Objectives

Adults Learning Mathematics (ALM) – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

The ALM International Journal is published twice a year.

ISSN 1744-1803
© Adults Learning Mathematics – An International Journal (2011)

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Adults Learning Mathematics – An International Journal

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Reflections on our first calculus undergraduate teaching assistant
Jessica M. Deshler
I am glad to introduce a new issue of *Adults Learning Mathematics: An International Journal*. This edition includes articles that focus on the realities and experience of adult learners with regards to the supports they need to become numerate and to solve problems in work and in life. They also provide some guidance for practitioners to help them support and transform, not only practice in mathematics, but the learners themselves. Transforming mathematics is a challenge for many adults who see this topic as a source of difficulties, anxiety, and negative feelings (Klinger, 2005). Current research in our field highlights the fact that some adults decide to change such feelings and adopt a positive disposition towards mathematics and mathematics learning. According Ramon Flecha (2000) it is possible to turn difficulties into possibilities (Flecha & Soler, 2013). In order to do that, we need to start from our dreams (Freire, 1998), but also from scientific evidence that inform us about what we need to do and how we need to do it (Flecha, Soler-Gallart, & Sordé, 2015).

Hassi and Laursen (2015), in a recent article, introduce the idea of personal empowerment as a form of *transformative learning*. According to these authors, empowerment is essential for adults to face the difficulties associated with learning mathematics; in particular, self-empowerment, cognitive empowerment, and social empowerment are critical areas of empowerment that adults need. In addition, many adult learners often need support from their own peers to help overcome their fears and doubts when going back to the “school.” This support becomes the cornerstone for them to develop their innate [mathematical] skills at least in two different ways: (1) from the cognitive point of view, as well as from (2) the affective dimension. The articles presented in this edition offer the readers some important perspectives for examining the supports adults need to advance their development in mathematics.

The first article in this issue explores the use of an educational mechanism used by practitioners in Ireland to help both traditional students and adult learners to improve their mathematics skills within the context of higher education. Fitzmaurice, Mac an Bhaird, Ni Fhloinn, and O’Sullivan discuss data from a survey on the evaluation of Mathematics Learning Support (MLS) in Ireland. In Ireland, the concept of MLS is a mechanism or strategy designed for students who want to receive a surplus to their regular courses. Many adult learners decide to take this type of support mainly because they feel they need extra-support due to the fact that they have been away from the school for many years; they stopped studying mathematics many years ago; or because they feel they are bad for mathematics. For many adult learners, the MLS becomes a real alternative resource that helps them to reinforce their mathematics’ background and develop some type of *mathematics* empowerment. The article address issues such as the motivational factors of adult learners who seek or decline MLS, and the authors provide consistent evidence to justify the value of such type of courses.

The second article in this issue focuses on the mathematics curriculum vis-à-vis rational numbers and problem solving. “Rational numbers” are difficult to understand, due to their mathematical polysemy or the multiple meanings they reflect (Behr, Lesh, Post, & Silver 1983, Bright, Behr, Post, & Wachsmuth 1988, Dufour-Janvier, Bednarz, & Belanger 1987, Mack 1995, Novillis 1979). In fact, the idea, “rational numbers” is not one that is, conceptually, easy to process. Many studies have attempted to define the rational numbers, (Vergnaud 1988) of rational numbers (Behr, et al., 1983, Kieren 1976, Noelting 1980a, 1980b), and the theoretical and intangible bases for
understanding what they are. Doyle, Dias, Kennis, Czarnoche, and Baker draw on Kieren’s (1976) presented a definition of rational numbers and discuss how a group of adult pre-algebra students deal with the idea of rational number as part-whole, ration, operator, quotient, measure, and fraction equivalence within the frame of problem solving activities. In that work, Doyle et al. (1976), addressed two different questions: (1) whether rational numbers are (or are not) sub-constructs providing a foundation for students to solve problems related to proportional reasoning, and (2) how students use such numbers to deal with ratios, proportion, and percentage when solving problems intuitively.

In this, current, edition, Doyle et al. build on previous work and present four different situations in which adults deal with problems related to proportional reasoning, and combined formal and informal approaches to solve the questions. Results show that adults use visual representations to make sense to the problems and even find suitable solutions for these problems. The results also suggest that visual representation is closely connected to informal reasoning, and in some cases, it acts as a [cognitive] bridge between the formal concept and the informal strategy used to solve the problem. This work not only validates that of Behr et al. (1983) extension of the Kieren sub-constructs to competency with problem solving and Lamon’s (2007) claim that rational number sub-constructs provide foundation for proportional reasoning; it also opens the window for further research focused on how rational numbers (or other mathematical constructs) may help adult learners to solve problems at the same time they are learning them or (in the terms of Doyle et al.) developing the concept of rational number.

The third article in this issue moves our attention towards the realm of numeracy in everyday contexts. Marks, Hodgen, Coben, and Bretscher introduce work based on the analysis of nursing students’ experiences of learning numeracy for professional practice. They discuss the disconnection between how numeracy is taught and assessed in the university versus real life situations where nurses must do calculations in order to conduct their professional practice—calculations that have real life implications and real consequences in terms of social impact of the practice. Oftentimes, the mathematical proficiencies suitable for the classroom is not sufficient for solving real situations in which professionals (in this case the nurses) make decisions that have life or death implications.

Marks et al. explore the tensions between numeracy in training/schooling and numeracy in practice, and set up a disjuncture between both “types of numeracy” to show that when school math lacks authenticity, it may drive students to see their mathematics courses as irrelevant or disconnected to real practice. In addition, Marks et al. provide evidence (from students) to show that the mathematical learning content in school is not always reflective of the mathematical realities of nursing; specifically, the mathematical learning content in many nursing programs is often not relevant for nursing, because it does not reflect the type of knowledge embedded in nursing practices. The students also found that calculators (machines) do the work for them, which is also discouraging. In this work, Marks et al. present some interesting analyses that have some important implications for how we teach our future professionals to address mathematical needs of their respective professions.

The fourth, and last, article in this issue is devoted to the topic of teacher training in higher education. Deshler presents an ethnography involving what she calls a first calculus undergraduate teaching assistant (UTA), Ann. This work involves the collection of qualitative data of Ann’s “steps” through her experience in being a graduate teaching assistant (GTA). Deshler explains how Ann became more and more engaged in teaching responsibilities, inside and outside the classroom, and how such engagement enabled her to consolidate her mathematical knowledge. Although Deshler’s work does not delve into specific mathematical content, per se, it presents an important example of how to build the mathematical teaching force in a way that benefits both the system and the individuals involved. For example, the role of any assistant is an apprenticeship that bodes well for the advancement of knowledge. In this case, the university and the program benefit from having Ann’s services as a teaching assistant, but Ann also benefits through the experience and learning derived from her role as an assistant. In essence, what Deshler presents is real life reflection on mathematical peer mentorship—where practice is modelled by a teacher, through a teaching assistant who provides support to her peers. This type of practices could be of great value for recent graduate students who would like to gain experience (and even confidence) using and teaching mathematics.
I hope that the readers will find all of the articles in this edition to be interesting. To touch back at the beginning of this editorial, all of the articles are particular snapshots of different aspects of adults learning mathematics. These works remind us that adult education is more than just a “second chance” for adults to further develop their academic skills; it also reminds us of the plethora of characteristics of adult learners, which include young adults who dropped out the system and adults who are in HE. The reader would notice that regardless of the learner characteristics, mathematics appears to present some challenges for all of them (in many different ways). Overall, what the four articles suggest is that we need more research to understand and meaningfully organize the cognitive, affective, and probably epistemological features embedded in the act of learning mathematics in adulthood. The four articles included in this issue open up the floor for readers to consider different approaches and concerns related to these matters. Enjoy!

References


Adult learners and Mathematics Learning Support

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Abstract
The provision of some level of Mathematics Learning Support is now standard in the majority of Higher Education Institutions in Ireland and the UK, and it is also available in many other countries. This provision is, in part, a response to the large numbers of students entering Higher Education who do not have the mathematical skills required and this cohort includes a significant number of adult learners. Research indicates that these students have different motivations and approaches to learning than traditional age learners. This paper considers the analysis of a large-scale student evaluation of Mathematics Learning Support in Ireland. In particular, it presents the responses and engagement levels of adult learners and compares these to those of traditional students. The findings are key to ensuring best practice in the provision of Mathematics Learning Supports for the wide variety of students who avail of it.

Keywords: Adult learners, engagement, evaluation, mathematics learning support

Introduction
The availability of some form of Mathematics Learning Support (MLS) is now what students can expect to find in the majority of Higher Education Institutions (HEIs) in Ireland and the UK. MLS is also available in HEIs internationally, for example in Switzerland, Canada and Australia (Gill et al., 2008; Perkin et al., 2012). MLS has been defined as a facility offered to students which is surplus to their traditional lectures and tutorials, the purpose of which is to offer non-
judgemental and non-threatening one-to-one support with mathematics (Ni Fhloinn, 2007; Lawson et al., 2003; Elliot and Johnson, 1994).

The main reason for the establishment and significant growth of MLS is as an approach to addressing the well documented ‘Maths Problem’. One of the ways O’Donoghue (2004) defines the ‘Maths Problem’ refers to the mathematical preparedness of incoming students in terms of their mathematical shortcomings or deficiencies at the university interface. Significant numbers of students entering HEIs are deemed as at-risk of failing or dropping out because they do not appear to be appropriately prepared for mathematics in HE and they often exhibit very weak mathematical backgrounds. This ‘Maths Problem’ is common place in HEIs in Ireland, the UK and internationally (Gill et al., 2010; Lawson et al., 2012). These at-risk students are main target of MLS.

One benefit of the economic downtown has been the welcome increase in adult learners returning to HE (Golding and O’Donoghue, 2005). In the Dublin Institute of Technology (DIT), adult learners constituted one fifth of the attendants at the Mathematics Learning Support Centre (MLSC) in its opening year (Ni Fhloinn, 2007). In 2012 Adult Learners accounted for 15.3% of full time students enrolled in HE in Ireland and 21% of full and part time students. Faulkner et al. (2010) stated that the presence of so many adult learners is one contributing factor to the increased numbers of at-risk students in first year courses.

In order to establish best practice in the successful provision of MLS, it is essential that it is comprehensively evaluated on a regular basis (Matthews et al., 2012). For example, quantitative research suggests that appropriate engagement with MLS can have a positive impact on student retention and progression (Lee et al., 2008; Mac an Bhaird et al., 2009). One of the initial aims of the Irish Mathematics Learning Support Network (IMLSN), which was established in 2009, was to conduct a large scale survey of student opinion on MLS.

Given the increasing proportion of adult learners in mathematics in first year courses, it was considered key that they should be identifiable in the survey so that their responses regarding the evaluation of MLS could be studied in detail. In particular in this paper, after a description of relevant literature and the methodology of the survey, we focus on adult learners’ responses, their backgrounds and we compare them to the overall cohort. The main research questions we are trying to address are:

1. What are the motivational factors of adult learners who seek MLS?
2. Why do some adult learners of mathematics not seek MLS?

**Literature review**

There is a concern that a lack of preparation in mathematics can lead to increased failure rates and low self esteem (Symonds et al., 2007) in HEIs. Aligned with that is a worry of impeding students in the study of other disciplines, e.g. engineering, science, etc. (Pell and Croft, 2008; Gill, 2006). Many students arrive in their HEI having chosen mathematics-intensive courses unbeknownst to themselves (FitzSimons and Godden, 2000). Most degree programmes, even non-specialist mathematics degrees, contain some mathematics and/or statistics component, as prospective employers require graduates to be proficient in mathematics, with some even setting numeracy tests as part of their selection process (Lawson et al., 2003). The mismatch between the knowledge of many students and the expectations of HEI teachers is one contributory factor to the problem and this mismatch arises partly through the increase in diversity of the backgrounds of students (Lawson et al., 2003; Faulkner et al., 2010). Diversity in the standards
of teaching and class size in HEIs tend to exacerbate the situation (Lawson et al., 2003; Gill, 2006).

One of the key responses to the ‘Maths Problem’ was the opening of Mathematics Learning Support Centres (MLSCs) to attempt to deal with the mathematical shortcomings of students (Pell and Croft, 2008; Gill, 2006). In 2004 in the UK it was reported that 62.3% of 106 surveyed universities offered some form of MLS (Pell and Croft, 2008, p168). In 2012, this number had jumped to 85% (Perkin et al., 2012). In 2008, an audit carried out by the Regional Centre for Excellence in Mathematics Teaching and Learning (CEMTL) in Ireland demonstrated that 13 out of 20 HEIs provided MLS in some form (Gill et al., 2008). Six years later, it is believed that this number is much higher. Most MLSCs are committed to servicing the needs of traditional and non-traditional (i.e. international and adult learners) students (Ní Fhloinn, 2007; Gill and O’Donoghue 2006). Carmody and Wood (2005) reported on the benefits of a drop-in MLSC for easing the transition to HE for first-year students. The drop-in centre caters for students from all faculties and has become a meeting place for collaborative learning. Tutors use a variety of teaching methods and resources, which is easier to do in a one-to-one situation than in front of a large class. Engagement with MLS has been shown (through mostly quantitative research) to impact positively on mathematics performance and grades and retention (Burke et al., 2012; Mac an Bhaird et al., 2009; Pell and Croft, 2008; Symonds et al., 2007). Pell and Croft (2008) state that while MLS is provided first and foremost for ‘at-risk’ students, it is more often the case that users tend to be high achievers working to attain high grades, a view supported by Mac an Bhaird et al. (2009) who have also shown that many ‘at-risk’ students still do not engage with MLS.

An adult learner, or Mature Student, is classified in the Republic of Ireland as a student that is 23 years of age or older on 1st January of the year of registration to HE (Ní Fhloinn, 2007). Entry for adult learners who have not got the minimum requirement for entry to their chosen course of study is usually gained via interview and is based on a number of factors including life experience and motivation, in addition to prior qualifications. Faulkner et al. (2010) studied the student profile in service mathematics programmes at the University of Limerick (UL) since diagnostic testing began there in 1997. The increase in adult learners of mathematics in these modules was quite pronounced. In 1997 there was one registered adult learner in Science and Technology Mathematics, two of the biggest service mathematics modules provided by this university; in 2008, there were at least 55 adult learners. This statistic is supported by Gill (2010) who states that in 2009/10, adult learners in UL constituted 14% of the entire cohort, a jump of 49% on the previous year. In 1997, 30% of students in service mathematics modules at UL were deemed to be at-risk. Fast forward to 2012 and 61% of students in the same modules are categorised as at-risk.

Adult learners in mathematics who return to education constitute a heterogeneous cohort. For example, participants on the ‘Head Start Maths’ bridging programme at UL range from 23 to over 45 years of age. A significant number of the students on the programme in 2008 had not studied mathematics in any formal sense for up to 20 years and 30% of participants had not taken the Leaving Certificate (LC) examination (Gill, 2010). The LC is the terminal examination taken by pupils at the end of secondary school in Ireland. Mathematics is compulsory for students and can be taken at three levels: Higher (HL), Ordinary (OL) and Foundation (FL). In DIT, Ní Fhloinn (2007) outlines how adult learners fall into the full-time, part-time or apprenticeship categories, with each type of student presenting with different characteristics and issues relating to their preparation, their approach to learning mathematics and confidence issues. For many adults returning to HE, mathematics presents an obstacle. Many find the idea of studying mathematics intimidating and this can have a potential negative impact on their mathematics confidence and subsequent performance (Golding and
O’Donoghue, 2005). Diez-Palomar et al. (2005) acknowledge the difficulty for adult mathematics education in efficiently addressing the needs of diverse cohorts. It can be very difficult for students to catch up with forgotten fundamentals and keep up with current studies simultaneously (Gill, 2010; Lawson et al., 2003).

Under-preparation of adults in mathematics is a grave issue in HE (FitzSimons and Godden, 2000) as students with an array of previous qualifications, on vastly different courses with a series of attainment and performance levels often present with a range of problems (Elliot and Johnson, 1994). Research tells us that many adult learners of mathematics exhibit maths anxiety when faced with mathematical tasks and can lack confidence in their mathematical abilities (Gill, 2010; Ni Fhloinn, 2007). This anxiety may impact adversely on their participation and performance in mathematics activities (Ashcraft, 2002). In fact Gill (2010) reported that mathematics is often the main worry/concern of students returning to university. Singh (1993) attributes this anxiety on the part of adult learners partly to examinations and a fear of failure. It has been well documented that mathematics learning is related to student confidence in their abilities (Coben, 2003). Many adults who are well capable of learning mathematics are inhibited from doing so because of their fear of the subject (Benn, 2000).

Diez-Palomar et al. (2005) and O’Donoghue (2000) acknowledge the difference between adult learners of mathematics and traditional learners. Adult learners carry with them an abundance of experiences that need to be considered in pedagogical practices. This view is supported by Tusting and Barton (2003) who add that adult learners have different motivations for studying than traditional learners and are more inclined to be autonomous and reflective learners. The decision to return to education has generally been their own decision and a deliberate one (FitzSimons and Godden, 2000). Though adult learners may lack confidence in their own abilities, they tend to be highly motivated (Ni Fhloinn, 2007; FitzSimons and Godden, 2000). Traditional lectures and assessments are not conducive to learning for many adult learners (Gordon, 1993 cited in FitzSimons and Godden, 2000) so many rely on MLSCs for support. In 2009/10 adult learners of mathematics at UL constituted 54% of the attendance at the drop in centre, even though they represented just 14% of the entire student population (Gill, 2010).

