Adult Numeracy in Australian Workplaces

Gail E. FitzSimons
Monash University
Australia

Abstract
During the last 18 months I have had the privilege of visiting several workplaces under the auspices of two research grants. Building on a burgeoning corpus of research into how mathematics/numeracy is used in workplaces around the world, and using activity theory as a research methodology, my goal in the first was to gain further insights in the Australian context. This is in preparation for the next phase of my major research project which is to design an evaluative framework for new learning technologies in adult numeracy. A second, smaller project has involved investigating how mathematics/numeracy is learned on the job in the case of chemical spraying and handling. Activity theory will again play a useful part in the analysis.

Introduction
Numeracy in the workplace is much more complex than the simple application of mathematical skills learned in school or vocational education. During the last 18 months I have had the privilege of visiting several workplaces under the auspices of two research grants (see Acknowledgements). Building on a burgeoning corpus of research into how mathematics/numeracy is used in workplaces around the world, and using activity theory as a research methodology, my goal in the first was to gain further insights in the Australian context. This is in preparation for the next phase of my major research project which is to design an evaluative framework for new learning technologies in adult numeracy (see FitzSimons, 2003). A second, smaller project involved investigating how mathematics/numeracy is learned on the job in the case of chemical spraying and handling. The activities of chemical preparation, application, transport, handling and storage undertaken by operative workers are high risk activities in terms of occupational health and safety of workers, their clients and in relation to environmental damage. They place high demands on workers' numeracy and literacy skills.

There are two aims for the first research project. One is to assist adult, community and vocational education to support students and industries through identifying the kinds of numeracy used in different workplaces and in the popular media. The other is to help designers and users of online courses and CD-ROMs in adult numeracy to develop and evaluate high quality products. I was looking for manufacturing and service industry workers who were willing to let me observe them at their normal work for half a day and to interview them about that work. I visited nine workplaces, including a fund-raising trivial challenge production office, a modular shed construction company, a local post-office, a short-term home rental company, a graphic design company using CNC [computer numerical controlled] machinery, a local playgroup, a small hairdressing salon, a garden equipment warehouse, and an aged-care hostel. In the second project observations and semi-structured interviews were also undertaken. Detailed observation of participants, that is operative workers undertaking numeracy tasks during chemical spraying occurred where possible. In both projects, where
permission was granted, the collection of artefacts took place, including actual samples of materials used (for example procedures, manuals, charts).

Although there have been many definitions of numeracy proposed over recent years, the definition of numerate behaviour by Coben (2003) which seems the most appropriate in the context of teaching and learning numeracy on the job is as follows:

"To be numerate means to be competent, confident, and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context. (p. 10)"

Workplace studies show that mathematical elements in workplace settings are subsumed into routines, structured by mediating artefacts, and are highly context-dependent. The mathematics used is intertwined with professional expertise at all occupational levels, and judgements are based on qualitative as well as quantitative aspects. Workplace numeracy education must also require a fundamentally different curriculum and pedagogy from that of school mathematics, yet encompassing underpinning mathematical knowledges and skills in ways that enable the generation of ‘new’ knowledge in order to solve problems, which cannot always be known in advance. For more details from the literature review, see FitzSimons (in press).

Activity Theory

In their introduction to an edited book on activity theory and social practice, Mariane Hedegaard, Seth Chaiklin, and Uffe Juul Jensen (1999) make the following observations on its history and development. Following the work of Vygotsky came the recognition that humans pass on tools and procedures for their use (knowledge) to the next generation. Thus, human development came to be seen as a social and cultural-historical process. In a further development,

"Leontiev conceptualized activity as a collective process, with actions as goal-oriented processes of individual subjects, and operations as psychic functions conditioned by the prevailing material conditions and available tools. ... Work was taken as the prototype of activity, and other types of activity were developed through human history as derived from work.” (p. 14)

Transcending the understanding of activity as solely or primarily social interaction led to “an understanding of activity as a societally and historically conditioned process” (p. 16). They authors claim that activity theory can be conceptualised as a collective process, dependent on interaction and communication, with social practice lying at the heart of the theory’s conceptual structure. The researcher is necessarily part of an activity system involving participants and practices, socially, culturally, and historically situated.

Yrjö Engeström (1999) describes Activity Theory as providing a worthy unit of analysis for enabling a theoretical account of the constitutive elements of an object-oriented,
collective, and culturally mediated activity system in all its complex interactions and relationships. The minimum elements of this system include the object, subject, mediating artefacts (signs and tools), rules, community, and division of labour (Figure 1).

Figure 1. The basic mediational triangle expanded (after Engeström, 1987)

Engeström (1999, p. 9) continues that: “the internal tensions and contradictions of such a system are the motive force of change and development.” Following Engeström (1987), there are primary contradictions between exchange and use values at each corner, as well as secondary contradictions between the corners, as exemplified by the strict hierarchical division of labour lagging behind the introduction of new technologies in the workplace. There is a tertiary contradiction of the introduction of a culturally more advanced form of the central activity. Finally, there are also contradictions between the central activity and its neighbour activities, namely: object activities, instrument-producing activities, subject-producing activities, and rule-producing activities.

Findings
Most, if not all, of the Australian Mayer (1992) Key Competencies were observed in all workplaces in the projects: (a) collecting, analysing and organising information, (b) communicating ideas and information, (c) planning and organising activities, (d) working with others and in teams, (e) using mathematical ideas and techniques, (f) solving problems, and (g) using technology. Similarly, most if not all of Bishop’s (1988) pan-cultural activities were also observed: counting, designing, explaining, locating, measuring, and playing.

The following were identified as the underlying mathematics concepts in chemical spraying and handling: addition, subtraction, multiplication & division of whole numbers and decimals; ratio & proportion; measurement: length, area, volume, capacity, mass [usually metric]; estimation; and approximation. The following were identified as processes used by workers to undertake these calculations: estimation, pencil & paper methods, use of basic 4-function calculator; verbal or written communication with other workers; consultation with prescriptive calculations sheets and with historical records;
completion of up-to-date records of chemicals used and their amounts; and consideration of other contextual factors, e.g., date/time; block area; crop; crop stage; weed/pest/disease targeted; chemical group; rate/ha; litre spray applied; method of application; temperature; wind speed; wind direction; rainfall; humidity; open advice in area.

Underlying mathematical concepts in the other nine workplaces included: algebraic thinking (for spreadsheets), calculations (with & without a calculator) and associated relevant estimation skills, geometric thinking, logic, measurement, and the accurate storage, retrieval, display, and interpretation of data. Clear communications with other stakeholders and creative problem solving were essential. Other mathematics-related competences for those in positions of responsibility included: high level skills in forward planning and organisation, as well as the ability to keep the operation financially viable and to meet other legal requirements (e.g., accurate and timely record-keeping) for accountability purposes.

How do workers learn to do these calculations? Most of these basic calculations are taught initially in school prior to the post-compulsory years. In the case of chemical spraying, most, if not all, of the workers have the Farm Chemical Users Certificate, or equivalent, and the relevant calculations are revised and practised here, in (semi-)contextualised settings. That is, the students get to observe and experience actual measurement skills, but what they lack are the ongoing records of any one particular site which provide a deep sense of meaningfulness to their calculations. For the other workplace observations, it seems that most relevant learning is done in the contextualised workplace, through observation, reflection, and creative adaptation to the artefacts and problems or goals at hand. However, these need to be supported by a firm foundation in school mathematics — beyond minimal grades necessary for school certification — together with a disposition to make sense of available data (present and historic), and a positive, creative approach to problem solving, especially in workplaces where timely and cost-efficient resolutions are imperative.

How did the workplace setting impact on how the calculations were done and how the processes were learnt by workers? In the chemical spraying workplace, calculations are always checked in some form by another person, whether the supervisor or the tractor driver, for example. Previous experience and historical data play a big role in determining reasonableness of answers. It also determines whether and how to approximate answers. Most importantly, the learning in the workplace varies from school mathematics education in that workers are always reminded to check their calculations for reasonableness, to ask repeatedly if they are not sure, and to consider their own and others’ personal safety.

In their studies of three companies or organisations in each of seven key UK sectors Hoyles, Wolf, Molyneux-Hodson, and Kent (2002) conclude:

“A key finding of this study is that ‘mathematical literacy’ is displacing basic numeracy as the minimum mathematical competency required in a large and growing number of jobs. Mathematical literacy is the term we have used to describe the application of a range of mathematical concepts integrated with a
detailed understanding of the particular workplace context. There is a need to
distinguish between numeracy, mathematics skill and mathematical literacy.”
(p. 3)

Of particular relevance to the first study, aspects of mathematics highlighted from
different sectors as being of significance in mathematical literacy include:
• Integrated mathematics and IT skills
• An ability to create a formula (using a spreadsheet if necessary)
• Calculating and estimating (quickly and mentally)
• Proportional reasoning
• Calculating and understanding percentages correctly
• Multi-step problem solving
• A sense of complex modelling, including understanding thresholds and
  constraints
• Use of extrapolation
• Recognising anomalous effects and erroneous answers when monitoring systems
• An ability to perform paper and pencil calculations and mental calculations as
  well as calculating correctly with a calculator
• Communicating mathematics to other users and interpreting the mathematics of
  other users
• An ability to cope with the unexpected (p. 5)

Most of these technical skills were explicitly or implicitly part of the mathematical
activities in the Australian workplaces which I visited.

