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Javier Díez-Palomar

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Objectives

Adults Learning Mathematics – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/ numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field, which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:
· Research and theoretical perspectives in the area of adults learning mathematics/numeracy
· Debate on special issues in the area of adults learning mathematics/numeracy
· Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Adults Learning Mathematics – An International Journal

In this Volume 6(1)

Editorial

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I am glad to be able to present this new edition of the *Adults Learning Mathematics: An International Journal* as chief editor. It is an honour to have the opportunity to contribute to scientific production in our area to make sure that adult learning mathematics achieves a relevant place in the International research sphere. *Adults Learning Mathematics: An International Journal* has followed a path of six years in which researchers around the world have worked and provided relevant contributions on aspects such as: training in the job place; the use of new methodologies and effort to find useful theoretical lines that clarify our understanding of how adults learn mathematics; and reflections of adults on mathematics, including its teaching and learning, and their experiences of resistance towards new methodologies.

This new edition of the journal deals with three key issues in adult learning mathematics:

First, there is the topic of anxiety. Christopher M. Klinger, from University of South Australia, presents an interesting analysis of the anxiety adults face when learning mathematics. In agreement with Klinger, one of the greater challenges for individuals who teach mathematics to adults is to overcome the resistances and negative attitudes that many have towards this area of knowledge. Adults face the feeling of math anxiety, but they also have stereotypes and negative beliefs that somehow affect both how they receive the mathematical contents as well as the strategies that teachers should use with them. In this article Klinger proposes "connectivism" as an approach or a means to overcome the behaviours and constructs that frame educational practices regarding the teaching mathematics to adults. This is an innovative and suggestive approach that has great potential to open new routes that will enable adults to overcome their fears of mathematics and to learn mathematics effectively.

In the next article, Rachael M. Welder, from Hunter College at The City University of New York, and Joe Champion from Texas A&M University in Corpus Christi focus on an understanding of graduate preservice elementary teachers as adult learners of mathematics. The authors state the problems faced by adults who return to school to attend graduate-level courses to become elementary teachers. As the authors indicate, these individuals experience targeted needs and specific struggles with mathematics. The authors analyse the perceptions of ten participants on their experiences as learners of mathematics. By means of their discussions, we see, again, the impact anxiety on mathematical learning. In this case, the debate leads us to learn some of the needs of adult mathematical learners, including the need to promote high quality mathematical education that will allow adult learners (regardless of their level) to have adequate resources to overcome mathematics anxiety, and therefore, the ability to effectively practice their role as teachers.

Finally, the third article in this edition is presented by Lisbeth Lindberg from Göteborgs Universitet and Barbro Grevholm, from the University of Adger. In this article the authors reflect on the integration between mathematics and vocational education, using data from a research project conducted in Sweden between 1998 and 2002. The focal point of this discussion is on the topic of "reform" in vocational programmes in upper secondary school and a new adopted syllabus of mathematics. The authors present a detailed analysis of the implications of diverse aspects of this vocational programme, including teacher training. In addition, they respond to the affordances and constraints that the involved participants in the project presented during their work. They point out the importance for the faculty to participate in mathematical activities and practices as part of their daily teaching in a community of practice, because it enables opportunities to reach targeted objectives.

The three articles illuminate current topics in adult learning mathematics which are unique to our area of work. I hope this edition will open new working and research paths and promote collective reflection that will lead us to find ways and guidelines to improve the practices that we are conducting in mathematics education oriented to the adult learners. Happy reading!

Dr. Javier Díez-Palomar
Chief Editor
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'Connectivism' – a new paradigm for the mathematics anxiety challenge?

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Abstract

A major challenge for practitioners in adult mathematics education is to achieve effective learning outcomes in the face of prevailing negative attitudes in their students, often present as a consequence of unsatisfactory early mathematics learning experience and flowing from the well-established connection between adult innumeracy and mathematics anxiety. Whether in non-specialist mathematics teaching in diverse disciplines such as economics, nursing, and teacher education, or in adult numeracy teaching, the issues are essentially the same: traditional approaches to mathematics teaching, including constructivism, do not work for math-averse students. The need to find new ways to tackle old problems is further fuelled by the impact of the digital age, with mounting evidence that many aspects of accepted teaching and learning practices are being generally undermined by learners' exposure to technology. Consideration of some stereotypes of traditional methodologies in the context of behavioural and cognitive characteristics common among math-averse and math-anxious students motivates a re-framing of the practitioner's approach and outlines strategies for effective practice that find alignment with connectivist approaches to learning, which is a more flexible approach than the constructivism currently favoured as a positive way of presenting mathematics to learners.

Introduction

The endemic adult innumeracy that is so deeply embedded in modern Western societies is inextricably linked with the spectrum of mathematics anxiety, negative mathematics attitudes and aversion to the learning of mathematics so often encountered by practitioners among adult learners. These aversive affective behaviours are typically founded on negative early mathematics learning experiences in the latter years of primary education, during the transition from instruction in concrete procedures to increasingly sophisticated and abstract concepts (Klinger, 2009). Pupils' difficulties during that period can be compounded, if not caused, by the 'guidance' they receive from teachers who may themselves be mathematically averse and even covertly innumerate in an integrative sense. Indeed, among undergraduate cohorts pre-service primary teachers who may be characterised by such negative attributes have been found to be disproportionately over-represented (ibid).

The challenge for teachers/practitioners in adult mathematics education, whether at the level of numeracy teaching or undergraduate non-specialist mathematics teaching (for instance, where mathematics is a service subject in non-mathematics disciplines such as economics, nursing, education, psychology, etc), is to find effective ways to break through the prevailing barriers of anxiety and disaffection so that their students may experience success in their mathematics learning, often for the first time, at a level that is at least sufficient for their immediate learning objectives. Ultimately, the greatest achievement will arise when students can overcome their anxiety and aversion to become independent learners with the capacity to extend their engagement with mathematics according to their inclination and intrinsic motivation rather than as a reaction to external drivers.

It will be argued here that, from adult numeracy teaching to undergraduate non-specialist mathematics teaching, the issues are essentially the same: 'traditional' approaches to mathematics teaching simply do not work for math-averse students. This will not surprise most practitioners but what, specifically, does not work? To respond to this, the basis of traditional pedagogies and their underlying epistemologies will be considered here in the context of behavioural and cognitive learning characteristics that typify math-averse and math-anxious students and which tend to undermine their learning goals. This will motivate a re-framing of the teacher/practitioner's approach with the aim of outlining strategies for effective practice.

Learner characteristics – the practitioner's challenge

Students who enter university with the aim of becoming mathematicians, scientists, engineers and the like do so in the rather obvious expectation that their courses will encompass substantial mathematical content. But these comprise a relatively small fraction of all commencing undergraduates. Most students in other disciplines rightly have no such expectation yet a rather large proportion of them will, nevertheless, encounter components of a mathematical or statistical nature that they will need to master if they are to succeed in their studies. However, previous studies (Klinger, 2006-2008 inclusive) have shown that, regardless of students' expectations as to content, mathematics anxiety and math-averse behaviours may be found, to varying extent, universally in all undergraduate student groups (but with greatest severity among student [pre-service] primary teachers) and particularly among pre-tertiary adult learners with aspirations of undertaking tertiary study.

The pervasiveness of mathematics anxiety in the community is well documented, with both practitioners and researchers reporting that perhaps a majority of adult learners exhibit at least some degree of anxiety when confronted with overtly mathematical tasks. This reality makes it apparent that many adult learners, and in particular those at the level of pre-tertiary adult education in numeracy and vocational mathematics, will bring to their mathematics/numeracy studies a strong affective load of negative preconceptions, both of mathematics and of their own capabilities. That is, they have only a vague concept of what mathematics is really all about, lack confidence in their own mathematical abilities, and often fail to appreciate the extent to which they actually and routinely engage in essentially mathematical thinking as they go about their daily activities. As a result, and notwithstanding that they may well display high-level competencies in other aspects of their lives (including study in subject areas other than mathematics), they display characteristics that will cause them to experience difficulty in their mathematics learning endeavours. Practitioners, then, are faced with the challenge of how best to respond.

Figure 1 (below) illustrates this challenge: the math-averse adult learner is surrounded, overwhelmed, by the task at hand of acquiring specific skills, dealing with math content, and developing effective learning strategies. The needed insights have to be facilitated from outside but they are blocked by the barriers of anxiety, negative beliefs and stereotypes. Therein lies the challenge – effective teaching strategies for these students go well beyond the 'usual' scope of mathematics teaching. Before specific content and techniques can even begin to be addressed, it is necessary to uncover the nature, extent, and source of this outer barrier. The wall has to be breached somewhere. Many of these students will state at the outset that they 'hate maths', either because they are 'no good at it and never have been' or because of negative early mathematics learning experiences. They tend to be quite entrenched in their views and may even be hostile and resentful at being confronted with the material since they feel they are being forced to do something they 'know' they're 'no good at'. Such students may appeal for help and yet, paradoxically, not expect to change their perceptions of mathematics and of their own ability and so the first task for the teacher/practitioner is to adopt an approach that allows them to admit the possibility that their negative views, no matter how valid they may once have been, need not be absolute and insurmountable.

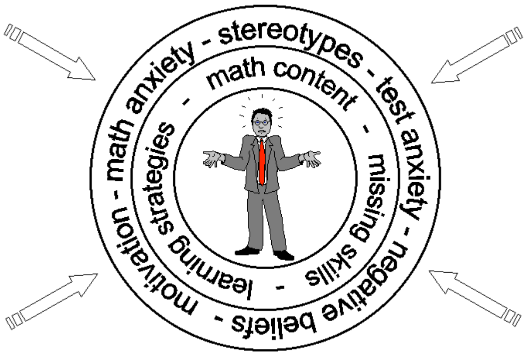

Figure 1

In this, both the students' and the practitioner's perceptions of their respective roles, the interaction between them, and the effect of these on student attitudes are particularly critical factors for successful outcomes and should, therefore, be examined carefully with the overarching goal being to educe transformative behaviours from an informative process that includes recognition of the following learner characteristics (Klinger, 2004):

- Confusion
- Lack of confidence
- Negative perceptions
- Lack of strategies
- Narrow focus
- Assessment-driven motivation
- Little or no appreciation of the concept of mathematics as language

When students are unclear about what is required of them, they tend to over- or under-estimate the difficulty of subject material and display little insight about the extent and relevance of their prior knowledge (mathematical and otherwise) and pertinent skills. Commonly, confusion dominates their behaviour: they do not know what it is that they 'don't know', have difficulty in organizing notes and other materials and make little use of text books or other resources. Repeated lack of success impacts on confidence, in which they are lacking, and manifests in low self-efficacy beliefs, often accompanied by strong negative emotions of embarrassment, self-deprecation, and helplessness (reported also by Karabenick and Knapp, 1988). Thus low expectations are common, as are lack of persistence and little interest in attempting to acquire deeper understanding (as opposed to 'quick fix' outcomes). Negative perceptions can also contribute to low self-esteem and may include their perceptions of self, particularly in terms of cognitive competence, extending to their instructor(s), topic organization, and reactions of their peers and family.

Although they may be very competent problem solvers in other contexts, confusion and negative perceptions and emotions are too strong an influence in this context, resulting in an obvious lack of strategies with which to negotiate or cope with their difficulties. The capacity to generalise is limited and there is little facility to synthesise knowledge connections to aid their understanding and this is often aggravated by a lack of attention to creative thinking and problem-solving skills (Kessell, 1997), which may stem from a narrow focus and strong tendency to see mathematics as 'different' from other intellectual activities. Consequently, motivation is likely to be assessment-driven rather than intrinsic – that is, these students pursue the 'extrinsic goals' described by Ryan and Pintrich (1997), with all the attendant consequences and implications. This may also extend from low self-efficacy beliefs.

In particular, while most will readily acknowledge that mathematics is loaded with specialised terminology, much of it involving common words whose definitions are much narrower than their everyday usage, and while they accept the use of specialised language to describe mathematical concepts, they fail to recognise the converse: that mathematics language describes specialised concepts. That is, there is little or no appreciation of the concept of mathematics as language.

It is common within teaching and learning literature to associate these characteristics with shallow, or surface, learning styles and their presence is frequently associated with avoidance behaviour such as that commonly observed in the math-averse. An important distinction should be made, however: while students may exhibit such characteristics in association with their mathematics learning, it should not be assumed that they more generally typify their learning style in other learning situations – I contend that in many instances the mathematics learning style is anything but intrinsic and is instead directed by a student's anxiety and prior mathematics learning experiences. In many respects, then, the above characteristics are reactive, manifesting in self-defeating behaviours that undermine the learning situation unless the teacher/practitioner can affect successful intervention.

Since the proximal cause is likely to be strongly related to past learning situations, forms of instruction that derive from a deficit model of remediation, echoing negative early encounters in the mathematics classroom, will be necessarily ineffective and serve to validate the student's poor perceptions. Rather, there is a need to establish an entirely different framework whereby students have a genuine opportunity to experience the epiphany they need to shed their history and construct new understandings. This is a matter of good practice (a value-laden term) for which 'it is vital to understand the epistemological basis that underlies the teaching of numeracy in the adult classroom.' (Swain, n.d.)

Epistemology and pedagogy in perspective

Behaviourism to social constructivism

'Skill and drill' teaching is the archetype of *behaviourism* in mathematics education. In this framework, the 'ideal' learning environment is one that maintains a focus on procedures and outcomes arranged hierarchically so that mastery of basic skills serves as a scaffold to more advanced activities in a linear and cumulative progression. Here, mathematical knowledge is external, absolute, and transmitted didactically with emphasis on learning as the correct application of appropriate algorithms to obtain correct answers, conditioned and reinforced positively by 'rewards' of success and concomitant approval and negatively by failure and disapproval (even to extremes of physical and psychological punishment, as commonly reported by sufferers of more severe mathematics anxiety). The practice of correct application is undertaken by studying worked examples and emulating the procedures in situations that are similar to the examples. While such practice might be termed 'problem solving', it is not thus in a modern sense since it lacks genuine creativity; rather, competence lies in the ability to find an appropriate mapping from a known example to a given scenario.

Orton (2004, p29) observes that, '…much of the teaching of mathematics has traditionally consisted of the teacher demonstrating a method, process, routine or algorithm to be used in particular circumstances, followed by the class attempting to solve routine questions using the set procedure. … Exposition by the teacher followed by practise of skills and techniques is a feature which most people remember when they think of how they learned mathematics'. He goes on to explain that while the objective is to establish strong stimulus-response bonds (if, indeed, such a construct is valid) teachers know well that these are frustratingly short-lived, with subsequent examination after several months interval revealing that 'most pupils demonstrate only that they cannot respond correctly'. Orton uses the example of the addition of fractions, the algorithm of which is taught, re-taught, and practiced throughout early schooling only to be forgotten repeatedly (*ibid*). This is not true understanding.

While the 'use it or lose it' impermanence of such mathematics learning is problematic in itself, this aspect is vastly more so because it admits no structured, regularised or intrinsic opportunities to promote understanding, which is redundant in the face of the imperative to elicit correct behavioural responses. Whatever learning takes place is discrete and disconnected, with each topic essentially partitioned from the body of discipline knowledge as a whole and considered often in isolation from, and generally without appeal to, the often considerable knowledge base that students may hold as a result of their learning in other disciplines and life experience.

There is a long tradition of teachers, from primary to tertiary levels of education, adopting an essentially behaviourist approach in their mathematics teaching when they would reject such methods in other curriculum areas. Those who are least prepared for the mathematics classroom are likely to have a limited procedural and rules-based view of mathematics. This accords with their own recollections of primary- and secondary-school mathematics classes and perhaps their everyday experiences as functionally numerate adults, where practical concerns often dictate a pragmatic reliance on procedures and algorithms (to the extent that these are remembered) that can be implemented mechanically and reliably to satisfy an immediate need. But it is not just inexpert teachers that employ this mode of instruction. At the other end of the spectrum, teachers with highly-developed mathematical ability may adopt a similar approach (Golding, 1990) as a result of their unconscious competence and lack of awareness of the actual complexity of their expertise, which may lead them to pursue perceived efficiencies in the transmission of 'obvious' knowledge. Moreover, it 'takes less time to state a well-established method than it does to guide students to its discovery… And, of course, the prevailing emphasis on skills tests influences many teachers toward the short-term goal of teaching rote procedures' (Golding, 1990 p46).

The foregoing is not to reject in its entirety all aspects of behaviourism; it is the means that are questioned rather than the ends. Clearly, the use of mathematics in the 'real world' – such as for vocational purposes – most often demands a focus on outcomes (the 'right answer') that can only be satisfied by the functional fluency that can only be found as a result of practiced ease.

Cognitivism arose largely in response to behaviourism. Whereas the latter ignored processes internal to the learner because they were not observable, the former argued that intentional action deriving from a learner's mental states contributed to behavioural and learning outcomes as the learner seeks to adapt to the learning environment. From a cognitive perspective, knowledge may be transmitted between individuals but is stored as internal mental constructs or representations. For mathematics education, the influence of cognitivism manifests most directly in an emphasis on learning by problem solving as a recursive process of assimilation and accommodation whereby a problem is interpreted by assigning it to existing internal representations or *schema* (Bartlett, 1932). This approach has been shown to yield superior learning outcomes for more experienced learners, for whom worked examples become increasingly redundant (Kalyuga, Chandler, Tuovinen, and Sweller, 2001). It must be stressed, however, that in traditional approaches to mathematics and numeracy teaching, the cognitivist approach augments rather than supplants behaviourist practices, which persist as the dominant mode of instruction when new (and, particularly, fundamental or foundational) procedures are introduced.

Social cognitivism, largely credited to educational psychologist Albert Bandura, fuses elements of behaviourism and cognitivism with social aspects of learning (Bandura, 1986). The theory recognizes that learning is at least as much a social activity as it is behavioural and cognitive and emphasizes the importance of observational learning, by which behavioural and/or cognitive changes are effected by the learner's comparative observations of others and of self, a process incorporated in the concept of self-efficacy beliefs (Bandura, 1997), which have been found to play a prominent role in the learning activities of math-averse students (Klinger, 2004-2009) through a complex interplay of motivational, behavioural, cognitive, and affective factors.

