Objectives

Adults Learning Mathematics (ALM) – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

· Research and theoretical perspectives in the area of adults learning mathematics/numeracy
· Debate on special issues in the area of adults learning mathematics/numeracy
· Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Editorial

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I’m glad to introduce this new issue of Adults Learning Mathematics: An International Journal. In the next few pages the reader will find three interesting articles focused on relevant issues in our field. This current issue of our journal is mainly focused on the field of college education from different approaches and topics of interest. This is a thought-provoking field that hasn’t been fully addressed in previous ALM journal editions. For this reason, I’m very glad to re-address this topic in this issue.

The notion of adult learner entails not just people who never or barely have had the opportunity to attend school. It refers also to young adults coming to the university level, and this may be one part of our target as researchers in mathematics education. I remember a discussion with a colleague of mine, from Radford University (USA), who talked about what it means to be an adult learner and the fundamental components of the definition that characterizes this target population. For sure, by adult learner we can neither refer to children nor teenagers going into high schools. What about younger persons attending College campuses? Do they fit within this category? I had a vivid discussion with my friend, arguing that adults are persons older than sixteen (or eighteen) years old (depending on the specific educational context in the world), that somehow have experienced drop-out or who have never had the opportunity to fully and formally participate in the school system. My friend taught me that people attending college essentially are adult learners as well.

I’m coming from the field of basic adult education. My background points me to focus in those persons who never had the opportunity to attend school; persons who barely have notions of what we call mathematics literacy or school mathematics (I prefer to use these terms, rather than numeracy, because all the persons are numerated in some sense, although could be a non-formal sense of being numerate, as I and other colleagues have argued elsewhere). This discussion was really useful to me, since expanded my view of what does it mean to be an adult learner. Maybe we, as scientific community, should have a more in depth discussion about this topic.

In the mean time, here is this issue, with two main articles focused on learners from College, and one based on students from an Art institute. Rebekah Lane brings us a fresh overview of how college students deal with visual imaginary. The author’s interest on visual imaginary came from her curiosity on determine what understanding College algebra students have about mathematical functions. In doing so, Lane brings up, for the reader, the question of how students rely (or not) on mental images to gain understanding of functional relationships. She draws back on a very well-know characterization of students/individuals, from a cognitive point of view: the ones who are more visual-oriented, and the ones who are more algebraic-oriented (Resnik). There has been a plethora of work done to demonstrate that many people have a better numerical / algebraic understanding, whereas other people prefer to use images, graphs, outlines or other visual artefacts to gain understanding on mathematical ideas.
This field has been very relevant also for basic mathematics educators, since many adult learners (and here I meant the broader definition of adult learner) are more likely to respond positively to teaching methods of mathematics based on visual images, rather than just larger sets of formulas and algebraic symbols on the board (Plaza). Often a picture makes more sense (and it is meaningfully) than just a series of numbers connected through an equation (Diez-Palomar, Menéndez, Civil, 2011). In this article Lane focus on a case study carried out in two College Algebra courses during fall 2005, in a black university placed in the South-Eastern part of USA. She used Presmeg’s Mathematical Processing Instrument (1995) to analyse her data and provide a vivid image of students’ reactions to visual imaginary. Her findings are somehow very suggestive and encourage readers to keep working to elucidate and clarify, in more depth, the relationship between visual imaginary and algebraic understanding.

The next article is based on innumeracy. In this work, Klinger provides a provoking reflection on how pre-service teachers address the issue of innumeracy in mathematics, as future teachers that will be dealing with students to teach them mathematics. Klinger has a deep understanding of the role that anxiety plays in the process of learning mathematics, based on his prior works (Klinger, 2011). This is always a recursive topic in Adult Mathematics Education, since many adults have been confronted with negative and unpleasant experiences dealing with Mathematics. Klinger is well aware of this fact, and in this article he explores the consequences of negative experiences in future teachers (of Mathematics). This article reminds to me how many times I have to work with my College students (who are going to become teachers themselves) to overcome all of their misunderstandings, fears and prejudices against Mathematics. One of the main arguments used by these students is that they experienced negative situations with Mathematics mainly because their teacher was not as helpful and supportive. This is not a banal discussion, and it has crucial consequences, as far as many of these students eventually will become teachers themselves, for transmitting their values, thoughts and feelings through Mathematics to their future students.

In this article Klinger draws on the data coming from the IMAES (Inventory of Maths Attitude, Experience and Self-awareness), an instrument developed by himself to identify the students’ perceptions of Mathematics, including their own capabilities; and the diagnostic test/tool (DT) corresponding to the level of Mathematical attainment expected of a year 8 pupil in a South Australian school. Looking on the results, Klinger ends with the conclusion that data coming from both instruments are congruent. As he states, “the relationship between a negative IMAES profile and low competency levels is complex and one does not necessarily imply the other; competency can be depressed by anxiety and when this is relieved more accurate assessments of competency can emerge.” This conclusion attracts readers’ attention to the crucial importance of the connection between anxiety and competency in Mathematics, and how this negative connection may switch from former teachers to future ones through life (school) experiences. For this reason, we need to make these connections explicit, to be able to assume seriously our responsibility teaching new members of the educational crew, free from the narrow view of Mathematics as a difficult topic, but centred on meaningful ways to explain a teach Mathematics to students.

Finally, the last article included in this issue of Adults Learning Mathematics: An International Journal presents the thoughts of Tim Glasser regarding the connection between Crossroads curriculum and the Standards principles with instructors’ approaches to Mathematics. This is a provocative work that may provoke some controversy. The author aims to investigate the notions of meaning and relevance in the frame of adult developmental math learning and instruction. Glasser conducted a large investigation at the Art Institute of
San Francisco, interviewing teachers and adult learners on different aspects of teaching and learning mathematics, to figure out what is mathematically relevant to adult learners and vocational instructors at the focal site. Glasser provides a rough analysis of his data, drawing on a constructivist methodological approach. He concludes with some inspirational statements that open a broad field for further work to elucidate how Crossroads and the Standards are interlinked; how constructivism may help us to better understand (or not) what adult learners and vocational instructors may feel relevant when doing mathematics; or how instructors and adult learners may consider using different teaching approaches, such as traditional ones (more focused on repetition), or more resources-based methods (using visual, verbal, spatial and textual representations). This article may have some connections with some important prior work conducted in our field regarding what elements should contain a curriculum of Mathematics for adult learners.

After reading these three articles, many questions surface regarding our work as mathematical educators/researchers. For sure, the term, adult learner, is a category that is broader than my conceptualization of it – in the context I work in. The articles provide much evidence on the huge diversity among adults as learners, which demand different strategies in terms of teaching from the perspective of the instructors. However, the “traditional” phenomena that has been largely studied and discussed in our field, such as the impact of emotions on adults’ feelings towards mathematics, are also valid situations that apply to students in Colleges, for example. College students also struggle with visual and more algebraic-look-like ways to introduce, discuss and understand mathematical ideas. This has important consequences: according to the teachers interviewed by Glasser, teachers should use more meaningful and relevant mathematics instruction. I just want to give the floor to the readers, to engage in these kinds of discussions, so that they can build their own opinion and draw on the evidence provided by these three articles presented here. Have a good reading!

Javier Díez-Palomar
Chief Editor
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How Visual Imagery Contributed to College: A Case of How Visual Imagery Contributes to a College Algebra Student’s Understanding of the Concept of Function in the United States

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Abstract
This investigation utilized the qualitative case study method. Seventy-one College Algebra students were given a mathematical processing instrument. This testing device measured a student’s preference for visual thinking. Two students were purposefully selected using the instrument. The visual mathematical learner (VL) was discussed in this article. VL was presented with mathematical tasks to complete over the course of a semester. Each task was given to the student individually. In order to thoroughly understand VL’s responses, task-based interviews were conducted and videotaped. In addition, the student was interviewed based on her response to the mathematical tasks. The tasks captured different types of mathematical functions. These included linear, quadratic, absolute value, and exponential functions.

As patterns emerged from the data, the researcher called them categories. Several emerging categories were examined in the article. Also, O’Callaghan’s (1998) translating component was present during the completion of linear, quadratic, absolute value, and exponential functions.

Introduction
“Algebra, whether at middle school level, high school level, or college level often strikes fear in the hearts of students. Generation after generation have passed down the opinion that algebra is not only difficult, but perhaps also boring” (Stephens & Konvalina, 1999, p. 483). In fact after teaching mathematics courses on the university level, the researcher has found the previous quote to be true for many students. How can anyone learn College Algebra or any other type of mathematics when one starts out afraid of it? This is one of the many obstacles that the researcher attempts to overcome each semester with students. One way to try to overcome the fear of algebra is by establishing a learning environment in one’s classroom where the students feel free to ask questions and share their opinions about mathematics, i.e., an algebra friendly learning community. This type of community of mathematical learners develops and grows throughout the semester.

In general, students are introduced to algebraic concepts in middle school and high school. “Some of the mathematical objects that are met for the first time in algebra are expressions, equations with unknowns, functions and variables, and monomials and polynomials” (Kieran, 1990, p. 99). Subsequently, students are expected to master the techniques of the course in college at a faster rate. Typically, Algebra I and Algebra II that are
taken for two years in high school become College Algebra at a higher education institution. (Some students enroll in Algebra I in middle school.) College Algebra is offered for one semester at a community college or university.

Algebra is seen as an abstract subject to most of the students that I have taught. According to Fey (1992), “many students do not become proficient in the skills of algebra…[and] very few students acquire the understanding of algebraic ideas and methods that is required to reason effectively with symbolic expressions” (p. 1). Furthermore, students learn at various rates and in different ways. For instance, some people are visual learners (Gardner, 1983). Others possess a kinesthetic learning style. Some students are tactile learners. Many students are auditory learners.

Presmeg (1986a) defines visualizers as being “…individuals who prefer to use visual methods when attempting mathematical problems which may be solved by both visual and nonvisual [sic] methods” (p. 298). According to Price (1996), visual preferences include support using “… pictures, filmstrips, computers, films, graphs, books, and magazines” (p. 10). Each of these methods may aid the understanding of algebraic concepts for the visual learner.

Presmeg (1986a) also defines non-visualizers as being “…individuals who prefer not to use visual methods when attempting…[mathematical problems which may be solved by both visual and non-visual methods]” (p. 298). Price (1996) explains that auditory preferences involve the inclusion of “…tapes, videotapes, records, radio, television, and precise oral directions when giving assignments, setting tasks, reviewing progress, using resources or for any aspect of the task requiring understanding, performance, progress, or evaluation” (p. 10). Tactile preferences include favoring the “…use [of] manipulative and three-dimensional materials; [in addition] resources should be touchable and movable as well as readable” (p. 10). Furthermore, Price (1996) states that kinesthetic learners prefer “…opportunities for real and active experiences for planning and carrying out objectives; site visits, seeing projects in action and becoming physically involved” (p. 10).

The research question for the study was developed from the following efforts. First of all, the researcher wanted to determine what understanding the algebraic concept of function meant to the visual College Algebra student. Did the students rely on mental images in order to gain understanding? If so, how did the students connect these mental images with understanding the concept of function? Could the students draw these mental images on paper?

Secondly, O’Callaghan (1998) developed a cognitive model for understanding functions. This framework included four components: “…modeling, interpreting, translating, and reifying” (p. 24). Translating was defined as “the ability to move from one representation of a function to another…” (p. 25). In addition, he explained that “the three most frequently used representations for functions are equations, tables, and graphs” (p. 25). In Lane (2006), the presence or absence of the translating component (O’Callaghan, 1998) was used to measure the mathematical learners’ understanding of functions. Therefore, the following research question was created.

**Research Question**

- How does visual imagery contribute to a visual College Algebra student’s understanding of functions?
Literature Review

This section examined research articles and studies that pertained to the concepts of functions and visualization. The section begins with a summary of literature on the concept of function, then, provides a summary of visualization studies relevant to this research.

Functions

Many of the studies regarding the concept of function examined the role of different cognitive models in the understanding of functions (Vinner, 1983; Vinner & Dreyfus, 1989; Thompson, 1994; Sfard, 1991; O’Callaghan, 1998; Breidenbach, Dubinsky, Hawks, & Nichols, 1992). For example, the concept image and definition of a function was presented as a cognitive model to help explain how students learn (Vinner, 1983; Vinner & Dreyfus, 1989; Thompson, 1994). In fact, Thompson (1994) believed that when the concept image and concept definition were balanced, then understanding was achieved. In addition, Thompson examined the understanding of functions in terms of developing an action conception, a process conception, and an object conception. Thompson also explored the definition of function in terms of the correspondence of variables and the co-variation of quantities.

Sfard (1991) presented a different conceptual framework pertaining to functions. Instead of a student’s understanding of functions being defined in terms of the concept image and concept definition (Vinner, 1983; Vinner & Dreyfus, 1989; Thompson, 1994), this model included an operational conception and a structural conception. The structural conception referred to seeing functions as “…abstract objects…” (p. 4). On the other hand, an operational conception included viewing functions as “…processes, algorithms and actions…” (p. 4).

Sfard (1991) described three ways to move from an operational conception of function to a structural conception. The first stage was interiorization. “At the stage of interiorization a learner gets acquainted with the processes which will eventually give rise to a new concept…” (p. 18). For example, “in the case of function, it is when the idea of variable is learned and the ability of using a formula to find values of the ‘dependent’ variable is acquired” (p. 19). The second phase was condensation. “The phase of condensation is a period of ‘squeezing’ lengthy sequences of operations into more manageable units.” Thus, the student would still use processes, however, the concept should become more concrete. For instance, “…the learner can investigate functions, draw their graphs, combine couples of functions (e.g. by composition), even to find the inverse of a given function” (p. 19). Thirdly, reification was the last stage. This third phase involved “…conceiving the notion as a fully-fledged object…” (p. 19). For example, “in the case of function, reification may be evidenced by proficiency in solving equations in which ‘unknowns’ are functions (differential and functional equations, equations with parameters)…” (p. 20).

O’Callaghan (1998) developed another type of cognitive model for understanding functions. This cognitive model did not measure understanding based on a concept image and concept definition (Vinner, 1983; Vinner & Dreyfus, 1989; Thompson, 1994). Instead, the framework included four components. The components were modeling, interpreting, translating, and reifying. First of all, modeling was referred to as “the ability to represent a problem situation using functions…” (O’Callaghan, 1998, p. 25). According to the author, interpreting was considered “the reverse procedure…” (p. 25) of the first component. “Problems could require students to make different types of interpretations or to focus on different aspects of a graph, for example, individual points versus more global features” (p. 25). Translating was defined as “the ability to move from one representation of a function to another…” (p. 25). Furthermore, he explained that “the three most frequently used
representations for functions are equations, tables, and graphs” (p. 25). Thus, translating could refer to moving from graphs to equations, equations to graphs, numerical tables to equations, equations to numerical tables, numerical tables to graphs, or graphs to numerical tables. “The final component of the model for functions is reification, defined as the creation of a mental object from what was initially perceived as a process or procedure” (p. 25). The fourth component was similar to Sfard’s (1991) third stage of moving from the operational conception to a structural conception of function.