Implications
The numeracy task of preparing and applying chemicals requires that a complex set of
variables must be taken into account by the person responsible. Numeracy in chemical
spraying and handling is always a social-historical and cultural practice, involving the
transformation of school-based mathematics. Estimation is always absolutely necessary,
based on prior experience of the kind of spraying needed, or even of just sensible results
for the novice. Common sense is of the essence. Judgements are needed as to when it is
appropriate to approximate the chemical mixture and when it is not, and how this
approximation may be usefully made. It is never acceptable to make a mistake in the
actual process — it may threaten public and environmental safety, also the livelihoods
of the operators and their managers. All calculations must be double-checked, and
asking questions where any doubt exists is strongly and repeatedly encouraged.
Confidence in undertaking the numeracy tasks comes from several sources, including
the support of ‘expert’ knowledge from the managers within the workplaces as well as
the internet. Team and group work is fostered as part of workplace practice. Artefacts
are used as resources to aid in formal calculations, or in other situations requiring
assessment and evaluation.

Mathematics teaching off-the-job, as a complementary educational context, is also
critical in supporting workplace numeracy. It is therefore essential that vocational
numeracy courses attend to the spectrum of mathematical activities listed above, and a
broad range of knowledges and skills beyond counting and measurement. It is
commonplace that pedagogy should attempt contextualise the skills and knowledges required, but this could be supported by addressing each of the Key Competencies, integrating them with mathematical skills and knowledges as described above. Designers of many kinds work in a three-dimensional, concrete world: somehow a bridge needs to be made so that vocational numeracy education does not simply produce more text. This implies that simulated work experience, through small self-contained episodes, would be a great advantage in preparation for work situations; also mini-projects with inbuilt uncertainties typical of the workplace.

Apart from specialised computer training, most learning seems to be ‘on-the-job’. However, this ability to learn requires a strong general mathematics education, in order that the workers can quickly adapt to the idiosyncrasies of their workplace contexts. Logical thinking and problem solving are examples of mathematical competences that can develop over the compulsory years and be refined by further formal education in numeracy as well as on-the-job. The ability to communicate is of the essence, and this incorporates mathematical understandings even though they may be largely invisible most of the time. The development of authentic communication skills should form an integral part of any vocational numeracy education.

Conclusion
This paper has given an account of workplace visits intended to illuminate the mathematical practices utilised by workers in their everyday practices. Activity theory was used as a theoretical framework in order to go beyond essentialist accounts common in the past (see FitzSimons, 2002). Any workplace activity is necessarily socially, culturally, and historically located. Numeracy education for the workplace must also recognise this fact. The challenge for the future is to adequately prepare numerate citizens for whatever life and work circumstances they confront.

Acknowledgements
This paper is based upon work funded by two grants: Australian Research Council Discovery Project, DP0345726; and the National Centre for Vocational Education Research, initiated and managed by Access and General Education Curriculum Centre, TAFE NSW, in collaboration with Monash University, with Susan Mlcek as co-researcher under contract to Access and General Education Curriculum Centre.
References


Investigating academic numeracy in non-mathematics courses at university

Linda Galligan and Janet A Taylor
University of Southern, Queensland

Changes in curriculum in both universities and schools, combined with increased diversity in university student populations has created a blurred picture of academic numeracy demands of courses, expectations of staff, and skills of students. To clarify this picture at the University of Southern Queensland, a survey of all academic staff teaching first year courses was conducted and an audit of 1st year courses was undertaken. It was clear from the results of the survey that there are some mismatches between expectation of mathematical skills present in commencing students and stated prerequisites. Further, academics were concerned about the mathematical abilities of entering students generally in the areas of critical thinking and problem solving and specifically in areas particular to their courses. The audit of courses identified many of the hidden academic numeracy concerns, some of which were also identified in the survey. These measures have been used to provide academic numeracy support for various courses.

Introduction
The changing nature of Australian universities means that today student populations are more diverse than ever before. This is especially true at the University of Southern Queensland (USQ) where in 2004 89% of students were not recent school leavers. One of the repercussions of this diversity is the widely reported concern about academic preparedness (McInnis & James, 1995; McInnis, James & Hartley, 2000). These concerns are prevalent in a range of subject areas, but mentioned particularly are students’ abilities to write and perform mathematics. Currently, mathematical preparedness is determined using topics and levels of mathematics achieved in secondary school, but there are difficulties with the implementation of these prerequisites. The problem appears to the understanding of the meaning of the term ‘mathematical skills’ as clearly ‘mathematics skills alone do not constitute numeracy’ (O’Donoghue, 2003). Others have also voiced these concerns in Australia (Kemp, 1995; Cousins & Roberts, 1995; Chapman, 1998) prompting the development of the following audit and survey (based on the work of Cousins & Roberts, 1995). These initiatives aimed to:

Part A
- review the mathematical skills and concepts needed by students enrolling in first year courses;
- assess academics’ perceptions of the students’ skill level;
- ascertain whether concerns about numeracy at USQ are widespread or isolated;
- make recommendations to the university, departments and individual course team leaders on the results of the above.

Part B
- undertake a document analysis (audit of text and study books from a case study undergraduate 1st year nursing courses) of academic numeracy.
- interview course leaders to confirm academic numeracy identified in above.
In the following paper academic numeracy is defined as:

Academic numeracy is a critical awareness which allows the student to situate, interpret, critique, use and perhaps even create mathematics in context, in this case the academic context. It is more than being able to manipulate numbers or being able to succeed at mathematics. Modified from Yatsukawa and Johnston (1994)

**Survey content and procedures**
The Queensland Junior Syllabus and the Senior Mathematics Syllabi for Maths A, B and C (Mathematics A is maths in society, Maths B and C contain algebra and calculus) were used to develop questionnaires detailing mathematics topics studied in the last three years of school (Years 10, 11 and 12). Questionnaires were designed to ascertain academics’ perceptions of mathematics topics expected for study in their respective courses and their perceptions of students’ performance in these topics. Individual comments were encouraged. Little to no diagnostic testing takes place within most courses so quantitative data were not requested.

All course team leaders responsible for first level courses were sent copies of the questionnaire and asked to pass copies to other course staff. A small number of informal interviews were conducted as requested (not reported).

In this paper only the mathematical skills required for first year courses will be discussed, and although the authors acknowledge that mathematics skills are also required in higher level courses, this is beyond the scope of this paper. Complete results are available from the authors (Taylor, Galligan & Van Vuuren, 1998).

**Results and Summaries**
- A total of 77 questionnaires were returned from a total of 136 sent to six faculties (Table 1). In five courses more than one lecturer from that course responded.

**Table 1: Number of surveys returned by Faculty**

<table>
<thead>
<tr>
<th>Faculty</th>
<th>Surveys</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commerce</td>
<td>Sent 4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Returned 3</td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td>Sent 10</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Returned 2</td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td>Sent 43</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Returned 28</td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td>Sent 25</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Returned 10</td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>Sent 6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Returned 5</td>
<td></td>
</tr>
<tr>
<td>Arts</td>
<td>Sent 48</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Returned 29</td>
<td></td>
</tr>
<tr>
<td>Total Sent</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>Total Returned</td>
<td>77 (57%)</td>
<td></td>
</tr>
</tbody>
</table>
 Forty six lecturers provided comments. The responses can be divided into two categories:

- academics’ perceptions of mathematical topics and skills required;
- academics’ perceptions of mathematical abilities in these topics and skills

While the initial questionnaire separated on and off campus students, most lecturers found it impossible to differentiate and all data were pooled.

**Mathematical proficiencies required by commencing students**

Within the questionnaire lecturers were asked to tick their perceptions of the skills needed for their course. Of the 77 respondents, 16 ticked no topics.

Lecturers were asked to indicate the expected mathematics background of students in their course by circling one or more of: None, Year 10, Maths A, Maths B, Maths C (see Table 2 for a selection of courses). In four courses there was a Maths A/Year 10 or None/Year 10 mismatch. There were also 15 lecturers who indicated Year 10 or 12 maths was not required, but then ticked a number of skills from Year 10 and 12 topics. As one lecturer commented: “Although maths is not stated as a pre-requisite or even as desired for entry into... we still expect (or rather assume) that students have a certain level of senior maths.”

