Objectives

Adults Learning Mathematics – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field, which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

• Research and theoretical perspectives in the area of adults learning mathematics/numeracy
• Debate on special issues in the area of adults learning mathematics/numeracy
• Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Adults Learning Mathematics – An International Journal

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Javier Díez-Palomar
Universitat de Barcelona
Barcelona, SPAIN
jdiezpalomar@ub.edu

Anestine Hector-Mason
American Institutes for Research
Washington, DC, USA
ahector-mason@air.org

We are pleased to introduce the first issue of volume 8 of *Adults Learning Mathematics: An International Journal*. In this current issue, readers will find articles focused in different topics, but with a common nexus: the relation between [school] mathematics and mathematical demand of activities conducted by persons in their workplace. There has been a plethora of research in the area of workplace mathematics, with some previous works by Gail E. FitzSimons playing an instrumental role in clarifying the “meeting places’ between mathematics and workplace (FitzSimons, 2002). Over the years, there has been some considerable efforts to clarify whether or not mathematics is embedded within routine activities related to different workforce roles (Coben, 2008; FitzSimons, 2002; Hoyles et al., 2002; Moreira & Pardal, 2012). Far from being just beautiful mathematical descriptions of the kind of mathematics implemented at the workplace, these studies really bring us an important body of knowledge to better inform vocational training programs, for example.

Now, a decade later, FitzSimons surprises us again with another review of the scientific literature in this area, advancing the contributions from her previous article on the topic. In the article included in this ALMIJ issue, FitzSimons uses a sociocultural approach to examine various types of literature that focus on the issue of workplace mathematics. FitzSimmons presents some essential perspectives on several key words and phrases that shape the meanings she conveys about the sociocultural aspects of workplace mathematics. For example, she defines *activity* as a unit of analysis, and drawing of this approach, she provides the reader with a comprehensive analysis of workplace behaviors and their relationship to mathematical practice. In addition, FitzSimons discusses the *nature* of workers’ mathematics, and uses concepts such as *boundary crossing* or *boundary objects* to examine different types of employments (plumbers, CAD/CAM technicians, financial services workers, electricians, chemical workers, etc.) that involve the use of mathematics. Essentially, FitzSimons brings to the field a critical examination of current knowledge in mathematics: the difference between mathematics in the workplace and mathematics in the school environment.

There is prevailing evidence to suggest that there is a social representation of workplace mathematics as “not looking as mathematics”; or, at least, the “type” of mathematics embedded
in routine procedures are not the same as the ones presented to students in the school (Needs a few citations here). FitzSimons provides a plethora of examples drawing of different researchers to illustrate issues inherent in the comparison of workplace mathematics to school mathematics.. Drawing on these evidence, it seems that the relevant point here is that mathematics present within the workplace are “mathematics in context”, which suggests that workplace mathematics are mathematics with meaning. This highlights a crucial difference between the mathematics taught in schools and the ones coming out from the employments. Financial service workers, for example, need to know the algorithms to calculate the monthly fractions for a mortgage, but they also need to understand what these algorithms mean in order to explain them in the context of a client’s personal economic plan. FitzSimons concludes that somehow mathematical knowledge has crystallized in black boxes through (1) automation and (2) the historical development of instrumentation. In other words, mathematics embedded within workplace activities are result of a historical process of accumulation based on how previous people used mathematical procedures to automatize actions or procedures when performing their work activity. Such socio-cultural approach provides us a splendid tool to visualize such type of mathematics.

There is a number of implications related to this kind of discussion, including the debate of what kind of curriculum plan should be implemented in vocational training courses, due the fact that professional activities in the real world entail more than the type of “formal” mathematics presented in a regular classroom. If this is true (and evidence provided by Dr. FitzSimons suggest so), then we need to introduce practical training within vocational training programs, since a crucial part of these trainings is to make “meaning” to the knowledge presented within the classroom.

In the second article in this edition, Lynda Galligan presents an interesting case of this debate: the case of nursing. Galligan also uses a socio-cultural approach to frame her work: Valsiner’s Human Development Theory. Valsiner further developed Vygotsky’s idea of zone of proximal development. He also proposed a more detailed conceptualization of the social process learning, using concepts such as macrogenetic processes or Method of Double Stimulation (MeDoSt) – some of them also present or suggested by Vygotsky himself. Galligan draws on the journey of “Sally,” a first year student in a University of South Queensland’s nursing program, and describes some decision-making at the Department of Nursing regarding reaccrediting student key academic skills that are aligned to the criteria of the Australian Nursing and Midwifery Council. Sally was not sure about her mathematical competence. She had unpleasant memories of mathematics (she has struggled with math many times).

Throughout the dialogues exemplified in the article, we can see how Sally moves from an initial state framed by these negative inputs towards mathematics towards a state in which she is able to demonstrate her learning of multiple different mathematical concepts (although in the “map of academic numeracy development” there is the possibility to forget some knowledge at the end of the trajectory). As is shown in Galligan’s work, learning appears to reflect an oscillating line between two different moments: the microgenetic and macrogenetic. These two profiles help us to envision the complexity of variables impacting on Sally’s ability to learn and understand mathematical concepts presented to her in the classroom. The use of learner trajectories, as is shown by Galligan, is a very interesting approach to analysing adult learner progress.

The third article in this edition is slightly different from the sociocultural perspectives presented by FitzSimons and Galligan, but it also addresses the issue of mathematics within the context of employment. In this article, Bhartgava focuses on evolutionary algorithms and their applications through a discussion of different elitist and non-elitist, multi-objective, evolutionary algorithms. This author presents a very compelling proposal related to the application of these algorithms in a range of fields, including financial time series, forecasting stock prices and stock ranking, risk return analysis, economic modelling, and so forth. Clearly, the article focuses on topics in “pure”, abstract, mathematics; however, all of the fields of application the article describes are closely related (in practical terms) to professional employment.
This article concludes on an issue focused on the connections between mathematics, adult learning and development of skills needed to perform activities within the labour market. It raises ideas about our current realities and our global experience in living in a special moment marked by the financial, economic and social crisis in the “old” Western-economies. Thousands of adults are going back to school looking for further training and credentials in order to fare better in social and economic opportunities. Across the globe, the labor force has also experienced a dramatic change due to the use of new Information and communication technologies (ICT), as well as other economic, social and political processes such as delocalization, among many others. Bhartgava’s article, as well as the works of FitzSimons and Gallager in this edition of the ALMJ, raises some very compelling questions for math practitioners and researchers: Namely, what kind of mathematics do we need to teach in our current courses for adult learners? How do these mathematics really match with the current [living, working, etc.] needs and trends of individuals in an age of globalization and advanced technological developments? What methodologies will provide us with concrete and more accurate tools to better analyse the “big picture” and provide the best answer to these questions? Presented below are three important pieces of work that give us insight into the world of workplace mathematics, which can help us to begin thinking of these seminal and important topics.

References


Doing Mathematics in the Workplace
A Brief Review of Selected Literature

Gail E. FitzSimons

University of Melbourne
<gfi@unimelb.edu.au>

Abstract
The aim of this review of selected literature on research into mathematics in the workplace is to offer researchers from the field of adult mathematics education an opportunity to become more familiar with this specialised area. In recent years much progress has been made, building on earlier research, but with increasingly nuanced understandings. In particular, sociocultural activity theory has played an important role, and several articles reviewed here have drawn on this theory. Their focus has been on gaining a comprehensive understanding of what workers actually do involving aspects of mathematics, set in the rich context of a functioning workplace. This encompasses taking account of workplace artefacts (technological & otherwise), various forms of communication (verbal & non-verbal), different forms of skills, and even the concept of boundary crossing; most importantly, how workers learn

Key words: workplace mathematics, technological competence, sociocultural activity theory

Introduction
The world of work has, throughout history and across cultures, incorporated mathematical thinking and communication into its tools, symbols, and organisational practices, as part of the production of goods and services. However, for the most part, the main objective is to get the job done as efficiently as possible, to satisfy a range of stakeholders, be they customers (external, internal, upstream, downstream), employers, shareholders, patients, audiences, and so forth.

There is no definitive workplace. Workplaces can range from a single trader (or tradesperson) right through to branches of multinational companies; workplace activities may be located at fixed sites, sites varying from job to job, or conducted virtually; work may be officially registered and recognised, or part of the informal economy. The work undertaken can vary in intensity, complexity, and responsibility; weekly hours can vary, and employment can be full-time, part-time, or casualised. Work regimes themselves are not necessarily desirable for

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1 This article is an extended version of a paper presented at ALM 19, Auckland, New Zealand, in June 2012.
individuals: too little or too much work, too many hours or too few, infrequent or unpredictable calls on their labours. Similarly, conditions of work can vary within and between workplaces.

Mathematics-containing workplace technologies in the form of tools or equipment, from everyday implements to highly technical robotic or other instrumented substitutes for human labour, often vastly exceed human capacities in their speed, accuracy, memory capacity, tolerance of dangerous or extreme physical conditions, and so forth. Besides being embedded in technological artefacts, mathematics is also rendered invisible in technologies of management, such as Quality Control [QC] regimes, standard operating procedures [SOPs], or other prescribed routines (FitzSimons, 2002). Formal mathematics — the academic mathematics familiar to most researchers — often only becomes visible in situations of breakdown.

Early research generally tried to identify and map workplace mathematics onto existing school mathematics curricula — itself an arbitrary selection from the discipline (Ernest, 1991). School mathematics, regarded as a proxy for academic mathematics, was taken as the norm, and workplace mathematical activity was accorded lower status. The earliest researchers tended to focus only on that mathematics which was visible (Harris & Evans, 1991), ignoring not only the crystallised mathematics embedded in workplace technological artefacts and routines, but also the entire sociocultural, political, and historical contextual settings of their observations.

In formal education, both general and vocational, the development of mathematical skills and techniques is the focus of attention and the object of the activity in the mathematics classroom; whereas, in the workplace, mathematics — if it is noticed at all — is almost always regarded as but one tool in the process of achieving a desired outcome. Workers, past and present, generally do not regard what they do as being mathematics nor themselves as being mathematical (e.g., Wedege, 1999).

Over a decade ago, I published a comprehensive review of research into workplace mathematics (FitzSimons, 2002). In recent years much progress has been made, building on earlier research, but with increasingly nuanced understandings. In particular, sociocultural activity theory — among others — has played an important role, and several articles reviewed here have drawn on this theory. Their focus has been on gaining a comprehensive understanding of what workers actually do involving aspects of mathematics, set in the rich context of a functioning workplace.

This article aims to provide a synthesis of selected workplace research over the last decade or so, with a view to answering the question of what knowledge has been gained since then. The focus is on the ‘world of work’ in relation to the mathematics that workers use mostly unconsciously, transform, or even create locally, and communicate through language, gestures, signs, and other non-verbal communication, in order to address the ever-evolving problems of their particular workplace activity.

**Research Methodologies and Important Theoretical Concepts Used in the Workplace Literature Reviewed**

The studies discussed in this literature review have adopted predominantly qualitative methods, with most adopting ethnographic approaches and many utilising socio-cultural activity theory, based on the work of Vygotsky and Leont’ev (see, e.g., Roth & Lee, 2007, for a comprehensive review). In these approaches, the unit of analysis is the activity itself which is generally undertaken by a group of people, (such as a work unit) in order to satisfy a motive (such as the production of a good or a service, in their broadest interpretation). Various actions are undertaken to achieve a range of goals supporting the activity, and, in turn, these depend on unconscious operations or skills. (For a further description of activity theory in relation to mathematics education, see FitzSimons, 2008.)

