Objectives

Adults Learning Mathematics – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field, which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:
· Research and theoretical perspectives in the area of adults learning mathematics/numeracy
· Debate on special issues in the area of adults learning mathematics/numeracy
· Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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We are glad to introduce a new issue of Adults Learning Mathematics: An International Journal. Over the years, our journal has analysed many different topics. Some of these topics include the connection between mathematics learning and vocational training in educational programs addressed to adults, mathematics anxiety, parental involvement, equity and justice in adults mathematics education, matters regarding the impact of gender issues in mathematics, and challenges that we still need to face in this sense, and proposals to theorize our field and bring a coherent corpus of knowledge based on scientific evidence. Always, Adults Learning Mathematics: An International Journal tried to offer possibilities and alternatives to managing challenges emerging from the discussions in our community. Many of these alternatives opened fruitful lines of research abroad.

Now, current debates are focusing on the idea of “synergies.” Namely, how can we work together to achieve more than what we would have otherwise achieved alone? Adult learning theories are full of examples demonstrating that collective efforts always produce further learning than individual ones (Flecha, 2000; Freire, 1998, Jarvis, 1995; Mayo, 2003). Vygotsky (1978) in his seminal work proposed the notion “zone of proximal development” to identify the capacity of an individual when such individual is supported by more capable partners. Previous research suggests that dialogue is a successful tool that adult learners use to share their thoughts, ideas, and strategies to solve mathematical problems (Diez-Palomar, 2009). Clearly, adults collaborate with each other to solve everyday problems, and this type of collaboration can be advantageous in the context of learning, in general, and in learning mathematics in particular.

Looking back, perhaps many of you can remember a time when you touched the shoulder of a classmate to ask him/her for clarifications on a concept that your teacher was trying to convey. However, from a research perspective, we know little about how this process of sharing can impact learning. Of course, interaction is productive, but how can interaction move learners beyond their individual boundaries? How can we, as educators, provoke this type of synergies among adult learners? What are some particular examples that can inform future teachers to design better [more productive] learning activities? The articles in this edition touch on answers to these questions.

The first article presented in this issue is an example of collective mathematical understanding, in an ironworking apprenticeship classroom. In this article Martin and Towers
use the notion of “improvisational coacting” to understand how learners share their individual ideas, understandings and contributions, in order to elucidate the correct answers to mathematical situations. According to the authors’ research, the context for these interactions among learners should not be directed, but be left to take shape organically or spontaneously. The authors discuss the “improvisational” as a process of “spontaneous action, interaction and communication,” where no one is directed to drive the process of answering/solving mathematical problems. This notion relates to the idea of “equality” emerging from theoretical approaches in adult learning such as dialogic learning (Flecha, 2000) or dialogical action (Freire, 1998), and connects strongly with the essence of adult learning: the autonomy of learners as active subjects of learning (Knowles, 1980). As the article shows, the case of Joe, Andy and Mike is a delightful example of how individuals work together to learn mathematics.

In the next article, Wes Maciejewski discusses a college-level foundational mathematics course involving the JUMP Math Program. Maciejewski’s starting point was the high rates of failure obtained by college students during their first semester of mathematics. Trying to address the recommendations of a previous project showing that colleges and college faculty should strengthen their commitment to student retention and success by adopting initiatives found effective in other colleges, Maciejewski was involved in implementing a JUMP Math curriculum to compare students’ gains in mathematics. According to Maciejewski, many complexities emerged from this work, and his article discusses these complexities in order to open more room for further research.

We close this issue with a last article focusing on masons’ professional practices related to mathematics. Using ethnographical methods, Moreira and Pardal provide a detailed analysis of how masons use mathematics in their workplace. The episodes reported by the authors illustrate clearly how mathematics is embedded in professional activities. The authors show that mathematics emerge as a transversal component (not always in a formal way), and individuals use non-formal strategies to deal with everyday situations that contain rich mathematics. Drawing on the Ethnomathematical approach, Moreira and Pardal analyse masons as a community of practice in which learning is based on everyday practices driven by the experienced who show newcomer/inexperienced how to do the job. Mario, Quim, Joao and Antonio are the main characters of the mathematical episodes introduced in this article. They explain how to build an “esquadro” to measure right angles, and how to make square holes with a machine that makes circular holes. Many mathematical ideas emerge from these episodes: perpendicularity, angles, squaring the circle, one of the three classical problems (coming from the ancient Greece), slope, etc. The examples remind us that mathematics sometimes could be invisible to the observer, but it really is embedded in our lives (Niss, 1994).

We just want to encourage the readers to swim into the articles, and benefit from the reading. Have fun!

References


“Some Guys Wouldn’t Use Three-Eighths on Anything...”:
Improvisational Coaction in an Apprenticeship Training Classroom

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Abstract
This paper presents some ongoing findings from a larger project exploring the growth of mathematical understanding in a variety of construction trades training programs. In this paper we specifically focus on the notion of collective mathematical understanding in an ironworking apprenticeship classroom. We identify the particular ways in which a group of three apprentices work collaboratively together to solve a workplace problem with a substantial mathematical element. Through drawing on the notion of ‘improvisational coactions’ (Martin & Towers, 2009) we detail the ways that individual ideas, understandings and contributions mesh together and are collectively built on by the group to allow a shared understanding to emerge. From this analysis we suggest that improvisational coactions can be a powerful means through which apprentices in the workplace-training classroom might effectively learn to tackle workplace problems that involve thinking and working mathematically. Although our conclusions are specific to this case, we would suggest that there are implications that may be relevant to other areas of workplace training.

Key words: mathematics; group work; improvisation; workplace; learning.

Introduction
FitzSimons, Micek, Hull, & Wright (2005) note that “current research positions numeracy as a social and cultural-historical process – to use terms drawn from activity theory” (p. 9). As a consequence of such a theoretical framing, increased attention is being paid to collaborative actions as an important aspect of both workplace learning and practice. For example, Boyer and Roth (2006), in researching the learning that emerged for participants in an environmental action group, noted “much of their learning occurs informally, simply by participating in the everyday, ongoing collective life of the chosen group” and “changing forms of participation are emergent features of unfolding sociomaterial inter-action, not determinate roles or rules” (p. 1028).
Drawing on the notion of ‘funds of knowledge’ (Baker & Rhodes, 2007; Gonzalez, Andrade, Civil, & Moll, 2001)—the informal, broader knowledge and experiences that adult learners often possess—Oughton (2009) talks of the need for learners to “admit doubt, challenge each other’s responses, and support each other in group activities” (p. 27). She further highlights that, in her study into learning in an adult numeracy class, “parts of the data show how the students share and pool metacognitive strategies such as eliminating easy possibilities first, and using different forms of visualisation” (p. 27).

O’Connor (1994) recognises that ethnographic studies also “emphasise that the nature of work itself is collective, and almost always requires the informal collective interaction and action among individuals” (p. 281), a view taken up by FitzSimons, Micek, Hull & Wright (2005) who state that “as an activity, work is a collective process, dependent on interaction and communication, using artefacts, such as tools, written materials, tables and charts, as an integral part of the process” (p. 9). More specifically, and of significance for learning, they suggest that “the communal model which operates has greater depth than any individual knowledge base; the group develops a communal memory of problems and solutions, and provides assistance to individuals—a valuable and relevant learning asset” (FitzSimons, Micek, Hull, & Wright, 2005, p.23). It should however also be noted that as FitzSimons, Micek, Hull, & Wright (2005) state “there has been little attention until now focused on how numeracy is learned in the workplace, taking into account the complex issues which surround apparently simple calculations, and the importance of social, cultural, and historical contexts” (p. 26).

In this paper, by discussing the case of three apprentice ironworkers, we explore the process through which a communal model is seen to emerge and be fostered from the social context and interactions of the group as they work together on a workplace task with a strong mathematical component. In doing this we draw on a framework of improvisation, in particular the notion of ‘improvisational coactions’ (Martin & Towers, 2009), a perspective and analytic tool that allows us to identify the emergence of a collective understanding of the workplace task and the associated mathematical ideas. This collective understanding is, we suggest, the means through which the three apprentices are able to successfully engage with and complete the task, in ways that would not perhaps have been possible individually.

**Improvisational coactions and collective understanding**

In recent papers (Martin, Towers, & Pirie, 2006; Martin & Towers, 2009) we offered the beginnings of a theoretical framework focussing on the collective mathematical activity of learners collaborating in small groups. We demonstrated that by using the analytic lens of improvisational theory it was possible to observe, explain, and account for acts of mathematical understanding that could not simply be located in the minds or actions of any one individual, but instead emerged from the interplay of the ideas of individuals, as these became woven together in shared action, as in an improvisational performance.

Improvisation is broadly defined as a process “of spontaneous action, interaction and communication” (Gordon Calvert, 2001, p. 87). Ruhleder and Stoltzfus (2000), in talking of the improvisational process, draw attention to “people’s ability to integrate multiple, spontaneously unfolding contributions into a coherent whole” suggesting that many of our everyday actions and interactions are improvisational in nature (p. 186). Influenced by the literature that focuses on improvisational action in the fields of jazz and theatre we have developed the theoretical construct of “improvisational coaction”. Sawyer (2003) talks of improvisational activity as being conceived of “as a jointly accomplished co-actional process” (p.38) and for us the use of the term coaction rather than simply interaction emphasises, in a powerful manner, the notion of acting with the ideas and actions of others in a mutual, joint way.

Improvisational coaction is a process through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built upon, developed,
reworked, and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual. We identify four specific characteristics of the phenomenon: No one person driving; An interweaving of partial fragments of images; Listening to the group mind; and Collectively building on the better idea (Martin & Towers, 2009). It is these characteristics that we employ in our analysis of extracts of videodata to explore the process through which collective understanding emerges for the three apprentice ironworkers. Although full definitions of the characteristics can be found in Martin & Towers (2009) we offer here brief descriptors that help to situate the interpretation of our data extracts through this lens.

No one person driving

In improvisational theatre, the term driving refers to an actor who is taking over the scene by preventing other actors from having an opportunity to contribute to the emerging direction of the performance (Sawyer, 2003, p. 9). In contrast, no one person driving suggests a more distributed kind of collaboration, where no single person dominates or controls the emerging action. In the context of mathematics we suggest that this is observed where no one learner in the group is individually able to contribute a coherent, clearly articulated solution to the mathematical problem. Instead, learners need to work collectively as their individual images are partial and they require offerings from others to complete them (see next section for an elaboration of this idea). Thus, there is an absence of a ‘driver’ (in the sense of a single, continually mathematically dominant person). This need for the collective and the lack of a dominant leader is in sharp contrast to some other data examples we have presented (Martin & Towers, 2010), wherein a mathematically stronger (or socially more powerful) learner dominates the process. While the learners in those episodes interact, they do not improvisationally coact.