While the importance of research in the teaching and learning of mathematics among adult learners has been duly recognised in recent years (Coben, 2003) it remains an ‘under theorised and under researched’ area (Galligan and Taylor, 2008, p99). Furthermore, research conducted on the teaching and learning within MLSCs is sparse (Galligan and Taylor, 2008).

**Methodology**

The IMLSN was established in 2009, and its guiding principles are similar, on a smaller scale, to the leading experts in the provision of MLS, the sigma (The Centre of Excellence in Mathematics and Statistics Support) network (http://sigma-network.ac.uk/) based in England and Wales. The IMLSN aims to support individuals and HEIs involved in the provision of MLS in Ireland. Once set up, the network decided it should promote the benefits of MLS to both staff and students on an institutional, national and international basis and agreed that a student survey was the best approach initially. The IMLSN asked the panel of researchers listed on this paper to undertake this student survey.

Student questionnaires are commonly used in the evaluation of MLS services (Lawson et al., 2003) in individual HEIs, so it was decided to create a student survey that could be used in all HEIs which provide MLS. HEIs who already distributed questionnaires on MLS were invited to submit them to the committee; these were amalgamated and a communal
questionnaire was formed as a result. This questionnaire was piloted in 4 HEIs with 100 students and subsequently refined based on analysis of the findings and expert statistical advice.

The resulting questionnaire had 17 questions, a combination of open questions and questions which required a response on a 5-point Likert scale. There were three main sections: Section A determined the students’ backgrounds; Section B focused on users of MLS; and Section C focused on non-users of MLS. First year service mathematics classes have the largest percentage of at-risk students and are the main target of MLS in terms of student retention and progression, so it was decided to issue the questionnaire to these cohorts only. Evaluation sheets are usually distributed within MLSCs but this can lead to bias as users already rate the MLSC to some extent if they attend it (Laws on et al., 2003). With this in mind, it was decided that the questionnaire should be issued in appropriate lectures to get a blend of user and non-user feedback and to reduce bias. The questionnaires were anonymous and there were no identifying characteristics. The questionnaire, in the first large scale survey of its kind, was issued to members of staff involved in the provision of MLS in HEIs in Ireland and they were asked to distribute paper copies in the appropriate first year service mathematics lectures during the second semester of the 2010-11 academic year. Service mathematics refers to users of mathematics (e.g. engineering, science, business etc.), rather than mathematics specialists (e.g. pure or applied mathematicians) (Burke et al., 2012).

The HEIs surveyed were Universities or Institutes of Technology (IoTs), and these have different and complementary roles and missions within HE in Ireland. At undergraduate level Universities focus on Level 8 (Honours Degree programmes), and IoTs emphasise career-focused HE offering Level 8 programmes but also programmes Level 7 (Ordinary Degrees) and Level 6 (Higher Certificates). IoTs also have a larger proportion of adult learners and students from disadvantaged areas and are stronger than the Universities in part-time and flexible provision (http://www.hea.ie/en/node/981). In the IoTs that participated in the survey, the ratio of Level 8: 7: 6 students was 49:38:11% which is similar to the 53:37:9% proportion of Level 8: 7: 6 students in IoTs nationally in the 2011-12 academic year.

A total of 1633 completed questionnaires were returned from 9 HEIs (5 Universities and 4 IoTs) comprising enormous quantities of both qualitative and quantitative data. Two graduate students were hired to input the data into SPSS, and SPSS was also used to analyse the quantitative data. NVivo was used to analyse the qualitative data. A general inductive approach was used to analyse the data guided by the specific research questions (Thomas, 2003). Data was read and analysed by two researchers independently, one from this panel of researchers and an external person to identify emerging themes. Further details on the analysis to date for all respondents (traditional students and adult learners combined) can be found in (Mac an Bhaird et al., 2013; Ni Fhloinn et al., 2014; Ni Fhloinn et al., to appear).

**Results**

In Section A of the survey questions were asked which focused on students’ backgrounds. Of the 1633 respondents, there were 221 (13.5%) adult learners, 73% of these were male and 91% were full-time students. In terms of students’ mathematical background, they were given the 4 options outlined in Table 1. Generally, a minimum of OL mathematics would be needed for most service mathematics courses in HEIs and this is reflected among respondents with only 18 of the 1563 students who provided their LC results in the survey having studied mathematics at FL. If they had not taken the LC, then they could select the “other” option.

A lower percentage of adult learners (than of the overall respondents) had taken HL, and higher percentages (compared with the overall) in the remaining three categories, with the majority studying mathematics at OL.
When the breakdown of the disciplines that students were in was considered, we found, for most discipline areas, the proportion of adult learners was in line with the overall proportions of survey respondents, see Table 2.

### Table 2 Degree Programmes of Adult Learners and of overall survey respondents.

<table>
<thead>
<tr>
<th>Subject</th>
<th>No. of Adult Learners</th>
<th>%</th>
<th>No. of Respondents</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science</td>
<td>80</td>
<td>36.2</td>
<td>583</td>
<td>35.7</td>
</tr>
<tr>
<td>Engineering</td>
<td>50</td>
<td>22.6</td>
<td>236</td>
<td>14.45</td>
</tr>
<tr>
<td>Business</td>
<td>55</td>
<td>24.9</td>
<td>484</td>
<td>29.64</td>
</tr>
<tr>
<td>Arts</td>
<td>7</td>
<td>3.2</td>
<td>67</td>
<td>4.10</td>
</tr>
<tr>
<td>Education</td>
<td>6</td>
<td>2.7</td>
<td>90</td>
<td>5.51</td>
</tr>
<tr>
<td>Computing</td>
<td>23</td>
<td>10.4</td>
<td>171</td>
<td>10.47</td>
</tr>
<tr>
<td>Total</td>
<td>221</td>
<td>100.0</td>
<td>1631</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Section B focused on MLS users. The majority of adult learners 136 (61.5%) availed of MLS, compared to only 32.2% of traditional learners. A Chi-Squared Test for independence indicated a statistically significant association exists (p<0.001) between type of student (i.e. Adult or traditional learner) and whether a student uses MLS, adult learners were more likely to seek MLS than traditional learners. In terms of gender, 68.3% of female adult learners compared to 43% of female traditional learners used MLS, and 59.4% of male adult learners in comparison to 23.3% of male traditional learners availed of MLS.

The mathematical backgrounds of both users and non-users of MLS among the adult learner sample were very similar, and the percentage breakdown was close to that of the adult learner population (see Table 1). When we considered subject discipline, the proportions of adult learners using MLS was very similar to the proportions of overall adult learners in each subject discipline (see Table 2).

Students who availed of MLS were asked, in an open-ended question, to comment on why they first decided to use MLS. There were 577 comments from attendees which were coded using GIA and the majority fell into 6 main categories as outlined in Table 3. This table contains comments from 122 of the 136 adult learners who responded.

### Table 3 Frequency of adult learner reasons for using MLS

<table>
<thead>
<tr>
<th>Categories of comments</th>
<th>Frequency of comments (n=122)</th>
<th>Sample comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra help</td>
<td>38.52%</td>
<td>“Needed help with maths”, “I had gone to the tutorials and still had trouble with a particular area”, “I wanted help with a maths problem and to understand where I was going wrong”, “Because the pace of the main lectures were too fast and I wasn’t keeping up”, “I had to catch up on missed lectures”</td>
</tr>
</tbody>
</table>
Table 3 Frequency of adult learner reasons for using MLS (…/…)

<table>
<thead>
<tr>
<th>Categories of comments</th>
<th>Frequency of comments (n=122)</th>
<th>Sample comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background/Ability:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comment about being</td>
<td>19.67%</td>
<td>“Hadn’t done maths in ages so I needed extra help”, “Because I haven’t studied maths in ten years and really felt quite daunted by the thoughts of returning to study maths”, “Coming back to study after a long break, needed all the help at hand!”, “Because I am not great at maths”.</td>
</tr>
<tr>
<td>from Maths for a while</td>
<td></td>
<td></td>
</tr>
<tr>
<td>prior to entry (from</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mature students) or</td>
<td></td>
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<tr>
<td>comment suggesting</td>
<td></td>
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<tr>
<td>poor confidence in</td>
<td></td>
<td></td>
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<tr>
<td>maths ability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assignments/Exams:</td>
<td>13.93%</td>
<td>“Struggling with maths assignments”, “I was stuck on understanding a part of an assignment and was spending a lot of time trying to figure it out”, “To help with revision”.</td>
</tr>
<tr>
<td>Looking for help with</td>
<td></td>
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<tr>
<td>specific aspect of</td>
<td></td>
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<tr>
<td>coursework assessment</td>
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<tr>
<td>during the semester</td>
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<tr>
<td>(upcoming test,</td>
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<tr>
<td>assignment) or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>attending for revision</td>
<td></td>
<td></td>
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<tr>
<td>or prep for end of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>term exams</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Struggling</td>
<td>9.02%</td>
<td>“I was struggling with the subject”, “Was lost with maths”.</td>
</tr>
<tr>
<td>Improve Understanding:</td>
<td>5.74%</td>
<td>“Because I thought it will be a great idea to use drop-in clinic if I want to get good grades.”</td>
</tr>
<tr>
<td>Positive comments about</td>
<td></td>
<td></td>
</tr>
<tr>
<td>attending to try to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>improve or gain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>better understanding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics Difficult</td>
<td>2.46%</td>
<td>“Because I find maths very difficult.”</td>
</tr>
</tbody>
</table>

A comparison of the frequency of responses in each category given by adult learners compared with the overall population of users provides some interesting differences. The frequency of responses from adult learners showed they are much more likely to make comments indicating that they:

- look for help as they have a long time away or suggesting poor confidence in their mathematical ability (19.67% as against 7.45% frequency of response),
- seek general extra help (38.52% as against 20.62% frequency of response)
- are struggling (9.02% as against 5.03% frequency of response).

In contrast, the frequency of responses from adult learners shows they are much less likely to make comments indicating that they:

- seek help specifically to get assistance with particular coursework assessment or revision for tests (13.93% as against 41.25% frequency of response)
• attend MLS to improve or gain better understanding (5.74% as against 15.94% frequency of response).

• state they find mathematics difficult (2.46% as against 9.71% frequency of response).

MLS users were asked to rate, on a 5-point Likert scale, the specific services available in their HEI and they were also given the opportunity to comment. The main support offered was a drop-in centre, so we will focus on that support in this paper. The distribution of ratings and responses from adult learners for the other services (e.g. ICT supports, workshops, support tutorials) are in line with that of the overall cohort.

All nine HEIs had a drop-in centre and 519 users rated them. 119 were adult learners and 89% of these rate it as worthwhile. There were 244 additional comments, 57 from adult learners and coding of responses placed them into the following three main categories:

• 20 (38.5%) relating to satisfaction levels with the service provided, 19 of which were positive, “Very helpful – I am even starting to enjoy maths now”, “Would not have a clue what I was doing if it was not for support”.

• 23 (40.4%) related to the physical resources, including staff and contact hours of the centres. Without exception, all comments stated that all of the above should be extended, “Class size was small for the amount of students”, “If there were more opening hours and people available as it is very busy” and “Sometimes a long waiting time; too busy”.

• 9 (17.3%) related to the quality of tutors; 5 positive, 1 negative and 3 which were positive and negative, “Always as helpful as they can be with the exception of one of the tutors who tends to be very rude and arrogant”.

In Questions 11-15, MLS users were asked about their perception of the impact of MLS on various aspects of their education, the questions had a 5-point Likert scale and they could also comment on their answers. Students were asked to rate the impact of MLS had on their confidence. 539 users responded, 124 were adult learners and 67% of these rated the impact as helpful in comparison to 56% of all users. There were 106 additional comments, 21 from adult learners with 20 of these positive, “It has helped me a lot. I don’t need to struggle alone to figure out things that I don’t understand”, “Still find it difficult but have a better understanding of maths”. For all users, approximately 75% of comments were positive.

Students were also asked if MLS had impacted on their mathematics performance in tests or examinations to date. There were 526 responses, 115 from adult learners and 65% of these stated that it had an impact, in comparison to 56% of all users. There were 103 additional comments, 21 by adult learners, 16 of which were positive (90% of overall comments were positive), for example: “I would have failed if the extra help had not been there”.

Students were asked to rate how MLS had helped them cope with the mathematical demands of their courses. There were 530 responses, 119 from adult learners and 72% of these indicated that MLS had been helpful in comparison to 65% of all users. There were 55 additional comments, 14 from adult learners, 12 of which were positive, for example “It has been a huge help”, “Wouldn’t be able to do maths without all the extra services and wouldn’t have a hope of passing the year. One of the (two) negative comments stated “Some of the tutors in the centre might be good at understanding maths but not good at teaching it”.

In Question 11 students were asked if they had ever considered dropping out of their studies for mathematics-related reasons. 128 of the 136 adult learners answered this question with 25 (19.5%) stating that they did consider dropping out, this is a similar proportion to that of
the overall student population. Question 12 asked (those who answered yes to Question 11) if MLS had been a factor in them not dropping out. 22 of the eligible 25 adult learners answered and 17 (77%) of these stated that MLS was an influencing factor in their decision not to drop out (compared to 62.7% of the overall population). Additional comments included: “Greatly. It has given me the confidence to turn maths as my worst subject into one of my best” and “Encouraged me to trust that my worries were normal and that practice would improve me”. 8 students left comments stating that they never considered dropping out because of the MLS that was available to them, “Never felt the need because of the support provided” and “No, but did worry about failing maths before using these facilities”.

Section C of the survey focused on students who had not availed of MLS. 85 (38.5% of) adult learners (compared with 67.8% of traditional learners) stated that they did not use the MLS facilities provided in their institution. In Question 16, non-attendees were asked to select from 7 fixed options, as to why they did not avail of MLS. For adult learners, the frequency of response in each category is interesting when compared with the overall 1041 students who did not use MLS, see Table 4 (note that students selected more than category).

<table>
<thead>
<tr>
<th>Category of response</th>
<th>% of adult learners who did not avail of MLS (n=85)</th>
<th>% of all students who did not avail of MLS (n=1041)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I do not need help with Maths</td>
<td>43.53%</td>
<td>48.13%</td>
</tr>
<tr>
<td>The times do not suit me</td>
<td>41.18%</td>
<td>28.34%</td>
</tr>
<tr>
<td>I did not know where it was</td>
<td>5.88%</td>
<td>17.87%</td>
</tr>
<tr>
<td>I hate Maths</td>
<td>3.53%</td>
<td>14.51%</td>
</tr>
<tr>
<td>Other</td>
<td>15.29%</td>
<td>12.78%</td>
</tr>
<tr>
<td>I was afraid or embarrassed to go</td>
<td>8.24%</td>
<td>11.43%</td>
</tr>
<tr>
<td>I never heard of the MLSC</td>
<td>11.8%</td>
<td>8.36%</td>
</tr>
</tbody>
</table>

In terms of individual respondents, it is worth noting that of the 85 adult learners who did not avail of MLS, 43.53% of these stated that they did not need help. In comparison, for the 941 (67.8%) traditional learners who did avail of MLS, 48.9% of these stated that they not need help. We can see in Table 4 that a larger percentage of responses from adult learners stated that the times did not suit and that they had not heard of the MLSC. The proportions of adult learners responding that they hated mathematics, did not know where MLS was or were afraid or embarrassed to go, were much lower than in the overall population.

There was also an opportunity to comment on answers to Question 16 and 34 adult learners did so. Twenty comments stated that they did not need help or were able to work it out by themselves; eight comments stated that the session timings did not suit them due to timetable or living circumstances; two stated that they never heard of the MLSC services; two comments related to a reluctance to attend: “Just felt a bit uncomfortable; felt the questions I had may seem a bit irrelevant”. These responses were consistent with overall student comments.
Table 5 Frequency of comments from adult learners who are non-users of MLS about what would encourage them to avail of MLS.

<table>
<thead>
<tr>
<th>Category</th>
<th>% of Responses (n=41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go if needed</td>
<td>46.34%</td>
</tr>
<tr>
<td>Results/Exams</td>
<td>0%</td>
</tr>
<tr>
<td>Better times</td>
<td>19.51%</td>
</tr>
<tr>
<td>More Information</td>
<td>19.51%</td>
</tr>
<tr>
<td>Resources/Location</td>
<td>4.88%</td>
</tr>
<tr>
<td>Advised to go</td>
<td>2.44%</td>
</tr>
<tr>
<td>Student Feedback</td>
<td>2.44%</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>4.88%</td>
</tr>
</tbody>
</table>

In Question 17, non-users of MLS were asked to comment on what would encourage them to use the MLS facilities. The responses were coded into categories using GIA and Table 5 above provides the breakdown of responses from the 41 Adult Learners who answered. Compared with the overall responses, adult learners were more likely to comment that they would access MLS if they needed. They were less likely to comment on resources/location or the need for student feedback or advice as reasons that would encourage them to avail of MLS. No Adult Learners mentioned exams or results as a prompt for them to access MLS.

Discussion and conclusion

In this paper we have considered the data concerning adult learners in our large-scale student evaluation of MLS. We also compared, where possible, these results with the overall cohort or with traditional learners. Our two main research questions were:

1. What are the motivational factors of adult learners who seek mathematics learning support (MLS)?
2. Why do some adult learners of mathematics not seek MLS?

When we considered the backgrounds of the respondents, we did not find a significant difference between adult learners and the overall cohort in terms of the disciplines that they were studying. This will be investigated further in the next stage of our analysis when we consider the breakdown of results in terms of the individual institutions that respondents attended. However, as one would expect, adult learners did present with a wider range of mathematical backgrounds than the overall cohort, with a smaller proportion taking HL and a higher percentage taking OL. This is consistent with research elsewhere, e.g. Gill (2010).

