Easily and Economically Overcoming Cognitive and Affective Obstacles to Competent Numeracy in a Wide Range of Adults

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Just as there are adults who seem normal under superficial observation but cannot even recognize letters, there are adults who seem normal under superficial observation but who know virtually nothing of the basic arithmetic facts and operations. Consequently, they cannot respond positively to elementary remedial instruction designed to develop fundamental mathematical skills. If they are undiagnosed, they are often interpreted as lazy or willfully resistant. Moreover, if undiagnosed multiple derivative difficulties will develop and never be properly remediated, leading to a practical lack of function in almost all quantitative applications, and the closure of a very wide range of employment options. Serious affective complications inevitably develop. These affective complications can be significantly exacerbated by cultural variables, such as the need for a "macho" image incompatible with working on the "skills of children". Fortunately, perhaps surprisingly, new fact generators and new algorithms and new presentation perspectives can remove both the cognitive and affective obstacles with great ease and economy. The paper will demonstrate each class (generators, algorithms, affects) of these new procedures using handout materials designed and intended for extensive audience involvement and participation.

Introduction

Since we are going to work with each other for a short period of time, I will try to assimilate what you already know about the teaching and learning of math to divergent adults in terms of job related math skills. For such assimilation to work, we must consider the following constraints:

1. The time limit we have together in this workshop.

2. The magnitude of our performance as expected by ALM, and above all by you as the participant who gave his/her time to learn something that is worth while. To achieve the height level of total quality management (TQM) our task must be manageable and reasonable to the limit that it will not create a level of anxiety that would affect our performance on the task.

3. Your learning style. For example, if you are a left brained passive or active learner, then you are expecting a well structured, highly organized lecturing format. On the other hand, if you are a right brained individual such conditions of teaching and learning would make
your life miserable. So, to address this constraint, a dialog between you and me is necessary to make your short stay with me productive as well as enjoyable.

Since my presentation is not a pre-canned one that lacks the desirable flexibility to meet your need, I am requesting your input on the following forms.

The first form is to help me in knowing you better. Such knowledge will help me to provide the best I can for your targeted population.

The second form is designed to collect from you the name of 3–5 fundamental mathematical skills that are impeding your students from learning job related mathematical skills at your targeted place of teaching/learning.

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**FORM 1**

**WHAT ARE YOU EXPECTING TO LEARN?**

**NAME:**

**IN THE SPACE PROVIDED BELOW PLEASE LIST THE SKILLS THAT YOU ARE EXPECTING TO LEARN FROM THIS WORKSHOP.**

1. ______________________________________________________________________

   ____

2. ______________________________________________________________________

   ____

3. ______________________________________________________________________

   ____

4. ______________________________________________________________________

   ____

5. ______________________________________________________________________

   ____

6. ______________________________________________________________________

   ____

7. ______________________________________________________________________

   ____

8. ______________________________________________________________________

   ____

9. ______________________________________________________________________

   ____

**PLEASE ADD MORE TO YOUR LIST IF YOU SO DESIRE.**
FORM 2
PROGRAMMATIC INFORMATION

NAME: ____________________________________________

IN THE SPACE PROVIDED BELOW PLEASE LIST 3–5 FUNDAMENTAL MATH-EMATICAL SKILLS THAT ARE EMPEDEING YOUR STUDENTS FROM LEARNING MATH-EMATICAL BASIC SKILLS.

1. ____________________________________________
   __________

2. ____________________________________________
   __________

3. ____________________________________________
   __________

4. ____________________________________________
   __________

5. ____________________________________________
   __________

________________________________________________________________________
   __________

Our procedures are very, very thoroughly researched and it is time for appropriately prepared and carefully monitored field applications. There is a very high likelihood that a properly implemented low-stress program would produce extraordinary large gains, very quickly and at low cost. The procedures quite literally have the potential for an unprecedented positive impact.

The catch is that because the procedures are quite different from anything else, those who will use them must be well trained. It is not enough to hand the teacher a manual and say “Do this”. Fortunately, because the procedures are inherently simple, training can be rather fast. Most teachers could be trained in comparatively very short period of time.

Low-stress methods are based on providing the relevant mathematical procedure with a complete and concise record, then using the options provided by this record to reorganize the procedure for greater efficiency and understanding. Virtually all topics lend themselves to this approach.

