will depend on the average time a patient takes to recover from the illness. Results from a run of the spreadsheet recurrence relation model are shown in Figure 11.

![Figure 11. Results generated by students in running the Simple Epidemic model.](image)

It is seen that the epidemic begins to decline when the number of recovered people in the population exceeds the number susceptible. This is due to the reducing likelihood of an infected person coming into contact with a susceptible person to spread the illness.

**Mini Scientific Research**

This approach represents the maximum level of student involvement in the planning, investigation and analysis of data for a substantial numeracy project related to their vocational area. An example of a project carried out by engineering students has been the investigation of the motion of a car when passing over a speed hump, in response to the springs and shock
absorbers of the car suspension system. Results from a run of the spreadsheet model are shown in Figure 12.

![Damped simple harmonic model for the motion of a car passing over a speed hump](image)

Figure 12. Damped simple harmonic model for the motion of a car passing over a speed hump

The model was developed by the students as a recurrence relation for small time intervals. At the start of the time interval, the current vertical velocity and acceleration of the car body were known. Initially these are zero for steady motion along the road.

As the car climbs the speed hump, it transmits a vertical force to the car body. In the case of the spring component, this is dependent on the shortening of the spring according to Hooke’s Law. For the damper component, however, a reverse force is generated which is proportional to the vertical velocity of the damper piston. Vertical acceleration of the car body can then be modelled from the resultant force through the suspension system and the mass of the car body.

Students were able to compare the results of spreadsheet modelling with video film which they produced of the actual motion of cars passing over speed humps at different speeds.

**Evaluation**

The researcher conducted observations of students who were undertaking numeracy tasks, and examined the solutions students produced before interviewing participants about their experiences during the project and their broader attitudes towards numeracy and mathematics. The project is ongoing, but it is clear that higher levels of interest and motivation have been generated by the various tasks, and students’ confidence in using mathematical techniques has been improved. In particular, problems identified early in the year have been addressed to a significant extent:

- Use of specialised mathematical vocabulary is more evident,
- Numerical and algebraic methods were being combined in the solution of problems,
- Skills in abstract reasoning have improved, and
- A deeper level of understanding of the mathematics used in problem solving is evident.
A difficulty which has not yet been fully resolved is the reconciliation of a problem solving and project based approach to numeracy, and the requirement by some Examination Board syllabuses to assess specific mathematical methods. As an example, we might consider the Essential Skills Wales Application of Number qualification.

From the description:

The aim of the Application of Number standards is to encourage candidates to develop and demonstrate their skills in using number to tackle a task, activity or problem by collecting and interpreting information involving numbers, carrying out calculations, interpreting results and presenting findings. (WJEC, 2013)

this qualification appears to closely embrace a real-world problem solving approach to numeracy. However, closer examination of the assessment requirements presented in Figure 13 shows that a large number of particular mathematical methods must be demonstrated in the work submitted by candidates.

It is evident from this list that no single realistic real-world project, or even small number of projects, is likely to come anywhere near covering all the stated requirements. There is therefore a tendency for tutors to revert to the bottom-up model of teaching mathematical topics individually to cover the syllabus, and vocational applications become limited and unconvincing to students.

A compromise approach is to primarily employ real-world problem solving as a means of motivating students, but also allocate time at the end of each project session to provide broader coverage of related mathematical methods and topics. Students are made aware that this is necessary in order to meet assessment requirements. For example, after solving a graphical problem which is discovered to involve an exponential function, students may be introduced to other related functions such as powers, inverse powers and logarithms. After the use of trigonometry to creatively solve an ill-defined circular motion problem, students might examine the use of trigonometric applications in other areas such as topographic surveying.

**Figure 13.** Extract from the specification for Essential Skills Wales Application of Number
Further Development

Creative problem solving can provide a structure for introducing new mathematical topics in a way which is motivating for students, demonstrates immediate relevance to vocational studies, and supports a deeper understanding of mathematical methods. As an example, a new approach has been used to introduce engineering students to calculus for the first time at the start of their course. Students are asked to estimate the volume of the centre cone of a jet engine (Fig.14):

### Aircraft engine

The front conical section of a new jet engine fan has a horizontal depth of 0.5m. The profile of the cone has the function

\[ y = \sqrt{x} \]

In order to model the rotation of the engine, the volume of the cone needs to be found so that its mass can be calculated.

![Aircraft engine image](image)

**Figure 14.** Volume calculation problem

In discussions between the tutor and the student group, it was agreed that the volume could be estimated by dividing the cone into a series of cylinders, with volumes then calculated and totalled using a spreadsheet (fig.14):

\[
\text{volume} = \sum \pi r^2 h
\]

\[
\int_0^{0.5} \pi r^2 \, dx = \int_0^{0.5} \pi (\sqrt{x})^2 \, dx = \pi \left[ \frac{1}{2} x^2 \right]_0^{0.5} = 0.125 \pi
\]

**Figure 15.** Solution to the volume calculation problem

Students were readily aware that the accuracy of the estimate would increase as the number of cylinders was increased, although in practice the number of cylinders would be limited by the capacity of the spreadsheet program. Integration was then introduced as an alternative quick and easy method of finding the total volume of an infinite number of infinitely thin cylinders – effectively providing the exact answer to the problem. The group were suitably impressed by
the power of mathematics, and the effort that can be saved by applying appropriate mathematical methods.

Overall, the development of numeracy through problem solving in vocational areas, either by naturally occurring applications or use of the framework of Tang et al. (2003), is seen as an effective way of increasing student motivation and creativity.

Conclusion

This project has examined approaches to improving the numeracy and problem solving skills of students in engineering, construction, computing, and environmental science courses at a further education college. It has been evident to the teaching staff that student motivation is critical to developing numeracy. Principal factors affecting student motivation were found to be: the relevance of numeracy tasks to students’ main courses, the realism and authenticity of tasks in the relevant vocational field, and the intrinsic interest and challenge of the problems presented.

A definite advantage of the use of open ended and ill-defined problem solving activities, apart from improving student motivation, was to encourage the development of wider numeracy skills. These include: communication of mathematical ideas in a form suitable for decision making, use of practical and common-sense techniques in obtaining solutions to problems, skills in data collection, and estimation of results to appropriate levels of accuracy. A particularly pleasing aspect observed by the researchers was an improved ability by students to inter-relate the numerical, graphical and algebraic representations of data sets.

The framework introduced by Tang et al. (2003) was considered by the teaching staff to provide a range of interesting opportunities for planning and structuring student activities which could be integrated into vocational courses. The student activities proposed by Tang et al. were seen to encourage: problem solving, group co-operative working, and independent learning.

Acknowledgements

Grateful thanks are due to numeracy tutors and students at Coleg Meirion-Dwyfor, Dolgellau, for allowing their work to be observed, and to college managers for allowing and encouraging practitioner research. Any opinions, findings, conclusions or recommendations are those of the author and do not necessarily reflect the views of Coleg Meirion-Dwyfor.

References


Counting or Caring: Examining a Nursing Aide’s Third Eye Using Bourdieu’s Concept of Habitus

Maria C. Johansson
Malmö University, Sweden
<maria.c.johansson@mah.se>

Abstract
This article is derived from analysis of observations and an interview with, Anita, a nursing aide, who was followed in her work in a semi-emergency unit in Sweden. Based on an analysis of this information, it is suggested that the process of going from school to a workplace can be viewed as a transition between different mathematical activities, which involve and require learning. Although it is easy to see transitions occurring between different contexts, they may also occur within the boundaries of a workplace and be connected to critical moments in the execution of work tasks. Adopting a social critical perspective, this article initiates a discussion about the transitions between potentially mathematical activities in work and how the values given to these different activities can be understood. It is further suggested there is difficulty in recognizing some activities in work, because, often, they are overshadowed by other competences and components needed in work, such as caring.

Key words: workplace mathematics; capital; habitus; transition; nursing aide

Introduction
Adults’ mathematics learning takes place in a wide range of settings throughout life. Notwithstanding this, many of us have school mathematics as a reference for learning mathematics. The focus on mathematics within the context of schooling may make it difficult for one to detect and understand what mathematics may become in other spheres in life. In this article, I investigate potentially mathematical activities in a nursing aide’s work using Bourdieu’s (1992, 2000) concept of habitus
11 while taking into account that my previous experience as a vocational teacher of mathematics has influenced what I am able to see and understand.

The feeling of having a limited understanding of mathematics in workplaces, and a curiosity to learn more about this has been the driving force for my work. I became aware of my limited understanding of workplace mathematics when I worked as a mathematics teacher in vocational education and prepared tasks on intravenous drips. The students were to become nursing aides

11 Habitus is at its most basic definition human’s dispositions to act in the social world.
and I produced tasks on different drip speed, drip size, and concentrations of active substances. Having received help from a nurse, the tasks seemed realistic, with proper substances and so on. I was very happy with the tasks, until the students reacted with stress, anxiety and fear. Almost crying, they asked me if tasks such as these were really the responsibility of nursing aides’. I tried to calm them down and apologized several times for giving them inappropriate tasks, but I was left with a feeling of how limited I was in understanding the practice they were heading for.