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2 Bear in mind that there is a considerable lag between the production of a manuscript and its eventual publication in hard copy, although this is now being reduced with online technologies.
Any breakdown in this system, including communication, is an opportunity for learning to take place — for workers and for researchers. If the worker’s necessary skills are not in place (e.g., needing to learn a new IT technique), this becomes an immediate goal in the ongoing activity. As the context changes, different goals and different actions come into play for the worker. A feature of workplace activity is that problems arise on a daily basis that cannot always be predicted in advance, and locally new solutions must be generated. For researchers, a breakdown in a routine activity offers the possibility for gaining deeper insights as participants are probed for more detailed explanations of things that were previously taken-for-granted.

Communication — in its many forms — is, of course, a central issue in activities associated with work; and the concepts of boundary crossing and boundary objects are raised by many researchers (see Akkerman & Bakker, 2011, for a comprehensive review). Workers at all levels interact with others within and beyond their designated practice: between the work unit and customers or suppliers, internal or external; or between various organisational layers. Communication can encompass dialogue, written texts, diagrams, signs, symbols, and gestures. People often need to communicate across the boundaries of their particular work setting and, since mathematics is often an integral part of the work process, mathematical thinking and reasoning is likely to be embedded in this communication. Workplace artefacts such as graphs, tables, spread sheets, historic records of production, data collection and output (including quality control protocols), are likely to be in use as well. As a means of communication between different groups, these boundary objects — as they have come to be known in the literature, building on the original work by Star in 1988 and 1989 (Star, 2010) — are often wrongly assumed by their producers and users to be transparent, leading to communication breakdowns.

In the studies reviewed here, data collection has generally included participant observation, interviewing, and collection of artefacts. Not surprisingly, many articles have focused on the use and the influence of technology as an integral component of contemporary workplaces.

Any formulation of skills is time- and place-specific, and is often embedded in labour relationships and broader social structures encompassing social actors, institutions, as well as social values and norms. Wedege’s (2000) understanding of workplace technology views mathematics as being integrated in four dynamically inter-related dimensions: (a) technique/machinery; (b) work organisation; (c) vocational qualifications; and (d) workers’ competences. In addition, as part of this multidimensional view of workplace mathematics, Wedege (2011, p. 3) provides a definition of technological competence, which she defines as “workers’ capacities (cognitive, affective, & social) for acting effectively, critically, and constructively” to challenges in the technological workplace.

Research Findings

The literature reviewed was drawn from over 50 refereed journal articles published over the last decade or so. Articles were selected on the basis of their capacity to illustrate: (a) differences in approaches to mathematics-related tasks between school students and workers, (b) graphic details of some mathematics-related activities actually carried out by particular workers, (c) differing structural resources available in the workplace, (d) workers’ attitudes to school mathematics or the vocational mathematics intended to be relevant to their future occupations, and (e) how technology-enhanced artefacts can help workers to uncover and find meaning in mathematics rendered invisible by automation.

Problem Solving: Plumbers vs. Students

One objective of many studies is to make comparisons between the workplace and the school setting. Jurdak and Shahin (2001) documented, compared, and analysed the nature of spatial reasoning by plumbers in the workplace and by school students while constructing the ‘same’
solids from plane surfaces. The authors drew on activity theory for its potential to explain the differences between the two settings in terms of motives, tools available and accessible, and constraints. Data were collected from a plumbing workshop with five experienced adult plumbers having little or no school experience, and five tenth-grade students (two girls & three boys), while constructing a cylindrical container of capacity one-litre and height of 20 cm.

Based on the method of structural analysis, Jurdak and Shahin (2001) identified differences in the types and sequence of each group’s actions as well as in the degree of complexity. Jurdak and Shahin contrasted the two groups:

In the course of constructing the task container, both the plumber and the environment changed. The plumber started with perceptual action, responded to what has been executed by continuously reviewing the model container, executed more actions, and so on. The goals for container construction changed as the container evolved and its actuality became possible. This interaction elicited critical skills such as recognizing opportunities or problem finding, knowing when and how to apply skills that have been learned in other contexts, and exploiting properties of the present situation. (p. 312)

On the other hand,

The students’ interaction with their physical environment was minimal and they approached the problem of constructing the task container by implementing the procedure that was derived directly from the classroom practice. ... they relied [almost] exclusively on mnemonic and cognitive tools by reading the problem, selecting the formula, calculating the unknown, and writing the answer. … The students showed little control over the problem solving process, they strongly believed in the power of formulas and algorithms and hence did not feel the need for self-monitoring other than checking the correctness of their calculations. (p. 312)

Jurdak and Shahin (2001) observed that, whereas in school mathematics is a conceptual tool, detached from the situations which give it meaning, in the workplace mathematics is a concrete tool which takes its meaning from the situation at hand to solve problems that may arise within that context. They concluded that although using mathematics in the workplace is more meaningful, school (i.e., formal) mathematics has more power and is more generalisable beyond the specific context of application.


Magajna and Monaghan (2003) conducted a case study of six CAD/CAM technicians who design and produce moulds for glass factories; their research focused mostly on volume calculations. The technicians’ work was in many ways related to mathematics (e.g., constructing shapes, calculating or managing the cutting operations on machine tools). To help describe the structure of emergent goals, Magajna and Monaghan followed Saxe’s four parameter model:

1. **Activity structures**: In order to understand a particular aspect of observed practice it is necessary to “consider the whole production cycle from the client’s initial order to the manufacturing of the mould and the use of the mould in the production of bottles in the (distant) glass factories” (p. 105).

2. **Social relations/interactions** influence the emergent goals.

3. **Conventions and artefacts**: Although some methods used to determine the volume may be traditional, the introduction of computers and CNC machines has also led to new methods.

4. **Prior understandings** brought by individuals, both constrain and enable the goals that individuals construct in practices.

Magajna and Monaghan (2003) found that “there was an evident discontinuity between the school mathematics used and the observed mathematical practices. This discontinuity was
evident at both a subjective and an objective level” (p. 117). At the objective level, school-like concepts and practices, such as linear equations, trigonometry and Pythagoras’s theorem, were used in the context of practice, even though the technicians did not necessarily understand all the mathematical background. School mathematics knowledge was used ‘as is’, without being questioned discussed or modified. However, discontinuity was also evident at the subjective level when the technicians claimed that there was no school mathematics in their jobs.

A second finding of Magajna and Monahan (2003) was that technology in this CAD/CAM workplace was so all-pervasive that it virtually structured the technicians’ activity and played a crucial role mathematically. Not only were their mathematical procedures shaped by the technology they used, but their mathematical understandings were used as a means to achieve technological results. Mathematical correctness was negotiated amongst the technicians and also with customers and contacts in the glass factory, in contrast to the school situation.

Third, reflecting the findings of Jurdak and Shahin (2001), Lave (1988), and others, Magajna and Monaghan (2003) found that in mathematical breakdown situations the practitioners either simply chose another mathematical or construction method or, more commonly, overcame the problem by technological means rather than mathematising the cause of the breakdown in the sense of reasoning about mathematical procedures. They “did not solve problems involving mathematical abstractions in this workplace” (p. 120). In both cases, the workers’ actions should be interpreted within the goal-oriented behaviour of their occupation: Technicians’ work is embedded within the imperatives and constraints of the factory’s production activity cycle which included: (a) time pressures demanding almost immediate solutions; (b) many mathematical procedures being frozen in the technology, often invisible, and poorly understood by practitioners who had limited control over them; (c) the final product and not the mathematics was what counted (i.e., mathematical correctness does not necessarily mean technological correctness); and (d) a lack of practitioner confidence in understanding the complex procedures involved.

Telecommunication Technicians: Tool Mediation

Triantafillou and Potari (2010) studied a group of technicians in a telecommunication organisation. Guided by an activity theory framework, they identified, classified, and correlated the tools that mediated the technicians’ activity, and studied the mathematical meanings that emerged.

The technicians’ typical daily activity was to fix a number of reported faults in the local underground copper-wiring network. Their major working tools were wire pairs that were bundled together into cables consisting of between 100 and 600 pairs. This wiring network started from the main organization building in the center of the city, went through specific boards, the telecommunication closets, and then distributed to a number of boxes around the area of the closet, and finally was directed to the local subscribers. The largest part of this network was underground. The technicians, in order to accomplish their daily main activity, had to perform a series of actions that were usually sequential. These actions were to: (a) trace the reported pair of wires on a set of physical tools (the telecommunication closet and the boxes around the area) by using the information given on an instruction sheet, (b) use a technical map to trace the underground wiring network from the closet to the subscribers’ boxes, and (c) use two measuring instruments ... to locate the exact point of fault. (p. 281)

In relation to these three actions, Triantafillou and Potari categorised the tools as (a) mathematical (communicative, processes, & concepts), and (b) non-mathematical (physical & written texts), which they presented in a comprehensive systemic network (see p. 290, Figure 5). They found that the technicians’ emerging mathematical meanings in relation to place value, spatial, and algebraic relations were expressed through personal algorithms and metaphorical and metonymic reasoning (e.g., using analogies or alternative representations such as graphs), indicating the situated character of their mathematical knowledge. Triantafillou and Potari identified differences from typical mathematical representations in the technicians’ explanations.
or their physical tools and written texts, in keeping with the findings of Magajna and Monaghan (2003). However, they also noted that a number of processes and strategies shared the same structure and characteristics with those developed in a school context.

From these relatively small-scale studies, I turn to a major UK research project where the concept of boundary crossing has been an important component, in addition to activity theory. Kent, Noss, Guile, Hoyles, and Bakker (2007) construed the term techno-mathematical literacies (TmL) to emphasise both the mediation of mathematical knowledge by technology and the breadth of knowledge required in the context of contemporary technology-rich workplaces that are both highly automated and increasingly focused on flexible responses to customer needs. They were also interested in how boundary objects “may facilitate effective communication between and within work teams and between work teams and customers” (p. 67). In the first phase of their research they carried out ethnographic case studies in 10 companies; in the second they carried out design experiments: I discuss findings from each, in the manufacturing and financial services sectors, respectively.

**Process Improvement in Manufacturing**

Kent, Bakker, Hoyles, and Noss (2011) conducted several case studies of process improvement in manufacturing companies, focusing on measurement aspects. Here, I discuss two: a production line bakery and a pharmaceutical packaging company.

*A process improvement (PI) project in a large cake production line bakery.* Kent, Noss, Guile, Hoyles, and Bakker (2011) observed the work of a PI team in a factory which involved cakes continuously moving on conveyor belts within linear ovens many metres long. The entire production line was several hundred metres long, with every stage of the baking process monitored using a combination of automated and manual measurements. In the PI process, the workers used measurements throughout the whole baking process to develop a capacity profile: a summary chart of speeds of the different machines and processes, intended to make visible bottlenecks in the process which could be targeted for improvement.

However, management had not been aware of the importance of creating a culture in which workers appreciated the need to measure and to take results seriously. Because the workers had not received any training in these requirements for PI, Kent et al. (2011) found that, initially, team members with less formal education were under-skilled in this kind of work. Measurements were taken inconsistently, from variable location points in relation to the cake and the machinery, and were not systematically recorded, hindering the optimisation techniques used in PI. After a 2-week project to improve performance on this production line, the workers’ views and actions were transformed through their participation in acts of measurement, reading the graphs, noticing things that were previously invisible to them, and thinking about solutions. Nevertheless, Kent et al (2001) noted that even when PI recommendations are made on logical (mathematical) grounds, there are often good practical reasons (e.g., cost) for management not accepting them.