**Table 2: Mathematics background expected by responding courses.**

<table>
<thead>
<tr>
<th>Faculty</th>
<th>None</th>
<th>Year 10</th>
<th>Maths A</th>
<th>Maths B</th>
<th>Maths C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commerce</td>
<td>Introduction To Law</td>
<td>Intro To Accounting</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Financial Markets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td>Aust Political Institution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td>Data Analysis</td>
<td>Foundation Psychology</td>
<td>Organic Chemistry</td>
<td>Foundation Chemistry</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intro Profess1 Computing</td>
<td>Climates - Past &amp; Present Computing</td>
<td>Inorganic &amp; Physical Chemistry</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nursing Foundations 2</td>
<td>Astronomy</td>
<td>Biophys. Science Foundations</td>
<td>Discrete Maths</td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td>Electronic Wshop &amp; Prod</td>
<td>Engineering Communications And Practices</td>
<td>Civil Engineering Materials</td>
<td>Electrical Technology</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Telecommunication Principles</td>
<td>Engineering Materials</td>
<td>Fluid Mechanics</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Telecommunications Systems</td>
<td>Aerodynamics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>Foundations Of Language</td>
<td>Soc-Cult Phys Ed &amp; Sport</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Health For Teachers</td>
<td>Computing And Design</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Learn Through Comp Program</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arts</td>
<td>Communication Scholarship &amp; Intro To Studio Practice</td>
<td>Voice And Movement 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Performance 1</td>
<td>Technology And Design</td>
<td>Sound And Lighting 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Music Craft 1</td>
<td>Radio Production 1</td>
<td>Intro To Public Relations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Total Courses | (n=72)                                    | 28                                           | 10      | 23      | 10      | 1
From Table 2 it can be seen that while 38 out of 72 (53%) courses expected no maths background or Year 10 mathematics, 34 out of 72 (47%) expected Mathematics A, B and/or C. However, while many individual courses expect a certain mathematical background, degree programs do not always specify Mathematics A, B, or C as prerequisites or even state them as desired for entry into their degree. This can lead to some confusions as occurred in the 3 following examples:

- Economics is a core course in the Bachelors of Business and Commerce where academics expect Maths B topics to have been completed, yet mathematics is not stated as a prerequisite in Bachelor of Business and is mentioned as desirable only in Bachelor of Commerce.

- In the Faculties of Education and Arts, 15 courses expected students to have a background in Mathematics A, yet no mathematics is stated as a prerequisite or desired.

- In the Faculty of Engineering and Surveying, Mathematics B is stated as a prerequisite, yet students enrolled in Electrical Technology are expected to have a Mathematics C background to undertake this course. However, Algebra and Calculus I is listed as a co-requisite in the course structure.

The skills assumed from the Year 10 were also separated (general skills, working mathematically, number, space, measurement, chance and data and algebra) and lecturers could tick topics/skills from each section. No Year 10 maths was required by 31 respondents although there were a number of skills and topics within Year 10 which were assumed, eg 11 lecturers assumed some knowledge of measurement, and 9 assumed skills in algebra.

The most important topic considered by lecturers was a skill in number work, with 71% of total respondents indicating at least one of the skills within this section. Moreover, 65% of respondents indicated general mathematical skills were needed in their course.

"In this course they don’t have to do any calculations but they have to be able to read psychology journal articles and draw conclusions about research..."

"This course is not at all mathematical and just assumes general numeracy knowledge and ability to do simple calculations......"

**Academics’ perceptions of mathematical abilities of commencing students**

The perceptions of academics about the mathematical abilities of commencing students were varied with the added complication that many academics found it difficult to rank large groups of students as either good, fair or poor because of the variation of abilities within any one class. Also although percentages were used to compare different groupings, these percentages were sometimes based on small numbers and caution is advised in their interpretation.
Despite these shortcomings, general perceptions can be gleaned from the data. Overall, students’ mathematical skills and abilities were thought to be fair with 682 out of 1210 (56%) of topics being marked this way (Table 3). However, the distribution of rankings between good, fair and poor was not balanced with 341 (28%) responses being ranked as poor and 187 (16%) responses as good. This indicates that there is some concern about students’ mathematics levels.

Table 3: Responding academics’ perceptions of student mathematical ability.

<table>
<thead>
<tr>
<th>Faculty</th>
<th>Poor</th>
<th>Fair</th>
<th>Good</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of Courses</td>
<td>No. of responses</td>
<td>% of Faculty</td>
</tr>
<tr>
<td>Arts</td>
<td>29</td>
<td>42</td>
<td>24</td>
</tr>
<tr>
<td>Business</td>
<td>2</td>
<td>17</td>
<td>50</td>
</tr>
<tr>
<td>Commerce</td>
<td>3</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>Education</td>
<td>5</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>Engineering</td>
<td>10</td>
<td>55</td>
<td>21</td>
</tr>
<tr>
<td>Science</td>
<td>28</td>
<td>210</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>341</td>
<td>682</td>
<td>187</td>
</tr>
</tbody>
</table>

These general perceptions were confirmed by written comments. Of these, 22% indicated in their comments that they had concerns with students’ basic mathematics and numeracy levels. For example:

“...students had great difficulty with pen and paper calculations...”

“Numeracy, in the sense of a feeling for the accuracy or not of an answer is generally missing...”

However, caution should be taken, in the view of one academic: “Judging student proficiency is very difficult because we are predominantly exposed to students with difficulties. Hence I suspect our perceptions are biased downwards...”.

It was apparent that overall, academics were concerned mostly about the general skills necessary for university study. In particular, the topics below were mentioned in over 50% of responses, with fractions being mentioned most frequently and use of language least.

- use fractions, decimals, percentages and ratios,
- ability to perform pencil and paper calculations,
- demonstrate an ability to use instruments, eg calculator, computer, measuring instruments,
- assemble, present and interpret data in tabular and graphical form,
- communicate mathematical ideas and reasoning,
- use mathematical language and terms accurately and appropriately;

According to the academics’ perceptions, courses using these topics contained students with a balance of good, fair and poor abilities except in the use of language and communication of ideas, where good students were few.
On the other hand within other topics over 50% of responses indicated that students were perceived to perform poorly. These topics included:

General skills
- making judgements as to the validity of mathematical reasoning
- using mathematical skills to analyse and solve unfamiliar problems
- generalise from one problem to another

Specific mathematical skills
- solve simultaneous equations
- recognise and represent linear, quadratic function, reciprocal and exponential function
- visualise, produce and describe translations, reflections, rotations and enlargements
- produce mathematical arguments to prove a proposition

The lack of general skills was reinforced by 13% of academics who commented on the essential nature of critical reasoning and logical thinking for commencing students.

"Of course the logical thinking and problem solving skills one learns in Maths are important."

Part B
Mathematics audit of Nursing courses
In many of the courses outlined in Table 2, no mathematics requirement was identified although some staff acknowledged that some skills were needed. This was particularly evident within Nursing courses. Closer examination revealed Nursing staff perceived difficulties especially in areas of working mathematically, algebra, general skills and number. In 2001, a detailed audit of the first year nursing program was undertaken as part of the development of a nursing support program.

The audit consisted of an analysis of the texts and assessments used in most courses, assessment of the numeracy skills needed followed by staff interviews to confirm importance placed on different sections of the courses. Table 4 lists the courses undertaken by 1st year nursing students. Note that Mathematics A (sound achievement) is a currently prerequisite for entry into the Nursing degree.
Table 4: Audited courses in 1st year nursing, with maths background

<table>
<thead>
<tr>
<th>Course</th>
<th>Maths background expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anatomy and Physiology</td>
<td>Year 10</td>
</tr>
<tr>
<td>Biophysical Science Foundations</td>
<td>Maths A (audit not taken)</td>
</tr>
<tr>
<td>Nursing Foundations 1</td>
<td>Not known</td>
</tr>
<tr>
<td>Nursing Foundations 2</td>
<td>None (audit not taken)</td>
</tr>
<tr>
<td>Physiology and Pharmacology Foundations</td>
<td>Not known</td>
</tr>
<tr>
<td>Social Sciences for Nursing</td>
<td>None</td>
</tr>
<tr>
<td>Nursing for Health</td>
<td>Not known</td>
</tr>
<tr>
<td>Medical Calculations</td>
<td>Year 10</td>
</tr>
<tr>
<td>Behavioural Science Foundations</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 5 summarises the mathematics skills and topics identified in three of the 1st year Nursing courses. Both the topics in ‘general topics and skills’ and ‘working mathematically’ were seen throughout both Nursing for Health and Medical Calculations, in the table only some have been highlighted.

Table 5: Mathematics skills and topics identified in three 1st year Nursing courses.

<table>
<thead>
<tr>
<th>Year 10</th>
<th>Social Sciences for Nursing</th>
<th>Nursing for Health</th>
<th>Medical Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>General skills</td>
<td>Make judgements as to Demonstrate an ability Use mathematical language</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working Mathematically</td>
<td>the validity of to use instruments eg, and terms accurately and mathematical reasoning measuring instruments appropriately</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space</td>
<td>Interpret congruence and similarity of shape</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>Ratios, percentages, Recording</td>
<td>compare operations, scientific notation</td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>basic operations &amp; calculate</td>
<td>&amp; Units, time, estimate area and volume</td>
<td></td>
</tr>
<tr>
<td>Chance and Data</td>
<td>timelines &amp; Recording weight &amp; height, units</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Algebra</td>
<td>Interpret data in tabular and graphical form; draw conclusions from surveys</td>
<td>Use formulas eg Translate from words to symbols, use formulas</td>
<td></td>
</tr>
</tbody>
</table>

In all courses students are expected to be able to interpret their assessment weightings (percentages) and make judgements relating to the progress of their assessment marks. They are also expected to be able to interpret such statements as “only students who have attended at least 80% of tutorials will be considered...” or “marks will be deducted if you are 10% under or over the word limit” All students are expected to read and create their own timetable.
Social Sciences for Nursing:
In this course the lecturer indicated no mathematics was expected. An audit of the material suggested that while students were not asked to perform any mathematics, they were expected to read and interpret quantitative information. In the statement: ‘In 1997, one study found that over three quarters of women under the age of 35 years of age felt that the most important role in life for a woman was to be a mother’ (quoted in student introductory book, from Bulbeck, 1997) students are expected to understand the meaning of the mathematical content but also be aware of the imprecise statements “most women” and ‘one study’. The texts in this course referred to randomized controlled trials as well as data comparisons using percentages and ratios, e.g.: “mortality from lung disease for men increased from 60% to 98% and mortality from diabetes for women increased from 209% to 249%” (Gernov, 2003: 77) or a table which shows the leading causes of death (standardized death rates per 100 000 persons) (Gernov, 2003: 97).