Furthermore, I became more and more in doubt about school mathematics as being useful outside of school, although its relevance is often justified in this way (Dowling, 2005). The differences between mathematics taught in schools compared to mathematics in workplaces may have consequences for how adults’ competence in workplaces is regarded (Gustafsson & Mouwitz, 2008, see also Björklund Boistrup & Gustafsson, in press). In addition, Wake (2013) suggests that workers sometimes do not even consider that what they are doing is related to mathematics, but rather to a goal-directed, workplace activity.

It is obvious that the use of mathematics in the work of nursing aides is situated in a certain context, influenced by many factors, such as the workplace organization, the well-being of patients, and possibly also the relationships that humans have with mathematics. Trying not to diminish these important aspects of the work situation, while remaining focused on the potentially mathematical activities, my aim is to consider how the transitions between different potentially mathematical activities can be understood through Bourdieu’s, concepts of *habitus* and *capital* (Bourdieu, 1992, 1996, 2000, 2004). In so doing, I ventured to investigate if it is possible to do two things: (1) capture the mathematical knowledge frequently labelled as tacit, and (2) identify what may be gained and lost in different transitions humans make when moving between contexts.

Mathematics in work

Activities involving mathematics in workplaces are not easily described, as they are often connected to the use of the technology and routines (e.g. FitzSimons, 2013; Hoyles, Noss, Kent, & Bakker, 2010; Jorgensen Zevenbergen, 2010; Wake &Williams, 2007; Wedege, 2000, 2004a). Consequently, the mathematics in these activities has been discussed as being “black-boxed”, both socially and technically (Wake & Williams, 2007). By this, they mean that the use of technology and automation has created a distinct genre of mathematics. With the increased use of technology, mathematics becomes more implicit, and hence technically black-boxed. Moreover, different groups and staff members in the workplace have different norms and rules, and this division of labour creates a social black-boxing. The notion of black-box derives from Latour (1993), and his networks of people, objects and ideas seemingly function as a whole, in which certain parts become invisible. Mathematics in work has also been labelled as tacit, with the possibilities for making it explicit considered difficult although not impossible (FitzSimons, 2002). These difficulties in identifying the mathematics may lead to a gap between adult learners’ perspectives on learning mathematics, compared to those of education policy makers and employers (Evans, Wedege, & Yasukawa, 2013). FitzSimons (in press) found that curriculum and vocational numeracy education mostly was about content knowledge, and hence based on narrow assumption of what vocational students may need. Instead if perceiving simple operations as sufficient for those students, FitzSimons further suggests that there is a need for a more holistic approach. With this approach not only the conceptual understanding is considered but also the creativity required in the workplace, and she notes:

However, in the workplace, as elsewhere in society, problems are ever-evolving and the development of new knowledge – locally new if not universally new – is an essential requirement.
for completing the task at hand within constraints of time and/or money, so workers often find themselves in ‘unthinkable’ territory, creating new knowledge. (FitzSimons, in press, p.1) 

Hence, there is a complexity surrounding the mathematics involved in workplace activities. Another complexity is that researchers use both mathematics and numeracy to describe activities in the workplace. In this article I avoid this sometimes value-based distinction. Instead I use the term mathematics in a wide sense when focusing on the transitions between different potentially-mathematical activities of nursing aides.

**Mathematics for nurses**

Similar to the case with other workplaces, previous studies have shown that the mathematics used by nurses is strongly interwoven with the practice of nursing, its routines and other important considerations (Coben, 2010; Pozzi, Noss, & Hoyles 1998). Consequently, there may be differences for nurses in what they learnt in their formal education and what they actually do. For example, Pozzi, et al. (1998) described how nurses considered that “knowing the drug” was a better safeguard against errors, than using an algorithmic calculation suggested in teaching text for nurses. The safety of patients is crucial in this kind of work, and mistakes in an emergency unit can have serious and fatal consequences (Coben, 2010). In such a context, numeracy is about being confident, competent, and comfortable in deciding whether to use mathematics and how (Coben, 2010).

Although the work of nurses has some similarities to that of a nursing aide, the fact that nursing aides do not have medical responsibility indicates that there are also some differences. For example, there are differences in the social and historical conditions around these professions and the division of labour involved. These kinds of differences are important because they may have an impact on Williams and Wake’s (2007) social black-boxing. Evertsson (1995) claims that the history of the profession of nursing aides is, to a large extent, neglected and over-shadowed by a focus on nurses. Caring institutions and hospitals reflect the power structures in society and in the educational system, with regard to class and gender, for example (Evertsson, 1995).

**A sociomathematical approach and Bourdieu’s concepts of habitus**

In the sociomathematical approach (Wedege, 2004b), mathematics is related to more than mathematics as an academic discipline, and also it takes into account mathematics as a social phenomenon and school subject (Wedege, 2004b). Examples of sociomathematical research interests are peoples’ relation to mathematics, and the function of mathematics education in society (Wedege, 2004b). Both humans and general structures are in focus in this approach, and in particular the interplay between *general* structures and *subjective* meaning is highlighted. The tension between humans and societal structures is captured by Wedege (2004b, 2010), who makes a distinction between *demands* made on humans, for example in school or as requirements for getting a job, and the mathematics *developed* by humans in certain practices.

Following Wedege’s (2004b, 2010) suggestion, I consider general structures of the workplace as having certain mathematical demands on nursing aides, but also that the nursing aides themselves develop their own mathematical competence. This individual creativity may be crucial when facing difficult tasks or demanding tasks (FitzSimons, in press). By this, I do not mean that official demands made on workers are necessarily more or less important than the developed competences, rather they are complementary. One example of a study accounting for both the demanded mathematics and what is developed by workers, could be given by the
findings of Pozzi, et al. (1998). They compare the common algorithmic procedures and formulas suggested for drug administration in teaching texts for nurses, with what nurses actually do. The formal and assumed calculations in the teaching texts for nurses are examples of the demanded mathematics. The authors further observed how the nurses instead used the specific concentration and more for each drug as the basis for their calculations. One example of such a calculation was “doubling it and put an extra zero”, which was the calculation used for a certain drug (Pozzi, et al., 1998, p.110).

This way of finding alternative arithmetic methods by “knowing the drug”, as described by Pozzi, et al. was also the safeguard against errors. This provides an illustration of the developed mathematics. In this article my focus is the transition between the demanded and the developed mathematics. The urge to capture the knowing to be found in those transitions has also guided my theoretical choice. In the framework of Bourdieu the mutual interplay between humans and society is captured in the concepts of habitus. Habitus is a system of dispositions, defined by Bourdieu (1992) as:

The conditions associated with a particular class of conditions of existence produce habitus, systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures, that is, as principles which generate and organize practices and representations that can be objectively adapted to their outcomes without presupposing a conscious aiming at ends or an express mastery of the operations necessary in order to attain them. Objectively ‘regulated’ and ‘regular’ without being in any way the product of obedience to rules, they can be collectively orchestrated without being the product of the organizing action of a conductor. (p. 53)

Thus, the habitus is to be found firstly in the conditions of existence, which commit humans to the social structures, sometimes without objective and conscious goal orientation. Moreover, habitus is a complex system of dispositions for acting in the social world, transposable but at the same time durable and carrying collective features. In Wedege’s (1999) research, a woman’s habitus was shown to have influenced her dispositions towards mathematics, and her dispositions for seeing herself as mathematically competent. The woman was born in a saddler’s family in Denmark at the beginning of the previous century, and failed in school mathematics. This outcome was seen as normal for a girl at this time. Later success in mathematics and involvement in mathematics at work and during leisure time, could not completely overcome how the woman perceived herself with regard to mathematics. With this case Wedege provides an example of the complexity of habitus carrying features of both class and gender, not as stereotypical labels but rather inscribed in the person as natural features in a specific context. It is also clear in this woman’s case how habitus is durable yet transposable or changeable, as she never fully ceased to see her failures in mathematics. If mathematics is the foundation of the sociomathematical approach (Wedege; 2004b, 2010), then, as discussed in the next section, Bourdieu’s different capitals act as the link between humans beings and social structures in a framework via habitus.

Bourdieu’s concepts of capital in relation to habitus

Bourdieu (1992, 1996, 2000, 2004), described different forms of capital, such as economic, cultural, and symbolic and saw them as the link between the individual and the social world. Humans act in the social world to convert one form of capital to another, according to which form is valued in the particular social space (Broady, 1998). Cultural capital has to do with education, as a consideration of both upbringing and the educational system. How the cultural capital is valued may differ according to different cultures and school systems. In Bourdieu’s work, the French system was in focus, and so his discussion of the impact of cultural capital
may not be valid in another context or at different points in time. However, as Williams (2012) notes that this kind of capital has an exchange value. By this, he means that a particular mark or degree in school mathematics becomes an entrance ticket to certain jobs or further education. Consequently, cultural capital can be considered the formal education for nursing aides in mathematics. Social capital is the social relations or contacts and can also give humans a kind of interest rate on their educational capital. The symbolic capital refers to what is valued in a certain social space. This kind of capital can grow into the body and become part of our habitus, sometimes unconscious and invisible, even to ourselves. For example, when we just know what to do in a given situation, often out an obvious necessity and make use of our embodied symbolic capital (Bourdieu, 2000). Corporal mechanisms and mental schemata in a person’s habitus can even erase the distinction between the physical and the spiritual world (Wacquant, 2004).