*Overall equipment effectiveness (OEE) in a pharmaceutical packaging company.* In this company, tablets were brought to be packed into foil and plastic blister packs, with a number of these packaged together into a cardboard box. Just prior to the research taking place, manually operated packing lines, each run by six operators, had been replaced by fully automated packing lines, now operated by one technician. The production manager described the activity as:

> Literally you pour tablets in one end and feed it cartons, film, foil, leaflets—and out of the other end comes a sealed pack, and groups of packs boxed and labelled—all automatically; all the operator does with that is to stack it on the pallet … the main thing is the ability to be able to tweak the machine to keep it going at the optimum speed we require. (Kent et al., 2011, p. 757)
Instead of using hand tools and having physical access to many parts of the old production process, the technician now had to operate the line through a computerised control panel—one level of abstraction away from the physical process, also from the measurement process. Kent et al. (2011) noted that there was more to tweaking and running machines well than recognised by management; especially managing product changeovers which varied significantly between operators. This required the technician: (a) to understand the physical mechanisms of the machine and to be able to identify and remediate problems, or (b) to be able to communicate what was wrong to the skilled maintenance engineers in the factory. It also required operators to be able to make sense of the measurements and (idiosyncratic) graphical data generated by the packing machine’s internal computers.

**Overall equipment effectiveness** (OEE) combines three generic variables for production, Performance, Quality and Availability, to construct a more abstract measure concerned with maintaining a balance between the three variables. Although OEE measures created a visibility about the system for senior managers, they were not discussed with the operational employees. Kent et al. (2011) concluded that technology brings an extra layer of complexity to measurement. Although it is commonly assumed that automation leads to a reduction in human error, many managers expressed the concern that it also leads to less engagement with the production process, and several preferred to have employees measure and report manually. As described above, the complexities faced by technicians in running the machinery highlight the importance of how their tacit knowledge interacts with their codified knowledge to get the job done effectively.

**Financial Services Workers**

One case study (Kent et al., 2007), design experiments was set in the financial services sector, and focused on the annual pension statement, a boundary object designed to facilitate boundary crossing between the company and customers. However, this pension statement routinely failed in its communicative role, largely due to the invisible factors of the underlying mathematical-financial models not being made available either to customers or to the Enquiry Team. Straightforward customer enquiries were to be resolved immediately by telephone, if possible. However, automation of the IT system, intended to ensure the accuracy of information sent to customers, had actually disempowered employees who lacked any real understanding of the models and calculations, inhibiting their communications with colleagues and customers.

Although the mathematics involved in finance seems superficially similar to secondary school mathematics, Kent et al. (2007) observed that in the workplace context every mathematical procedure, no matter how simple, is part of a whole range of decisions and judgments about complex processes or products. Employees need to be able to mathematically appreciate computer outputs, interpreting them in their context, and recognizing which components are hidden by the IT system. They also need to be able to reason about the mathematical models embedded in the system in terms of the key relationships between product variables (e.g., percentage rates, management fees, sales commissions) and their effect on outputs presented in the form of graphs or tables.

Bakker, Kent, Hoyles, and Noss (2011) detailed an intervention in the same industry where they designed technology-enhanced boundary objects (TEBOs) in order to improve employees’ understandings of the mathematics behind the mortgages they sold (i.e., their technomathematical literacies). Their goal was not to teach employees the mathematics behind boundary objects such as the mortgage package, but to engage with the different aspects of their underlying mathematical models. They developed software to model or reconstruct actual practice, using data drawn from a current account mortgage (CAM) which integrated a regular current account with the property mortgage. The company had previously published a booklet containing a standard repayment graph which was actually mathematically unrealistic. Bakker et
al. (2011) described it as *pseudo-mathematical*: mathematical in form but not in function. Interviews revealed that

\[ \text{... the sales agents were largely unaware of any but the simplest relationships between interest rates and repayment schemes, even though they were able to explain mortgages in general financial terms (e.g., capital and interest), ... sales agents saw the different interest rates as labels for instruments: annual rates as labels attached to a mortgage arrangement or a savings account, monthly rates as labels attached to credit card or loan debts. (p. 29)} \]

In other words, sales agents had not understood the mathematical meanings or relationships. One TEBO reconstructed the mortgage graph in a spreadsheet with all the input variables and calculations made explicit in order that the graphs could be produced according to the input variables that matched different customer scenarios. Another TEBO modelling credit-card debts revealed to participants the problems with paying back only the obligatory monthly charge. Anecdotally, this intervention seemed to have improved employees’ understanding and confidence, but they were forbidden by management for legal reasons to use the TEBOs in actual practice.

**Black Boxes in Industry**

Williams and Wake (2007a), drawing on their earlier work from a major study (Wake & Williams, 2001), sought to expose contradictions between College and work in order to explain the apparent disappearance of mathematics in the workplace. They were attempting to answer two questions:

- How is mathematics shaped, indeed often hidden, by workplaces?
- What processes serve to shape mathematics in workplaces differently from that in Colleges, and hence cause a ‘gap’ for students when presented with mathematics from workplaces? (p. 318)

Instead of focusing specifically on the differences between the two systems of workplace and College, they sought to engage with these differences as contradictions in practice. Grounding their research in cultural-historical activity theory, and conducting multiple case study visits to workplaces by college students and teacher-researchers, they observed that mathematical processes have been crystallised in *black boxes* shaped by workplace cultures. They identified two key processes through which this happens: automation and the historical development of instrumentation.

According to Williams and Wake (2007a), automation occurs when the work of mathematics is crystallised in instruments, tools, and routines that control operations, and these operations are then automated — for example, in the design of a machine, or in the writing of a program to perform precision cutting operations in three dimensions. These artefacts serve as boundary objects between the mathematics performed historically, in their original design, and their current use by operatives who use them on a daily basis, but without needing to know or understand how they work. Williams and Wake described this as a distribution of mathematics in *time*. As a consequence, they concluded, this workplace mathematical genre is mediated by idiosyncratic conventional forms, the mathematical origin and significance of which may have vanished over time. As such it is also opaque to outsiders, even to the mathematically trained researchers themselves.

Williams and Wake (2007a) also described how the historical development of instrumentation could result in sub-units of the workplace community being protected from mathematics by a division of labour, supported by communal rules, norms, and expectations. These are often regulated by boundary objects (e.g., data collection forms) which seem to distribute and crystallise work *horizontally*, in a division of labour across social *space*. Workers collecting data, for example, may have no understanding of the importance of the data collection, the uses
to which it is put, or even the need for accuracy — in the manner comparable to research by Kent et al. (2011) on process improvement on the cake production line.

As identified in the work of Triantafillou and Potari (2010), the use of metaphor and metonymy is a topic of growing interest for researchers involved with school mathematics teaching and learning. Williams and Wake (2007b, p. 346) asked: “How can models and metaphors mediate communication between students, workers, and teacher-researchers?” Explanations by workers that were productive sometimes drew upon cultural models, including metaphors and mathematical models, that made connections with relatively more concrete, well understood resources such as commonly used metaphors of communication, time and space, supported by gestures and/or words making reference to time, place, and so on. They gave an example of such a conversation with an engineer, which showed how they, as outsiders, could come to make sense of what at first appeared to be a very mysterious and opaque spreadsheet formula and to understand the mathematics inside. Williams and Wake (2007b) used the expression workplace genres of mathematics because workplaces, technologically mediated or otherwise, tend to develop their own conventions and terms. However, more technological workplaces may also draw on elements of other widely recognised social and cultural forms of mathematical language, such as those known as engineering mathematics, spreadsheet mathematics, and so on.

**Apprentice Electricians Learning to Bend Pipes**

In an empirical study framed by cultural-historical activity theory, specific differences between college and workplace were identified and theorised, not by boundary crossing but by the concept of personality (discussed below). Roth (2012) investigated (a) the geometrical practices of electrician apprentices learning to bend electrical conduits in college and on the job, and (b) how they handled the relation between differing practices. The requirements for doing well in the two activity systems were very different: exhibiting knowledge of trigonometry in one, and doing a good job that makes bending and subsequent pulling of wires practical in the other. Formal trigonometry was the reference in the classroom, whereas the codified rules of practice were the main reference on the job.

In college, students intending to become electricians are taught conduit bending theory, and are required to study basic trigonometry. The textbook provides “magic circles” to help calculate such functions as the sine, cosine, and tangent. Apprentices carry out extensive calculations and measurements to determine angles, their positions on the tubing, and the distances between the angles. Once a student has calculated the distances, s/he uses measuring tape and bender to produce the tubing such that it properly bypasses the obstacle provided. However, in their practical conduit bending class, apprentices encounter a specialised conduit bender on which much of the required information is inscribed, rendering the trigonometric calculations superfluous (see Fig. 4, Roth, 2012, p. 7 [online]).

Roth’s (2012) study illustrates the radical differences between the (mathematical) practices of bending electrical metal tubing, in college and in the workplace, calling into question the usefulness of vocational courses which emphasise formal mathematics that is treated as irrelevant in the workplace. Nevertheless, the electrical apprentices managed to move between the different activities with a sense of coherence, integrating these differing experiences as part of the electrician personality they develop. Having gone to college allows the electricians to work according to the formal and legal requirements of the electrical code, while meeting the practical demands and operating within the workplace constraints.

Roth (2012) noted that in addition to the experienced differences between college and workplace, the electricians’ discourse about those differences was both topic and resource in their workplace conversations. However, in the case of mathematics, there was little cross-reference between college and workplace, in story telling or in practice. According to Roth,
stories encode not only practical knowledge but also the very process of subjectification, which describe the changes within an activity system (school, work); and personality, the changes the individual undergoes as s/he moves repeatedly between systems of activity. Being able to talk about the contradictions between the two systems is as much part of being a recognized practitioner as is competent practice.

Chemical Spraying and Handling

The processes of preparation, application, handling, storage and transport of chemicals are key elements of a range of economically significant industries, and place high demands on workers’ literacy, and especially numeracy skills. Many of these skills are acquired during employment on-the-job or in associated off-the-job training. However, a substantial body of research evidence demonstrates that such skill transfer is achieved only with difficulty, and that numeracy skills are highly context-dependent. FitzSimons, Mlcek, Hull, and Wright (2005) undertook 13 case studies of enterprises which used chemicals extensively in industries including rural production, amenity horticulture, local government, outdoor recreation, and warehousing.

The mathematical processes and strategies used by workers to undertake calculations included: estimation, written methods or basic calculator use; oral or written communication of mathematics to other workers, and the interpretation of their mathematics; consultation with prescriptive calculations sheets and with historical records or online data; and completion of up-to-date records of chemicals used and the corresponding amounts. Complex contextual factors included date/time of spraying; block area; specific crop to be sprayed; crop stage; weed/pest/disease targeted; chemical group, rate per hectare, litres of spray applied, method of application; temperature, wind speed, wind direction, rainfall, humidity; and variations in equipment used, from small-scale backpacks to broad-acre mechanised spraying. Considerations such as these demand that the workers have a broad understanding of the entire work process of the area in which they are involved.

The accuracy of calculations was highly dependent on these factors as well as the degree of accuracy available on the equipment used. Importantly, economic and legal contingencies are strongly implicated within these contextual factors. Safety standards are critical, both in terms of the risk of spillage or misapplication of chemicals which may be harmful to people (workers & consumers) and/or the environment. Errors or carelessness may even cause spoilage of the end-product itself (such as an entire crop or vintage or sports ground) or the imposition of large fines for environmental pollution, thereby risking the company’s financial viability.

For these workers involved in chemical handling and spraying, it was a requirement that they already held a basic vocational certificate in the area. Learning on the job was largely experiential, with opportunities for them to become enculturated into communities of practice through interrelationships with other employees. Supervisors were often involved in initial training and check regularly on work practices; in some cases novices were given trial areas to spray and then asked to account for any discrepancies in expected area coverage and/or spray consumption. Most workplaces placed a strong emphasis on ensuring that workers are prepared to check before acting, of being unafraid to ask a seemingly dumb question.

