Adults Learning Mathematics

An International Journal

Chief Editor
Javier Díez-Palomar

Editor
Anestine Hector-Mason

Guest Editor
Graham Griffiths

Volume 9(2)
November 2014

ISSN 1744-1803
Objectives

Adults Learning Mathematics – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum bringing together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members an enormous contribution has been made to making available theoretical and practical research in a field, which remains under-researched and under-theorised. Since 2005 ALM also provides an international journal.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

Research and theoretical perspectives in the area of adults learning mathematics/numeracy
Debate on special issues in the area of adults learning mathematics/numeracy
Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

The ALM International Journal will be published twice a year.
ISSN 1744-1803

Editorial Team:

Dr. Javier Diez-Palomar, Universitat de Barcelona, Spain [Chief Editor]
Dr. Anestine Hector-Mason, American Institutes for Research, Washington, DC, USA [Editor].
Graham Griffith, Institute of Education, University of London [Guest Editor]
Dr. Katherine Safford-Ramus, Saint Peter’s College, Jersey City, New Jersey, USA
Dr. Chris Klinger, University of South Australia, Adelaide, Australia
Kees Hoogland, APS - National Center for School Improvement, Utrecht, the Netherlands

Editorial Board:

Prof. Alan Bishop, Monash University, Melbourne, Australia
Prof. Marta Civil, University of North Carolina, Chapel Hill, U.S.
Prof. Diana Coben, Kings College London, UK
Dr. Jeff Evans, Middlesex University, London, UK
Dr. Gail FitzSimons, Monash University, Melbourne, Australia
Prof. Gelsa Knijnik, Universidade do Vale do Rio dos Sinos, Brazil
Prof. John O’Donoghue, University of Limerick, Ireland
Prof. Wolfgang Schloeglmann, University of Linz, Austria
Prof. Ole Skovsmose, Aalborg University, Denmark
Dr. Alison Tomlin, Kings College London, UK
Prof. Lieven Verschaffel, University of Leuven, Belgium
Prof. John Volmink, Natal University Development Foundation, Durban, South Africa
Prof. Tine Wedege, Malmö University, Malmö, Sweden
Adults Learning Mathematics – An International Journal

Special Issue

Critical Moments in Adult Mathematics Education

In this Volume 9(2)

Editorial
Graham Griffiths and Anestine Hector-Mason

More… or less? Towards a critical pedagogy of adult numeracy
Aileen Ackland

“I remember the whole board being full of different calculations and trying to make some sense of it.” The influence of significant moments in adult numeracy teachers’ own learning experiences on their teaching practice
Carolyn Brooks

Beyond questionnaires – Exploring adult education teachers’ mathematical beliefs with pictures and interviews
Sonja Beeli-Zimmermann

Critical issues in adult numeracy practice – contradictions and strategies
David Kaye

Numbers talk – words count: Language policy and adult numeracy education in Wales and New Zealand
Diana Coben and Barbara Miller-Reilly

Accreditation not aggravation
Cath Moss and Judith Archer

Implications of social practice theory for the development of a numeracy programme for the Gusilay people group in Senegal
Elisabeth Gerger

Provoking mathematical thinking: Experiences of doing realistic mathematics tasks with adult numeracy teachers
Janette Gibney
Editorial

Graham Griffiths
Institute of Education, University of London
London, UK
graham.griffiths@learningunlimited.co

and

Anestine Hector-Mason
American Institutes for Research
Washington, DC, USA
ahector-mason@air.org

The 20th International Conference of Adults Learning Mathematics – A Research Forum was held in Caerleon at the University of South Wales. The title of the conference was ‘Critical Moments in Adult mathematics,’ a topic that is sufficiently broad to allow for a range of contributions from researchers and practitioners in adult mathematics. Two volumes of articles were derived from this effort, the first of which was published on ALM-online.net in June, 2014. The articles in this second volume address the title of the conference in a variety of ways, which reinforces the notion of the criticality of the varied mathematical moments that participants and submitters experienced.

In the articles for this edition we see an undergirding motif reflecting cases made for an education based on a critical understanding of the world. Some articles reflect on critical moments in the lives of individuals – both teachers and learners, for example; in other articles, readers are made to reflect on the case for policy and practice to be more critically informed. In all cases, it appears very obvious that the authors offer adult numeracy practitioners a number of significant and meaningful ideas that could potentially reshape their thinking about mathematics, if not drive their efforts in advancing mathematical policy, practice, and research.

The first article More... or less? Towards a critical pedagogy of adult numeracy by Aileen Ackland argues for a new look at the socio-cultural understanding of mathematics. Ackland posits that in Scotland, like many places across the world, a ‘social practice approach’ is often argued for in the context of mathematical practice, and adult numeracy practitioners are often encouraged to use ‘more context, more activity’, as she puts it. Ackland draws upon studies primarily around the idea of mathematical practice and she presents a strong argument indicating that the corpus she selected, display not only a limited perspective on mathematics in use, but they also form an essentially neo-deficit perspective in terms of approaches to mathematics pedagogy. Contending with these perspectives, Ackland argues for a critical mathematics education that uses real situations derived
from the adult learner perspectives. Ackland’s work presents several implications for teacher education.

The next article is a contribution from Carolyn Brooks entitled “I remember the whole board being full of different calculations and trying to make some sense of it:” The influence of significant moments in adult numeracy teachers’ own learning experiences on their teaching practice. In the paper Brooks reports on the testimony of personal mathematics histories of two adult numeracy teachers including some critical moments in their personal developments. Brooks compares and contrasts the two experiences and argues that these are intertwined with their teaching practices. In line with the motif of both the conference and the articles in this edition of the journal, Brooks work appears to be an exemplification of Ackland’s notion around using real situations derived from the adult learner perspectives.

In the next article, Sonja Beeli-Zimmermann, brings to light some of the similar concerns in both Brook’s and Ackland’s articles, though in her discussion of the issue of ‘beliefs,’ Zimmerman’s article reflects some of the same matters, and perhaps coheres more closely with Brook’s. Zimmerman’s Beyond questionnaires – Exploring adult education teachers’ mathematical beliefs with pictures and interviews is an exploratory investigation of the use of pictures in analysing the beliefs of teachers. The author poses a thought provoking, if not provocative question to participants, namely “Imagine you were an artist and agreed to do the following contract work: What is mathematics? Analyses of the images participants generated from this question involves the use of a scheme, including deductive codes, developed by Rolka and Halverscheid (2011). Like the other articles in this volume, this is also a unique contribution to mathematics in general and specifically to mathematical thinking.

In Critical issues in adult numeracy practice – contradictions and strategies, David Kaye reflects on his experience working within the context of professional development in mathematics, and uses selected cases to generate three key areas for investigation. Kaye argues that three significant perspectives: ‘multiple intelligences’, ‘a profound understanding of fundamental mathematics’ and ‘how the mind creates mathematics’ provide a useful framework for practitioners to reflect critically on practice. The approach taken is supported by the Open University’s guide to action research. Indeed, one of the best ways for math practitioners to reflect on practice is through action research. Kaye’s work offers practitioners and researchers alike, some very interesting viewpoints vis-à-vis the diverse ways in which meaning in mathematical research and practice can be generated, processed, and shared.

From a very different perspective, Diana Coben and Barbara Miller-Reilly investigate and compare language policy and adult numeracy education in New Zealand and Wales. In Numbers talk – words count: Language policy and adult numeracy education in Wales and New Zealand the authors review the Māori and Welsh languages as a part of their international comparative study of adult numeracy education. Coben and Miller-Reilly argue that while language and literacy has been explored in the literature, more work is needed around the area of language and numeracy to ensure that math practitioners are abreast of practices for non-native speakers. In particular, they propose that policy needs to be informed from a critical linguistic perspective, particularly in relation to human rights. The issue of language and mathematics are within the same theoretical borders as ethnomathematics, and both ideas illuminate matters pertaining not only to language equity, but also mathematical equity. Clearly, by itself, their discussion creates a critical moment in adult mathematics.

Other articles in this edition include an article from Cath Moss and Judith Archer, as well as one from Elizabeth Gerger. In their article, Accreditation not Aggravation, Moss and Archer report on an action research project designed to improve mathematical communication. The authors detail how technology can assist in communication and help students develop confidence in a changing learning environment. This idea of communication in mathematics, ties into Coben’s and Miller-Riley’s work.
on language and mathematics, but Moss and Archer presents this issue from a different angle – from the perspective of advancing student confidence. Confidence is a behavioural attribute that could be mediated by the quality and capacity of the mathematics instructor. Advancing student confidence is an interesting role for mathematics instructors, but it is also a relevant one that could help shape student interest in mathematics.

From an ethnomathematical perspective, Elisabeth Gerger reports on the Implications of social practice theory for the development of a numeracy programme for the Gusilay people group in Senegal. Much like the other authors in this edition, Gerger’s work is a very unique and interesting one that drives attention to several of the themes captured by the other authors (e.g., critical moments, in mathematics, perspectives, language and culture, to name a few). Gerger presents research on some “traditional” numeracy practices of the Gusilay people in Senegal and makes recommendations for developing a numeracy programme specifically for women. She recommends that the developing programme be based on a strong foundation of traditional knowledge and practices so that it meets the needs of women who are faced with new numeracy related challenges within a changing society.

Finally, Janette Gibney presents a report of an action research project which looks at the development of mathematical ideas with a group of adult numeracy trainee teachers. In Provoking mathematical thinking: experiences of doing realistic mathematics tasks with adult numeracy teachers, Gibney describes three cycles of study in which different tasks are introduced to the group of trainees, and investigates the move between ‘real worlds’ and ‘maths worlds’. As a math teacher, the author exemplifies the meaning of these two concepts by presenting a number of significant moments for trainee teachers and reflecting on how critical those concepts are for herself as a teacher trainer. Using tasks developed by advocates of the ‘Realistic Mathematics Education’ movement, Gibney reminds us of a truth that the field needs to begin to play closer attention to: how some trainee teachers struggle to move between ‘real worlds’ and ‘maths worlds’, not trusting their own intuition and tending to privilege formal (classroom-based) mathematical approaches.

Below, we present these eight articles. Through a variety of approaches to investigation, these authors offer the research community and practitioners important insights that could enable them to move the field of adult mathematical education forward. As you will see, all of the articles in this edition present readers with some very unique perspectives, unique ideas, and unique approaches to uncovering various types of meanings in mathematics that are valuable for practitioners, researchers, and policymakers. Together, they remind us that these are, indeed, critical moments in mathematics!
More… or less? Towards a critical pedagogy of Adult Numeracy

Aileen Ackland
School of Education, University of Aberdeen, Aberdeen, Scotland
<a.ackland@abdn.ac.uk>

Abstract
The development of socio-cultural understandings of mathematics combined with policy interest in adult numeracy as a result of international studies, which compare skill levels in different countries, have impacted adult education practice in recent years. In Scotland, a ‘social practice approach’ is espoused and adult numeracy tutors are encouraged to add more to the learning experience – more context, more activity. But is this a sufficient pedagogic response to the insights from social practices theory and a socio-cultural perspective of mathematics? This paper draws on evidence from studies of practitioners’ understandings of social practices theory to argue that these responses are limited and potentially limiting of adult learners and represent a neo-deficit approach. Instead a critical pedagogy is required, which may require that the tutor bring less, not more, to the learning experience. Critical pedagogies would involve exploring with learners the powerful uses of mathematics in society. Adult numeracy learners could learn not only to understand the mathematics in use but to use mathematics for their own projects. The paper concludes with some thoughts on the kind of teacher education required to support tutors to become more critical in their pedagogy.

Key words: Adult numeracy, social practices, critical pedagogy

Introduction
This paper argues that the changes to adult numeracy practice evident in recent years are an insufficient response to current theoretical perspectives on mathematics in society and potentially limiting of adult learners. It arises out of a number of personal and professional concerns: to promote education for social justice, to synthesise literacy and numeracy issues, to share insights about practice across different sectors of education. It is informed by my experience in schools, further education colleges and as a tutor and organiser for a voluntary sector provider of adult education for democracy. The argument has been developed through my work as teacher educator involved in designing and delivering professional development for Adult Literacies practitioners in Scotland between 2005 and 2012. A research project undertaken in this period, with practitioners participating in the work based professional development programme, provides evidence in support of my arguments.

Writing for a journal with mathematics in the title, I am however, beset by imposter syndrome. An arts graduate, teacher of English, literacy tutor and qualitative researcher, I find myself subject to doubts about my capability and credibility to contribute to academic debate about maths. Boaler (2009) provides one explanation for my feelings of unworthiness. She describes as ‘the elephant in the

---

1 Between 2005 and 2012, I was Curriculum and Research Leader for the Scottish TQAL Consortium which consists of the Universities of Aberdeen, Dundee & Strathclyde. The Consortium was contracted by Scottish Government to develop and deliver a teaching qualification for adult literacies tutors (TQAL), which became the Professional Graduate Diploma in Education (Adult Literacies).
classroom’ the assumption that success in maths is a sign of general intelligence (p.2). Such an assumption breeds adult insecurities as they assess their own value in relation to the value accorded their different kinds of knowledge. Boaler reminds us that the mathematics afforded such power – school maths - is a narrow subject restricted to classroom contexts: ‘a strange mutated version’ (ibid) of real mathematics’. Nevertheless, ‘people who teach maths’, the voice in my head goes, ‘are much cleverer than me’. I share this reflection because power is at the core of my argument about the teaching of adult numeracy. My proposal is for pedagogies, which confront the elephant and expose assumptions about different forms of knowledge to examination.

Although I will base my argument primarily on the situation in Scotland, I believe it has validity internationally at a time when a culture of global comparisons of educational outcomes drive harmonisation of educational practice in different countries. Whilst claims have been made about the distinctiveness of Scotland’s approach to what is termed here, adult literacies (Scottish Government, 2011), the direction of practice developments is similar to many other countries and related to the hegemony of the neo-liberal capitalist project.

The paper begins with a brief description of the Scottish context for adult literacy and numeracy. Since the ‘social practice approach’ endorsed in Scotland appears to derive from socio-cultural understandings of literacy and numeracy as social practices, these perspectives are presented and their implications for educational practice considered. Drawing on data from studies of Scottish practitioners’ understandings of a ‘social practice approach’ I suggest that much adult literacy and numeracy practice does not in fact reflect these implications. The assumptions behind some new practices of numeracy teaching and learning are then examined. Following an assertion that these assumptions could indicate what Auerbach (1995) labels a neo-deficit approach, I propose alternative critical pedagogies, providing some examples from the school sector as well as adult education.

Finally, I conclude with a consideration of the need to develop critical pedagogies of teacher education.

The Scottish Adult Literacy and Numeracy context

In 2001 the Scottish Executive responded to the International Adult Literacy Survey results (OECD, 2000) with a policy initiative for Adult Literacy and Numeracy (Scottish Executive, 2001). The strategy acknowledges that ‘Literacy and numeracy are skills whose sufficiency may only be judged within a specific social, cultural, economic or political context’ (Scottish Executive, 2001 p. 7) and defined literacy in broad terms as

The ability to read and write and use numeracy, to handle information, to express ideas and opinions, to make decisions and solve problems, as family members, workers, citizens and lifelong learners. (Ibid)

The socio-cultural perspective implicit in these statements was further reflected in a subsequent shift in terminology in policy and in practice - from Adult Literacy and Numeracy (ALN) to ‘adult literacies’ (plural), an umbrella term which encompasses literacy, numeracy, ICT and ESOL. The plural term recognises the diversity of literacies as well as the interconnectedness of these four domains in social practices. The Curriculum Framework (2005) advocated ‘a Scottish approach to adult literacy and numeracy learning’ described as a ‘social practices approach’ (Scottish Executive, 2005, p.5).

The broad definition and the use of the concept of adult literacies blurred the boundaries in practice between, in particular, literacy and numeracy and many tutors who had previously taught one or the other were now required to teach both. There was concern, however, that in contrast with literacy, adult numeracy may be overlooked and that practitioners, some of whom may be themselves ‘maths anxious’, required support to provide effective numeracy learning opportunities. Professor
Diana Coben was commissioned to produce a report making recommendations for the development of an effective adult numeracy strategy for Scotland. Adult Numeracy: shifting the focus (Coben, 2005) elaborated the Scottish definition of literacies with a working definition of numeracy:

To be numerate means to be competent, confident, and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (Coben, 2000, 35, emphasis in the original)

Whilst emphasising the importance of numeracy not being overshadowed by literacy, the report recognises correspondences between literacy and numeracy as social practices. Social practices theory and the implications for teaching and learning have, however, been ‘articulated more clearly with respect to adult literacy than to numeracy’ (Coben, 2005, 28). Coben articulates a ‘social practices approach’ to numeracy as critical numeracy requiring connectionist teaching and realistic mathematics (p.8; p.22).

This group of approaches starts from the position that adults are active agents in the world, rather than seeing them as inadequate individuals with a numeracy deficit.

Adult education … is seen as a tool for social justice, aiming to equip people with knowledge and tools to examine, criticise and seek to change the economic, political, and social realities of their lives. (Coben, 2005, p.23).

She recommended a number of ways in which this approach could be progressed in Scotland.

To summarise, in Scotland adult numeracy is encompassed within the term adult literacies. A social practices approach is advocated, in contrast to a ‘…deficit approach…where the individual is encouraged to take a test that will demonstrate a failure to meet a set of standards…’ (Scottish Government, 2011, p.14). Coben articulated this Scottish social practice approach to numeracy as critical numeracy. In the years since these key texts, ‘the social practice approach’ has become the doxa of practice in Scotland.

**Literacy and numeracy as social practices**

As Coben acknowledged, the conceptualisation of literacy as social practices has received more attention than similar socio-cultural understandings of numeracy. The social practices theory of literacies in society advanced by the New Literacy Studies (NLS) (see for example Street, 1984; Barton, 1994; Barton and Hamilton, 1998; Gee, 2008) emphasises the inherent power relationships affecting uses of literacy in a social context and illuminates the situated nature of literacies acquisition. The NLS view of literacy as situated, socially constructed and inherently ideological challenges what Street refers to as the autonomous model (Street, 1984), which assumes literacy to be a value and context free individual cognitive competence. Crowther et al highlight the ideological dimension in the title of their edited book, Powerful Literacies (2001). They demonstrate in a variety of practice contexts how work with literacies learners requires practitioners to be aware of power relations and to critically examine with learners sociocultural literacies practices. Gee (2008, p.45 - 49) provides a very clear illustration of such a critical approach in his examination of the ‘aspirin bottle problem’. His analysis of the warning text on an aspirin bottle demonstrates how teaching the ‘reading’ of such a text must go beyond decoding to engage with questions about drug companies, social relations and the structure of society. As Freire and Macedo (1987) put it, literacy requires ‘reading the word and the world’.

Understandings of numeracy as social practices have mainly been explored in ethnomathematics (see for instance Powell and Frankenstein, 1997) and through ethnographic research (for example, The new Mathematics project in Liberia, Cole, 2000, pp72 -80). Benn (1997) explored the
implications of this perspective for adult education. She states the fundamental tenet of this perspective as ‘that mathematical knowledge is a social construct… created by human beings whose thinking is influenced by a historical and political context’ (p.27). Chapter 3, ‘Mathematics: a peek into the mind of God?’ provides a historical and social analysis of mathematics and traces a move from an absolutist to a fallibilist view. ‘The absolutist views mathematics as neutral and value free as opposed to subjective and value-laden’ (p.31). This shift corresponds with Street’s distinction between the autonomous and ideological views of literacy. A social practices view recognises that numeracy as well as literacy practices exclude, position, implicate people in relation to ideological assumptions (Kerka, 1995). Benn (1997, p.37) argues that in adult numeracy learning values must therefore be made overt within a critical pedagogy.

According to Ernest (2002, p.5), a critical mathematics education would develop the following aspects of understanding and awareness:

- Critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements to government and interest group pronouncements.
- Being aware of how and the extent to which mathematical thinking permeates everyday and shop floor life and current affairs.
- Having a sense of mathematics as a central element of culture, art and life, present and past, which permeates and underpins science, technology and all aspects of human culture.
- Being aware of the historical development of mathematics, the social contexts of the origins of mathematical concepts, symbolism, theories and problems.
- Understanding that there are multiple views of the nature of mathematics and controversy over the philosophical foundations of its knowledge.

Although Ernest here refers to a critical mathematics education, increasingly the term numeracy is used in adult education to indicate the wider perspective, to acknowledge mathematics as a discourse and to distinguish between real mathematics and school maths (Yasukawa et al., 1995). Numeracy here is more than mathematics and it is only numeracy if it is political (p. 816); if, in other words, it recognizes the power dimension.

**Practitioners’ understandings of a social practices approach**

Above I have sketched out the theoretical views apparently influencing adult literacies practice in Scotland. In these views, the espoused Scottish ‘social practice approach’ entails critical pedagogies. Maclaclan and Tett (2006) found little evidence of critical practice, however, and in 2008 Hillier questioned the extent to which the social practice perspective of literacies was actually transforming practice in Scotland (Hillier, 2008, p.6). Involved at this time in the development of the new teaching qualification for adult literacies tutors, I was concerned with how the theory of social practices was currently being construed in practice to better appreciate the challenge of how the radical socio-cultural understandings could be translated into changes in practice. Between 2008 and 2010 I undertook research with a group of practitioners undertaking the professional qualification. The research used a variety of methods inspired by Personal Construct Theory (Kelly, 1955) - including individual and group reflective activities, and structured interviews - to explore how practitioners were construing ‘a social practice approach’. (For full details of this research and its methodology see Ackland, 2013.)

Practitioners in this study associated the social practices approach with two main characteristics – learner-centeredness and relevance. The following quotes, from an activity in which students had 5
minutes to write their definition of the approach, are representative of how these two characteristics are repeated throughout the data:

My understanding of the Social Practice of literacies is that it's directed by the needs of the learner; Learner-centred….making the learning process relevant; Creates a relevant link to the learner’s life. It individualises learning; Social practices is you’re asking learner what they want to improve; It’s taking the learner’s perspective into account and, if appropriate, adapting my practice to their social norms.

Within this discourse, the learner (singular) tends to be isolated in the learning environment but linked to their individual everyday life, which is unquestioned. The relationship between teacher and learner is generally interpreted as one of service. Teachers should be ‘empathic’ and ‘non-directive’. Care for the learner is paramount and summed up in the notion of practice being ‘non-threatening’ (these terms appeared frequently in the interview data). In all the data there is little trace of the critical pedagogy implied by the social practices perspectives as examined previously, or with the articulations by Benn (1997), Papen (2005) and Coben (2008) of the implications for adult literacy and numeracy. Yet one practitioner reflected that ‘you don’t really need to have a wonderful theoretical grasp of it, it’s just… to me it’s natural’.

My findings are supported by research by Swinney (2013), which included analysis of Scottish adult literacies practitioners’ narratives of practice. She found that a marked consistency of discourse of ‘social practice’ and ‘literacies’ masked significant differences in underlying philosophies of adult education. She describes how practitioners use the term ‘social practice approach’, ‘as encapsulating an array of ‘learner centred’, ‘informal’ and ‘contextualised’ approaches to learning and teaching which placed an emphasis on emotional and relationship aspects of learning’ (p.241). She concludes that ‘there was no evidence to suggest practitioners, in using ‘literacies’, intended to convey a politicised understanding that…is implied by a ‘social practice’ analysis…’. The practitioner in her study who reflected that ‘the social practice approach was nothing new to most of the workers that we had here’ (p. 239) may be expressing a similar taken-for-grantedness about characteristics of practice as the student quoted above.

**Characteristics of practice – more, more, more**

From both studies it is clear that practitioners in Scotland believe that their practice conforms to the orthodoxy of the Scottish ‘social practice approach’ and is distinct from a deficit approach. In the following section, quotes from practitioners in my study (Ackland, 2013) indicate some of the elements of their practice that they associate with the new approach. What this appears to mean in the detail of practice is more…particularly more work for the tutor:

I feel the social practice model is so important to literacies as it is creates a relevant link to the learner's life. It individualises the learning, which makes for a lot more preparation for us, but demonstrates the difference between adult and children's learning.

More individual learning planning is evident:

…after some discussion with the learner about what they need their literacy/numeracy for, what areas of their life is their need for literacy most necessary, then a program of learner led literacy/numeracy would be developed and worked on. This will enhance inclusion, confidence, and employability and allow the learner to participate more confidently in everyday life, whether it is shopping, going for a meal, paying bills and so on.

To support individual learning planning, an interactive tool, ‘The Curriculum Wheel’, was developed (Scottish Executive 2005) and its use ‘rolled out’ to all adult literacy and numeracy partnerships. Individual learning planning was given even greater priority as the focus of a Practitioner-Led Action Research project in 2008 (St Clair et al, 2009). The project aimed ‘to support
practitioners in leading a research project looking at the individual learning planning (ILP) process. ILPs are central to the literacies field in Scotland, as they are used for defining objectives, planning instruction, and assessing achievement by learners’ (p. 1). The shift in this introductory statement from the singular ‘process’ to the plural ‘ILPs’ is indicative. It is plans that have proliferated, in many cases as quite extensive textual artefacts incorporating a degree of initial diagnosis as well as identification and evaluation of relevant learning. They are sometimes experienced by tutors as more paperwork, ‘not part of the ‘real’ work of literacy teaching and learning’ (Hamilton, 2009). Hamilton demonstrates how ILPs can shape the relationships between tutors and learners and align both their identities with system goals. Although in Scotland the guidance on developing ILPs is perhaps even more permissive than in England where Hamilton was studying ILPs, her analysis contains important insights about how the forms can constrain the possibilities for learners. For instance, as more formal accreditation is required to demonstrate learner progress (HMIE, 2010), the learning outcomes of the qualifications influence learners’ goals. Learning goals written on ILPs are also identified within discourses about literacy and numeracy ‘needs’, mainly linked to economic participation, and in Scotland, inside the discourse of ‘relevance’.

The planning process focuses learning on the individual needs and requires tutors and learners to identify learning goals associated with their everyday lives:

- It [social practices] is when people work on the things they need to be able to do for specific activities in their lives, for example if they have a new job and need to be able to convert metric and imperial measurements or be able to use an electronic till – it’s a sort of ‘functional literacy.

Another more then is contextualisation:

Social Practices is about ensuring that all learning that goes on in our service is contextualised and embedded in the learner’s chosen vocational area or interests.

In relation to numeracy the contexts drawn upon tend to be shopping, budgeting, work and sometimes leisure interests such as sewing or DIY. There can be a number of problems, in my view, with this interpretation of ‘contextualised’ in relation to numeracy. In some cases the contextualisation is superficial and may mathematise a problem that in ordinary circumstances would not involve the use of mathematics. For example, the following problem is set in an everyday context.

There is, however, no purpose to the problem, other than to demonstrate the ability to create a graph.

A gardener plants a variety of bulbs for the spring in both his front and rear gardens. Detailed below:

<table>
<thead>
<tr>
<th>Bulb Type</th>
<th>Front Garden</th>
<th>Rear Garden</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tulip</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Snowdrop</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Daffodil</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Crocus</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Hyacinth</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Create a suitable graphical form (table, graph, chart or diagram) to illustrate his planting.

Figure 1. Numeracy assessment, Core Skills Numeracy modules

Contextualisation such as this, I suggest, is a mere dressing up of school maths with the backdrop of everyday life. Problems are not structured in the context of real life purposes, rather only elaborated with textual detail. This can lead to another more… more words. Numeracy tasks become literacy tasks. Whilst mathematics in societal contexts is often entangled with language, text that is not crucial to the purpose of a classroom task can serve simply to create an additional barrier. For example:

http://www.aloscotland.com/al&viewresource.htm?id=2819
Jimmy needs to wallpaper his bedroom. The dimensions of the room are 3m by 4m by 2.5m. There is one door - 2m x 1m - and one window 1.5m x 1m. A standard roll of wallpaper is .53m x 10m. How many rolls of wallpaper does Jimmy need?

In real life, Jimmy is unlikely to make a narrative of the problem in this way. He is more likely to tackle it physically and with concrete objects. An extra dimension of difficulty is introduced into the spatial and numerical problem by its translation into the language of the maths problem. The requirement to decode language such as ‘dimension’ and to understand the conventions of giving width and height measurements adds a superfluous literacy task.

Even where the context suggested has a real purpose, the adult literacies tutor’s commitment to not challenging the everyday life of their learners can pose problems. The following is the transcription of an excerpt from a recording of practitioners discussing a social practice approach as part of the research project:

A For the sake of argument… if a learner comes and says I want to learn weights and measures coz I want to be a drug dealer… do you say ‘no problem, we’ll start here then…. ’ or start questioning their motives?

B (laughing) I once did ratios and the topic was…the question was…‘where would you use ratios in everyday life?’… and it actually came up. And I’m going ‘I’m no teaching this!’ …. but that’s what they are doing anyway …

C ….if that’s what they can relate to…

D …but if you go against your employer…

A ….we’re agents of the state…

B Well see as long as I wasn’t advocating that this was a good thing I would take their scenario…

C I’ve done that too

B…coz at the end of the day if you can get them to learn………………(laughs) to dae it properly… (all laugh) …no but….

An extreme example, perhaps, but one that raises serious issues about the dangers of carrying contextualisation too far without critical consideration of purpose.

In many cases the commitment to contextualisation brings with it the requirement for tutors to furnish their groups with realia. More stuff. From my experience of lesson observations, this frequently involves food – pizzas and chocolate bars to cut up and share, supermarket goods to compare prices. Shopping catalogues are also popular. As well as providing artefacts for the contextualisation, these are often justified in relation to the need to accommodate different learning styles. In the new numeracy classroom, teaching must be adapted to more learning styles and particularly for kinaesthetic learners. The allegiance to the concept of learning styles and to the necessity of tutors catering to different styles continues to be very strong despite critiques of learning styles theory and practice (Coffield et al, 2004) which draw attention to the ways in which it can restrict rather than empower. It has become part of the culture of care, in which not identifying an individual’s style and providing for it is considered disrespectful.

I have enumerated a number of ways in which I believe tutors are adding more material to the adult learning experience with a focus on meeting individual needs with ‘relevant’ activities. In adult numeracy ‘relevance’ concentrates on arithmetic calculations in everyday activities such as budgeting; indeed literacies now encompasses a further literacy - financial literacy. Financial capabilities are referred to repeatedly throughout the refreshed Scottish adult literacies strategy (Scottish Government, 2011). Whilst these changes to approach are a positive shift from a culture of ‘death by worksheet’, I believe they can be limiting of adult learners’ possibilities and represent a
limited response to socio-cultural understandings which emphasise the power relations inherent in social practices.

Firstly, there is evidence that learners have motivations for wanting to attend numeracy classes beyond the everyday application. Swain (2005) concludes that ‘mathematics does not have to be ‘functional’ to capture students’ interest, involvement’. He found that one of the main reasons they want to learn ‘is …to prove their ability to learn a high status subject which they believe to be a signifier of intelligence’ (p. 305). This is congruent with Boaler’s point mentioned in the introduction. As she implies, the assumed link between maths and intelligence has great power, and not just in the classroom. Ernest (2002, p. 4) explores how success in mathematics can ‘give students advanced power through enhanced life chances in study, the world of work and social affairs… Qualifications in mathematics are accorded a privileged role and have unique social significance as gatekeepers’. The concentration on everyday application may ignore the broader empowerment issues Ernest identifies. Whilst numeracy teachers and researchers may use the term to denote more than maths, it may be experienced by learners as less than maths – a basic functional level of the subject without status and power. There are class issues also in the embedding of numeracy in vocational interests, as Bernstein warns: ‘Vocationalism appears to offer the lower working class a legitimation of their own pedagogic interests and in doing so appears to include them as significant pedagogic subjects yet at the same time closes off their own personal and occupational possibilities’ (Bernstein, 2004, p.213).