When students who engaged with MLS were considered, there was a statistically significant association (Chi-Squared Test, p<0.001) between student type (i.e. adult learners or traditional) and whether a student uses MLS, demonstrating that adult learners are more likely to seek support than traditional learners. This supports other research, e.g. Ni Fhloinn (2007) who states that adult learners in DIT seek support much earlier than traditional learners, even as early as the first day of term. However, in our study, we found no significant difference in the mathematical backgrounds of adult learner users and non-users of MLS.

Partial answers to our first research question are provided when the reasons why students engaged with MLS were investigated. Analysis suggests that adult learners in our study were more likely than traditional students to mention the following reasons for engaging: having been a long time away from education; poor confidence in their mathematical ability; seeking general extra help; struggling with mathematics. In contrast, adult learners were much less
likely than traditional students to mention the following reasons: to get help with specific coursework assessment or as revision for tests; to improve or gain better understanding; to state they find mathematics difficult. Being an adult learner, having not studied mathematics in any formal sense for a long time lends itself to having gaps in knowledge due to forgotten or perhaps never learned material.

Lawson (2008) states that some students avoid support due to a fear of embarrassment or feeling that they just have too many mathematical problems to deal with. This gap in knowledge appears to act as an impetus rather than an obstacle for the Adult Learners in our study to engage with support “As I have been out of the education system for many years I felt I needed the extra support”. These adult learners were motivated to engage because of their worry about gaps in their mathematical knowledge and the length of time they had been away from studying mathematics “As a mature student I needed a refresher”. Wolfgang and Dowling (1981) may partially explain this finding as they maintain that traditional and adult learners have different motivations and approaches to study. Safford (1994, p50) supports this stating that while adult learners may carry ‘intellectual baggage’, they are generally self-directed and making the decision to return to education implies a motivation for change and growth.

A significantly smaller proportion of adult learners did not avail of MLS when compared to the overall cohort. In terms of our second research question, we considered the reasons given by students for non-engagement with MLS. According to Ashcraft and Moore (2009) avoidance is often the consequence of mathematically anxious students. Bibby (2002) reports that math anxiety and shame of own mathematics ability are reasons that students fail to seek help with mathematics. In a study carried out by Grehan et al. (2011, p.79) at NUI Maynooth, the reasons divulged for lack of engagement with MLS included ‘fear; lack of personal motivation; the anonymity of large classes; and to a lesser extent the lack of awareness of support services’. Symonds et al. (2008) list a fear of embarrassment and a lack of information regarding the whereabouts of the mathematics support as reasons why students do not engage. Our findings largely contrast with those just mentioned. The largest proportion of responses from both adult learners and the overall cohort who did not engage with MLS indicated that they simply did not need to: “Good service for students – just didn’t need to avail of it”; “I would definitely find time to attend if I needed to”. It is reassuring that many of those who do not utilise the resources provided simply do not feel the need. Only 13% of adult learners stated that they did not know where it was and/or had not heard of the support and just 4 stated that they were afraid or embarrassed to go “Just felt a bit uncomfortable, felt the questions I had may seem a bit irrelevant”. As we discussed earlier, fear and embarrassment were more of a motivation to attend rather than not attend MLS.

Overall, respondents were very positive about the MLS experience they received in their institution, with adult learners especially so, e.g. users of MLS reported increased confidence in the mathematical abilities and finding it easier to cope with the mathematical demands of the courses “I’ve had a fear of maths all my life so with MLC help I’ve become more confident”. It is clear from the comments that MLS provides a mathematical lifeline, so to speak, for many adult learners: “I would be seriously lost without the MSC and the extra maths classes ran. Now I actually like maths”; “Excellent and I credit the help I receive here to me passing all my maths tests so far”.

Many of the comments highlighted the important role of MLS tutors. Lawson (2008) states that students attend MLSCs precisely because they offer emotional and MLS to students who suffer from mathematics anxiety. FitzSimons and Godden (2000), and Safford (1994) recommend the provision of this warm supportive environment in which individual needs are met and adult learners of mathematics can thrive. The quality of staff is crucial to the success of
MLS (Lawson, et al., 2003) and in particular in relation to the education of adult learners (FitzSimons & Godden, 2000). Gill (2006) states that the one-to-one attention students receive in MLSCs is most highly favoured. Some of the responses in this study referred to how they preferred the teaching approach used in the MLSCs to those in their regular tutorials “People in the MLSC explain the questions or doubts you have the way the people in the tutorials should”.

However, Lawson et al. (2003) states that not everyone will make a good MLS tutor and this is reflected by the small number of negative comments about certain MLS tutors, e.g. “Possibly some training in social skills for some of the tutors”. Benn (1994) encourages teachers to tread carefully when dealing with Mature Students of mathematics as it will influence how students perceive the subject. It is in the nature of MLS evaluation that both positive and negative comments can be used constructively. To this end, the IMLSN is in the process of developing and collating MLS tutor training materials which can be used to help ensure best practice in the recruitment and training of tutors. There were some other negative comments, e.g. in relation to the timing of the drop in centre or classes, the volume of students in attendance and hence the lack of one-to-one attention at busy times: “It’s sometimes very crowded and the instructors cannot get to you”, “Sometimes the wait for assistance is 30-45 minutes”. These findings resonate with those of Lawson et al. (2003) who state that MLSCs are inclined to be very busy at certain times, such as at examination time, and there will be waiting times as a result. Again, these comments were not standard across the survey and will be of more relevance to the individual institutions when further analysis is presented.

It is very difficult to claim that MLS is responsible for increases in retention or student success rates in mathematics (Lawson et al., 2003). Mac an Bhaird et al. (2009) tell us that we cannot take full credit, because a number of connected factors are in play when it comes to student progress, such as motivation, among others. However, the findings from this study indicate a high level of adult learner satisfaction with the services provided by the MLSCs throughout Ireland, and many adult learners indicated that MLSCs are responsible for their not dropping out of their studies. “It was a very valuable experience, whereby without it I would have certainly failed”.

References


The Rational Number Sub-Constructs as a Foundation for Problem Solving

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Abstract

One of the many roles of two year community colleges in the United States is to bridge the gap between secondary school and college for students who graduate from high school with weak mathematics skills that prevent them from enrolling in college level mathematics courses. At community colleges remedial or developmental mathematics courses review the pre-algebra and/or algebra skills required for college level mathematics. Fractions are often cited as the most difficult topic for students due to their abstract nature (Wilensky, 1991). This study with adult pre-algebra students is based upon a teaching research experiment in which the Kieren’s fraction sub-constructs of part-whole, ratio, operator, quotient, measure and the fractional equivalence were used as foundational concept knowledge during problem solving. In the first quantitative part of this study, students’ proficiency with Kieren's rational number sub-constructs are used as independent variables in a multiple linear regression model to
predict or explain students' competency in formal problem solving. This part of the study supplies hypothetical or statistical suggested pathways for students learning and transition from fraction concepts to proportional reasoning. Then in the second qualitative part of this study, transcripts from classroom lectures during the teaching research experiment are reviewed in order to understand how students used these rational number sub-constructs during problem solving with ratio, quotient, proportion, and percent.

Keywords: Adult remedial mathematics, fractions, sub-construct, ratio, operator, quotient, measure, informal and formal proportional reasoning

Introduction

Proportional reasoning is often cited as a critical component in the transition from informal to formal mathematical thought. In the pre-algebra curriculum proportion typically come after fraction and ratio, however many educators believe it should be introduced earlier and the connections between these topics should be emphasized (Streefland, 1984). The claim that instruction in proportions should be based upon and connected to students’ understanding of fractions puts more emphasis on this important concept. Fractions represent a difficult concept for many students. Almost every instructor has heard a student proclaim, “I hate fractions.” In an effort to clarify the relationships between various fraction concepts the Kieren (1976) model of fraction and the extension of this model by Behr, Lesh, Post and Silver (1983) was studied using quantitative analyses by Charalambous & Pitta-Pantazi (2007), with children, and Baker, Czarnocha, Dias, Doyle and Prabhu (2009) with adults. The Behr et al. (1985) extension of Kieren’s work was used as a theoretical foundation to study the relationship between procedural and conceptual knowledge for adult students reviewing fraction concepts in Baker, Czarnocha, Dias, Doyle, Kennis and Prabhu (2012).

The first objective of this study is to test an underlying hypothesis inherent in the Behr et al. (1983) extension that the rational number sub-constructs provide a foundation for problem solving in the realm of proportions. This is done by using student proficiency with these constructs and fractional equivalence as independent variables in an analysis of variation (ANOVA) linear regression model to predict student competency with problem solving.

The second objective involves analyzing classroom transcripts during the teaching research project in order to determine how these rational number concepts are used during student informal reasoning with ratio, proportion and percent problems.

Literature Review

Problem-Solving

Cognitive theorists suggest that all learning takes place in a problem solving or goal directed environment. A problem solver acquires methods and strategies to obtain a goal in one of three manners. The first is through direct instruction, the second is by discovery and the third is using analogy to previous solutions. Learning and increased proficiency in a domain is characterized by the ability to recognize chunks or patterns of elements which repeat over problems of a similar structure (Anderson, 1995). These chunks or patterns can be identified with problem solving schema which are triggered whenever “an individual tries to comprehend, understand, organize or make sense of a new situation” (Steele & Johanning, 2004, p. 67) The ability of a
student to recognize elementary schema that relates to previous situations is viewed by math educators as the first (recognition) stage in the development of problem solving (Cifareli, 1998).

Direct instruction in problem solving in a mathematics classroom frequently takes place through modeling correct problem-solving behavior. Then students are given problems with similar structure to strengthen their skills at recognition and the use of analogy. That is educators employ repetition, recognition and generalization often by adapting problem solving sequences with increasing difficulty and generalization (Steele & Johanning, 2004). Unfortunately, weak problem solvers tend to employ strategies dominated by superficial aspects of a problem and in a classroom situation their ability to recognize a pattern and transfer knowledge is heavily influenced by what cognitive psychologist refer to as “temporal proximity,” that is whatever type of problem they are solving in class is what they expect to use (Anderson, 1995). Another frequently observed trait is referred to by Lamon (2007) as “non-conservation of operation” this behavior is characterized by the choice of an operation that is easy to perform given the numerical values presented without consideration of problem structure. A student who replies that “when 3 lbs. are divided into 9 packages the result is 3,” would be exhibiting such problem solving behavior.

The inability of many students to assimilate information about the problem structure into their choice of operation(s) makes an over reliance on modeling correct problem solving behavior ineffective. The insight that these students need to directly engage in the process has lead to reforms that emphasize student discovery during problem solving. For cognitive psychologist the discovery or formation of new methods and techniques for problem solving are built upon a “rich conceptual knowledge base” (Byrnes & Wasik, 1991, p. 778).

Concept development and problem solving are frequently treated as separate branches of mathematics. However, several educational researchers suggest a dynamic interaction between them (Steele & Johanning, 2007; Lesh, R., Landau, M. & Hamilton, E., 1983). Tracy Goodson-Epsy (1998) uses both the stages of problem solving introduced by Cifareli based upon the ability to recognize and mentally represent solution strategies to a given problem and the stages of concept development based upon the work of Piaget. She concludes that students in the lower stages of problem solving, “recognition and re-presentation, typically held weak conceptions of variable and equality” (p.244).

The Kieren Model and Behr et al. Extension

Kieren proposed that the concept of a fraction can be viewed as the composition of five related but distinct sub-constructs, the primary sub-construct of part-whole knowledge and the four secondary sub-constructs of ratio, operator, quotient and measure. An extension of this model to corresponding fraction operations, equivalence and problem solving was developed by Behr et al. (1983).

In Figure 1, the primary sub-construct of part-whole and the row of the secondary sub-constructs: ratio, operator, quotient and measure can be viewed as conceptual knowledge. The bottom row, added by Behr et al. (1983), includes, problem solving which is the focus of this study, as well as the procedural knowledge of multiplication and addition, which were the focus of an earlier related study Baker et al. (2012).

In Figure 1 neither procedural knowledge nor fractional equivalence is given a role in promoting problem solving. Educational researchers consider fractional equivalence and
equivalence schemes as “basic constructive mechanisms for rational number knowledge-building” (Pitkethly & Hunting, 1996, p.8). The results of Baker et al. (2009) corroborate the idea that fractional equivalence is considered as conceptual knowledge and its role in determining student competency with problem solving is analyzed in this study.

The arrows in Figure 1 from all four sub-constructs pointing to problem solving represent an underlying hypothesis that knowledge of these concepts lead to competency with problem solving. Lamon (2007) uses the Kieren sub-constructs as a foundation to promote proportional reasoning and thus agrees that solving proportion and related problems should be based upon these rational number concepts, in particular, she notes that students develop rational number sense through encounters with different representations of rational numbers.

In the first quantitative component of this study, equivalence and the other rational number sub-constructs are used to investigate the hypothesis that competency with these sub-constructs promotes proficiency with problem solving based upon ratio, rates and proportion. In the second qualitative component student use of these rational number concepts during the transition from informal to formal proportional reasoning in the math classroom is analyzed.

**Proportional Reasoning**

Proportional reasoning has been described as a foundation or core of algebra and higher mathematics (Berk, Taber, Gorowara & Poetzl, 2009; Lo & Watanabe, 1997). Despite the importance of proportional reasoning in subsequent math courses, educators point out that, many college students fail to manifest effective formal proportional reasoning (Adi & Pulos, 1980). Lamon (2007) affirms that the lack of ability to reason proportionally is widespread when she notes, “a sense of urgency about the consistent failure of students and adults to reason proportionally… my own estimate is that more than 90% of adults do not reason proportionally…” (p.637)

**Informal Proportional Reasoning**

Fischbein (1999) noted that there is no commonly accepted definition for intuitive knowledge or informal reasoning. However, informal reasoning is frequently used in mathematics education.
to refer to problem solving strategies demonstrated by children before formal instruction in mathematics. Carpenter (1986) found that children who used informal strategies were fairly successful at solving word problems. His characterization of children’s strategies as informal is reminiscent of Vygotsky’s (1997) notion of “spontaneous concepts” that children develop before instruction as opposed to the “scientific concepts” characterized by a hierarchy of connections which is the structure they learn during formal instruction.

Many educators share the view that formal instruction in (proportional) reasoning should be based upon informal reasoning in real life situations and this has lead them to lament the lack of this connection in formal schooling, “…too often, we ignore the child’s experience with ratio and proportions outside of formal mathematics lessons and teach children algorithms, which utilize techniques that are alien to them…..” (Singh, 2000, p.291)

In this study informal reasoning strategies were presented during math instruction, therefore a characterization of informal reasoning based upon processes and elementary schema is more appropriate than one based upon spontaneous or pre instructional thought.

Transition from informal to formal Proportional Reasoning

Intuitive reasoning has been studied within the domain of proportions (Fernandez, Llinares, Modestou, Gagatsis, 2010) in particular during the transition from informal to formal proportional reasoning (Karplus, Pulos, & Stage, 1983). Nahors (2003) relates educational studies of children’s schema with rational number concepts to the work of educators who have mapped out the transition from informal to formal proportional reasoning and serves as an excellent framework to define and analyze the intuitive reasoning exemplified in the classroom transcripts.

The example used by Fischbein (1999) to illustrate informal proportional reasoning is, “if one liter of juice costs 5 shekels then how much does 3 liters of juice cost?” (p. 15) Nahors (2003) outlines the steps an individual might use to solve this proportion problem at different levels of conceptual development. These steps include observing the two referents (liters and shekels), the rate or equivalence between them, and the understanding this equivalence is invariant under multiplication. At the initial level an individual begins an additive process of counting or iterating by the given composite referent quantities. In this case 1 liter to 5 shekels, 2 liters to 10 shekels…. Using the schema terminology of Steffe and Olive (1988) Nahors refers to this reasoning as a “coordinate unit-coordinating scheme.” (p.137) In a second level of development an individual understands that the new amount of 3 liters is three times the original 1 liter and then multiplies the cost times 3. Nahors refers to the process involved in this approach as “iterable composite units coordinating scheme.” (p.138) Nahors considers this a slightly more sophisticated and powerful version of the coordinate unit-coordinating scheme due to its multiplicative nature.

The third level is an intermediate step in proportional reasoning and is often described by educational researchers as the unit rate approach (Karplus, Pulos & Stage, 1983; Nahors, 2003). In a proportion problem, it involves first finding the unit rate between the given referents and then a multiplicative based iteration strategy as described in the iterable composite units coordinating scheme to solve the proportion. This level of concept development is considered by educators to begin formal proportional reasoning.
In the qualitative part of this study, the analysis of classroom transcripts is based upon the work of Nahor. The objective is to identify the processes and elementary schemes students use when applying rational number concepts during informal reasoning with ratio, quotient, proportions, and percent problems and the difficulties they experience.

The Sub-constructs of Rational Number Sense

The definitions of the fraction sub-constructs are taken as in Charalambous & Pitta-Panzini (2007). The part-whole sub-construct interprets the symbol notation \( \frac{p}{q} \) to represent the partitioning of a whole entity into \( q \) equal shares and then taking \( p \) out of the \( q \) shares. The part-whole sub-construct is used as a foundation for developing rational number sense in the mathematics curricula. However, the part-whole sub-construct is limited in that it does not readily illustrate the concept of an improper fraction. The measure sub-construct is frequently evaluated through placement of a fraction on the number line. Measure involves an application of the part-whole concept by determining the placement of \( \frac{p}{q} \) on an interval with a designated unit. The unit is partitioned into \( q \) equal parts and the resulting sub-unit \( \frac{1}{n} \) is iterated \( p \) times.