For at least three decades, *constructivism* has been the darling of many in the education community. Alenezi (2008) claims that mathematics and science instruction is 'increasingly grounded in constructivist theories of learning' (p17). Essentially, the so-called 'math wars' – the debate between traditional versus reform mathematics education – derives from philosophical differences between behavioural and cognitive approaches and those of constructivism. The central tenet of constructivism is that knowledge cannot be transmitted but is a construct of the mind as a consequence of experiential learning. *Information* may be transmitted but its transformation to knowledge is an internal process affected by learners discovering relationships between new information and their inner knowledge representations and constructions of reality.

That is, the learner is not a passive vessel to be filled (the behaviourist's standpoint) but has an active role in building understanding so as to make sense of the world. Learning results from an ongoing process of hypothesizing, rule-creation and reflection, with new information being evaluated in the context of existing rules that are themselves subject to revision or rejection if found incapable of accommodating the data or otherwise discovered to lack internal consistency. The teacher is dispossessed of the role of didactic authority to become instead an information conduit and facilitator of the learning process, charged with finding ways to present information that is relevant to the student's learning needs in a manner that makes it meaningful. This is to be achieved by providing students with opportunities to discover, explore and apply ideas that will satisfy learning objectives.

Social constructivism adds the further proposition that there can be no sensible definition of knowledge that ignores its social context. That is, knowledge must necessarily be grounded in the social values, standards, mores, language and culture by which the learner acquires an understanding of the world. It is also to be developed socially in the classroom via open communication between instructor and student and by peer interactions in common exploration activities. In this sense then, while knowledge is in one sense individual and internal, learning is a social activity and social interaction extends the location of knowledge via communicated and shared understandings.

While constructivism dominates current pedagogy, I suggest that there are profound flaws in the context of mathematics and numeracy education. First, the principal required curriculum outcome is identical to that of behaviourists and cognitivists – a demonstrated ability to *perform* by applying the appropriate procedures to a given situation, in a manner that is consistent with the formalism, to arrive at a correct result that may be communicated to and verified by others (that is, the formulation, solution, and answer must be presented according to agreed conventions). Second, there are implicit assumptions that self-directed learners have 'sufficient prior knowledge and skills (particularly basic literacy, numeracy and study skills) to engage effectively and productively for generating new learning' (Rowe, 2006 p101). Elementary or foundational formalism and conventions, in particular, are not reasonably accessible to exploration and discovery; they need to be learned in essentially the same way as vocabulary and rules of grammar – no matter how they arrive at their ideas, students must know *what* to write and *how* to write it in order to reliably record and communicate them.

Last (and far from least), the word 'basic' is often applied to the everyday arithmetic of addition, subtraction, multiplication, division, fractions, decimals, and percentages. There is a tendency for those who 'can do' to use the word in a dismissive or belittling fashion with those who 'can't do' – "You should be able to do *that* [at least]… it's just basic arithmetic". Math-averse learners have heard statements like this throughout their mathematics learning history and in the work-force, often accompanied by expressions (both verbal and non-verbal) of disparagement, derision, frustration, or anger and, sometimes physical chastisement. While concepts of number, number representation, and arithmetic operations are certainly *fundamental,* there is nothing *basic* (in the sense of 'simple' or 'obvious') about them – it has taken humankind millennia to invent/discover these concepts and to formalize them in terms of definitions, rules, procedures, and algorithms. One might wonder how many of those with greater facility in the use of the procedures of 'basic' arithmetic can actually sensibly talk about their use without resorting to parroting rules once learned. It is neither reasonable nor sensible to expect students at any level to *actually* discover 'basic' mathematical concepts and corresponding procedures by pursuing a literal constructivist agenda.

Elementary mathematics instruction texts appear to have changed considerably in the period since constructivism came to the fore of educationalists' thinking. Modern texts tend to be colourful and inviting, with many diagrams. They contain numerous 'real world' scenarios to accompany and supplement more formal aspects of the text; these are usually provided to motivate students' engagement by providing social grounding for the presented concepts and worked examples. In addition, practical exercises presented as 'experiments' identify opportunities for learners to verify various aspects by quasi-independent enquiry.

It takes little more than a cursory examination to discover that these apparent changes are largely superficial. The core material (definitions, rules, procedures, algorithms and 'drill' exercises) continues to be present and largely indistinguishable from that of texts compiled during pre-constructivist eras. That is, constructivist principles appear to have been applied as a veneer that mostly serves to augment cognitivist problem-solving activity. Ultimately, and fundamentally, little has really changed; teachers must still be able to demonstrate, via skills tests and league tables, that they are engaged in practices that satisfy benchmarked standards and learners must demonstrate, through standardized assessments, that they have acquired sufficient mastery of the curriculum.

There are other problems, which constructivist thinking appears to mask. Figure 2 shows a 'gifted' Grade 5 student's work, taken from Davis and Maher (1990). In a typically constructivist approach, the student, Ling, has been given a problem to solve (stated in the figure) and a range of manipulable materials, including Pattern Blocks, which she used to explore and correctly solve the problem. Her teacher had then asked if she could write her solution and she first recorded the diagram.

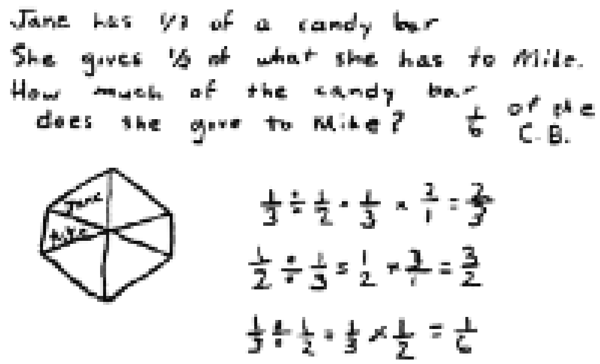

Figure 2 The Candy Bar Problem (Davis and Maher, 1990, p.75)

That Ling started her diagram by drawing a hexagon indicates that she *knows* the correct answer – she's 'done the maths'. The teacher asked if she could do it in figures and this is where her troubles began; she responded by writing the first line in the figure, 1/3 ÷ 1/2 = 1/3 × 2/1 = 2/3. Davis and Maher write:

> 'Ling is clearly a good student; she has learned the "invert and multiply" rule correctly. But she has produced an incorrect answer! How come? Because she has not called upon the correct algorithmic solution procedure. There is no reliable way to go from a problem statement to a solution procedure unless you get a correct representation of the problem.' (Davis and Maher, 1990 p75)

The first attempt at writing the problem and solution being not in accord with her experience, Ling wrote the following two lines with some frustration. Eventually she was apparently satisfied that she had finally written something that produces the answer she 'knows' is correct. But of course, no doubt out of desperation, she has claimed that division and multiplication are the same thing in this instance.

Now, this example is interesting not because of the student's approach or, directly, because of the constructivist pedagogy. Rather, it is interesting here because of the authors' observations, which focus on the student not having called upon 'the correct algorithmic procedure' (*ibid*) instead of identifying that this example is not one of problems with mathematics but, rather, problems for Ling with the language of mathematics – a different thing entirely. Thus Davis and Maher entirely misses the real point! Ling's mistake was not in calling upon the wrong *procedure* but in her failure to correctly *translate* the English expression, 'half of what she has' (i.e. one half of one third), into the corresponding mathematics expression. This is not a procedural error nor is it a retrieval error (as in faulty memory recall of a rule), it is a *language* error. No-one with a fluent grasp of mathematics as *language* could make such an error without seeing the mistake immediately or on reflection – which is something that Ling never did, apparently.

This simple example is profound in its illustration of a very common feature of students' mathematics work (even those operating at advanced levels). They write to create, largely by mimicry when working at the elementary level, the *appearance* of mathematics rather than writing to express meaning using mathematical language. Advanced-level students will frequently write mathematics by manipulating symbols according to the rules that they have acquired rather than, again, writing to express meaning so that the written mathematics tells a 'story'. That the teaching of mathematics and numeracy generally focuses on the doing of mathematics, without explicitly attending to the language of mathematics and students' fluency therein, is no particular fault of any of the 'isms' discussed thus far as these epistemological and philosophical standpoints are equally deficient in this regard.

Connectivism

The latest contender in educational theory has been termed *connectivism*, a 'learning theory for a digital age' advanced by George Siemens (2005) in response to an awareness that technology is increasingly undermining many aspects of accepted teaching and learning and that prevailing learning theories are inadequate in the present era. In fact, the term, 'connectivism, was not coined by Siemens and has been used by others in a different sense – for instance De Geest, Watson and Prestage (2002, p22) describe connectivism as a desire to 'view mathematics as a connected, holistic way of working rather than as separate topics'. Siemens' connectivity, on the other hand, is considerably different, being motivated by the exponential growth of knowledge (in the sense of a hypothetical audit of what is known, which is a debatable concept in its own right and beyond the scope of the present work) and the observations that, in an increasingly technological and networked world in the 'digital age', 'know-how and know-what is being supplemented with know-where' (p4) and the 'capacity to know more is more critical than what is currently known' (p7).

In proposing connectivism, Siemens borrows, somewhat speculatively (even extravagantly), concepts from the science of complexity, including chaos theory, networking, and self-organization. A significant flaw, though, is the absence from his 'model' of the essential criterion for self-organization, which is the presence of randomness (self-referential noise) as a driver. One might speculate that cognition in a living organism is sufficiently 'noisy' to provide the missing component. Still, despite this omission, there is considerable appeal in many aspects of the proposal. Self-organization and emergent higher-order phenomena from self-referential complex systems is a universal principle (Klinger, 2005) that is strongly reflected in the behaviour of networks and random graphs. The complex and self-referential nature of the human mind is, perhaps, obvious, and in simplistic terms the human brain is certainly a network of interconnected neurons with memory and other cognitive activities being associated with changes to neural connectivity.

However, rather than further pursuing what is, essentially thus far, just an analogy, it will be useful to focus attention on one specific aspect of connectivism that provides a good illustration for the purpose of reframing adult numeracy practice (and, indeed, fundamental mathematics education more generally). This is encapsulated in Siemens' statement that connectivism 'posits that knowledge is distributed across networks and the act of learning is largely one of forming a diverse network of connections and recognizing attendant patterns' (Siemens, 2008 p10).

Reframing practice – a connectivist approach

What I am suggesting here, as a conceptual discussion based on many years experience of working with adult learners with mathematics anxiety and math-averse behaviours in pre-tertiary and tertiary settings, is that the particular value of the connectivism paradigm in mathematics and numeracy teaching lies in exploiting the properties of network connectivity in complex systems. By actively pursuing opportunities for students to forge links that promote an understanding of mathematics as *language*, they may establish connections that permit mappings between mathematical concepts and their various skills and understandings of the world. That is, mathematics language is to be understood in terms of things and language that the learner already knows (through appeal to common-sense and intuition by metaphor and analogy).

This view posits that the connectivity attained by forming links between mathematical know-how, language and other skills from the student's existing knowledge base serves to build understanding so that dependence on mathematical rules becomes redundant (in that, as a consequence of increasing fluency, they come to be seen as obvious consequences of the mathematics language rather than algorithmic procedures to be applied mechanistically). As the internal and self-referential (reflective) knowledge network grows by the formation of new connections that incorporate more and more nodes of both congruent and disparate knowledge and experience, it undergoes periods of self-organizing criticality whereby there are cognitive phase transitions (to borrow physics terminology) that spontaneously yield flashes of emergent deeper understanding (epiphanies or 'ah-ha' moments). Increasingly, the learner is empowered to undertake self-directed learning according to need or inclination.

It is to be emphasized that the foregoing is entirely speculative. Nevertheless, in terms of pedagogical practice, there is substantial merit in the approach of considering mathematics first and foremost as language and focussing on ways and means to develop students' fluency therein while utilising their existing skills and knowledge-base as leverage. For the teacher/practitioner working with math-averse and mathematically anxious adult learners, this will most often demand a reframing of the learning situation away from traditional practices towards techniques that explain and demonstrate how the context and methods of mathematics are revealed through its application as *language*, mapping these onto concepts and language with which the student is otherwise familiar and confident. Of paramount importance is the need to first expose and debunk the preconceptions that underpin negative self-efficacy beliefs, recognizing that these stem predominantly from prior unsatisfactory mathematics learning experiences that were likely grounded in pedagogy derived from behaviourist principles.

Every new mathematics learning activity should be approached from a language perspective, first identifying a common base of understanding with which students can connect so that concepts can be discussed in natural language before proceeding to translate them into the formalism of symbolic mathematics language. This methodology can, and should, be openly explained so that students understand explicitly that they are engaged in learning a language so that if their initial efforts are frustrated they will recognize that many of their difficulties are, in reality, language difficulties and that these will abate as the mathematics language becomes less unfamiliar. Fortunately, students may be reassured, the vocabulary and grammar of mathematics is small compared to natural languages and very literal. There must be particular emphasis that any mathematics that the student reads or writes *must* make sense – it must 'say something'; that is, it must always be possible to translate freely in either direction: mathematics language to natural language and the converse.

As with the learning of natural languages, students must be guided to cultivate an 'ear' (or eye in this case) for dissonance between what is understood and that which is written or read and to develop the ability to self-correct. Whenever meaning is obscured by ambiguity or lack of semantic clarity, or whenever an attempted procedure fails despite apparent correct application, both the instructor and the learner should be alert to inappropriate language construction or interpretation. This may prompt the need to unpack whatever is being attempted to examine, test, and remedy underlying language weaknesses or misunderstanding. Because the mathematics will be shown increasingly to 'make sense' and be something other than obscure procedures and rules, this attention to language is essential as a first step to reducing confusion and anxiety and to broaden students' focus.

While maintaining this concentration on language, the introduction of new concepts should be effected by referral to prior concepts that are properly understood (that is, the student has gained fluency at that level) and, wherever possible, by seeking to identify analogous or parallel ideas in non-mathematical every-day domains. The aim is to establish, wherever possible, connections between what students already know and that which they seek to learn. Often, skills and procedures that appear alien and intractable in a mathematical context may be shown to be intrinsic (metaphorically if not literally) to many 'ordinary' adult activities and situations – seeking to embed new mathematical material by forming connections with students' existing knowledge network creates familiarity by association, adds leverage to the learning task, and brings learners closer to achieving the cognitive phase transition that transforms information into knowledge and understanding. With each advance, students gain confidence, overcoming their negative perceptions to discover intrinsic motivation for actively pursuing their further learning as an end in itself, rather than simply to satisfy assessment requirements. The outer barrier depicted in Figure 1 is thus penetrated. The math-aversion and anxiety may never disappear entirely but future encounters with mathematics can at least be directed by informed, adult decisions rather than dominated by attitudes and emotions imposed by an unfortunate history.

Conclusion

The needs and characteristics of math-averse and mathematically anxious adult learners present teachers/practitioners with a significant challenge for which the traditional 'isms' – behaviourism, cognitivism, constructivism – underpinning conventional pedagogy in mathematics and numeracy teaching are in not only in deficit but can be seen to be directly associated with aversive affective behaviours that typically stem from students' prior experiences of mathematics schooling. A review of the learner characteristics and their association with surface-learning styles, together with the recognition that these stem from past mathematics learning experiences, provides both insight into the nature of the challenge and a context from which to consider the epistemological foundations of traditional mathematics teaching and, more particularly, to give consideration to the utility of the latest 'ism', connectivism, and its value as a means to effect behavioural and cognitive transformations.

Aspects of connectivism resonate with techniques and approaches known from professional practice to be broadly successful in alleviating mathematics anxiety and achieving effective learning outcomes. The concept of invoking the properties of network connectivity in complex systems (of which the human brain is an archetypal example) as a means to explain learning provides a potential theoretical framework to support the advocated reframed adult numeracy practice: specifically, the pursuit of opportunities to approach the learning of mathematics as language and to form multiple links to connect new information with students' existing knowledge networks.

The paradigm of connectivism, at least along the lines indicated here, needs considerably more research to become established as significantly more than a rhetorical device. Nonetheless, there is obvious intuitive appeal in the insights suggested by this new 'ism' that surely warrants further investigation.

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Toward an Understanding of Graduate Preservice Elementary Teachers as Adult Learners of Mathematics

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Abstract

Career-switchers who are returning to university for training as future elementary school teachers join an important and increasing group of adult learners of mathematics. These graduate preservice elementary teachers often place a high value on learning mathematics because of its prominent role in their prospective careers, but their learning often requires overcoming histories of personal struggles with mathematics. This inquiry used a mixed methods designed and focused on ten participants' perceptions of their experiences as learners of mathematics, anxiety regarding mathematics, and graduate mathematics coursework. All of the participants left professional careers to join a full-time graduate teacher preparation program in New York City. Bridging research from adult education and teacher education, we analysed interview transcripts, mathematics anxiety measures, and survey data using mixed methods both qualitative and quantitative research approaches. The findings show that, starting in adolescence, and continuing into adulthood, participants described negative experiences with exam performance, tracking, and instruction during formal school mathematics. Participants perceived these struggles as contributing to patterns of avoiding mathematics coursework and experiencing moderate-to-high levels of test mathematics anxiety. In spite of, or perhaps partly because of, their personal struggles with mathematics, the participants described urgently wanting additional high-quality mathematics preparation focused on upper-elementary content, manipulative-based pedagogy, and current elementary mathematics curricula.

Key words: mathematics; preservice elementary teachers; graduate education.

Introduction

Non-traditional teacher preparation programs were initially established in the United States during the 1980s in response to projected teacher shortages and have steadily grown to become an important source of teachers in the U.S. (Humphrey, Wechsler, & Hough, 2005). For instance, non-traditional programs produce nearly a third (6,590 of 20,839) of the newly certified teachers in New York City, or about 20% of the approximately 33,000 new U.S. teachers prepared through non-traditional programs annually (United States Department of Education, 2009).