Whilst I am by no means suggesting that adult numeracy teaching does not have an empowerment agenda, I am worried that the ‘functional’ discourse is erasing the more critical ingredients. In Scotland, literacies were described as complex capabilities consisting of knowledge, skills and understanding (Scottish Executive, 2005, p.35), understanding being linked to the more critical dimension. In texts between 2000 and 2012, the three terms gradually reduce to two – ie knowledge and skills, with understanding disappearing. At UK level, The National Institute of Adult Continuing Education’s (NIACE) report on adult numeracy presents a variety of case studies of effective practice. Effective practice is represented as ‘…relevant, interesting and enjoyable’ focusing on ‘….practical and relevant skills’ (Southwood and Dixon, 2012, p. 3). The case studies provide examples of learning for:

- Shopping for bargains
- Budgeting
- Measuring for DIY
- Decorating

The ‘vital ingredients’ of adults learning maths do not appear to include any critical approaches. Here too there is silence about the political dimensions of numeracy. I have no doubt that examples of critical approaches exist across the UK, but where this approach is invisible in authoritative documents, it may cease to be a legitimate practice, even in Scotland where the rhetoric of the social practices approach persists.

Echoing Freire and Macedo (1987), Frankenstein (1998) refers to ‘reading the world with math’. Like Gee in his aspirin bottle problem (2008, p.45 - 49) she argues that it is insufficient to support learners to merely calculate budgets or best deals correctly - budgeting tasks can make ‘money and family finance ‘neutral’’ (Frankenstein, 1998): ‘Even trivial math applications like totaling grocery bills carry the ideological message that paying for food is natural and that society can only be organized in such a way that people buy food from grocery stores…’. A critical pedagogy is required which would explore the power relations of supermarkets, global corporations, consumers and capitalism. The discourses of financial literacy and employability so prevalent now in adult numeracy conceal the assumptions of a neo-liberal ideology. Skills for ‘employability’ are predicated on the
requirements of the global economic race. A functionalist approach does not question these requirements.

Allman (1999) discriminates between normalizing/limited reproductive praxis and critical/social transformation praxis. Despite the discourse in Scotland of the ‘social practice approach’, the findings of research with practitioners (Maclaclan and Tett, 2006; Ackland, 2013; Swinney; 2013) suggest that praxis tends to be of the first kind; learner-centeredness and relevance is mainly concerned with supporting people to operate within the structures - ‘a better fit for the world’ (Freire 1972, p.57). This falls short of the critical stance implied, as argued above, by a social practices perspective.

The rhetoric of ‘a social practice approach’ and its apparent anti-deficit stance has become an orthodoxy of literacies practice in Scotland (Ackland, 2013). This masks underlying differences in values, ideologies and pedagogical approaches. In the main Scottish practitioners’ construe ‘a social practice’ approach as similar to the learner centred approach which prevailed in adult basic education prior to the emergence of the new literacy studies (Hamilton and Hillier, 2006, pp. 109-124); the language has changed but this is experienced as a new way of talking about what was already accepted and ‘natural’ practice. Writing in relation to family learning, Auerbach (1995) describes this ‘post-deficit’ situation as dangerous; she examines how discourses which apparently reject a deficit model, as ‘the social practice approach’ claims to do, continue to be based on traditional deficit assumptions of the requirement for individual change to adapt to unquestioned social structures. As she does with family learning discourses, I have tried to problematize some of the claims of ‘a social practice approach’ to adult numeracy with the concern that unexamined the rhetoric may serve ‘a rationalising function, masking underlying deficit views with an aura of credibility’ (Auerbach, 1995, p.651). In Adult Literacies in Scotland 2020 (ALIS 2020, Scottish Government, 2011) the neoliberal economic project is explicitly connected to the language of socio-cultural theory. Adult Literacies for economic participation is ‘most successfully taught using a “social practice” approach’ (Scottish Government, 2010, p.7). Auerbach (1995) describes this as a ‘neo-deficit’ discourse in which the emphasis on power relations and the requirement for critical pedagogies implied by social practices perspectives are excluded.

**Less is more – a critical pedagogy of numeracy**

How then can numeracy education reflect more effectively the implications of social practices perspectives? Such pedagogy, I believe, requires that tutors bring less to the learning experience, taking a problem-posing stance in which they examine with learners mathematics in use in society. Even a superficial skim of the daily media demonstrates how contemporary life is saturated with numbers and statistics entangled with text. Numbers have a powerful effect on language: ‘The power of numbers is such that they render visible and hence incontestable the complex array of judgements and decisions that go into measurement, a scale, a number’ (Rose, 1999, p. 208). A critical pedagogy would engage with the power of numbers, explore calculation as ‘qualculation’ (Callon and Law, 2005), infused with values and ideologies.

It may begin, as practitioners often say, with the motivations and interests of learners but in this case the question of what and why they want to learn - maths, mathematics or numeracy - would be discussed critically, with the elephants clearly visible in the room. Within a critical pedagogy, rather than merely reflecting on their own mathematics history, learners might investigate the history of mathematics in different cultures as a means of engaging with a fallibilist perspective. It is sometimes believed that learners must master the basic processes of a subject before they can engage critically at a meta level. Based perhaps on Bloom’s taxonomy, critical thinking, it is assumed, comes higher up the hierarchical triangle of capabilities. Although a critical pedagogy of numeracy should also
develop learners’ capacity to use mathematical processes for their own projects, its starting point might reverse the hierarchy to assume adults’ capabilities as critical thinkers.

Hierarchical thinking can also permeate numeracy curricula as a building block mentality where simple rote operations build systematically to more complex problems. Realistic mathematics is much messier and Frankenstein (1998, p.56) argues that ‘problems with neat pared down data and clear cut solutions give a false picture of how mathematics can help us ‘read the world’’. She advocates studying mathematical topics through deep and complicated problems and outlines a number of examples of investigating mathematics in use – such as unemployment statistics – which could meet the 4 goals of a critical mathematical curriculum, which she defines as:

1. Understanding the mathematics
2. Understanding the mathematics of political knowledge
3. Understanding the politics of mathematical knowledge
4. Understanding the politics of knowledge (ibid, p.53)

As in a Freirian pedagogy (1972) the complex problems can come from the learning group’s observations of their own surroundings. In this case the tutor does not bring the context to the classroom but takes the learning process into the context. Numeracy groups might use ethnographic techniques to examine numeracy practices or numeracy in the media in similar ways as have been used in literacy (see for example, Roberts and Prowse in Papen, 2005, pp. 143-146 who describe how literacy learners explored their favourite soaps critically).

Gutstein (2003; 2006; 2008; 2012) describes the development of mathematics curricula for social justice in an urban Latino school. Here current community issues, such as the redrawing of a school catchment area, provide the problems to be investigated mathematically with school pupils. This goes beyond exploring other people’s uses of mathematics to develop the capabilities to use mathematics to address and represent community projects.

Gutstein (2006) also engaged parents in a critical dialogue about the school maths curriculum. Given that Swain’s research (2005) noted that another powerful motivator for adults attending numeracy classes is to help their children with maths, this is an important extension of developing new curricula. Some adult numeracy courses of the ‘Keeping up with the kids’ variety, respond to parents’ fear of confusing their children by not knowing the ‘right way’ to do school maths, by giving them a better knowledge of the school’s approach so that what they do will ‘fit’ with this. E.g.

All parents want their children to do well at school and to succeed. However, many simply don't know where to start. Everything seems to have changed since your own schooldays, and you don't want to confuse your children by using different methods to their teacher. Family Learning can help. (Family Learning website³)

The danger is that this may reinforce an absolutist view of mathematics. It is not congruent with Boaler’s (2009, p. 138 -40) assertion that the children who are most successful at maths are those who can decompose and recompose problems using a variety of strategies. In her view, parents should not only be supported to help their children ‘play’ with maths problems developing confidence in a variety of strategies, parents can also have a powerful role in challenging narrow maths curricula and traditional pedagogies in schools (pp195 -206). A critical pedagogical approach with parents whose motivations are to help their children, then, would problematize schools curricula. It might involve critical dialogue about changing fashions in teaching and learning and about the relationships between maths, mathematics and numeracy. Rather than a relationship being established in which the school tells parents about how they teach maths, as is sometimes the case, the relationship could be one of critical dialogue between parents and schools in which adults are empowered to challenge the authority of school curricula.

³http://www.familylearning.org.uk/
Other aspects of family life are ripe for critical numeracy investigations. Critical approaches to examining text for author, purpose, values, positioning etc. are standard in literacy groups. A model for applying a similar critical ‘reading’ process to numeracy texts has been developed from Freebody and Luke’s (1990) Four Resource Model of Critical Literacy. This model could be used with adults to examine commonplace texts and question how numbers, measurements and statistics are being used within power relations. Below is an example using a numeracy text from a common family breakfast cereal box.

Figure 2. Breakfast cereal box

Dr Terry Maguire’s work on developing maths eyes with children, parents and in communities⁴ explores the power of large numbers in public discourse. The question of ‘how big is a billion?’ might lead, in a critical pedagogy, to investigations about relative spending at international level and the values implicit in government budgets for war, welfare and poverty. Investigations on such issues might make use of Internet sources such as, what can $611 billion buy?⁵ or Information is Beautiful⁶ which provide different representations, including info graphics, about economic decisions.

Info graphics are becoming a significant feature of multimedia and appear daily in news stories in print, on television and on the Internet. A critical pedagogy might explore why this is the case, what makes them powerful communicators and communicators of power. As new communication technologies shift to support authorship rather than readership, learners could create their own info graphics to represent their own analyses of issues⁷. In this section, I have sketched out just a few illustrations of a critical pedagogy of numeracy drawing from examples relating to schools as well as adult education and making links between literacy and numeracy. This problem-posing stance engages not just with the personal concerns of learners but with political issues. Gutstein insists that such curricula require teachers to build political relationships with students (2008), in which they do not merely meet people’s needs but support their projects. This may be challenging for some numeracy tutors. I share Freire’s view that education is never neutral (1972) but this view is not shared by all teachers, some of whom see this position as containing the threat that they will unduly influence learners or appear judgmental of alternative political or cultural perspectives. Being ‘non-judgemental’ and ‘non-threatening’ are linked in the adult education culture of care. De Freitas (2008, p. 205) suggests that some maths teachers may even be drawn to the subject by a perception of its

---

⁴ http://www.haveyougotmathseyes.com/
⁶ http://www.informationisbeautiful.net/visualizations/the-billion-dollar-gram/
⁷ http://www.educatorstechnology.com/2012/05/eight-free-tools-for-teachers-to-make.html
neutrality. Perhaps if we concentrate on teachers bringing questions rather than answers, respect for learners may become decoupled from the need to remain non-judgemental and non-threatening. A critical pedagogy is one in which all are challenged to examine the taken for granted. It inevitably involves discomfort.

**Critical teacher education**

What kind of professional development might, then, support a shift to more critical pedagogies of adult numeracy? I believe its starting point must be that education is never neutral. It must explore the purposes of adult education and examine why educational content, curricula and pedagogies are the way they are not merely how to teach (Ackland and Wallace, 2006). As in any critical pedagogy of adult learning, questions, not answers, should initiate practitioners’ own investigations in which theory, policy and practice are regarded as contingent and power-laden. The process might involve Freirian decoding of representations of education. Throughout this paper I have used the three terms maths, mathematics and numeracy at different times and to denote different things according to my understandings. The significations of these three terms are fluid and they are in use in education loaded with quite different assumptions. As stated previously, numeracy is favoured by some adult educators and researchers to encompass the social practices and fallibilist perspective. In my experience it is used by many tutors, and in policy, to relate to a more functionalist view of mathematics. Recently listening to some maths colleagues in teacher education discuss their development of a new course for the primary school sector, I was intrigued to hear them insist that they would not use the term numeracy at any point. Their use of the term was pejorative, implying that it carried associations of a ‘dumbing down’ of maths. Adult learners may also perceive numeracy as more basic than maths and perhaps as a more current form of the old term arithmetic which was used in Scottish schools for a subject taught to pupils in ‘lower sets’. Numeracy is a contentious term. In teacher education, exploring the distinctions between the terms can stimulate reflexivity as practitioners surface their own assumptions but also dialogue about the power of different forms of knowledge. I have found a triad activity drawn from Personal Construct Theory (Kelly, 1955) very powerful. The three terms are presented on cards as so:

<table>
<thead>
<tr>
<th>Maths</th>
<th>Numeracy</th>
<th>Mathematics</th>
</tr>
</thead>
</table>

Participants are asked to group them as two that they think are similar and one that they think is different. They are then asked to explain why they have arranged them in this way. Questions are used to prompt an exploration of the assumptions implicit in the arrangement.

Other taken for granted concepts of adult education must also be examined. The shift in language from education to learning has been adopted almost universally, but it is not without critiques (e.g. Biesta, 2005). Learner-centeredness, as I have explored above, is not automatically empowering. These shibboleths and what is held to be ‘natural’ (see practitioner quote in Practitioners’ understandings of a social practices approach section above) and accepted practice should form the core of deconstructive dialogues in a critical pedagogy of teacher education. Assumptions about the practice of other sectors, schools for example, as opposed to community based learning, and popular contrasts between andragogy and pedagogy could be deconstructed. In Scotland, the claims to the distinctive of ‘the social practice approach’ should be examined within the bigger picture of the political and ideological hegemonies that frame practice within the UK, Europe and internationally. This would entail critical discourse analysis of the policies which shape practice (such as in Oughton, 2007) combined with critical analyses of how numbers are used in policy (Hamilton, keynote...
presentation ALM20). We should not forget too to discuss how numbers get used in practice for justification and recognition. (In Scotland, statistics from the International Literacy Survey – 23% of Scots have insufficient skills – quickly achieved factual status as they were repeated at all levels of practice to substantiate the need for adult literacies work at local level.)

An understanding of learners is a requirement in most standards of teacher competence. A narrow interpretation of this requires teachers to know about the circumstances and characteristics of learners to consider the barriers they might experience to learning. Whilst knowledge of communities is important, and can be developed through trainee teachers’ ethnographic investigations, it is insufficient to support critical pedagogies. ‘Community knowledge’ should be combined with ‘critical knowledge’ (Gutstein, 2012, p. 301) – a sociological appreciation of history, economics and political relations - and would entail examining roots as well as manifestations, perhaps using problem solving tree diagrams. Learning styles for example, might be examined not merely as a phenomenon to react to but one, which may have cultural roots, mechanisms of reinforcement and a role in the maintenance of existing socio-cultural privileges.

Fundamentally, the culture of cares in which being non-judgemental and non-threatening are held as sacred principles must be questioned as potentially limiting. When participants in teacher education are themselves challenged to move out of their comfort zone, they easily see the parallels with their learners’ experience:

We ask learners to go outside their comfort zones in their learning… so why should we not be pushed outside ours? And it’s good to see how this feels (on reflection of course!) and be reminded of how valuable it is to face challenge. (TQAL participant comment8)

Risk, discomfort and uncertainty, reflexively explored, can build confidence in teachers to confront challenge for themselves and with their learners.

Conclusion

In this paper, I have explored the implications of social practices theory for adult numeracy and examined some current practices in relation to these. I argue that the discourse of ‘the social practice approach’ in Scotland may mask neo-deficit ideologies implicit in practices focussed on learner-centeredness and relevance. I have proposed some alternative practices towards more critical pedagogies of adult numeracy. Professional development, which also commits to a critical pedagogy, is key, I believe, and might write back the critical into current discourses of practice.

Acknowledgements

This paper developed from a keynote presentation at the Adults Learning Mathematics research group conference of July 2013. I am very grateful to the conference organisers for the invitation to address the group and for the opportunity to meet with researchers, many of whom have influenced my thinking as I have traversed the boundaries between literacy and numeracy.

References


---

8 Professional development through professional enquiry http://www.nrdc.org.uk/content.asp?CategoryID=1548


Gutstein, E. (2006). The real world as we have seen it’: Latino/a parents’ voices on teaching mathematics for social justice. Mathematical Thinking and Learning, 8(3), 331–358.


HMIE. (2010). Improving adult literacy in Scotland. Livingston: HMIE.


Websites

Eight Free tools for Teachers to Make Awesome Infographics
http://www.educatorstechnology.com/2012/05/eight-free-tools-for-teachers-to-make.html[accessed 29/01/14]

Family Learning: http://www.familylearning.org.uk/ [accessed 29/01/14]

How big is a billion: http://www.boston.com/news/nation/gallery/251007war_costs/ or http://www.informationisbeautiful.net/visualizations/the-billion-dollar-gram/[accessed 29/01/14]

Maths Eyes: http://www.haveyougotmathseyes.com/ [accessed 29/01/14]

Numeracy assessments, Adult Literacies Online resource:
http://www.aloscotland.com/alo/viewresource.htm?id=2819[accessed 29/01/14]

Numeracy in the News /4 resource model for critical numeracy

Professional development through professional enquiry http://www.nrdc.org.uk/content.asp?CategoryID=1548 [accessed 29/01/14]

“I remember the whole board being full of different calculations and trying to make some sense of it.”

The influence of significant moments in adult numeracy teachers’ own learning experiences on their teaching practice.

Carolyn Brooks
Anglia Ruskin University
Chelmsford and Cambridge, England
<carolyn.brooks@anglia.ac.uk>

Abstract
In researching how adult numeracy teachers actively motivate and enable learners to apply the numeracy skills they learn to their own real life practices, a case study of two adult numeracy teachers and their learner groups was undertaken. This paper compares the teachers’ contrasting personal experiences of mathematics learning to consider how significant moments in their own learning experiences may have influenced their beliefs about, and approaches to, their adult numeracy teaching practice. Similarities and differences between teachers’ approaches are explored and compared, to conclude that teachers’ learning experiences greatly influenced their beliefs and teaching practices.

Key words: mathematics; numeracy; learning; experience; teaching; practice

Introduction
The transfer of mathematics skills from classroom to real life contexts is both complex and problematic (Kanes, 2002; Kelly, 2011; Nunes & Lave, 1988; Schliemann & Carraher, 1993), but that is what teachers of Functional Mathematics⁹, in the UK, are required to facilitate in order to “help people to gain the most out of life, learning and work” (Qualifications & Curriculum Authority, 2007, p.3). When researching the specific ways in which adult numeracy teachers actively motivate and help learners to apply the numeracy skills they learn to their own real life practices, a case study of two adult numeracy teachers and their learner groups was undertaken. Prior to the research both teachers had successfully completed their specialist teaching qualification, the Numeracy Diploma in Teaching in the Lifelong Learning Sector, with me, the researcher, as their tutor, on a part-time basis over two years. The data identified both similarities and differences in teachers’ practices and approaches. Having asked the teachers about their own mathematics learning experiences, I became curious about the ways in which their previous experiences may have shaped their numeracy teaching practices, and that is the focus of this research paper.

---

⁹ Functional Mathematics is a qualification available in compulsory and non-compulsory education in the UK, which is designed for learners to acquire skills, which enable them to apply mathematical skills in work and life situations.
Research identifies beliefs, contexts, thought processes and development programmes as influences on teachers practices, suggesting that teachers’ own learning experiences are just one factor in shaping their teaching approaches. Ernest (1994) suggests the practice of teaching mathematics depends primarily on the teacher’s system of beliefs about mathematics and mathematics’ learning and teaching, the constraints and opportunities provided by the social context of the teaching situation, and the teacher’s reflection and level of thought processes, including their ability to reconcile their practice with their beliefs. The social context includes the adopted curriculum and assessment systems as well as learners’, peers’, and managers’ expectations, and these factors can be highly influential in affecting a teacher’s approaches in the classroom. The teacher’s level of consciousness of his or her own beliefs about and approaches to mathematics, and the learning and teaching of mathematics, is also important; Ernest (1989) suggests that having an awareness of what the alternative beliefs and approaches might be, as well as the teacher’s ability to “reconcile and integrate classroom practices with beliefs” (p. 253), are key to determining a teacher’s practice.

Swan and Swain (2010) used a teacher development programme and resources, which became available nationally as Thinking Through Mathematics (Swan & Wall, 2007), to challenge and change adult teachers’ beliefs and teaching practices. The programme’s aims were to engage teachers in using challenging and ‘connected’ approaches in which students were more collaborative and active in their learning, focussing on activities which enabled learners to develop their conceptual understanding. Some teachers on the programme reported that pressures from management hindered their use of the approaches, which supports Ernest’s (1994) suggestions about the influence of the social context. Overall, the teachers developed a more connectionist orientation, moving away from transmission and discovery views of teaching and learning mathematics. Swan and Swain (2010) concluded that changes in teachers’ beliefs were instigated by the changes in practice which teachers tried out, discussed and reflected upon.

The approaches and activities promoted within Swan’s Thinking Through Mathematics resources were modelled by the tutor/researcher on the case study teachers’ qualification courses, and the teachers were actively encouraged to try out the activities in their own practice, and to reflect on their effectiveness. In addition a social practice approach was also promoted, and teachers were encouraged to take account of their learners’ real-life uses of numeracy, and to use authentic materials (Appleby & Barton, 2008) to help learners make links between the concepts they were learning and their potential application outside of the classroom. Therefore, during their courses, their tutor challenged and may have influenced the two case study teachers’ beliefs about numeracy and about numeracy teaching and learning.

But, how were their beliefs formed in the first place? Who else shaped their ideas and beliefs about numeracy and about numeracy teaching and learning? What other experiences may have helped determine the kind of teacher they have become? These are some of the questions that arose during the data analysis stage of the primary study.

The following section considers a range of researchers’ views about what, in a person’s background and experiences, affects their beliefs and practices as teachers. Then, the research methodology is outlined before the findings are presented in terms of teachers’ experiences and their methods and approaches. Finally the findings are analysed to draw conclusions about the way in which personal learning experiences influence teachers’ practice.
The effect of teachers’ backgrounds on their teaching practice: a literature review

Guillaume and Kirtman (2010) undertook a study of 144 pre-service elementary (primary) teachers in the USA, to investigate how previous mathematics “experiences contribute to teachers’ images of themselves as teachers and notions of what it means to teach well” (p.121). The authors point out that mostly, “teachers are products of the school systems that they pass through as students and re-enter as professionals” (p.124) and that both school and non-school experiences influence the beliefs and values they have. The research showed that most of the trainees’ stories identified both “powerfully positive” and “poignantly negative” (p.128) peaks and troughs in their self-reported performance levels in, and attitudes towards, mathematics over time. The peaks and troughs were often linked to participants’ reactions to: particularly powerful teachers, specific content, or by significant experiences such as examinations or phases in their own social or emotional development. A subset of the respondents received messages, whether intended or unintended, about their own intelligence and ability (or lack of it) to do mathematics. Ninety-eight (68%) of the participants identified the power that a teacher had on their self-esteem and their outlook on mathematics. Of these, 73 (around half the total sample) identified that a teacher had changed their views of mathematics in a way which had a long lasting effects. Guillaume and Kirtman (2010) suggest that because out-of-school experiences (for their participants) were limited in mathematics, the role of teachers in shaping students’ attitudes towards mathematics, and their ability to learn it, was particularly prominent.

Williams (2011) also considers teachers’ experiences of learning throughout their schooling, as well as the influence that other teachers may have had on their teaching approaches, as trainees and as qualified teachers. Both the teachers Williams researched worked as ‘A’ Level\(^{10}\) mathematics teachers in sixth form colleges, and were recommended as highly successful teachers.

In Williams’ research, John (pseudonym for a participant), focusses primarily on “finishing the syllabus and getting the grades” (2011, p.133). His lessons focus on presenting the mathematics and examples followed by learners practising questions, giving one-to-one support as well as further group explanations where necessary. In addition, John supports struggling students on a one-to-one basis outside the classroom. Williams’ findings are that John’s beliefs about learning and teaching are significantly shaped by his experiences as a learner. For example, a personal tutor helped John to realise that with practice he could master the procedures involved (p.134); and his own A-level (Mechanics) teacher, who had a very traditional approach, inspired him. At university John became unable to understand much of the mathematics he was taught and instead explains that he learnt by “tricks” (p.135). Williams discovered that although during his career John spent some time being a more innovative teacher, focussing on implementing approaches encouraged by his teacher education course, over time his focus reverted to a more traditional approach, influenced by both his social context and by his personal role models or “heroes” as Williams calls them.

Sally (pseudonym of another participant in Williams, 2011) focusses primarily on conceptual development. Her lessons involve “group work, problem-solving and discussion as well as whole-class activity where group work is communicated” (2011, p133). Sally explained that when she was a learner, she would take home her notes from lessons taught by rote, to work on until she was able to make sense of the ideas herself. Her experiences as a private tutor helped her understand how essential self-confidence and self-belief are to enable learners to think things out for themselves. As a sixth form teacher she went back to her working class roots to offer herself as something of a role model to her learners: someone who can achieve success despite their low socio-economic background. Williams’ (2011) findings are that Sally, too, draws powerfully on her own learning experiences, and also that both teachers teach in a way that would support their own approaches to

\(^{10}\) Advanced Level (‘A’ Level) qualifications follow compulsory schooling in the UK, and prepare learners to go on to further academic study (e.g. university degrees).
learning as learners of mathematics themselves. Williams explains: “the stories crucially figure learners-in-general as being like the learners they used to be” (p.140). Sally does not draw much on her own teachers’ styles; the few teacher experiences she does draw on shows them as “anti-heroes” (Williams, 2011) i.e., figures she does not wish to emulate.

Amin (2012) explores the stories of three mathematics teachers’ experiences of their own mathematics learning in South Africa, where they grew up “in the shadows of apartheid” (p.2), subject to socio-political and racial adversity, and economic deprivation, resulting in limited access to education and career choices. Amin explores their memories of their own education and the significant others that helped them achieve success in mathematics, and also explores their own approaches to teaching.

In her research, Amin analyses the stories of one male and two female teachers. Aziz, male, had a father who pointed out the mathematics in the everyday things around them. For example, whilst shopping they had to calculate the sales tax on items, and he would explain “how mathematics was used in the construction of roads and buildings” (2012, p.5). Aziz explains that due to his father’s teaching, rather than his school teachers’ lessons, he sailed through mathematics at school. He now sees parents rather than teachers as those who most affect the outcome of the children he teaches, and Amin suggests that as a result of this, Aziz “resisted the role of inspiring, creative teacher” (p.7), instead teaching “vague and abstract” mathematics (p.6). In the absence of parental involvement, he does not have high expectations for his learners.

Sindiswe, one of the female teachers in Amin’s (2012) research, was taught by a teacher who “spoke a lot about Pythagoras” (p.4) but never told his learners what made Pythagoras such a great mathematician. He told the learners that only he (the teacher) knew about mathematics and that they “were too stupid to do mathematics” (p.4). Sindiswe recounts: “I believed him, for a while” (p.4). However a change of job for her father meant the family moved to another city and Sindiswe went to a new school where she was able to learn mathematics and be successful in her examinations. Therefore Sindiswe experienced both poor and good teaching and, perhaps as a result of this, sees the role of the teacher as crucial to learner success.

Nisha, the second female teacher in Amin’s (2012) research, explained that she was, overall, a very good learner, but weak in mathematics because of the “unkind and very unsocial” (p5) maths teachers she had: one who spent much time focussing on just writing numbers, another “killed [them] with mental tests” (p.5), stressing the importance of speed, and another was unable to explain how he solved the problems he gave them. At the interview, Nisha reflects that she is probably trying to be the kind of teacher that her own teachers were not (i.e., she spends time explaining things and she tries to make maths fun so that she is not a bad maths teacher herself). Nisha was a self-reliant and independent learner (2012); she explains that she basically taught herself, and as a result of this, she, unlike her own teachers, wants “to make it work for kids” (p.5).

Skovsmose (2012), on whom Amin (2012) draws, suggests that whilst social, economic, political and cultural factors influence a person, the “person’s experiences and interpretations of possibilities, tendencies, propensities, obstructions, barriers, hindrances” (p.2) also shapes their ever-developing “foreground”. I interpret Skovmose’s concept of a person’s foreground to mean someone’s self-perceived opportunities or potential. Skovsmose explains that both external and subjective factors shape a person’s foreground. I suggest that those subjective factors might be a mixture of both cognitive and affective responses, and it is these which define a person’s perceptions. Skovsmose proposes that the construction of meaning may be supported by creating classroom activities which relate to scenarios relevant to learners’ backgrounds, but that showing an active interest in a student is more powerful in helping establish meaningfulness.
Skovsmose explores the idea of “ruined foregrounds” (p.5) where a lack of social and economic resources, and stereotyping, crush the opportunity of a person reaching their aspirations. In terms of a learning situation it could simply be something that appears unattainable to a learner or to a group of learners, and Skovsmose (2012) suggests that such a “ruined foreground[s] can be the most direct cause of failure in school” (p6).

Wedege explores Bourdieu’s concept of habitus, which she describes as “a system of dispositions which allow the individual to act, think and orient him or herself in the social world” (1999, p211). Skovmose’s concept of ‘foreground’ has some resemblance to this. Wedege points out that the system of dispositions that Bourdieu explores are durable (Bourdieu, 1980 – translated by R. Nice, 1990 – as cited in Wedege, 1999), so although they are strong, they can nonetheless be changed, i.e. they are not permanent. Skovsmose’s concept of foreground is that of an ever-developing perception of one’s own potential, i.e. something that is changeable, although at times potentially quite impenetrable. Therefore, this suggests that teachers’ beliefs, although strong, can change; Guillame and Kirtman (2010), and Williams (2011) suggest that other teachers can effect such change.

The literature described above suggest that teachers’ beliefs, social teaching contexts, and level of thought processes all influence a teacher’s practice. Teachers’ own experiences of the school system, as learners, and in identifying teaching approaches, are thought to help shape their beliefs about teaching and learning. Research also suggests that use of alternative teaching approaches, through development programmes, can influence a change in teaching beliefs and approaches. These ideas were drawn upon when trying to analyse and interpret whether, in the case study data, there appeared to be any connections between teachers’ learning experiences and their teaching approaches. The methodology is outlined below before the findings and analysis are discussed.

**Methodology and Methods**

A collective case study of two adult numeracy teachers from the further education sector was undertaken, as part of a wider MA study. The two teachers (Anne and Katie) were selected by purposive sampling (Cohen, Manion & Morrison, 2011), i.e., they were specifically chosen because I had previously observed them actively seeking to make links between mathematical concepts and real-life contexts in which the mathematics could be used. Having obtained ethics approval from Anglia Ruskin University, permission from the participants’ learning organisations, and informed consent from the two teachers and their learners, data were collected from discrete numeracy classes in two different Adult and Community Learning settings during May-June 2012. One learner group was working towards an adult numeracy qualification, the other was a family learning group, there to learn how to support their children’s mathematics learning.

Semi-structured interviews were held with each teacher to discuss their backgrounds, aims and methods; these were audio-recorded then fully transcribed for the purpose of analysis. The use of open questions (e.g., ‘what was your own experience of learning maths at school?’) helped maximize data integrity. For each teacher, two two-hour observations of their teaching were carried out, to observe the methods teachers use to help learners make links between their numeracy learning and the use of numeracy outside the classroom. The purpose of carrying out the observations was to enable me to verify what teachers said they did at the interview stage (Robson, 2011), and to capture approaches that may not have been voiced. Field notes were made during the observation to record non-audio information and audio-recordings were made using a digital recorder. Relevant parts of the audio recordings were transcribed for the purpose of analysis, and integrated with the field notes.

During analysis of the primary study I was interested to note that the teachers’ own experiences of learning mathematics contrasted, and therefore for this paper I used the data I had already collected to investigate this further. As identified in the Introduction, questions which arose were: How were the
Teachers’ beliefs formed in the first place? Who else shaped their ideas and beliefs about numeracy and about numeracy teaching and learning? What other experiences may have helped determine the kind of teacher they have become? The interviews did not explore the teachers’ personal backgrounds (e.g., how their beliefs about mathematics and their views of themselves as learners may have been influenced by their own parental role models, and their childhood and adult experiences outside of school), therefore the first question remains outside the scope of this paper. Given the available data, the focus was to investigate how previous mathematics teachers and mathematics and numeracy learning experiences may have shaped these teachers’ beliefs and practices. The findings of the research are intended to contribute to the work of teacher educators and teachers in considering the development of teaching practice.

A thematic coding approach was used as the basis for analysing the transcribed interviews and observation notes, to identify themes arising (Robson, 2011). To identify the types of activities used in practice, some pre-determined codes were identified at the outset, informed by a review of the literature (e.g. Kelly, 2011) and prior experience, but these were amended and other codes arose during data analysis. Corbin and Strauss (2008, p66) liken the process of coding data to “‘mining’ the data, digging beneath the surface to discover the hidden treasures contained within data.” This approach was essential in minimising researcher bias and in seeking to represent the data as truly as possible. The types of teaching and learning activities were categorized according to how ‘abstract’ they were, i.e., devoid of any non-mathematical context, and, at the other extreme, how ‘situated’ they were, i.e., immersed in a real-life context. Categories that sat between these two extremes were also identified during coding and analysis, e.g., ‘Quasi’ methods, which include the kinds of mathematical word problems which are included in mathematics and numeracy textbooks, worksheets and test/exam questions, but which commonly bear little resemblance to real life (Dowling, 1998). The number of occurrences of different types of activities that were either observed or outlined by teachers during their interviews was used to establish the extent to which the two teachers used similar or different types of activities. The order in which different types of activities were sequenced, during classes, was also analysed.