Through this process, the measure sub-construct extends the part-whole concept to include improper fractions. The quotient sub-construct interprets \( \frac{p}{q} \) as the amount obtained when \( p \) quantities are divided into \( q \) equal shares. The quotient sub-constructs support a dual interpretation of \( \frac{p}{q} \) as the number of equal shares obtained when a quantity \( p \) is divided into \( q \) equal sized shares. The ratio sub-construct interpretation of \( \frac{p}{q} \) involves a comparison between two quantities \( p \) and \( q \) and thus it extends the part-whole interpretation to include part-part.

Operator is synonymous with the process of taking a fraction of some quantity, thus the operator sub-construct interpretation of \( \frac{p}{q} \) involves multiplication by \( p \) and division by \( q \). The operator concept is associated with the input-output box in which the output is a fractional amount of the input quantity. The exercises used to evaluate the part-whole and ratio sub-constructs are mostly pictorial, measure is evaluated through the number line, operator through the input-output box, and quotient through problem situations often involving sharing a pizza. Exercises used to evaluate the equivalence sub-construct are based primarily upon translation between part-whole pictorial representations i.e., identifying the fraction associated with a picture containing 2 out of 5 objects shaded and then shading the appropriate number of boxes out of 15 objects that corresponds to the equivalent fraction.

Also included are solving missing value problems that can be solved through scalar multiplication i.e., the second level of intuitive reasoning an example would be, find \( x \) in the proportion \( \frac{2}{5} = \frac{x}{20} \). The exercises used to evaluate the rational number sub-constructs are included in the appendix and are essentially identical to those of Charalambous & Pitta-Pantazi (2007). The results of factor analysis and reliability tests on the exercise sets used to evaluate these sub-constructs are given in this appendix as well.

Research Questions

Research question 1: To what extent do the Kieren’s rational number sub-constructs predict or explain students’ competency with formal problem solving based upon proportional reasoning?

Research question 2: How do students use Kieren’s rational number concepts when reasoning informally during proportion and percent problem solving? Specifically what
schemes are observed during student use of this reasoning and what difficulties do students experience?

**Setting**

The quantitative data in this study came from the same source as Baker et al. (2012) and like this article involves student proficiency with fraction concepts. However, unlike the earlier article is also includes data from these students with proportional reasoning. Thus, in both articles the data was collected over several semesters from 334 adult students enrolled in pre-algebra courses taught by six professors of Mathematics at Hostos Community College (HCC) and Bronx Community College (BCC) both urban community colleges in the City University of New York (CUNY) system.

The teaching research experiment (1) from which this data was collected was designed on an educational approach in which the rational number sub-constructs served as a basis to develop competency with problem solving involving ratio, rates, proportion and percent. The classroom sessions were focused on problem solving with an emphasis on guidance and encouragement rather than direct instruction. In this sense the common methodology of the instructors could be described as constructivist instruction i.e. based upon discovery learning. Classroom transcripts of several of these professors during this teaching research project are analyzed for student reasoning with the rational number concepts during informal proportional reasoning.

The assessment of the original teaching research project contained a control group (n=34), and experimental group (n=46), using a pre-test and post-test that focused on problem solving with ratio, rates, proportions and percent. The same professors taught sections of each group. There was no significant difference between the mean scores of the pre-test between the groups but the experimental group significantly outperformed the control group on the post-test at the p < 0.001 level.

As noted in Baker et al. (2012), “the student body at these community colleges is predominately female (70%-80%) and minority (85%-95%) and is the mathematically weakest group of students applying to the CUNY system. These students have failed both the algebra and pre-algebra placement exams in mathematics and are not eligible to take college level mathematics course until they pass these courses. At these community colleges the pre-algebra course lasts fifteen weeks, it covers real numbers such as decimals and fractions, proportions, percent and an introduction to algebra.”

**Methodology**

The exercises sets for these sub-constructs were adapted from those used by Charalambous & Pitta-Pantazi (2007) and except for problem solving are identical to those used in Baker et al. (2012). Problem solving was evaluated through application problems involving ratio, rates and proportions that were taken from the adult curriculum. Principal factor analysis and reliability tests were conducted on the exercise sets (Cramer, Post & dellMa, 2002) in order to determine the components within each set and the reliability of the set of exercises. All problem sets and the results of these analyses are listed in the appendix.

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1 This study was partially funded by the grant. Problem Solving in Remedial Mathematics: A jump start to reform, CUNY College Collaborative Incentive Research Grant Program (2010), Czarnocha, Prabhu, Dias & Baker.
Results and Discussion

Quantitative Analysis

The quantitative analysis of correlations between variables used in this study is identical to that used in Baker et al. (2012) and is based upon the assumption that the mean scores of two variables are significantly different (T-test) and there is a positive and significant correlation between them. As noted in Baker et al. (2012) in such a situation, “the underlying premise is that students’ knowledge of the easier concept will precede and be used to acquire knowledge of the more difficult concept. Thus knowledge of easier concept X will imply knowledge of more difficult Y this will be written as, X ⇒ Y. Furthermore, the square of the correlation coefficient \( r^2 \) indicates the percent variation of Y explained by X. This will be written as X⇒Y, \( (r^2\%) \).” (p.47) For example, if X⇒Y, (40%) then, given a class of students proficient in X one can expect 40% to be competent with Y. The first research question involves quantitative analysis of student competency with the rational number sub-constructs, fractional equivalence, and problem solving. The means and correlations between these sub-constructs are listed.

Student Performance: Mean and Standard Deviation

A two sided T-test confirms that the mean score of part-whole is significantly easier than the other sub-constructs. In a second tier are equivalence and ratio. The third tier is operator and quotient, then measure and finally problem solving or proportional reasoning.

Table 1 Mean scores and standard deviations on sub-constructs  (n=334)

<table>
<thead>
<tr>
<th>Sub-construct</th>
<th>( \overline{x} )</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Part-whole</td>
<td>0.74</td>
<td>0.18</td>
</tr>
<tr>
<td>2) Equivalence</td>
<td>0.68</td>
<td>0.28</td>
</tr>
<tr>
<td>3) Ratio</td>
<td>0.67</td>
<td>0.24</td>
</tr>
<tr>
<td>4) Operator</td>
<td>0.62</td>
<td>0.27</td>
</tr>
<tr>
<td>5) Quotient</td>
<td>0.55</td>
<td>0.25</td>
</tr>
<tr>
<td>6) Measure</td>
<td>0.49</td>
<td>0.28</td>
</tr>
<tr>
<td>7) Problem Solving</td>
<td>0.41</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Correlations between Sub-constructs

The correlations in Table II confirm Lamon’s (2007) statement that the rational number sub-constructs are very connected to one another and suggest the Behr et al. (1983) hypothesis that fraction concepts leads to problem solving is valid. In order to determine the extent to which the rational number sub-constructs predict proportional reasoning we employ multiple linear regression with Kieren’s rational number sub-constructs and equivalence used as independent variables and students’ competency with formal problem solving based on proportional reasoning as the dependent variable.

Baker et al. (2012) worked with an underlying assumption for a linear regression or analysis of variance (ANOVA) model that, “each independent variable correlates significantly
with the dependent variable and there is a significant difference between the mean score of each independent and dependent variable” (p.47). Each of these assumptions has been demonstrated in either Table I or II. As noted in Baker et al. (2012), “In an ANOVA the F-value indicates the strength of the relationship between the independent variables and dependent variable and the p-value determines whether the model is significant. When the p-value of the model indicates it is significant the relevant question becomes what is the interaction between the independent variables as they predict or explain the dependent variable.” (p.47) As explained in Baker et al. (2012) the interaction between the independent and dependent variables is quantified by first the significance or p-value of each variable and second the beta value of each independent variable which determines how much of the dependent variable it explains in the given ANOVA model.

<table>
<thead>
<tr>
<th>Predictor Variable</th>
<th>Beta</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ratio</td>
<td>0.13</td>
<td>p&lt;0.026</td>
</tr>
<tr>
<td>2) operator</td>
<td>0.27</td>
<td>p&lt;0.001</td>
</tr>
<tr>
<td>3) quotient</td>
<td>0.18</td>
<td>p=0.001</td>
</tr>
<tr>
<td>4) measure</td>
<td>0.14</td>
<td>p=0.001</td>
</tr>
<tr>
<td>5) equivalence</td>
<td>0.17</td>
<td>p=0.002</td>
</tr>
</tbody>
</table>

In response to the first research question, the foundational factor of part-whole is not significant in predicting problem solving. However, all the other rational number sub-constructs and equivalence are significant. The means in Table II and the beta values and p values in Table III indicate that the more difficult concepts of measure, quotient and operator are more influential in explaining student competency with problem solving than the easier concepts of ratio, equivalence and part whole.
This result, combined with the high correlations exhibited in Table II suggest that the rational number sub-constructs development in a hierarchical manner with the more difficult concepts built upon the foundation of earlier concepts in much the way Vygotsky (1997) would describe scientific concepts and schema theorist like Sfard (1991) would describe structured schema. Furthermore, the more developed an individual’s hierarchy of concepts the more competent they will be with problem solving. In this case 44% of student competency with proportional reasoning can be explained by their proficiency with the rational number sub-constructs. This statistical analysis validates educators who stress a connection between concept development and problem solving.

In the qualitative part of this study classroom transcripts are reviewed to understand how the rational number sub-constructs are used when engaged in informal reasoning during problem solving with ratio, rate, quotient, proportion and percent.

**Qualitative Data Review of Transcript**

A student asks to review the following problem given on an earlier test. Two students participated in the dialogue (GT & SP).

6.2.1 A taxi charges $6.50 for the first quarter mile and $0.50 for each additional mile. What is the charge for 1 ¾ mile?

The teacher (T) calls upon a struggling yet determined student who had the problem correct on the test (GT).

GT: I made a line with quarters 0 ¼ ½ ¾ 1 ¼ ½ ¾ 2

T: (The teacher draws out the number line)

GT: First is $6.50, then $7.00 she counts slowly and are carefully until she reaches 1¾ and proclaims the answer $9.50.

T: Very good, did anyone do it differently?

Silence

T: Do we have to count out each $0.50 (Directs this question to the class—a second student SP answers)

SP: No, we know there are 6 additional quarters so we can multiply 6($0.50) = $3.00 and add this to the $6.50 to get the answer.

This is one problem (other than percent) that students independently constructed and used the number line consistently to solve. GT understood the initial rate 1 quarter mile to $6.50, distinguished this from the subsequent rate 1 quarter mile to $0.50 and she successfully represented this rate on the number line. SP also distinguished between the rates 1 quarter mile to $6.50 and 1 quarter mile to $0.50 and she also represented these rates on the number line. However, SP iterated the quarter miles 6 times and then coordinated this with the appropriate dollar amount through multiplication. The reasoning of GT is an example of a “coordinate unit coordinating scheme” (Nahors, 2003) while the multiplicative reasoning of SP is a good example of an “iterable composite units coordinating scheme.” Example 6.2.1 shows that students can and will return to informal strategies effectively and independently when a problem situation is amenable (Nahors, 2003).
The teacher (T) presents the following quotient problem during a lecture on fractions after multiplication and addition of fractions had been discussed. Students who participated include (JM, YM, GT & JA).

6.2.2 Each package of meat is to contain $\frac{2}{3}$ lb. There are 8 lb. of meat to be made into packages. How many packages will there be?

T: What do we do?
JM: We multiply: $\frac{8}{1} \times \frac{2}{3}$
T: Class, do we all agree? Can anyone explain why?
JM: We divide. (JM changes his mind after the instructor-teacher questions his reasoning)
T: Okay, why? (Asks the entire class to see if someone can supply an answer as to why division—second student answers)
YM: Because we are taking the $\frac{2}{3}$ lb as a part from the whole.
T: But why is this division? I mean this could indicate subtraction or even perhaps some other operation why division?
YM: Because we are dividing the 8 lb. into parts
T: Good, we are partitioning or dividing the 8 lb. into parts each $\frac{2}{3}$ lb. So how do we divide?
JM: $\frac{2}{3} \div \frac{8}{1}$
T: Okay let’s work this out: $\frac{2}{3} \div \frac{8}{1} = \frac{2}{3} \times \frac{1}{8} = \frac{2}{24} = \frac{1}{12}$ so what is the answer? (Asks entire class. It was not clear who called out the following two responses)
Class: The answer is 12
T: Okay but why did we get $\frac{1}{12}$?
Class: It’s basically the same answer
T: Well it’s not exactly the same and it can be confusing if you forget to reduce the $\frac{2}{24}$ to $\frac{1}{12}$ you would probably get this wrong on a test. Okay, guys we did something wrong can anyone tell me what is wrong?
JM: We should have divided $\frac{8}{1} \div \frac{2}{3}$
T: Yes, we divide and the 8 goes first because it is the 8 lb. that is being partitioned or divided up into groups. (Teacher works out the problem)
T: I want to look at this same problem using a number line. Where does the fraction $\frac{2}{3}$ go on the number line, between what two whole numbers?
GT: Between 2 and 3.
T: No anyone else?
YM: Between 0 and 1.
T: Good (draws a number line) Let’s count 1 package is $\frac{2}{3}$ lb. counting over two more thirds we have (points to 1 $\frac{2}{3}$) 2 packages then counting over two more thirds we have 2 lbs. is 3 packages.

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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{2}{3}$</td>
<td>1</td>
<td>$\frac{3}{3}$</td>
<td>2 lbs.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3 packages</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

T: Class if 2 lbs. is 3 packages then 8 lbs. requires how many?
YM: 12
GT: Why, I don’t see this?
T: Okay, in proportion form: $\frac{2lb}{3pac} = \frac{8lb}{X}$ what is the value of X? How do we find X?
GT: Okay I get it.
T: Class does everyone get this? How do we solve this problem? (Teacher works out the solution to this proportion)
T: Did anyone do this problem differently?

JA: I drew out 8 figures each 1 lb. and took $\frac{7}{3}$ lb.
Wait let me draw this on the board (draws 8 rectangular pieces representing 1 lb. each and partitioned them into two-thirds for each package)

T: Okay what did you do next?

JA: There are 8 packages of ⅔ lb. each so I take these. Then there are 8 parts of ⅓ remaining. She counts these up in pairs (1,2,3,4) for a total of 12.

GT: I don’t get it. What is she counting?

T: After taking the ⅔ lb. there is ⅓ lb. remaining (pointing to the un-shaded part) correct?

GT: Yes I see it.

T: We now count in pairs we do this because we need two of these parts to make a ⅔ lb. package Correct?

GT: Okay

T: So we count by pairs, how many pairs are there?

GT: Okay now I see it, there are 4 pairs.

T: So we add these 4 to the original 8 to get 12.

The student JM was guessing which operation and when he understood it was division he then had trouble setting up the division correctly, confusing the dividend with the divisor. This is an example of non-conservation of operation. After the 2/3 lb. to 1 package rate was placed on the number line, the students followed the additive iteration of this equivalence as the teacher counted up to, 2 lbs. to 3 packages. Then after understanding that 2/3 lbs per package is equivalent to 2 lbs per 3 packages the students followed the informal scalar multiplication to arrive at the solution.

In this example the teacher used the number line or measure concept to coordinate the referents in much the same way as GT and SP had in the previous example 6.2.1. As indicated by the incorrect response of the student GT as to where the fraction 2/3 goes on the number line and the relatively low mean score of measure (0.49) in Table I the use of a number line-measure concept is difficult for students when a fractional referent in this case 2/3 lb. is involved. Thus, the measure concept, while very useful for students during informal reasoning can be an area of difficultly when fractions are involved. This provides a partial answer to research question 2.

The quotative method used independently by the student JA employs the part-whole concept to take 2/3 of each 1 lb. iterating the result to 8 lbs. while coordinating this process with the remainder 1/3 lb. (un-shaded areas) and the referent number of packages. Thus it represents a slightly more sophisticated version of the “coordinate unit coordinating scheme.” (Nahors, 2003) In this way it directly addresses the issue of non-conservation of division which is clearly a big problem for students as JM’s response indicated.