Breidenbach, Dubinsky, Hawks, and Nichols (1992) investigated how 59 math majors developed a process conception of functions that differed from Thompson (1994) and the previous authors. According to Thompson (1994), “from the perspective of students with a process conception of function [the second part of the cognitive model], an expression stands for what you would get by evaluating it” (p. 26). On the other hand, Breidenbach, Dubinsky, Hawks, and Nichols (1992) explained, “a process conception of function [the third phase of the cognitive model] involves a dynamic transformation of objects according to some repeatable means that, given the same original object, will always produce the same transformed object” (p. 251). Breidenbach, Dubinsky, Hawks, and Nichols presented the following three phases: pre-function conception, action conception, and process conception. According to the authors, attaining the third phase represented understanding functions.

Some of the function studies investigated college students (Vinner & Dreyfus, 1989; Thompson, 1994; O’Callaghan, 1998; Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Dreyfus & Eisenberg, 1983). For instance, Dreyfus and Eisenberg (1983) analyzed 84 college students’ understanding of the concept of function. Dreyfus and Eisenberg (1983) specifically examined three functional characteristics: linearity, smoothness (differentiability), and periodicity. Dreyfus and Eisenberg (1983) investigated how the college students’ understanding of these characteristics affected their construction of graphs of functions. In fact, “…28% of the responses were (piecewise) linear continuations while 62% were smooth (differentiable) continuations. The remaining 10% were neither piecewise linear nor smooth” (p. 129).

Other studies examined how high school students understood functions (Moschkovich, 1999; Schwarz & Dreyfus, 1995; Sajka, 2003; Monk & Nemirovsky, 1994). Moschkovich (1999) presented two case studies depicting how ninth and tenth grade algebra students understood the concept of function as they worked in pairs. This study focused on high school “…students’ use of the x-intercept in equations of the form y = mx + b” (Moschkovich, 1999, p. 169). Schwarz and Dreyfus (1995) examined how ninth grade students understood functions after receiving instruction in a computer software environment. Sajka (2003) reported on how an average high school mathematics student understood functions using case study research. Sajka referred to a students’ understanding in terms of developing a procept [sic] of function. Monk and Nemirovsky (1994) presented a case study of a twelfth grader who used visual characteristics of graphs and a technology component, called the Air Flow Device, to understand functions. Monk and Nemirovsky found the student to be concerned with the steepness of a straight line.

Some additional function studies included Eisenberg and Dreyfus (1994), Yerushalmy (2000), Slavit (1997), and Karsenty (2002). The purpose of the first study “…was to help students think of functions in a visual way, and to help us understand the obstacles they must overcome in doing so” (Eisenberg & Dreyfus, 1994, p. 47). Yerushalmy (2000) investigated the problem solving strategies of two middle school students in a function approach algebra course. Slavit (1997) analyzed how understanding was attained through the property-oriented
view of function. In the final function-related study, Karsenty (2002) examined 24 adult’s long term cognitive abilities on linear functions.

The gap in the literature appeared to pertain to College Algebra visual and non-visual learners. What does understanding the concept of function mean to these visual and non-visual mathematical students? Do visual and non-visual algebra learners translate from one representation of a function to another (O’Callaghan, 1998)?

Visualization. Most of the studies on visualization that were included encompassed Abstract Algebra, beginning Calculus, Calculus III, Engineering, middle school, and high school students (Zazkis, Dubinsky, & Dautermann, 1996; Aspinwall & Shaw, 2002; Aspinwall, Shaw, & Presmeg, 1997; Lean & Clements, 1981; Kirshner & Awtry, 2004; Krutetskii, 1976; Presmeg, 1986a; Presmeg, 1986b; Presmeg, 1989; Presmeg, 1992; Vinner, 1983).

For instance, Krutetskii (1976) examined research concerning 34 visual learners. Krutetskii (1976) reported the following two perspectives based on the results of the study. First of all, the absence or presence of showing a preference for the visualization of abstract mathematical concepts and having a strong development of spatial abilities “…does not determine its type” (p. 314). The second perspective was that the presence of the previously mentioned components “…showed a very high intercorrelation in our experiments” (p. 314).

Based on these perspectives, Krutetskii (1976) constructed a framework of three types of mathematical learners. They were analytic, geometric, and harmonic. The analytic mathematical learner was “…characterized by an obvious predominance of a very well developed verbal-logical component over a weak visual-pictorial one” (p. 317). For example, when given a choice between using equations and graphs, this kind of learner generally would solve mathematical problems using equations. The geometric mathematical learner was “…characterized by a very well developed visual-pictorial component, and we can tentatively speak of its predominance over a well developed verbal-logical component” (p. 321). For instance, when given a choice between expressing a mathematical relationship using equations or graphs, this student would have chosen graphs or diagrams. The harmonic mathematical learner feel confident with both approaches.

In fact, Presmeg (1989) states, “visual imagery which is meaningful in the pupil’s frame of reference may lead to enhanced understanding of mathematical concepts at primary and secondary levels” (p. 21). What happens on the collegiate level? How does visual imagery impact College Algebra learners? Apparently, the gap in the research was how visualization affects College Algebra visual and non-visual students.

Methodology

This study uses a qualitative research methodology (including case study) to study the College Algebra participant’s use of visual imagery in understanding functions. Two case studies of College Algebra students were investigated (Lane, 2006), however, one of the students will be discussed in this paper, the visual College Algebra student.

Participants

The population in the study came from two College Algebra courses in the fall of 2005. Together, the two sections were comprised of 71 students. The researcher taught both sections. The students attended a large four-year historically black university in the South-Eastern portion of the United States.
The research involved the purposeful sample\(^2\) (Patton, 1990) of two students using Presmeg’s (1985) Mathematical Processing Instrument. This testing device measured a student’s preference for visual thinking in mathematics. Therefore, one non-visual mathematical learner and one visual mathematical learner were chosen; findings from the latter are the focus of this paper.

**Instruments**

This investigation used the Mathematical Processing Instrument and the Mathematical Processing Questionnaire by Presmeg (1985). These tools were chosen because they measure how a student prefers to process mathematical information, i.e., visually or non-visually.

Presmeg’s Mathematical Processing Instrument included three sections (A-C) of mathematics problems for students to solve. The author recommended section B only or sections B and C for college-level students. All 71 students were provided with section B of the instrument. Section B had 12 mathematical word problems to solve. Each question could be solved numerically, algebraically, and graphically. Graphical solutions or drawing diagrams were considered as visual solutions. Numerical and algebraic solutions were considered as non-visual solutions. The test was scored by adding the total of two for every visual solution, one if the problem is not attempted, and zero for every non-visual solution. The highest score possible was 24/24 (24 out of 24). The lowest score possible was 0/24 (0 out of 24). If the student’s visualization score was 12/24 or higher, then he or she was considered as having a preference for visual thinking in mathematics. On the other hand, if the participant’s visualization score was 10/24 or lower, then he or she was considered as having a preference for non-visual thinking in mathematics. The students were required to show their work for the solution(s), however, they were not required to use a specific method of solution over another. The participants were also asked to choose their own method of solution and turn in their papers. (See Appendix A for a copy of this instrument.)

In addition, each student was supplied with a Mathematical Processing Questionnaire (Presmeg). The questionnaire was a follow-up to the participants’ responses to the Mathematical Processing Instrument. This questionnaire provided three or more solutions for the students to choose the one that was most similar to their response. After the participants completed the questionnaire, they were asked to turn in their responses. (See Appendix B for a copy of this questionnaire.)

After completing Presmeg’s Mathematical Processing Instrument, 52 College Algebra students scored from 0/24 to 10/24. As a result, these 52 students were considered to have a preference for non-visual thinking in mathematics. In addition, after completing the instrument, 19 College Algebra students scored from 12/24 to 20/24. As a result, these 19 students were considered to have a preference for visual thinking in mathematics.

The Visualizer (VL) (Presmeg, 1986a) for the present study was purposefully selected from the 19 students. The participant scored 16/24 on Presmeg’s (1985) Mathematical Processing Instrument. In addition, VL was extremely detailed regarding the student’s answers on the instrument.

**Data collection**

“Qualitative data consist of quotations, observations, and excerpts from documents” (Patton, 2002, p. 47). Therefore, the data sets for the present study were interviews and document reviews. The interviews were based on the participant’s responses to mathematical tasks.

The functions examined in this study included first degree, second degree, and higher order functions. For example, first degree or linear functions can be written in symbolic form
as \( f(x) = ax + b \) where \( a \) and \( b \) are real numbers. The graph or pictorial form of a linear function is a straight line. Second degree functions can be expressed in symbolic form as \( f(x) = ax^2 + b \) where \( a \) and \( b \) are real numbers. The pictorial form of second-degree functions is called a parabola. A basic parabola can look like a bowl that is shaped like the letter “u”. In addition, higher order functions can be written in symbolic form as \( f(x) = ax^3 + b \) where \( a \) and \( b \) are real numbers. Higher order functions have an exponent of three or higher in their equations. Odd numbered higher order functions can be expressed in pictorial form in three parts that are connected. One portion of the graph is curved downward or decreasing. The second part of the graph remains constant. The last portion of the graph is curved upward or increasing (Lial, Hornsby, & Schneider, 2001).

The documents in this investigation were College Algebra Writing Journals, tests, College Algebra Web Homework, and a researcher journal. Bogdan and Biklen (1998) posited that documents “…can be used as supplemental information as part of case study whose main data source is participant observation or interviewing” (p. 57). The College Algebra Writing Journals included the students’ feelings, beliefs, and interpretations about mathematics in their own words. They also recorded their struggles and concerns regarding College Algebra over the semester. Specifically, the struggles and concerns pertained to class assignments and/or algebraic concepts that were introduced in class. Since the course was held three days a week, the students were expected to complete a minimum of three entries per week. Each entry was at least one-half of 8 1/2 inches by 11 inches of a page. In addition, the visual and non-visual learners wrote an additional entry after every interview session. The interview session entries included the participants’ feelings, beliefs, and interpretations about the mathematical tasks. They also recorded any struggles encountered in completing the tasks. The entry was at least one-half of 8 1/2 inches by 11 inches of a page.

The participants’ College Algebra tests were a second source of extant data. Four chapter tests and one final examination were given. The concept of function permeated the last three tests and the final examination. As part of the directions, the students were asked to represent functions numerically, algebraically, and/or graphically on these exams.

A researcher’s journal was the third document data source. Specifically, the journal entries began in the fall of 2005 after the students completed Presmeg’s (1985) visualization instrument. It included the time, place, and length of each interview session. The entries also included the researcher’s feelings, beliefs, and interpretations regarding the mathematical task interview sessions. The idea of a researcher’s journal was supported by Lincoln and Guba (1985), who indicated that “each investigator should keep a personal journal in which his or her own methodological decisions are recorded and made available for public scrutiny” (p. 210).

**Data Analysis**

Data analysis began through an examination of the participant’s initial interview session. Each transcript was analyzed before the next interview session occurred in order to look for any possible emerging patterns or themes. As any patterns/themes were found, they were investigated in the next interview session.

Measures were taken in order to ensure trustworthiness in the data analysis and thus in the findings. “The four terms ‘credibility’, ‘transferability’, ‘dependability’, and ‘confirmability’ are, then the naturalist’s equivalents for the conventional terms ‘intended validity’, ‘external validity’, ‘reliability’, and ‘objectivity’” (Lincoln & Guba, 1985, p. 300).
In the study, credibility was established by using triangulation and member checking (Lincoln & Guba). No assertion was considered valid unless it could be supported by two or more pieces of data. In the study, triangulation of the interview sessions and documents (College Algebra Writing Journals, tests, College Algebra Web Homework, and a researcher journal) occurred. According to Lincoln and Guba, “the concept of triangulation by different methods thus can imply either different data collection modes (interview, questionnaire, observation, testing) or different designs” (p. 306).

“The member check, whereby data, analytic categories, interpretations, and conclusions are tested with members of those stakeholding groups from whom the data were originally collected, is the most crucial technique for establishing credibility” (Lincoln & Guba, 1985, p. 314). The member checking technique was applied to the study by allowing the participant to read the results section. The student was asked to pay special attention to the overall written interpretations of her responses to the various mathematical tasks, which helped to build the case study. After that, the student provided reasons for the response. In addition, the student was encouraged to indicate any information she felt was left out of the case study that might be pertinent. All of the member check responses were reported. The biggest difference between these two forms of establishing credibility was that “member checking is directed at a judgment of overall credibility, while triangulation is directed at a judgment of the accuracy of specific data items” (p. 316).

The researcher had the “…responsibility to provide the data base that makes transferability judgments possible on the part of potential appliers” (p. 316). In the study, the database included descriptions of the time and context of the case study. In fact, the case study was written with thick description in order to make transferability possible.

To promote the study’s dependability criterion for ensuring its integrity, the overlap methods of triangulation were used (Lincoln & Guba, 1985). This technique was chosen based on the authors’ following claim which emphasized, “since there can be no validity without reliability (and thus no credibility without dependability), a demonstration of the former is sufficient to establish the latter” (p. 316). Thus, triangulation, which was discussed earlier, was used to help establish credibility and in turn as an overlap method to establish dependability. As an overlap method, the focus was on triangulation of multiple and different methods.

In order to help establish confirmability, an audit trail was maintained throughout the study. The audit trail included five of the categories that have been suggested by Halpern, 1983 cited in Lincoln and Guba, 1985. They are raw data; data reduction and analysis products; data reconstruction and synthesis products; process notes; and materials relating to intentions and dispositions. First of all, the raw data in the present study was the results from Presmeg’s (1985) visualization instruments, audio and videotaped interview sessions, and the College Algebra Writing Journals and tests. Secondly, the data reconstruction was utilized in order to condense information and identify any common patterns or relationships. Thirdly, data reconstruction and synthesizing products occurred in the study by identifying and organizing common categories and/or themes, examining the participants’ interpretations and my interpretations, reporting findings, and identifying connections to existing literature and/or theory. Next, the process notes, which included methodological decisions, were recorded in the researcher’s journal. The fifth category of materials relating to intentions and dispositions of the study were also recorded in the researcher’s journal.