Having gained a picture of the similarities and differences between teachers’ approaches, instances where teachers had talked about their own mathematics learning experiences were analysed to identify themes and connections arising from these memories, which might inform teachers’ beliefs, and possibly their practices. These instances were considered alongside those where they talked about their learners, again helping to discover their underlying beliefs. Having identified very contrasting personal learning experiences, I analysed the possible relationship between teachers’ learning experiences and their respective teaching approaches, as arising from the data, then turned to the literature to support further analysis of this.

The next sections outline the findings, starting with teachers own learning experiences, then considering their teaching approaches, before moving onto analysis of possible relationships between them.

**Teachers’ experiences of learning mathematics**

**Anne**

Anne is a numeracy tutor in an adult community college, working on a sessional basis, and she supports family learning classes in the community as well as general adult Functional Mathematics classes in her college. Anne left school at sixteen with qualifications which include CSE

---

11 The Certificate of Secondary Education (CSE) was, prior to 1988, a qualification available for those learners in compulsory education who were not deemed sufficiently academic to achieve the alternative ‘O’ Level qualifications.
Arithmetic, to work in the investment banking sector in London. During her parental career break she gained a GCSE\(^{12}\) in Mathematics and an AS\(^{13}\) level in English Literature, before achieving her teaching qualification, and is currently undertaking an MA\(^{14}\) in Education.

Anne found her experiences of learning mathematics at school largely abstract, explaining: “I don’t ever remember it being anything to do with…everyday life. I can only ever remember chalk and talk type approach – on the board and then you do it”. She recalls a mental image of one of her teachers:

She was going through multiplication…the column method…through the rules of this is how you do it. I remember, kind of, the whole board being full of different calculations and trying to make some sense of it. That’s…my real memory of maths at school.

This negative memory of the impenetrable wall of abstract algorithms is an important one for Anne.

Anne says she “lagged behind” at primary school and she specifically remembers feeling “quite worthless”, explaining: “I remember actually sitting there thinking I really don’t get it, I really don’t get it, but I can’t put my hand up…” . This compared to a more positive experience in secondary school, where for the first three years she explains: “I had quite a good teacher, and I quite enjoyed it, and even though it was quite abstract, she was kind of gentle, and it was a different approach”. However, as a rebelling teenager, other factors affected her learning and ultimately she was entered for a CSE rather than an ‘O’ Level\(^{15}\) exam. Her school explained to her that “because [she] wasn’t clever enough in maths, [she] had to take arithmetic…” (p2); Anne feels she could have done better.

As an adult learner she became a much more independent learner, seeking information from books and websites in addition to her teachers.

Anne expanded on the idea of feeling quite worthless:

…that feeling of not being good enough. You’re not good enough for that set, or you’re not good enough for that level…It’s not a positive message about learning in general because I think it does go to other things. People think if you’re good at maths then you’re good at everything, don’t they?

Here Anne is drawing on the idea of maths as a signifier of intelligence, suggesting that because she wasn’t very good at maths, people (including her teachers and perhaps herself) thought she wasn’t very clever generally. Her words also suggest that being told she wasn’t good enough in maths shaped her beliefs about herself more generally. In her role as a parent she supports her children with their learning to try to ensure they do not experience the same negative feelings.

Katie

Katie is a numeracy tutor in an adult community college, supporting adult Functional Mathematics groups within her college, working on a sessional basis. She stayed on at school to take Physics, Chemistry, Mathematics and Further Mathematics at A level, before achieving a BA\(^{16}\) in Engineering from the University of Cambridge. Following graduation she became a management consultant and then had several management roles in a large international company. Following a parental career break, she achieved her teaching qualification.

\(^{12}\) General Certificate of Secondary Education (GCSE). GCSEs are currently the most common form of qualification available in compulsory schooling. They replaced the CSE and ‘O’ Level qualifications.

\(^{13}\) AS Level qualifications form the first half of an Advanced Level (‘A’ Level) qualification. These are academic qualifications aimed at 16-19 years olds to prepare them for university. (They follow on from GCSEs).

\(^{14}\) Master of Arts degree

\(^{15}\) The Ordinary Level (‘O’ Level) was, prior to 1988, an academic qualification available to learners in compulsory education which prepared them to go onto further study.

\(^{16}\) Bachelor of Arts degree
Katie’s overriding memory of school mathematics was also that it was largely abstract: “I remember learning rules and routines, with very little application to real life”, with the possible exception of primary school where she remembers: “doing basic measurement and things like that”. She describes her later experience of Further Mathematics as “horrendously abstract”, explaining: “I actually knew that I was having difficulty, thinking how on earth can this be used in real life?”. Nonetheless, Katie’s qualification choices and successes suggest that she was very good at mathematics throughout her schooling, and that she had few problems learning it.

A particularly memorable event for Katie was her entry test for university. She explains:

I can remember very specifically one question; it was about working out moments and momentum, and we’d done that in a purely abstract fashion, and the question was if you twiddled a Rubik’s cube into different shapes and formats how that changed it…It was a real, real challenge to actually apply it to a real object that you were familiar with.

During her engineering degree, which started to “put things in a practical context”, this difficulty continued. She explains:

I actually found it very difficult to link the kind of pure mathematics with an actual application. It’s not that I couldn’t see the purpose of it, but I actually found doing the maths was hard in that context…it was difficult to see why that bit of maths was the relevant maths to use.

Katie found her mathematics knowledge useful to her in many of her jobs, including as a management consultant, where she was required to analyse company accounts and project present value into the future along with other types of modelling. In a later job, as a Marketing Manager for a multinational organisation, she was required to undertake market research and explains that she “had to learn maths on the job to be able to do that” as she had not previously studied statistics. She describes this as “a very practical application of maths, in order to make very expensive decisions based on the analysis”.

**Teachers’ methods and approaches: findings**

*Similarities and differences: methods*

Two types of activities stood out as being the most commonly used by both teachers. The most common was where teachers and/or learners made links between real-life contexts and the mathematics they were exploring in the classroom, through discussion. The next was where the focus was on the abstract, either the numbers themselves, or on the underlying patterns, relationships and concepts.

The third most frequent category differed between the two teachers. Katie used ‘Quasi’ methods, which include the kinds of mathematical word problems which are included in mathematics and numeracy textbooks, worksheets and test/exam questions, but which commonly bear little resemblance to real life (Dowling, 1998). In contrast, ‘Situated’ methods was the next category in Anne’s practice, where learners and/or teachers provided examples of their actual uses of mathematics within a real life context, e.g. a learner, who is a care worker, unable to check her pay-slip because it was presented to her in hours and decimals of hours rather than in hours and minutes.

The sequences of activities observed in the classroom and discussed at interview were also analysed to identify any patterns emerging from the order of different types of activities. What emerged from this analysis was that the teachers have converse approaches to teaching overall. In general, Anne tends to start with real life contexts, using these to identify the maths within, and then addresses the mathematics identified using more abstract activities. In contrast, Katie generally starts
with more abstract concepts and calculations and then makes the links between these concepts and real life contexts.

**Approaches: Anne**

Themed analysis of the interview data provided a deeper exploration of these differences. Anne always starts with what her learners know, explaining “I don’t see that I can make any connections if I don’t do that”, regularly getting learners to produce mind maps to pool their knowledge. Her approach is for the learning to come from the learners, rather than her, or from text books. She avoids “being autocratic”, and explained her dismay when one of her volunteers ‘took over’ an activity that a group of students were doing, saying: “before I know it, he’s writing different fractions on the board and it’s really busy, and I didn’t even want to write a fraction – I wanted them to write the fraction!”. She explained that this incident upset a student as well as herself.

Her drive is to help learners to feel good about themselves and their ability to learn mathematics. She takes time to value her learners: “Each week without fail I always make sure I share what students are doing, to look at different approaches, to praise the diversity of what students bring in”. She articulates: “it’s about how they [learners] feel about themselves…about their own self concept…that’s what I feel”, and later she draws on this idea again, relating it to her own experiences, saying: “It’s about what’s inside and how you feel about yourself”.

Anne discussed the importance of being learner focussed, and identifying contexts that are relevant to all learners. She identified that each learner has different numeracy needs and contexts, e.g. one learner needs to understand her wage statement, others wish to learn the mathematics they couldn’t master at school so they can help their own children, and others do not live independently. She explains it is challenging, but possible, to find contexts that are relevant to all learners. It is evident she knows all her learners and their life contexts well. Her overall style for her scheme of work is topic-based teaching (Ness & Bouch, 2007), mapping the learning to the curriculum, within the umbrella topic. For example, using the broad topic of Energy Use, her learners chose to explore aspects that were of specific interest to them such as electrical units used by a hairdryer or a kettle, or hot water used in a bath.

**Approaches: Katie**

Katie identifies the difficulty of balancing her own aims for learners with her organisation’s aims:

I know I won’t have a job if they don’t get funding, so it’s absolutely in my interest for people to pass the qualifications and take them. Whereas…I know that some of the learners don’t actually need the qualification; they’re only in the class for their own personal gain.

Here Katie voices some of the conflicting aims that teachers seek to mediate. This perhaps explains why, although Katie is sensitive to learners’ individual needs, the curriculum drives the organisation of her schemes of work. Her style is to spend periods of three to four weeks developing knowledge, methods and concepts of some aspects of the curriculum, then to consolidate this by using a carousel of activities, which are designed to stretch learners to apply their learning to real-life problem solving scenarios. The time pressure of covering all the necessary mathematics skills and concepts drives the pace of learning, which at times may be too fast for some learners, which Katie is uncomfortable with. However she tends to revisit subjects throughout her scheme of work, in different ways, to enable learners to gradually make their own sense of concepts.

Katie suggests that starting with the abstract concept, rather than a situated context can help learners. As an example she refers to the place value system and using it to make sense of
multiply and dividing by 10, 100, etc., explaining that sometimes “putting it into a context isn’t necessarily a helpful thing to do straight off…[because] metric measurement is so confusing for some people”. She also suggests that when starting off in a context-free way, learners are often motivated to relate it to a context they know, e.g. money, to help explain their thinking, during discussions.

Nonetheless Katie tries to regularly incorporate real life contexts that are meaningful to learners, but suggests that this is not straight forward: “You pick any one thing that’s right for one person and it might not be right for anyone else”, so although she tries to put things into context, “whether it’s the learners’ own contexts is another question”. She admits that “if I understand everyone well enough I can try and do that [put it into their contexts], but I don’t really understand all their backgrounds and what they’re doing”. Her solution to this is to give multiple examples that people are likely to be familiar with in some way, to hopefully prompt them to make their own links. An example of this is using the weather forecast and temperature, as well as a bank balance and a profit and loss account, to show how positive and negative numbers can be related to real-life contexts.

Analysis of the links between teachers’ own mathematics learning experiences and their approaches to teaching numeracy

Following data analysis it occurred to me that the different course outcomes (I observed Anne’s family learning class and Katie’s general numeracy class) may account for the teachers’ different methods and approaches. However a brief follow up telephone interview with Anne identified that her approaches are consistent across both her family learning and her general adult numeracy classes. This suggests that the differences are more to do with personal approaches to teaching than course outcomes.

Anne

Anne experienced both peaks and troughs in her performance and attitude towards mathematics during her schooling, perhaps not dissimilar to some of the other teachers in Guillame and Kirtman’s (2010) research, and like those teachers, the changes were influenced by her reactions to teachers as well as to phases in her own development. Despite being told she wasn’t clever enough to take higher level mathematics, she retained a perception of herself that she could have done better, so perhaps the balance of positive as well as negative experiences prevented her foreground from being ruined (Skovsmose, 2012).

Nonetheless the feeling of not being good enough formed part of her foreground. She acknowledges that it was not solely her school experiences that prevented her from reaching her full potential, but clearly her teachers had a significant impact. Perhaps this taught her that she could not rely totally on her teachers to help her reach her full potential, which made her a more independent learner as an adult. However, like Sindiswe (Amin, 2012), Anne sees the role of the teacher as very important to her learners’ success. It is evident that she also sees her learners themselves as a very important learning resource.

Her most prominent memory of learning mathematics from school is of struggling to make sense of a board full of multiplication calculations. In contrast, a teacher she liked she described as ‘kind of gentle’. In her own teaching practice I would argue that Anne draws on the former teacher as an “anti-hero” (a figure she does not wish to emulate) and the latter as a “hero” (Williams, 2011). This is demonstrated by her learner-led approach throughout her practice, including her high attentiveness to her learners’ feelings, and her dismay at her volunteer writing fractions all over the board.
Katie

Katie was successful throughout her mathematics education, and was likely to have experienced a consistently high level of performance and a positive attitude towards mathematics (Guillame & Kirtman, 2010). Although not explicit in the data, I suggest that in contrast to Anne, Katie was identified as ‘clever’, contributing to a very positive foreground (Skovsmose, 2012) which helped her to become a high achiever.

Nonetheless she was aware of the limitations of her knowledge and her teachers, as her “horrendously abstract” Further Mathematics learning meant that at times she struggled to apply the mathematics to real objects and to her Engineering degree. Therefore, like Sindiswe (Amin, 2012) and Anne, I think Katie also sees the role of teacher as important to her learners’ success.

It is clear that Katie is very aware of the contrasting experiences her learners had to her own, with most unsuccessful in achieving mathematics qualifications at school. She acknowledges that she doesn’t understand all her learners’ backgrounds, but she genuinely believes that “everybody has a lot of capability in numeracy, but they don’t always realise they’ve got it” and she sees it as her role to help learners to realise this and to believe in themselves. In this way Katie is different to Aziz (Amin, 2012), who was also very successful in mathematics learning, but who saw the family rather than the teacher as having the main role in facilitating mathematics learning.

Katie did not mention specific teachers in her interview, but she makes significant efforts in her own practice to help learners relate the abstract concepts to practical, real life problem solving. Therefore I suggest that to some extent she draws on her Further Mathematics teacher, at least, as a kind of anti-hero (Williams, 2011), i.e. as the kind of teacher she does not wish to emulate. This is demonstrated in the way that she gives her learners multiple examples of real-life uses of concepts such as negative numbers, and also in the problem solving carousel activities which are a regular feature in her schemes of work.

Both teachers

In contrast to Anne, Katie is perhaps less resistant to the influences of her organisation and management goals, or perhaps the culture of qualification success is more prevalent in her college than in Anne’s (Ernest, 1994). Consequently Katie tries to maintain a balance between situating the learning in real life contexts and preparing her learners for their test.

Williams (2011) suggests that teachers see “learners-in-general as being like the learners they used to be” (p140), and there is evidence to suggest that, to some extent, this is the case for Anne, who seems to understand, and empathise with, her learners on several levels, but less so by Katie, who recognises that the learners she is working with are different to the kind of learner she was. Nonetheless, as a learner, being able to apply the mathematics was important, and she tries to help her learners to do this, so perhaps in this way she sees them as like her.

Conclusion and Recommendations

This paper has explored ways in which teachers’ own mathematics and numeracy learning experiences have influenced their system of beliefs about numeracy and numeracy learning and teaching. It seems that their own mathematics learning experiences have been highly influential in shaping their beliefs about mathematics learning and teaching, and in cultivating their underlying approaches to their own teaching practice, including the influence of previous teachers providing roles as ‘heroes’ and ‘anti-heroes’. There was also some evidence to support the idea that teachers teach
learners in a way that would support teachers’ own approaches to learning, and that their work context may influence their practice.

Therefore it is important to raise numeracy teachers’ awareness of the influences on their practice. Teachers should be supported to explore the links between their own life and learning experiences and their teaching practice approaches, to discuss and challenge their beliefs, and to support them to test and evaluate alternative approaches, to see which work best for their own learners. In this way it might be possible to enhance teachers’ practices, despite their seemingly deep-rooted beliefs, to find a balance that maximizes learners’ learning as well as meets their organisation’s goals.

References


Sonja Beeli-Zimmermann
University of Bern
Bern, Switzerland
<sonja.beeli@edu.unibe.ch>

Abstract
Because of the impact that mathematical beliefs have on an individual’s behaviour, they are generally well researched. However, little mathematical belief research has taken place in the field of adult education. This paper presents preliminary results from a study conducted in this field in Switzerland. It is based on Ernest’s (1989) description of mathematics as an instrumental, Platonist or problem solving construct. The analysis uses pictures drawn by the participants and interviews conducted with them as data. Using a categorising scheme developed by Rolka and Halverscheid (2011), the author argues that adults’ mathematical beliefs are complex and especially personal aspects are difficult to capture with said scheme. Particularly the analysis of visual data requires a more refined method of analysis.

Key words: adult education teachers, mathematical beliefs, qualitative methods, content analysis

Introduction
Beliefs and their influence on an individual’s actions have been researched for over a century. In the field of education belief research has gained momentum after the cognitive revolution, which led to more interest in teachers’ thinking and decision-making processes (Thompson, 1992). More specifically, beliefs relating to mathematics and the teaching of mathematics have been investigated in a number of contexts, and while in some areas concrete results have been produced, open questions remain. The “unevenness” of mathematical belief research not only applies to the geographic distribution and to particular thematic areas, as identified by Pehkonen (2004), but also to specific target groups. The field of adult education, more specifically adult basic education, seems to be particularly neglected. While Taylor (2002; 2003) or Dirkx and Sprugin (1992) discuss general beliefs of adult educators, only a limited number of studies on mathematical beliefs of adult educators could be identified. The study presented in this paper, therefore, aims at contributing to this neglected field and describes the mathematical beliefs of five Swiss adult education teachers. In line with its exploratory nature, its overall approach is a qualitative one. On the basis of the work of Rolka and Halverscheid (2011) pictures created by the participants themselves are used in combination with interview data to explore these adult education teachers’ mathematical beliefs. In addition to interpreting these data, the usefulness of different data sources as well as the suitability of the employed methodology is assessed.

17 Among these few two contributions from previous ALM proceedings are worth mentioning: Henningsen and Wedege (2003) discussed the issue of values and mathematics, and Stone (2009) looks at how the institutions in which teachers work affect their beliefs and practice in the classroom.
Theoretical framework

The need for a generally accepted definition for beliefs has already been postulated more than twenty years ago, for example by Pajares (1992) and Thompson (1992). Yet even in the seminal work on mathematical beliefs by Leder, Pehkonen and Törner (2002), no generally agreed upon definition has been identified or proposed. Beliefs are considered “a messy construct” (Pajares, 1992) and the wide variety of definitions is occasionally considered one of the reasons for the lack of progress in this research field (Pajares, 1992). Very often terms such as conceptions, attitudes, values, judgements or personal theories – to name a few – are used synonymously. Some of them are broader than others and they stress cognitive and affective aspects to different extents (Pehkkonen, 2004). As this study is based on a specific methodological approach presented by Rolka and Halverscheid (2011), it also follows these authors’ understanding of beliefs, namely that beliefs are considered to be a person’s world view (Rolka & Halverscheid, 2011, p. 521). It therefore adopts a very broad and inclusive approach, integrates cognitive as well as affective aspects and accommodates the view that beliefs entail both conscious and sub-conscious elements (for a more extensive discussion of these issues see Pajares, 1992; Furinghetti & Pehkonen, 2002; Pehkonen, 2004).

When it comes to mathematical world views, there is more agreement, particularly regarding the fact that there is not one right view of mathematics. Ernest (1989) describes three contrasting views of mathematics, namely (1) a problem-solving, (2) an instrumentalist and (3) a Platonist view. Again they have also been used as a reference by Rolka and Halverscheid (2011) and are therefore also an integrated part of this study. These views can be summarised as follows:

1. “a view that mathematics is a useful but unrelated collection of facts, rules and skills” (Ernest, 1989, p. 21)

2. “a view of mathematics as a static but unified body of knowledge, consisting of interconnecting structures and truths” (Ernest, 1989, p. 21) and


In addition to beliefs about mathematics, beliefs about the teaching and learning of mathematics are relevant for any person teaching mathematics. However, as they are not the focus of this paper, these areas of beliefs will not be discussed any further.

Methodological framework, procedure and participants

A broad variety of research instruments has been employed when researching beliefs, including large scale questionnaires, various types of interviews or the analysis of specific materials such as lesson plans or journals kept by teachers (see for example Leder & Forgasz, 2002; Speer, 2005; Forgasz & Leder, 2008). One key aspect of educational research is that the issues under investigation are often not easily accessible, something which also holds true for beliefs: people are not always conscious of their beliefs – a fact which presents specific methodological challenges. In her discussion of photo elicitation, Rose (2012) argues that “elicitation interviews with participant-generated visual materials are particularly helpful in exploring everyday, taken-for-granted things” (Rose, 2012, p. 306). This aspect combined with the experience that language can slow down the creative process, as it requires higher cognitive demands, provide important arguments for the use of drawings not only with children, but also with adults (Burton, 2010). This study therefore chose to adopt an approach used by

There are other authors who present very similar threefold perspectives, among them Dionne who called them the traditional, the formalist and the constructivist perspective (Dionne, 1984) and Törner and Grigutsch (1994) who presented the toolbox, system and process aspect of mathematics.
Rolka and various colleagues who have used students’ drawings to explore children’s world views (Rolka & Bulmer, 2005; Rolka & Halverscheid, 2006; Halverscheid & Rolka, 2007). Their experiences are summarised in one article, in which they also present the classification scheme developed for the analysis of the drawings (Rolka & Halverscheid, 2011). The authors present two critical characteristics for each of Ernest’s (1989) three views of mathematics and identify key questions and essential points for all of them (see Table 1 for details of this scheme).

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Characteristics</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrumental</td>
<td>Non-coherent sequences (1a)</td>
<td>Are there several objects within the work which belong to a particular field of mathematics but do not show any relation with one another?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does the text consist of an enumeration or a classification of items rather than showing the parallels in-between?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: the items instead of their characteristics are considered to be important, that is, the items are more important than their meaning in a wider context</td>
</tr>
<tr>
<td></td>
<td>Facile conception of usefulness/application of mathematics in the course of life (1b)</td>
<td>Is there a slight evidence of the importance of mathematics and its application?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Is the attention drawn to the fact that applications are useful rather than in which way?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: the central motivation point for practicing mathematics is the convenience one can gain where the character of usefulness always comes to the fore</td>
</tr>
<tr>
<td>Platonist</td>
<td>Display of mathematical coherence (2a)</td>
<td>Are there any references drawn between any mathematical items in the work?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does the text show a cohesive character within the implementations?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: relations are identified but not necessarily self-drawn</td>
</tr>
<tr>
<td></td>
<td>Theory/history of mathematics (2b)</td>
<td>Is the development of mathematics referred to as a determined, somewhat stable construct of knowledge?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Are scholars who once made mathematics crucial to the work?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: mathematics as a static entity predetermined by nature</td>
</tr>
<tr>
<td>Problem-solving</td>
<td>Autonomous mathematical activities (3a)</td>
<td>Does the setting of tasks offer the occasion for using mathematics actively and self-dependently?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Do certain actions enclose mathematical items as well and are not mentioned without any reference?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: not only meta-mathematical explanations, but something inventive; an extract out of a mathematical process allowing not only to counterfeit, but also permitting independent thinking</td>
</tr>
<tr>
<td></td>
<td>The development of mathematics (3b)</td>
<td>Is the development of mathematics indicated by being a process?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does the description transcend the image of mathematics being a complete and static product?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Essential: dynamic of mathematics (through the author or somebody else) is described as a process</td>
</tr>
</tbody>
</table>
As Rolka and Halverscheid (2011) have pointed out, using pictures as sole data sources entails highly subjective interpretations, which is why they have additionally asked their participants to write a text and, in some cases, also conducted interviews. Similarly, Rose refers to Collier who as one of the first to use photo elicitation already argued that the information entailed in an image can only be accessed through interviews (Collier, 1967 as cited by Rose, 2012, pp. 300-301). This study therefore follows the idea of data triangulation and also makes use of interviews conducted with the participants about the creation and content of their picture. The semi-standardised interviews allows enough flexibility with respect to the individual pictures, but also ensures that key issues are addressed in all interviews (see annex for a list of the questions used). The verbal data was analysed using the method of qualitative content analysis as described by Mayring (2010). At the heart of this approach is a set of categories (codes) which are defined and revised in the course of the work (feedback loop).

The codes used in this study are on one hand derived from the scheme by Rolka and Halverscheid (2011) as it is described in Table 1 (deductive codes), on the other hand they were developed from the available data (inductive codes). In addition to the six characteristics listed in the presented table above, the inductive codes consist of two large groups of codes, namely ‘Mathematics’ and ‘Other issues’. The category ‘Mathematics’ consists of two subcodes, that is ‘Characteristics of mathematics’ and ‘Mathematical terms and fields’ which were mainly used to specify particular aspects of the six characteristics presented in the table above. The category ‘Other issues’ identifies themes frequently mentioned in the interviews, but which cannot easily be integrated into the deductive codes. It encompasses a wide variety of themes, namely: ‘Personal issues’ (subdivided into ‘Emotions’ and ‘Personal experiences’), ‘Language’, ‘Definition of everyday mathematics’, ‘Education’ and ‘Nature’. The system of inductive codes was established in three steps, first on the basis of the pictures, secondly on the basis of the interviews and thirdly the two systems were integrated into one encompassing scheme. As the full interview material collected in this study goes beyond the issue of the created pictures, the presented code scheme will be altered once the rest of the interview material is analysed.

The data for this study were collected in the summer of 2012 in Switzerland. All participants are members of the Swiss network for everyday mathematics through which they were recruited. Ten days before the first interview they received a letter asking them to create a picture answering the question what mathematics is for them. Together with this task they received an A3-format piece of paper, which they had to use for the creation and presentation of their picture. They were asked to return the picture to the author no later than two days before the first meeting, as it was the basis for the first interview. A second interview focused on the participants’ biography and teaching. The average time between the two interviews was one month and on average they lasted 82 minutes (minimum 58 min., maximum 138 min.). Most interviews took place on the premises of the participants’ work place. They were conducted in Swiss German, recorded digitally and later transcribed in standard German. Only a small part of the interview data is included in the analysis on which this paper is based, namely the first section of the first interviews where the participants talk about the pictures.

Twelve individuals volunteered to participate in the study after being informed about the project and the corresponding requirements, eight of them were selected for the interviews. Out of this

19 The network is called “Netzwerk Alltagsmathematik” and is comprised of some 100 people from German speaking Switzerland who are interested in numeracy. See http://www.netzwerk-alltagsmathematik.ch/ (last accessed June 17, 2013).

20 The literal translation of the task is as follows: “Imagine you were an artist and have accepted the following contract work: What is mathematics? A personal view. Present your views in a pictorial, creative manner, working with materials and techniques of your choice (coloured pencils, watercolour, collage, etc.).”

21 The only requirement was that applicants were teaching adults and address mathematical topics in their classes. They were informed that they would need to invest a maximum of four hours of their time, during
group, five were selected in order to have as homogenous a group as possible. All of these five participants have attended one of the first two numeracy\textsuperscript{22} trainings for adult education teachers in Switzerland\textsuperscript{23} and none of them studied mathematics at the tertiary level. Furthermore, all are currently working as adult education teachers, though to different extents, in different subjects and with different students. Three of them are teachers of German as a second language, one of them is a self-employed public relations worker (and teaching part time), the last is working as a course leader in the social affairs department of a large Swiss city. They have an average age of 50 years (between 43 and 57), three of them are male, two female. Their educational backgrounds are very diverse and many of them have been educated and trained in different jobs. Table 2 below presents a short overview of the five individuals labelled P1, P2, P3, P4 and P5 in the remaining text.

Table 2. Overview of the participants.

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>F</td>
<td>M</td>
<td>F</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>Age (in years)</td>
<td>57</td>
<td>53</td>
<td>52</td>
<td>43</td>
<td>45</td>
</tr>
<tr>
<td>Education</td>
<td>High school, commerce diploma, translator, music school, adult educator</td>
<td>Compulsory school, chemical technician, forestry manager, adult educator</td>
<td>High school, teacher training, speech therapy</td>
<td>High school, teacher training, public relations and communication specialist</td>
<td>Compulsory school, commercial apprenticeship, social work, adult educator</td>
</tr>
<tr>
<td>Work experience</td>
<td>Secretary, own company, translator, mother, adult education teacher</td>
<td>Chemical technician, craftsman in France and Spain, outdoor social worker, father, adult education teacher, case manager, trainer</td>
<td>Teacher, various small jobs, speech therapist, mother, adult education teacher</td>
<td>Radio journalist, self employed consultant, adult education teacher, trainer</td>
<td>Banking, IT coordination, social work, adult education teacher</td>
</tr>
<tr>
<td>Current position</td>
<td>German as a second language teacher with low qualified people</td>
<td>Course leader</td>
<td>German as a second language teacher, speech therapist, trainer</td>
<td>Self employed public relations consultant; trainer</td>
<td>German as a second language teacher, developing numeracy material</td>
</tr>
<tr>
<td>Time spent working on mathematics (P's own estimation)</td>
<td>0% *</td>
<td>10%</td>
<td>0% *</td>
<td>10%-15%</td>
<td>40%</td>
</tr>
</tbody>
</table>

* 0% indicates that these two people were not teaching specific numeracy classes at the time of the interviews. Most of these classes do not take place due to low enrolment rates, but both address related issues in their other classes.

which they would have to complete a small non-mathematical task and participate in two interviews in a location of their choice.

\textsuperscript{22} Without going into the details of conceptual discussions of mathematics and numeracy, the latter term is used in the context of this paper whenever the work situation of the informants is referred to, as they clearly see themselves as teachers of everyday mathematics (numeracy) and not mathematics in general. Numeracy is considered to be the appropriate translation for the German term everyday mathematics in this context.

\textsuperscript{23} Specific numeracy training for adult education teachers is new in Switzerland. Currently the fourth training course aimed at trained adult educators with „no reservations towards mathematics“ is on-going (see also: http://www.alice.ch/index.php?id=1330, last accessed January 6, 2014). Some 45 individuals have attended the eight day course so far.
Findings

Visual data

Before analysing the pictures with the scheme presented in Table 1, they are first described in terms of the techniques used in their creation to provide a better understanding of their diversity as well as the shared characteristics. Four out of the five informants have chosen to present their view of mathematics in form of a collage: They put together various small bits and pieces and in one case combined these items with paintings and drawings (see figures 1 to 5 for reprints of the five pictures). Only one person (P5) presented an integral painting. Incidentally this was one of two who gave a title to their picture, namely “Chaos and Order” (the title of the picture of P4 roughly translates to “One Day Mathematics”, see figure 2). Looking more specifically at the items used for the pictures, the following types can be distinguished: (i) artefacts such as receipts, timetables, programmes, money, stamps, or a playing card (mainly used by P4); (ii) newspaper or magazine clippings (mainly used by P3) or pictures printed from the internet (mainly used by P1); and (iii) own writings and drawings, printed and cut out (mainly used by P2).

When asked why they chose this particular mode of creating the picture, the informants said it suited them or that it was the only way such as task could be approached. While all informants said to have thought about the task and how to approach it, before completing it, only one of them went about it systematically: P1 wrote down a list of words she associated with mathematics and then systematically looked for images which illustrated each term. The others worked more spontaneously, for example emptying his wallet (P2) or using those pictures from magazines that appealed to her (P3). P5 relied on a painting technique he was familiar with and liked. During the interview the informants had the chance to add to a copied version of their picture, however, none of them chose to do so and in spite of some of them identifying specific gaps when explicitly asked for them, they all were satisfied with their creation in the end and said that the picture represents them well as a numeracy teacher.

In the following paragraphs each of the characteristics presented in Table 1 will be discussed briefly in order to analyse the five pictures systematically. If necessary, references to interview statements will be included, and some specific methodological observations will be made.