Learning in these kinds of workplace differs significantly from formal institutionalised education in that it is rich in context, supported by historical records, and mediating artefacts such as tools, equipment, manuals, charts, and so forth, as well as communication of a qualitatively different kind from the classroom. In this workplace environment the object is satisfactory task completion, with much more at stake than appropriately accurate calculations. (See FitzSimons & Wedege, 2007, for a more comprehensive review of related literature on workplace numeracy and workplace learning.)
Conclusion and Implications

It is pleasing to see a burgeoning of research in this subfield of mathematics and work, especially in journals such as this, ALM’s own journal, and in many international conference proceedings. Several conclusions may be drawn, each with implications for further research. It is abundantly clear that school and workplaces are, in general, completely different activity systems, even though learning permeates both. In the language of activity theory, their motives and goals rarely intersect, even though their operations may in fact appear to do so. School and workplace have two different logics, as shown by the data with respect to the purposes and uses of technology, behaviours expected of workers and students, and relative values placed on cognitive and practical skills. As pointed out by Evans (1999), among others, the issue of transfer remains problematic.

Automation has had a major impact, especially on primary and secondary industries, with a consequent reduction in the visibility of the mathematical processes it envelops. The term black box originated in the 20th century with the development of cybernetics, but Maaß and Schläglmann (1988) argue that as a process it dates back to the Stone Age: Its success relies on its relatively context-free transfer of results, where knowledge without understanding can be passed from developers to end-users relatively easily. However, it is this hidden complexity—not only in technology but permeating many workers’ daily activities—which presents a great challenge to workplace mathematics researchers and vocational mathematics educators. It is worth reiterating the importance of the interaction between the tacit knowledge of workers and the codified knowledge of the workplace—see Eraut (2004) for a comprehensive analysis.

A recurrent question concerns the relationship between formal vocational mathematics education and actual workplace practice. On the one hand, in traditional trade-related vocational mathematics programs there are contradictions between school and workplace practice—as shown by Roth (2012); see also FitzSimons (2002). On the other hand, competency based training programs, which require only the atomised mathematical skills immediately visible, can disempower workers if they have no real mathematical understanding to build upon. Yet, the very nature of contemporary work and society requires people being able to cope with, and even contribute towards addressing the inevitable unforeseen changes and ever increasing degrees of complexity—and this needs real meaning making in mathematical aspects and appropriate skills in communication.

As outsiders, researchers interested in workplace mathematics have to learn something of the work process (norms, rules, & division of labour), and the workplace technology and jargon, if they are to make sense of the practices they observe there. There is also a need for them to respect the mathematics of workers that may not conform to the traditional school model. Williams and Wake (2007a) recognised that this demands inquiry skills and appropriate predispositions, as well as social confidence, which many of their College students—as prospective workers—seemed to lack. They made a series of recommendations for college students and trainee workers:

1. to understand the mathematics of others, including comprehending a diversity of unfamiliar technical media;
2. to become proficient at using spreadsheets, including decoding particular spreadsheets, taking into account contextualisation within the given work process;
3. to experience mathematics as a communal activity as is common in workplace situations;
4. to encounter a diversity of mathematical conventions and methods; and
5. to develop mathematical thinking within realistic, complex workplace contexts.
In order to make sense of workplace mathematics, outsiders need to develop flexible attitudes to the way mathematics looks, to the way it is rendered invisible by tools and artefacts and by the division of labour. Sociocultural activity theory offers one methodological framework for researchers to investigate the social, cultural, and historical factors that contribute to the complexity of work environments where adults do mathematics: where they use the mathematics they already know, transform it, possibly create (locally) new mathematics; always with the intention of solving problems that are meaningful and important to them personally or professionally, embedded in practical constraints, and with serious, sometimes life and death, consequences.

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References


Becoming Competent, Confident and Critically Aware:

Tracing Academic Numeracy Development in Nursing

Linda Galligan

University of Southern Queensland

Abstract

This paper describes the mathematical journey of a mature aged nursing student as she struggles to become more academically numerate. Within the paper, academic numeracy is defined around three features: competence, confidence and critical awareness of both the context of mathematics and students’ own relationship with mathematics. It then uses this definition to describe a course for 1st year nursing students to develop their mathematics skills needed for their degree. A conceptual framework, based on Valsiner’s Human Development Theory, is used to trace students’ developing understanding of academic numeracy. Finally the paper describes one student, Sally, as she struggles to become more numerate.

Key words: nursing, academic numeracy, Valsiner, human development

Introduction

This paper is based on research situated in a first year course in an Australian university, University of Southern Queensland (USQ), where about 200 students were learning/relearning mathematics in the context of a nursing degree. The aims of the research (Galligan, 2011a) were to: (1) investigate nursing students’ current knowledge of academic numeracy; (2) investigate how nursing students’ knowledge and skills in academic numeracy was enhanced using a developmental psychology framework; and (3) utilise data derived from these investigations to develop a theoretical model to embed academic numeracy in university programs. This paper will concentrate on the second of these aims.

Within this research, I defined academic numeracy as:

- mathematical competence in the particular context of the profession and the academic reflection of the profession at the time;
• critical awareness of the mathematics in the context and in students’ own mathematical knowledge and involves both cognitive and metacognitive skills; and
• confidence highlighting its deeply affective nature (Galligan, 2011a)

The paper first outlines the course. This is followed by a summary of the theoretical framework, then the journey of one student, Sally to exemplify the framework. The paper then concludes with future directions.

The Course

Prior to 2006, a number of approaches had been taken to develop nursing students’ numeracy levels at the USQ (Galligan & Pigozzo, 2002). In 2006 USQ’s nursing program was reaccredited with the Australian Nursing and Midwifery Council (ANMC). This meant courses, especially those offered in first year, were revised. When planning the reaccreditation, it was decided by the Department of Nursing, that nursing students needed to develop some key academic skills in first semester of first year, as a separate course. Two new integrated first year nursing courses were developed (Lawrence, Loch, & Galligan, 2010) that included Information Technology and mathematics (one course) and literacies skills (second course). The aims of first course were to develop students’ numeracy and Information Technology skills directly linked to their degree. These skills were addressed by embedding aspects of the other courses taken in the students’ first semester and course content encountered later in the program.

The course briefly described in this paper, consisted of $10 \times 2$ hour tutorial style sessions, six of which were numeracy related. Table 1 highlights some of the mathematics needed and the context in which it is seen, and each tutorial session concentrated on one of these modules. While the concepts of decimals, fractions, percentage, proportion, measurement, and scale are all studied at the primary level at school, many adults have not mastered these concepts and this has implications in nursing (Hilton, 1999; Pirie, 1987).

<table>
<thead>
<tr>
<th>Module</th>
<th>Mathematics content</th>
<th>Nursing examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic and Formulas</td>
<td>basic operations, fractions, decimals, squares, and order of calculations and use formulas</td>
<td>Body Mass Index, Lean Body Weight and Ideal Body Weight</td>
</tr>
<tr>
<td>Graphs and Charts</td>
<td>read single scale graphs; read and construct patient charts; draw graphs with appropriate scale, title and labels and units; and interpret graphs</td>
<td>patient charts; drug profiles; graphs found in nursing articles</td>
</tr>
<tr>
<td>Rates and Percentage</td>
<td>calculate percentages of given values; convert to &amp; from decimal fractions to percentages; express two quantities as a rate; determine quantities from given rates;</td>
<td>determine drip rates; pay rates; use of % burns calculations; % concentrations of drugs</td>
</tr>
<tr>
<td>Ratio and Proportion</td>
<td>manipulate equivalent fractions and ratios;</td>
<td>read a percentile chart; drug calculations</td>
</tr>
<tr>
<td>Measurement</td>
<td>identify the units used in the metric system; convert between units of measurement; convert from</td>
<td>read syringes</td>
</tr>
</tbody>
</table>
ordinary to scientific notation;
multiply and divide by powers of
10 and
multiply and divide by decimals

| Drug Calculations | problem solving | read drug calculation problems correctly; recognise the different types of drug calculations; recognise the solutions and units needed in drug calculation problems |

*Table 1. Mathematical content of course*

The course took the definition of academic numeracy, outlined in the introduction, and incorporated these features into teaching and assessment sections of the course (Galligan, 2011b) as summarised in Table 2

<table>
<thead>
<tr>
<th>Item</th>
<th>Course components</th>
<th>Numeracy component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maths Relationship scale</td>
<td>Critical Awareness</td>
</tr>
<tr>
<td>2</td>
<td>Discussion Forum</td>
<td>Critical Awareness</td>
</tr>
<tr>
<td>3</td>
<td>Self-Test</td>
<td>Competence &amp; Confidence</td>
</tr>
<tr>
<td>4</td>
<td>6 maths tests</td>
<td>Competence</td>
</tr>
<tr>
<td>5</td>
<td>Post-test</td>
<td>Confidence, Competence &amp; Critical Awareness</td>
</tr>
</tbody>
</table>

*Table 2. Course components and links to definition of numeracy*

In class, students were asked to discuss and rate their past relationship with mathematics (item 1). They were also directed to read two articles on the relationship between mathematics and nursing. Using these two exercises as a basis they were asked to reply to the questions “Describe your previous experiences with mathematics in a couple of sentences” and “How do you think mathematics...will be important for you as a nursing student, and later as a professional” in an online forum (item 2).

As part of a first assignment ask, students were asked to complete a 30 item maths test (Galligan, 2011b) where the marks they received was on the completion of the test, not whether they got it right or wrong (item 3). For each question students’ were asked to rate their confidence, and reflect on their answers in relation to their competence, confidence, and critical awareness. They also completed six short competence tests through the semester (item 4) and one Post-test (item 5) where they were asked to reflect on their current numeracy status. For item 5, marks were also allocated to their competence.

The quantitative data give a static view of numeracy at a point in time. The reflective comments, while providing a deeper understanding of students’ numeracy, particularly in terms of critical awareness, still gave little insight into students’ development. In order to see how students progressed numerically (if they are), a qualitative approach was needed. The following section now describes the theoretical human development frame (Valsiner, 1997) used for this investigation.
Theoretical Framework

Adult students learning mathematics are developing their confidence and competence in the subject. In the context of this research, development is seen as a ‘change in an organisational system in time which is maintained (rather than lost) once the condition of its emergence disappears’ (Valsiner 1997, p. 3). Using this view, understanding where these students are (before the numeracy learning experience), where they are going and how they are going to get there is of critical importance. Valsiner argues that:

A person involved in mastering a skill is no longer lacking that skill, nor is the skill present in its fully-fledged form. The skill is coming into existence. The phenomenon here is quasi structured. Rudiments of the skill can be detected in the flow of conduct, yet nobody can say for sure that the skill as such already exists. (2000, p. 105)

For this paper, three aspects of Valsiner’s Human Development Theory are now highlighted: the Zone of Proximal Development; macrogenetic processes that influence development; and the Method of Double Stimulation (MeDoSt) to help capture any development. These will be briefly described (details in Galligan, 2011a, b) and then exemplified in one student’s journey which follows.

Vygotsky’s Zone of Proximal Development (ZPD) utilised in this study, is seen within a Valsinerian approach which defines ZPD more narrowly as the “set of possible next states of the developing system” (Valsiner, 1997, p. 200) which emerge from an interaction with both the environment and scaffolding provided by another (e.g. a teacher) or self-scaffolding. In this context, questions that need to be answered are: what do students do next when trying to understand some mathematical concept, or what could students do next? Why do they get “stuck?” What so teachers do? What scaffolding is used? The actual development emerges from the negotiation process within this set of possibilities. This negotiation process is researched in microgenetic contexts, i.e. short episodes where there is potential development. Over time, a string of these microgenetic episodes may lead to change and development and a stable pattern with a more ontogenetic flavour may emerge.