We have developed applications and programmatic sequences from nursery school through graduate school; our research has been distributed across the United States and has invariably provided enormous positive gains (usually order of magnitude gains) at very low cost. These gains are extremely gratifying, but they carry with them a subtle hazard.

Because these gains are so unusual and dramatic, it is extremely important that credibility be unarguably established by keeping records of those students who were taught the procedure, what they were taught, and exactly what the results were. We are in the process of implementing curriculum change at certain familiar locations in the United States and have found that an
incremental, carefully evaluated approach (with plenty of time for teacher and/or parents adjustment) leads easily and peacefully to great permanent improvements.

There are five major benefits to be developed and evaluated through this material and training:

1. A very large reduction in mastery time occurs. Students acquire these procedures in a small fraction of the time required for mastery of the conventional procedures. This allows the teacher to give more precious curriculum time to the difficult concepts.
2. There is a great increase in the number of students who are successful. Virtually all students can become extremely effective with these procedures. There will be now “slow” group requiring extra teacher time and suffering embarrassment or frustration.
3. Stamina, power, and distraction resistance will be very much extended with no additional effort. Because the procedures make virtually no use of short term memory, a great deal of work (more problems or larger problems) can be done with ease, and if concentration is momentarily lost, the child can return to his work place without confusion or repetition (because every step of the work he has performed has been concisely recorded.)
4. Greatly improved diagnosis and prescription will occur. This represents both a large saving of teacher time and a large increase in the amount of corrective information flowing back to students. Because every step in a low-stress algorithm is expressed in special concise notation, the teacher, or an aide, or a family member, or a student helper, or the student who did the work can find any possible error simply by comparing the student’s work to a teacher provided sheet displaying the correct steps and the correct answers.
5. Significantly improved motivation and morale always occur. Rapid, frustration-free mastery and wide spread success sharply increase the positive feelings of students toward mathematics. Improved students’ attitudes increase a teacher’s feelings of work and satisfaction.

Serendipity of these procedures is their extreme economy, both of implementation and evaluation. Implementation requires only student activity sheets (which might be even mimeographed) and the assessments measure large unambiguous effects with standard problem formats and simple semantic differentials.

In a sense, there is (and will continue to be) ever increasing pressure for students and workers to be more technologically prepared. Therefore, as mathematics controls technological development and arithmetic modulates mathematical growth, it is likely that a dramatically successful basic math or applied math program would eventually attract extraordinarily large and diverse markets (schools, adult education, industrial training, military training, etc.). This would be especially true for an easily partitioned program, where independent, highly effective techniques could be presented as an increasing set of supportive supplements that might incrementally secure markets, without prematurely alarming the publishers of established conventional programs that have limited effectiveness.

Please note that the procedures apply at any level and to any topic in mathematics or the quantitative applications of subtraction. Often effects of the low-stress approach are even more dramatic on higher level procedures, but the principles are always exactly the same:

1. All operations component to a procedure are explicitly expressed in concise notations. This drastically reduces demands upon memory, for both initial learning and operation, and greatly simplifies the diagnosis of error.
2. The procedure itself is reorganized for greater efficiency, taking advantage of these explicitly expressed components. This streamlined restructuring can provide significant advantages procedurally, conceptually and logistically.

You have a brief description of low-stress theory. Please read it, try its procedures in your classes, and give us your feedback for further possibility of collaboration. Please understand that our well researched procedures are tools for success. You, and only you, are the ones that will determine its usefulness and benefits to your students.

The diagnosis and identification of the 27 Most Common Barrier to Success in Elementary Mathematics and How to Remove Them Through Low-Stress Math is the frame of reference of our basic training. These barriers and Low-Stress Math techniques to remove them are correlated with the skills required to be mastered by the State and Federal mandates. The new adopted textbooks by the State will be an integral part to be used as reference material in the help of diagnosing and the removal of these barriers.

Program Description:

The general topics considered are intersections of certain curriculum standards published by state or national agencies. These include number, operations, relations and functions, geometry, measurement, probability and statistics. However, all of these things are approached with a focus on new alternative mathematical procedures (full record algorithms, which are usually called “low-stress”) and upon new learning-teaching strategies that accompany these procedures for the diagnosis of the mathematical barriers that are keeping the students from the mastery of basic and fundamental mathematical skills:

Major Intended Learning Outcomes:

At the end of our training institute each participant will be able to:

1. Identify the error pattern involved in the wrong answer given by the student. That will include the 22 patterns of computational errors as identified by Robert B. Ashlock *Error Patterns in Computation*, 6th Edition. 1994.
2. Teach Low-Stress Math techniques to overcome such barriers to success.
3. Develop, using all available sources, and reinforcement activities to overcome such barrier(s).
4. Correlate all classroom activities to the adopted framework and Standardized tests selected by the district. Participant will have the chance to use the new material provided by the new text books adopted by his/her school.