An interweaving of partial fragments of images

In talking about musicians, Monson (1996) wrote:

When you get into a musical conversation, one person in the group will state an idea or the beginning of an idea and another person will complete the idea or their interpretation of the same idea, how they hear it. So the conversation happens in fragments and comes from different parts, different voices. (p. 78)

We see a similar evolution of mathematical ideas in the growth of collective mathematical understanding, where fragments of mathematical ideas (or images) initially offered by individuals become acted upon by others and coalesce into a shared (or distributed) image for the group. It is just not the fact that different learners are offering partial (or incomplete) images that is important for the collective growth of understanding, but also that others in the group choose to take these up and add to them. The following two sections elaborate this adding to and building upon in greater detail.

Listening to the group mind

Although it might appear that improvisational performances are unscripted and that ‘anything goes’ this is not the case, and a number of conventions govern the ways in which an improvisational performance develops. Becker (2000) notes the importance that everyone pays attention to the other players and be willing to alter what they are doing “in response to tiny cues that suggest a new direction that might be interesting to take” (p. 172). In terms of mathematics, such a cue would likely involve the offering of some new (perhaps partial) mathematical idea or a possible strategy or approach to take to a problem in order to proceed.
The notion of listening to the group mind (Sawyer, 2003, p. 47) places a responsibility on those who are positioned to respond to an offered action or innovative idea as much as on the originator, and it is this process wherein the group collectively determines whether, and how, the idea will be accepted into the emerging performance, that we suggest is the key to the emergence of a collective understanding.

Collectively building on the better idea

One implication of listening to the group mind is that when one person does or offers something new that, in the view of the group, is likely to be a useful idea or strategy to collectively pursue, then “everyone else drops their own ideas and immediately joins in working on that better idea” (Becker, 2000, p. 175). This collectively building on the better idea is key to how collective understanding actually grows through coaction. Of course this requires some understanding of what ‘better’ might look like and of how to recognise it. In mathematics a ‘better idea’ is one that (at that moment) seems to be a different, new, more powerful way to proceed. Again, the better idea might be a new piece of mathematics or an alternative way to think about the problem—this is established through listening to the group mind and is then collectively acted upon.

The case of Joe, Andy, Mike, and the ironworking task

The larger study, from which the data in this paper is drawn, is made up of a series of case studies of apprentices training towards qualification in various construction trades in British Columbia, Canada. From these case studies we are continuing to develop a series of ‘stories of understanding’ which seek to identify and elaborate the often complex process through which apprentices engage with mathematics in the workplace training classroom (e.g., Martin & Towers, 2007; Martin, LaCroix, & Fownes, 2005, 2006). The case studies involved video recorded observations, together with field notes and interviews with selected apprentices. Data was collected in the training classroom and workshop. Both whole classes and smaller groups of learners were observed, depending on the structure of the session. In observing and analysing the ways in which the apprentices used their mathematical knowledge in the context of workplace tasks we drew on the approach proposed by Powell, Francisco, & Maher (2003) allowing us to construct a series of emerging narratives about the data, of which this perspective on collective action is one.

To illustrate the role, and power, of improvisational coacting in the growth of understanding in the workplace-training classroom, we will consider the case of three apprentices, known as Joe, Andy and Mike, and their engagement with a workplace task. The apprentices are in the second year of an apprenticeship-training program to become credentialed ironworkers. Their training course is part time, and, when not in college, they are employed full time for various companies in different parts of British Columbia, Canada. The taught program is based in an Institute of Technology in Burnaby, BC, and involves classroom and practical sessions. In this session Joe, Andy and Mike have been posed the task of establishing the size of choker sling required to lift an assembled structure of four large iron beams into an upright position, and later of determining where the crane should be positioned to accomplish this. Figure 1 illustrates this actually being carried out.
Figure 1. The four beams, assembled and being lifted.

The structure consists of two upright beams, one top crosspiece, and one middle beam, which are welded together on the ground and then hoisted into an upright position. As can be seen in the photograph, this T-shape piece is lifted into position using two chokers in a sling arrangement around the top beam. It is the size of these chokers (i.e., the diameter of the cable, in inches) that the apprentices have been asked to calculate, something that is dependant on the total weight of the structure to be lifted (i.e., the total weight of the four beams).

The data being discussed here were drawn from a classroom session where the apprentices worked with technical plans to determine the appropriate configuration prior to its practical implementation. The apprentices were in a larger class of about twenty students and Joe, Andy, and Mike worked closely together, for about one hour, at a desk, where they were video and audio recorded. A researcher acted as an observer to the session and made appropriate notes. The extracts of data offered here, analysed through the interpretive framework of improvisational coactions, illustrate the way in which a collective understanding (of both the task and the mathematics required by it) emerges and grows for Joe, Mike, and Andy, and how this allows them to successfully choose a choker of the appropriate size.

Extract One – Length of the choker

The first part of the task required that the group calculate the total weight of the structure to be lifted (i.e., the sum of the weights of the four beams). Just prior to the start of the extract below the apprentices have completed this calculation, finding firstly the total weight in kilogrammes (seven hundred and twenty-seven kilogrammes) and the converting this to pounds by multiplying by two point two (giving one thousand, six hundred, and twenty-one point four pounds). They are now moving on to determine the size of choker required to safely lift the structure. They have chosen to use a rigging structure involving two chokers. This kind of choker hitch is shown in figure two.
They now need to decide where to place the chokers on the top beam, choose the length of each cable, calculate the vertical height of the triangle created by the two chokers (commonly known as the “vert”), and then find the stress in each choker. To find this stress they employ a commonly used formula: 

\[ \frac{m}{n} \times \frac{lv}{l} \]

where \( m \) is the total mass to be lifted; \( n \) is the number of chokers; \( l \) is the length of each choker; and \( v \) is the vertical height. This stress then determines the choker size, i.e., the diameter of cable that is safe to use for lifting the structure.

1. Andy: The length, eh? We can find out the length.
3. Andy: Yeah.
4. Joe: ‘Cos we’re going to choke it here and here (he indicates two points near the end of the top beam)
5. Mike: We can just give it a length if we want to
6. Andy: No you can’t
7. Mike: Why not?
8. Joe: Yeah for a scenario, just give it a length of…what’s this distance from here to here again? (He is referring again to the length of the beam)
9. Mike: Three thousand and forty eight (this is the distance in millimetres between the two points).
10. Andy: Three metres.
11. Mike: Three and a half metres.
12. Joe: Three and a half metres.
13. Mike: So we want to be about three and a half metres up?
14. Mike: All sides we want to equal around the same, right?
15. Joe: Exactly. Yeah. Right, have to make a triangle.
16. Mike: So you can make that into footage, three point zero four eight.
17. Andy: Ten feet.
18. Mike: Yeah. Ten-foot chokers it is.
19. Andy: Ten feet chokers.
20. Mike: How many fee..
22. Andy: How many feet in a metre? Three point..
23. Mike: Yeah. It is ten feet times three point two eight oh eight. Yeah. Ten feet.
25. Mike: Ten-foot chokers.
The apprentices start with clarifying among themselves exactly what they are being asked to do. Andy points out that they can find the length, Mike refines this to choker length and Joe indicates how finding this will help in the actual lifting of the structure. Mike then suggests that they could just choose a length. In theory, this is not incorrect as different lengths will simply lead to different angles being created between the top beam and the two chokers (different shaped triangles is another way to visualise this). Andy rejects this idea, but it is then picked up on by Joe who suggests choosing a length equal to that of the top beam (three thousand and forty-eight millimetres). As Mike then expresses it, “all sides we want to equal around the same, right?” Although not essential, it is common practice in the workplace, for safety reasons, to choose cable lengths that will generate an equilateral triangle, as this minimizes the stress in each cable, and it is this idea that they then use as they calculate the length of each choker. They decide to switch to using imperial measures, and so work with ‘ten foot’ as the chosen length. Following a brief check of this conversion they agree that ten-foot chokers will be their chosen length. This decision, and agreement, to use ten-foot chokers arranged to give an equilateral triangle is not one that is instantly arrived at, nor merely stated by one apprentice in the starting of the task. Instead we see all three contributing ideas about possible ways to proceed and these partial ideas and offerings interweave as the conversation develops, leading to a collective image for an appropriate rigging design. No single apprentice dominates the conversation or is able to simply calculate the choker length. The notion of forming an equilateral triangle, although initially posited by Joe (line 8) is one that is collectively built on as a better idea (lines 9-16), and frames the calculation that follows, which is also carried out by an interweaving of partial contributions, leading to the accepted answer, by the group, of ‘ten foot’.

Extract Two – Finding the vert

The next stage in determining the correct size of choker is to use the formula \(\frac{m \times l}{n \times k}\). They have now calculated \(m\), \(n\) and \(l\), and need to find \(v\), the vertical height of the equilateral triangle created by their chosen choker arrangement.
At the start of this extract Joe initially thinks that ten feet is the vertical distance. Mike says no, and Andy elaborates. In talking about ‘top’ and ‘bottom’ distance we suggest he is referring to the base and hypotenuse of the created triangle. Joe (line 31) shows that he accepts and is willing to build on this better idea by introducing the required formula for calculating the stress on each choker and they consider which parts of this formula they know (the total mass, the number of chokers and the length of each). Andy asks what the missing number, the “vert”, would be and Joe points out that they have to calculate this. Mike then proceeds to start the calculation—he mentions trigonometry (though he is going to use Pythagoras’ Theorem) but gets Joe involved in performing the actual calculation. Although initially Joe simply seems to be ‘number crunching’, this is not the case. In line 41 he demonstrates that he understands the need to find the square root of seventy-five and offers the answer of eight point six. The process of deciding what to find, and then the subsequent calculation is not performed by one apprentice, but instead through the collective offering and building on partial elements of what is needed. Again, we see the other two apprentices reflecting on the answer and ensuring that they know what they have found and that it is appropriate. It is not simply accepted as correct, but instead there is an explicit and articulated checking that there is a collective, shared image here. Andy clarifies that this is “our vertical” (line 43) and Joe and then all three, in different ways, offer statements to confirm that the answer “sounds right” or “sounds good”. As shown in the transcript, Mike expresses his satisfaction slightly differently (line 46).