In addition to these microgenetic contexts, macrogenetic processes are also of crucial importance. That is, “cultural beliefs, personal beliefs, past experiences, memories and future anticipations” (Joerchel & Valsiner, 2003para. 10) may have an effect on the microgenetic episodes.

The process of uncovering the possible development utilizes MeDoSt (Figure 1). In this method, there are both stimulus means, and stimulus objects. In interviews conducted in this study, the researcher asked the nursing student to solve a drug, or a maths question (stimulus object). The student aimed to answer the question. In the room, there may be books, pens, formulas, calculator, other students, the tutor and, at times, even models, such as syringes or bottles of tablet (study setting). The student thinks about how to solve the problem (stimulus means), also drawing upon previous meaning memories such as fear, anxiety and self-efficacy be used for adult learning mathematics (the macrogenetic processes). In the study, the material was also designed to promote students to act in certain ways, e.g., doing exercises and looking at screen casts, reflecting on answers etc. The student may remember or see the formula (action tool). She thinks to use the formula to solve the problem. The three functions (a, b and c in Figure 1) are utilised: (a) the formula chosen has some relation to the drug problem, (b) If it is successful it may be used again (c).
Within this setting, the microgenetic episode of interest is at the point of any meaning blocks. Here, the data captured may be studied to see any possible trigger and the set of possible next states. For example, will students abandon the goal and start again; will they exit from the situation; will they get around the meaning block (and how)?

This theoretical frame can best be understood in the context of a student, whose journey is now described.

**Sally’s Journey**

Sally was a first year student in the over 45 age group who volunteered to be part of the study and came to five small group and individual sessions. Sally’s results in the Self Test and Post-test were the worst of those who completed the course (from 5/32 to 15/32). Her confidence levels were lower than the average as well (2.93/5 to 3.65/5). Her competence and confidence did vary depending on the mathematics context. For example, while she obtained almost 90% for the section on percentages and rates, her confidence levels were very low (Figure 2)

![Figure 1. Vygotsky’s Method of double stimulation (from Valsiner, 2007, p. 371)](image)

**Figure 1.** Vygotsky’s Method of double stimulation (from Valsiner, 2007, p. 371)

In a post-test survey completed before results were posted, Sally strongly disagreed that her
mathematics skills were good enough before the start of the course. In a web posting she said, “Maths was not my favourite subject at school so I can’t say that I was very interested in that at all.” She also strongly disagreed that she would pass the course (over 96% of students surveyed strongly agreed or agreed they would pass the course). She passed with a “C” grade. Sally did not complete a reflective diary entry at the beginning of the course. However, comments during the course give some details. While she is fluent in English, her first language is Italian and she had completed some schooling in Italian. Working in a town about 1.5 hours from the university and working up to 10 shifts per fortnight in an allied industry, she could only come to the university two days. These pictures of Sally, gleaned from her comments and my subjective interpretations, create a macrogenetic story (Figure 3).

Her past mathematical experiences were typical of many mature aged students who struggle with mathematics, but she had the added pressures of time away from university and perhaps a second language issue. In addition, her inexperience with higher education expectations made university life bewildering, particularly at the beginning of semester. Sally’s numeracy issues and development, however, could be investigated from interviews which took place during the semester. A closer study of some short periods of time within those interviews, particularly where meaning blocks became apparent, assisted in understanding the issues.

The complete story of Sally (Galligan, 2011a), identified a number of mathematical issues and proposed possible learning development. The following are two examples: one on fractions and the other on squares and square roots.

Interview four focussed on multiplication of fractions. Sally often confused multiplication and division. Three issues emerged. One was evidence of an incomplete state of understanding what to do and why. There was also the importance of affect, and an issue of an approach to study (after a little prompting to reflect by the teacher).

After a few attempts with some examples, she tried to evaluate $27/3 \times 26/4$. She was able to...
articulate that it would be, “Sorry, I think it should be, I don’t know. 27 times 26, divided by 12. Yeah”. Next we tried $\frac{27}{13} \times 26 \div 42$. Even though it looks like she tried it three times before she got it right, (lines 148, 149) she was able to do it without any help, as the facilitator did not prompt her that it was wrong. Her excitement was good to see and she wanted to do more.

138. Sally Can I just do it without saying anything?
147. Facilitator Please.
148. Sally Oops do it again now the answer is 1 point 1 point 7.
149. Sally Multiplied the 13 by the 42 and then…That would be the same answer. [Unclear] Yes I got it right, I’m so excited. Doesn’t happen! Can we have this every day? This is exciting…I got it right 1.285.

Why was she was persistent in multiplying instead of dividing? She did suggest her Italian background more than once, but it may also be her approach to tackling problems. In this interview she said to Tania (the other student), “I wish I could be like that, analyse everything ‘cause I go that is what it means and off I go like I don’t…”.

In interview 5, on talking about the formula for radiation intensity $I = \frac{1}{d^2}$, she did not appear to have any knowledge of square or square root, but thought it was doubling (coming from the superscript “2”). After trying both a visual (line 25) and undoing approach (lines 17, 19 and 30) she got stuck between lines 31 and 35, she may have been OK at line 35. Even after some explanation, her last statement, “a kind of doubling” (line 59), suggested that she did not want to let go of this link to doubling (lines 16, 20 and 33):

13. Facilitator They wouldn’t ask it the other way round would they? If the intensity was 64, how far away would it be?
14. Sally 32.
15. Facilitator Square root do you know square root?
16. Sally No, thought it was double.
17. Facilitator No. Now they may not ask this, I think it would be too hard, but see how you’ve got, how I’ve said the distance was two, the intensity was a quarter. So you’ve got a quarter was the intensity, which is one over two squared. So if the intensity was a $64^{th}$, that’s one over what squared? So it’s the square root of 64, the opposite operation to squaring.

18. Sally No, I don’t get it.
19. Facilitator That’s okay. Hopefully you won’t be asked this but if he did ask it, so if you’ve got, let’s just take the basics, if you’ve got five squared, that’s 25. So another way of saying that is, the square root of 25 is five. Square root means going back to the origin.
20. Sally So can I just do that, this is my own way of, 50? No 100.
21. Facilitator Don’t double it.
22. Sally So.........10?
23. Facilitator 100
24. Sally Ummmm (doesn’t get it)
25. Facilitator Let me show you another one, maybe visually.
If I’ve got a square, and one side is two units, so I’ve got two units and the other side is two units, the area of that is four square units (OK). So if I’ve got a square and the area is four, then what’s the side? The side is the square root of four, which is two. It’s the opposite operation.

26. Sally  It’s the opposite
27. Sally  I should feel right at home with this it’s the other way around. (laughter) I’m serious
28. Facilitator  You can use your calculator as well. There’s the square root and there’s the square. So if you press five, five squared equals 25. Now cancel that and press the square root sign. Square root 25 equals 5. Do square root of 64 equals **eight**. Now do eight squared.

29. Sally  Equals 64. Okay.
30. Facilitator  What we’re doing is the opposite operations all the time.
31. Sally  Can I? Would that be 6?
32. Facilitator  No. What’s six squared?
33. Sally  I’m trying, I’m working so hard. ..... I’m thinking. No, I want to get one on my own. (long pause) It’s not 12, see I’m getting mixed up here.

34. Facilitator  You are. It’s common. Use your calculator.
35. Sally  I don’t want to, No, no, no I’m frustrated with myself. 36 OK Six by six is 36, basically that’s what it means. ... so if I do...(pause) maybe not..... eight OH that one?

36. Tania  So square root of 25 is 5 how do you do square on the calc so 2 and that equals so press the square... work out how to use the sc calculator (unclear)
37. Sally  Is that right? (yes) its an easy one but yeah OK.
38. Facilitator  Yep. Did you learn your times tables when you were little?
39. Sally  In Italian.
40. Facilitator  ..... Try seven.
41. Sally  Seven squared? OK?
42. Facilitator  Seven squared, seven sevens.
43. Sally  Is that right?
44. Facilitator  Yep. Squared means multiplied by itself. Keep that in your mind.
45. Sally  So it’s kind of doubling.

In the pre- and post-test there were two questions, one finding the square root of a number and the other was finding \( BMI = \left( \frac{w}{h^2} \right) \). In the pre-test, Sally did not answer either question. In the post-test, she got the square root correct, but in the BMI she divided \( h \) into \( w \) but did not square the \( h \). Sally appears to be oscillating in this developing state and could easily revert without more reinforcement.

If these episodes are seen in terms of MoDoSt (see Figure 4 based on Figure 1) then Sally’s self-scaffolding “I-positions” at times influence her actions. She has at her disposal, tools to assist her in solving the question at hand and it appears these two forms of stimulus means, work both with and against her. When we look at the next possible future actions, while the researcher may promote certain next actions (guide), Sally may choose just to stop.
While Sally’s past mathematics story is not fully revealed, there appeared to be gaps in her mathematics understanding from a relatively young age. Sally’s constraints were outside factors, which hindered her progress in her mathematics journey. In addition, working and travelling that much in a week would have contributed to her lack of success. She just did not have time to reflect on how to be successful at university and she often did not do tasks that were asked of her. The university culture appeared to be alien to her. She may have been aware of the possible future actions in front of her, but just was not able to address many of these. A possible reason for her relatively poor numeracy performance is that she was not able to consolidate on the things discussed in class and in the small group sessions. She came to an assignment and just did it to move on to the next piece of work. From her actions, (Figure 5) trying to move forward to a new goal (B), in a typical setting, she does not appear to be able to access the new tools of checking and taking care (area to the right labelled ‘missing’); and when she gets something wrong she will often revert to past, inaccurate memories (area to the left).

At the end of semester Sally wrote:

It is imperative for me to get this mathematical side of things such as, measurements, percentages, multiplications, divisions, fractions, down pact [sic] as it is a huge responsibility to have as a future Registered Nurse, to administer the right dosages to other people in need, so that they can feel better, they rely on that, and my aim in life is to help others in need, and I cannot let anyone down or hurt anyone in the process.

Sally is now a registered nurse.
Discussion

Upon reflection on Sally’s and other students’ numeracy journeys in this research, it is clear that the journeys are not direct. There are many points along the way where students stop. These ‘nodes of stuckness’ (Figure 6) may result in the staying at this point (or moving backwards) or staying until something (a trigger) helps them to move forward, either scaffolded from a tutor, themselves or another. It may be that there are no definitive observable triggers. The pathway may become more familiar each time a concept is revisited, as long as there is active recognition of parts of the concept that were unfamiliar, parts that are now more in focus and parts that are still to be made clear, and being comfortable with that uncertainty. While there may not be one particular AHA moment, there should be a realisation by students that they are more numerate (i.e., more competent, critical and confident) now than at the beginning of the semester.

Figure 6. Map of academic numeracy development (Galligan, 2011a, p. 339)

Along the journey they may move around, lose their way or go back to where they were before, oscillating between one point and another without really moving forward or moving forward as they are questioning their understanding. With the oscillation there are many potential paths to possible futures. A student’s final state may be anywhere along the oblique line. ‘Final’ is relative. In this state a student may still forget knowledge (e.g., how to find a percentage increase in a value), but retain thinking mathematically; think to check; think of consequences; think I could do this; etc. Some people’s way of thinking has been transformed and each may be different.