In this study, the focus will be on habitus, cultural capital, and symbolic capital. The concepts of Bourdieu have also been used for earlier studies concerning mathematics in work, and were found to be useful tools for the theorization of the world of work, and how mathematical dispositions may promote or hinder workers (Zevenbergen Jorgensen, 2010). The concepts of Bourdieu were in this study useful for understanding the younger workers’ skills, instead of seeing the young workers as having limited numeracy. Zevenbergen Jorgensen found significant differences in habitus, and also how these differences created tensions between old and young workers based on their ways of seeing and enacting numeracy. The younger generations’ habitus were to a larger extent influenced by digital technology, while the older had more manual arithmetic frames of reference.

Bourdieu does not explicitly mention mathematics as a form of capital. In his later work, he did emphasise how there was a shift from Latin to mathematics as a selection tool in the educational system (Williams, 2012). The importance given to mathematics in the education system influences its value. What is considered as important and relevant with regard to mathematics also changes over time as shown by Zevenbergen Jorgensen (2010). Therefore, it is likely that different mathematical activities are valued differently and hence hold various amounts and forms of capital. This is important in a study about nursing aides, where the hierarchical workplace organisation as a practical and rational matter, may conceal other power relations (Evertsson, 1995). These relations affect the valuing and attention paid to certain activities. The focus in this article is potentially mathematical activities, and when nursing aide may need to make transitions between these in critical situations.

**Transitions**

In transitioning from school to work, for example, it is important to recognise the transformation and creation of new relations between knowledge and social activities, and how this could contribute to an understanding of mathematics in the workplace (Wake, 2013). Meaney and Lange (2013) see transitions between contexts as always involving learning, with contexts being defined as systems of knowledge enacted in social practices. The notion of transition could also be seen as a way around the issue of transfer (Beach, 1999). Beach claims that transfer derives from educational psychology and refers to cognitive matters. From a purely cognitive approach, transfer is seen as relatively unproblematic (Evans, 1999). Evans notes that from a situated perspective, transfer instead should be considered impossible. Beach (1999) suggested an alternative stance from a sociocultural viewpoint, namely that of consequential transitions, which means transitions that are reflected upon from a sociocultural perspective. Thus, the social and historical context of the activity is taken into account as well as the artefacts involved. Beach also identifies several forms of transition. In this article, I make use of
Beach’s lateral and encompassing transitions. The former occurs when individuals move between contexts such as school and work, and the latter when change occurs within the boundaries of a social activity.

Encompassing transitions I suggest can be related to the moving between the demanded and developed mathematics in the sociomathematical approach. By this I mean that the official ways to handle a work task are related to the demanded mathematics. Workers also develop complementary ways of completing the potentially mathematical tasks. In other words it is likely that workers in general, and nursing aides in particular, need to make transitions between what is demanded and what they develop. I find it important to shed light on the dichotomy between developed and demanded, as the transition between these has and holds learning opportunities. I suggest that this can be done by understanding different ways of being engaged in potentially mathematical activities, the transitions between them, and the value the activities are given.

For this purpose I have chosen the concept of habitus (Bourdieu, 1992, 2000), firstly because of its possibility to grasp the interplay between individuals and structures. Secondly, my reason for choosing habitus and capital is the fact that habitus has a clear corporal component and different capitals can grow into the body. Hence, there is a possibility that the body becomes itself a black box. It is important to try to understand the significance of the bodily understanding when people transition between the demanded mathematics and the developed. This I see as crucial for reducing the gap between adult learners’ perspectives on learning mathematics and those of education policy makers and employers. The incorporation of capitals is also connected to learning. Bourdieu (2000) describes learning as a durable bodily change (see also Wacquant, 2004). Habitus as a theoretical choice calls for methodological explanation and justification.

Methodology

My intention with this small scale case study (Bryman, 2008) is not to produce a truth, but rather to understand and construe the transitions made within the boundaries of a workplace not familiar to me. This is done through my interpretation of the work of Anita (a pseudonym). As a matter of reflexivity (Hammersly & Atkinson, 1995; Malterud, 2001), my lack of previous experience was used as an advantage. What was obvious for a person with Anita’s long experience was not at all evident to me. This made it possible to pinpoint tacit knowing. Anita is a nursing aide, with more than twenty years of experience, both in her home country in Eastern Europe and in Sweden. The empirical part of the study was conducted with inspiration from ethnography (Hammersly & Atkinson, 1995). However, in a study of this format it is not possible to provide the descriptive thickness normally associated with ethnography. In addition, there is the ethical dilemma of construing another person’s habitus. Bourdieu (2000, p. 128) writes: “Even among specialists of the social sciences, there will always be those who will deny the right to objectify another subject and to produce its objective truth.”

Access was facilitated by a research team member having personal contact with a nurse. From this contact we were introduced to a physician, also head of the ward. He gave us permission to enter the ward with a video-camera. On the ward the nursing aide in charge picked a colleague for us to follow. First our intention was to follow the nursing aide in charge, but she wanted, as she said, to give this opportunity to a colleague of hers. This she told us was because the nursing aides were so rarely paid attention. Two video-recorded visits were made in a hospital in Sweden 2012, each lasting for about an hour. These were then transcribed. After an initial analysis, an informal interview was held with Anita. Having in mind that that
Mathematics in the workplace might be black-boxed, both technically and socially, the topics of the interview were to a large extent introduced by Anita. She talked much about how the profession of nursing aides had developed from formerly being about assisting nurses to nowadays being what she called “its own profession”. This is aligned with Evertsson’s (1995) historical analysis of the profession overshadowed by a focus on nurses.

The interview was tape-recorded and partly transcribed. The sound was of good quality except a short part which was difficult to hear as Anita and I watched the video together. In qualitative research the reliability is often referred to as dependability (Bryman, 2008), and the data loss when we watched the video was compensated by what was gained by Anita explaining what had happened during the observation. As the interview was conducted a couple of months after the observation, looking at the video were also crucial for refreshing our memory. Another way of ensuring dependability was to look at the video together with researchers in the team before conducting the interview. From the individual case of Anita alone it is not possible to make any generalisations, frequently labelled as transferability in qualitative research (Bryman, 2008; Malterud, 2001). With the concepts of Bourdieu it is, however, possible to connect the individual case to the social structures in society. This is aligned with the methodology proposed by Salling Olesen (2012). He notes that workers or groups of workers invest their body and soul, knowledge and commitment when entering a workplace, but they do so against the background of a life history that is a part of a wider societal context (Salling Olesen, 2008, 2012). About, using an individual case Salling Olesen (2012) claims:

It is to use this individual case to theorize learning as an aspect of the social practice, a moment in a subjective life history embedded in the symbolic and social environment, and contributing to societal processes of reproduction as well as innovations. (p. 5)

This methodological view – taking the connection between individuals and society into account – is aligned with the framework of Bourdieu. The possibility for including the bodily manifestation of habitus was facilitated by the use of video and the opportunity to watch it several times. Thereafter, it was possible to raise questions about issues not understandable by the observation alone. My intention with carrying out firstly observations and then the interview was to grasp the complexity of habitus, in which my own habitus, with its own connection to mathematics education, was also considered. Therefore, I have tried to be attentive to and reflect on my own relation to mathematics, and to school. School mathematics will have influenced our perceptions of mathematics; both regarding what should be included as mathematics, and also as a personal relation and experience of it. Thus, our understanding of mathematics and emotions related to it are likely to be connected to the school mathematics incorporated into our habitus (Lundin, 2008).

The analysis makes use of the sociomathematical concepts of demanded and developed mathematics. By this, I refer to the demanded mathematics as what is required in this kind of work, in relation to the mathematical activities that are developed in work. I start with a description of the observation, then I analyse what could be seen as demanded. Then the interview with Anita is described. The analysis is supported by the concepts of habitus and capital, and the notion transition.

**Visiting Anita at work**

The first meeting with Anita was made at the semi-emergency unit where she works. To blend in with the environment, those of us in the research team had to wear the same white clothes as the staff. I followed and observed Anita, while a research colleague was video-recording. This
made it possible for me to ask questions in order to understand what was going on, which had to be done in a manner that did not disturb the work.