In addition, the data were analyzed by using O’Callaghan’s (1998) translating component for understanding functions. O’Callaghan’s (1998) model contributed to the theoretical framework of the study.
Findings

Visual Imagery

In the study, the Visualizer used visual imagery in five of the mathematical tasks. Specifically, VL relied on visual imagery during the completion of tasks one, four, eight, nine, and ten. These tasks included linear, quadratic, absolute value, and exponential functions.

During the completion of mathematical task #1 (Figure 1), visual imagery contributed to the VL’s demonstration of understanding functions because she relied on visual imagery regarding the linear function $y = x$.

![Figure 1. Mathematical task #1.](image1)

VL drew her mental image and used it to complete the task (Figure 2).

![Figure 2. Visualizer’s mathematical task #1 graph.](image2)
During the completion of mathematical task # 4 (Figure 3), visual imagery contributed to the VL’s demonstration of understanding functions because she relied on visual imagery regarding the quadratic function $y = x^2$.

![Figure 3. Mathematical task # 4.](image)

VL drew her mental image and used it to complete the task (Figure 4).

![Figure 4. VL’s image of $y = x^2$.](image)

During the completion of mathematical task # 8 (Figure 5), visual imagery contributed to the VL’s demonstration of understanding functions because she relied on visual imagery regarding the absolute value functions $y = |x|$, $y = |x + 1|$, and $y = |x + 1| - 2$.

![Figure 5. Mathematical task # 8.](image)
The participant also shared her personal viewpoint of the task. “It was simple, not at all difficult. I observed the shifts in the original graph \( y = |x| \) to the present graph [provided in task # 8] and wrote the equation \( y = |x + 1| - 2 \) that I believe the graph illustrates”. (Even though the graph of \( y = |x| \) was not sketched on the task eight sheet, VL reported that \( y = |x| \) as the original graph of an absolute value function.)

During the completion of mathematical task # 9 (Figure 6), visual imagery did not contribute to the VL’s demonstration of understanding functions because she did not construct an accurate equation (symbolic form) to match the given graph (graphic form) of the function.

![Graph](image)

Figure 6. Mathematical task # 9.

However, VL used visual imagery to solve the problem because the participant relied on the visual imagery of the exponential function \( y = 2^x \). In addition, VL shared her personal viewpoint about this task in the College Algebra Journal.

This task was a bit challenging to me. It reminded me of the previous task but I could not remember how I shift the graph on the x – axis instead of the y – axis.

During the completion of mathematical task # 10 (Figure 7), VL used visual imagery to solve the problem.

![Graph](image)

Figure 7. Mathematical task # 10.
Visual imagery contributed to VL’s demonstration of understanding functions because the student relied on the image of the exponential function $y = e^x$ during the completion of the tenth task.

**Understanding of functions**

In the study, the visual mathematical learner’s understanding of functions was measured by the presence or absence of the translating component (O’Callaghan, 1998) for understanding functions. In addition, O’Callaghan’s (1998) translating component was present during the completion of linear, quadratic, absolute value, and exponential functions.

Specifically, the VL translated the following linear functions: $y = x$ and $y = x + 5$. First of all, VL translated the given (symbolic form) equation of $y = x + 5$ to its (numeric form) table of numerical values. Secondly, VL translated the numeric form of $y = x + 5$ and the given symbolic form of $y = x$ to their (graphic forms) graphs.

During the beginning of the fourth mathematical task, by relying on visual imagery VL translated the symbolic form of $y = x^2$ to its graphic form. VL also translated the following quadratic functions during the completion of the fourth mathematical task: $y = x^2 + 1$, $y = -x^2 + 1$, $y = x^2 - 1$, and $y = -x^2 + 1$. VL translated from equations (symbolic form) to tables of numerical values (numeric form). In addition, VL translated $y = x^2 - 1$, $y = -x^2$, and $y = -x^2 + 1$ from equations (symbolic form) to graphs (graphic form).

Explicitly, VL translated the following absolute value functions: $y = |x|$, $y = |x + 1|$, and $y = |x + 1| - 2$ during the completion of the eighth mathematical task. By using visual imagery, VL translated $y = |x|$ and $y = |x + 1|$ from equations (symbolic form) to graphs (graphic form). In addition, by using visual imagery, VL translated $y = |x + 1| - 2$ from a graph (graphic form) to an equation (symbolic form).

Specifically, VL translated the following exponential functions during the ninth mathematical task: $y = 2^x - 5$, $y = 2^x + 5$, $y = (2^x + 1) + 5$, $y = -5^x + 1$, and $y = (2^x + 5) + 1$. VL translated each function from an equation (symbolic form) to a graph (graphic form). Overall, VL did not construct an accurate equation to match the graph of the function that was provided in mathematical task # 9. This showed the absence of O’Callaghan’s (1998) translating component in the final solution of task nine because the participant did not translate the given graphic form of the function in task nine to its symbolic form of $y = 2^x + 5$.

During the tenth mathematical task, VL translated the given equation (symbolic form) of the exponential function $f(x) = e^{x-1}$. Explicitly, VL translated the symbolic form of the function to its graphic form.

**Conclusions**

**Emerging Categories**

In the report of the case study of the visual mathematical learner, as patterns emerged from the data the researcher called them categories (Lane, 2006). In this article, the categories discussed will include the patterns that emerged from the data collected pertaining to mathematical tasks one, four, eight, nine, and ten.

First of all, VL used the following three categories during the completion of mathematical task # 1. Category A was substituting specific values for the variables x and y.
into the equations. Category B was plotting specific points of a function on a graph. Category C was detecting a relationship between the concepts of slope and steepness.

Secondly, VL used the following four categories during the completion of mathematical task # 4. Category A was substituting specific values for the variables x and y into equations. Category E was looking for a relationship between the symbolic form and graphic form of a function. Category F was using the graphing calculator for arithmetical operations. Category G was using the graphing calculator to construct a relationship between the symbolic form and graphic form of a function.

Next, VL used two categories during the completion of mathematical task # 8. Category E was looking for a relationship between the symbolic form of a function and the graphic form. Category I was focusing on specific visual features of the graph of a function.

In addition, VL used the following three categories during the completion of mathematical task # 9. Category E was looking for a relationship between the symbolic form of a function and the graphic form. Category I was focusing on specific visual features of the graph of a function. Category G was using the graphing calculator to construct a relationship between the symbolic form of a function and the graphic form.

Finally, VL used the following four categories during the completion of mathematical task # 10. Category E was looking for a relationship between the symbolic form of a function and the graphic form. Category G was using the graphing calculator to construct a relationship between the symbolic form of a function and the graphic form. Category B was plotting specific points of a function on a graph. Category H was using various features of the graphing calculator.

To summarize, all of the Visualizer’s (VL) emerging categories were listed using alphabetical letters with the corresponding mathematical task or tasks in Table 1.

- Category A: substituting specific values for the variables x and y into equations
- Category B: plotting specific points of a function on a graph
- Category C: detecting a relationship between the concepts slope and steepness
- Category D: misinterpreting the graphical representation of a function after multiplying and adding specific values to the symbolic form of a function
- Category E: looking for a relationship between the symbolic form of a function and the graphic form
- Category F: using the graphing calculator for arithmetical operations
- Category G: using the graphing calculator to construct a relationship between the symbolic form of a function and the graphic form
- Category H: using various features of the graphing calculator
- Category I: focusing on specific visual features of the graph of a function

<table>
<thead>
<tr>
<th>CATEGORIES</th>
<th>MATHEMATICAL TASKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, 4</td>
</tr>
<tr>
<td>B</td>
<td>1, 10</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2 (was not discussed in this article)</td>
</tr>
<tr>
<td>E</td>
<td>4, 8, 9, 10</td>
</tr>
</tbody>
</table>

Table 1: VL’s Emerging Categories
Discussion

The results discussed in this article suggest the participant’s reliance on visual imagery correlated with Presmeg (1989). The author stated, “visual imagery which is meaningful in the pupil’s frame of reference may lead to enhanced understanding of mathematical concepts at primary and secondary levels” (p. 21). In the study, how the College Algebra student used visual imagery contributed to her understanding of functions.

Thompson (1994) stated, “…a concept definition is a customary or conventional linguistic formulation that demarcates the boundaries of a word’s or phrase’s application” (p. 24). Thus, the concept definition of a specific function describes the attributes of that function using words. “On the other hand, a concept image comprises the visual representations, mental pictures, experiences and impressions evoked by the concept name” (Thompson, 1994, p. 24). The author believed that students could be more successful in their understanding of functions when the concept image and concept definition are balanced.

In the study, VL’s concept image was based on her reliance on visual imagery. When the student’s use of visual imagery (concept image) and concept definition were balanced, understanding of functions was demonstrated which correlated with Thompson (1994).

Sfard (1991) mentioned three ways to move from the operational conception to the structural conception. The first stage was interiorization. “At the stage of interiorization a learner gets acquainted with the processes which will eventually give rise to a new concept…” (p. 18). For example, “in the case of function, it is when the idea of variable is learned and the ability of using a formula to find values of the ‘dependent’ variable is acquired” (p. 19). The second phase was condensation. “The phase of condensation is a period of ‘squeezing’ lengthy sequences of operations into more manageable units.” Thus, the student would still use processes, however, the concept should become more concrete. For instance, “…the learner can investigate functions, draw their graphs, combine couples of functions (e.g. by composition), even to find the inverse of a given function” (p. 19).

In the study, VL depicted Sfard’s (1991) condensation stage. This occurred through the student’s demonstration of understanding linear, quadratic, absolute value, and exponential functions.

In the study, VL demonstrated many of the traits of Krutetskii’s (1976) geometric learner. These included mathematical images from the student’s mind that were described and/or drawn on paper.

Arcavi (2003) stated, “visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings” (p. 217). The Visualizer in the study displayed Arcavi’s (2003) definition of visualization. The student reflected upon pictures, images, and diagrams in her mind and on paper throughout the study. In addition, VL’s purpose was to try to communicate the mathematical information that she was thinking about. Specifically, this occurred on mathematical tasks one, four, eight, nine, and ten.

Significance of the study
One of the major goals in mathematics education is to ensure the success of all students in mathematics. A way of accomplishing this goal is by incorporating different kinds of learning experiences for the variety of learners in the College Algebra classroom. By determining what understanding the algebraic concept of function means to the visual College Algebra student this goal may be achieved.

It is vital that College Algebra learners see mathematics as meaningful and relevant. “Mathematics instruction must reach out to all students: women, minorities, and others who have… differing learning styles… faculty must provide a supportive learning environment and promote appreciation of mathematics” (Writing Team and Task Force of the Standards for Introductory College Mathematics Project, 1995, p. 3).

Acknowledgements

This manuscript is based on the dissertation, How Graphing Calculators and Visual Imagery contribute to College Algebra Students’ Understanding the Concept of Function. The professor directing the dissertation was Dr. Leslie Aspinwall.

Notes

1 Approximately, 13,000 students attended the university.
2 The participants are selected based on a specific characteristic. In this case, the characteristic is a student’s preference for visual thinking in mathematics.
3 The researcher made entries in the journal that pertain to all of the data collection activities.
4 The taped interview sessions were transcribed.
5 Triangulation of data involves validating particular pieces of information with another source or method.
6 Member checking procures confirmation that a case includes the information constructed by the participant(s) and makes corrections, if needed.

References


Appendix A
Mathematical Processing Instrument

Important:
(a) Do not write on this problem sheet. Write your solutions on the solution sheet provided.
(b) For each problem, show your working as much as you can.
(c) You are required to attempt all problems.

SECTION B:
B-1. A track for an athletics race is divided into three unequal sections. The length of the whole track is 450 meters. The length of the first and second sections combined is 350 meters. The length of the second and third sections combined is 250 meters. What is the length of each section?

B-2. A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its starting point?

B-3. A mother is seven times as old as her daughter. The difference between their ages is 24 years. How old are they?

B-4. In an athletics race John is 10 meters ahead of Peter. Tom is 4 meters ahead of Jim and Jim is 3 meters ahead of Peter. How many meters is John ahead of Tom?

B-5. At first, the price of one kg of sugar was three times as much as the price of one kg of salt. Then the price of one kg of salt was increased by half its previous price, while the price of sugar was not changed. If the price of salt is now 30 cents per kilogram, what is the price of sugar per kilogram?

B-6. Some sparrows are sitting in two trees, with each tree having the same number of sparrows. Two sparrows then fly from the first tree to the second tree. How many more sparrows does the second tree then have than the first tree?

B-7. A saw in a sawmill saws long logs, each 16 meters long, into short logs, each 2 meters long. If each cut takes two minutes, how long will it take for the saw to produce eight short logs from one long log?

B-8. A jar of kerosene weighs 8 kilograms. Half the kerosene is poured out of it, after which the jar and contents weigh $4 \frac{1}{2}$ kg. Determine the weight of the jar.

B-9. A passenger who had traveled half his journey fell asleep. When he awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire journey had he been asleep?

B-10. If you place a large, entire cheese on a pan of a scale and three quarters of a cheese and a $\frac{3}{4}$ kg weight on the other pan, the pans balance. How much does an entire cheese weigh?

B-11. There was twice as much milk in one can as in another. When 20 liters of milk had been poured from both cans, then there was three times as much milk in the first can as in the second. How much milk was there originally in each can?

B-12. Ten plums weigh as much as three apricots and one mango. Six plums and one apricot are equal in weight to a mango. How many plums balance the scales against one mango?
Appendix B
Mathematical Processing Questionnaire

IMPORTANT:

On this questionnaire you are asked to consider how you did the mathematical processing problems that you were recently asked to do. Every problem has three or more possible solutions.

SOLUTIONS

SECTION B:

B-1. Solution 1: I solved this problem by imagining the track for the race and then working out the length of each section.

- Length of third section = 450-350 = 100 metres
- Length of first section = 450-250 = 200 metres
- Thus length of second section = 150 metres.

B-1. Solution 2: I drew a diagram that represents the track and then worked out the length of each section.

B-1. Solution 3: To solve this problem I drew conclusions from the information given, and did not imagine or draw any picture at all:

\[
\begin{align*}
\text{Length of whole track is 450 m} & \quad x + y + z = 450 \\
\text{First and second sections combined is 350m} & \quad x + y = 350 \\
\text{Conclusion: Length of third section = 450-350 = 100m} & \quad z = 100 \\
\text{Second and third sections combined is 250m} & \quad y + z = 250 \\
\text{Conclusion: Length of first section = 450-250 = 200m} & \quad x = 200 \\
\text{Thus length of second section = 450-200-100 = 150m} & \quad y = 150
\end{align*}
\]

B-2. Solution 1: I imagined the path taken by the balloon, and then worked out the distance between the starting and finishing places. I found the distance to be 100 + 50 = 150 meters.