Non-coherent sequences (1a)

While collages can be presented as integral pictures, as it has been done by P3 and P4, they can also be seen as illustrative of presenting non-coherent sequences, the first of the classification criteria described by Rolka and Halverscheid (2011). A prototypical example of such a presentation is the picture created by P1, which consists of eight individual elements, laid out systematically:
Similarly when talking about their pictures, the participants in some cases moved from one element to the next (particularly P1), while others talked in more coherent and general perspectives about mathematics. However, it is worth noting, that there are hardly any “objects […] which belong to a particular field of mathematics” (Rolka & Halverscheid, 2011, p. 528) depicted in the pictures. Again the creation by P1 is a good example: None of the eight pictures can be identified as an object belonging to a field of mathematics. The pictures by P2 and P3 contain some elements which can easily be allocated to fields of mathematics (for example equations belonging to the field of algebra), but they clearly do not constitute large shares of their pictures. The majority of the specific elements show mathematics in a wider and applied context, such as shopping or time keeping. In the remarks made by P1 it becomes clear, that her elements are actually representing characteristics of mathematics rather than objects of mathematics – each item stands for a specific aspect of mathematics, such as regularity, symmetry or confusion. And even though they all seem unconnected and unrelated, P1 stresses that “They [the eight small pictures] are related to me, to my experience of mathematics. […] Amongst themselves they are not directly – they represent the same in different ways. They represent mathematics.” (P1).

In short, while many of the pictures present unrelated objects, the stories the informants tell convey a different message, namely that the specific elements do belong together and can be seen as different sides of the concept mathematics. Furthermore they indicate that the wider context is emphatically important – an aspect that is highlighted in the next point.

**Facile conception of usefulness/application of mathematics in the course of life (1b)**

When it comes to the usefulness and application of mathematics it is interesting to note that applications are dominant in most of the pictures and that in the interviews many informants stress how present mathematics is in everyday life. The picture created by P4 is an illustration of the presence of mathematics in everyday life, as it consists mainly of real objects such as money, a stamp or a lottery ticket. The creator of the picture also stresses, that one cannot choose to do mathematics respectively numeracy, one has to do it: “It is part of our everyday life, well, yes. But also it is smashed into your face. One has to do it. […] It is not simply, just flowing along somehow.” (P4)
This focus on mathematical aspects of everyday life is not surprising, considering the participants’ background as numeracy teachers. Their pictures present rich illustrations of the many contexts in which mathematics is found: in games, timetables or receipts, measuring time or money. These applications also illustrate one characteristic highlighted by many, namely that mathematics is frequently hidden and needs to be identified first. It is therefore interesting to note that in their talks the participants often refer to basic activities such as counting, categorising or organising when speaking of mathematics, rather than identifying specific operations such as adding, multiplying or calculating percentages which stand behind these applications. Furthermore, they all very clearly divide mathematics into two areas – applied mathematics, which is what they know and do, and abstract mathematics, which is beyond them. This duality is very often referred to in the context of their own educational experiences:

To a certain extent, math was playful for me. I understood it well, I liked doing equations. […] But, when it was too abstract or so, at some point maths became a problem zone [pointing to the same word she had glued on her collage]. (P3)

Or: “Being a linguist, I’ve quite a high affinity for math. I had that at school as well. I was left behind when it was not connected to everyday life.” (P4)

There is a strong sense of the relevance of applications in everyday life and at the same time a division of mathematics into an applied and an abstract or theoretical part, an issue which will be discussed in the next section.

One last point which needs to be mentioned in this context is an illustrative example of the difficulty of interpreting pictures, as it has also been pointed out by Rolka and Halverscheid (2011). P3 has drawn a series of artefacts on her picture and commented:

At some point it occurred to me that actually all inventions that we have such as a car, a plane, a submarine and here the mouse and the computer – all of them have to do with math. […] Our modern cities would not be possible without math. (P3)

---

24 As Swiss German is predominantly an oral language in which the term mathematics does not exist, the English equivalent term math is used in all quotations to reflect this aspect.
Both the mentioned images as well as this statement can easily be interpreted as illustrations of mathematics being useful. But when asked about the usefulness of mathematics, P3 answered:

Pfffft, that is a question I have never asked myself. I think that, well categorising and sorting, those are excellent skills that one needs to have in order to do math. I think. And at the same time they are also cognitive functions. (P3)

The interviewee does not provide a direct answer; also she never used the word useful in her answers herself. So while her picture clearly could be interpreted as showing the usefulness of mathematics, the informant does not stress that aspect herself at all.

In short, when looking at the aspects constituting the instrumental view of mathematics, it can be said that visually a few of the pictures imply the presentation of non-coherent sequences, but that this view is modified by their verbal explanations. Furthermore, the application of mathematics is dominant, in both the pictures, and the interviews. There is also a particularly strong notion of a division of mathematics into an applied and an abstract part.

Display of mathematical coherence (2a)

This characteristic is one that is most difficult to analyse in the five pictures. As stated above, there are very few mathematical items are presented, therefore it is difficult to identify references between them. Particularly interesting in this context are the two pictures by P3 and P4. At first glance they depict a coherent image, but when looking closer both consist of various individual elements. And while many of these elements could be sorted or grouped (for example visual presentations, formulas or measurement tools in the image included below), this is not done by the creators of the image.

![Figure 3. Picture created by P3, presenting a mix of technologies used.](image)

When examining the statements made during the interviews, all of the pictures can be considered to contain at least some cohesive elements, as the interviewees identify references between the elements. Since coherence was visually not always present, the participants were specifically asked for it and only P2 says that he is not sure that the elements on his picture have any connections, except to him as a person and his thinking: “They are really the thought clouds that I associate with it [mathematics]. In the end it should illustrate a little bit the ambivalent attitude that I have towards math.” (P2)
And while this sentiment of the creating person itself as the connecting aspect of the picture is partially reflected by others, two people said that all shown elements “have something to do with math [...] it meets on the meta level of math” (P3). This and other statements imply that the participants do see mathematics as a unified (and unifying) body comprised of different parts or levels: “The depth is reflected in the colour blue, but also in the sea and the waves [...] also from the easy to the difficult, always this, from the profound to the superficial, from the simple to the complex.” (P3)

Or:

For me, math as a concept is not really interesting. Until the point, when I reach some limit and then I have to go one [level] deeper and say, okay, what does it look like more abstractly, what are the structures that are behind it. (P4)

These statements again reflect the profound duality of mathematics that has been identified before, namely the two parts of applied mathematics – or in terms of the participants those aspects of mathematics they are able to do – and the abstract, conceptual side of mathematics.

*Theory/history of mathematics (2b)*

This characteristic is clearly absent in all pictures – there are no references to any mathematical scholars or historic events. Similarly in the oral discussion, history of mathematics or its development are very rarely touched upon and are definitely not a reoccurring theme. In some statements development is implied, for example: “I believe that in math we are currently at this point, most likely also, because time and again we were curious.” (P3)

This statement implies not only a history of mathematics, but also leaves other possibilities for the future. At the same time it shows clearly that this development is not determined and stable. Similarly the following statement: “At some point there was a decision for the decimal system.” (P4)

This observation additionally implies that there are actors who actively influence the development of mathematics by taking specific decisions. Mathematics is therefore clearly not seen as being "a static entity predetermined by nature" (Rolka & Halverscheid, 2011, p. 528). However, it is worth noting, that though largely absent in the pictures, nature is a key issue in the interviews. It is mentioned by four participants and all identify a close relationship between mathematics and nature, namely that there is a lot of regularity and symmetry in nature and that this can be described with mathematics. However, it is clearly people who “put patterns over nature. [...] It [picture of a daily routine] stands for the human who has made this division of something that is determined by the sun” (P1). But this active use of mathematics is the core theme of the problem solving view of mathematics, which is described after a short paragraph summarising the key elements of the Platonist perspective.

When it comes to assessing the aspects constituting the Platonist perspective it can be said that virtually no aspect of this view is represented in the participants’ pictures: Coherence in their pictures is strongly linked to the participants themselves as the pivotal point of the presented elements, rather than being identified between the items in the work. Scholars or historic issues are not depicted at all and the only element, which is implicitly present, is a methodical part of mathematics, namely

---

25 Interestingly P5 does not refer to nature itself, he refers to the environment in cities, namely people and public transport and he assigns mathematics a very similar role in this context, namely that it describes and explains patterns in our environment, in this case the man-made: “If you look around, if you walk around the city, you get a feeling that people walk around chaotically. And if you have the correct algorithm, you find an order in it, how it all works and so on. Or also a train, how the entire thing works. It looks very chaotic, but it is all planned meticulously.” (P5)
everything that is not applied mathematics. However, that aspect remains very vague and needs to be explored further.

**Autonomous mathematical activities (3a)**

The focus of this aspect lies on the independent thinking by the person using mathematics. The picture by P2 is an example which – upon closer inspection – is very illustrative of this process: It consists of creative adaptations of word problems (sentence in the centre at the bottom), an own presentation of the number Pi (little square in the upper middle) as well as other changed and adapted quotations or principles (the line around spider web is a free adaptation of the Haiku, a Japanese form of poetry where the precise number of phonetic units is decisive):

![Figure 4. Picture created by P2, representing some autonomous mathematical activities.](image)

So while the issue of autonomous mathematical activities is not always easily accessible in the pictures, the explanations the participants have given clearly illustrate its importance at different levels: “It [mathematics] makes independent, from other people. It empowers. It enables you to participate in society without depending on other people.” (P1)

While this person stresses a fundamental idea of independence facilitated by mathematical abilities, another makes the link between specific mathematical items and individual activities: “Anything that is related to measuring. [...] Measuring time, measuring weight, ehm presenting things in graphs, translating it into curves.” (P3)

Another stresses the relevance for everyday life:

There are larger themes such as shopping, where a lot of mathematical things come into play. Dates, prices, weight, being able to compare. The rule of three always comes into play relatively quickly. And this is the hidden [math], that needs to be carved out. (P5)

For many of the participants the setting of the task or identifying the mathematical aspects in a given situation is the starting point for autonomous mathematical activities – people need to solve a problem and mathematics is considered to be helpful:

And if a mathematician is looking at what a person, who understands nothing of math, is doing, he could probably very easily formalise that and present it with numbers or transfer it into graphs or so. And the person himself only wanted to have a larger harvest or make a dress or a shoe or something. (P3)
It is interesting, that in the context of these participants, it is not always easy to distinguish between the instrumental and problem-solving perspective, as the idea of application and usefulness is very closely related to or even a prerequisite for independent thinking.

**The development of mathematics (3b)**

This issue has already been touched upon when discussing the aspect of history of mathematics, where it was shown that if history is talked about by the participants it is understood as a process. And while no names of people producing mathematics are mentioned, there are references which imply that such people exist:

And I think math is something to be developed. Even though one can define laws and the like, they are defined on the basis of our own boundedness. That means, because we are limited, what we define here can also only be a form of boundedness. […] But it doesn’t really reflect anything complete, because we are not complete, so it can’t be complete either. (P5)

Many of the participants mention the possibility of being creative with the application of mathematics, for example cheating when keeping scores in games, but are at the same time aware of the fact that there are clear limits as to what one can do:

I can’t, suddenly, well, I can claim something absurd in math, but then to prove it is a lot harder than for example in other areas. […] In math that is quite difficult. There is relatively little room for manoeuvre. And I don’t really know whether that has already been exhausted. (P2)

Out of the three perspectives of viewing mathematics, the perspective of problem solving is undoubtedly the most present one in the analysed data. The pictures very clearly present views of mathematics implying a strong personal engagement, particularly in specific applications of mathematics. Furthermore, the statements indicate that mathematics is dynamic rather than complete and static – even though the participants recognise that these developments are clearly beyond their own capabilities.

Summarising and very broadly speaking, it can be said that on the basis of the dominant characteristics in their pictures, the participants’ views of mathematics are clearly mixed. It is equally clear, that the least relevant view is the Platonist, as neither mathematical coherence nor its theory or history feature prominently in the presented pictures. The analysis has also shown that it is indeed very difficult to work with pictures only, when trying to understand and describe their creators’ view of mathematics. The picture created by P5 is a good example of this fact that visual data on its own can be very hard to interpret. Without the accompanying interview this picture could not have been classified. However, once formulated, its key message comes across very clearly:

For me, the circle represents absolute order […] Order also means to be able to orient yourself in the chaos and if one takes the relation to math, math to me is like a language with which you can create order in a chaos. So if from the outside something looks chaotic and you then look closer, with the help of math you can explain certain things or identify new dimensions within and a new understanding.” (P5)

![Figure 5. Picture created by P5, representing an integrated presentation of mathematics.](image)
In order to complete this sketchy analysis based on the pictures, some select issues which only surface in the interviews will briefly be presented in the next section. This will help to get a better understanding of the complex views that the interviewed adult education teachers have.

**Verbal data**

While the analysis of the pictures has particularly highlighted the relevance of application for the creators of the pictures, and in some instances also their autonomous mathematical activities, they not only raised a number of questions with respect to the interpretation of specific elements, but also left out a number of issues which only emerged from the analysis of the interviews. As a first element of this section the issue of language, which does not arise at all from the pictures will be discussed, afterwards a number of issues, which are only marginally or implicitly present in the pictures will be presented.

**Language**

One issue which was addressed by all interviewees at least once, but cannot be identified in the pictures is language. Mathematics was seen as complementary to language in mastering everyday life: “And numeracy is more like [...] let’s say decisions and organising or I don’t know exactly what. Either way, it is not only language, I have to deal with something else as well.” (P2) And: “It [mathematics] is a help, like language. Language helps mastering your life and communication, sure, and math also helps mastering your life, doesn’t it?” (P1)

As this statement already indicates, the participants identified similarities between language and mathematics, not only that both have an instrumental function for mastering everyday life, but also at a more fundamental level: “For me math is like a language where one can create order in a chaos. [...] And math is the same for me, it is like a language for me to also explain something, phenomena.” (P5)

More specifically, two interviewees mentioned that mathematics is like the grammar in language which explains how things work and by contrast, “numeracy is in this sense the expression of what I do in daily life. Like I can speak without ever wasting any thought on grammar or linguistic structures or on metalinguistic condition.” (P3) Other points mentioned include differences between mathematics and language, for example the fact that language has developed more organically, without seemingly arbitrary decisions such as the naming of numbers (P4), or difficulties of non-native speakers with word problems (P2).

**School**

There is only one element on the picture by P2 which can be clearly attributed to a school context, namely a piece of an exercise book with several crossed out attempts of solving a problem. Contrary to this single artefact, school was mentioned several times in the interviews: On one hand in the context already mentioned when discussing the pictures, namely the experience that all of the participants at some point of their educational career reached a point where mathematics became too abstract. On the other hand, several references were made to the way mathematics is taught and experienced at school in general. In their role as adult educators the interviewees are often confronted with students who had very negative experiences with school in general and particularly mathematics classes. They are very aware of this aspect and clearly state that in adult education (or in their classes) other principles apply to the teaching of mathematics:
But it was stressful at some point and I can imagine that my participants, that they are simply stressed. They get somewhere where they can’t continue, it just doesn’t go any further with their imagination. [...] And that as a course leader you are always aware of that, that each individual has his or her limits. I notice precisely if a person simply can’t, that doesn’t work and then I don’t insist, because I don’t want them to panic. [...] As adult educator I can do that. I have the liberty of simply stopping. (P1)

**Personal experiences**

The interviewees’ own educational experiences with respect to mathematics teachers and classes are an important aspect that is not directly reflected in the pictures, but is constitutive of how they see mathematics. In addition to their specific school experiences which did not leave any of them “a typically traumatised person” as P5 said, many positive memories connected to mathematical activities were mentioned, for example solving puzzles, mental arithmetic when shopping, playing cards or being particularly good in a specific field of mathematics area such as solving equations or probability. Again, some of the participants make a direct link from their experiences to their students’ by stating that one of their goals in their courses is that their participants are also able to experience mathematics as something fun, that they are able to enjoy doing it – like they themselves did at some point.

**Affective issues**

Personal experiences are closely related to emotions which can be equally difficult to express in images. Even seemingly explicit presentations, such as the stick man wedding couple in the picture by P1, can stand for something very different, as the interviewee explains:

> Here [pointing to the picture] I actually googled stick man, this represents a stick man. [...] It is probably not the right image, [...] but it is a symbol for dumbing down. And for the two dimensional presentation of something that is three dimensional and has so many sides. Well, and if you just want to flatten something on to a plain and reduce it to some lines which do not conform to reality [...] a simplified presentation [...] And it came to my mind that also with math – we are sometimes dumbed down by the mindless calculations and by how we learn math in school. (P1)

Overall it is interesting to note, that the participants view their own experiences with mathematics generally positively and those of others, mainly their students, predominantly negatively.

**Characteristics of mathematics**

One last aspect, which is represented indirectly in many pictures, particularly in the picture by P1 who argues that each of the small images stands for one characteristic or aspect of mathematics, is its nature. When asked to describe the nature of mathematics the use of adjectives is predominant and while the participants were not directly asked to do so, many of them used numerous adjectives when talking about mathematics, for example: mathematics is useful, explaining, precise, organising, abstract but also hidden, playful, reliable or contradictory. The one feature, which was named most often and in each interview at least once, can be summarised with the adjective fundamental. It includes statements like “I think, yes, maths is like always included” (P3) or “It is always and everywhere” (P4). This is one aspect which seems central to the belief of the interviewed teachers, but which cannot easily be reconciled with any of the three perspectives described in the used scheme.

Overall, a number of additional aspects of mathematics, such as its close relationship with language, how it is experienced in school and other everyday situations as well as the connected affective issues gain more clarity in a verbal exchange than in visual representations. Furthermore, it
seems that they are better suited to provide insight into dynamic aspects of beliefs, namely their development or how they influence teaching practices. The combined results of the visual and verbal descriptions will briefly be discussed in the next paragraph before some concluding remarks are presented.

**Discussion**

Many characteristic features of mathematical beliefs and other issues prominently discussed in mathematics education can be recognised in the beliefs of the five adult education teachers presented in this study. Among them the relevance of affective issues (Evans, 2000), the invisibility of mathematics (Wedge, 2010) and findings described in many other studies such as the early formation of beliefs, or their influence on an individual’s behaviour. One aspect which is completely absent is the issue of gender: Lim (1999) has found that mathematics is generally perceived to be a male domain and Pehkonen (1994) has identified gender differences as one area of belief research which is well documented – however, it has not been an issue in the data used for this paper. In the next paragraphs, two aspects of the findings will be discussed, namely how the described beliefs fit into Ernest’s categories and methodological issues.

**Instrumental, Platonist or problem solving views?**

On one hand, it seems easy to allocate specific views of mathematics to the participants. Both their pictures as well as the interviews illustrate the relevance of the application of mathematics in the course of life – one aspect of the instrumental view – as well as the importance of autonomous mathematical activities – one key aspect of the problem solving view. Both these aspects can be explained with the context in which they work and their identity: They see themselves as numeracy teachers and the issue of autonomous applications in real life, in specific contexts, is a key aspect of numeracy. On the other hand, there are also some issues, which seem to be unique to these five individuals, among them the relevance of language when discussing mathematics. Potential explanations for this aspect are on one hand the informant’s background – all of them also work as German as a second language teacher – on the other hand their training: In their education as numeracy teachers, comparing language and mathematics was a prominent theme (oral information by the respective course leader).

Two other dominant aspects of the described mathematical beliefs include firstly the fact that the participants see mathematics as being divided into what they know and are able to do and the rest – an aspect that is not reflected in either Rolka’s and Halverscheid’s (2011) or Ernest’s (1989) original presentation of the three views. Again, this can partially be explained by the teachers’ identity as well as their experiences: they see themselves as numeracy teachers and not mathematicians therefore needing only specific knowledge in the former field. Furthermore, during their education they all have experienced that there are issues in mathematics that they do not understand. One could argue that this division of mathematics into what they understand and are able to do, that is above all numeracy, and the rest, namely “abstract mathematics”, is not only based on their experiences but also allows them to see themselves as competent numeracy teachers. The second aspect is that the participants see mathematics as something fundamental and universal and in this function they provide it with a trait that goes beyond the strict instrumentality of specific procedures, namely that mathematics per se explains and organises the world. One could argue that this characteristic is constructivist rather than instrumental, however the notion of usefulness is still strongly linked to this function of explaining
and organising the world, therefore the relevance of the instrumental view for these adult education teachers is justified.

Methodological issues for future research

Overall, it can be said that using pictures to explore adult education teachers’ beliefs has worked very well. Many of the participants commented positively upon the task of creating the picture and were engaged in the research process. Furthermore, data collected from the pictures and interviews have proven to be somewhat complementary – two positive aspects of visual methodologies as they are also identified by Rose (2012). The complementarity of issues raised has also been reflected in the elaboration of the codes, where it can be seen that certain codes predominantly occur in the visual respectively verbal data (for example time and money respectively language). However, looking at the diversity and richness of the pictures presented and the classification reached according to the three views of mathematics, a sense of inadequacy remains. The benefits of working with visual data seem to vanish if one of the main results is a categorisation that hardly goes beyond what could also be attained by using questionnaires. The obtained results therefore not only underline the relevance of the question raised by Halverscheid and Rolka (2007) whether the identified categories describe the works extensively, but point to the more fundamental question of whether these categories which focus on one aspect of beliefs are adequate to analyse the richness of visual data. Particularly if beliefs are understood in the broad sense of world views taking into account affective as well as cognitive aspects, a more encompassing analysis which allows capturing these diverse components of beliefs is needed.

Kress and van Leeuwen (2006) argue that verbal and visual communication both have their specific possibilities and limitations in constructing meaning and in the shift towards a new visual literacy the corresponding abilities of reading visual communication are essential. They present a system of categories which can be used to critically analyse images and which could be a starting point for a more fundamental and inclusive analysis of the pictures presented here. For example, the layout of images: While Rolka and Halverscheid (2011) reduce this aspect to the issue of connectedness (point 1a in Table 1), Kress and van Leeuwen (2001) suggest various types of narrative and conceptual representations of different types of layout – an aspect which could enrich the presented analysis. Moreover, as the created images were discussed extensively, a multimodal perspective taking the specificities of and interaction between the verbal and visual data into account as described by the same authors (Kress & van Leeuwen, 2001) could be even better suited for a more in depth analysis of the presented data. Such an approach could most likely help to address some of the challenges encountered when using the scheme developed by Rolka and Halverscheid (2011), namely that adults’ perspectives of mathematics seem to be somewhat richer (for example including less mathematical objects, but more characteristics of mathematics) and are therefore sometimes more difficult to capture with the provided characteristics only. Or the specific difficulty, which these authors also encountered and which became even more prominent in the analysis of pictures created by adults, namely that of separating and identifying the extent of each of the three perspectives in mixed world views. As mentioned before, it has proven to be particularly challenging to differentiate between the aspect of application of mathematics in everyday life and autonomous mathematical activities. Once again, the question that Rolka and her colleague (Halverscheid & Rolka, 2007) ask might need to be reformulated, namely that using visual data is not best suited to estimate the extent of each of the three perspectives, but rather how they are related.

Furthermore, it would be interesting to explore, how the perspectives relate to the division of mathematics into what the participants can do and the rest, as it has been described above. As Rolka and Halverscheid (2011) have observed an overwhelming dominance of the instrumentalist view
amongst younger students, a move towards more autonomous mathematical activities might also be explained with increasing life experience. Another emerging question in this context is what factors facilitate a change from an instrumental use to an autonomous use of mathematics.

In short: the performed analysis has proven a valuable starting point for exploring adult education teachers’ beliefs, but other more suitable analytic methods are needed in order to find answers to some of the questions raised or formulate alternative questions.

**Conclusion**

When taking stock of the experience of using pictures to explore mathematical beliefs of adult education teachers, the overall assessment is a positive one. Furthermore, combining visual and verbal data (data triangulation) has proven to be beneficial as the two sets not only confirmed the centrality of particular themes, therefore validating each other; they also complemented each other, as some issues only emerged in one of the two sets, indicating that data triangulation also helped to improve the quality as the two sets enriched each other thematically. These benefits gained from data triangulation could be exploited even more, if triangulation was also applied to the methods of analysis. More specifically, if a particular visual method of analysis was used or if content analysis was adapted to reflect the specificities of both the visual and verbal data. Each of them has their characteristics and unique qualities that need to be respected and taken into consideration in the analytic process.

**Annex: Interview questions**

List of questions used for talking about the creation and content of the picture. While the first question was always the same, the order of the following questions was adapted depending on how the interview developed.

1. You received the written task and what happened then?
2. Why have you decided to use this mode of presentation?
3. How is mathematics presented in your picture?
4. What is the connection between the elements of your picture?
5. Are there aspects of mathematics that you wanted to present but could not do so?
6. Is there anything you would like to add to what we have said?

**References**


Beeli-Zimmermann, Exploring adult education teachers’ mathematical beliefs


Critical issues in adult numeracy practice – contradictions and strategies

David Kaye
Learning Unlimited, Institute of Education
<david.m.kaye@btopenworld.com>

Abstract
This paper discusses the critical situations I have been asked to ‘improve’ by providing professional development for teams of adult numeracy and functional mathematics teachers in the post-16 sector in London. These situations have not been identified through any research process, but arise from internal management reviews of course outcomes and staff development provision. The assessment by the institution’s management of these situations is often very different from that of the teaching staff. And my view as a teacher trainer is probably different again. The main focus of my intervention is to suggest changes to planning and teaching strategies. However, organisational structures have also to be considered. The author argues that three significant theories, ‘multiple intelligences’, ‘a profound understanding of fundamental mathematics’ and ‘how the mind creates mathematics’ provided guidance for the reflection of practice. The approach taken is supported by the Open University’s guide to action research.

Keywords: numeracy, mathematics skills, adult mathematics learning, critical issues, strategies.

Introduction
This paper provides a review of a series of interventions into adult numeracy teaching in London, United Kingdom (UK) over a two-year period from 2011 to 2013. The interventions were made at the request of Further Education Colleges to improve the standard of teaching. With reflection, the concerns of the local management have been identified as critical issues in the teaching of numeracy to adults. Similar issues were identified in a number of Colleges and contradictions between the teaching aims and methods were also identified. To help improve the outcomes some activities were suggested. These have since been reviewed and can now be examined as a set of strategies to improve teaching and learning. This paper recounts this journey from support for professional staff to a set of key theories that underpin innovative interventions in practice.

Though I aim to analyse a range of research sources that are relevant to this journey, I will follow a narrative that is founded on the experiences of giving support both in structured sessions and during teaching practice observations. This set of experiences was not designed at the time as a research project, but do now form the basis for a, retrospective, critical review of strategies for improving adults learning mathematics.

Critical Situations
The critical situations that form the teaching practice, core to this analysis, arose out of the formal provision of professional development to improve teaching and learning standards. Over a number of
years the UK government funded support to educational and training institutions on a national basis. Such support reflected various formats, and included partner institutions supporting each other, banks of on-line resources or the provision of specialist trainers. There was, for a short period, a particular focus on adult numeracy, and it was this situation that provided the opportunity for staff development visits to be made. See, for example, the pages on “Whole Organisation Approach to literacy, language and numeracy (LLN) Framework” on the Excellence Gateway site for Supporting Skills and Improving Practice.

In retrospect, I identified the following as the main situations that concerned the institutional management about their numeracy teaching, and for which they requested some specialist help:

- Working with students on vocational courses
- Working with ESOL students
- Raising students’ level from Level 1 to Level 2
- Preparing for functional mathematics assessments
- Making the numeracy class more interesting

Let us look at these in a little more detail.

Working with students on vocational courses comes out of a long history of adult numeracy and mathematics being seen as one of the basic skills that underpin success in almost all vocational education and training. Those familiar with the policy issues in this field in the UK since 2000 might be aware of the debates that have developed over the issues of integration, embedding and context. (See for example the NRDC report on embedding literacy, language and numeracy [Casey, H. et al 2006]). The iColleges were concerned about attendance and outcomes on numeracy and mathematics support classes.

Teaching ESOL students is a particularly large part of the work of adult numeracy practitioners working in London. The expression, ESOL, a contraction for “English for speakers of other languages” is used as shorthand for students who do not have English as their first language, whether or not they are attending language classes. Many numeracy teachers work with classes that are largely or entirely comprised of ‘ESOL’ students and so institutions are concerned with how best to serve this cohort.

Raising students’ level from Level 1 to Level 2 is with reference to the levels of the Adult Numeracy Core Curriculum (ANCC) and to the more recently introduced Functional Mathematics. The content of these levels can be explored on the Excellence Gateway site for Skills for Life Core Curriculum, particularly in the “numeracy progression overview” document26. For some teachers and curriculum managers the change of content from Level 1 to Level 2 is seen as a much larger challenge than movement between other levels, when planning teaching and learning.

Preparing for functional mathematics assessments is particular to the English situation, as this was a new form of assessment for the sector introduced in pilot form in 2007. It has, only quite recently, become the main form of assessment for adult students. It is a very different form of assessment compared to the national tests that were used previously. The national tests were multiple-choice questions, whereas functional mathematics aims to measure process skills and requires more writing and explanation. The mathematical content however, is very similar.

Making the numeracy class more interesting is a very broad category that in practice covers issues in which the curriculum managers considered that the mathematics teaching was too traditional, and

---

26 http://www.excellencegateway.org.uk/node/14938
the teaching staff were not open to new approaches. See for example, the approaches associated with collaborative learning, such as described by Malcolm Swan (Swan, 2007)).

The practice for this ‘reflection-on-action’ [Schön quoted in OU (2005) p24] comprised staff development sessions devised by the author. These took place in colleges of further education and local adult education services in the London (UK) area. The courses at these institutions were for students aged 16 or above. However for organisational and funding purposes the courses are usually organised separately for young adults, aged 16 to 19 and adult classes for those aged 20 and above. The courses can generally be classified under three headings: vocational, functional mathematics and English language (ESOL). The teachers attending the staff development sessions included mathematics and numeracy specialists, support teachers (for literacy and numeracy) and specialist vocational teachers.

Contradiction and strategy 1 – Order of numeracy topics

The impact of reports about poor numeracy, particularly the Moser report (DfEE 1999), led to the publication of the Adult Numeracy Core Curriculum (ANCC) in 2001 (Basic Skills Agency). There remains considerable dispute whether this document is properly described as a curriculum, despite its name. However, what is certain is that it set out a list of topics divided into sections, sub-sections and curriculum elements. These curriculum elements were presented across three levels: Entry Level, Level 1 and Level 2. The Entry Level was itself sub-divided into three, Entry 1, 2 and 3.

The three sections of the ANCC are number, measure, shape & space and handling data. These were based on a model established by the National Curriculum for primary school mathematics. An example of an element from the number section at Entry 3 is “add and subtract using three-digit whole numbers”. An example of an element from the measure shape & space section at Level 1 is “work out the perimeter of simple shapes”. The Core Curriculum is now available as an on-line document, however, for about 5 years the printed document was the only version available and the order of the elements tended to be followed as a syllabus by many numeracy teachers. This close adherence to the printed document was further compounded by the common practice in many institutions to encourage (at the very least) numeracy teachers to identify the numeracy elements covered in their lesson plan by their distinct element reference number. The ANCC itself encourages this.

The curriculum elements must be clear and used with learners. The aim must be that learners develop the concepts and the language that will help them make sense of their learning and go on doing it. Evidence shows that the inclusion of explicit curriculum targets in learning programmes has resulted in a clearer identification of outcomes by learners, and in better attendance and progression by learners (BSA, 2001, p. 8).

As the core curriculum has been used as a syllabus, schemes of work begin with the four arithmetic operations, proceed through whole numbers and then decimals and fractions and percentages. Here is the contradiction. All of these teachers are aware that it is considered good practice to take the students experience into account and place calculating techniques into context. Yet the part of the curriculum most removed from any context is introduced first and can easily take up half of the course time.

The ANCC itself emphasises the need to take into account the students’ past experiences.

The skills and knowledge elements in the adult numeracy core curriculum are generic. They are the basic building blocks that everyone needs in order to use numeracy skills effectively in everyday life. What is different is how adults use these skills and the widely differing past experience that they bring to their learning. This is the context that the learner provides . . . (BSA, 2001, p.8)
The strategy I propose is simple and straightforward, yet experience has shown it is frequently condemned and rejected. The ANCC is divided into three sections, Number, Measure, shape & space and Handling data. That strategy is simply to start the course in a section other than Number. To begin the course with some aspect of measuring or collecting data. There are three main advantages to be gained from this strategy.