During this first visit, Anita was monitoring patients or “tog kontroller” (which means “took controls” in English) on the patients, as said it is described on the ward. The patients were connected to digital supervision monitors. Controls, she told us, were made every four hours and included collection of the physiological parameters: respiratory rate, heart rate, blood pressure, temperature, urine output, and alertness. Different values of these parameters were given colours and scores on a chart. The chart (see Figure 1) was coloured outwards, from green in the middle (0), then yellow (1), orange (2), and, finally, red (3) at either end. The red columns indicated the most critical values. A total score $\geq 5$ required immediate attentions from a doctor and the emergency team. There is also an additional text in the chart, which says that deep concerns about a patient or acute deterioration are other reasons for contacting the doctor and the emergency team.

During the control of one patient, a doctor was summoned to take a blood sample for a blood gas analysis. The blood gases are connected to several of the physiological parameters in the chart, but give another kind of description of the patient’s condition. The doctor arrived quickly and took a blood sample and disappeared after a few minutes. Anita then took the sample to a digital laboratory where the analysis was performed and automatically transferred onto digital patient records. In the digital laboratory, Anita said: "This will take a minute, but one minute is a long time so I can do other things instead, so I will not wait for the test results". After this, Anita returned to other patients to encourage, console and chat, while further controls were made. None of the controls were apparent to an observer, but these were explained by Anita afterwards. The observation clearly noted that Anita was devoted to caring and comforting.

![Figure 1](image.png)

*Figure 1.* The coloured chart, with normal and critical values.

After having collected the values from the patients Anita took out a piece of scrap paper from her pocket (Figure 2) and typed in values in the patients’ digital hospital record. The scrap paper shows the data collected from two patients.
While these routines appear to be very structured, and even have numerical and mathematical content, the nursing aides on the ward did not seem to have these perceptions. On several occasions, they said "you know" or "you feel" or even "it's the third eye". We were told, during the observation, that the “third eye” was an important characteristic of the nursing aides’ skills. During the interview, Anita told me more about the third eye. She said that “everybody can have it, but not from the beginning,” suggesting that it develops from experience. There also seemed to be a tension between the rational chart, based on numeric values, and the more elusive feeling of just knowing. This feeling is discussed later in more detail.

While the observation clearly noted that Anita was devoted to caring and comforting, it was not noted that she also, for instance, counted breaths minute. This became clear during the interview, when she gave an example of how she is counting breaths. It was also visible on her paper, which she showed us after she had taken the controls. The respiratory rate or breaths per minute is noted as “24 A” on the paper in Figure 2. Otherwise, the explicit use of numerical values seemed absent, as these were probably not very interesting for the patients. Only once did Anita explicitly talk about values, and it was to convince a patient about her recovery: "Your values are much better today than they were yesterday, so much better".

**Analysis of the demands**

After having visited Anita at work, and also interviewing her, it was obvious that taking the controls on the patients was one of her regular and important work tasks. The coloured chart with its columns and scores can be considered as part of the explicit and demanded mathematics. So, from the sociomathematical viewpoint, handling the chart can be seen as one mathematical requirement for nursing aides. The coloured chart was used for facilitating the judgment of a patient’s condition, and to identify patients at risk of catastrophic deterioration. In order to take the different parameters measured into account, these are given different scores, and a total score of five or more is defined as a risk, which needs attention from a doctor. Understood in mathematical terms, nursing aides need skills in:

- Reading a chart
- Understanding distribution of values, facilitated by colours
- Comparing values
Adding values

This could be considered as basic mathematics by for example a mathematics teacher. By giving the values different scores which need to be added, it can be reduced to simple calculations. This work can be considered as needing only a limited amount of cultural capital, or education.

**The interview with Anita makes clear that “17 is not always 17”**

When I met Anita we had a conversation about working as a nursing aide. With her many years of experience, also in different countries, she had a lot to tell. Due to her experience and by having a mother tongue other than Swedish she also volunteers as an interpreter on the ward. This she told me has a certain value for doctors and of course for patients, in a semi-emergency unit where quick decisions are crucial. Over and above this she talked about her presence as having a calming influence on immigrant patients because they felt confident with her. She had difficulties in explaining this but phrase it as: “She is like us”. The conversation also covered much about workplace education but did not turn to mathematics, which I wanted to understand more about it in relation to this workplace. (The Swedish original transcript is given in brackets)

**Maria:** I was thinking about this technical, technical education, and such. All the things you do with the tests, reading the monitors, using the charts, and so on. To me it seems somehow like mathematics. (Jag tänkte på det där, det där med teknisk utbildning och sånt. Alla saker du gör med tester, avläsa moniter, använda tabeller och sånt. För mig verkar det på något sätt som matematik.)

**Anita:** Yes, it is! (Ja, det är det!)

**Maria:** But I don’t know… if I think about school… how could school provide this education? (Men jag vet inte… om jag tänker på skolan… hur skulle skolan kunna utbilda för detta?)

**Anita:** Ah, okay you mean like that … one should have basic mathematical skills, absolutely, like percentages, one should have the basic knowledge but nobody will ask about sine and cosine, nobody will … but I have learnt it and everybody has but for our profession I mean that we need the basic stuff. It has to do with percentages, addition and subtraction. That is necessary but I take for granted that everybody knows that. (Ah, okej, du menar så… man ska ha, man ska ha grundläggande matematiska kunskaper, absolut, som procent, man ska ha grundläggande kunskap men ingen kommer att fråga efter sinus och cosinus, det kommer ingen att göra, men jag kan det och alla kan det men för vårt yrke är det vi behöver det grundläggande. Det har att göra med procent, addition och subtraktion. Detta är nödvändigt men det tar jag för givet att alla kan.)

It worth noting that Anita considered trigonometry as something that everybody has learnt. As I perceived that there was something, from my own school mathematical habitus “vaguely mathematical” in her work, I tried to find out more about it:

**Maria:** It is difficult to explain, but all the judgments you make and all the priorities you have, and the fact that you do it differently…like when you read from the monitor and taking the pulse manually. (Det är svårt att förklara men alla bedömningar du gör och alla prioriteringar, och just det att du gör det annorlunda…till exempel när du avläste monitorn och tog pulsen samtidigt.)

**Anita:** I think it is normal and obvious. Such thing cannot be learnt in school. (Jag tycker att det är normalt och självklart. Sånt kan man inte lära sig i skolan.)

**Maria:** For you it is obvious but for me coming from school it is very interesting. (För dig är det självklart men för mig som kommer från skolan är det väldigt intressant.)

**Anita:** For example I count the respiratory rate. All people breathe differently and when they are ill even more differently. Such things you don’t learn … so I count the respiratory rate of one
patient and get 17 ... let’s say 17 but I have learnt that this patient has 17 because s/he is ill. I can judge that, I can judge that the other has 17 because s/he really doesn’t feel well, yet another has 17 because s/he is hyperventilating, and that one has 17 by pretending in order to get more morphine than s/he has already got, and that one has 17 because... (Till exempel så räknar jag andningsfrekvensen. Alla människor andas olika och när de är sjuka ännu mer annorlunda. Såna saker lär du inte dig i skolan...så jag räknar andningsfrekvensen hos en patient och får 17, säg 17, men jag har lärt mig att hon har 17 därför att hon är sjuk. Jag kan avgöra det. Jag kan avgöra att en annan har 17 därför att han eller hon verkligen inte mår bra, och en annan har 17 för att han eller hon hyperventilerar, och den har 17 för att den låtsas för att få mer morfin än den redan har fått, och den har 17 för att...)

Maria: I think it is really interesting that 17 can mean so many things compared to school, where it means 17. (Jag tycker verkligen att det är intressant att 17 kan betyda så olika för i skolan betyder det ju 17.)

Anita: This is something completely different, and everybody here would have told you exactly the same. (Detta är något helt annat och vem du än skulle fråga så skulle du få samma svar.)

Being a bit confused by 17 not being 17, I return to this issue again:

Maria: This I find really interesting ... even this you are saying about the respiratory rate. Because you mean that we have different lungs and you cannot know how much oxygen a breath contains neither measure the volume of the lungs, so this is replaced by a feeling ... that you feel what 17 means in this case. (Det här är ju verkligen intressant...även detta du säger med andningsfrekvensen. För att du menar att vi har olika lungor och du kan inte veta hur mycket syre varje andetag innehåller eller mäta lungornas volym, så detta ersätter du med en känsla, att du ser och känner vad som menas med 17 för just den patienten.)

Anita: Yes, it is the same as with the woman I just looked after. On our ward we have a machine that is connected to the patient and from that I have learnt to read how much air that is getting into the lungs. (Ja, det är samma som med den kvinnan jag nyss tittade till. På vår avdelning har vi en maskin som kopplas till patienten och där vi har lärt oss att avläsa hur mycket luft som kommer in i lungorna.)

Maria: Aha... (Aha...)

Anita: Such things are learnt here in work (Såna saker lär man sig här på arbetet.)

Maria: This machine...now I must try to understand...this machine can measure what you have a feel for? (Alltså den här maskinen...nu måste jag försöka förstå...den här maskinen mäter det du känner på dig?)

Anita: Yes, something like that. (Ja, någonting sånt.)

Maria: It is actually quite... (Det är ju faktiskt ganska....)

Anita: Yes, something like that... (Ja, någonting sånt...)