Nursing for Health
While the lecturer did not complete the survey, an interview was conducted and an audit of the text and assessment items undertaken. In one assessment item students measured distant vision, while in an assignment calculated the average daily intake of kilojoules, water, fat and fibre; compared this with the recommended daily intakes.

Medical Calculations
Two lecturers completed the survey and generally perceived students’ skills to be poor. While in the survey, neither lecturer thought any topic from the ‘space’ or ‘number’ sections was specifically needed, in interview they indicated that students were expected to compare the volumes of two fluid samples. Assessment items expected students to calculate when a drip finished and to convert 3.2 hours to hours and minutes. Neither lecturer recognised that students had to translate from words to symbols, although most of the drug calculations require students to interpret word problems.

Discussion and Conclusions
Academic numeracy is more than just mathematics and academic numeracy concerns have become an increasing priority within higher education over the past 10 years. This study confirms that staff at USQ do have concerns about academic numeracy. Such concerns are being voiced throughout the world and in the Higher Education Sector are typified by the results found in a similar report conducted by the University of Adelaide (Cousins & Roberts, 1995).

Our concerns are centred on the following:

- mismatches do occur between stated mathematical prerequisites and expectations of the teaching staff,
- many courses across all faculties contain students who are believed to possess poor mathematical skills especially in areas of general skills, working mathematically and algebra,
- it is believed that students’ abilities to understand, interpret and communicate logical arguments are poor,
Many courses contain mathematical topics and skills which students may not understand or are unaware that they need.

Mismatches
It is apparent that there were a number of mismatches between written prerequisites and the expectations of teaching staff. This is not unexpected, as it is difficult for all academics to keep abreast of changes in secondary school mathematics. The task is made even more difficult by the diversity of educational experiences that are accepted as entry requirements, especially with mature age students. This investigation supports a long held belief by the authors that a statement of “no mathematics” or the omission of any mention of mathematics from an entry requirement is interpreted differently by different people. Fifteen lecturers who ticked no mathematics for their course went on to detail particular mathematical skills required for their course. It is the authors’ experience that a statement of “no mathematics” or the omission of maths skills from a course description also leads a number of students to believe that absolutely no numeracy skills will be required.

Mathematical Abilities
Perceptions of mathematical abilities were varied and caution needs to be used as results are based on academics’ impressions rather than quantitative testing of students.

However, bearing this in mind it appears academics are concerned about students’ abilities. Fractions, decimals, percentages and ratios and ability to do pencil and paper calculations are topics that are mentioned most frequently but are not the topics that academics believe students have the most difficulty with. These topics revolve around general skills and skills associated with working mathematically, eg making judgments as to the validity of mathematical reasoning, using mathematical skills to analyse and solve unfamiliar problems, generalising from one problem to another and communicating mathematical ideas and arguments. Such concerns often revolve around the inability of many of our students to think critically and solve problems. Faculties and courses were also found to have further specific mathematical concerns, eg Bachelor of Science (Nursing) with number work, Bachelor of Engineering with algebra. Routine mathematics testing and support which has taken place in some courses over the past 5 or more years confirms this (Galligan, 1998; Taylor & Morgan, 1999; Galligan, 2001; Galligan & Pigozzo, 2002).

The results of this study support the concept that academic numeracy is more than just working with numbers it is a complex interaction between mathematics, literacy and thinking skills in context and as such any programs involved in improving the academic numeracy need to integrate all three components within curriculum developments.

References


Galligan, L. (1998). ‘Students' confidence with their answers to a diagnostic mathematics test in economics for external students’. In Bridging the Distance: Proceedings of the 8th Annual Conference of the Bridging Mathematics Network (pp. 65-70). Toowoomba, Queensland: University of Southern Queensland.


Mathematics integrated in a job-function

Torkel Haugan Hansen
Adult Learning Research, Norwegian University of Science and Technology
Trondheim, Norway

Introduction
Adults in the labour market today will often face a situation where further education is a necessity as a result of major restructuring and technological alterations in many companies and businesses. Many semi-skilled workers and workers with short education are being ‘forced’ into some form of training, and many with a specific vocational education into retraining to manage new challenges in the workplace.

The aim of my Ph.D.-project is (1) to establish and visualize mathematics as a general competence in the labour market in Norway today and (2) to analyze and characterize the mathematical competences of adults with significant work experience, in order to better adapt adult education to this group returning to the educational system.

This paper discusses a small qualitative study which has been made as a pilot-study for this research. The study involves one single worker that has been observed at work and interviewed in order to localize the mathematical competences in her job-function.

Adults and mathematics
“Adults and mathematics” is a new field of research in Norway. The necessity for a common frame of reference with theories and concepts connecting the research areas of the didactics of mathematics, adult education and research in qualifications, led me to Tine Wedege and her use of theories and concepts as expressed through her doctoral-dissertation (Wedege, 1999), as well as through her research with Lena Lindenskov in the project “People's mathematical knowledge in technologies undergoing change” (1998-2002).

The basis of my study is the assumption that functional mathematical competence is needed on the labour market in Norway. It is also assumed that these functional mathematical skills that all in the labour force need in principle, are possible to locate and describe. The pilot study wants to document and visualize that the worker in question makes use of mathematical ideas and techniques in her work. This is done through a detailed description of performed tasks which explicit or implicit involves mathematical activity, manipulation of numbers of figures, quantification and assessment of quantitative measures or spatial matters.

Based on Wedege (1999: 120), the study has two working hypothesis.

1. In any job problems that only can be solved by quantification and/or use and assessment of quantitative measures appear.
2. There are systematic differences between mathematics/calculations on the workplace and mathematics/calculations in traditional education, both in form, contents and competences.
Mathematics is often an implicit tool for solving a specific work task, and based on this it is reasonable to assume that much of the mathematical competence one is likely to encounter in a workplace is in form of tacit or practice-based knowledge. That proposes some challenges worth discussion.

**Tacit knowing**

Michael Polanyi (1967) reconsiders human knowledge by starting from the fact that we can know more than we can tell. He explains this as the outcome of an active shaping of experience performed in pursuit of knowledge.

I find it useful to divide tacit knowing (as defined by Polanyi) into two dimensions. (1) A *technical dimension*, which is characterized by the kind of informal skill and handiness which can be labelled as ‘knowhow’, and that is very hard to pinpoint what actually endure. Through years of experience the craftsman develops an abundance of expertise “at his fingertips”, but is often unable to articulate the scientific or technical principles behind what he knows. Tacit knowing also contains (2) a *cognitive dimension*, consisting of mental models, beliefs and perceptions so ingrained that we take them for granted. The cognitive dimension of tacit knowing reflects our image of reality and our vision for the future, and even if it can not be easily articulated, is fundamental for how we perceive the world around us.

According to Polanyi, the basic structure of tacit knowing consists of two terms, which combines two ways of knowing something. We focus our attention towards the second term, and hence the subject is specifically known. But we know the first term only by relying on our own awareness of them for attending to something else, the second term (Polanyi, 1967:10). For example, a craftsman has at all times his attention directed towards a particular task that shall be done. He is not aware of the variety of skills and knowledge used for solving this task, and often unable to specify the elementary acts, because his focus is somewhere else. He is attending *from* these elementary movements to the achievement of their joint purpose. These elementary acts remain tacit, and Polanyi calls this the *functional structure* of tacit knowing.

Generally speaking, the reintroduction can not replace its implicit counterpart, and therefore tacit knowing is a vital part of all knowing which can not be replaced by something else, according to Polanyi. In other word, he claims that we can not reconstruct the tacit knowing as explicit, and thereby he is leaving us with a concept that is very hard to analyze and characterize.

**Between tacit and explicit knowing**

In an article from 1998, Ikujiro Nonaka and Hirotaka Takeuchi (Nonaka & Takeuchi, 1998) starts with Polanyi, but where Polanyi leaves a concept that is static, closed and unsuitable for analysis, Nonaka and Takeuchi reopens the concept. They admit that tacit knowing can not be formalized and communicated easily, but claims that tacit knowing can be acquired directly from others without use of language. For example, apprentices work with their masters and learn craftsmanship not necessarily through language, but through observation, imitation and practice. And if this knowledge can be shared, it must also to some extent be identifiable and findable.