1. It avoids presenting adult students with the mathematical techniques, such as mental calculations, that they probably find the most difficult, if not impossible, right at the start of the course. This is often countered with the argument that they ‘have to know how to multiply – to know their tables’. Perhaps they do. However, I question whether this traditional approach can work with most adult students. The students are in the adult numeracy class because they have not achieved previously. If they have completed secondary school they will have been shown the techniques for multiplication at least 10 times; if they have already had additional help in school and attended other post-16 classes this is probably closer to 15 times. Why should this occasion be any different?

2. Measure shape & space and handling data provide ‘in-built’ context for working with numbers. Something must always be measured or a shape must be a shape of something, and have dimensions. If data is being collected it must be about something. Starting a programme of study with topics drawn from these sections provides the opportunity for the examples used to be relevant to the students’ lives or the other courses they are studying. All of the calculating techniques from the ‘Number’ section occur when manipulating problems within these topics. Over time, appropriate support can be developed where it is necessary.

3. By starting with measuring or data not only are we avoiding starting with topics that are likely to be the most challenging for students – there is the opportunity to start with topics that the students are more familiar with. Very often the students themselves do not identify what they can do as mathematics at all (Colwell, 1997). For example, a student may have poor multiplication skills, and therefore have considerable difficulty in converting measurements. However they may have excellent estimating skills, demonstrating a thorough understanding of measurement, but the students may well consider this ‘just common sense’.

Contradiction and Strategy 2 – Numeracy for speakers of other languages

The problem as it is posed is ‘what we have to do to teach mathematics to the students who do not have English as their first language?’ In discussing this further with the teachers concerned, there appears to be a contradiction between what the teachers think they should be teaching and what the students need to learn. For teachers in stand-alone adult numeracy classes, this problem has been compounded by the recent introduction of Functional Mathematics. As was described above the new Functional Mathematics assessments require more writing to explain why a particular solution has been chosen. Part of the strategy here is to know about and understand the background of these students. The term ‘ESOL’ is used to refer to a very wide range of students. Many of the students will have lived in Britain for a comparatively short period of time, and therefore their schooling or education would have taken place elsewhere. In many institutions ESOL students are placed in classes according to the level they have been assessed at in English, and these are often at Entry Level. In the mathematics class the teaching is likely to begin with calculating methods, as discussed above. This may well be totally unnecessary and even cause confusion.

The students may well (currently) have a low level of English, but that does not mean that they cannot calculate; they may well have a good knowledge of mathematics. If a student has completed their secondary education in another country they are likely to be fully competent in their calculating skills. They may well be proficient using other methods, and this is where confusion can occur. If a
different way to calculate is demonstrated, they may well think that they are doing something wrong using the one they have been taught previously in school. Given they may have a low level of English it will be difficult to discuss this, and so care needs to be taken to ensure previously acquired skills are recognised and supported. It is important to recognise that the skill some students need to learn is the language of mathematics. There is quite a complex relationship between the language in which mathematics was learnt and the current learning medium of English. Dhamma Colwell (1997) gives examples of the processes people experience as they move from one language to another in their mathematical practices. For example M changed from Cantonese speaking school to an English-speaking one at the age of eleven. She found that maths was the only subject that she could understand easily, because the symbols used were the same in both languages (Colwell, 1997 p67).

In this example skills in mathematics are compensating for the lack of skills in English, by depending on familiar symbols.

The other part of the strategy is to ensure that connections are repeatedly made between the mathematical items, saying and writing the words that describe it, and the symbols used to represent it. An example of this is ‘ratio’. This is the mathematical item. It is written as ‘3:2’ and said as “three to two”. The concept of manipulating quantities in ratio may well be understood, but to discuss it and ask questions it is necessary to have the written and spoken language of ‘3:2’ and “three to two”.

This can be represented by an image using the concept of a number.

![Figure 1. Representations for number three](image)

Explanations given in a numeracy or mathematics lesson usually use all three - representing the concept in some form, saying or writing in words a definition or explanation and presenting the concept in symbolic form. These are very often not presented at the same time and moving from one to the other, with the intention to explain more clearly, can cause confusion.

**Contradiction and strategy 3 – Numbers with and without context**

The need to consider the context is particularly relevant to working with students on vocational courses. The pressure on institutions to ensure students have the mathematical skills to achieve their primary learning goal on a vocational course has long been an issue. For example Gail FitzSimmons discusses this in the Australian context in the late 1990s (FitzSimmons 1997). It is still a very live issue. At the time of writing, August 2013, the UK Government has just announced new measures for 17 year olds to continue to learn English and mathematics. Professor Alison Wolf, who headed a government review of vocational qualifications, described continuing in the two subjects as the most important recommendation of her inquiry. “Good English and maths grades are fundamental to young people’s employment and education prospects,” she added. “Individuals with very low literacy and numeracy are severely disadvantaged in the labour market.” (Wolf, 2013)
The contradiction is that mathematics can be presented differently in a vocational class to how it is introduced in the mathematics support class. An example of this was observed during an LSIS support session (see LSIS Support Programme – Barking and Dagenham College). In a painting and decorating class the students had to make a six-colour wheel on the doors they were decorating. Under the instruction of the painting and decorating teacher the students drew a circle. They then marked out the length of the radius of the circle six times around the circumference of the circle and drew lines from these marks to the centre. They completed the activity by painting in the three primary colours of red, yellow and blue, and by overlapping creating the three secondary colours of green, orange and purple (or violet). The mathematical solution to this problem would involve considering that a circle can be divided into 360 degrees and that to divide the circle into six equal parts then requires the calculation of 360 divided by six. To complete the practical task angles of 60° would then need to be measured or constructed.

This situation raises many more questions about the purpose of certain problems, and the reasons given for doing certain calculations. However, for the purposes here the strategy to be noted is that practical solutions are used in vocational classes that are different from those a mathematics teacher is likely to use. If this is not taken into account the numeracy / mathematics support classes are likely to be seen as irrelevant. The recognition of different sorts of mathematics in vocational and cultural contexts has been developed far more deeply, practically, pedagogically and theoretically under the heading of ‘ethnomathematics’, particularly in South America. (See for example Knijnik’s (2007) study of the mathematical practices in the Brazilian Landless Movement).

Theoretical inspiration

The contradictions and strategies I have been discussing arose out of my own practice in teaching adult numeracy, in teacher education and in professional development. This practice was informed by reflection on the feedback received from teachers and discussions with colleagues and also on a whole body of theories and research on teaching adults mathematics. In reflecting on my own practice, I realised that I was concerned that such reflection and evaluation should lead to change. This was associated with certain approaches to action research, such as that described in the Open University guide for action research:

The second approach has other attractions. As noted, it draws upon Schön’s (1983; 1987) ideas of ‘the reflective practitioner’ and ‘reflection-on-action’: the active and critical consideration and reflection by us, as practitioners, on such aspects as the motives behind and the consequences of our professional practice. This is achieved through a process of action-reflection-action and is what permits us as teachers to analyse our practice, both for ourselves and for others, and thus to change and develop. (OU, 2005, p.24).

The next section provides a summary of my thoughts about the contradictions and strategies in teaching adults under three headings:

• Different ways of thinking about a problem . . . and solutions
• A deeper understanding of how people calculate
• Considering how the brain manipulates numbers

These ideas have been inspired by the work of three very different researchers, whose work has helped to explain the contradictions and inspire the new strategies. The first is the theory of ‘Multiple intelligences’. Howard Gardner first published this in 1983 in Frames of Mind. Since then he has updated the theory by taking into account how others have used this theory and adding one more intelligence to the original seven (Gardner, 2006). Gardner’s theory, in its current form, identifies eight different sorts of intelligences. Two of these are linguistic and logical-mathematical, and in his
debate with the psychometricians (those who work with intelligence tests) he argued that the traditional tests primarily measured these two only.

The other intelligences that Gardner describes are musical, spatial, bodily kinaesthetic, interpersonal, intrapersonal and naturalist. What I found particularly helpful from this theory is that it provides a theoretical basis for recognising that people can be poor at some tasks but very good at others. Particularly they may have poor mathematical skills, or mathematics approached in a particular way, but have many other talents. If that is the case, then these talents can be used to build their numeracy experience, rather than continuing to focus most on the parts of mathematics they cannot do.

The second source is that of the researcher Liping Ma, in her study Knowing and Teaching Elementary Mathematics (Ma, 1999). The main focus of this study is to compare the mathematical knowledge and teaching practices of teachers trained in the USA and China. The examples she focuses on are very instructive, such as looking at how teachers understand the rules for dividing a fraction by a fraction. However, what I found particularly instructive was the section entitled: ‘Profound Understanding Of Fundamental Mathematics’ (Ma, 1999, pp118-124).

What this provides is an argument for having a deep understanding of the concepts that underpin the processes involved in basic calculating. This, once again, provides support for the development of alternative strategies. With this ‘profound understanding of fundamental mathematics,’ a teacher would be easily able to adapt a calculating process to suit a particular student, and would have the personal skills to evaluate a different method used by a student. Without such understanding, the teacher is left with only the method they have learnt, which they may be able to perform by rote, but cannot be explain or deviate from.

The third source is the work of Stanislas Dehaene (1999). His ideas are summarised in the book, The Number Sense which is sub-titled ‘how the mind creates mathematics’. There are three things that give me inspiration from this book. The first is the introduction the author presents to the neuroscientific approach to understanding mathematics. He introduces the reader to studies of the brain, which show where, and possibly how, numbers and quantities are manipulated. Much of the work of the neuroscientists Dahaene showcased has been to work with patients who have lost specific number skills, after an illness or an accident, and identify which parts of the brain have been damaged. The second is the introduction to the concept of ‘subitizing’. This is described as a particular ability that enables one, two or three (and possibly four) objects to be recognised and distinguished without one to one counting. It is used to show there is a number sense in very young babies and animals and to support arguments for some aspects of understanding numbers as being innate.

The third is introducing the term ‘numerosity’. This is the attribute of a group of things that gives it countable quantity. It is recognising amounts. This I found useful in discussing at a fundamental level what we mean when we talk about ‘a number’ or ‘numbers’. The word ‘numbers’ has so many meanings that having a specific word which refers to the concept of ‘amounts’ rather than how a number is written or said can help clarify thinking and from that, how number concepts are explained and demonstrated.

Finally there is one more source that needs to be noted. I have spoken briefly about the importance of collaborative work with colleagues. Over recent years my reflections and self-evaluation of staff development initiatives have been supported by discussions and joint work with my colleagues and by the initiatives in teacher training for adult numeracy specialists. This body of knowledge and experience can be found summarised in ‘Teaching in Adult Numeracy’ (Griffiths & Stone, 2013).
Conclusion

In this paper the experience of working with a wide range of adult numeracy professionals is reflected upon in order to identify the key changes to teaching strategies that were being promoted. The key changes to teaching strategies are recognised as being underpinned by three diverse theories: Howard Gardner’s ‘multiple intelligences’, (2006); Liping Ma’s ‘a profound understanding of fundamental mathematics’ (1999) and Stanislas Dehaene’s ‘how the mind creates mathematics’ (1999). The process has been seen to have similarities with the reflective practices associated with action research.

References


**Resources from the Excellence Gateway**

Functional mathematics: Standards: [http://www.excellencegateway.org.uk/node/20517](http://www.excellencegateway.org.uk/node/20517)

Teaching and learning support material [http://www.excellencegateway.org.uk/node/22188](http://www.excellencegateway.org.uk/node/22188)

LSIS Support Programme – Barking and Dagenham College Collaboration between functional skills specialists and vocational specialists

[http://repository.excellencegateway.org.uk/fedora/objects/eg:5390/datastreams/DOC/content](http://repository.excellencegateway.org.uk/fedora/objects/eg:5390/datastreams/DOC/content) accessed September 2013

Whole Organisation Approach to literacy, language and numeracy (LLN)

Numbers Talk – Words Count:
Language policy and adult numeracy education in Wales and New Zealand

Diana Coben
University of Waikato, New Zealand
<dccoben@waikato.ac.nz>

Barbara Miller-Reilly
University of Auckland, New Zealand
<brbara@math.auckland.ac.nz>

Abstract
In this paper we review and compare language policy in relation to adult numeracy education in Wales and New Zealand with respect to the Māori and Welsh languages in the latest stage of our international comparative study of adult numeracy education. While much has been written about the relationship between language and literacy, the relationship between language and numeracy - especially adult numeracy - has been less explored, especially from a policy perspective, despite evidence of the importance of language for learning. We seek to shed light on the policy context in which adult numeracy education is set in Wales and New Zealand with respect to these languages, viewed from a critical linguistic human rights perspective.

Key words: numbers, language policies, ethnomathematics

Introduction
In both Wales and New Zealand policies are in place to raise levels of adult numeracy. Both countries have also developed measures to revitalise the Welsh (Cymraeg or y Gymraeg) and Māori (Te Reo Māori) languages respectively and afford them some protection from the language spoken by the majority of the population: English; sizeable numbers of people in both countries speak other languages.

In this paper we seek to shed light on the policy context in which adult numeracy education is set in Wales and New Zealand with respect to these languages. We begin by setting out our argument, from a critical linguistic human rights perspective, on why language matters – or should matter - to adult numeracy educators. We then give a brief review of research on language in relation to mathematics education, and ethnomathematics before setting the scene with a brief language-focused description of the current legal contexts, demographics, principal policy drivers, strategies and policies on language, education and adult numeracy in Wales and New Zealand. Finally, we compare current policies and strategies with respect to language and adult numeracy education.

27 In this paper we use ‘Te Reo Māori’ or ‘Te Reo’ (literally: ‘the Māori language’ or ‘the language’) and ‘Welsh’, respectively, to refer to these languages.
Why should language matter to adult numeracy educators?

Language is important in relation to adult numeracy education because, in a world where, as Barwell, Barton and Setati (2007, p. 113) point out, “Multilingualism is no longer an extraordinary case”, the language in which mathematics (or numeracy) is learned and practised has a bearing on learning. They identify “three good reasons for focusing on multilingual issues in mathematics education” which we summarise as follows:

1. Increasing movement of populations across international borders.
2. Widespread demand for access to English internationally, whether or not this is desirable or promoted.
3. The rise of minority and indigenous peoples’ movements, usually incorporating a strong educational focus with immersion and bilingual settings on the agenda, as the means to political and economic emancipation and cultural renaissance. (Richard Barwell et al., 2007, p. 114)

We concur; Barwell, Barton and Setati’s third reason is particularly pertinent to this paper since Welsh and Te Reo Māori, the indigenous languages of Wales and New Zealand respectively, are the focus of this paper.

We start from the position that language rights are human rights. As François Grin states:

being a native Welsh speaker in Cardiff or a Māori speaker in Auckland (instead of a native speaker of English) cannot, in a liberal society, be construed as a failing for which one should have to atone through a lifetime of denial of one’s identity, culture and language. (Grin, 2005, p. 455)

Accordingly, we approach our exploration of language policy and adult numeracy education in Wales and New Zealand from a critical linguistic human rights perspective, echoing Stephen May’s argument for a more nuanced sociolinguistic and wider socio-political approach to the issues of language, inequality and social justice with which minority language rights are centrally concerned (May, 2005, p. 339). While a full exposition of our perspective is outside the scope of this paper, we are broadly aligned with May’s and Grin’s positions in their works cited here and mindful of the United Nations’ Declaration on the Rights of Indigenous Peoples (United Nations, 2007) to which both the United Kingdom and New Zealand are committed. We characterise our perspective as ‘critical linguistic human rights’ because a linguistic human rights approach per se “is a necessary but far from sufficient argument for advocating the protection and promotion of minority languages and/or of linguistic diversity” (Grin, 2005, p. 458). Ultimately we are interested in the practical and pedagogical implications of language policy for adult numeracy teaching and learning in situations of linguistic diversity and a critical approach allows us to consider these.

By ‘language policy’ we mean:

a systematic, rational, theory-based effort at the societal level to modify the linguistic environment with a view to increasing aggregate welfare. It is typically conducted by official bodies or their surrogates and aimed at part or all of the population living under their jurisdiction. (Grin, 1996, p. 31)

We turn next to look at research on language in relation to mathematics education, including work to explicate the ‘mathematics register’ (Meaney, 2006), and ethnomathematics, in order to outline the ‘state of play’ in these fields as this may be relevant to our study.
Research on language and ethnomathematics in relation to mathematics education

The role of language in mathematics learning

There is growing recognition from around the world that language (and bilingualism/multilingualism) plays a key role in mathematics teaching and learning (R. Barwell, 2003).

Research on Gaeilgeoirí learners (students who learn through the medium of Irish) in transition from learning mathematics through Gaeilge (Irish) to learning it through English at the third level “highlights that mathematical understanding is influenced by language and […] the students’ cultural background and experiences” (Ni Riordáin & O’Donoghue, 2008, p. 248).

Findings from a larger study of the Irish context by Ni Riordáin & O’Donoghue are consistent with those found in bilingual contexts in other countries:

- Studies in these contexts [also] found that students learning through the medium of English (their second language of learning) experienced problems with syntax, semantics, and mathematics vocabulary in the English mathematics register, with language playing a key role in their mathematical performance. (Ni Riordáin & O’Donoghue, 2011, p. 62)

A study of EAL students studying mathematics in English-medium classes at secondary school and university in New Zealand found that:

- EAL students suffer a disadvantage in mathematics learning due to language difficulties. […] Prepositions and word order were key features causing problems at all levels. So also were logical structures such as implication, conditionals, and negation, both at senior secondary and third year university levels. (Neville-Barton & Barton, 2005, pp. 13-14)

- Similarly, a study of English-Chinese language differences by Galligan (2001, p. 112) found “large differences in orthography, syntax/semantics, and phonetics” which “may have consequences in the processing of mathematical text”. Ni Riordáin (2013, p. 6) has also examined the differences between the Irish and English languages and points out the difficulties of how to interpret “whether differences between the languages have a differential impact on cognitive processing”.

The mathematics register

Yore, Pimm and Tuan (2007, p. 599) discuss the “importance of general cognitive and metacognitive abilities […] and discipline-specific language” for both scientific literacy and mathematical literacy. They emphasize the importance of “not overlook[ing] or underemphasiz[ing] the fundamental literacy component of mathematical and scientific literacy for all students”. This ‘fundamental literacy component’ for mathematics, the ‘mathematics register’ of a language, “includes both the terminology and grammatical constructions which occur repeatedly when discussing mathematics” (Meaney, 2006, p. 39). Yore, Pimm and Tuan (2007, pp. 565-566) stress that “natural language is only a starting point toward acquiring the disciplinary discourses or languages of mathematics and science”, that it is essential to engage with the “three-language problems of moving from home language to school language and onto scientific and mathematical language” to become “mathematical and scientific literate”. For English as an additional language (EAL) students there is the complication of a further language.

In New Zealand, the development of a Māori mathematics register (Tikanga reo tātai) has been described by Barton, Fairhall and Trinick (1997, p. 3). They describe a “self-conscious process of mathematics discourse production […] highlight[ing] both the complexity and the multiple directions of language evolution” (Barton et al., 1997, p. 8).
Ethnomathematics

The ‘father of ethnomathematics’, Ubiratan D’Ambrosio, has defined ethnomathematics as “the maths practised among cultural groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on” (Ubiratan D'Ambrosio, 1985, p. 45) and later as “the arts or techniques developed by different cultures to explain, to understand, to cope with their environment” (Ubiratan D'Ambrosio, 1992, p. 1184).

The inclusion of ethnomathematical perspectives into the mathematics education of indigenous students is often described as being politically essential and culturally, linguistically and pedagogically beneficial (see, for example, Powell & Frankenstein, 1997). A New Zealand study is pertinent here: Colleen McMurchy-Pilkington explored Māori women’s mathematical thinking in marae28 kitchens. She explored the “complex mathematical thinking and reasoning skills” that Māori women have constructed outside of school and “embedded in their everyday cultural practices” (McMurchy-Pilkington, 1995, p. 21). She argued that despite policies that have contrived to alienate Māori girls from mathematics over 150 years, Māori women have been a driving force behind Māori educational initiatives (McMurchy-Pilkington, 1995, p. 7).

More recently, the Māori Adult Learners (Te Pakeke Hei Ākonga30) project aimed to capture the perspectives of learners, tutors and providers on how language, literacy and numeracy in foundation learning programmes can be optimised for adult Māori learners. It was found that the “PTEs30 and iwi31 providers expect to deliver more than literacy and numeracy skills; they aim to celebrate the Māori identity of their learners and usually teach Māori tikanga32 and sometimes Māori language as well” (McMurchy-Pilkington, 2009 p.1).

The findings with regard to adult numeracy education and Te Reo Māori are encouraging:

The Māori learners felt their tutors were teaching them and their needs rather than a set curriculum. This was in contrast to their school days. They acknowledged that they were learning more than numeracy and literacy. They were learning social skills (how to get along with other people), survival skills, how to study more effectively, cultural skills and knowledge […] work employment skills, self-confidence, te reo Māori (in some instances), self-respect and respect for others. Their learning was more interactive, it related to everyday life, and in maths it was more hands-on. Their tutors explained and clarified things and made learning fun. (McMurchy-Pilkington 2009, p.2)

However, ethnomathematical approaches alone may not be enough. As Meaney, Fairhall and Trinick (2008) found, cultural practices, including ethnomathematical ones, cannot be separated from the language in which they were developed. Hence, changing the language or the linguistic register in which practices are discussed will have an impact on how the practices are perceived by students and could result in a loss in the fundamental values that would normally accompany the practices. They conclude that without proper consideration of this issue many of the benefits associated with using ethnomathematical approaches may be nullified.

We turn now to consider the demographic and policy contexts in Wales and New Zealand in order to situate our discussion of language policy in these countries in relation to adult numeracy education.

28 Marae: an area of land where the Wharenui (meeting house) sits – the community focal point.
29 https://akoaotearoa.ac.nz/community/recommended-resources-ako-aotearoa/resources/pages/te-paceke-hei-%C4%81konga-m%C4%81ori-adult-learne
30 PTEs are Private Training Establishments.
31 Iwi: the largest everyday social units in Māori society, analogous to that of tribe or clan. Māori who know their iwi connections typically value them highly and take pride in knowing their genealogy.
32 Tikanga Māori refers to general behaviour guidelines for daily life and interaction in Māori culture http://www.korero.maori.nz/forlearners/protocols
Wales

Status of the Welsh language in Wales

Welsh is the oldest language in Britain, dating back possibly 4000 years. Early in the ninth century, when Wales enjoyed a cultural and political autonomy that lasted until the Norman invasions in 1066, the Latin alphabet was adapted for writing Welsh and Welsh literature emerged. In more recent centuries there has been a long history of the suppression of the Welsh language in Wales in favour of English. Following the Laws in Wales Acts of 1535 to 1542, Wales was treated as part of England; English was the only language of the courts and all use of Welsh was banned from public office.

Literacy began to spread throughout Wales in the 18th century, greatly aided by the circulating schools established by the preacher Griffith Jones. He devised an efficient three-month system to teach children and adults to read in their mother tongue using religious texts. The language taught was usually Welsh, although English was used in areas with an English-speaking majority. Almost half the population had attended these schools by the time Jones died in 1771 and Wales became one of the few European countries to have a literate majority.

While there were very few Welsh national institutions in 1850, in 1896 the Central Welsh Board was established to inspect grammar schools and a separate Welsh Department of the Board of Education was established in 1907. Although speaking Welsh in schools was not illegal, it had no government support and was actively discouraged; the medium of instruction was English and pupils could be punished for speaking Welsh until as late as the 1930s. Nevertheless, the BBC’s Welsh service began introducing some Welsh-language radio programming in 1937; Welsh was allowed in the courts from 1942; and Welsh-medium education was authorised in 1944. The Welsh Language Act 1967 legitimised the use of Welsh in the courts and in 1988 the Welsh Language Board was established to advise the British Secretary of State for Wales on language issues. From the 1990s, legislation required equal treatment of the Welsh and English languages in Wales: The Welsh Language Act 1993; The Government of Wales Act 1998; The Welsh Language Measure 2011; and The National Assembly for Wales (Official Languages) Act 2012. Under this legislation: public bodies prepare and implement a Welsh Language Scheme; local councils and the Welsh Government use Welsh as an official language, with official literature in Welsh and English; and the Welsh language is visible in public. Welsh language media are important instruments of language revitalisation; these comprise: a Welsh language TV station (S4C) and a radio station: BBC Radio Cymru; a weekly national paper, magazines and regional monthly papers. This is important because the availability of broadcast media, especially television, in the minority language strengthens other language revitalisation measures, as Grin and Vaillancourt (1998, p. 114) point out.

Through a process of devolution within the UK, the Government of Wales Act 1998 created the National Assembly for Wales which determined how the UK government’s budget for Wales is spent and administered. Education is a devolved power under this legislation. More recently, the Government of Wales Act 2006 created the Welsh Assembly Government (the executive), separate from the National Assembly for Wales (the legislature).

Welsh language speakers

Wales has a diverse population with 78 languages spoken in 2006. In 2011, 19 percent of usual residents aged 3 or more could speak Welsh, a reduction from 21 percent in 2001, despite an increase in the total population (3.06 million in 2011). The population with no Welsh language skills increased

33 http://www.bbc.co.uk/wales/history/sites/themes/language.shtml
34 http://www.bbc.co.uk/wales/history/sites/themes/society/language_literacy.shtml
between 2001 and 2011. Welsh is now a minority language in the Welsh-language heartlands of Carmarthenshire and Ceredigion and the proportion of Welsh speakers has also dropped in Gwynedd and Anglesey, mainly due to inward-migration by non-Welsh speakers (in 2011 26 percent of the population was born outside Wales). However, there are more Welsh speakers aged 3 to 14 and 20 to 44 years but fewer in other age groups\(^{36}\) (ONS, 2013).

The Welsh language in schools

The Welsh Assembly’s strategy, The Learning Country, set out a ten-year vision and associated actions to transform education and lifelong learning in post-devolution Wales (Welsh Assembly, 2001). Welsh-medium schools were included in the strategy and the government’s Welsh-Medium Education Strategy was published in 2010 with a vision to have an education and training system that responds in a planned way to the growing demand for Welsh-medium education (Welsh Assembly Government, 2010, p. 4). In English-medium schools Welsh is taught as a second or additional language, compulsory to age 16 in alignment with the Welsh Second Language Action Plan (Welsh Government, 2012).

The Welsh Assembly Government’s literacy and numeracy strategy: *Words Talk – Numbers Count*

Post-devolution, the Welsh Government’s commitment to raising standards in literacy and numeracy has been expressed in a series of reports with an emphasis on basic skills in work. These include Skills that Work for Wales (Welsh Assembly Government, 2008), the first annual report of the Wales Employment and Skills Board (2009) and the Employer Pledge\(^{37}\), awarded to employers who support staff with essential skills needs. The government’s second Basic Skills Strategy, Words Talk – Numbers Count (National Assembly for Wales, 2005), covers 2005 to 2010 and takes forward the agenda of The Learning Country (Welsh Assembly, 2001). Words Talk – Numbers Count covers all ages, and includes the aim that “the number of adults with poor basic skills should be diminished significantly” (National Assembly for Wales, 2005, p. 6). From 2007 the National Assembly for Wales (since 2011, the Welsh Government) has promoted Wales as a “bilingual and multicultural nation” but “there has been less of a strategic emphasis on the development of basic skills in the medium of Welsh” (Miller & Lewis, 2011, p. 6).

Adult numeracy in Wales

Adult numeracy in Wales has been measured in a series of national and international surveys in recent years. All four countries of the UK took part in the second round of the International Adult Literacy Survey (IALS) in 1996; England, Scotland and Wales took part as one jurisdiction (Carey, Low, & Hansbro, 1997) and Northern Ireland was assessed separately (Sweeney, Morgan, & Donnelly, 1998)\(^{38}\). The UK did not take part in the successor to IALS, the Adult Literacy and Lifeskills (ALL) survey. Instead, the National Survey of Adult Skills in Wales 2010 (Miller & Lewis, 2011) assessed literacy and numeracy skills of adults aged 16 to 65\(^{39}\) through the medium of English and the literacy


\(^{38}\) IALS surveyed ‘quantitative literacy’ rather than ‘numeracy’.

\(^{39}\) The survey used the latest available mid-year population estimate from the Office for National Statistics at the point the survey was carried out in 2009 and was 1,927,000 (Llywodraeth Cymru/Welsh Government, personal communication, 7 February, 2014).
(but not numeracy) skills of Welsh-speaking adults through the medium of Welsh. The 2010 Survey replicated similar surveys undertaken in 2004 (BMRB, 2004; BSA, 2004).

The 2010 survey found that progress on numeracy was slower than literacy, with 50 percent of respondents assessed at Level 1 or above, three percentage points higher than in 2004 but five percentage points below the Strategy target of at least 55 percent to achieve at least Level 1 numeracy by 2010. Assessment results for Welsh literacy declined between 2004 and 2010, from 67 to 63 percent at Level 1. The report notes that “The gap between literacy and numeracy assessment levels also widened, so that four times as many people were classified at Entry Level for numeracy than for literacy in 2010 and some 918,000 working age adults across Wales were estimated to have numeracy skills below Level 1” (Miller & Lewis, 2011, p. 9). Numeracy levels were higher amongst the employed and those with greater household income, higher qualifications and amongst the older age groups. The survey also revealed a substantial gender gap in numeracy: 60 percent of women and 41 percent of men were assessed at Entry level; and 29 percent of men were assessed at Level 2 or above, as compared to 13 percent of women (Miller & Lewis, 2011, p. 10).

England and Northern Ireland (but not Wales or Scotland) have taken part in the successor to the ALL survey, the Programme for the International Assessment of Adult Competencies (PIAAC) (OECD, 2013).

**Aotearoa** New Zealand

The precise date of the first human settlement in New Zealand is debated, but current understanding is that Māori arrived in the 13th century. From 1642, when Abel Tasman named the country New Zealand, Māori had increasing contact with European and other seafarers and then with settlers, the latter mainly from the British Isles. The Treaty of Waitangi⁴¹ (1840) is the nation’s founding document, setting out the terms of a political compact between Māori and the British Crown. Following the Treaty, from 1841 to 1907 New Zealand was termed a British colony, then, from 1907, a Dominion of the British Empire and in 1931 became a founder-member of the British Commonwealth⁴². In 1986 residual British legislative powers ended and New Zealand became formally (rather than just de facto) self-governing. Until the ‘Māori Renaissance’ of the 1980s, New Zealand government policies favoured the Pākehā⁴³ majority. Since then biculturalism has been emphasised, based, since 1992, on Treaty of Waitangi claims and settlements.

**Languages spoken in New Zealand**

New Zealand has three official languages: English; Te Reo Māori; and New Zealand Sign Language. New Zealand is one of the world’s most super-diverse societies, with 160 languages spoken (Spoonley & Bedford, 2012). Despite its linguistic diversity, 76.6 percent of New Zealanders continue to be monolingual in English (NZHRC, 2008).

**A proposed national languages policy for New Zealand**

2013 marked the 21st anniversary of the publication of the Ministry of Education report Aotearoa: Speaking for ourselves (Waite, 1992), widely understood to be a precursor to a national languages policy (East, Shackleford, & Spence, 2007; Holmes, 1997; Spence, 2004). However, while various

---

⁴⁰ Aotearoa is the Māori name for New Zealand; in this paper we use the country’s official name: New Zealand.

⁴¹ http://www.nzhistory.net.nz/politics/treaty-of-waitangi

⁴² In 1949 the British Commonwealth was re-named the Commonwealth of Nations.

⁴³ Pākehā refers to non-Māori New Zealanders, primarily those of European descent.
language initiatives (outlined below) have emerged, so far there is no such policy. Meanwhile, independently of government, the New Zealand Human Rights Commission (NZHRC, 2010) has proposed a national languages policy, recently summarised in priorities for action (quoted in RSNZ, 2013, p.3).

Te Reo Māori in New Zealand

Use of Te Reo Māori in New Zealand was actively suppressed and discouraged in favour of English until the later half of the 20th century (NZHRC, 2012, p. 3). The Māori Language Act 1987 brought Te Reo Māori into the public policy domain in New Zealand and established the Māori Language Commission to promote and support the growth of the Māori language. A 1991 Amendment to the Act expanded the range of legal settings in which Te Reo could be used. As in Wales, the broadcast media are important instruments of language revitalization. There are two TV channels (Māori Television and Te Reo) and there are Māori language programmes on mainstream TV and Māori radio stations. The promotion of Te Reo has resulted in the public acceptance of the use of some Māori language and protocol as a part of New Zealand culture. For example, by 2011 most government agencies had Māori and English names and traditional Māori welcome and farewell ceremonies are often performed at official functions.