This extract of the interview with Anita suggests that there is more going on than merely the collection of the patients’ values and comparison of these with the values on the coloured chart. Instead, Anita has developed an experienced-based abstract feeling for the patients’ condition – a third eye. As an example of this feeling she takes 17 breaths per minute to illustrate what differences in meaning an isolated and discrete figure can have.

Analysis of the developed mathematics and the “third eye”

Taking the respiratory rate as an example, the chart gives concrete and decontextualized values. This I suggest is in contrast to what Anita says about counting to 17, as an abstract value related to many other parameters as, for example, the depth of each breath, or about a patient’s
simulated illness. When the rate of breaths is connected to, for example, the volume of lungs, the depth and the pressure, it is closer to a function of oxygen saturation of the blood than to discrete and concrete values. Anita’s explanation of counting to 17 does not explicitly refer to mathematics, although many different parameters and the relations between them are taken into account. Instead I suggest that it refers to “the feeling” the nursing aides have, which they also label as “the third eye”.

I interpret the “third eye” as what is developed from a sociomathematical perspective. So, having the third eye could be seen as a symbolic and also embodied capital shared by competent nursing aides, a disposition to understand the patients’ conditions and act accordingly. In critical situations there is no time for reasoning. Instead, having the habitus of a skilled nursing aide involves the corporal or sensual component of the “third eye”, allowing for judgements and decisions. Furthermore, relating the respiratory rate to many different parameters requires a higher level of abstraction than just calculating a total sum of 5. By this I do not mean that one form of knowing is preferable to the other, but rather how both forms of knowledge can be seen as complementary.

What is, instead, interesting to note is the transition between the explicitly demanded mathematics on the chart and the developed but elusive feeling. This transition I suggest requires a habitus with the “third eye,” but also the demanded mathematics as cultural capital, and hence formal education. Moreover, it requires confidence and some power to, if needed, go against the coloured chart. The mathematics demanded in this case is rather basic, but facilitates the workplace routines and probably increases patient security. However, it seems crucial to be competent, confident, and comfortable about how to and when to use mathematics, as Coben (2010) suggests in her definition of numeracy for nurses. I suggest that the skill of making these transitions should be paid more attention, and also the connection to reasoning and to being critical in general.

Discussion

Although mathematics in work is different from school, it could be relevant to consider the similarities between the mathematics demanded in work, such as the coloured chart, and school mathematics. In doing so it is also necessary to pay attention to the transitions that have to be made in critical situations. An example of this is when Anita compared her capacity, or her “third eye,” to a number on the chart. Therefore, it would be a mistake to consider the explicit demanded mathematics as what is needed purely in terms of school mathematics. In connection to this it is also worth mentioning the limitation that my school mathematical habitus places on understanding what is actually going on. My focus when doing observations was on the chart and technical tools. It was not until I had the conversation with Anita that I became aware of what else was going on.

It would be naïve to believe that this activity can easily be contextualized in school mathematical tasks. Furthermore, the development of a “third eye” could not possibly happen in school, as Anita noted. A third-eye is certainly not gained from so-called real world problems. A misplaced contextualization can even make the task less accessible for several reasons. It may be that it makes students worried because it is not in line with the work they are heading for, as was the case with my intravenous drip task. Another risk is that the contextualization restricts or overshadows the mathematical content, which gets less space and probably becomes insufficient for vocational students. Therefore, the benefit of contextualized tasks should be further investigated, although it is still important to take into account the complexity in work and influencing factors other than mathematics.
Viewing learning as a bodily change (Bourdieu 2000, Wacquant, 2004) the embodied knowledge that Anita developed as a feeling of what 17 means in a particular case is, to some extent, related to mathematics and a crucial competence in Anita’s work. The symbolic capital of the “third eye” grows into and becomes a part of the body and senses and thus a part of habitus, as a form of bodily knowing. However, it is less likely to be acknowledged than the explicit use of mathematics found in the demands and in the chart. So, ironically, in this work transitioning from a lower level of abstraction to a higher leads to a loss of the visible need for cultural capital. The embodied knowledge is rendered invisible as it becomes a part of an experienced nursing aide’s habitus with the “third eye” as an important characteristic. The third eye is consistent with what Wacquant (2004) writes about corporal and mental schemata of habitus erasing the distinction between the physical and the spiritual world. The difficulties in detecting this knowing, together with the historical subordination of nursing aides, may lead to an assumption that abstractions or mathematical reasoning are neither needed nor used by nursing aides.

When observing the semi-emergency unit with a video camera, I could not by any means perceive that Anita was focusing on counting breaths per minute and judging the rate in relation to other conditions. What I perceived was, instead, how she cared for the patients and gave them comfort. I suggest that this could also be seen as an example of the black-boxing, mentioned by Williams and Wake (2007). She was apparently doing both caring and counting simultaneously in order to make the patient feel comfortable while she was counting. This is also aligned with common requirements in a workplace where conceptual knowledge and creativity are mutually dependant when completing work tasks (FitzSimons, in press).

Making the assumption about the profession of nursing aides as being mostly about caring is misleading. Instead, I see a need to further investigate the kind of knowing that is frequently labelled as tacit, or as being black-boxed either technically or socially. It is important to take into account how the profession of nursing aides has been viewed in the past, how it has developed and is viewed in our current society. Certainly, the transitions between the chart and the “third eye” require a particular competence and experience. From a sociomathematical viewpoint, it is an act of balancing between the demanded mathematics and the developed. Seen as a reflected transition these acts of balancing require learning. If vocational students are provided with short courses, or restricted curricula, it will have serious consequences. These will not only affect the possibilities of gaining access to higher education, but may also lead to the presumption that the work force is easily educated and replaceable. This is just the opposite of what Anita explains about her work.

What is happening in the transition between the demanded mathematics and the developed in terms of corporal understanding needs to be further researched. I believe that it is also highly relevant to consider the transitions adult learners have undertaken, such as, for example, moving from school to work, or moving between other kinds of contexts such as different countries. An example is when Anita refers to trigonometry as something that everyone knows, but which is different to how I view it. It seems as if this cultural capital gets lost in her transition to a new country. This could also be seen in relation to how Anita volunteers as interpreter on the ward. Whether she knows trigonometry, or not, is not of interest in this work. Instead, she had made use of her language skills, and probably gained a position on the ward through doing so. There also seem to be parts of her habitus shared by immigrant patients who feel comfortable and secure with Anita.

The question of whether it is common or rare to know trigonometry is however hanging in the air. It is not possible to know retrospectively if this knowledge could have been an advantage in the Swedish education system. Some parts of habitus or certain capitals may be
lost or rendered invisible in different transitions, and others may instead be rendered visible. The question is who benefits from what is gained and lost in these transitions, and the overall question that I think needs further investigation is: What can we learn from the different transitions learners make, and how can these be related to mathematics? For this purpose, the concepts of capital and habitus, and, more specifically, the changes habitus undergoes in transitions, could be useful as analytical tools. An underlying question is if the label tacit knowledge is more relevant for work than for school, or if it could be that certain forms of knowing are silenced?

Acknowledgements

This article was written as a part of my Ph.D. project within the project "Adult's Mathematics: In work and for school", funded by the Swedish Research Council and initiated by professor Tine Wedege at Malmö University. I would like to thank Gail FitzSimons, Tamsin Meaney, and Lisa Björklund Boistrup for constructive comments on earlier versions of this paper, and I would also like to thank Lisa Björklund Boistrup, Lars Gustafsson and Marie Jacobson, with whom I gathered and discussed the data. Furthermore, I would like to thank the anonymous reviewers for their valuable comments.

References


A Workplace Contextualisation of Mathematics: Measuring Workplace Context Complexity

Knowing what you know, as distinct from what you do, can facilitate re-contextualisation for change

John J. Keogh
The Institute of Technology Tallaght, Dublin 24, Ireland
<john.keogh@ittdublin.ie>

Theresa Maguire
National Forum for the Enhancement of Teaching and Learning, Ballsbridge, Dublin 4, Ireland
<terry.maguire@teachingandlearning.ie>

John O'Donoghue
University of Limerick, Limerick, Ireland
<john.odonoghue@ul.ie>

Abstract
Recent research undertaken by the authors (Keogh, 2013; Keogh, Maguire, & O'Donoghue, 2010, 2011, 2012), identified the mathematics activity that underpinned what may be regarded as low-skilled, low paid jobs, and aligned it with the National Framework of Qualifications in Ireland. In the course of this research, it emerged that although the mathematics expertise deployed was modest in terms of complicatedness, it was used by workers in circumstances that were both sophisticated and volatile in varying degrees. To this extent, it was discernable that mastery of routine mathematics alone was a poor indicator of a person’s ability to ‘do the job’. Furthermore, a National Survey of People at Work in Ireland, while confirming the Mathematics use/denial paradox, revealed that work was not perceived to be ‘straightforward’ despite widespread adherence to processes, procedures and routines. The authors argue that there exists a spectrum of factors that operate to ‘complexify’ otherwise routine mathematics, with the possible consequence of concealing the role of mathematics and intensifying its invisibility in the workplace and all that that entails. This paper describes these affective factors
as comprising a workplace contextualization of mathematics which elaborates the complexity of the workplace context in which mathematics at varying levels of complicatedness may be expressed. In this way, workers, employers and providers of learning opportunities may be better informed regarding employability and worker mobility in the long term.