The following example was presented during an introduction to percent lecture. The previous example was how to take a percent of a given number. It had been discussed using both informal iteration on the number line and formal reasoning with proportion. Typically after exposure to formal techniques students prefer to translate into an equation using the structural identification of the phase ‘% of’ as multiplication or they set up a proportion using the phrase ‘% of’ to identify the base-whole, and the phrase “is” to identify the amount-part. The teacher uses the number line to clarify the amount and the base and expected the class to use a proportion to solve this problem. Students involved in discussion are (AH, EZ & JA).
6.2.3 30% of some number is 900 find the number?
T: Any ideas?
AH: We can put this on the number line like the previous one.
T: Okay, draws the following line (The line was decomposed into 10% as familiar reference points, however these were not labeled)
\[
\begin{array}{cccccc}
0% & 30% & 50% & 100% \\
|-----------------------------|-----------------------------|-----------------------------|
0 & & & & &
\end{array}
\]
T: Where does the 900 go? Is with the 100% or with the 30%?
EZ: It goes with the 30%
T: Why?
EZ: Because, we have 30% of some number is 900, so 30% is 900 and X goes with the 100%.
T: Okay, (places 900 on the number line as shown)
\[
\begin{array}{cccccc}
0% & 30% & 50% & 100% \\
|-----------------------------|-----------------------------|-----------------------------|
0 & 900 & & & &
\end{array}
\]
AH: Why does the 900 go with the 30%?
T: 30% of some number is 900. Thus, we take a 30% of an unknown number, this means we take a part of this number and get 900. The number is bigger than the 900 we take 30% of a bigger number that we don’t know. When we don’t know a number we label it X so this unknown X is 100%. Do you understand?
AH: I think so.
T: How do we find 100% which is the X? (The expectation was that students would suggest a proportion as in the problem done before this one)
JA: We can find 10% which is 300.
(Teacher was surprised although in previous examples the number line had been used to give meaning to setting up a percent proportion and the informal iteration strategy had been used the decomposition with percent problems had not been shown in class).
T: (Marks the equivalence 10% is 300 on the appropriate scale point) Does everyone see how she got this? (The class nods agreement, it is clear to them)
\[
\begin{array}{cccccc}
0% & 10% & 30% & 50% & 100% \\
|-----------------------------|-----------------------------|-----------------------------|
0 & 300 & 900 & & &
\end{array}
\]
T: Okay, what should we do next?
JA: 100% is 3000. (The speed in which she answers indicates the use of scalar multiplication of 10 on the equivalence of 10% is 300)

The student EZ successfully identified the amount and the base with the assistance of the number line and yet could not immediately set up a proportion when the teacher asked. Perhaps this is because the base was unknown. Despite having been exposed to formal proportions JA continue to effectively use decomposition into an appropriate rate (10% is 300) represent this on the number line and then used scalar multiplication to solve this percent problem. The sequence of first decomposition of 30%—900 to the equivalent 10%—300 followed by scalar multiplication by JA contains the essence of the unit rate approach and thus documents a student transitioning between informal and formal reasoning independent of the teacher’s guidance.

In general, representation of percent through measure is easier for students than for fraction and the instructors frequently observed students independently using the measure sub-construct as for reasoning proportionally when solving percent problems during this project.

The following example was done near the end of a percent lecture after students had been exposed to formal proportion and percent equations. It mixes the fraction operator concept with percent. The student involved in the class dialogue was (RG, JM & EZ).
6.2.4 Find 60% of $\frac{2}{3}$ of 600.

T: How should we do this? (Silence) Okay how about we find $\frac{2}{3}$ of 600 first. (Silence)

T: If you had to describe the fraction $\frac{2}{3}$ how would you do this? (Silence) Suppose you wanted to explain $\frac{2}{3}$ to a child who did not know what it meant how would you do this?

RG: I would draw it. (Teacher draws a circle.)

RG: Then make a peace sign.

T: (After drawing the circle divides it into 3 parts with a peace sign).

What would you do next? How do you represent the 2?

RG: Take 2 of these parts.

(Teacher shades in two of the three parts.)

T: How would you relate this picture of $\frac{2}{3}$ to the 600?

RG: The 600 is the total.

T: So how much is the $\frac{2}{3}$?

RG: It is 400.

T: Class how did he get the 400?

JM: Each part is 200 so two of them are 400.

T: Good, does everyone see this? Can anyone tell me how to do this mathematically or formally in a faster way without pictures?

JM: $\frac{2}{3} \div 600$

T: $\frac{2}{3} \div \frac{600}{1} = 2 \times \frac{1}{600} = \frac{1}{900}$ is this correct?

JM: No

T: When taking a fraction of something the word ‘of’ indicates multiplication.

(Writes out $\frac{2}{3} \times 600/1$ and works it out to obtain 400)

T: What is the next step?

JM: We find 60% of 400

T: How do we do this?

JM: We set up a proportion $\frac{60}{100} = \frac{n}{400}$

T: good. (Solves the proportion) Did anyone else do it differently? What operation is indicated by the word “of”?

EZ: Multiplication

T: Good (Writes out 60% $\times$ 400 $\Rightarrow$ 0.60 $\times$ 400 and solves)

The responses of JM are an example of non conservation of operation. JM had forgotten how to take a fraction of a quantity and even after the teacher demonstrated an informal solution to the problem he appeared to be guessing which operation corresponded to this informal reasoning process. The teacher employed the part-whole sub-construct to represent $\frac{2}{3}$ as a visual picture. The students RG & JM were able to relate the 600 to the total and decompose this to find the unit rate or equivalence of 200 to a $\frac{1}{3}$ part before taking twice this as the answer. These processes make up an elementary schema that corresponds to the operator concept. The operator concept is closely related to adult students’ part-whole knowledge and proficiency with multiplication, Baker et al. (2012).

In answer to research question 2, the reasoning schemes used effectively in these examples were basically of two types, one that corresponds to the operator concept supported by part-whole and the other involving iteration and decomposition into equivalent rates supported by the measure concept. The informal strategy based upon the elementary schemes of iteration and decomposition were those presented by Nahors (2003). The students (GT & SP) used such iterative reasoning supported by measure in 6.2.1 to solve the taxi problem. The teacher also used iterative reasoning supported by the number line during the quotient problem (6.2.2) when 2/3 lb. per package was iterated to 2 lb. per 3 packages.
The elementary schema associated with the operator concept of taking a fraction or percent of a quantity involves the processes of identify and distinguishing between the base-whole and partial-amount. The visual for this schema was represented by a circle-fraction or number line—percent and the student related problem information to the corresponding visual concept. This elementary schema supported by part-whole pictures was used by the teacher to help students understand the process of taking \( \frac{2}{3} \) of 600 (6.2.4). Example 6.2.3 demonstrates the coordination of these schemes. First, EZ used the operator/amount-base schema to represent 30% to 900 on the number line. Then JA applied decomposition to an equivalent rate of 10% to 300 and finally she iterated to find the solution.

These examples demonstrate how concepts in visual form stimulate student engagement in problem solving through the process of relating problem information to a relevant picture whether a circle, rectangular bar or number line. These concepts also help shape and formulate the reasoning process. For example, RG and JM intuitively know to divide the 600 by three and double the result after relating it the whole circle in 6.2.4. In like manner JA intuitively decomposed the 30% to 900 to its equivalent 10% to 300 and iterated to solve after EZ had correctly represented it on the number line.

**Conclusion**

In the first part of this research study it was demonstrated that the rational number sub-constructs explain about 44% of a students’ problem solving based upon proportional reasoning. This validates the Behr et al. (1983) extension of the Kieren sub-constructs to competency with problem solving and verifies Lamon’s (2007) assertion that the rational number sub-constructs provide a foundation for proportional reasoning. The generalization of this result connects two areas of mathematical research, concept development and problem solving and suggests the interaction should be of significant interest to mathematics education.

In the second part of this study transcript of classroom lectures in which the rational number sub-constructs serve as a foundation for informal reasoning are analyzed for evidence of how these concepts were used. An analysis of student reasoning shows the concepts of part-whole-circles and number line-measure when represented in visual form acted as a catalyst for students reasoning. That is students became engaged in the reasoning process when they related problem information to these pictures. These picture-concepts also supported informal reasoning as students formulated strategies and applied processes based upon how problem information was represented.

Students used an elementary operator schema that involved identifying the partial-amount and base-whole with problem information and representing this on a diagram. They also used these picture-concepts especially measure to support the processes of iteration and decomposition. These results support educational research on the benefits of concept-pictures or visuals during problem solving (Goodson-Epsy, 1998; Steele & Johanning, 2004; Caddle & Brizuela, 2011).

The application of the processes of iteration and decomposition to rates on the number line-measure concept supported student transition to formal proportional reasoning. However, the measure concept was difficult for many students to grasp especially when a fractional quantity was involved. One exception was the use of whole number percent. The adult students independently applied their part-whole knowledge to represent given percent amount and base information on the number line and then applied the processes of iteration and decomposition.
Most adult textbooks and curriculum are based upon the same sequence of topics as that presented for children. Thus, percent is introduced after proportion and rates which are introduced after decimals and fractions. An effort to arrange the topics for adults according to the informal reasoning observed in this study would suggest the integration of whole number percent earlier in the adult curriculum with fraction, rate and proportion.

**Reflection upon Learning Theories**

In mathematics education there are separate branches for and corresponding models of concept development and problem solving. In the APOS (action-process-object-schema) model, concept development begins with an individual’s actions upon existing concepts and then with reflection upon these actions they become internalized processes that eventually lead to new concepts-objects and ultimately schema (Czarnocha, Dubinsky, Prabu & Viadokovic, 1999). On the other hand, the development of a problem solving schema for Cifareli (1998) focuses on recognition of strategies from problems with similar structure and reflection upon the processes involved.

The informal reasoning processes described in this study are dictated by the problem structure and reflection upon such structure is an essential component in the development of problem solving (Cifareli, 1998). On the other hand, these processes can be viewed as actions upon conceptual knowledge of part-whole and measure and in the APOS model reflection upon such processes leads to concept development. This suggests that such reflection is a point of commonality between these models of learning. Although an analysis of how these models are connected is beyond the scope of this study, cognitive theories that situate learning, including concept development within the framework of problem solving remind us that such a link exists.

**References**


Appendix

Principal factor analysis and reliability tests were conducted on the exercise sets (Cramer, Post & dellMas, 2002). The Kaiser-Meyer-Olkin measure of sampling adequacy was 0.6 or more for all sets of exercises, thus indicates the factor analysis was appropriate. The exercises sets are listed by the factor-component they fall in. The Cronbach’s alpha value of sampling reliability was 0.6 or more for all sets of exercises except quotient thus, all exercises sets are considered reliable except quotient.

**Ratio**

**Component 1**
1) In a History class there are 2 male to every 3 female students, use fraction notation to write the ratio of male to female students in the class. (*)
2) In a History class there are 2 male to every 3 female students, use fraction notation to write the ratio of female to male students in the History class. (*)
3) Use fraction notation to write the ratio of female to total students in the History class. (*)

**Component 2**
4) Write the ratio 4 to 36 in simplest terms. (*)(†)
5) Write the ratio 48 to 16 in simplest terms. (*)(†)

**Component 3**
Juan and María are making lemonade. Given the following recipes whose lemonade is going to be sweeter?
6) Juan uses 2 spoons of sugar for every 5 glasses of lemonade
    María uses 1 spoon of sugar for every 7 glasses of lemonade (*)
7) Juan uses 2 spoons of sugar for every 5 glasses of lemonade
    María uses 4 spoon of sugar for every 8 glasses of lemonade (*)

**Component 4**
Jose jogs each morning before work. Determine which of the following days he jogged at a faster rate. Please answer a) Monday b) Tuesday or c) Not able to determine from information given.
8) On Tuesday he jogged a longer distance than he did on Monday. On both days he jogged exactly the same amount of time. (*)
9) On Tuesday he jogged a shorter distance than he did on Monday. On both days he jogged exactly the same amount of time. (*)
The Kaiser-Meyer-Olkin measure of sampling adequacy for these 9 questions was 0.64. These 4 components explained 76% of the variation of the exercise set. The Cronbach’s alpha value for this exercise set was 0.67 and thus it is reliable. (*) commonality with other exercises in this set is more than 0.5
(†) Added by present authors and not found in Charalambous and Pitta-Pantazi (2007).

**Operator**

**Component 1**
There were two exercises that evaluated the operator concept through functional input-output boxes.
1) The following diagram represents a machine that outputs \( \frac{2}{5} \) of the input number. If the input number is 200 then what is the output number? (*)

2) An input-output machine has outputs that is \( \frac{1}{5} \) of the input. If the input number is 480 then that is the output number? (*)

3) Find half of \( 1 \frac{1}{2} \) hours (*)

4) Find \( \frac{4}{5} \) of \( \frac{7}{8} \) of 40,000 (†)

5) Find \( \frac{3}{5} \) of \( \frac{5}{8} \) of 4000 (†)

Component 2
6) Taking \( \frac{2}{5} \) of a number is the same as dividing the number by 5 and multiplying this result by 2, True/False? (*)

7) If we divide a number by six and multiply by twenty-four this the same as multiplying by the fraction \( \frac{1}{4} \) True/False? (*)

Component 3
8) A recipe calls for \( 1 \frac{1}{2} \) cup of flour. Which of the following expresses the amount of flour required for \( \frac{1}{3} \) of this recipe? (*) (†)
   a) \( \frac{3}{2} \div \frac{1}{3} \)  b) \( \frac{1}{2} + \frac{1}{3} \)  c) \( \frac{3}{2} \times \frac{1}{3} \)  d) \( 1\frac{1}{2} - \frac{1}{3} \)  e) not given

9) Find \( \frac{3}{4} \) of \( \frac{1}{5} \) (*)
   The Kaiser-Meyer-Okin measure of sampling adequacy was 0.69 with 58% of the variation explained by these three components.
   The Chronbach’s alpha was 0.69 and thus operator is a reliable set of exercises.
   (*) Commonality was at least 0.5
   (†) Added by present authors and not found in Charalambous and Pitta-Pantazi (2007).

Measure
Component 1
Locate the following numbers on this number line:

\#1) \( \frac{1}{6} \)  \#2) \( \frac{4}{3} \)  \#3) \( \frac{5}{6} \)

\( 0 \) \( \frac{1}{3} \) \( 1 \)

\[ \text{←---------------------→} \]

Component 2
\#4) Locate the number one “1” on the number line below:

\( 0 \) \( \frac{4}{9} \)

\[ \text{←---------------------→} \]
#5) Locate the number one “1” on the number line below:

\[ 0 \quad \frac{2}{4} \]

\[ \leftarrow \ldots \rightarrow \]

The Kaiser-Meyer-Okin measure of sampling adequacy was 0.63 with 72% of the variation explained by these two components. Chronbach’s alpha 0.69 thus measure is a reliable set of exercises. Commonality was at least 0.5 or above for all exercises. All exercises were used in Charalambous and Pitta-Pantazi (2007).

**Quotient**

Component 1

1) Three pizzas are shared equally among four students what fraction of a pizza will each receive? (*)

2) It takes \( \frac{3}{4} \) kg of apples to make one pie. How many pies can be made using 20 kg? (*)

Component 2

3) A beach \( \frac{3}{2} \) miles long is divided into 6 equal parts. How long is each part? (*)

4) Two pizzas were shared equally among a group of students. If each student received \( \frac{2}{5} \) of a pizza then how many students were there?

5) If 3 pizzas are shared evenly among seven girls while 1 pizza is shared evenly among three boys. Who gets more pizza, a girl or boy? (*)

(*) Commonality was 0.5 or above

All exercises were used in Charalambous and Pitta-Pantazi (2007).

The Kaiser-Meyer-Okin measure of sampling adequacy was 0.621 and 52% of the variation was explained by these two components. The Chronbach’s alpha was 0.49 thus quotient was not a reliable set of exercises.

**Equivalence**

Component 1

1) A 2x3 rectangular array of equal squares is given with 1 square shaded. Next to this are 24 un-shaded identical objects the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

2) Four identical objects are presented with 1 circled. Next to this is a 2x8 rectangular array of equal squares the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

3) A 2x3 rectangular array of equal squares is given with 4 squares shaded. Next to this are 24 un-shaded identical objects the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

4) Four identical objects are given with 3 circled. Next to this is a 4x4 rectangular array of equal squares the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

5) 16 un-shaded identical objects are presented. Next to this is a 2x2 rectangular array of equal squares the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

6) A 2x16 rectangular array of equal squares is given with 4 squares shaded. Next to this are 6 un-shaded identical objects the student is asked to shade in the appropriate number of objects to represent an equivalent fraction.

Component 2

7) \( \frac{2}{3} = \frac{?}{12} \)  

8) \( \frac{25}{40} = \frac{5}{?} \)  

9) \( \frac{7}{9} = \frac{42}{?} \)

All exercises were used in Charalambous and Pitta-Pantazi (2007).
The Commonality was 0.5 or above for all exercises. The Kaiser-Meyer-Okin measure of sampling adequacy was 0.86 and 67% of the variation was explained by these two components. The Chronbach’s alpha was 0.85 thus equivalence was a reliable set of exercises.

**Part-Whole**

Component 1
1a) Given a picture of four triangles and five circles; the question is what fraction of the objects are triangles? (*)
1b) Given two triangles; what fraction of the total triangles do these represent? (*)

Component 2
2) Given a picture of a circle with 2 out of 5 equal parts shaded; the question is what fraction of the circle is shaded? (*)
3) Given figure composed of seven squares, three of which are shaded; the question is what fraction of the squares are shaded? (*)
4) Given five equivalent objects three of which are circled; the question is what fraction of the objects are circled? (*)

Component 3
5) Given a rectangle array composed of six equal squares one of which is shaded; the question is what fraction of the squares are shaded? (*)
6) Given four identical objects one of which are circled; the question is what fraction of the objects are circled? (*)
7) Given a figure composed four equivalent objects three of which are circled; the question is what fraction of the objects is circled? (*)

Component 4
8) The fraction $\frac{2}{3}$ corresponds to taking a chocolate bar, dividing it into three equal parts and taking two of these parts. T/F? (*)
9) The fraction $\frac{2}{3}$ corresponds to taking a set of objects dividing it into three equal parts and taking two of them. T/F? (*)

Component 5
10) Given a 3x2 rectangle array composed of six equal rectangles four of which is shaded; the question is, does the shaded region corresponds to the fraction $\frac{2}{3}$? (*)
11) Does the shaded part of this rectangle correspond to the fraction $\frac{2}{3}$? (*)
12) Given a 2x6 array of circles 8 of which are shaded; the question is does the shaded objects represent the fraction $\frac{5}{6}$? (*)
13) Given a 1x5 rectangular array of equal squares with 2 shaded; the question is does the shaded part of the rectangle correspond to the faction $\frac{2}{5}$?

All exercises were used in Charalambous and Pitta-Pantazi (2007).