97
Nonaka and Takeuchi make this explicit in what they call the *four modes of knowledge conversion*. They assume that knowledge is created through the interaction between tacit and explicit knowledge. As we have already seen, tacit knowing is personal, contextual and therefore hard to formalize and communicate, whereas explicit knowledge means knowledge that is expressible and possibly transferable through formal and systematic language. The authors postulate four different modes: (1) from tacit knowing to tacit knowing, which they call socialization; (2) from tacit knowing to explicit knowledge, or externalization; (3) from explicit knowledge to explicit knowledge, or combination; and (4) from explicit knowledge to tacit knowing, or internalization.

*Socialization* is a process of sharing experiences and thereby creating tacit knowing such as shared mental models and technical skills. Without some form of shared experience, it is extremely difficult for one person to project himself into another individual’s thinking process. The key to acquiring tacit knowing is experience. (ibid:220.)

*Externalization* is a process of articulating tacit knowledge into explicit concepts, and Nonaka and Takeuchi regards this process as essential in knowledge-creation. When trying to express the tacit knowledge, we express its essence mostly in language. Yet, expressions are often inadequate, inconsistent and insufficient (ibid: 221.). In my opinion, these gaps and discrepancies do not necessarily represent something solely negative. It can be a trigger for reflection and interaction between the individuals. When adequate expressions can not be found through analytical methods, we must use a non-analytical method. Externalization is often driven by metaphors and/or analogies (ibid.).

*Combination* is a process of systematizing concepts into a knowledge system. This involves combining different bodies of explicit knowledge, and is according to Nonaka and Takeuchi the form of most knowledge creation carried out in formal education and training at schools. (ibid: 222.) *Internalization* is a process of embodying explicit knowledge into tacit knowing, related to “learning by doing”. For explicit knowledge to become tacit, it helps to have knowledge become verbalized or diagrammed into documents, manuals or oral stories. (ibid: 223)

This gives us a knowledge spiral where knowledge creation is a continuous and dynamic interaction between tacit knowing and explicit knowledge, shaped by shifts between different modes of knowledge conversion, which are in turn induced by several triggers.

**The pilot study**

The concept of numeracy and methodology

I find the definition of ‘numeryc’, developed and reconstructed by Lindenskov and Wedge as an analytical concept in relation to adults’ mathematics-containing everyday competences, particularly useful for my work. Their general definition of *numeracy*...
describes a mathematics-containing everyday competence that everyone, in principle, needs in any given society at any given time. A person’s numeracy can not be
determined solely as a collection of skills and understandings. It is not restricted to
managing the four basic arithmetic operations and other mathematical subjects
(Lindenskov & Wedege, 2001).

They also develop an operational working model for studying adult numeracy, where
numeracy is given four dimensions. One dimension is media: numeracy depends on
whether it is to be used on oral or written information, a wooden beam or change, even
if the numbers and the arithmetic operations are the same. Media is therefore divided
into (1) written information and communication, (2) oral information and
communication and (3) concrete materials, time and processes. The second dimension is
situation context: what you actually know depends on whether the activity takes place in
school, in work, in the supermarket etc. The third dimension in the model is personal
intentions: it is vital if the intention is to obtain information, plan production or to kill
time. The fourth dimension is skills and understandings, i.e. estimating, mathematical
modelling, sense of geometry etc. (ibid.).

Some of the mathematical competence of the workplace is tacit. This is a vital
assumption in my study, based on earlier empirical studies as well as theories on tacit
knowing. An indication of this is that “No” is a common answer among workers when
they are asked whether they use mathematics in their job. The workers’ ability to
explain and tell about their mathematical competences is probably limited.

It is necessary to visit the workers in their environment and ‘share’ experiences with
them at the workplace. This gives the researcher a chance to reconstruct the mental
models and mathematical competences that lies behind the technical or practical skills
that the workers perform, through a form of socialization. Interviews and conversations
will be a useful continuation to try and externalize the tacit knowing to explicit
concepts. This design will also make a good foundation for observing and interviewing
the workers about mathematics more explicitly used in their job.

The progress of the empirical data acquisition is inspired by and follows the operational
model of Wedege (1999 & 2004):

A. The researcher follows the worker for half a day making notes continuously,
taking photographs and collecting relevant material.
B. The researcher uses a couple of hours reading, editing and complementing his
notes, rethinking the observation and preparing the interview.
C. The worker is interviewed for approximately ½ hour at the end of the working
day to go thoroughly into episodes of special interest to the researcher. It is also
investigated whether it has been a ‘normal’ day at work, or if the tasks were
special on this particular day.
D. The researcher can make a new appointment to come back and sort out
obscurities in the observation if this is necessary.

A report is made after the observation and the interview, where particularly interesting
episodes are described. The researcher’s and the worker’s analysis and remarks about
the mathematical ideas and techniques used, as well as photographs supplement this. Possible disagreements between the researcher’s analysis and the worker’s perceptions are noted.

**The sample**
This pilot study consists of one single case. The subject of research is a 23-year-old woman that I will call Anne, working permanent part time at a cafeteria. Anne has nine years of experience working in cafeterias, most of the time besides school or studies. Her education has involved some mathematics, and her last involvement with formal mathematical education was three years prior to the observation. She has no education as a cook, and no specific responsibility for making food.

The cafeteria is situated in the same building as a nursing home which is the centre of a larger area of apartments for elderly people that need moderate care. The cafeteria is open to the general public, but the vast majority of costumers are old people living in these institutions. They use the cafeteria for being social over a cup of coffee and eating dinner. This is reflected in the assortment, which roughly consists of buns and sandwiches, cakes and waffles, warm and cold drinks, as well as one traditional course for dinner every day. The costumers help themselves to food and drink of their desire (excluding dinner which the personnel serves) and pays at the cash register at the end of the counter.

Anne’s day was roughly divided in three. Before opening hours, most of the work was buttering buns and sandwiches, making waffles and coffee etc. In the period after opening hours until dinnertime, she mainly operated the cash register serving costumers, occasionally interrupted by tidying and washing dishes. The last part of the day consisted of heating, serving and receiving payment for dinner. The interview showed that it was a quiet day with unusually few costumers, but normal regarding which job-functions was performed.

**Processing data**
We will now look at some situations of particular interest using the operational working model of Lindenskov and Wedege (2001):

*Situation 1: Working with the cash register and receiving payment*
Situation context: Anne stands at the cash register serving costumers. This consists mainly of price-information, receiving payment and giving back change. This was her only task in busy periods, all other tasks were taken care of by colleagues.

Personal intentions: Giving correct information about prices, demanding the correct amount of money and giving back the correct amount of change, in order for the amount of money in the cash register to be correct at the end of the day.

Media:
(1) Price lists, display and keyboard on the cash register, book for writing the total amount for regular costumers that pays on a monthly basis.
(2) “12 kroner” – “here’s sixty-four back” – “unfortunately I have nothing smaller than a 500-bill”
(3) Money of different values.

Skills and understandings: Simple arithmetic calculations, mental addition and subtraction.

The cash register was "semi-automatic". Anne had to manually enter the amount for the different items in different categories, but the register calculated the total amount. It could also be used for calculating change. The assortment is limited and the prices are fixed, and she knows most of them by heart. She consistently used the cash register to calculate the total amount, but added/multiplied identical items mentally before entering it, making for instance one punch for two coffees. She never used the register to calculate the change, this was done mentally.

I: Can you say something about how you use the cash register?
A: I don’t calculate much mentally before punching. I just punch, and it’s got something to do with if it's a rush. It’s not only that it’s a problem calculating mentally, but it's a better chance for it to be correct when it goes pretty fast.

I: So you punch every
A: (interrupting) Yes. Of course, if it’s two waffles and two coffees I calculate it mentally and then punch

I: But I noticed that you didn’t use the register to calculate change
A: No. I could have to be safer, but I don’t

I: You just calculate it fast in your head?
A: Yes. (pause)

I: Why?
A: (3 sec silence) Well, I quite simply think it’s fairly easy when the total amount is 34 kroner and you get a hundred-bill. It’s quite easy to count up from thirty-three to a hundred. (Laughs)

I: You mentioned when I was there that if something summed up to for instance forty-two kroner and the costumer gave you a hundred, but then some seconds later said “Hold on, I’ve got two in coins here”, you found it easier to calculate the change if you hadn’t punched a hundred.
A: Yes. I think so. It’s easier to take it mentally from forty-two to hundred-and-two than to start calculating in relation to the amount in the display having to think whether to add of to subtract two when you’re in a hurry.

Anne’s first answer shows an expectation regarding what the researcher wants to know something about, and shows through her answer an understanding for that mathematics is involved in this situation at work. She points out that she uses the available appliance to solve the mathematical problem, and she argues that when she is in a hurry she feels there is a larger probability of making a correct calculation. It is not the difficulty of the mathematical problem itself, but the time pressure and the demand for accuracy that makes the task so challenging that she prefers to use appliances for help.

Calculating change is perceived as an easier operation. The situation is clearer, there are only two numbers that is to be subtracted, not a chain of additions. Mental calculation is the most time efficient way to do it, because the answer is calculated almost immediately.
Situation 2: Calculation on dinner-portions

Dinner was delivered by a driver. It consisted of salmon, delivered in eight large aluminium tins, potatoes, carrots and cucumber-salad that was vacuum-packed. The food was cold, but prepared only to be heated in a steam oven before serving. The delivery was 60 portions.