Around the world, education faculty employ standards for mathematical competency to selectively admit students into teaching programs (Burton, 1987). However, because of the immense need for teachers in inner city and under-served schools, teachers are often recruited without an acceptable level of mathematics competency and are expected to develop this knowledge through their teacher preparation program (Ashun & Reinink, 2009). Yet, in non-traditional programs like those in New York City, the mathematical preparation of elementary teachers is particularly challenging because typically students are only required to complete one mathematics course of any kind; and, analyses of syllabi have revealed huge variation in the content covered across these single mathematics courses (Boyd, et al., 2008).

Improving the mathematical preparation of elementary teachers is a major concern of policy makers and educational researchers (Adler, Ball, Krainer, Lin, & Novotna, 2005). This concern extends to a need to better understand better the beliefs preservice elementary teachers have about their own K-12 mathematics instruction, together with potential anxiety the teachers experience around mathematics, because these beliefs have been linked to their future instructional practices (Fang, 1996; Thompson, 1984). Consequently, "an important goal of every teacher education program should be to help pre-service teachers develop beliefs and dispositions that are consistent with current educational reforms and to assist them in addressing their mathematics anxiety" (Gresham, 2008, pp. 171-172).

In contrast to the strong track-record of success and high self-efficacy experienced by most preservice secondary mathematics teachers (Champion, 2010), many preservice elementary teachers are tasked with learning to teach mathematics in spite of long personal histories of struggling to learn mathematics. Ashun and Reinink (2009) underscore the global scale of the typical nature of this phenomenon:

> Casual conversation with these adults reveal that many do not fondly recall learning mathematics when they were younger. Many of these adults are now choosing to start second careers as elementary teachers who must, by virtue of the Ontario classroom system, teach mathematics as part of their core curriculum. ... [T]here is no doubt that [adult preservice elementary teachers'] inability to block out previously held (and inherently erroneous) beliefs in mathematics education is a contradiction in terms (Wedege, 1999) since these learners bring a wealth of experience and common sense competence from their everyday lives. (Ashun & Reinink, 2009, p. 33)

With a multitude of factors potentially impacting the mathematical content preparation of adults switching to careers in elementary education, it is vital that graduate elementary teacher preparation programs address the unique needs of this population. Concomitantly, basic research into the mathematical experiences and perspectives of these prospective teachers is needed to (1) better understand the mathematical preparation and beliefs of preservice elementary teachers, and (2) inform the design and delivery of graduate mathematics courses which are offered to preservice elementary teachers. Our study specifically addresses adult preservice teachers' affect towards, and beliefs about, mathematics by exploring their prior learning experiences and mathematical preparation in the context of a graduate teacher education program.

Review of Literature

One aim of our study was to explore potential bridges between two areas of mathematics education research - the mathematical preparation of preservice elementary teachers and the mathematical experiences of adult learners. Besides a few recent exceptions (Ashun & Reinink, 2009; Wheeler, 2009), there has been limited research into the important subpopulation of adult preservice elementary teachers. Consequently, our review of literature constructs a conceptual framework by synthesizing two often separate lines of research relating to the mathematics preparation of graduate preservice elementary teachers. First, we summarize the literature regarding the mathematical beliefs, motivations, and obstacles to success that often face adult learners of mathematics. Second, we describe several themes within the large body of research into the mathematical beliefs and mathematics anxiety of preservice elementary teachers. To better contextualize our findings, we reserve additional research into sources of math anxiety and as well as coping mechanisms among preservice elementary teachers for the discussion following our results.

Adult Learners of Mathematics

As career-changers, who in most cases have been previously active in non-teaching careers, graduate preservice elementary teachers often approach their mathematics training with a unique set of challenges. According to Schloglmann's (2006) survey study of adult career-changers, it is important to distinguish between the larger group of career-changers who choose to participate in adult education and the smaller group who report feeling compelled to participate in adult education because of, for instance, losing their job. While Schloglmann found adults in the larger group were often highly motivated to learn mathematics and saw their courses as an opportunity for personal and financial improvement, those who felt compelled to take mathematics often viewed mathematics as a threat to completing their education. In many cases, these adult education participants perceived multiple reasons to fear mathematics as an obstacle to their educational attainment:

> [Adult students] often feel unable to learn, because their learning processes have occurred over a long time, whereas they recognize that mathematics learning requires intense processes of abstraction and generalization. A frequent consequence of this is a students' fear that his or her memory is unable to hold all the abstract concepts required. Formal learning often brings back memories of mathematics learning in school and it is generally accepted that for many adults these memories are bad. (Schloglmann, 2006, pp. 14-15)

Though many face a unique set of challenges to learning formal mathematics, there is substantial evidence that adults are able to succeed in learning postsecondary mathematics. Elliott (1990) found that non-traditional students (ages 25 and older) scored at the same level as traditional students (ages 18-24) on a test of basic mathematics skills. Interestingly, Elliott discovered a negative association between non-traditional female students' beliefs that luck played a role in mathematics achievement and their subsequent course grades, further underscoring the idiosyncratic ways in which personal beliefs and affect can influence mathematics performance among adult learners of mathematics. Similarly, Richardson's (1994) synthesis of research on non-traditional students suggests that older students typically have the basic skills to productively study and progress in higher education, but often contend with increased personal and financial concerns that can cause them to withdraw from school.

Although enrolment by adults over the age of 24 in higher education grew in the U.S. by nearly 150% from 1970 to 2000, these non-traditional students remain less likely than traditionally aged (18-24) students to complete degree programs; while approximately 54% of all traditionally aged U.S. college students complete a bachelor's degree in six years, only 41% of non-traditional males and 35% of non-traditional females tend to reach that goal (Taniguchi & Kaufman, 2005). Bean and Metzner's (1985) conceptual model for non-traditional student attrition identified four primary factors influencing the decision to drop out of a program of higher education: (1) current academic performance (e.g., GPA), (2) intent to leave, (3) past educational performance, and (4) environmental factors like family commitments. In highlighting the effects of the external environment on non-traditional student attrition, the authors contrasted the demands on adults' time outside of school to the prominent role that adolescent variables, such as identity formation and social integration, have in traditional student attrition. Some of the external demands faced by adult learners include employment, commute time (Schuetze & Slowey, 2002), financial aid, childcare, and remedial coursework (Bundy & Smith, 2004).

Adult learners certainly face challenges to succeeding in higher education, but many of the external factors influencing their lives (e.g., family, career) can be seen as sources of motivation. For example, Chao and Good (2004) found many adult students drew motivation from support provided by friends, family, and faculty, and from a more mature understanding of the financial investment associated with higher education and the prospects for career development. Similarly, Blair, McPake, and Munn (1995) found myriad reasons supporting adults' choices to return to school, including "gaining qualifications or skills to secure a better job, enjoyment, learning for its own sake, getting out of the house, making new friends, and gaining a place on a more advanced course" (p. 637). Their study participants mentioned grants, student loans, and support systems as necessary aids in returning to school. Jane, one of the participants, shared her reasons for returning to school that included bettering her children's future and socializing:

> This is me trying to get back. [I wanted] to provide my youngest son with some type of nursery education and give myself something to take my mind off everything…It gives me the chance to meet other people—otherwise I would be totally isolated…I was given encouragement and support…It helped me get myself sorted out and it's given support for me and my children. (Blair, et al., 1995, pp. 644)

The Mathematical Knowledge and Beliefs of Preservice Elementary Teachers

Ball, Thames, and Phelps (2008) have recently claimed that there may be nothing more fundamental to teacher competency than how well teachers know the subjects they teach. Relying on the rationale that "improving the mathematics learning of every child depends on making central the learning opportunities of our teachers" (Ball, 2003, p. 9), many elementary education teacher preparation programs emphasize building content-specific understanding of foundational mathematics such as representations of numbers and arithmetic operations (Lee, Meadows, & Lee, 2003; Welder, 2007). This has been a challenging goal, because preservice elementary teachers tend to enter their coursework less mathematically proficient and express more negative views of mathematics than the general population of college students (Rech, Hartzell, & Stephens, 1993). With limited opportunity (sometimes as little as a single mathematics course) to experience change in their beliefs and understanding of mathematics, many U.S. elementary teachers work with a persistent lack of the profound understanding of mathematics that serves as the base for successful mathematics teaching (Ball, Hill, & Bass, 2005; Ma, 1999)

Learning mathematics is not solely a cognitive activity (Schloglmann, 2006). Beliefs about the purposes for and underlying structures of mathematics, combined with emotional and physiological responses to encountering mathematics, can have a profound influence on the mathematical development of preservice elementary teachers (Wheeler, 2009) and the teaching practices of elementary teachers (Thompson, 1984). Not surprisingly, researchers have identified links between prospective teachers' beliefs and attitudes about mathematics and their mathematics performance (Harding-DeKam, 2005). Preservice elementary teachers' affective reactions to mathematics, especially reports of mathematics anxiety, are particularly prominent as potential sources for poor performance in the foundational mathematics courses offered in teacher preparation programs.

In his seminal meta-analysis of math anxiety literature, Hembree (1990) identified math anxiety as a profound barrier to people's ability to perform well in math and to continue their formal math education. People with math anxiety experience fear "when they are called on to do math—whether it is working through a problem at the chalk board as an entire class looks on, taking a math test, or even calculating a restaurant bill" that can "prevent them from using the math knowledge they possess to show what they know" (Ashcraft & Kirk, 2001 as cited in; Beilock, Gunderson, Ramirez, & Levine, 2010, p. 1). People with math anxiety may exhibit intense worries and self-doubt that can compromise their mathematical thinking and reasoning (Beilock, 2008); and therefore, math anxious students tend to "perform more poorly than their abilities would suggest when they are exposed to math" (Beilock, et al., 2010, p. 1).

Not surprisingly, Hembree (1990) found increases in math anxiety correlate to lower achievement in mathematics across grade levels. However, among college students, those who majored in elementary education had the highest levels of math anxiety, compounding the obstacles already faced by preservice elementary teachers. Math anxiety is especially concerning among preservice elementary teachers because it has negatively associated with content knowledge for teaching mathematics (Gleason, 2008) and because it can result in:

> (a) a potential on the part of mathematics anxious teachers to pass their anxiety on to their students thus inhibiting their future mathematics attainment; and (b) engagement, on the part of teachers, in mathematics avoidance behaviours and the use of less than effective instructional techniques. (Brady & Bowd, 2005, p. 44).

Background

Setting

All of the study participants were enrolled in a non-traditional childhood education program at a large, public, urban university in New York City. This university offers an accredited and nationally recognized graduate program in childhood education leading to initial certification in childhood education (grades 1-6) in the state of New York. As a graduate program specifically designed for career-switchers, it is comprised of students with varying educational backgrounds and work experiences. Most adults enter the program having had substantial work experience after the completion of their undergraduate programs, but they typically start the program with very little or no prior teaching experience. During this one and a half to two-year program, participants take a pre-planned sequence of courses, which includes one required course in the teaching and learning of mathematics in the elementary school. This three-credit, semester-long course meets face-to-face during 15 weekly 150-minute sessions. In addition, the students are required to spend at least ten hours observing mathematics instruction at a public New York City elementary school of their choice.

According to the syllabus prepared by the course coordinator, the elementary mathematics course is designed to provide students an opportunity to (1) become knowledgeable of fundamental skills and concepts related to the elementary school mathematics curriculum, (2) develop teaching strategies and appropriate assessment techniques related to elementary school mathematics instruction, and (3) discover and/or enrich an enjoyment of learning and teaching mathematics. Several sections of the course are offered every term, including summer sessions, and the sections are taught by a small set of mathematics education professors (including the primary author) and a rotating set of adjunct instructors.

Research Questions

Within the context of the conceptual framework and prior findings identified by the review of literature, our study addressees two research questions:

1. In what ways do graduate preservice elementary teachers perceive prior learning experiences as influencing their beliefs and anxiety about mathematics?
2. To what extent do graduate preservice elementary teachers perceive their childhood education program as adequately preparing them to teach elementary mathematics?

Methods

Research Design

We used a mixed-methods design with special emphasis on qualitative methods because we believed a focus-group methodology afforded opportunities to develop detailed views of participants' experiences with and perceptions of their preparation to teach mathematics. Our interview protocol incorporated open-ended questions in an effort to limit potential to bias students' responses and allow for themes and variations to emerge from the participants' responses during the analysis of transcripts.

Two semi-structured interviews (Merriam, 1998) allowed us to tailor questions to session participants and to gather data on similar aspects of the participants' experiences while leaving room for dynamic questioning to probe the participants' unanticipated responses during the interview (Creswell, 2009). In addition to limiting the potential for constrained responses due to participants' feeling intimidated by discussing their mathematics experiences with a mathematics educator in their program, we believed the focus-groups would give study participants a chance to openly discuss their mathematics experiences within a group of their peers and to learn from one another. The design also allowed us to compare and contrast responses in the two interviews and to better understand the contexts and scope of emerging themes.

Our research design has several limitations that are important to acknowledge. First, the inquiry included a small sample of graduate preservice elementary teachers at a single urban alternative teacher education program. Secondly, seven of the ten interview participants had previously completed a section of the mathematics course taught by the primary researcher. Though this increased familiarity of the interview participants with the interviewer afforded opportunities, it also may have subtly influenced participants' interview responses. Third, the exploratory nature of our study and relatively limited duration provided for to construct only preliminary conclusions regarding the research questions. As in most studies of this scale, we invite readers to consider the context surrounding our study and characteristics of the participants when considering the potential transferability of the findings to other settings (Patton, 2002).

Data Collection and Analysis

The data sources for our study were records from two extended group interviews and three surveys. The interviews included ten participants divided into two focus groups of five, based on their availability. The interviews lasted two hours each and were audio and video recorded, transcribed verbatim by the research team, and analyzed using the professional qualitative data analysis software program NVivo 8 (QSR International, 2008).

In addition to the focus group interview data, we collected participants' responses to three surveys. First, a background survey asked participants to provide demographic and contextual personal data. Background variables collected included age, years since completing secondary school, undergraduate major, sex, race/ethnicity, and employment history. In addition, participants were asked to describe their prior secondary and university-level mathematics experience as well as plans for working while in the program and teaching after completing the program. Secondly, participants completed a survey regarding their experiences with and perceptions of mathematical manipulative before, during, and after completing the elementary mathematics course. LastIn addition,ly, students completed the Mathematics Anxiety Rating Scale–Shortened Version (MARS-SV), developed and validated by Suinn and Winston (2003) as a shortened version of the original and widely used MARS survey instrument (F. C. Richardson & Suinn, 1972).

The MARS-SV contains 30 brief descriptions of behavioural situations. For each item (e.g., "thinking about a mathematics exam - five minutes before"), participants are rated their anxiety level on a Likert scale from 1 (not at all anxious) to 5 (very much anxious). Mean responses for the 15 items in each of the two subscales in the MARS-SV, namely test math anxiety and numerical math anxiety, can be interpreted on the same 5-point anxiety scale. Normative data has not been published for the MARS-SV instrument that might allow for diagnostic assessments of the levels of mathematics anxiety experienced by our study participants (Plaisance, 2007; Suinn & Winston, 2003). We used participants' scores on the test math anxiety and numerical math anxiety subscales – each ranging from 1 (low anxiety) to 5 (high anxiety), in combination with interview responses to develop holistic descriptions of participants' levels of math anxiety. Collectively, the interview participants reported moderate-to-high test math anxiety ($M = 3.4$, $SD = 0.6$, range = 2.2 to 4.1) and low numerical math anxiety ($M = 1.8$, $SD = 0.5$, range = 1.1 to 2.6). At the two ends of the test math anxiety spectrum, Aidan and Mika reported low levels of test math anxiety (scoring 1.6 and 2.2, respectively), while Olivia and Bailey described the highest levels (each scoring 4.1).

We open-coded (Patton, 2002) the interview transcripts with descriptive phrases based on our review of literature and research questions, then inductively synthesized the codes into categories and themes among responses. Throughout our analysis, we placed special emphasis on the variety of participants' experience and confirming and disconfirming evidence regarding emergent themes. To add credibility to findings, we triangulated themes (Creswell, 2007) with the quantitative and qualitative data collected through the three survey instruments and member-checked (Merriam, 1998) our results with one of the study participants.

Description of Study Participants

Study participants were recruited from various sections of the mathematics for elementary teachers course offered at the research site during the fall semester of 2009. Seven of the ten interview participants completed the course together in one section taught by the primary author. A single participant came from a second section, taught by a fellow mathematics education professor; the remaining two participants took the course from an adjunct instructor. All were graduate students in the second semester of their first year of a non-traditional elementary teacher preparation program. Each student had earned a passing grade in the course during the previous semester. Since this course was not only the sole required, but also the sole offered, mathematics course in the program, the students had finished their mathematics coursework in the program.

Most of the study participants (8 of 10) were female and all but one described themselves as White (the remaining participant was Asian-American). Study participants ranged in age from 23 to 41 (M = 27.7, SD = 6.1), with five participants over the age of 24. The participants had each completed a U.S. bachelor's degree program up to 13 years prior (M = 4.0, SD = 4.9) to enrolling in the elementary certification program, and had professional work experience prior to joining the graduate program. The wide variety of reported occupations included flight attendant, publicist, assistant to an attorney, coordinator of an English language development program, substitute elementary school teacher, data analyst, teaching assistant at an elementary school, and activities coordinator at a small museum.

Five participants continued to work while they completed the graduate program, retaining jobs such as nanny, server at a restaurant, and data analyst, and reported working a mean of 31.5 hours per week (SD = 11.4) while completing their teacher preparation coursework. Though most of the prospective teachers (8 of 10) planned to teach in the New York City area after obtaining certification, they had substantial commutes to the central urban campus location (M = 48.5 minutes each way, SD = 12.3, Range = 23 to 68). All of the ten participants' undergraduate majors were in non-teaching fields and had no specific mathematics requirements (e.g., Urban Studies, Spanish, Art History, Communication Studies). Consequently, the participants reported limited college mathematics experience (just two participants completed a college calculus course), and three of the participants were required to complete an additional college-level mathematics course as a condition of admission into the teacher preparation program.