Extract Three: Size of the choker

With the four required measurements the apprentices then quickly calculated the stress for the rig. Having obtained the (correct) answer of 936 lbs they now move to determine the size of the choker. This is a piece of information read from a published chart which provides the correct choker size for a given rigging configuration and stress.
In the above extract, using the rigging chart Joe reads off the value of “three-eighths” (of one inch). Mike agrees with this, and it might seem the task is solved and the appropriate size choker found. Interestingly though, Joe offers instead the better idea of “one half”—challenging the mathematically correct answer of “three-eighths” (and his own suggestion). This is a situation perhaps unique to the workplace, where it is not uncommon practice to choose a size larger than that required—for reasons of safety. Thus a correct answer is not necessarily an appropriate one. Mike listens to Joe’s suggestion but doesn’t agree and points out how simple the configuration is, and that the structure is not that heavy. At this point, it is not clear whether the group mind will accept the larger size as appropriate. Although Joe agrees with Mike’s statement that the structure is not that heavy, he remains convinced of the need for half-inch chokers and re-presents this to the group, looking for the others to agree and to build on his suggestion. The coacting here is somewhat different to that seen in the previous extracts, as it is not strictly about the solving of a mathematical problem or the completion of a calculation. Instead, it is about the collective establishment of what is appropriate in the workplace.

Here too, the building on the better idea is also changed. It is not that Joe wants the group to add to his suggestion mathematically or to elaborate this. Instead it is more of a desire for there to be a shared understanding of why they should choose a choker of this size. He doesn’t seem to be comfortable or willing to simply dictate to the group (and be a driver), but instead wants to enable a group consensus, to which all are committed. Andy then agrees that he doesn’t like the smaller size of choker and when Sarah (a visiting tutor to the class) asks “why?” it is Joe who explains that it is safer “in the field” to use a larger size—even though the mathematical calculation suggests the smaller size will be adequate. Andy agrees, but again notes the weight of the structure (line 63). Joe accepts this, and also that this is usually a decision made by ironworkers as something of a matter of personal preference, and that “some guys wouldn’t use three-eighths”. It is then Mike who draws these ideas together for the group in a way that allows them to proceed through essentially acknowledging both choices as valid, but suggesting that here (in the context of a contrived, classroom task) three-eighths is a sufficient answer and the group continue to the next step. Although the data only allows us insight into this group’s in-classroom mathematical practices, it does suggest that their choice might have been different were this an actual workplace task rather than a pencil and paper problem, and that a workplace choice might err on the side of safety rather than simply mathematical correctness.

**Improvisational Coactions and Collective Understanding in the Workplace**

As noted earlier, the nature of work is collective—involving and requiring individuals to be able to work together and to draw on each other’s expertise and experience. In the data extracts above the group act with the contributions of other group members in a mutual, joint way that we characterise and explain through the lens of improvisational coaction. The way in which the group is able to mesh together fragments of each individual’s knowing, through listening to the
group mind, is what enables their collective mathematical understanding to grow, ultimately enabling them to complete the task successfully.

Their coaction occurs in two significant contexts. The first relates to the way the apprentices engage with, and generate, the mathematics needed in the task. Martin and LaCroix (2008) talk of “images of visible mathematics”—referring to the “conventional mathematical content that is taught and learned in the workplace training setting” (p. 127)—here this would include the formulas and calculations they employ in determining the choker length, the ‘vert’ and the choker size. No single apprentice ‘knows’ or ‘provides’ all of this mathematics, instead it emerges from an interweaving of partial fragments of images. The three apprentices need to share and build on partial mathematical ideas offered by each other to ensure that the group both uses appropriate mathematical formula and complete the required calculation correctly.

Apprentices in trades like plumbing and ironworking are often not mathematically strong or confident, though this is not to say they are in any way mathematically incompetent. Our research suggests that their understandings seem to be partial, fragmented, specific and often deeply contextual. Collaborative working and coaction is one way through which such understandings become more complete, more general and more flexible, and as with Andy, Joe, and Mike, lead to correct and appropriate solutions to tasks and problems.

Also in these data we see a kind of coacting that is perhaps unique to the workplace—the process through which collective judgements are made about the reasonableness of the mathematical solution. The workings of Andy, Joe, and Mike are always underpinned by the overall purpose of the task—the safe lifting of a heavy, iron structure. As the extracts illustrate, there are many ways that the lifting of the iron beam structure could be accomplished, involving different kinds of hitches as well as different sizes and arrangements of the chokers. There is not just one correct solution, and it is the collective coaction that allows an appropriate decision and way forward to emerge from the working of the group—through listening to the group mind and collectively building on better ideas. However, these choices are not just mathematical questions and the context of the workplace is especially significant, as safety is always of paramount concern. The use of an incorrect choker would be dangerous, and thus ensuring both the choice of appropriate mathematics, the accuracy of any calculation and an awareness of the appropriateness of the answer is vital. As noted by Martin, LaCroix, and Fownes (2005), “in the school classroom, an incorrect answer will likely result in nothing more than a mark on a piece of paper, whereas in the workplace are real costs associated with such errors” (p. 23). On a building site involving large constructions, such costs may be human, as well as financial. Determining the appropriate choker size is not a task to be taken lightly.

**Conclusion**

Although it is recognised that work is a collective process, much of the instruction in the workplace-training classrooms (certainly those in which we worked) remains traditional in form and individual in focus. Some practical tasks are done collaboratively, but the kind of classroom described in this paper is an unusual one, having an organizational structure that requires group work and discussion. However, it is clear that such an approach to workplace learning can foster and stimulate powerful improvisational coaction, both around mathematical ideas and the way these are employed in the workplace. Such ways of working and thinking, as with Andy, Joe, and Mike, can generate a collective understanding that, as noted by FitzSimons, Micek, Hull, & Wright (2005), “has greater depth than any individual knowledge base” and through which a “communal memory” (p. 23) that also supports the idea that individual knowledge can emerge and be a powerful resource for learning and working. Groups that improvisationally coact not only have the capacity to work mathematically, and to solve problems, in ways that individually may not be possible, but also, through a process of continual verification and shared image making, to generate solutions that are less prone to incorrectness and error.
Acknowledgements

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References


Martin. “Some Guys Wouldn’t Use Three-Eights on Anything...”: Improvisational Coaction in an Apprenticeship Training Classroom.


A College-Level Foundational Mathematics Course: Evaluation, Challenges, and Future Directions

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Abstract

Recently in Ontario, Canada, the College Math Project brought to light startling data on the achievement of students in Ontario’s College of Applied Arts and Technology System related to their performance in first-year mathematics courses: one-third of the students had failed their first-year mathematics course or were at risk of not completing their program because of their performance in such a course. Here I present the results of an attempt to address the findings of the College Math Project. A foundational mathematics course, based on the JUMP Math program, was designed and implemented at a college in Toronto, Ontario. Although the students who took this program made appreciable gains in their achievement, it is difficult to assert its effectiveness over other programs because of the absence of studies profiling college math education practices either in Canada or internationally. The intention of this article is to help establish a datum for research into specific college math education programs.

Key words: college mathematics education.

Introduction

In 2007 a group of colleges in Ontario, Canada (Centennial, Humber, George Brown, Georgian, Seneca, and Sheridan) sponsored the College Mathematics Project (CMP), designed to gain insight into the mathematics achievement of first-year college students in Ontario. The CMP analyzed the records of over 10,000 students who had enrolled in a first-semester mathematics course at one of the participating colleges. The results of the project were startling: 34% of students received a grade of D or F in their first-semester mathematics courses, jeopardizing their progress (Assiri, Byers, Orpwood, Schollen, & Sinclair, 2008). The study was replicated in 2008 and 2009 including data from all Ontario colleges, with similar results (Assiri, Orpwood, Schollen, & Sinclair, 2009; Orpwood, Schollen, Marinelli-Henriques, & Assiri, 2010).

To conclude the 2007 CMP, the participating colleges came to a consensus on two points: that student achievement in first-semester mathematics courses in Ontario colleges needs to be significantly improved, and that the attainment of this goal requires concrete action by all stakeholders (Assiri et al., 2008). To address these points, the CMP colleges proposed a list of suggestions for action. The CMP recommends that “colleges and college faculty strengthen their commitment to student retention and success by adopting initiatives found effective in other colleges”, and that “the Government of Ontario adopt a ‘K-16’ vision of student success and continue to support research into the interface between levels”. The conclusions of the CMP 2008 and 2009 are of the same spirit as these (Assiri et al., 2009; Orpwood et. al., 2010).
At this point, it is important to clarify what a college is in the Canadian context. The community college systems in Canada are quite diverse and vary from province to province (Gallagher & Dennison, 1995). Throughout the present article I take 'college' to mean an Ontario College of Applied Arts and Technology.

The purpose of the present study is to report on an implementation of a college-level foundational mathematics course design intended to address the recommendations of the CMP. The course is based on the materials and methods of the JUMP Math program (Mighton, 2003; Mighton, 2008). JUMP Math, a charitable organization dedicated to increasing general numeracy, is in a unique position to answer the call of the CMP. Since its incorporation in 2001, JUMP Math has produced course materials for grades 1 to 8 which cover the entire Ontario mathematics curriculum and has developed a corresponding method of delivery and instruction. The JUMP Math approach has produced favourable anecdotal results in Vancouver, Canada, and London, England (“JUMP Math-Brock University Pilot Study”, 2005; “Lambeth Pilot Programme”, 2006). A more recent controlled-group study comprising 272 students from 29 classrooms across 18 schools in a rural Canadian school board found that the mathematics knowledge of those students assigned to the JUMP Math classes grew twice as much as those in the control group (Solomon, Martinussen, Dupuis, Gervan, Chaban, Tannock, & Ferguson, 2011). The school boards that have adopted JUMP Math methods indicate a significant improvement in their students' academic performance, a significant decrease in math anxiety, and an overall increase in the positive attitudes among students toward mathematics (Hughes, 2004; “Jump for Joy!”, 2004). As I will discuss, these latter factors can greatly affect a student's performance in mathematics.

In 2009, I was involved in implementing a JUMP Math curriculum in three separate courses at a college in Ontario. Initially, the study was designed to compare student gains over the term to historical data. This approach was abandoned, however, once the complexities of the college environment emerged. It is the goal of the current article to discuss these complexities so that future research may benefit.

**Background**

**The College Learning Environment**

Adapting a program intended for primary school students to the college learner is a challenging endeavour, in large part due to the overall insufficient understanding of the latter demographic (Coban, 2006; de Brouker & Myers, 2006). As will be discussed at the conclusion of this study, very little research has been preformed on college math learners, especially in the Canadian context. There is, however, a solid body of research on affects of the adult learner which we will take as relevant to this study. Among these, attitudes toward mathematics, specifically mathematics anxiety, appears to be a vital indicator of student success (Hembree, 1990; Ma, 1999; Tobias, 1978). Students with high math anxiety perform significantly lower on any evaluation of math ability than those comfortable with math. Ashcraft and Krause (2007) argued that math anxiety occupies the working memory leaving less devoted to the mathematical task. As the difficulty of a math problem increases, requiring more working memory, those with high anxiety perform increasingly worse than those with low or no anxiety. Clute (1984) recognized the significance of anxiety in student learning and studied the effects of two different instructional approaches on college math learners. Those students with high math anxiety benefited, in terms of higher achievement on a standardized test, from an approach based on explicit instruction, heavily reliant on tightly scaffolded lessons.