I: I overheard you saying 8 times 8 is 64.
A: Yes.
I: What were you doing?
A: It was to find out approximately how many portions there was in one tin, and since there was eight tins there were about eight or somewhat less than eight in each.
I: Ok, so that was what you were doing.
A: Yes, to know so that you don’t take way too much or too little in a portion. It’s given that a portion of fish is so-and-so many grams, but weighing it is unnecessary and bothersome.

Situation context: Anne is in the kitchen by the delivered trolley with dinner together with her colleague, discussing the size of one portion of dinner.

Personal intentions: To find out how much salmon constitutes one portion.

Media:
(2) “eight times eight is sixty-four” – “yes, seven times eight is fifty-six”
(3) Eight aluminium tins with salmon.

Skills and understandings: Coarse estimation, proportional reasoning.

Anne and her colleague use mathematical estimation to find out approximately how much salmon corresponds to one portion. She emphasizes that this is much easier than weighing it on the available scale. The accuracy of the estimate was not very good, but adequate, particularly on a day with few costumers. During the interview she says that they have never experienced running out of food, so the consequences of doing an error during the estimation are small. But still they need some clue to the size of the portions, to avoid heating (and destroying) a lot more food than they sell.

They use a combination of experiences to determine how much food they should heat. One of the factors to be taken into account is whether the dinner of the day is a popular dish. Potential costumers call the cafeteria throughout the day to know what is for dinner, and bread salmon is not popular among old people in Norway. Together with external factors like the weather, whether there is exciting sports on television and their estimation of how large a portion is, they decide how much food to heat. This joint judgement ought to fit quite good. It is not completely accurate, but drawing from their experience regarding all the other factors that affects the situation, it “will do”.

102
Conclusion
The observation and the interview show generally that mathematics/calculations are used to solve problems in this job-function, and thereby working hypothesis 1 is supported. However I find it interesting to observe that technical appliances are available for solving most of these problems. Anne’s use of these appliances seems to be based on a judgement of demand for accuracy and time-efficiency. The fastest solving method seems to be chosen, provided that this satisfied the required accuracy.

When I am to draw conclusions after this pilot, I realise to have set up a theoretical framework about tacit knowing, which was not something I observed in this case. In the situations where mathematics was observed, the mathematics was explicit to me as a researcher. It is however interesting that Anne herself in advance claimed that she did not use any mathematics in her job, so the mathematics might be invisible to the worker even if it is explicit to a researcher. The relationship between visible and invisible, implicit and explicit mathematics and tacit knowing and explicit knowledge is therefore an issue of further investigation.

Acknowledgement
I thank Tine Wedege for her helpful comments during my work with this paper.

References


Flexible learning
Egil Peter Hansen
VUC Syd, Copenhagen, Denmark

Flex Ring
What's that?
A Flex Ring is a tool to learn something new.

Flex comes from "flexible", and Ring from "ring/circle". Flexible, because the tool gives you the opportunity to learn new things with different techniques. Ring, because the tool visually appears as a circle. You could see the ring as a running track with different lanes.

A Flex Ring in Math could consist of a theme from everyday life to which different mathematical techniques naturally belong. While you work with the theme, you learn to use math in these everyday life situations.
The Flex Ring shows possibilities of working with the material and the cases in different ways. Maybe in some situations you prefer to learn using a video with matching exercises. In other situations you might prefer a traditional work book or a training session on the computer.

The Flex Ring suggests different ways to work with the same problems. It means you do not have to work with all the material in the ring.

Why at all a Flex Ring?

Learning new things it is often experienced that you already has some knowledge about it.

You may find out as you are learning new things that there are bigger gaps of knowledge than you had first anticipated.

While learning new things you may often find that the new stuff is so interesting and exciting that you are tempted to dig deeper – deeper than the teacher and material might expect.

This is what the Flex Ring tries to help structure.

The Flex Ring shown here has two lanes. The inner ring is level 1, the outer ring level 2. You many choose the lane that fits the test level you want to end with. But you can easily choose from the other lane if you need to. If you are heading for a test at level 2 you might lack some basic math knowledge and you can pick it up through the instructions at level 1. If you are heading for a test at level 1 and you feel challenged in a way that gives you the desire to learn more, then you could move your focus for a while to lane 2.

Open exercises

Some of the suggestions lead to open exercises. In an open exercise you decide the scope and the level yourself. It is an important aspect. Here one really senses how math can be applied in everyday life.
Check before and/or after a theme

In the digital version of the Flex Ring you find an active link to a test. You can take the test before you start working with the theme or you can take the test after. One of the themes "Shapes and figures" has much geometry. If you feel on top of geometry, you may have a go at the exercises right away.

Once you have finished a theme, it might be a good idea to consult the specific exercises found in the active link.
Teaching innumerate adults: using everyday life experience to develop proceptual thinking

Jens Langpaap
Universität Hamburg, Germany

My research focuses on adults who have acquired only rudimentary mathematical knowledge, which is not always adequate to manage simple everyday problems. Their innumeracy is described as a consequence of developmental dyscalculia. Nolte (2002) calls them “Nichtrechner” (non-calculators). Non-calculators can be seen in analogy to illiterate persons. In the PISA study the definition of mathematical literacy “revolves around the wider uses of mathematics in peoples lives rather than being limited to mechanical operations” (OECD 2001a, 22) and includes “the ability to put mathematical knowledge and skills to functional use rather than mastering them within a school curriculum” (OECD 2001, 22). Mathematical literacy also implies the ability “to pose and solve mathematical problems in a variety of situations as well as the inclination to do so, which often relies on personal traits such as selfconfidence and curiosity” (OECD 2001, 22).

The practical outcome of my work is a teaching concept for non-calculators. In the last ten months I taught ten female students (26 to 51 years old) with little mathematical knowledge. The teaching took place in one-to-one lessons. The aim was an individual access to mathematics and the development of mathematical literacy by including the students’ mathematical experiences in everyday life situations. One theoretical question of my work is: What relevance do these mathematical everyday life experiences have for the learning process within the scope of my concept?

The teaching concept

Even non-calculators do not enter settings of mathematical education without mathematical experiences. Their knowledge, especially experiences of everyday life, can be used as starting point for mathematical instruction. They deal with money, they hang up pairs of socks on the clothesline, they are involved in explicit or implicit mathematical actions and questions. It is necessary to give flexibility to their knowledge to make it transferable for use in everyday life situations. Many researchers as for example Schlöglmann assess mathematical everyday life experiences as suitable starting point for reducing learning obstacles and for development of mathematical comprehension (Schlöglmann 2002).

Every lesson begins with a talk about the students’ mathematical experiences made during the previous week. The student tells the story of an everyday life event. This may be a report about a situation doing shopping, about the exercise of planning of a train journey, a story about mathematical demands during a game of cards at home, about information given by a newspaper or the petrol consumption of his or her own car. Drawing on this story, the student and the teacher generate a mathematical problem together. For example: “Calculate your car’s petrol consumption of last Monday“. Solving this everyday life problem is the main job of the lesson. Dealing with special mathematical demands that occur during the solving process may be object of additional instruction, for example problems with division or proportional
relations. Every lesson ends with a summary part, where student and teacher review the learning outcome and the rational and emotional contentment with the lesson.

Generating an everyday life story

A lesson is introduced by the question: What situations with mathematical aspects have you experienced this week? Examples: I was unable to give a 10 percent tip to the waiter, I misunderstood the quantity of 10 decilitres of a swedish cooking recipe, I was not on time for my appointment with the doctor because of incorrect calculations of time.

This introduction has two intentions. At first it supplies the context for the everyday life problem, that has to be solved in the next step. Secondly it should encourage the student to become aware of the spectrum of his or her own mathematical experiences. This is necessary since parts of mathematical knowledge are implicit and unconscious (see Coben 2000).

If the student’s report doesn’t supply usable aspects for the ongoing lesson, the student focus on the entire activities of one special day of the week. The student and the teacher look for mathematical aspects there. Alternatively, they make up fictional but realistic or conceivable aspects of this day. In every case the story has to be part of the student’s experience or imagination.

Solving an everyday life problem

Proceeding on the real or fictional everyday life story student and teacher together generate a mathematical problem. For example: “Compare two supplies of a supermarkets brochure” or “Calculate your car’s petrol consumption of last Monday”. My students often applied procedures and fragments of school knowledge without reflecting them. So during the solving process the teacher motivates the student to special behaviour concerning problem solving aspects. The student should be encouraged to “make sense” of her or his doing and to support it by argumentation instead of using unreflected procedures. Therefore during the solving process of an everyday life problem the students are encouraged to become aware of their mathematical doings and to scrutinize it constructively. At that point the ability of knowledge and skill transfer is essential. Bauersfeld (1983) discussed that knowledge is bounded to subjective areas of experience. The common structure of different contexts has to be detected to allow a transfer between these areas of experience. Through the training of transferring a mathematical problem into a familiar or understood context the students are encouraged to refer contexts to each other.