Numbers of Te Reo Māori speakers in New Zealand

The most recent New Zealand Census for which figures are available (2006) showed a total population of 4,027,947, with Māori at 15.4 percent. Māori were counted in two ways: by ‘ethnicity’ (i.e., cultural affiliation: 15 percent) and ‘descent’ (i.e., ancestry: 18 percent)44. The figures for those of Māori descent are rising: in 2006 they were up 26 percent from 1991. The Māori population has a predominantly young demographic profile: in 2006, 35 percent of people of Māori descent were under 15 years old, while only 4 percent were 65 or over (Statistics New Zealand, 2007b). The Census shows that Te Reo is spoken by 4 percent of the New Zealand population and English is spoken by 96 percent; all adult Māori speakers can also speak English. Amongst the Māori population, 25 percent of those aged 15 to 64 and 49 percent of those aged 65 or over could hold a conversation in Te Reo Māori. However, there is a decline in the use of the language by those aged under 15: of these, 17 percent could hold a conversation in Te Reo Māori in 2006, compared with 20 percent in 2001 (Statistics New Zealand, 2007a).

However, the Survey on the Health of the Māori Language in 2006 (Research New Zealand, 2007) shows significant increases in Māori adults’ proficiency in Te Reo. Speakers of the language are up 8 percent, readers of it up 10 percent, writers up 11 percent and the numbers of those who can listen with understanding are up 9 percent, compared with the 2001 survey (Statistics New Zealand, 2002). This increase comes alongside progress in re-establishing intergenerational language transmission, with more Māori adults speaking Te Reo Māori to children in their homes and in community domains. The greatest increases have been recorded in the higher proficiency levels, which have more than doubled in the 15-24 and 25-34 year age groups. More young people now have some degree of speaking proficiency, with increases of 13 percent, 16 percent and 10 percent across the 15-24, 25-34 and 35-44 year age groups respectively (Research New Zealand, 2007).

44 Intermarriage between Māori and non-Māori in New Zealand has been commonplace since early colonial times.
Te Reo Māori in schools

By the 1830s Māori were attending mission schools in large numbers and becoming literate in English and Māori. By the early 1860s Pākehā were in the majority and English became the dominant language. Te Reo became confined to Māori communities and was suppressed in schools (including government-funded Native Schools), either formally or informally, in the name of assimilation with the wider community.

Since the mid-1980s the Māori Renaissance has led to the emergence of different levels of Māori language and cultural immersion in education: ‘Māori-medium’ schools; ‘Māori language in English medium’ schools; and ‘no Māori language in education’. ‘Māori-medium’ schooling comprises kōhanga reo (pre-school); kura kaupapa Māori (primary school); and wharekura (secondary school) wherein students are taught the curriculum in the Māori language for at least 51 percent of the time. In ‘Māori language in English-medium’ schools students learn Māori as an additional language, or are taught the curriculum in the Māori language for up to 50 percent of the time. Thirdly, ‘no Māori language in education’ ranges from an introduction to the Māori language via Taha Māori to no involvement in Māori language education at any level. In addition, whare wānanga provide Māori Tertiary options and most of the major universities and technical institutes teach Te Reo Māori as a language subject. In principle, parents can send their children either to an English-medium school or to a Māori bilingual or immersion school but in practice many parents have no local access to Māori-medium programmes.

Māori students’ outcomes of schooling in Mathematics

The New Zealand Ministry of Education’s Best Evidence Synthesis (BES) for mathematics, Effective Pedagogy in Pāngarau/Mathematics, paints a disturbing picture of low achievement in mathematics by Māori students, with one in three leaving school without any formal qualifications: “the harsh reality is that average achievement, as shown in PISA and other mathematics assessments (e.g., the National Education Monitoring Project), is lower for Māori and Pasifika”. (Anthony & Walshaw, 2007, p. 9).

Recent efforts to improve this situation with respect to Māori include Te Poutama Tau project (Trinick & Keegan, 2007) which aimed: to develop the discipline to support Māori-medium mathematics; to raise Māori student achievement in mathematics by improving the professional capability of teachers; and to continue the revitalisation of Te Reo Māori. The authors found that 11 year-old pupils performed significantly above the national norms for Māori-medium schools (Trinick & Keegan, 2007, p. 20).

Adult numeracy, literacy and language in New Zealand

In the ALL survey New Zealand adults consistently scored average on all four scales of the survey. The main variation in the literacy and numeracy skills of 25 to 65 year olds was due to three key factors: completed education; language background; and computer use. People whose first language was English had a considerable advantage, especially in English literacy, but also in numeracy tested

---

45 Missionaries first transcribed the Māori language in 1814 and the written language was systematised by 1820.
46 Taha Māori means ‘the Māori perspective’, in this context, this entails the use in teaching of simple Māori words, greetings or songs.
47 Whare wānanga: ‘house of learning’ or ‘house of teaching’ (referring to higher learning or teaching).
49 Pasifika refers to people of Pacific Nations heritage.
50 The ALL survey measured prose literacy, document literacy, numeracy and problem solving.
in English. Non-first language speakers of English were less disadvantaged if their main home language was English (Lane, 2010, p. 1). Interestingly, when ethnic identification was examined as a secondary factor for English first-language speakers, “an advantage for Europeans and a disadvantage for Māori and Pasifika, especially in numeracy, [was found] within this language subgroup” (Lane, 2010, p. 4). New Zealand is taking part in the second round of PIAAC, the successor survey to ALL, with results due in 2016.

New Zealand’s strategies, policies and goals with regard to Te Reo Māori and Māori students

New Zealand has instituted various strategies, policies and goals with regard to Māori students and Te Reo Māori; these are outlined below.

The Māori Language Strategy 2003

The Māori Language Strategy 2003 set out five goals to be achieved by 2028: the majority of Māori will be able to speak Māori to some extent and proficiency levels in speaking, listening to, reading and writing Māori will increase; Māori language use will be increased at marae, within Māori households and other targeted domains; all Māori and other New Zealanders will have enhanced access to high-quality Māori language education; iwi, hapū51 and local communities will be the leading parties in ensuring local-level language revitalisation; iwi dialects of the Māori language will be supported; the Māori language will be valued by all New Zealanders and there will be a common awareness of the need to protect the language (Ministry of Māori Development, 2003). In 2011 Te Pūnini Kōkiri, the successor to the Ministry of Māori Development, published a Review of the Māori Language Sector and the Māori Language Strategy (Te Pūnini Kōkiri, 2011). A proposed national Māori Language Strategy (Te Puni Kōkiri, 2013) was released in December 2013 with public consultation in February 2014. The Strategy proposes strengthening the focus on whānau52, hapū and iwi and consolidating Māori leadership.

Ka Hikitia

Ka Hikitia is the New Zealand Ministry of Education’s (2008, 2013) strategy for Māori achieving educational success as Māori, currently being refreshed for the period to 2017. In the first stage of the Ka Hikitia strategy, from 2008 to 2012, Māori language in education was identified as one of four areas in which coordinated activity would have most impact (Ministry of Education, 2008, p. 24).

Adult literacy, language, and numeracy education policy in New Zealand

There has been a strong development of adult literacy, language, and numeracy education policy in ‘foundation learning’ in New Zealand, consistently in literacy, although numeracy and language appear and disappear in official policy documents. For example, More Than Words, the policy document that launched the New Zealand Adult Literacy Strategy in 2001, is silent on numeracy but does include “the opportunity to achieve literacy in English and Te Reo Māori” (Walker et al., 2001, p. 3).

51 Hapū: A named division of a Māori iwi (tribe) consisting of a number of whānau (extended families); membership is determined by genealogical descent.

52 Whānau: extended family.
The subsequent *Literacy, Language and Numeracy Action Plan 2008–2012* (TEC, 2008) includes both language and numeracy, as the title suggests. However, Te Reo Māori is not mentioned and language is subsumed within literacy (TEC, 2008, p. 7).

Two years later, *Getting Results in Literacy and Numeracy* (TEC, 2010) provided an update on what had been achieved since the Action Plan and outlined the next steps for the tertiary education sector. The main focus is on literacy and numeracy, not language.

The title of the most recent document associated with New Zealand’s adult literacy and numeracy policy, the *Adult Literacy and Numeracy Implementation Strategy* (TEC, 2012), reflects the focus on literacy and numeracy but once more language is not included. Te Reo Māori is mentioned, but only weakly, in the New Zealand adult Learning Progressions in listening, speaking, reading and writing in New Zealand English (introduced in 2005 and a key element in the New Zealand adult literacy and numeracy infrastructure).

The current Tertiary Education Strategy 2010–2015 (TES) specifically includes Te Reo Māori: “tertiary education plays a major part in promoting the revitalisation of te reo Māori” (New Zealand Office of the Minister for Tertiary Education, 2010, p. 7). At the same time, TES Priority 4 is “improving the literacy, language and numeracy and skills outcomes from levels one to three study” (New Zealand Office of the Minister for Tertiary Education, 2010, p. 10) and the Tertiary Education Commission’s (TEC’s) priority groups are Māori, Pasifika and Youth learners.

The TEC’s operational approach and implementation of the government’s Draft Framework for Māori Learners, *Tū Māia e te Ākonga* 2013-2016, is informed by the *Tū Māia Working Group*. This Group advises the TEC and shares best practice in the tertiary sector to ensure that Māori enjoy success as Māori in tertiary education, particularly at higher levels.

### Te Reo Māori in adult numeracy education practice and provision in New Zealand

Foundation programmes in tertiary education are provided by universities, polytechnics, whare wānanga, private training establishments (PTEs) and iwi providers. There is targeted government funding for ‘embedded’ (i.e., integrated) literacy and numeracy provision at Levels 1 to 3 of the National Qualifications Framework but, as noted above, Te Reo Māori barely features.

Against this background, the Māori Adult Learners (Te Pakeke Hei Ākonga) project aimed to capture the perspectives of learners, tutors and providers on how language, literacy and numeracy in foundation learning programmes can be optimised for adult Māori learners (McMurchy-Pilkington, 2009). Findings from this project are discussed earlier in this paper in the Ethnomathematics section.

### Comparing New Zealand and Wales

As we have seen, New Zealand and Wales have similar-sized populations in which the majority of people speak English, a minority speak Te Reo Māori or Welsh (albeit a larger minority in Wales than in New Zealand) and adult members of these linguistic minorities generally also speak English. Both countries have a history of the suppression of their native languages by successive British governments, followed by relatively recent attempts at language revitalisation, including through education, with varying but limited success to date.

In both countries the native language is a powerful symbol of a resurgent national, cultural and/or ethnic identity. The main policy driver in this respect in Wales is devolution, with the Welsh language

---

53 http://www.literacyandnumeracyforadults.com/resources/354991
an important outward and visible sign of increased independence within the UK. In New Zealand, the main policy driver is biculturalism (Māori and Pākehā), with the revitalisation of Te Reo Māori strongly bound up with issues of ethnicity and culture, played out against the backdrop of post-colonial redress for Māori through Treaty of Waitangi settlements. This is happening at the same time as New Zealand is becoming increasingly linguistically (and ethnically and culturally) diverse.

Both countries are liberal democracies, currently governed by coalitions of political parties. Rata (2011) warns that institutionalising ethnicity in New Zealand is leading to the re-racialisation of society, thereby subverting the claim to universalism upon which liberal democracies are based. It is notable that debates around language and culture in Wales have largely abjured issues of ethnicity (although perhaps language is a proxy for ethnicity for some) and there is no equivalent in Wales to the emphasis in New Zealand on Māori kaupapa and tikanga (Māori world view and protocols).

At the level of policy, Wales’ language policy is enshrined in law, while New Zealand has no formal language policy. Nonetheless, the revitalisation of the minority language is a live issue in both countries. In both countries, also, education in the native language is available as an additional language and (though to varying extents) Welsh-medium and Māori-medium education are available in compulsory schooling.

From the late twentieth century there is also evidence of similar levels of numeracy amongst adults of working age, similar concern by both governments to raise levels of numeracy and literacy for economic and social reasons and similar government responses in terms of the creation of an infrastructure to support adult literacy and numeracy learning and achievement, together with associated research and professional development. As a result, adult numeracy education in both New Zealand and Wales is widely available. However, in both countries it appears to be conducted almost exclusively through the medium of English. This requires some explanation, given the efforts to revitalise Welsh and Te Reo Māori outlined above and the “growing recognition that language (and bilingualism/multilingualism) plays a key role in mathematics teaching and learning” noted by Ní Riordáin and O'Donoghue (2011, p. 45). It may simply be that adult numeracy tutors are thin on the ground in both countries, with Welsh-speaking or Māori-speaking bilingual tutors especially rare. If so, this bespeaks the need for tutor training and professional development – in language and adult numeracy pedagogy - to fill the gap. However, that is unlikely to happen without a clear policy direction.

The absence of a robust link between language policy and adult numeracy education indicates the need for further policy development, alongside research and development focusing on language issues in relation to both existing adult numeracy learners and those outside provision; for some in either camp the language of instruction may be a barrier rather than a tool for learning.

A way forward? Policy evaluation from a critical linguistic human rights perspective

Since the adoption of the United Nations’ (1992) Declaration on the Rights of Persons Belonging to National or Ethnic, Religious and Linguistic Minorities, states must positively engage in the protection of minorities rather than just refrain from discrimination. Grin (2004) points out the need for fairness and presents a framework for evaluating the monetary and non-monetary costs and benefits of minority language policy, including important social non-monetary effects, from a critical linguistic human rights perspective. Beneficial social non-monetary effects include “harmonious community relations or the positive value placed on diversity in its own right” and “long term benefits such as building capability, competitiveness and reducing costs”, while social non-monetary costs include the unused skills of speakers of non-majority languages (Royal Society of New Zealand, 2013, pp. 3-4, citing Grin, 2004). While a full policy analysis along Grin’s lines is beyond the scope
of this paper, the findings of the Māori Adult Learners project, outlined above, indicate beneficial social non-monetary effects of adult literacy and numeracy education, with some inclusion of Te Reo, tikanga and kaupapa Māori and a whānau atmosphere, all of which were valued by Māori learners. It behoves us as educators not to squander the unused and undeveloped numeracy skills, knowledge and understanding of speakers of non-majority languages.

Finally, Stephen May (2005, 2011) looks forward to ways in which minority languages may be reconstituted as instrumentally useful, rather than simply as ‘carriers’ of identity. He hopes that minority language rights will continue to develop a more nuanced sociolinguistic and wider socio-political approach to the issues of language, inequality and social justice with which it is centrally concerned.

We echo May’s hope and share his and Grin’s critical linguistic human rights perspective. We believe May is right to emphasise the practical advantages of implementing minority language rights within nation-states (May, 2005, p. 327). In further work we plan to explore what these advantages (and any disadvantages) may be (and for whom) with respect to adult numeracy education and how these rights might be articulated within relevant policy frameworks in linguistically diverse societies. We see minority languages as something to celebrate rather than just accommodate in adult numeracy education. Language is too important for the language dimensions of adult numeracy learning to be ignored in policy, teaching and the organisation of provision. Numbers and words are both important in adult numeracy teaching and learning. To invert the title of the adult literacy and numeracy strategy in Wales: numbers talk and words count.

Acknowledgements

We acknowledge the support of our institutions in writing this paper but wish to make clear that the ideas contained herein are our own and do not represent the policy of either of our institutions.

We wish to thank the following people who discussed aspects of this paper with us, all of whom did so in a personal capacity, rather than as representatives of their institutions:

• Theresa Aucamp, Te Wānanga o Aotearoa, New Zealand
• Conny Huaki, Te Wānanga o Aotearoa, New Zealand
• Niki McCartney, National Centre of Literacy and Numeracy for Adults, University of Waikato, New Zealand
• Tamsin Meaney, Malmö University, Sweden
• Pania Te Maro, National Centre of Literacy and Numeracy for Adults and Te Whare Wānanga o Awanuiārangi, New Zealand
• Jenny Young-Loveridge, Faculty of Education, University of Waikato, New Zealand

We also extend our thanks to those who attended our presentation at the ALM20 conference for their insightful comments.

References


Abstract
This paper describes an action research project that investigated a range of activities to improve learners’ mathematical communication skills. It also gives details of a subsequent case study that illustrates how technology can provide a means of overcoming some of the difficulties learners and tutors face in communicating about numeracy, while developing confidence and enhancing skills in a rapidly changing learning environment.

Key words: accreditation, communication, confidence, evidence, and technology

Background
For many, especially older, adult learners, the experience of mathematics is that of rote learning and being asked to complete pages of ‘sums’ independently of one another. In recent years the shift in accreditation of numeracy skills towards communicating ideas and meaning has been an awkward one for both learners and assessors and Chinn suggests that ‘problems and confusions are both with the vocabulary of maths and with the language (semantics) of maths’ and that difficulties are a ‘consequence of the way word problems are written and constructed’ (Chinn, 2012).

Improving communication skills within the context of, and with specific application to, numeracy is a key part of adult numeracy learning and tutors of adult learners are faced with the challenge of providing evidence of learners’ descriptions and explanations of mathematical processes and outcomes for accreditation or other purposes. This paper describes an action research project that investigated a range of activities to improve learners’ mathematical communication skills. It also gives details of a subsequent case study that illustrates how technology can provide a means of overcoming some of the difficulties learners and tutors face in communicating about numeracy, while developing confidence and enhancing skills in a rapidly changing learning environment.
**Action Research**

Learners within Monmouthshire Adult & Community Education took part in an action research project between October 2012 and February 2013. The purpose of the project was to investigate ways of developing the communication skills of adult numeracy learners, with specific relevance to fulfilling the Essential Skills Wales (ESW) qualification requirements at Level 1 and 2 of ‘describing a practical problem or task’ and ‘describing and explaining the results of calculations’.

At the start of the action research, learners from two small adult numeracy classes assessed their own confidence in four key communication skills: identifying facts and ideas; choosing appropriate vocabulary; verbal presentation skills and writing skills. A range of interventions was trialled over a three-month period. These included: peer review of work completed by previous learners; use of writing scaffolding sheets and mini practice tasks. The learners as a group and, following the trial then discussed the interventions; they were required to repeat a self-assessment of confidence in these skill areas. (Fig. 1 shows an example of the results collated from the skill ‘choosing appropriate vocabulary’)

![Figure 1](image.png)

*Figure 1. Finding words to describe what I mean.*

Although the results of the action research showed that the interventions had developed communication skills overall, it was noted that it would be beneficial to investigate alternative methods of supporting and developing skills, which may be particularly important to learners with specific learning difficulties such as dyslexia.

Following the action research project and through discussion it became apparent that one form of intervention that had not already been explored was the use of technology. Although learners in both of the classes involved in the research have regular access to computers and word processing software and use these when writing about mathematics, the process of typing a word processed document is much the same as hand-writing onto a page with no inherent support for the organisation of the writing.
The availability of Apple iPads for use within numeracy classes in Monmouthshire Adult & Community Education led to the discovery of the application ‘Story Creator’, an education tool that allows iPad users to create simple e-books using video, photos, text and audio.

**Accreditation**

The question that could be asked by both teachers and learners is why, when learners are being assessed for their numeracy/mathematics skills, are they being asked to provide explanations of their choice of methods and results? The reason for this requirement is that in real life, mathematical thinking and processes do not begin and end with manipulation of numbers (Ball, 2007; Schoenfeld, 1992). If learners are to apply their learning then they need to be able to consider how they are going to address problems, where they will find the data necessary to solve problems, which computational methods should be employed and how to evaluate obtained results. If accreditation is to reflect a learner’s ability to use mathematics purposefully then this whole process needs to be assessed.

While computational expertise can be easily evidenced without the need for any words, explanations and evaluations often need to be verbalised. Assessment of these skills may take place in the classroom during discussions or by integration within other projects or activities but evidence that the learner has achieved these skills needs to be recorded in some way to ensure the rigour of outcomes. Traditionally the learner providing written accounts, accompanied by documentation of teacher assessment, records explanations and evaluations; however, the exclusive use of this method of evidence needs to be challenged with regard to its validity (Scottish Qualifications Authority, 2009). This is particularly so where the learners have little or no literacy, even though their cognitive skills mean that they are able to use numbers within everyday applications and may be developing appropriate evaluative skills. We need to make clear distinctions between assessment of skills and means of providing evidence.

Unless awarding organisations specify an assessment method it is up to the teacher/assessor to select how that is to be achieved. Internal quality processes need to be in place to ensure that the method chosen meets the usual assessment requirements of reliability, authenticity, validity etc. Where learners have literacy difficulties, it could be argued that the validity of the assessment may be compromised if learners are asked to record their ideas in writing. Are we assessing the learner’s ability to write or their ability to evaluate?

Endeavouring to minimise assessment issues which arise as a result of a learner’s ability (or inability) to write does not mean that communication skills should not be addressed in our mathematical teaching. This project does not aim to put writing skills to one side but to look at how the development of communication skills can be embedded within mathematical teaching.

In this project the accreditation chosen was a credit-based qualification (Agored Cymru, 2013). This type of qualification enables learners to target specific skills determined by their personal, academic and vocational needs and allows them to combine units of communication, maths and ICT within a qualification. Even though the units chosen address specific skills, these skills are not taught or used in isolation. In this instance the focus of the unit was on measure but it also required the learner to use the four rules of number, ratios, estimation, 3D shapes, knowledge and use of mathematical language, problem solving and interpreting the results of calculations. In addition to this the learners were developing their communication and ICT skills.
The Learners

The ‘Story Creator’ application was used with a group of three adult learners. Two of the learners, both female, had progressed to the class having previously attended local Family Learning provision and the third was a new learner, male, wishing to work towards achieving a level two numeracy qualification in order to access further education. All three learners were assessed as currently working at level one of the Essential Skills Wales standards (Essential Skills Wales, 2009). Both female learners had engaged in Family Learning as a means of improving their own skills whilst also being able to support their children. It was acknowledged early in the course that all three had recollections of negative mathematical experiences in school but that, for the two women, the Family Learning class had provided an opportunity to begin to change their perceptions of, and build confidence in, their own mathematical abilities.

Since this class commenced quite late in the academic year, it was considered more appropriate to offer a short numeracy course using Agored Cymru accreditation. This gave the learners the opportunity to demonstrate the skills required for measure within the context of a garden design project. The specific assessment criteria include ‘outline problems to be tackled’ and ‘present and explain the results of calculations using measurements’ giving the learners an opportunity to use their numeracy communication skills.

Issues surrounding the use of mobile technology for evidencing the assessment of skills

Whilst the education centre used for the study was in the fortunate position of having the use of iPads within the classroom, it did not have access to Wi-Fi which meant that learners were not able to download the graphics which could have further enhanced the presentation of their work. In this instance the problem was overcome by the learners taking photographs or drawing. In effect they were creating their own graphics.

Not all learners initially embraced the use of iPads, which were seen as another barrier to overcome within the learning process. The benefits of keeping up with new learning and leisure tools, particularly for those with children, were explored with the learners. Any anxieties were soon overcome as they became more adept with their iPads.

The experience of the teacher in using mobile technology was also a limiting factor. Story Creator may not be the most appropriate application and the time to trial others would be beneficial. For example, it would be desirable to use an application which included graphical backgrounds to facilitate multiplication using the lattice method or to aid the drawing of 3d shapes. In this instance, complex calculations were carried out on paper and photographed for inclusion in the Story Creator work. This was not ideal but did demonstrate the learners’ problem-solving skills in overcoming the issue. (The original paper based calculations were then submitted for accreditation alongside the e-portfolios.)

Maintaining copies of learners’ work also created a challenge, especially with no immediate Wi-Fi access. This was easily overcome by the tutor using standard internet storage devices once the lesson was complete. Another possible solution would be to use a tablet device that allows the attachment of USB storage devices.
Outcomes

Apple iPads with the Story Creator application were available for the class to use in the eighth session of a ten-week course and learners created e-books to record evidence of how they had calculated the cost of soil to fill their plant tubs and how much water they needed in order to fill a circular pond. Learners studied the mathematical skills required to complete the garden design task in the preceding weeks and, during these sessions, evidence for assessment criteria concerning computational skills was collected using non-ICT based methods and presented in a traditional paper based portfolio.

The learners had no experience with the application before the session and after an initial reluctance to try it, possibly caused by a lack of confidence in using new technology, they engaged with this method of learning and collecting evidence quickly and fully. Learners expressed a strong desire to complete their e-books regardless of how much additional time it would take outside of the usual session duration.

An unexpected outcome of using the e-books as a means of recording evidence was the way in which it supported learners to organise their thoughts and calculations. Since there is a limited amount of text space available (a maximum of one line), and only one photograph or diagram per page, learners were encouraged to consider the order in which they tackle the stages of a practical numerical problem. All three learners, one of whom has been formally assessed as having dyslexia, were able to organise their e-evidence in such a way as to introduce the problem, explain the information to be collected and used, describe the calculations step-by-step and both describe and explained their results. This was in contrast to the results of the action research project, where several learners still had immense difficulty organising their numerical problems even after the interventions. Additional research using this or a similar application is needed in order to test this hypothesis. This would be of particular value to learners for whom organisation of ideas is a problem.

One of the original aims of using a multi-media storybook was to alleviate anxiety around writing down ideas and to encourage learners to make audio recordings as an alternative. In fact, all three learners refused to use the audio recording tool, feeling too self-conscious to do so and preferring to revert to writing. However the quality of the text produced was high and the structure of the e-book, allowing only one line of typed text per page, meant that learners were required to focus on the main topic of each stage of the calculation and to express this succinctly.

With the restriction in the amount of text allowed, learners were keen to fill each page with visual evidence to support their text. They did this by using photographs taken during the session and simple drawings (not to scale), which were labelled to show the measurements they needed and had taken. The photographs included learners taking measurements, showing measurements on measuring instruments, screen shots of web information, idea boards from group discussions and shots of written calculations. The combination of visual and text based evidence appeared to give the learners confidence in what they were describing and explaining.

During the final session the e-evidence books were shared with the rest of the class and learners were given an opportunity to discuss their experiences using the iPads as a tool within the classroom. All agreed that it had been a more fun way of producing evidence than simply writing on paper and that they had enjoyed the task more because of the availability of mobile technology and its multi-media approach. The global consensus was a sense of pride in the style, quality and completeness of what had been achieved in such as small space of time and a sense of excitement at the possibilities of using the tool again in learning and exploring its uses in other settings.

Since the main aim of the e-books is to provide an alternate method of presenting evidence for accreditation, it has been imperative that the e-evidence created by the learners undergo standard Agored Cymru quality processes (Agored Cymru - Quality Assurance). Reactions from internal quality assessors have been mixed – in fact there appeared to be some reluctance to internally verify
the assessment of this work. This was at odds with the excitement shown by the learners, and teacher, at the opportunity to explore and use new technology. The lack of traditional paper evidence appeared to unnerve some internal verifiers although once they were shown how to access the e-evidence they were reassured that the learners had indeed fulfilled all the criteria.

The question of authenticity arose since it could not be proven purely from the e-evidence that it was the learner’s own work. It can be argued that this is the same for traditional paper based portfolios of evidence. Internal quality processes need to be in place to ensure that authenticity can be guaranteed whichever form of evidence is used. In fact, the e-book provides very personalised evidence of mathematical task organisation, implementation and evaluation. The use of photographs of written calculations meant that some assessors found it difficult to read, and therefore check, the calculations; however these could be validated by comparison with the accompanying paper based calculations.

Conclusion
Overall the experience of using mobile technology to record and present evidence has been a very positive one for the learners in terms of enjoyment of learning and development of organisation, communication and numeracy skills. In addition, it demonstrates that evidence-gathering methodology need not impede the assessment process and this innovative approach removed the often perceived toil of portfolio building. There are still issues that need to be addressed; most notably the engagement of those internally verifying the assessment process and it is considered that they would benefit from observing the use of such technology within the classroom in order to fulfil the complete quality assurance process and to build their own confidence in alternative methods of evidencing skills.

References
Implications of social practice theory for the development of a numeracy programme for the Gusilay people group in Senegal

Elisabeth Gerger
SIL, Dakar, Senegal
<elisabeth_gerger@sil.org>

Abstract

In this article, I present research on some traditional numeracy practices of the Gusilay people group in Senegal and make recommendations for developing a numeracy programme for women. Based on a strong foundation of traditional knowledge and practices, the programme will aim to meet felt needs of women who are faced with new numeracy related challenges due to changes in society. My research is placed in the framework of social practice theory, which emphasizes the fact that numeracy is not a set of skills that are learned and used in isolation, but rather practices that happen in context and vary with it. After a brief outline of social practice theory and the methodology I have chosen for my research, I analyze my findings from that perspective and suggest some practical implications for developing a numeracy programme for Gusilay women.

Key words: adult numeracy; social practice; ethnography; Africa

Introduction

In my work as coordinator for adult literacy programmes in several Senegalese languages I am often asked by literacy class participants why we do not offer a numeracy programme. Many learners, especially women, feel the need for acquiring more numeracy skills. For example, some women sell charcoal or cookies, but have no idea how to fix the selling price. Teaching numeracy to women within the context of a literacy programme will increase their understanding of basic mathematical concepts, strengthen their ability to mathematize and give them skills and confidence to better face numeracy related challenges. I decided to review relevant literature and to research traditional numeracy practices of one people group, the Gusilay, in order to be better equipped to help meet these felt needs. I set out to investigate the following questions: What are the felt needs of Gusilay women in the area of numeracy? What are some of their traditional numeracy practices?

Literature tells me that each society develops mathematical ideas in a different way due to various factors, based on the needs of the group (Zaslavsky, 1999). In order to build a curriculum for a numeracy programme that is designed specifically for the target group, relating to their cultural values, practices and needs, a thorough analysis of the situation, including linguistic and social research into existing numeracy practices and felt needs is required (Dalbéra, 1990). Moreover, basing the curriculum on traditional practices and beginning with what adults already know should build motivation, help learners overcome their fears of not being able to learn numeracy and enable them “to develop their ability to cope with their problems themselves” (p. 11).
This article begins with a brief summary of social practice theory and the methodology I have chosen, followed by some background information on the Gusilay people group in Senegal, their number system and a description of my participant observations of three numeracy “events” (Heath, 1983) from harvesting rice to cooking, and selling vegetables. A discussion of the implications of my findings, from a social practice theory point of view, leads to various suggestions for the development of a numeracy programme for Gusilay women. This time could be seen as a critical moment in adult mathematics for the Gusilay people group, when important questions are raised that will influence the development of a relevant numeracy programme.

**Social practice theory**

Developed in the context of literacy (Barton & Hamilton 1998; Gee, 1990; Street, 1984;), social practice theory emphasizes the fact that literacy practices are embedded in broader social and cultural practices and are influenced by the context in which they happen. Moreover, the purposes and meanings a reader brings to the text vary. Literacy is therefore not just a set of mechanical skills that, once acquired, can be used in other situations. Street criticized what he termed the “autonomous” view of literacy, which “works from the assumption that literacy in itself – autonomously – will have effects on other social and cognitive practices”, and suggests that “in practice literacy varies from one context to another and from one culture to another and so, therefore, do the effects of the different literacies in different conditions.” (2003, p. 77).

Even more so than literacy, mathematics had for a long time been viewed as decontextualised and value-free, an abstract code with unlimited power of transfer. This idea was challenged through research in the 1980s (Carraher, Carraher & Schliemann, 1985; Lave, 1988; Saxe 1991), which led Lave and Wenger (1991) to propose the concept of “situated learning”, viewing learning as a social process that happens in a specific context and involves relationships, motivation and values.

The implications of social practice theory for numeracy have been researched and discussed (Baker, 2009; Baker, Street & Tomlin, 2008; Evans, Wedege & Yasukawa, 2013; Tett, Hamilton & Hillier, 2006;). Baker (2009) adapted social practice theory to numeracy, emphasizing the fact that “numeracies”, like “literacies” vary with the social context and have different associated uses and meanings. According to him, mathematics as social practice implies “being aware that mathematics takes place in contexts with values, beliefs and social relations” (p. 6) and using a constructivist approach that “takes a broad vision of learners’ funds of knowledge for mathematics”, including not only skills, but also “processes of engaging with mathematics” and relationships etc. (p. 7).