A correlation does not, however, appear to exist between math anxiety and ability in elementary mathematics. Ashcraft and Krause (2007) found essentially no difference between the scores of a group of highly math anxious undergraduate students and a group of low anxiety students on the elementary arithmetic questions on a standardized test. A difference did emerge...
as the questions became more difficult; the high-anxiety group answered fewer questions accurately than those with low anxiety. This characteristic impedes objective assessment of a learner's mathematical ability and may stem the progress of students who are capable, but who do not conform to certain assessment practices.

Other factors have been shown to affect demonstrable mathematical ability. Among these are self-concept, value, enjoyment, and motivation (Aiken, 1976; Bandalos, Yates, & Thordike-Christ, 1995; Csikszentmihalyi & Schiefele, 1995). Language ability has also been indicated as a predictor of success in math (Abedi & Lord, 2001) and may be an especially important variable in the college environment as many college students have low language ability.

The three courses involved in this study are 'foundational' or 'remedial'. That is, the students have, at some point in their prior education, been exposed to the course material. The students in these courses may have been classified as 'at risk' (AR) prior to enrolling in the college. Indeed, many of the students involved in the current research were classified as learning disabled (LD). There is a growing body of research on effective instruction for AR and LD adolescents which may be relevant to the current study. One method of instruction that has consistently provided strong results is explicit instruction (Kroesbergen & van Luit, 2003; Kroesbergen, van Luit, & Maas, 2004; Swanson, Carson, & Sachse-Lee, 1996; Swanson & Hoskin, 1998; Swanson & Hoskin, 2001) where a task is decomposed into small sub-tasks and extensive guidance and individual practice is offered.

In terms of course content, there is a tendency in college, and adult, education to present the students with 'real world' problems, even though these may not draw on the student's real-world experiences (Oughton, 2009). Emerging research, however, indicates that students may learn concepts more deeply and have a greater transference of the concept to novel situations if they learn from abstract instantiations of the concept. Kaminski, Sloutsky, and Heckler (2008) presented groups of undergraduate students with an abstract mathematical concept. One group received instruction via an abstract instantiation of the concept while other groups received instruction through concrete examples. The abstract group significantly outperformed the concrete groups in transferring the concept to a novel situation. In a further experiment, one group learned through the abstract representation and then were provided with a concrete example of the concept while another group were exposed to only the abstract representation. Again, the purely abstract group outperformed the abstract-concrete group. This suggests that students may benefit from an instructional approach that de-emphasizes concrete examples. This may be particularly relevant to the college environment since not only do language barriers appear to play a role in student success in mathematics (Abedi & Lord, 2001), such examples are situated with language that often may be colloquial or unfamiliar.

The JUMP Math program was specifically designed around all of the factors above. It is for this reason that JUMP Math was deemed appropriate for the college learner.

**A Primer on JUMP Math**

JUMP Math was founded in the 1990's by John Mighton, mathematician and playwright, as a tutoring system for children (Mighton, 2003; Mighton, 2007). Since that time the organization has grown significantly and currently produces workbooks that correspond to the K-8 curricula across Canada. The JUMP Math method is geared toward reducing students' anxiety toward mathematics. This is accomplished by guiding the students through a series of problems, all relating to one topic and predominately abstract in nature, that become only slightly more difficult from one to the next. As Mighton (2007) writes, “Children become very excited when they succeed in meeting a series of graduated challenges, and this excitement allows them to focus and take risks in their work.” For an example of a graduated challenge, Mighton (2007) recounts his experience with Matthew, an autistic child with high levels of math anxiety:
...‘Matthew, you’re very smart. Could you add these fractions?’ and I wrote: \(\frac{1}{17} + \frac{1}{17}\). When he had written the answer, \(\frac{2}{17}\), I said, “You’re amazing! Could you add these?” [and I wrote] \(\frac{1}{39} + \frac{1}{39}\). When he had answered that question, I said, “You’re in big trouble now. I’ll have to give you these.” [writing] \(\frac{1}{73} + \frac{1}{73}\). As I continued to increase the size of the denominators, Matthew became more and more excited. After he had successfully added a pair of fractions with denominators in the hundreds, he was beside himself. (Mighton, 2007, p. 16)

Although Matthew was a learner with exceptionalities, this lesson exemplifies the JUMP Math method. The questions provided to Matthew are all identical in terms of their mathematical content, but they do differ in difficulty from Matthew’s perspective. Students relate difficulty of an arithmetic question with the size of the numbers (Ashcraft & Krause, 2007). By increasing the size of the numbers without altering the question, the tutor was able assist Matthew in overcoming his fear of numbers through helping him realize that number size does not affect the operation.

The primary component of the JUMP Math course is the printed workbooks, a main feature of which is their pared-down style. Lessons are presented succinctly, using few words and extraneous pictures. The light-on-language approach should assist the college learner with lower language ability (Abedi & Lord, 2001; Ciancone, 1996). Also, there is evidence that excessive diagrams and pictures in textbooks can serve as a distractor to students (DeLoache, 2005).

JUMP Math appears to be a good fit with what the literature identifies as effective instruction for AR and LD students, which comprises the majority of the student demographic included in this study. The JUMP Math method of instruction may be classified as explicit instruction following (Swanson and Hoskin, 2001). The incremental steps in the lessons, coupled with copious feedback and extensive practice, are intended to reduce anxiety and increase self-concept in and enjoyment of mathematics. It is for these reasons that JUMP Math was chosen for the college setting. Previous studies on JUMP Math have provided favourable results (Hughes, 2004; “Jump for Joy!”, 2004; “JUMP Math-Brock University Pilot Study”, 2005; “Lambeth Pilot Programme”, 2006) which lends support to the use of JUMP Math. In addition, the material covered in the college foundational courses corresponds to the Ontario grade 8 curriculum, which is covered entirely by the corresponding JUMP Math material.

A Description of the Courses

Three classes were included in this study, one first-year foundational course in a General Arts and Science program (GAS), one in the traditional College Vocational program (CV+), and one in the College Vocational program from lower-achieving students (CV). The GAS course is for students enrolled in a liberal arts program. Students who receive a grade of less than 50% on a computer-based placement exam are required to enrol in the GAS course. Students are also able to place themselves into the course if they feel unprepared for college study. The CV and CV+ courses are a part of a program designed to prepare for meaningful employment students who are not typically considered for college admission.

All three courses cover basic number facts and operations—addition, subtraction, multiplication, and division—with rational numbers, including positive and negative integers and fractions; linear algebraic equations; the order of operations—brackets, exponents, division, multiplication, addition, and subtraction; ratio, proportion, and percent; basic graphical techniques; and, basic manipulation of equations containing variables. The level of difficulty of the GAS course is comparable to that of the Ontario grade eight curriculum while the CV and CV+ course are at a grade six level.
Instructors volunteered for the course and underwent a three-day training prior to the start of the course with a refresher training mid-term. The training session consisted of two main components: a description of the student affects of which an instructor should be cognisant, and the pragmatic implementation of a JUMP Math lesson. In addition, instructors received teacher resources from JUMP Math, including a teacher's guide comprising detailed lesson plans.

Methodology

As outlined above, a list of factors was employed to evaluate the course: change in student's technical math ability; change in student's attitudes toward math; student's enjoyment of, and general comments about, the course; the instructor's perceptions of how well the students performed, in terms of their engagement and mastery of concepts, relative to previous years; and instructor's feelings and general comments about the course. The first two are quantitative in nature and were measured with two tests, one on basic math concepts and operations covered in the course, and one designed to measure the student's attitudes toward mathematics, both offered once at the start and once at the conclusion of the course. The math test was the Canadian Adult Achievement Test, a standardized test. The second was the Mathematics Attitude Inventory (MAI) (Sandman, 1980).

The MAI was administered to gauge the student's attitudes toward math. The test is designed to measure six constructs of attitudes toward mathematics: (a) Perception of the mathematics teacher; (b) anxiety toward mathematics; (c) value of mathematics in society; (d) self-concept in mathematics; (e) enjoyment of mathematics; and, (f) motivation in mathematics (Sandman, 1980). The first construct was worded to refer to prior instructors for the start of the term and for the current instructor at the end. As outlined in the introduction, all these factors may influence a student's performance in mathematics.

Before the conclusion of the course, an anonymous survey was distributed to the students to record their thoughts about the course. The students were asked to respond to the questions: Compare this course to other math courses you have taken. (Was it too easy/difficult? Did you have enough time for practice?) What was the best thing about this course? What did you not like? What did you like about the JUMP Math workbooks? What did you not like about the JUMP Math workbooks? What helped you learn the most in this class? How could this course be improved? How long has it been since you were in school last? What was the highest level of math you studied before coming to [the college]? Before taking this class, how would you rate your math ability (option to circle 'Advanced', 'Average', or 'Low')? After taking the class, how would you rate your math ability (option to circle 'Advanced', 'Average', or 'Low')? The first several questions are designed to gain insight on what improvements should be made in future offerings of the course. The remaining questions attempted to elaborate on who the students are.

After the conclusion of the course the instructors shared their experiences and thoughts on the courses.

Results

The results of the project are mixed. The CV+ students exhibited the most marked improvements while the CV had more modest gains. Most interestingly the GAS instructor decided to discontinue the exclusive use of the JUMP Math materials and methods in their course. An analysis of this decision is present in the discussion section.