How can non-calculators be encouraged to make sense of their own mathematical solving action? The student can learn to use his everyday experiences. A formal calculating problem can be thought as calculating with money or dealing with
quantities of everyday life. The intention is the use of existing competences in these familiar contexts. For example: Abstract numbers like 4.6 and 0.25 can be interpreted as quantities of everyday life like 4.60€ or ¼ litre of milk. Dealing with such familiar quantities may be easier to the students.

Another technique to understand a formal mathematical problem is to weave a story around it. The abstract problem 30*8 may be calculated by a procedure like “multiply 3 and 8 and afterwards add zero”. This procedure leads to the result but reduces the aspects of numbers to operations with digits. Alternatively the problem can be translated into a story that deals with quantities instead of formal numbers: “At first 3 children take 8 lollipops, together 24 lollipops. Afterwards the group swallowed by the factor 10, so the quantity of 24 lollipops is needed 10 times”. The story implies in a senseful way the effect of factor 10 to the result. Radatz and Schipper see the advantage that such “number stories” make aspects and relationships between numbers clearer and meaningful (Radatz/Schipper 1983).

Also pictures can describe mathematical problems. Lorenz points out the importance of visual representations for the development of flexible mental representations (Lorenz 1992). An example: A student had problems to calculate periods of time when the time of departure and arrival of a train was given. The student helped herself by drawing an inner representation of 6.49 pm as reduced picture of a clock. By this she was able to realize the meaning of 6.49 pm as 11 minutes to 7 pm.

![Clock](image)

**Additional instruction**

Additional instruction is part of every lesson but not the main job. Whenever lack of basic mathematical knowledge is revealed, special instruction units are created. The additional instruction is characterized by the fact, that it is not part of a special curriculum planned ahead. The need of additional instruction arises from problems during the work on everyday life problems. For example: During a calculation of petrol consumption costs substantial problems in the concepts of and the dealing with decimal numbers could be revealed. Also additional instruction is led by the idea of attaching meaning to numbers.

**Change to procedural thinking**

The majority of my students tends towards the unreflected use of procedures. Standardized procedures are fast in application and suitable for relieving the brain but applicable without understanding its inner logic. 54*30 can be counted as sequence of operations with digits:

“4x3=12, 5*3=15, hang up 0 to 15, 150+12=162, hang up 0 to 162 -> 1620”

A more meaningful way uses the nature of numbers and their relationships:

“50*3=150, 150*10=1500, 4*3=12, 12*10=120, 1500+120 -> 1620“

For primary schools Krauthausen (1995) describes calculating problems as often unconnected to processes of understanding if the aim is an early automation of
standardized procedures. Krauthausen compares standardized and half-standardized calculating procedures and sees in the learning of half-standardized principles possibilities to stimulate creative and flexible thinking whereas for standardized procedures understanding is not necessary (Krauthausen 1995).

**Standardized procedure:**

\[
\begin{align*}
34 & \times 19 \\
34 & \\
306 & \\
346 &
\end{align*}
\]

(dealing with digits)

**Half-standardized procedure:**

\[
\begin{align*}
34 & \times 19 \\
34 & \times 20 = 680 \\
34 & \times 1 = 34 \\
680 - 34 &= 646
\end{align*}
\]

(Dealing with numbers)

Anderson (1983, 1988) created a cognitive model that describes how and when (and when not) procedural knowledge is activated in situations where this knowledge is needed. He divided the memory into three parts: the declarative memory, the procedural (production) memory and the working memory. “Declarative knowledge refers to knowledge about facts and things; procedural knowledge refers to knowledge about how to perform various cognitive activities. Procedural knowledge fundamentally has a problem-solving organization” (Anderson 1990, 219). The activation of a procedure depends on a process of pattern matching. If in the working memory data of the outside world and the declarative memory exist a searching process identifies fitting procedures. If the data doesn’t fit with any procedure procedural activation does not happen. Besides procedural knowledge could stay inactive even if from an outside point of view the data factually fits with a procedure. This could happen because the activation of a procedure depends on aspects like preceded experiences and learning processes, perception and attention. Incomplete information has to be completed by pattern searching routines or the use of general criteria for problem solving (Nolte 1991). The Anderson model gives an explanation for the fact that procedures may stay inactive although the student possesses them. If the student prefers a one-sided focus on procedures without reflecting and understanding them and without relating it to declarative knowledge problem solving may fail.

![Diagram](image)

**Figure 1.2** A general framework for the ACT production system, identifying the major structural components and their interlinking processes.

*(Anderson 1983, 19)*
Dealing with questions of numeral knowledge Gray&Tall (1994) introduced the idea of “procepts” (derived from “procedure plus concept”). Procepts mean the connection of procedural and conceptual knowledge. Gray, Pitta & Tall (1997) distinguish between procedural and proceptual thinking. Procedural thinking implies the activation of mainly procedural knowledge. An input „58-19“ may activate counting strategies and calculation routines. The proceptual thinker instead uses meaningful, declarative knowledge and connects it in flexible ways with procedural knowledge. Concerning the input „58-19“ knowledge about the neighbourhood of 19 and 20 could be activated and could lead to the procedure 58-20+1. Gray (1991) researched addition and subtraction solving strategies of children aged from 7+ to 11+. Low performing students preferred simple counting strategies, so ways of solving on procedural level. They had much more troubles with problem solving than high performing students, who thought in proceptual ways by using conceptual knowledge in conjunction with procedures. This leads to the hypothesis that adult non-calculators should be motivated to change from procedural to proceptual thinking. This implies for the lesson to discuss together with the student the problematic nature of unreflected procedures.

A case study

In the following example procedural and proceptual thinking will be focused on. The scenes have been interpreted by sequential text analysis following concepts of interpretative instruction research (Krummheuer&Naujok 1999, Maier 1991, Voigt 1984).

The female, 26 aged student of the case study works as secretary in a pharmaceutical company. The scene is part of the second lesson, so less instruction has taken place yet. The student formulates a typical counting problem in her job:

_A hospital orders 300 (so called) nose adapters with unit price 19 Cent (net) and 25% discount._

A partial problem was the calculation of 300*0.19€. At her first attempt this she calculated by a standardized procedure of multiplication. So she made use of procedural knowledge.

\[
\begin{array}{c}
300 \times 0.19 \varepsilon \\
\hline
0 \varepsilon \\
30 \varepsilon \\
27 \varepsilon \\
\hline
1570.0 \varepsilon
\end{array}
\]

This procedure was well done but the routine itself required no understanding. To develop the students proceptual thinking part of the instruction concept is the solving of a problem on different ways. At this point the scene starts. The teacher asks the student to solve the partial problem 300*0.19€ again but in another way. For the present he keeps open which alternative approach could be chosen.
Scene I: activation of procedural knowledge

S (=student): “surely you could calculate this in such a “kettensatz” [some calculation procedure] again, but then I don’t know why I come to this and then why this works. That’s why I must look again. But naturally you tend towards to write x is equal to (laughs) how much. But finally that’s no use to me. There I don’t know why. Again, what was your suggestion? Yes?”

Interpretation: The student’s idea for solving the problem is the use of a procedure called "kettensatz". This notion was not part of the instruction and indicates extern acquired knowledge (perhaps from school). The word “kettensatz” implies a meaning like “sentence”, “rule” or “procedure” and so indicates procedural knowledge. She rejected the usage of “kettensatz” by herself with the words “but then I don’t know why I come to this and then why this works”. She characterizes this procedure as usable but not meaningful. The procedure leads her to a result but without understanding the functioning of the procedure. So the student indicates that for her not only the result is important but also the understanding of her own action. She sees the necessity of further reflection: “That’s why I must look again”. On the other hand she sees a general preference to follow a special procedure: “But naturally you tend towards to write x is equal to (laughs) how much.” So the student describes an inclination to procedural thinking. But she also characterizes the use of the routine “to write x is equal” as in the end useless because of her lack of understanding.

Then L suggests a strategy called „Nachbaraufgabe“ (neighbourhood strategy). This strategy was a minor subject in the previous lesson. Neighbourhood strategy means the use of a similar but easier arithmetical problem. His intervention has to be seen as an attempt to confront the student with an alternative calculation strategy that supports proceptual thinking. L proves whether S remembers the neighbourhood strategy and whether she is able to transfer this strategy to the actual problem. In the following scene S chooses this approach:

\[ 300 \times 0.19 = 300 \times 0.20 - 300 \times 0.01 \quad (0.20 \text{ is neighbour of } 0.19) \]

First she tries to calculate 300 \times 0.20€.

\[
\begin{array}{c}
300 \\
\times 0.20 \\
\hline
0.00
\end{array}
\]

This picture develops by the following steps:

1. \(300 \times 20 = \) then a pause of 5 sec
2. \(300 \times 0.20 = \) then a pause of 10 sec
3. \(300 \times 0.20 = 6 \)
4. \(300 \times 0.20 = 6 \)

\[1\] original text: „man könnte das auch bestimmt wieder in som kettensatz rechnen, aber dann weiß ich letztendes wieder nicht warum ich da so hingekommen bin und dann wieso das so funktioniert. deswegen muss ich noch mal gucken. aber man ist natürlich geneigt zu schreiben x ist gleich (lacht) wieviel aber das das bringt mir letztenendes nichts. da weiß ich ja nicht warum. ähm nochmal, was war der vorschlag? ähm ja/"
Interpretation: First S writes „300*20=“ (1). Here it is uncertain whether her thinking is related to the context money or the problem is seen as formal operation with numbers. At least the context money is involved in thinking when she adds „0,“ (2) and decides to write „0,20“ instead of „20“. This becomes clear at a later point when she says: “and then I thought yes that are only 20 cent”\(^2\)\(^\ast\). Immediately S counts \(3^\ast2=6\) (3). Her calculation follows a strategy of leaving zeros and commas out. A digit-by-digit-strategie can be assumed. Then S crosses the whole line out (4) and started a second attempt:

\[
\begin{array}{c}
300 \times 0,20 = 6900
\end{array}
\]