(*) The commonality value was at least 0.5
The Kaiser-Meyer-Okin measure of sampling adequacy was 0.72 and 60% of the variation was explained by these three components.
The Chronbach’s alpha value was 0.7 thus part-whole is a very reliable set of exercises.

**Formal Proportional Reasoning**

Component 1
1) Hank drove 500 miles in $8 \frac{1}{3}$ miles, what was his average speed or rate in miles per hour? (*)
2) If \( \frac{3}{4} \) cup of coleslaw contains 120 calories. How many calories are there in \( \frac{2}{5} \) cup? (*)

3) If the ratio of a:b is 2:3 and b is 4200 then find the value of a. (*)

4) If \( \frac{2}{5} \) of 4000 is equal to \( \frac{1}{4} \) of some number then find the number. (*)

5) If 0.5 ml of medicine are mixed with 2 ml of water to form a solution then what is the ratio of drug to water in simplest terms?(*)

Component 2

6) Which of the following fractions is closest to 1? (*)
   a) \( \frac{2}{3} \)  
   b) \( \frac{3}{4} \)  
   c) \( \frac{4}{5} \)  
   d) \( \frac{5}{6} \)

7) Circle the smallest fraction: (*)
   a) \( \frac{2}{11} \)  
   b) \( \frac{3}{13} \)  
   c) \( \frac{4}{23} \)  
   d) \( \frac{5}{6} \)

All exercises were taken from end of year departmental exit exam.
(*) The commonality value was at least 0.5

The Kaiser-Meyer-Okin measure of sampling adequacy was 0.75 and 51.3% of the variation was explained by these two components.

The Chronbach’s alpha value was 0.70 thus the formal proportional reasoning exercises formed a very reliable set of exercises.
Nursing Students’ Experiences of Learning Numeracy for Professional Practice

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Abstract

This paper examines nursing students’ experiences of the teaching and assessment of numeracy for nursing. Data from interviews with eight student nurses at a large school of nursing in the United Kingdom are analysed using a constructivist grounded theory approach to explore their perceptions of any disjunctures between the ways in which numeracy is taught and assessed in universities and the broader context in which calculations are carried out by nurses in practice. This paper makes an original contribution through providing an in-depth qualitative exploration of how these disjunctures may arise and hence proposes a change to the focus of numeracy courses and assessments which may begin to resolve some of these tensions.

Key words: numeracy; nursing; student nurses; medication calculation; ‘numeracy for nursing’.

Numeracy for Nursing: Context and Background

There has been long-standing concern about the safe administration of medications within clinical settings. Accurate drug calculation is seen as fundamental to safe medication administration and the reduction of associated risk. There has been a strong focus on nursing students’ numerical knowledge and skills, including the development of drug dosage calculation competence, and since 2008 the UK nursing regulatory body, the Nursing and Midwifery Council (NMC) has required students to achieve 100 percent in a numeracy in practice test in order to register as nurses. This requirement is articulated in the NMC’s Essential Skills Cluster.
for Medicines Management (NMC, 2007) updated in 2010 (NMC, 2010). In response, courses in ‘numeracy for nursing’ have been added to pre-Registration training and a plethora of numeracy assessments has developed. However, despite some noteworthy attempts to inject authenticity into these courses and assessments, some appear to be far removed from the real world calculation contexts and methods used by practising nurses. Further, there are serious questions as to whether some current approaches towards increasing students’ mathematical performance actually support them in terms of the calculations they encounter in clinical practice (e.g., Hutton, 1998). On placement, students may encounter and develop numeracy skills in an arbitrary manner dependent on factors such as the clinical setting, ward specialism and mentorship. The calculations students witness in clinical settings may use different methods from the academic, formulaic focus of some numeracy for nursing courses. This paper analyses data from interviews with student nurses (N=8) to explore tensions between numeracy in training and numeracy in professional practice, as perceived by the student nurses. The paper, through providing an in-depth qualitative exploration of any perceived tensions between numeracy in training and numeracy in professional practice, speculates on how such tensions arise and proposes a new and original change to the focus of numeracy courses and assessments which may begin to resolve these tensions.

Studies have identified that, as is common with the population more generally, many nursing students struggle with calculations involving ratios, decimals and unit conversions (e.g. Brown, 2006; Galaverna et al., 2015; Pierce et al., 2008; Weeks et al., 2000). Many students have a weak understanding of the decimal number system producing errors out by a factor of 10 or 100. Clinically, this can have profound implications for under- or over-dosing (e.g., Arkell & Rutter, 2012). The predominant response to these findings has been to increase numeracy teaching on pre-Registration courses and require assessment of student nurses’ numeracy competencies. However, there are issues with this, not least that “there is no international consensus on the nature and scope of numeracy for nursing” (Coben & Hodgen, 2009, p. 18). As such, the numeracy taught may not reflect, or increase competencies in, calculations required in the clinical setting, with some studies suggesting a false association between competency in academic mathematics and competency in calculations in clinical practice (Dyjur, Rankin, & Lane, 2011). Importantly, a narrow focus on calculation skills fails to account for the nature of nurses’ work. Errors made in practice are usually “multi-factorial” (Sabin, 2001, p. 6), with mathematical skills being just one component within these.

**Teaching and Assessment of ‘Numeracy for Nursing’**

In the United Kingdom and elsewhere nurse training institutions and/or regulatory bodies are increasingly requiring that nursing schools assure themselves of the numeracy competence of prospective students at the point of selection (see, for example, NMC, 2008). However, nursing schools may use inappropriate proxy measures of the numerical competence required for nursing. Entrants to pre-Registration nursing programmes in the UK now face: numeracy testing on entry to training; numeracy courses/tutorials; numeracy-based mentoring on placement; and computer-based or written assessments of numeracy in practice. As noted above, since 2008 the NMC has required all nursing students to achieve 100 percent in a numeracy in practice test. Failure to achieve this standard bars students from registering as nurses (Coben, 2010).

In 2010, so therefore after the interviews conducted for this study, and part-informed by research to establish a benchmark in numeracy for nursing (Sabin et al., 2013), medication dosage calculation – problem solving (MDC-PS) competencies were specified within the NMC’s (2010) Essential Skills Cluster (ESC) for Medicines Management. In addition, the NMC’s “Advice and Supporting Information for Implementing NMC Standards for Pre-Registration Nursing Education” (NMC, 2011) states that:
“Programme providers may wish to take the following information into account when determining assessment criteria:
1. An ESC assessment strategy for medication-related calculation that demonstrates competency across the full range of complexity, the different delivery modes and technical measurement issues.
2. Assessment that takes place in a combination of the practice setting, computer lab and simulated practice that authentically reflects the context and field of practice.
3. Diagnostic assessment that focuses on the full range of complexity, identified at each stage, and recognizing the different types of error (conceptual, calculation, technical measurement), which can then be linked to support strategies.”

(NMC, 2011, pp. 60-61)

While the 2010 and 2011 NMC specifications go some way to addressing concerns with the requirement to achieve 100 percent in a test of numeracy in practice, these concerns remain. Studies have raised concerns about the validity and reliability of the ‘numeracy in practice’ tests set in some training contexts, querying what the tests assess and the extent to which the 100 percent pass mark actually reflects mastery of the numeracy needed for nursing (Coben & Hodgen, 2009). As long as the NMC Guidelines are not a mandatory requirement, ‘numeracy in practice’ tests may still be somewhat removed from practice. For example, while they may assess calculation skills, they may not account for the incumbent technical issues that nurses face in practice such as “failing to displace air bubbles and air boluses from syringes” (Coben et al., 2010, p. 100), hence failing to reflect the multi-faceted errors identified by Sabin (2001) and the concept of competence in medication dosage calculation problem-solving as comprising conceptual, calculation and technical measurement competence set out by Weeks et al. (2013). Dyjur et al. (2011, p. 200) also examine these concerns, suggesting that “there is a serious disjuncture between educators’ assessment and evaluation work where it links into broad nursing assumptions about medication work.” This paper explores this disjuncture between the teaching and assessment of numeracy skills in universities and the broader context in which calculations are carried out by nurses in practice, examining nursing students’ experiences of the teaching and assessment of numeracy for nursing in both academic and clinical settings.

Data and Methodology

This paper is part of an interdisciplinary research project (see acknowledgements) investigating aspects of the teaching, learning and assessment of numeracy for nursing in the undergraduate/Diploma nursing programme in a large School of Nursing in England. There is currently limited literature directly examining nursing students’ experiences and perceptions of the teaching, learning and assessment of numeracy for nursing. Much research draws data from written sources (i.e., questionnaires and surveys) and retains a strong reliance on statistical methods (e.g., Wright, 2011). Therefore, this part of the larger project sought to address the question: “How do nursing students experience the teaching and assessment of numeracy for nursing in both academic and clinical settings?”

In this paper, data are drawn from individual interviews with eight student nurses conducted in 2008. Interviews were conducted by a research assistant outside of the School of Nursing to ensure participants’ confidentiality. Each interview, lasting approximately one hour, was semi-structured, ensuring aspects of interest to the study were covered but allowing the student to bring up aspects that were important to them. As part of the interview students were presented with four numeracy questions (some in a nursing context – see Appendix A) to focus the discussion about assessment of numeracy for nursing courses. Students were under no pressure to complete these questions and calculators were available if required.
All eight interviewees were second or third year students (who were the students impacted by the changes set out in this paper) in either the adult or mental-health branches who self-selected and volunteered to take part in these interviews following an invitation to the cohort. As the sample consisted of volunteers it is likely that the eight respondents were relatively confident with mathematics compared to students generally. The study was conducted according to ethical procedures in place at the time. During the interviews, students were asked to look back on changes in their courses and reflect on a range of placement experiences. The full sample is given in Table 1. It is important to note that none of the sample were paediatric/child branch students where numeracy requirements can reasonably be expected to be more involved.

Table 1 Sample of students interviewed within this study.

<table>
<thead>
<tr>
<th>Pseudonym</th>
<th>Gender</th>
<th>Year of Course</th>
<th>International or Home Student</th>
<th>Highest Maths Qualification</th>
<th>Area of Specialism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caroline</td>
<td>Female</td>
<td>2</td>
<td>International</td>
<td>GCSE</td>
<td>Adult</td>
</tr>
<tr>
<td>Derek</td>
<td>Male</td>
<td>2</td>
<td>Home</td>
<td>GCSE</td>
<td>Mental Health</td>
</tr>
<tr>
<td>Georgia</td>
<td>Female</td>
<td>2</td>
<td>Home</td>
<td>GCSE</td>
<td>Adult</td>
</tr>
<tr>
<td>Jennifer</td>
<td>Female</td>
<td>3</td>
<td>Home</td>
<td>AS Level</td>
<td>Mental Health</td>
</tr>
<tr>
<td>Judith</td>
<td>Female</td>
<td>3</td>
<td>International</td>
<td>A Level</td>
<td>Adult</td>
</tr>
<tr>
<td>Nicole</td>
<td>Female</td>
<td>2</td>
<td>Home</td>
<td>GCSE</td>
<td>Adult</td>
</tr>
<tr>
<td>Peter</td>
<td>Male</td>
<td>2</td>
<td>Home</td>
<td>A Level</td>
<td>Adult</td>
</tr>
<tr>
<td>Rose</td>
<td>Female</td>
<td>2</td>
<td>Home</td>
<td>GCSE</td>
<td>Mental Health</td>
</tr>
</tbody>
</table>

The interviews were transcribed and then collated in NVivo. Analysis was conducted using techniques drawn from constructivist grounded theory (Charmaz, 2006). In this approach, in contrast to ‘traditional’ grounded theory, analytical concepts or categories are derived from reading the data alongside existing theoretical analyses. Constructivist grounded theory therefore responds to criticisms of grounded theory as narrowly empiricist and atheoretical and is an appropriate approach for this study, allowing the authors to build upon their prior work in this area while simultaneously allowing students’ perceptions to be foregrounded rather than dictated through prior theorisation or policy literature. Analytical categories were developed into broad themes responding to the research questions of this aspect of the project and the analysis tools of NVivo were used to examine relationships within the data and produce thematic data-based models. These formed the basis of the results presented below with data extracted to illustrate the discussion.

**Student Nurses’ Experiences of Numeracy Training and Assessment**

This section of the paper draws on the major themes identified in the student interviews to examine their experiences of the numeracy course and opportunities to develop their numeracy skills – as taught within the university setting – in clinical practice.

There can be a tendency when planning numeracy teaching outside of the context in which it is used to fall back on traditional or school-based mathematics. Some numeracy for nursing courses appear to fall into this trap, relying heavily on written and formulaic methods. Wright (2007) suggests that such formal methods and mathematical terminology are inappropriate for the clinical environment, not reflecting the oral expression and everyday language used in drug calculations in practice. Students in this study expressed similar concerns, stating that writing everything out longhand was far removed from what they encountered on placement or expected to be doing in practice. It was generally felt that the numeracy course
within their pre-Registration training was a ‘maths’ course rather than a ‘numeracy in practice’ course:

Derek: Well sometimes it’s weird because you’re experiencing someone who has come from a nursing background but who won’t show you in a nursing way, they’ll show you in an, an imitation of a mathematical way, like they’ll show you the equation way, because that’s the way maths does it, but they won’t show you, you know real, placement, or working, experience way. With the actual maths [course], there’s no practical example given of that, not in my recollection … the mathematics seems apart or separate.

Derek suggests how, despite coming from a nursing background, there is a tendency for the course tutors to deliver a mathematics programme rather than support the development of skills as they will be used in clinical practice. He also begins to highlight a tension between ways of working in training and ways of working in practice. As a result of the perceived mathematical focus of the numeracy course, students talked about seeing it as mathematics and often irrelevant to clinical practice. They talked about questions being written from a mathematical standpoint in mathematical language, using mathematical expressions, rather than as they would be encountered in practice. This resulted in many questions being identified by the students as unauthentic and unrealistic. Where questions were delivered in nursing contexts, students were particularly astute in identifying unrealistic situations that they would be unlikely to encounter in practice:

Georgia: It’s 1.5 milligrams per kilo so I’m going to times 23 kilograms which is her weight, which she’d be dead if she was.

Georgia did note later in the interview that this could relate to paediatric medicine, but this highlights a difficulty in maintaining reality for all students across pathways. Coben (2010, p. 16), drawing on Hutton (1997), has suggested that “there should be an element of differentiation between the requirements for each of the branches of nursing” yet this appears not to be the experience of these student nurses. Other students also highlighted pathway issues, including questions referring to equipment such as micro-droppers only found in paediatric practice and deemed irrelevant to adult-branch training. In addition, writing from a mathematical standpoint resulted in answers to questions that, although mathematically sound, made little sense in reality. Some ‘correct’ answers involved half-drops or miniscule quantities of parenteral medications (e.g. those delivered through injection or infusion rather than by mouth) that it would be impossible to draw up accurately:

Nicole: In practice I would go and check it, firstly, because it just seems like under 1ml which is like, that much. It depends what you’re giving as well, it really does, I suppose if you’re giving something incredibly potent, erythromycin is an antibiotic, and I wouldn’t, I can’t imagine that you’d give less than 1ml, I just can’t, it doesn’t seem realistic.

Nicole highlights the inauthenticity of the answer in relation to practice and the equipment – for instance syringe capacities – used. Other students also suggested the pseudo-nursing context with “Smarties [brightly coloured sugar-coated chocolates] and fake drugs” (Georgia) and “older charts that they’d had from previous times” (Derek) to be problematic, resulting in greater student scepticism rather than the assumed intention of increased realism. The current literature strongly supports the need for questions to be “derived from authentic settings” (Coben et al., 2010, p. 11), giving students a sense of the drugs and drug strengths they are making calculations on (Wright, 2011) in order to understand their relevance and develop skills appropriate to the clinical environment. However, this ambition seems somewhat removed from students’ current experiences. The need for realism was highlighted in students’ (limited) positive comments about the numeracy course or workbook; every response referred to
authentic and applied situations and real-life scenarios. These were noted to be more prevalent in the second year numeracy course/workbook in comparison with the first year course/workbook that was heavily mathematised, with Rose suggesting that, as a result, “in a way the second year is actually easier than the first year.”

The need for authenticity relates not only to the question format and answers but also to a comprehension of the wider context of nursing calculations. Meechan et al. (2011, p. 730) caution that “medication administration should not be and must not be a mechanical process” and that it must take account of the contextual features involved such as patient history, reflecting the multi-faceted nature of drug administration discussed earlier. Numeracy workbook questions, and particularly assessments undertaken on an individual basis, are unable to account for much of this context, being “isolated from embodied reality and the sights, sounds, smells, and other cues that place the nurse in the everyday world of practice” (Dyjur et al., 2011, p. 206). Students noted this as an issue, highlighting the “nice little paraphrase or a three-line question” (Derek) as far removed from the artefacts and equipment they will work with in practice. Further, the need to work individually immediately reduces authenticity in that it removes a key safeguard in clinical practice: double-checking. In reality, checking is mandated in the NMC (2010) Standards for Medicines Management. Standard 8 “Administration” states that: “You may administer with a single signature any prescription only medicine (POM), general sales list (GSL) or pharmacy (P) medication.” (p. 7). However, for controlled drugs: “It is recommended that for the administration of controlled drugs, a secondary signatory is required within secondary care and similar healthcare settings”; and two signatories are required when checking and confirming the stock of controlled drugs (p. 7). With respect to drug calculations the NMC recommend that: “Some drug administrations can require complex calculations to ensure that the correct volume or quantity of medication is administered. In these situations, it is good practice for a second practitioner (a registered professional) to check the calculation independently in order to minimise the risk of error. The use of calculators to determine the volume or quantity of medication should not act as a substitute for arithmetical knowledge and skill” (p. 26). Of note also is Standard 20, in which the NMC recommend that for Intravenous Medication (administered into a vein): “Wherever possible, two registrants should check medication to be administered intravenously, one of whom should also be the registrant who then administers the intravenous (IV) medication” (p. 34); and that “At a minimum, any dose calculation must be independently checked” (p. 34).