Conclusion

This paper proposed that Valsiner’s work be investigated more closely as an approach to understanding adult learners’ development. He advocates methodologies that centre on careful observation in microsettings, where development may be observed. His approach to the total environment; scaffolding and self-scaffolding; and the investigation of sets of possible future actions that result from the environment/scaffolding interaction (both successful and not), all fit in the methodology of double stimulation.

Broadly, this study outlined an example of support in academic numeracy aligned to students’ needs, as the student population demands more flexible relevant learning material in their busy complex worlds (Lawrence, Galligan, & Loch, 2008). More particularly, in the context of this
paper, it opened a lens to view a here and now setting and the next possible moments. This was also powerful lens for the teacher as it also suggests development: to learn when to speak and when to remain silent; to learn when to promote past experiences and when to leave them aside; when to move forward to push understanding and when to stop.

Finally, this research has promise to transfer this model of embedding academic numeracy to both new contexts (Galligan, 2012 (accepted 2011)). This approach to learning development is not restricted to academic numeracy. It can be used wherever a phenomenological view of learning development is needed and is particularly useful in adult learning where the study of student action, reflection on action and the environment around action and consequential development of the learner needs to be observed.

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A Note on Evolutionary Algorithms and Its Applications

Shifali Bhargava

Dept. of Mathematics, B.S.A. College,
Mathura (U.P)- India.
<shifalibhargava@gmail.com>

Abstract

This paper introduces evolutionary algorithms with its applications in multi-objective optimization. Here elitist and non-elitist multiobjective evolutionary algorithms are discussed with their advantages and disadvantages. We also discuss constrained multiobjective evolutionary algorithms and their applications in various areas.

Key words: evolutionary algorithms, multi-objective optimization, pareto-optimality, elitist.

Introduction

The term evolutionary algorithm (EA) stands for a class of stochastic optimization methods that simulate the process of natural evolution. The origins of EAs can be traced back to the late 1950s, and since the 1970’s several evolutionary methodologies have been proposed, mainly genetic algorithms, evolutionary programming, and evolution strategies. All of these approaches operate on a set of candidate solutions. Using strong simplifications, this set is subsequently modified by the two basic principles of evolution: selection and variation. Selection represents the competition for resources among living beings. Some are better than others and more likely to survive and to reproduce their genetic information. In evolutionary algorithms, natural selection is simulated by a stochastic selection process.

Each solution is given a chance to reproduce a certain number of times, dependent on their quality. Thereby, quality is assessed by evaluating the individuals and assigning them scalar fitness values. The other principle, variation, imitates natural capability of creating “new” living beings by means of recombination and mutation. Although the underlying principles are simple, these algorithms have proven themselves as a general, robust and powerful search mechanism. Moreover, EAs seem to be especially suited to multi-objective optimization because they are able to capture multiple pareto-optimal solutions in a single simulation run and may exploit similarities of solutions by recombination.

The application of evolutionary algorithms (EAs) in multi-objective optimization is currently receiving growing interest from researchers with various backgrounds. Most research in this
area has understandably concentrated on the selection stage of EAs, due to the need to integrate vectorial performance measures with the inherently scalar way in which EAs reward individual performance, i.e., number of offspring. The first pioneering studies on evolutionary multiobjective optimization appeared in the mid-1980s (Fourman, 1985; Schaffer, 1984; Schaffer, 1985). After that a few different MOEA implementations were proposed in the years 1991–1994 (Fonseca & Fleming, 1993; Hajela & Lin, 1992; Horn et al., 1994; Srinivas & Deb, 1994; Kursawe, 1990). Later, these approaches (and variations of them) were successfully applied to various multiobjective optimization problems (Cunha et al., 1999; Fonseca & Fleming, 1998; Ishibuchi & Murata, 1997; Parks & Miller, 1998).

The question is which EA implementations are suited to which sort of problem and what are the specific advantages and drawbacks, respectively, of different techniques.

- In contrast to SOPs, there is no single criterion to assess the quality of a trade-off front; quality measures are difficult to define. This might be the reason for the lack of studies in that area. Up to now, there has been no sufficient, commonly accepted definition of quantitative performance metrics for multiobjective optimizers.

- There is no accepted set of well-defined test problems in the community. This makes it difficult to evaluate new algorithms in comparison with existing ones.

- The various MOEAs incorporate different concepts, e.g., elitism and niching that are in principle independent of the fitness assignment method used. However, it is not clear what the benefits of these concepts are. For instance, the question of whether elitism can improve multi-objective search in general is still an open problem.

The above issues sketch the scope of the present work and result in the following research goals:

1. Comparison and investigation of prevailing approaches.
2. Improvement of existing MOEAs, possible development of a new, alternative evolutionary method.
3. Application of the most promising technique to real-world problems in the domain of system design.

The first aspect aims at finding advantages and disadvantages of the different approaches and yielding a better understanding of the effects and the differences of the various methods. This involves the careful definition of quantitative performance measures which ideally allow for different quality criteria. The last goal is important for identifying those problem features which cause the most difficulty for MOEAs to converge to the pareto-optimal front. The comparison also includes the examination of further factors of evolutionary search such as populations size and elitism.

As a result, these investigations may either contribute to the problem of sampling the search space more efficiently by improving existing methods or lead to the development of a new evolutionary approach. Usually, these applications are by far too complex to be handled by exact optimization algorithms.

This paper reviews current evolutionary approaches to multi-objective optimization discussing their similarities and differences. It also tries to identify some of the main issues raised by multi-objective optimization in the context of evolutionary search, and how the methods discussed address them. From the discussion, directions for future work, in multi-objective evolutionary algorithms are identified.

### Evolutionary Approaches to Multi-objective Optimization

The family of solutions of a multiobjective optimization problem is composed of all those elements of the search space which are such that the components of the corresponding objective
vectors cannot be all simultaneously improved. This is known as the concept of Pareto optimality.

A more formal definition of pareto-optimality is as follows:

Consider without any loss of generality, the minimization of $n$ components $f_k, k = 1,2,...,n$ of a vector function $f$ of a vector variable $x$ in a universe $U$, where $f(x) = (f_1(x), f_2(x),...,f_n(x))$.

Then a decision vector $x_u \in U$, is said to be pareto-optimal if and only if $\exists \ x_v \in U$ for which $v = f(x_v) = (v_1, v_2,...v_n)$ dominates $u = f(x_u) = (u_1, u_2,...u_n)$

The set of all pareto-optimal decision vectors is called the pareto-optimal efficient or admissible set of the problem. The corresponding set of objective vectors is called the non-dominated set.

Evolutionary algorithms seem particularly suitable to solve multi-objective optimization problems, because they deal simultaneously with a set of possible solutions, the so-called population. This allows to find several members of the pareto optimal set in a single run of the algorithm instead of having to perform a series of separate runs as in the case of traditional mathematical programming techniques. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the pareto front as they deal easily with discontinuous or concave pareto fronts, whereas these two issues are of real concern for mathematical programming techniques. MOEA are very attractive MOP solution techniques because they address both search and multi-objective decision making. Additionally they have the ability to search partially ordered spaces for several alternative trade-offs. They find a wide range of non-dominated solutions close to the true pareto-optimal solutions.

A MOEA defining characteristic is the set of multiple objectives being simultaneously. Otherwise task decomposition clearly shows little structural difference between the MOEA and its single objective EA counterparts.

### General EA Tasks

1. Initialize population
2. Fitness evaluation (vector/ fitness transformation)
3. Recombination
4. Mutation
5. Selection

### Non-Elitist Multi-objective Evolutionary Algorithms

Non-Elitist Multi-Objective Evolutionary Algorithms (MOEA) are algorithms which do not use any elite-preserving operator. Some important Non-Elitist MOEA includes the following:

**1. Vector Evaluated Genetic Algorithm (VEGA)**

This is the simplest possible multi-objective GA and is a straightforward extension of single-objective GA for multi-objective optimization. Schaffer implemented this first multi-objective GA to find a set of non-dominated solutions (Schaffer, 1984). This GA evaluated an objective vector, with each element of the vector representing each objective function and emphasizes
solutions which are good for individual objective functions. To find intermediate trade-off solutions, Schaffer allowed cross-over between any two solutions in the entire population. Also a VEGA has the same computational complexity as that of single-objective GAs. The main advantage of a VEGA is that it uses a simple idea and is easy to implement. Only minor changes are required to be made in a simple GA to convert it to a multi-objective GA and this does not incur any additional computational complexity. But here, as each solution in a VEGA is evaluated only with one objective function, thus every solution is not tested for other objective functions, all of which are also important in the context of multi-objective optimization.

2. Vector-Optimized Evolution Strategy

In this approach, the basic self-adaptive evolution strategy for single-objective optimization is modified to handle multi-objective optimization problems. This algorithm performs a domination check to retain non-dominated solutions and a niching mechanism to eliminate crowded solutions. The simulation results were shown on a single problem and no further work has been pursued, hence this is not used by current researchers (Kursawe, 1990).

3. Weight Based Genetic Algorithm

The key issue in WBGAs is to maintain diversity in the weight vectors among the population members. In WBGAs the diversity in the weight vectors is maintained in two ways. In the first approach, a niching method is used only on the substring representing the weight vector, while in the second approach, carefully chosen subpopulations are evaluated for different pre-defined weight vectors, an approach similar to that of the VEGA. Since a WPGA uses a single-objective GA, not much change is needed to convert a simple GA implementation into a WPGA one. Moreover, the complexity of the algorithm is smaller than other multi-objective evolutionary algorithms. As the WPGA uses a proportionate selection procedure on the shared fitness values, for mixed type of objective functions (some are to be minimized and some are to be maximized), complications may arise in trying to construct a fitness function. WPGA may also face difficulties in finding pareto-optimal solutions in problems having non-convex pareto-optimal region (Hajela et al., 1993).

4. Multi-objective Genetic Algorithm

Fonseca and Fleming first introduced a multi-objective GA (MOGA) which used the non-dominated classification of a GA population (Fonseca & Fleming, 1993). This explicitly caters to emphasize non-dominated solutions and simultaneously maintains diversity in the non-dominated solutions. The MOGA differs from a standard tripartite GA in the way fitness is assigned to each solution in the population. The rest of the algorithm is the same as in classical GA. Since niching is performed in the objective space, the MOGA can be easily applied to other optimization problems. This algorithm may be sensitive to the shape of the pareto optimal front and to the density of solutions in the search space.

5. Non-Dominated Sorting Genetic Algorithm

In non-dominated sorting GA, the dual objectives in a multi-optimization algorithm are maintained by using a fitness assignment scheme which prefers non-dominated solutions and by using a sharing strategy which preserves diversity among solutions of each non-dominated front. The computational complexity of the fitness assignment procedure is mainly governed by the non-dominated sorting procedure and the sharing function implementation. The main advantage of an NSGA is the assignment of fitness according to non-dominated sets. An NSGA progresses towards the pareto-optimal region frontwise (Srinivas & Deb, 1994).
6. Predator-Prey Evolution Strategy
This strategy does not use a domination check to assign fitness to a solution but uses the concept of predator-prey model. The main advantages of this method are its simplicity and that it does not emphasize non-dominated solutions directly. The disadvantage of this strategy is that no explicit operator is used to maintain a spread of solutions in the obtained non-dominated set. Instead, each predator is assigned the task of eliminating the worst neighboring solution with respect to a different objective. Also there is no special care taken to maintain the intermediate solutions (Laumanns et al., 1998).

7. Distributed Sharing GA
In this approach, the distributed island model is used to maintain diversity among non-dominated solutions. The GA population is divided into a number of subpopulations and independent genetic operations are performed to each island. Subpopulations from all islands are collected together and the non-dominated solutions are recorded (Hiroyasu et al., 1999).