Results

Themes from our analysis suggest study participants desired additional coursework in mathematics content and methods, improved support and guidance for field experiences, and more opportunities to overcome mathematics anxiety stemming from their prior experiences as learners.

Theme #1 – Prior Experiences and Mathematics Anxiety: "You just get through it until you don't have to do it anymore."

Our inquiry revealed five primary sources of math anxiety among graduate preservice elementary teachers relating to the participants' primary and secondary school experiences.

These included (1) experiences of ability-level tracking in elementary and middle school, (2) comments from previous mathematics teachers, (3) high expectations for performance in advanced coursework, (4) poor performance in previous mathematics classes, and (5) struggles during the transition to formal mathematics in middle and high school. In the following narrative, we provide specific examples of participants' accounts of these perceived sources of mathematics anxiety with the aims of highlighting the variety of the participants' experiences and stressing the profound ways in which the participants appeared to have internalized their early mathematical experiences.

Several participants traced experiences of mathematics anxiety to being tracked by ability into separate mathematics courses from their peers during early adolescence. When asked to describe the roots of her math anxiety, Cassidy pointed first to tracking: "I think it was when everything started to split up into the smart class, the middle-ground class, and the slow class. That was a huge factor." Scarlett's experience with tracking was also personally meaningful. She recalled doing well in mathematics in elementary school, even volunteering to attend extra mathematics sessions during recess in the fourth grade. In seventh grade, however, she recalled being transferred to a lower-level mathematics class halfway into the year: "I remember it being, to me, a huge deal, and since then I always felt really timid about math because of that one experience of being bumped down as a kid." Even though she was later tracked into advanced secondary classes, Scarlett reported avoiding higher-level mathematics coursework in high school and college, saying, "Anything to not touch a number."

In addition to describing social messages associated with tracking in mathematics courses, some participants' described perceptions of inadequate or hurtful teachers as affecting their mathematics anxiety. Logan recalled a teacher that put a refrigerator box around his desk because he was often distracted during class. Natasha recalled a teacher who taught exclusively from her chair, asking students to write main points on the board for her during her lectures. Olivia, described a particularly difficult year in her first secondary mathematics class:

> In ninth grade, I didn't do well at all. I had a horrible teacher and I cried after every class, she was really unsupportive and I didn't really get what was going on… There was a lot of shame involved, and with something like math… So many people experience this… you know, to have someone tell you you're an idiot, you're so stupid for not getting it…I don't even remember specific comments. I just remember the feeling like, whatever it was she said, it made me feel like I was just, that I'm just not a math person, that math is just not for me.

At 41 years of age, Natasha was the oldest participant in the study. She described being "full of math anxiety," but also recalled being an average mathematics student until a very difficult freshman mathematics class in high school:

> I hit high school and I had this horrendous algebra teacher freshman year - Mrs. Johnson… She made you feel very stupid if you didn't get it. I remember she was discussing I think it was distributive property and I remember her going like this [makes a forceful grabbing and dropping gesture with her hand]. Really angrily, "It's distributive, what don't you understand? Why don't you get it?!" Going just really angry, showing that she was distributing something. I'm like, "I have no idea what you're doing with this, what does this mean?" And I failed. I had to take it again in my next year.

Mika, Logan, Belle, Scarlett, and Olivia each described experiencing pressures to perform well in advanced mathematics coursework during secondary school. The internal and external pressures to meet the high expectations for success in mathematics led to experiences of math anxiety among Scarlett, Belle, and Olivia in particular. Belle, who struggled in mathematics during high school, summarized the strain associated with a culture of success among her schoolmates:

> I went to a preparatory school that was competitive and really hard, and I remember just feeling terrible about not being as smart as my friends and not doing as well in math. There was shame and embarrassment. And I think part of it is not wanting to admit that you don't get something and so my strategy was to put it off.

Belle's experience of wanting to avoid mathematics as a result of episodes of poor performance was a consistent theme across nearly all study participants. Scarlett deliberately avoided an advanced placement course as a senior in high school and took a philosophy logic class in college to satisfy her general mathematics requirement. Aidan, who reported low mathematics anxiety and a strong affinity toward mathematics, said he still felt shame years later because of dropping out of an engineering program due to failing a college calculus class. Cassidy described taking a view of mathematics as a kind of academic waiting game, "You just get through it until you don't have to do it anymore. I think since I was probably in seventh grade... I've been mentally counting down until the day that I don't have to do math anymore." Natasha, who believed her dyslexia may have interfered with her ability to learn secondary mathematics, put it most clearly: "If I can avoid [mathematics], I will."

A final source of mathematics anxiety reported by participants regarded academic and cognitive struggles with learning mathematics during the transition to secondary mathematics, especially while studying formal algebra. Bailey, for example, remembered doing well in mathematics until high school, when she felt a shift in her perceptions of whether she was able to do mathematics:

> As soon as I got to high school, it was like, you can't really coast by... I had some really good teachers, but it was weird that not until you're a teenager do you realize that there's something I'm really, really bad at. I'm good in school all around, so that was kind of a shock. And it's continued to this day.

Natasha likened the introduction of algebraic notation to doing mathematics in an unfamiliar language, saying, "It's like here, do this math in Spanish." Aidan described the sudden need for algebraic competency in high school as "a shock to the system" that required a shift in thinking for students as they made the transition to formal mathematics.

Theme #2 – Quantity of Mathematics Instruction: "It's a good stepping stone, but it's just one."

In order to address the second research question, we asked the interview participants about their perceptions of the single required mathematics course in the program. The participants responded by talking about their initial reactions to taking mathematics courses as part of their elementary teacher preparation program, their experiences in the course they completed, and whether they would benefit from additional mathematics coursework.

Several participants described efforts to avoid taking multiple mathematics courses in their teacher education programs. Scarlett, for example, said she did not know what to expect in mathematics courses, feared not being able to succeed in them, and even avoided other teacher education programs because they required multiple mathematics courses. After taking the course, however she expressed a consensus among participants that more mathematics courses are needed in the teacher preparation program:

> Coming into the program, I thought one semester…a refresher would be fine. I really had this thought that you could look at the textbook also and if you didn't get something, you'll have the textbook to help teach you. And now it's so clear how important it is for you to have a full understanding in order to impart it to kids with different ways of teaching math. I would feel way more prepared to have at least another semester of delving into [math] more.

Natasha, Olivia, and Cassidy also noted a shift in their thinking about elementary mathematics after beginning the mathematics for elementary teachers course. They described feeling overwhelmed by the breadth of elementary mathematics content and agreed that a one-semester course is not enough to properly prepare them to effectively teach all levels of elementary mathematics. Natasha described a strong feeling of nervousness surrounding having to teach mathematics, and Cassidy said the instructor "did a really fantastic job giving us a good primer for teaching math...It's a good stepping stone, but it's just one. I definitely don't feel ready to go into a classroom."

Although some participants said they initially wanted as little mathematics coursework as possible in their elementary teacher preparation programs, after completing the one required course of their program, several students echoed Scarlett's desire for additional mathematics instruction. As future teachers, they perceived they may need to teach mathematics differently from the ways in which they were taught mathematics as children. Sasha compared the teachings of the class to her elementary learning experiences saying, "Yeah, we never learned with manipulatives, we didn't have problem-based lessons so it's a totally new thing." Natasha and Olivia's comments exemplify the apparent consensus during the two group interviews:

> I think another class would absolutely do it. Because the foundation I was getting in the class was great and I know those things backwards and forwards and I feel confident about them. But we only got through a certain amount of [content] so there's a lot more out there that I'm just not comfortable with yet. –Natasha

> I think a full year with one teacher, one course, designed...doing exactly what we did in the beginning but then continuing on to the higher level stuff. A lot of people in the class aim to teach fourth through sixth grade…I think it would make sense to focus there...I have a goal to teach the upper levels, fourth or fifth grade, and I'm feeling anxiety about that content. –Olivia

Participants described wanting at least one additional course to continue exploration of ideas beyond where their course had ended. They suggested a sequence of two consecutive courses taught by the same instructor that could be comprehensively designed to continue the current class onto topics relevant to upper elementary and middle grades, such as fractions, decimals, and algebra. There was much agreement in the focus group interview after Natasha confessed, “I'm really not comfortable with fractions at all. Fractions are frightening, and I feel like I need an algebra class.”

Unfortunately, there are currently no other graduate mathematics courses offered at the research site that would be appropriate for preservice elementary teachers, even if they elected to take one in addition to their required coursework. In lieu of additional courses being offered, the participants suggested rearranging the program’s pre-planned sequence of courses so that full-time graduate students are not enrolled in their only mathematics course during the first semester of their program. Natasha proposed:

> If they can't add another course, if they could at least put the course we did have later on in the program. Because first semester we hadn't had any real field experience, we didn't even know what questions to ask. And I'm just starting to formulate them now. But I didn't even know what I didn't know…it was just a little too soon to have it first semester. Ideally, I would like to have this first semester and then another one later on. If that's not going to happen, then at least move it on a little bit. Because I was unprepared for what math looked like.

Theme #3 – Quality of Mathematics Content and Instruction: “It's how in touch you are with what it's actually like to be an elementary school teacher, teaching math.”

We asked the interview participants about their perceptions of the curriculum and instruction they received during their mathematics for elementary teachers course. The participants, who had each taken the course from one of three different instructors, specified various elements of their course that were beneficial to their learning and offered critiques of, and suggestions for improving, course content and instruction.

The participants agreed on a need for instructors with specialized experience in the field of elementary education. Comparing her experience with other students in the program, Scarlett noted:

> I remember talking to other people in other classes…they said that they had experiences where it was just someone who was a mathematics professor…just teaching math and not focusing on how are you going to teach this to kids and how does this fit in to education at large.

She also commented that institutions of higher education need to, “ensur[e] that instructors for these courses are people that are directly involved in education and mathematics education... it's how in touch you are with what it's actually like to be an elementary school teacher, teaching math.” Moreover, Aidan believed the ever-changing set of adjunct professors who teach the mathematics course at the research site could also negatively affect the quality of instruction.

Besides variations in the quality of instructors across sections of the course, participants made several comments about the mathematics content that had been covered in their courses. Bailey, who took the course from a mathematics education professor not involved with the research, described a perception that the content covered in her class was not relevant to elementary mathematics.

> [The professor] would pick a topic and then go really into detail about it. And we would just kind of be like, okay, I don't really know what that has to do with elementary math…"what you [professor] are talking about, I have no idea what that is." And the things that I've actually seen used in the classroom in elementary school, we did not even talk about it. We didn't even talk about ways to teach math to children. We did some math that was maybe for high school and college kids, just actually did the math...so I really didn't learn anything.

Bailey elaborated on her perceived disconnect between course content and elementary teaching when discussing manipulatives:

> We had one day when we talked about [base ten blocks]...He just sort of gave them out for us to play with them, and he was like, "Do this problem," but then we went onto something else. And it was like, okay, but how does that relate to elementary students? How would we use this in a lesson? I would have appreciated more discussion about how this is going to help [children] continue their learning and build on their prior knowledge of math.

Bailey thought the course did not address the mathematics currently taught in the elementary school curriculum, especially the many different representations and procedures she perceived as being commonplace in local curricula. When Olivia mentioned that other preservice teachers in the program hadn't even see the lattice method for multiplication, Bailey interjected, "I've never seen that before." Aidan identified this variation in content as forming a "huge disconnect" in the course curriculum. In addition, some participants voiced concerns and fears about integrating the teaching strategies modelled in the mathematics for elementary teacher course with school and state-mandated mathematics curricula:

> I'm kind of nervous about reconciling the curriculum with what we've been learning about…spending a lot of time working with manipulatives and allowing that time for students to discover connections and understand...and from what I understand, that's not necessarily aligned with the curriculum used in New York City. I'm just nervous about adapting the curriculum to fit my beliefs about education, and how math should be taught. And I don't know what kind of support I'm going to have in modifying the curriculum if I'd like to. –Olivia

In contrast to the dissatisfaction expressed by participants regarding the quality of course content, Riley described finding value in an activity in which the students in her class analyzed procedural errors made by children:

> One particular thing that was helpful that I've never actually done in a math class before is looking at student errors and figuring out what went wrong, how they were thinking about the problems, how they got that wrong answer, and then talking about the different ways that we came up with, like, what they could have done… I never considered doing that…I thought about what questions the students might ask me and how I would respond.

In addition to the course content, participants described pedagogical elements of their mathematics for elementary teacher courses that they found beneficial. Olivia said she learned from watching her instructor model effective teaching methods, and Cassidy said she found it "refreshing" that her instructor focused on conceptual understanding throughout the course:

> I would say it kind of blew my mind just to think about math from a completely different perspective…[The instructor] focused so much on really understanding what was going on, and not necessarily getting the right answer. [The instructor] just reinforced that throughout the entire semester, you know, "why did you get what you did,"…the process and the whole thing. That was refreshing because that was the first time that I think I had really ever thought about math like that.

Moreover, several participants said they enjoyed working together in small groups during class time. These participants were all members of the same class, where the students worked in consistent, self-selected groups throughout the semester. Scarlett described feeling a sense of community and positive interdependence during group work:

> You feel like you're entering into a community more. I think most people in the program really feel dedicated to education on the whole and so you are invested in helping your peers do well because it's only benefiting kids and the education community at large. I felt a sense of that.

Belle said that working in groups helped her to better understand how others learn, and Olivia added that she enjoyed the spirit of collaboration established in the class:

> It wasn't competitive. I think we were all really collaborating. For a teacher preparation program, I think if you have that dynamic, the learning environment is really effective. I liked it a lot more than other classes, which were just round tables. Pretty much in every class, the best experience is doing group work. At this level.

Finally, Cassidy identified the pace of the class as being an important element in her learning:

> I felt so comfortable with the pace that we were moving. It was like we were making progress and chopping away at things, but if we also were allowed time to really let things sink in...I felt like it gave us an opportunity to really feel comfortable with the material and not just in taking it in, but in spitting it back out.

Discussion

Summary of Findings

The majority of our participants described experiencing mathematics anxiety and identified an array of experiences they believed led to feelings of inadequacy and fear surrounding mathematics. The participants seemed to have internalized their early mathematical experiences from primary and secondary school, allowing prior events in their education to have remarkably direct influences on their learning of mathematics as adults. Their stories provided insight into the variety of elements of mathematics education at all levels, both content and pedagogical, that can stimulate and perpetuate mathematics anxiety.

In addition, participants described taking a negative view toward mathematics prerequisites when they were selecting a teacher preparation program. Some had purposely avoided programs that would have required them to take higher-level mathematics courses to make up for missing prerequisites, mentioning fears of not being able to succeed and feelings that such courses would not be relevant to their futures as elementary teachers. However, after completing the one required course of their current program, many students described wanting additional mathematics instruction. The results support the claim that effective elementary mathematics teacher preparation courses can change the way preservice teachers think about mathematics, and they highlight needs for (1) qualified instructors who can make content relevant to teachers, (2) additional opportunities for preservice teachers to continue developing their mathematics skills, and (3) program structures that support the pedagogical development of preservice teachers.

Connections to Mathematics Anxiety Literature

As mentioned in the review of literature, our study contributes to the large body of research on mathematics anxiety among preservice elementary teachers by focusing on the important subpopulation of adult career-switchers enrolled in non-traditional teacher preparation programs. Our results suggest that participants' perceived five primary sources of mathematics anxiety; we will now discuss how our findings compare to the related literature on origins of mathematics anxiety among preservice elementary teachers.

First, our work supports several studies that have indicated mathematics anxiety experienced by preservice teachers in collegiate content courses is largely rooted in the teachers' early experiences as mathematics learners (Brady & Bowd, 2005; Gresham, 2007; Uusimaki & Nason, 2004). As Philippou and Natashatou (1998) suggested, "teachers' formative experiences in mathematics emerge as key players in the process of teaching since what they do in the classroom reflects their own thoughts and beliefs" (p. 191). Second, our finding that participants traced anxiety to negative experiences with mathematics teachers and coursework during their transition to high school supports Brady and Bowd's (2005) conclusion that preservice elementary teachers' mathematics anxiety often stems from negative experiences in secondary school. The nature of the negative experiences described by our participants was consistent with the those reported by Brady and Bowd, whose participants encountered (1) fast-paced instruction, (2) comments from teachers that made them feel inadequate because of difficulty in understanding content, and (3) in one instance, being given a passing grade under the condition that the student take no further mathematics courses. In addition, we found that several participants avoided mathematics as a way of coping with feelings of mathematics anxiety and poor performance in school mathematics, and this underscores the importance of what Brady and Bowd (2005) term a "cyclical phenomenon" in mathematics education:

> Negative experiences with formal mathematics instruction led many participants to discontinue their study of the subject, or discouraged them from pursuing formal mathematics instruction beyond that which was necessary to fulfil high school graduation or university admission requirements. This led to the perception on the part of many respondents that their mathematics education had not prepared them to teach the subject confidently, a condition that has the potential to be replicated in their students. (Brady & Bowd, (2005, p. 45)

Our results regarding mathematics anxiety do, however, contrast with some findings in the mathematics anxiety literature. For instance, Uusimaki and Nason (2004) found that

Australian preservice elementary teachers with high mathematics anxiety mostly attributed it to negative experiences with primary school teachers rather than to specific content or social factors, such as messages received from family or peers. However, with our participants, we heard more references to secondary school as a source of mathematics anxiety than elementary school; furthermore, our participants identified several social factors as primary sources of their anxiety, including messages associated with tracking and high expectations established by family members and peers. Our findings did align with Uusimaki and Nason's indications that specific mathematical content and social factors contributed to some participants' math anxiety, including (1) being asked to communicate their mathematics knowledge, (2) efforts to learn algebra, and (3) experiences with space and number sense content.