Instructional Techniques and Strategies:

This institute focuses on full record (low-stress) procedures. Some of these special techniques are procedural. That is, they are improvements in the processes employed by a student when performing the operations and activities associated with each sub-topic. These procedural improvements have been thoroughly researched formally, clinically, and in the field.

Some of those special techniques are pedagogical. That is, they are improvements in teaching procedures made possible by, and capitalizing on, the Low-Stress math process improvements. These are refined by the latest learning accelerators and have been thoroughly researched clinically and in the field.
These procedural and pedagogical techniques are used for the identification and removal of the barriers to success in elementary mathematics. Each teacher is taught those low stress procedures relevant to her grade level and probable student population (Proportion of able vs. challenged …etc.). After mastering the procedures the teacher is asked to develop a series of lesson cores. These consist primarily of the material the teacher will use in teaching a particular topic, and its related identified barriers, with low-stress procedures. After the developed materials are approved by the instructor the teacher reproduces enough of them for the class. The Curriculum responsibilities have been divided across the class, so that when material are exchanged, each teacher acquires a notebook of materials covering a broad range of topics and learning situations.

The Assessment Techniques or Strategies That Will Be Used to Determine the Achievement of the Intended Learning Outcomes:

Formative measures will organized around the Framework adopted and the Standardized test adopted by the district and /or the State. In addition, participants produced material will be reviewed by the consultant before they are duplicated for distribution. Participants will be allowed to use their school new adopted text books and related material for the production of such handouts. A daily feedback form will be given to all participants to identify any area that is not clear to any one of them as well as a request for review of a specific topic.

Program Agenda

Conceptual Framework

The rationale and organizing principles that guide our program are based on a trifocal model which mirrors the Framework goals for our teachers. We believe that our participants must be KNOWLEDGEABLE about learners, content, and pedagogy. They must be REFLECTIVE as they plan, implement, and evaluate pedagogical and curricular issues. Participating teachers must be COLLABORATIVE in developing and honing communication and leadership skills necessary to work with colleagues, participants parents, and community leaders to plan and implement efficient and effective educational programs and to initiate change when needed. We believe that CRITICAL THINKING must be a strong cognitive strand connecting these three elements. Critical thinking is a process that involves assessment, analysis, synthesis, evaluation, and appropriate action. It is our goal to prepare the optimum Professional Educator for the 21st Century.

Conceptual Codes

A code for each of our program’s organizing principles appears at the left of each goal or objective below. “K” refers to “knowledgeable”, “R” refers to “reflective”, “C” refers to “collaborative”, and “T” refers to “critical thinking”, that is, “T” refers to circumstances where this connecting strand is especially relevant. Combination of codes, e.g. “R;T” refer to situations where more than one principle applies. Any combination of principles is possible.

Goals and Objectives

There are two goals for this program. The first is that participants who successfully complete the program become highly effective mathematics for teachers for the learning divergent who is suffering from any barriers to succeed in mathematics. The second is that all enrolled successfully complete the program.

The first of these goals may be regarded as two broad objectives:

The first objective says that at the end of the program each participant will be able to present lucid written explanations and clear iconic or concrete demonstrations of a concept and its related procedures that are reviewed in this institute and are assigned to the participant based on his/her grade level that he/she will be teaching during the school year 2001–2002. As even the simpler
The second objective says that at the end of the program, participants will be able to prepare highly effective children’s mathematics activity also with corresponding check sheets. These sheets must be both feasible and appropriate for the class where they are to be used and relevant to the removal of a specific barrier to success in mathematics. This aspect of the Resource Manual is prepared by groups of class members, but its intellectual requirements resemble those of first objective activities. Consequently, this objective relates to all three programs principles (knowledge, reflection, collaboration) and the connecting strand (critical thinking). It is coded “K,R,C,T”.