8. Distributed Reinforcement Learning Approach
Maraino and Morales suggested a distributed reinforcement learning approach, where a family of agents is assigned to different objective functions (Mariano & Morales, 2000). Each agent proposes a solution to optimize its objective function. All such solutions are combined and a non-dominated compromised set of solutions are identified. Each non-dominated solution is rewarded. In the context of solving continuous search space problems, an agent considers solutions in a particular search direction from its current location. The solution is evaluated by the agent’s corresponding objective function. The rewarding mechanism provides a direction for the algorithm to move towards the pareto-optimal region and the non-domination check maintains a diverse set of solutions, while simultaneous creation of multiple solutions by a directional search method helps to find new solutions in the search space.

9. Nash GA
This GA is motivated by a game theoretic approach in which one player is allowed to get associated with each objective function and it tries to optimize its objective function while keeping other objective functions unchanged. In a periodic sequence of operations, the Nash GA is terminated when no more improvement is recorded. At this steady-state scenario, the resulting solution is a Nash-Equilibrium solution and is a candidate pareto-optimal solution. Although the investigators claim better convergence properties of this GA compared to the NSGA, it is clear that an explicit niche-performing operator must be used to maintain multiple pareto-optimal solutions (Sefrioui & Periaux, 2000).

Elitist Multi-objective Evolutionary Algorithms
These are evolutionary algorithms which use elite preserving operator. Elite preservation or emphasizing currently elite solutions is an important operator in an EA. An elite preserving operator favors the elites of a population by giving them an opportunity to be directly carried over to the next generation. Elitism can be implemented to different degrees in an MOEA. The presence of elitism should improve the performance of a multi-objective EA, but care must be taken to control the effective degree introduced in the progress. Now we present some algorithms that attempt to achieve a controlled elitism in multi-objective evolutionary optimization:
1. Rudolph’s Elitist Multi-Objective Evolutionary Algorithms

Rudolph suggested a multi-objective evolutionary algorithm which required the introduction of a diversity preservation mechanism (Rudolph, 2001). This algorithm with a positive variation kernel of its search operators allows convergence to the pareto-optimal front in a finite number of trials in finite search space problems. The positiveness of the variation kernel makes sure that the probability of creating any offspring from an arbitrary set of parent solutions in a finite number of trials is one. Thus in any population, if no pareto-optimal solution exists, the positiveness of the variation kernel of the combined search operators ensures that one such member will be created in a finite number of trials. With the elite preserving strategy of the above algorithm, this member cannot ever be deleted from the population. The main disadvantage of this algorithm is that it does not ensure any diversity among the obtained solutions.

2. Elitist Non-Dominated Sorting Genetic Algorithm

Deb suggested an elitist non-dominated sorting GA (termed NSGA-II) which uses an explicit diversity-preserving mechanism (Deb, 2000). The overall complexity of the NSGA-II is at most $O(MN^2)$. The diversity among non-dominated solutions is introduced by using the crowding comparison procedure which is used with the tournament selection and during the population reduction phase. Since solutions compete with their crowding distances, no extra niching parameter is required here. In the absence of the crowded comparison operator, this algorithm also exhibits a convergence proof to the pareto-optimal solution set similar to that in Rudolph’s algorithm, but the population size would grow with the generation counter. The elitism mechanism does not allow an already found pareto-optimal solution to be deleted. This algorithm loses its convergence property when the crowded comparison is used to restrict the population size. In latter generations when more than N members belong to the first non-dominated set in the combined parent-offspring population, some closely packed pareto-optimal solutions may give their places to other non-dominated yet non-pareto-optimal solutions. Although these latter solutions may get dominated by other pareto-optimal solutions in a later generation, the algorithm can resort into this cycle of generating pareto-optimal and non pareto-optimal solutions before finally converging to a well distributed set of pareto-optimal solutions.

3. Strength Pareto Evolutionary Algorithm (SPEA)

Zitzler and Thiele (1998) proposed Strength Pareto Evolutionary Algorithm (SPEA) which introduced elitism by explicitly maintaining an external population $\overline{P}$. This population stores a fixed number of the non-dominated solutions that are found until the beginning of a simulation. At every generation, newly found non-dominated solutions are compared with the existing external population and the resulting non-dominated solutions are preserved. The SPEA not only preserves the elites but also uses these elites to participate in the genetic operations along with the current population in the hope of influencing the population to steer towards good regions in the search space. In the SPEA, clustering ensures that a better spread is achieved among the obtained non-dominated solutions. This clustering algorithm is parameter-less, thereby making it attractive to use. The fitness assignment procedure in the SPEA is more or less similar to that of Fonseca and Fleming’s (1993) MOGA and is easy to calculate. In SPEA, if a large external population is used, the selection pressure for the elites will be large and the SPEA may not be able to converge to the pareto-optimal front. On the other hand, if a small external population is used, the effect of elitism will be lost. Moreover, in the SPEA fitness assignment, an external solution, which dominates more solutions, get a worse fitness. This assignment is justified when all dominated solutions are concentrated near the dominating solution.
4. Pareto-Archived Evolution Strategy (PAES)

PAES uses an evolution strategy and its main crux lies in the way that a winner is chosen in the midst of multiple objectives. The PAES has a direct control on the diversity that can be achieved in the pareto-optimal solutions. The PAES performs better when compared to other methods in handling problems having a search space with non-uniformly dense solutions. The disadvantage of PAES is that change of the depth parameter changes the number of hypercube exponentially, thereby making it difficult to arbitrarily control the spread of solutions. In order to give the PAES a global perspective the concept of multi-membered ES is introduced. Since offspring are not compared against each other and only compared with the archive, this method does not guarantee that the best non-dominated solutions among the offspring are emphasized enough (Knowles, & Corne, 2000).

5. Multi-objective Messy Genetic Algorithm (MOMGA)

This is the extension of the original messy GA in which the use of m different template strings in an era is suggested (Veldhuizen, 1999). In the level-1, MOMGA each partial string is filled from m template strings chosen randomly before the era has begun. Each filled string is evaluated with a different objective function. The objective vector obtained in this process is used in the selection operator. At the end of an era, the best solution corresponding to each objective function is identified and is assigned as the template string corresponding to that objective function. A concurrent MOMGA was also proposed by suggesting parallel applications of the MOMGA with different initial random templates. At the completion of all MOMGAs, the obtained external sets of non-dominated solutions are all combined together and the best non-dominated set is reported as the obtained non-dominated set of solutions of the CMOMGA. Although the solutions obtained by this procedure do not indicate the robustness associated with an independent run of an MOMGA, this parallel approach may be desirable in practical problem solving. A study using a probabilistically complete initialization of MOMGA population to reduce the computational burden is an improvement over the past studies (Zydallis et al., 2001).

6. Non-dominated sorting in Annealing GA (NSAGA)

This non-dominated sorting in annealing GA (NSAGA) uses a simulated annealing-like temperature reduction concept along with the Metropolis criterion. The first-stage probability calculation is along the lines of finding the transition probability of creating the offspring population from the parent population. The second probability calculation is based on the Metropolis criterion, which uses an energy function related to the number of non-dominated solutions in a population. In an elitist sense, an offspring population is accepted only when the probability of creating such a population and accepting it with the Metropolis criterion with an updated temperature concept is adequate. Clearly the goal of this work is to modify the NSGA procedure with a simulated annealing-like acceptance criterion, so that a proof of convergence can be achieved.

7. Multi-objective Micro-GA

This Multi-objective Micro-GA maintains two populations. The GA population is operated in a similar way to that of the single-objective micro-GA, whereas the elite population stores the non-dominated solutions obtained by the GA. The elite archive is updated with new solutions in a similar way to that achieved in the PAES. The search space is divided into a number of grid cells. Depending on the crowding in each grid with non-dominated solutions, a new solution is accepted or rejected in the archive (Coello & Toscano, 2000).
8. Elitist MOEA with Coevolutionary Sharing (ERMOCS)

This multi-objective GA (ERMOCS) is based on Goldberg and Wang’s coevolutionary sharing concept (Goldberg & Wang, 1998; Neef et al., 1999). For maintaining diversity among non-dominated solutions, the coevolutionary shared niching (CSN) method is used. The elite preservation is introduced by using a pre-selection scheme where a better offspring replaces a worse parent solution in the recombination procedure. In the coevolutionary model, the customer and businessman populations interact in the same way as in the CSN model, except an additional imprint operator is used for emphasizing non-dominated solutions. After both customer and businessman populations are updated, each businessman is compared with a random set of customers. If any customer dominates the competing businessman and the latter is at least a critical distance away from other businessmen, it replaces the competing businessman. In this way non-dominated solutions from the customer population get filtered and find their place in the businessman population. On a scheduling problem, ERMOCS is able to find well-distributed customer as well as businessman populations after a few generations.

Constrained Multi-objective Evolutionary Algorithms

In most practical search and optimization problems, constraints are evident. Often the constraints are many in numbers and are nonlinear. Now we deal with several multi-objective evolutionary algorithms which have been particularly suggested for handling constraints.

1. Penalty Function Approach

In the penalty function approach, the constraint violation in an infeasible solution is added to each objective function. Thereafter, the penalized objective function values are optimized. For relatively large penalty terms (compared to objective function values), this method practically compares infeasible solutions based on their constraint violations. Again for the same reason, a feasible solution will practically dominate an infeasible solution. Both of these characteristics together allow the population members to become feasible from infeasible solutions and, thereafter, allow solutions to converge closer to the true pareto-optimal solutions.

2. Jimenez-Verdegay-Gomez-Skarmeta’s Method

This work suggested a careful consideration of feasible and infeasible solutions and the use of niching to maintain diversity in the obtained pareto-optimal solutions. This algorithm uses the binary tournament selection in its core. Here feasible and infeasible solutions are carefully evaluated by ensuring that no infeasible solution gets a better fitness than any feasible solution (Jimenez et al., 1999). Only inequality constraints of the lesser-than-equal-to type are considered in their study, whereas any other constraints can also be handled by using the procedure. The disadvantage of this algorithm is that by preserving diversity among infeasible solutions explicitly, the progress towards the feasible region may be sacrificed. Also there exist a couple of additional parameters which a user must set right. In order to make the non-dominance check less stochastic, a large comparison check is needed. Furthermore, the algorithm does not explicitly check the domination of participating solutions in a tournament.

3. Constrained Tournament Method

Here the definition of domination is modified. Before comparing two solutions for domination, they are checked for their feasibility. If one solution is feasible and the other is not, the feasible solution dominates the other. If two solutions are infeasible, the solution with the smaller normalized constraint violation dominates the other. On the other hand, if both solutions are
feasible, the usual domination principle is applied. The advantage of this method is that in addition to the constraint violation computations, this strategy does not require any extra computational burden. The constraint domination principle is generic and can be used with any other MOEAs. Since it forces an infeasible solution to be always dominated by a feasible solution, no other constraint handling strategy is needed.

4. Ray-Tai-Seow’s Method

Ray, Tai, and Seow (2001) suggested a more elaborate constraint handling technique, where the constraint violations of all constraints are not simply added together; instead, a non-domination check of the constraint violations is made. Here, three different non-dominated sorting procedures are used. In addition to a non-dominated sorting of the objective functions, a couple of non-dominated sortings using the constraint violation values and a combined set of objective function and constraint violation values are needed to construct the new population. This algorithm handles infeasible solutions with more care than any other of the constrained handling techniques and diversity is maintained in the population. But the disadvantage is that in a later generation, when all population members are feasible and belong to a sub-optimal non-dominated front, the algorithm stagnates. Also during the crossover operation, three offspring are created. The first one is created by using a uniform crossover with an equal probability of choosing one variable value from each parent. The other two solutions are created by using a blend crossover, which uses a uniform probability distribution over a range that depends on a number of threshold parameter values. The difficulty arises in choosing parameter values related to each of these operators. Another difficulty arises because five solutions are accepted after each crossover operation. This process will cause the population to soon lose its diversity. Three non-dominated ranking and head-count computations make the algorithm more computationally expensive than the other algorithms.