B-2. Solution 2: I drew a diagram representing the path taken by the balloon, and then worked out the distance between the starting and finishing places.

\[
\begin{align*}
\text{Distance} & = 100 + 50 = 150m.
\end{align*}
\]
B-2. Solution 3: In order to solve this problem, I noticed only the information which was important for the solution (without imagining the path of the balloon). Then the distance between the starting and the finishing places was 100m + 50m = 150m.

B-3. Solution 1: I solved this problem by trial and error:

<table>
<thead>
<tr>
<th>Daughter’s age:</th>
<th>Mother’s age:</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 years</td>
<td>26 years</td>
<td>No</td>
</tr>
<tr>
<td>3 years</td>
<td>27 years</td>
<td>No</td>
</tr>
<tr>
<td>4 years</td>
<td>28 years</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Thus the daughter’s age is 4 years and the mother’s 28 years.

B-3. Solution 2: I solved this problem by using symbols and equations, e.g.,

Let daughter’s age be x years.

Then mother’s age is 7x years.

Difference between their ages is 6x years.

Therefore 6x = 24. Thus x = 4.

Thus the daughter’s age is 4 years and the mother’s age is 28 years.

B-3. Solution 3: I drew a diagram representing their ages:

```
\[ \text{Daughter’s age} \]
\[ \text{Mother’s age} \]
\[ \text{Difference between their ages} \]
```

From the diagram, difference between their ages is 6 equal parts, totalling 24 years.

Thus each part represents 4 years. The daughter’s age is 4 years and the mother’s age is 28 years.

B-3. Solution 4: I imagined the diagram as in solution 3, and then reasoned that 6 parts represents 24 years, so one part represents 4 years (with or without using symbols). Thus the daughter’s age is 4 years and the mother’s 28 years.

B-4. Solution 1: I imagined the four people and then worked out the distance between John and Tom. John is 3 meters ahead of Tom.

B-4. Solution 2: I drew a diagram representing the four people, and then worked out the distance between John and Tom.
John is 3 meters ahead of Tom.

B-4. Solution 3: I solved this problem merely by drawing conclusions from the sentences in the problem:

- Tom is 4m ahead of Jim and Jim is 3m ahead of Peter.
- Conclusion: Tom is 7m ahead of Peter.
- John is 10 meters ahead of Peter.
- Conclusion: John is 3 meters ahead of Tom.

B-5. Solution 1: I solved this problem by drawing a diagram which represented the prices of the sugar and the salt.

![Price Diagram]

Previous price of 1 kg of salt (30c).

In the diagram it can be seen that after the price of salt was increased, the price of 1 kg of sugar was twice the price of 1 kg of salt (now 30 cents).

Thus the price of 1 kg of sugar is 60 cents.

B-5. Solution 2: I used the same method as for solution 1, but I drew the diagram “in my mind” (and not on paper).

B-5. Solution 3: I solved the problem by reasoning. The price of 1 kg of salt is now 30 cents. This is 1 ½ times the previous price; thus the previous price was 20 cents per kg. Thus the price of sugar is 3x20 cents, that is, 60 cents.

B-5. Solution 4: I solved the problem using symbols and equations, e.g.,

Suppose the previous price of salt was x cents per kg.

Then the price of sugar was 3x cents per kg.

After the increase, price of salt is 1 ½ x cents per kg.

Thus the price of 1 kg sugar is twice the present price of salt, that is, 2x30 = 60c.

B-6. Solution 1: I solved the problem by reasoning. After two sparrows flew from the first to the second tree, the first tree had two less than before, while the second tree had two sparrows more. Thus the second tree had four more than the first.

B-6. Solution 2: I drew a diagram.

![Bird Diagram]
The second tree has four more sparrows than the first.

B-6. Solution 3: Same method as for solution 2, but I drew the diagram “in my mind” (and not on paper).

B-6. Solution 4: I solved this problem by using an example, e.g., suppose at first there are 8 sparrows in each tree. After 2 sparrows fly from the first to the second, the first tree has 6 sparrows and the second 10. Thus the second tree has 4 more sparrows than the first.

B-6. Solution 5: I solved this problem using symbols and equations, e.g.,

Let the number of sparrows in each tree at first be \(x\).

After two sparrows fly from the first tree to the second, the first tree has \(x-2\) and the second tree has \(x+2\) sparrows. The difference in the number of sparrows is \((x+2) - (x-2) = 4\).

B-7. Solution 1: To solve this problem I drew a diagram representing the long log being cut into small logs.

16m

From the diagram, 7 cuts are needed to produce 8 short logs. Thus time required is \(7 \times 2 = 14\) minutes.

B-7. Solution 2: As in solution 1, but I “saw” the diagram in my mind.

B-7. Solution 3: I solved the problem by reasoning. If the long log were more than 16 meters long, one would need 8 cuts to produce 8 short logs. But the last cut is not needed, so 7 cuts are required. Time taken is \(7 \times 2 = 14\) minutes.

B-8. Solution 1: I solved this problem using symbols and equations, e.g.,

Let the weight of the jar be \(x\) kg.

Then the weight of kerosene is \((8-x)\) kg.

So the weight of half the kerosene is \(\frac{1}{2}(8-x)\) kg.

Then \(x + \frac{1}{2}(8-x) = 4 \frac{1}{2}\). Thus \(x = 1\).

Thus the weight of the jar is 1 kg.

B-8. Solution 2: I drew a diagram representing the respective weights.

\[
\begin{align*}
\text{Weight of kerosene} & \quad 8 \text{ kg} \\
\text{Weight of half the kerosene} & \quad 4 \frac{1}{2} \text{ kg} \\
\text{Weight of jar} & \quad \text{Weight of jar}
\end{align*}
\]

From the diagram, weight of half the kerosene is \(8 - 4 \frac{1}{2} = 3 \frac{1}{2}\) kg.

Thus weight of kerosene is 7 kg, and weight of jar is 1 kg.

(Or directly: Weight of jar is \(4 \frac{1}{2} - 3 \frac{1}{2} = 1\) kg.)

B-8. Solution 3: As in solution 2, but I “saw” the diagram in my mind.

B-8. Solution 4: As in solution 2, but without any diagram or image at all.
B-9. Solution 1: I drew a diagram representing the distance traveled.

Half his journey  Distance he slept  Half distance he traveled while sleeping

From the diagram, if the whole journey is 6 parts, he slept for 2 parts, that is, one third of the entire journey.

B-9. Solution 2: As in solution 1, but I “saw” the diagram in my mind.

B-9. Solution 3: I solved this problem using symbols and equations, e.g.

Let the distance for which he slept be x units.
When he awoke, the remaining distance was $\frac{1}{2}x$ units.
Then $(x + \frac{1}{2}x)$ constitutes half the journey.
So the whole journey was $2(x + \frac{1}{2}x) = 3x$ units.
Thus he slept for one third of the journey.

B-10. Solution 1: I solved this problem by drawing a diagram representing the objects.

Removing three quarters of a cheese from both scale pans, one quarter of a cheese balances a $\frac{3}{4}$ kg weight. Thus a whole cheese weighs $4x \frac{3}{4}$, i.e., 3 kg.

B-10. Solution 2: As in solution 1, but I “saw” the diagram in my mind.

B-10. Solution 3: I solved this problem using symbols and equations, e.g.,

Let the weight of a cheese be x kg.
Then $x = \frac{1}{4}x + \frac{3}{4}$. Therefore $x = 3$
Thus the weight of a cheese is 3 kg.

B-10. Solution 4: I reasoned without using a diagram or image:

One quarter of a cheese weighs $\frac{3}{4}$ kg. Thus a cheese weighs 3 kg.

B-11. Solution 1: I solved this problem using symbols and equations, e.g.,

Let original amounts of milk be x liters and 2x liters.
Amounts after pouring out are (x-20) and (2x-20) liters.
Then $3(x-20) = 2x-20$.
$x = 40$.
Thus the original amounts of milk were 40 liters and 80 liters.

B-11. Solution 2: I drew a diagram representing the amounts of milk.
From the diagram, for the first can to contain three times as much as the second after pouring, amount remaining in second can must be 20 liters. Thus original amounts were 40 liters and 80 liters.

B-11. Solution 3: As in solution 2, but I “saw” the diagram in my mind.

B-12. Solution 1: I used symbols and equations, e.g.

Let weight of plum be $x$ units and weight of apricot be $y$ units.

Then weight of a mango is $(6x+y)$ units.

Thus $10x = 3y + (6x+y)$

So $x = y$

Then weight of a mango is $6x+x$, i.e., 7 units.

Thus one mango balances the scales against 7 plums.

B-12. Solution 2: I solved this problem by drawing diagrams representing the weights.

balanced

(10 plums) (3 apricots, 1 mango)

balanced

(10 plums) (3+1 apricots, 6 plums)

From each scale pan remove 6 plums. Then 4 plums will balance 4 apricots. Thus 1 plum will balance 1 apricot. One mango balances 6 plums and one apricot, which is thus equivalent in weight to 7 plums.

B-12. Solution 3: As in solution 2, but I “saw” the diagram in my mind.

B-12. Solution 4: I solved this problem by reasoning (without imagining any picture).

One mango balances 6 plums and 1 apricot.
Thus 10 plums balance 3 apricots, 6 plums, and 1 apricot.
Thus 4 plums balance 4 apricots.
Thus (from first line) one mango balances 7 plums.
Addressing Adult Innumeracy Via an Interventionist Approach to Mathematics Aversion in Pre-service Primary Teachers

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Abstract

Student primary teachers tend to have pervasive and frequently severe negative attitudes, low mathematics self-efficacy beliefs, and anxiety of mathematics that are more extreme than those of any other undergraduate student group. If unaddressed, such mathematics aversion will be carried into primary school classrooms, presenting a tangible and substantial risk to the mathematics learning experiences of generations of primary pupils and perpetuating the relationship between adult innumeracy and mathematics anxiety. Here, the role of primary teachers is considered via a specific focus on the mathematics attitudes and competencies of pre-service (student) primary teachers commencing their first year of Bachelor of Education (Junior Primary and Primary) degree studies. This involves, firstly, a review of an earlier study using the IMAES instrument developed by the author, which identifies the students’ perceptions of mathematics, including their own capabilities; and secondly, analysis of the results of a short but comprehensive skills test administered to a different (but comparable) cohort so as to identify facets of the students’ understanding of, and capacity to carry out, fundamental mathematical tasks. Finally, this latter cohort’s attitudes towards, and perceptions of, mathematics in the context of their experiences are examined from a qualitative perspective via an open questionnaire. The results provide evidence that the impact of mathematics aversion in primary teachers may be reduced considerably by appropriate interventions during pre-service teacher preparation programs.

Introduction

The endemic adult innumeracy found in most Western societies is inextricably linked with various levels of maths-anxiety and negative mathematics attitudes, together with an often-profound aversion to the learning of mathematics. An earlier study (Klinger, 2009) that sought a better understanding of the innumeracy problem, examined the connection between adult innumeracy and mathematics anxiety. This showed the origin of both to be located in the area of primary education, with a particular focus identified for the role of primary teachers, particularly in the middle to late years of primary education. While matters of curriculum content, pedagogy, and time allocated to mathematics teaching and learning activities were identified as problematic areas, of particular relevance to the present work were the dimensions relating specifically to characteristics of teachers. These were:

1. Teachers’ expertise and preparedness to teach mathematics effectively; and
2. The attitudes and anxieties of teachers and prospective teachers.

If one accepts that attitudes and perceptions are defining character attributes, it is perhaps an obvious conclusion that student teachers will ultimately carry theirs into primary school classrooms. If their views of mathematics are profoundly negative, there are deep implications
for the perpetuation of poor early mathematics learning experiences and hence, ultimately, for adult numeracy concerns.

The role of primary teachers in the relationship between adult innumeracy and mathematics anxiety is examined here via a specific focus on the mathematics attitudes and competencies of pre-service (student) primary teachers commencing their first year of Bachelor of Education (Junior Primary and Primary) degree studies. First, the mathematics attitudes, self-efficacy beliefs, and mathematics anxiety of one cohort, surveyed in an earlier study using the Inventory of maths attitude, experience, and self-awareness (IMAES) instrument developed by the author, are reviewed to identify the students’ perceptions of mathematics, including their own capabilities. The existence of severe negative affective influences among pre-service primary teachers is again identified and it is postulated that these might reasonably be expected to manifest in a concomitant deficit of functional numeracy skills. Second, the results of a short but comprehensive skills test, administered as a diagnostic intervention to a different (but comparable) cohort, are analysed to identify facets of the students’ understanding of, and capacity to carry out, fundamental mathematical tasks. The test is a principle component of a multi-faceted approach to the challenge of preparing math-averse primary and junior primary student teachers for their future classroom careers. Thirdly, this latter cohort’s attitudes towards and perceptions of mathematics in the context of the intervention are examined from a qualitative perspective via an open questionnaire.

While the findings support the functional deficit hypothesis, it is also demonstrated that negative attitudes, poor self-efficacy beliefs and anxiety are ‘...plastic, not steel’ (Klinger, 2006). That is, they may be positively modified with appropriate interventions during teacher preparation programs.

Recap – primary teachers and the IMAES instrument

According to the 2003 IEA’s Trends in International Mathematics and Science Study (TIMMS), internationally about a quarter of those teaching mathematics at fourth-grade level have a post-secondary specialization in the subject – but the statistic is strongly skewed by very high proportions (ranging from 48-62%) in Latvia, Russia, Moldova, Iran, and Singapore. In the UK and the USA only 8% of primary teachers have a mathematics major and in Australia the proportion is reported as 17%. However, they have an average of 16 years teaching experience and more than 90% of fourth-graders participating in the TIMMS study were taught by teachers ‘who felt ready to teach the topics in number, algebra, measurement, and data’ (Mullis, Martin, Gonzalez, & Chrostowski, 2003, p. 255). The report also compared qualitative responses from fourth-grade primary school pupils and eighth-graders recently transitioned to secondary school. This indicated that over a four year period there was a substantial decline in the proportion of pupils who agreed ‘a lot’ that they ‘enjoy learning mathematics’ and a corresponding doubling of the numbers who disagreed with the statement. Similarly, High SCM (Self-Confidence in Learning Mathematics) assessments declined greatly, while Low SCM assessments doubled (Mullis et al, 2003).