Tett, Hamilton & Hillier (2006) point out various implications for practice regarding the curriculum, learning, teaching as facilitating and supporting rather than transmitting information, the roles of curriculum managers and other key programme personnel. They appreciate the fact that a social practice view provides a framework that even allows a numeracy programme itself to be analysed and understood as a set of numeracy practices. According to the authors, a social practice perspective on numeracy “is not just meant to be descriptive but engaged – it changes the situation it analyses by articulating new understandings and learners and teachers to actively ‘take hold’ of adult literacy, numeracy and language and shape it for their own purposes” (p. 13).

Evans, Wedege & Yasukawa, with regards to social justice issues, point out the fact that adult mathematics education that starts with learners’ numeracy practices and therefore with different social situations, means that there is a tension between affirming learners’ roots and “the generalizing views of mathematics that smooth out these differences” (2013, p. 225). In the context of a numeracy programme for Gusilay women, this could mean that even though the numeracy programme helps
women to be more efficient in their traditional activities as market vendors, it enables them at the same time to run a shop or eventually become a financial consultant, which are male dominated domains.

**Methodology**

There are several reasons for my choosing social practice theory as the framework for my research. My review of literature on social practice theory (Papen, 2005; Street, Baker & Tomlin, 2008) and on numeracy practices in non-Western countries and ethnomathematics (D’Ambrosio, 2001; Gebre et al. 2009; Nabi, Maddox, 2001; Rogers & Street, 2009) have made me aware of the importance of context. Moreover, in my opinion, social practice theory matches the holistic worldview that is characteristic of Senegalese society, one of many cultures that “value contextual understanding rather than decontextualization and objectivity” (Ascher, 1991, p. 6). Another rationale for taking into account the insights of social practice theory is provided by adult education theory, emphasizing the wealth of knowledge and experience adults bring to the classroom and the importance of relevance to daily life (Knowles, Holton III & Swanson, 1998).

An ethnographic approach, commonly used in anthropological research, seemed by far the most suitable method for studying numeracy as social practice, observing and describing numeracy practices in the context in which they occur. During a total of four weeks of research in the town of Thionck-Essyl, between October 2011 and January 2012, I participated in various numeracy events. The research tools I used were participant observation, unstructured discussions and visual methods, especially photographs and where possible artefacts. I made an effort to choose people without school experience, but found that some of them had formal schooling.

**Numeracy practices of the Gusilay ethnic group in Senegal**

**Background information**

The Gusilay live in Thionck-Essyl, a town with a population of about 15,000 inhabitants, situated in the Casamance region in the south of Senegal. Traditionally, they have been agriculturalists, mainly growing rice but also millet and peanuts. Preparing the rice fields, before and at the beginning of the rainy season, is men’s work. Sowing, transplanting and harvesting traditionally is women’s work, with women often working together. Each married woman automatically joins an age group when she gets married. Women of the same age group, who got married during the same period in the same part of town, often do activities together. Moreover, women frequently organise themselves by forming associations, often with the goal of earning money, e.g. by harvesting other people’s rice fields as a group. Most women work hard every day, with highlights in the hardships of life being celebrations that include singing and dancing. Relationships are characterized by solidarity and much teasing and laughter.

Overall, less than 40% of Senegalese women over the age of 15 are estimated to be literate (UNESCO, 2012). Many younger Gusilay women have gone to school for a few years. The language of instruction there is French, a language not understood by most children when they enter school, and repetition and drop-out rates in Senegal are high. The Gusilay are motivated to learn to read and write in their language and especially to become more proficient in the area of numeracy since life has changed drastically in recent decades. There are many new numeracy related challenges women have to face nowadays. Whilst growing rice is still the occupation of many inhabitants of Thionck-Essyl, an increasing number of women run small businesses to meet the needs of growing family expenses. Money is used in more domains, as can be seen with payment for school equipment and electricity bills, since the town recently got electrified.
Banks attract a growing number of clients. New technologies like cell phones, scales, calculators or computers are introduced. With these changes comes a need for acquiring more and different skills and knowledge. In conversations with members of our partner organisations and also through questioning women in the literacy classes I coordinate, I found that women’s main goal in wanting to learn numeracy is improving the financial situation of their families. Areas of special interest that were mentioned were learning to count and calculate better, dealing with money and calculate profit, understanding written documents (e.g. bank statements or children’s health records), keeping track of family expenses and knowing how to weigh in order to sell in bigger quantities or in income generating projects like soap making.

The Gusilay number system

Whilst an in-depth discussion of the number system is beyond the scope of this article, I will mention some basic facts that I consider most relevant for the development of a numeracy programme. Here is an excerpt of Yashina’s (2011) list of the main cardinal numbers in Gusilay.

<table>
<thead>
<tr>
<th>Number</th>
<th>Gusilay Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 yanur</td>
<td>1 guñen (lit. ‘hands’)</td>
</tr>
<tr>
<td>2</td>
<td>2 síruba</td>
<td>15 guñen gaat (lit. ‘hands foot’)</td>
</tr>
<tr>
<td>3</td>
<td>3 sìfaajir</td>
<td>20 gafaakan (lit. ‘the end of a person’)</td>
</tr>
<tr>
<td>4</td>
<td>4 sìbaagir</td>
<td>30 gafaakan n’guñen</td>
</tr>
<tr>
<td>5</td>
<td>5 futok</td>
<td>40 gafan gúruba</td>
</tr>
<tr>
<td>6</td>
<td>6 futok n’yanur</td>
<td>60 gafan gúfaajir</td>
</tr>
<tr>
<td>7</td>
<td>7 futok n’síruba</td>
<td>80 gafan gubaagir</td>
</tr>
<tr>
<td>8</td>
<td>8 futok n’sìfaajir</td>
<td>100 eceme</td>
</tr>
<tr>
<td>9</td>
<td>9 futok n’sìbaagir</td>
<td>1,000 éwuli</td>
</tr>
</tbody>
</table>

There are distinct number words for the numbers 1 to 5, 10, 15, 20, 100 and 1000. All other numbers are mathematical calculations. Number words get long as numbers get higher. 100 and 1000 are loanwords from a dominant language in the area. There are two traditional bases, 5 and 20, and a more recent one, 10 (Kané, 1987). Vocabulary for new concepts like ‘plus’, ‘minus’, ‘tens’, ‘hundreds’ or ‘book-keeping’ would need to be developed, ideally based on traditional concepts. For example, there is the expression ‘gassaŋar’ that might be used to explain the concept of units and tens: Traditionally, the fruit of a certain palm tree is cut, one or two bunches per tree, and put into piles of ten on the ground. For example, you will say that you have five ‘gassaŋar’, five piles of ten.

There is a difference between numbers in general and numbers in the context of money. Calculating with money in Gusilay is based on the ‘ékori’, with 1 ‘ékori’ equalling 5 F CFA. Therefore when numbers are used in the context of money, the value is actually five times as much as the word itself (e.g., 25 F CFA is called ‘futok’ [five, understood 5 ‘ékori’], even though the print on the coin says 25). There are various proverbs that contain numbers and riddles, for example: “I have two children and two adults. They have gone fishing and caught three fish. Each of them has got one fish.” (Solution: Son, father and grandfather).

---

55 Gafän’ is a contraction of ‘gafaakan’. ‘Siruba’ becomes ‘gúruba’, since it adapts to the noun ‘gafan’, Gusilay being a noun class language where attributes, including numerals, change according to the noun they define.

56 773 is ‘siceme futok síruba guñen gafak bugan gúfaajir guñen sìfaajir’.

57 Franc de la communauté africaine.
Participant observation of various numeracy practices

Working in the rice field

Since it was harvest time, I spent a day doing participant observation on harvesting rice. Here is an account of my observations. I join seven women, who belong to the same age group, to harvest a ‘ñikin’ (rice field) that belongs to one of them. We walk on small paths for about an hour, the women carrying baskets with water, knives, some food and rolled-up strips of leaf from a certain tree that are used for binding the sheaves. After a quick breakfast of rice and curdled milk we start cutting the rice. The women work in rows, each one cutting the rice at the bottom of the stem with a knife.

The woman beside me shows me how to hold the rice I have cut, with the stems in the palm of my hand and the leaf outside my thumb. From time to time I tear off the leaves. The women talk whilst working, mainly about their work for the next few weeks. Each stem is cut by hand with a knife, the length being assessed by visual judgment, maybe 20-30 cm according to my guesses. When a ‘galiten’ (handful) of rice is cut, it is laid on the ground on top of a row. The ‘galiten’ do not seem to be of a definite size.

One woman tells me: “You lay it down so that your hand does not get tired because the rice is too heavy”. One of the women binds the ‘galiten’ together into an ‘ekok’ (sheaf). She wraps a strip around the first ‘galiten’, then adds the next one, pushing down hard and wrapping the strip tightly. When asked how many ‘galiten’ make up the ‘ekok’ she is binding, the woman does not know the answer. My colleague has counted: There are ten ‘galiten’. She wonders whether it is the length of the strip that decides the size of the ‘ekok’. One of my informants tells me later that in the past sheaves used to be much smaller. We finish the ‘ñikin’ in about two hours, having harvested eight sheaves of rice.

Each woman brings her own rice for lunch, whilst the owner of the rice field has to contribute the fish. She also pays 1,000 CFA and gives two sheaves of rice to the association they all belong to. The women tell me that women’s associations can be hired to work on anybody’s field, but then the price is higher. Reflecting on my participant observation, I realized that the main strategy used in working in the rice field is estimation and approximation by visual judgment. The length of the rice stems, the measure of ‘handfuls’ of rice as well as the size of a sheaf are all determined by visual judgment based on experience. I am told that visual judgment is also used in sowing and transplanting, e.g. for determining the distance between individual seeds and between the seedlings when transplanted to the rice field into three lines on each row. Estimation seems to be a much more useful skill in this society than measuring with exactness.

The women I observed are of the same age group and at the same time organised in an association, together with other women. There are clear regulations as to the contributions of each person involved. As a newcomer, I joined in the activity, learned by doing and was shown how to do it correctly by one of the women.

Cooking the daily rice dish

I spend a day with Khady, my host. We cook a dish called ‘etoj ekaama’, rice with a sauce of ground peanuts and manioc leaves. Khady is 35 years old and is pregnant with her eighth child. She lives in the same house with her husband, a second wife and their children. The kitchen is a square house made out of dried earth, with a roof and several small rooms. We sit in the little square porch area on little stools while cooking. When I ask Khady at what time she starts cooking, she says, “It does not matter when exactly”. The children come home from school between 1 and 1.30pm, and the food
should be ready shortly after. Khady tells me that she has a cell phone that she could consult to check the time. Around 10 am - I check my watch - Khady sends a boy to the market with money and instructions as to what he should buy. I see him come back with six packets of peanuts, a hot pepper, a piece of dried fish, four green bitter tomatoes, four stock cubes, a piece of dried sea snail and a packet of small beans. Khady must have calculated the amount of money she needed to give him in advance. She has been to school and speaks some French.

We pick a bucket full of manioc leaves in a nearby garden. Another boy is sent with 200 CFA to have them shredded at a mill. We could have pounded them ourselves, but I am told that this would take too long. These days Khady buys three sacks of rice per month, all at once, since her own rice is already finished and the harvest on her rice fields has not yet started. She tells me that she uses three nescafé tins of rice per meal. I know from another informant that in the past, woven baskets were used in the kitchen, ‘gáfankum’ for storing rice and ‘funip’ for storing peanuts or salt, but nowadays empty nescafé tins have replaced the baskets for measuring rice.

Having crushed the peanuts in a small mortar, several handfuls at a time, we sieve them through a strainer with fairly big holes and afterwards one with a finer mesh. The rice is first steamed, on a strainer on top of a pot of boiling water, with a cloth ribbon wrapped around the gap between the pot and the lid to seal it. Khady takes a handful of salt from a big mustard jar and puts it on top of the rice. Then she dumps the rice into the boiling water. She says she just knows how much water to put into the pot and how long she needs to let the rice steam and then boil. Then she puts the cooked rice aside, adds more charcoal to the fire and puts on a new pot, about 1/3 full with water, for the ‘etoj’ sauce. She washes the green tomatoes in a bucket of water, takes the stems off and puts them into the boiling water. Two of Khady’s smaller children sit with us, and the baby is on her back. A chicken runs on top of the roof beams from time to time, and dirt falls on me. We chase it away several times.

The boy comes back with the ground manioc leaves; they now cover only maybe 5 cm of the bucket. Khady puts them into the boiling water. She washes the dried fish and the piece of snail and adds it. Then she puts the peanut powder into the now green water. From time to time she stirs the sauce with a big metal spoon. I notice that she lets it simmer for at least two hours. Finally Khady adds the hot pepper, the stock cubes and two spoonfuls of shrimp powder, which she takes from a big mustard jar. Her daughter Awa, who has just come back from school, is putting the rice onto two platters, when Khady suddenly realises that she has forgotten to add the beans. Awa crushes them, using pestle and mortar, and Khady puts them into the sauce and lets it boil for another 30 minutes or so according to my reckoning.

Reflecting on my observations, it struck me that my way of categorising is different from the traditional Gusilay way. ‘Time’, for example, is numerical information for me, whilst for a Gusilay, it might be in the category of ‘the way relationships are used’ or ‘duration of an activity’. Traditionally, numbers have not played a big role in many mathematical concepts in Africa (Zaslavsky, 1999).

Moreover, I realised that women all over the world cook using estimation and approximation, techniques honed through experience. And certainly women all over the world, and specifically in Senegal, are not aware of how much mathematical knowledge is applied when cooking. The amounts of water or salt are assessed by estimation and approximation, as is the time needed for cooking. Strategic planning is involved in deciding what ingredients to buy for the daily meal or how many bags of rice to purchase per month. Many of these strategies are applied largely unconsciously due to daily routine and experience.

Measuring capacity is based on the body, e.g. a handful, or on varying containers, e.g. spoonful, bucketful and various pots. Khady knows from experience what size cooking pots and platters to use. There is no interrelated system for measuring capacity. Some of the strategies include the use of
number. Mental calculation skills were put into practice as Khady decided how much money to give to the boy to buy ingredients at the market. Khady knows money, and she knew how much money to give to the boy to go shopping, so she must have some mental calculation skills. I did not ask her where she learned those. Her assessing the value of time versus money when sending the boy to the mill is an example of numeracy being linked to values. It might be interesting to find out whether her appreciating time over money was influenced by my presence.

**Selling condiments at the market**

The market place of Thionck-Essyl is a big square of roughly 40 x 40 m, bordered by mango trees. About 20-25 women vendors sit on small wooden stools or plastic buckets under the trees, some with wooden tables on which their goods are displayed, some with plastic mats on the ground. I join them for several days to observe them and learn from them.

The president of the market, an elderly lady named Mariama Sambou, sits on a bucket behind her goods, which are laid out on a big plastic sheet on the ground. She tells me that she has never gone to school and has traded for 30 years. This reminds me of Bhola’s emphasizing the fact that “oral numeracy” (1994, p. 89) is a cognitive process unrelated to the learning of reading and writing. Mariama tells me that she has never taken a pen to calculate, but does it all mentally. She tells me that she prefers to not make errors, because it is she herself who has a problem if she does, and that she makes fewer mistakes than others who write.

Mariama sells slices of cabbage, carrots cut in halves, manioc, dried fish, fresh hot peppers, onions, hot dried pepper, tomato paste in little plastic bags, stock cubes and other ingredients that women need for their daily cooking. Women buy these each day in small quantities, which is why the market vendors can make a profit by selling their goods in small quantities. The market women chat with each other and their clients. A young woman with a baby on her back buys a pile of onions and hands Mariama a coin. Mariama asks her neighbour to exchange the coin into smaller ones; having small coins is a challenge in Senegal.

Mariama sells three kinds of dried fish: ‘con’, ‘bërr’ and ‘dëggërbopp’. She tells me that she bought 1 kg of ‘con’ for 600 CFA; 1 kg equals six fish. She cuts each fish into three pieces and sells the piece for 50 CFA. This is how her mental calculation works: She takes two fish, which makes six pieces in total. Again two, that makes 12 pieces; again two, that makes 18 pieces. She sells 12 pieces at 50 CFA each, which makes 600 CFA, then the six that are left: 300 CFA, so she sold for a total of 900 CFA. I reflect on the fact that multiple additions seem much slower and more complicated than multiplications. She takes off the 600 CFA and sees that she has a profit of “only 300 CFA”, as she says. For the ‘bërr’, she paid 1,500 CFA for 1 kg. She sells a piece for 100 CFA. There are 18 pieces in 1 kg. She sells for a total of 1,800 CFA and knows that she again has a profit of 300 CFA.

She bought 1 kg ‘dëggërbopp’ for 800 CFA. 15 fish weigh 1 kg. First she sorts the fish. She sells the big ones for 100 CFA, the middle ones for 75 CFA and the small ones for 50 CFA. She gives me an example:

She sells five fish at 100 CFA, which makes 500 CFA. She has ten fish left. Then she sells four fish at 75 CFA. For this she calculates in her head “75 times 2 makes 150, and again 75 times 2 makes 150, so the total is 300 CFA”. She has six small ones left, which she will sell for 50 CFA. She calculates with five. If she sells five, she gets 250 CFA. The last one makes 50 CFA, so the total is 300 CFA. Five for 500 CFA and four for 300 CFA makes 800 CFA. She knows that that is the price she bought them for. “So I know that my profit is identical with the six little fish, that is 300 CFA”, she says. It strikes me that she does not add up all the income and then
deduct her cost, but states that her profit is identical with the fish that are left, a concrete object rather than an abstract number.

Mariama also explains how she calculates the profit she makes from selling onions. She buys a 25 kg bag of onions for 7,500 CFA in the nearby town of Bignona. She pays 200 CFA for the transport of the bag and 900 CFA for herself. She calculates only one way transport expenses into the cost of the onions, and the return trip she will calculate with the other goods she bought. When she gets home, she first weighs the sack with the scales she owns. Sometimes there are 26 or 27 kg in a bag. She weighs per kg and then counts how many onions there are in 1 kg. She gives me an example: There are eight onions per kg. She wants to sell the kg at 500 CFA. If they are all the same size, she sells one for 75 CFA. She puts the onions in groups of four, which makes 300 CFA per group. She knows that she can sell the kg for 600 CFA if she sells the onions one by one or per group. If she sells them by kg, she will have 500 CFA only for the kg. She earns 100 CFA more per kg if she sells one by one. Afterwards she thinks some more about the bag. She takes 20 kg. Each kg she will sell for 500 CFA, so 500 times 20 makes 10,000 CFA for 20 kg. The rest is 5 kg × 500 = 2,500 CFA. The total is 12,500 CFA. She takes off the 7,500 CFA that she spent on buying the bag. Then she takes off the transport cost of 1,100 CFA, which leaves her with a profit of 3,900 CFA. If she sells each onion separately, she has more profit, between 5,000 CFA and 6,000 CFA total.

Mariama puts the money she earns in her bank account. I ask how she reads the bank statements. For each payment, she gets a receipt, and her children will read the amount for her. Sometimes she checks and looks at all the receipts and calculates in her head.

Reflecting on my observations, I realize that market vendors frequently buy by weight and sell by number. This might be the case because people buy small quantities of vegetables, which are not easily weighed in units of 100 g or so. Bigger scale trade uses international measurement systems, e.g. some goods like fish and rice are mostly sold using scales.

Mariama is an expert at mental calculation. She seems to know a lot of calculations by heart, e.g. multiples of 5, which is 25 CFA in the context of money. I have noticed that many food items cost 25 CFA. Strategies I observed include regrouping of concrete objects, counting, and mental calculation strategies of decomposition and repeated grouping. She has a fairly good knowledge of addition, subtraction and some multiplication, but sometimes used multiple additions rather than multiplication. Many women I know have limited or no mental calculation skills. I even talked with a young vendor who sells fresh fish at the common price, without knowing how to calculate well. Moreover, most women do not know how to read scales and have no notion of Western measurements of weight.

Implications of social practice theory for the development of a numeracy programme

Analysing my observations of traditional numeracy practices from a social practice viewpoint has led me to various suggestions for the development of a numeracy programme for women.

Choice of language

The logical conclusion for the choice of language in the classroom is the language used in everyday life. It is people’s first language in which they can express themselves best, and in which all the numeracy practices I observed happened. Meaney, Fairhill & Trinick emphasize the fact that “cultural practices including ethnomathematical ones cannot be separated from the language in which they were developed”, since the language used impacts how students perceive the practices (2008, pp. 62-63).
The Gusilay number system responds to the needs of society and has been developed and adapted to new demands. The challenges posed by the length of higher number words, the lack of vocabulary for new concepts and written calculations with numbers in the context of money will need to be addressed. Traditional concepts should be used where possible when developing new vocabulary, e.g. the expression ‘gassanja’ could be introduced to denote tens. In order to be immediately relevant, I propose that the programme begins with numbers in the context of buying and selling. Therefore the different value of numbers in the context of money will be discussed in the class, in order to avoid confusion. “25 CFA + 25 CFA = 50 CFA” will be written as “E 5 + E 5 = E 10”. This might need to be tested.

The language issue could lead to a discussion of values, goals and conflicts, since the younger women and children in Thionck-Essyl learn mathematics in school in French. Often local languages are viewed as inferior, and it might surprise some people to realise that they can say everything they need and want to express in their language.

Relationships and power relations

The issue of relationships needs to be addressed at various levels. A class constituted of women belonging to the same age group or association has the advantage of group dynamics, relationships and rules within the group matching traditional standards and being already established. The teacher should be a Gusilay woman, since women will feel less threatened and more inclined to trust another woman than a man. The choice and training of teachers should happen bearing in mind Street’s observation, made in the context of literacy: “The way in which teachers or facilitators and their students interact is already a social practice that affects the nature of the literacy being learned and the ideas about literacy held by the participants, especially the new learners and their position in relations of power” (Street, 2003, p. 77). The group could be viewed as an already established “community of practice”, in the sense of being involved in “a more encompassing process of being active participants in the practices of social communities and constructing identities in relation to these communities” (Wenger, 1999, p. 4, emphasis in the original). Class activities will include discussions, role play, dancing and singing.

Another issue that needs to be addressed is the questions of who establishes the curriculum, who is responsible for developing and running the programme, and in which ways do teachers and learners participate in decision making, the governance of the class, etc.

Power relations are also influenced by the fact that attending a numeracy class empowers women by increasing their understanding of basic mathematical concepts and their ability to mathematize and by giving them skills and confidence to better face numeracy related challenges. As Mellin-Olsen put it, mathematics “is also a structure of thinking–tools appropriate for understanding, building or changing a society.” (1987, p. 17).

A strong foundation of traditional knowledge

The envisaged numeracy programme will aim at building on participants’ “funds of knowledge” (Moll, Neff & Gonzalez, 1992) rather than focussing on their deficits (Baker, 2009). Traditional numeracy practices have great value and will serve as a strong basis for an adult numeracy programme. Building on and giving value to these practices makes learners aware that they already know and use a lot of numeracy skills, strengthens their roots and self-esteem and increases their motivation for learning more. For example, discussing the numeracy skills used when cooking a meal will help women realise how much they know already. Making existing mental calculation strategies explicit and available to all learners will enable them to use a very practical skill that fits well into the
context of a largely oral society. Other funds of knowledge in the larger sense include ways of categorising, traditional wisdom expressed in proverbs, games and riddles, and traditional ways of measuring time and capacity with their inherent values.

Baker encourages going “beyond a limited focus on number and also include concepts from shape, space, data, patterns, ways of thinking etc.” (2009, p. 14). The Reflect method, with its focus on development of maps, matrixes, calendars and diagrams that “represent local reality, systematise the existing knowledge of participants and promote the detailed analysis of local issues” (Archer and Cottingham, 1996, p. 5) represents a helpful approach.

Teaching numeracy as practices

Fourthly, the aim is to teach numeracy as practices rather than skill (Baker, 2009), which in turn encourages a teaching style of facilitating (Tett, Hamilton & Hillier, 2006). Baker (2009) suggests seeking to work from everyday practices towards formal numeracy practices and to be explicit when switching between the two. Gebre et al. (2009) emphasizes the importance of making numeracy taught and practiced in the classroom similar to real life in order to facilitate transfer.

Ideally an income generating project accompanies the numeracy teaching and learning, so that the women can practice their newly acquired knowledge and skills immediately. For example, the group could meet in their communal garden and learn how to weigh by weighing their harvested vegetables or calculate their profit when selling them. They could count and calculate in the context of selling their produce to their clients. The introduction of new forms of numeracy could include the teaching of written numeracy, introduced with challenges like writing income-expenditure lists, opening a bank account and learning how to read the bank statements, reading children’s health booklets or the instructions for taking medicine.

“Discover, Discuss and Develop”

Finally, teaching and learning will be relevant to learners’ daily experiences, with discussion and reflection as important components of classroom practice. The approach “Discover, Discuss and Develop” (Gebre et al., 2009) could be used with the group. The teacher and learners, in this context all from the same ethnic group, identify together what people know about numeracy practices in the community, and the group then discusses the issues raised. The third step is to build on to the first two steps, for example by introducing new forms of numeracy according to what is relevant to the learners.

A discussion on differences in values and ways of classification of Gusilay and Western culture could be introduced with the question whether known strategies need to be replaced by new ones. I doubt that the strategy of estimation in cooking or in field work needs to be replaced by more precise measurements. In contrast, more precision is needed when weighing ingredients for making soap or taking medicine at specific times. The technique of estimation, traditionally not used in the context of numbers, could be discussed and maybe applied to the context of money in situations where exactness is not required, for example in estimating roughly whether the change received is correct.

Problems encountered in learners’ daily lives will form the basis of learning in the class room, with teachers using a problem-solving approach (Fordham, Holland & Millican, 1995). The use of different strategies to get to a solution is encouraged, discussed and taught explicitly (Ginsburg & Gal, 2000), since in everyday practices a variety of strategies and approaches are used also. Discussions with the
whole group should foster the ability to analyse and reason rather than imitate and learn by heart. Investigation and cooperation will be encouraged by working on problems in pairs or small groups.

The class could organise a small project like buying, roasting and selling peanuts and plan, discuss the processes, results and challenges encountered as part of the learning experience. A presentation of the history of mathematics could serve to show learners that mathematics is by no means an import from the West, but has some of its roots in Africa.

Conclusion

My research on some numeracy practices of the Gusilay shows the existence of a variety of traditional practices, techniques, values and concepts. An analysis of my observations from a social practice theory viewpoint has led me to suggest five basic considerations for the development of a numeracy programme for women:

The first language of the participants, used in everyday numeracy practices, will be the language of instruction. Attention needs to be given to the issue of relationships, including power issues, on various levels including that of programme designers, teachers, classroom practice and the resulting changes of power relations as an outcome of the learning experience. The numeracy programme will attribute value to and build onto traditional knowledge. Discussing and making knowledge and techniques available to all learners will form the basis of the programme, at the same time increasing learners’ self-esteem and motivation by affirming their roots and identity. Numeracy will be taught as practices, with the teacher seeking to work from everyday practices towards formal numeracy practices. Finally, learning is facilitated through discovery, discussion and reflection. The goal of the numeracy programme is to see women grow in dignity and self-confidence, prospering by actively increasing their knowledge, using literacy, numeracy and language to meet their felt needs and to develop in areas that are important to them.

References

Gerger, Implications of social practice theory for the development of a numeracy programme.


**Provoking mathematical thinking: Experiences of doing realistic mathematics tasks with adult numeracy teachers**

Janette Gibney

University of South Wales  
<janette.gibney@southwales.ac.uk>

**Abstract**

This action research project looks at what happened when a small group of adult numeracy teachers with widely different experiences of learning and teaching mathematics explored their own informal numeracy practices and undertook a series of collaborative mathematical tasks. Evidence from qualitative data collected during the enquiry suggests that realistic tasks can provoke a range of mathematical thinking and learning responses which allow us to identify ways in which procedural and conceptual thinking is being used, and to track learning journeys through different stages of problem-solving. Although more experienced numeracy teachers could move between and within their ‘real worlds’ and ‘maths worlds’ with intent and ease, others had less integrated experiences, often valuing perceived mathematical powers over their own intuitive powers, with mixed success.

Key words: mathematical thinking, action research, adult numeracy teachers, realistic, realisable, mathematisation, collaborative classroom, intra- and extra-mathematical.

**Introduction**

Historically, within the UK, adult numeracy teaching is a field that many people move into sideways, often from teaching other disciplines. The requirement for practitioners to have a set level of personal mathematics skills was introduced only relatively recently and it is not untypical to find teachers of numeracy who lack confidence in their own mathematical ability (Cara et al., 2010). Personal mathematics development is therefore an important component within many pre- and in-service adult numeracy teacher education programmes. Teachers are encouraged to develop their mathematical thinking throughout their training, both by participating in class activities and pursuing private study. As a tutor and course leader on such a programme, I have observed that when it comes to building a personal mathematics portfolio, many teachers exhibit fairly mechanistic and unreflective ways of working. This is true not only in terms of the approaches they adopt, but also the sorts of independent tasks they choose to undertake - often a surprisingly narrow diet of content-driven and competence-based exercises. The purpose of this research project was to explore how to better support adult numeracy teachers to develop and extend their own mathematical thinking. The rationale for this extends beyond the perceived need for adult numeracy teachers to ‘upskill’ and is based on the underlying assumption that developing teachers’ confidence, awareness and insight into their own mathematical thinking, will better equip them to develop and extend the mathematical thinking of their learners.
Method of enquiry

The classroom, tutor and teachers

The twelve teachers in the group participating in this enquiry were aged 25-55, from socially and ethnically diverse backgrounds and included two teachers whose first language was not English. They were all undertaking professional development in adult numeracy teaching and consistently demonstrated high levels of motivation and engagement although their personal experiences of mathematics, both as teachers and learners varied tremendously.

Within the group, we had negotiated a shared sense of adult numeracy as involving more than basic mathematical skills, or the application of mathematics in everyday life but rather numeracy as a way of negotiating the world through mathematics, “not less than maths but more” (Johnston & Tout, cited in Coben, 2004, p3). In the course of working together during the year, we had tried to develop a co-operative and conjecturing classroom - a milieu that explicitly challenged deficit models of adult numeracy. This ethos was influenced by the idea of funds of knowledge describing the informal knowledge, skills and experiences that adult learners can draw on but may not be evidenced by formal qualifications (Moll et al, 1992; Baker, 2005), a concept that can be broadened to include interpersonal and metacognitive skills.

Responses to initial classroom probes into their mathematical thinking suggested that few of the teachers moved flexibly between different representational modes. The most mathematically experienced wanted to adopt a symbolic or algebraic response whenever possible, with few trying out more practical approaches. The least experienced saw this use of ‘formal’ mathematical methods as their ultimate goal, placing less value on other approaches. This apparent lack of variety on the teachers’ own mathematical journeys was often in contrast to the active learning and multi-sensory approaches they were developing to support mathematical thinking with their own learners. The initial focus of the enquiry was to explore how to provoke adult numeracy teachers to think and act less mechanistically as ‘doers’ of mathematics themselves.

Methodology

The enquiry adopted an action research approach based on the idea “that a practitioner is involved in analysing a situation, planning an alternative action, carrying out that action, and then evaluating the effects of what they have done” (Mason, 2002, p172). The research was broken down into three smaller cycles or phases of enquiry and reflection. These were undertaken over an eight week period in the final semester of the course.

All participants within the group were involved in research design tasks for about an hour a week in class with some out-of-class time required for auditing, self-reflection and write-ups. Although an essentially social constructivist perspective informed the research focus and the design of classroom interventions, the research methodology itself was mixed. Data collection from tutor field-notes and audio-recordings of semi-structured group discussions focussed on teachers’ interpretations and evaluations of tasks undertaken in and out of the classroom, suggestive of an ethnographic approach. Other sets of data, however, were generated from audio-recordings of pair discussions, stimulated recall interviews, written work and tutor observations which aimed to capture responses to paired and individual tasks. Though more typical of positivist methodologies, these provided rich qualitative data which allowed me as a practitioner researcher to experience more fully what happened as teachers engaged in tasks.