Another contrast we identified was between our participant's views of manipulatives and some reports indicating that the introduction of manipulatives in collegiate mathematics courses can be a source of mathematics anxiety for preservice elementary teachers (Gresham, 2007, 2008; Vinson, 2001). Vinson (2001) found that during their engagement with mathematics materials during methods courses, some study participants found that their math anxiety increased, largely due to a lack of prior experience using manipulatives; these students "were struggling with relearning mathematics at the same time that they were learning to use the manipulatives" (p. 93). In addition, one of the participants in Gresham's (2008) study described the apprehension felt by many preservice elementary teachers who encounter manipulatives for the first time:

> I don't ever remember using manipulatives when I was in school. I remember my teacher giving us lots of worksheets or problems from our math book and that's how we learned. Either you got it or you didn't. It was horrible. During the math methods class, the instructor brought in all sorts of manipulatives. I freaked out and was so terrified at first, but after using them in class and learning how to use them to help us teach, I actually started enjoying the math I was learning. (Gresham, 2008, p. 179)

In comparison, all of the participants in our study described their interaction with manipulatives positively. Given the different study populations and sample sizes, our results do not necessarily contradict Gresham or Vinson's findings, but the contrasting accounts from our participants do suggest a potential need for future research in this area.

Implications

For mathematicians and mathematics educators involved in the preparation of elementary teachers, non-traditional aged students and career-switchers present a unique set of challenges. Our study, in combination with existing literature, suggests that many of these prospective teachers have a history of negative experiences as learners of mathematics, feel moderate-to-high levels of mathematics anxiety (especially in regards to tests), and can view mathematics as a threat to their ability to be a successful teacher. However, we also found our study participants to be remarkably self-aware of their mathematics anxiety and to hold well-reasoned views on what they need from their mathematics preparation to overcome these feelings. Collectively, our results suggest graduate preservice elementary teachers may particularly benefit from well-articulated, reform-oriented mathematics instruction (i.e., in the spirit of recent recommendations by the National Council of Teachers of Mathematics) and supplemental opportunities to apply and extend their emerging understanding of mathematics.

One of the most interesting outcomes of our study was that many of our participants described a history of difficulties achieving in and anxiety pertaining to mathematics, yet many specifically requested more mathematics coursework as part of their teacher preparation program. We found this result extremely encouraging and reflecting indications that our participants had positive experiences in their single required mathematics courses. If practical, it might well be beneficial to graduate preservice elementary teachers if there were an additional mathematics course with emphasis on, as suggested by the participants, upper-elementary topics such as fractions and algebra. Such a course might also be bolstered by concurrent field experiences for preservice teachers to engage with elementary students, especially early in a teacher education program when classroom experiences are particularly valued by prospective teachers. In cases where additional mathematics requirements are impractical, our participants suggested optional mathematics courses, professional development opportunities, and local conferences for teachers as welcome supplements. The integration of voluntary mathematics opportunities could be particularly attractive to non-traditional students who might be put-off from enrolling in a teacher preparation program with heavy mathematics requirements.

The primary method for coping with mathematics anxiety among our study participants appears to have been simply avoiding mathematics whenever possible. However, preservice teachers need to engage in accessible, yet challenging mathematics in order to be prepared to teach elementary mathematics effectively. Our interviews suggested several avenues for accomplishing this goal, including (1) communicating clear expectations for assignments and assessments, (2) modelling effective strategies for teaching mathematics, (3) explicitly connecting course content and local mathematics curricula, and (4) offering repeated opportunities for preservice teachers to demonstrate mastery of conceptual understanding. Logan (self-described as highly math anxious) reported a major reduction in his level of anxiety after realizing he could succeed in the elementary mathematics course. He stressed that this success led to new-found confidence: “I think the ability to do the math gave me a confidence to teach it as well.”

Our data suggested that participants perceived the use of manipulatives Our data suggested using manipulatives to represent elementary mathematics content was perceived by our participants to be pedagogically relevant, accessible, and cognitively helpful. Consequently, our study supports the common practice among mathematics educators to model mathematical content with physical manipulatives to emphasize a conceptual understanding of foundational mathematics (Vinson, 2001). As Levine (1996) found, “using manipulative materials to teach concepts underlying mathematical operations was different from the procedural focus of learning mathematics during [the preservice teachers'] elementary school experience and facilitated their conceptual understanding” (p. 7). Vinson’s (2001) study of 87 preservice elementary teachers enrolled in manipulative-rich mathematics courses reported that participants experienced better understanding of mathematics concepts and procedures when they were presented in pictorial or concrete forms rather than formal symbolic form alone. Our study supports Vinson’s findings in suggesting that elementary educators may benefit from teacher education programs that model effective manipulatives-based mathematics instruction.

Finally, our study identified several characteristics of a graduate elementary teacher preparation program that could potentially support the preparation of effective teachers of mathematics. Our participants stressed the importance of intra- and inter- disciplinary coordination across their coursework, especially consistency across sections of mathematics classes and communication between instructors delivering courses during a given semester. This would serve to allay some participants’ concerns that they received instruction that may have been disconnected from local mathematics curricula. Program coordination could also include improved integration between preservice teachers' field experiences in local schools and their content coursework. As Bailey put it, much of the content learning for a teacher can be motivated by the demands arising from authentic teaching experiences: "I've learned more [math] in two days of being in a third grade classroom than I did all last semester."

Directions for Future Research

Since our study was primarily exploratory in nature, one strength of our findings is that they suggest several potentially fruitful avenues for future research. Some important questions flowing from our work include:

- How can teacher preparation courses/programs mitigate potentially negative effects of preservice elementary teachers' mathematics anxiety on their future teaching?
- How havehas adult preservice elementary teachers' life experiences afforded them with support networks and coping strategies for overcoming mathematics anxiety?
- What effect does manipulative-based instruction have on preservice elementary teachers' levels of mathematics anxiety?
- Do preservice elementary teacher preparation programs with varying mathematics requirements recruit, retain, and/or produce teachers with differing levels of mathematics mastery or anxiety?
- What impact could applying the recommendations of this article have on graduate elementary teacher preparation programs, in terms of student enrollment and demographic composition of students?

Summary

The primary reason for conducting the current study was to better understand graduate preservice elementary teachers as adult learners of mathematics. We found our participants to be attentive, reflective, and highly-motivated career-switchers who viewed mathematics as a necessary and important component of their preparation to effectively teach in elementary schools. The participants often recalled substantial mathematics anxiety and a history of avoiding mathematics whenever possible as adolescents, a pattern they attributed to experiences in formal school mathematics, including tracking, negative experiences with secondary mathematics teachers, and poor performance in one or more mathematics classes. Along the way, our participants provided meaningful insights into potential ways to effectively prepare non-traditional preservice elementary teachers to understand and teach mathematics in a reform-based, conceptual manner. As mathematics educators, this work has provided us a better understanding of adult career-switchers enrolled in elementary teacher preparation programs and will influence our teaching and advising of these students. We hope our readers have also been inspired to further consider ways in which we can effectively serve this population of preservice elementary teachers.

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Mathematics in vocational education: Revisiting a developmental research project, Analysis of one development research project about the integration of mathematics in vocational subjects in upper secondary education in Sweden

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Abstract
In this article we describe and discuss the analyses of a developmental research project that took place in Sweden during 1998 to 2002. We carried out four different analyses in order to explore the learning outcomes from the project that could inform long term curriculum change and teacher collaboration in vocational education in Sweden. The analyses led to condensed descriptions of the project, including an exposition of its constraints and affordances as well as its developmental cycles. Also revealed was a comparison between the aims of the project and the actual outcomes. Based on the analyses and discussions, we suggest possible implications for practice and for future studies on vocational education.

Introduction
A reform of the upper secondary school in Sweden GY2011 will take place in 2011 and the preparatory discussions about the reform triggered the authors of this article to revisit and explore one developmental research project that was carried out during 1998-2002. We started this process of revisiting earlier data with current perspectives in 2007. The project KAM was initiated because of the former curriculum reform in 1994, Lpf94 (Utbildningsdepartementet, 1994). There were minor revisions of the reform during the time of the project (Skolverket, 2000). The questions that were asked focused on whether the project can inform us about how schools and teachers could improve mathematics learning experiences in vocational programmes and benefit from the results of the delayed analysis of the KAM-project.

The most critical issue in the discussion is whether pupils in vocational education benefit from studying the same first course in mathematics in upper secondary school as all other pupils. The argument for the first course to be the same for all students is that society is demanding more mathematics in almost all vocations and that all pupils should be prepared for that and be given the right to be able to directly build on to their upper secondary school education at a later stage in life. The arguments against are that pupils in vocational education need a different kind of mathematics course that is directly related, and seen by the students to be directly related, to their vocational studies and that it is the vocational interests that should guide the selection of topics and content.

The situation and problems behind the KAM-project

Over the years 1998 to 2002 a developmental research project, called the KAM-project, was carried out in an upper secondary school in a major city in Sweden. KAM stands for Karaktärsämnenas matematik, the mathematics of the vocational subjects. In this article we intend to describe, analyse and discuss this project in order to answer a number of research questions. We start by presenting the situation that gave rise to the project.

In 1994 a new curriculum, Lpf 94 (Utbildningsdepartementet 1994; Skolverket 2005, 2006), was selected for the Swedish upper secondary school. Under this curriculum, the organisation of upper secondary school meant, first of all, that pupils in the theoretical programmes and the vocational programmes were to study in the same school system and have the same study time of three years. Earlier, vocational education had taken place in separate schools, vocational schools, and followed a different curriculum over two years rather than three. Secondly, a compulsory mathematics course for all pupils, course A was introduced. Earlier pupils in vocational education did not study mathematics as a separate subject but were taught vocational calculations by the vocational teacher. Thirdly, the new curriculum Lpf 94 demanded collaboration between the teachers of mathematics and the main vocational subject in the programme, in order to ensure that the mathematics learnt was influenced by the needs of the pupils in the vocational part of the education. Additionally, it was the intention to ensure that the education would not prevent pupils who later wanted to continue to study at tertiary level from doing so.

All these demands were new to school leaders, teachers of mathematics and vocational subjects. The education of teachers of mathematics and vocational teachers were traditionally very different in nature and length and the teachers from the different areas had ways of working which were different and not well known to each other (Lindberg, 2009). Teacher education was not changed as a consequence of the new curriculum. Thus teachers were inquiring about how the new plans could be implemented and how to meet the new demands according to Lpf 94.

It was this demanding and confusing situation for teachers in vocational programmes that led some teacher educators to initiate what was going to grow to a developmental research study on the teaching of mathematics in those programmes. One of the authors of this article, Lindberg, took part as teacher educator and researcher in all the developmental cycles of KAM, while the other author, Grevholm, acted as researcher and scientific leader of the project between 1999 and 2002. Thus, being aware of our double roles in relation to KAM, we will carefully explain how we have tried to avoid subjectivity in the present analysis of the project.

In this article we will first present and discuss the new syllabus for mathematics, and after that discuss earlier research of relevance and related scientific literature. The research questions will be formulated, followed by the theoretical framework we intend to use for the analysis of the KAM-project. We then present and discuss the methods used, the data collection and the different stages of the analysis. Stage 1 is the analysis that resulted in the condensed description of the project. Stage 2 of the analysis consists of the findings of affordances and constraints in the project and stage 3 is an account of the developmental cycles we found in the study. Stage 4 is an analysis of the aims in the KAM-project in relation to the results of the project.

The vocational programmes in upper secondary school

The aim of the vocational programmes in upper secondary school is to provide an education that leads to the level where the pupil can have achieved acceptable vocational knowledge (Skolverket, 2006) that is to get their first job in a specific vocation. But the programme must also provide preparation that can be used as a basis for further studies later on. Most of the vocational programmes include practical components, where students are working in smaller groups (less than 15 per group) in a practical work situation, similar to an authentic work environment. These components include a main vocational subject taught by a vocational teacher. For example in the vehicle programme, pupils are working in a workshop similar to an authentic mechanical workshop, but most of the time without customers, time stress and the responsibilities for economic transactions.

The new syllabus of mathematics, especially course A

As mentioned, all pupils in the upper secondary school have to study the first course in mathematics, course A, as it is a core subject course and is included in all programmes. The content in the subject of mathematics is selected from a number of areas. Much of this content is covered in the mathematics courses of the compulsory school and the different parts are deepened and developed in the upper secondary school. Besides the content from the compulsory school new areas are introduced, deepened and gradually extended in the upper secondary school as the pupils take on more mathematics.

The directions of the studies are general so pupils should be able to master situations for themselves and for societal purposes. Emphasis is placed on mathematics providing knowledge to be used in the orientation of the students' programme of study, for example in a vocational programme. The exact wording is as follows:

> Both in everyday and vocational life, there is an increasing need to understand the meaning of and be able to communicate on issues with a mathematical content. … The power of mathematics as a tool for understanding and modelling reality becomes evident when the subject is applied to areas that are familiar to pupils. Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course. Knowledge of mathematics is a prerequisite for achieving many of the goals of the programme specific subjects. (Skolverket, 2000)

The written document indicates that there are many good opportunities to see the relevance of mathematics and the connections between mathematics and work. One can see that there is an emphasis on using mathematics within the study orientation and the vocational programmes. This means that the mathematics teacher could use content and applications from different vocations. The aims of the mathematics curriculum indicate a progression and extension from "to solve concrete problems in their immediate environment" to "solve problems that occur regularly in the home and society, which is needed as a foundation for further education" and further to "formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation" (Lpf94, 1994).

The curriculum is written in such a way that mathematics from the compulsory school is a base for applications and further development in course A. In algebra the goals in year 9 are "to be able to interpret and use simple formulae, to solve simple equations, as well as to be able to interpret and use graphs for functions describing real relationships and events" (Lpo94, 1994). This can be compared with the extension expressed in course A "to be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation", and "to be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems", and finally "to be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society". The pupils should "be accustomed, when solving problems, to using computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes" (Lpf, 1994). Thus the same kind of mathematical objects are treated but in somewhat different ways.

Here the study orientation is the same as the character of the specific programme the pupils are studying. It is obvious that it is intended that there is progression, extension and development over the years (See appendix 1). This intention does not necessarily reflect the development of knowledge in mathematics for each pupil.

The new Lpf94 curriculum was unique as it had a uniform post-16 educational system for the first time, and especially unique as the VET (Vocational Education and Training) was integrated in this system. The aim of the syllabus was a raised educational level in mathematics in Sweden for all citizens. The content of the course A was designed to strengthen the competence of the individual to function as a good citizen in a democratic society. At the same time there was a requirement that the course supported the students' understanding of the components of education that aim towards their future profession. National documents require the teachers to collaborate between the subjects and that the teaching of mathematics should demonstrate that the students can benefit from their mathematical knowledge when studying for the profession. The aim was to provide the student with opportunities to achieve the competence needed for citizenship and the labour market. The challenges for teachers were to make the connection between the content of the studies in mathematics and the reality of the post-school workplace visible. This requires more insight and knowledge for the teachers about different areas of content from mathematics and the vocational subjects that the pupils will meet within the vocational programme.

Earlier research and related literature on mathematics in vocational education

It seems to be unique for Sweden to offer vocational education as part of the upper secondary school system. Thus, we have not been able to find any international studies dealing with teaching mathematics in the kind of school programme we are investigating. We have reviewed methods used in studies on vocational education and found that they are of minor interest for this article.

The findings of this review were presented at a research conference and documented in Lindberg (2009). As the title, *To search for mathematics in the vocational teaching and learning- an overview of theories and methods,* indicates, the author searched for documents and research to find examples of theories and methods. The sources included databases such as MATHDI, proceedings from the Psychology of Mathematics (PME) and Adults Learning Mathematics (ALM) conferences. Most of the results presented came from research areas close to vocational education such as workplaces. The result shows that, over the years as technology became more accessible and easy to use, there was a trend towards using a multimodal approach when collecting data. It has, most often, not been obvious in the documentations what kind of theories the researchers were using, because theoretical underpinnings are rare.

The historical development of vocational education in Sweden was also investigated and documented in the paper *Historisk bakgrund till matematikens betydelse i yrkesprogrammen (Historical background to the importance of mathematics in vocational programmes)* (Lindberg, 2007). This paper is written in Swedish and gives an overview of the vocational education and training concerning mathematics up to the situation of today. Document analysis was carried out and based within a theoretical framework of hermeneutics (Gadamer, 1960).

In the ALM forum for researchers, practitioners and policy makers, Lindberg was one of the initiators of the topic group of mathematics education for the workplace. Over the years (2000-2004), when Lindberg was facilitator of the group, different aspects were discussed as concepts of workplace mathematics, data collection, differences between countries, lack of communication between mathematics educators and vocational educators (Lindberg, 2005).

The research questions

In revisiting the project from 1998 to 2002 we decided to try to answer the following questions:

- What are the characteristic features of the school based developmental research project KAM?
- What characterises the affordances and constraints that the involved participants in the project met during the work?
- What were the actual outcomes of the KAM-project compared to the aims?
- What are the conclusions and implications that can be drawn from this kind of developmental research project?

Theoretical considerations and framework

The implementation of a new curriculum and its syllabuses is a challenge for any school, school system and nation. The steps taken and signals given are not easy to see and interpret.

A development project, as described in this article, can illuminate and provide further insights to processes that can emerge out of changes and highlight outcomes that are both expected and unexpected. In order to answer the questions posed we have chosen to carry out several analyses based on complementary theoretical frameworks. Stage 1 and 4 in this process are based on a hermeneutic analysis of published documents. Stage 2 is based on the theoretical concepts of affordances and constraints and stage 3 is founded on theory of developmental research. We will in the following three subsections present the theoretical constructs and views that we used.

Hermeneutic analysis of documents

In a hermeneutic analysis the interpreter moves from parts of the text to smaller units, or vice versa, in trying to clarify meaning of the content. This movement between part and the whole, which leads to a deeper understanding, is called the hermeneutic spiral. The pre-understanding of the reader provides a certain holistic view as a starting point. The interpretation of different parts leads to a change of this holistic view and a new round of interpretation can take place (Ödman, 1979; Gadamer, 1960). There are three demands to fulfil: the system of interpretations should be coherent logically; the interpretations must have a connection with the object of interpretation; and the interpretation and its results must be disseminated to the reader in an appropriate way. From the presentation it should be explicit how the interpreter has reached the conclusions.