Given this substantial decline in pupils’ confidence and enjoyment of mathematics learning, the teachers’ perceptions of their readiness are incongruent with actual practice and the classroom experience. Perhaps they really mean that they feel prepared to deliver the curriculum. Perhaps their uncertainties and anxieties are such that they lack an appreciation of the distinction between teaching procedures and promoting an understanding of the language and process of mathematics. In that sense, many of them could be regarded as being covertly innumerate at the level of Maguire and O’Donoghue’s (2002) integrative phases.
In one study of pre-service teachers, 72% of the subjects perceived their own negativity to be particularly attributable to the primary teachers who taught them (Uusimaki & Nason, 2004). Much more has been written about the consequences this can have in the primary classroom and it is clear that those new to teaching are particularly swayed by their past learning experiences (Stables, Martin, & Arnhold, 2004). Schuck and Grootenboer put it succinctly, stating that the negative beliefs about mathematics generally held by student primary teachers ‘prevent them from teaching mathematics that empower children’ (Perry, Way, Southwell, White, & Pattison, 2005, p. 626). While this has long been acknowledged, the literature is dominated by qualitative and descriptive methodologies in studies that report the attributes of pre-service teachers.

In contrast, the IMAES instrument provides quantitative profiles that permit comparison with other groups. The details are well documented (Klinger, 2006) but, in brief, it is a multi-part questionnaire that uses (mostly) 5-point Likert scales for responses to statements about maths-attitude, maths-anxiety, mathematics self-efficacy beliefs, and past/early mathematics learning experiences. Empirical results showed that pre-service primary teachers scored lower than other students in the three chief constructs of maths-anxiety, maths-attitude, and mathematics self-efficacy beliefs. The results are reproduced in Figure 1 below (zero on each scale indicates neutrality). Compared to all other students, they had stronger responses to negative statements on the questionnaire and weaker responses to the positive statements, and very low p-values provided strong (α=5%) evidence to infer that the observed differences were indeed real effects with some systematic cause (Klinger, 2009).

In summary, student primary teachers tend to have pervasive mathematics anxiety, negative attitudes, and low mathematics self-efficacy beliefs that are more extreme than those found in any other undergraduate group. ‘Drilling down’ to the level of individual questionnaire statements revealed internally consistent responses that identified strong apprehension of the mathematics classroom experience (whether reflective or anticipatory), fearful perceptions of mathematics itself and the challenges it presents, disinterest in mathematics as an occupation or intrinsically enjoyable activity, and lack of problem-solving confidence. However, it was observed that, on the whole, the subjects’ disaffection with mathematics was more a reaction to mathematics learning than to mathematics itself. The findings are consistent with those of other researchers reporting the common occurrence among primary education students of negative attitudes towards mathematics and science, including many who are overtly maths-anxious and even maths-phobic as a result of their past mathematics learning experiences (see, for instance, Taplin, 1998; Schuck, 1999; Trujillo & Hadfield, 1999; Hawera, 2004). Indeed, some twenty years ago (that is, in the era when the average primary teacher reported in the TIMMS study was in training) the so-called Speedy Report (Speedy, Annice, Fensham, & West, 1989) in Australia stressed the importance of high-order mathematical knowledge and competency while noting serious concerns that many student primary teachers were entering their teaching courses with a very poor knowledge of mathematics.
For nearly a decade, the University of South Australia’s School of Education has responded to the challenge of math-averse primary and junior primary student teachers by adopting a proactive approach within the core course/topic, ‘Mathematics Curriculum for Early and Primary Years 1’, undertaken by commencing undergraduate and graduate-entry students. The tailored approach consists of several components, described to the students as ‘diagnostic tools’ presented to provide them with opportunities to recognise and expand their mathematical knowledge base, taking into consideration their curriculum needs as future primary (‘Reception to Year 7’ or ‘R-7’) teachers. The components comprise:

- a non-standardised timed (1 hour) diagnostic test in four sections covering:
  1. number: place value, arithmetic operations, money, fractions, decimals, and percentage;
  2. space and measurement;
  3. data (including tables, graphs and diagrams) and chance;
  4. patterns (including simple algebraic relations), number theory (e.g. prime numbers), and order of operations (‘BODMAS’);
- supplementary lectures and tutorials, extending to 1:1 support as required; and
- a reflective questionnaire.

The questions presented in the diagnostic test/tool (DT) correspond to the level of mathematical attainment expected of a Year 8 pupil in a South Australian school. The test is initially undertaken during the first week of the course/topic without prior preparation and, although presented as a means of functional evaluation for the benefit of the students, it also serves a gatekeeper function: students must demonstrate mastery by attaining a minimum 80% pass rate for each section of the test in order to progress. While they are permitted three
attempts (with different questions each time), those who remain unsuccessful after the third attempt are obliged to repeat the entire course/topic.

After the initial administration of the DT, additional (extra-curricular) lectures and tutorials are provided. Attendance is not compulsory but students are encouraged to reflect on their diagnostic results and to seek tuition and support according to their individual needs, which may include making their own arrangements. At the start, tutorial groups are quite large (30-50 students) but well staffed with casual tutors, drawn (increasingly) from schools and/or professional organisations; many of them, as teachers themselves, are recognised for their expertise in the field of providing mathematics support at this level. Tuition in these classes is said to be directed towards promoting understanding rather than being merely functionally remedial and they appear to be sufficient for the majority of students to demonstrate mastery on their second attempt, following which those who remain unsuccessful self-identify to attend small-group tutorials (12-20 students) and also have access to 1:1 tutorial advice before undertaking their final attempt at the DT.

The non-standardised protocol for the DT was adopted so as to afford the lecturer the opportunity to adapt or modify content on the basis of experience informed by practice. While this is desirable pedagogically, it tends to limit the extent to which results from successive cohorts can be directly compared in any longitudinal analysis, although the tendencies exhibited in the data (Table 1) are considered to be generally representative.

<table>
<thead>
<tr>
<th>1st attempt (n=132)</th>
<th>2nd attempt (n=105)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pass</td>
</tr>
<tr>
<td></td>
<td>n</td>
</tr>
<tr>
<td>1. Number</td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>83</td>
</tr>
<tr>
<td>2. Space &amp; Measurement</td>
<td>48</td>
</tr>
<tr>
<td>3. Data &amp; Chance</td>
<td>88</td>
</tr>
<tr>
<td>4. BODMAS, patterns &amp; number theory</td>
<td>44</td>
</tr>
<tr>
<td>One section</td>
<td>31</td>
</tr>
<tr>
<td>Two sections</td>
<td>30</td>
</tr>
<tr>
<td>Three sections</td>
<td>24</td>
</tr>
<tr>
<td>Four sections</td>
<td>25</td>
</tr>
<tr>
<td>OVERALL</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 1 Summary of Diagnostic Test (undergraduate, 2007)

The results in Table 1 obtained from a cohort of 132 undergraduate students who began the course/topic in 2007. The summary data from the performance of individual cohorts such as this is illuminating on several levels, not least in that very low mastery rates in first attempts reveal the lack of preparedness observed by various researchers noted previously, while subsequent attempts demonstrate the value of perseverance. Moreover, feedback from the reflective questionnaire illustrates the value that the vast majority of students ascribe to the process (see below).
Twenty-five students representing 18.9% of the cohort demonstrated mastery over all sections in their first attempt, with more than 80% identifying weaknesses in one or more sections. Some two-thirds of the students encountered difficulties in the sections on space and measurement concepts and with order of operations, patterns and number theory. Questions involving more straightforward arithmetic and finding information from tables and graphs were somewhat less problematic, though each corresponding section provided difficulties for about one third of the students, which is a far from inconsequential outcome. Of particular interest is the proportion of students (some 63%) who were troubled by more than one section of the test, indicating very clearly that their lack of success cannot be explained as a mere aberration, oversight, or simple memory lapse but instead reveals a much broader dysfunction in their lack of preparedness.

Following the first instance of the test, two students withdrew from the course leaving 105 to undertake the second attempt. The majority of these (71.4%) were successful, taking to 75.8% the proportion of the cohort now having exhibited mastery of the material. Of the remaining 30 students who had failed to demonstrate overall mastery at this point, 12 had failed just one section, 16 had failed two sections, one had failed three sections and another individual failed all four sections. Again, the greatest difficulties were encountered with the sections on space and measurement concepts and with order of operations, patterns and number theory, the former being the most problematic. At the third attempt, all but four students achieved the required results and they were invited to re-enrol in the course/topic at the next opportunity, being unable to obtain a passing grade in this instance.

A separate cohort of graduate-entry students in the same year, 2007, followed essentially the same protocol of diagnosis and support. An abridged summary is shown in Table 2:

<table>
<thead>
<tr>
<th>Section</th>
<th>Fail</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Number</td>
<td>21</td>
<td>38.9</td>
</tr>
<tr>
<td>2. Space &amp; Measurement</td>
<td>18</td>
<td>33.3</td>
</tr>
<tr>
<td>3. Data &amp; Chance</td>
<td>8</td>
<td>14.8</td>
</tr>
<tr>
<td>4. BODMAS, patterns &amp; number theory</td>
<td>25</td>
<td>46.3</td>
</tr>
<tr>
<td><strong>OVERALL</strong></td>
<td>37</td>
<td>68.5</td>
</tr>
</tbody>
</table>

Table 2 Summary of Diagnostic Test (graduate-entry, 2007)

Here, although a substantial majority (68.5%) of the students were unable to demonstrate mastery, the proportion of successful students was almost double that of the undergraduate cohort. While Section 4 was the most problematic for this group, too, it was rather less so at 46.3% compared to 66.7%. Section 2 performance compared favourably, also, with the proportion of students encountering difficulties in this area being almost half that of the undergraduate group (33.3% compared to 63.6%) and similarly for Section 3. Only in Section 1 were the respective proportions roughly equivalent. Following their second attempt, 89.5% of the graduate-entry group had demonstrated mastery of the test material compared to 71.4% of the first group.
An explanation for these differences might be considered to lie in the greater experience of these graduate entrants, gained by undertaking a first undergraduate degree. Although a tempting speculation, it could, however, be quite misleading without first considering the nature of those studies and particularly the discipline in which they were undertaken. It may be at least as tempting to speculate that, as former undergraduate students in non-teaching degree programs, an IMAES profile would reveal them to be rather less math-averse and anxious and thus in a relatively advantaged position (though apparently not to any great extent).

What is clearest in these differences is that the intervention opportunities afforded by the diagnostic tools and procedures appear to be highly effective in raising students’ awareness of their strengths and weaknesses and guiding them to much more successful outcomes. The students themselves express an appreciation of this in the feedback provided via completion of reflective questionnaires. Students were almost unanimous in their agreement that the diagnostic tool was effective in this regard, with comments that (for instance):

“It was great to see from the beginning what we knew and needed help with.”
“I think the remedial classes helped me immensely.”
“I had completely forgotten these maths concepts and I now look at my own everyday activities in a very different light.”

Around 80% of respondents agreed that their strengths and weaknesses had been identified early and their insights as to their needs show broad correspondence with the test results. While many reported that they had discovered the need to review and revise material, similar numbers felt that they had encountered concepts that they had not learned adequately at school. One student commented, “Maths has always been a weakness and this is probably due to primary school teaching as well as lack of review now.” Compared to those who felt otherwise, almost twice as many students believed that the diagnostic tool identified areas that needed improvement of which they were previously unaware. Their comments included:

“I had unrealistic ideas about my abilities in all areas.”
“I thought I was doing the question correctly in the original DT however got them wrong.”
“It took the DT for me to realize what areas needed revision and I probably would not otherwise have been aware of them.”

The provision of supplementary tuition was generally appreciated, with a good take-up rate, and students were also proactive with more than 50% of them obtaining outside assistance from family, friends, and former mathematics teachers. They also identified a range of benefits that arose as a consequence of the diagnostic tool and intervention – on a personal level, in terms of improved confidence and greater self-awareness (“I thought I knew it all; boy was I wrong”; “I was very anti maths but now I am much more questioning in a mathematical way”); to their mathematics knowledge base, and as a prospective primary teacher (“Confidence that I know the ‘fundamentals’ of each area/topic or at least where/how to find more information”; “I can see how I can teach children to love maths without my past negative experiences of maths filtering in”).

Finally in these reflective questionnaires, students gave suggestions on how to improve the DT. These focussed overwhelmingly on a desire for more notice and preparation time before the administration of the first test and for the test time to be increased from 1 hour to 1½ hours. Numerous comments suggested that many students experienced test anxiety to which they then attributed their poor outcomes from the initial DT.
Discussion and Conclusion

Although findings for different IMAES and DT cohorts were reported here, experience with the IMAES instrument in numerous contexts has demonstrated that the resulting profiles are robust in their ability to characterise affective, cognitive, and behavioural attributes. There is no evidence that the DT cohorts are atypical in any significant respect and the DT results in fact confirm that the postulated behaviours are manifested in these groups. They also reveal that after two decades the concerns expressed in the Speedy Report (Speedy et al., 1989) remain topical. The relationship between a negative IMAES profile and low competency levels is complex and one does not necessarily imply the other; competency can be depressed by anxiety and when this is relieved more accurate assessments of competency can emerge, as Ashcraft (2002) pointed out, observing that highly math-anxious individuals ‘do not have a global deficit in math competence’ (p. 182). Lack of motivation, though, has been found to contribute to low attainment (Mitchell, 1993) and the Third International Mathematics and Science Study (TIMSS) found a widespread positive correlation between liking mathematics and math achievement (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996).

Highly relevant to the present context are the observations that:

1. children’s attitudes towards mathematics tend to decline as they progress through primary to secondary education (McLeod, 1994); and

2. children’s conceptions suffer from the destructive effects of ‘unimaginative instruction and non-positive teacher attitudes’ and the pressure to ‘cope with highly demanding tasks, frequently at a pace beyond their ambition’ (Philippou & Christou, 1998, p. 192).

It is highly likely that the majority of pre-service primary teachers represented in this study are themselves casualties of these phenomena. Moreover, it is likely that, if left unchecked, they will themselves become perpetrators of such ills in their future classroom. Of this there can be little doubt: the extremes of the negative profile revealed by the IMAES results coupled with the very poor competency levels diagnosed by the first DTs present a tangible and substantial risk to the mathematics learning experiences of generations of primary pupils.

If one assumes that 100 of the 130 undergraduate students in this study actually enter the profession and further supposing an average career span 20 years with an average conservative teaching load of 25 pupils per year (primary school teachers usually have responsibility for a single class for one year), one teacher might influence (for better or worse) some 500 pupils. On such a basis, this one cohort of student teachers might be expected to reach 50,000 individuals whose early mathematics learning experiences may well determine their future as numerate or innumerate adults over the ensuing 60 or so years. From such a perspective, it is reassuring that the results reported here demonstrate that positive interventions within teacher education programs are not only possible but necessary. However, there remains considerable scope for far greater progress in efforts to beat the numeracy problem. As a matter of public policy, it should be unacceptable that so many prospective teachers should begin their professional education from such a low mathematics base.
Acknowledgement

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References


Investigating Meaning in Learning: A Case Study of Adult Developmental Mathematics

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Abstract
The objective of this article is to investigate meaning and relevance in the context of adult developmental math learning and instruction. In this case study, at the Art Institute of San Francisco, 12 vocational instructors and four math learners are interviewed on their early and current math experiences. During the semi-structured interviews, the adult math learners and vocational instructors reminisce on math in their learning and in their work. The interview transcripts are later analyzed for constructivist themes or codes.