Keywords: complicatedness, complexify, invisibility, employability, mobility

**Introduction**

A National Survey of People at Work in Ireland, augmented by several case studies, produced strong evidence regarding the character and role of workplace mathematics. It seems that although procedures and routines proliferate in the workplace, and workers adhere to their procedures, they soundly reject the suggestion that their work is straightforward (Keogh, 2013; Keogh et al., 2010, 2011, 2012). The survey substantially confirmed the mathematics use/denial paradox, while the case studies identified hundreds of instances of numerate behaviour in encounter with all Mathematics Domains, but at quite a modest level. The implication is that Mathematics Knowledge Skill and Competence (MKSC) in the workplace was not captured by identifying the level of complicatedness alone, which suggest that all jobs with the same levels of MKSC may not be considered equivalent.

Further analysis of the discourse surrounding the case studies, revealed that work is a social activity, having multiple properties and facets, is performed under pressure of time and accuracy, with attendant materiality, depth, scope and peer to peer accountability, not necessarily aligned with the authority conferred by seniority or role status. These dimensions arise across a range of spectra and combinations such as may differentiate the MKSC required in one job when compared to another, the novice from the expert, between what a worker ‘knows’ and what s/he ‘does’, and may not feature in the official accounts of a case study’s Standard Operating Procedures (SOP). Whether learnt formally, informally, non-formally, tacitly or through analogical rationality (Gustafsson & Mouwitz, 2010), they are highly valued in the workplace. The descriptor ‘work experience’ is used as a unitary concept, implying a depth of understanding that is commensurate with the quantity of time served. However, the case studies, and the findings of the National Survey of People at Work in Ireland, provided the basis for a workplace characterization that is rather more profound and may be described more completely in terms of its Complexity (Keogh et al., 2011). The authors now elaborate these themes as a contextualization of the workplace, in 5 dimensions namely, Accountability, Clarity, Familiarity, Stressors and Volatility in which mathematics knowledge skills and competence, regardless of level of complicatedness, are deployed. Each of these characteristics in turn comprises constituent strands as described in the following sections.

**Accountability**

A common dictionary definition of the term ‘accountability’ is having to do with taking responsibility or being in some way culpable, connoting a degree of power and control as might be associated with a supervisory or management role. The corollary is that the ‘ordinary’ worker, for whom there are no official levers of power, is unaccountable and completely free of responsibility. The case studies suggest that accountability is a more immediate and tangible concept, comprising a range of components, each defining part of its context namely **Audit Materiality, Decision Making, Initiative, Concreteness, Judgment, Planning and Responsibility** in degrees of intensity that vary from job to job, as elaborated in the following sub-sections.
Audit Materiality

This facet refers to the impact of error, ranging from the negligible to the catastrophic. For example, a worker in a supermarket may use the same MKSC as a person packing parachutes. This contrast highlights that workers can, by making a simple mistake, compromise the service provided by the employer and expose the organization to embarrassment, loss of business, reputation and the risk of complete failure, despite the presence of appropriate procedures and SOPs.

Decision making

Whether the worker is permitted or expected to make decisions, to what extent, and under what conditions, extends the remit of that worker beyond simply executing a sequence of tasks. This may be further nuanced by the influence of other stressors which may produce both formal and informal interpretations of the decision-making rules or guidelines.

Initiative

A worker may have complete latitude to assess a novel situation and respond accordingly, or be required to apply the SOPs to the letter. There may be a ‘fuzzy’ understanding of when the worker is expected to use his/her initiative and when not. A worker who assumes responsibility for having acted *ultra vires*, adds an extra tier to the dimension of Initiative component of a job, with a possible consequence of placing his/her continued employment at risk.

Concreteness

It is plain that the lowest level of manual work e.g. digging soil, comprises elements that are fully recognisable, physically present and few, whereas, at the opposite end, some or many work components may be abstract, theoretical or imagined. In the central range of concreteness, a tradesperson may handle elements that are concrete and specific, but expected to take into account other factors such as the appearance of the finished product and its aesthetic fit with work accomplished by other people.

Judgment

From time to time, a worker may have resolve conflicting variables. Such an intervention may form part of the job specification, may be conditioned or may require knowledge and expertise from elsewhere. In this way, the exercise of judgement, in what circumstances and to what extent, adds to the fabric of the context in which MKSC are deployed in work.

Planning

Planning, as a component of context at the highest end of the spectrum, is typically associated with optimising the likelihood of a satisfactory outcome. Low-grade jobs may have little or no involvement in planning, although this may not be the case in the strictest sense. The authors argue that every job contains some element of sequencing tasks with the benefit of local knowledge, keeping in mind tasks that follow, for example, loading goods on a truck while being conscious of the delivery sequence and / or load stability. In this way, the planning
dimension of a job may be learned explicitly or tacitly, and may be subject to rules and guidelines that vary in specificity.

**Responsibility**
Responsibility has become synonymous with guilt and the definition of who pays compensation when something goes wrong. While it is associated with high status and the power to command resources, the authors suggest that it trickles down through the hierarchy, depositing degrees of responsibility at every identifiable level, including those at the lowest level. Each worker has some degree of responsibility to his/her peers, regardless of their principal activities, to produce work on time and in line with specifications.

Each of these sub-dimensions of Accountability interacts in unique combinations and may be influenced by the degree of clarity with which the context is perceived by the worker and his/her colleagues.

**Clarity**
Clarity around the aims and objectives is a desirable feature of the workplace, and one that is obtained in varying degrees. It is a difficult concept to describe succinctly, as its meaning is dependent on the situation it intends to describe, particularly so in a rapidly changing workplace. At every level in an organization, it is critical that everybody has a clear understanding of their purpose, whether in anticipation of an outcome in the near-, mid- or long-term. The authors suggest that the extent of clarity in the workplace is a combination of the interaction of several factors namely, Distracters, Priorities, Reflectivity, Information Sources, Vision and Information Completeness.

**Distracters**
This refers to the likely presence of elements that may distract the worker from their purpose, or add the potential for confusion and error. Simple, tightly defined jobs, involving one or few elements would seem to be free of distracters, except perhaps boredom born of narrow, repetitive cycles. Other distracters may be explicit and easily identified and discarded. Towards the upper end, it may become more difficult to discriminate between pertinent factors and distracters that are embedded and plausible.

**Priorities**
The setting of priorities is a function of the control and command structure in organisations, but not exclusively so. In the more project-mature organizations, such milestones are agreed amongst the individuals with the relevant expertise, each of whom must juggle their local resources. Discretion regarding priorities is not necessarily aligned with job status, especially in global enterprises that commission very specific outcomes from their plants spread across the World. To this extent, the exposure to competing priorities, however set, is another descriptor of workplace context.
Reflectivity

Reflective practice in industry is common, although it may be realised as project review, strategic planning, periodic reports, performance review, and systems and financial audits. It is pervasive and hierarchical insofar as the outcomes tend to flow upstream. It may be initiated in reaction to a costly error, to identify a systemic flaw, in which case the remedies flow downstream. Reflection, in pursuit of continuous improvement may inject a force for change in the metrics and methods employed in, and therefore, constituting, work practice.

Information Sources

The sources of work information may range from single, simple source, expressed in job specific terms at the lower end, to multiple sources in various formats, referencing concrete, abstract and theoretical data on familiar and unfamiliar topics. It may be verbal and non-specific, requiring interpretation and locally-attuned inference. It may be deduced from dialogue and rumour, or adduced from relevant experience and may vary in reliability. Dealing with multiple information sources would seem to describe a crucial element of any job, and could impinge on other context strands such as clarity, and accountability.

Vision

Vision, in this sense, has to do with the meaningfulness of the job to the individual. It alludes to the sense of purpose, beyond the boundaries of the job and how the output of the job integrates with surrounding activity to produce something that is whole in itself. For example, the collection of meter readings for input to a spreadsheet is a limited experience in the absence of further explanation. In contrast, acquiring a broad view of an organisation’s aims and position within the market can influence the way in which work is done and the utility of the supporting artefacts, including MKSC.

Information Completeness

Work information is likely to be complete in circumstances that are tightly controlled and closely monitored, although not necessarily so. Incomplete or imprecise information, imports guesswork and uncertainty, however informed, and tends to increase the risk of error. At the leading edge of industrial research and development, complete information is the object being pursued. Creative and innovative activities feature aspects that are known and unknown in extent, and the recognition that there may be other unknown-unknowns, and perhaps even the unknowable. That this is a facet in the workplace that varies in impact on how work is done is another workplace context attribute.

Exposure over time may contribute to the extent to which the characteristics and properties of the workplace become familiar.