Student Nurses’ Experiences of Numeracy in Practice

A feature of pre-Registration nurses’ numeracy development, which should bridge the interface between university and practice, is numeracy training and consolidation on placement. However, the literature suggests that this can be inconsistent with placements being “rich or poor in numeracy terms, depending on the exigencies of the situation” (Coben, 2010, p. 14). This reflects the experiences reported by the students in this study. When asked about their encounters with calculations in practice on placement, many noted that this was highly dependent on the assigned clinical setting. While some settings, such as Accident and Emergency (The Emergency Department), provided access to a range of calculation situations, many, for instance those on psychiatric liaison in the mental-health branch, provided little or no perceived opportunity to implement the numeracy training from university into a practical context. The ward specialism tends to dictate access to numeracy development. As Judith noted, while much of the numeracy encountered in university involved drip-rates and infusions, “very few wards have infusions going, like all the time, these kinds of infusions.” Students may have had access to these on cardiac or surgical wards, but placements on less acute wards, such as
ENT (Ear, Nose and Throat), left students feeling that patients were “generally fine and well and not needing much … I found that actually in a lot of cases there wasn’t much need for calculations” (Peter).

An additional issue with developing numeracy skills in the clinical setting is that institutional policies bar student nurses from specific clinical procedures, for instance the giving of intravenous medications and accessing cannula (a thin tube inserted into the vein to administer intravenous medication) sites. While they could still be involved in the calculations aspect of an infusion, the students in this study found that, in practice, they tended to be excluded from the whole procedure:

Peter: I find it actually really quite surprising how much people [mentors] just sort of are just ‘oh you’re just a 1st year student, get out of the way, I’ll do this, you can’t do that’ which is a little bit discouraging and also meant that any skills that I’d learn in sort of calculations weren’t really being applied.

As Peter highlights, this has clear implications for students’ numeracy development. Many students noted that they were only involved in the most basic calculations involving only enteral (by mouth) route medications, such as being asked how many 500mg tablets a patient requiring 1g of Paracetamol required. They expressed some exasperation at not being involved in or being able to learn or apply complex skills and some regret at the perceived missed opportunities for this.

The earlier section examining students’ experiences of numeracy training and assessment highlighted the tendency for numeracy course constructors and tutors to mathematise the content, resulting in students being taught in a way, as Derek highlighted, that was not perceived to be the “real, placement, or working, experience way”. In what ways was the “placement way” perceived to be different from university methods? It was noted earlier that there is no agreed consensus on what numeracy for nursing should involve. Wright (2012) notes that the numeracy skills taught to student nurses are often based on assumptions about how problems such as drug calculations should be solved – often in a traditional mathematical sense – rather than being based on how nurses actually go about solving such questions in practice. This point is also echoed by Dyjur et al. (2011), who suggest that university numeracy teaching methods are removed from the materiality of the clinical setting. The students in this study made similar observations, noting the gulf between the formulaic written methods expected in the university setting and the methods used by practising nurses:

Peter: You never really write calculations down in practice, that’s never really expected

In particular, the student nurses commented on how practising nurses seldom relied on their own calculations but drew on a range of methods and resources in producing and checking any numeracy element. For instance, many students mentioned the protocol books “telling them how to make up drugs to a certain, I guess concentration or percentage” (Georgia) and Jennifer produced a ready reckoner giving drip flow rates for varying stock strengths, volumes and infusion times. Hoyles, Noss, and Pozzi (2001) found that nurses used a range of methods, including calculators, which were often quite different from the methods taught in university.

A key difference between university and practice, mentioned by all students and often many times, was the use of a calculator, banned in university and numeracy assessments but used regularly in practice:

Derek: The actual calculations, you would always double check with a colleague or a calculator and I don’t see why you wouldn’t kind of, because in reality that’s what you would do, so why wouldn’t we do that even in the tests, they might be better if we could actually use a calculator.
Working longhand, using methods such as an algorithm for long division, fails to reflect methods used in clinical practice (or in most areas of daily functioning). This leads to the question of whether assessments involving written methods are assessments of numeracy in practice or assessments of calculation techniques. The use, or otherwise, of calculators in university pre-Registration numeracy courses and assessments has been long debated (Hunter Revell & McCurry, 2012) and it is the finding that many practising nurses are unable to perform calculations without a calculator (Pentin & Smith, 2006) that has led to an increased focus on written methods in numeracy courses. However, the students in this study reported using some written methods without understanding, rendering the answers as meaningless as those obtained via a calculator and increasing the possibilities for human error.

In relation to the above, a further key contention students voiced was the feeling of being barred from taking university mathematics into placement by the attitudes of practising nurses who did not recognise university mathematics as something they did on a daily basis:

Caroline: But they always say ‘but they don’t do calculations in practice’.

Other students made similar comments. Some expressed frustration that their attempts to adhere to course requirements and use or observe numeracy in practice were met with dismissal and derision. However, such responses may not be the fault of practising nurses, who may not see what they do as mathematics, making the calculations they are engaged in “invisible mathematics”. The notion of “invisible mathematics” is developed by Coben (2002, p. 55) from earlier work and refers to “the mathematics one can do, which one does not think of as mathematics – also known as common sense.” Mathematics becomes invisible because the naturalness, familiarity and regularity with which someone carries out a procedure reconstitute the mathematics as common sense. Further, for many people, mathematics is seen as the algorithmic, formulaic subject of school mathematics; as such, mathematics conducted in everyday life using methods far removed from the classroom cannot be conceived by the individuals partaking in it as mathematics, hence the mathematics becomes invisible. In their interviews, the students in this study referred to incidents on placement where nurses were engaged in familiar natural practices rendering the mathematics invisible:

Nicole: If they’ve been doing, it’s like any job I suppose that you get used to doing, if you’ve given the same drug 300 times you will know how much to reconstitute it with automatically and that’s what I think happens a lot of the time, so they know that if they’re given Vancomycin [an antibiotic], it’s reconstituted with this and away they go, just do it, so I don’t think they, it’s just there like, if you’ve done something over and over again.

Peter: She just whacked the stuff up and I don’t know whether she just knew what she was doing, so you know a bag of saline being over 8 hours or something, whether she just knew instinctively, I expect she probably did, but she would just sort of whack it up, assess the drip rate herself with the infusion set and just sort of let it run and walk off, not explain anything to me, not tell me what she’s done, not explain what drip-rate she’d used or anything like that.

In both recalled incidents, and in others students talked about, the mathematics was there – and in many cases drew on quite complex mathematical ideas – but familiarity with the situation led to the actions being seen in a non-mathematical way. Neither the doer (the practising nurse) nor the observer (the student on placement) recognises the inherent mathematics; in essence the mathematics becomes invisible. This is potentially problematic in that by not recognising the mathematics, it becomes difficult to transfer skills (Coben, 2002), or in this case, pass on skills, and so develop students’ understanding of numeracy in practice.

In fact, nurses are involved in mathematics. There is quite an extensive literature examining the mathematics practising nurses are involved in on a day-to-day basis, much of
which reveals the invisible nature of their mathematical practice. For instance, Cartwright (1996) found that nurses made regular use of estimation skills, yet the only way they could talk about the estimation process was to talk about experience rather than the mathematical processes involved. Experience was also found to be the main point of reference when experienced nurses talked about setting and maintaining intravenous infusions. Other nurses have been found to use scalar methods to find required dosages (Wright, 2012); again these are not thought of by practitioners as mathematics as they do not use a formulaic method; in an earlier paper Wright (2007, p. 830) describes nurses’ practices as “street mathematics” in reference to the seminal study by Nunes, Schliemann, and Carraher (1993) in which Brazilian street sellers used highly contextualised yet efficient and effective calculation methods. Dyjur et al., (2011, p. 207) also look at nurses’ mathematics in these terms, drawing on Lave and Wenger’s (1991) notion of situated learning to show how practising nurses develop “experiential knowledge” allowing them to “discontinue their reliance on formulas” which they may have developed as students. This has implications for how students are taught numeracy in practice. The formulaic method does not represent what happens in practice, yet what happens in practice is not easily expressed or transferred to students. The notion of common-sense may be one way into this; the students in this study were aware of the need to think about how reasonable their answers were and appeared to have covered this in their numeracy course:

Jennifer: But in practice you’d always have a look at how much you’d actually been giving if it turned out that you were giving 7 vials of something then you’d probably know that something was a bit amiss, that you’d made a mistake. That’s what we were always taught, like the common sense.

It may be questioned whether common sense can be taught, but this is potentially a way in which practice and university mathematics could be brought together. Discussing situations such as giving seven vials (small glass containers holding liquid medicines) may make the mathematics involved more visible, and also brings in the authenticity and relevance discussed previously.

Bringing students’ learning of numeracy more into line with the methods practising nurses use may make courses seem more real in that they could better reflect the methods students see practising nurses using on placement:

Caroline: You don’t have to calculate it, the machine does it for you. An ordinary nurse will actually say ‘oh you don’t need to do it, the machine will do it for you’.

Instinctiveness, familiarity and estimation (camouflaged as a reliance on machines), in addition to non-formulaic methods, serve to hide the mathematics from practitioners. Getting students to identify the mathematics involved in such incidents, rather than collude with practising nurses in seeing it not as mathematics may provide a way in to teaching numeracy for nursing in a more contextual and meaningful way. Certainly, what happens in university needs to be brought into line with what happens in practice and all – students, practising nurses, course tutors – need to be able to identify the mathematics involved.

In the extracts above, the practising nurses many students referred to would have been their placement mentors. The mentor plays a particularly important role in terms of numeracy development, becoming a ‘significant other’ to the students. While ‘significant other’, taken from Coben’s (2002) work on adults’ mathematical life histories, would usually refer to someone with more prolonged contact with the individual, the very specific role of the nursing mentor places them in a position in which they fulfil the role of a significant other in developing numeracy in placement. Students repeatedly discussed in their interviews how mentors had either helped, or acted as a barrier to, developing their understanding of numeracy in practice.
Students identified a conflict in the mentor’s role and the challenges inherent in balancing their responsibilities as a practising nurse with their responsibility to support the development of student nurses. Difficulties in achieving this balance have previously been identified in the literature, with Wright (2011) commenting on the frustration felt by student nurses in the lack of opportunity to put into, or see in, practice, the use of taught calculation strategies, in part because mentors could not identify or explain the mathematics themselves. Further, the very nature of the nursing environment means that the situations where more complex calculations are being carried out which would be useful for students to be involved in, are very often emergency situations or those where patients are acutely unwell. In such situations, the nurse’s priority is the patient and the mentor is unable to support the core skill development of the student (Meechan et al., 2011). The students in this study felt that the lack of time and the busyness of departments such as Accident and Emergency, made it difficult to question calculations or to ask for explanations and as such reported they “found it hard so far to get the necessary practice when I’m in clinical practice” (Georgia).

In addition to the wider opportunities afforded, or restricted, by the clinical environment, characteristics specific to individual mentors also impacted significantly on students’ numeracy development. Students referred to ‘luck’ in terms of their assigned mentor and the subsequent implications this had for numeracy development. In some cases, particularly for those students on the mental-health training branch, they were significantly constrained through being allocated non-dual trained mentors who had not undergone adult nurse training and as such were in a weak position to develop students’ nursing calculation skills. For other students, mentors were assigned who had completed their nursing training before calculations training had come into force. Although hospital trusts implement their own calculations tests, these nurses, who may be mentors, are able to practice without having passed such a test. In practice, students noted that this resulted in these nurses having all their drug work conducted by a senior nurse, increasing the pressures on that nurse and reducing the possibility for the student to engage with any calculations conducted as mentoring was not part of the role of the senior nurse. Sabin (2001, p. 37) talks about the need for clinical staff, in this case nursing mentors, “to articulate their problem solving strategies, and to encourage discussion regarding alternative strategies.” However, practicalities of the clinical environment, combined with the potential lack of skills or training in some mentors, not only in the actual mathematics but in the skills required to communicate and transfer this knowledge (Coben, 2010), result in a weak mentoring relationship in terms of developing numeracy skills in practice.

The Reality of Numeracy in Practice: Safeguards and Double-checking

The previous sections have highlighted a key disjuncture between the individualistic written calculation approach of numeracy training and assessment, and approaches used in clinical practice, not least the implementation of the NMC’s Standards requiring the key safeguard of double-checking. The reality of clinical practice, which imposes such safeguards on practice, seems to sit at a disjuncture with university training, something Derek took further in his interview:

Derek: We have to do a number of questions with the maths and there was one question which related to a child and it said this baby is being prescribed, blah blah blah, it was a normal question and I straightaway was thinking, well hold on, in reality, if I had to give that I’d be really, really cautious about that because it’s a child. Is that an appropriate amount? What does the BNF [British National Formulary – a pharmaceutical reference book giving prescribing details of all medicines including appropriate doses] say about that? You know, what’s the situation? You have to consider loads of other factors and I thought what was more appropriate there was that you actually didn’t give an answer, because if you just blindly went through it could kill the child. What would be more appropriate would be putting in that stopgap of saying in a certain circumstance, actually find out and double, triple check.
Derek’s interview quote highlights an important disjuncture between the ways in which numeracy is taught and assessed in universities and the broader context in which calculations are carried out in practice. This quote particularly brings to the fore the role of safeguards which appear absent in university numeracy training. As Derek observes, while there is usually one correct mathematical answer to numeracy questions they are presented with in the university context, some of these answers would need to be checked or applied in the clinical setting before a ‘solution’ could be reached. This has two outcomes; the calculation skills required by universities are not being followed-up or practised in the clinical setting and students are not being prepared for the ways in which calculations actually take place in practice.

Many of the contextual features which change the ways in which numeracy happens in practice have resulted from safeguards being built in to reduce the risks associated with calculation errors. While positive in terms of patient safety, this has implications for how students are taught in that methods not taking into account such safeguards are inauthentic and unreliable in the clinical setting. As Dyjur et al. (2011, p. 205) note, “one unintended consequence is that these advances may make it more difficult to maintain currency in calculation ability.” Jukes and Gilchrist (2006) take this issue further, suggesting, perhaps provocatively, that with so many safeguards in place and machines to calculate and administer medications automatically, there may be no need for students to retain calculation skills. However, the experiences of the students reported in this paper would suggest that mathematics is required in practice but needs to be recognised, and it is perhaps a different type of contextual mathematics that students need to be involved in. Students note that the calculations they are asked to perform are not what would be required in practice due to built in safeguards:

Peter: Partly because like things are, I suppose made to be bullet-proof, I suppose and sort of idiot proof, which is, I think, quite useful and therefore you’re not really expected to make that sort of calculation.

Such safeguards occur across the drug administration process and can involve several individuals in double and triple checking. Prescriptions are written by doctors, usually in unit doses, and charts are reviewed and, if necessary, clarified, by pharmacists to “make the prescription clearer in an attempt to reduce the possibility of administration error” (Arkell & Rutter, 2012, p. 202). These safeguards were witnessed by students who subsequently referred to them in suggesting that being asked, as pre-Registration nurses, to make such calculations was irrelevant to practice:

Nicole: If they’re a good doctor and they know that it’s erythromycin [an antibiotic] that they want, then they’ll put it in a unit that’s available, does that make sense, and if they don’t then the pharmacists are always coming round checking the drug charts and they will put it in the correct units, you see green pen over everything, so I don’t, you might have to if something, I don’t know, if the unit wasn’t available, I don’t know, I don’t know, but I do find them a bit irrelevant.

While Nicole does note that calculations may be required if a particular unit dose was unavailable, several students emphasised that the calculations they are being asked to complete tend to be in the doctors’ or pharmacists’ domain, rather than that of the nurse. Further, Nicole refers to units that are available, a built in safeguard which many students referred to:

Nicole: In practice because a lot of drugs come in their varying doses so if you’ve got 5mg or 2mg or 0.5mg whatever it happens to be then in theory it should cover all shapes and sizes ... they don’t usually prescribe things in a unit that it doesn’t come in.
The majority of medications are supplied in specific units reflecting usual prescribing patterns. Where a patient requires a non-unit dose, this can usually be achieved through addition of the units available and is written as such by the pharmacist on the drug chart. While nurses are involved in (invisible) mathematics in administering these medications, the mathematics is not that which is taught in universities.

A further disjuncture between university and clinical setting mathematics comes in infusion rate calculations which make up a substantial proportion of nursing students’ numeracy courses. Every student in this study noted that, in clinical practice, infusions, particularly where quantities were crucial, were set through equipment such as an IVAC ® Infusion Pump (an electronic pump delivering a continuous rate of infusion of a drug over a specified time). In assessment questions, students were presented with images of these but were still asked to calculate drip-rates. However, in practice they noted that this was not necessary and was in fact calculated by the machine:

Peter: Anything where you need a strict infusion or you’ve got someone who is on a strict fluid balance chart or whatever, you use some sort of electronic infusion devise anyway, like an IVAC pump or something which you literally just dial up how much you use, so how many millilitres per hour you want infused and they’re fairly easy to use so in that sense you don’t need to calculate drip-rates for those.