I begin with the results of the MAI. Three factors, perception of the mathematics teacher, self-concept in mathematics, and motivation in mathematics, showed appreciable increases; one, anxiety toward mathematics, a slight increase; one, enjoyment of mathematics, remained effectively unchanged in the CV+ course and decreased in the CV course; and one,
value of mathematics in society, decreased (Table 1). This is the first time the MAI was used in this setting and no previous data are available which makes interpretation difficult. At most we can observe change but say nothing about the degree of change. An increase in the values for factors 1, 3, 4, 5, and 6, and a decrease in the factor 2 value, is favourable. This is not what was observed. Anxiety toward mathematics appears to have increased over the duration of the course. However, it is not known exactly when the follow-up MAI was taken; it may have been offered after the instructor mentioned an anxiety-inducing event (e.g., a test). The enjoyment of mathematics factor may be heavily correlated with the individual instructor teaching style and the value of mathematics in society factor may correspond to the nature of the course curriculum. Expanded use of the MAI will provide benchmark data that can be used to evaluate any one course.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Start GAS</th>
<th>CV+</th>
<th>CV</th>
<th>End GAS</th>
<th>CV+</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perception of the Mathematics Teacher</td>
<td>19.32</td>
<td>20.02</td>
<td>22.97</td>
<td>N/A</td>
<td>27.29</td>
<td>23.90</td>
</tr>
<tr>
<td>Anxiety toward Mathematics</td>
<td>8.39</td>
<td>8.15</td>
<td>9.21</td>
<td>N/A</td>
<td>8.64</td>
<td>9.72</td>
</tr>
<tr>
<td>Value of Mathematics in Society</td>
<td>20.57</td>
<td>22.76</td>
<td>22.05</td>
<td>N/A</td>
<td>21.47</td>
<td>20.65</td>
</tr>
<tr>
<td>Self-concept in Mathematics</td>
<td>13.90</td>
<td>13.83</td>
<td>16.63</td>
<td>N/A</td>
<td>16.03</td>
<td>17.14</td>
</tr>
<tr>
<td>Enjoyment of Mathematics</td>
<td>21.00</td>
<td>22.04</td>
<td>23.34</td>
<td>N/A</td>
<td>21.99</td>
<td>21.65</td>
</tr>
<tr>
<td>Motivation in Mathematics</td>
<td>7.97</td>
<td>8.96</td>
<td>9.49</td>
<td>N/A</td>
<td>9.82</td>
<td>10.53</td>
</tr>
</tbody>
</table>

*Table 1. Mathematics Attitudes Inventory results.*

The results of the CAAT tests are in Table 2. The numbers are average primary/secondary school grade level equivalents and the 'PS' stands for 'post-secondary'. Both CV and CV+ courses demonstrate gains in student achievement. Again, historical data are not available and the values can only be taken to indicate change. The GAS students tested, on average, at a post-secondary level of mathematical ability at the start of the term, well beyond the intended level of the course. This is certainly a factor that influenced the GAS instructor's decision to discontinue the exclusive use of the JUMP Math approach.

<table>
<thead>
<tr>
<th>Beginning</th>
<th>End</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV+</td>
<td>6.47</td>
<td>7.46</td>
</tr>
<tr>
<td>CV</td>
<td>5.22</td>
<td>5.68</td>
</tr>
<tr>
<td>GAS</td>
<td>PS</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Table 2. Average CAAT grade level equivalency scores for the CV and GAS courses.*

**Survey Responses for CV and CV+**

The survey results are quite promising. First, consider the CV+ course. The question what did you like about the JUMP Math workbooks? largely received positive comments. Responses to the follow-up what did you not like about the JUMP Math workbooks? were even more positive: two-thirds were 'N/A' or 'nothing'. Other positive support for JUMP Math includes 'more JUMP math books' in response to how can this course be improved? Responses to the question what did you not like [about the course]? were largely about math in general and nothing specific about the course.

For the CV course, responses to all of the survey questions are mixed. For example, what is the best thing about this course? received 'there was nothing I liked about this math course it was the same math work I did in high school' and 'I think this course is very good'.
And the follow-up question what did you not like? received 'everything was too [sic] easy and boring and we kept on doing the same work for half the semester' and 'I pretty much liked everything'.

A peculiarity emerges when contrasting the responses in the two CV courses. Many of the CV+ students indicate that the course was difficult, while many CV students stress that it was easy. This is difficult to interpret since both courses used the same workbooks, yet the CV students at the start of the course performed at more than a grade level lower than those in CV+ and gained only half a grade level on average by the conclusion of the semester.

Despite the generally positive outcomes observed in the two CV implementations, there are issues that need to be addressed. As we will see, class heterogeneity was a major issue in the GAS implementation of JUMP Math materials. It is an issue here, too, but to a lesser extent. Although a few students with higher ability may not be a concern in the CV+ course, it may be in the CV program, since the students self-selected for enrolment. Based on the CAAT scores, there appears to be at least one student who is well beyond the material being presented. This is most apparent in the responses to the survey questions. Some are quite articulate while others are marked with misspellings, letter reversal, and low quality penmanship. This may indicate that there are some fundamental literacy and cognitive issues that must be addressed before, or concurrent with, math instruction.

Survey Responses for GAS

In stark contrast to the success of JUMP Math in the CV program, the response to the JUMP Math General Arts and Science (GAS) implementation was mixed, tending to a rejection of JUMP Math. The instructor of the GAS course felt that the methods and materials were not entirely appropriate and decided to abandon JUMP Math mid-semester. Despite this, the results should not be viewed as negative. Much was learned from this experience that will be able to guide the GAS Foundational Math program much more effectively, I believe, than if positive effects were observed. An analysis is called for.

In consultation with the GAS instructor, three factors were identified as contributing to the halting of the JUMP Math implementation: heterogeneity of the student body, the inappropriateness of the JUMP Math materials and methods, and attendance. I consider each of these in turn.

The first is the biggest factor. Severely mathematically deficient students were grouped into the GAS class alongside those deemed only slightly deficient, if deficient at all. The diversity of ability became apparent in the student's attitudes toward the class. The instructor conducted an informal survey mid-semester. The responses are as follows, grouped according to perceived ability of the student:

Intermediate to Advanced Students:
- I like the instructor’s step-by-step explanations. Now I remember it. Let’s move on.
- When am I going to learn something new? What about algebra and graphing? Not blaming instructor, just course content.
- This stuff is too easy!
- I got 80s in math in high school. Why are we covering such basic material?
- I feel like I’m in Grade 6.

Weaker Students:
- I like the JUMP math exercises in the workbooks, especially the fractions unit.
- I’m understanding math better than before.
- I’m glad I’m taking this level; it provides a good review of material I already know but forgot because I was away from math for a long time.
Heterogeneity is implicated in student survey responses as well. Throughout the survey, 'move quicker' and 'advance more' responses indicate that the entire class is not able to progress as a unit.

The second factor encompasses two concerns. First, the JUMP Math workbooks were found to be excessive. This resonates with some of the students: 'very repetitive' and 'I didn't like the repeating of the questions' were common survey responses. However, one student indicated that 'there was nothing I did not like' about the JUMP Math workbooks and, in response to "how can this course be improved?", wrote 'do more work in the JUMP workbooks'. Second, the workbooks did not align entirely with the course syllabus. As will be discussed, there is often a tension between the course outcomes and what an instructor can deliver effectively.

Attendance was low and marked with tardy arrivals into class—a rampant problem across the college campus. This, however, may be attributed to the early class time, 8:00 A.M.—a factor mentioned in the student surveys.

The course offering was not entirely unsuccessful, as is indicated in the year-end surveys. Some of the responses are worth analysing. Consider the question “what did you like about the JUMP Math workbooks?” Although one student responded with 'I didn't really like them, they made me feel like I was in grade 2', others wrote 'they were helpful' and 'useful', and that they 'laid everything out well'. I interpret these mixed responses as indicating that the materials are not far off base and that refining them may make materials appropriate for this type of course.

Discussion

This study was designed to evaluate the effectiveness of a JUMP Math-based college-level foundational mathematics course. During the study, however, more was learned about the college math environment than about the program itself.

Despite potential limitations, this study represents a step toward addressing the recommendations of the College Mathematics Project. Indeed, this study could be viewed as more than pragmatic; it is attempting to break ground by addressing a demographic of students in Canada that have been neglected and allowed to fall by the wayside. Very little research, if any at all, has been preformed on the college math learner in Canada. A cursory search through the major channels for education research in Canada (ERIC, College Quarterly, Canadian Journal of Education, Google Scholar, Canadian Journal of Higher Education) reveals only few articles on the Canadian college demographic. Much more research has been conducted in the United States. Also, there has been increasing activity recently in the field of Adult Numeracy (Condelli, 2006), a domain that has significant overlap with college math education. A few international organizations exist that promote research in adult numeracy and mathematics education; of note, Adults Learning Mathematics (ALM) has emerged as a preeminent research forum on the issues of adult numeracy—the group publishes a bi-annual journal and hosts an annual conference. Despite the interest in, and the appearance of, a well-established body of research on adult numeracy, very little focus has been placed on pedagogical practices (Tout & Schmidt, 2002). A report by the National Research and Development Centre for Adult Literacy and Numeracy in the United Kingdom sought to address the questions “what is known from research about effective pedagogy?” and “what factors in teaching cause adult learners to make progress in adult literacy and numeracy?” The report analysed over 4,500 publications and concluded that “there were very few studies that provided quantitative evidence to answer these questions,” (Brooks et al., 2004). Adult numeracy, however, seems too broad a term and may encompass too diverse a demographic for the purposes of this study or future studies in this vein. It is evident that further research is needed not only in the field of college math education, but on the connections of this field to adult numeracy.
It is not clear how well the research that has been performed on the college math learner outside of Canada transfers to the Canadian context. For example, the term “college” has a very different meaning in the United States than in Canada, and within Canada “college” varies from province to province (Gallagher and Dennison, 1995). Initial research should be performed to explore the Canadian college mathematics environment—Who are the students? What are their backgrounds, abilities, and aspirations? What mathematical abilities do employers expect of college graduates? Who are the instructors? —and how this relates to those in other countries.

Data was gathered during the course of this study for both the Mathematics Attitudes Inventory and the Canadian Adult Achievement Test. Without more data from a larger set of students, the numbers present in the data have no meaning; at most a change in the values can be observed. I suggest gathering data on a larger scale so that instructional practices can be gauged against a benchmark. Adopting such an approach can help college educators evaluate what gains are being made. It may also help evaluate the some of the claims made in the CMP 2010, specifically, “Most students learn best when mathematics is embedded in the context of a practical field of interest to them,” (Orpwood et al., 2010).

Perhaps the largest confounding variable in the GAS course offering of JUMP Math was the skill heterogeneity present in the class. Many capable students where present in the class alongside those that need extensive foundational work. Although some students opted to enrol in this course, most were placed by a computer-based skills assessment. Recognizing that all assessments have inherent limitations, this type of assessment may be particularly inappropriate since it may unintentionally use the affects of the examinees against them. As van Groenestijn (2001) states, “Criteria for placement tests on math skills of adult basic education students are needed to develop tests that are not too 'school-like' because the ABE students are often blocked by math anxiety due to past negative school experiences, students may encounter language problems that affect their math skills, simple math problems do not measure practical problem-solving skills, and a placement test having only right and wrong answers does not provide insight into mathematical procedures of adults.” Use of the computer-based assessment should be re-evaluated considering the majority of the students in the GAS course—a course intended to correspond to the first years of the provincial high-school curriculum—were independent determined to be at a post-secondary level in their mathematics ability by CAAT. This heterogeneity creates tension for the instructor who must simultaneously satisfy the college-mandated curriculum and maintain the students' interest.

The GAS course was offered twice a week at 8:00 A.M. The instructor indicated that student attendance was highly variable but often low. He mentioned that this is understandable, considering who the students were as individuals. Many had great responsibilities outside of college, with families and jobs, say, and a foundational math course, however necessary the college viewed it, was not at the top of their priorities. The instructor felt that the foundational students often have failed or have a poor record of achievement in math and that a foundational course only serves to remind them of their failures. Being sensitive to who the students are appears to be a major point to consider when offering a foundational math course (Miller, Pope, & Steinmann, 2004).