From instructor interviews, there appears to be a strong correlation between instructor views of meaning and learning and constructivist principles. There is a weaker correlation of these themes with the views of the adult learners, and there is evidence these developmental learners show signs of cognitive overload on certain constructivist tasks. These adult developmental learners appear to derive mathematical meaning from behaviorist learning and instruction involving step by step processes, the linking previous concepts, and the repetition of key ideas. It may be that this is the way these informants learned math in the pre-constructivist days of mathematics instruction.

Introduction
The primary objective of this study is to find what is mathematically relevant to adult learners and vocational instructors at the focal site. The focal site in this case is the Art Institute of San Francisco (Art Institute), a privately owned vocational college.

A mathematically imaginative workforce is crucial because of advancing methods of production, especially the introduction of computer-based tools such as Photoshop, Illustrator and AutoCAD. Workers are less and less expected to carry out mindless, repetitive chores. Instead they are engaged actively in team problem-solving, talking with their co-workers and seeking mutually acceptable solutions (Bailey, 2001). Workers must ask the right questions, assimilate new information and solve unfamiliar problems in new ways.

Throughout their lives learners will need to be adaptable – to continue to explore the world, accommodate changing conditions and actively seek and create new knowledge. This need or flexibility implies that a mathematics education must emphasize a dynamic literacy that is centred on the problem being solved (Swan, 2005).

In the enlightened postcolonial period after 1945, anthropologists started to present interviews as scholarship using journalistic techniques (MacLeod, 1987/1995), rather than using the historically compromised scientific approach (Burawoy, 1991). Similarly, the ultimate objective of this study is not to just establish a relationship between two pre-ordained variables, but to use the instructor and adult learner insight into mathematics instruction and learning to help our learners and inform our instructors at the focal site (Duncan-Andrade,
1999). The interview data is being taken as a whole of the case and the interview data is triangulated with observations, student work. The data is also triangulated with the secondary indicators of learning outcomes such as the quantitative pass rates, persistence, average class grade and absences.

**Literature Review**

“Some learners find it pointless to do their mathematics homework; some like to do trigonometry, or enjoy discussions about mathematics in their classrooms. Some learners think that mathematics is useless outside school; other learners are told that because of their weakness in mathematics they cannot complete a Bachelor’s degree. All these raise questions of meaning in mathematics education” (Kilpatrick, Hoyles, and Skovsmose, 2005b p.5). Meaning is used in a rather personal sense of the learner relating to relevance and personal significance. For example, “What is the point of this for me?” On the other hand, meaning can also be used in a rather objective way when describing “an agreed, common meaning within a community” (Kilpatrick et al., 2005b, p. 9).

We may claim that an activity has meaning as part of the curriculum, while students might feel that the same activity is totally devoid of meaning” (Kilpatrick et al., 2005b, p. 9). One can, however, even go a step further by saying that although a learner might think that a certain activity is totally devoid of objective meaning, the learner still sees a personal meaning in relation with the activity. This personal meaning, then, can be of different kinds. The first issue for meaning is that personal meaning is subjective and individual. This means that every person has to construct her or his own meaning with respect to a certain object. There is no given objective meaning; meaning cannot just be endowed. Also, as the construction of meaning is not collective but individual, different students sitting in the same lesson can also construct different meanings (Kilpatrick, Hoyles and Skovsmose, 2005a, p. 2).

To summarize, meaning can be reflected on and it can also be subconscious. This means that the process of meaning making can sometimes be dominant in the situation, so that as one is aware of what is going on; the meaning enters consciousness. Meaning does not have to be conscious but can be constructed sub-consciously, so that it is there without awareness. From a constructivist perspective, Kilpatrick states that the problem of construction of meaning itself is not really tackled. This is an evasive problem, because it is difficult to know what each partner (the learner and the teacher) thinks; we can only hypothesize this by interpreting what they do and say (Kilpatrick et al., 2005b p.19). Math has no implicit meaning. This implies that everyone has to construct his or her own personal meaning, so that it is probable that learners develop different kinds of meaning concerning the same mathematical task or problem.

**Mathematics Learning**

There is also a plethora of recent literature, which supports meaningful mathematics learning. The National Council of Teachers of Mathematics (NCTM) released a landmark document *Curriculum and Evaluation Standards for School Mathematics (The Standards)*, which posits that increasing meaning in mathematics instruction, curriculum and assessment better prepares the learner for employment: The fastest growing jobs require much higher math, language and reasoning capabilities than current jobs, while slowly growing jobs require less (Cohen, 1995, p.7).

Clearly influenced by the foundational philosophies of constructivism and
incorporating elements of Knowles’ (1980) concepts of adult centered learning, the Standards suggests all high school learners are capable of succeeding in mathematics, but courses must link-in and interweave the Big Ideas of mathematics. The Big Ideas include balance, number sense, proportional reasoning, variable, representation, measurement, relations and inductive and deductive reasoning. According to The Standards, these Big Ideas should be introduced as early as possible in the mathematics syllabus to allow learners to gain familiarity with these concepts of mathematics and ultimately to prepare the learners for subsequent and more advanced algebra courses.

Several assumptions shape the vision of The Standards. Firstly, mathematics is something a person does. Knowing mathematics means being able to use it in a purposeful way. To learn mathematics, learners must be engaged in exploring, conjecturing and thinking rather than the rote learning of rules and procedures. In other words, mathematics is not a spectator sport. When learners construct a personal knowledge derived from meaningful experiences, they are much more likely to retain what they have learned. This underlines the new role of the teacher in providing and explaining these experiences. Another assumption is that mathematics has a broad content encompassing many fields. Learners can benefit from exposure to a broad range of content that reveals the usefulness of mathematics. Through this exposure learners can build a foundation of relevance and meaning to their learning of mathematics. Another assumption of The Standards is that mathematics instruction can be improved through the appropriate use of evaluation. Evaluation should concentrate not only on assessing what the learners knows, but how they think and approach problems.

Crucially, according to The Standards, the teaching and learning focus should be on reasoning, rather than rote learning. The teacher of mathematics should orchestrate discourse by posing questions and tasks that elicit, engage, and challenge each learner's thinking. The teacher of mathematics should:

- Listen carefully to learners’ ideas.
- Ask learners to clarify and justify their ideas orally and in writing.
- Identify, from among many ideas that learners bring up in a discussion, ideas to study in depth.
- Decide when and how to attach mathematical notation and language to learner's ideas.
- Decide when to provide information, when to clarify an issue, when to model, when to lead, and when to let a learner struggle with a difficulty.
- Monitor the learner’s participation in discussions and deciding when and how to encourage each learner to participate.

The teacher of mathematics should promote classroom discourse in which learners:

- Listen to, respond to, and question the teacher and one another.
- Use a variety of tools to reason, make connections, solve problems, and communicate.
- Initiate problems and questions.
- Make conjectures and present solutions.
• Explore examples and counterexamples to investigate a conjecture.
• Convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers.
• Rely on mathematical evidence and argument to determine validity.

Another seminal work in this area is *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* (Crossroads) by the American Mathematical Association of Two Year Colleges. *Crossroads* has established goals and standards for preparation for college-level mathematics and is guided by these fundamental principles:

- All learners should grow in their knowledge of mathematics while attending college.
- Learners should study mathematics that is meaningful and relevant.
- Mathematics must be taught as a laboratory discipline.
- The use of technology is an essential part of an up-to-date curriculum.

*Crossroads* is highly influenced by *The Standards*, but is yet another highly influential work in its own right, because it is primarily directed to adult developmental mathematics. *Crossroads* reflects many of the same principles found in *The Standards*. These standards place emphasis on using technology as a tool and as an aid to instruction, developing general strategies for solving real-world problems, and being actively involved in the learning process. In particular, developmental math courses at the post-secondary level should include traditional topics, but should also incorporate technology and project-based learning.

**Research Question**

The research question addressed in this article is:

Are the principles of constructivist math learning reflected in the math recollections of adult learners and vocational instructors at the *Art Institute*?

In the next section will describe the methodology used to answer this question.

**Research Methodology**

The research draws from participatory action research methods, (Kemmis, and McTaggart, 1988) which position the researcher simultaneously as a practitioner at the focal site where the study takes place, and also as a collaborator with the learners themselves with the aim of improving their own educational practices. The research goals of this research are as follows:

- The improvement of practice through continual learning and progressive problem solving.
- A deep understanding of practice and the development of meaningful mathematics learning.
- An improvement in developmental mathematical learning outcomes in the community of the *Art Institute*.

According to the above sources, participatory action research as a method is scientific in that it changes something and observes the effects through a systematic process of
examining the evidence. The results of this type of research are practical, relevant, and can inform theory.

The design of this case study is based on a seminal work by Yin, in that a single case study is used to confirm or challenge a theory, or to represent a revelatory or extreme case (Yin, 1994). In this instance, the case study at the Art Institute is ideal for a revelatory case study, as the researcher has access to a phenomenon that was previously inaccessible. For example, many developmental mathematics courses occur in the space of public education and are devoid of context and relevance, but at the Art Institute developmental mathematics is studied simultaneously with the learner’s vocational degree and as such is ideal for a revelatory case study of meaning in mathematics.

Each individual case study consists of a whole study, in which facts are gathered from various sources and conclusions drawn on those facts. Consideration has been given to construct validity, both internal and external. Yin suggests using multiple sources of evidence as the way to ensure the construction of validity. He lists six sources of evidence for data collection in the case study protocol: documentation, archival records, interviews, direct observation, participant observation, and physical artifacts.

Art Institute informants were also interviewed on the general theme of mathematics in their education and, in the case of the instructors, their current instructional positions, in order to answer the research question. The outline of these semi-structured interviews can be found in Appendix D. Vocational instructors, in addition to learners, were chosen as interviewees from the focal site because they provide an acute insight into the meaning and especially the relevancy of mathematics to the learner’s chosen career. There are some insights the learner can never provide. One of the goals of the interview design was to keep the interviews with the instructors comparable to the learners by including similar questions for both.

The semi-structured interviews with vocational instructors at the Art Institute yielded 12 digitally recorded interviews with representing various vocational categories such as Advertising, Animation, Fashion Design, Fashion Marketing, Interior Design, Culinary Arts, Graphic Design, Liberal Arts, Game Programming and Game Design. The outline of this study, the recruiting script and consent form for the instructor interviews were initially e-mailed to all the vocational instructors at the focal site. This yielded only four replies, indicating it would be extremely difficult to recruit instructors from all the vocations at the focal site using this scattershot approach. Further instructor interviewees were eventually selected by snowball sampling, which is a special non-probability method used when the desired sample characteristic is rare. While this technique dramatically lowered search time, it comes at the expense of introducing bias because the technique itself reduces the likelihood that the sample will represent a good cross section from the population of vocational instructors.

The interviews lasted approximately twenty minutes in length; they took place in the instructor’s classroom during a break in class or in specialized interview rooms provided by the Art Institute and were digitally recorded for later transcription and analysis.

Recruitment of adult math learners for interview required a different approach. On the last day of the developmental math course, the study was introduced with a recruiting script and consent form and all the learners were invited to participate in an interview. Four learners responded to the request by returning the signed and dated consent form. These students were
then informed that they would receive an e-mail at the start of the next developmental math course, letting them know that they were selected for an interview and setting up a time and place for the interview. Ultimately these interviews took place in a specialized private interview rooms, they lasted approximately twenty minutes in length each and were digitally recorded for later transcription.

**Descriptive Categories for Analysis**

The interview transcripts of both adult math learners and vocational instructors at the *Art Institute* are coded based on the *Crossroads* principles as categories for analysis:

<table>
<thead>
<tr>
<th>Crossroads Principle</th>
<th>Theme</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>All learners should grow in their knowledge of mathematics while attending college.</td>
<td>The Big Ideas of math</td>
<td>BI</td>
</tr>
<tr>
<td>Learners should be taught mathematics that is meaningful and relevant</td>
<td>Meaningful and relevant mathematics instruction</td>
<td>MI</td>
</tr>
<tr>
<td>Developmental mathematics should be taught as a laboratory discipline.</td>
<td>Relevant constructivist projects</td>
<td>LD</td>
</tr>
<tr>
<td>The use of technology is an essential part of an up-to-date curriculum.</td>
<td>Technology in adult math learning</td>
<td>TI</td>
</tr>
<tr>
<td>Other</td>
<td>Miscellaneous</td>
<td>MT</td>
</tr>
</tbody>
</table>

The research analysis is based on the frequency of the categories for analysis in the transcripts of the semi-structured interviews with learners and vocational instructors. The construction of these categories for analysis, or codes, is not arbitrary, but based on the principles of the *Crossroads* report as shown above.

<table>
<thead>
<tr>
<th>The Big Ideas of Math Theme</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>Pro</td>
</tr>
<tr>
<td>Number Sense</td>
<td>Nus</td>
</tr>
<tr>
<td>Measurement</td>
<td>Mmt</td>
</tr>
<tr>
<td>Variable</td>
<td>Var</td>
</tr>
<tr>
<td>Relations</td>
<td>Rel</td>
</tr>
<tr>
<td>Representation</td>
<td>Rep</td>
</tr>
<tr>
<td>Induction/Deduction</td>
<td>Idu</td>
</tr>
<tr>
<td>Balance</td>
<td>Bal</td>
</tr>
</tbody>
</table>

The first of these principles is *All learners should grow in their knowledge of mathematics while attending college* (BI). This theme is further detailed by the sub-themes of the *Big Ideas*. The second principle of *Crossroads* is that *Learners should be taught mathematics that is meaningful and relevant* (MI). In addition, these instructional themes were also sub-themed with the cognitive, behaviorist and constructivist instructional styles, such that:

<table>
<thead>
<tr>
<th>Meaningful Instruction Sub Theme</th>
<th>Code</th>
</tr>
</thead>
</table>

The third and fourth principles are *Mathematics should be taught as a laboratory discipline* (LD) and *The use of technology is an essential part of an up-to-date curriculum* (TL). The miscellaneous themes (MT) were reserved for meaningful learning and instruction themes that were not based on the *Crossroads report*. Finally, all of the above themes in meaningful mathematics learning were cross-coded with vocational and learner identities.