Peter expresses a common belief that, due to the automated pumps available, there is no need for the calculations they are expected to engage in. While this may or may not be the case, and is explored further in the discussion below, facing this mismatch between what is taught in university and what is experienced on placement potentially leads to students seeing the numeracy course as irrelevant, making it harder for any positive aspects to reveal themselves. It is worth noting that Noss, Hoyles, and Pozzi (2002) give an example where operating automated pumps required greater mathematical understanding than in ‘ordinary’ drug dosages.

**Discussion**

This paper, through a grounded analysis of student interviews, has suggested that pre-Registration trainee nurses face distinct conflicts in terms of numeracy teaching and assessment. The numeracy skills students learn and that are assessed in the university setting are not reflected in practice. Likewise, the mathematical methods seen in practice are not taught or discussed in the university setting. Students are faced with a chasm or disjuncture between what they are taught in university and what they witness and how they are expected to carry out mathematical procedures in the clinical setting. The clinical environment is set up to be as safe as possible, but numeracy methods taught in universities do not reflect working under these safeguards and hence disregard the adapted calculation practices and procedures of practising nurses.

While there is an argument that practising nurses should be able to cope and work manually if, for instance, a volumetric pump is not available, it seems unlikely that one-off teaching of this at pre-Registration level will be sufficient for it to be implemented if such a situation should arise. It seems more realistic to support pre-Registration students in developing an understanding of the range of methods used by practising nurses, and a more secure ‘feel’ for the numbers and measures they encounter, increasing the likelihood that they will spot errors and can quickly tell whether a manually calculated dosage is likely to be correct. A key issue relates to calculators. In our study, nursing students highlighted the presence of calculators in clinical practice and their absence in training. Yet, evidence suggests that nurses perform better with calculators (Hutton, 1997). Calculators are, however, not a panacea and the students in our study appeared to view calculators as largely unproblematic and always giving the correct
answer. The evidence from school mathematics suggests that learners need to understand that the calculator is only a tool and need to be taught how to use this tool and interpret the results correctly (Ruthven, 1998).

A further key area for change, brought up across the student interviews, is the need for much stronger collaboration and understanding between universities and clinical settings. Current provision can come across as fragmented; this needs to be brought together and the connections, not just between numeracy courses and placement but with all aspects of pre-Registration training, made explicit. Bringing university and clinical practices closer together would provide students and mentors with an opportunity to explore practising nurses’ methods, to investigate the mathematics underlying these and to develop a deeper appreciation of the mathematics that is conducted on a daily basis in clinical settings, including the differences between the various clinical specialisms.

**Conclusions and Recommendations**

This paper has provided an in-depth analysis of nursing students’ experiences of the teaching and assessment of numeracy for nursing in both academic and clinical settings. In doing so, it has revealed the tension or disjuncture between practices in these two sites as experienced by nursing students. Pre-Registration numeracy courses tend to mathematise the content and rely on formal written methods, while numeracy in clinical practice – responding to the need for safe-guards – is embedded in protocol books, calculator use, and familiarity with standard unit-doses. Importantly, clinical practice takes place in a social and collaborative context rather than individually as is common on many pre-Registration courses. As things currently stand, lack of congruence between numeracy courses and expectations in the university setting and in clinical practice, appears to result in a number of pre-Registration students viewing the whole numeracy course as irrelevant and little more than an element to be passed as part of their pre-Registration training.

It would appear from the students’ experiences in this study that there is a need to reconsider the structure and content of numeracy courses in pre-Registration nurse training. While current approaches to the development of numeracy for nursing often propose hierarchical models with numeracy in clinical practice at the pinnacle (e.g. Wright, 2011), we suggest that, when teaching numeracy for nursing, the relationship to clinical practice needs to be made explicit from the initial stages of training. Removing the authentic context may create and extend the gap between university and clinical setting numeracy practices, even where the intention is the opposite. Many of the safeguards, and resultant changes to calculations in practice, are a result of the context in which nurses must work, and student nurses should be trained, in authentic ways, to work within this. Mathematical fundamentals are of course still central. Studies have demonstrated substantial weaknesses in this area and some widely reported errors have resulted from deficits in understandings of the decimal number system and an inability to recognise when derived answers are out by a factor of 10 or 100. Many nursing students will not have encountered numerate subjects since GCSE level (General Certificate in Secondary Education – taken at the age of 16) so there is a need to ensure a firm foundation in these elements. However, it is also important that students recognise why they are being taught apparently context-free ‘school’ mathematics and there is space here, from the beginning of the course, to engage students in an understanding of risk and the potential implications of fundamental errors in a nursing context. Context is vital to students’ authentic development of numeracy skills and their willingness to participate in such learning. Many students in this study indicated that the ways they were taught and assessed bore little resemblance to what they encountered in practice. The use of artefacts needs to be stronger and their use needs to reflect how they will be encountered in practice, for instance: through the use of complete drug charts
rather than carefully extracted elements; working with real medications and using tools such as
protocol books and equipment readily available to practising nurses in the clinical setting. Such
an approach would be in keeping with the model of competence in medication dosage
calculation problem-solving as comprising conceptual, calculation, technical measurement
competence described by Weeks et al. (2013) and referred to in the Standards for Pre-
Registration Nursing Education (NMC, 2010). Students need the space and time to engage with
questions around what would happen in reality, such as applying common-sense, double
checking and looking up quantities in the BNF (British National Formulary), rather than
providing quick mathematical answers in pseudo-nursing contexts.

Important work has already been done to investigate the numeracy competencies and
training of pre-Registration nurses. This has resulted in important changes including an increase
in the authenticity of training and assessment. However, there is still work to be done in
increasing authenticity and reducing the gap between university and practice, particularly in
relation to the rapidly changing technological environment. This study has drawn heavily on
students’ experiences and suggests that really listening to these should be an important starting
point in bringing about positive change.

Acknowledgements

This paper is part of an interdisciplinary research project entitled ‘Numeracy for Nurses’,
Principal Investigators: Diana Coben and Jeremy Hodgen, with Meriel Hutton and Sherri
Ogston-Tuck, funded by King’s College London.

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Appendix A

Four numeracy questions presented within individual interviews. Questions 1 and 4 have a nursing context while questions 2 and 3 are abstract.

1. A girl with an infection is prescribed 1.5 mg per kg of an antibiotic. The girl weighs 23 kg. What is the dose in mg for her weight to be administered?

2. What is 5% expressed as a decimal?

3. What is 0.04 mcg expressed as ng?

4. A dose of 22.5 mg is prescribed. The stock dose of oral suspension for erythromycin is 250 mg per ml. What volume should be administered?
Reflections on Our First Calculus Undergraduate Teaching Assistant

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Abstract

This article describes some reflections from the first Calculus I undergraduate teaching assistant in our department as she explored the various ways in which she was able to support both novice and experienced Calculus teachers and the effect of her experience on her academic and career plans.

Keywords: Undergraduate teaching assistant; instructional support; calculus.

Introduction

There is a growing body of literature (across academic disciplines) that supports the use of undergraduate teaching assistants (UTAs) in college classrooms for a variety of reasons. The use of UTAs provides benefits for all involved in the process—the faculty members who work with UTAs in the form of support in teaching; the students enrolled in the courses in which the UTAs assist in the form of additional support in (and sometimes outside of) class; and the UTA in the form of experience teaching and mentoring from more experienced supervisors.

Over the last few years there has been a steady rise in the general undergraduate population at our institution, a large land-grant public research university, as well as in the number of students taking undergraduate mathematics classes. In addition to this, in response to calls for more active learning models in our classrooms to support improved student understanding, our department implemented changes within course structures in several undergraduate mathematics classes to allow for more in-class participation of students in an effort to move away from the traditional lecture format seen in many undergraduate mathematics classes to allow students to explore active learning. Specifically, in Calculus I, class sizes were lowered and graduate teaching assistants (GTAs) added as instructional support in the classroom to aid more experienced instructors in implementation of active learning strategies. GTAs have been called a “crucial intermediary in the classroom” between faculty members and the undergraduates they teach (Stoecker et al., 1993, p. 334). This belief has been extended to UTAs (Fingerson & Culley, 2001) and is the perspective of this article’s author.
To facilitate the use of UTAs in the classroom, the department recently began allowing undergraduate students to earn credit by enrolling in a teaching practicum course. For this credit, they are to offer instructional support to Calculus instructors who are willing to supervise them. This paper describes the first instance of using a former Calculus I student as a UTA in Calculus I during subsequent semesters. I will present my own reflections on the process of supervising her over the course of several semesters as well as the reasons I (and the instructional team for Calculus I) initially sought the assistance of a UTA, the benefits and drawbacks of the experience, as well as some logistics of the entire process and lessons learned to aid us in improving the experience for both faculty who supervise UTAs and for the UTAs themselves. Most importantly I describe the effect of the process on the UTA and how it has affected her collegiate and post-collegiate career plans.

Institutional Context

With simultaneous growth in enrolment and efforts to provide a more active, personalized learning environment for students, there has been a parallel increase in the demand on instructional resources. Indeed, between fall 2008 and fall 2014 we increased our offerings in lower level mathematics courses (Liberal Arts Mathematics through Calculus II) from 60 sections to over 130 sections. The existing instructional pool in our department was not sufficient to implement desired pedagogy and curriculum changes to shift toward more interaction and groupwork in our Calculus I classrooms. In an effort to relieve the instructional need of Calculus instructors, the notion of UTAs was considered and the first, Ann, was enrolled in the teaching practicum course.

Other departments at our institution had successfully been enrolling undergraduates in their own undergraduate teaching practicum courses for years. While there are currently no existing university guidelines for this university-wide course, many departments had developed their own guidelines and requirements for UTAs. The Department of Mathematics had not yet developed guidelines or requirements for UTAs so we treated our experience with Ann as a learning experience wherein we could define both her role and that of her supervisor as her teaching practicum progressed. The only guideline instructors did agree upon was an equivalence of three hours work per week for each credit earned. Therefore, during Ann’s first semester, she enrolled in 3 hours of credit and was expected to work roughly 9 hours per week in the support of Calculus I.

While existing UTA programs generally use more systematic processes for selection of assistants and assigning faculty supervisors (Fingerson & Culley, 2001; Mendenhall & Burr, 1983) these processes were more spontaneous in our case. Ann learned of the teaching practicum opportunity by discussing with her GTA (during calculus) how he became a class assistant. From there, she was referred to her instructor then to the course coordinator and her assignment to a faculty supervisor was made based solely on the times she was available to attend calculus classes the next semester.

Ann’s Experience

Ann’s Experience as a Student of Calculus

During Ann’s semester as a student in Calculus I, both the instructor of record for her class and the class assistant were GTAs (one more experienced than the other). One of the primary
responsibilities of the class assistant is to work with students during group work activities. Near
the end of the semester, Ann was frustrated in her class assistant’s ability to facilitate effective
groupwork. As a disgruntled student, she decided to ask her class assistant how he got his job.
He explained the position of GTA to her, and informed her of how she might become an
assistant and earn credit for this as an undergraduate.

While Ann didn’t have any mathematical preparation beyond the course in which
she would be assisting, we felt comfortable using her as our first UTA for two reasons. First, she
had just successfully completed the course and was therefore aware of what was expected of the
students in the course. Second, throughout the semester she spent as a student in calculus, she
was seen by her instructor assisting a classmate who struggled throughout the semester. Her
skill to work well with others was thought to be an asset in a class with a focus on active
learning and relatively frequent group activities.

Ann’s First Day as a UTA

Ann’s class assignment was determined by the course coordinator the night before classes began
that semester. Therefore, Ann showed up to class on the first day without clearly defined
expectations for appropriate classroom behaviour. During that first class, Ann (having just
successfully completed the course) was eager to play the role of student in class and answer
questions as they arose, already knowing the answers. It became evident that a discussion about
her role in the classroom was needed. This was just the first of several instances throughout the
semester where it was apparent that the instructional team had made assumptions about the
knowledge Ann had with regards to her responsibilities and roles as a UTA.

While there were no predetermined guidelines for the undergraduate teaching practicum
in mathematics, there was an initial need of support for grading. Our calculus course was multi-
section with hundreds of students each semester. Students use an online homework system for
some homework problems, but approximately 50% of homework and all exams and quizzes are
completed by hand, requiring many hours of grading. Exams and group work projects are group
graded, meaning one grader grades only a subset of problems, but grades these for each student
enrolled in the course regardless of which section they are enrolled in. Despite the ever-present
need for grading help, the instructional team was aware that responsibility for supervision of a
UTA extended beyond grading oversight. However, as this was the first experience for any of
the instructors, we decided to learn to navigate the additional responsibilities as they arose
throughout the semester.

Ann’s Past, Present and Future

Both of Ann’s parents were secondary school teachers during her childhood, which greatly
affected her career choice. She was always interested in science, but was unsure about what she
would do with that career choice. The one thing she was sure of, though, was that she did not
want to become a teacher. She maintained this belief during her first two years in college but
was still uncertain about her future plans. With an interest in science, she began her college
career as a chemistry major and eventually decided that a career in the medical field would be
more satisfying. She applied to and was accepted into the university’s Medical Laboratory
Science program while working as a calculus UTA.
Ann assisted in Calculus I classes for over two years. While the initial need and easiest way to implement additional support was in grading, that was only a small part of her role as a UTA. Over the first few months, and subsequent semesters, Ann took on more and more responsibilities in and outside of the classroom. She proctored exams, graded homework and quizzes, ran help sessions, answered questions related to the online homework platform students used, and most importantly, she assisted in the facilitation of group work in class with the assistance of the instructor and GTA. Ann came to realize one significant contribution she was able to make as a UTA. While working over several semesters, she worked under instructors with various levels of experience, from faculty with over 12 years teaching experience to GTAs in their first or second year of graduate school. After her first semester, she assisted in multiple classes each semester. Classes met most days of the week, and most classes followed roughly the same pace throughout the semester. Therefore, on any given day Ann could have worked with students in multiple classes on the same activity or topic. She soon learned to watch for different ways in which different instructors taught the same material. She also learned how to take information from the classes of the more experienced instructors to the less experienced ones. She began to regularly meet with the more novice instructors before class to discuss what the more experienced instructors had been doing in their classes. Ann felt this to be her greatest contribution to newer instructors.

The experience of assisting others to learn mathematics had significant effects on her career plans. During her first year assisting, her interest in mathematics grew and by the end of the second year, she had changed her major to mathematics, and had decided she wanted to pursue teaching. The experience of working with students had shown her a side of teaching that she didn’t get to see from her parents. She realized that not all teaching would be the same and that it was a career she could enjoy. She graduated with her undergraduate degree in mathematics, completed her senior capstone in mathematics education and is currently enrolled in a graduate program in mathematics education.

**Discussion**

Undergraduate Teaching Assistants are used throughout institutions in various disciplines to provide instructional and administrative support for laboratory or recitation sections and serve as class assistants. Their roles are as varied as their disciplines and institutions. Being a UTA is a way for undergraduates to solidify their content knowledge, gain confidence, gain classroom experience and prepare for leadership positions. For an instructor, having a UTA in the class can provide both administrative (attendance tracking, grading, data entry) and instructional (non-traditional pedagogy implementation, active learning assistance) support in the undergraduate classroom.

One benefit of having Ann as an assistant for several semesters was that she was well prepared to assist instructors as needed with little direction after her first semester whereas GTAs who are new(er) to the course each semester required more time spent by instructors informing them of course policies and procedures and in implementation of course-specific curriculum. Roughly half of our GTAs are Master’s students, many of whom spend two years or less in our program, so their progression through GTA assignments moves rather quickly to enable them to teach their own class before degree completion. This results in a relatively quick turnover in GTAs assisting within any given course. The availability of a reliable individual each semester who needed little oversight was an asset to the instructional team.
However, there were also unanticipated benefits to both Ann, and to the instructors who supervised her. Her greatest instructional contribution was her ability to take information from experienced instructors’ classes to novice instructors. The UTA can present this information from the perspective of a student in the class, as well as from that of an assistant working with students providing the novice instructor with information that would not be present if the more experienced instructor were to simply tell the novice instructor how they teach certain topics or lead certain activities. This role has been emphasized as a role of UTAs in subsequent semesters when UTAs assist multiple instructors, including novice GTAs.

The greatest unanticipated benefit Ann received from her experience cannot be understated. While she entered college not knowing what she wanted to do for a career, she was sure about what she didn’t want to do—teach. However, as she gained confidence in her mathematical ability, her interest in mathematics increased as well. This increased interest led to an eventual change in academic and career plans, leading to one completed degree in mathematics and an advanced degree in progress.

While the opportunity to work with this specific UTA presented itself to course instructors and coordinators in this case, future endeavours to recruit, select and assign UTAs will be more systematic. Instructors should be involved in choosing their own UTAs as opposed to having them assigned to their classes based on schedules. They should be chosen based on personality traits which the instructors feel will most benefit the instructor, the UTA or the students depending on the ultimate goal of the instructor in overseeing the UTA throughout the semester. Clear expectations of classroom behaviour and conversations about the goals of the UTA and the goals of the instructor will occur before the assignment begins.

In addition, many of our majors do not declare mathematics as an academic major until well past the level of calculus, usually in their third year after taking many upper level mathematics and science classes. Indeed, very few students enter the university with a declared major in mathematics. While Ann made her decision roughly during her third year (the same time that most of our majors do), she had only taken Calculus I. With a more systematic approach to recruiting UTAs early in their mathematical careers, specifically from within the most successful Calculus I students, we have the opportunity to attract more mathematics majors earlier in their academic career providing them with more opportunities to explore mathematics and mathematics education at deeper levels and to pursue undergraduate teaching.

**References**