Even if this study established the JUMP Math college course as an effective system of instruction, it would remain to show how effective it is relative to common instructional practices. I feel, however, that such research is not ready to be performed. No commonalities exist within college math curriculum, instructional and assessment practices, textbook choice, and course workload. In addition, other factors can vary wildly from instructor to instructor. An evaluation of what constitutes common practice in college math education is in order. It is also time to take stock of college remedial courses. There is no consensus in the literature as to the efficacy of remedial course work. Horn, McCoy, Campbell, and Brock (2009) found that placement in a foundational-level English course negatively impacted a student's future success,
while Waycaster (2001) found that a significant proportion of students who successfully graduate from community college have taken a developmental or remedial mathematics course. Perhaps the largest issue that this study raises is that of the dearth of research in this domain. Throughout the study, it was necessary to draw on research from other demographics or domains that may turn out to be tenuously relevant to the demographic studied. Concrete, foundational research must be performed if we are ever to introduce research-based practice into our college math classrooms.

References


APPENDIX: A SAMPLE JUMP MATH LESSON

NS8-25: Multiplying Fractions by Fractions

Figure 1 presents a sample of the JUMP Math material used in the college foundational courses. This lesson is part of a series on learning basic operations with fractions. A key feature of this lesson is worth highlighting. The lesson focuses on only one concept with very gradual incremental progress in difficulty. This is a hallmark of the JUMP math approach; even the most seemingly “obvious” steps are not assumed by the instructor to be assimilated by the student. This fine-grained approach allows the instructor to assess where any difficulties reside.

Figure 1. A sample JUMP Math lesson.
Mathematics in Masons’ Workplace

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Abstract

This paper presents masons’ professional practices, which are related to mathematics. It aims to contribute to the area of adult mathematics education and to enlarge knowledge about how mathematics is used at the workplace. Methodologically it was followed an ethnographic approach. The key informants of the study were four masons aged between 40 and 60 years old. Observations and interviews were carried out at the workplace in a civil construction setting in the Lisbon area. Firstly we present masons’ views about schooling and mathematics, as well as the importance that masons confer to mathematics in their profession. Then the paper describes and discusses three episodes observed at masons’ workplace in their professional practices. As a whole these episodes illustrate how mathematical knowledge is imbedded in professional activities. Independently of schooling, masons daily and implicitly apply mathematics. It is this practical knowledge of mathematics that, after being uncovered, is for us educators of most relevance to adult-learning mathematics contexts, because not only does it connect school content and curricula to labour-market necessities, but it also makes use of adult learning experiences to support new mathematical learning.

Key words: mathematics education; adults education; masons; professional practices; mathematics in workplaces; ethnomathematics; community of practices.

Introduction

Training has been recognized as a principal mean to achieve professional development and with it to promote better social conditions. Moreover the results of research have been contributing to news ways of thinking about professional training and to recognize as indispensable in future training both the educational value of the work environment and adults’ knowledge acquired through work and experience.
In addition, to the extent that alternative forms of learning processes of knowledge acquisition are at the core of adult education’s pedagogy it is important to deepen the understanding of techniques and skills that adults seed in their professional context and how they cope with mathematics knowledge and take advantage of it to face new problems and situation. As Wedge & Evens (2006, p.30) point out:

The subject area of adult education encompasses formal adult mathematics education as well as adults’ informal mathematics learning in the communities of everyday practice, for example in the workplace.

Workplaces are, thus, part of the adults’ learning experience and present rich contexts to educational investigation. Moreover, from the viewpoint of mathematics education, professional knowledge as well as cultural mathematics are nowadays both considered as sources of wisdom and inspiration in designing curricular activities and seen as important components of the pedagogy and didactics of the field. In addition, mathematics education can really determine a person’s success in professional careers and job opportunities.

The research presented in this paper addresses the issue of masons’ professional practices that involve mathematical knowledge. Masons, sometimes illiterate and mostly with very little formal education, build houses, and monuments that stand for centuries. In their professional activity they do calculations and reasoning, and use mathematics daily. It is this practical knowledge of mathematics that, after being uncovered, is for us educators of most relevance to adult-learning mathematics, because it might be used in of adult learning experiences to support new mathematical learning. As educational researchers we need to understand workplaces as contexts of professional practices where implicit mathematical knowledge is performed, because they help to deepen our understanding of how mathematics is used and thought in practical applications required by specific work practices. The investigation of masons’ activities, where mathematical reasoning and tools processes are present gives the researcher insight and wisdom to understand the dynamics of the relationships between the knowledge applied by masons in their workplace and mathematical knowledge. Thus, the investigation presented in this paper collects and describes situations observed in masons’ workplaces that illustrate the use of implicit as well as explicit mathematics. In particular, it addresses the following questions:

- In what ways do masons use mathematics in their professional activity?
- What kind of relationship exists between that professional use and formal mathematics instruction?
- What are the mathematical concepts that masons use in their professional practices?
- How and where did masons learn the mathematics that they use?

This paper aims to contribute to the area of adult mathematics education, and to enlarge knowledge about how mathematics is used at the workplace. For us, the relevance of studying masons’ professional knowledge is to put it to good curricular use in the context of mathematics education either to design general courses in adults’ mathematics education or to think about specific curricular vocational courses.

In addition, by investigating what masons know in practice, we intend to explore their mathematical reasoning and calculations to enlarge, strengthen, and develop their mathematical competencies. Simultaneously, with this study we hope to highlight, on the one hand, the value of masons’ work, with its professional specificities, practices, social prestige, and role in society, and on the other hand, the universal component of work, its role and place in society, and its intrinsic meaning for most social groups.
Theoretical Background

The theoretical framework that guides this research derives from three major fields: ethnomathematics, adults’ education and the concept of “community of practices” and its relationship to learning. Next we address which one of these fields.

Adults’ Education

Adults Education is a complex subject. If in one hand the term 'adult' varies both in terms of defining age, and according to social representations, on the other hand, "adult education" is also a polysemic term that encompasses a myriad of different situations as adults are illiterate, with little formal education, with a school trajectory marked by failure and also the situations of life-long learning that increasingly tend to be part of adulthood. Thus, education and training needs in the adult world are hugely variable and dependent on each adult specific circumstances, prior training and education needs.

The field of research of Adults Education has been asserting especially after the World War II. A milestone in the development of the field was the report submitted by Gerald Bogard, in 1991, following a research project commissioned by the Council of Europe. It is not our intention to examine the report here; it is nevertheless necessary to say that it presents an innovative perspective on adult education based on both the significance of cultural learning and the value of transversal competencies. Moreover this report highlights three key aspects in the field of adult education, which are the following:

i) The place and time of education, that discusses it as a long term process of socialization and therefore articulates pedagogy with both an institutional and social field

ii) The appreciation of the uniqueness of the educational process that takes in consideration the diversity of adults’ knowledge and the different ways of how it was acquired which is hardly harmonized with the compartmentalized situations of school subjects.

iii) The acting role of the learner within his/her pedagogical relationship (Canario, 2008, pp. 22-26)

Although there are several trends in the area of adults education, the trend that is for us most significant is the one that highlights "the development of training integrated in the work actions" (Canario, 2008, p. 29). This trend is based on the idea that "the exercise of the work is, in itself a producer of competencies" (Canario, 2008, p. 30). That is, research has pointed out how, when the process of adult education is integrated with work actions, the valorisation of the “human factor” as a set of non-technical skills allows workers to know the overall process of production where he/she is inserted (Berger, 1991; Bogard, 1991). In addition the developed competencies have a much more complex nature, in the sense that they are not uni-disciplinary, but holistic and polyvalent and in addition it helps to develop the professional identity.

Ethnomathematics

Ethnomathematics is considered by D’Ambrósio (2002, p.9) as:

…that mathematics which is performed by cultural groups such as urban and rural communities, groups of workers, professional groups, children within a certain age,
indigenous societies, and many other groups that identify themselves by common goals and traditions.

Thus, Ethnomathematics not only provides a valuable framework for researching mathematical activity from all over the world, but also offers the possibility to study and analyse the mathematics of professional groups such as masons. Moreover, it also affords for discussing and reflecting upon its findings and relating them to educational practices and aims. In fact, since the 1970s, research findings from the field of ethnomathematics have demonstrated (1) that different professional or cultural groups possess particular ways of approaching mathematics, (2) the social, cultural and political nature of the variables and processes involved in mathematics education, and (3) the complexity of the articulation between mathematical knowledge based in primary culture and that promoted by schools, highlighting the dissociation of formal mathematics education from daily life (Moreira, 2007).

Thus, to understand professional practices that involved mathematical knowledge provide insightful contexts that contribute to develop new perspectives about social and cultural aspects implicated in the mathematical learning processes. Moreover they reveal different ways in which the use of implicit mathematics is connected with its explicit use. In fact, since the seminal works of Carreher, & Schliemann (1993) with child vendors on the Brazilian streets who daily performed complicated mathematical calculations and the one of Abreu (1998) on the methods of production of cane sugar that shows the existence of proper mathematical procedures to measure, several studies have been focused in the mathematics practised by professional groups. In Portugal, Fernandes (2004) conducted a study focused in the mathematics of locksmiths; Costa, Nascimento & Catarino (2006) developed a study about the mathematical practices used in two traditional jobs tanoaria (barrel maker) and latoaria (tin worker), and Pardal (2008) conduct a research on masons’ professional practices. In Brazil, mathematical practices of civil construction were investigated by Duarte (2003). In addition, the theoretical grounds of Ethnomathematics has been used to understand the mathematics of young adults (Fantinato, 2003)

Community of practices

Lave & Wenger (1991/1997) understand learning as intimately related to the notion of belonging to the group where it takes place. This is, membership in a community of knowledge and practice implies the attainment of the necessary knowledge attached to this community through a process of legitimate peripheral participation that enables newcomers to become members with full participation in the community. As Lave and Wenger state “learning is an integral and inseparable aspect of social practice” (1991/1997, p. 31). Thus, both the notion of belonging and identity are linked with the process of learning inside the group. As these authors argue:

Viewing learning as legitimate peripheral participation means that learning is not merely a condition for membership, but is itself an evolving form of membership. We conceive of identities as long-term, living relations between persons and their place and participation in communities of practice. Thus identity, knowing, and social membership entail one another. (Lave, & Wenger, 1991/1997, p. 53)

Moreover learning, as a component of a social practice, involves the whole person and the activity that she/he is working on – tasks, functions and comprehension do not exist separately. As the same authors point out:
Learning thus implies becoming a different person with respect to the possibilities enabled by these systems of relations. To ignore this aspect of learning is to overlook the fact that learning involves the construction of identities. (1991, p.53)

“Communities of practice” (Lave & Wenger, 1991/1997; Wenger, 1998) is an important concept to understand how mathematics is present in the workplace. As communities which gather people informally based in their interest in learning and doing a common activity, although each person might have different responsibilities and functions, community of practices are not only a set of people with common interests but also a set of persons that learn, construct and manage knowledge (Wenger, 1998).