Theory of affordances and constraints

Using affordances and constraints to evaluate collaboration between teachers of different subjects in vocational education will give us one tool to analyse the project. Kennewell (2001) writes about use of the *concepts affordances and constraints* in didactical activity. According to Kennewell, affordances are the attributes that provide potential for action and the constraints are the conditions and relationships amongst attributes which provide structure and guidance to the course of actions. Affordances and constraints must be considered in relation to the abilities of the participants of the activity they support. Kennewell was studying use of ICT (Information and Communication Technology), but in our case this is replaced by integration of mathematics and vocational subject studies. In the KAM-project both pupils' abilities and teachers' abilities were studied. According to Kennewell (2001) the teacher's role is to orchestrate the supporting features in an attempt to make it possible for the learners to bridge the learning gap. Achieving learning effort on the side of the pupils is required to overcome the gap between their existing abilities and the intended abilities in the setting. Kennewell traces the supporting features of affordances to Gibson (1986) and of constraints to Greeno (1998). In the developmental project, KAM, the reports make use of similar expressions to describe what is going on. Thus we find it rewarding to try to use the *concepts affordances and constraints* in the analysis of the reported developmental work.

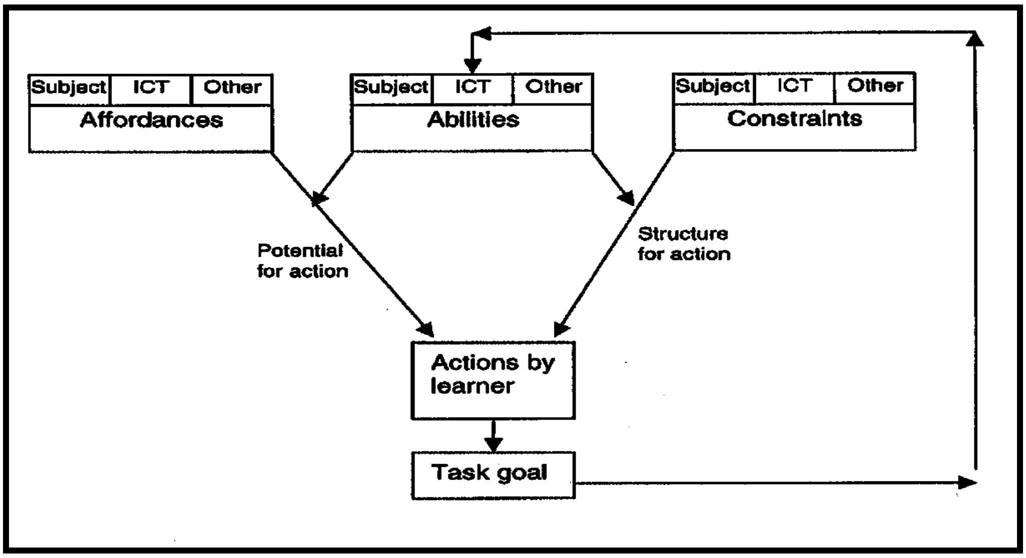

Figure 1. Kennewell's framework (Kennewell, 2001, p. 107)

As figure 1 shows, the teacher's intentions for the didactical activity are concerned with reflection and the development of abilities (Kennewell, 2001). Thus the framework used, incorporates task goals and outcomes according to the figure (, 2001).

Developmental research, the philosophy of the KAM-the project

An aim of the research project was to promote educational development, expressed as developing mathematics teaching and improving pupils' learning in mathematics by the integration of the studies in mathematics and the vocational subjects. Thus development of education is closely intertwined with the research project. Koeno Gravemeijer calls this developmental research and describes in some detail how he sees that (1994). He references Freudenthal (1988), who claimed that thought experiments are important in educational development. The developer envisions the teaching and learning process and, after implementing it, will try to find evidence to see if the expectations he had are confirmed or not. The feedback of practical experience into new thought experiments creates an iteration of development and research. Gravemeijer claims that this cyclic process is at the centre of Freudenthal's concept of developmental research (1994). Practice depends on a cyclic alternation of development and research and the cyclic process is more efficient when the cycle is shorter. The cyclic process can be seen as the learning process of the developer. A global theory, which Freudenthal, according to Gravemeijer, would prefer to call philosophy, guides the developmental work. This theory functions as the basis for the learning process by the developer and is nurtured by the alternation between thought experiment and practical experiment. That learning process can be interpreted as theory development.

Developmental research has some similarities with what is called design research. Eric Wittman is one researcher who sees mathematics education as design science (1998). He emphasizes that an important element in studying didactics of mathematics is building theory or theoretical frameworks related to the design of empirical investigations. Kelly (2003) writes about design research as a research dialect that attempts to support arguments constructed around the results of active innovation and intervention in classrooms. Di Sessa and Cobb

(2004) point out that in design research, theory must do real design work in generating, selecting and validating design alternatives. They also emphasise that development of theory should be one of the primary goals of design research.

We see the KAM-project as developmental research and the cyclic alternation process of development and research is crucial and we intend to illustrate parts of this process. What Freudenthal calls the philosophy of the work, the global theory behind the study, is in our case based on two important theoretical constructions, namely: integration of mathematics and vocational subjects; and a co-learning community of teachers. The integration of the subjects is both theoretical and practical. The theoretical part is the analyses of syllabuses and work material. The practical part is the performances in the school-settings. The co-learning community (COL) or professional learning community (PLC) is as Hord (1997, p. 1) describes it ”a powerful staff-development approach and a potent strategy for school change and improvement”.

Methods and methodology

The KAM-project, which was an empirical case study, had the character of developmental research project and has been reported to The National Agency for Education (Skolverket) in three parts. The project can be seen as a mainly qualitative case study and it ended in year 2002. We decided to explore the project again in an interpretative mode and learn more from it from a research perspective. Because of our involvement in the empirical study during 1998-2002 and risk for subjectivity we decided to use only the printed material from the project as sources in this part of the research. The documents available are the three KAM-reports (KAM1:1, 1999; KAM 1:2, 1999; KAM 2, 2001; KAM 3, 2002), the DUGA-report, printed working material, stored working notes and published conference papers presented about the project during the years. See table 1.

Report	Date of publication	Number of pages	Target group
DUGA	August 1996	35	National Agency for Education Gothenburg University
DUGA, attachment	August 1996	28	National Agency for Education Gothenburg University
KAM 1 part 1	January 1999	11	National Agency for Education
KAM 1 part 2	June 1999	42	National Agency for Education
KAM 2	September 2001	31	National Agency for Education
KAM 2 attachment	September 2001	78	National Agency for Education
KAM 3	November 2002	41	National Agency for Education

Table 1 Written documents used in the data collection

We started in 2007 by collecting all the available printed material and reading it. In order to be able to answer the research questions in the article we decided to divide the analysis in four parts or stages.

- Stage 1 is the analysis that resulted in the condensed description of the project. This step is needed both to make explicit our understanding of the project and for the reader to be able to follow the further presentation.

- Stage 2 of the analysis consists of the findings of affordances and constraints in the project. In the project reports challenges, problems and obstacles of different kinds became visible and the concepts of affordances and constraints seemed to be a helpful tool to use for this part.
- Stage 3 is an account of the developmental cycles we found in the study and the analysis of the developmental cycles of the project that we found by investigating the documents.
- Stage 4 is an analysis of the aims in the KAM-project in relation to the results of the project. Here we want to compare the actual outcome of the project as it was reported with the aims that were set.

For the condensed description in stage 1 of the KAM-project we have relied on the written reports from the different parts of the project. We have tried not to rely on our memories of how the work progressed but used the written reports from the different parts of the project as our source material. The analysis process started by reading and re-reading the reports. The interpretation of the text and the understanding of the project were developed in a hermeneutic spiral, where the relation between the parts and the holistic picture was used (Gadamer, 1960). The first purpose was to listen to the story told and to get a description of the work. The first reading activates one's memory but not in detail as the intention is to gain an overview. The second reading starts the reflection on the different parts of the project. The readers must return to the different project reports to check if the description still fits into their overall image of the project. Much time was spent on reading, rereading, checking and discussing. In the time when not focusing directly on the texts, reflection starts and questions are formulated for the next return to the text. The mode of reading thus changes. The answers are not easy to find at once, so the reading starts all over again with a new focus. The process goes in circles or as in a spiral. In the condensed description we decided to include the purpose and aim of the parts of KAM, the processes and data collection made, and the results.

For the analysis in stage 3 we use the theory of developmental research also presented earlier. In stage 4 again, we interpret the report texts in a hermeneutic way in order to reach understanding of presented aims and outcomes both in detail and from a more holistic perspective.

Methodological discussion and justification of methods used in this article

In order to interpret and create understanding of such a large and long-lasting project as KAM there are no self-evident methods to use. We had to struggle before we made decisions about use of theories. Our discussions will be made at least partly visible here.

The theory of legitimate peripheral participation in a community of practice (Lave & Wenger, 1991) could be considered to be used in this study. The discussion led us to realise that there was no existing community of practice with stable conditions for the teachers to enter into in the kind of school we are investigating. For both the mathematics and the vocational teachers the new situation means that they have to face new demands and there are no established practices for them to enter. The concept of apprenticeship might be used as an instrument for the analysis, but the project we are analysing is not organised as an apprenticeship model so this would not be adequate in our study. In reading the reports we found that the concept constraints had been used spontaneously and it was thus natural to use the theory of affordances and constraints, particularly as there were strong design aspects to the project. The reports from KAM talked about problems, challenges and opportunities, which seems to align well with Kennewell's theoretical views of affordances and constraints (Kennewell, 2001).

From the overview of the whole of the KAM-project it was clear that the project is a developmental research study. This was of course not obvious at the beginning of the project when the development parts dominated and the practical aims were in the major focus. Later the alternation between developmental work and theoretical influence became more explicit. We find it productive to interpret the process in the parts of the project in terms of developmental cycles, where empirical parts nurture the research influence of the project.

It is rare to find such delayed analyses of a developmental research project as the one we have made in this article. In the light of discussions about the usefulness and justifiability of educational research (Burkhard & Schoenfelt, 2003) we find this kind of investigation helpful and worthwhile. Especially as the structure for vocational education in upper secondary school is going to change again in 2011, this article can inform teachers and policy makers about possible actions in order to improve the mathematics in vocational education.

Participants in the different parts of the KAM-project

The KAM-project took place in an upper secondary school with vocational programmes. Involved in the project were university lecturers, a scientific leader, the project leader who was one of the mathematics teachers, other mathematics teachers, teachers in Swedish language, vehicular mechanics teachers and transport teachers. Furthermore, classes from the Vehicle programme and occasionally student teachers took part. All the teachers from the upper secondary school were the normal teachers in the school for the pupils in those programmes. The teachers who took part were practitioners that worked with the same groups of pupils. See table 2.

Project Part	**KAM 1**	**KAM 2**	**KAM 3**
Year	98-99	99-01	01-02
Mathematics teachers	3-5	3	3
Vocational teachers	2-5	5+3	3
Other teachers	2	1+1	1
Student teachers	2		
Pupils	2 classes (30 pupils)	1 class (14 pupils)	120 pupils
Researchers	2	2	2

Table 2 Number of participants in KAM 1-3 parts

Results from the analysis in step 1, the condensed descriptions of the project parts

In this section we describe in a condensed way the different parts of the KAM-project. In each case we include the purpose, aims, carrying out, data collection and results. Our main sources are the KAM project reports, i.e. the KAM 1-report (two reports, KAM 1:1 and KAM 1:2), the KAM 2- and KAM 3-reports.

Attention has been paid to research ethical aspects during the entire project and the following quote from the report of KAM 3 (p. 8) has among other things been a guiding star in the work:

> The project has been carried out and been accounted for in a fair manner, i.e. the scientific demands and accepted ethical norms are respected. The researchers also observe an ethically acceptable behaviour in their relations with the surrounding world. This includes for instance to avoid making public statements in the name of science regarding issues for which one lacks scientific competence. The equality between women and men is a moral value to protect. Equality in the scientific community is to be viewed as an ethical issue when it comes to research. The research activity should be documented in an open way to enable outside examiners to follow the entire process (Forsman, 1997).

The theoretical founding idea for the three parts of the KAM-project was collaboration between teachers in order to be able to integrate the course mathematics A and vocational subjects. The students were supposed to experience relevant knowledge as one unit to get a holistic impression of the knowledge for their vocation.

Report	Theoretical framework/ tacit or communicated	Methods /Data collection	Number of pages	Authors
KAM 1:1 KAM 1:2		Tests Analyses of syllabi, materials, text-books, etc	11 42	Kilborn & Maerker
KAM 2	Situated cognition and learning, ethnography	Study visits, Classroom Observations, Field notes, Interviews, Recordings, Tests, Questionnaires	31	Grevholm, Lindberg, & Maerker
KAM 2 attachment		Summary of test results, examples of planning- and teaching material, examples of pre-tests, teaching analyses, classroom observations, reflections, recordings and diaries	78	Grevholm, Lindberg, & Maerker
KAM 3	Action research and ethnography	Renewed analyses, of materials, text-books, etc Tests, Interviews, Observations, Field notes, Questionnaires, Surveys with quantitative and qualitative data	41	Grevholm, Lindberg, & Maerker

Table 3 Overview of printed reports from KAM

We now present the short accounts of the three parts of KAM following the structure sketched in table 3.

KAM 1

Purpose

According to the reports from KAM 1 (KAM 1:1, 1999, KAM 1: 2, 1999), one purpose was to work out mathematical models, where the students could apply and develop the mathematics, via co-operation between mathematics teachers and vocational teachers in a systematic way. Those models should be more appropriate and better suited to the students' preknowledge and abilities. Another purpose was to analyse what content in mathematics that is required to be used as a tool for students in vocational programmes where only mathematics course A is compulsory. One more aim was to analyse how mathematics teacher education could better prepare teachers to adapt to the reality of the schools, especially for vocational programmes.

Aim

In KAM 1, one aim was to make the pupils feel that mathematics is meaningful by starting with problems directly related to the vocational courses. In the long run the aim was to see mathematics more intertwined with the vocational subject. Another aim was to help the pupils to see the relevance of mathematics to manage to solve tasks in the vocational subject that they had previously had difficulty mastering. One more aim was to start from the pupils' level and to let them work in their own pace. In achieving those aims the pupils would get a better chance to achieve a pass grade in mathematics.

Carrying out and data collection

The project had scheduled time for meetings and individual work for the project members. In the beginning the vocational teachers informed the mathematics teachers about the content and how they taught this to the pupils. Then the mathematics teachers analysed this content and compared it to the content in mathematics, course A. After that they came back and discussed this with the vocational teachers to find out if the vocational teachers thought that the ideas of changing the explanation and procedures could work for the pupils. From the work of gear ratio and changes in revolutions for example there was a need to work with fractions, proportionality and estimations via mental calculations (KAM 1:1, 1999, p. 6). The teachers trialled the teaching material with some groups of pupils. Much of this work was done in order for the teachers to use the same words/concepts when teaching the pupils. The focus group during KAM 1 was mainly the pupils. Textbooks, teaching material as tools and manuals were part of the data collection. These data were used for the analyses in the project. Both the tests used and the results for testing the students' standard were collected.

Results

The project produced material (KAM 1:1, 1999, p. 5) for in service training for the teachers in a school. Thus the vocational teachers would be able to show the pupils the relevance of using mathematics in the vocational subjects and vice versa. This material was later used as teaching material in one class. Another class was used as a control group. The pupils' results from the tests showed an improvement (on short term basis) when using the mathematical tool to understand the element gear ratio and their motivation increased (ibid, p. 18).

KAM 2

Purpose

In KAM 2 (KAM 2, 2001) one purpose was to continue in the same way as in KAM 1 to build a coherent platform out of content and teaching/learning in some vocational courses as Vehicle, basic course, Pneumatics and Electricity year 1 and the core subjects Mathematics, course A and Swedish, course A and B. Another purpose was to have in-service training for the teachers in the programme enabling the pupils to feel this coherence and to become more motivated to study parts in their programmes where mathematics was involved.

Aim

KAM 2 continued from KAM 1 to analyse mathematical models and teaching methods in one upper secondary school in vocational programmes regarding the mathematics that is present in course A, using a process oriented approach. The aim was to use cooperation between teachers and together develop the teaching content and new approaches to improve the interest and success of the pupils.

Carrying out and data collection

Part 2 of the KAM-project was an extension of part 1 of the KAM-project and mainly aimed at the teachers carrying out a competence development programme through working together as a team to plan the programme for the pupils. When analysing the Vehicle electricity course many of the goals of mathematics, course A, became very visible. For example, "to have deepened and extended their understanding of numbers to cover real numbers written in different forms" and "with and without technical aids, be able to apply with judgement their knowledge of different forms of numerical calculations", and "to be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems" (Lpf94, 1994).

Through the classroom observations questions were raised about the teaching both in relation to the subject and the didactics and these were discussed in sessions with the teachers. To facilitate the subsequent discussions the teachers used tape recorders to report about and reflect on the classroom activities. The teachers also maintained a written diary periodically documenting the work during the project. During the development of the KAM-project the need for evaluation arose, both of the pupils and of the teachers. The pupils' evaluation was carried out in cooperation with the teacher in Swedish. The teacher evaluation took place through portfolios. Based on these, the teachers received feedback about their own teaching which otherwise could be hard to obtain. Qualitative evaluation completed involved interviews of teachers and students. There were even visits and studies out in the motor work shops.

Results

The collaboration between teachers of the core subjects and the vocational subjects improved during the course of the project. The teachers were interested in taking part in the planning of each others' subjects. The results show that if the subjects are taught with mutual support then the pupils will be more successful in their studies of the theoretical parts in all of the involved items. But it is important that the pupils understand the coherence in the content and in the organization of the education for the results to be a success. The teachers involved in the project noticed that it brought a better instruction and improved learning, but that the process was slow (KAM 2, p. 23). A great deal of the time was used for discussions, and the communication process was further developed from the KAM-project part 1 as the mutual language developed. Some of the teachers expressed pride in the work that had been done in the project in their evaluation of this project (ibid., p. 23). The work with discussions and carrying out change, demands proceeding with caution as the project takes place at the same time as all the other activities that teachers work with on a daily base. At the same time, it is important to be purposeful and to have the goal visible during the work even when the process takes a long time (ibid, p. 27).