Familiarity

Familiarity is a gauge of what has become known as the ‘comfort zone’. This is a concept rooted in Adventure Education which indicates an anxiety-neutral, risk-free environment conducive to steady performance (White, 2009). It may be realised in the workplace as a state in which the worker is well practiced in the performance of a sequence of tasks, in unchanging
surroundings, in encounter with stable, recognised components. Beyond the ‘comfort zone’, lies the ‘stretch zone’ in which it is thought there exists a fundamental disequilibrium which promotes intellectual development and personal growth (Panicucci, 2007). Such a workplace presents challenges to the worker that are nonetheless within their capacity to achieve.

An overall sense of familiarity, or otherwise, may be the product of Specificity, the nature of the Principal Activity, the range of job-related Elements, their associated Facets, the impact of Groups in work and Routine.

**Specificity**

This refers to the extent to which components of a job are specific, recognised and unvarying at one extreme, in contrast with the abstract, theoretical, and widely varying at the other, with gradations in between to account for degrees of transformation from one to the other. The implications for the context in which the experience of work occurs are clear, encapsulating a factor which presents more challenges as specificity diminishes in proportion to the advance towards the abstract.

**Principal Activity**

The worker’s principal activity adds a determining context characteristic. A single, closely defined and monitored, solitary activity has a simplifying effect on the worker’s job. In contrast, a professional person, at the leading edge of his/her discipline is likely to encounter a wide variety of familiar and unfamiliar situations, diagnose problems, develop creative solutions and implement them, in multiple interacting activities. In the interim, individuals may switch between increasingly varied activities in response to workplace demands.

**Elements**

A job may comprise a single element at the basic level, or progress through an unvarying sequence of tasks, to one that is moderately, or extensively influenced by internal or external factors, some of which may be unfamiliar. This reflects complexity in the sense of the number of elements and ways in which the elements can be combined. As these quantities increase so too does the degree of complexity.

**Facets**

Not to be confused with Elements, Facets, in this case, deals with the extent to which elements may be nuanced, and not solely an empirical count. This connotes a capacity to detect and interpret a particular instance of an element and to act accordingly. Facets may become familiar over time, but that may not preclude the emergence of a novel occurrence, all of which conjures up an influential consideration of the workplace context. For example, the job’s SOP addresses each Element i.e. work order, delivery address, delivery type (document, computer media, property deed etc.), related security and operating principles. However, each individual client may have formal and informal preferences or Facets, to which the worker must adhere to retain their custom,
Group

Solitary activity can be challenging to those unsuited to working alone, but may be appropriate to a person unsuited to working in a group. Engaging with a small group, becoming familiar over time, may present less challenges than belonging to a larger group that is mainly co-located. The ability to participate in an unfamiliar group, which may be large and partially or substantially distributed across a number of locations in geography, time and culture, implies a maturing set of knowledge skills and competence, and confidence in one’s mathematics and other capabilities at their point of use.

Routine

Following a familiar set of tasks in the same sequence, repeatedly, may be a product of the constraints imposed by procedure or a set of procedures, conditioned by internal or external factors. However, as the survey findings have shown, procedure accounts for just over half of workplace activity, the balance being evoked by unspecified factors such as this present workplace contextualization is seeking to capture. Routine is a ubiquitous dimension in work, and is not completely positive in its implications, but is worth regarding for its descriptive qualities.

However, many workplaces may differ in the range of factors, including routine, that could contribute to stress experienced by workers.

Stressors

The uniqueness of the individual makes it impossible to be definitive about the causes and effects of stress in the workplace. The authors do not presume to comment on the possible effect of ‘distress’ in the workplace, but rather to introduce a range of factors that either singly or in combination, may change the experience of work, while using the same level of MKSC or other skills. The suggested factors are: Constraints, Pressure, Problem-potential range, Solutions, Sources of stress, and Structure of the workplace.

Constraints

In the unlikely event of limitless resources, constraints are imposed to optimize output minimize the input, in terms of time, materials and labour. Ranging from the clear and simple at one end of the spectrum, to those which are broad, imprecisely defined and inferred from internal and external conditions at the other, constraints have the potential to simplify or complexify work. The presence of a few clear and fixed constraints is characteristic of a job at the lower end of the scale, whereas, multiple, flexible, interrelated and mutually regulating constraints may add substantially to the performance of work towards a specific outcome.

Pressure

Workplace pressures come in many guises including the cultural, temporal, personal, professional, philosophical and political. Most common of these has to do with priority, urgency, accuracy and expectations. For example, completing a set of calculations under
extreme and continuous time pressure is quite a different proposition to performing the same mathematical activity at leisure. In this way, the experience of work may be described by levels of pressure ranging from none, through loosely defined expectations, to issues of volume throughput targets, compliance, quality, accuracy, culminating in extreme pressure as may feature in cases of emergency.

**Problem - potential range**

Simple jobs exhibit little or no potential for problems, excepting equipment breakdown. Even then, the worker may be required, or permitted only, to report the situation by triggering a call for attention. Jobs may increase in complexity in line with the number and possible range of familiar problems, through to levels of expertise needed to deal with multiple, mutually dependent, independent and/or novel problems.

**Solutions**

Similarly, the range of available responses to problem situations escalate from there being one response to all problems, through a continuum of the application of familiar solutions to familiar problems, progressing to mainly unfamiliar problems to that requiring novel responses and creative solutions to unfamiliar problems. Each of these levels of expertise, adds to the palette with which to discriminate between the experience value of different jobs, and the selection of the appropriate mathematics-based response.

**Sources of Stress**

There may be few or many centres from which workplace stress may arise. They may be internal or external to which the individual is exposed partially, moderately or broadly. They may be avoidable, or an integral part of the work, having a relentless and cumulative effect. A more complete treatment of stress in the workplace is beyond the scope of this document, however, dealing with multiple sources of stress in work, is, potentially, very challenging to the individual, and may affect deeply, the environment in which MKSC finds expression.

**Structure**

Working in a highly structured, tightly defined organization, lends simplicity to its functions, albeit at the cost of flexibility, which itself might cause stress. Clarity concerning demarcation, rules, accountability and so on, may cause lower levels of stress. Loosely structured, broadly defined, matrix-configured organizations, may give rise to increased levels of stress as a result of their fluid, inherently unstable nature, which could be described in terms of volatility.

**Volatility**

Volatility is the property of frequent and unanticipated change that may be short-lived. The extent of volatility in the workplace necessitates the capacity to respond to sudden and new developments in the market or the customers’ demands. It may be characterized as occurring over 5 transitions namely, completely stable, mainly stable, moderately unstable, mainly unstable, and totally unstable.
Organizations and their embedded jobs are subject to change with varying degrees of need and urgency, as may be profiled by *Conditionality, Demands, Diversity, Predictability, Range and Risk.*

**Conditionality**

The performance of work may be subject to a variety of conditions, the state of which may be determined by known or unknown, internal or external factors, themselves being influenced by other conditions. The range of affective conditions may differ in quantity and power. Other jobs may be immune to conditions, requiring the same response every time. The recognition of conditionality and the extent to which it pertains to a job, reflects the set of appropriate knowledge and skills and the competence, in the broadest sense, that it develops.

**Demands**

The demands on a job justify its existence insofar as it has been created to fill a perceived need. Simple jobs have few demands that are clearly defined and relatively easily met. More complex jobs feature multiple demands that may not easily coalesce and may compete for resources. At this extreme, the worker sequences his/her activities, and may deploy innovative methods to cope. The effect of multiple, competing demands, may de-stabilize the job to an extent that is unlikely in a job profiled by one or few demands.

**Diversity**

Diversity is the property of difference, rather than breadth. In the workplace, it refers to the extent of heterogeneity, and coherence of the tasks. While it makes sense to gather together mutually dependent tasks, requiring elaborations of related sets of knowledge, skills and competence, there are jobs that occupy the boundaries of other specialities enabling cooperation and communication. For example, a change-management specialist may need to communicate with engineers, accountants and computer software developers, in order to ensure cohesion and the desired outcome. In contrast, a completely homogenous workplace implies little scope for diversity that may not be accounted for otherwise.

**Predictability**

Complete predictability in a job engenders familiarity, stability, clarity, and the establishment of routine. Complete unpredictability adds depth to many of the other factors including stress, accountability, familiarity and the absence of clarity. The majority of jobs probably lie between these two poles, as evidenced by the survey findings and case studies.

**Range**

The breadth of components associated with a job confers the potential for complexity commensurate with its range. Single-issue jobs are simpler and more straightforward when compared to those encompassing several issues distributed a broad, yet coherent, landscape.
Risk

In this context, risk alludes to certainty of outcome and the extent to which it is confined. Jobs for which the outcome is almost certain e.g. attending a machine that cuts metal forms with a die, have quite a different character from stock-brokering. The risk associated with the former has more to do with the wellbeing of the machine operator rather than whether the die will produce the expected form. The activities of the latter risk the organization’s resources in the expectation of substantial gain, while at the same time exposing it to potentially catastrophic loss. Risk may be classified as that component of a decision-making process for which there is insufficient information. It may not be permanent and pervasive and may be conditioned and limited. Most jobs are located along a continuum between these extremes, exerting concomitant influence on the context in which Mathematics and other knowledge, skills and competence are used.