Lave & Wenger’s (1991) viewpoint allows for the concept of community of practice as socially permeable, in the sense that the community accepts among its members different interests and knowledge in regard to the performed activity. As such, it not only includes a set of persons who develop a specific relationship with the culture of the group, namely because they have common goals and needs, which in order to be achieved involves the development of certain practices, but also entails “a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice” (p. 98). It is the relationship among people, their organization, and the knowledge that they produce as well as their interactions with other communities that, as a whole, constitute the community of practice.

Wenger (1998) emphasizes what he terms the three dimensions of a community of practice. They are: a mutual engagement, a joint enterprise and a shared repertoire. Practices unfold themselves in a social world where interest, powers and status are present. As Wenger states:

A community of practice is neither a haven of togetherness nor an island of intimacy insulated from political and social relations. Disagreement, challenges, and competition can be forms of participation. (1998, p. 77)

Being shared, a social practice ends by connecting people in diverse and complex forms that constitute themselves as participants in the community of practice. In order to achieve community coherence, Wenger proposes, the people in question engage in the “negotiation of a joint enterprise” (Wenger, 1998, p. 77).

**Methodology**

One special characteristic of ethnographic research is the long-lasting presence of the researcher at the place of investigation, to create familiarity with the “native” culture and to develop interpersonal relationships with locals. This life experience allows the researcher to move toward the comprehension of the other. In addition ethnography has been pointed out as an appropriated methodology to be used in research that aims to understand mathematics from the participants’ viewpoint (Barton, 1997) and specially to be aware of the role of the participants’ workplaces in the production of mathematical meanings because of its suitability for getting proximity to the context of everyday operations (Zevenbergen, 2000). Thus, to better comprehend how mathematics emerged in masons’ professional practices we decided to use an ethnographic approach to be acquainted with activities, behaviours, perspectives and masons’ daily routines.

The ethnographic research demanded our continuing attention as researchers playing the role of the participant observer, with daily proximity and direct involvement in the masons’ social
setting. We have to interact with them, namely to slow down their practices in order to bring into speech their realities that were intertwined with mathematics. As Bourdieu pointed out,

The relationship between informant and anthropologist is somewhat analogous to a pedagogical relationship, in which the master must bring to the state of explicitness, for the purposes of transmission, the unconscious schemes of his practice”. (1977, p. 18)

We were very consciousness about the fact that if we failed by not creating a propitious ambiance to elicitation, masons would not trying to bring into speech their explanations and we would not experience the practices and its dynamics, and get as close as possible to their rational. In other words we strongly felt how “the ethnographer is the ultimate instrument of fieldwork (Heath & Street, 2008, p. 57).

Data was collected from September 2006 to July 2007 in a civil construction setting located in the Lisbon area. Usually we visit the setting two times a week. There were eight full-time masons at the jobsite.

Because we were acquainted with the owner of the building company, it was easy to get permission to carry out participant observation and to interact with the masons during work time. Before the beginning of fieldwork we first spoke with masons openly about the research aims and processes and about the possibility of having to tape some conversations. The key informants were selected mainly for their predisposition to collaborate, as well as their ability to communicate with the researchers. Another important criterion was to have at least five years’ experience in the trade, to ensure that they knew their profession in detail. Finally four male masons between forty and sixty years old were chosen.

We started by making informal visits to the setting and explaining our research goals to the masons. Our first aim was to meet the masons and get to know their daily routines. Later we focused our conversations on the most common routines that we, as researchers in mathematics education, recognized as involving mathematics. With this focus in mind we encouraged our key informants to talk about their professional procedures.

In this way we recorded information about several episodes that show how mathematics is embedded in masons’ professional practices. The observations and interactions were done in the professional context, highlighting mathematical processes and ideas in a contextualized and interrelated way with professional activities.

Semi-structured interviews were also conducted to learn about the masons’ school experience and professional trajectories, as well as their personal thoughts about mathematics. All the interviews were done in the professional setting according to the masons’ availability and in accord with their job routines. With our informants’ agreement the interviews were taped.

Participants’ perspectives on schooling and mathematics

The data collected in this study shows that masons did not go through any kind of formal education to learn their profession. In a process of apprenticeship, newcomers in this kind of profession learn in the community of practice of their professional jobsite: the older and more knowledgeable masons demonstrate how to do the job to the newcomers, who learn by observation and imitation, starting by executing the less complex tasks. The key informants in this research have different levels of formal education- two of them completed the 6th grade, one the 4th grade and other 3th grade all of them dropped out of school during basic education (1st to 9th grade) either because their family economic status did not allow them to continue schooling, or because they wanted to be independent from their families.

The four share a similar conception of mathematics. That is, they all assert the importance of mathematics for their professional practices as well as the great importance of
mathematics for life, but they separate the mathematics that they learned in school from the mathematics they use in their professional community. More specifically, they consider school mathematics to be more difficult and “higher status” than the mathematics that they apply day-by-day.

We present our key-informants: Mario, Quim, Joao e Antonio and their views on schooling and mathematics.

Mario is 45 years old and has been a mason for thirty-one years. When he was a little boy his dream was to be a physician. However, because of his family’s economic needs, early in his life he needed to look for a job. He finished his formal education in 6th grade. When he started his apprenticeship as a mason, his work was his great support. As Mario says:

I was a little boy and sometimes I felt like a fish out of water. If not for some of my companions, I would not have learned to enjoy my profession.

Mario considers school mathematics essential to his profession, but asserts that experience is even more important. Moreover, he emphasizes that he applies mathematics even without being aware of it. As Mario reports:

I was never good in mathematics. I only knew how to do the basics, and to do divisions I need a calculator. My teacher used to say that I was a “donkey” in mathematics.

Quim, 42 years old, decided to drop out of school in 6th grade against his family’s will. Quim wanted to be independent from his family, at least economically. It was a friend who spoke to him about the possibility of being a mason. In Quim’s own words:

In the beginning it was not easy at all. To be an auxiliary mason is even harder than to be a mason. You have to do the hardest tasks, like carry heavy containers full of concrete.

In regard to mathematics he affirms the importance of school mathematics, although he tells us that in his profession he does calculations in a more practical and unconscious way, as he learned from his master masons. He highlights that a mason needs to know how to calculate proportions, percentages, areas and volumes, to measure angles, etc. As he says:

The mathematics that I learned in school is much more technical and harder to apply than the mathematics that we use daily in building. The latter is so easy that we even forget that we are applying it.

Joao is 52 years old and dropped out of school in 4th grade because his mother passed way and he needed to help his family. He remembered that it was his mother who both used to instigate him to move on in school and to help him with homework. Joao started to work in civil construction when he was 13 years old. He started as an auxiliary mason auxiliary and learnt at the workplace, in the construction building itself. It was the elder and more knowledgeable masons who taught Joao the techniques of his profession and how to use them. As Joao said:

It was not difficult to learn. I’m a smart guy and when it was possible I tried to do things
by myself.

With 16th years old Joao already dominated the techniques and possessed the necessary basic skills to move on from auxiliary mason to a full mason.

In regard to mathematics Joao considered that what he learnt during basic education was important, although experience and observation of elder masons at work were most significant. Joao agrees that mathematics is crucial in his profession and he highlights that a mason needs not only to know basic arithmetic but also how to calculate areas, perimeters, how to read, apply and compute scales, how to estimate quantities, for example of blocs and to work out budgets. However Joao highlighted that such activities are now routines for him and thus he does not remember that he are doing mathematics.

Antonio is our elder key-informant and he has been a mason for 46 years. He finished 3th grade long time ago and dropped out of school because as he belonged to a large family it was necessary to care for his brothers and later on to have an income. It was Antonio’s father who suggested him to work in civil construction. The fact that he has only 3th grade does not restrain him to apply mathematical calculations and even to teach the newcomers. Antonio says that school was not necessary to learn mathematics because his life was his school. As he says, joking:

I deal with numbers even with my eyes closed.

Antonio points out that in his profession mathematics is fundamental although its learning does not require attending school because one can learn in daily situations. In fact he uses several mathematical processes and performs mental calculations to solve problems, saying with proud that:

My calculator machine is my head. And the results are always correct.

Mathematical Episodes

Throughout the fieldwork, distinct mathematical content easily emerges in masons’ professional contexts, clearly showing possibilities of working out mathematics in connection to masons’ real work needs, especially in the areas of Geometry and Arithmetic. Next, we will describe three episodes: “The construction of an “esquadro” (a tool used to verify that walls are perpendicular to each other); “Adapting tools” (the process followed to solve a situation using the technical tool that masons possessed); and the “Roof’s inclination”, (the process developed by masons to calculate the inclination of the roof necessary to build up the roof itself).

The construction of the “esquadro”

One of the masons’ daily routines is to measure angles, especially right angles. In order to do this, masons often use a tile because they know that a tile contains four right angles rigorously measured. However, to build up perpendicular walls, as in rectangular rooms, and be sure that the walls will keep the right angle between them, masons need a more appropriate tool than tiles. For this purpose they construct what they name “o esquadro” (the esquadron).

To construct the “esquadro,” masons use long, thin wood strips that they nail together following precise procedures. The following dialogue, recorded while our informant Mr. Antonio was making an “esquadro,” describes the process of making and confirming the accuracy of the “esquadro.”
R(esearcher): Mr Antonio, I see that you already divided the inside of the house into smaller rooms. How do you know that this wall (internal wall) is perpendicular to the exterior wall?

Antonio: Well. I know because I made my measurements.

R: How did you take these measurements?

A: I used the “esquadro” to be sure that the wall was in “esquadria” (perpendicular).

R: How do you make an “esquadro”? Do you mind showing me?

A: Yes. Come here so I can show you.

Antonio placed two long, thin wood strips on the floor and joined two of the ends with nails, making an angle between them of roughly 90°.

![Figure 1. First step in the construction of the “esquadro”](image1)

R: You told me that the angle between the strips is 90° more or less. However, the “esquadro” needs to be a precise instrument. You need to have an angle with a rigorous measure…

A: Yes! Of course! I need to be sure that between the two strips there is a right angle. We will get there.

R: So, what is the size of the angle?