At the end of the lesson participants will be able to:

<table>
<thead>
<tr>
<th>Code</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>K,R,C,T</td>
<td>Prepare with their group, activity sheets and check sheets for each of four distinct differentiation procedures.</td>
</tr>
<tr>
<td>K,R,T</td>
<td>Explain, individually, the appropriate assignment of differentiation procedures to distinct divergences.</td>
</tr>
</tbody>
</table>

Tentative Institute Outline:

This is somewhat indefinite with respect to order, content, and emphasis because adjustments are made for each particular class. However, it is likely to be a close approximation listed below if the group is typical. Topics and major sub-topics are shown.

**Agenda**

**Day 1:**

<table>
<thead>
<tr>
<th>Time</th>
<th>Topic(s) covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30-9:00 a.m.</td>
<td>Registration</td>
</tr>
<tr>
<td>9:00-10:30 a.m.</td>
<td>An introduction and overview of Low-Stress program</td>
</tr>
<tr>
<td></td>
<td>Removing the constraints of learning mathematics</td>
</tr>
<tr>
<td></td>
<td>Immediate Success in mathematics: Is it achievable?</td>
</tr>
<tr>
<td></td>
<td>Why Low-Stress math program works?</td>
</tr>
</tbody>
</table>

I. Basic Facts of Addition

A. Definition

B. Usual Procedure in Developing Mastery

1. Referencing
2. Generating
3. Memorizing

C. New Procedure

1. Japanese
2. DOOF
3. Activity Sheet Format
4. Sheet Examples

II. Column Addition

A. Special Record Notation

B. Procedure

C. Diagnosis by Matching

D. Advantages

E. Procedure (Multicolumn Addition)

F. Motivating Practice

1. Big Problems
2. Special Effects

G. Simple Discrete Form and Fast Discrete Form
I. Identification of error patterns in addition.
J. Generating work-sheets for participants.

III. Subtraction
A. Basic Facts
B. Notation (Quasi-digits)
C. Subtraction Procedure
   1. Borrowing without zero
      a. Scientific format
      b. Discrete format
   2. Borrowing over zero
      a. First format
      b. Second format
   3. Teaching Sequence
   4. SC 2000 Frame Work and SC state testing program.
   5. Identification of error patterns in subtraction.
D. Motivating Practice
E. Generating work-sheets for participants

IV. Multiplication
A. Basic Facts
   1. Definition
   2. Generation
   3. Easy Counts
   4. Major Trick
   5. Universal access (MOOF)
B. Single Place Multiplier
   1. Split-Place Notation
   2. Procedure
C. Multi-Place Multipliers
   1. Discrete Format
   2. Scientific Format
D. Teaching Sequence
E. SC 2000 Frame Work and SC state testing program.
F. Identification of error patterns in multiplication.
G. Generating work-sheets for participants.

V. Division
A. Overview and Expressions
B. Process
   1. Structure
   2. Estimation
      a. Finding Place
      b. Finding Face
C. Problem with Conventional Materials
D. Overestimation
   1. Structural
   2. Procedural
E. Underestimation
F. Multi-Cycle
   1. Cycle
      a. Three Components of a Cycle
   2. Determining Number of Cycles
   3. Instructional Strategies: Teaching Format
G. SC 2000 Frame Work and SC state testing program
H. Identification of error patterns in division.
I. Generating work-sheets for participants.

12:00-1:00 p.m.  Lunch

1:00-3:00 p.m.  Fractions Using LSM

VI. Fractions
A. Commentary
B. Terms
C. Least Common Expressions and Procedure
D. Dividing for LCM with Large Numbers
   1. Divisibility Rules
   2. Factoring Division
E. MS 2000 Math Frame Work and CTBS Terra Nova applicable questions.
F. Identification of error patterns in fractions.
G. Generating work-sheets for participants.

VII. Factoring Models and Concepts
A. Overview
B. What does a Fraction mean?
C. SC 2000 Frame Work and SC state testing program.
D. Identification of error patterns.
E. Generating work-sheets for participants.

VIII. Statistical Models
A. Overview
B. SC 2000 Frame Work and SC state testing program.
C. Identification of error patterns.
D. Generating work-sheets for participants.

DAY Two:
8:30-9:00 a.m.  Questions & answers on day one work
Reflection on day one work

9:00-10:30 a.m.  Measurement

IX. Measurement
A. Time
   1. Clock Time
   2. Calendar Time
B. Other English Units
   1. Overview
   2. Scales
   3. Shortcuts
   4. Time Revisited
   5. Metric Conversion
C. SC 2000 Frame Work and SC state testing program.
D. Identification of error patterns.
E. Generating work-sheets for participants.