KAM 3

Purpose

The purpose of the work in KAM 3 was to extend the work of KAM 1 and KAM 2 in all the six classes in grade 1 at the school thus scaling up the project.

Aims

From the results from KAM 1 and KAM 2, one aim was to study the strengths and weaknesses of the intervention when up-scaling. Another aim was to widen the coordination with core courses and mathematics. A further aim was to revise the earlier produced teaching material and to further investigate teaching methods and teachers' competences.

Carrying out and data collection

More and more the project was carried out as an action research project (Rönnerman, 2010) with a flavour of ethnography (Cohen & Manion, 1994; Bessot & Ridgeway, 2000) where the focus was the activities related to the subjects visible in the teaching/learning situation. The lessons were implemented as developed in KAM 2. In the course Vehicular Electricity Course A, some of the aims for Mathematics Course A were the same, i.e. number sense, to interpret and handle formulas and they were carried out there. In Vehicular Mechanics Basic Course simple solving of equations was illustrated with the concept of torque. In Transport Vehicle

Basic Course the aim was to work with the concept of velocity and units for length and time were in focus (KAM 3, pp. 10-11).

The lessons were followed up in seminars where teachers representing the different subjects took part. This was partly for revision of the material from the lessons but also as planned in-service training / professional learning for the teachers where they learnt more about the didactics of the different subjects. Observations in classrooms and workshops were documented as field notes and annotated after discussions with the respective teacher (ibid., p. 12). Qualitative analysis was used here and where the pupils were asked about their attitude towards the subject (ibid., pp. 20-22). The scientific leader interviewed the teachers and the principals of the school during the project (ibid, pp. 28-36). The same analysis method was used for the teachers' evaluation where they wrote in an open ended form about their views of the project and the result was summarized (ibid, p. 23). Quantitative analysis was used where the aim was to test the students' knowledge before and after a specific unit of work (ibid, pp. 13-20).

Results

The increase in scale of the project occurred in fewer classes than planned mainly because the school reorganised and the school leaders became less supportive. The survey showed that the students improved in mathematics, but even more in the vocational subject, when they were exposed to the KAM-model (ibid, p. 19). The interviews with the teachers indicated that the project required support from a solid organisation and support from the head of the school (ibid, p. 23). The lack of support in carrying out the project could be viewed as a weakness. The strength was the teachers' beliefs and engagement in the project (ibid, p. 23).

Results from stage 2 of the analysis - Affordances and constraints in the project

Using Kennewell's model we can now analyse the project. In the table we have listed affordances and constraints mentioned in the reports.

	Affordances	**Constraints**
1	New curriculum and new syllabus	Pupils' prerequisite knowledge
2	School based	Different teacher cultures –different didactical contract/ teacher role
3	Teacher education	Didactical competence
4	School management	New school leader
5	Schedule/Timetable	Schedule/timetable
6	Routines	Organisation
7	Teacher learning	Substitution of teachers
8	Money	Lack of money or resources
9	Time	Lack of time or simultaneuous time

Table 4 Affordances and Constraints

We will start by giving examples of affordances, i.e., potential or possibilities for action. The affordance is that the curriculum (Lpf94, 1994) gives every pupil opportunities "to solve mathematical problems of importance for vocational and daily life" (Lpf94, 1994), and "Upper secondary school mathematics should thus be linked to the study orientation chosen in such a way that it enriches both the subject of mathematics and subjects specific to a course." (Lpf94, 1994). In the teaching situation there should be cooperation between the mathematics and vocational teachers to give unity or wholeness for the pupils.

In chapter 1.2, Common tasks for the non-compulsory school system, it says: “Developments in working life mean inter alia that traditional boundaries between vocational areas need to be revised and that demands are imposed on our awareness of not only our own but also the competence of others. This in its turn imposes demands on the school/s working structure and organisations” (Lpf94, 2006, p.6) Further in the section Knowledge and Learning it says “The pupil’s acquisition of knowledge is dependent on developing the ability to see interconnections.” And in the section Development of the Individual School: “the school shall attempt to arrive at flexible solutions for its organisation, range of courses and working structures. Co-operation with the compulsory school, and universities and university colleges shall be developed as shall the co-operation between non-compulsory schools.” (Lpf94, 2006, p. 7). The affordances in this paragraph are related to the curriculum but as the implementation of the curriculum was the aim of the project they also became affordances in the project.

There is one specific chapter of the curriculum stating the responsibilities of the school head: “The school head is responsible for the school results and thus has, within certain limits, special responsibility for ensuring that: Teachers and other personal receive opportunities for the development of competence required for them to be able to carry out their tasks professionally” (Lpf94, 2006, p. 18) In reports this came to be an example of the constraints in this project. “Half of the teachers who were scheduled to teach in the courses involved in the project had not taken part in the in service courses for the project” (KAM3, 2002, p. 24).

The constraints arose mostly during the daily work of the project. Critical aspects were the tensions between the project leader and the school management, when the project leader asked for pedagogically motivated changes, which were uncomfortable for the school leaders (ibid., p. 35). Further organisational changes decided by the school management became obstacles to achieving the project aims and this also created tensions and frustration.

One constraint, i.e. the hindrances and problems for the projects, as well as being an affordance is the new curriculum. The pupils were not aware that they had to study more mathematics when going to a vocational programme. This could show as lack in motivation. In the former programme the mathematics had only been calculations that were relevant to do in the workshop or directly related to the vocational subject and this was taught by the vocational teacher. Now the pupils had to be taught by a mathematics teacher and this could be a problem because of the students’ expectations (KAM 3, 2002, p.32-33).

Many of the students also had poor prerequisites in mathematics as documented by tests (KAM 1:2, 1999, pp. 8-14) at the beginning of the project part KAM 1. The teachers came from different cultures and had different didactical contracts which gave rise to different teacher roles (KAM 1:2, 1999, p. 20). The teachers had different didactical competence as they had taken different teacher education programmes with different content (KAM 3, 2002, p. 27). During the project there were exchanges among teachers some of whom were not aware of the project and its purpose (KAM 3, 2002, p. 24. p. 28).

The school leaders did not support the organisation of the project by supporting changes in schedule/timetable when needed. “These changes of the leaders gave rise to many changes of the organisations at the school.” (KAM 3, 2002, p. 26) The school management was not part of the project and did not feel involved. The project took more time than expected. Here is one statement about that:”The project is so enormously big, so many involved. There is a KAM-project meeting every week.” (ibid., p. 31).

To summarize this section it is fair to say that the constraints related to the teachers were expected and taken in consideration, but the reactions and restrictions from the school leaders had not been predicted. Among the affordances at the school were that all the teachers, but one, who were participating in the project, were qualified teachers with many years of practice in the school system in their specific subjects. One teacher had many years of teaching and he obtained his teacher qualification during the project (KAM 2, attachment 3, p. 11). He joined the project during his studies. The school had access to computers, tape recorders and on the premises there were well equipped workshops for car mechanics. In the workshops and garages there was much material to use to demonstrate and build models that could be used in mathematics education. The school had recently been involved in an ICT project to develop the ICT competence among the teachers and the school had a well equipped computer room (KAM 1:1, p.10). There was also one teacher who was responsible for this room who taught in the computer subject (ibid., p. 1; KAM 1:2, p.5).

From the table it is obvious that some features can be seen as both an affordance and a constraint. For example, the new curriculum is the affordance that initiates and inspires the project as such. But the curriculum also represents some limiting structures and creates obstacles. When pupils' prerequisite knowledge is too far from what is expected it becomes difficult for the teachers to reach the goals of the curriculum. Thus affordances and constraints can in some cases be seen as two sides of the same coin.

Results from the analysis in stage 3. The developmental cycles in the KAM-project

This section shows how the KAM-project became visible as a developmental research project. The iteration of empirical and practical development work intertwined with theoretical discussions and reflections is illustrated below.

A pre-project of KAM, called DUGA (1996), was initiated and driven by teacher educators together with student teachers. Teacher educators felt the need to find out how the integration of mathematics and vocational subjects could be organised with the help of the mathematical content in course A and what the needs were for mathematics in vocational subjects. The opportunities for integration of subjects and to teach the new content in course A to pupils in vocational programmes were to be considered and given form in practice. There was a wish to document what kind of mathematics was needed in the vocational subjects. Another aim was to construct new teaching materials and suggest relevant applications in the vocational programmes. The planned development was inspired by the theoretical conditions given by the new syllabus in Lpf94. The demands for changes can be seen as theory based prescriptions in the form of the new curriculum, which had to be taken into practice by both teacher education staff and in teachers' daily teaching in school. The DUGA-project was not fully completed but left a number of questions to be answered by the teacher educators. Some of these questions (KAM 1.1, 1999, p. 2) seem to have triggered the first part of the KAM-project.

In KAM 1 the developmental research character of the project becomes explicit. The aim was to study whether the pupils would succeed better in the studies when the content of mathematics was integrated into the vocational subjects and vice versa. In order to answer that question some systematic investigations were completed and documented. Another part of the aim was to construct developmental material to be used both for competence development of the teachers and for teaching in one class to gain experience, while another class was used as a control group. In the construction process, theoretical insights were used and inspired the development component. The empirical and practical knowledge from teachers combined with the theoretical knowledge that teacher educators brought from research in mathematics education. In the developing learning community (Jaworski, 2002) the shared knowledge of all parts contributed to the construction of teaching material. That learning process was a learning situation for both the teachers in school and the teacher educators.

In this part of the KAM-project the major focus was the improvement of the students' learning of mathematics with the teachers of mathematics and vocational subjects collaborating to achieve this through the creation of learning opportunities.

It seemed as if the theoretical goals expressed in the new curriculum were accepted by the teachers and they were prepared to strive to reach the goals with the tools that were suggested in the plan that is integration of the subjects. In the official booklet about programme goals (1997) it is stressed that the applications in mathematics should be locally adapted to the pupils' study direction/ programme and to the pre-knowledge, interest and needs of the pupils (Skolverket, 1997, p. 107). This met the requirements of the Lpf94 to adapt the mathematics to the students' study programme and for the local work plans to respect mathematical knowledge in the vocational subjects and ensure that the applications in mathematics arise from the vocation thus making mathematics meaningful for the students (Skolverket, 1997). Thus we implicitly see theoretical foundations inspiring the development work in this first developmental cycle of the project. In the report from KAM 1 there is a list of references but these are not mentioned specifically in the text as such. Rather, we interpret it to be a list of theoretical sources that were used in the project. Most of the references are non-scientific works, such as popular scientific papers or books, or books used in mathematics teacher education. One doctoral thesis is mentioned.

In the report from KAM 2 the theoretical foundations were more explicit. The report included a section that discussed the theoretical background and referred to relevant research reports. Situated knowledge and learning was discussed and mathematics in vocational education was seen as activity (KAM 2, 2001, p. 9). The teacher's role as guide and builder of bridges was emphasised as was the importance of the knowledge of the teacher when it comes to motivate the pupils for studies that is to make the studies meaningful for them (ibid., p. 10; Pehkonen, 2001). These theoretical views influenced the work of the project and became visible, for example, in the process-oriented work-forms that were used (ibid., p. 17). Among the experiences mentioned as an important component was giving the teachers opportunities at several occasions to discuss changes and let their thoughts mature and develop over time. As a growing community of practice the teachers engaged in competence development, where they discussed the working material that had been developed and visited the motor workshops (ibid., p. 20-21; Jaworski, 2002).

In the next cycle of the KAM-project, reported in KAM 3 (2002), the main aim was to upscale the project, that is to involve all six classes in the school in the kind of work that earlier had been carried out in smaller proportions. In this process the importance of documenting the results was recognised and thus multimodal data collection was carried out. Both quantitative and qualitative methods were used in order to analyse observations, test results, questionnaires and interviews. The report indicates that the methods were similar to the ones used in action research with an ethnographic view (ibid., p. 7). The activities in the respective subjects were in focus in the study. The choice of methods was guided by a wish to report the outcomes of the project from many perspectives. Up-scaling of smaller case studies is not well documented in the research but had been requested by the teachers (Adler et al, 2005). Not much guidance about how the up-scaling can be carried out can be found in the literature. In the case of KAM 3 a number of complicating factors evolved and created obstacles for the final phase of the project. The practical outcomes that were documented in this up-scaling process can inform theories in mathematics education research about some of the possible affordances and constraints that influence the up-scaling process. The conclusion is still that it is possible to carry out local school development but it demands that both school leaders and teachers have a strong will to create change and a conviction that they work for the best of the pupils (ibid., p. 36). Developmental projects of this kind have to be allowed to take time, demand careful planning and have funding for some resources.

It was not until we had an opportunity to revisit the project and study the reports and documents again from a more long term perspective, that it became clear to us that the KAM-project must be seen as a developmental research study. While we were closely involved in the KAM- project and when the data and analysis was not complete it was difficult to see the character of the project. Thus we find it rewarding to be able to study all the documents and describe and analyse what was actually going on, now from a post-project reflective research perspective. The KAM-project over time evolved to become a school project that was more and more research based and it is clear that the reports developed to become more systematic, structured and linked to earlier documented research.

Results from the analysis in stage 4, Results of the KAM-project related to the aims

Stage 4 is an analysis of the results of the KAM-project in relation to the aims. The KAM-project was, as mentioned before, completed in three parts: KAM 1, KAM 2 and KAM 3. The overarching aim was to teach mathematics in ways that could enhance the pupils' mathematical knowledge in vocational programmes. Pupils' grades were expected to improve with the result of an improvement in the pass rate. The purpose was to make stronger connections between mathematics and vocational subjects to convince pupils of the relevance of mathematics.

Aims and results of the first part, KAM 1

According to the reports from KAM 1 (KAM 1:1, p. 1) the aims were

1. "to increase and deepen the pupils´ mathematical knowledge in the vocational programmes in the upper secondary school in a national perspective" and
2. "to analyse the real need of mathematics in different vocational subjects". One important part in the project was
3. "to bring about a systematic co-operation between the vocational teachers and the core subjects teachers".

The aim number 1 was to help pupils to get higher grades in mathematics and better understanding of the role of mathematics in the vocation. The tool to get there as stated in number 2 was to analyse the vocational subjects and compare with the content of mathematics, Course A. This process was pursued in collaboration between vocational teachers and mathematics teachers as stated in aim number 3. An implicit aim number 4 was also to come up with a model for in-service training for teachers in a school. One model was worked out to give strategies and methods to work together and some teaching material was produced. The teachers tested out teaching material in some groups of pupils. Most of this work was done in order to educate the teachers to use the same words/concepts when teaching the pupils. Gradually this cooperation became a routine that worked. Some of the aims as number 3 and 4 are very wide-ranging and it is hard to see if they have been achieved.

Concerning aim number 2, some analysis of the need for mathematics in vocational subjects was carried out during this phase but there was no clear outcome from this other than that the mathematics in course A was used in different ways in the vocational subjects.

For aim 1, some tests showed progress for pupils but the tested group was very small and it is hard to see whether pupils really increased and deepened their knowledge in this short period of time. Additionally the simple written tests evaluated only limited aspects of the mathematical learning process. There is no convincing evidence of pupils' lasting, improved knowledge. Some evidence of fruitful cooperation between vocational and mathematics teachers exists. The intention to develop a course to be used in teacher education was not fulfilled in this part.

Aims and results of the second part, KAM 2

In KAM 2 (2001) the aim (1.) was to get a common platform for three vocational courses and mathematics, course A, and in some cases Swedish, course A and B. This meant that all teachers were expected to work via integration in the subjects involving mathematics teachers, vocational teachers and one Swedish teacher. The cooperation was intended to occur in the vehicle programme, year 1, in one school. The aim was to work with a process orientation. Another aim

(2.) was to develop competences for the teachers in this work. The pupils were intended to perceive their studies as something united and holistic and to feel more motivated to study mathematics in their programme.

In KAM 2 the intention was to continue in the same way as in KAM 1, i.e. to analyse mathematical models and teaching methods in one upper secondary school in vocational programmes regarding the mathematics that was present in course A using a process oriented approach. The aim partly differed from that of the initial plan because the project received less funding than was requested. In order to evaluate the results the questions below were posed in the report of KAM 2 (p. 11-13). We try to answer the questions in connection to this text.

- *What change was there in the pupils′ understanding of the vocational subjects if the mathematics teaching was integrated in the content of the core subject?*

  The report claims a small enhancement but there is no significant evidence for that. It is pointed out that it is necessary for the pupils to realise the connection between the subjects. The analyses of the project indicates that there is no significant evidence for pupils' improved understanding due to the fact that the time that elapsed in the project was very short and the number of participants was small.

- *What were the changes for the teachers when the mathematics teaching was integrated with the content of the core subject?*

  Obstacles for the cooperation between vocational and mathematics teachers were observed. The teachers declared that they gained better insight in the content of the different subjects and were positive to the working procedures. This outcome was demonstrated only in teachers' written and verbal reports and not in any actions observed in the classrooms. The teachers were offered opportunities to exercise collaboration. The fact that teachers declared the positive outcome of the project in terms of gaining better insight in the different subjects is very challenging. That this was not observed is not because it might not exist, but there was not enough recourses in the project to follow up with more classroom observations.

- *What was the effect of tutoring the teachers in connection to the activities taking place in the classroom for the developmental change of teachers?*

  The teachers were able to reflect more about the effect of integration. Their reflections were documented by using portable recorders. This recording was more efficient and convincing than for them to write down the reflections.

- *How did the teachers and the leaders in the school consider the project and the work with integration between the subjects?*

  Teachers who were involved in the project from the start said that the project resulted in a better teaching and learning situation, but that the process was slow. They claimed that they were proud of what they achieved. The leaders in the school supported the project but there were changes in the organisation. The new leaders were not able to understand the process and how the development sometimes created conflicts in the daily work. This result is very important as it points out the school leaders' role as part of development in the schools. When there is a change of school leaders it becomes crucial that they commit themselves to what has been decided and to follow through.