This picture develops by the following steps:

(1) \(300 \times 0,20 =\)
(2) \(300 \times 0,20 =\)
(3) \(300 \times 0,20 =\) then a pause of 4 sec
(4) \(300 \times 0,20 = 0,\) then a pause of 1 sec
(5) \(300 \times 0,20 = 0,6\) then a pause of 0,5 sec
(6) \(300 \times 0,20 = 0,600\) then a pause of 5 sec
(7) \(300 \times 0,20 = 0,6000\) then a pause of 2 sec
(8) \(300 \times 0,20 = 0,60,00\)
(9) \(300 \times 0,20 = 0,60,00\)

Interpretation: S writes down the problem again (1). The crossing out (2) and the restoring of null (3) demonstrates her irresolution whether she should use 300*20 oder 300*0,20. She decides on using the comma notation. The use of 0,20 causes her to write the result also (and in her words "simply\(^\ast\))(\(^\ast\)) in null-comma notation (4). As in the previous approach the calculation is done by a digit-by-digit-strategy. First she deals with the digits 3 and 2 (\(^\ast\)) and writes „6“ as result. Then step by step the nulls are considered, what is shown by the pauses (see 5-7 and \(^\ast\)). Later on S characterizes her procedure as not understood but learned knowledge (\(^\ast\)). An other later on statement characterizes here knowledge as procedural knowledge concerning the dealing with isolated digits: „whether I have learned this to put these [nulls] there again“\(^3\)\(^\ast\). Finally S writes a second comma (8) probably to transfer the numbers of after-comma-digits. Later on she cancels the first comma and separates the first null by a stroke (9).

Summary: The whole line of action can be characterised as procedural. The procedural action is accompanied by insecurity on the side of the student.

**Scene II: activation of procedural knowledge**

In the following part S continues the neighbourhood strategy:

\[S\text{ writes „}300\times0,01\text{“, then 3 sec. pause, then she writes „}=\text{“ und finally she underlines it:}\]

\(^\ast\) original text: „und dann dachte ich ja das sind nur 20 cent“

\(^\ast\) The following interpretation additionally depends on explanations by S, that are made in later scenes. References to those scenes are marked by (\(^\ast\)).

\(^4\) original text: „ob ich das einfach mal gelernt hab dass man sie da wieder aufführen muss“
\[ 300 \times 0.01 = \]

S: "something like this for example I can’t manage. 0 comma 0 that I have to do in the written mode [S means: by standard computing procedure]. I can’t multiply this."\(^5\)

Interpretation: S continues her comma-notation. It is open whether she focususses the decimal notation or the context of money. Her pause indicates that she has problems to go on or that she reflects her previous calculation step. She calls the problem "something like this" that indicates a certain distance to the problem. Her statement "something like this for example I can’t manage. 0 comma 0" indicates that S sees the problem as a representative of a special kind of arithmetical problem that is difficult to her. To solve the arithmetical problem S again chooses a standard procedure for multiplication. So she activates procedural knowledge.

\[ 300 \times 0.01 = \]

\[ 300 \]

L: "(...) hm this you like to calculate. (S: yes) 300 multiplied by 0,01. (L shows to this) or how do you formulate... formulate this problem for you (L shows to this)"\(^6\)

S: "(pause 5 sec) now I remember. But when you follow it up and you don’t have to think twice. How much is 1 how much is 300 multiplied with 1 cent. And then I know 100 cent is 1 euro. So 300 cent should be 3 euro. (...)"\(^7\)

Interpretation: The writing direction indicates that S indeed follows a standard procedure of multiplication. So she counts digit bei digit. Because she uses a procedural strategy L interrupted her at that point. He tries to stimulate once more a thinking related to the starting problem. In the student spontaneously arises a thought. She calls the first factor "1 cent" and so she activates declarative knowledge. The focus on the money activates further declarative knowledge that is the equality of 100ct and 1€. Finally procedural knowledge is activated that is the transfer from "1€ is equal to 100ct" to "3€ is equal to 300ct".

The following makes clear that the activation of the declarative knowledge does not has happened naturally. This is so much more remarkable because S shows at least at

\(^5\) original text: "sowas kriege ich zum Beispiel nicht hin. 0 komma 0 das muss ich schriftlich machen. das kann ich nicht mal."

\(^6\) original text: "(...) hm dieses willst du ausrechnen. #ja# 300 mal 0 komma 0 1. (zeigt darauf) oder wie formuli formulier dir die aufgabe mal für dich (zeigt darauf).

\(^7\) original text: "(5 Sek.) obwohl jetzt jetzt weiß ichs wieder aber. wenn man genauer nachhakt und nicht genauer drüber nachdenken muss. ähm wieviel sind 1 also wieviel sind 300 mal 1 cent. und da weiß ich 100 cent sind 1 euro. 300 cent müssten dann 3 euro sein. (...)"
two points of the whole sequence that she has the ability to include into her considerations the focus on the context of euro.

\[ L: \text{,,yes d'accord.\textsuperscript{8}}\]

\[ S: \text{,(laughs) but I really have problems with this. (writes an asterisk above the cross sign) there\textsuperscript{9}} \]

\[
\begin{array}{c}
300 \times 0.01 = \\
300.
\end{array}
\]

\[ L: \text{,,you have taken an important step. (S: hm/) you rummaged in your experience what do I know. And then you said yes I remember and then you used this.\textsuperscript{10}} \]

\[ S: \text{,,But I'm saddened about the fact that I don't get it on my own. Yes you always have to push me towards this because naturally it is more comfortable than to calculate twice.\textsuperscript{11}} \]

Interpretation: S needs a person that "push"es her to special reflections. The activation of declarative knowledge within the context money doesn't happen on her own but needs an external impetus: "you always have[!] to push me to this". S is unsatisfied ("saddened") with this situation that means dependence on others. Again this shows that the activation of contextual declarative knowledge is not her preference. The preference to use (unreflected) procedures is strengthened by emotional aspects: "it is more comfortable".

**Summary and hypothesis**

In the run-up to the scenes presented above the student shows her preference to use standardized procedures for calculation. She was able to apply the standardized procedure for multiplication in a correct way. But problems with decimal numbers arised when S tried to go forward step-by-step following the neighbourhood strategy. It is here that she also shows her preferences to solve 300*0,20€ and 300*0,01€ by using digit-by-digit-procedures. These were characterized by unsureness. The student herself remarked a lack of understanding related to her own doings. Besides, she expressed her urge to reach understanding. The activation of declarative knowledge about the context money and derived from this the activation of efficient procedures of calculation were initiated by the teachers input. The turn from procedural to proceptual thinking proved to be fruitful but required external impulse.

---

\textsuperscript{8} original text: "ja d'accord."

\textsuperscript{9} original text: "(lacht) aber ich hab da wirklich probleme mit. (S malt einen Stern über das Multiplikationssymbol) da"

\textsuperscript{10} original text: "du hast jetzt einen schritt getan der sehr wichtig ist. #hm/# du hast in deinem erahrungsschatz gewählt was kenn ich. und hast gesagt aha ich erinnere und hast das dann verwendet."

\textsuperscript{11} original text: "aber ich finde das traurig dass ich von alleine da immer nicht drauf komm (lacht), ja man muss mich immer drauf schubsen weil (uv) ist natürlich auch bequemer als dann noch mal nachrechnen."
An inclination to procedural thinking is typical for all of my students. Furthermore, all of them express their urge to reach an inner understanding of the own mathematical procedures and the mathematical world itself. The student of the case illustrated above expressed her learning improvements by statements like these:

- She felt pleasure with the experience that a problem has several ways of solving.
- She experienced reasonable halfstandardized procedures as more pleasant than unreasonable standardized ones. Mindless learned knowledge got meaningful.
- During an arithmetical quantity problem the student found herself satisfied not to depend on the unreflected learned procedure “rule of three” and to have found an alternative and reasonable way of solving.
- In the progress of instruction the student became able to count more and more without using her fingers and to get rid of her fear for numbers.
- For simple multiplication problems like 6*8 the student started to avoid counting up strategies as in the past. She turned to the use of number relationships as demonstrated in the neighbourhood strategy.

Finally, my hypotheses are the following:

- Non-calculators are characterized by procedural thinking.
- The development of proceptual thinking is nessessary and needs to be triggered by an external impetus.
- Non-calculators possess declarative knowledge that is related to everyday life experience. This knowledge can be used as starting point to develop proceptual thinking.
- Non-calculators want to understand their own mathematical action. They are open for a turn from procedural to proceptual thinking.

Bibliography