<table>
<thead>
<tr>
<th>Informant</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertising Instructor</td>
<td>Adv</td>
</tr>
<tr>
<td>Animation Instructor</td>
<td>Anim</td>
</tr>
<tr>
<td>Fashion Design Instructor</td>
<td>FaDes</td>
</tr>
<tr>
<td>Fashion Marketing Instructor</td>
<td>FaMar</td>
</tr>
<tr>
<td>Interior Design1 Instructor</td>
<td>IntDes1</td>
</tr>
<tr>
<td>Interior Design2 Instructor</td>
<td>IntDes2</td>
</tr>
<tr>
<td>Culinary Arts Instructor</td>
<td>CulArt</td>
</tr>
<tr>
<td>Graphic Design Instructor</td>
<td>GrDes</td>
</tr>
<tr>
<td>Liberal Arts Instructor</td>
<td>LibArt</td>
</tr>
<tr>
<td>Game Programming Instructor</td>
<td>GaPrg</td>
</tr>
<tr>
<td>Game Design Instructor</td>
<td>GaDes</td>
</tr>
<tr>
<td>Learner 1</td>
<td>L1</td>
</tr>
<tr>
<td>Learner 2</td>
<td>L2</td>
</tr>
<tr>
<td>Learner 3</td>
<td>L3</td>
</tr>
<tr>
<td>Learner 4</td>
<td>L4</td>
</tr>
</tbody>
</table>

Finally, artifacts such as learners’ projects, exercises and class observations, complete the data gathering. These were also used as a data in order to perform the analysis.

**Results**

Graph 1a shows there are approximately seventy instances of *all learners should grow in their knowledge of mathematics while attending college* themes in the 12 instructor interviews and less than five for the four learner interviews. This also contains references to project based learning themes and twelve references to technological co-themes within the primary theme. There are surprisingly small frequencies of themes combined with meaningful and relevant instruction. Overall this theme has a powerful resonance amongst the instructors, but less so amongst the learners.
Graph 1a. Distribution of the principle all learners should grow in their knowledge of mathematics while attending college (code BI)

The Graph 1b shows the approximately seventy instances of the *all learners should grow in their knowledge of mathematics while attending college* (BI) themes in the 12 instructor interviews and four learner interviews broken down into the sub-themes of the *Big Ideas* of mathematics. The frequency shows a large tendency amongst instructors to themes of proportion, number sense and measurement, with considerable references to variable, relations, representation and balance. Learners are hardly represented in this category.

Graph 1b. Distribution of the sub-themes on the all learners should grow in their knowledge of mathematics while attending college

The Graph 2a shows there were approximately forty instances of the theme *all learners should be taught mathematics that is meaningful and relevant* in the 12 instructor interviews and less than ten for the four learner interviews. There were a surprisingly small frequency of co-themes in meaningful instruction from the *Big Ideas*, project learning and in technology.

Graph 2a. Distribution of the principle all learners should be taught mathematics that is meaningful and relevant (code MI)

Graph 2b shows the approximately forty instances of all learners should be taught mathematics that is meaningful and relevant themes in the 12 instructor interviews and four learner interviews broken down into the instructional sub-themes. From instructor interviews, there appears to be a strong correlation between instructor view of meaningful instruction and the principles of The Standards and Crossroads reports. We can see from Table 2b that the adult developmental learners appear to derive much more mathematical meaning from behaviorist and cognitive instructional tasks involving step by step process, the linking previous concepts and the repetition of the Big Ideas in different contexts.

Graph 2b. Distribution of the sub-themes on the principle all learners should be taught mathematics that is meaningful and relevant

Graph 3 shows a high frequency of the principle; developmental mathematics should be taught as a laboratory discipline, with approximate forty-five references, with seventeen of these also referencing the Big Ideas and five also referencing technological co-themes.
Graph 3. Distribution of the principle developmental mathematics should be taught as a laboratory discipline (code LD).

Graph 4 shows the frequency of the use of technology is an essential part of an up-to-date curriculum themes, with approximately twenty two references, with eight also co-referencing the Big Ideas of mathematics. These themes proved useful in generating relevant and meaningful projects and example problems.

Graph 4. Distribution of the principle the use of technology is an essential part of an up-to-date curriculum (code TL)

**Limitations of the Results**

The traditional perspective of research methods literature typically puts most of the emphasis on the role of the researcher in the interview process. Interviewers are purported to be "instruments" in the research process, and the researcher is encouraged to build rapport and trust with the interview subjects by being an attentive listener and having a "sympathetic understanding" of, and profound respect for, their thoughts, opinions, and perspectives (Angrosino, M. and Mays de Pérez, 2000). While some have warned about the potential dangers of over-rapport, such as the lack of objectivity on the part of the researcher, others have encouraged the establishment of some type of relationship between the interviewer.

Perhaps the most important development away from this traditional model of the interview in recent years has been the articulation of the constructivist approach to the interview. Constructivism emphasizes the dialogic nature of the interview and the mutuality of the research. In contrast to the traditional approach in which the interviewee is viewed as a repository of answers and the interview process itself is visualized as a conduit or pipeline of information that the researcher seeks, constructivism understands the interview as a meaning-
making experience and as a site for producing knowledge through the "active" collaboration of both interviewer and interviewee (Fontana, A. and Frey, J., 2000). The interview is no longer defined as a question-and-answer format, social scientific prospecting, or a search-and-discovery mission, but a "special performance" involving interviewer and interviewee both eliciting and representing an interpretive relationship of the world. Some may be suspicious of this journalistic and non-scientific approach, but we have to remember that the supposedly objective scientific approach to anthropology has been historically compromised (Burawoy, M., 1991).

**Findings**

From the analysis, came this set of findings:

- The vocational instructor interviews are liberally sprinkled with the constructivist learning and instruction principles of *Crossroads*.
- Alternatively, the learner interviews show a quantitative and qualitative tendency to an interactive, behaviourist, step-by-step instructional approach, with each step logically linked to its predecessor, resonates strongly with the adult learner informants.
- The use of repetition to create meaning in various contexts is a common theme in the instructor and adult learner interviews, “I had to do the problems over and over to check my solution and this repetition made me get the concepts”.
- Instruction presenting mathematics in visual, verbal, spatial and textual representations of meaning is an important theme in the vocational instructor interviews.
- The interview results in the miscellaneous themes and especially the artifact evidence the appendices; indicate that developmental learners show signs of cognitive overload on certain constructivist tasks.

**Conclusions**

The principles of constructivist math learning are strongly reflected in the math recollections of vocational instructors at the *Art Institute*, but not so much in the recollections of adult math learners. The vocational instructor interviews suggest a strong linkage between the principles of *Crossroads and The Standards* to relevance in specific vocational training. According to the interview themes, the **Big Ideas** of mathematics can be directly linked to relevant vocations, practices and ubiquitous software design tools such as *Illustrator, Photoshop* and *AutoCAD*.

In addition, the repetition of mathematical ideas in various contexts is a common theme in the learners’ interviews. Likewise, learners in the Confucian Heritage Countries (CHC) are known to practice memorizing and repetition, which, if one equates memorizing with surface learning, brings into question the amount of meaningful learning that takes place. However in CHC memorizing is not synonymous with rote learning (Biggs, 1996). Repetition is carried in order to create meaning, or as a contextual response to the critical need to pass exams. As far as the learner in the CHC is concerned, memorizing is a means of becoming
thoroughly acquainted with the subject, to facilitate reflection and understanding. This same phenomenon may exist in the adult learner community.

The interview results in the miscellaneous themes and especially artifact evidence indicate that developmental learners show signs of cognitive overload on certain constructivist tasks. If a learner has acquired appropriate automated schemas, cognitive load will be low and substantial working memory resources are likely to be free (Sweller, 1994). In contrast, if the elements of material that require processing must each be considered as a discreet element in working memory because no schema is available, cognitive load will be high. Working memory may be entirely occupied in processing large numbers of individual elements. Secondly, the characteristics of the instructional material are important. Some material can be learned element by element, without relating one element to another. Learning multiplication of decimals is a good example. Each multiplication operation on a decimal number can be learned without reference to any other item in the schemas. Such material is low in element interactivity and low in intrinsic cognitive load. It imposes minimal demands on working memory.

Alternatively, situations where a number of elements must be considered simultaneously for the successful execution of a task are called high element interactivity tasks. A learner competent in elementary algebra will treat the distribution, c(a + b) as a single, automated schema requiring limited working memory resources. A novice who has just commenced learning algebra may need to treat each symbol and relations between symbols as individual, interacting elements, resulting in a working memory overload. These adult learners may have been previously accustomed to a traditional and rote approach to mathematics instruction. These developmental learners appear to derive more mathematical meaning from behaviorist tasks involving step-by-step processes; the linking of previous concepts, and the repetition of key ideas are effective approaches.

Finally, from the contradictions between learner and instructor perspectives, we must conclude that mathematics meaning is subjective and individual. The concept of multiple representations of meaning, rather than Gardner’s lamely termed multiple intelligences (Gardner, 1983), avoids the controversial definition of intelligence and emphasizes the subjectivity of meaning.

Implications and Recommendations

This study has important implications for secondary and tertiary math instructors, academic directors, and principals. The resulting recommendation would be to limit cognitive load in developmental learners by utilizing carefully controlled and highly structured constructivist projects ensuring the concept or schema is in place before embarking on ambitious constructivist exercises and projects.

The learner survey and interview results show that an interactive, step-by-step, visual instructional approach, with each step logically linked to its predecessor may be a suitable approach to developmental mathematics. According to instructor interviews, developmental learners appear to derive mathematical meaning from visual and verbal approaches, which I have termed Multiple Representations of Meaning. Multiple meanings may be a more helpful concept than the concept of multiple intelligences (Gardner, 1983).

Another implication is that instructor-training programs might shift the emphasis from teaching universal concepts of objective meaning to those of subjective meaning. Courses for prospective instructors should provide an awareness of what research reveals about project
based learning, how we learn mathematics, models for effective learning, and an understanding of the power and limitations of the use of technology in the classroom.

According to Crossroads, further research is necessary to understand what works specifically in adult developmental mathematics. In particular, we need to understand more precisely which approaches do promote positive learning outcomes. Is it because of the material covered, the instructional methods used, challenges outside of the classroom such as financial or family constraints, or some combination of all of these factors?

This research is of a single case and can be expanded to multiple cases or extended over time with different instructors at the same focal site. These studies could then be scaled up to multiple focal sites and multiple time lines.

References


APPENDIX A. Results

There are two distinct ways in which the results of the qualitative analysis of the adult learner and instructor interviews may be reported. One way is to look at the data in a quantitative way. The interview transcripts provide the opportunity to count the frequencies in which each of the codes was attributed to learners and instructor transcripts. Another method of analysis tends to the qualitative, being based on the analysis of actual responses by interviewees. The following graphs show how these meaningful themes were quantitatively distributed among the transcripts of the learners and instructor interviews. In contrast, the quote tables apply a qualitative lens to the interview questions.

Graph 1a. Distribution of the principle all learners should grow in their knowledge of mathematics while attending college (code BI)

<table>
<thead>
<tr>
<th>Themes</th>
<th>Instructor</th>
<th>Learner</th>
</tr>
</thead>
<tbody>
<tr>
<td>BI</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>MI</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LD</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TL</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MT</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

“I’m making something little and dealing with scale, proportions and perspective all the time”

“I get my learners to draw a circle and divide it into 12 equal parts using just a ruler, protractor and compasses”

“This distance ratios on a guitar fret board relate to pitch and the harmonics relate to factors of those ratios”
Table 1a: Transcript examples of the principles all learners should grow in their knowledge of mathematics while attending college (code BI)

Graph 1b. Distribution of the sub-themes on the all learners should grow in their knowledge of mathematics while attending college

<table>
<thead>
<tr>
<th>FaDes</th>
<th>Code: BI,LD</th>
<th>Sub:nus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“I have to add four fractional measurements”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FaDes</th>
<th>Code: BI,LD</th>
<th>Sub:mmt, pro</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“I also have to scale the measurements from medium down to small sizes and up to large sizes”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FaDes</th>
<th>Code: BI</th>
<th>Sub:mmt, var, rel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“I know a lot of them cannot do a circumference from a radius”</td>
<td></td>
</tr>
</tbody>
</table>

Table 1b. Example transcripts of the sub-themes on principle all learners should grow in their knowledge of mathematics while attending college

Graph 2a. Distribution of the principle all learners should be taught mathematics that is meaningful and relevant (code MI)
Table 2a. Example transcripts of the principle all learners should be taught mathematics that is meaningful and relevant

<table>
<thead>
<tr>
<th>Code: MI,LD</th>
<th>CulArt Sub:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“This is a technical art college so we need to stress both skills. I would bring in some graphical or visual elements. Also steer away from just formulas and stress the why behind the problem. Like science, do the experiments and show them if you can”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code: MI</th>
<th>GraDes Sub:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“It was completely dependent on the teacher. I was either completely successful or not successful at all”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code: BI</th>
<th>IntDes2 Sub:</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Not so much, this is how you are, but let’s fix this. A little less of writing it off as just a weakness. I see a difference now how my child is taught, we are not going to write you off, you can do it, we want you to do it! Now she is getting A’s and B’s”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code: MI</th>
<th>Lmn2 Sub: cog</th>
</tr>
</thead>
<tbody>
<tr>
<td>“The class was good for me because you go over the easy stuff first, get that solid and than move on to the next step, instead of jumping right into algebra. I mean, I was screwing up with division and I have division down now. It was much better”</td>
<td></td>
</tr>
</tbody>
</table>

Graph 2b. Distribution of the sub-themes on the principle all learners should be taught mathematics that is meaningful and relevant
Table 2b. Example transcripts of the sub-themes on the principle all learners should be taught mathematics that is meaningful and relevant

<table>
<thead>
<tr>
<th>Code</th>
<th>Sub</th>
<th>Learner</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI</td>
<td>cog</td>
<td>Lrn2</td>
<td>“Moving gradually step by step definitely helps”</td>
</tr>
<tr>
<td>MI</td>
<td>con</td>
<td>Lrn4</td>
<td>“The teaching style was a little different here, a little more flow and intuition”</td>
</tr>
<tr>
<td>MI, LDBI</td>
<td></td>
<td>GaPro</td>
<td>“95% of people think visually. I would have them plotting Cartesian graphs and making a picture. An equation can make a picture. Like with my sister in-law, x cubed doesn’t mean anything, but an S on its side might have more meaning”</td>
</tr>
<tr>
<td>MI</td>
<td></td>
<td>GraDes</td>
<td>“Working lots of problems, project them on to the wall, working them through step by step”</td>
</tr>
<tr>
<td>BI</td>
<td></td>
<td>FaMar</td>
<td>“And I always repeat it over and over so they get it”</td>
</tr>
<tr>
<td>LD</td>
<td></td>
<td>FaDes</td>
<td>“I love figuring out the geometric shapes. I would have been great at math if I was designing cargo pockets!”</td>
</tr>
</tbody>
</table>