- *How could a portfolio be used as a method of evaluation in subject integrated work? In what way could the Swedish subject support the pupils?*

  When the pupils were using the portfolios the teachers received spontaneous feedback from the pupils' writing. The students' writing indicated small improvements. This was a very good help for the teachers to see how the pupils could make the transfer from one subject to another. The whole idea of the cooperation and integration was to help the pupils to see the connections between the subjects and as a result get a better understanding.

What was the result of this part of the project compared to the aim?

The report showed that co-operation and collaboration worked to a greater extent than what was normally the case in schools. Teachers shared their knowledge and competence within and between the groups and felt that they achieved wider competence. The result was better shared understanding of the connection between core subjects and the vocational subjects. The teachers also produced new integrated teaching material. One aim was to develop competences for the teachers in this work so that the pupils experience their studies as something united and holistic and for them to feel more motivated to study mathematics in their programme. There is evidence that the pupils felt more motivated as they asked for more KAM-mathematics. They experienced what they called KAM-mathematics as something different and more useful.

The report also concluded (KAM 2, 2001, p. 27) that it is possible to achieve the aims and goals of the curriculum for mathematics in the vocational programmes. This demands that pupils have a pass result from the compulsory school. The integration of subjects facilitated pupils' opportunities to achieve the goals. During the project much time and effort was spent on communicating the project at conferences and meetings to raise the awareness of the needs in this area as well as to share findings. This was not articulated as an aim in the project, but it grew naturally to discuss and value the relevance for this kind of project. It was of course expected of the main funding body 'Skolverket' that dissemination from the project would take place.

Aims and results of the third part, KAM 3

One of the aims in this part was to build further on the experiences from KAM 1 and KAM 2. The ideas and material produced earlier were to be tested in all classes in grade 1 (year 10 of school but grade 1 in the Vocational Programme) at this school. The second aim was to use the same model in grade 2 (year 11 of school). This part included the making of an inventory of the material, and documenting the teaching methods and teacher competences in a similar way to the approach in KAM 2. The first aim was to scale up the developmental work and the second aim was to implement the work model again.

The first aim was partially achieved and the second one was just executed to a low degree. The challenges and problems which arose during KAM 3 are presented and discussed in the report. Among them were:

- a reorganisation of the school,
- several new school leaders,
- changed timetable and class- and group-structure, and
- new teachers not informed about the earlier parts of the project.

Still the teachers had faith in the project and fought to continue it. The processes and ideas from the earlier work in KAM 1 and 2 could be implemented in a larger group of pupils but not in all classes in year 1 as was the aim. The work planned in year 2 could not be carried out.

Implications of the results in KAM 3

Cooperation between teachers of mathematics and vocational subjects promotes pupils' development in vocational education. Motivation was raised and better learning outcomes in both mathematics and the vocational subjects were achieved according to the teachers' evaluations. Cooperation demanded from teachers a widened competence and a willingness on the part of a teacher to both enter into the work of colleagues and allow them to enter into the teacher's own work. The reward was greater work satisfaction and a stronger feeling of being able to support pupils' learning. There was a demand for new evaluation methods and flexibility. Teachers needed to be open to a great variety in methods, tools and models of explanation. An important condition was support from the school management. Long term and sustainable planning is also important if such change is to become part of the culture of the school.

The KAM-project was disseminated at different meetings and conferences. (See appendix 2).

Discussion and conclusions

In revisiting the project in which we participated between 1998 and 2002 we have tried to answer the following questions:

- What are the characteristic features of the school based developmental research project KAM?
- What characterises the affordances and constraints that the involved participants in such a project meet during the work?
- What were the actual outcomes of the KAM-project compared to the aims?
- What conclusions and implications can be drawn from this kind of developmental research project?

One evident characteristic feature of the KAM-project is that the project started as a school based development project initiated by teacher educators, grew into a more teacher-driven project and became more and more a developmental research project, where theory and practice were interwoven in a fruitful iteration process in the different components of the project. The teachers and researchers collaborated in all aspects of the project. In the beginning, the theoretical driving force was the new curriculum and the integration of vocational education programmes into the general upper secondary school system. In later stages, when more refined questions about integration of mathematics and vocational subjects were studied, mathematics education research provided the theory that inspired the project. The character of the reports developed from more descriptive narratives (without scientific references) in the early reports into papers and reports which followed the structure of a research paper, establishing theory and referencing literature.

Another feature is that unexpected affordances and constraints became noticeable as the project progressed. The established goals and aims of the project were not all realistic. When teachers are expected to do work that they never have done before (like for example developing textbook material) the completion of the task and hence the aim may be unattainable. But when teachers are expected to develop and reflect upon their daily teaching in a community of practice with other teachers the outcome is valuable and important and the knowledge thus gained can be used by them. The work in the community of teachers of mathematics and vocational subjects has shown that it is possible to integrate the teaching in those subjects. It also shows that if such integration is practised, it improves the outcomes for the students and increases the efficiency of the work and its value for the teachers. The small scale case of integration was convincing for the teachers and researchers but in trying to up scale of the intervention to all classes in a school there were too many unexpected obstacles and factors which impeded the project.

The project also shows that the decision from a historical point of view to combine the more theoretical programmes and the vocational programmes in upper secondary school in one school in a form with the same length and structure (Lindberg, 2009) can create a basis for different groups of teachers to collaborate and share each others' knowledge. In this combined school organisation different kinds of knowledge (sometimes called theoretical and practical) are visibly given the same value. In fact the vocational subjects are valued more highly than the theoretical as they may cost much more to teach, are taught in smaller groups (15 instead of 30 pupils) and have a visible impact on the students' future vocations. The integration of mathematics and vocational teaching would be more difficult to implement if the programmes were more separate in the school organisation. The groups of teachers confirmed that the mathematics content in course A is useful and needed for the learning of the vocational subject. Together they convinced the pupils that they were guiding them in the same direction, towards learning for adult life, both in their vocation and for full participation in a democratic society. It must be seen as both appropriate and possible to develop theoretical and vocational paths of education to become equivalent in length and structure as in Lpf94.

The parts of the project that dealt with identifying pupils' learning show that most of the pupils lack the expected pre-knowledge from the compulsory school. For example the teaching material produced as appropriate for the learning of fractions was about content which was part of teaching in year five and six in compulsory school. It seems clear that many of the pupils in the vocational programmes have received a pass in mathematics from compulsory school implying that they have attained knowledge from the curriculum but which does not equate to the knowledge they carry with them to upper secondary school. The fact that pupils do not acquire the expected knowledge in compulsory school then leads to the creation of problems in upper secondary school. The mathematics course A is blamed for the failure of the pupils, but in fact the failure happened already early in compulsory school. The wise thing to do is not to reduce the mathematics in course A, but to focus on the compulsory school ensuring that the pupils will be offered adequate learning opportunities so they enter upper secondary school with relevant pre-requisites. Pupils need appropriate mathematics for their vocational education and later for professional life.

The most influential affordances and constraints for the work in the KAM-project were the opportunities that were created by the new curriculum and school organisation, the support from school management and leaders, the organisation of time schedule and other resources for teachers, and the competence of teachers. It is obvious that the new curriculum meant that teachers were faced with new expectations and thus were ready to take action. Other studies indicate the upper secondary mathematics teachers in many cases tend to be loyal to the educational system (Kleve, 2007; Hundeland, 2010). The KAM case also convinces us that the full support of the school management is absolutely crucial for the success of a project. When during KAM, part 3, the organisation was changed in ways that did not support the project, this effectively prevented some of the work on the project, and hence aims of the project could not be met.

The competence of the teachers is central for the success of a development project and in this case the professional learning that was offered inside the project was well received and seen as meaningful learning for the teachers. Their experience in the project was that they could implement in the classroom what was learnt directly through their work on the KAM project. They had influence on the content of the teaching programme and had control of what was offered in the teacher seminars. The ownership of the project work is thus important.

Integration between mathematics and the vocational subjects does not occur naturally. There is a need for focussed professional learning opportunities for teachers involved in both the mathematics Course A and the vocational teachers. The KAM-project convinces us that teachers are willing and eager to be part of such competence development related to their daily teaching and they are able to design collaboratively challenging interventions and involve their pupils in a constructive manner.

Thus, one overarching theme of the project was the collaboration. The models for collaboration can differ depending on the purpose. Some models have been described in the report "Villkor och vägar för grundläggande yrkesutbildning" (2001), ("Conditions and paths for basic vocational education"). Collaboration does not necessarily occur automatically. But in periods of change as in the implementation of a new curriculum, an important way of working is to involve teachers in discussing the consequences of this new curriculum for education. Accordingly, the teachers can perceive their profession not to be a one-person profession but a profession where people work together daily. In the KAM-project another underlying motive was to encourage the pupils to act cooperatively. Some collaboration also took place in occasional activities in the form of project work and thematic work. These ideas have emanated from the cooperation which was established between the teachers from the separate subjects. This was an essential part of the KAM project. Rather than emanating just from one subject didactical point of view, the planned curriculum in the particular vocational programmes in KAM was designed cooperatively.

What were the actual outcomes from the KAM-project compared to the set aims and goals? From the analysis in stage 4 the most important conclusion is that there is convincing evidence that it is possible to integrate the teaching of mathematics with the teaching of vocational subjects. When teachers collaborate in such ways it is more demanding for them but they perceive it as more fruitful and meaningful. There is no clear indication that the learning outcome of pupils improved, mainly because the studied groups were too small and there was no opportunity to follow up the development over time. The short term tests indicate some improvement, but this might have been the case even if no intervention had taken place. More clear is the evidence that the pupils' motivation has improved and that they subjectively experience what they call "KAM-mathematics" as more meaningful and relevant. They are also able to see the coherence of the studies in mathematics and vocational subjects. As research has shown, pupils' beliefs about mathematics influence their learning opportunities (Pehkonen, 2003). This is an important finding.

The collaborative production of teaching materials was most important as a tool to trigger the teachers to open for discussion about their actual teaching. Teachers' reflections about their teaching and the written documentation created opportunities for deeper investigations of how the subjects could be better connected and seen by pupils as a united whole rather than isolated parts. There is very little research on mathematics in vocational education and more studies are needed and necessary for the development of the teaching and learning, which is perceived by pupils as a meaningful experience. The perspectives of pupils, their results and experiences from instruction and learning must be included in such studies.

Implications from the KAM-project

What are the conclusions and implications that can be drawn from this kind of developmental research project for future school based projects? A project of this kind needs support in two ways. One is support and encouragement from the leadership within the schools to facilitate the project and ensure local issues such as timetable do not impede the project. The school management should be involved in the project. This includes scheduling time for meetings. The other is finance to enable time for teachers, researchers and project participants to carry out the work. It is not possible to run such a project within normal school settings without the extra resources. Even with extra resources the demand on teachers is that they devote more time and effort than in the normal case to their work. The project indicates that teachers are willing to do so in order to get a more rewarding working situation and see increased motivated in pupils.

Skolverket (The National Agency for Education) funded the KAM-project but as far as we know they have not promoted the knowledge and results obtained by the project. Even if the participants in the project, as has been shown, have disseminated the project widely, it seems as if most Swedish teachers in vocational programmes know little about the findings. In the preparatory discussions for the new curriculum during 2008-2010, there has been no mention of the outcome as far as we know. Thus changes in the mathematics courses will be taken without any reference to what teachers learnt about opportunities to integrate mathematics and vocational subjects. One of the political suggestions, though, seems to be in line with the conclusions from KAM. Mathematics in compulsory school will be given one more teaching hour per week in years 7-9. Hopefully this can lead to a situation where the pass grades from compulsory school have real meaning demonstrating pupils' acquired knowledge.

The KAM-project also shows that, in order to gain deeper knowledge about what is actually going on in classrooms, there is a need to involve teachers in the research studies as equal partners offering them real opportunities to influence the project work. For that kind of study a developmental research design would be fruitful, where teachers and researchers collaborate closely, and let theory and practice inform each other in the cycles of the work. Future changes of curriculum could be based on this kind of developmental research studies. It is also a suitable way for carrying out analysis of the curriculum and creates a more long-term discussion about the content of the curriculum.

This project can also have an impact on how to organize teacher education when students are doing the practical studies or practice. There students can meet and discuss the relations between their different subjects and what impact it has on pupils' understanding if the students know more about the bridges and connections between different subjects. There could be specific tasks for the students to carry out in the practice period that focus on collaboration and integration for better learning opportunities for the pupils. This approach could also help teacher educators to be more involved in the school development.

Because this project pointed out the importance of the role of the school leader, it may be appropriate to research how they can support the implementation of the mathematics courses in the new curriculum GY2011.

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Appendix 1

Syllabus Mathematics Course A, for upper secondary school, from Lpf94

Below are the goals that pupils should have attained on completion of the course A. Pupils should:

- be able to formulate, analyse and solve mathematical problems of importance for everyday life and their chosen study orientation
- have deepened and extended their understanding of numbers to cover real numbers written in different forms
- be able to apply with judgement their knowledge of different forms of numerical calculations linked to everyday life and their study orientation, with and without technical aids.
- have an advanced knowledge of geometric concepts and be able to apply these to everyday situations and in the different subjects of their study orientation
- be sufficiently familiar with basic geometrical propositions and reasoning so that they understand and are able to use concepts and different ways of thinking in order to solve problems
- be able to interpret, critically examine and with discrimination illustrate statistical data, as well as be able to interpret and use common co-ordinates
- be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation
- be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems
- be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society
- be accustomed when solving problems to use computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes
- be familiar with how mathematics affects our culture in terms of, for example, architecture, design, music or the arts, as well as how mathematical models can describe processes and forms in nature. (Skolverket, 2005-11-02) (http: www.skolverket.se/skolfs?id=637)

Development of Mathematics from the compulsory school to course A

In table 1 below different aspects of mathematics, as it is written in the syllabus of mathematics, can be followed from school year five via school year 9 to course A.

Goals from the Syllabus	Goals that pupils should have attained by the end of the fifth year in school	Goals that pupils should have attained by the end of the ninth year in school	Goals that pupils should have attained on completion of the course A
A. Aims	Pupils should have acquired the basic knowledge in mathematics needed to be able to describe and manage situations, and	Pupils should have acquired the knowledge in mathematics needed to be able to describe and manage situations, as well as solve problems that occur regularly	Pupils should: be able to formulate, analyse and solve mathematical problems of importance for everyday life and their

Goals from the Syllabus	**Goals that pupils should have attained by the end of the fifth year in school**	**Goals that pupils should have attained by the end of the ninth year in school**	**Goals that pupils should have attained on completion of the course A**
	also solve concrete problems in their immediate environment.	in the home and society, which is needed as a foundation for further education.	chosen study orientation be accustomed when solving problems to use computers and graphic calculators to carry out calculations and use graphs and diagrams for illustrative purposes.
B. Numbers and operations on numbers	Within this framework, pupils should – have a basic understanding of numbers, covering natural numbers and simple numbers in fractions and decimal form, – understand and be able to use addition, subtraction, multiplication and division, as well as be able to discover numerical patterns and determine unknown numbers in simple formulae.	Within this framework, pupils should – have developed their understanding of numbers to cover whole and rational numbers in fraction and decimal form.	have deepened and extended their understanding of numbers to cover real numbers written in different forms.
C. Calculations in different modes	– be able to calculate in natural numbers – in their head, and by using written calculation methods and pocket calculators.	– have good skills in and be able to make estimates and calculations of natural numbers, numbers in decimal form, as well as percentages and proportions in their head, with the help of written calculation methods and technical aids.	with and without technical aids, be able to apply with judgement their knowledge of different forms of numerical calculations linked to everyday life and their study orientation.
D. Measuring, Geometry	– have a basic spatial understanding and be able to recognise and describe some of the important properties of geometrical figures and shapes, – be able to compare, estimate and measure length, area, volume, angles, quantities and time, as well as be able to use drawings and maps.	– be able to use methods, measuring systems and instruments to compare, estimate and determine length, area, volume, angles, quantities, points in time and time differences, – be able to reproduce and describe important properties of some common geometrical objects, as well as be able to interpret and use drawings and maps.	be sufficiently familiar with basic geometrical propositions and reasoning so that they understand and are able to use concepts and different ways of thinking in order to solve problems.

Goals from the Syllabus	Goals that pupils should have attained by the end of the fifth year in school	Goals that pupils should have attained by the end of the ninth year in school	Goals that pupils should have attained on completion of the course A
E. Handling of data, Statistics	– be able to read off and interpret data in tables and diagrams, as well as be able to use some measures of location	– be able to interpret, compile, analyse, and evaluate data in tables and diagrams, - be able to use the concept of probability in simple random situations.	be able to interpret, critically examine and with discrimination illustrate statistical data, as well as be able to interpret and use common co-ordinates.
F. Algebra		– be able to interpret and use simple formulae, solve simple equations, as well as be able to interpret and use graphs for functions describing real relationships and events.	be able to interpret and deal with algebraic expressions, formulae and functions required for solving problems in everyday life and in other subjects in their study orientation be able to set up and interpret linear equations and simple exponential equations, as well as use appropriate methods and aids to solve problems be able to set up, interpret and illustrate linear functions and simple exponential functions and models for real events in private finances and in society.

Table 5 Goals from the syllabus in mathematics year 5, year 9 and course A.

Appendix 2

The table below indicates some of the dissemination points of the KAM-project

Place	Year	Event	Mode
Manchester	1999	Section of ALM 6	Oral presentation
Boston	2000	ALM 7	Published paper
Tokyo	2000	ICME 9	Oral presentation
Gothenburg	2000	Mathematics biannual conference	Paper
Östersund	2003	Swedish Mathematical Society	Oral Presentation
Kungälv	2004	ALM 11	Topic Group
Melbourne	2005	ALM 12	Published Paper

Table 6 Dissemination at conferences of KAM