X. Measurement Decisions
A. Overview
B. Classes
   1. Kind of Unit
   2. Dimension
C. SC 2000 Frame Work and SC state testing program
D. Identification of error patterns.
E. Generating work-sheets for participants

10:30-10:45 a.m.  Break

10:45-12:00 Noon

XI. Names of Numbers
A. History
B. Logical Numeration
C. Structured Concept of a Period
D. Names of Periods
E. SC 2000 Frame Work and SC state testing program
F. Identification of error patterns.
G. Generating work-sheets for participants.

XII. Decimals
A. Vocabulary and Concepts
B. Converting Fractions to Decimals
C. Decimals to Fractions
D. SC 2000 Frame Work and SC state testing program
E. Identification of error patterns.
F. Generating work-sheets for participants.

12:00-1:00 p.m.   Lunch
1:00-3:00 p.m.

XII. Operations in whole Number Mode
A. Overview
B. Addition
C. Subtraction
D. Multiplication
E. Division
F. Conversion of Mixed Numbers to Decimals
G. SC 2000 Frame Work and SC state testing program
H. Identification of error patterns.
I. Generating work-sheets for participants.

XIV. Roman Numerals
XV. Modeling Numbers and Operating
A. Theory and Vocabulary
B. Overview
C. Models of Numbers
   1. Pictorial Models
   2. Real Time Patterns
D. Simple Modeling of Operations
   1. Addition
   2. Multiplication
E. Immediate Applications
F. SC 2000 Frame Work and SC state testing program
G. Identification of error patterns.
H. Generating work-sheets for participants.

DAY Three:
8:30-9:00 a.m.  Questions & answers on day one and day two work
Reflection on day one and day two work
9:00-10:30 a.m.

XVI. Geometry
A. Shape
B. Congruent Matching
C. Perimeter, Area, Volume, and Formula Relations
   1. Perimeter
   2. Area
   3. Volume
D. Geometric Models of Relation
E. SC 2000 Frame Work and SC state testing program.
F. Identification of error patterns.
G. Generating work-sheets for participants.

10:30-10:45 a.m.   Break
10:45-12:00 Noon

XVII. Measures of Central Tendency
A. Mode
B. Median
C. Mean
D. SC 2000 Frame Work and SC state testing program (box and whisker)
E. Identification of error patterns.
F. Generating work-sheets for participants.

XVIII. Estimation
A. What if front end estimation and when to use it?
B. Is there a difference between rounding and estimating and front end estimation?
C. When rounding and estimating is used?
D. SC 2000 Frame Work and SC state testing program.
E. Identification of error patterns.
F. Generating work-sheets for participants.

12:00-1:00 p.m. Lunch
1:00-2:45 p.m.

XIX. Equations and Number Sequences
A. Background
B. Basic Procedure
C. Dealing with Signs
D. Sign Rule for Multiplication and Division
   1. General Rule
   2. Consolidation Rule
E. Equations with Integers
F. SC 2000 Frame Work and SC state testing program.
G. Identification of error patterns.
H. Generating work-sheets for participants.

XX. Mixed Numbers
A. Regrouping and Subtracting Mixed Numbers
B. All Operations
C. SC 2000 Frame Work and SC state testing program.
D. Identification of error patterns.
E. Generating work-sheets for participants.

XXI. Word Problems
A. General Strategy
B. Nature of analysis
C. Practice
D. Graph Problems
E. SC 2000 Frame Work and SC state testing program.
F. Identification of error patterns.
G. Generating work-sheets for participants.

2:45-3:00 p.m. Foundation Program Closure
                Foundation Program Evaluation

Day 4: Optional as desired by the district
8:00-9:00 a.m. Questions & answers on the first three days
               Reflection on the first three days
9:00-10:00 a.m. Grade level small group activity for reflection and production of required material.
               Consultants will work with grade level groups.
10:00-10:15 Break
10:20-12:00 p.m. Validating school curriculum to meet Mississippi State Frame work in Mathematics using Hutchings-Bishara LSM analysis. (Each participant is asked to bring with him/her a copy of Terra Nova and Grade Level Testing results for the students that they will be teaching in 2001-2002 school year). e.g., 4th grade teachers bring 3rd grade results.
12:00-1:00 p.m. Lunch
1:00-2:30 p.m. Using Curriculum validation data to increase test scores in mathematics on the ITBS Using LSM techniques. LSM lesson Planning and implementations.
2:30-3:00 p.m. Reflection and program evaluation.