Graph 3. Distribution of the principle developmental mathematics should be taught as a laboratory discipline (code LD)
Table 3. Transcript examples of the principle developmental mathematics should be taught as a laboratory discipline (code LD)

<table>
<thead>
<tr>
<th>Code</th>
<th>Sub:</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>FaDes</td>
<td>mmt, var</td>
<td>“With a circle skirt we have to do a radius at the top for the waist and a radius at the bottom for the hem. Anything in the round, we also have to do funnel shaped sleeves”</td>
</tr>
<tr>
<td>AuPro</td>
<td>pro, rel</td>
<td>“Can you create a ratio? What is a ratio? We went though that together and he realized 120 BPM is twice as fast as 60 BPM. We were looking at rates and time. If I have 2 beats per second how many BPM do I have?”</td>
</tr>
<tr>
<td>Adv</td>
<td></td>
<td>“Sure. I show them a lot of ads in magazines. And of course they hadn’t seen them before, because nobody under forty reads a newspaper. So we investigate how much it is to go on Facebook or YouTube or whatever”</td>
</tr>
<tr>
<td>Adv</td>
<td></td>
<td>“One of the projects is that class is to find a location for their business. They have to go online and find the cost for rent of the place their business is going to be”</td>
</tr>
<tr>
<td>FaMar</td>
<td>BI, BI</td>
<td>“Your students have to memorize the basic formulas, then they have to apply them to Excel”</td>
</tr>
</tbody>
</table>

Graph 4. Distribution of the principle the use of technology is an essential part of an up-to-date curriculum (code TL)
Table 4. Transcript examples of the principle the use of technology is an essential part of an up-to-date curriculum (code TL)

<table>
<thead>
<tr>
<th>Code: TL</th>
<th>Sub:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lrn4</strong></td>
<td>“I’m looking at graphing and how the x and y coordinates of Photoshop fit into it, like position”</td>
</tr>
<tr>
<td><strong>Lrn4</strong></td>
<td>“I would just paint over the basic forms. Once I started doing certain models that requires a high degree of symmetry, you have to mirror it with the origin over here with a certain number of grid points”</td>
</tr>
<tr>
<td><strong>Lrn4</strong></td>
<td>“No, Photoshop. I use Illustrator sporadically. In logo design, I might create a stencil in Illustrator and shoot that to use in Photoshop. I don’t use Illustrator nearly as much as I use Photoshop”</td>
</tr>
</tbody>
</table>

Graph 5. Distribution of miscellaneous learning and Instruction themes (code MT)

<table>
<thead>
<tr>
<th>Code: MT, LD</th>
<th>Sub:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>IntDes1</strong></td>
<td>“Math just seemed to make sense to me, part of that was because I was dyslexic. I had to do the problems over and over to check my solution and this repetition made me get the concepts”</td>
</tr>
<tr>
<td><strong>AuPro</strong></td>
<td>“He was like…. explosions going off in his head! Frankly, I don’t think he understood my lecture until that moment [of doing a project]”</td>
</tr>
</tbody>
</table>
Table 5. Transcripts of miscellaneous learning and instruction themes (code MT)

<table>
<thead>
<tr>
<th>Code</th>
<th>Sub</th>
<th>Transcript</th>
</tr>
</thead>
<tbody>
<tr>
<td>AuPro</td>
<td>MT</td>
<td>“At this point they don’t have the skills to research their own materials”</td>
</tr>
<tr>
<td>IntDes1</td>
<td>MT,BI</td>
<td>“I had to do the problems over and over to check my solution and this repetition made me get the concepts. When it got to balancing equations it was a piece of cake”</td>
</tr>
<tr>
<td>LibArt</td>
<td>MI</td>
<td>“Working lots of problems, project them on to the wall, working them through step by step”</td>
</tr>
<tr>
<td>FaMar</td>
<td>MI</td>
<td>“And I always repeat it over and over so they get it. My challenge with math was, they wrote it on the board and they said it, but I missed it! It was like, Oh, what do I do?”</td>
</tr>
<tr>
<td>CulArt</td>
<td>MI,LD</td>
<td>“This is a technical art college so we need to stress both skills. I would bring in some graphical or visual elements. Also steer away from just formulas and stress the why behind the problem. Like science, do the experiments and show them if you can”</td>
</tr>
<tr>
<td>IntDes2</td>
<td>BI,TL</td>
<td>“Later they get into the whole three dimensional thing in AutoCAD. It is important that their brain can see something from one view, but it can also be visualized from another perspective. Looking at a space from different angles. Imagine it, without having to draw it first”</td>
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Table 6. Transcripts of Visual, Verbal and Spatial Representations of Meaning
Appendix B: Interview Results

Results of the themes on the principle *all learners should grow in their knowledge of mathematics while attending college* (BI)

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Sub-Themes on the *Big Ideas* of Math

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Themes on *all learners should be taught mathematics that is meaningful and relevant* (MI)

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Mathematics must be taught as a laboratory discipline (LD)

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The use of technology is an essential part of an up-to-date curriculum (TL)

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APPENDIX C: Observation and Participation in an Adult Developmental Tutoring Session

Pre-Observation
This is the classroom with the huge arched windows. As I entered the room, the instructor, who was hosting the tutoring session, was grading papers. 2 learners worked on their College Algebra homework (I has asked what they were working on).

I noted two learners from the previous class who were intently staring out of the huge window onto the street below. They had noticed that some of the simple looking transactions being played out in crowded streets below were, in fact covers for drug deals. We talked for a while about how animators were observers. The two animators disappeared into the streets.

Physical Description
This horseshoe arrangement allows many tutors to easily access each learner's work.

Artifacts: A student tutor-tutoring list. Many of my learners have complained of poor tutoring from student tutors. Perhaps more training for the tutors?

Narration of Events
As I entered the room, the instructor, who was hosting the tutoring session, was grading papers. 2 learners worked on their College Algebra homework (I has asked what they were working on). The instructor, who had an Italian background, began discussing Rosetta Stone (language learning software) with a learner who wanted to learn Chinese.
Exercise: simplify $t + \frac{1}{4}s + 6t + \frac{3}{5}s$

Learner: I don't get the $\frac{1}{4}s + \frac{3}{5}s$ part

Researcher: First choose the similar terms you can combine
Learner: $t$ and $6t$
Researcher: Ok, let's combine the $t$s, that's the easy part
Learner: $t - 6t = -5t$
Researcher: Good. Now let's do the $s$ terms: $\frac{1}{4}s + \frac{3}{5}s$
Researcher: You can't combine these right now, as they have a different denominator.
Researcher: What would be a suitable common denominator for these terms?
Learner: How about 20?
Researcher: Ok, let's see what that looks like $\frac{5}{20}s + \frac{6}{20}s$
Researcher: How much bigger is 20 than 4?
Learner: 5 times as big
Researcher: 5 times good! To make the fraction the same, we must also multiply the numerator by 5
Learner: $\frac{5 \times 1}{20}s + \frac{6}{20}s$
Researcher: Let's do the other part. How much bigger is 20 than 5?
Learner: 4 times as big
Researcher: 4 times ok! To make the fraction the same, we must also multiply the numerator by 4
Learner: $\frac{5}{20}s + \frac{6}{20}s = \frac{5}{20}s + \frac{12}{20}s$
Learner: So that's $\frac{17}{40}s$ right?
Researcher: No. The common denominator stays the same. You only need to add the numerators together
Learner: So it's $\frac{17}{20}s$
Researcher: That's right. So what is the final simplification?
Learner: $-5t + \frac{17}{20}s$
Researcher: That's right. We have given it a haircut.

Other learners are beginning to drift in for the 6 o'clock College Algebra class. It's time to go.
APPENDIX D: Semi Structured Vocational Instructor and Adult Learner Interview Questions

Your own math education

Looking back on your own education, did you experience an interest and engagement with Math? If so, ask questions 1 to 4. If not ask question 4 to 8.

1. What was it that sparked your interest?
2. Has this continued to be your motivation, or have any other factors emerged either during or since your school days?
3. Did your engagement with Math ever falter? If so, when and why and how did you regain it?
4. At school did you fully appreciate the meaning and relevance of Math?
5. What hampered/promoted your own Math learning?
6. When did your math education cease to engage you and why?
7. What could have been changed that would have improved your experience and made Math more relevant and meaningful for you?
8. Have you been able to improve your understanding of Math since leaving school? If so, when and how?

Math in your experience as a vocational instructor

9. Are there any elements of Math used in your formal curriculum? If so, what are they and how are they used?
10. Do you think your curriculum would benefit from additional Math? If so, what form would you like them to take?

Your own opinions on the Meaning and Relevance of Math in Adult Education

11. Your own opinion or comment on any aspect of this subject that may not have been covered by this interview would be of value:
APPENDIX E: Completed Constructivist Learning Exercises

PATTERN BLOCK FRACTIONS

1. If the hexagon pattern block is one-whole:
   a. What is the value of a trapezoid?
   b. What is the value of a rhombus?
   c. Write an addition statement that shows that a relationship between the triangle and the rhombus.

2. If the rhombus is one-whole:
   a. What is the value of a trapezoid?
   b. What is the value of a triangle?
   c. Write a subtraction statement that shows a relationship between the triangle, trapezoids, and rhombi.

3. If the trapezoid is one-whole:
   a. What is the value of the triangle?
   b. What is the value of the rhombus?
   c. Write a multiplication statement that shows a relationship between the rhombus and the hexagon.

4. If the hexagon is one-whole:
   a. What is the value of the sum of the trapezoid and a triangle?
   b. What is the value of the sum of the rhombus and a trapezoid?
   c. Write a division statement that shows a relationship between the triangle and the trapezoid.
The Hexagonal Table Problem

In the Square Tables Problem you figured out the number of people who could sit in terms of the number of tables that were available.

What if instead of squares, the tables were hexagons?

**Question 1:** How many people can sit if there are 2, 3, or 4 tables?

- 2 tables = 12 people
- 3 tables = 18 people
- 4 tables = 24 people

**Question 2:** What pattern do you see? Describe the pattern in words:

*Add 6 people for every table added.*

Let \( T \) represent the number of tables. Express the number of people who can sit in terms of \( T \).

\[
P = 6T
\]

**Question 3:** Use the expression you just wrote to figure out how many people can sit if there are 8 tables, 10 tables, and 100 tables.

- \( P = 48 \) for 8 tables
- \( P = 60 \) for 10 tables
- \( P = 600 \) for 100 tables

**Question 4:** How many tables do you need for 90 people?

- 15 tables

Thanks to Mara Landers
The Hexagonal Table Problem: Part 2

The situation: Your guests push the tables together.

**Question 5:** How many people can sit at each table in the picture at the right?

1 table: 6 people
2 tables: 10 people
3 tables: 14 people
4 tables: 18 people

**Question 6:** *Without drawing the tables,* figure out how many people can sit when 10 tables are pushed together. How did you figure this out?

\[ T = 10 \]

\[ P = 4T + 2 \]

10 tables = 42 people

**Question 7:** *Without drawing the tables,* figure out how many people can sit when 20 tables are pushed together. How did you figure this out?

\[ T = 20 \]

\[ 4T + 2 = 82 \text{ people} \]

**Question 8:** How many people can sit at \( T \) tables?

\[ P = 4T + 2 \]

\[ P - 2 = 4T \]

\[ \frac{P - 2}{4} = T \]

Thanks to Mara Landers
APPENDIX F: Example Discovery Learning Projects

Mr. Glasser
9/23/09
Final Project

Question #1.
You’re doing some manual film editing and you have three strips of mm film to clip together. Keeping in mind that 32 frames equal a second of animation, measure the three strips and solve the movies collected length if each 1/16 of an inch is 32 frames.

\[
\begin{align*}
7 + 4 + 6 &= 17 in. \\
\frac{11 \times 1}{1} &= \frac{272}{60} = 4.53 \\
60 \text{ seconds} &= 1 \text{ min.}
\end{align*}
\]

Answer: 4 min. 32 seconds
or 4.53

Question #2.
You have a project due in 60 days and progress is measured in percentages at this job. It is 50 days in from the start date, what percentage of work should be finished?

\[
\begin{align*}
\frac{x}{100} &= \frac{50}{60} \\
x &= \frac{50 \times 100}{60} \\
&= \frac{5000}{60} \\
&= \frac{500}{6} \\
&= \frac{83.33}{1}
\end{align*}
\]

Answer: 80%
1. What is the Distance Between Enemy1 and Enemy2?

2. James is designing a 2d Sidescrolling game. But he needs to create a level on a [30] [400] grid with 64 x 64 tiles. How many tiles will he need?

3. What are the X, Y coordinates for Enemy 1 and 2?
Final Project: F. of Math

Optical Disc capacity comparison for Console games of yesteryear and today.

Legend
KB (Kilobyte)
MB (Megabytes)
GB (Gigabytes)

Playstation One: CD-ROM = 700 MB
Playstation 2: DVD-ROM = 4.7 GB
Xbox: DVD-ROM = 4.7/8.5 GB
Gamecube: DVD-ROM = 1.5 GB
Wii: DVD-ROM = 4.7 GB
Xbox360: DVD-ROM = 8.5 GB
PS3: Blu-Ray = 25/50 GB

1.) How much a DVD-ROM (4.7) in percent can it go in a Blu-Ray Disc (single layer 25GB)?

2.) How many CD-ROMs (700 MB) can you fit in a DVD-ROM (8.5 GB)?

3.) What is the percent of a Gamecube disc (1.5 GB) of a Wii Disc (4.7GB)?
APPENDIX G: Discovery Learning Projects Showing Signs of Cognitive Overload

**Dr. Horrible with a Vengeance** (1995)

The clip starts with John’s father in the park opening a laptop. A riddle must be answered to defuse a bomb.

Exactly 4 gallons of water must be placed on a scale to defuse the bomb using only a 2-gallon bottle and a 5-gallon bottle. How can this be accomplished?

1. How much does the bottle weigh?
2. How long does it take to fill the bottle?

Exactly 2 gallons, leaving exactly 1 gallon of empty space.

If you have a full 5 gallons, you pull 1 gallon out of 5 gallons, you have exactly 4 gallons.
For every 1 rectangle there are 2 black squares. They double as they scale.

11 = 11 white rectangles would equal 22 black squares.
Final Math Project

1) On a grill that is 8 ft. long. It can cook up to 32 lbs. of meat. How pieces of meat could fit if one piece of meat weighs an average of 2 lbs.?

2) I am making a wedding cake for a celebrity couple. They want the cake to be very decorative. What is the width and height of the cake in inches?

3) I’m cater for a party of 180 people. The group wants round tables. The average round table seats 6 people. How many round tables do I need?