Mason (2002, p52) suggests that in researching one’s own practice, it is useful to differentiate between giving a brief-but-vivid “account-of what was seen, heard, experienced” and analysing,
explaining or “accounting-for” incidents. Accounts-of will be used to illustrate salient incidents and experiences, along with excerpts from edited transcripts of audio-recordings and examples of teacher responses to tasks. Data analysis will be through a mixture of event sampling using and adapting pre-specified categories from wider theoretical and empirical research, and accounting-for recurring phenomena using key constructs and frameworks which are reported within each of the three action research cycles.

This paper will now outline key findings from cycle 1 of the enquiry before going on to focus in particular on significant moments arising from data generated within naturally occurring peer-peer discourse between two pairs of teachers during the second and third cycle of the enquiry.

**Cycle 1 Awareness raising**

Gattegno (1988, p167) highlights the importance of teachers sensitising themselves to their own behaviours, emotions, and awarenesses:

> Teachers need to make themselves vulnerable to the awareness of awareness, and to mathematization, rather than to the historical content of mathematics. They need to give themselves an opportunity to experience their own creativity and when they are in contact with it, to turn to their students to give them the opportunity as well.

In considering what sorts of mathematical activities to use within this action research, I wanted tasks that would support teachers to take the initiative and become more fully engaged in their own mathematical thinking. Schoenfeld (1994) developed a broad and age-independent description of what learning to think mathematically means:

1. Developing a mathematical point of view – valuing the process of mathematisation and abstraction and having the predilection to apply them.
2. Developing competence with the tools of the trade and using these in the service of the goal of understanding structure – mathematical sense-making.

But what did mathematical thinking and mathematisation look like ‘outside formal mathematics classrooms’? Research has demonstrated that adults have access to many informal numeracy practices (Street, 1984; Nunes, Schliemann & Carraher, 1993a; Baker & Rhodes, 2007). The idea that teachers need to become aware of learners’ innate or natural powers to think mathematically (Mason and Johnston-Wilder, 2006) is echoed in a number of recent research reports (Swan, 2006; Swan and Swain, 2007). Indeed, much official discourse now actively encourages adult numeracy teachers to “build on the knowledge learners already have” (Swain et al, 2007, p. 7).

The belief in the importance of teachers’ recognising their own funds of knowledge and exploring innate mathematical sense-making powers themselves, provided the initial impetus for considering everyday contexts and numeracy in the task design. By exploring what we as adult numeracy practitioners noticed about our own numeracy practices, would any shared characteristics, prior knowledge or behaviours related to mathematical thinking emerge to inform the design of tasks for subsequent action research cycles, for both experienced and less experienced participants?

**Task design 1**

During the first week of the enquiry, teachers and tutors made diary notes about what they identified as their numeracy practices over the course of a ‘work-day’ and a ‘non-work day’. These were mostly handwritten on two large A3 diagrams resembling a clock face. A further record sheet was completed
during the second week. This required us to identify and classify mathematical behaviours we noticed according to what Bishop (1988) identified as six universally occurring activities: counting, locating, measuring, designing, playing and explaining.

**Analysis**

Each week, findings were shared with peer partners. Subsequent whole group discussion were animated, as numerous and at times conflicting *accounts-of* and *accounts-for* were generated:

**Accounts 1**

The supermarket does all the price comparisons – I just read the labels.

We’re on a really tight budget so I’m working out stuff with money all the time.

I get the kids to help with the adding up when we’re in the supermarket.

I was quite shocked – I do more maths out of work than when I’m teaching.

It took ages to park this morning. Usually there are a few places left, but today it was practically deserted.

I didn’t realise how much time I spend in the car at the moment – there’s journey times, buying petrol, using maps and Google directions, speeds and signs, even working out the best lane to be in where all the road works are.

I’m totally addicted to Sudoku at the moment – my son and I try to see who can finish first.

Teacher and tutor *accounting* suggested that as well as becoming more sensitised to our own numeracy practices, we engaged in a diverse range of socially and culturally situated mathematical behaviours. Although some of us identified possible mathematical topics and themes related to particular situations or times, others discussed what they actually did. Many omitted or ignored things they did not consider mathematical but “just common sense”. This illustrates how difficult it is to design learning tasks tailored to each individual’s particular experiences.

Data from this part of the enquiry did however suggest some common characteristics of the group’s everyday numeracy practices which tended to involve purposeful activities which were often collaborative e.g. family activities involving playing, cooking, shopping or constructing. These were often linked to particular roles and could be dependent on and shaped by particular tools or realia e.g. maps, self-service checkouts, petrol pumps, Sat Navs. This is in line with findings from similar studies into everyday numeracy practices (Lave, 1988; Harris, 2000; FitzSimons, 2005). For example, in reviewing a range of empirical research some 20 years ago, Resnick (1987) noted that much activity outside classrooms is socially shared. She contrasted examples of shared knowledge and understanding, tool manipulation, contextualised reasoning and situation specific competencies from everyday numeracy practices with the sorts of individual knowledge and skills, abstraction, symbolic manipulation and generalised learning more likely to be experienced in many formal mathematics classrooms.

**Implications from Cycle 1**

This initial analysis suggested that the teachers’ informal numeracy practices could be drawn on more effectively by providing tasks which afforded:

- Opportunities for them to work together on problems.
- Access and use of cognitive tools.
• Direct engagement with objects and situations rather than purely symbolic thinking.
• Use of situation-specific competencies (adapted from Resnick, 1987).

However, the overall goal was to further develop and extend these teachers’ mathematical thinking; to build on existing knowledge and ensure those with little or less successful experience of learning maths were empowered to operate successfully within formal mathematics classrooms too. To this end, the framework above merely presented possible points of departure.

In terms of identifying an actual topic base for the mathematical tasks to be used in the next cycles of the research, I was particularly struck by the relatively infrequent use made of ‘standard’ measures or indeed measuring devices during awareness raising activities in Cycle 1. Discussions with teachers revealed resonated experiences and generated additional complex, contingent and subjective strategies for measuring and estimating everyday phenomena:

**Accounts 2**

In the morning I know when the bath’s getting full … I can hear how long I’ve got to drink my coffee

I can estimate how much it’ll cost by how full the trolley is.

I know how much squash to add by the colour – not dilution ratios!

I measure how crowded a place is by how far I have to go to get to an uncrowded place.

Buying petrol has nothing to do with gallons or litres…

Don’t need an alarm clock… my dogs tell us when it’s time to get up.

**Cycle 2 Plausible estimates**

Subsequent research and review of potential mathematical thinking tasks which could be adapted in accordance with the research focus and findings to date, uncovered a number of suitable open-ended tasks based on estimation and measure. A set of classroom assessment tasks (CATs) which had already been field-tested were chosen for cycle 2 of the research. These involved “Making plausible estimates” based on Fermi-type problems (Ridgeway and Swan, 2010).

**Task design 2**

Figure 1 details the task objectives presented to the whole group:

The aim of this task is to provide the opportunity for you to work with your partner to:

• make sensible assumptions
• develop a chain of reasoning
• choose suitable units
• communicate your assumptions and reasoning effectively to peers

**Extension:** Identify upper and lower bounds i.e. what range of values would you give in order to be pretty certain that you have included the true value being estimated?
Tahta (1981) makes a useful distinction between inner and outer tasks which helps here to distinguish between the explicit outer task of finding a plausible estimate and the intended inner task which would allow both teachers and tutor to gain experience of what mathematical thinking and communicating might look, behave and feel like.

By building on a range of theoretical and empirical research, Goos et al (2004, p100) identify five assumptions they argue are crucial to creating a culture and ‘community of mathematical inquiry’:

1. Mathematical thinking is an act of sense-making, and rests on the processes of specialising and generalising, conjecturing and justifying.

2. The processes on mathematical inquiry are accompanied by habits of individual reflection and self-monitoring.

3. Mathematical thinking develops through student scaffolding of the processes of enquiry.

4. Mathematical thinking can be generated and tested by students through participation in equal-status peer partnerships.

5. Interweaving of familiar and formal knowledge helps students to adopt conventions of mathematical communication.

Mindful of the desire to value and develop teachers’ informal and formal mathematical experiences, I found the first of these assumptions resonated strongly with the notion of accessing learners’ innate powers and the last two strongly influenced my decisions to conduct the plausible estimation sessions with particular peer partners, and to require teachers to present and justify their findings to the whole group. The focus of analysis within this cycle of the research also moved onto data generated by two pairs of teachers within the group who fulfilled certain contrasting characteristics related to previous experience of teaching and learning mathematics.

Teachers M and N were confident in using higher level mathematical skills, had studied mathematics at university level and each had at least five years’ experience of teaching mathematics to adult learners mainly within further education settings. They were given the ‘mummies’ task in Figure 2.

An unravelling roll of paper is 33 metres or 100 feet long.

Will one roll be enough to wrap a person up?

![Figure 2. Mummies](image)

Teachers R and S were less confident in their mathematics skills and knowledge, had no formal mathematics qualifications beyond a foundation level and had quite recently become involved in teaching adults numeracy within their respective work-based training organisations. They worked on the ‘briefcase of pennies’ task in Figure 3.

Suppose you filled a briefcase with one penny coins.

How much money would you have?
Before considering in more detail what unfolded as these teachers engaged with their plausible estimation tasks over the next two week period, it is important to outline further theoretical frameworks which significantly impacted on both the conduct and analysis of data from this second cycle of enquiry.

**Realistic maths and mathematisation**

The idea of relevance and realism within mathematics teaching is complex and contested. Many authentic mathematics and real problem solving approaches advocate settings and situations which try to motivate and engage learners by using topics relevant to their immediate concerns. However Swain et al (2005) argue that is the quality of an individual's engagement with a problem that makes math meaningful rather than its utility or everydayness. Others, like Cooper and Dunne (2004) highlight the hidden rules younger learners must negotiate when tackling contextualised word problems and how these can adversely impact on learners from different cultural or social backgrounds.

**Realistic mathematics** is a term which better describes the sorts of tasks adopted within this enquiry and relates to an approach developed by Freudenthal (1991) which accentuates the actual activity of doing mathematics and advocates the power of learners to make things real for themselves by using their imagination. Such realistic tasks require learners to mathematise subject matter from real or realisable situations and reinvent mathematical insights, knowledge and procedures in the course of “their own mathematical activity rather than from the traditional view of presenting mathematics to them as a ready-made system with general applicability” (Gravemeijer cited in Barnes, 2004, p5). These situations can include contextual problems or mathematically authentic contexts for learners where they experience the problem presented as relevant and real. Such a process- rather than content-driven approach was consistent with the research focus of developing more relational and creative mathematical thinking and built on earlier project findings. The ‘plausible estimation’ tasks are examples of such realistic tasks.

**Horizontal and vertical mathematisation**

Additionally, in responding to the challenge that both learners and teachers often experience in trying to distinguish between concepts and procedures in mathematical thinking, Treffers (1987) developed the idea of horizontal and vertical mathematisation within this realistic maths framework. According to Freudenthal (1991, p41) horizontal mathematisation “leads from the world of life to the world of symbols” which Barnes (2004) suggests happens when learners use their informal strategies to describe and solve a contextual problem. On the other hand, vertical mathematisation occurs when the learners' informal strategies lead them to find a suitable algorithm or to solve the problem using mathematical language. For Freudenthal (ibid), this is where “symbols are shaped, reshaped, and manipulated, mechanically, comprehendingly, reflectingly”. For example, in the case of the ‘plausible estimation’ tasks, the process of establishing the important information required and using an informal strategy such as trial and improvement to arrive at an estimate would be horizontal mathematisation. Translating the problem into mathematical language through using symbols and later progressing to selecting an algorithm such as an equation could be considered vertical mathematisation, as it
involves working with the problem on different levels. This framework will be used within the analysis of data generated during teacher work on the ‘plausible estimation’ tasks.

**Analysis**

When first presented with these Fermi-type problems, teachers responded with surprise and some uncertainty:

**Extract 1**

R: Make it pound coins and I might have a go.

S: How are we going to do this then?

R: Haven’t a clue.

This intrigue right at the start of the tasks worked effectively to harness teachers’ emotions. Teachers M and N worked flexibly between mainly *iconic* or visual and *symbolic* modes of representation. They modelled a human being as a cylinder, deciding this would be the most effective way of minimising surface area rather than use a collection of smaller cylinders; agreed symbolic formulae for this surface area; assigned variables and established a direct proportional relationship between foot length and height.

R and S initially worked in an *iconic* mode with mental images of their own bags and briefcases, before looking around the room for more immediate concrete items which approximated their mental image of the briefcase on the task sheet. They stayed in this *enactive* mode, using a ruler to measure an actual briefcase and the diameter and depth of a 1p coin. Cooper & Dunne (2004) might argue that this is an example of teachers not understanding conventions for estimating and rounding. However, taking time within this mode allowed R and S to feel confident in their estimates before moving on to iconic and symbolic approaches. M and N, in contrast showed throughout the task that they knew and understood ‘the rules’ of the mathematics classroom:

**Extracts 2**

M: We’ve worked to the nearest whole number throughout so we should use 2m for the height.

And later:

N: The task just wants us to decide yes or no, so we’ve done it. We don’t need to work out how much more paper we’d need.

Ironically, their numerical estimates were the weakest chain in their initial argument as they got caught up in the mathematical conventions and opted for a height of 2m rather than their original 1.8m. The extension task which M and N completed required them to establish upper and lower bounds and brought the idea of *dimensions-of-possible-variation* (Marton and Booth, 1997) into play. By considering the *range-of-permissible-change* (op cit) for the heights of adults and children, constraints on their first model were established and a new model created which would incorporate a baby’s particular body shape and size. This extension task had extended the teachers’ mathematical thinking by requiring them to look for *invariance in the midst of change* (op cit). Overall, M and N demonstrated sophistication in their relational understanding and worked together to integrate relevant real-life experiences as they engaged flexibly between horizontal and vertical *mathematisation* of the problem (Treffers, 1987). The ease with which I was able to identify and analyse M and N’s responses to this task using the frameworks provided is also significant as it may provide an *account of* what I readily recognise and implicitly value as ‘mathematical thinking’.
R and S had a significantly different journey through the task. S began to *mathematise vertically* using an algorithmic procedure involving volume. She wanted to work out the volume of the case and then to divide by the volume of a single coin and remembered that $V = l \times w \times h$ “maybe”. R, on the other hand, began with *horizontal mathematisation* - drawing lines of pennies as he built up a sense of volume through layering. His approach required him to find out how many coins were needed to layer out the briefcase and he struggled to make sense of the approach adopted by S:

**Extract 3**

R: I’m just using a practical approach that I understand but S’s solution is more mathematical.

R’s implicit value judgment here about some sorts of mathematical thinking being more valuable than others resonates powerfully. Barnes (2004, p59) argues that using a formula does not necessarily imply better conceptual understanding and warns of the “danger of focusing too much on vertical mathematisation”. In fact, when R and S discovered that their initial results did not match, it was by using R’s *horizontal mathematisation* that they were able to establish that an error must have been made with S’s conversion rates. Indeed, R had a very solid conceptual understanding of volume whereas S had adopted *vertical mathematisation* mechanically but without Freudenthal’s (op cit) other two requirements: comprehension and reflection. This prevented her from spotting the common misconception made in converting between units of volume. It was by reverting back to R’s layering approach that both were able to work out that $1 \text{ cm}^3 = 10\text{mm} \times 10\text{mm} \times 10\text{mm}$ and come to an agreed plausible estimate. Having to articulate for the whole class their chains of reasoning, initial assumptions and ways of validating results helped both R and S strengthen their understanding of the general algorithms they had adopted, although more time to consider upper and lower bounds may have consolidated this further.

Although R and S were less confident in terms of their *intra-mathematical* skills, it is important to note that they brought a range of social, communication and meta-cognitive skills and experiences to the task process which allowed them to discuss, peer check and when necessary seek help from peers and tutor. They were tenacious, supportive of each other and prepared to take their time, progressing with small steps along repeated cycles of what Mason, Burton and Stacey (1985, p156) describe as “the helix of manipulating – getting a sense of – articulating” when thinking mathematically.

Both sets of teachers were provoked by the tasks to fall back on their own experiences and access a range of personal ‘everyday’ or *extra-mathematical* knowledge in diverse ways. M and N drew confidently on their experiences of child birth to establish estimates for the width of the head and the length of a new born baby. However, when asked how they might check their findings, neither M nor N wanted to try out their solutions. The intrinsic motivation and interest was in the *intra-mathematical* process – the accuracy of their final estimation all but redundant. M and N had quickly moved from the *real* world to their own *mathematical* world and intended to stay there. In contrast, when asked how they would check their estimate, R and S went straight back to *extra-mathematical* knowledge of their *real* world:

**Extract 4**

S: You’d not use volume at all – you’d empty the case and weigh them.

R: Just like they do in the bank.

This willingness (or not, in the case of teachers M and N) to re-engage with the *real* world scenario in order to evaluate not only the plausibility of the estimates found but the validity of the *mathematisation* process itself may have significant implications for the teachers’ own professional practice. Arguably, failing to reinterpret and validate mathematical results within *real* situations can
result in leaving unrealistic modelling unexposed. For some learners this sort of uncritical mathematical thinking does nothing to close ‘the gap’ between real and maths worlds.

When asked whether the ‘Pennies in a Briefcase’ task had been useful, R and S responded:

**Extract 5**

S: Yes, it got us thinking – we had to use lots of different sort of maths.

R: We’d forgotten lots. I think I understand units of volume better now.

Although these 'Plausible Estimation' tasks required only a basic knowledge of geometry, numeric skills and units of measure, the teachers did engage in more relational and connected thinking. Misconceptions related to conversions of units, use of appropriate formulae and rounding errors were identified through self and peer monitoring and teachers seemed to develop a more intrinsic feeling for the plausibility of their estimates. The value of developing conceptual and procedural knowledge in tandem seemed clear to all participants, and some teachers were also able to reflect more confidently and critically on their chains of reasoning.

**Coding framework for plausible estimates**

A more analytical comparison of the mathematical thinking and specific problem solving strategies the two pairs of teachers adopted in moving from real worlds to their maths worlds and (sometimes) vice-versa is difficult, not least because they were undertaking two different tasks. However, by adapting a framework devised by Arleback (2009), I was able to encode data from recordings of peer-peer discussions during the ‘Pennies in a briefcase’ and ‘Mummies’ tasks:

1. Reading: reading the task and getting an initial understanding of the task
2. Making model: simplifying and structuring the task and mathematising
3. Estimating: making estimates of a quantitative nature
4. Calculating: doing maths - performing calculations, solving equations, drawing diagrams
5. Validating: interpreting, verifying and validating results: calculations and the model itself
6. Summarising: summarising the findings and results in writing or orally

![Figure 4](image-url)

*Figure 4. Mathematical behaviours during 'plausible estimations'*
Figure 4 aims to capture a macroscopic and fairly dynamic picture of how teachers were *heard* to move between different ‘behaviours’ during the audio-recordings of the first 30 minutes of paired work on these tasks. Coded activities are identified within blocks, representing approximately 30 second time intervals. A whole group tutor intervention (WGI) took place after 15 minutes, and tutor interventions (TI) for particular pairs are also identified. X indicates where teachers have explicitly used *extra-mathematical* knowledge and experiences in diverse ways as outlined earlier.

Interestingly, although R and S had divergent calculation strategies during their tasks, the actual mathematical behaviours displayed in the diagram were similarly categorised within this framework as was the modelling stage which did not differentiate between horizontal and vertical mathematisation.

**Implications from Cycle 2**

Although the main value of these diagrams to me as a practitioner comes from the actual process and challenge of coding and categorising the peer-peer discussions, they do also provide some triangulation of earlier observations on how and when *extra-mathematical* knowledge is used, some new insights into the timescale of comparative progress through the tasks by both pairs, the frequency with which the teachers validated results and the time taken to summarise findings in preparation for articulation to the whole group. Arleback (2009) noted similar phenomena with his learners and observed that validation of results involved checking calculations, estimations *and* the initial model. However although both pairs of teachers here used articulation to summarise *and* peer validate their calculations, results and decision making processes throughout, M and N were more reluctant to ‘re-enter’ the messier *real* world once they had found a comfortable place of abstraction in their *maths* world.

**Cycle 3 Creating measures**

For the final cycle of the research, a second set of field-tested mathematical tasks were used. These aimed to prompt teachers “to evaluate an existing measure of an intuitive concept and then create and evaluate their own measure of this concept” (Ridgeway and Swan, 2010). A key component within this cycle would be the requirement for both pairs of teachers to test and evaluate any measures created back in the *real* world.

**Task design 3**

Requiring teachers to start from everyday concepts – steep-*ness, sharp-*ness, awkward-*ness, compact-*ness, crowded-*ness and square-*ness – to mathematise phenomena by creating their own measures seemed even more closely related to the experiences of awareness-raising in the first cycle of the enquiry and consistent with the sort of mathematisation and guided re-invention advocated by a *realistic* maths approach. As well as provoking mathematical thinking, I hoped these tasks would afford meaningful two-way connections between *real* and *maths* worlds.

Experiences during cycle two of the enquiry suggested that peer partners worked well together. This time however, I provided more scaffolding in the form of prompts in teachers’ work packs, so that tasks could be sustained and worked on independently. These included regular self-monitoring and reflection opportunities, consistent with the second and third assumption identified earlier as crucial to a *community of mathematical inquiry* (Goos et al, 2004). Teachers worked on these extended tasks in class each week for an hour over a three week period. Although they had individual work packs, pairs were expected to work collaboratively to reach a point where they would be able to go out on campus to test whether their measures actually worked. A written summary of findings ‘so far’ with commentaries, photographs and individual reflections on the creating measures process would provide evidence for teachers’ personal mathematics portfolios.
Before tasks were distributed, an introductory activity was undertaken to encourage teachers to consider themes, processes and specific features evoked by particular concepts:

- With your partner, take a few minutes to discuss what the concept of ‘sharp-ness’ means to you both.
- This might include thoughts, images, experiences, associations, special words or phrases, contexts or feelings.
- Use a concept map to record your initial responses.

Figure 5. Example of introductory activity for ‘creating measures’ task

When finally presented with their actual tasks, several teachers experienced what Mason & Johnston-Wilder (2006, p96) describe as “a contradiction of expectation” which they argue is a useful disturbance to provoke activity:

Extract 6

M: Oh, it’s nothing to do with pain or needles …

The actual ‘creating sharp-ness’ activity presented to teachers M and N is shown here:

Without measuring anything, put the four bends in order of "sharp-ness”.

Figure 6. ‘Sharp-ness’ Activity 1 Warm-up

This first activity specifically invited teachers to engage with iconic modes of representation. By inviting them to ‘look first, and act later’, I hoped that the teachers would use their own mental imagery and innate sense-making powers to identify similarities and differences between images, to specialise and generalise, order and classify and begin to become aware of some of the properties of the bends, or in the case of teachers R and S, the staircases which they might be able to explore later:
Without measuring anything, put the staircases in order of "steep-ness".

Figure 7. ‘Steep-ness’ Activity 1 Warm-up

Although these two dimensional images were less life-like than those used in the earlier ‘plausible estimation’ tasks, they were not conventionally mathematised to one-dimensional lines. Another feature of the classroom at the start of this third cycle of enquiry was the availability of mathematical equipment – tools for measuring, different sorts of paper including square, graph and dotty, calculators, counters, centicubes, etc. Indeed, all tasks required teachers to undertake some hands-on measurement, ensuring that everyone got involved at an enactive level, quite literally manipulating, constructing and measuring particular properties of their task concepts. Nunes et al (1993b) identify the significance of such measuring tools in supporting mathematical reasoning in younger learners and increasingly adult learners are being re-introduced to the power of multi-sensory approaches to mathematical sense-making. These tasks required that my teachers did the same.

Figure 8 shows how the learning objectives for the ‘steep-ness’ task were introduced to teachers R and S:

**Objectives**

This problem gives you the chance to:

- criticise a given measure for the concept of "steep-ness"
- invent your own ways of measuring this concept
- examine the advantages and disadvantages of different methods.

*Figure 8. ‘Steep-ness’ Task objectives*

**Analysis**

In their initial discussions on the staircases, R and S identified a range of factors influencing their perceptions of steepness: personal preferences about heights and depths of steps, fitness and stamina, carrying shopping bags, going up or down, taking single or multiple steps, individual heights and builds, disabilities, indoor or outdoor steps, surfaces, ‘length’ of staircases. Their considerations were very much rooted in the social context of the staircase journeys – who, why, when, where, how often. Rather than a straightforward exercise in finding gradients, R and S were tackling a much more complex modeling task within the real-world scenario they had created.
M and N on the other hand again moved almost immediately to abstract mathematisation, exploring how they could use trigonometry to create a measure of ‘sharpness’, focusing solely on angles and width with no consideration of other contextual factors. When prompted, they were able to generate other variables: roads, lanes, vehicles, weather, surface, speed, visibility, gradient, etc. but the relevance of these only really became apparent to them when they went outside to test their new measure in the messier real world. Figure 9 provides a brief account-of their measure for ‘sharpness’:

This may also convey some the unconscious assumptions and value judgements that I, as someone more comfortable within the abstract maths world of algebra myself, make about what mathematical thinking looks like. It certainly accounts-for some of my confidence that such realisable tasks can provide effective points of departure for diverse groups of teachers to engage in doing, thinking and communicating mathematically and to recognise what this engagement entails.

Although data from this third cycle of the enquiry provided many other textured examples of ways in which the ‘creating measures’ tasks provoked teachers to engage in mathematical thinking, I will finish by focusing on one further incident that was particularly significant and indeed disturbing:

**Extract**

S: Before today I thought I could look at a slope and know how steep it was. But when you do the measurements, you realise it’s different. I’ll never decide about steepness by eye again.

What had happened for this teacher to conclude that her intuitive understanding and experience of steepness in the real world was wrong? Data from the audio-recording of S and R’s work and stimulated recall interviews suggest that S drew this puzzling conclusion as a result of some very ‘logical’ deduction:

**Account 8**

R and S measure the height and slope of staircases on campus.

Back in the classroom, they use Pythagoras to calculate length.
They produce scale drawings - ‘staircase triangles’.
They measure the angles.
The steepest staircase isn’t the one they thought it would be.
You can’t trust your eyes to measure steepness.
Do it by measuring in future!

Ironically, R and S had no need to use Pythagoras at all but had been so excited in “finally understanding how to do it” that they built it into what was otherwise a reasonable algorithmic approach to measuring steepness, believing their calculations would be more accurate if they only had to use two real-life measurements. However, rather than consider that they might have made a calculation error, S instantly gave up her own internal sense of what a reasonable result should look like, trusting to the “power of mathematics” and in particular, the power of formulae, over-riding the evidence of her own eyes. R who was much less critically engaged in the process, was happy to concur with S and seemed unconcerned that evidence from calculations totally contradicted his initial observations ‘by eye’.

This episode suggests that for S, the world of formal maths although exciting was still very external to her own internal world. It also suggests something about how she valued different sorts of knowledge – with formal mathematical powers at the top of the hierarchy and her own at the bottom. It took time, considerable peer checking and more experiences of measuring and testing staircases around the campus before S’s mathematical and personal worlds began to reintegrate. R and S may have recovered from this incident but it continues to resonate strongly with previous personal experiences.

If learners override extra-mathematical understanding, how can they develop their ability to judge whether their answers are sensible and how often do they leave classes not knowing any more how to do something that made sense to them at the start of the lesson? In the case of R and S, the incident actually provided a sort of dissonance that generated another very fruitful point of departure. However, in a short time-restrained session where curriculum and assessment demands may prevent teachers and learners taking the time to move within this horizontal phase of mathematisation to deal constructively with misconceptions and bridge gaps between real and maths worlds, how damaging might this sort of mathematical experience be to learners’ self-confidence and self-concept?

While researching this phenomenon further, I found an article in which Meissner (2006) suggests that we have a number of internalised representations or micro-worlds which inform our subjective domains of experience. He identifies a reflective and subjective domain of experience (SDE) and argues that although both are important for flexible thinking one can often be more dominant over another, particularly when a new problem or conflict arises:

The individual prefers to ignore the conflict rather than modify the SDE or adopt another SDE. In mathematics education it is quite natural that an ‘analytical-logical’ behaviour remains dominant and that conflicting, common-sense experiences or spontaneous ideas get ignored.

(Meissner, 2006, p3)

This is an interesting theoretical construct with which to try to understand why R seemed relatively unperturbed by cognitive dissonance, while S was so easily enticed to relinquish her own common sense experiences.

At this stage of the enquiry then, my initial disappointment that carefully selected and adapted mathematical tasks had resulted in some teachers dismissing rather than valuing their own intuitive mathematical powers, was tempered by the fact that engagement with these same tasks had generated
phenomena that provided insight into another interesting and valuable point of departure related to how we move between and within our formal and informal, real and mathematical worlds.

**Summary discussion**

The initial focus of this action research project was to improve practice in supporting adult numeracy teachers develop and extend their own mathematical thinking. At each stage of this inductive process, as a participant observer I have collected, reflected on and evaluated data related to teachers’ responses to a series of research design tasks. In particular, using audio-recordings to reflect on classroom discourse during collaborative work on mathematical tasks and in oral presentations to peers generated evidence of rich, cyclical and non-linear problem solving and mathematical thinking processes. It was a real privilege to listen to teachers interacting together with energy, trust, humour, perseverance, intelligence and humanity.

During this enquiry, I hoped to gain insights into a group of adult numeracy teachers’ mathematical thinking but learned a great deal more about my own assumptions, beliefs, and expectations. In focusing on the quality of my own interventions and interactions with teachers, I need to recognise that I can be just as mechanistic and instrumental in supporting work on mathematical tasks as they can be in solving them. I also recognise, value and am more likely to favour mathematical thinking and behaviours which mirror my own formal mathematical experiences and interests and need to be fully conscious of this if I am to further develop my own inclusive practices in supporting teachers to develop mathematical thinking.

Teachers and tutors come to formal mathematics classrooms with funds of knowledge, which include diverse and contingent informal numeracy practices which are culturally and socially situated. These often go unrecognised, are not valued or are held subconsciously. Raising awareness of these through systematic reflection can provide valuable insights into hidden personal and interpersonal resources and propensities which can be harnessed or challenged to support teachers’ own mathematical thinking and, hopefully, their professional practice.

More enactive and iconic approaches can open up or close down possible lines of inquiry in unexpected ways. Similarly, tasks which specifically require teachers to take more time in manipulating and getting-a-sense of the mathematical structures of a problem, though often more time-consuming, are less likely to result in teachers adopting mechanistic or instrumental approaches.

**Conclusion**

What unfolded during this small-scale practitioner enquiry suggests that doing realistic mathematics tasks within a community of inquiry can provoke a range of mathematical thinking and learner responses. These allow us to identify ways in which procedural and conceptual thinking can be used within horizontal and vertical mathematisation, and how learner journeys can be tracked through different stages of problem solving. Such tasks can also provide meaningful starting points to teachers with varying levels of prior mathematical experience. However, teacher and tutor beliefs and assumptions about what constitutes mathematical behaviour can support or constrain the intent and ease of movement within and between their real and mathematical worlds, and vice versa. While teachers with more experience of mathematics could do this flexibly, despite preferences and predispositions to reside in more formal mathematical mental environments, others with less confidence or less well developed intra-mathematical knowledge and skills dismissed their own innate sense-making and extra-mathematical knowledge too readily, with mixed success.
Recommendations for future practice

Adult numeracy and mathematics teacher education courses need to support students to engage regularly in a variety of sustained, open-ended and realistic mathematical tasks, with further extended tasks signposted for independent study.

If teachers are to develop greater awareness of what mathematical thinking looks, feels and sounds like, more self and group reflection and evaluation tasks need to take place with explicit reflections on inner, outer and meta-tasks encouraged within personal maths portfolios and group discussions.

New mobile technologies are being used increasingly and naturalistically within sessions: listening to, watching and analysing targeted audio- and video-recordings of engagement in their own mathematical thinking tasks will support teachers to develop awareness of awareness further.

The key literature, frameworks and constructs which informed the context and conduct of this enquiry along with the specific mathematical tasks used could be shared and contribute to reading lists used on other adult numeracy teacher education courses.

Throughout this paper, there has been an underlying assumption that developing teachers’ confidence, awareness and insight into their own mathematical thinking, will better equip them to develop and extend the mathematical thinking of their learners. Adult numeracy teacher educators need to identify and value further opportunities for students to explicitly evidence and reflect on how they are using their own experiences of thinking and acting mathematically to inform their practice with learners.

References