Salient issues of Multi-objective Evolutionary Algorithms

With the success of MOEAs in different problem domains, many new techniques have been suggested. This demands a proper method of assessing the performance of a newly suggested algorithm. Since an MOEA is supposed to perform the tasks of converging close to the true pareto-optimal front and maintaining a diverse set of non-dominated solutions, an algorithm must be assessed with respect to both of these tasks. Ironically, it is difficult to have one performance metric to evaluate both of the above issues adequately.

For evaluating a new algorithm, there is also a need to test it with problems possessing known complexities of the search space and with a known pareto-optimal set. Knowledge of the exact locations of the pareto-optimal solutions is helpful in investigating the search abilities of an algorithm. With the development of a number of MOEAs over the past few years, there have been some studies in comparing them systematically. Those MOEAs which properly implemented elite preservation, emphasized non-dominated solutions, and maintained diversity among non-dominated solutions, all performed well. In several studies it was clear that elite preservation is an important operation in converging as well as sustaining a good diverse set of non-dominated solutions.

An important aspect of maintaining diversity among non-dominated solutions is the space in which the diversity is required. The diversity preserving operator must treat the proximity of the solutions in the decision variable space. On the other hand, if the diversity in the objective space is more important, the proximity must be measured in the objective space. It is important to keep in mind that the proximity in one space may not mean that a proximity in the other space would be automatically obtained. This has been found to be particularly true in certain nonlinear and complex problems.
The concept of multi-objective optimization can also be used to solve other kinds of optimization problems in an efficient way. For example, a constrained single objective optimization problem can be considered as being a multi-objective optimization problem of optimizing the objective function and minimizing all constraint violations. The principle of finding multiple optimal solutions can also be extended to other similar problems, such as goal programming. Because of the lack of an optimization algorithm which can find multiple optimal solutions simultaneously, goal programming approaches traditionally use relative weights of objectives and resort to finding one solution corresponding to one weight vector at a time. Also, an interesting aspect is that as the number of objectives increase, a large proportion of a randomly chosen population becomes non-dominated. When this happens, introduction of elitism becomes tricky. This is because a large number of population members are candidate elite solutions, therefore not allowing many solutions to be accepted in any generation. Moreover, there are ways to choose an adequate population size such that a reasonable proportion of the population members belongs to the dominated fronts for initially introducing variability in the population.

In MOEAs, convergence to the pareto-optimal front and simultaneous maintenance of a good distribution are both important. Although there exist a number of MOEAs with theoretical convergence properties to the true pareto-optimal front, they do not guarantee maintaining any spread of solutions. More studies to develop MOEAs with properties of convergence as well as spread of solutions remain as an imminent challenge to the researchers of MOEAs.

**Applications of Multi-objective Evolutionary Algorithms**

Here we discuss concisely some real life application examples of multi-objective evolutionary algorithms. These application examples are conducted in the context of real-life problems. Some important applications are in computational finance, economics, engineering design, encryption and code breaking etc. Each example shows different particularities of the MOEA design, implementation and usage.

1. **Financial Time Series**
   Niched Pareto Genetic Algorithm has been used to find patterns in financial time series such that predictions can be made regarding the behavior of a certain stock (Horn, Nafploitis & Goldberg, 1994). The methodology has also been used for the identification of significant technical analysis patterns in financial time series (Ruspini & Zwir, 1999). Two objectives are considered i.e. quality of fitness and its extent. Fitness measures the extent to which the time series values correspond to a financial uptrend, downtrend or head and shoulders interval.

2. **Forecasting Stock Prices**
   Although long term forecasting is not possible for the stock market, it is normally possible to perform short term forecasting with heuristics. The use of genetic programming in this area has become increasingly popular, since GP can be used for symbolic regression, emulating the tasks traditionally performed by ANNs.

3. **Stock Ranking**
   The aim of this problem is to classify stocks as strong or weak performers based on technical indicators and then use this information to select stocks for investment and for making recommendations to customers. Many MOEAs has been reported in this application area. Mullei and Beling (1998) used a GA with a linear combination of weights to select rules for a classifier system based on profitability.
4. Risk Return Analysis

It is slightly different from risk-return trade up which is made in investment portfolio. Credit portfolios handled by banks operate under different rules and therefore they are not modeled using the original Markowitz approach. Schlottmann and Seese (2002) used an approach similar to the NSGA-II for solving portfolio selection problems relevant to real-world banking (Deb, Pratap, Agrawal & Meyarivan, 2002). In the problem studied by the authors, a bank has a fixed supervisory capital budget. There is an upper limit for investments into a portfolio consisting of a subset of assets (e.g., loans to be given to different customers of the bank), each of which is subject to the risk of the default (capital risk). So, in this case, besides having an expected rate of return (as in the original Markowitz problem), each asset also has an expected default probability and a net exposure within a fixed risk horizon. The resulting problem has a discrete constrained search space with many local optima and two conflicting objective functions. Unlike the original NSGA-II, the authors adopted an external archive containing the non-dominated solutions found along the search. For validating the approach, the authors adopted data from the Credit - Metrics Technical Document.

5. Economic Modelling

Mardle uses a GA with a weighted goal programming approach to optimize a fishery bio-economic model (Mardle et al., 2000). Bio-economic models have been developed for a number of fisheries as a means of estimating the optimal level of exploitation of the resource and for assessing the effectiveness of the different management plans available.

6. Model Discovery

This is an interesting area in econometrics in which non-parametric models are assumed and one tries to use an evolutionary algorithm to derive a model for a certain type of problem (e.g., forecasting nonlinear time series). Normally, artificial neural networks (ANNs) have been used for the model itself, but several researchers have used evolutionary algorithms to find the most appropriate ANN that models the problem of interest.

7. Data Mining

The use of data mining techniques for learning complex patterns is a very promising research area in economics and finance. For example, the mining of financial time-series for finding patterns that can provide trading decision models is a very promising topic (Chen, 2002).

8. Investment Portfolio Optimization

One of the most promising fields of application is investment portfolio optimization. It can vary from simple portfolios held by individuals to huge portfolios managed by professional investors. The portfolio contains stocks, bank investments, real estate holdings, bonds, treasury bills etc. The motto of it is to find an optimal set to invest on, as well as the optimal investment for each asset. This optimal selection and weighting is a multi-objective problem where total profit of investment has to be maximized and total risk is to be minimized. There are also different constraints, depending on the type of problem to be solved. For example, the weights normally have lower bounds, upper bounds and many other constraints. This is the so-called optimal investment portfolio that one wishes to obtain by using optimization techniques. This problem is traditionally studied using the Markowitz portfolio selection model (Markowitz, 1952)
9. Risk Management
The study of risk and the reaction of an agent is a very interesting research area. Some researchers have studied, the formation process of risk preferences in financial problems (Chen, 2002).

10. Coevolution
The use of co-evolutionary approaches for certain problems in economics and finance (e.g. for studying artificial foreign exchange markets) is a very interesting topic that certainly deserves attention. Co-evolutionary MOEAs are still not too common, but their potential use in financial areas may boost the interest of researchers in paying more attention to them. Many other possible areas include, the study of consumers patterns, credit scoring, economic growth and auction games.

11. Air Operations Mission Planning
Air operations mission planning is a complex task, growing ever more complex as the number, variety, and interactivity of air assets increases. Mission planners are responsible for generating as close to optimal taskings of air assets to missions under severe time constraints. This function can be aided by decision-support tools to help to ease the search process through automation. Several applications of multi-objective evolutionary algorithms for discovering suitable plans in the air operations domain, including dynamic targeting for air strike assets, intelligence, surveillance, and reconnaissance (ISR) asset mission planning, and unmanned aerial systems (UAS) planning have been presented (Rosenberg, Richards, Langton, Tenenbaum & Stouch, 2008).

12. Survival Analysis
A multi-objective evolutionary algorithm for the extraction of models for survival analysis has been proposed and evaluated. To use multi-objective evolutionary algorithms for survival analysis has several advantages. They can cope with feature interactions, noisy data, and are capable of optimizing several objectives. This approach is capable of producing accurate models, even for problems that violate some of the assumptions made by classical approaches (Setzkorn, Taktak & Damato, 2006).

13. Engineering Design
Getting the most out of a range of materials to optimize the structural and operational design of buildings, factories, machines, etc. is a rapidly expanding application of GAs. These are being created for such uses as optimizing the design of heat exchangers, robot gripping arms, satellite booms, building trusses, flywheels, turbines, and just about any other computer-assisted engineering design application. There is work to combine GAs optimizing particular aspects of engineering problems to work together, and some of these can not only solve design problems, but also project them forward to analyze weaknesses and possible point failures in the future so these can be avoided.

14. Trip Traffic and Shipment Routing
New applications of a GA known as the "Traveling Salesman Problem" can be used to plan the most efficient routes and scheduling for travel planners, traffic routers and even shipping companies. The GA gives shortest routes for traveling, timing to avoid traffic tie-ups and rush
hours, most efficient use of transport for shipping and including pickup loads and deliveries along the way. The program models all this in the background and improve productivity, while the human agents do other things.

15. Encryption and Code Breaking
On the security front, GAs can be used both to create encryption for sensitive data as well as to break those codes. Encrypting data, protecting copyrights and breaking competitors codes have been important in the computer world ever since there have been computers, so the competition is intense. Every time someone adds more complexity to their encryption algorithms, someone else comes up with a GA that can break the code. It is hoped that one day soon we will have quantum computers that will be able to generate completely indecipherable codes.

16. Optimizing Chemical Kinetic Analysis
GAs are proving very useful toward optimizing designs in transportation, aerospace propulsion and electrical generation. By being able to predict ahead of time the chemical kinetics of fuels and the efficiency of engines, more optimal mixtures and designs can be made available quicker to industry and the public. Some computer modeling applications in this area also simulate the effectiveness of lubricants and can pinpoint optimized operational vectors, and may lead to greatly increased efficiency all around well before traditional fuels run out.

17. Reservoir System Optimization
This study presents a novel approach for solving multiobjective reservoir system optimization problems using Differential Evolution (DE). The proposed methodology for Multi-objective Differential Evolution (MODE) combines pareto dominance criteria with DE for nondomination selection and crowded distance comparison operator for promoting solution diversity, and incorporates elitism in its evolution to improve the performance of the algorithm. The optimization involves minimization of flood risk, maximization of hydropower production, and minimization of irrigation deficits while properly evaluating other constraints. The MODE resulted in many Pareto optimal solutions in a single run, by specifying the reservoir releases and storage policy for each solution. The interdependence among the decision variables is better exploited using MODE. It is also found that the performance of MODE is better than NSGA-II for the reservoir system optimization problem. Thus, the obtained results suggest that the MODE approach is robust, and converging to the true Pareto optimal front with a good solution spread and coverage (Reddy & Kumar, 2007).

Summary
This paper gives a brief overview of multi-objective evolutionary algorithms. This paper describes need of multi-objective evolutionary algorithms, development of non-elitist and elitist multi-objective evolutionary algorithms, constrained multi-objective evolutionary algorithms with their salient issues. This paper also discusses the application of multi-objective evolutionary algorithms in several areas such as finance, engineering, economics, chemistry, transportation etc.
References