A: We know that is 90°. Now to be sure that the “esquadro” has a right angle, we measure 30cm along one strip and 40cm on the other; or, another possibility is to measure 60cm on one strip and 80cm on the other. (Fig.2)

![Figure 2. Second step in the construction of the “esquadro”](image2)
R: Is it finished?
A: No. Now comes the most important step.
R: Why?
A: Because now I need to measure 50cm between the first pair of marks or 1m between the second. (Fig. 3)

![Figure 3. Final step in the construction of the "esquadro"](image)

R: Why are you using the measures of 30cm and 40 cm, or 60cm and 80 cm?
A: Because this is my “scale” and I’m sure that with these measures, the other side of the “esquadro” will measure 0.5m or 1m, and thus I know that my “esquadro” has 90°.
R: Do you always use all these measures?
A: No! I use the measures 30cm and 40cm or the measures 60cm and 80cm, depending on what I’m doing.
R: So the “esquadro” is for measuring right angles. Can you prove that the angle in the “esquadro” is a right angle?
A: Of course I can prove it. With my calculations I have no doubts, but even so I can put a tile on the “esquadro” to prove that the angle has 90°.

The construction of the “esquadro” shows an empirical use of the Pythagorean Theorem as a way to obtain right angles. We asked Antonio where he acquired this knowledge. Antonio said that he learned it a long time ago, with experience and with the help of elder masons. When we spoke with Antonio about the Theorem of Pythagoras he was not impressed by our explanations. From Antonio’s professional point of view the Theorem of Pythagoras does not hold much interest, mainly because the professional context does not require it (Pardal, 2008; Pardal & Moreira, 2009)

**Adapting tools**

Frequently, to solve a problem at the workplace requires that masons reasoning about the solution in connection to a specific technical tool. The episode that we are going to describe shows this situation. This is, because Mr. Antonio’s boring machine only makes circular holes and it was necessary to perforate a hole on a surface with a quadrangular shape, Mr. Antonio faced a new problem, that he solved applying basic geometric knowledge (Pardal, 2008, pp.122-123).

R(esearcher): Do you mind to show me what are you doing?
A(ntonio): I’m drawing squares to incrust the lamps in the roof.
R: How are you going to make it?
A: Then, I draw four circles inside the square.
R: Why are you drawing squares?
A: Because my boring machine is circular and I want quadrangular holes. Thus, I need to make four circles.
R: Why four?
A: I need to do a square hole with a 18cm side, but the machine has a diameter of 9cm. Thus, if I make two circles side by side I get the 18cm as the side of the square.

![Figure 4. Square divide in four equal squares](image)

R: Where are you going to perforate?
A: After drawing the square, I’m going to divide it in four small squares each one with 9cm side, which is the measure of the diameter of the circle of the machine.
R: And then…

![Figure 5. Circles inscribed in the square (each circle represents the perforation of the machine)](image)

A: Then, I put the machine inside each square and I do the holes.

**Calculating the roof’s angle of inclination**

“Roof” is a universal name for covertures. Generally the roof is constituted by the composition of inclined plans. The simplest roof is the one that has only two inclined plans. It is called the gable roof. If the roof has four inclined plans it is called a hip roof.

The function of the roof is to protect people and their belongings against exterior factors such as the snow, the rain, etc. Another additional function is the caption and distribution of water rains. The roof’s inclination depends on the type of the covertures as well as on local climatic conditions. Thus, according to the angle of inclination of the plans that constitute the roof, waters rain might be more or less flew off and the heaviness of the snow more or less bearded. For example, in some places in Europe, like the Alps, roofs have a high root ridge in
order to hold better the heaviness of the snow and, therefore the angle of inclination of the roof needs to be more than 60º.

Figure 6. The interior and exterior sigh of the inclination of a gable roof.

In the following episode masons explain to us how they build a gable roof (Pardal, 2008, pp 106-108).

R: Do you mind to explain to me how do you build the roof Mr. Antonio?
A: Of course!

Next Antonio explains how he does the calculations to construct a 35% inclination roof.

R: How do you calculate the inclination of the roof? What does it mean a 35% inclination?
A: An inclination of 35% means that to each horizontal meter we should have 0.35m in the vertical. Thus, the roof has an inclination of 35%

R: Do you mind to explain it better?
A: Of course! If the roof measures 4m in the horizontal, and I know that for each meter in the horizontal it needs to have 0.35m in the vertical I multiply four metros by 0.35. Thus I know that its root ridge will be 1.4m (4mx0.35).

R: If the roof has 3m in horizontal and 1.5m in the vertical, what will be the inclination?
A: The solution is identical. The only difference is that in the former situation I did a multiplication in the later I should do a division.

R: So, what are you going to divide?
A: In the former situation I knew the measures of the horizontal line and the inclination, so I multiplied the two values. Now I know the root ridge’s height (1.5m) and the value of the horizontal line (3m), thus, I divide the root ridge by the horizontal line (1.5m: 3m) and the inclination is 50%

R: How do you obtain 50%?
A: Well, the result of the division is 0.5. Then I multiply it by 100 to get the value of the percentage, and it is 50%.

The following image illustrates a gable roof as well as the triangle that it forms and the calculations, as we do it in mathematics education, to compute the percentage of the roof’s inclination.
Figure 7. The roof’s inclination and the process of its calculation

Discussion

As Barton (1996, p. 214) states:

Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices, which we describe as mathematical, whether or not the cultural group has a concept of mathematics.

It is in some aspects certainly the case of masons. In fact, during fieldwork, we witnessed several situations where masons did not recognize that they were using mathematical knowledge, in spite of its relationships to school mathematics. That is, frequently masons apply mathematical knowledge in a practical and intuitive way, using specific strategies to solve problems, without being aware that mathematical ideas are involved.

Moreover Wedge (2002, p.25) presents five hypotheses about the mathematics used by semi-skilled workers. From mason participants’ perspectives on mathematics and from the above-described episodes, we can verify these five hypotheses. Thus:

- The three above described episodes are problems “arise(d) that can only be solved by quantification” (hypotheses 1).
- In the three episodes, the situations required “relatively simple formal skills and understanding of mathematics” (hypotheses 2, p. 25)
- The use of mathematics is differently from its use in traditional teaching (hypotheses 3).
- “Workers think mathematics is important in the labor market, they do not regard mathematics as something important of personal relevance to them” (p.25) (hypotheses 4)
- Workers are not fully conscious of their mathematical activity in their daily work and, thus, of their ‘mathematical’ competence (hypotheses 5).

In addition Wedge & Evans, (2006, p.53) argue about the use of mathematics in the workplace:

… in the workplace what could be called the situation-context (Wedge, 1999a) throws up problems which may (or may not) require the use of mathematical ideas and
techniques. These problems result from the need to solve a working task where the numbers are to be found or constructed with the relevant units of measurement (e.g., hours, kilograms, and millimetres). It is the working requirements and functions, in a given technological context, that control and structure the process, not a narrowly defined task. Some of these problems may look like school tasks (the procedure is given in the work instruction) but the experienced worker has his/her own routines, and methods of measurement and calculation.

The episodes described above: “The construction of the esquadro”, “Adapting tools” and “The roof’s inclination” are a clear illustration of the above argument, in the sense that they are tasks carried out to solve problems posed by situations-context at the workplace and they involve numbers and measures that get meaning in the process of solving the problems. That is, it is the context itself that dictates what numbers and measures are going to be used. Moreover, calculations and reasoning present in the three episodes are similar to some word problems that one might find in mathematics textbook, however, for masons they are routine problems that obey to precise procedures and norms.

From the analysis of the first episode “The construction of the esquadro” we observe that although the Theorem of Pythagoras is taught in school, what emerges from the above dialogue is that masons have been familiar with the application of this theorem for a long time. In fact when we said to Anthony that we can use the Theorem of Pythagoras to calculate faster he noted that:

I do not know that name, but Mr. Pythagoras uses his scale and I use mine, as well as my colleagues use them.

Thus, Antonio does not know the formal name of the Theorem of Pythagoras, but he knows how to apply it in his professional context in the particular case of two Pythagorean triplets – 30, 40, 50 and 60, 80, 100. Fernandes (2004) relates a similar finding in a study about mathematical knowledge in a workshop of locksmiths. The apprentice locksmiths also use the Theorem of Pythagoras to verify if the cover of a chair is in “esquadria”—that is, if it has a rectangular shape—and they use the Pythagorean triplet 6, 8, 10.

Research conducted by Duarte (2003) in Brazil also pointed out that masons use proportions to solve math problems in the course of their work. In fact, several professional practices used by Brazilian masons are similar to the ones observed in our study. For example, to calculate the perimeter of a circumference a popular technique is to add the value of four diameters of the circumference, this is the perimeter of the square inscribed in the circumference. This way of doing the calculation does not indicate the exact value of the perimeter of the circumference but avoids the more complicated procedure of to measure the wire around the circumference.

The episode that we named Adapting tools is an example of what Bessot (2000) argues. As she says:

In the professional area being studied, the part of the device made up of technical tools determines professional practice in the same way as do the succession of actions. Mathematical knowledge is preserved in these tools, and adapted to specific situations. (Bessot, 2000, p.225)

This is, the available technology is used in association with mathematical techniques, in creative ways, to solve professional problems, expressing this professional group’s difference in approaching to specific professional situations.. In the episode described above it was the
characteristics of the technical tool combined with their mathematical empirical knowledge that they can performed with it, that determined the creative approach that masons used to solve the practical problem they faced, this is, according to different contexts masons improve their practices by figuring out new ways of using their professional technology with the support of mathematical processes that they already know.

**Final Consideration**

To the extent that it is necessary to perform or assume new roles in professional life it is necessary further education that includes learning specific techniques as well as social and theoretical skills. Thus, professional education requires constant lifelong learning as well as the adaptation of former learning.

In order to update professional education as well as to acquire new vocational training it is indispensable to be aware of what the particular mathematical requirements of a given profession are (Bessot, 2000). Masons’ knowledge collected by means of fieldwork throughout several real episodes might constitute a set of significant materials that could provide more examples to use educational contexts. As FitzSimons notes (2009, p.350)

> The different kinds of experiences brought by the learners – educational, practical and multicultural – can provide important resources for contextualizing their mathematics education, provided that the learners are willing to share them and feel confident to do so.

To study masons’ practices and describe them, highlighting their mathematical process and contents, allows us to make use of their practical examples for situations in lifelong education. In this way it is possible both to construct curricula aimed at adults learning mathematics and to develop curricula suitable for labour markets.

Thus we argue that these practical applications of mathematical concepts may constitute, for us mathematics educators, material to work with in order to certify mathematical competencies in an official process.

As a final remark we highlight that educational researchers should understand, explain and propose courses where the dynamics of the relationship between mathematics and the workplace are considered as resources for cultural improvement and further mathematical competences.

Finally, it is important to say that this research does not claim to exhaust the understanding of the phenomenon in study. As in any work that attempts to mine reality, data is infinite. Humility to deeply recognize that fact is always necessary.

**References**


Wedege, T. (1999). ‘To know or not to know – mathematics, that is a question of context.’ Educational Studies in Mathematics, 39, 205-227