Workplace Context-Complexity Protocol

The Workplace Contextualization of Mathematics described in preceding paragraphs, represents an extensive range of parameters with which to differentiate between jobs, regardless of the level of complicatedness of their mathematics knowledge skills and competence. The unique nature of each job may be reflected by the extent to which these parameters are present in the job specification and profile. That these workplace characteristics shaped the context in which MKSC were used, inspired the authors to develop an appropriate framework to capture the essence of the workplace namely a Workplace Context-Complexity Protocol, to enable the context in which MKSC are used in the workplace to be more fully reported.

Protocol Structure

Each of the main context headings, Accountability, Clarity, Familiarity, Stressors and Volatility, is listed with its attendant properties as sub-headings, in the attached Appendix 6.1. Each property of the protocol is scaled and described across 5 transition states, and assigned a two-step scoring range to permit interpretation toward the lower or upper end of the scale. For example, The Volatility property, Predictability, may be scored at 5 or 6 to indicate that a job may feature moderate unpredictability that is more than the lower adjacent category (4) but somewhat less than would justify the next higher category (7), i.e. mainly unpredictable. This scoring system recognizes that there is no empirical scale to measure these things yet, and that the boundaries are not sharp and clear cut. Nevertheless, guided by the evidence available and by working through each heading and sub-heading in turn, it is possible to produce a detailed profile of the workplace context. In this way, the Workplace Contextualisation of Mathematics can be used as a protocol for profiling the Context-Complexity of a workplace. The idea is that it is possible to capture the complex circumstances in which fairly routine mathematics knowledge skills and competence are used in many workplaces. The possibility that an individual may deny their use of mathematics, or dismiss it as common sense, argues in favour of a mechanism that is capable of making the mathematics more visible and more fully accounted for. The structure and application of the National Framework of Qualifications in Ireland (NFQ)(QQI, 2012) and its alignment with formally established complicatedness of mathematics at different levels is reported elsewhere (Keogh et al., 2010).

This present work suggests an augmentation to the NFQ to facilitate the recognition of, and communication about, mathematics activity in work for the benefit of mathematics teaching,
learning and assessment.

**Extended NFQ Illustrated**

The provisions and structure of the NFQ reflect its provenance and purpose namely to set and maintain the standards expected as learning outcomes, formally acquired, across 10 levels, and detailed in terms of Knowledge, Skills and Competence. In contrast, the learning outcomes required in the workplace are dynamic, in reaction to change to meet its own needs, regardless of domain, and untrammeled by the depth and breadth necessary for progression in formal learning environment. Competence in the workplace is a concept that bears little resemblance to that accounted for in the NFQ, and presents a more ‘spikey’ profile that is tuned to local conditions rather than conformance with a remote, generalized description. In this way, it seems that mathematics in the workplace may not be accounted for fully in terms of complicatedness alone.

The Workplace Contextualization of Mathematics, described herein, and the Context-Complexity Protocol which it underpins, are the products of in-depth case studies comprising doctoral research. At the time of writing, no comparable frameworks had been located to capture similar facets of the modern workplace. Nevertheless, these tools offer the prospect of extending the provisions of the NFQ, to enhance mathematics visibility, and to recognize the sophisticated circumstances in which MKSC is used in the workplace. The application of the Context-Complexity Protocol to a sample case study discussed in the next section, demonstrates the added power to communicate an extended NFQ would offer for the benefit of the individual, employer, recruiter and curriculum developer.

**Sample Case Study**

‘R’ is a Warehouse Picker. He is required to retrieve 60 items per hour from a warehouse that stores approximately 3 million separate documents, files, deeds, legal briefs, and computer media. He is guided by a ‘work order’, one for each customer, issued by his line manager. The ‘picks’ are distributed across 2 buildings, each of 4 floors, fitted with up to 26 storage racks on each floor, each with 52 bays, each with 3 shelves, each of which may contain 27-30 boxes of documents. There are several fireproof safes and secure vaults to contain sensitive documents and electronic media. Each ‘pick’ is tagged with a barcode which indicates its location by referencing building, floor, row, bay and shelf, but no more. R is provided with a scanning device which lists the barcodes in alphabetical sequence – not by optimal route. It is not feasible to accumulate picks as his work progresses from beginning to end. Instead, he deposits parts of the pick at strategic locations around the warehouse to be gathered at the end of the session. The pick route is planned by his taking account of the locations of cargo lifts, stairs and access points between buildings, and the next ‘nearest neighbour’. He must decide how much time to devote to a pick that is not in its reported location, bearing in mind the need to complete his work within the time allotted and the impact on the delivery person and customer service of omitting the requested document.

This warehouse picker has little formal education. The level of mathematics he actually displays in the performance of his job scarcely meets the learning outcomes at level 1. He has the lowest status in the workforce, yet he bears ultimate responsibility for picking the correct item and making it available for delivery to the correct customer. The work instructions he is provided with are clear in general, but surrounded by distracters, competing priorities, and moderately incomplete information. He reflects on his work and introduces unauthorised ‘work-
arounds’ to compensate for the shortcomings in the warehouses’ design. He is informed by different but familiar information sources, although constrained, partially, by the preceding and following picks. Several years of experience has resulted in mid-range familiarity and subject to stressors generally in the middle range. In arriving at complexity level indicators, each item of the protocol was considered in turn and matched to the band that most closely described his work.

The outcome of this matching process is shown in the next section.

**Extended NFQ – Sample Case Study**

The standard NFQ approach to the accreditation of learning when applied to a sample case study, represents the identified mathematics knowledge and skills at level 1, having met the criteria detailed in the relevant Significant Learning Outcomes (SLO) set out in the assessment criteria. In keeping with standard custom and practice, the same level is credited to the four Competence sub-strands, namely Context, Role, Learning to Learn and Insight, each shown separately in Figure 1.1 on the assumption that these properties are somehow embedded in the learning process.

![Figure 1.1. Company A, Case Study 1, Mathematics Knowledge, Skills and the Competence Strands of Context, Role, Learning to Learn and Insight - standard interpretation.](image)

However, an evaluation of the Competence in Context and Role, based on the evidence of observations and on interpreting the formal provisions of the NFQ, exceed that of the Mathematics Knowledge and Skills levels identified in the Case Study – Job Shadowing phase. Figure 1.2.
The scale of Learning to Learn-recognition is biased in favour of the formal learning structures, leaving no scope for the recognition of tacit, informal and non-formal learning. The apparently extreme score recorded for Insight, reflects intelligent exposure to the workplace and its capacity to promote tacit rationality and analogical thinking.

The impact of having applied the Context-Complexity Protocol to Case Study 1, is shown in Figure 1.3. While a separate trace is shown for each mathematics domain for consistency with the broad aims of the research, their confluence would seem to indicate their interdependence rather than distinct and discrete behaviour. The plots shown represent the mean score of the factors comprising each dimension of workplace context-complexity.

When these data are combined, the resultant graphic, Figure 1.4, captures not only how complicated the Mathematics Knowledge and Skills deployed in this particular workplace are, (plotted at NFQ level 1), it also shows the observed, rather than assumed, levels of competence in Context, Role, Learning to Learn and Insight, appropriate to each Mathematics Domain. The
key additional job profile information reports Context-Complexity dimensions which effective performance in the workplace demands.

![Figure 1.4. NFQ extended to account for Context-Complexity.](image)

The profile of the context-complexity communicates new information about the workplace and a sense of what the individual ‘knows’ in addition to what s/he ‘does’. While the complicatedness of the MKSC at NFQ level 1 may be characterized as routine, the circumstances under which they are used requires their deep understanding, in support of thinking and as a guide to action.

**Implications for Mathematics Teaching & Learning**

The implications for mathematics teaching and learning for and by adults, may be profound, especially when considered in tandem with other outcomes of this research. The findings of the national survey associated with this research tended to confirm the importance of the context in which mathematics knowledge, skills and competence are realised. While this is not new information, it contributed to the formation of the Workplace Contextualisation of Mathematics detailed in the attached appendix, 6.1. By embracing this contextualisation of the workplace, teachers of mathematics have the opportunity to mould the learning environment accordingly. For example, a problem could be posed that, while requiring the application of mathematics techniques, might invite an answer expressed in terms of the ‘least-worse’ outcome. While this strategy may challenge the teacher’s imagination, it could highlight the idea that mathematics can support strategic thinking and need not be an end it itself. In a forthcoming companion document, the authors introduce the concept of a subject-centric perspective of cultural historical activity theory as a possible explanation of mathematics invisibility in the workplace and suggest a mechanism by which it may be measured. Taken together, these devices may offer an holistic approach to establishing a realistic starting point for adults learning mathematics, and a road map for future action.
References


